THE COMPLEX LANGEVIN METHOD FOR ULTRACOLD FERMIONS

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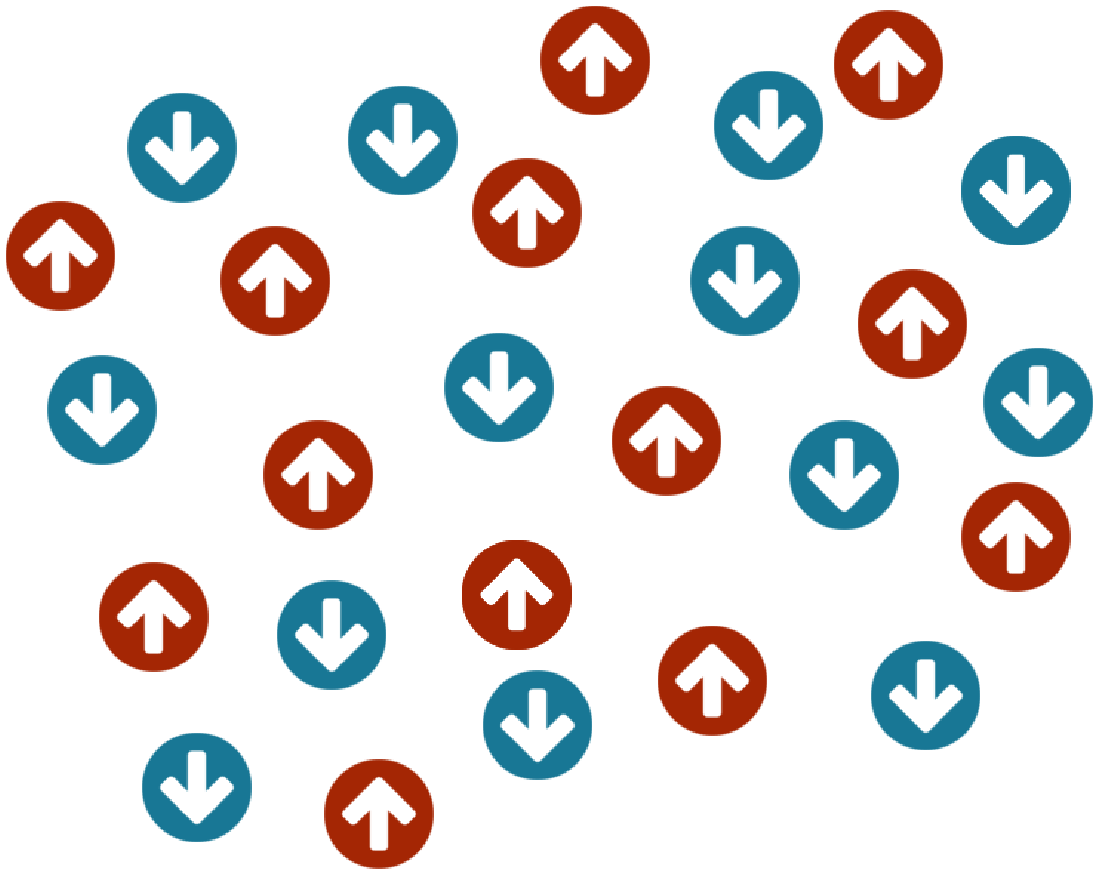
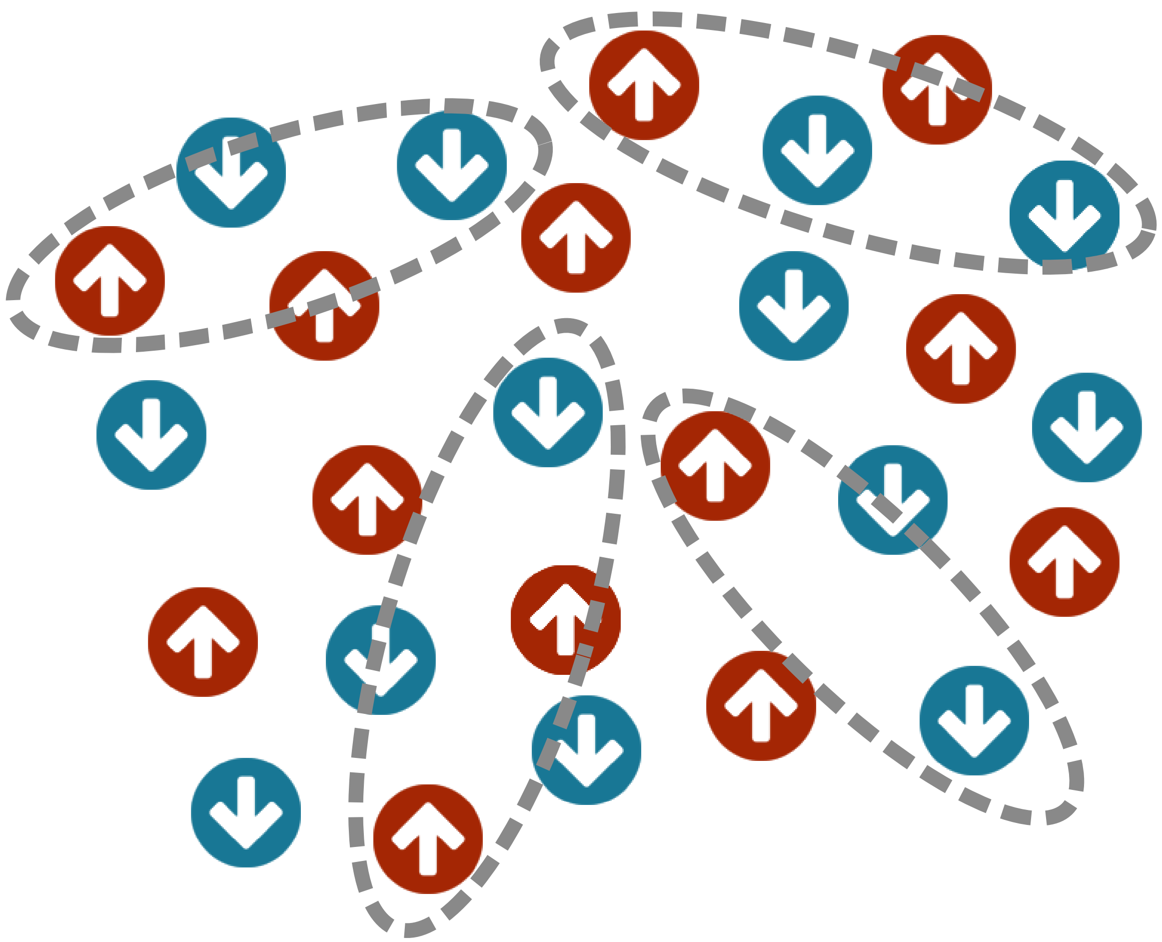
Marburg, FOR 1807 Winter School 2018

[LR, Loheac, Drut, Braun *in preparation*]
[LR, Porter, Drut, Braun *Phys. Rev. D* 96, 094506, 2017]
[LR, Porter, Drut *Phys. Rev. A* 93, 033639, 2016]

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research
CSC Center for Scientific Computing Frankfurt
HIC for FAIR
Helmholtz International Center
TECHNISCHE UNIVERSITÄT DARMSTADT



(electrons in metals, nuclear physics,
neutron stars, controllable experiments, ...)



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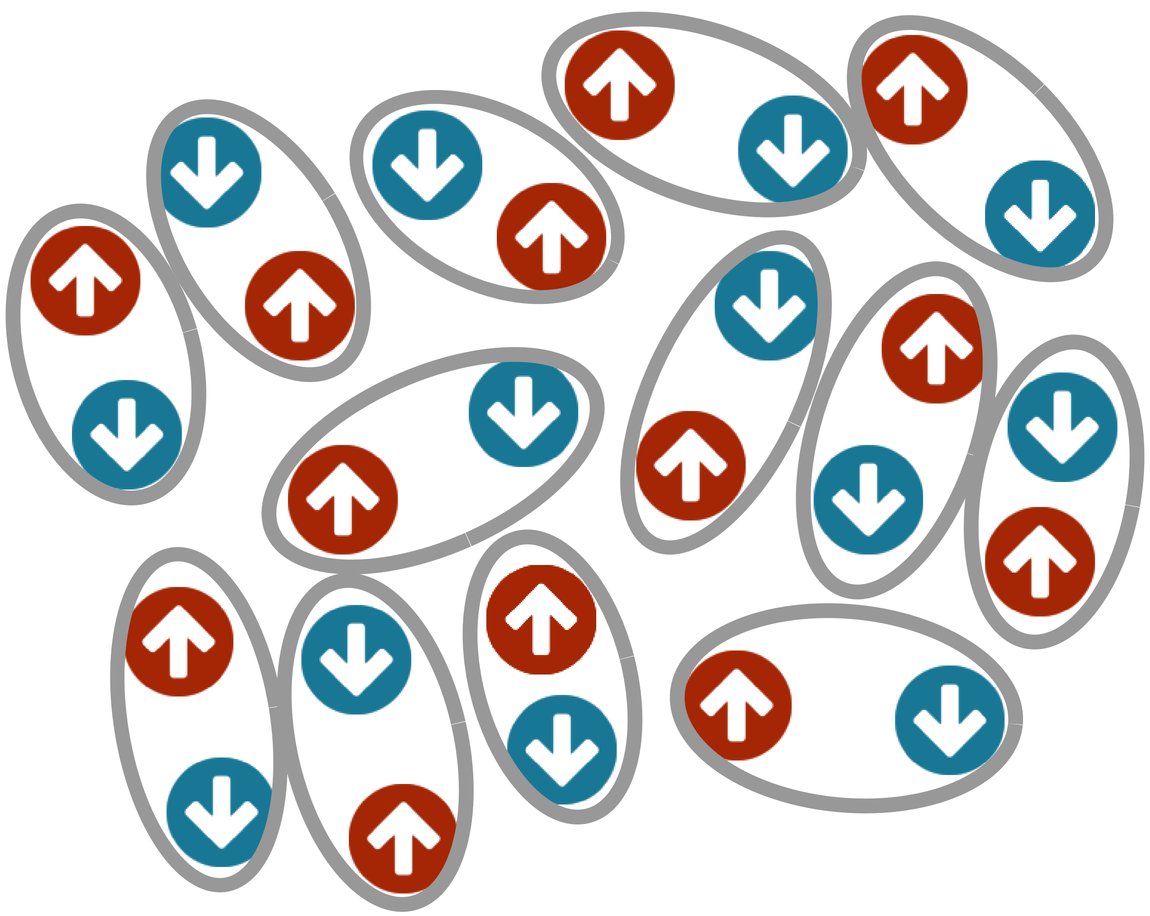

