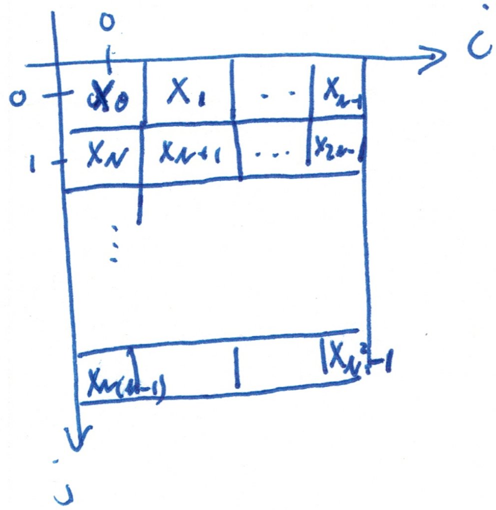
2-D



$$I(i,j) = \{X_{i+N\cdot j}\} \qquad I = \{X_{i+Nj}\}_{i,j=0}^{N-1}$$

Projections:

in $\vec{i}$ direction : $\text{RowSum}(j) = \sum_{i=0}^{N-1} I(i,j) = \boxed{\sum_{i=0}^{N-1}\left(X_{i+N\cdot j}\right)}$

in $\vec{j}$ direction : $\text{ColSum}(i) = \boxed{\sum_{j=0}^{N}\left(X_{i+N\cdot j}\right)}$

2N equations

$\{X_{i+Nj}\}_{i,j=0}^{N-1}$ - N^2 unknowns

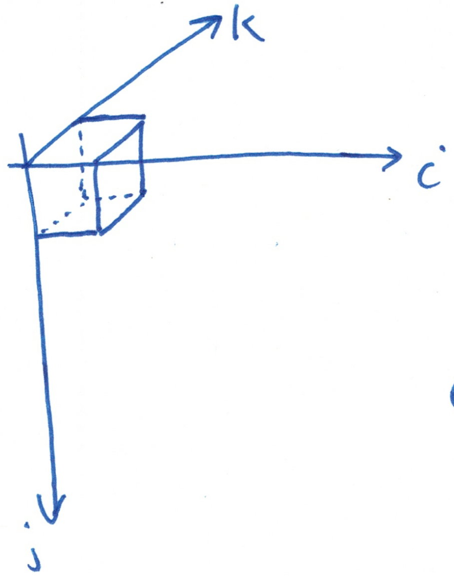
2-D conservation of Mass rules

$$\sum_{j=0}^{N-1} \sum_{i=0}^{N-1} I(i,j) \stackrel{\Delta}{=} M = \begin{pmatrix} \text{image} \\ \text{mass} \end{pmatrix} =$$

$$\boxed{\begin{aligned} &= \sum_{j=0}^{N-1} \mathrm{RowSum}(j) \\ &= \sum_{i=0}^{N-1} \mathrm{Col\,Sum}(i) \end{aligned}}$$

(immediate to see).

3 - D



each Voxel indexed by $\{i, j, k\}_0^{N-1}$

Indexing of the Voxels:

$$\boxed{V(i,j,k) = X_{i+N\cdot j+N^2\cdot k}}$$

Projections:

$$I_1(i,j) = \text{in } \vec{k} \text{ direction} = \sum_{k=0}^{N-1} X_{i+N\cdot j+N^2 k}$$

$$I_2(i,k) = \text{in } \vec{j} \text{ direction} = \sum_{j=0}^{N-1} X_{i+Nj+N^2 k}$$

$$I_3(j,k) = \text{in } \vec{i} \text{ direction} = \sum_{i=0}^{N-1} X_{i+Nj+N^2 k}$$

$3\cdot N^2$ equations

$\{X_{i+Nj+N^2k}\}_{i,j,k=0}^{N-1}$ - N^3 unkno

3-D Conservation of Mass Rules

$$\sum_{k}\sum_{j}\sum_{i=0}^{N-1} V(i,j,k) \stackrel{\Delta}{=} \underline{M} = \begin{pmatrix}\text{cube} \\ \text{mass}\end{pmatrix} =$$

$$\boxed{= \sum_{i,j} I_1(i,j) = \sum_{i,k} I_2(i,k) = \sum_{j,k} I_3(j,k)}$$

(again, immediate).

"slices":

$$RowSum\left(I_1(i,j)\right) = \sum_{i=0}^{N-1} I_1(i,j) = \sum_{i=0}^{N-1}\sum_{k=0}^{N-1} X_{i+Nj+N^2k} =$$

RowSum is when we sum over first Arg.

$$= \sum_{k=0}\sum_{i=0} X_{i+Nj+N^2k} = ColSum\left(I_3(j,k)\right)$$

colsum is when we sum over second

$$\Rightarrow \boxed{\begin{aligned} RowSum\left(I_1(i,j)\right) &= ColSum\left(I_3(j,k)\right) \\ ColSum\left(I_1(i,j)\right) &= ColSum\left(I_2(i,k)\right) \\ RowSum\left(I_2(i,k)\right) &= RowSum\left(I_3(j,k)\right) \end{aligned}}$$

and similarly