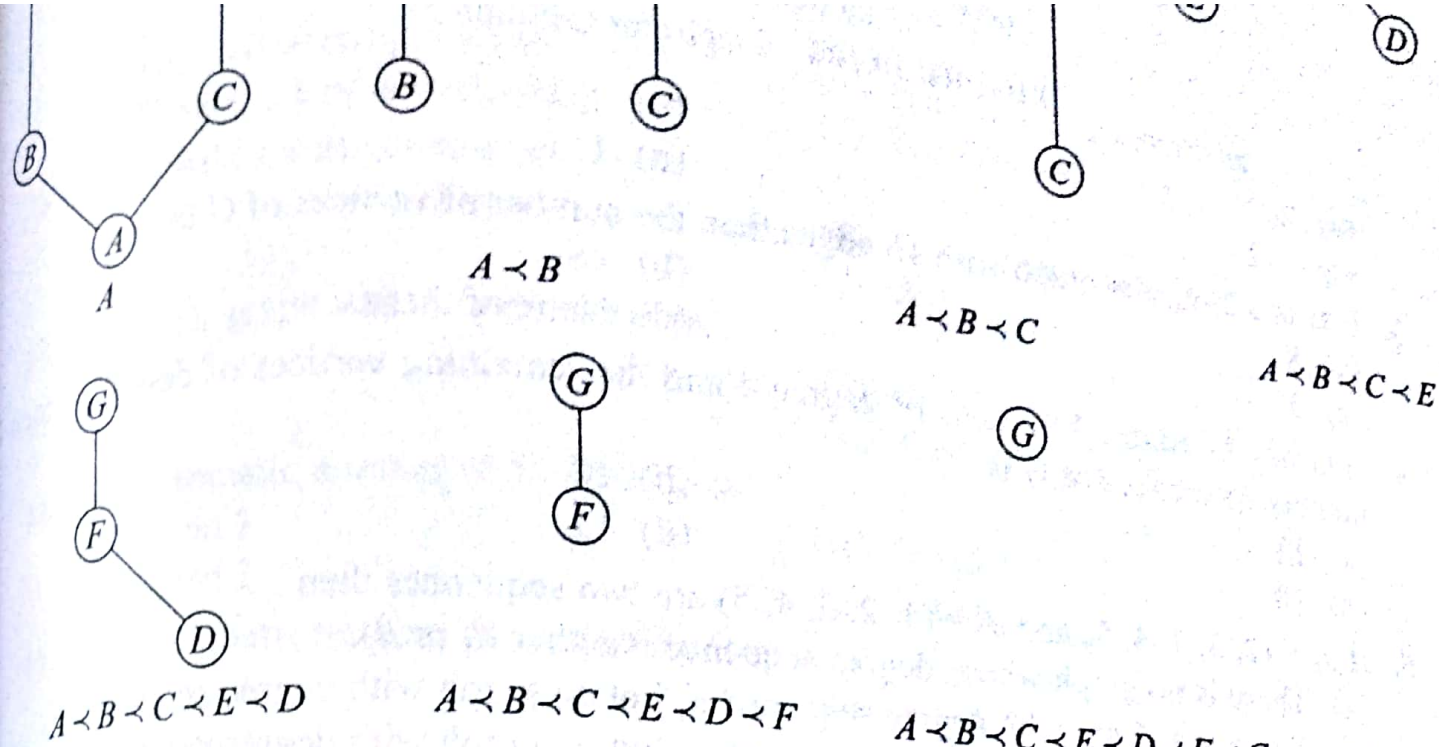


FIGURE 10.102 Topological sorting of Example 10.24.

LF-TEST

oose the correct answer to Questions 1–42

1. The number of components of G_1 and G_2 given in Figure 10.103 are

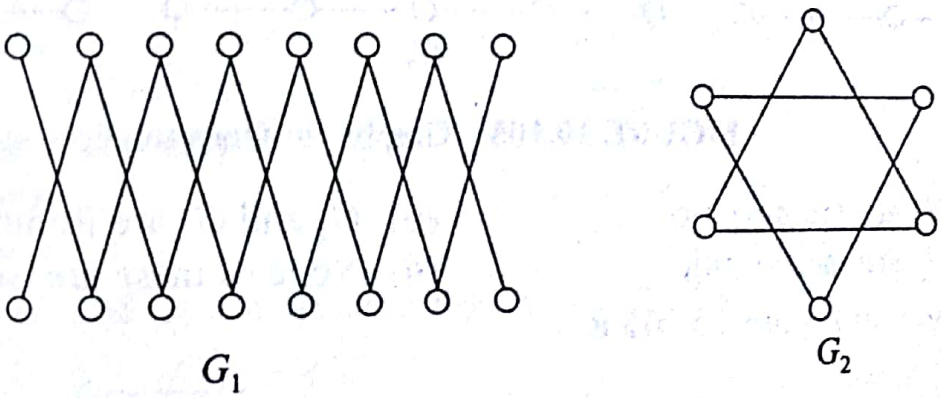
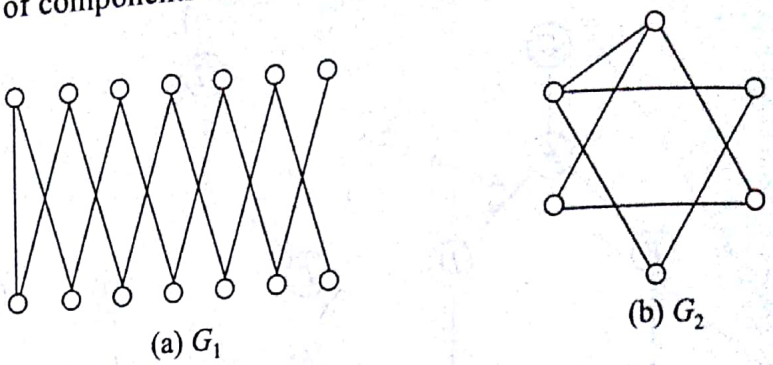


