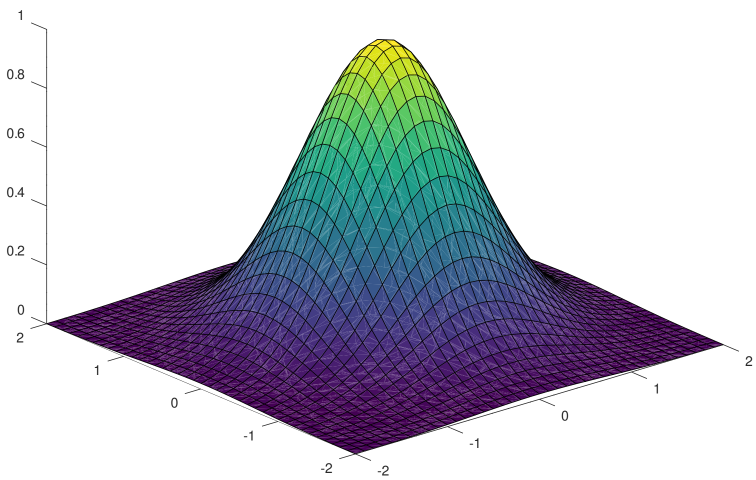
EN2040 - Random Signals and Processes

University of Moratuwa

Department of Electronic and Telecommunication Engineering

Simulation Assignment



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Index No: **170221T**

Date: May 20, 2019

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Note: The figures included in this document were generated using Octave and saved as vector images

1 Introduction

In this Assignment, rectangular pulses of $\pm A$ amplitude are simulated. These pulses carry binary equiprobable data over a communication channel. In each section which follows in this report, I intend to find out the effect of factors such as noise, interference, and amplification on the received signal.

Visualizations including Histograms, Stair plots are included to get a better intuition of these effects.

2 Generating Equiprobable Rectangular Pulses

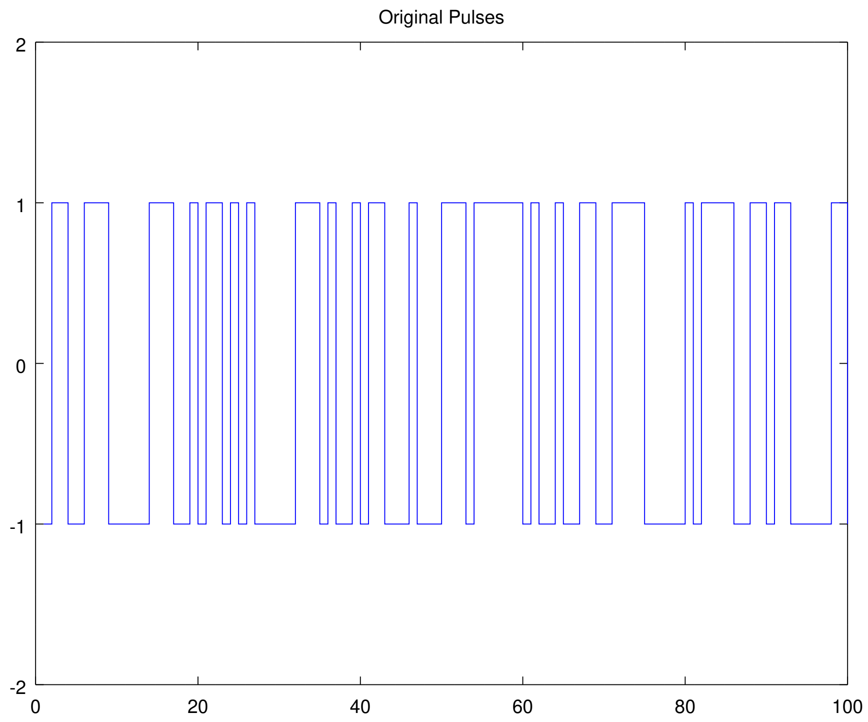

Figure 1: First 100 elements of the Rectangular Pulse Sequence (S)

```
L=1000;  %length of the sequence
A=1;     %variance 1

binary_seq = mod(reshape(randperm(1*L),1,L),2);%equiprobable random
pulses=zeros(1,L);

%%% Generating Rectangular pulses based on the binary seq %%%
for i=1:L
    if(binary_seq(i)==1)
        pulses(i)=A;
    else
        pulses(i)=-A;
    end
end

stairs(pulses(1:100)); %plot first 100 samples
title('Original Pulses')
axis([0 inf -2*A 2*A])
hold on
```

3 Generating an AWGN Sequence $\sigma^2 = 1$

Figure 2 shows the first 100 samples of the Additive White Gaussian Noise(AWGN) which will be considered as the noise of the channel.

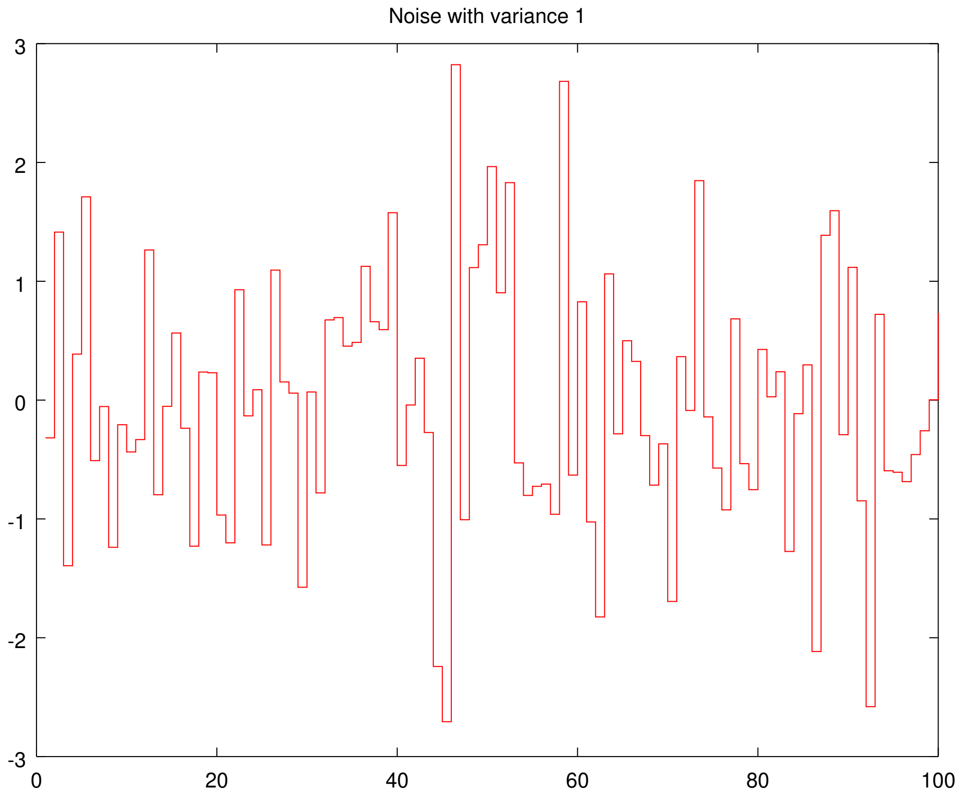

Figure 2: First 100 samples of the AWGN Noise with $\sigma^2 = 1$

```
L=1000;     %length of the sequence
variance=1; %variance 1
N1=sqrt(variance)*randn(1,L);   %%noise
```

4 Sequence of the Recieved Signal (R)

4.1 Impact of the variance of noise on R by varying $\sigma^2 = x$

From the plots, it is evident that when the noise variance increases, the received signal becomes more and more distorted.
The transmitted signal of amplitude:1 is completely dominated by high noise variances.