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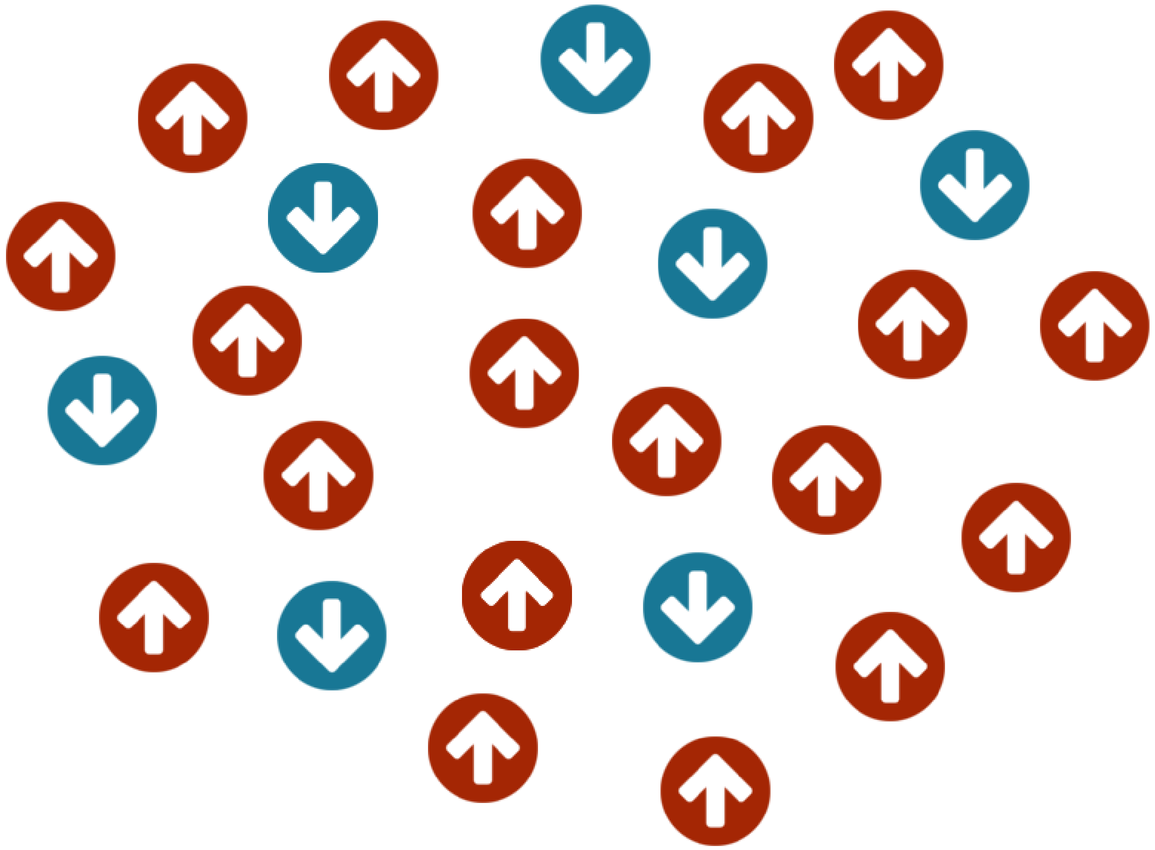
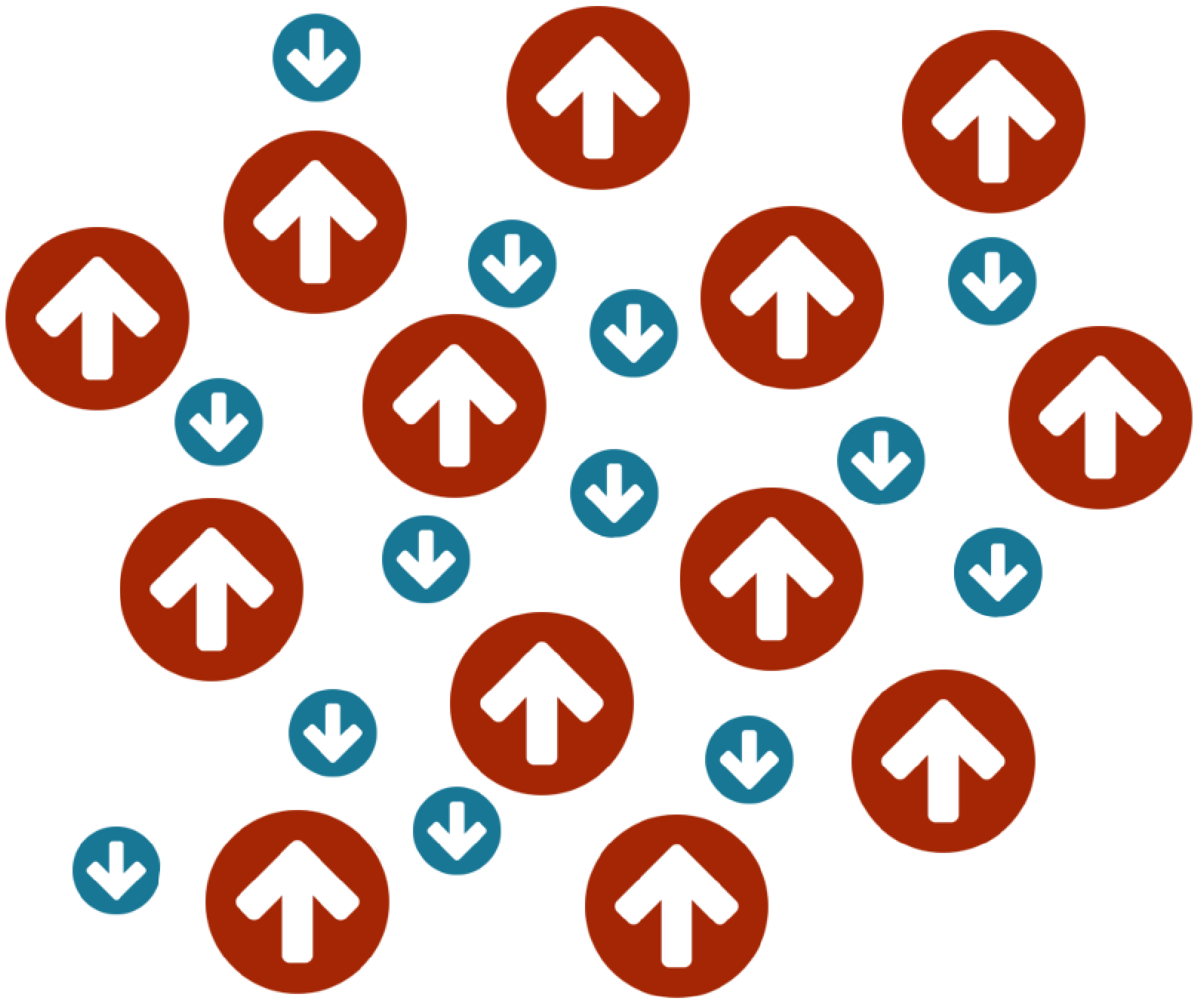


spin polarized systems

(inhomogenoeus phases?)



mass imbalanced systems

(inhomogenoeus phases?)

model: contact interaction

$$\hat{H} = -\sum_{s=\uparrow,\downarrow} \int d^d x \, \hat{\psi}_s^\dagger(\vec{x}) \left(\frac{\hbar^2 \vec{\nabla}^2}{2m_s} \right) \hat{\psi}_s(\vec{x})$$

$$+ g \int d^d x \, \hat{\psi}_\uparrow^\dagger(\vec{x}) \, \hat{\psi}_\uparrow(\vec{x}) \, \hat{\psi}_\downarrow^\dagger(\vec{x}) \, \hat{\psi}_\downarrow(\vec{x})$$

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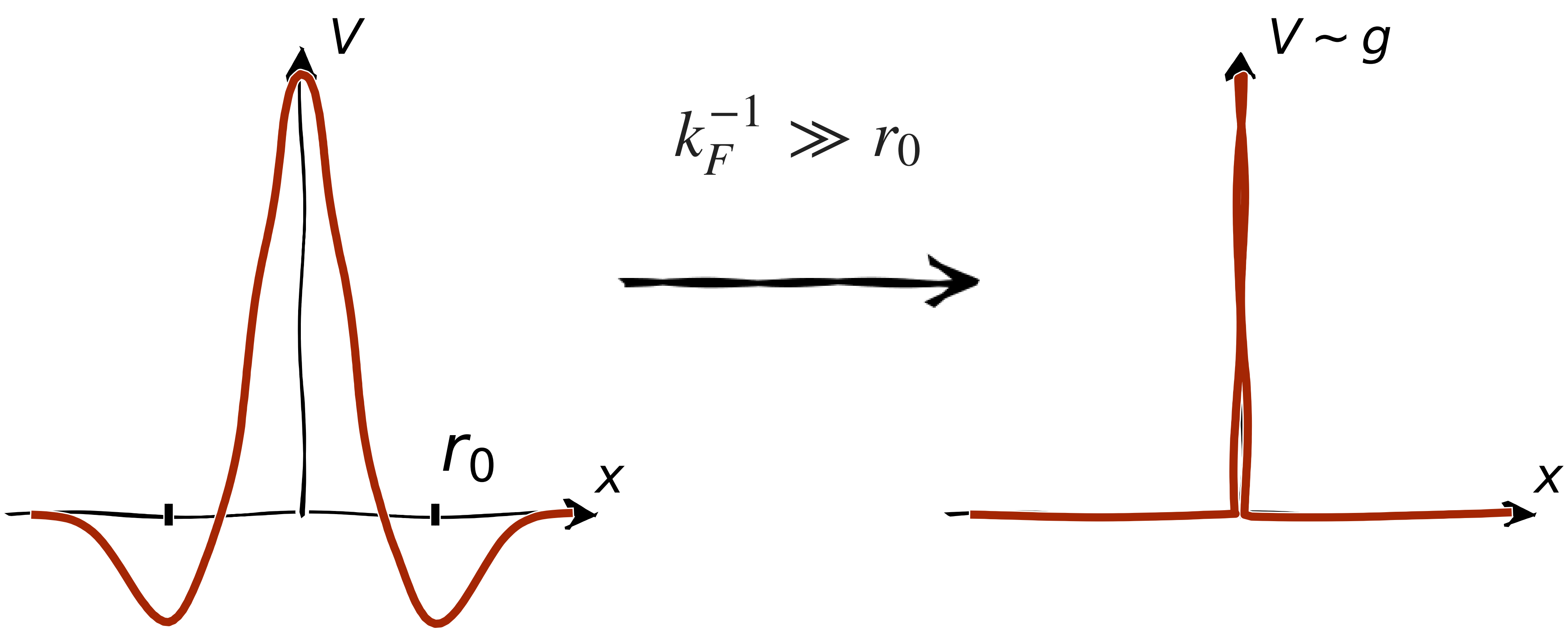

what do we need to compute?

