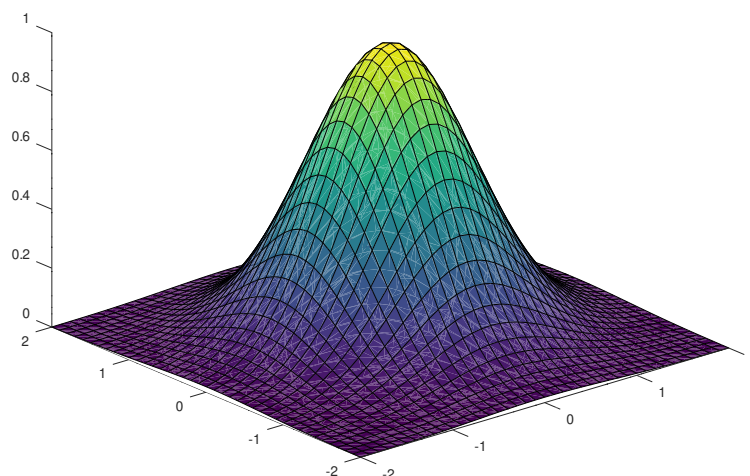


EN2040 - RANDOM SIGNALS AND PROCESSES

UNIVERSITY OF MORATUWA

DEPARTMENT OF ELECTRONIC AND TELECOMMUNICATION
ENGINEERING

Simulation Assignment



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Date: May 20, 2019

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1 Introduction

In this Assignment, rectangular pulses of $\pm A$ amplitude are simulated. These pulses carry binary equiprobable data over a communication channel. In each section which follows in this report, I intend to find out the effect of factors such as noise, interference, and amplification on the received signal.

Visualizations including Histograms, Stair plots are included to get a better intuition of these effects.

2 Generating Equiprobable Rectangular Pulses

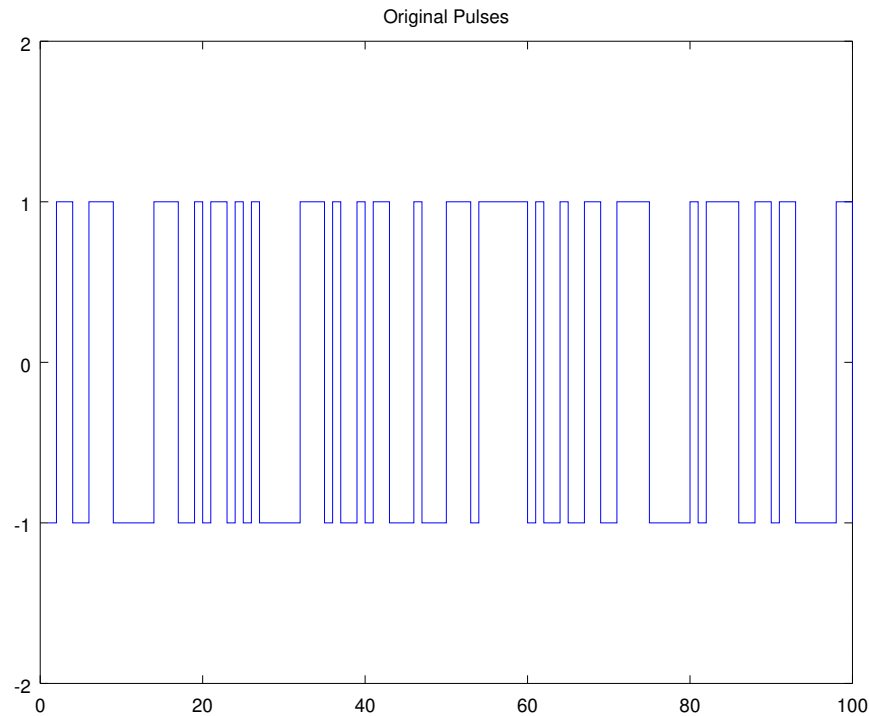


Figure 1: First 100 elements of the Rectangular Pulse Sequence (S)

```
1 L=1000; %length of the sequence
2 A=1; %variance 1
3
4 binary_seq = mod(reshape(randperm(1*L),1,L),2);%equiprobable random
5 pulses=zeros(1,L);
6
7 %%% Generating Rectangularpulses based on the binary seq %%%
8 for i=1:L
9     if(binary_seq(i)==1)
10         pulses(i)=A;
11     else
12         pulses(i)=-A;
13     end
14 end
15
16 stairs(pulses(1:100)); %plot first 100 samples
17 title('Original Pulses')
18 axis([0 inf -2*A 2*A])
19 hold on
```

3 Generating an AWGN Sequence $\sigma^2 = 1$

Figure 2 shows the first 100 samples of the Additive White Gaussian Noise(AWGN) which will be considered as the noise of the channel.

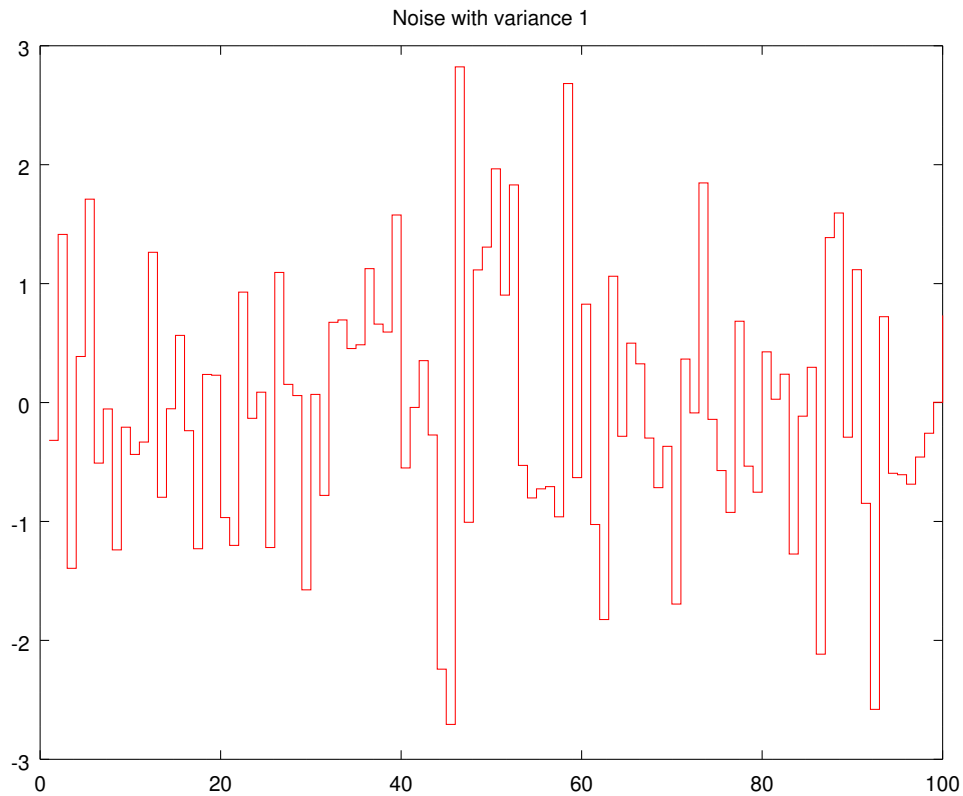


Figure 2: First 100 samples of the AWGN Noise with $\sigma^2 = 1$

```

1      L=1000;      %length of the sequence
2      variance=1; %variance 1
3      N1=sqrt(variance)*randn(1,L);  %%noise

```

4 Sequence of the Recieved Signal (R)

4.1 Impact of the variance of noise on R by varying $\sigma^2 = x$

From the plots, it is evident that when the noise variance increases, the received signal becomes more and more distorted.

The transmitted signal of amplitude:1 is completely dominated by high noise variances.