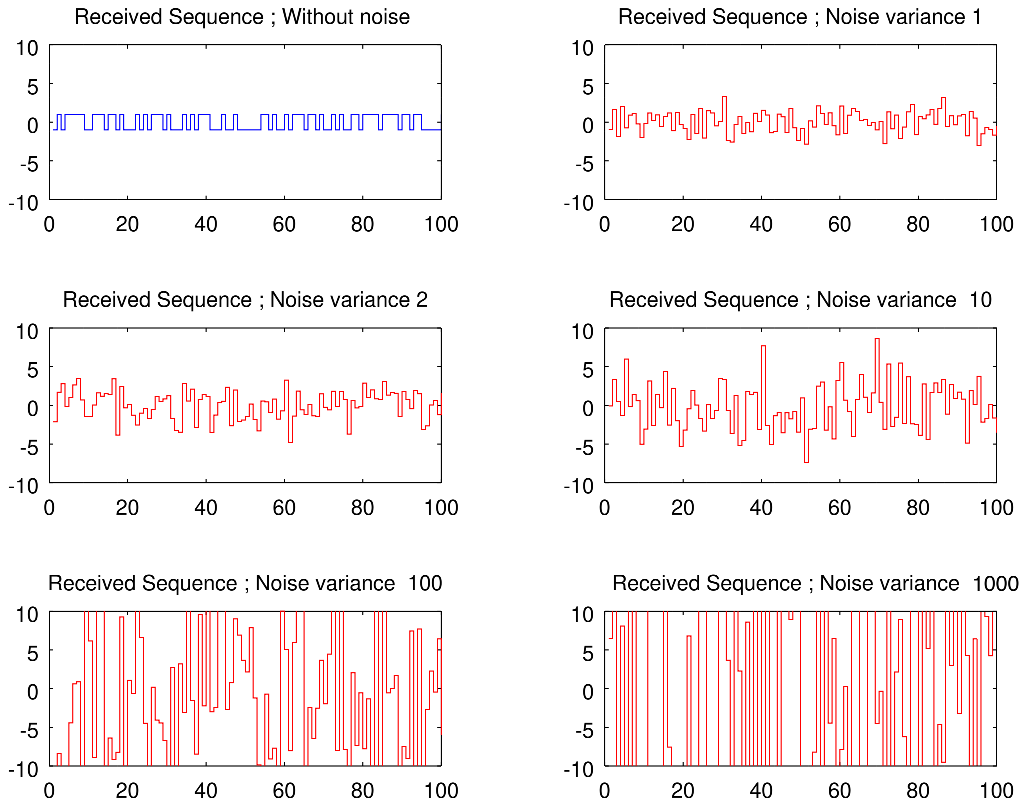

Figure 3: Q3: Varying the variance σ^2

5 Decoded signal (Y) vs Transmitted signal

Shown below is the first 100 samples of the transmitted pulses. Comparing the transmitted sequence with the Decoded sequence(Y), there seems to be certain bit changes.

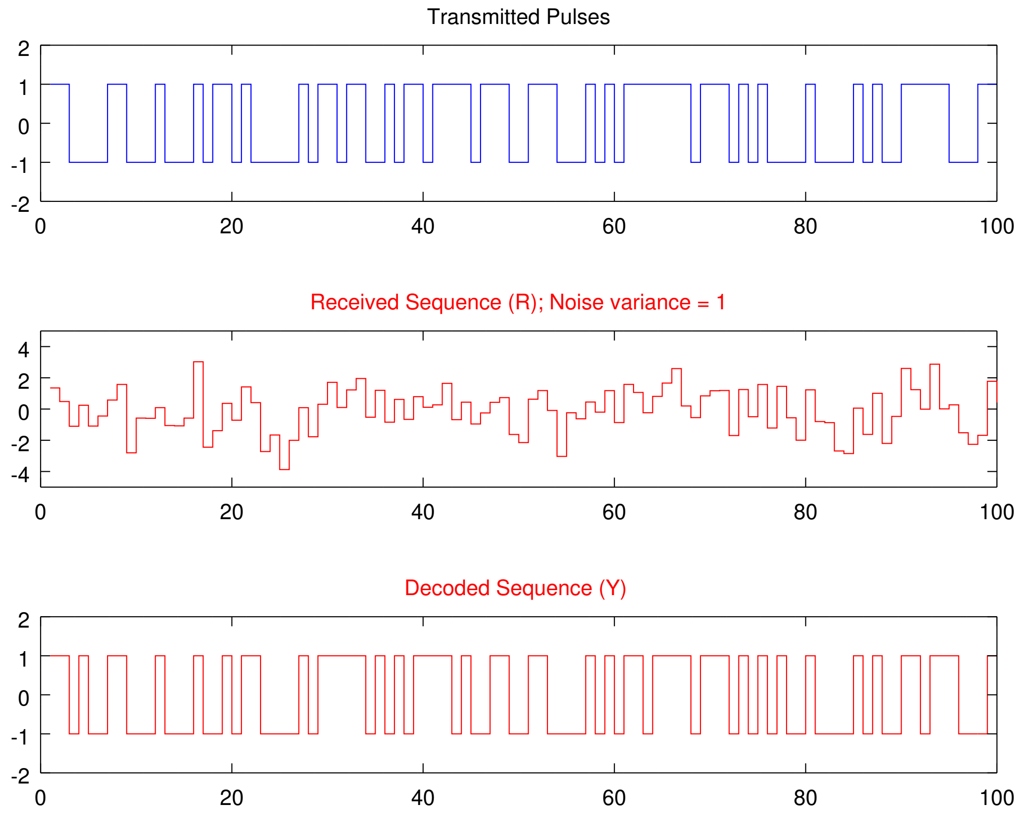

Figure 4: Q4: Sketch and compare the sequence of Y with the transmitted signal

6 Code to generate histogram of the received sequence

```
function g = draw_histogram(R,bins)  %my function
    lower_bound=min(R);
    upper_bound=max(R);

    gap=(upper_bound-lower_bound)/bins;
    x =lower_bound:gap:upper_bound;
    y=zeros(1,length(x));

    for i=1:length(R)
        id=floor((R(i)-lower_bound)/gap)+1;
        y(id)=y(id)+1;
    end

    bar(x,y, 'BarWidth', 1);
    title('Histogram of the Received Signal')
end
```

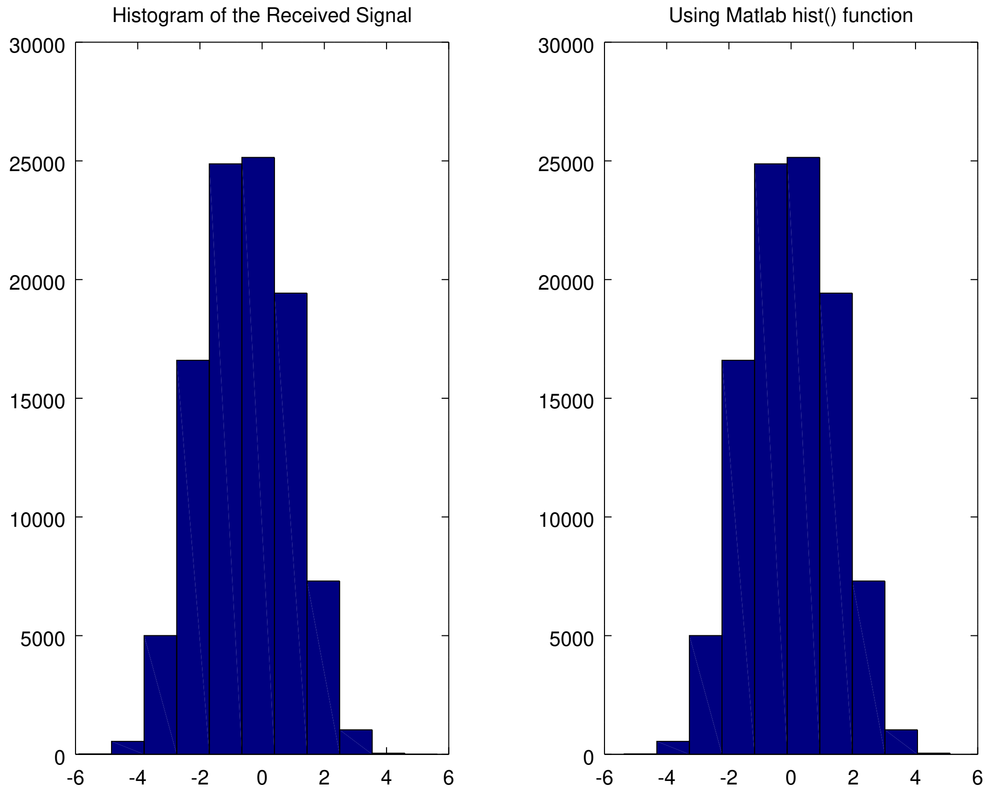

Figure 5: Q5: Function to draw histogram (Bin Size=10)

```
%main program
    subplot(1,2,1)
    draw_histogram(R,bins); %calling draw_histogram.m

    subplot(1,2,2)
    hist(R,bins);
    title('Using Matlab hist() function')
```

The function, **draw_histogram** divides the range of values to the given number of bins and calculates the frequency of each bin.
Each frequency is recorded to an array and then visualized by the **bar** function of matlab.

