# Approaching Fermi mixtures with complex Langevin: EOS and pairing behavior

Lukas Rammelmüller[1], Joaquín E. Drut[2], Jens Braun[1]

[1]Technische Universität Darmstadt, IKP Theory Center, [2]University of North Carolina at Chapel Hill, Department of Physics and Astronomy

Theoretical treatment of imbalanced Fermi systems is challenging. Exact analytic methods, if available, are limited to 1D setups and thus numerical treatment is often the only viable option. Among the most successful methods for balanced Fermi gases, in particular for systems beyond the few-body regime, are Quantum Monte Carlo (QMC) approaches. For imbalanced Fermi systems, however, these approaches suffer from an exponential scaling with system size: the infamous sign-problem. A way to circumvent this issue is provided by the complex Langevin method which we employ to 1D imbalanced fermions as well as the 3D unitary Fermi gas with finite spin asymmetry. [Parisi,Wu '81; Parisi '83; Aarts '09; Seiler et al. '17; Loheac, Drut '17; LR, Porter, Drut, Braun '17; LR, Loheac, Drut, Braun '18]

## complex Langevin in a nutshell

- after discretizing space and (imaginary) time and performing a Hubbard-Stratonovich transformation, we can write the partition sum $\mathcal{Z}$ as a path integral over the auxiliary field $\phi$:

$$\mathcal{Z} \equiv \mathrm{Tr}[e^{-\beta\hat{H}}] \rightarrow \int \mathcal{D}\phi\, e^{-S[\phi]}$$

- similarly, we can compute observables:

$$\langle\hat{\mathcal{O}}\rangle = \frac{\mathrm{Tr}[\hat{\mathcal{O}}e^{-\beta\hat{H}}]}{\mathrm{Tr}[e^{-\beta\hat{H}}]} \rightarrow \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\, \mathcal{O}[\phi] e^{-S[\phi]}$$

- key idea of stochastic quantization: a $(d+1)$-dimensional random process is used to sample the measure of a $d$-dimensional euclidean path integral

$$\frac{\partial\phi}{\partial t} = -\frac{\delta S[\phi]}{\delta\phi} + \eta$$

- with a discrete Langevin equation we can generate a Markov chain of complexified auxiliary fields $\phi$ that can be used to compute observables stochastically

## 1D fermions in the ground state: polarization & mass imbalance

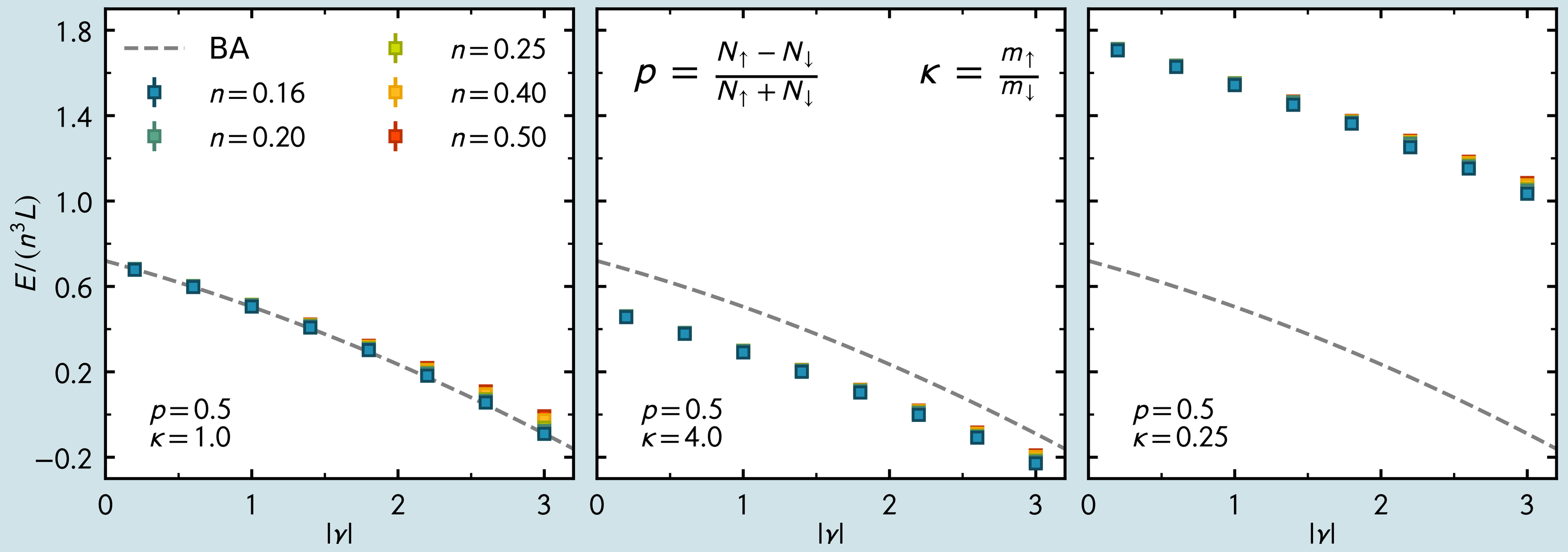
[LR, Porter, Drut, Braun '17; LR, Drut, Braun '18; LR, Drut, Braun *in preparation*]

