

# 1 Robust portfolio optimization using second-order cone programming

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## Executive Summary

Optimization maintains its importance within portfolio management, despite many criticisms of the Markowitz approach, because modern algorithmic approaches are able to provide solutions to much more wide-ranging optimization problems than the classical mean–variance case. By setting up problems with more general constraints and more flexible objective functions, investors can model investment realities in a way that was not available to the first generation of users of risk models.

In this chapter, we review the use of second-order cone programming to handle a number of economically important optimization problems involving:

- Alpha uncertainty
- Constraints on systematic and specific risks
- Fund of funds with multiple active risk constraints
- Constraints on risk using more than one risk model
- Combining different risk measures

### 1.1 Introduction

Despite an almost-continuous criticism of mathematical optimization as a method of constructing investment portfolios since it was first proposed, there are an ever-increasing number of practitioners of this method using it to manage more and more assets. Given the fact that the problems associated with the Markowitz approach are so well known and so widely acknowledged, why is it that portfolio optimization remains popular with well-informed investment professionals?

The answer lies in the fact that modern algorithmic approaches are able to provide solutions to much more wide-ranging optimization problems than the classical mean–variance case. By setting up problems with more general constraints and more flexible objective functions, investors can model investment realities in a way that was not available to the first generation of users of risk models.

In particular, the methods of cone programming allow efficient solutions to problems that involve more than one quadratic constraint, more than one

quadratic term within the utility function, and more than one benchmark. In this way, investors can go about finding solutions that are robust against the failure of a number of simplifying assumptions that had previously been seen as fatally compromising the mean–variance optimization approach.

In this chapter, we consider a number of economically important optimization problems that can be solved efficiently by means of second-order cone programming (SOCP) techniques. In each case, we demonstrate by means of fully worked examples the intuitive improvement to the investor that can be obtained by making use of SOCP, and in doing so we hope to focus the discussion of the value of portfolio optimization where it should be on the proper definition of utility and the quality of the underlying alpha and risk models.

## 1.2 Alpha uncertainty

The standard mean–variance portfolio optimization approach assumes that the alphas are known and given by some vector  $\alpha$ . The problem with this is that generally the alpha predictions are not known with certainty—an investor can estimate alphas but clearly cannot be certain that their predictions will be correct. However, when the alpha predictions are subsequently used in an optimization, the optimizer will treat the alphas as being certain and may choose a solution that places unjustified emphasis on those assets that have particularly large alpha predictions.

Attempts to compensate for this in the standard quadratic programming approach include just reducing alphas that look too large to give more conservative estimates and imposing constraints such as maximum asset weight and sector weight constraints to try and prevent any individual alpha estimate having too large an impact. However, none of these methods directly address the issue and these approaches can lead to suboptimal results. A better way of dealing with the problem is to use SOCP to include uncertainty information in the optimization process.

If the alphas are assumed to follow a normal distribution with mean  $\alpha^*$  and known covariance matrix of estimation errors  $\Omega$ , then we can define an elliptical confidence region around the mean estimated alphas as:

$$(\alpha - \alpha^*)^T \Omega^{-1} (\alpha - \alpha^*) \leq k^2$$

There are then several ways of setting up the robust optimization problem; the one we consider is to maximize the worst-case return for the given confidence region, subject to a constraint on the mean portfolio return,  $\alpha_p$ . If  $w$  is the vector of portfolio weights, the problem is:

$$\text{Maximize } (\min(w^T \alpha) - \text{portfolio variance})$$

subject to

$$(\alpha - \alpha^*)^T \Omega^{-1} (\alpha - \alpha^*) \leq k^2$$

$$\alpha^{*T} w = \alpha_p$$

$$e^T w = 1$$

$$w \geq 0$$

This can be written as an SOCP problem by introducing an extra variable,  $\alpha_u$  (for more details on the derivation, see Scherer (2007)):

$$\text{Maximize } (w^T \alpha^* - k\alpha_u - \text{portfolio variance})$$

subject to

$$w^T \Omega w \leq \alpha_u^2$$

$$\alpha^{*T} w = \alpha_p$$

$$e^T w = 1$$

$$w \geq 0$$

Figure 1.1 shows the standard mean–variance frontier and the frontier generated including the alpha uncertainty term (“Alpha Uncertainty Frontier”). The example has a 500-asset universe and no benchmark and the mean portfolio alpha is constrained to various values between the mean portfolio alpha found for the minimum variance portfolio (assuming no alpha uncertainty) and 0.9. The size of the confidence region around the mean estimated alphas (i.e., the value of  $k$ ) is increased as the constraint on the mean portfolio alpha is increased. The covariance matrix of estimation errors  $\Omega$  is assumed to be the individual volatilities of the assets calculated using a SunGard APT risk model. The portfolio variance is also calculated using a SunGard APT risk model.

Some extensions to this, e.g., the use of a benchmark and active portfolio return, are straightforward.

The key questions to making practical use of alpha uncertainty are the specification of the covariance matrix of estimation errors  $\Omega$  and the size of the confidence region around the mean estimated alphas (the value of  $k$ ). This will depend on the alpha generation process used by the practitioner and, as for the alpha generation process, it is suggested that backtesting be used to aid in the choice of appropriate covariance matrices  $\Omega$  and confidence region sizes  $k$ . From a practical point of view, for reasonably sized problems, it is helpful if the covariance matrix  $\Omega$  is either diagonal or a factor model is used.