FIGURE 10.103 Graphs for Question 1.

(a) 3, 3
(b) 3, 2
(c) 2, 3
(d) 2, 2.

2. The number of components of G_1 and G_2 given in Figure 10.104 are



(a) G_1 (b) G_2

FIGURE 10.104 Graphs for Question 2.

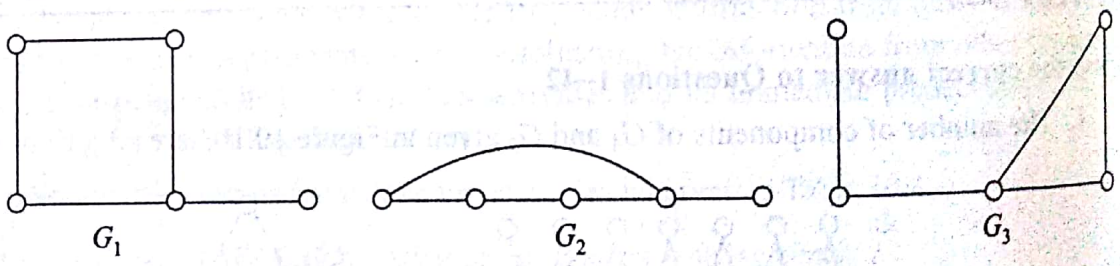
(a) 2, 2 (b) 2, 1
(c) 1, 2 (d) 1, 1.

3. If G is a 2-regular graph with 16 edges then the number of vertices of G *is*
(a) 8 (b) 16
(c) 32 (d) None of these.

4. If G has 21 edges, 3 vertices of degree 4 and the remaining vertices of degree 3, then the number of vertices of G is
(a) 13 (b) 17
(c) 18 (d) 12.

5. If $d_1 = (2, 3, 4, 4, 5)$ and $d_2 = (1, 2, 3, 4, 5)$ are two sequences then
(a) There is no graph whose degree sequence is either d_1 or d_2.
(b) There is a graph with degree sequence d_1 but no graph with degree sequence d_2.
(c) There is a graph with degree sequence d_2 but no graph with degree sequence d_1.
(d) There is a graph with degree sequence d_1 and a graph with degree sequence d_2.

6. If G_1, G_2 and G_3 are given in Figure 10.105, then



G_1 G_2 G_3

FIGURE 10.105 Graphs for Question 6.

(a) G_1 and G_2 are isomorphic (b) G_1 and G_3 are isomorphic
(c) G_2 and G_3 are isomorphic (d) None of these are isomorphic.

7. The graph given in Figure 10.106 is

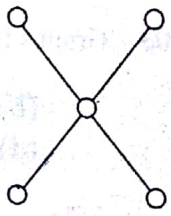


FIGURE 10.106 Graph for Question 7.

(a) C_5 (b) D_5
(c) W_5 (d) Bipartite graph.

8. The graph given in Figure 10.107.

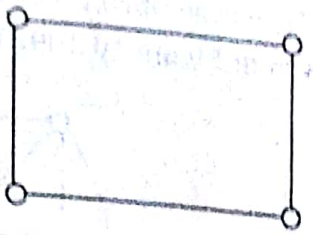


FIGURE 10.107 Graph for Question 8.

(a) Is bipartite but not a tree (b) Is a tree but not bipartite
(c) Neither a tree nor bipartite (d) Bipartite and a tree.

9. A tree with 5 vertices can be
(a) D_5 (b) P_5
(c) C_5 (d) W_5.

10. A regular graph with 5 vertices can be
(a) D_5 (b) P_5
(c) C_5 (d) Tree.

11. The chromatic number of P_5 and C_5 are
(a) 2 and 3 (b) 2 and 2
(c) 3 and 2 (d) 3 and 3.

12. The chromatic number of C_5 and W_5 are
(a) 4 and 4 (b) 3 and 3
(c) 4 and 3 (d) 3 and 4.

13. A complete bipartite graph $K_{m,n}$ is a tree when
(a) $m = 1, n = 2$ (b) $m = 2, n = 2$
(c) $m = 2, n = 3$ (d) $m = 3, n = 2$.