$$\mathcal{Z} = \mathrm{Tr}[\mathrm{e}^{-\beta \hat{H}}] = \mathrm{Tr}[\mathrm{e}^{-\beta(\hat{T} + \hat{V})}]$$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \mathrm{Tr}[\hat{\mathcal{O}}\, \mathrm{e}^{-\beta \hat{H}}]$$

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discretize space & time, decouple the interaction
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discretize space & time, decouple the interaction with a Hubbard Stratonovich transformation

Rewrite the problem as a path-integral:

$$\mathcal{Z} = \int \mathcal{D}\phi \;\det M_\phi^{\uparrow} \;\det M_\phi^{\downarrow} \equiv \int \mathcal{D}\phi \; \mathrm{e}^{-S[\phi]}$$

calculating observables

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\phi \; P[\phi] \, \mathcal{O}[\phi]$$

produce a set of **random auxiliary-field configuations** $\{\phi_i\}$, compute observables & average for expecation values

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produce a set of **random auxiliary-field configuations** $\{\phi_i\}$, compute observables & average for expecation values

probability measure:

$$P[\phi] \;\propto\; \det M_\phi^{\uparrow} \; \det M_\phi^{\downarrow} = \begin{cases} \text{attractive, balanced case} \rightarrow \text{OK} \\ \text{otherwise} \rightarrow \text{sign problem} \end{cases}$$

Complex Langevin (in a nutshell)

stochastic quantization: equilibrium distribution of a $(d + 1)$-dimensional random process is identified with the probability measure of our d-dimensional path integral

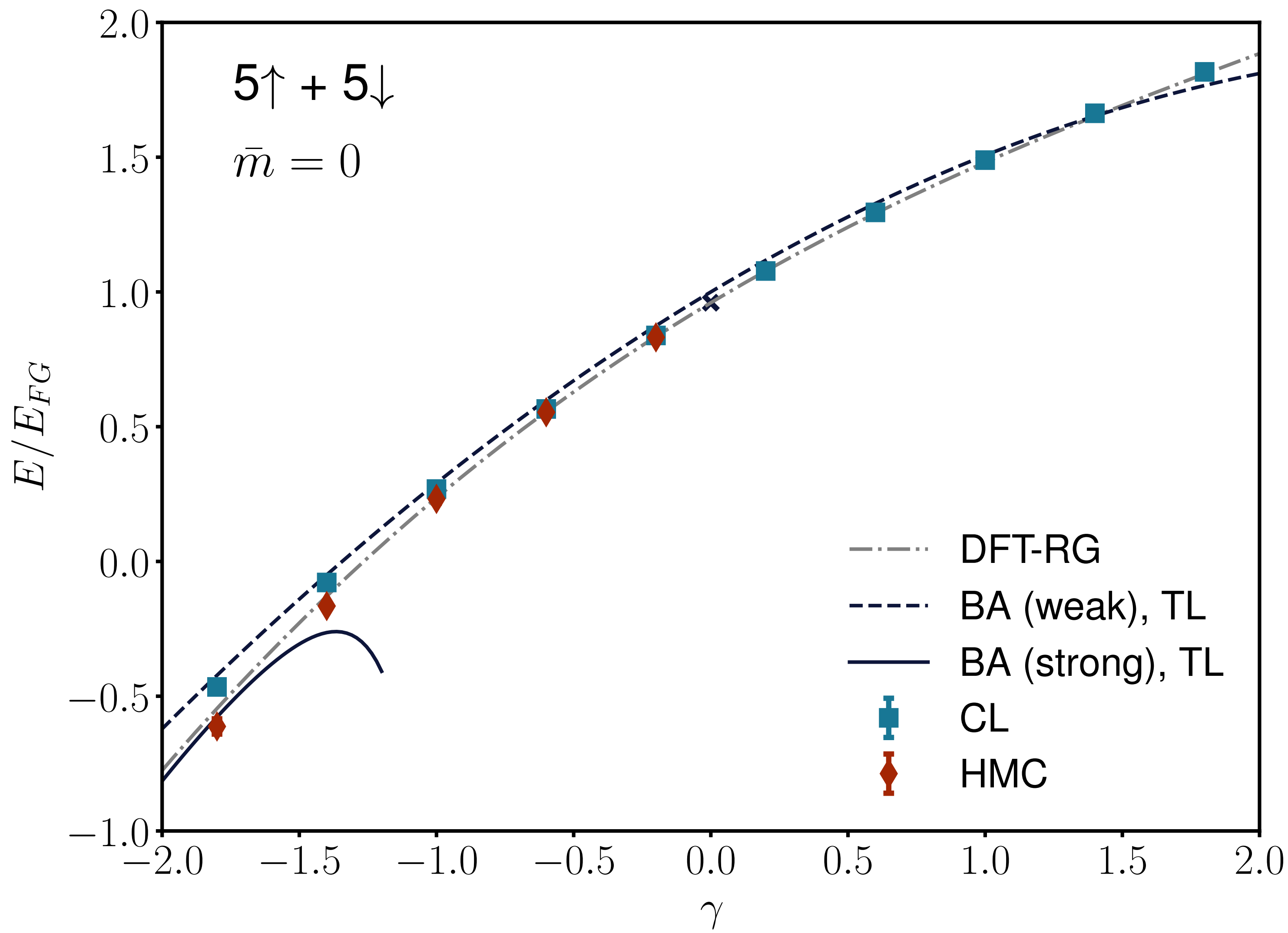
random walk governed by Langevin equation (Brownian motion):

$$\frac{\partial \phi}{\partial t} = -\frac{\partial S[\phi]}{\partial \phi} + \eta(t)$$

[Parisi, Wu '81], [Aarts '09; Seiler '17]
[Loheac, Drut '17; LR, Porter, Drut, Braun '17]

first step: compare to other methods

[LR, Porter, Drut, Braun '17]



$\gamma = g/n$

[BA: Iida, Wadati '07; Tracy, Widom '16]
[DFT-RG: Kemler, Pospiech, Braun '16]
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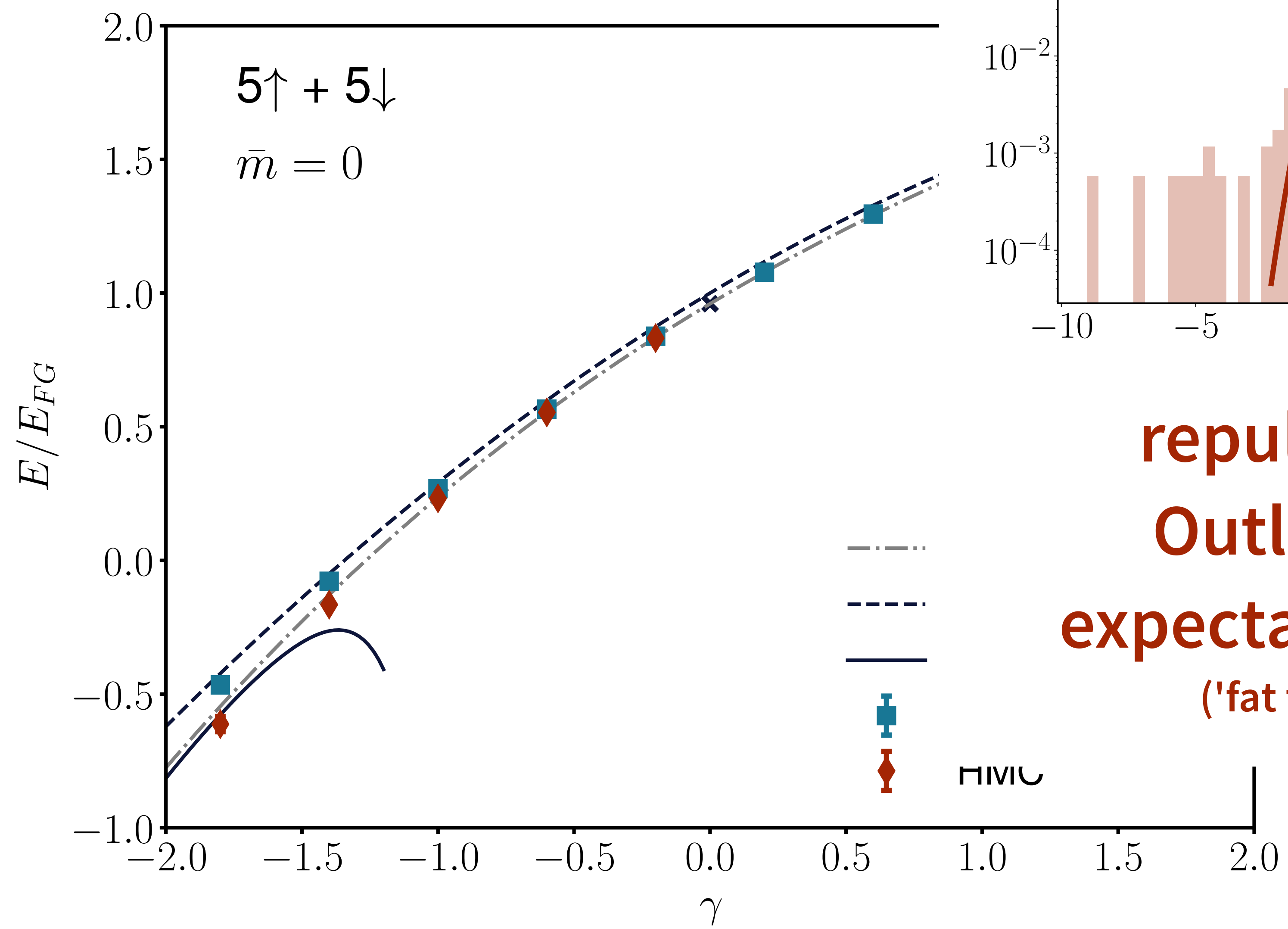

