GENERAL ASSEMBLY

Markov Chains Monte Carlo - An Introduction

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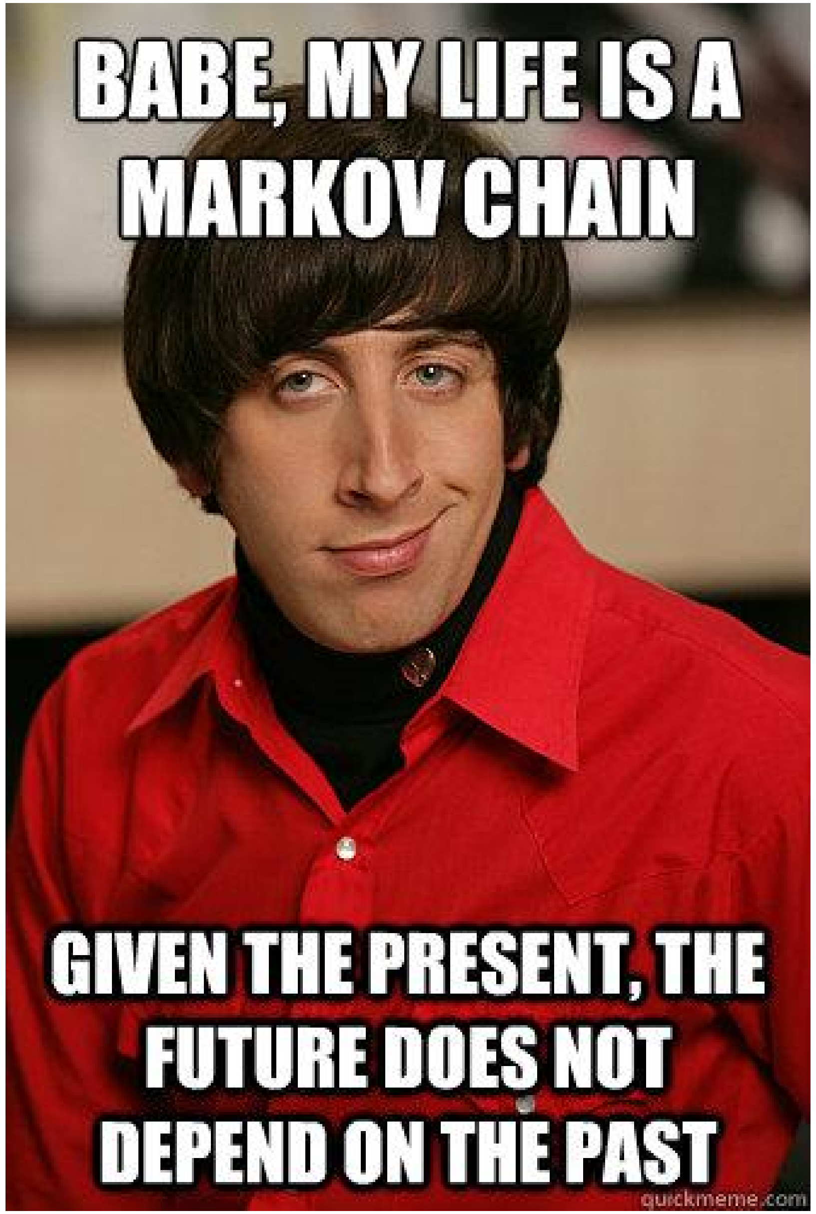
OPENING

MCMC - An Intro



Markov Chains - An Intro

MCMC - An Intro

Markov Chains are mathematical systems that go between two states.

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These chains contain possible states of an occurrence, as well as the probabilities of the states associated with those occurrences.

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The chains use a matrix and a vector to model and predict the behavior of a system of states

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Consider this case:

Suppose a taxi company has divided the city into three regions: Northside, Downtown, and Southside. The company has been keeping track of pickups and deliveries and has found that of the fares picked up Downtown, 10% are dropped off in Northside, 40% stay Downtown, and 50% go to Southside.

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Of the fares that originate in Northside, 50% stay in Northside, 20% are dropped off in Downtown, and 30% are dropped off in Southside. Of those fares picked up in Southside, 30%end in Northside, 30% are delivered to Downtown, and 40% stay in the Southside region.

