GENERAL ASSEMBLY

Intro to Bayes

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LEARNING OBJECTIVES

- Understanding the basics of Bayes' Therom
- Introduction to the "ingredients" of the Bayesian "world view"
- Do some exercises (conceptual/mathematical) to build greater intuition/ facility with Bayesian thinking

What is Bayes’ Therom?

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A way to figure out conditional probability

What is conditional probability?

Conditional probability is the probability of an event happening, given that it has some relationship to one or more other events.

Bayes’ Therom gives you the actual probability of an event given information about tests.

Events and tests are different!

Ex: Testing positive for cancer does not mean you in fact have cancer

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- Tests are flawed
- Tests don’t give REAL probabilities
- False positives skew results

Breast Cancer test results

	Cancer (1%)	No Cancer (99%)
Test Pos	80%	9.6%
Test Neg	20%	90.4%

If you have a positive test result, what are the chances you actually have cancer?

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	Cancer (1%)	No Cancer (99%)
Test Pos	True Pos: 1% * 80%	False Pos: 99% * 9.6%
Test Neg	False Neg: 1% * 20%	True Neg: 99% * 90.4%

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The chance of an event happening is the number of ways it could happen given all possible outcomes

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Probability = desired event / all possibilities
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	Cancer (1%)	No Cancer (99%)
Test Pos	True Pos: 1% * 80%	False Pos: 99% * 9.6%
Test Neg	False Neg: 1% * 20%	True Neg: 99% * 90.4%

- Chance of getting true positive: 0.08
- All possibilities: (true positive + false positive) 0.008 + 0.09504 = 0.10304
- Chance of cancer: 0.008/0.10304 = 0.0776 or about 7.8%

Hence, Bayes' Therom:

$$\Pr(\mathrm{A}|\mathrm{X}) = \frac{\Pr(\mathrm{X}|\mathrm{A})\Pr(\mathrm{A})}{\Pr(\mathrm{X}|\mathrm{A})\Pr(\mathrm{A}) + \Pr(\mathrm{X}|\mathrm{not\ A})\Pr(\mathrm{not\ A})}$$

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— **Pr(A|X)** = Chance of having cancer (A) given a positive test (X). This is what we want to know: How likely is it to have cancer with a positive result? In our case it was 7.8%.
— **Pr(X|A)** = Chance of a positive test (X) given that you had cancer (A). This is the chance of a true positive, 80% in our case.
— **Pr(A)** = Chance of having cancer (1%).
— **Pr(not A)** = Chance of not having cancer (99%).
— **Pr(X|not A)** = Chance of a positive test (X) given that you didn't have cancer (~A). This is a false positive, 9.6% in our case.

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Even simpler: The chance of a true positive result divided by all positive results

$$\Pr(A|X) = \frac{\Pr(X|A)\Pr(A)}{\Pr(X)}$$

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$$\boxed{\pi(\theta|x_1, x_2, \ldots, x_n)} = \frac{\boxed{f(x|\theta_i)}\boxed{\pi(\theta)}}{\boxed{\sum_{i=0}^{n} f(x|\theta_i)(\pi(\theta_i)}}$$