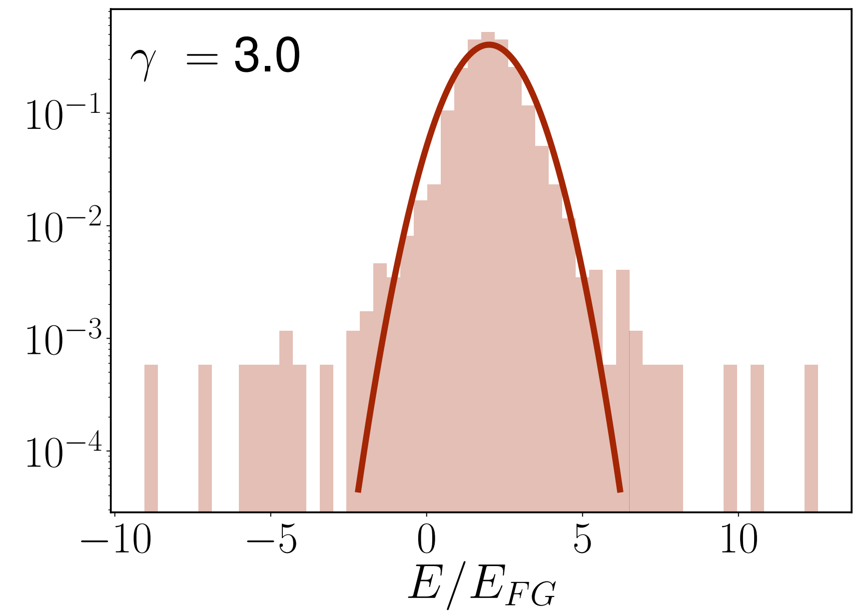

repulsive side:
Outliers skew
expectation values!
('fat tail' problem)

$$\gamma = g/n$$

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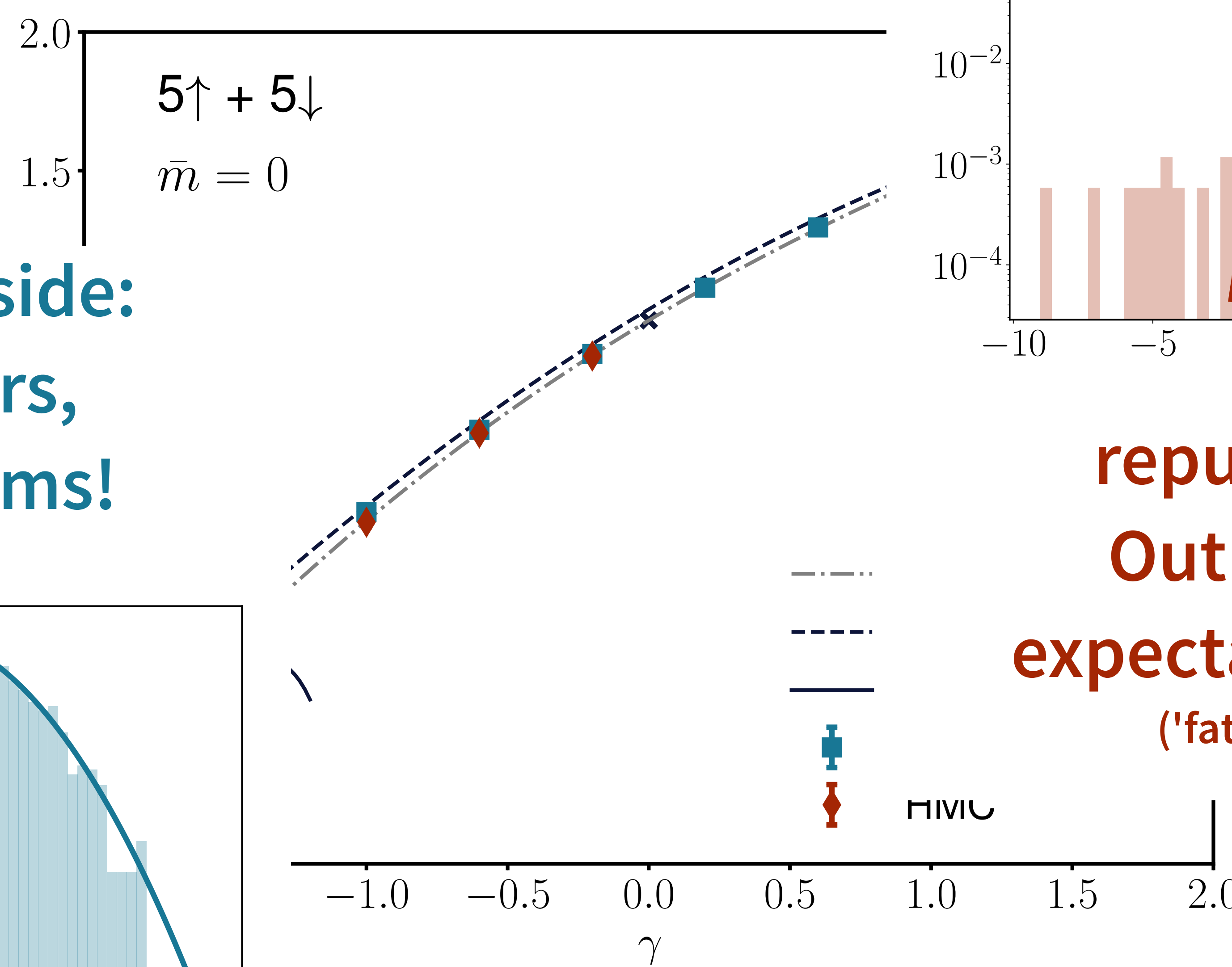
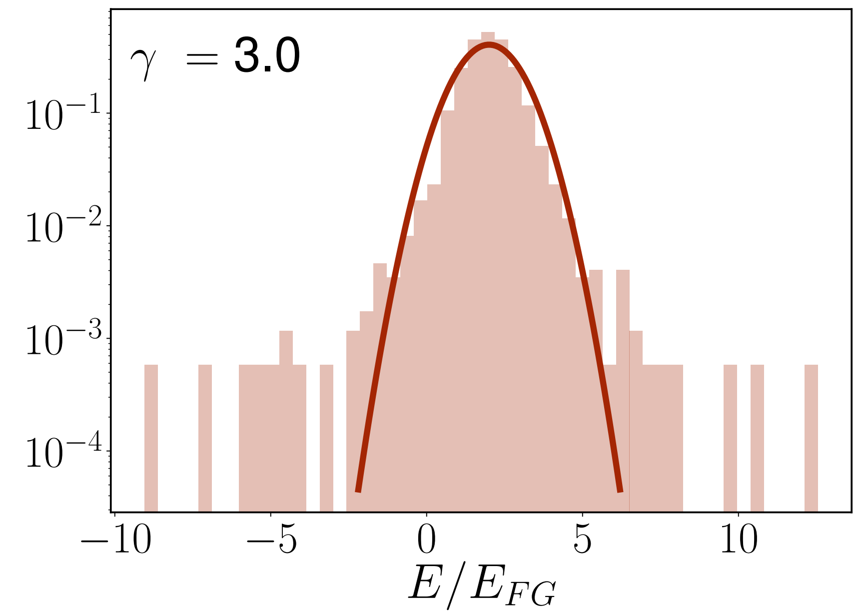
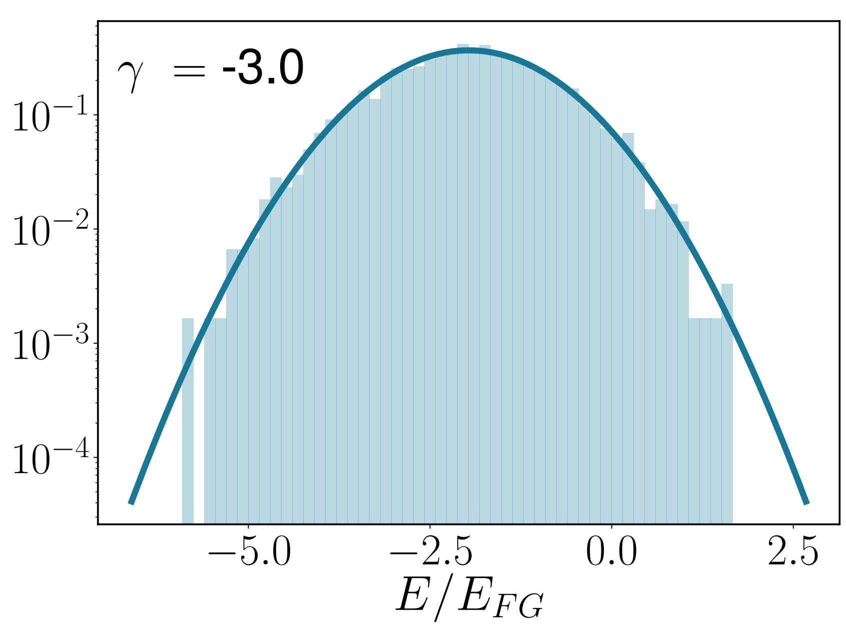




attractive side:
no outliers,
no problems!



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polarized 1D fermions: equation of state