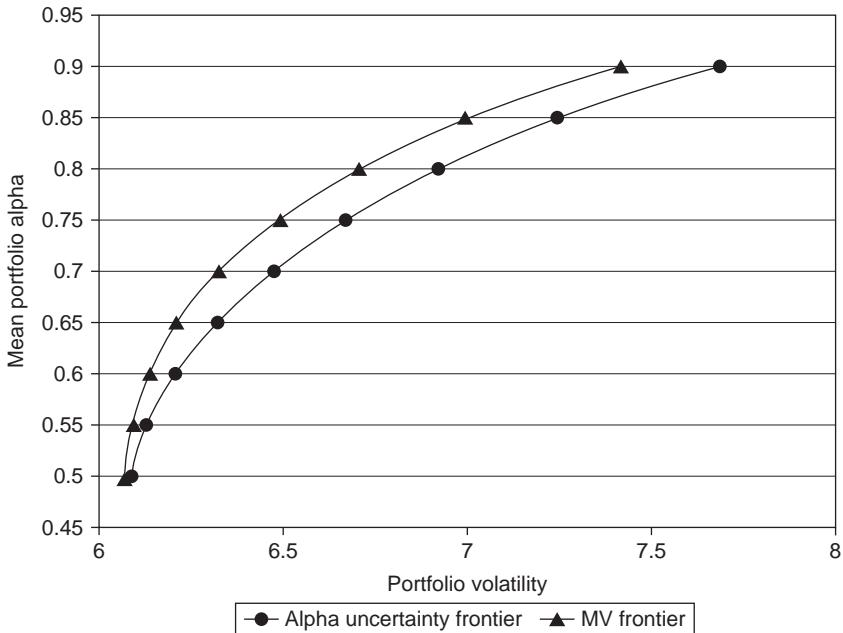


Figure 1.1 Alpha uncertainty efficient frontiers.

### 1.3 Constraints on systematic and specific risk

In most factor-based risk models, the risk of a portfolio can be split into a part coming from systematic sources and a part specific to the individual assets within the portfolio (the residual risk). In some cases, portfolio managers are willing to take on extra risk or sacrifice alpha in order to ensure that the systematic or specific risk is below a certain level.

A heuristic way of achieving a constraint on systematic risk in a standard quadratic programming problem format is to linearly constrain the portfolio factor loadings. This works well in the case where no systematic risk is the requirement, e.g., in some hedge funds that want to be market neutral, but is problematic in other cases because there is the question of how to split the systematic risk restrictions between the different factors. In a prespecified factor model, it may be possible to have some idea about how to constrain the risk on individual named factors, but it is generally not possible to know how to do this in a statistical factor model. This means that in most cases, it is necessary to use SOCP to impose a constraint on either the systematic or specific risk.

In the SunGard APT risk model, the portfolio variance can be written as:

$$\mathbf{w}^T \mathbf{B}^T \mathbf{B} \mathbf{w} + \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}$$

where

$w = n \times 1$  vector of portfolio weights

$B = c \times n$  matrix of component (factor) loadings

$\Sigma = n \times n$  diagonal matrix of specific (residual) variances

The systematic risk of the portfolio is then given by:

$$\text{Systematic risk of portfolio} = \sqrt{(w^T B^T B w)}$$

and the specific risk of the portfolio by:

$$\text{Specific risk of portfolio} = \sqrt{(w^T \Sigma w)}$$

The portfolio optimization problem with a constraint on the systematic risk ( $\sigma_{\text{sys}}$ ) is then given by the SOCP problem:

$$\text{Minimize } (w^T B^T B w + w^T \Sigma w)$$

subject to

$$w^T B^T B w \leq \sigma_{\text{sys}}^2$$

$$\alpha^{*T} w = \alpha_p$$

$$e^T w = 1$$

$$w \geq 0$$

where

$\alpha^* = n \times 1$  vector of estimated asset alphas

$\alpha_p = \text{portfolio return}$

One point to note on the implementation is that the  $B^T B$  matrix is never calculated directly (this would be an  $n \times n$  matrix, so could become very large when used in a realistic-sized problem). Instead, extra variables  $b_i$  are introduced, one per factor, and constrained to be equal to the portfolio factor loading:

$$b_i = (Bw)_i, i = 1 \dots c$$

This then gives the following formulation for the above problem of constraining the systematic risk:

$$\text{Minimize } (b^T b + w^T \Sigma w)$$

subject to

$$\mathbf{b}^T \mathbf{b} \leq \sigma_{\text{sys}}^2$$

$$\alpha^{*\top} \mathbf{w} = \alpha_p$$

$$\mathbf{e}^T \mathbf{w} = 1$$

$$\mathbf{b} = \mathbf{B} \mathbf{w}$$

$$\mathbf{w} \geq 0$$

Similarly, the problem with a constraint on the specific risk ( $\sigma_{\text{spe}}$ ) is given by:

$$\text{Minimize}(\mathbf{b}^T \mathbf{b} + \mathbf{w}^T \Sigma \mathbf{w})$$

subject to

$$\mathbf{w}^T \Sigma \mathbf{w} \leq \sigma_{\text{spe}}^2$$

$$\alpha^{*\top} \mathbf{w} = \alpha_p$$

$$\mathbf{e}^T \mathbf{w} = 1$$

$$\mathbf{b} = \mathbf{B} \mathbf{w}$$

$$\mathbf{w} \geq 0$$

Figure 1.2 shows the standard mean–variance frontier and the frontiers generated with constraints on the specific risk of 2% and 3%, and on the systematic risk of 5%. The example has a 500-asset universe and no benchmark and the portfolio alpha is constrained to various values between the portfolio alpha found for the minimum variance portfolio and 0.9. (For the 5% constraint on the systematic risk, it was not possible to find a feasible solution with a portfolio alpha of 0.9.) Figure 1.3 shows the systematic portfolio volatilities and Figure 1.4 shows the specific portfolio volatilities for the same set of optimizations.

Constraints on systematic or specific volatility can be combined with the alpha uncertainty described in the previous section. The resulting frontiers can be seen in Figures 1.5–1.7 (the specific 3% constraint frontier is not shown because this coincides with the Alpha Uncertainty Frontier for all but the first point).

