Research Statement

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1. Motivation

Since my childhood I could sense a need for automation in the basic forms of mathematical operations that we were taught, like addition and subtraction. It felt like there was a clear logical entity being repeated to get the job done. Our examinations were mostly to recall factual information which felt no different from a query search on google. The system seemed a little meaningless to me. Through these experiences I was developing two "needs" inside of me - 1. my need to generate new information that google wouldn't be able to answer and 2. my need to automate my processes as much as I could push it to. During my engineering, to seek out experiences of this sort, I had built line followers - a clear and precise logical system that repeated behaviours encoded into it. I got to experience how the theory of algorithms interfaces with reality.

During my masters at CMI, the introduction to logic, functional programming languages, etc had an immense impact on me. It felt like logic was the core circuitry on top of which mathematics was built. While this may or may not be the truth, but this feeling led me to explore Theory B leaving my initial penchant towards the core ideas of algorithms which are a part of Theory A. But, I also felt like those blind men trying to understand the elephant. Perhaps the demarkations are one and the same - the nature of computation and its many facets. Leading me full circle to my initial naive goals.

Towards fulfilling these goals, I choose to first do some projects with Prof. R. Ramanujam in the interplay between automata games like parity games and proof theory for my masters thesis. This experience led me to choose jam as my guide for my PhD journey where we deviated from the goals a little and looked into the intersection of logic and game theory.

In the PhD, we try to come up with different logics whose models would capture different classes of games. PhD was a training which was vital for me to gain some experience in :

(1) Setting up relevant logics for **formalising** the object in mind.
(2) Being able to show **expressibility** of properties in the vocabulary of our logic.
(3) Have a brush with the methods that test the **limits of expressibility of logics** - like EF games, Compactness and Zero-One Laws.
(4) The arguments involved in proving the **decidability** of a logic.

2. Research Experience

I have had the fortune to work with Professor R. Ramanujam and my esteemed seniors, Prof. Sunil Simon and Dr. Anantha Padmanabha, on trying to come up with logics that can formalise the reasonings done in strategic interactions. This research continues humanity's efforts of formalising different facets of Game Theory. In this area researchers previously had been able to formalise the idea of **"common knowledge"**, with the use of **epistemic logics** that emerge from the **Modal Logic literature**. If one follows, Giacomo Bonanno's concise summary of this area in his published manuscript **"Modal Logic and Game Theory: two alternative approaches"**, one can see that our work would lie in formalising aspects of both *descriptive* and *prescriptive* game theory.

We work over Normal Form Games, $\mathcal{G} = \langle (\Sigma_i)_{i \in [n]}, (\pi_i)_{i \in [n]} \rangle$ for n - players where $n \in \mathbb{N}$ and $n > 1$ and Σ_i represents the non-empty set of strategy choices for player i and π_i denotes the payoff function for player i. The solution concepts from game theory that we sought to express in our logics are well-studied properties such as the Nash Equilibrium (NE) of a finite game description, the Finite Improvement Property (FIP), Weak Acyclicity (WA), etc. In this endeavour, I had picked up the use of First Order Logic with a monadic least fixed point operator, (**MLFP**) and also worked with variants of Propositional Dynamic Logic (**PDL**).

In the context of the work involving **MLFP** logic, **Improvement Graphs** form the key mathematical object over which the solution concepts listed above are defined. We try to work over both the **Improvement Graphs** and the given **normal form game description** as the **models** over which we would want our logics to capture the solution concepts. Also since the **game theoretic properties** mentioned above happen to coincide with the **path properties of the improvement graph**, we needed an operator that could capture paths and do analysis on them. Our work here, tends towards capturing the **computational aspects of reasoning** in game theory. Alternatively, in the distinctions made by Bonanno, mentioned above, it would lie in the realm of formalising the *prescriptive* aspects to game theory. Since the computational aspects formed our centrepiece we strived to incorporate the ideas from descriptive complexity theory which is a rich area where logic and complexity mesh. This had also been the starting point for our foray into **MLFP**. The choice of monadic over arbitrary arity least fixed points or even a transitive closure operator, was from a perspective of minimal description resource usage following up on the **arity hierarchy** work done by Dr. Martin Grohe.

