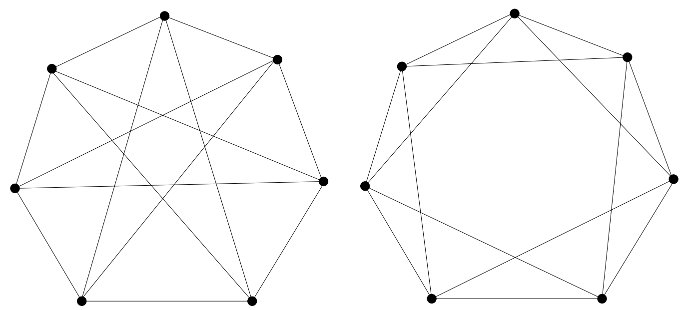
National Institute of Technology Karnataka, Surathkal
Department of Mathematical and Computational Sciences
Odd Semester (2019 – 2020)
Course Code: MA602 Course Title: Discrete Mathematical Structures
Course Instructor: Dr. Srinivasa Rao Kola

Problem Sheet – 4

1. Prove that a connected graph G is an Eulerian graph if and only if it can be decomposed into cycles.
2. Whether the following graphs are isomorphic?



3. Prove that a simple graph with n vertices must be connected if it has more than $[(n-1)(n-2)]/2$ edges.
4. If the intersection of two paths is a disconnected graph, show that the union of two paths has at least one cycle.
5. Draw a graph having cycles that has a Hamiltonian path but does not have a Hamiltonian cycle?
6. Prove that a graph G with n vertices always has a Hamiltonian path if the sum of the degrees of every pair of vertices v_i, v_j in G satisfies the condition $d(v_i) + d(v_j) \geq n - 1$.
7. Prove that any subgraph H of a connected graph G is contained in some spanning tree of G if and only if H contains no cycle.
8. Can you construct a graph if you are given all its spanning trees? How?
9. Suppose that you are given a set of n positive integers. State some necessary conditions of this set so that the set can be the degrees of all the n vertices of a tree. Are these conditions sufficient also?
10. Let v be a vertex in a connected graph G. Prove that there exists a spanning tree T in G such that the distance of every vertex from v is the same both in G and in T?
11. In a given connected weighted graph G, suppose there exists an edge e_s whose weight is smaller than that of any other in G. Prove that every minimum weighted spanning

tree in G must contain e_s?

12. In a connected graph, a cut-set is a set of edges whose removal from G leaves G disconnected, provided removal of no proper subset of these edges disconnects G. Then prove the following.
    (a) Every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.
    (b) In a connected graph G, any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set.
13. Prove that the maximum vertex connectivity one can achieve with a graph G of n vertices and e edges ($e \geq n-1$) is the integral part of the number $\frac{2e}{n}$; that is $\lfloor \frac{2e}{n} \rfloor$.
14. Prove that the maximum flow possible between two vertices u and v in a network is equal to the minimum of the capacities of all cut-sets with respect to u and v.
15. Prove that every connected graph with three or more vertices has at least two vertices which are not cut-vertices.
16. Show that a graph with n vertices and with vertex connectivity k must have at least $kn/2$ edges.
17. Construct a graph G with the following properties: Edge connectivity of $G = 4$, vertex connectivity of $G = 3$, and degree of every vertex of $G \geq 5$?
18. Suppose that a singles tennis tournament is to be arranged among n players and the number of matches planned is a fixed number e (where $n - 1 < e < n(n-1)/2$). For the sake of fairness, how will you make sure that some players do not group together and isolate an individual (or a small group of players)?
19. Let us define a new term called edge isomorphism as follows: Two graphs G_1 and G_2 are edge isomorphic if there is a one-one correspondence between the edges of G_1 and G_2 such that two edges are incident (at a common vertex) in G_1 if and only if the corresponding edges are also incident in G_2. Discuss the properties of edge isomorphism. Construct an example to prove that edge-isomorphic graphs may not be isomorphic?
20. Prove that an Eulerian graph cannot have a cut-vertex with an odd number of edges.