Rate Feasibility in Wireless Networks

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Introduction

- Simple abstraction of a wireless network:
  Vertices: wireless nodes
  Edges: possible communication links

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  Vertices: wireless nodes
  Edges: possible communication links
- Assume each link has a unit capacity (without interference)
- But Interference $\Rightarrow$ Not all links can be simultaneously active
- A fundamental question: Can a given vector of link demands be satisfied *simultaneously*?

Rate Feasibility

- Λ - set of all Rate Vectors for which there exists a TDMA schedule.
- $\mathbf{r}$ - the query Rate Vector
- Problem: Does $\mathbf{r} \in \Lambda$?

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- Λ - set of all Rate Vectors for which there exists a TDMA schedule.
- $\mathbf{r}$ - the query Rate Vector
- Problem: Does $\mathbf{r} \in \Lambda$?
- T - the incidence matrix for the set of all subsets of non conflicting links

$$
\begin{aligned}
z = min\ \mathbf{1}^T\mathbf{x} \\
\mathbf{Tx} = \mathbf{r},
\end{aligned}
$$

Then $\mathbf{r} \in \Lambda$ iff $z \leq 1$

Is *r* "feasible"?

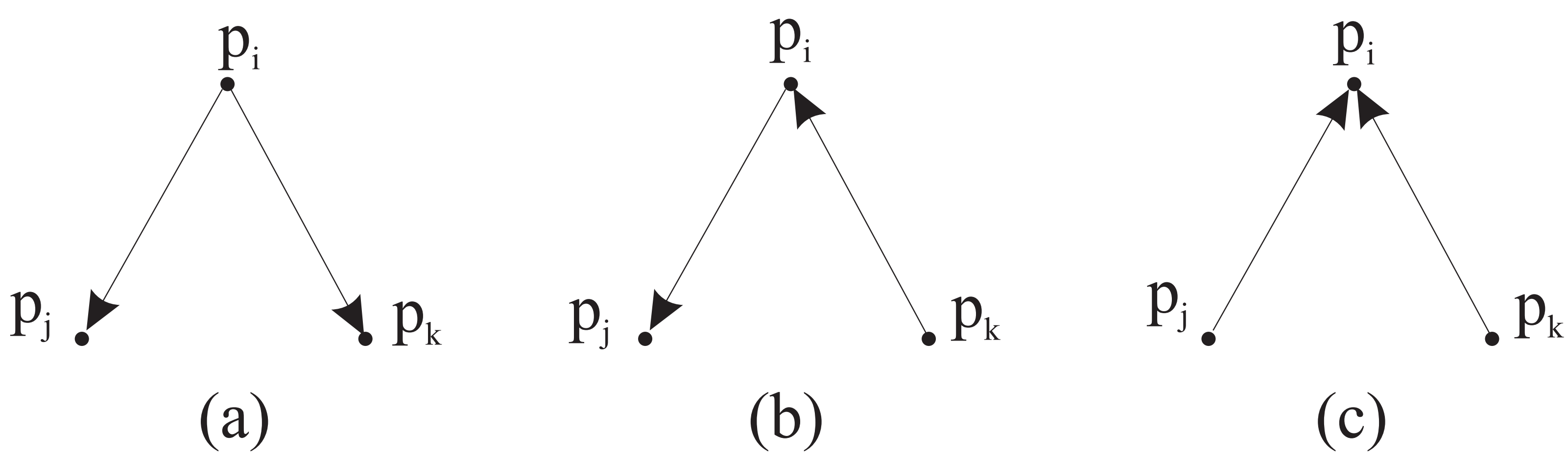
- An answer to whether a given rate vector is feasible depends on the following:

1. Graph structure *(the specific problem instance)*

2. Interference constraints *(the problem model - critical for the computational complexity)*

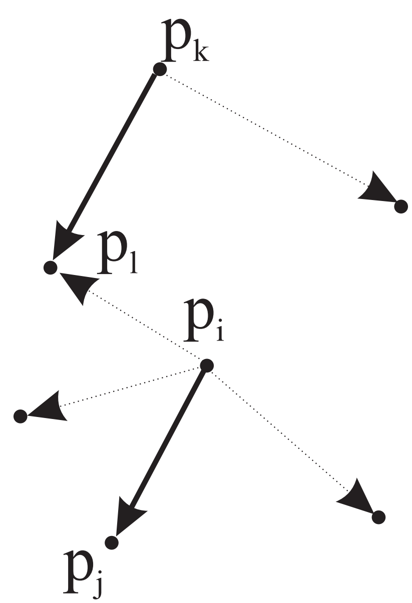
Interference

Primary Constraints: Links conflict when they share a node.



Interference

Secondary constraints: Links can conflict even while not sharing a common node.



Links (p_i, p_j) and (p_k, p_l) conflict

Background . . .

- Primary and Secondary constraints (appropriate for *wireless networks*): problem is NP-hard [Arikan,'84]

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- Primary and Secondary constraints (appropriate for *wireless networks*): problem is NP-hard [Arikan,'84]
- Primary alone: Polynomial scheduling algorithms exist [Hajek, Sasaki '88]
- Question: *Are there restricted subclasses of wireless networks that are tractable?*

Our Results . . .

