Forces between Multiple Charges

According to the parallelogram law of addition. Experimentally it is verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. This is termed as the ***principle of superposition*.**

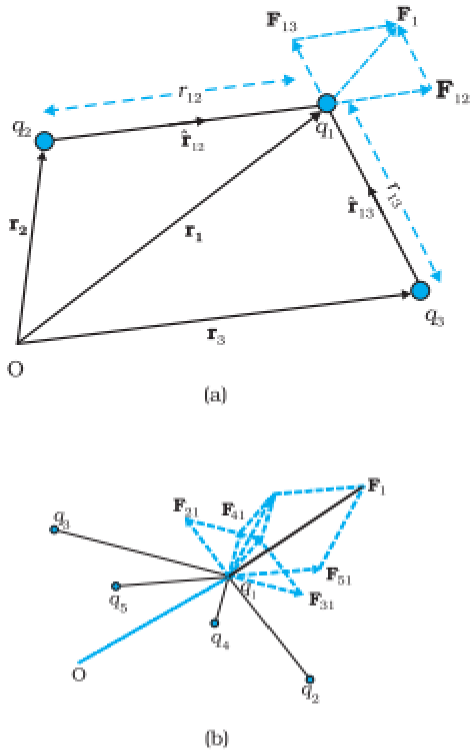

Fig.2 A system of (a) three charges (b) multiple charges.

To better understand the concept, consider a system of three charges q_1, q_2, q_3, as shown in Fig.2. The force on one charge, say $q1$, due to two other charges $q2$, $q3$ can, therefore, be obtained by performing a vector addition of the forces due to each one of these charges.

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

Thus the total F_1 on q_1 due to the two charges q_2 and q_3 is given as

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

The above calculation of force can be generalized to a system of charges more than three, as shown in fig.1(b)

The **principle of superposition** says that in a system of charges $q_1, q_2, \ldots, q_n$, the force on q_1 due to q_2 is the same as given by Coulomb's law, i.e., it is unaffected by the presence of the other charges $q_3, q_4, \ldots, q_n$. The total force $\mathbf{F}_1$ on the charge $q1$, due to all other charges, is then given by the vector sum of the forces $\mathbf{F}_{12}$, $\mathbf{F}_{13}, \ldots, \mathbf{F}_{1n}$:

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i}$$

The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of the electrostatics is basically a consequence of Coulomb's law and the superposition principle.