The shape of the specific risk frontier for the alpha uncertainty frontier (see Figure 1.7) is unusual. This is due to a combination of increasing the emphasis on the alpha uncertainty as the constraint on the mean portfolio alpha

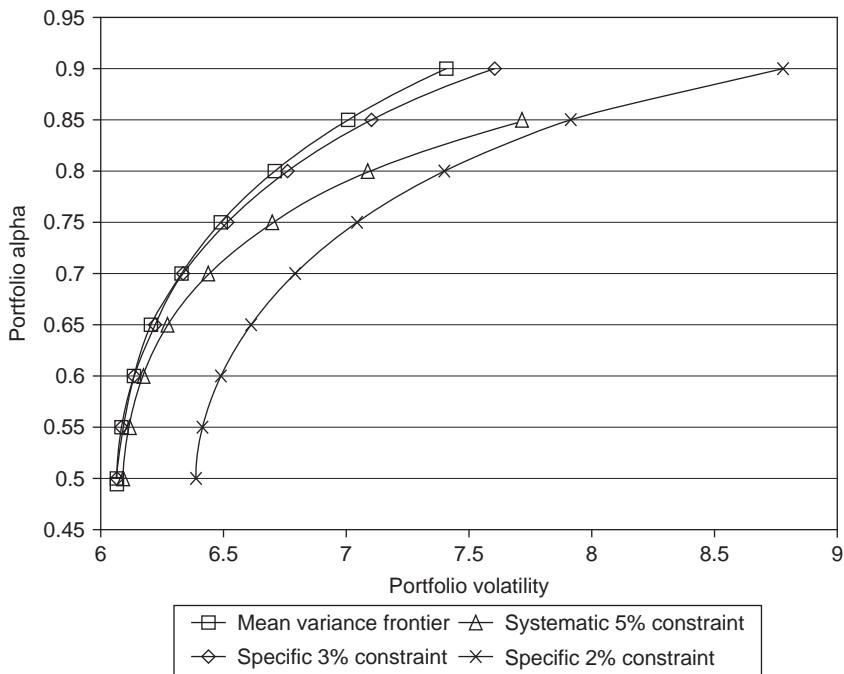


Figure 1.2 Portfolio volatility with constraints on systematic and specific risk.

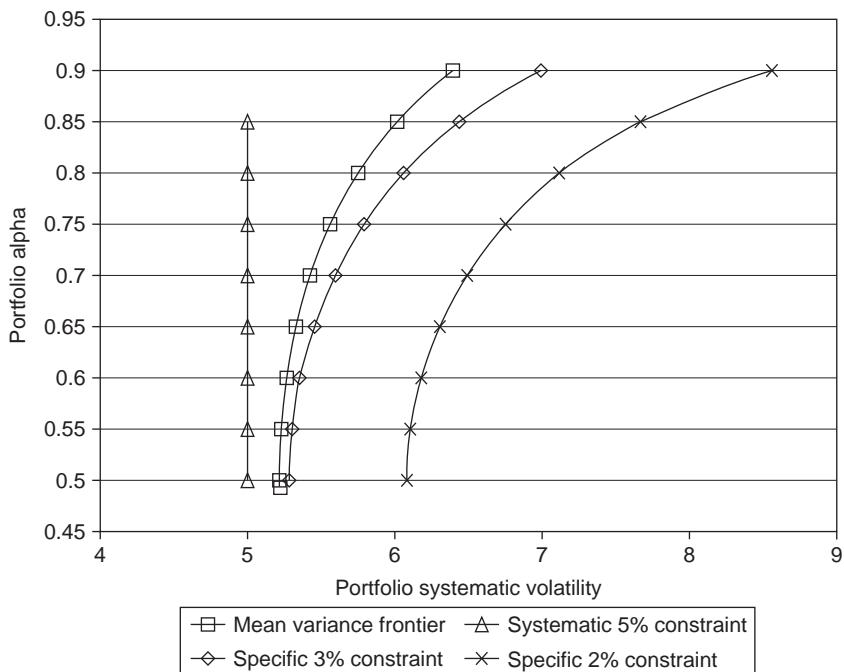


Figure 1.3 Portfolio systematic volatility with constraints on systematic and specific risk.

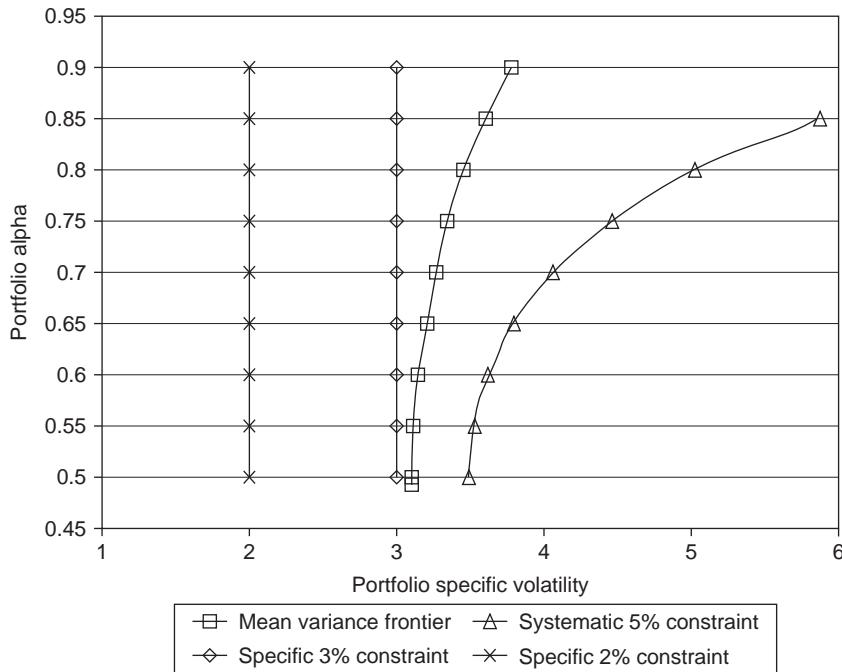


Figure 1.4 Portfolio specific volatility with constraints on systematic and specific risk.