We also take strides into formalising the *descriptive* aspects of game theory. For this venture, we tweak **PDL** to accommodate the local aspects of the class of games we wanted to formalise (in particular, **Social Network Games** and **Large Games**). In the case of *social network games*, where each player affects another players' strategies and this mutual strategization between the players becomes relevant, we have the logic as described below :

The formulas of the logic are presented in two layers: **local player formulas** and **global outcome formulas**. The logic is parameterised by n, the number of players, and the strategy set Σ and $p \in P$ be a proposition from an infinite set of propositions P.

The syntax for **local** formulas is given by:

$$\alpha \in L_i ::= a \in \Sigma \mid e_j \mid \neg\alpha \mid \alpha \vee \alpha' \mid N_{rel\ r}\, \alpha$$

where $rel \in \{\geq, \leq, <, >\}$, $i \in [n]$ and r is a rational number, $0 \leq r \leq 1$.

And the syntax for **global** formulas is given by:

$$\phi \in \Phi ::= p@i \mid \alpha@i, \alpha \in L_i \mid \neg\phi \mid \phi \vee \phi' \mid \langle i \rangle \phi \mid \Diamond^* \phi$$

The $N_{rel\ r}$ operator incorporated the ability to investigate the neighbourhood of a particular player and the $\Diamond^*$ operator helped us with assessing strategy profiles reachable from another.

In Table 1, the columns are of central interest in our investigations. We have a class of models that pertains to a description of normal form games or a particular game form; we have a logic with which we try to express game theoretic properties over said model those that correspond to a subclass of the class of models in that row; and, we have results highlighting the usefulness or a feature of the logic in question. A row of the table should be read as follows. The class of models and the logic aforementioned appear in columns 1 and 2 of the row respectively. In column 3, a property $\mathbb{P}$ is mentioned which is expressible by the corresponding logic mentioned in column 2. In column 4, we have some of the results obtained on the properties of the logic.

For instance in the first row, we work over the structure of improvement graphs with the MLFP logic. We are able to capture the game theoretic notions of Nash equilibrium and Finite Improvement Property (FIP) using our logic. By which we mean, we capture subclasses of finite strategy n player games (where n is fixed) that have a pure strategy Nash Equilibrium (N.E.) or the property of FIP. And, it turns out that the model checking question for a formula of MLFP, ϕ, with an instance of an improvement graph of a given n player game, A, is decidable in time, $|A|^{\mathrm{qr}(\phi)}$, where qr is defined to be the quantifier rank of a formula. In particular, the formulas restricted to the domain of game theory that we investigated, can be model checked in polynomial time given that their quantifier ranks are bounded above by 2.

TABLE 1. Research Highlights

Model	**Logic**	**Properties**	**Results obtained**
Improvement Graph	MLFP	N.E , FIP, Acyclicity	Model Checking
Game Models (fixed n- number of players)	MLFP	Priority separable payoff	same as above
Generalised Game Models	extended MLFP	same	same as above
Social Network Game	PDL variant	Payoffs of the Majority and Public Goods Game	Decidable logic
Large Games	MSO-STRAT	Payoffs of the large games	Finite unsatisfiability
Large Games	Implicit modal logic variant	Same as above	Decidable Logic

3. SOME CONCRETE PROBLEMS

We have some concrete problems that I came across during my PhD. And I would like to work on them along with the collaborations at TIFR. I present the list of problems here.

- Currently I am trying to extend my first paper into its journal version with my collaborators Prof. Sunil Simon and Prof. R. Ramanujam. For this, we are attempting a proof to show that a formula in the MLFP logic variant (we extend by introducing binary arity second order quantifiers with which we are able to express functions like the strategy profiles), is able to capture the property of Nash Equilibrium in a game called priority separable game, inspired from separable games like **polymatrix games**. We have shown that the **model checking problem** for this extended MLFP logic we have is **PSPACE-complete** (unpublished as of now).
- For any formula, $\phi \in$ FO(lfp), when interpreted over finite structures, we know that there is a structural result that converts ϕ into a logically equivalent formula ψ which is structurally of the form where the outermost quantifier is the lfp operator with the inner most structure being a FO-fragment. ψ is said to belong to LFP$_0$. I ask a similar question for MLFP. Can a formula in MLFP have a **logically equivalent** MLFP$_0$ formula? Such a structural transformation would be beneficial for the model checking complexity. My hunch though, is that it cannot be done. A crucial step in the transformation is to have a