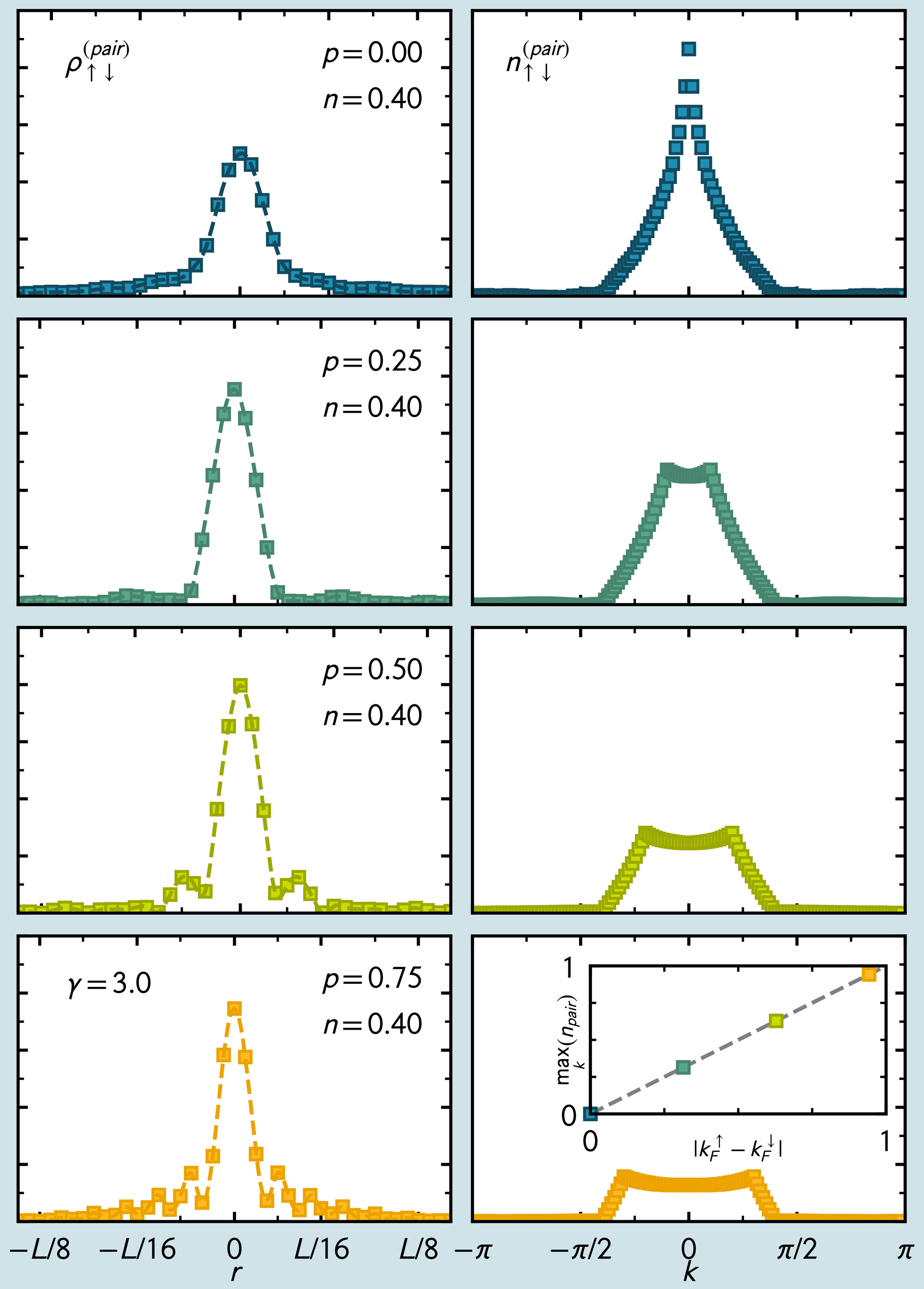
$$\hat{H} = -\sum_{s=\uparrow,\downarrow}\int \mathrm{d}^d x\, \hat{\psi}_s^\dagger(\vec{x})\left(\frac{\hbar^2\vec{\nabla}^2}{2m_s}\right)\hat{\psi}_s(\vec{x}) + g\int \mathrm{d}^d x\, \hat{\psi}_\uparrow^\dagger(\vec{x})\,\hat{\psi}_\uparrow(\vec{x})\,\hat{\psi}_\downarrow^\dagger(\vec{x})\,\hat{\psi}_\downarrow(\vec{x})$$