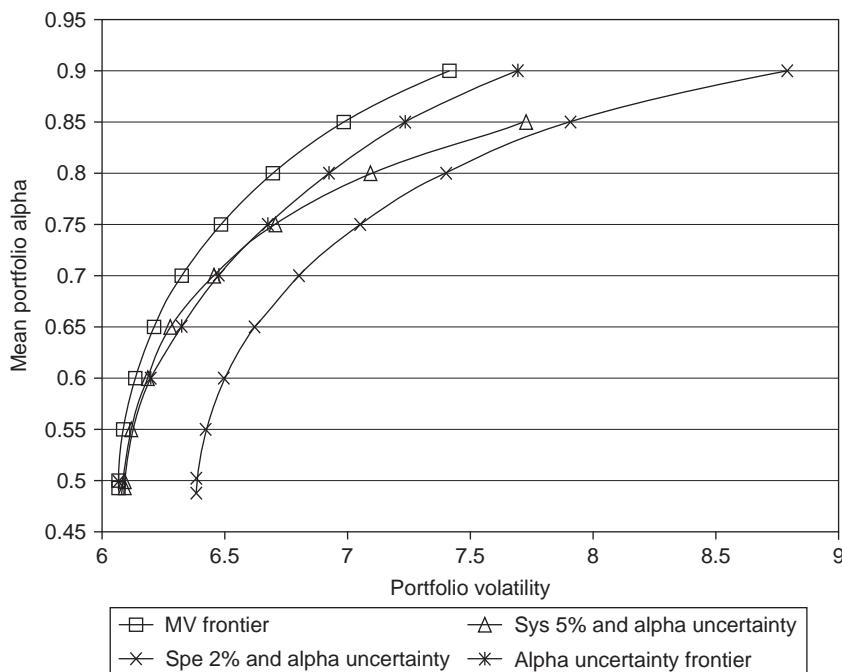


Figure 1.5 Portfolio volatility with alpha uncertainty and constraints on systematic and specific risk.

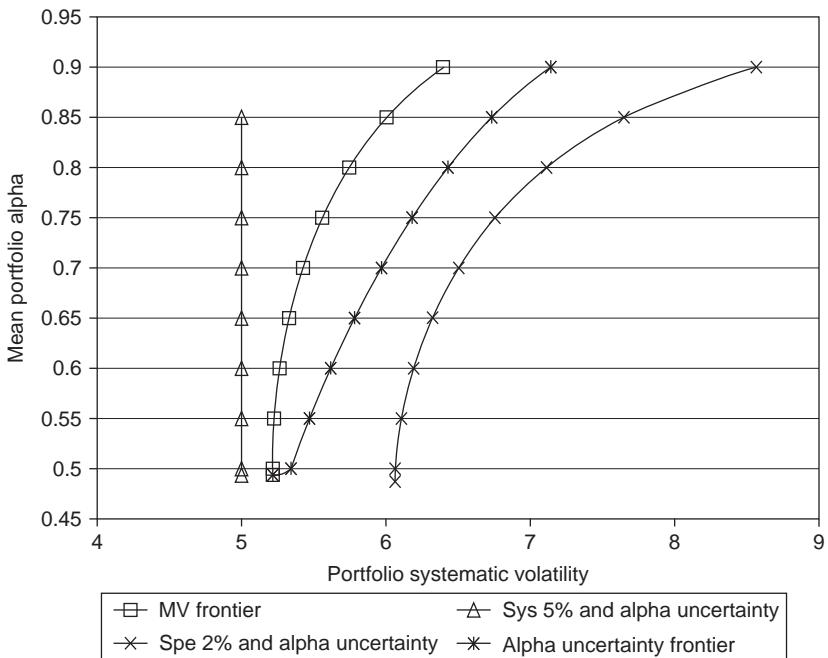


Figure 1.6 Portfolio systematic volatility with alpha uncertainty and constraints on systematic and specific risk.

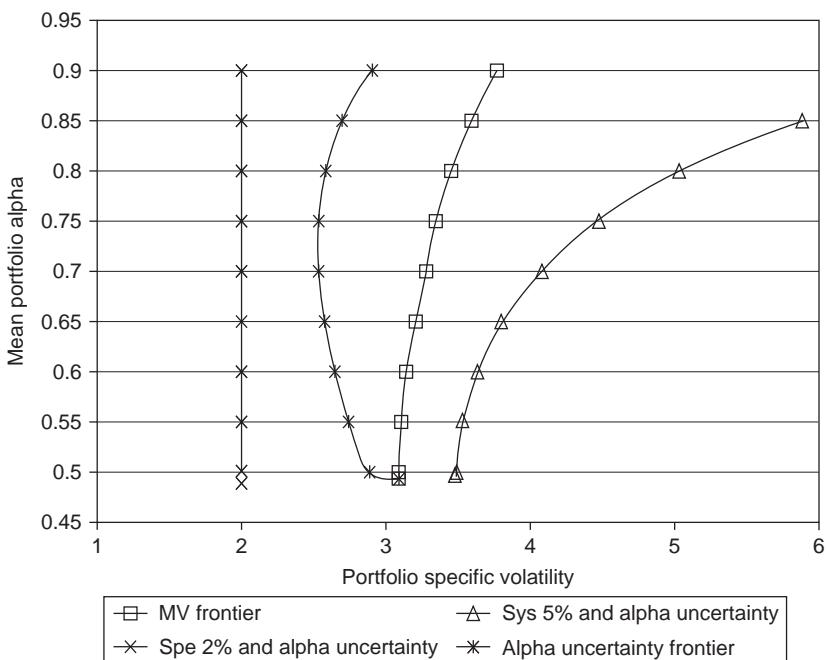


Figure 1.7 Portfolio specific volatility with alpha uncertainty and constraints on systematic and specific risk.

increases, and the choice of covariance matrix of estimation errors. In a typical mean-variance optimization, as the portfolio alpha increases, the specific risk would be expected to increase as the portfolio would tend to be concentrated in fewer assets that have high alphas. However, in the above alpha uncertainty example, because the emphasis increases on the alpha uncertainty term, and the covariance matrix of estimation errors is a matrix of individual asset volatilities, this tends to lead to a more diversified portfolio than in the pure mean-variance case. It should be noted that with a different choice of covariance matrix of estimation errors, or if the emphasis on the alpha uncertainty is kept constant, a more typical specific risk frontier may be seen.

Whilst the factors in the SunGard APT model are independent, it is straightforward to extend the above formulation to more general factor models, and to optimizing with a benchmark and constraints on active systematic and active specific risk.

## 1.4 Constraints on risk using more than one model

With the very volatile markets that have been seen recently, it is becoming increasingly common for managers to be interested in using more than one model to measure the risk of their portfolio.

