Equation of state & pair-correlations in one-dimensional Fermi systems

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Understanding the LHC

637. Wilhelm und Else Heraeus-Seminar

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Outline

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- Motivation: from QCD to ultracold gases
- Method: brief overview
- Results: discussion of EOS & pair-correlations
- Summary, conclusions & future work

From QCD to ultracold gases

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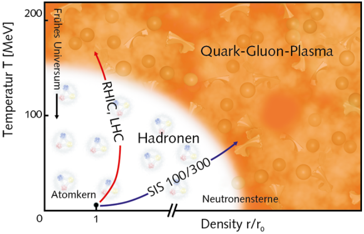
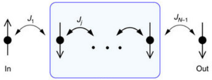
- explore & understand phases of strongly-interacting theories
- universal behavior: *unitary fermi gas* (UFG)
- ultracold quantum gases are a versatile tool to probe strongly-interacting systems (in experiment and theory)

  A.N. Wenz *et al.*, *Science* **25**, 2013.

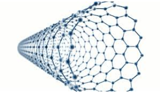
FAIR @ GSI



- many available methods, exchange with lattice QCD
- here: 1D, often a benchmark for new methods, *but also interesting physics!*



O.V. Marchukov *et al.*, *Nature Comm.* **7**, 2016.



Model:
Fermions on the lattice

- Gaudin-Yang model for fermions in 1D

$$\hat{H} = -\sum_{s=\uparrow,\downarrow} \sum_{x} \hat{\psi}_s^\dagger(x) \frac{\hbar^2\nabla^2}{2m_s} \hat{\psi}_s(x) + g\sum_{x} \hat{\psi}_\uparrow^\dagger(x)\, \hat{\psi}_\uparrow(x)\, \hat{\psi}_\downarrow^\dagger(x) \hat{\psi}_\downarrow(x).$$

- problem: $\hat{H} = \hat{T} + \hat{V}$ not diagonalizable.
  ($\hat{T}$ diagonal in momentum space, $\hat{V}$ diagonal in real space)
- in some cases exactly solvable with Bethe ansatz
- Monte Carlo methods more versatile and extendible

M. Gaudin, *Phys. Lett. A* **24**, 1967.
C.N. Yang, *Phys. Rev. Lett.* **19**, 1967.

Method:
Hybrid Monte Carlo (HMC) in a nutshell

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- rewrite Hamiltonian with time-discretization & Hubbard-Stratonovich transform: path-integral over *auxiliary fields* $\mathcal{Z} = \int \mathcal{D}\sigma \; P[\sigma]$
- instead of one very complicated many-body problem: many single-particle problems in an external field.
- integration with MC (Metropolis):
  1. start with random configuration
  2. update configuration
  3. accept/reject, calculate observables
  4. repeat from step 2 until satisfied with statistics
- "hybrid": instead of local updates, *global* updates with molecular dynamics (on-shell propagation)

S. Duane, A.D. Kennedy, B.J. Pendleton, D. Roweth, *Phys. Lett. B* **195**, 1987.
A. Bulgac, J.E. Drut, P. Magierski, *Phys. Rev. Lett.* **96**, 2006.

Results: Equation of state

ground-state energy vs. coupling strength

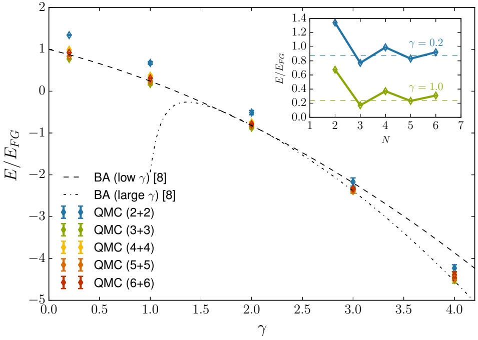
- ground-state energy experimentally accessible
- computed via

$$\langle \hat{H} \rangle = -\frac{\partial}{\partial \beta} \log \mathcal{Z}$$

- excellent agreement with Bethe ansatz

M. Wadati, T. Iida, *Phys. Lett. A* **360**, 200

- limit of large γ: bosonic behavior



$$\gamma = g/n$$

LR, W.J. Porter, A.C. Loheac, and J.E. Drut, *Phys. Rev. A* **92**, 2015.