6.1 Effect of changing number of bins

When increasing the number of bins, the histogram takes the features of a Normal Distribution.

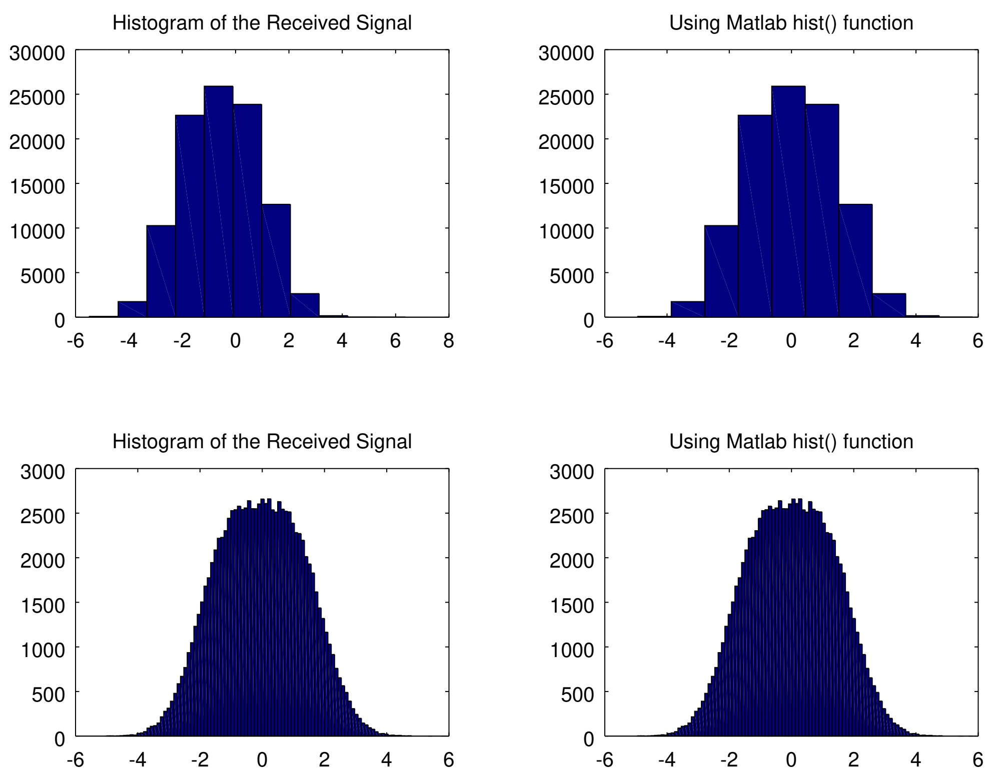

Figure 6: Q5 a): changing bins from 10 to 100

6.2 Conditional PDF

We fix variance(σ^2=1) of noise and change the Amplitude of the transmitted signal.

6.2.1 A=1; $f_{R|S}\ (r \mid S = A)$ $f_{R|S}\ (r \mid S = -A)$

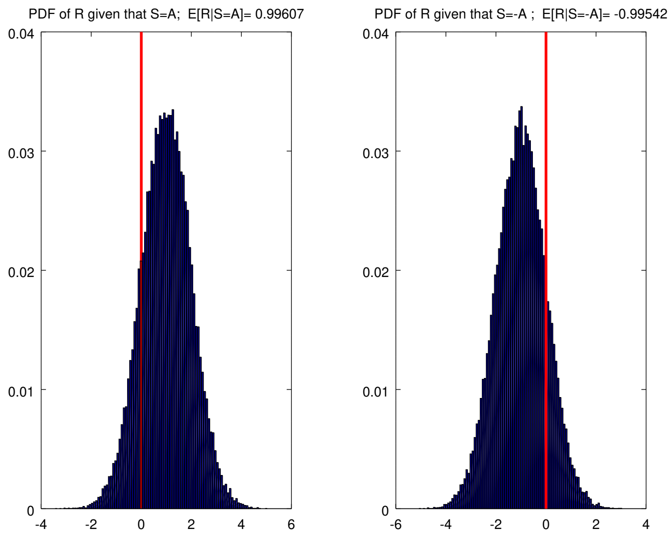

Figure 7: Conditional PDFs with A=1

6.2.2 A=10; $f_{R|S}\ (r \mid S = A)$ $f_{R|S}\ (r \mid S = -A)$

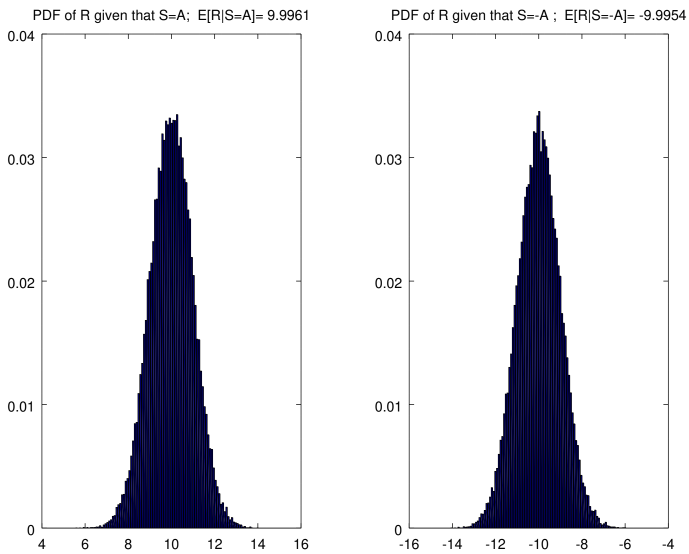

Figure 8: Conditional PDFs with A=10

When the amplitude of the transmitted signal increases, the conditional pdfs move towards $\pm A$ according to **Figures 7,8,9**.
This is because the effect of $\sigma^2 = 1$ variance is outperformed by the high amplitude of the transmitted signal.
According to **Table 1**, the conditional distribution's mean nearly equals to the amplitude.

