– Recursive form of $P_n^m(\cos\theta)$ and $\dfrac{\partial P_n^m(\cos\theta)}{\partial\theta}$ based on

Fundamentals of Spacecraft Attitude Determination (Markley + Crassidis), Springer, 2014 p 384

$m=0$ $$P_n^0(\cos\theta) = \frac{1}{n}\left[(2n-1)\cos\theta\, P_{n-1}^0(\cos\theta) - (n-1)P_{n-2}^0(\cos\theta)\right] \qquad (10.93a)$$

$0<m<n$ $$P_n^m(\cos\theta) = \cos\theta\, P_{n-1}^m(\cos\theta) + (n+m-1)\sin\theta\, P_{n-1}^{m-1}(\cos\theta) \qquad (10.93b)$$

$m=n$ $$P_n^n(\cos\theta) = (2n-1)\sin\theta\, P_{n-1}^{n-1}(\cos\theta) \qquad (10.93c)$$

$\rightarrow P_0^0(\cos\theta) = 1$

$\rightarrow P_1^0(\cos\theta) = \cos\theta$

$$P_1^1(\cos\theta) = (1-\cos^2\theta)^{\frac{1}{2}}\frac{d}{d\cos\theta}P_1(\cos\theta)$$

$$P_1(\cos\theta) = \frac{1}{2^1 1!}\frac{d}{d\cos\theta}(\cos^2\theta - 1) = \frac{1}{\cancel{2}}(\cancel{2}\cos\theta) = \cos\theta$$

$\rightarrow P_1^1(\cos\theta) = \sin\theta$

<u>By inspection:</u>

$m=0$ $$\frac{\partial P_n^0(\cos\theta)}{\partial\theta} = \frac{1}{n}\left[(2n-1)\left\{\cos\theta\frac{\partial P_{n-1}^0(\cos\theta)}{\partial\theta} - \sin\theta\, P_{n-1}^0(\cos\theta)\right\} - (n-1)\frac{\partial P_{n-2}^0(\cos\theta)}{\partial\theta}\right] \qquad \text{RJB 1}$$

$0<m<n$ $$\frac{\partial P_n^m(\cos\theta)}{\partial\theta} = \cos\theta\frac{\partial P_{n-1}^m(\cos\theta)}{\partial\theta} - \sin\theta\, P_{n-1}^m(\cos\theta) + (n+m-1)\left[\sin\theta\frac{\partial P_{n-1}^{m-1}(\cos\theta)}{\partial\theta} + \cos\theta\, P_{n-1}^{m-1}\cos\theta\right] \qquad \text{RJB 2}$$

$m=n$ $$\frac{\partial P_n^n(\cos\theta)}{\partial\theta} = (2n-1)\left[\sin\theta\frac{\partial P_{n-1}^{n-1}(\cos\theta)}{\partial\theta} + \cos\theta\, P_{n-1}^{n-1}(\cos\theta)\right] \qquad \text{RJB 3}$$

$$\frac{\partial P_0^0(\cos\theta)}{\partial\theta} = 0$$

$$\frac{\partial P_1^0(\cos\theta)}{\partial\theta} = -\sin\theta$$

$$\frac{\partial P_1^1(\cos\theta)}{\partial\theta} = \cos\theta$$