negation normal form equivalent, which comes as a consequence of the **Stage comparison Lemma** from the LFP literature. I believe this particular step requires the use of a binary least fixed point operator. So the following question remains open : whether we can carry out such a crucial stage of the transformation in the monadic least fixed point operator.

- A pure strategy $s_i \in S_i$ is a **never best response** if $\forall s_{-i} \in S_{-i}$ we have $\exists s' \in S_i$ such that, $\pi_i(s', s_{-i}) > \pi_i(s_i, s_{-i})$, where, π_i is the payoff for player i and $S_{-i} = \underset{j \neq i}{\times} S_j$. Given a game $\mathcal{G}$, let $G^0 = \mathcal{G}$, let G^k be the game obtained by deleting all the pure strategies of all the players that are strictly dominated in G^{k-1}. Let $G^\infty = \bigcap_{k=1}^{\infty} G^k$ be the game obtained by applying this iterative procedure of elimination. This particular process is known as the iterated elimination of never best responses, IENBR. I wish to know if G^∞ is **expressible** in the FO(lfp) logic. The core of the problem lies in understanding what sorts of iteration procedures can the least fixed operator perform versus the ones it cannot.
- Most of the game properties we expressed required us to work over a **two variable MLFP logic**. I need to check if 2-variable MLFP is **decidable** or not.
- Given two lists of integers, $\vec{a}, \vec{b} \in \mathbb{Z}^n$ what is the **minimum arity** of the **least fixed point operator** I would require to express the summed up sequence, $\vec{c}$, where $c_i = a_i + b_i$, for $i \in n$, when addition is given to us in the vocabulary.
- **Bisimulation characterisation** for the extended PDL logics for the social network and large games that I had worked on.
- This is more of a long term project. We know that **fixed point logics with counting** can capture the **ellipsoid method**. I would like to see how far the logic can be pushed to capture the algorithms that are used for searching the mixed nash equilibrium for games, like the classical path following algorithms of **Scarf** or the **Lemke-Howson method**.

4. Statement of Purpose

The area I worked in would be in the realm of Game Theory and Logic. As highlighted earlier, the previous works branched out from the attempt at **formalising** the notion of "common knowledge" towards which the natural choice was **Modal Logic**. We of course use the Modal Logic variants and give logical descriptions for the two classes of games - the **Social Network Games** and **Large Games**. Both add a unique flavour to the logical descriptions needed. We also extend the literature when we work over **Improvement Graphs** and use the least fixed point logic, **MLFP**. I would like to collaborate with Dr. Shibashis Guha with whom I have been corresponding. I like his work on the intersection between **Games and Automata**. It would be an interesting area to branch out to and know further about from my perspective.

As highlighted above in the list of concrete problems, many of them are from the areas of **descriptive complexity** which is a subdomain of **Finite Model Theory**. I corresponded with Dr. Abhisekh Sankaran, who has been working on finite model theory for a while now. He is currently working at TRDDC, Mumbai. I feel before I can try to attempt the problems I have cited above, I need more of a foothold on the basic techniques and results in this area. And thus the guidance of Dr. Abhisekh Sankaran would be really fruitful for my growth in this field.

I would like to suggest Dr. Shibashis Guha and Dr. Abhisekh Sankaran as my guides if I be accepted to continue my academic journey into TIFR, Mumbai. Also, since TIFR is a leading TCS research institute in the area of **complexity theory**. I would be glad for all the help I can get in this venture of building bridges between logic and complexity theory of game theoretic problems.

5. Future Directions

I am open to working on problems and collaborate with researchers in these fields. I have the overall objective of polishing my skillsets. I believe TIFR is a natural continuation to my academic journey where I would be able to interact with the foremost experts in areas that I am interested in.

THANK YOU!