- Bounded density
  - A randomized approximation algorithm
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- Bounded density
  - A randomized approximation algorithm
  - More generally relevant to membership in complex convex sets
- Fixed width slab
  - Generalize results on fractional coloring unit disk graphs to a more general class
  - Deterministic, Exact solution.

Adjoint Graph

- Adjoint Graph:
  - Vertices - Link midpoints
  - Edges - defined by link conflicts (both primary and secondary)
- Communication (Interference) radius - $r_C(r_I)$ for wireless nodes $\Rightarrow$ Structure of its adjoint.
- Link Rate-feasibility $\equiv$ Node rate-feasibility in adjoint graph with independent sets.

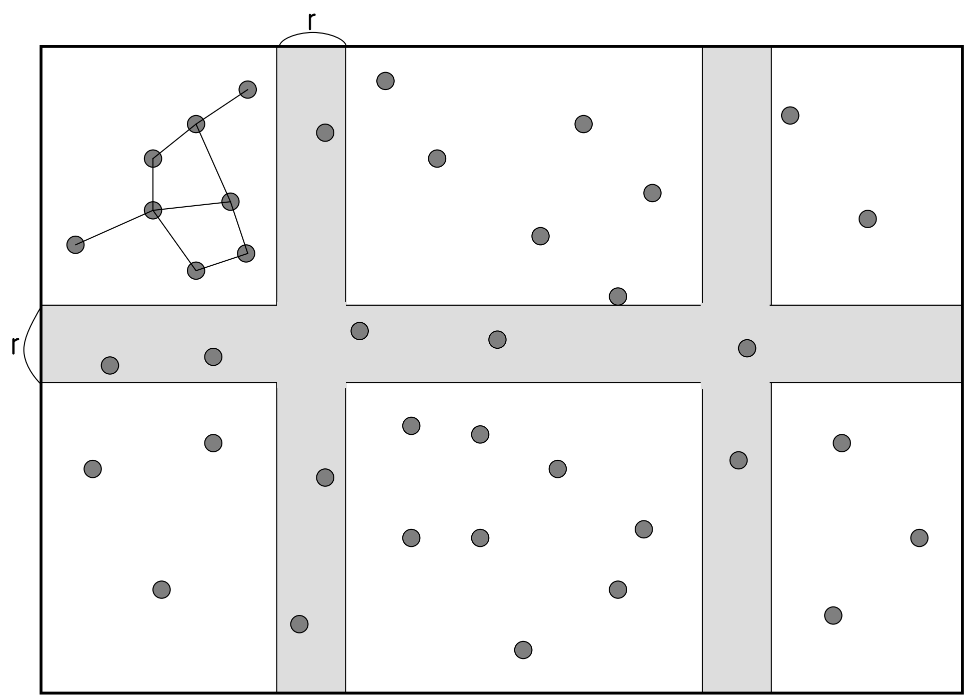
Bounded density

- Let $B(v, R) = |\{u \in V : u \neq v, d(u, v) < R\}|$
  Then, G has *bounded density* $D > 0$, if for all $v \in V$
  $$\frac{B(v, R)}{R^2} \leq D$$
- Bounded density of wireless graph $\Rightarrow$ Bounded density of adjoint
- $\exists R > 0$ such that two vertices farther than R in the adjoint have no edge.

Algorithm overview

Partition the graph into:

- Regions, small enough to be efficiently solved
- Boundaries, thick enough separate them



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- Solve for feasible schedules in each region
- Merge them to get a global schedule – (*schedule satisfies everyone except nodes in boundary*)
- Randomize the boundary and compute an average schedule obtained over sufficiently large iterations. – (*such that its unlikely for a node to fall in the boundary.*)

Main Properties

(w.h.p.) Given $\epsilon > 0$:

(1) If $\mathbf{r} \in \Lambda$, algorithm outputs a TDMA schedule $\hat{T} = (\alpha_k, I_k)_{k \leq M}$ (with $M =$ poly(n)), such that:

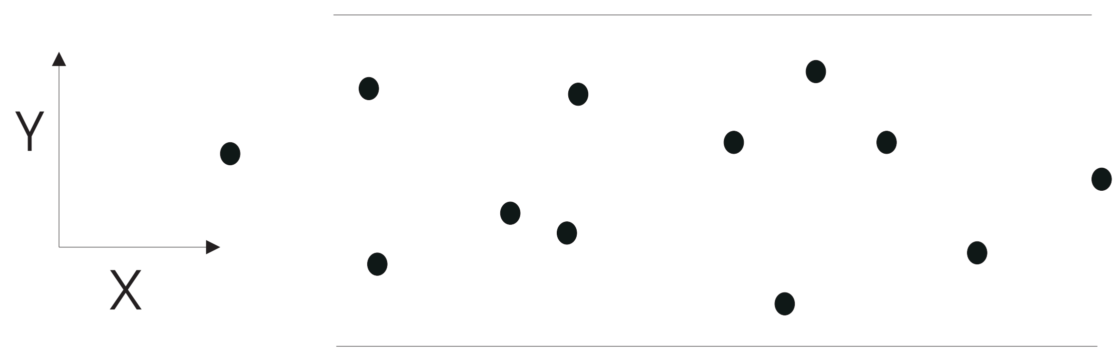
$$(1-\epsilon)\mathbf{r} \leq \sum_k \alpha_k I_k,$$

(2) If $(1-\epsilon)\mathbf{r} \notin \Lambda$, it declares NOT FEASIBLE.

