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Assignment -2

Q1. (i) R: $S \rightarrow 0S11 \mid \varepsilon$ $\xrightarrow[\text{form}]{\text{C.N.F.}}$ R: (Set of rules)

$S \rightarrow OU \mid \varepsilon$
$P \rightarrow NN$
$T \rightarrow NP$
$U \rightarrow ST$
$O \rightarrow 0$
$N \rightarrow 1$

$G \equiv (\{S, P, T, U, O, N\}, \{0, 1\}, R, S)$

$G \equiv (V, \Sigma, R, S)$

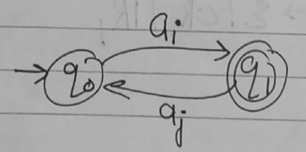
$V \equiv \{S\}$, $\Sigma = \{0, 1\}$,
S = start variable

(ii) R:

$A \rightarrow \{A\} \mid (A) \mid [A] \mid \varepsilon$

$G = (V, \Sigma, R, S)$
$V \equiv \{A\}$, $\Sigma = \{ `\{', `\}', `(', `)', `[', `]' \}$, $S \equiv A$.

Q2. (i) DFA:



, $\forall a_i, a_j \in \Sigma$ [a_i, a_j => i/p symbols of alphabet, ~~can be the~~ may or may not be the same]

CFG:

$R_0 \rightarrow a_i R_1$, $\forall a_i \in \Sigma$. [for all input symbols, there will be a rule of the form $R_0 \rightarrow a_i R_1$]

$R_1 \rightarrow a_j R_0 \mid \varepsilon$, $\forall a_j \in \Sigma$

(ii) $L(G) = L_1 \cup L_2$, $L_1 = \{a^i b^i c^k, i, k \geq 0, i \neq k\}$,
$L_2 = \{a^i b^j c^i, i, j \geq 0, i \neq j\}$

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R:

$S \rightarrow XY | W$

$X \rightarrow aXb | \varepsilon$

$Y \rightarrow cY | \varepsilon$

$W \rightarrow aWc | Z$

$Z \rightarrow bZ | \varepsilon$

CFG: $\equiv (V, \Sigma, R, S)$

$V = \{S, X, Y, W, Z\}$

$\Sigma = \{a, b, c\}$, S: start variable

Q3. (i) R:

$R_0 \rightarrow \varepsilon | 0R_1 | 1R_1$

$R_1 \rightarrow 0R_2 | 1R_2$

$R_2 \rightarrow 0R_3 | 1R_3$

$R_3 \rightarrow \varepsilon | 0R_1 | 1R_1$

(Converted from DFA)

CFG $\equiv (V, \Sigma, R, R_0)$

$V \equiv \{R_0, R_1, R_2, R_3\}$

$\Sigma \equiv \{0, 1\}$

(ii) R:

$A \rightarrow$ formal B methods | methods B formal | $a_i A$

$B \rightarrow \varepsilon | a_i B$, $\forall\ a_i \in \{a, b, c \dots z\}$

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Q4. (i) R: $S \rightarrow \varepsilon \mid aSb \mid aSbb$

$CFG = (V, \Sigma, R, S)$, $V = \{S\}$, $\Sigma = \{a, b\}$

(ii) R: $S \rightarrow aSc \mid \varepsilon$, $CFG = (V, \Sigma, R, S)$,
$S \rightarrow S_1$, $V = \{S, S_1\}$
$S_1 \rightarrow bS_1c \mid \varepsilon$, $\Sigma = \{a, b, c\}$

Q5. (i) R: $S \rightarrow aSa \mid bSb \mid a \mid b$

$CFG = (V, \Sigma, R, S)$, $V = \{S\}$, $\Sigma = \{a, b\}$

(ii) $L(G)$

$k <= i$ $\downarrow$ $k <= j$ $\downarrow$

~~$S_1 \rightarrow aS_1S_1'c \mid aS_1S_1'$~~ $S_1 \rightarrow aS_1c \mid aS_1 \mid S_1'$
~~$S_1' \rightarrow b \mid \varepsilon$~~ $S_1' \rightarrow bS_1' \mid \varepsilon$