In the SunGard APT case, the standard models produced are medium-term models with an investment horizon of between 3 weeks and 6 months. However, SunGard APT also produces short-term models with an investment horizon of less than 3 weeks. Some practitioners like to look at the risk figures from both types of model. Most commercial optimizers designed for portfolio optimization do not provide any way for them to combine the two models in one optimization so they might, for example, optimize using the medium-term model and then check that the risk prediction using the short-term model is acceptable. Ideally, they would like to combine both risk models in the optimization, for example, by using the medium-term model risk as the objective and then imposing a constraint on the short-term model risk. This constraint on the short-term model risk requires SOCP.

Other examples of possible combinations of risk models that may be used by practitioners are:

- SunGard APT Country and SunGard APT Region Models
- Risk models from two different vendors, or a risk model from a vendor alongside one produced internally
- Different types of risk model, e.g., a statistical factor model, one such as those produced by SunGard APT, and a prespecified factor model

One way of using both risk models in the optimization is to include them both in the objective function:

$$\begin{aligned} \text{Minimize } & [x_1((w - b)^T B_1^T B_1 (w - b) + (w - b)^T \Sigma_1 (w - b)) \\ & + x_2((w - b)^T B_2^T B_2 (w - b) + (w - b)^T \Sigma_2 (w - b))] \end{aligned}$$

subject to

$$\alpha^{*\top} w = \alpha_p$$

$$e^\top w = 1$$

$$w \leq w_{\max}$$

$$w \geq 0$$

where

$w$  =  $n \times 1$  vector of portfolio weights

$b$  =  $n \times 1$  vector of benchmark weights

$B_i$  =  $c \times n$  matrix of component (factor) loadings for risk model  $i$

$\Sigma_i$  =  $n \times n$  diagonal matrix of specific (residual) variances for risk model  $i$

$x_i$  = weight of risk model  $i$  in objective function ( $x_i \geq 0$ )

$\alpha^*$  =  $n \times 1$  vector of estimated asset alphas

$\alpha_p$  = portfolio return

$w_{\max}$  =  $n \times 1$  vector of maximum asset weights in the portfolio

This is a standard quadratic programming problem and does not include any second-order cone constraints but does require the user to make a decision about the relative weight ( $x_i$ ) of the two risk terms in the objective function. This relative weighting may be less natural for the user than just imposing a tracking error constraint on the risk from one of the models. Figure 1.8 shows frontiers with tracking error measured using a SunGard APT medium-term model (United States August 2008) for portfolios created as follows:

- Optimizing using the medium-term model only
- Optimizing using the short-term model only
- Optimizing including the risk from both models in the objective function, with equal weighting on the two models

The same universe and benchmark has been used in all cases and they each contain 500 assets, and the portfolio alpha is constrained to values between 0.01 and 0.07.

Figure 1.9 shows the frontiers for the same set of optimizations with tracking errors measured using a SunGard APT short-term model (United States August 2008).

It can be seen from Figures 1.8 and 1.9 that optimizing using just one model results in relatively high tracking errors in the other model, but including terms from both risk models in the objective function results in frontiers for both models that are close to those generated when just optimizing with the individual model.

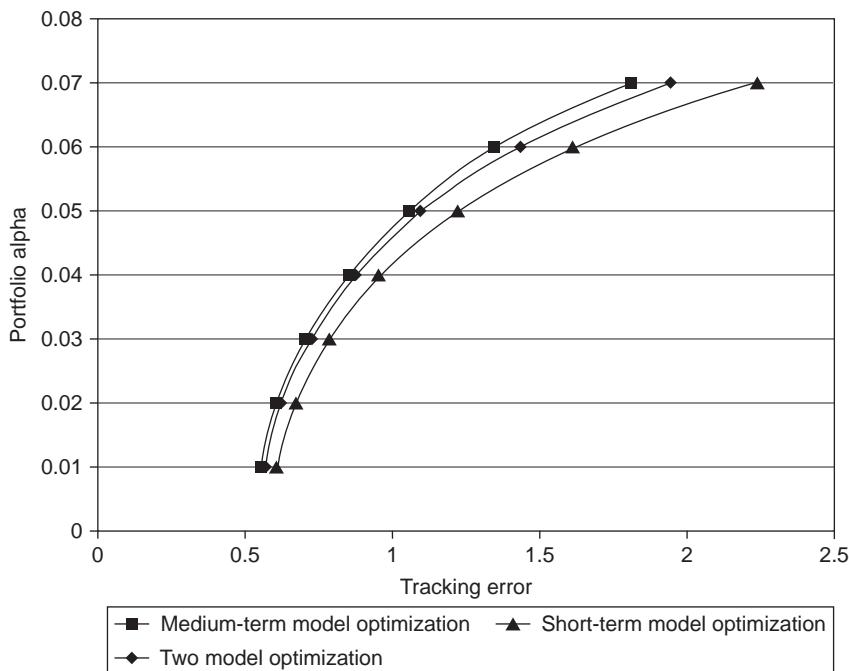


Figure 1.8 Tracking error measured using the SunGard APT medium-term model.

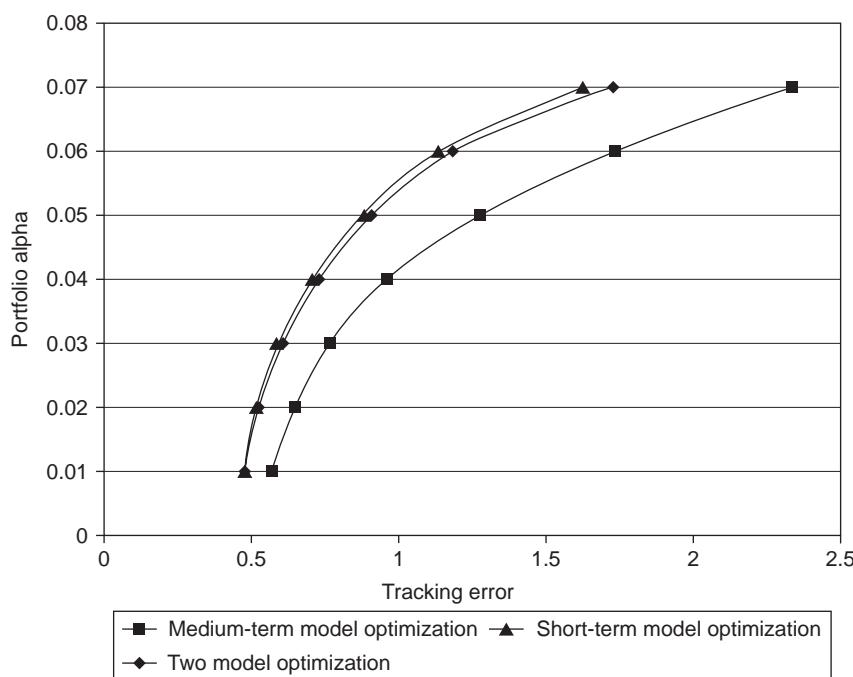


Figure 1.9 Tracking error measured using the SunGard APT short-term model.