14. A 3-regular graph with four or more vertices is
(a) An Euler graph (b) Not an Euler graph
(c) A wheel (d) A cycle.

15. The graph K_n is planar if and only if
(a) $n \le 3$ (b) $n = 3$
(c) $n \le 4$ (d) $n = 4$.

16. $K_{m,n}$ is planar if and only if
(a) $m \le 2$ or $n \le 2$ (b) $m \le 2$ and $n \le 2$
(c) $m = 2$ or $n = 2$ (d) $m = 2$ and $n = 2$.

17. Two graphs G_1 and G_2 are given in Figure 10.108. Then

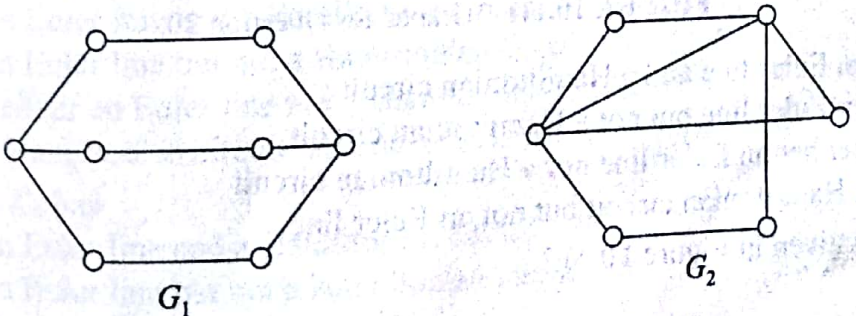


FIGURE 10.108 Graphs for Question 17.

(a) G_1 and G_2 have Euler line
(b) G_1 and G_2 have open Euler line
(c) G_1 has an Euler line and G_2 has an open Euler line
(d) G_2 has an Euler line and G_1 has an open Euler line.

18. Two graphs G_1 and G_2 are given in Figure 10.109. Then

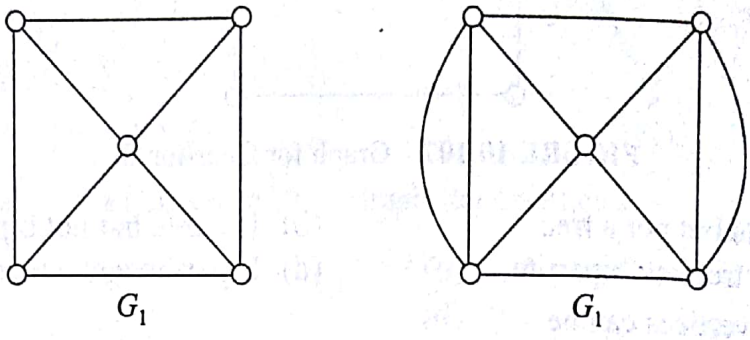


FIGURE 10.109 Graphs for Question 18.

(a) Neither G_1 nor G_2 has an Euler line
(b) Neither G_1 nor G_2 has an open Euler line
(c) Both G_1 and G_2 have Euler line
(d) Both G_1 and G_2 have an open Euler line.

19. The graph given in Figure 10.110.

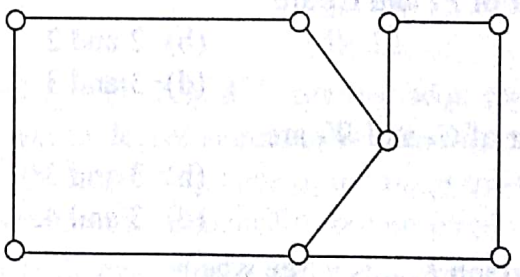


FIGURE 10.110 Graphs for Question 19.

(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit.
(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

20. The graph given in Figure 10.111

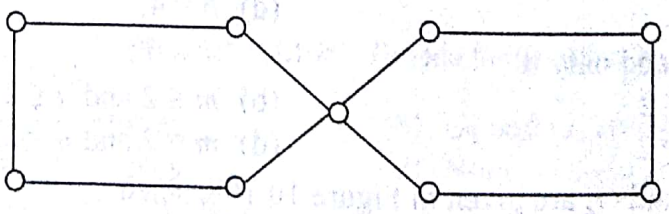


FIGURE 10.111 Graphs for Question 20.

(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit
(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

21. The graph given in Figure 10.112.

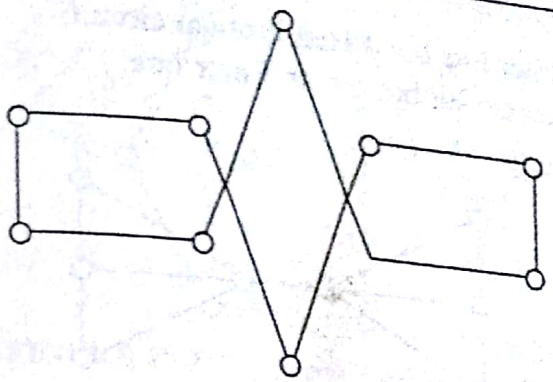


FIGURE 10.112 Graphs for Question 21.