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The taxi company would like to know what the distribution of their taxis would be over a certain amount of time

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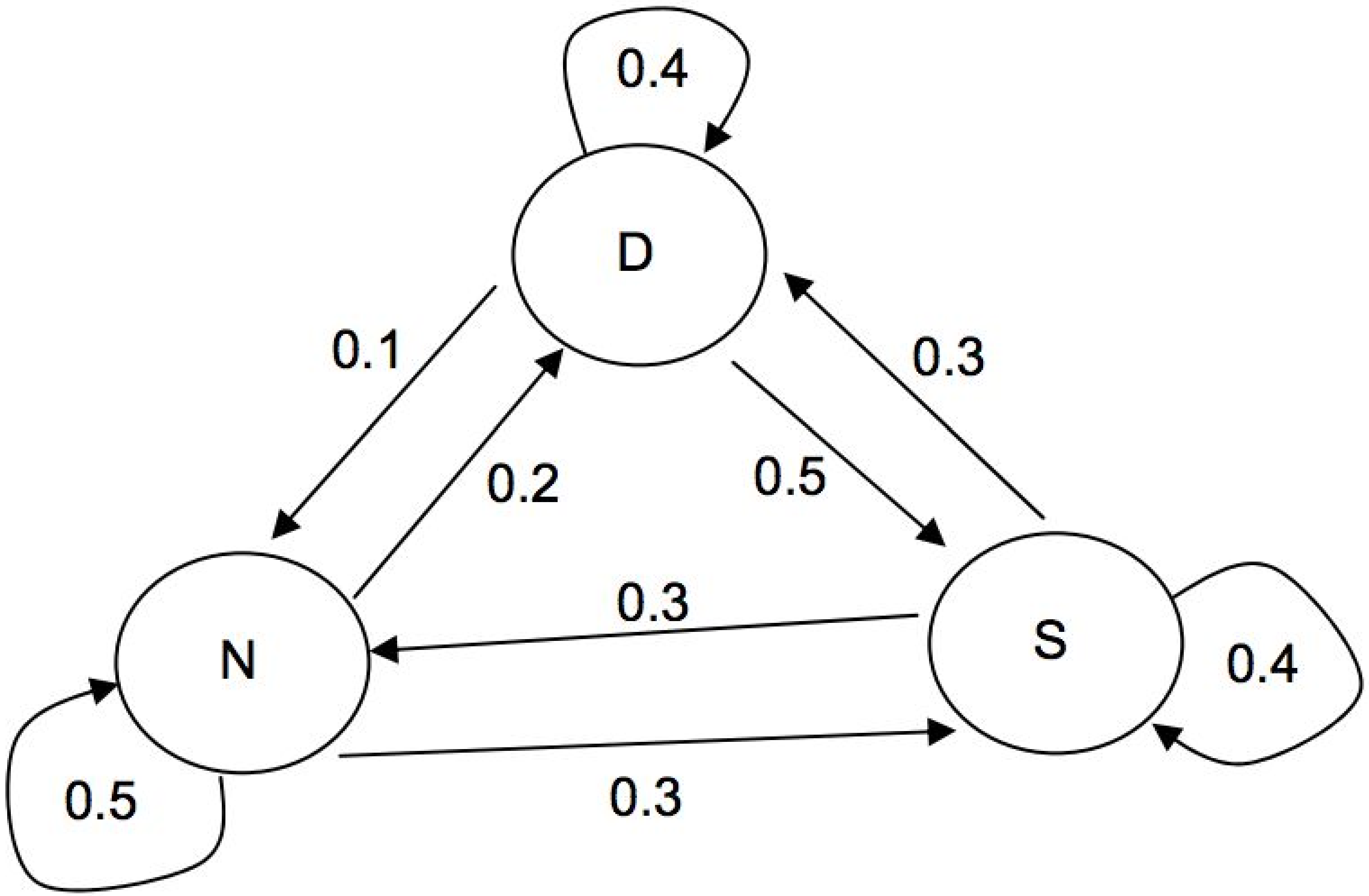

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“A Markov chain is a special process in which the transition probabilities are constant and independent of the previous behavior of the system.”

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Markov Chains have the following properties:

1. For each time period, every object (person) in the system is in exactly one of the defined states. At the end of the time period, each
   object either moves to a new state or stays in that same state for another period.
2. The objects move from one state to the next according to the transition probabilities, which depend only on the current state (they do not take any previous history into account). The total probability of movement from a state (movement from a state to the same state does count as movement) must equal one.

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Markov Chains have the following properties:

3. The transition probabilities do not change over time (the probability of going from state A to state B is the same as it will be at any time in the future)."

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In Markov chains, while each state has it’s own probability, we can calculate the transitional probability to creating a *transition matrix*.

