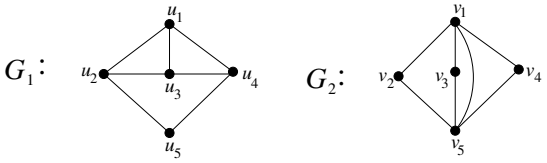
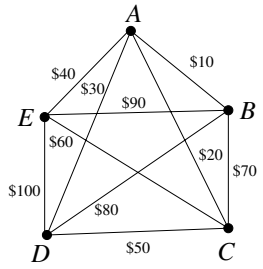
National Institute of Technology Karnataka, Surathkal
Department of Mathematical and Computational Sciences
Odd Semester (2019 – 2020)
Course Code: MA602 Course Title: Discrete Mathematical Structures
Course Instructor: Dr. Srinivasa Rao Kola

Problem Sheet – 3

1. Show that a simple graph G cannot exist with vertices of degrees 1, 3, 3, and 3.
2. Give an example of a graph having exactly one odd vertex?
3. A simple graph of order $p(\geq 2)$ is called perfect if no two of its vertices have equal degrees. Prove that "no graph is perfect". If we replace "(≥ 2)" by "(≥ 1)," does the conclusion change?
4. Suppose you and your husband attended a party with three other married couples. Several handshakes took place. No one shook hands with himself (or herself) or with his (or her) spouse, and no one shook hands with the same person more than once. After all the handshaking was completed, suppose you asked each person, including your husband, how many hands he or she had shaken. Each person give a different answer.
   (a) How many hands did you shake?
   (b) How many hands did your husband shake?
5. Let G be a graph of order $(p \geq 2)$, and suppose that for every vertex v of G, $deg(v) \geq (p-1)/2$. Prove that G is connected?
6. Let G be a connected graph containing only even vertices. Prove that G cannot contain a bridge.
7. Whether the graphs below are Hamiltonian? Justify.



8. True or false? Every Eulerian graph is Hamiltonian. Explain.
9. True or false? Every Hamiltonian graph is Eulerian. Explain.
10. Consider the Traveling Salesman Problem for the network shown below. Starting at each of the five cities, determine the cost of Hamiltonian cycle obtained by always proceeding to the next city using the cheapest possible route. Show that in no case will this produce a Hamiltonian cycle of least cost?



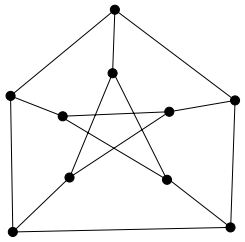
11. Let G be a forest of order p and size q having k components. Obtain an expression for q in terms of p and k?

12. Let G be a regular graph of even order $P \geq 4$. Prove that at least one of G and $\overline{G}$ is Hamiltonian?

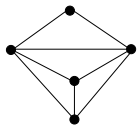
13. Prove that Q_n is Hamiltonian for $n \geq 2$?

14. True or false? If G is a connected (p, q) graph and $n = 3p - 6$, then G is planar. Explain.

15. The figure below shows a famous graph called the Petersen graph. Is the Petersen graph planar? Justify.



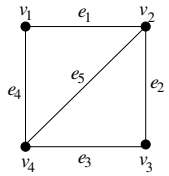
16. Prove that $\chi(G) = 4$ for the graph G below.

G: 

17. Prove that every planar graph G contains a vertex v such that $deg(v) \leq 5$.

18. Prove that if G is a planar graph, then $\chi(G) \leq 5$.

19. Determine the adjacency matrix A of the graph G below. Without performing any matrix multiplication, determine A^2 and A^3?



20. If A is the adjacency matrix of a graph G and $A^2 = [a_{ij}^{(2)}]$, describe $a_{ij}^{(2)}$, $i \neq j$, and $a_{ii}^{(2)}$ without using the word "walk"?