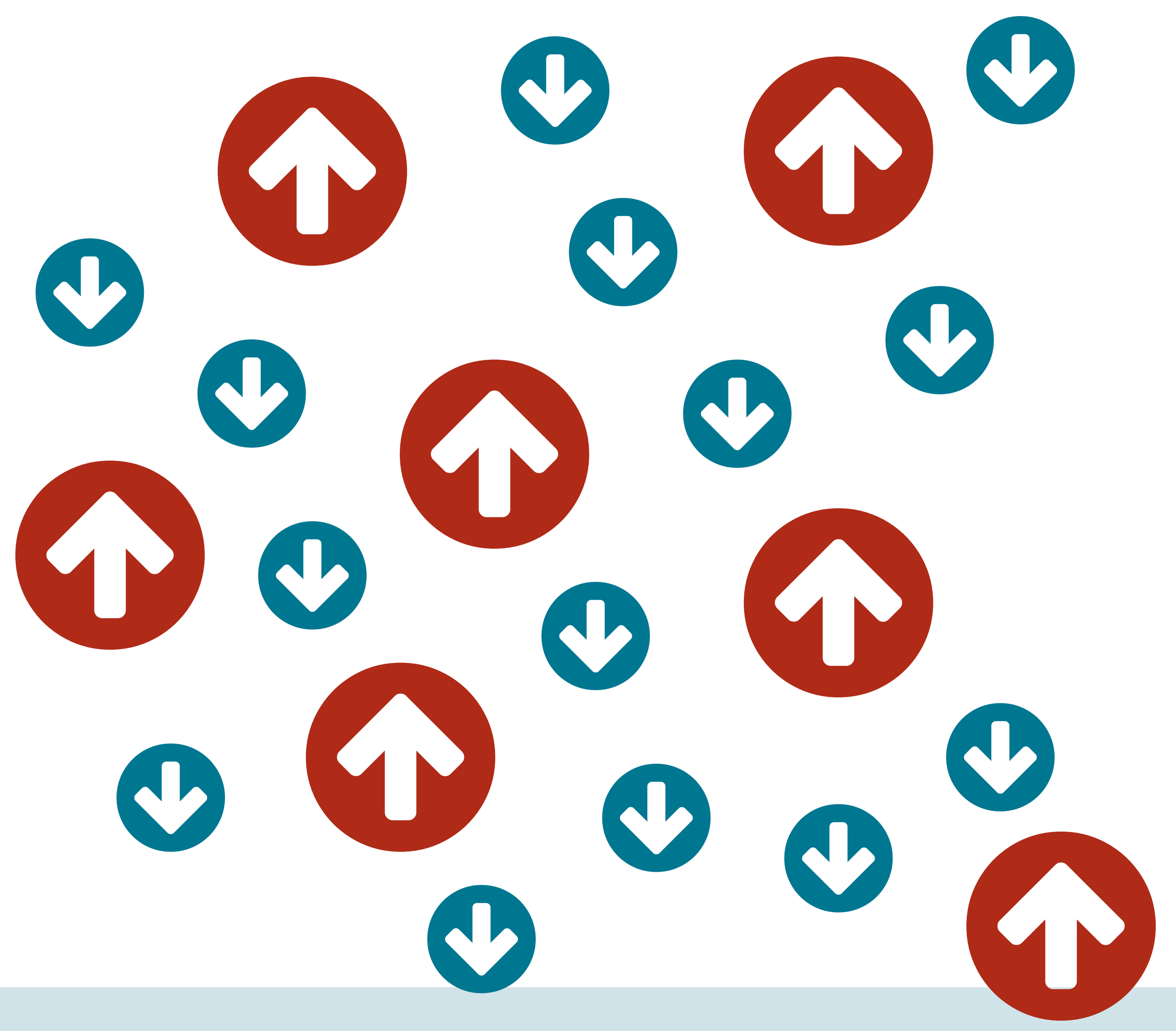


Imbalanced Fermi gases & the complex Langevin approach

Lukas Rammelmüller[1], Joaquín E. Drut[2], Jens Braun[1]