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Transition Matrix

A *transition matrix* associated with a Markov chain with n states is an $n \, X \, n$ matrix T with entries a_{ij}

$$\begin{array}{cc} & \text{Current State} \\ \text{Next state} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{bmatrix} \end{array}$$

Next state

$$a_{ij} = P(\text{state } i \mid \text{state } j)$$

Current state

1. $a_{ij} \geq 0$ for all i and j.
2. The sum of the entries in each column of T is 1.

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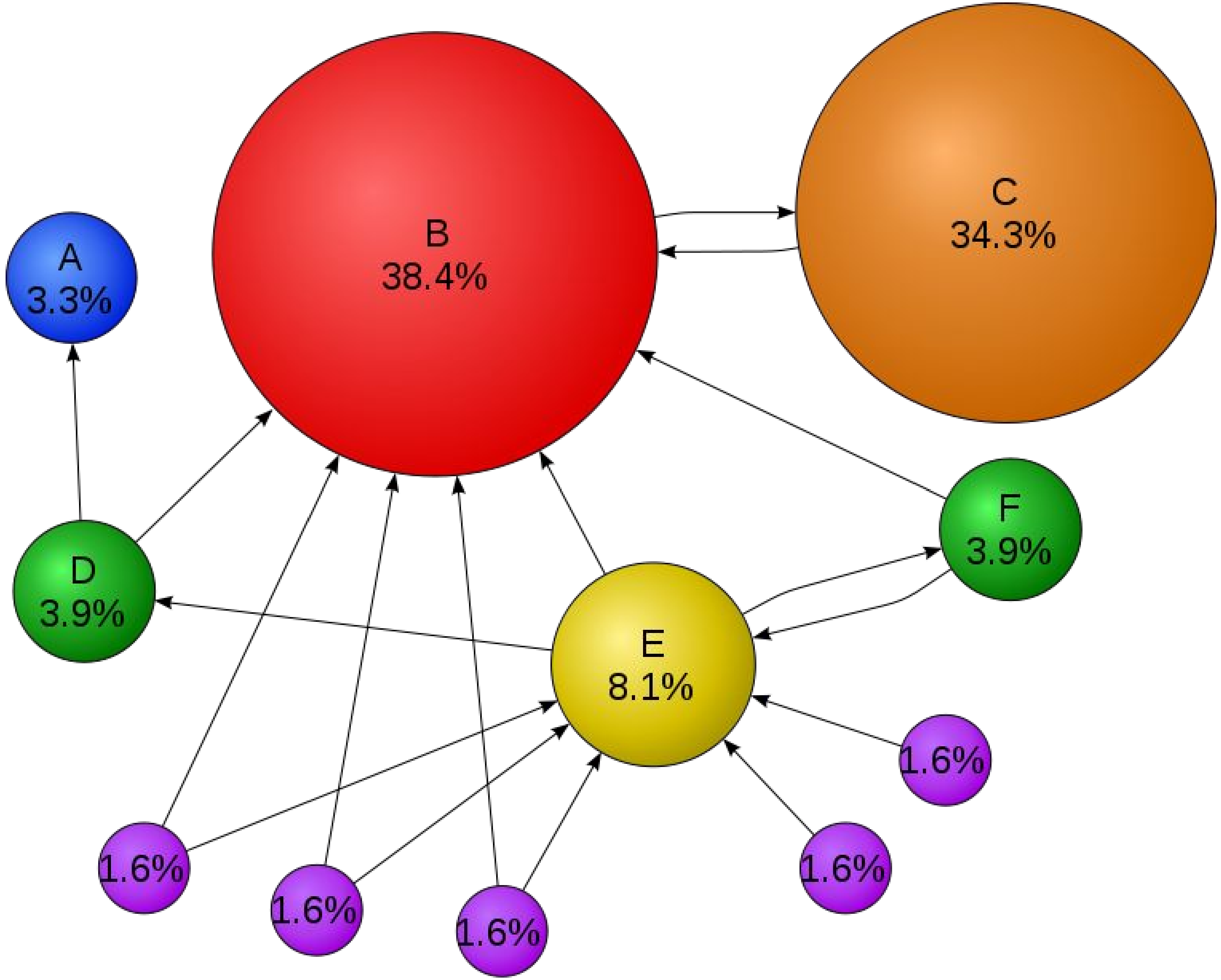
How to we use Markov Chains? Google Page Rank



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All of what we have just seen is a *neat* Markov Chain, which satisfies *several* conditions that we will not discuss today (AKA the transition probabilities satisfy these conditions)

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Irrespective of the initial starting state we will eventually reach an equilibrium probability distribution of states.