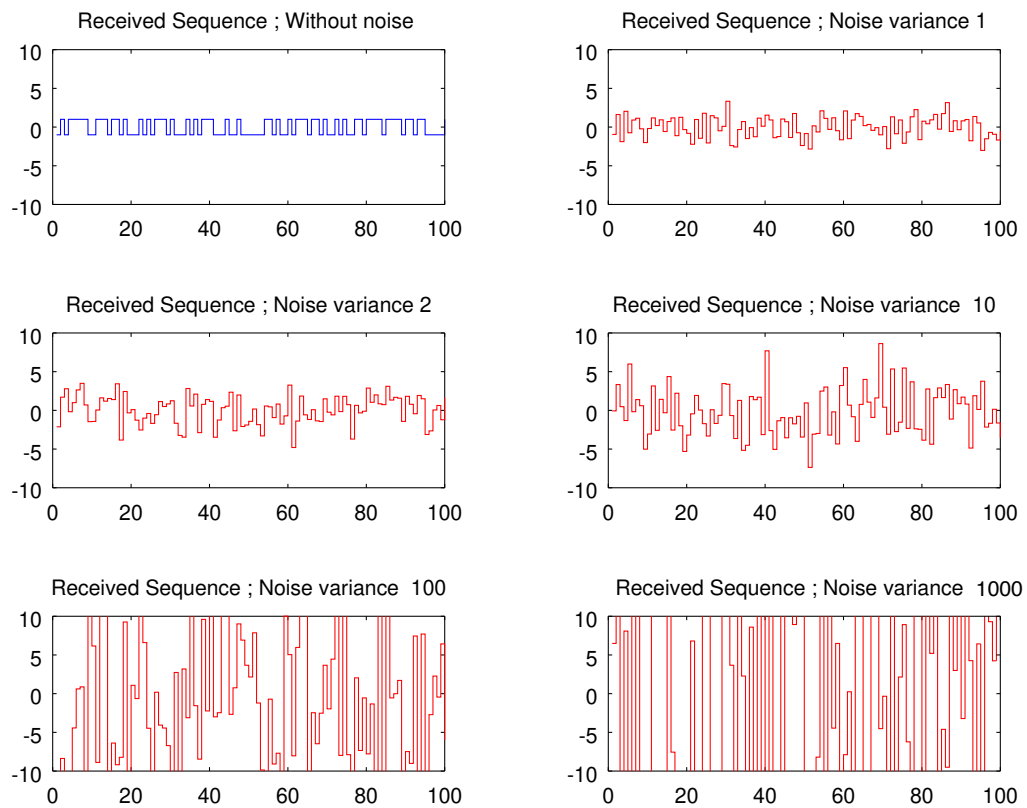


Figure 3: Q3: Varying the variance σ^2

5 Decoded signal (Y) vs Transmitted signal

Shown below is the first 100 samples of the transmitted pulses. Comparing the transmitted sequence with the Decoded sequence(Y), there seems to be certain bit changes.

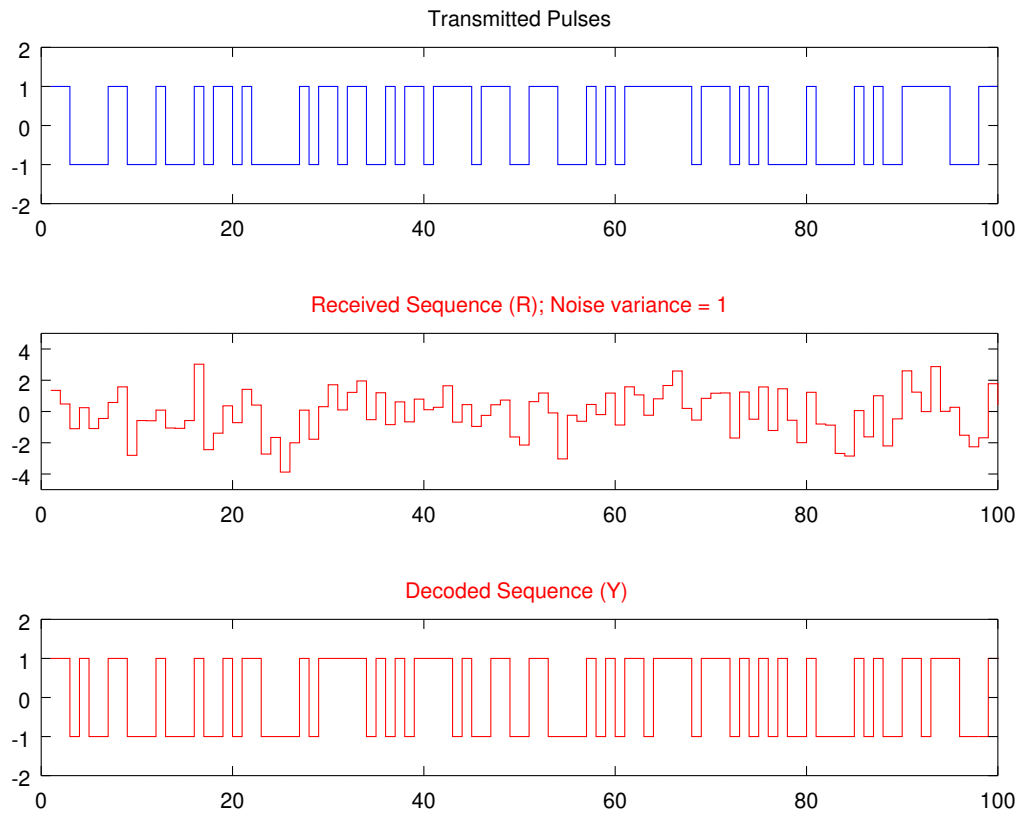


Figure 4: Q4: Sketch and compare the sequence of Y with the transmitted signal

6 Code to generate histogram of the received sequence

```
1 function g = draw_histogram(R,bins) %my function
2     lower_bound=min(R);
3     upper_bound=max(R);
4
5     gap=(upper_bound-lower_bound)/bins;
6     x =lower_bound:gap:upper_bound;
7     y=zeros(1,length(x));
8
9     for i=1:length(R)
10         id=floor((R(i)-lower_bound)/gap)+1;
11         y(id)=y(id)+1;
12     end
13
14     bar(x,y, 'BarWidth', 1);
15     title('Histogram of the Received Signal')
16 end
```

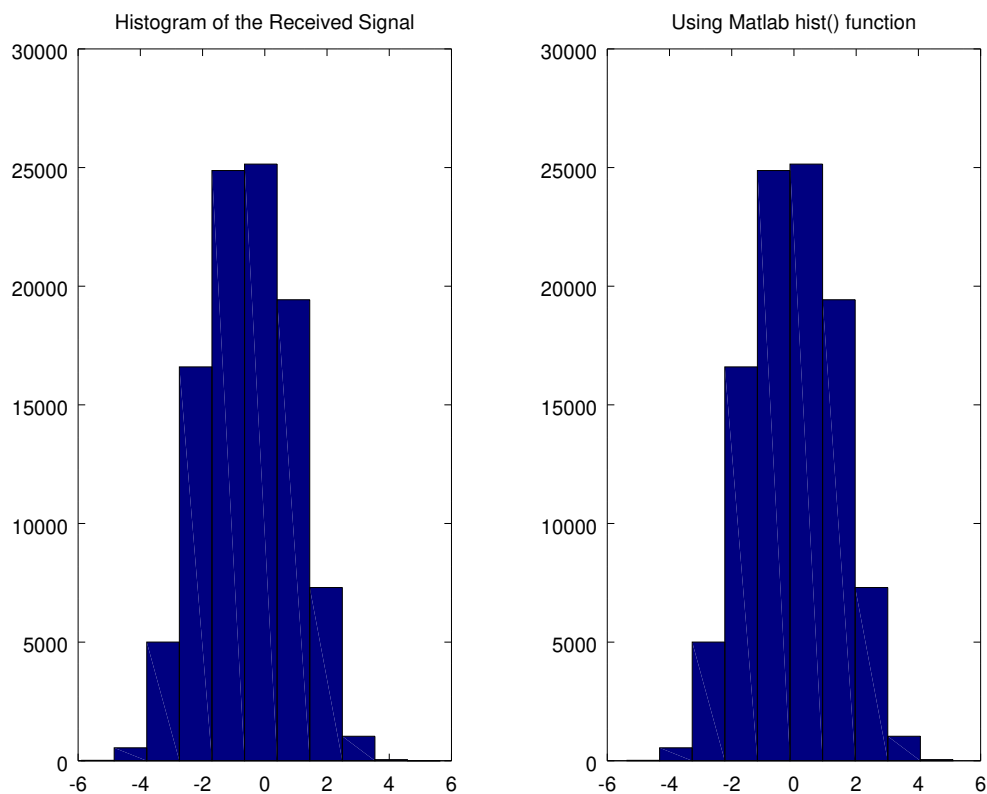


Figure 5: Q5: Function to draw histogram (Bin Size=10)

6.1 Effect of changing number of bins on the histogram of the received sequence

```
1 %main program
2 subplot(1,2,1)
3 draw_histogram(R,bins); %calling draw_histogram.m
4
5 subplot(1,2,2)
6 hist(R,bins);
7 title('Using Matlab hist() function')
```

The function, **draw_histogram** divides the range of values to the given number of bins and calculates the frequency of each bin.