(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit
(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

22. The graph given in Figure 10.113.

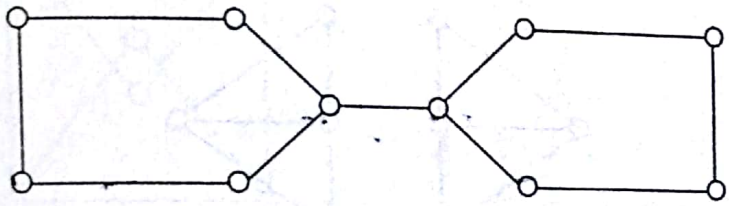


FIGURE 10.113 Graphs for Question 22.

(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit
(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

23. A tree with 3 or more vertices
(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit
(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

24. The graph Q_3
(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit
(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

25. The graph Q_4
(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit
(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

26. The graph K_5 has
(a) Has an Euler line and a Hamiltonian circuit
(b) Has an Euler line but not a Hamiltonian circuit

(c) Has neither an Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an Euler line.

27. The graph given in Figure 10.114

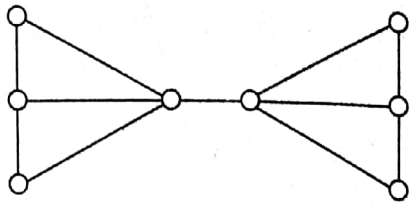


FIGURE 10.114 Graph for Question 27.

(a) Has an open Euler line and a Hamiltonian circuit
(b) Has an open Euler line but no Hamiltonian circuit
(c) Has neither an open Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an open Euler line.

28. The graph given in Figure 10.115

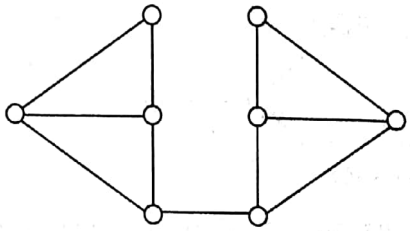


FIGURE 10.115 Graph for Question 28.

(a) Has an open Euler line and a Hamiltonian circuit
(b) Has an open Euler line but no Hamiltonian circuit
(c) Has neither an open Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an open Euler line.

29. The graph given in Figure 10.116

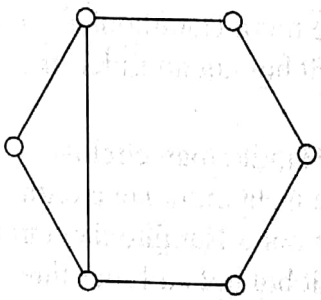


FIGURE 10.116 Graph for Question 29.

(a) Has an open Euler line and a Hamiltonian circuit
(b) Has an open Euler line but no Hamiltonian circuit
(c) Has neither an open Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an open Euler line.

30. The graph given in Figure 10.117

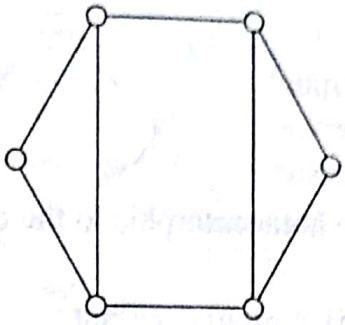


FIGURE 10.117 Graph for Question 30.

(a) Has an open Euler line and a Hamiltonian circuit
(b) Has an open Euler line but no Hamiltonian circuit
(c) Has Neither an open Euler line nor a Hamiltonian circuit
(d) Has a Hamiltonian circuit but not an open Euler line.

31. The graphs G_1 and G_2 are given in Figure 10.118.

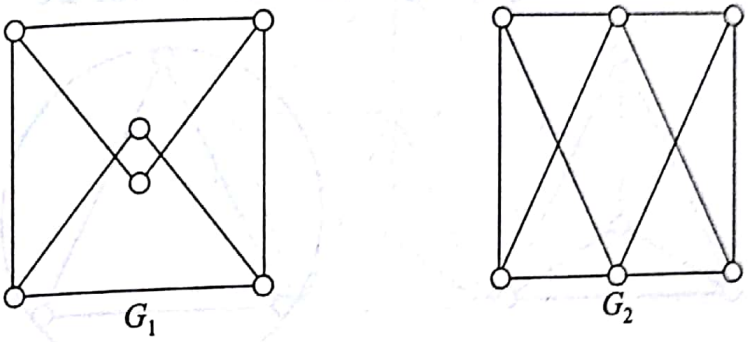


FIGURE 10.118 Graphs for Question 31.

(a) G_1 and G_2 are planar
(b) G_1 and G_2 are nonplanar
(c) G_1 is planar but G_2 is nonplanar
(d) G_2 is planar but G_1 is nonplanar.

