Theoretical Justification for Noise Testing

0. Definitions

- We will denote by $(\mathbf{X})_{c,i,j}$ our original image with dimensions $n \times m$ indexed by i and j and h channels indexed by c ,
- We denote by $\tilde{\mathbf{X}}$ the noisy version of $\mathbf{X}$
- Let $N = n \times m \times h$

1. Gaussian Noise

1.1 Model

We have $\tilde{\mathbf{X}}_{c,i,j} = \mathbf{X}_{c,i,j} + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma)$

So we have:

$$\mathbf{S} = \sum_{c,i,j} \frac{1}{\sigma^2} (\tilde{\mathbf{X}}_{c,i,j} - \mathbf{X}_{c,i,j})^2 \sim \chi^2{}_N$$

1.2 Confidence Interval

let $p \in [0, 1]$. we will construct a symmetric interval $\mathcal{I}$ centered at $\mathbb{E}[\mathbf{S}] = n$ from which $\mathcal{P}(\mathbf{S} \notin \mathcal{I}) = p$

We will choose $p = 10^{-6}$ and we will call it the likelihood limit.

1.3 Test

- If the calculated term $\mathbf{S} \in \mathcal{I}$. We will say that the result is consistent with the noise model, and it will pass.
- Otherwise, the calculated noise is not consistent with the noise model, and the test will fail within a confidence of $1 - p$.

2. Impulsive Noise

Inspired from the Wilson score on binomial distribution

3. Speckle Noise

Similar to Gaussian Noise test, but with:

$$\mathbf{S} = \sum_{c,i,j} \frac{1}{\sigma^2} \left(\frac{\tilde{\mathbf{X}}_{c,i,j}}{\mathbf{X}_{c,i,j}} - 1 \right)^2 \sim \chi^2{}_N$$