Each frequency is recorded to an array and then visualized by the **bar** function of matlab.

6.1 Effect of changing number of bins

When increasing the number of bins, the histogram takes the features of a Normal Distribution.

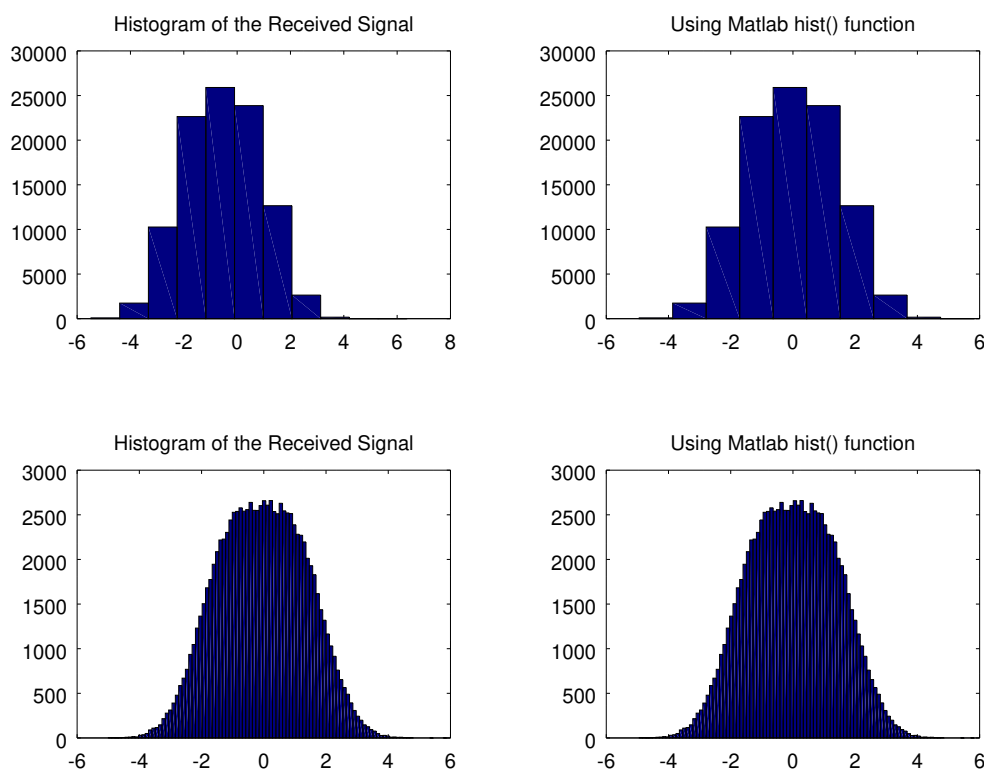


Figure 6: Q5 a): changing bins from 10 to 100

6.2 Conditional PDF

We fix variance($\sigma^2=1$) of noise and change the Amplitude of the transmitted signal.

6.2.1 $A=1$; $f_{R|S}(r | S = A)$ $f_{R|S}(r | S = -A)$

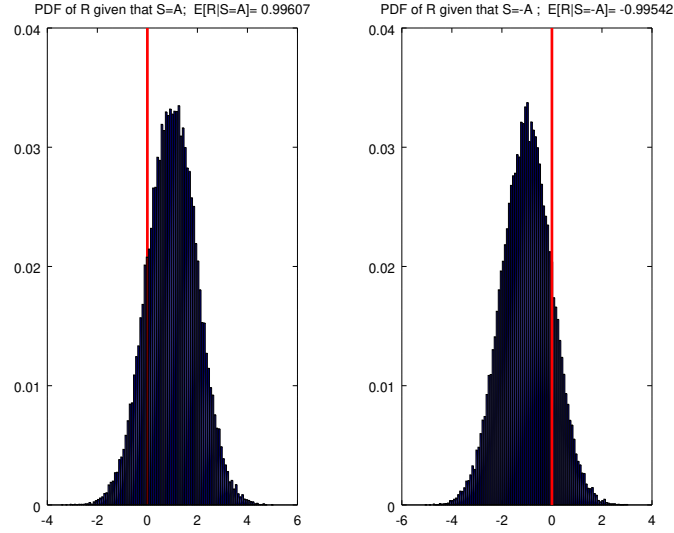


Figure 7: Conditional PDFs with $A=1$

6.2.2 $A=10$; $f_{R|S}(r | S = A)$ $f_{R|S}(r | S = -A)$

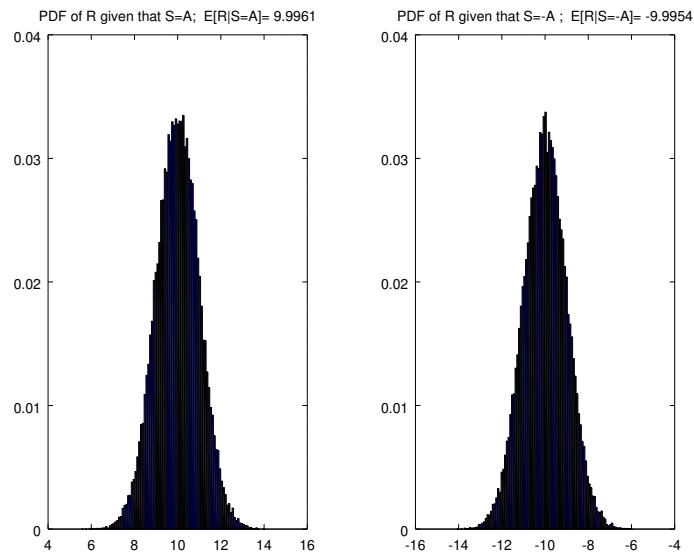


Figure 8: Conditional PDFs with $A=10$

6.3 Expected values of Distributions

When the amplitude of the transmitted signal increases, the conditional pdfs move towards $\pm A$ according to **Figures 7,8,9**.

This is because the effect of $\sigma^2=1$ variance is outperformed by the high amplitude of the transmitted signal.

According to **Table 1**, the conditional distribution's mean nearly equals to the amplitude.

6.2.3 $A=100$; $f_{R|S}(r | S = A)$ $f_{R|S}(r | S = -A)$

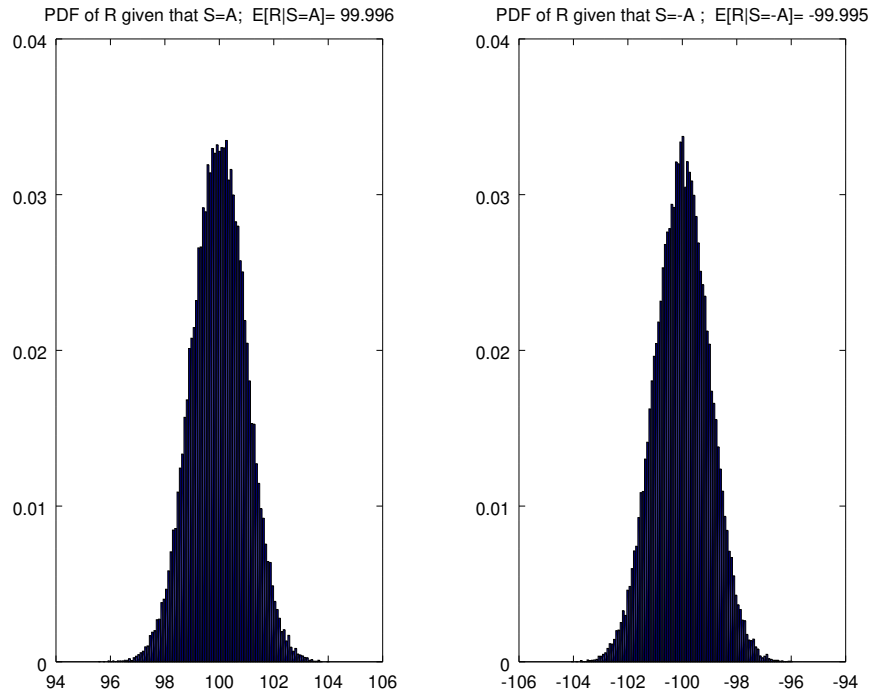


Figure 9: Conditional PDFs with $A=100$

6.3 Expected values of Distributions

Table 1: Impact on the Expected value with the Amplitude of the transmitted signal