Using SOCP, it is possible to include both risk models in the optimization by including the risk term from one in the objective function and constraining on the risk term from the other model:

$$\text{Minimize}[(w - b)^T B_1^T B_1 (w - b) + (w - b)^T \Sigma_1 (w - b)]$$

subject to

$$(w - b)^T B_2^T B_2 (w - b) + (w - b)^T \Sigma_2 (w - b) \leq \sigma_{a2}^2$$

$$\alpha^{*T} w = \alpha_p$$

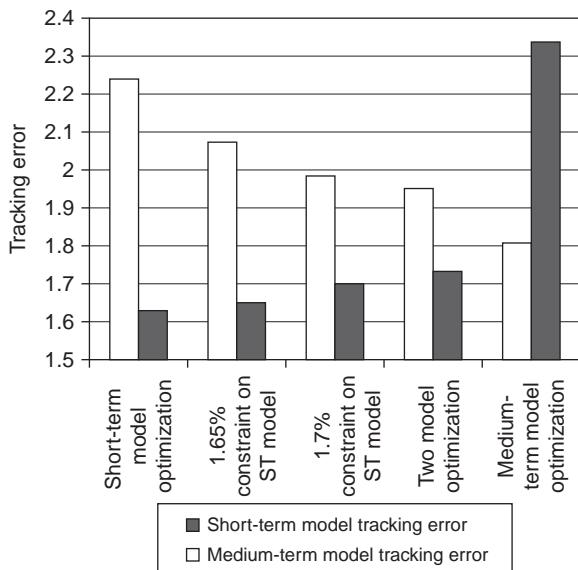
$$e^T w = 1$$

$$w \leq w_{\max}$$

$$w \geq 0$$

where  $\sigma_{a2}$  = maximum tracking error from the second risk model.

Figure 1.10 shows the effect of constraining on the risk from the short-term model, with an objective of minimizing the risk from the medium-term model, with a constraint on the portfolio alpha of 0.07. The tracking errors from just optimizing using one model without any constraint on the other model, and optimizing including the risk from both models in the objective function, are also shown for comparison.



**Figure 1.10** Tracking errors with constraints on the SunGard APT short-term model tracking error.

Whilst the discussion here has concerned using two SunGard APT risk models, it should be noted that it is trivial to extend the above to any number of risk models, and to more general risk factor models.

## 1.5 Combining different risk measures

In some cases, it may be desirable to optimize using one risk measure for the objective and to constrain on some other risk measures. For example, the objective might be to minimize tracking error against a benchmark whilst constraining the portfolio volatility. Another example could be where a pension fund manager or an institutional asset manager has an objective of minimizing tracking error against a market index, but also needs to constrain the tracking error against some internal model portfolio.

This can be achieved in a standard quadratic programming problem format by including both risk measures in the objective function and varying the relative emphasis on them until a solution satisfying the risk constraint is found. The main disadvantage of this is that it is time consuming to find a solution and is difficult to extend to the case where there is to be a constraint on more than one additional risk measure. A quicker, more general approach is to use SOCP to implement constraints on the risk measures.

The first case, minimizing tracking error, whilst constraining portfolio volatility, results in the following SOCP problem when using the SunGard APT risk model:

$$\text{Minimize}[(w - b)^T B^T B(w - b) + (w - b)^T \Sigma(w - b)]$$

subject to

$$\alpha^{*T} w = \alpha_p$$

$$w^T B^T B w + w^T \Sigma w \leq \sigma^2$$

$$e^T w = 1$$

$$w \leq w_{\max}$$

$$w \geq 0$$

where

$w = n \times 1$  vector of portfolio weights

$b = n \times 1$  vector of benchmark weights

$B = c \times n$  matrix of component (factor) loadings

$\Sigma = n \times n$  diagonal matrix of specific (residual) variances

$\sigma$  = maximum portfolio volatility

$\alpha^*$  =  $n \times 1$  vector of estimated asset alphas

$\alpha_p$  = Portfolio return

$w_{max}$  =  $n \times 1$  vector of maximum asset weights in the portfolio

An example is given below where an optimization is first run without any constraint on the portfolio volatility, but with a constraint on the portfolio alpha. The optimization is then rerun several times with varying constraints on the portfolio volatility, and the same constraint on the portfolio alpha. The universe and benchmark both contain 500 assets. The resulting portfolio volatilities and tracking errors can be seen in Figure 1.11.

The second case, minimizing tracking error against one benchmark, whilst constraining tracking error against some other benchmark, results in the following SOCP problem when using the SunGard APT risk model:

$$\text{Minimize}[(w - b_1)^T B^T B(w - b_1) + (w - b_1)^T \Sigma(w - b_1)]$$

subject to

$$\alpha^{*T} w = \alpha_p$$

$$(w - b_2)^T B^T B(w - b_2) + (w - b_2)^T \Sigma(w - b_2) \leq \sigma_{a2}^2$$

$$e^T w = 1$$

$$w \leq w_{max}$$

$$w \geq 0$$

where

$b_1$  =  $n \times 1$  vector of weights for benchmark used in objective function

$b_2$  =  $n \times 1$  vector of weights for benchmark used in constraint

$\sigma_{a2}$  = maximum tracking error against second benchmark

An example of this case is given below where an optimization is first run without any constraint on the tracking error against the internal model portfolio, but with a constraint on the portfolio alpha, minimizing the tracking error against a market index. The optimization is then rerun several times with varying constraints on the tracking error against the internal model portfolio, and the same constraint on the portfolio alpha. The universe and benchmark both contain 500 assets. The resulting tracking errors against both the market index and the internal model portfolio can be seen in Figure 1.12.

