1. How many edges are there in an undirected graph with 10 nodes (vertices) each of degree six?

2. Show that the following graphs are isomorphic.
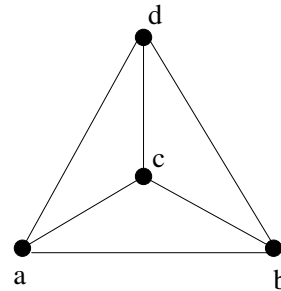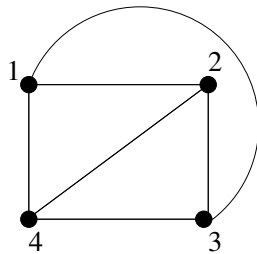
3. Show that the following graphs are not isomorphic.
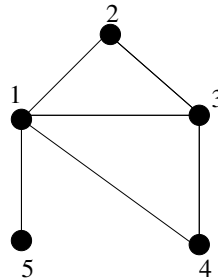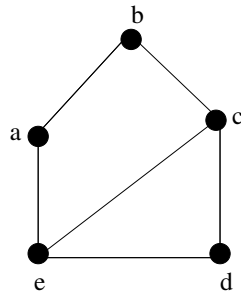
4. How many nonisomorphic simple graphs are there with 3 vertices?

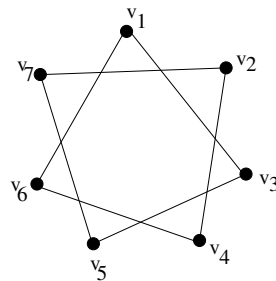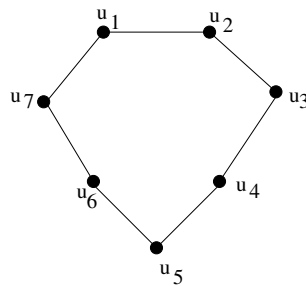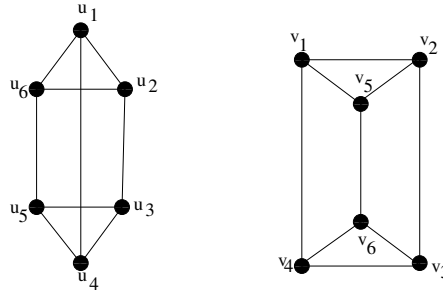5. Determine whether the given pair of graphs are isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.
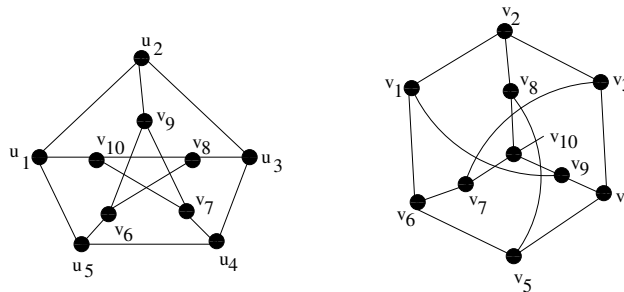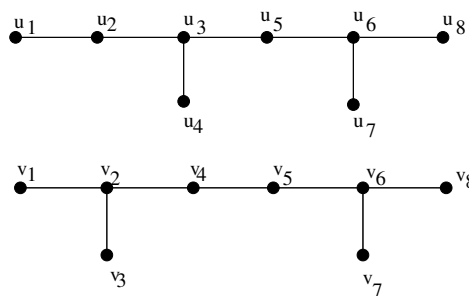
(a)

(b)

(c)

(d)

6. Draw an undirected graph represented by the following adjacency matrix.

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix}$$

7. Find the number of walks of length 3 between two different vertices in K_4.

8. Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a cycle.

9. Determine whether the graph below decomposes into copies of P_4.
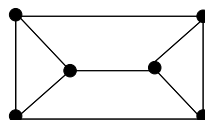
10. Determine whether the statements below are true or false.

    (a) Every disconnected graph has an isolated vertex.

    (b) A graph is connected if and only if some vertex is connected to all other vertices.

    (c) The edge set of every closed trail can be partitioned into edge sets of cycles.

(d) If a maximum trail in a graph is not closed, then its endpoints have odd degree.

11. Prove that a bipartite graph has a unique bipartition (except for interchanging the two bipartite sets) if and only if it is connected.

12. Prove or disprove:

    (a) Every Eulerian bipartite graph has an even number of edges.

    (b) Every Eulerian simple graph with an even number of vertices has an even number of edges.

13. Prove or disprove: If G is an Eulerian graph with edges e, f that share a vertex, then G has an Eulerian circuit in which e, f appear consecutively.

14. Prove that the minimum number of edges in a connected graph with n vertices is $n - 1$.

15. Prove or disprove: If u and v are the only vertices of odd degree in a graph G, then G contains uv-path.

16. In the class of nine students, each student sends valentine cards to three others. Determine whether it is possible that each student recieves cards from the same three students to whom he or she sent cards.

17. Prove that every simple graph with at least two vertices has two vertices of equal degree. Is the conclusion true for loopless graphs?