[1]Technische Universität Darmstadt, IKP Theory Center, [2]University of Noth Carolina at Chapel Hill, Department of Physics and Astronomy

Theoretical treatment of imbalanced Fermi systems is challenging. Exact analytic methods, if available, are limited to one-dimensional setups and thus numerical treatment is often the only viable option. Among the most successful methods for balanced Fermi gases, in particular for systems beyond the few-body regime, are Quantum Monte Carlo (QMC) approaches. For imbalanced Fermi systems, however, these approaches suffer from an exponential scaling with system size: the infamous sign-problem. A way to circumvent this issue is provided by the complex Langevin method, originally applied to relativistic field theories. Here, we show recent advances in non-relativistic systems achieved by adapting the complex Langevin method for ultracold fermions. [Aarts '09; Seiler et al. '17; Loheac, Drut '17; LR, Porter, Drut, Braun '17]

complex Langevin - a lattice approach to ultracold fermions

- after discretizing space and imaginary time and performig a Hubbard-Stratonovich transformation we can write the partition sum $\mathcal{Z}$ as a path integral over the auxiliary field ϕ:

$$\mathcal{Z} \;\equiv\; \mathrm{Tr}[e^{-\beta\hat{H}}] \;=\; \cdots \;=\; \int \mathcal{D}\phi \, \det M_\phi^{\uparrow} \, \det M_\phi^{\downarrow} \;\equiv\; \int \mathcal{D}\phi \, e^{-S[\phi]}$$

- similarly, we can compute observables: $\langle\hat{\mathcal{O}}\rangle \;=\; \frac{1}{\mathcal{Z}}\mathrm{Tr}[\hat{\mathcal{O}}e^{-\beta\hat{H}}]$

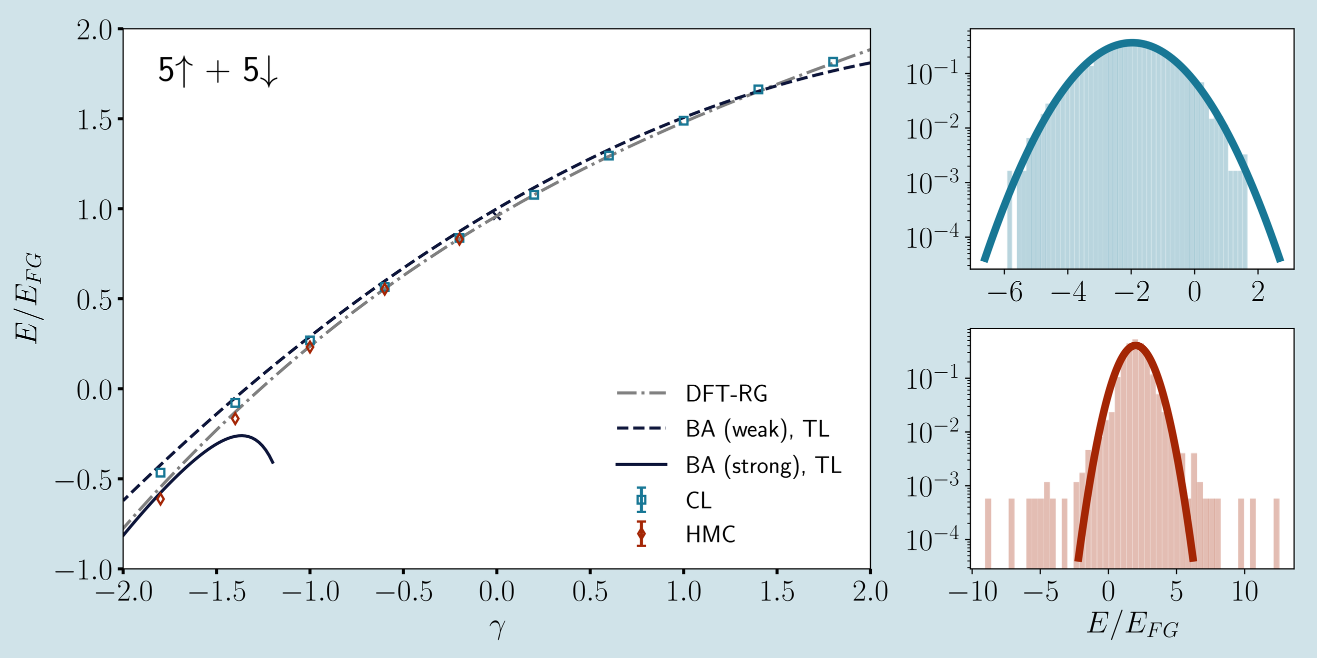
- key idea of stochastic quantization: a $(d+1)$-dimensional random process is used to sample the measure of a d-dimensional euclidean path integral [Parisi, Wu '81]