The Prior

Posterior Distribution

Marginal / Total Probabilities

The likelihood function

Independent Practice

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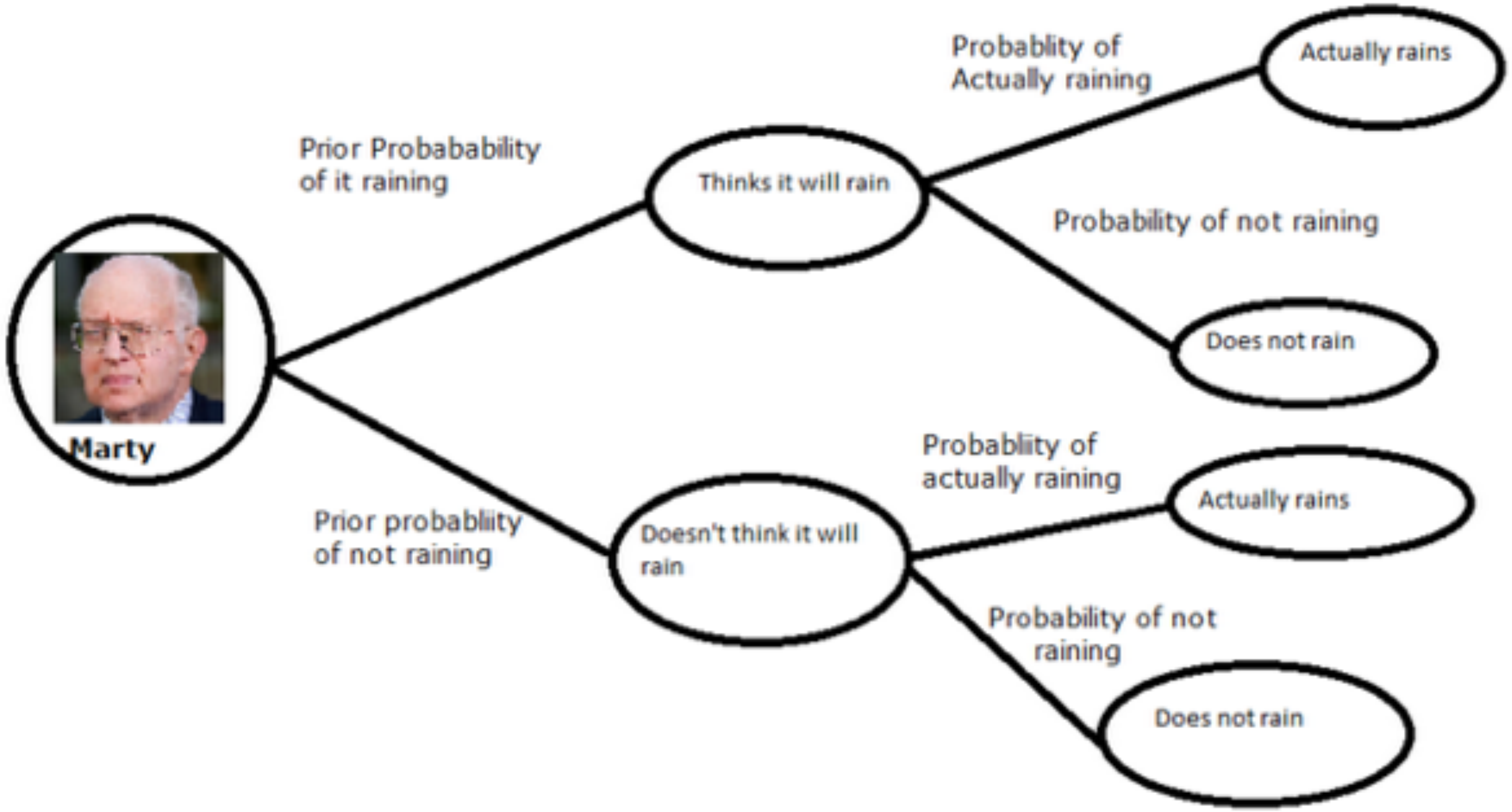
Observe the following:



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Review Probability: http://stattrek.com/probability/probability-rules.aspx?Tutorial=AP

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1.1.1 Finger Exercise 1 - Review of computing the conditionals

Suppose we assume the prior probability distribution can be thought of like a simple bias coined toss, where the probability of it raining is .4. Similarly, assume that the 'likelihood' probability of it raining again can be thought of as a simple biased coin toss, where the probability of it not raining is .9 if Marty thinks it will rain, and the probability of it actually raining is .7 if Marty doesn't think it will rain.

- What is the probability of it actually raining if Marty thought it would rain?
- What is the probability of it not raining if Marty didn't think it would rain?

1.1.2 Finger Exercise 2 - Computing numerically with the basic Bayes formula

Following the scenario above, what is the Probability of it actually raining, given the *total probability* of the prior? (**hint**: We are asking for P(A|B)