Amplitude	$E[R S=A]$	$E[R S=-A]$	$E[R]$
$A=1$	0.99607	-0.99542	0.004498
$A=10$	9.9961	-9.9954	0.004498
$A=100$	99.996	-99.995	0.004498

6.4 Sketch the PDF $f_R(r)$

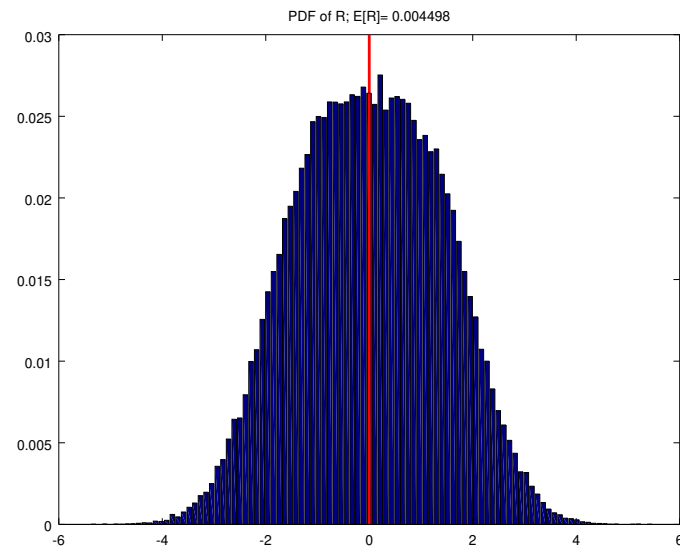


Figure 10: PDF $f_R(r)$ with $A=1$

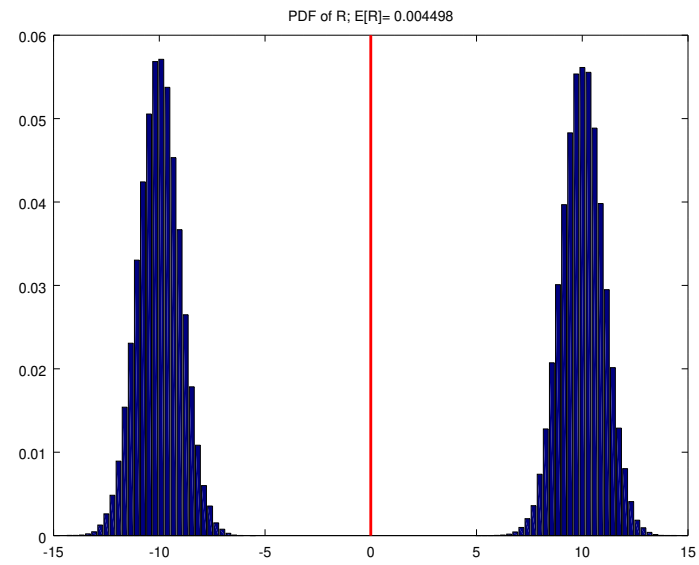


Figure 11: PDF $f_R(r)$ with $A=10$

6.4 Sketch the PDF $f_R(r)$ GENERATE HISTOGRAM OF THE RECEIVED SEQUENCE

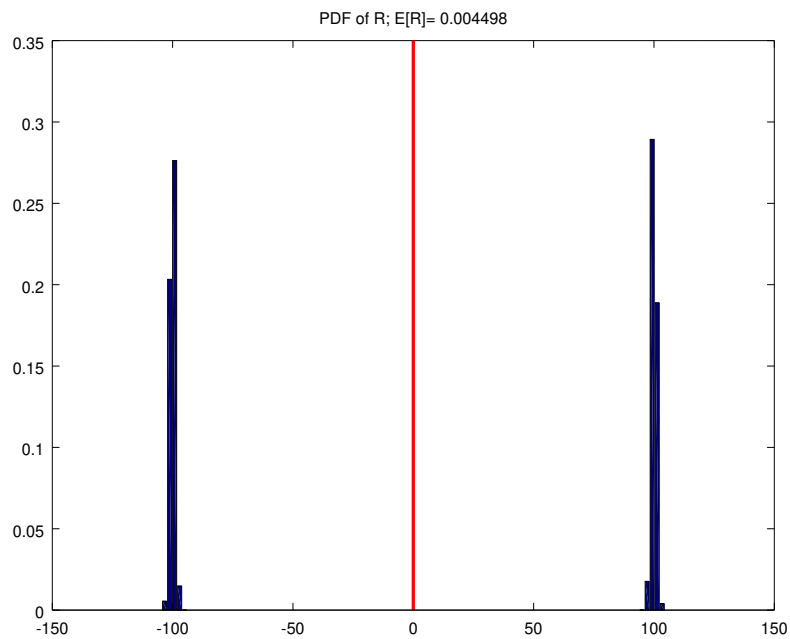


Figure 12: PDF $f_R(r)$ with $A=100$

Because of the clear separation of the distribution, it will be much easier to decode the signal.

7 Effect of Interference

7.1 PDF comparison w/without interference

It can be observed that adding interference to the signal increases the variance of the received signal. This is clear when observing the area to the left of the Red lines drawn at $x=-4$.