32. The graph K_5
(a) is planar
(b) is nonplanar after the removal of any edge
(c) is planar after the removal of any edge
(d) None of these.

33. K_n is nonplanar
(a) for $n \leq 4$
(b) for $n \geq 4$
(c) for $n \leq 5$
(d) for $n \geq 5$.

34. If a connected 4-regular planar graph has 6 vertices then the number of regoins determined by the graph is
(a) 4
(b) 6
(c) 8
(d) 10.

35. If G_1, G_2 and G_3 are given by Figure 10.119

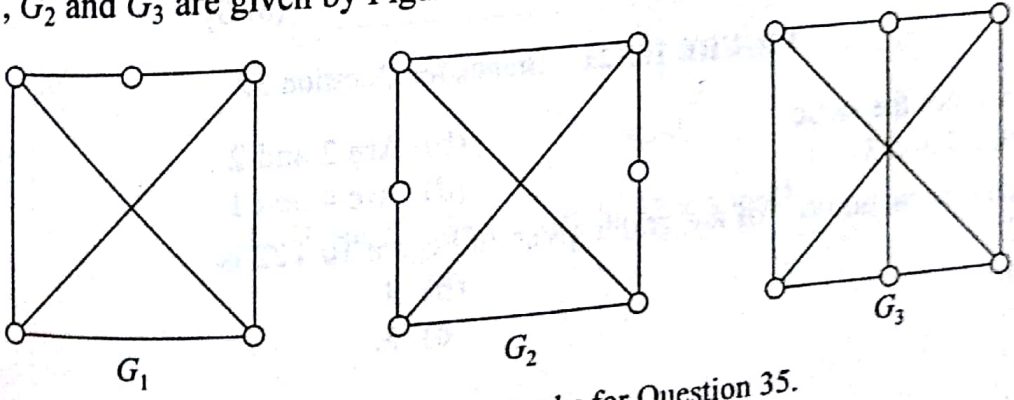


FIGURE 10.119 Graphs for Question 35.

then
(a) G_1 and G_2 are homeomorphic
(b) G_1 and G_3 are homeomorphic
(c) G_2 and G_3 are homeomorphic
(d) None of these graphs are homeomorphic to the other two graphs.

36. Petersen Graph
(a) Has an Euler line and a Hamiltonian circuit
(b) Has neither an Euler line nor a Hamiltonian circuit
(c) Has an Euler line and planar
(d) Has a Hamiltonian circuit and nonplanar.

37. Petersen graph has no n-circuit for
(a) $n = 4$ (b) $n = 5$
(c) $n = 6$ (d) $n = 8$.

38. The chromatic numbers of the graphs given in Figure 10.120 are

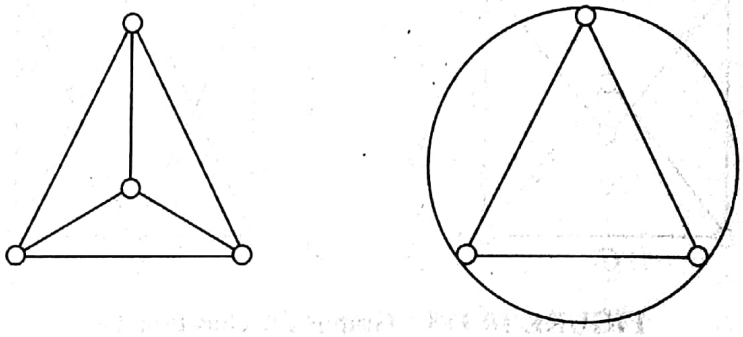
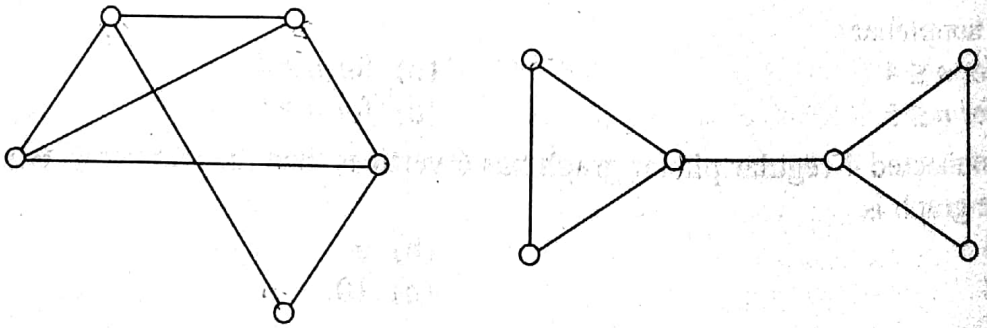


FIGURE 10.120 Graphs for Question 38.

(a) 3 and 3 (b) 4 and 4
(c) 3 and 4 (d) 4 and 3.

39. The chromatic numbers of G_1 and G_2 given in Figure 10.121.



(a) G_1 (b) G_2

FIGURE 10.121 Graphs for Question 39.