$$\frac{\partial\phi}{\partial t} = -\frac{\delta S[\phi]}{\delta\phi} + \eta$$

- with a discrete Langevin equation we can generate a Markov chain of complexified auxiliary fields ϕ that can be used to compute observables stochastically

1D fermions in the ground state

$$\hat{H} = -\sum_{s=\uparrow,\downarrow}\int d^dx\,\hat{\psi}_s^\dagger(\vec{x})\left(\frac{\hbar^2\vec{\nabla}^2}{2m_s}\right)\hat{\psi}_s(\vec{x}) + g\int d^dx\,\hat{\psi}_\uparrow^\dagger(\vec{x})\,\hat{\psi}_\uparrow(\vec{x})\,\hat{\psi}_\downarrow^\dagger(\vec{x})\,\hat{\psi}_\downarrow(\vec{x})$$



balanced case: compare to other methods

[LR, Porter, Drut, Braun '17]

- excellent agreement of ground-state energies among all tested methods [BA: Iida, Wadati '07; Tracy, Widom '16; DFT-RG: Kemler, Pospiech, Braun '17; Kemler '17; HMC: LR, Porter, Loheac, Drut '15]
- convergence to thermodynamic limit visible already at $N = 5\uparrow + 5\downarrow$
- distributions well-behaved at attractive couplings, fat tails at strong repulsion

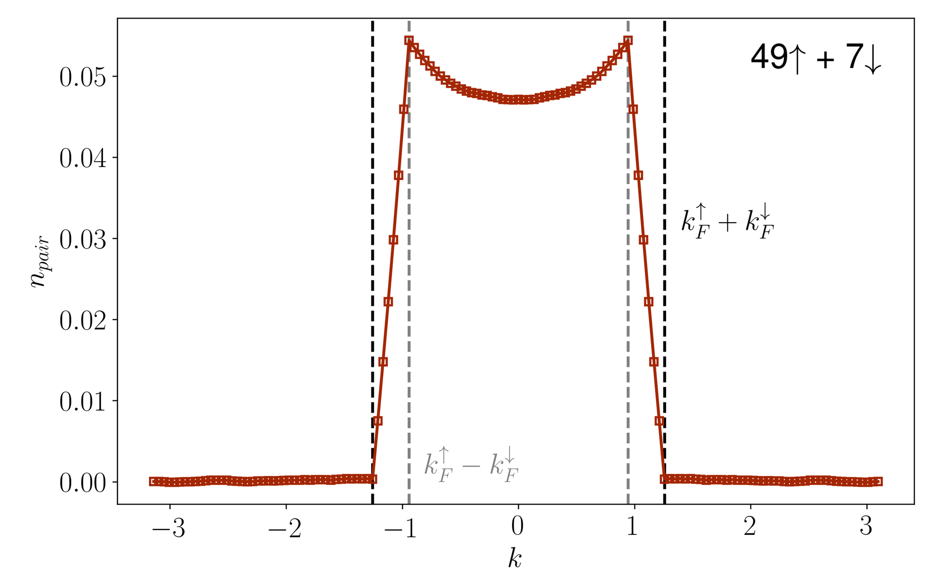
mass- & spin-imbalanced systems: two-body quantities