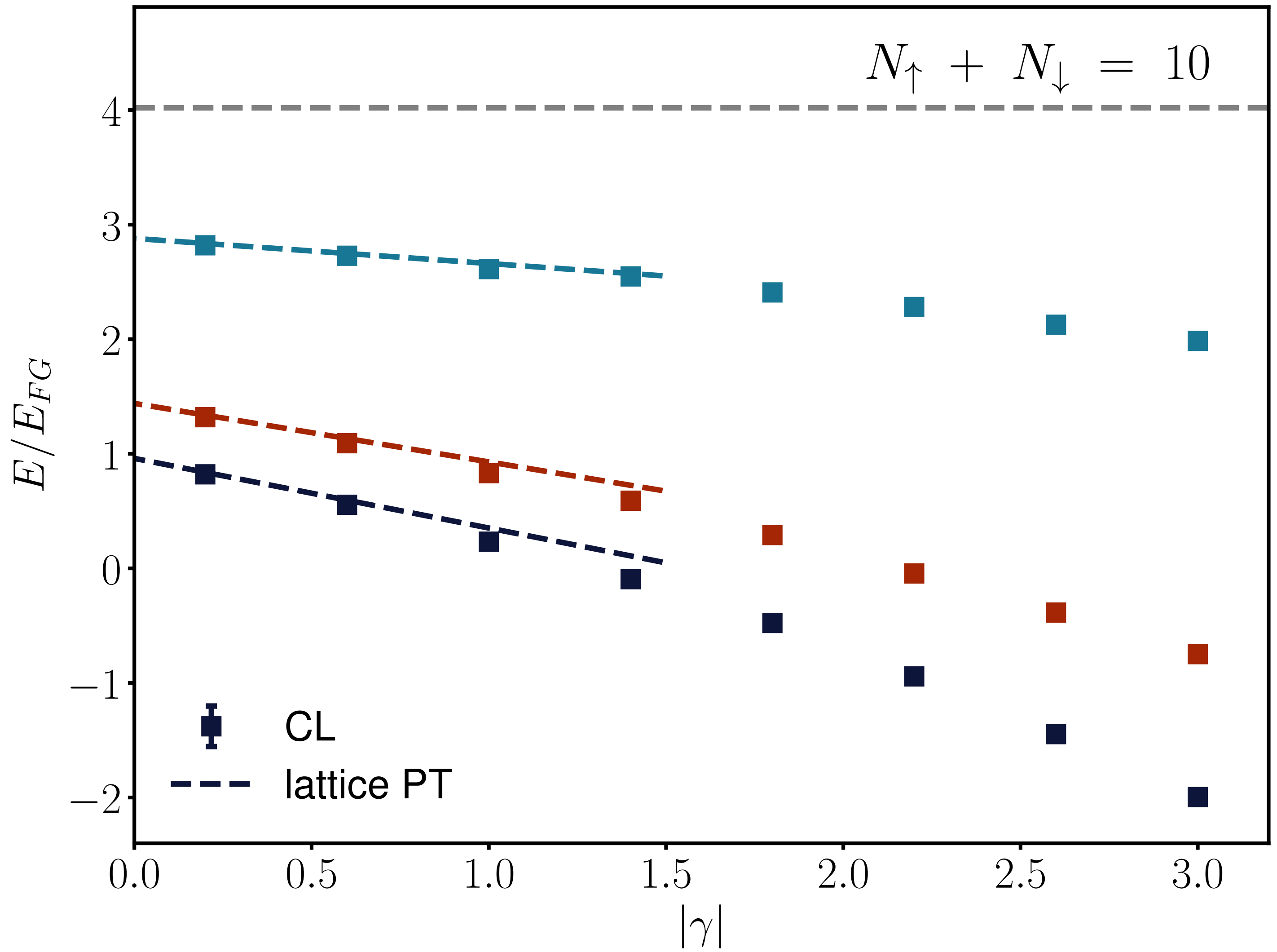
[LR, Drut, Braun *in preparation*]

$p = 1.0$

$p = 0.8$

$p = 0.4$

$p = 0.0$



$\gamma = g/n$

$p = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

polarized 1D fermions: pair correlation

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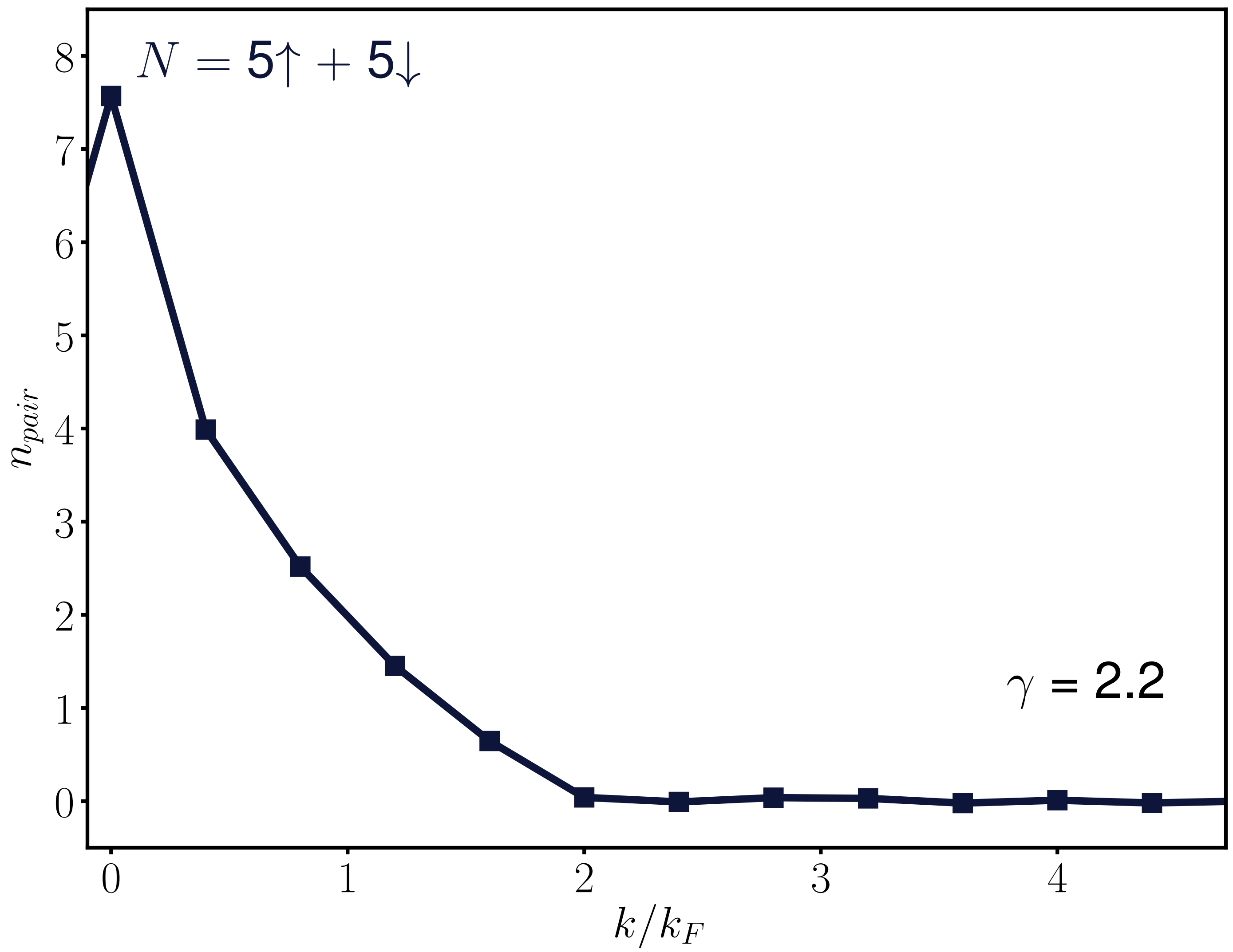

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$$n_{pair}(x, x') = \langle \hat{\psi}^\dagger_\uparrow(x)\, \hat{\psi}^\dagger_\downarrow(x)\, \hat{\psi}_\downarrow(x')\, \hat{\psi}_\uparrow(x') \rangle$$

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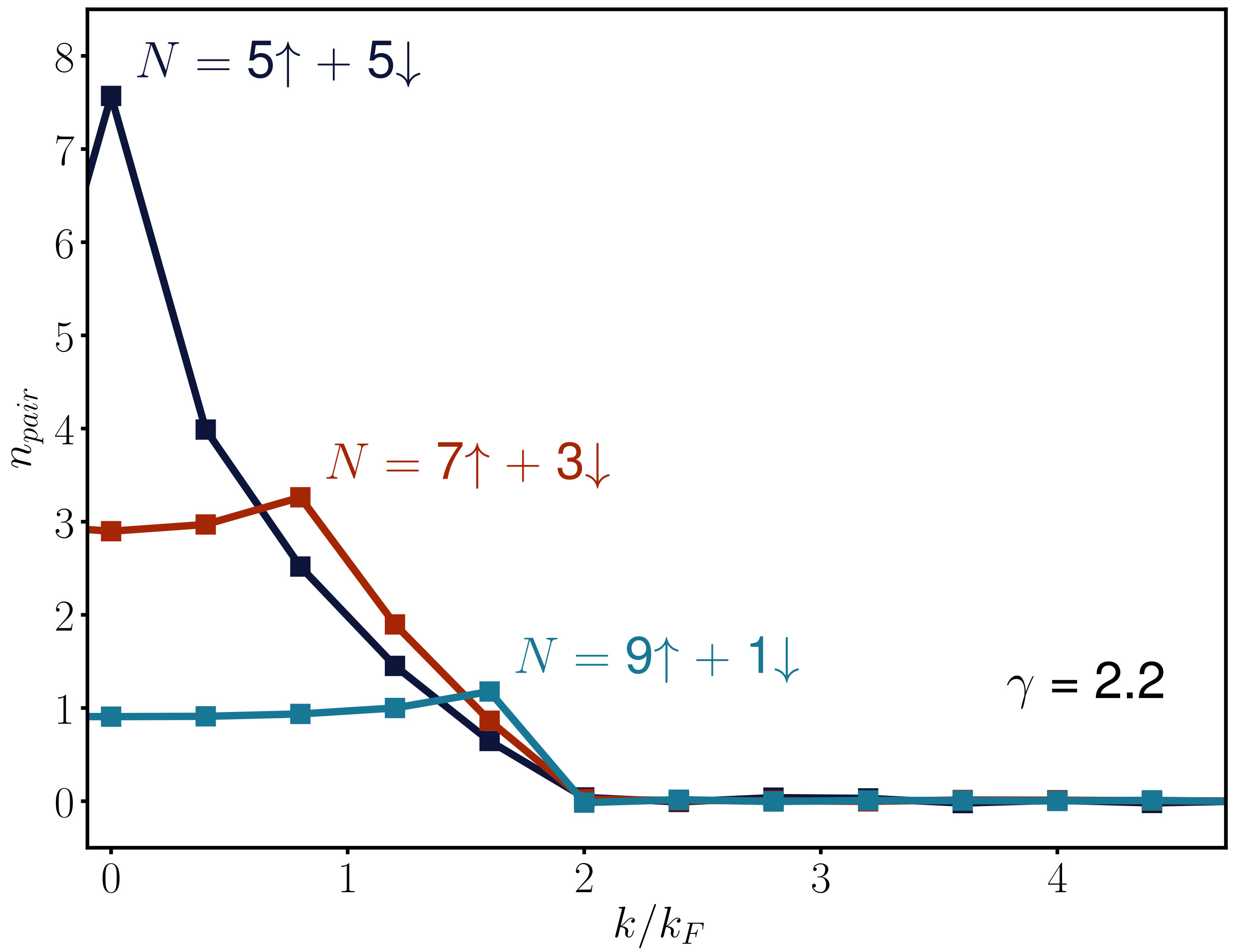