$S_2 \rightarrow bS_2c \mid bS_2 \mid S_2'$
$S_2' \rightarrow aS_2' \mid \varepsilon$

taking union, we get:
R: $S \rightarrow S_1 \mid S_2$
$S_1 \rightarrow aS_1c \mid aS_1 \mid S_1'$
$S_1' \rightarrow bS_1' \mid \varepsilon$
$S_2 \rightarrow bS_2c \mid bS_2 \mid S_2'$
$S_2' \rightarrow aS_2' \mid \varepsilon$,

$V = \{S, S_1, S_1', S_2, S_2'\}$,
$\Sigma = \{a, b\}$

Q6. $S \rightarrow AT_1 \mid AB \mid BT_2 \mid BA$ new variables: T_1, T_2, A, B
$S_1 \rightarrow AT_1 \mid AB$
$S_2 \rightarrow BT_2 \mid BA$
$T_1 \rightarrow S_1B$
$T_2 \rightarrow S_2A$
$A \rightarrow 0$
$B \rightarrow 1$

Q7.

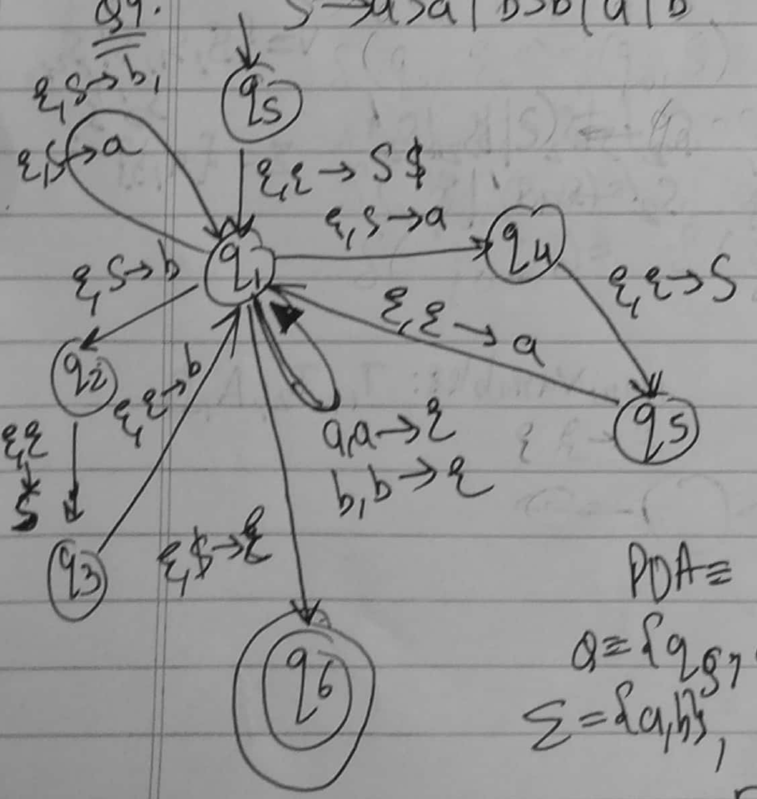
$S_0 \to AE \mid SB \mid AS$
$S \to AE \mid SB \mid AS$
$A \to TY \mid a \mid TS$
$B \to SC \mid XX \mid TY \mid a \mid TS$
$C \to XS$
$E \to SB$
$T \to a$
$X \to b$
$Y \to AS$

Q8.