### energy equation of state

- excellent agreement of GS energies with exact solution in the thermodynamic limit (Bethe Ansatz) [BA: Iida, Wadati '07; Tracy, Widom '16]
- convergence to zero-range limit with decreasing density $n$
- experimentally accessible, e.g. Fermi-Fermi mixtures of

$$^{6}\mathrm{Li} + {}^{40}\mathrm{Ka} \quad\rightarrow\quad \kappa \simeq 6.67$$
$$^{40}\mathrm{Ka} + {}^{161}\mathrm{Dy} \quad\rightarrow\quad \kappa \simeq 4$$




### on-site pair-correlation function:

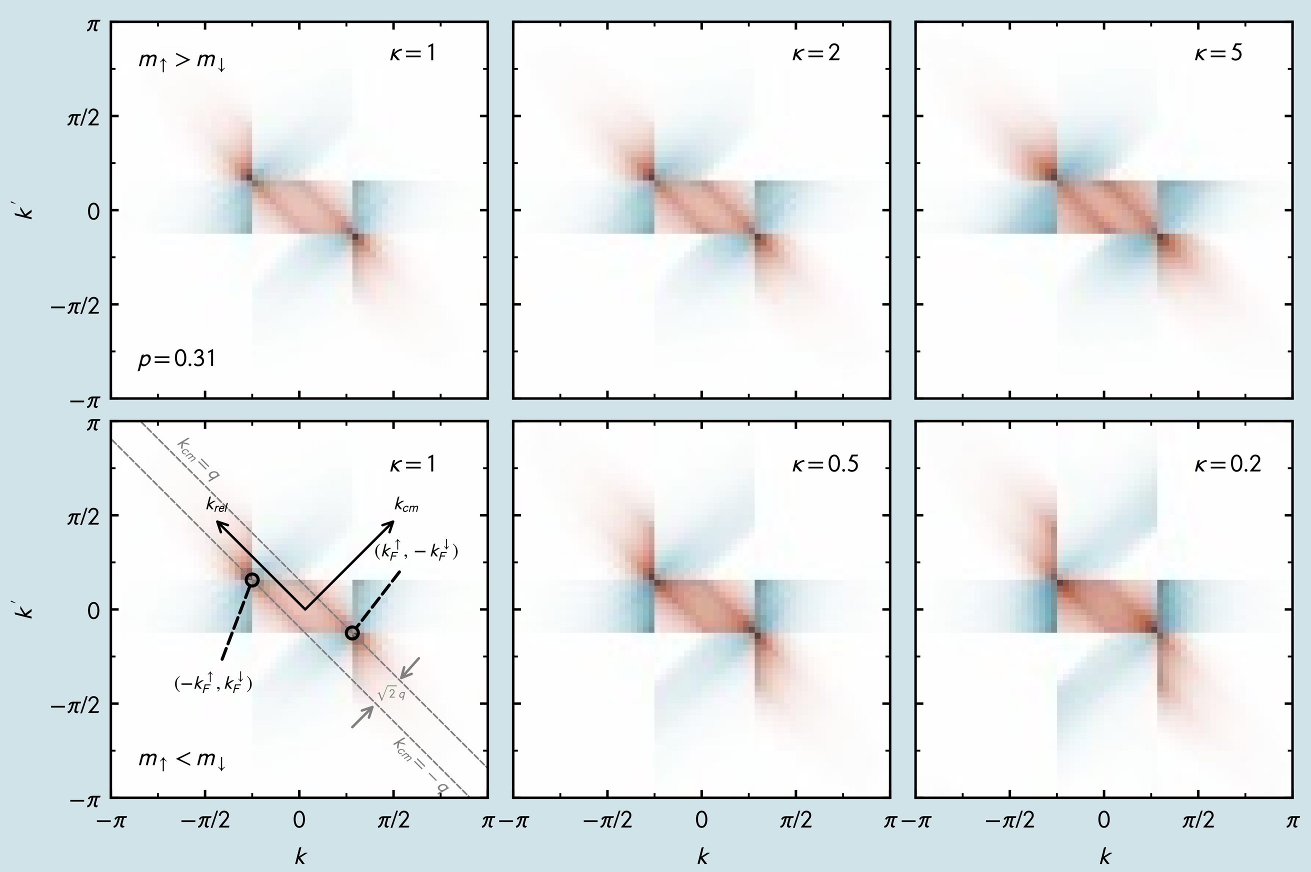
$$\rho_{pair}(|x-x'|) = \langle\hat{\psi}_\uparrow^\dagger(x)\,\hat{\psi}_\downarrow^\dagger(x)\,\hat{\psi}_\downarrow(x')\,\hat{\psi}_\uparrow(x')\rangle$$