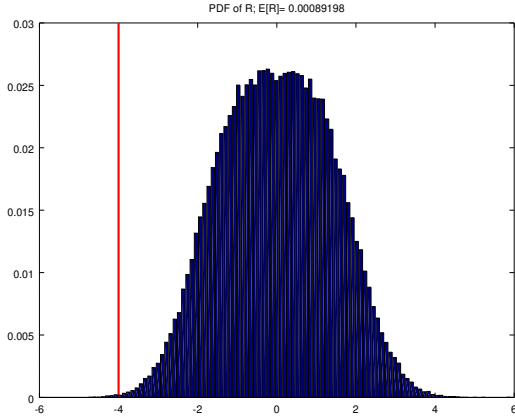


Figure 13: PDF Without Interference

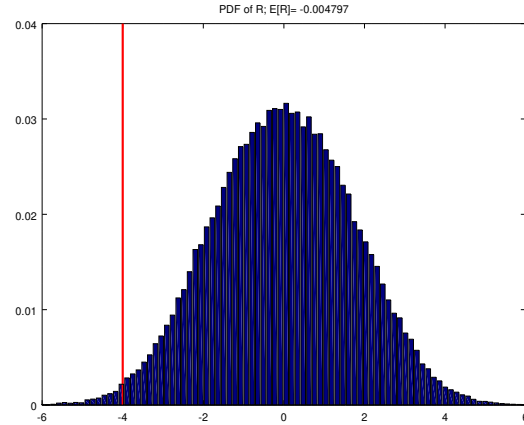


Figure 14: PDF With Interference

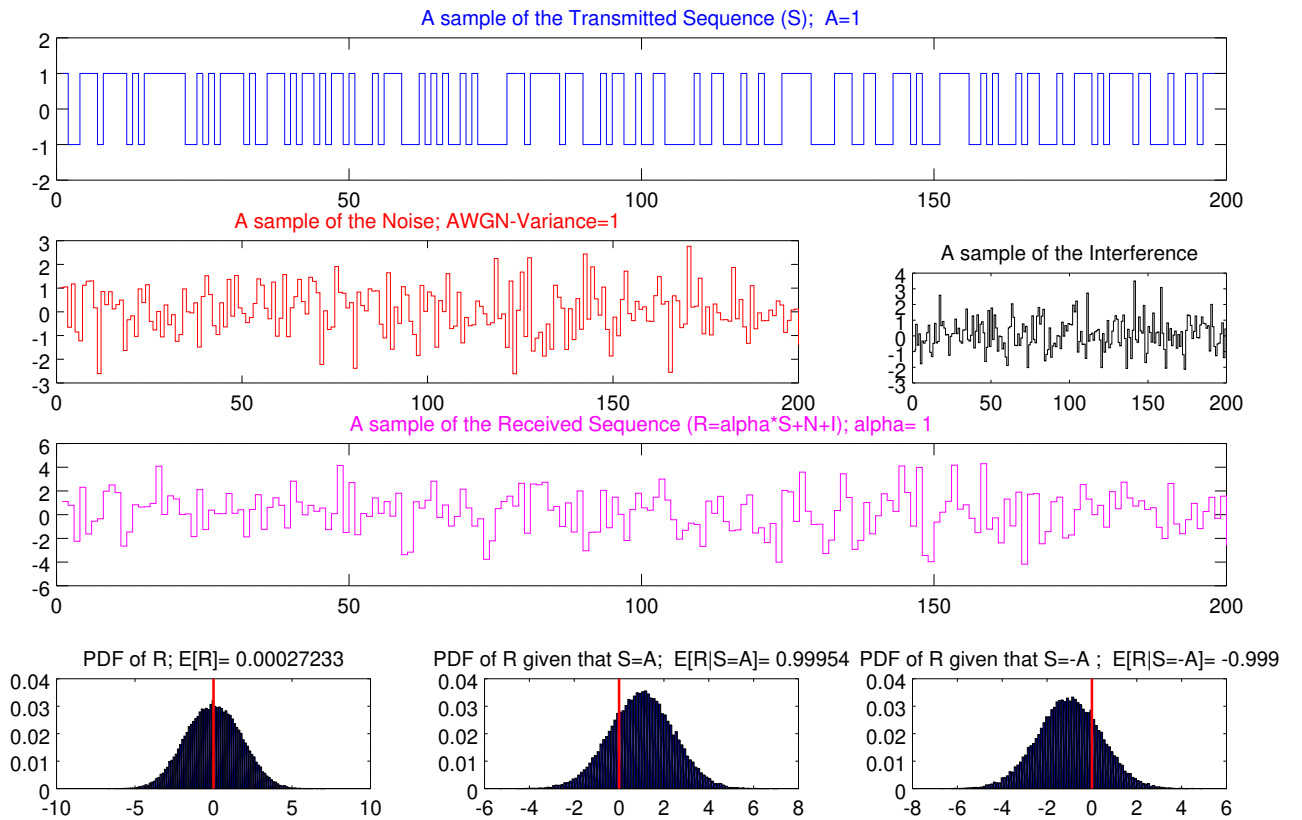
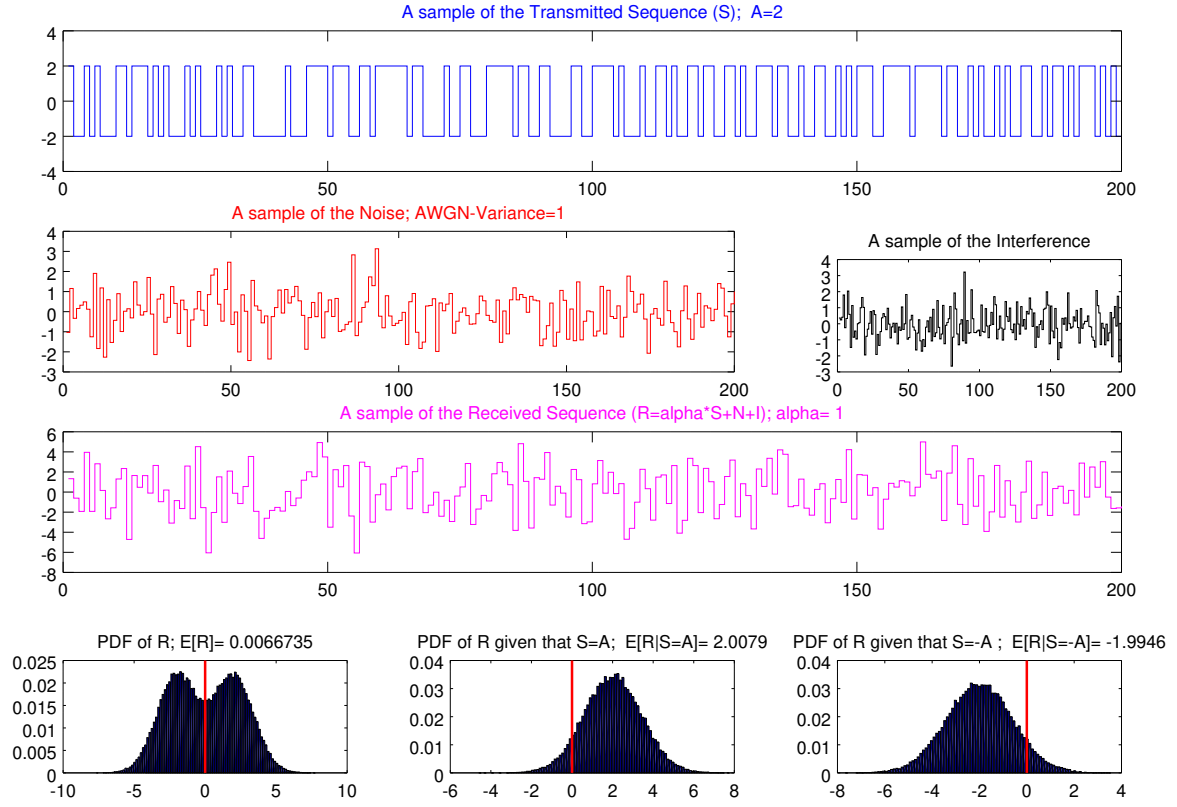
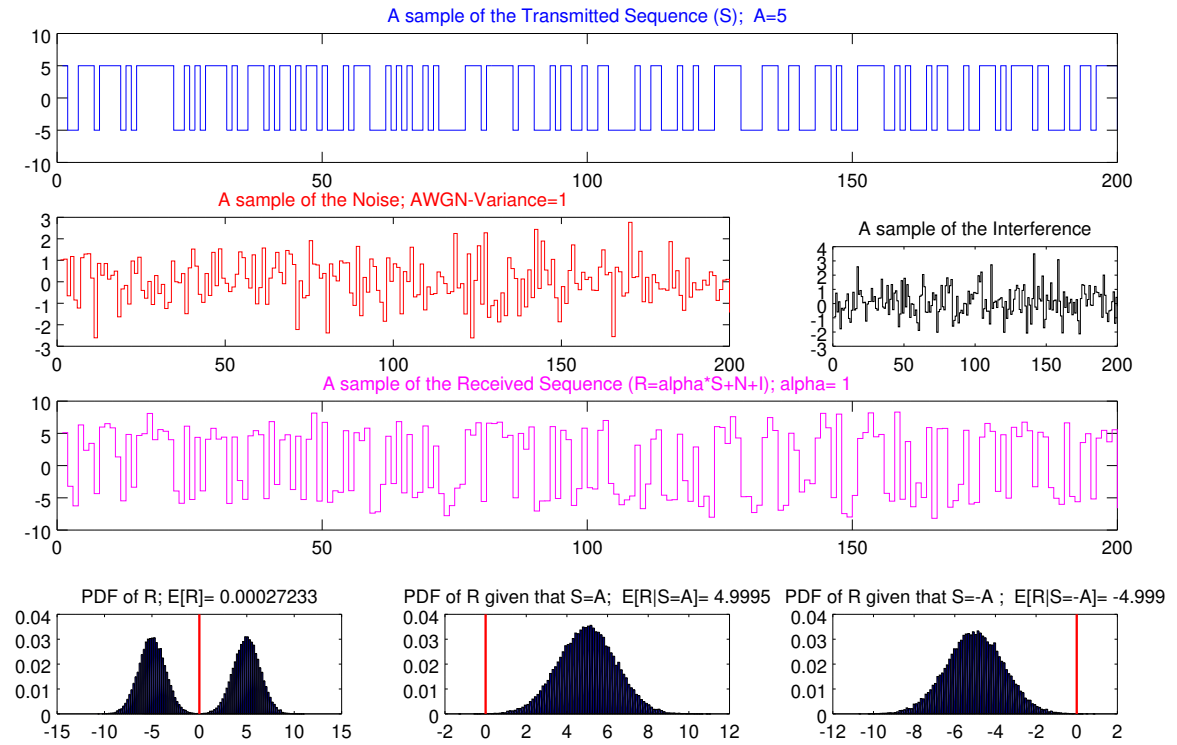