6.2.3 A=100; $f_{R|S}\ (r \mid S = A)$ $f_{R|S}\ (r \mid S = -A)$

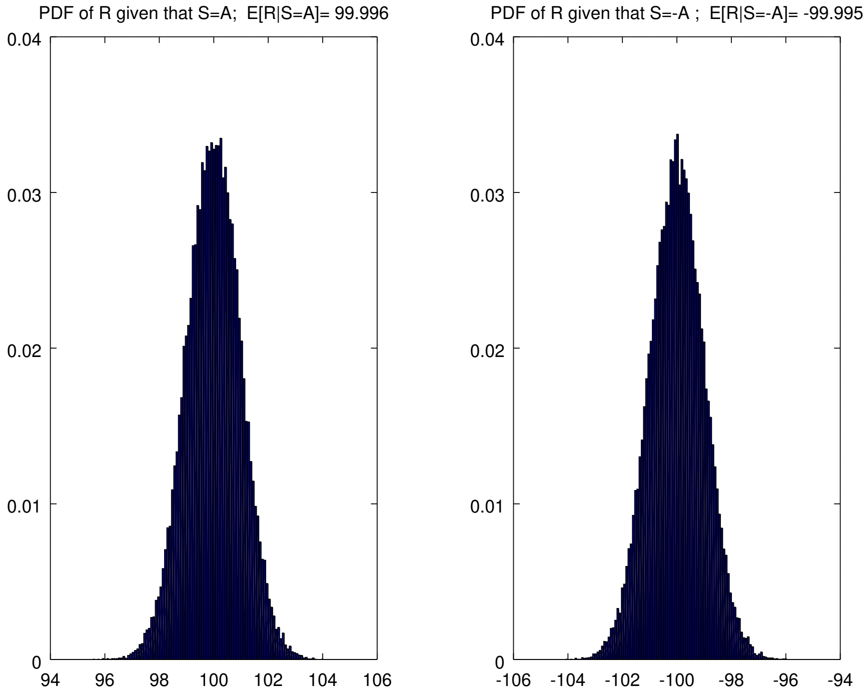

Figure 9: Conditional PDFs with A=100

6.3 Expected values of Distributions

Table 1: Impact on the Expected value with the Amplitude of the transmitted signal

Amplitude	E[R|S=A]	E[R|S=-A]	E[R]
A=1	0.99607	-0.99542	0.004498
A=10	9.9961	-9.9954	0.004498
A=100	99.996	-99.995	0.004498

6.4 Sketch the PDF $f_R(r)$

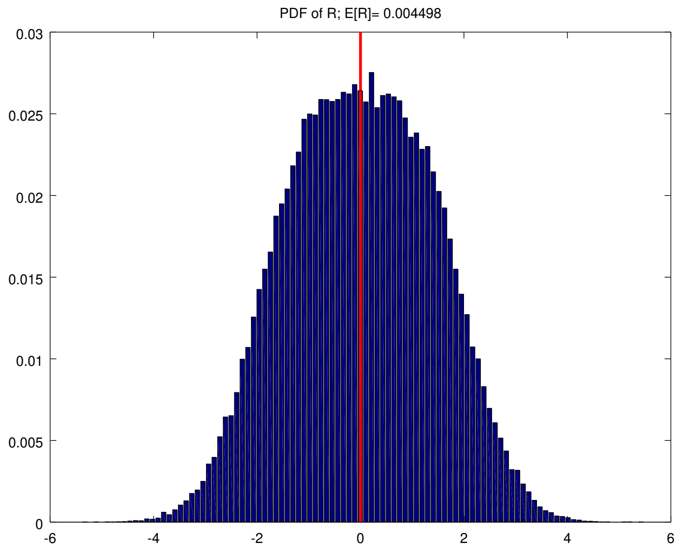

Figure 10: PDF $f_R(r)$ with A=1

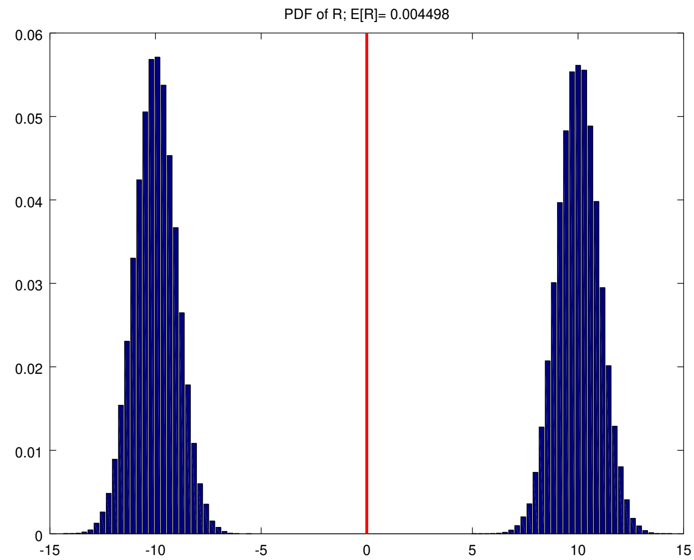

Figure 11: PDF $f_R(r)$ with A=10

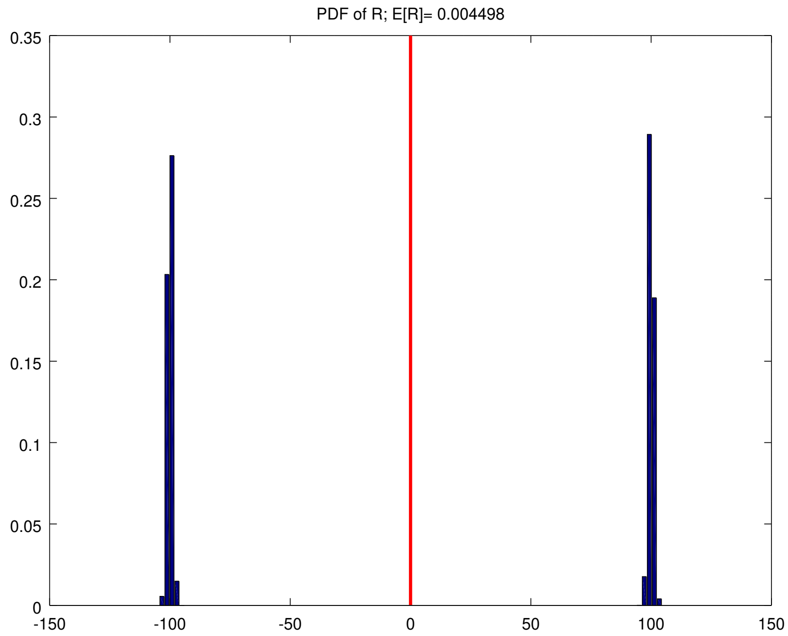

Figure 12: PDF $f_R\ (r)$ with A=100

Because of the clear separation of the distribution, it will be much easier to decode the signal.

7 Effect of Interference

7.1 PDF comparison w/without interference

It can be observed that adding interference to the signal increases the variance of the received signal. This is clear when observing the area to the left of the Red lines drawn at x=-4.