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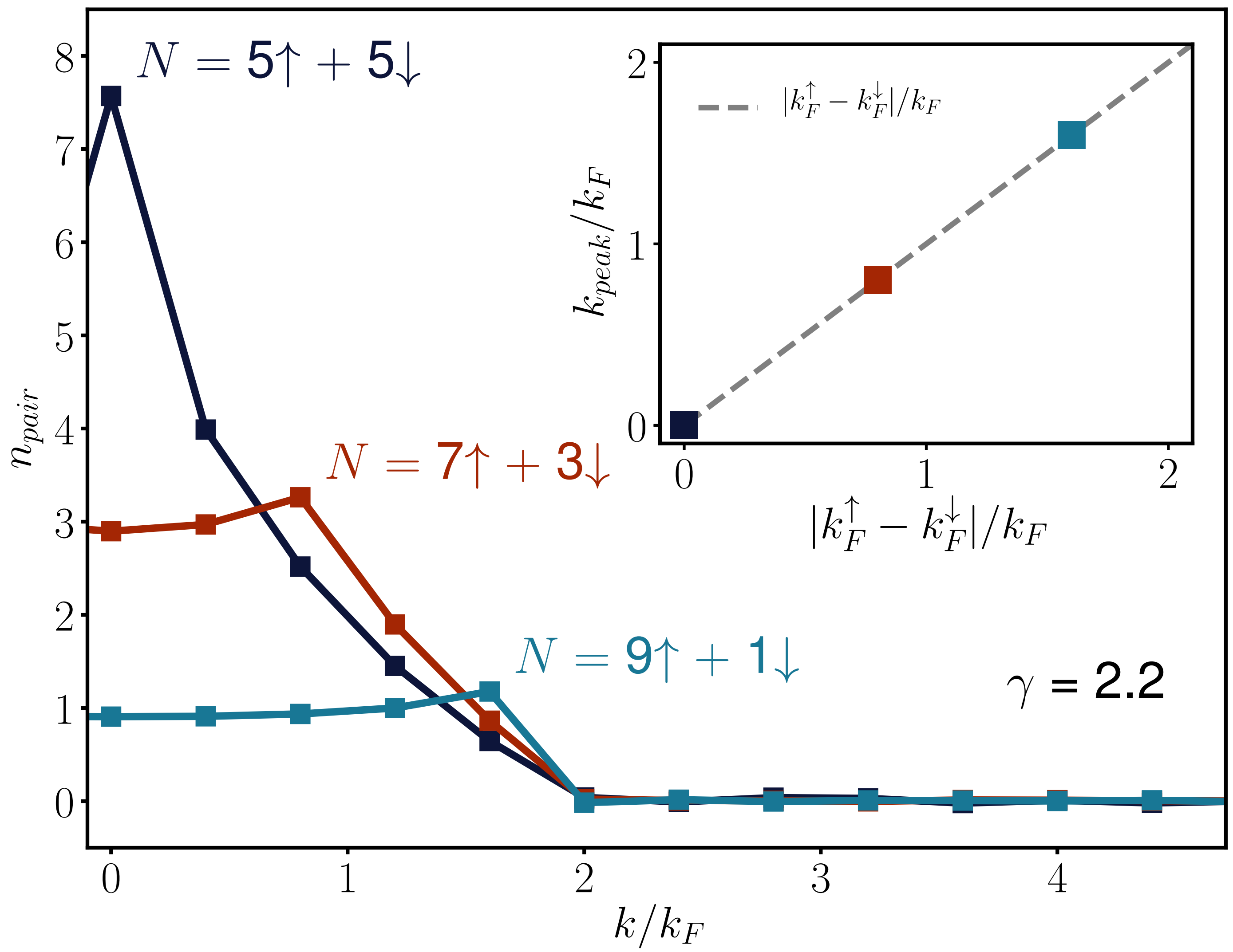

signature of FFLO type pairing

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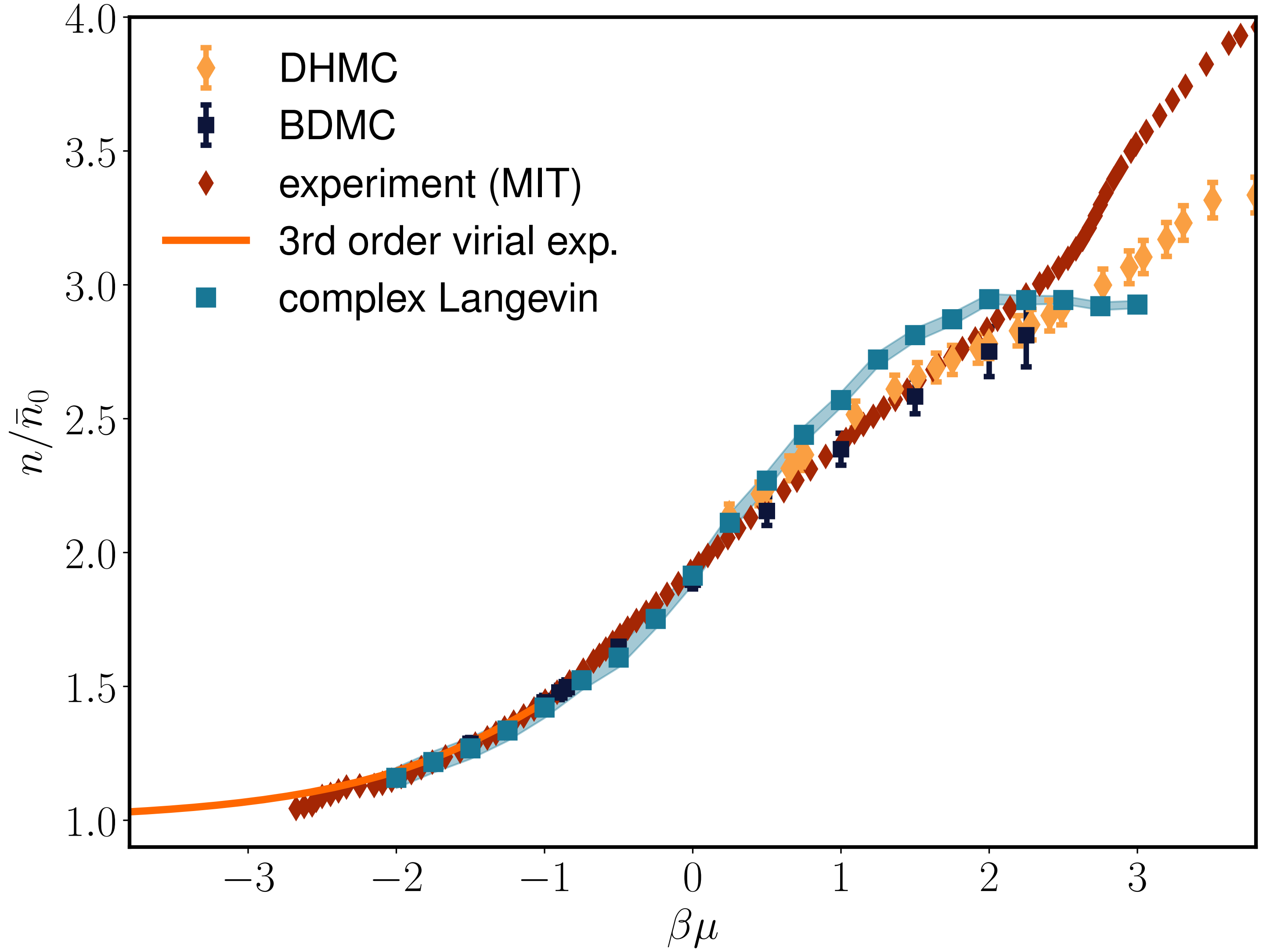
UFG at finite T: equation of state

[LR, Loheac, Drut, Braun *in preparation*]



[experiment/BDMC: van Houcke et al. '12]
[DHMC: Drut, Lähde, Wlazlowski, Magierski '12]

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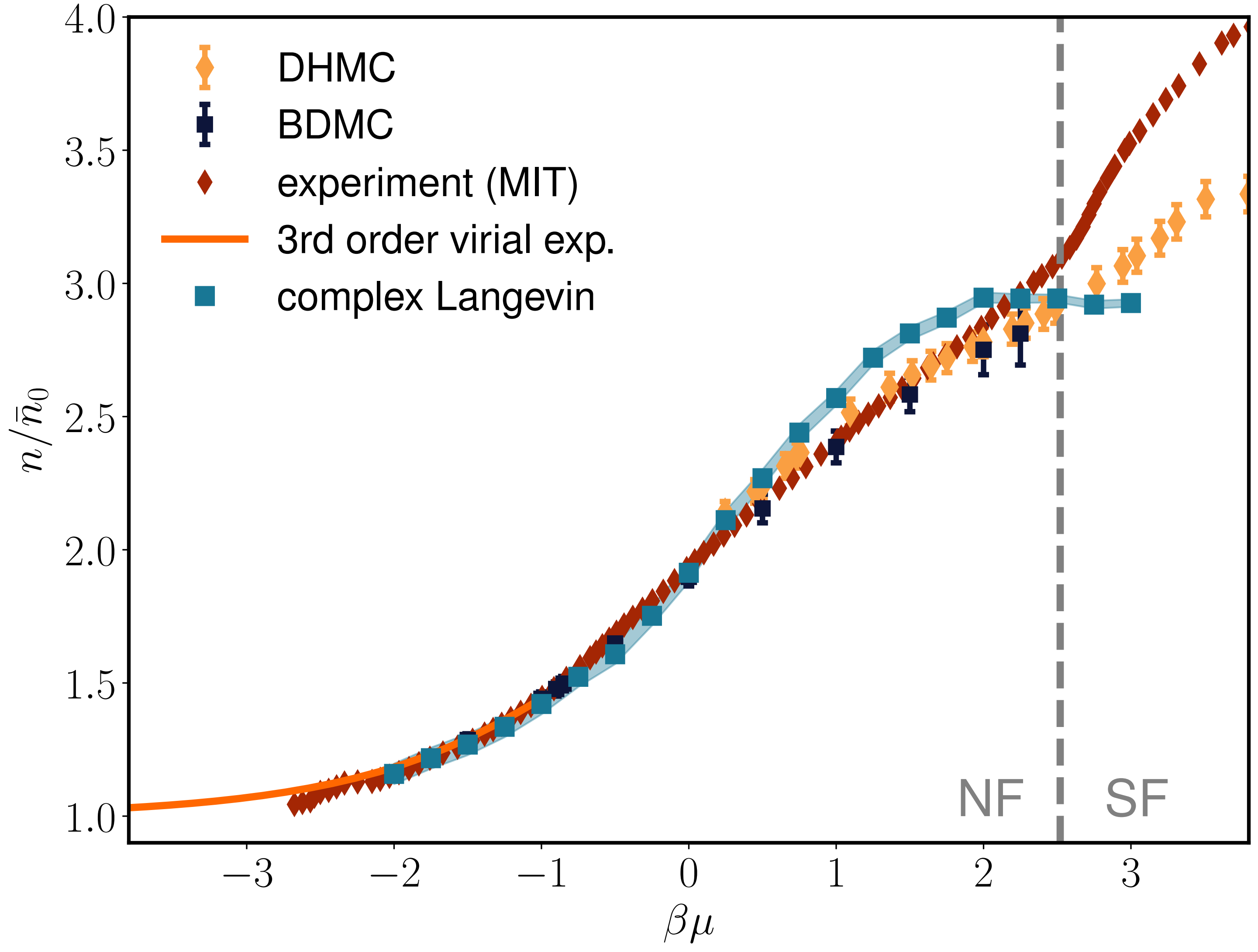
$T_C \sim 0.17\, T_F$