Figure 15: Effect of Interference $A=1$; $R = S + N + I$

Figure 16: Effect of Interference $A=2$; $R = S + N + I$ Figure 17: Effect of Interference $A=5$; $R = S + N + I$

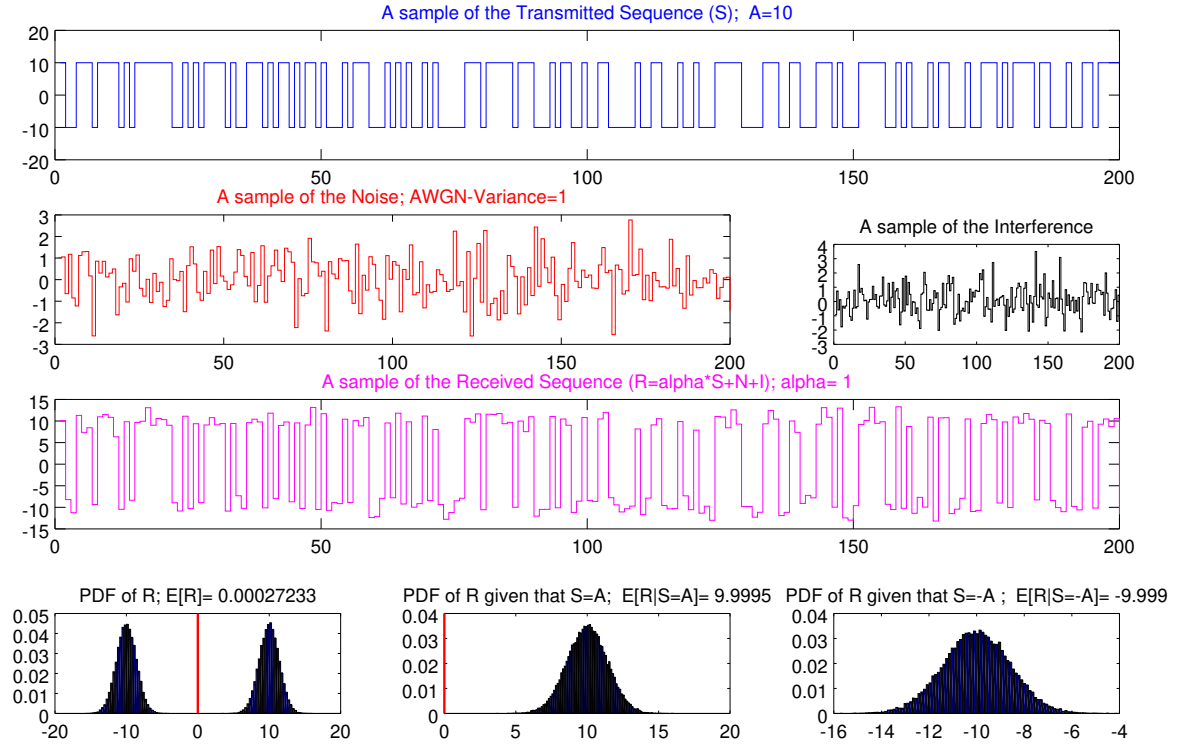


Figure 18: Effect of Interference $A=10$; $R = S + N + I$

7.2 Effect of Changing Amplitude

Discussion

It can be observed that when the amplitude increases, the pdfs split in to two different areas.

Reason

- This is due to the fact that the noise variance is $\sigma^2 = 1$, and the effect it does to the received signal is minor when the amplitude is increased.
- If we observe Figure 18, (left bottom image) we can clearly see that the distribution has been concentrated into two distributions centered around -10 and +10.
- Therefore the effect of $\sigma^2 = 1$ noise and interference has become very low on the transmitted signal

8 Signal Amplification

In this section we consider a situation where Interference is not present.

- $\alpha=2$ (Figure 19)
- $\alpha=5$ (Figure 20)
- $\alpha=10$ (Figure 21)