[LR, Porter, Drut, Braun '17; LR, Drut, Braun *in preparation*]

on-site pair-correlation function:

$$\rho_{pair}(|x-x'|) = \langle\hat{\psi}_\uparrow^\dagger(x)\,\hat{\psi}_\downarrow^\dagger(x)\,\hat{\psi}_\downarrow(x')\,\hat{\psi}_\uparrow(x')\rangle$$

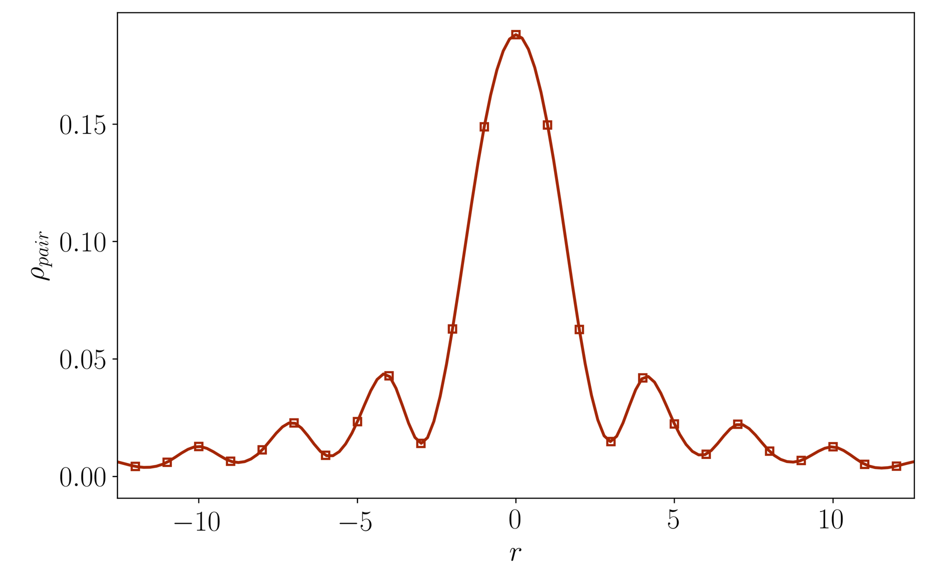
- off-center peak in pair-momentum distribution at $q = |k_F^\uparrow - k_F^\downarrow|$
- consequence: spatially oscillating "order parameter" (inhomogeneous pairing)