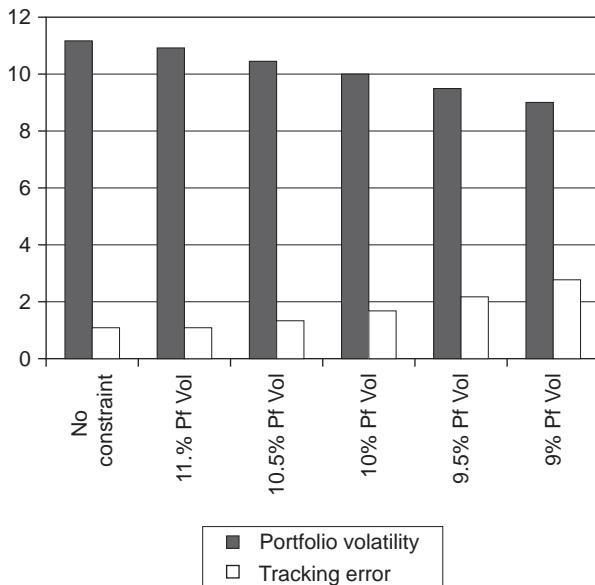


Figure 1.11 Risk with portfolio volatility constrained.

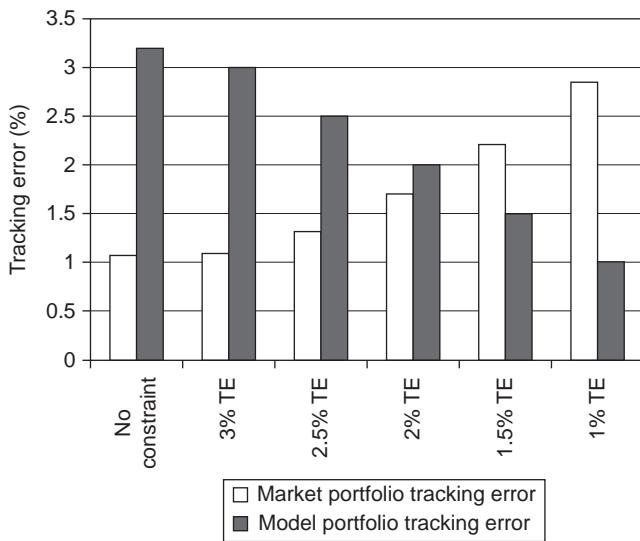


Figure 1.12 Risk with tracking error constrained against a model portfolio.

## 1.6 Fund of funds

An organization might want to control the risk of all their funds against one benchmark, but give fund managers different mandates with different benchmarks and risk restrictions. If the managers each individually optimize their

own fund against their own benchmark, then it can be difficult to control the overall risk for the organization. From the overall management point of view, it would be better if the funds could be optimized together, taking into account the overall benchmark. One way to do this is to use SOCP to impose the tracking error constraints on the individual funds, and optimize with an objective of minimizing the tracking error of the combined funds against the overall benchmark, with constraints on the minimum alpha for each of the funds. Using the SunGard APT risk model, this results in the following SOCP problem:

$$\text{Minimize } (w_c - b_c)^T B^T B (w_c - b_c) + (w_c - b_c)^T \Sigma (w_c - b_c)$$

subject to

$$w_c = \sum_i f_i w_i, \sum_i f_i = 1, f_i \geq 0$$

$$(w_i - b_i)^T B^T B (w_i - b_i) + (w_i - b_i)^T \Sigma (w_i - b_i) \leq \sigma_{ai}^2, i = 1 \dots m$$

$$e^T w_i = 1, i = 1 \dots m$$

$$w_i \geq 0, w_i \leq \max_i, i = 1 \dots m$$

$$\alpha_i^{*T} w_i \geq \alpha_{pi}, i = 1 \dots m$$

where

$m$  = number of funds

$w_i$  =  $n \times 1$  vector of portfolio weights for fund  $i$

$b_i$  =  $n \times 1$  vector of benchmark weights for fund  $i$

$w_c$  =  $n \times 1$  vector of weights for overall (combined) portfolio

$f_i$  = weight of fund  $i$  in overall (combined) portfolio

$b_c$  =  $n \times 1$  vector of overall benchmark weights

$B$  =  $c \times n$  matrix of component (factor) loadings

$\Sigma$  =  $n \times n$  diagonal matrix of specific (residual) variances

$\sigma_{ai}$  = maximum tracking error for fund  $i$

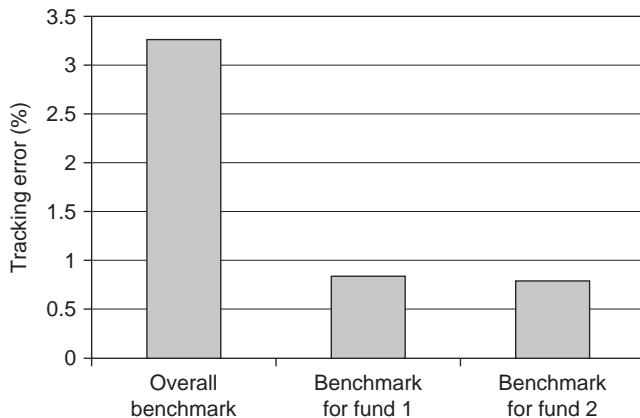
$\max_i$  =  $n \times 1$  vector of maximum weights for fund  $i$

$\alpha_i^*$  =  $n \times 1$  vector of assets alphas for fund  $i$

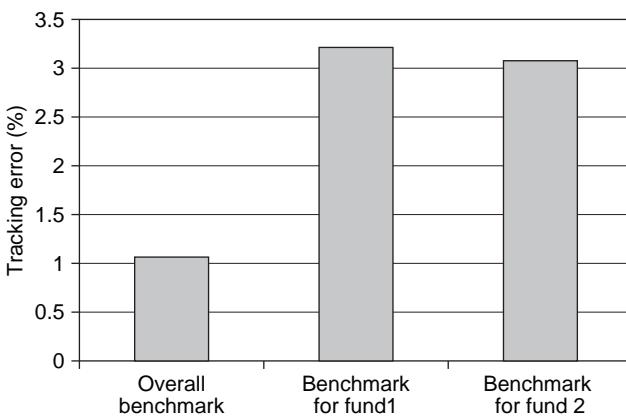
$\alpha_{pi}$  = minimum portfolio alpha for fund  $i$

In the example given below, we have two funds, and the target alpha for both funds is 5%. The funds are equally weighted to give the overall portfolio. Figure 1.13 shows the tracking error of the combined portfolio and each of the funds against their respective benchmarks where the funds have been optimized individually.