$S \to WT \mid AT \mid AB \mid WB$
$X \to AX \mid a$
$Y \to BY \mid b$
$A \to a$
$B \to b$
$T \to BY$
$W \to AX$

Q9. $S \to aSa \mid bSb \mid a \mid b$

stack string: $S_1 S_2 \cdots S_k$
top ⟵ bottom



$\delta(q_1, a, a) = q_1, \varepsilon$
$\delta(q_1, b, b) = q_1, \varepsilon$

$\delta(q_1, \varepsilon, S) = q_1, b$
$\delta(q_1, \varepsilon, S) = q_1, a$

$PDA \equiv (Q, \Sigma, \Gamma, \delta, q_s, F)$
$Q \equiv \{q_s, q_1, q_2, q_3, q_4, q_5, q_6\}$
$\Sigma = \{a, b\}$, $\Gamma = \{\$, a, b\}$,
$F \equiv \{q_6\}$

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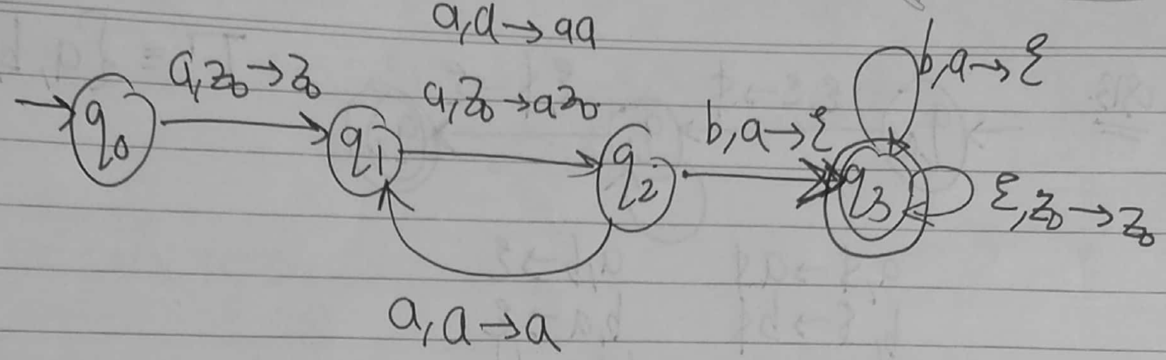
Q10.