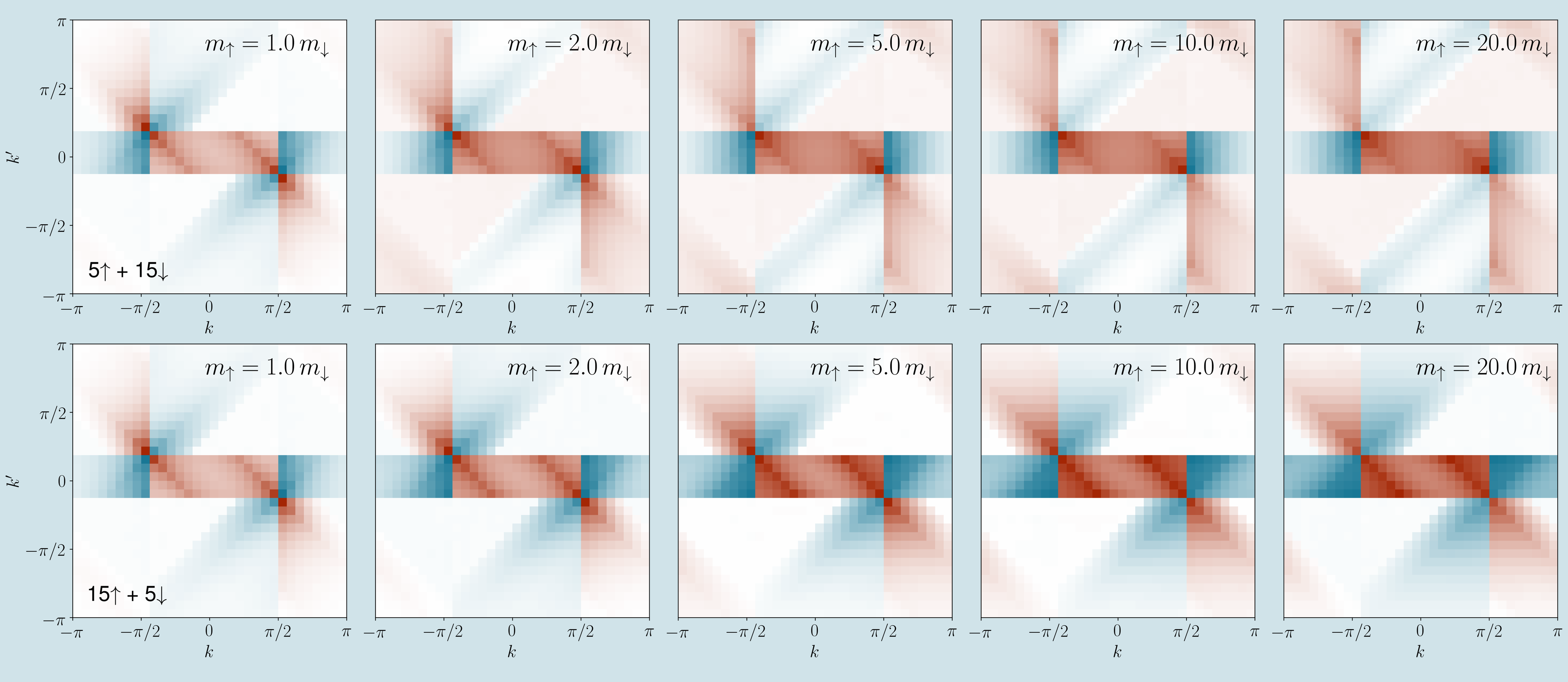


density-density correlation function (momentum space):

$$n_{\uparrow\downarrow}(k,k') = \langle\hat{\psi}_{k\uparrow}^\dagger\,\hat{\psi}_{k\uparrow}\,\hat{\psi}_{k'\downarrow}^\dagger\,\hat{\psi}_{k'\downarrow}\rangle - \langle\hat{\psi}_{k\uparrow}^\dagger\,\hat{\psi}_{k\uparrow}\rangle\langle\hat{\psi}_{k'\downarrow}^\dagger\,\hat{\psi}_{k'\downarrow}\rangle$$

- clean signal of FFLO-type pairing at $(\pm k_F^\uparrow, \mp k_F^\downarrow)$
- peak position in pair-momentum distribution constant despite increasing mass imbalance, density-density correlator shows difference in structure
- experimentally accessible, e.g. for Fermi-Fermi mixtures of ^{6}Li and ^{40}Ka or ^{40}Ka and ^{161}Dy

