National Institute of Technology Karnataka, Surathkal
Department of Mathematical and Computational Sciences
Discrete Mathematical Structures (MA602)
Odd Semester (2019 - 2020)
Problem Sheet 2

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1. Prove or disprove: Dijkstra's algorithm for shortest path always gives a unique shortest path between any two vertices of a graph G, if and only if it is a tree.
2. Prove that C_5 is the only self complementary cycle.
3. True or False : Every graph with fewer edges than vertices has a component that is a tree.
4. Prove that petersen graph has no cycle of length 7.
5. Let G be a graph which is isomorphic to its complement G. Prove that G must have $4k$ or $4k+1$ vertices for some integer k.
6. A connected graph with n vertices has exactly one cycle if and only if it has exactly n edges.
7. Prove or disprove: a planar graph has a cut vertex if and only if its dual has a cut vertex.
8. True/ False?
   (a) Every subgraph of a planar graph is planar
   (b) Every subgraph of a nonplanar graph is nonplanar
   (c) Every simple planar graph has a vertex of degree at most 5.
9. Prove that in a connected k-regular graph G with n vertices if $k \geq \frac{n}{2}$ then eccentricites of all vertices are same.
10. Prove or disprove: If G is a connected graph with all edges having all distinct weight then the minimum weighted spanning tree is unique.
11. Prove that a forest of k trees which have a total of n vertices has $n-k$ edges.
12. Prove that every connected planar graph with less than 12 vertices has a vertex degree at most 4.
13. Prove that a tree with maximum degree $n > 1$ has at least n vertices of degree 1.
14. Below is a network with the arcs labeled with their lengths. Using Dijkstra's algorithm find shortest path from source node 0 to all nodes.