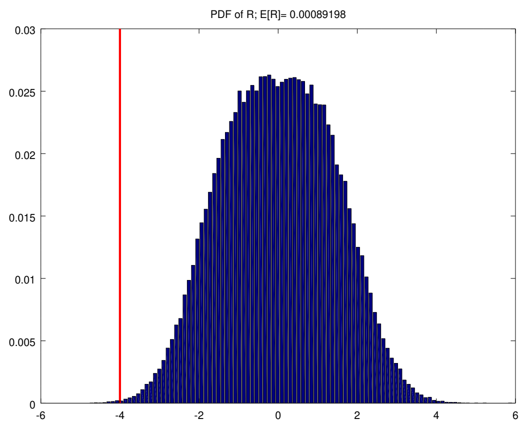

Figure 13: PDF Without Interference

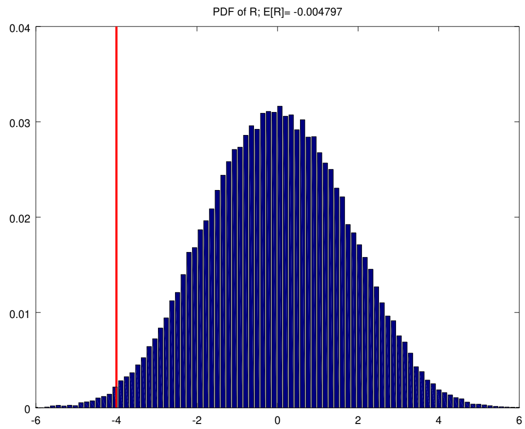

Figure 14: PDF With Interference

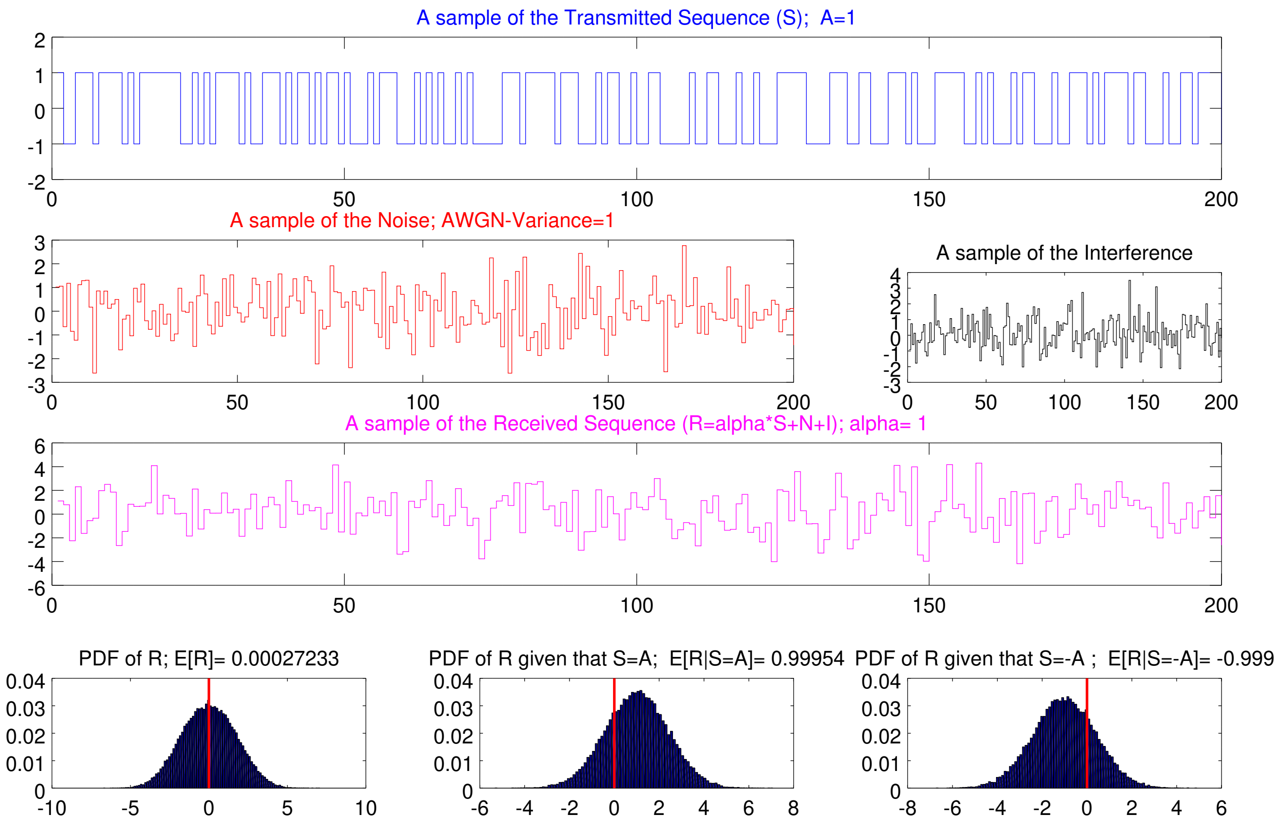

Figure 15: Effect of Interference **A=1 ;** $R = S + N + I$

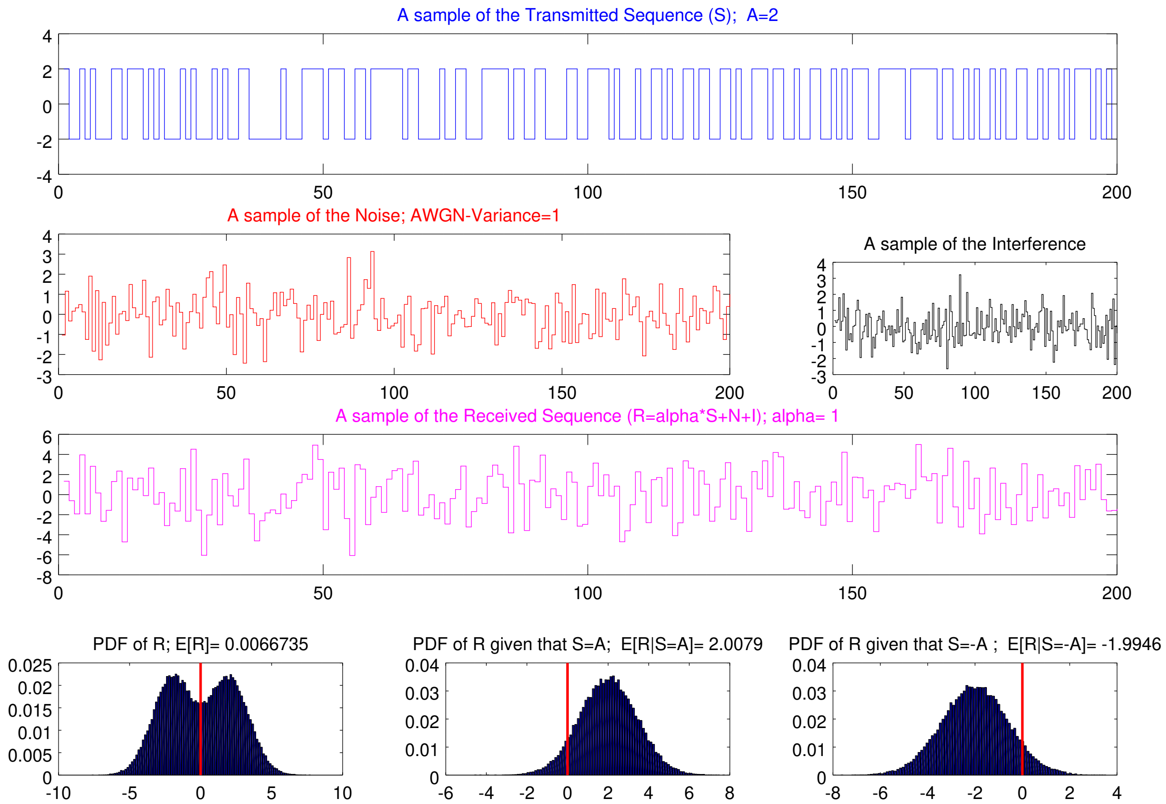

Figure 16: Effect of Interference **A=2 ;** $R = S + N + I$

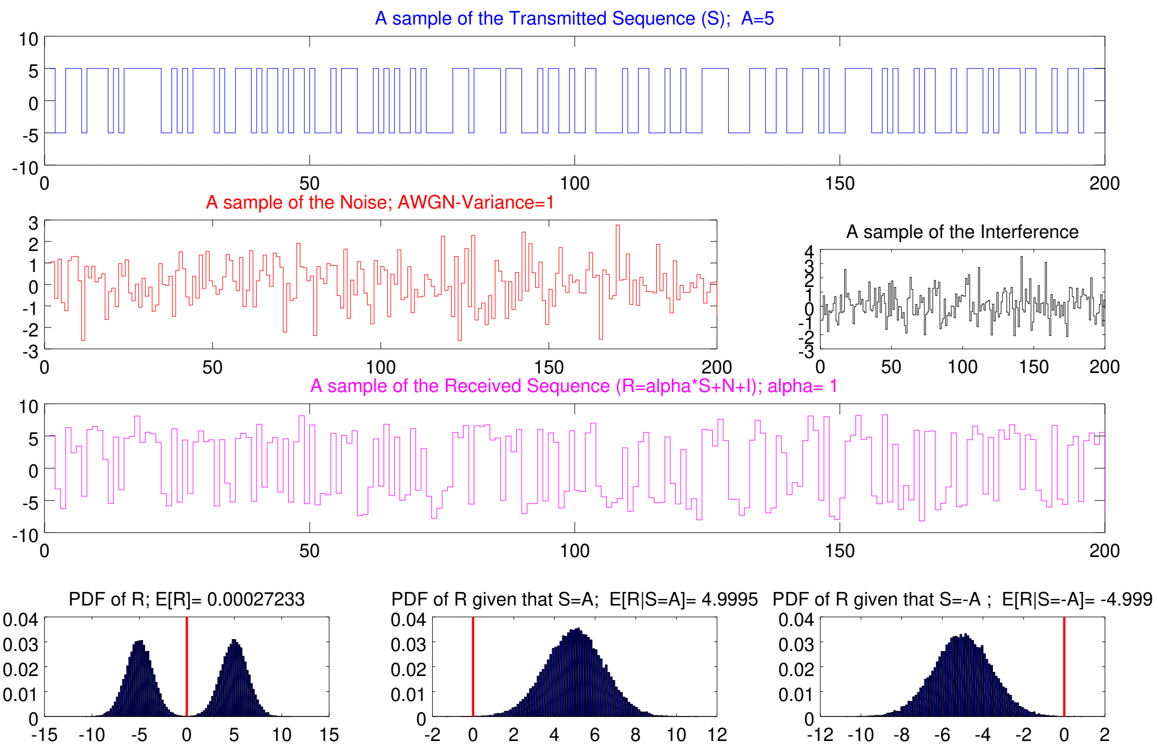

Figure 17: Effect of Interference **A=5 ;** $R = S + N + I$

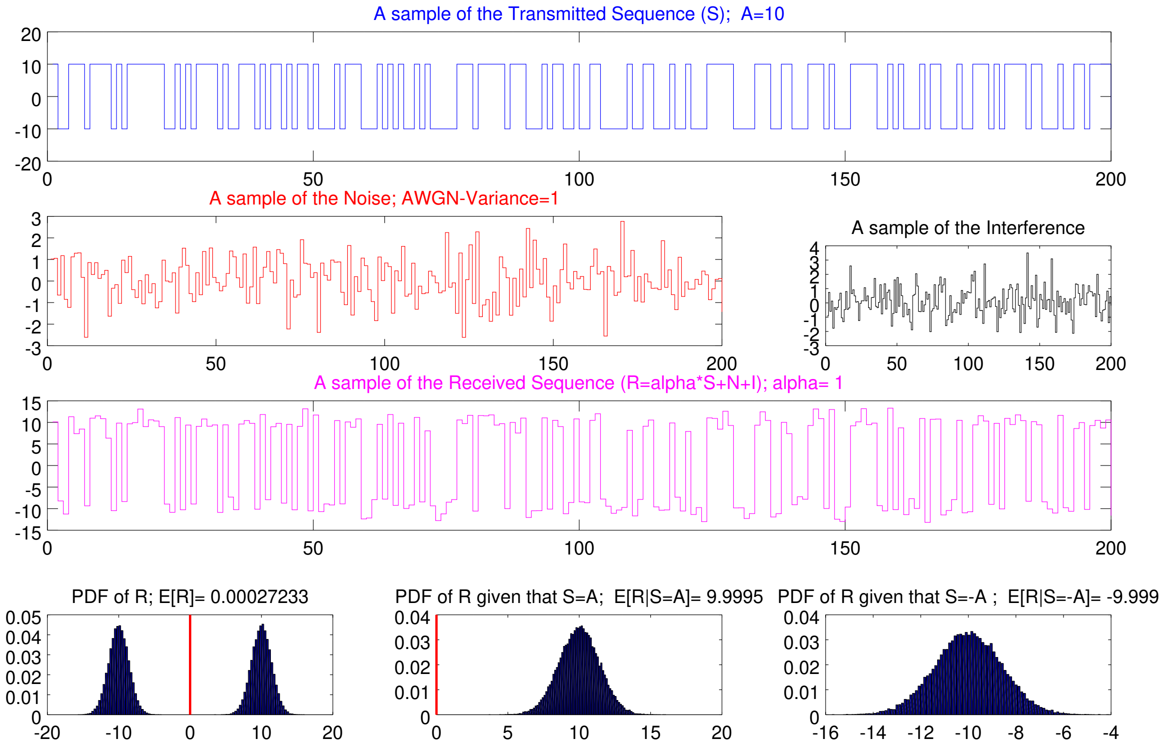

Figure 18: Effect of Interference **A=10** ; $R = S + N + I$

7.2 Effect of Changing Amplitude

Discussion
It can be observed that when the amplitude increases, the pdfs split in to two different areas.
Reason

- This is due to the fact that the noise variance is $\sigma^2 = 1$, and the effect it does to the received signal is minor when the amplitude is increased.
- If we observe Figure 18, (left bottom image) we can clearly see that the distribution has been concentrated into two distributions centered around -10 and +10.
- Therefore the effect of $\sigma^2 = 1$ noise and interference has become very low on the transmitted signal

8 Signal Amplification

In this section we consider a situation where Interference is not present.