received signal is amplified by a factor of α , such that $R = \alpha S + N$

8.1 Effect of changing α

8.1.1 $\alpha=2$

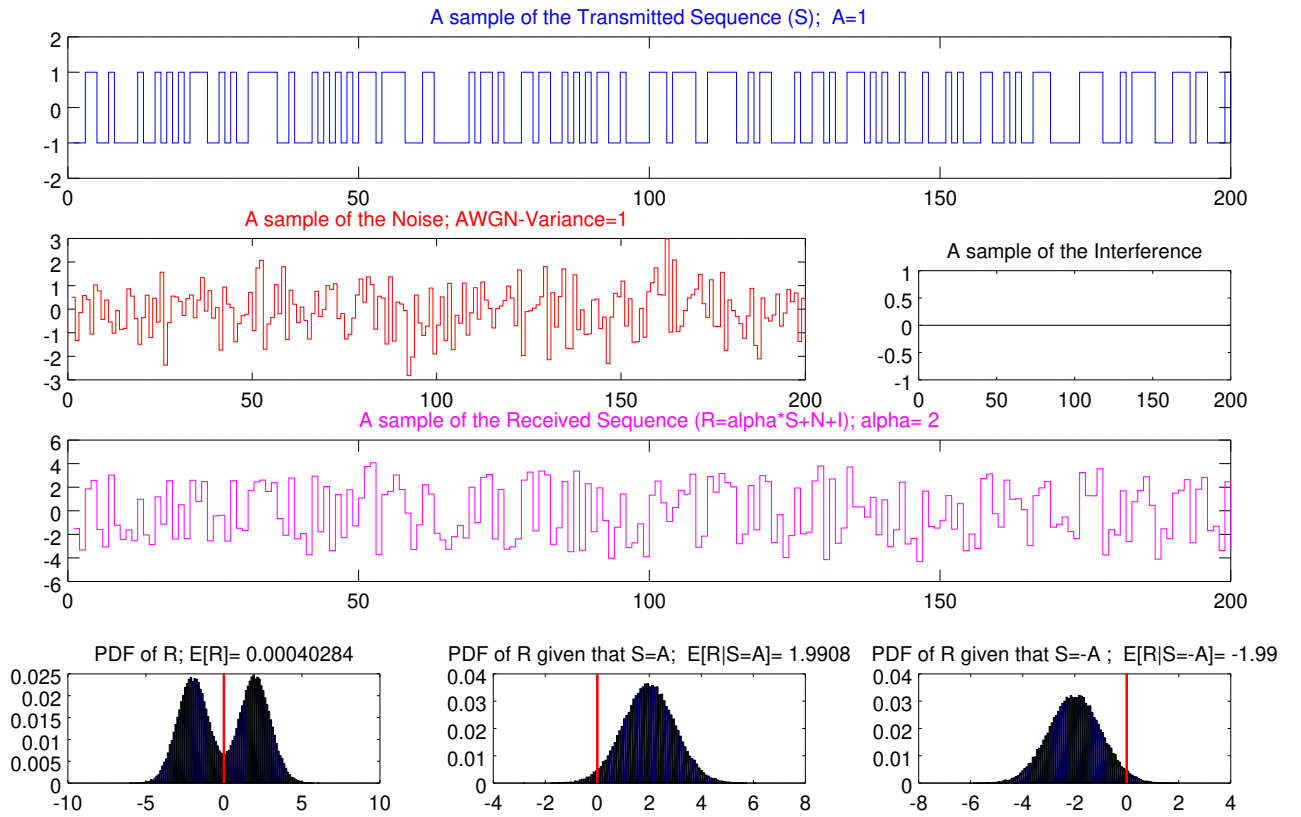
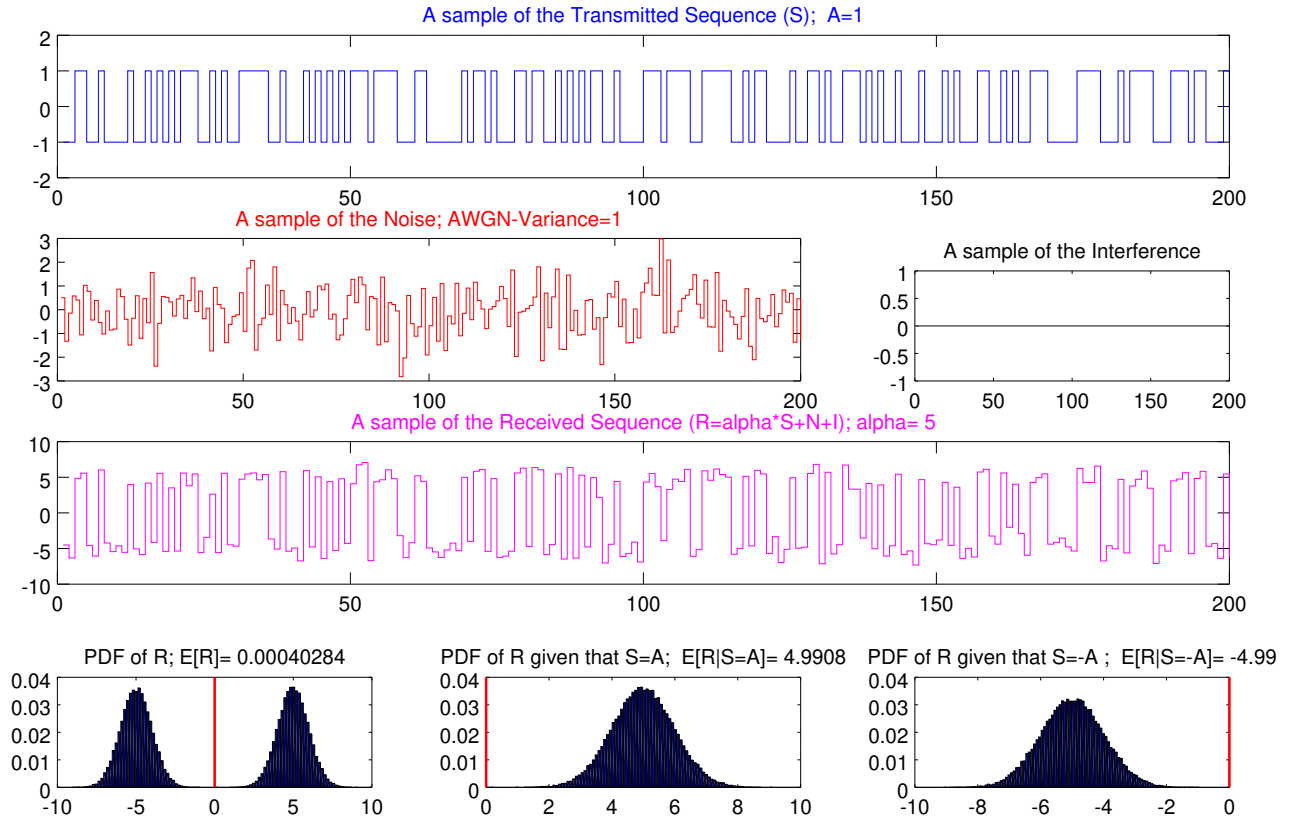
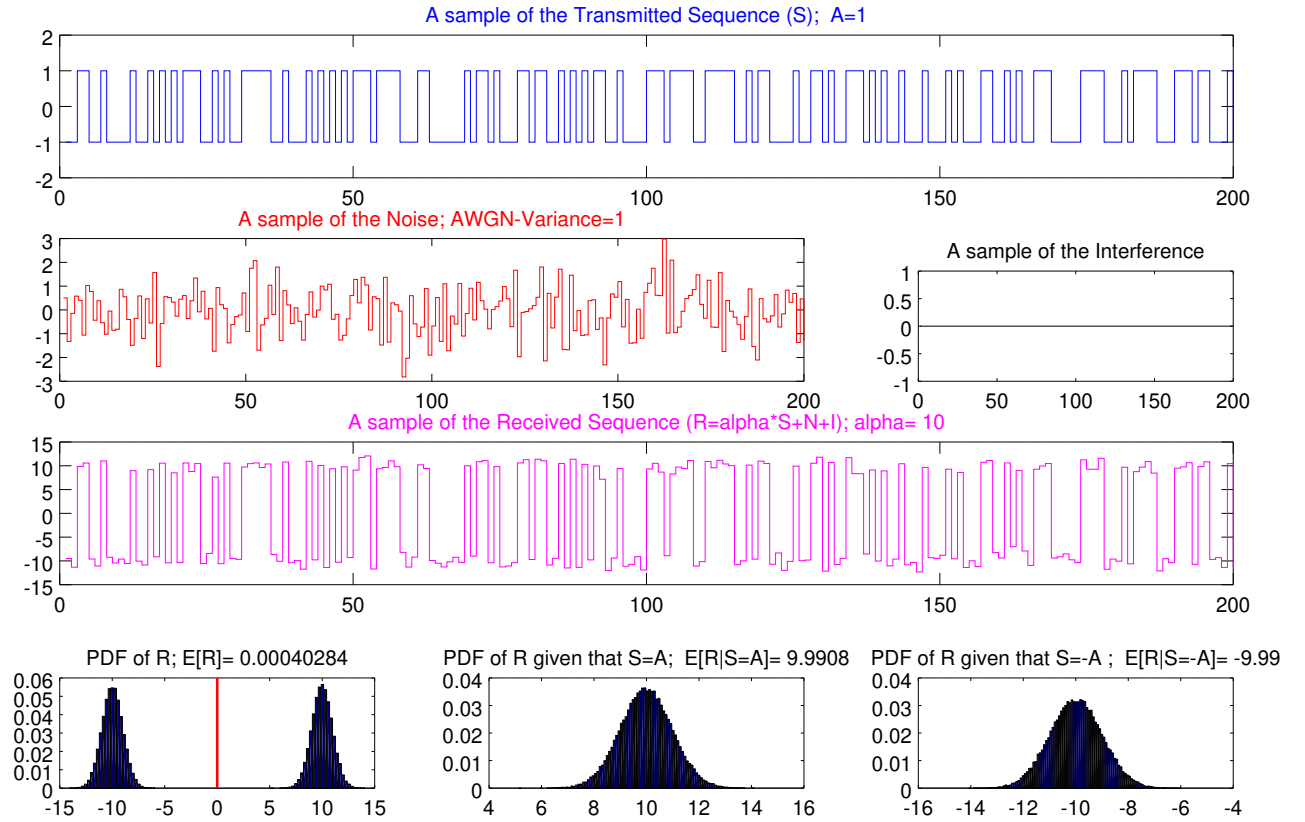


Figure 19: Amplifying factor, $\alpha=2$; $R = \alpha S + N$

8.1.2 $\alpha=5$ Figure 20: Amplifying factor, $\alpha=5$; $R = \alpha S + N$

It is clear from the figures that when α increases, there is a clear separation of the PDF of R . Thus, decoding the signal will be much accurate.

Futhermore, the means of the conditional distributions become nearly equal to $\alpha \cdot A$

8.1.3 $\alpha=10$ Figure 21: Amplifying factor, $\alpha=10$; $R = \alpha S + N$

9 Conclusion

By simulating the practical constraints of a channel such as noise and interference, it becomes clear that, when designing a communication system we should have a prior knowledge on the extent which noise and interference could span. After doing such research on it, we can come to a conclusion about the signal strength to be transmitted.

This will ensure that the signal will be received with a required accuracy.