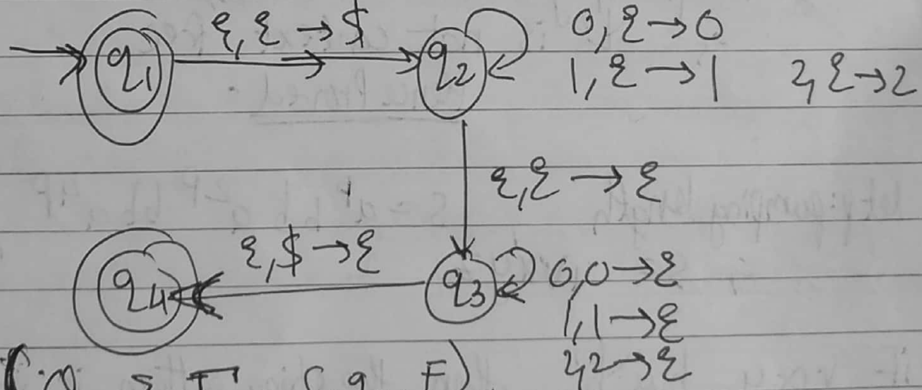


$PDA = \{Q, \Sigma, \Gamma, \delta, q_0, F\}$

$Q \equiv \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, Z_0\}$,

δ: as shown in figure, $F = \{q_3\}$

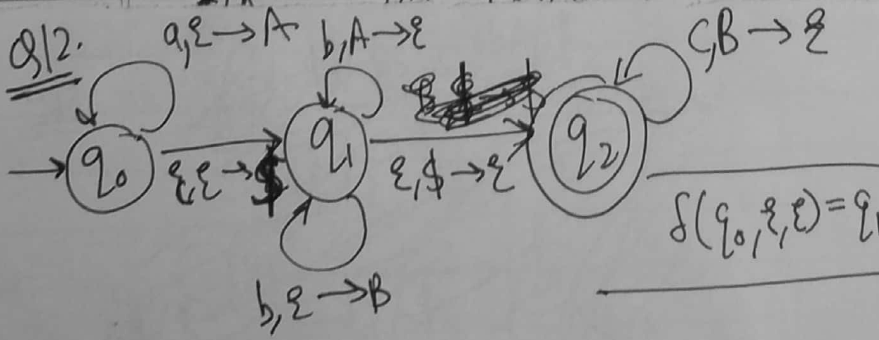
Q11. $L = \{w^r w \mid w \in \{a,b,c\}^*\} \equiv \{w^r (w^r)^r \mid w^r \in \{a,b,c\}^*\}$
$\equiv \{z z^r \mid z \in \{a,b,c\}^*\}$



$PDA \equiv (Q, \Sigma, \Gamma, \delta, q_0, F)$,

$Q \equiv \{q_1, q_2, q_3, q_4\}$, $\Sigma \equiv \{a, b, c\}$, $\Gamma = \{$

Q12.

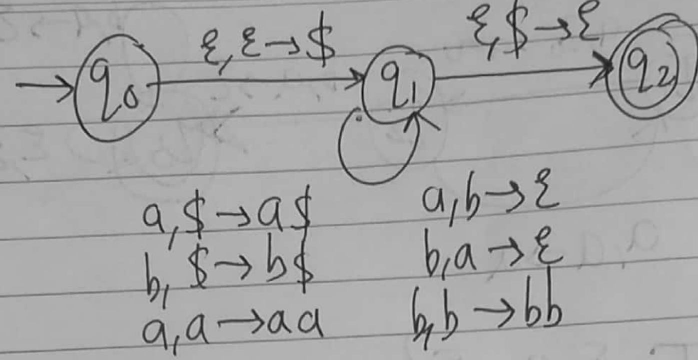


$\boxed{\delta(q_0, \varepsilon, \varepsilon) = q_1, \$}$

$Q \equiv \{q_0, q_1, q_2\}$
$\Gamma \equiv \{A, B, \$\}$
$F \equiv \{q_2\}$,

$T = \{a, b, \$\}$

Q13.



Q14. (i) $L = \{a^n b^n a^n b^n \mid n \geq 0\}$, let p = pumping length.
$s = a^p b^p a^p b^p$, let $s = uvxyz$ (following the pumping lemma).
If either 'v' or 'y' contain more than one type of alphabet symbol, uv^2xy^2z does not contain the symbols in correct order, hence can't be a member of A.

If both v & y contain atmost 1 type of alphabet symbol, uv^2xy^2z contains a's & b's of unequal length. Hence uv^2xy^2z is also not a member of A.

We have shown that 's' cannot be pumped without violating the pumping lemma conditions,
∴ 'L' is not context free

Hence Proved.

(ii) let p: pumping length, $s = a^p bb\, a^{2p}\, bb\, a^{4p}$, $|s| \geq p$,
∴ $s = uvxyz$

if vxy has 'b', then the string pattern is violated in some way, after we pump it.

If 'b' is absent in them, then there is an incorrect number of a's in the pumped up string, in violation of either the given no. of the given order of the generalised string.

i.e. if $\underbrace{uvxy}_{a^p}\underbrace{z}_{bba^{2p}bba^{4p}}$, then uv^2xy^2z definitely has the 1st block of a's with length definitely $> p$.