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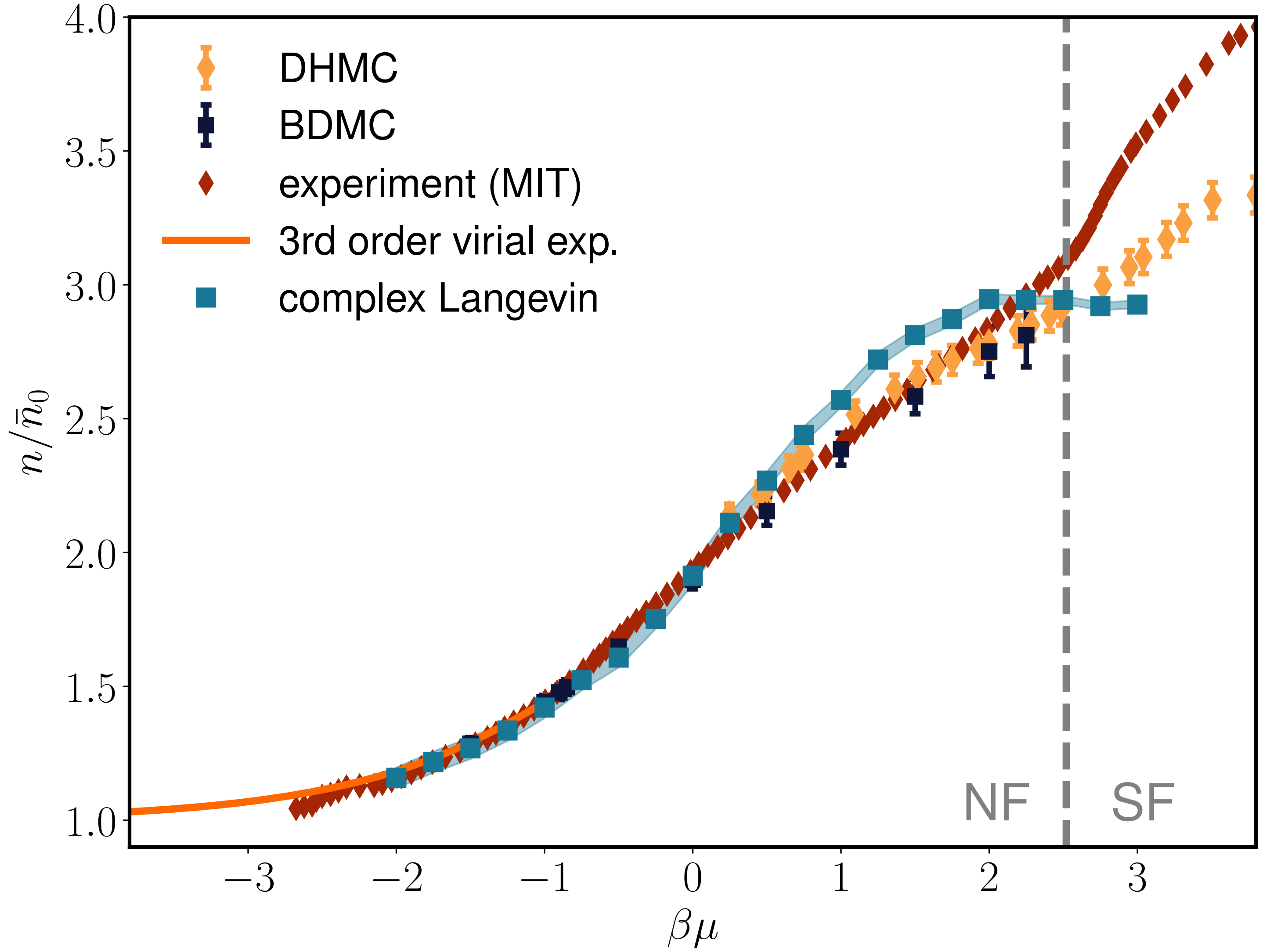

CL results:
finite lattice!
$(V = 9^3)$

low temperatures: λ_T increases
($\lambda_T \ll V^{1/3}$ must be fulfilled)

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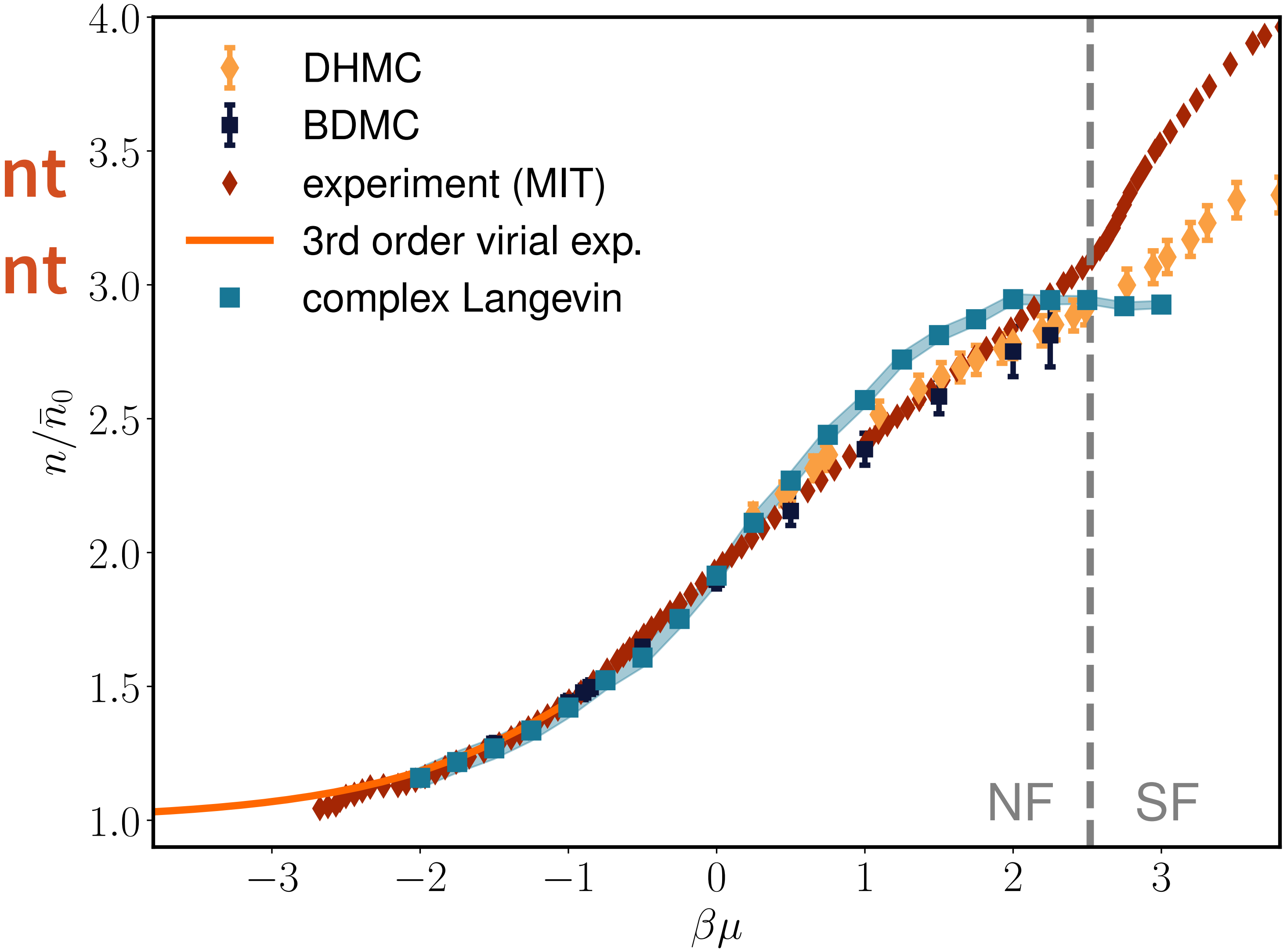
good agreement with experiment and other methods!