- α=2 **(Figure 19)**
- α=5 **(Figure20)**
- α=10 **(Figure 21)**

received signal is amplified by a factor of α, such that $R = \alpha S + N$

8.1 Effect of changing α

8.1.1 α=2

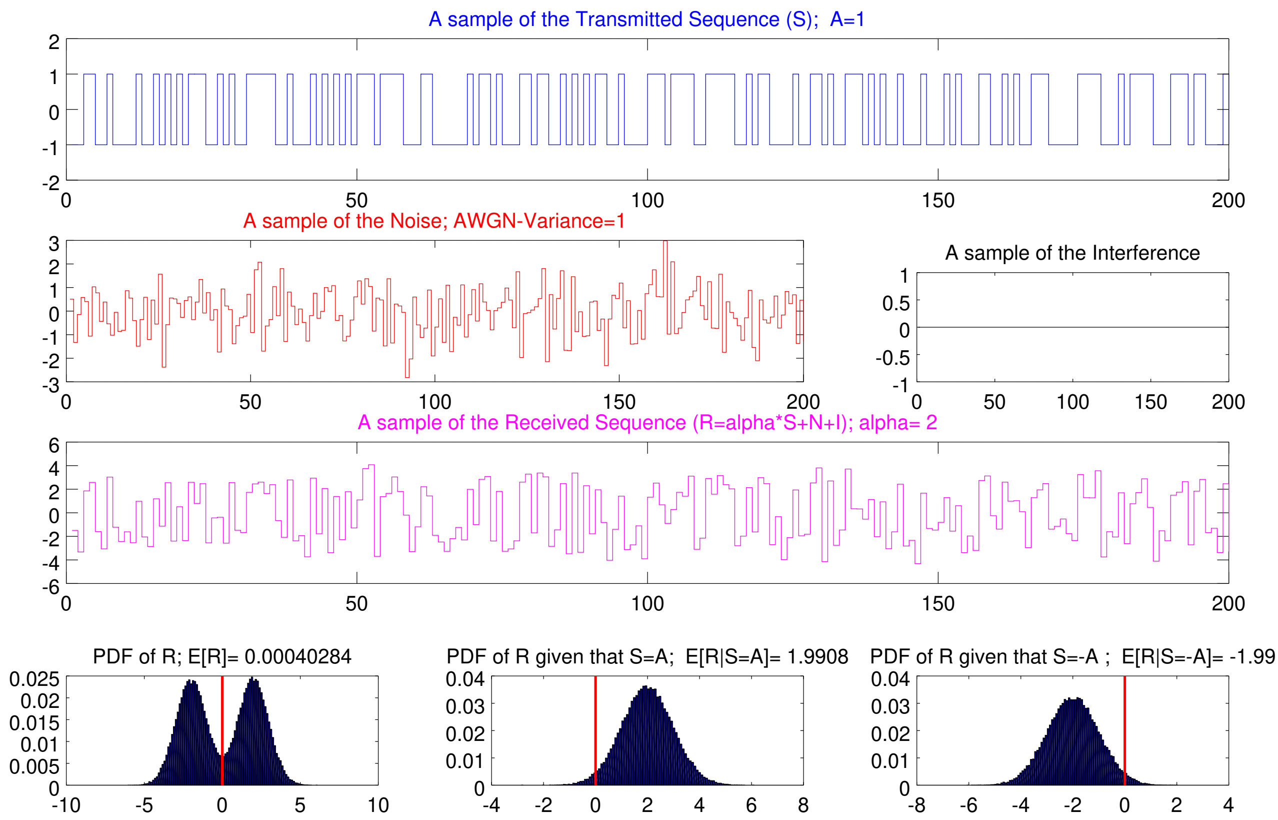

Figure 19: Amplifying factor, α=2 ; $R = \alpha S + N$

8.1.2 α=5

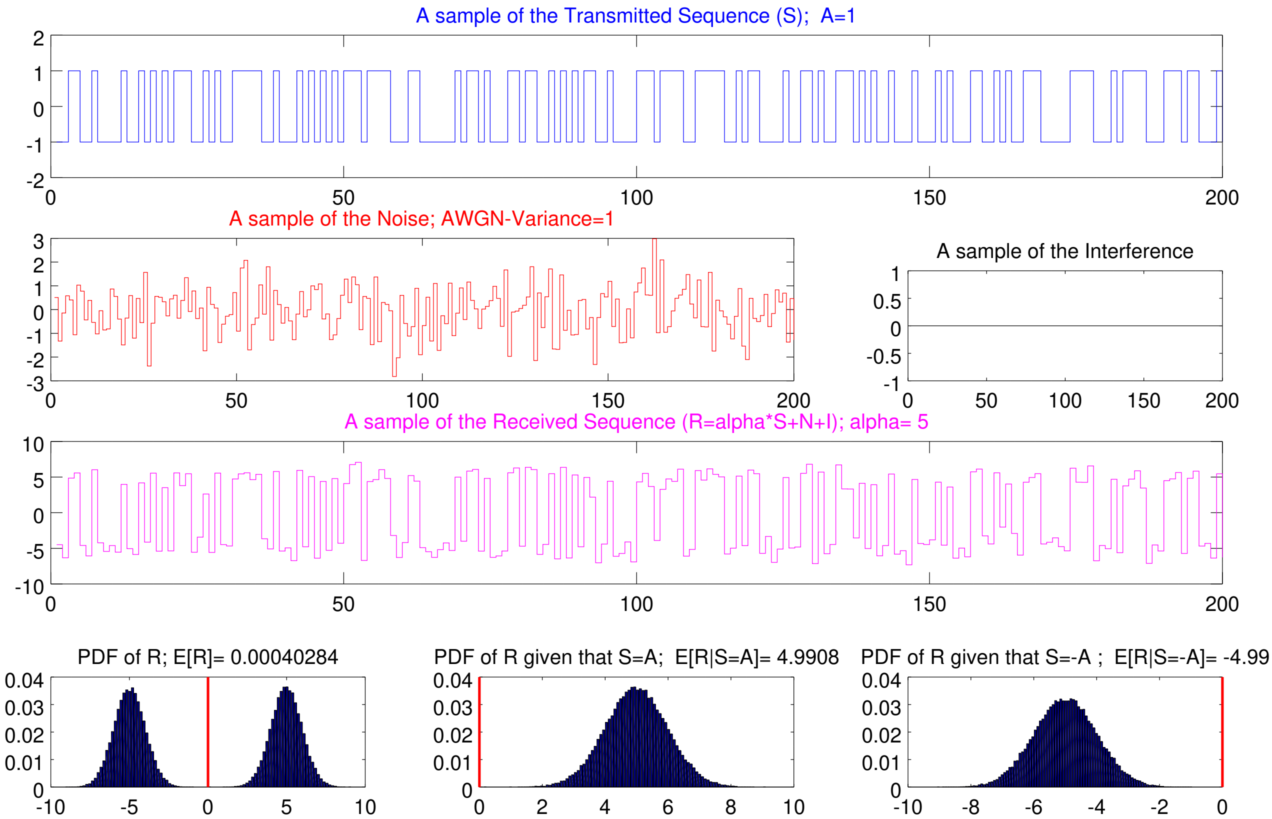

Figure 20: Amplifying factor, α=5 ; $R = \alpha S + N$

It is clear from the figures that when α increases, there is a clear separation of the PDF of R. Thus, decoding the signal will be much accurate.
Futhermore, the means of the conditional distributions become nearly equal to α*A

8.1.3 $\alpha = 10$

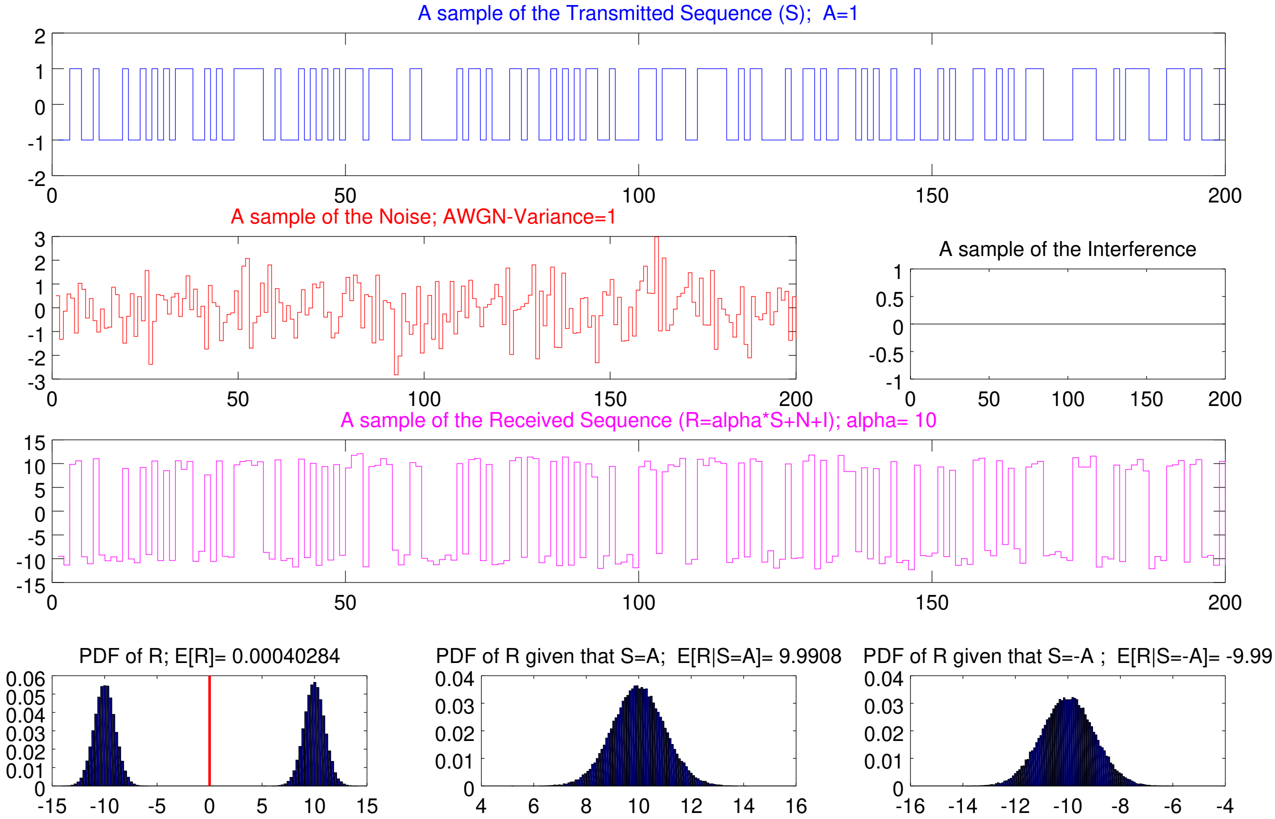

Figure 21: Amplifying factor, $\alpha = 10$; $R = \alpha S + N$

9 Conclusion

By simulating the practical constrains of a channel such as noise and interference, it becomes clear that, when desgining a communication system we should have a prior knowledge on the extend which noise and interference could span. After doing such research on it, we can come to a conclusion about the signal strength to be transmitted.
This will ensure that the signal will be received with a required accuracy.