(a) Are not the same (b) Are 2 and 2
(c) Are 3 and 3 (d) Are 4 and 1.

40. The chromatic number of the graph given in Figure 10.122 is
(a) 2 (b) 3
(c) 4 (d) 5.

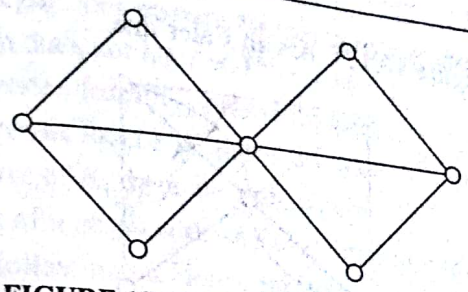


FIGURE 10.122 Graph for Question 40.

41. An example of a 6-chromatic graph is
    (a) D_6 (b) C_6
    (c) W_6 (d) K_6.

42. The chromatic number of the graph given in Figure 10.123 is

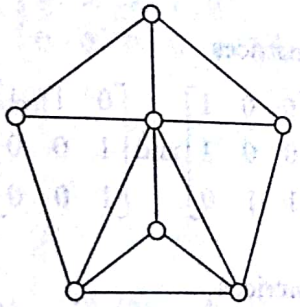


FIGURE 10.123 Graph for Question 42.

    (a) 3 (b) 4
    (c) 5 (d) 6.

Determine whether the following statements are true or false (Questions 42–63):

43. There exists a graph with degree sequence (3, 4, 3, 4, 3).
44. There exists a graph with degree sequence (1, 1, 1, 1, 1).
45. There exists a graph with degree sequence (4, 4, 4, 4, 4).
46. There exists a graph with edge sequence (2, 3, 3, 4, 5, 6, 7).
47. The graph given in Figure 10.124 is bipartite.

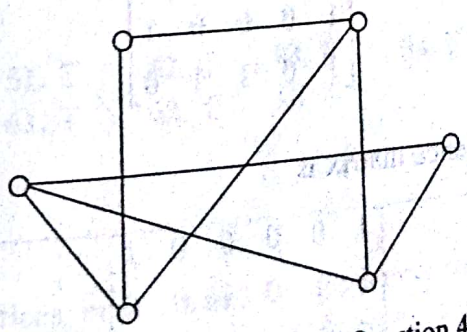


FIGURE 10.124 Graph for Question 47.

48. The graph given in Figure 10.125 has an Euler line.

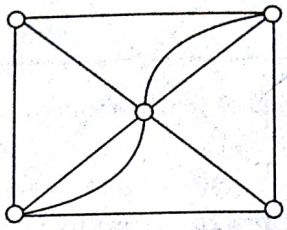


FIGURE 10.125 Graph for Question 48.

49. The graph given in Figure 10.125 has an open Euler line.
50. The graph given in Figure 10.125 has a Hamiltonian circuit.
51. Every symmetric binary matrix with 0's on the principal diagonal is the adjacency matrix of some graph.
52. The graphs with adjacency matrices

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

are isomorphic.

53. The graphs with adjacency matrices

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

are isomorphic

54. The graph whose adjacency matrix is

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

has an Euler line.

55. The graph whose incidence matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

has no Euler line.

56. Most of the parallel algorithms operate through a hypercube.
57. The de Bruijn digraph does not have an Euler line.
58. The removal of any vertex from $K_{3,3}$ makes it planar.
59. The removal of any vertex from $K_{3,4}$ makes it planar.
60. In the adjacency matrix of K_n there are only n zeros.
61. The incidence matrix of a graph with an Euler line has even number of 1's in every row.
62. If a graph G has the following incidence matrix,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

then it has a Hamiltonian circuit.
63. Every tree is bipartite.

Answers to Self-test

1. (d) 2. (d) 3. (b) 4. (a) 5. (a) 6. (a) 7. (d) 8. (a) 9. (b) 10. (c)
11. (a) 12. (d) 13. (a) 14. (b) 15. (c) 16. (a) 17. (d) 18. (b) 19. (d) 20. (b)
21. (a) 22. (c) 23. (c) 24. (d) 25. (a) 26. (a) 27. (b) 28. (c) 29. (a) 30. (d)
31. (a) 32. (c) 33. (d) 34. (c) 35. (a) 36. (b) 37. (a) 38. (d) 39. (c) 40. (b)
41. (d) 42. (b)
43. F (Three vertices of odd degree)
44. F (Three vertices of odd degree)
45. T
46. F (A graph with 7 vertices cannot have a vertex of degree 7)
47. F (Chromatic number is 3)
48. F 49. T 50. T 51. T 52. T 53. F 54. F 55. T 56. T 57. F
58. T 59. F 60. T 61. T 62. T 63. T

Iteration 3

Combine the weights of U_2 and S and add a new vertex U_3. Label the edges.

Figure 11.56(c) gives the Huffman tree. The codes of M, P, I and S are 100, 101, 11 and 0.