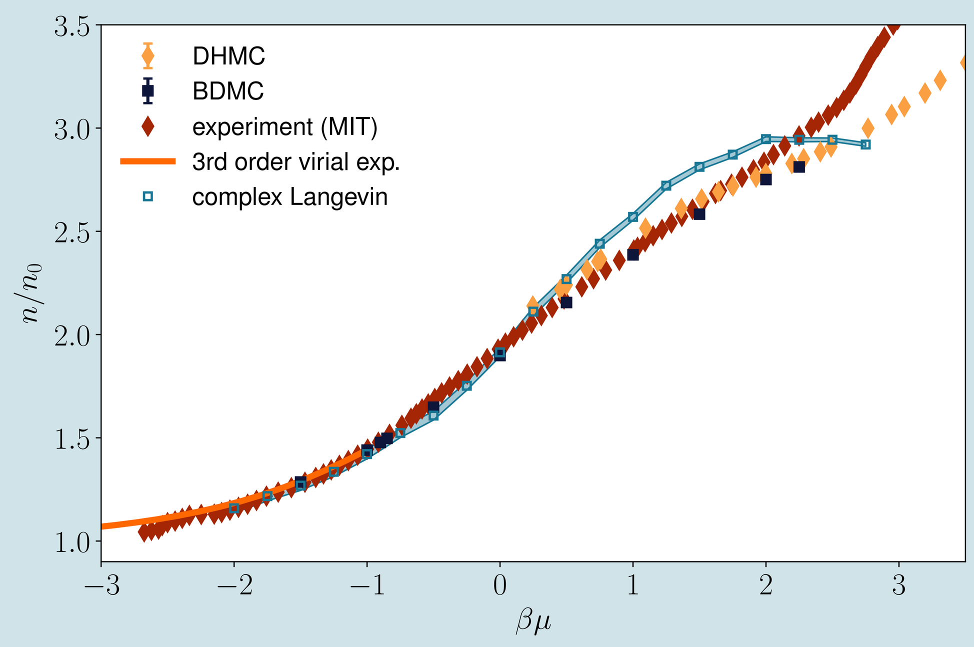


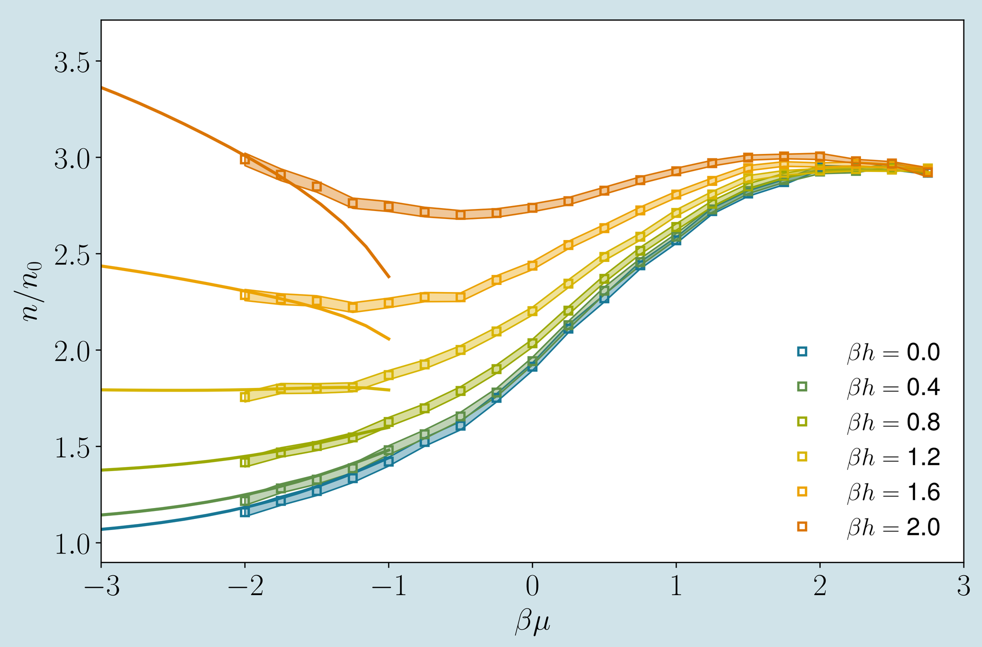


3D unitary Fermi gas at finite temperature

[LR, Loheac, Drut, Braun *in preparation*]

- balanced case: excellent match to experimental and numerical values at high temperatures [experiment/BDMC: VanHoucke et al. '12; DHMC: Drut, Lähde, Wlazlowski, Magierski '12]
- low temperatures: thermal wavelength λ_T increases, finite volume effects visible (currently $V = 9^3$)





- chemical potential asymmetry $h \equiv \frac{\mu_\uparrow - \mu_\downarrow}{2}$
- good agreement with virial expansion at large fugacity
- key question: what is the critical asymmetry for superfluidity (Clogston limit) as a function of temperature?

stay tuned!

future studies will include:

- larger lattices for the unitary Fermi gas at finite polarization
- thermodynamics and pair correlations in polarized 2D/3D Fermi gases
- effect of finite mass imbalance on the pairing mechanism in 2D/3D
- long-term goal: *ab-initio* phase-diagrams of spin- and mass-imbalanced Fermi gases

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