10 Appendix

10.1 main.m

```
close all;
clear all;

L=100000;
A=1;
variance=1;
threshold=0;
bins=100;
alpha=1;

binary_seq = mod(reshape(randperm(1*L), 1, L), 2 ); %equiprobable
    random binary sequence%
pulses=zeros(1,L);

%% Generating Rectangular pulses based on the binary seq %%
for i=1:L
  if(binary_seq(i)==1)
    pulses(i)=A;
  else
    pulses(i)=-A;
  end
end

N1 = sqrt(variance)*randn(1,L);   %%noise
N2 = sqrt(2*variance)*randn(1,L);
N3 = sqrt(10*variance)*randn(1,L);
N4 = sqrt(100*variance)*randn(1,L);
N5 = sqrt(1000*variance)*randn(1,L);


R =pulses+N1;  %% received signals with different
R1=pulses+N1;  %% noise variances
R2=pulses+N2;
R3=pulses+N3;
R4=pulses+N4;
R5=pulses+N5;
disp_n=100; %%display 100 samples


%%% Plot Variance curves %%%%%%%
figure(1)
```

```
subplot(3,2,1)
stairs(pulses(1:disp_n));
title('Received Sequence ; Without noise ')
axis([0 inf -10*A 10*A])
hold on

subplot(3,2,2)
stairs(R1(1:disp_n),'LineStyle','-','color','r');
title(['Received Sequence ; Noise variance ',num2str(variance)])
axis([0 inf -10*variance 10*variance])

subplot(3,2,3)
stairs(R2(1:disp_n),'LineStyle','-','color','r');
title(['Received Sequence ; Noise variance ',num2str(2*variance)])
axis([0 inf -10*variance 10*variance])

subplot(3,2,4)
stairs(R3(1:disp_n),'LineStyle','-','color','r');
title(['Received Sequence ; Noise variance  ',num2str(10*variance)])
axis([0 inf -10*variance 10*variance])

subplot(3,2,5)
stairs(R4(1:disp_n),'LineStyle','-','color','r');
title(['Received Sequence ; Noise variance  ',num2str(100*variance)])
axis([0 inf -10*variance 10*variance])

subplot(3,2,6)
stairs(R5(1:disp_n),'LineStyle','-','color','r');
title(['Received Sequence ; Noise variance   ',num2str(variance)])
axis([0 inf -10*variance 10*variance])

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Write Plot as EPS file%%%%%
print -depsc part3.eps
close all;


%%%%% DECODING ######
decoded=zeros(1,L);

for i=1:L
  if(R(i)>threshold)
    decoded(i)=A;
  else
```

```
    decoded(i)=-A;
  end
end


figure(2)

subplot(3,1,1)
stairs(pulses(1:disp_n));
title('Transmitted Pulses')
axis([0 inf -2*A 2*A])


subplot(3,1,2)
stairs(R(1:disp_n),'LineStyle','-','color','r');
title('Received Sequence (R); Noise variance = 1','LineStyle','-','
    color','r')
axis([0 inf -5*variance 5*variance])

subplot(3,1,3)
stairs(decoded(1:disp_n),'LineStyle','-','color','r');
title('Decoded Sequence (Y) ','LineStyle','-','color','r')
axis([0 inf -2*variance 2*variance])

%%  Write Plot as EPS file %%
print -depsc part4.eps
close all;

%%%%%% For L=100,000
%%   code to generate and plot the histogram of the received sequence
    taking the no of bins as 10.
figure(3)

subplot(2,2,1)
draw_histogram(R,10); ## my function ##

subplot(2,2,2)
hist(R,bins);
title('Using Matlab hist() function')

subplot(2,2,3)
draw_histogram(R,100); ## my function ##

subplot(2,2,4)
hist(R,100);
```

```
title('Using Matlab hist() function')

print -depsc part5_a.eps

%% conditional pdf of Recieved signal       #####
%%%%%   given that transmitted with A amplitude #####

I1 = zeros(1,L);

for i=4:7
  A_=10^(i-4)
  pulses_=pulses.*10^(i-4);
  R_=pulses_+N1; %%%% received signal
  figure('name',['Effect of changing Amplitude to ',num2str(A_)])

  conditional_pdf_plot(R_,alpha,N1,I1,pulses_,A_,disp_n,variance,bins)
      %%pulses,disp_n,A,R,

  cond(R_,alpha,N1,I1,pulses_,A_,disp_n,variance,bins)
end

close all;

figure('name','Effect of introducing Interference')
A=1

N1 = sqrt(variance)*randn(1,L);   %%noice
I1 = sqrt(variance)*randn(1,L);   %%interference

R  = pulses+N1+I1;
conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)

A=5
pulses=pulses.*5;
R  = pulses+N1+I1;
conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)

A=10
pulses=pulses.*2;
R  = pulses+N1+I1;
conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)

```

```
A=1
pulses=pulses./10;


%%  EFFECT on amplification

alpha=2;
figure('name',['Effect of changing Alpha to ',num2str(alpha),' with no
   Interference'])
R  = pulses*alpha+N1;
I1 = zeros(1,L);
conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)

alpha=5;
figure('name',['Effect of changing Alpha to ',num2str(alpha),' with no
   Interference'])
R  = pulses*alpha+N1;
I1 = zeros(1,L);
conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)

alpha=10;
figure('name',['Effect of changing Alpha to ',num2str(alpha),' with no
   Interference'])
R  = pulses*alpha+N1;
I1 = zeros(1,L);
conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)

close all;
```

10.2 conditional_pdf_plot.m

```
function g = conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,
   variance,bins)
  inter=yes
  if(sum(I1)==0)
    inter=no
  end
  close all;

  figure(15,position,get(0,screensize))

  subplot(4,3,1:3)
```

```
stairs(pulses(1:2*disp_n),'LineStyle','-','color','b');
title(['A sample of the Transmitted Sequence (S);  A=',num2str(A)],'LineStyle','-','color','b')
axis([0 inf -2*A 2*A])

subplot(4,3,4:5)
stairs(N1(1:2*disp_n),'LineStyle','-','color','r');
title(['A sample of the Noise; AWGN-Variance=',num2str(variance)],'LineStyle','-','color','r')
axis auto;

subplot(4,3,6)
stairs(I1(1:2*disp_n),'LineStyle','-','color','k');
title(['A sample of the Interference'],'LineStyle','-','color','k')
axis auto;

subplot(4,3,7:9)
stairs(R(1:2*disp_n),'LineStyle','-','color','m');
title(['A sample of the Received Sequence (R=alpha*S+N+I); alpha= ',num2str(alpha)],'LineStyle','-','color','m')
axis auto;

subplot(4,3,10)
[f,x]=hist(R,bins);
bar(x, f / sum(f));
title(['PDF of R; E[R]= ',num2str(mean(R))]);
hold on;
line([0, 0], ylim, 'LineWidth', 2, 'Color', 'r');

subplot(4,3,11)
pdf_A1=conditional_pdf(R,pulses,A);
[f,x]=hist(pdf_A1,bins);
bar(x, f / sum(f));
title(['PDF of R given that S=A;  E[R|S=A]= ',num2str(mean(pdf_A1))]);
hold on;
line([0, 0], ylim, 'LineWidth', 2, 'Color', 'r');

subplot(4,3,12)
pdf_A2=conditional_pdf(R,pulses,-A);
[f,x]=hist(pdf_A2,bins);
bar(x, f / sum(f));
title(['PDF of R given that S=-A ;  E[R|S=-A]= ',num2str(mean(pdf_A2))]);
hold on;
```

```
  line([0, 0], ylim, 'LineWidth', 2, 'Color', 'r');

  name=strcat(Summary-A=,num2str(A),-Var=,num2str(variance),-alpha=,
     num2str(alpha),-Interference=,inter,.eps)

  %% Write Plot as EPS file%%%%%%
  print (15,name, -depsc,-S1180,720,-F:8)




end
```