- Complexity is $O\left(\frac{n \log n 2^{O\left(\frac{R^2 D}{\epsilon^2}\right)}}{\epsilon}\right) \approx O(n \log n)$.

Fixed width slab

- Assume wireless nodes are restricted in one coordinate. (e.g. Y- dimension bounded, while X can range all over: IVHS)



Adjoint Structure

- Represent the vertex corresponding to the link between $\mathbf{p}_i, \mathbf{p}_j$ as $(\mathbf{p}_i, \mathbf{p}_j)$. Then:

1. There cannot be an edge between vertices $(\mathbf{p}_i, \mathbf{p}_j)$ and $(\mathbf{p}_k, \mathbf{p}_l)$ if $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 \geq r_c + r_i$

2. There will be an edge between vertices $(\mathbf{p}_i, \mathbf{p}_j)$ and $(\mathbf{p}_k, \mathbf{p}_l)$ if $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 < r_i - r_c$

Useful Results

- Let M be the incidence matrix for the set of all independent sets of a graph G. Fractional coloring on G is to solve:

$$
\begin{aligned}
z = min\ \mathbf{1}^T\mathbf{x} \\
\mathbf{M}\mathbf{x} \geq \mathbf{1} \\
\mathbf{x} \geq \mathbf{0}
\end{aligned}
$$

- Fractional coloring problem on a Unit Disk Graph has polynomial solution if nodes are within a fixed width slab. [Matsui 02]

Contrast with Rate Feasibility

- Fractional coloring algorithms solve a Node based rate feasibility problem with equal rates.

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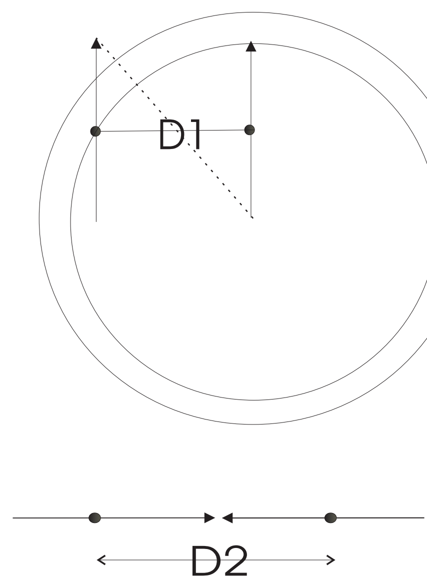
- Fractional coloring algorithms solve a Node based rate feasibility problem with equal rates.
- Wireless adjoints do not have unit disk graph structure.



Key Observations

1. The results for unit disk graphs can be extended to a more general class called (d_{min}, d_{max}) graphs, which includes our wireless adjoints.

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2. The *equal rates* in fractional coloring can be generalized to *arbitrary rates*.

(d_{min}, d_{max}) - graphs

$G(P) = (P, E)$ induced by point set $P \subseteq \{(x, y) \in \mathcal{R}^2\}$ satisfies the following properties:

1. *Property P1*:
   $\forall \mathbf{p}_1, \mathbf{p}_2 \in P, \|\mathbf{p}_1 - \mathbf{p}_2\| \geq d_{max} \Rightarrow (\mathbf{p}_1, \mathbf{p}_2) \notin E$
2. *Property P2*: $\|\mathbf{p}_1 - \mathbf{p}_2\| < d_{min} \Rightarrow (\mathbf{p}_1, \mathbf{p}_2) \in E$

- If $d_{min} = d_{max}$, we get Unit Disk Graphs.
- Note: Between d_{min} and d_{max} we have no restriction.

(d_{min}, d_{max}) - graphs

- Theorem: Matsui's algorithm can be generalized to (d_{min}, d_{max}) graphs.

- Proof idea: The properties $P1$ and $P2$, though implicitly buried together in the specification of the unit disk graphs, are actually only needed separately.

Rate Feasibility on Fixed Width Slabs

- Wireless adjoints are $(r_I - r_C, r_I + r_C)$ graphs.
- Apply the Fractional Coloring algorithm, generalized to arbitrary rates.
- Works in polynomial time, due to our generalization from unit disk graphs to (d_{min}, d_{max}) graphs.

Conclusion and Future interests

- Have shown two relevant subclasses where rate feasibility is tractable for wireless networks, even though the general problem is NP-hard.
- Negatives: These are quite centralized descriptions of the algorithms.
- Potential interest for practicality: Finding even lower complexity distributed versions.
- Integrate Rate Feasibility checking seamlessly into cross layer design.

Thank you for your attention!

. . . Questions?