~~Q13~~ Q14. (iii) Consider $S = 0^k 1^k 1^k 0^k 0^k 1^k$, $w = 0^k 1^k$, ; $w^R = 1^k 0^k$
$= 0^k 1^{2k} 0^{2k} 1^k$

$S = uvxyz$

Case 1: v and/or y contain 0's from 0^k starting block,
consider $uv^0xy^0z = uxz = 0^{i_1} 1^{i_2} 0^{2k} 1^k$, where $i_1 < k$ & $i_2 \leq 2k$.
~~if~~ Suppose ~~we~~ $uxz \in L$, then $uxz = 0^{i_1} 1^{i_2} 0^{2k} 1^k$ must be of the form $\alpha\alpha^R\alpha$, & the 1st α must begin with a block of i_1 ($i_1 < k$) 0's followed by some 1's. Thus $\alpha^R\alpha$ must contain a block of atmost $2i_1$ (which is $< 2k$ 0's).
But uxz contains a block of 2k 0's, hence a contradiction is observed, ∴, $uxz \notin L$.

Case 2: v and/or y contain some symbols from 1st block of 2k ~~k~~ 1's ; $|vxy| \leq k$, v & y can't contain any 1's from 2nd block of k 1's. [1^k]: ~~a~~ let us assume $S = uv^0xy^0z = uxz \in L$.
$uxz = 0^{i_1} 1^{i_2} 0^{i_3} 1^k$, $i_1 \leq k$, $i_2 < 2k$, $i_3 \leq 2k$. ; $s \in L$,
~~s~~ s has the form: $\alpha\alpha^R\alpha$, with the latter α ending with a block of k 1's preceded by some 0's.
Thus $\alpha\alpha^R$ must contain a block of 2k 1's.
But uxz contains a block of $i_2 < 2k$ 1's, which is a contradiction.
; $s \notin L$.

Case 3: v and/or y contains some symbols from 2nd block of 2k 0's
; $|vxy| \leq k$, v & y can't contain any 0's from 1st block of k 0's.
Consider $uv^0xy^0z = uxz = 0^k 1^{i_1} 0^{i_2} 1^{i_3}$, $i_1 \leq 2k$, $i_2 < 2k$, $i_3 \leq k$.
Assume $uxz \in L$, ∴, it is of the form $\alpha\alpha^R\alpha$. Hence the 1st α must begin with the block of k 0's followed by some 1's. Thus, $\alpha^R\alpha$ must contain a block of 2k 0's. But uxz contains a block of $i_2 < 2k$ 0's, which is a contradiction. ; ~~uz~~ $uv^0xy^0z \notin L$.

Case 4: v and/or y contains some symbols from the 2^{nd} block of k 1's.
$\because |vxy| \le k$, v, y can't contain any 1's from 1^{st} block of 2k 1's.
Consider $u\cancel{x}^0 x y^0 z = uxz = 0^k 1^{2k} 0^{i_1} 1^{i_2}$, $i_1 \le 2k$, $i_2 < k$.
Assume $uxz \in L$, $\therefore$ it must be of the form $\alpha\alpha^R\alpha$. Hence, the 2^{nd} α must end with the block of i_2 $(i_2 < k)$ 1's preceeded by some 0's.
Thus, $\alpha\alpha^R$ must contain a block of atmost ~~2i~~ $2i_2$ $(<2k)$ 1's.
But uxz contains a block of 2k 1's (because it belongs to L). Hence, a contradiction is observed. $\therefore$ $uxz \notin L$.

In all the cases, the pumping lemma is violated, language L is not context free.

Hence Proved

Q15. $L = \{a^i b^{2i} a \mid i \ge 0\}$, let pumping length be 'p'.
$S = a^p b^{2p} a$, $\therefore$ $|s| \ge p$, it can be split into $s = uvxyz$.

(i): Let $v = \varepsilon$, $\therefore$ $y \ne \varepsilon$. $S = uxyz$, This condition can have 1 sub-cases, in which y has only a's. i.e. $xy = a^p$ $[\because |xy| \le p]$.
$\therefore$ if we pump S with y^i, $S' = u a^j (a^{p-j})^i b^{2p} a$, and clearly $2p \ne 2 \times [j + pi - pj]$; $\therefore$ $s' \notin L$. It cannot happen that y contains some a's & some b's, because then $|xy| > p$ which will violate the pumping lemma as well.