10 Appendix

10.1 main.m

```
1 close all;
2 clear all;
3
4 L=100000;
5 A=1;
6 variance=1;
7 threshold=0;
8 bins=100;
9 alpha=1;
10
11 binary_seq = mod(reshape(randperm(1*L), 1, L), 2 ); %equiprobable
    random binary sequence%
12 pulses=zeros(1,L);
13
14 %% Generating Rectangular pulses based on the binary seq %%
15 for i=1:L
16     if(binary_seq(i)==1)
17         pulses(i)=A;
18     else
19         pulses(i)=-A;
20     end
21 end
22
23 N1 = sqrt(variance)*randn(1,L); %%noise
24 N2 = sqrt(2*variance)*randn(1,L);
25 N3 = sqrt(10*variance)*randn(1,L);
26 N4 = sqrt(100*variance)*randn(1,L);
27 N5 = sqrt(1000*variance)*randn(1,L);
28
29
30 R =pulses+N1; %% received signals with different
31 R1=pulses+N1; %% noise variances
32 R2=pulses+N2;
33 R3=pulses+N3;
34 R4=pulses+N4;
35 R5=pulses+N5;
36 disp_n=100; %%display 100 samples
37
38
39 %%% Plot Variance curves %%%%%%%%%%
40 figure(1)
```

```

41
42 subplot(3,2,1)
43 stairs(pulses(1:disp_n));
44 title('Received Sequence ; Without noise ')
45 axis([0 inf -10*A 10*A])
46 hold on
47
48 subplot(3,2,2)
49 stairs(R1(1:disp_n),'LineStyle','-','color','r');
50 title(['Received Sequence ; Noise variance ',num2str(variance)])
51 axis([0 inf -10*variance 10*variance])
52
53 subplot(3,2,3)
54 stairs(R2(1:disp_n),'LineStyle','-','color','r');
55 title(['Received Sequence ; Noise variance ',num2str(2*variance)])
56 axis([0 inf -10*variance 10*variance])
57
58 subplot(3,2,4)
59 stairs(R3(1:disp_n),'LineStyle','-','color','r');
60 title(['Received Sequence ; Noise variance ',num2str(10*variance)])
61 axis([0 inf -10*variance 10*variance])
62
63 subplot(3,2,5)
64 stairs(R4(1:disp_n),'LineStyle','-','color','r');
65 title(['Received Sequence ; Noise variance ',num2str(100*variance)])
66 axis([0 inf -10*variance 10*variance])
67
68 subplot(3,2,6)
69 stairs(R5(1:disp_n),'LineStyle','-','color','r');
70 title(['Received Sequence ; Noise variance ',num2str(variance)])
71 axis([0 inf -10*variance 10*variance])
72
73 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
74 %% Write Plot as EPS file%%%%%%%%
75 print -depsc part3.eps
76 close all;
77
78
79 %%%% DECODING #####
80 decoded=zeros(1,L);
81
82 for i=1:L
83     if(R(i)>threshold)
84         decoded(i)=A;
85     else

```

```

86     decoded(i)=-A;
87     end
88 end
89
90
91 figure(2)
92
93 subplot(3,1,1)
94 stairs(pulses(1:disp_n));
95 title('Transmitted Pulses')
96 axis([0 inf -2*A 2*A])
97
98
99 subplot(3,1,2)
100 stairs(R(1:disp_n), 'LineStyle', '-', 'color', 'r');
101 title('Received Sequence (R); Noise variance = 1', 'LineStyle', '-', '
    color', 'r')
102 axis([0 inf -5*variance 5*variance])
103
104 subplot(3,1,3)
105 stairs(decoded(1:disp_n), 'LineStyle', '-', 'color', 'r');
106 title('Decoded Sequence (Y) ', 'LineStyle', '-', 'color', 'r')
107 axis([0 inf -2*variance 2*variance])
108
109 %% Write Plot as EPS file %%
110 print -depsc part4.eps
111 close all;
112
113 %%%%%% For L=100,000
114 %% code to generate and plot the histogram of the received sequence
    taking the no of bins as 10.
115 figure(3)
116
117 subplot(2,2,1)
118 draw_histogram(R,10); ## my function ##
119
120 subplot(2,2,2)
121 hist(R,bins);
122 title('Using Matlab hist() function')
123
124 subplot(2,2,3)
125 draw_histogram(R,100); ## my function ##
126
127 subplot(2,2,4)
128 hist(R,100);

```

```

129 title('Using Matlab hist() function')
130
131 print -depsc part5_a.eps
132
133 %% conditional pdf of Recieved signal #####
134 %%% given that transmitted with A amplitude #####
135
136 I1 = zeros(1,L);
137
138 for i=4:7
139     A=10^(i-4)
140     pulses=pulses.*10^(i-4);
141     R=pulses+N1; %%% received signal
142     figure('name',['Effect of changing Amplitude to ',num2str(A)])
143
144     conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
145         %%pulses,disp_n,A,R,
146
147     cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
148 end
149 close all;
150
151 figure('name','Effect of introducing Interference')
152 A=1
153
154 N1 = sqrt(variance)*randn(1,L); %%noice
155 I1 = sqrt(variance)*randn(1,L); %%interference
156
157 R = pulses+N1+I1;
158 conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
159 cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
160
161 A=5
162 pulses=pulses.*5;
163 R = pulses+N1+I1;
164 conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
165 cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
166
167 A=10
168 pulses=pulses.*2;
169 R = pulses+N1+I1;
170 conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
171 cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
172

```



```

173 A=1
174 pulses=pulses./10;
175
176
177 %% EFFECT on amplification
178
179 alpha=2;
180 figure('name',['Effect of changing Alpha to ',num2str(alpha),' with no
    Interference'])
181 R = pulses*alpha+N1;
182 I1 = zeros(1,L);
183 conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
184 cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
185
186 alpha=5;
187 figure('name',['Effect of changing Alpha to ',num2str(alpha),' with no
    Interference'])
188 R = pulses*alpha+N1;
189 I1 = zeros(1,L);
190 conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
191 cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
192
193 alpha=10;
194 figure('name',['Effect of changing Alpha to ',num2str(alpha),' with no
    Interference'])
195 R = pulses*alpha+N1;
196 I1 = zeros(1,L);
197 conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
198 cond(R,alpha,N1,I1,pulses,A,disp_n,variance,bins)
199
200 close all;

```

10.2 conditional_pdf_plot.m

```

1 function g = conditional_pdf_plot(R,alpha,N1,I1,pulses,A,disp_n,
    variance,bins)
2     inter=yes
3     if(sum(I1)==0)
4         inter=no
5     end
6     close all;
7
8     figure(15,position,get(0,screensize))
9
10    subplot(4,3,1:3)

```

```

11 stairs(pulses(1:2*disp_n), 'LineStyle', '-', 'color', 'b');
12 title(['A sample of the Transmitted Sequence (S); A=', num2str(A)], '
    LineStyle', '-', 'color', 'b')
13 axis([0 inf -2*A 2*A])
14
15 subplot(4,3,4:5)
16 stairs(N1(1:2*disp_n), 'LineStyle', '-', 'color', 'r');
17 title(['A sample of the Noise; AWGN-Variance=', num2str(variance)], '
    LineStyle', '-', 'color', 'r')
18 axis auto;
19
20 subplot(4,3,6)
21 stairs(I1(1:2*disp_n), 'LineStyle', '-', 'color', 'k');
22 title(['A sample of the Interference'], 'LineStyle', '-', 'color', 'k')
23 axis auto;
24
25 subplot(4,3,7:9)
26 stairs(R(1:2*disp_n), 'LineStyle', '-', 'color', 'm');
27 title(['A sample of the Received Sequence (R=alpha*S+N+I); alpha= ',
    num2str(alpha)], 'LineStyle', '-', 'color', 'm')
28 axis auto;
29
30 subplot(4,3,10)
31 [f,x]=hist(R,bins);
32 bar(x, f / sum(f));
33 title(['PDF of R; E[R]= ', num2str(mean(R))]);
34 hold on;
35 line([0, 0], ylim, 'LineWidth', 2, 'Color', 'r');
36
37 subplot(4,3,11)
38 pdf_A1=conditional_pdf(R,pulses,A);
39 [f,x]=hist(pdf_A1,bins);
40 bar(x, f / sum(f));
41 title(['PDF of R given that S=A; E[R|S=A]= ', num2str(mean(pdf_A1))])
    ;
42 hold on;
43 line([0, 0], ylim, 'LineWidth', 2, 'Color', 'r');
44
45 subplot(4,3,12)
46 pdf_A2=conditional_pdf(R,pulses,-A);
47 [f,x]=hist(pdf_A2,bins);
48 bar(x, f / sum(f));
49 title(['PDF of R given that S=-A ; E[R|S=-A]= ', num2str(mean(pdf_A2)
    )));
50 hold on;

```

```
51 line([0, 0], ylim, 'LineWidth', 2, 'Color', 'r');
52
53 name=strcat(Summary-A=,num2str(A),-Var=,num2str(variance),-alpha=,
54             num2str(alpha),-Interference=,inter,.eps)
55
56 %% Write Plot as EPS file%%%%%
57 print (15,name, -depsc,-S1180,720,-F:8)
58
59
60 end
```