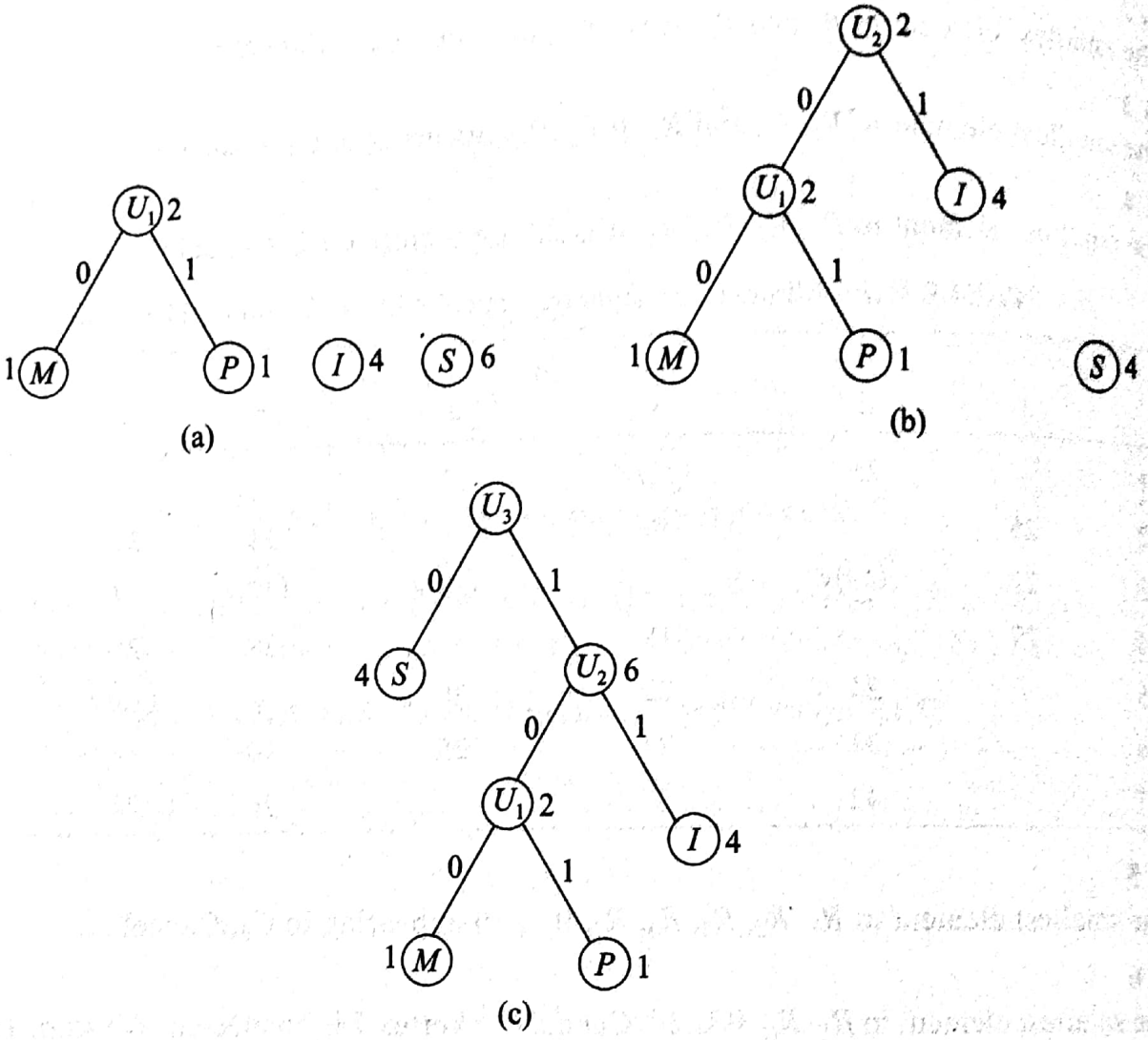


FIGURE 11.56 Huffman code of Example 11.54.

SELF-TEST

Questions 1–5 related to the rooted tree given in Figure 11.57.

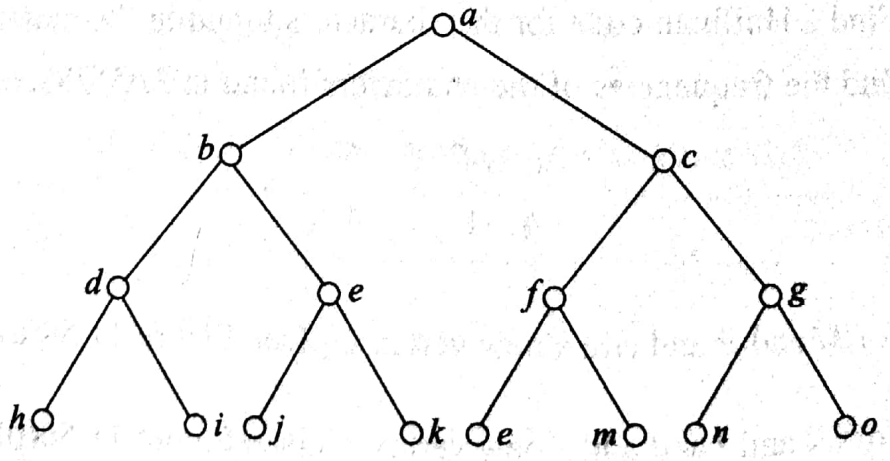


FIGURE 11.57 Rooted tree of Self-test 1–5.