Similar case occurs when $y = \varepsilon$ & $v \ne \varepsilon$, on checking $s = uv^i xz$, the condition that b's should be twice as many a's is violated.
if v contains a's & b's, then either $|vxy| \le p$ is violated, or on pumping v, the order is disrupted, i.e. new strings are of the form aa... b---- a-... b-.... a.

for both being non ~~not~~ empty, either the order of a's & b's is violated on pumping, or the no. of a's & b's are not in accordance with the rule $a^p b^{2p}$, or that $|vxy| \le p$ is violated.

Q16. $L = \{a^n, n \text{ is prime}\}$, let $s = a^p$, p: pumping length,
now we can split 's' into, $s = uvxyz$,
S.t.:
$v = a^q, y = a^t$, let $r = |uxz| = p - q - t$
$(q + t > 0)$

then $|uv^r xy^r z| = r + rq + rt = r(1 + q + t)$, this is the length of string which should have been present in L, according to the pumping lemma. But for all such strings in L, the length was supposed to prime number. and here it comes out to be composite.

$\therefore uv^r xy^r z \notin L$, $\therefore$ pumping lemma is violated.

let $r = 0 \Rightarrow$ length of string $= 2p$ = composite, $\therefore uxz \notin L$
let $r = p+1 \Rightarrow$ length of string $= p^2$ = composite, $\therefore uv^{p+1}xy^{p+1}z \notin L$

$\therefore$ pumping lemma is violated, hence this is not a context free language.
Hence Proved

Q17. $L = \{1^{n^2}, n > 0\}$, let $s = 1^{p^2}$, $s = uvxyz$, such that $|uvxyz| = p^2$
after pumping by i, $s' = uv^i xy^i z$, $|s'| = p^2 + (i-1)t \leq p^2 + (i-1)p$
[we assume $|vy| > 0$ & $|vxy| = t \leq p$
in accordance with the pumping lemma]
let $i = 2 \Rightarrow p^2 < |uv^2xy^2z| \leq p^2 + p$, but the next possible length is $(p+1)^2$, & if we consider $(p+1)^2 = p^2 + p$, then $p \neq$ natural no.

We can use this to prove, for any i, if $|uv^i xy^i z| = k^2$, then for $|uv^{i+1}xy^{i+1}z|$ to be a perfect square [i.e. $(k+1)^2$, because it is the next possible square of natural no.], $|v| + |y| = 2k + 1$, here lengths of v & y depend upon 'k', which should not be the case.

Hence the language is not context free. Hence Proved

Q18. let $L_1 = \{a^n b^n a^m \mid n, m \geq 0\}$, $\Rightarrow S \to XA$

$X \to aXb \mid \varepsilon$

$A \to Aa / \varepsilon$

$L_2 = \{a^n b^m a^m \mid n, m \geq 0\}$, $\Rightarrow S \to AX$

$X \to aXb \mid \varepsilon$

$A \to Aa / \varepsilon$

$\therefore L_1 \cap L_2 = \{a^n b^n a^n \mid n \geq 0\}$, $\equiv$ not context free because

let $s = a^p b^p a^p$, p: pumping length, $\therefore |s| \geq p$, $s = uvxyz$,

here, by pumping up or down, either ~~either~~ the form $a^p b^p a^p$ is violated ~~by~~ due to unequal no. of a's & b's or the change in their ordering

i.e. a... b... a... b... a, in such fashion.

If this does not occur, then definitely $|vxy| > p$, which is again a violation of pumping lemma.

The former violation occurs when both v and/or y contain only 1 kind of symbol. The latter violation occurs when either of them (v ~~&~~ or y) contains ~~the same~~ both symbols

$\therefore L_1 \cap L_2$ is not context free.