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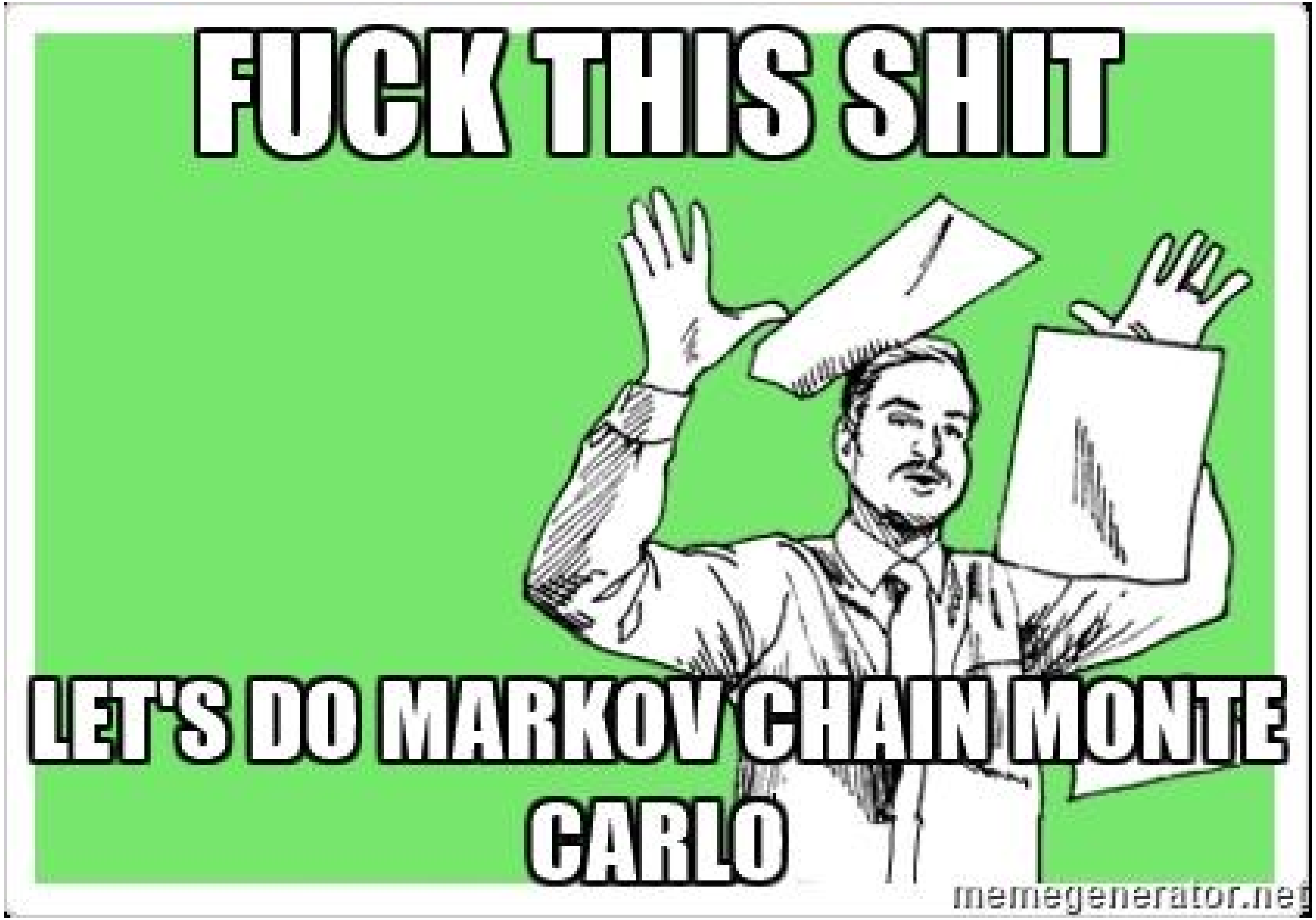

Markov Chain Monte Carlo

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With Markov Chain Monte Carlo, in a sense we used a “reversed” Markov Chain.

We identify a way to construct a 'nice' markov chain such that its equilibrium probability distribution is our target distribution

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If we start at a random point in the Markov Chain, and continue to make probability draws from the chain randomly, at some point we would eventually make it seem like those draws are coming from the target distribution.

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To boil this down, Markov Chain Monte Carlo is a means of taking a sample from a complicated distribution

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It's essentially a complicated term for a *random walk* on a graph

Markov Chains Monte Carlo - Practice

Conclusion

Q & A