In this case, the tracking error against the overall benchmark is much larger than the tracking errors for the individual funds against their own benchmarks.



**Figure 1.13** Tracking errors when optimizing funds individually.



**Figure 1.14** Tracking errors when optimizing funds together.

This sort of situation would arise when the overall benchmark and the individual fund benchmarks are very different, e.g., in the case where the overall benchmark is a market index and the individual funds are a sector fund and a value fund. It is unlikely to occur when both the overall and individual fund benchmarks are very similar, for instance, when they are all market indexes.

Figure 1.14 shows the tracking errors when the combined fund is optimized with the objective of minimizing tracking error against the combined benchmark, subject to the constraints on alpha for each of the funds, but without the constraints on the individual fund tracking errors.

Figure 1.15 shows the results of optimizing including the SOCP constraints on the tracking errors for the individual funds.

From the organization's perspective, using SOCP to constrain the individual fund tracking errors whilst minimizing the overall fund tracking error should achieve

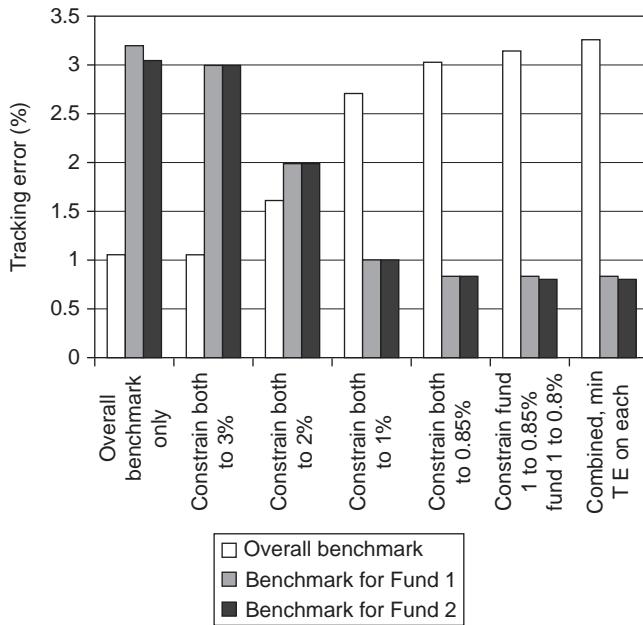


Figure 1.15 Tracking errors with constraints on risk for each fund.

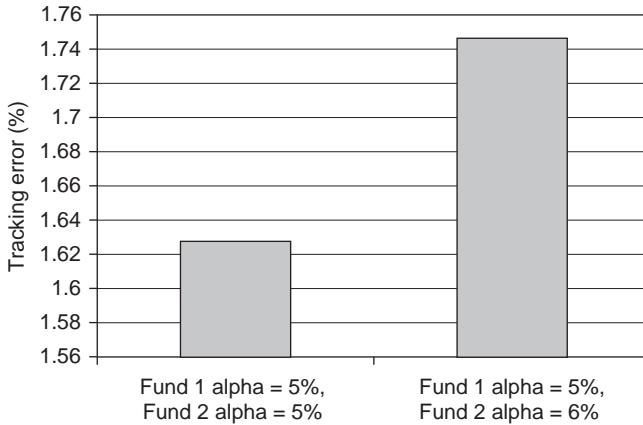


Figure 1.16 Tracking error with different alpha constraints on Fund 2.

their goal. However, there is a question as to whether this is a fair method of optimization from the point of view of the individual managers. Suppose that instead of both managers in the above example having a minimum portfolio alpha requirement of 5%, one of the managers decides to target a minimum portfolio alpha of 6%. If they are still both constrained to have a maximum individual tracking error against their own benchmark of 2%, it can be seen from Figure 1.16 that the tracking error for the overall fund against the overall benchmark will increase.

The organization might decide that this new tracking error against the overall benchmark is too high and, to solve this problem, will impose lower tracking error restrictions on the individual funds. This could be considered to be unfairly penalizing the first fund manager as the reason the overall tracking error is now too high is because of the decision by the second manager to increase their minimum portfolio alpha constraint. It is tricky to manage this issue and it may be that the organization will need to consider the risk and return characteristics of the individual portfolios generated by separate optimizations on each of the funds both before setting individual tracking error constraints, and after the combined optimization has been run to check that they appear fair.

## 1.7 Conclusion

SOCP provides powerful additional solution methods that extend the scope of portfolio optimization beyond the simple mean–variance utility function with linear and mixed integer constraints. By considering a number of economically important example problems, we have shown how SOCP approaches allow the investor to deal with some of the complexities of real-world investment problems. A great advantage in having efficient methods available to generate these solutions is that the investor’s intuition can be tested and extended as the underlying utility or the investment constraints are varied.

Ultimately, it is not the method of solving an optimization problem that is critical—rather it is the ability to comprehend and set out clearly the economic justification for framing an investment decision in terms of a trade-off of risk, reward and cost with a particular form of the utility function and a special set of constraints. There are many aspects of risky markets behavior that have not been considered here, notably relating to downside and pure tail risk measures, but we hope that an appreciation of the solution techniques discussed in this chapter will lead to a more convincing justification for the entire enterprise of portfolio optimization, as the necessary rethinking of real-world utilities and constraints is undertaken.

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