Results: Equation of state

ground-state energy for few-body systems

- ground-state energy experimentally accessible
- computed via

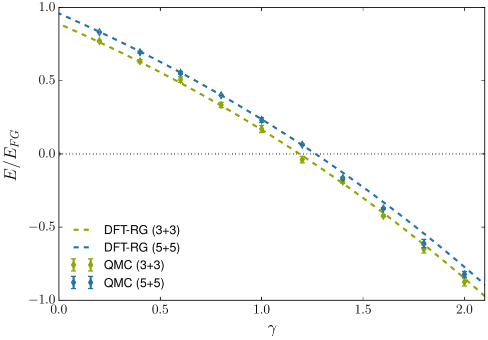
$$\langle \hat{H} \rangle = -\frac{\partial}{\partial \beta} \log \mathcal{Z}$$

- excellent agreement with Bethe ansatz

M. Wadati, T. Iida, *Phys. Lett. A* **360**, 200

- limit of large γ: bosonic behavior
- perfect agreement with few-body DFT-RG

S. Kemler, M. Pospiech and J. Braun, *in preparation*

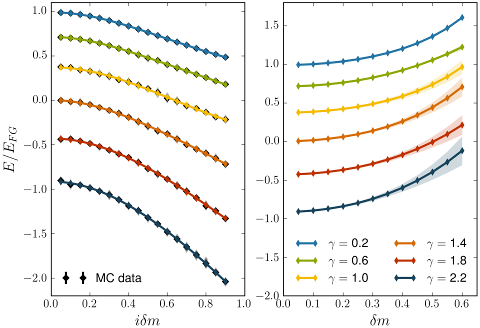


$$\gamma = g/n$$

Results: Equation of state

Extension to mass-imbalanced systems

- experimental realization of ^{6}Li and ^{40}Ka mixture
- typically subject to sign-problem
- sign-problem avoided by rewriting $m_\uparrow = m_0(1 + i\delta m)$ and $m_\downarrow = m_0(1 - i\delta m)$
  J. Braun, J.E. Drut, D. Roscher. *Phys. Rev. Lett.* **114**, 2015.
- real mass-imbalance through analytic continuation



LR, W.J. Porter, J. Braun, J.E. Drut, *in preparation.*

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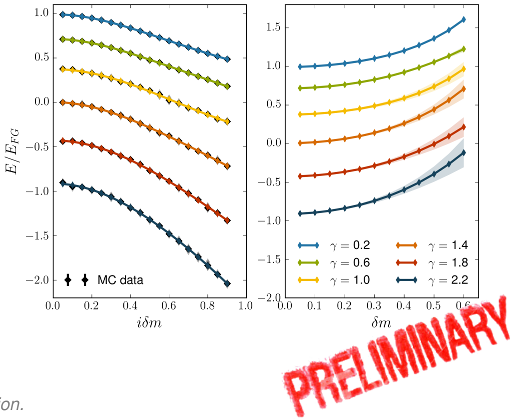
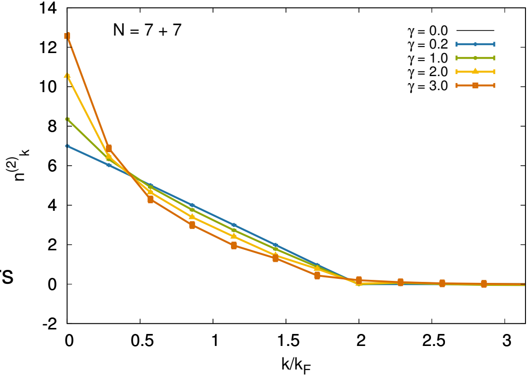


LR, W.J. Porter, J. Braun, J.E. Drut, *in preparation.*

Results: Pair-correlation

- pairing properties through pair-correlation function
  $$\rho_{(2)}(|x - x'|) = \langle \hat{\psi}^\dagger_\uparrow(x)\, \hat{\psi}^\dagger_\downarrow(x)\, \hat{\psi}_\downarrow(x')\, \hat{\psi}_\uparrow(x') \rangle$$
- smooth increase of peak at $k = 0$ with interaction strength
- formation of tightly-bound pairs
- crossover from BCS-pairing *Bose-Einstein condensate*



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Summary, conclusion & outlook

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Summary

- precise calculation of energies and correlations
  agreement for few-body systems, convergence to TL
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Future work

- spin- and mass-imbalance (FFLO behavior expected)
- long-term goal: nonperturbative phase-diagrams of 3D unitary Fermi gas