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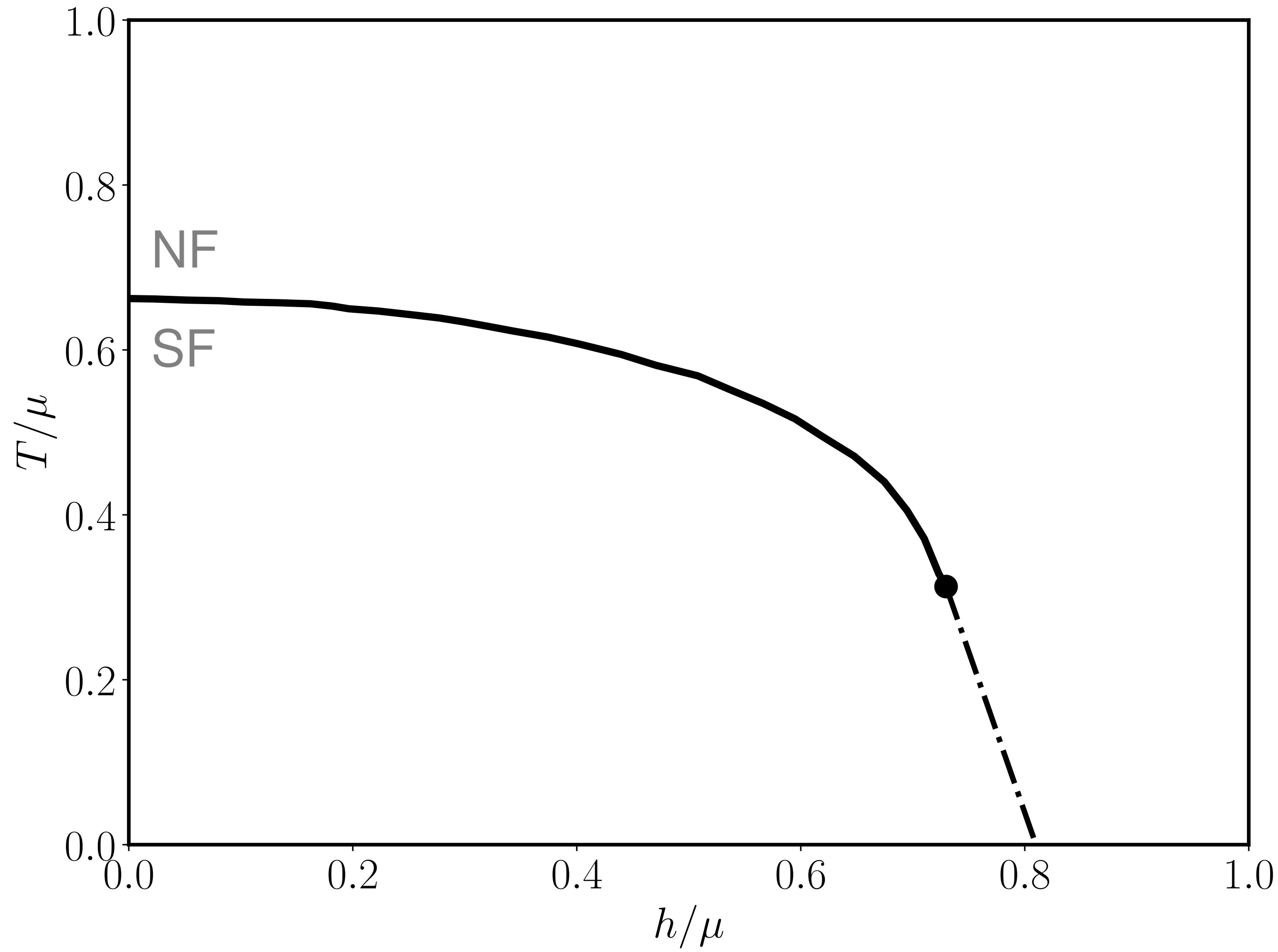
UFG at finite T: phase diagram (mean field)



$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$

[mean-field phase diagram: e.g. Chevy, Mora '10; Braun et al. '13]
[beyond mean-field:: e.g. Boettcher et al. '14; Roscher, Braun, Drut '15]

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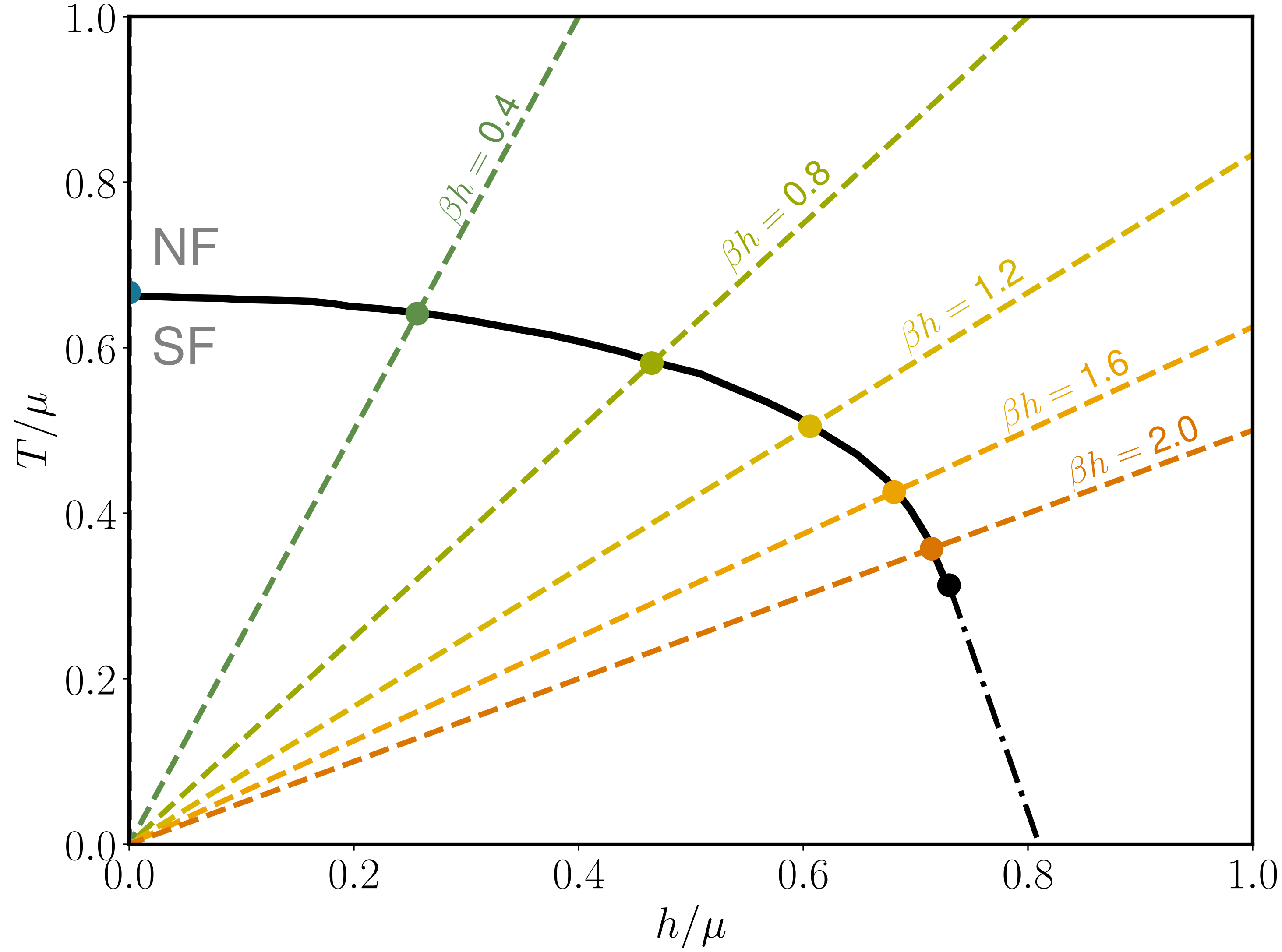

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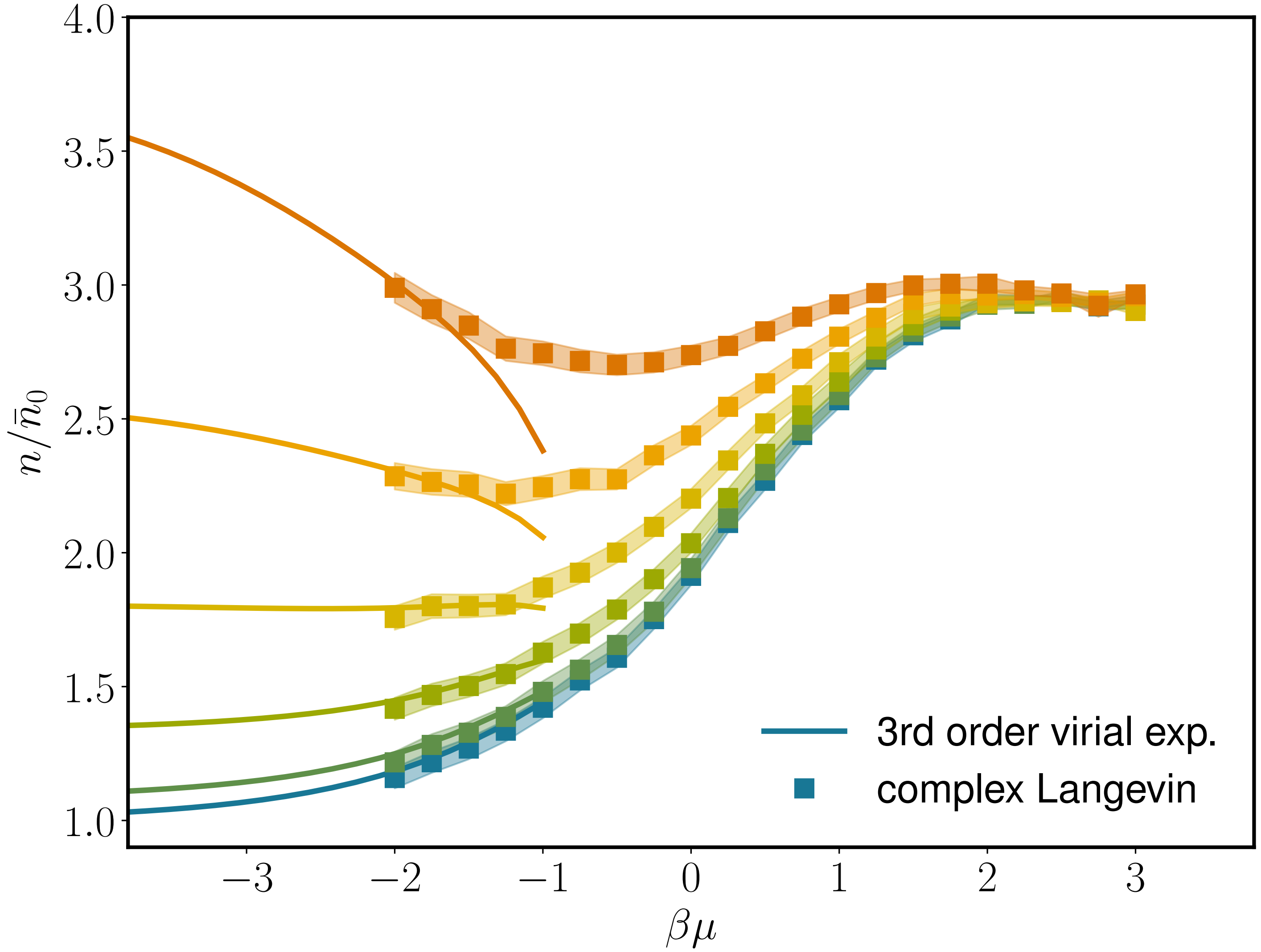

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virial expansion: $\ln \mathcal{Z} = \mathcal{Q}_1 \sum_n z^n b_n$

RECAP & OUTLOOK

imbalanced Fermi gases are hard to treat:
accessible with the **complex Langevin** method

CL compares well with other methods wherever possible

works in the ground state & at finite temperature
in any spatial dimension

Up next: investigation of pair correlations in $d > 1$,
mass imbalance at finite temperature, phase diagram in 2D/3D, ...