Hence Proved

Q19. ① Union: Let L_1 be generated by $G_1 = (V_1, T_1, P_1, S_1)$ & L_2 be generated by (V_2, T_2, P_2, S_2)

let S_1 be a nonterminal of G_1 (subscripted by 1), S_2 be that of G_2 (subscripted by 2)

$V_1 \cap V_2 = \phi$.

G generates $L_1 \cup L_2$, such that $G = (V_1 \cup V_2 \cup \{S\}, T_1 \cup T_2, P_1 \cup P_2 \cup \{S \to S_1 | S_2\}, S)$

each rule starts with $S \to S_1 | S_2$

subsequent rules are either entirely from G_1 or G_2.

~~each word is thus either in L_1 or L_2~~ ~~generated~~ each word thus generated is either in L_1 or L_2.

$\therefore$ if L_1, L_2 are 2 CFLs then $L_1 \cup L_2$ is also a CFL.

Hence Proved

(2) Concatenation: let L_1 & L_2 follow the previous definitions.

CFG that generates L_1L_2 is: $G = (V_1 \cup V_2 \cup \{S\}, T_1 \cup T_2, P_1 \cup P_2 \cup \{S \to S_1S_2\}, S)$

this CFG generates words which can be divided into 2 parts, with the 1st part existing in L_1 & the 2nd part in L_2.

eg. L_1: $S \to aSa \mid bSb \mid a \mid b \mid \varepsilon$ L_2: $\{a^nb^n \mid n \geq 0\} \to S \to aSb \mid \varepsilon$

$L_1 \cdot L_2$: $S \to S_1S_2$

$S_1 \to aS_1a \mid bS_1b \mid a \mid b \mid \varepsilon$

$S_2 \to aS_2b \mid \varepsilon$

Q20. Assumption: complement of a CFL is also a CFL.

~~$\therefore L_1, \overline{L_1}$ & $L_2, \overline{L_2}$ are~~ if L_1, L_2 are CFL, so are $\overline{L_1}, \overline{L_2}$

$\therefore \overline{L_1} \cup \overline{L_2}$ is a CFL (closure under Union), $\therefore \overline{\overline{L_1} \cup \overline{L_2}}$ is also a CFL (according to our assumption)

$\therefore$ but $\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2$, and

$L_1 \cap L_2$ was proven to be non-context free.

Hence, a contradiction is found. $\therefore$ our assumption is wrong.

Q21. if L_1 is a CFL, then L_1^* is also a CFL.

let L_1 be generated by $G = (V_1, T_1, P_1, S_1)$

L^* is generated by $G' = (V_1 \cup \{S\}, T_1, P_1 \cup \{S \to S_1S \mid \varepsilon\}, S)$

each word generated by G' is either ε or some sequence of words in L_1.

each word in L_1^* can be generated by G'

eg. $L_1 = \{a^nb^n, n \geq 0\}$, L_1: $S \to aSb \mid \varepsilon$

L_1^*: $S \to S_1S \mid \varepsilon$

$S_1 \to aS_1b \mid \varepsilon$

Q22. It is a non-context free grammar, because of the ~~term~~ missing term $a^{100}b^{100}$.

Consider $s = a^{101}b^{101} = \underset{u}{a^{100}}\,\underset{v}{a}\,\underset{x}{\epsilon}\,\underset{y}{b}\,\underset{z}{b^{100}}$; $uv^ixy^iz \equiv a^{100}.(a)^i(b)^i(b)^{100}$

$= (a)^{100+i}(b)^{100+i}$

$\approx a^N b^N$.

each string of the form uv^ixy^iz, $\forall i \geq 0$ should be present in this language.

consider the case $i=0$:

$$uxz = a^{100}b^{100}, \text{ but this } \notin L.$$

hence ~~we~~ by pumping down, a violation of pumping lemma was found.

$\therefore$ this Language violates the pumping lemma, and hence is non-context free.

Hence Proved