1. T is
   (a) a binary tree (b) a full binary tree
   (c) a ternary tree (d) a full ternary tree.
2. The number of internal vertices of T is
   (a) 6 (b) 7
   (c) 8 (d) 9.
3. The number of vertices of T at level 1 or 2 is
   (a) 2 (b) 4
   (c) 6 (d) 8.
4. The number of vertices each having 3 ancestors is
   (a) 2 (b) 4
   (c) 6 (d) 8.
5. The inorder traversal of the subtree with root at c is
   (a) $l f m c n g o$ (b) $f l m c n g o$
   (c) $l f m c n o g$ (d) $l m f c n g o$.

Choose the correct answers to Questions 6–17.

6. If $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ are two trees, $|E_1| = 17$ and $|V_2| = 2|V_1|$, then
   (a) T_1 is a tree with 16 vertices (b) T_2 is a tree with 35 vertices
   (c) T_2 is a tree with 35 edges (d) T_2 is a tree with 34 edges.
7. If $G = (V, E)$ is a forest of seven trees and $|E| = 40$, then $|V|$ is
   (a) 33 (b) 47
   (c) 39 (d) 41.
8. A rooted tree T is an m-ary tree if
   (a) Every vertex of T has at most m children
   (b) Every vertex of T has exactly m children
   (c) Every internal vertex of T has at most m children
   (d) Every internal vertex of T has exactly m children.
9. The number of rooted trees with three vertices is
   (a) 2 (b) 3
   (c) 4 (d) 5.
10. The number of binary trees with 5 vertices is
    (a) 2 (b) 3
    (c) 4 (d) 5.
11. The number of rooted binary trees with 5 vertices is
    (a) $C(10, 5)$ (b) $C(10, 5)/6$
    (c) $C(10, 5)/5$ (d) $C(10, 5)/10$.
12. For the algebraic expression $a + b * (c/d)$ the polish and the reverse polish notations are
    (a) $a + bcd/*$ (b) $a * bcd/+$
    (c) $abcd/* +$ and $+ a * b/cd$ (d) $+ a * b/cd$ and $abcd/ * +$.
13. The number of spanning trees of a connected graph with n vertices and n edges is
    (a) n (b) $n - 1$
    (c) $n + 1$ (d) None of these.

14. The number of chords of a spanning tree T of a connected graph G with n vertices and e edges is
(a) $e-1$ (b) $n-1$
(c) $n-e+1$ (d) $e-n+1$.

15. The number of spanning trees and minimal spanning trees of the complete graph with 4 vertices with distinct weights is
(a) 16 and 16 (b) 16 and 1
(c) $C(6, 3)$ and 1 (d) $C(6, 3)$ and $C(6, 3)$.

16. The chromatic number of a tree with n vertices is
(a) n (b) $n-1$
(c) 3 (d) 2.

17. A tree with 3 or more vertices remains a tree when
(a) Any vertex is removed. (b) Any edge is removed.
(c) A pendant vertex is removed (d) None of these.

State whether the statements 18–30 are true or false.

18. If $T=(V, E)$ is a tree then the removal of any edge from T disconnects T.
19. A Hamiltonian path in a connected graph is a spanning tree.
20. A pendant vertex in a connected graph is contained in every spanning tree.
21. An open Euler line in a connected graph is a spanning tree.
22. A minimal spanning tree of a connected graph need not be unique.
23. If T is a minimal spanning tree of a connected graph G then the path from u to v in T is the shortest path from u to v in G.
24. While applying DFS algorithm, the vertices at the bottom of the graph are visited first.
25. While applying BFS algorithm beginning with vertex v_i, all the nodes adjacent to v_i are visited in the order from left to right.
26. {0, 1, 01, 001} is a prefix code.
27. A full binary tree with n vertices has $(n+1)/2$ leaves.
28. There exist binary trees for which preorder and postorder coincide.
29. There exist binary trees for which preorder and inorder coincide.
30. There exist binary trees for which postorder and inorder coincide.

Answers to Self-test

1. (b)	2. (b)	3. (c)	4. (d)	5. (a)	6. (c)	7. (b)	8. (c)	9. (b)	10. (d)
11. (b)	12. (d)	13. (d)	14. (d)	15. (b)	16. (d)	17. (c)	18. T	19. T	20. T
21. F	22. T	23. F	24. F	25. T	26. F	27. T	28. F	29. T	30. T

For Question 23, consider