- off-center peak in pair-momentum distribution at $q = |k_F^\uparrow - k_F^\downarrow|$
- spatially oscillating "order parameter" (inhomogeneous pairing)

### density-density correlation function (momentum space):

$$n_{\uparrow\downarrow}(k,k') = \langle\hat{\psi}_{k\uparrow}^\dagger\,\hat{\psi}_{k\uparrow}\hat{\psi}_{k'\downarrow}^\dagger\,\hat{\psi}_{k'\downarrow}\rangle - \langle\hat{\psi}_{k\uparrow}^\dagger\,\hat{\psi}_{k\uparrow}\rangle\langle\hat{\psi}_{k'\downarrow}^\dagger\,\hat{\psi}_{k'\downarrow}\rangle$$

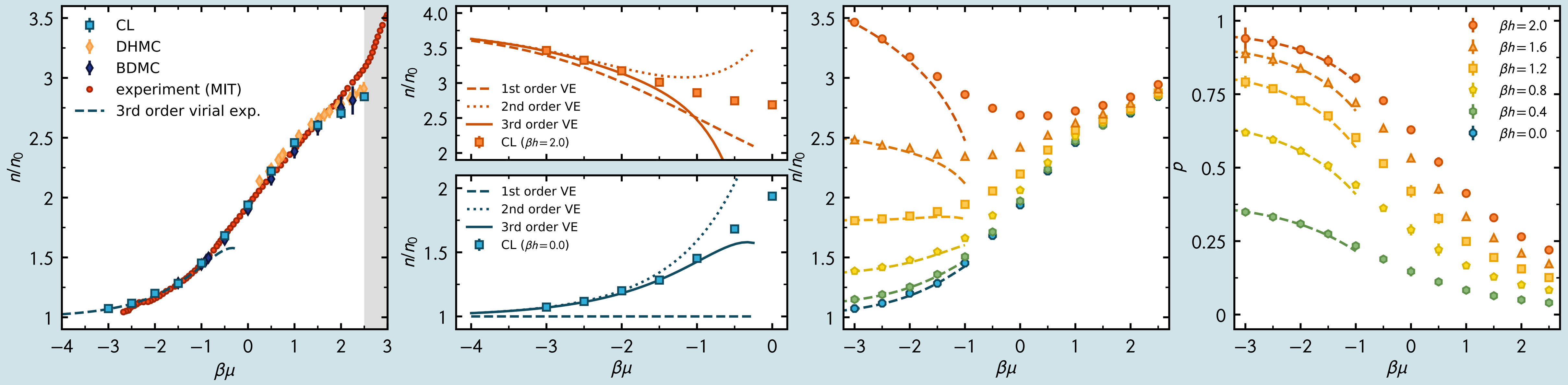
- clean signal of FFLO-type pairing at $(\pm k_F^\uparrow, \mp k_F^\downarrow)$
- peak position in pair-momentum distribution remains constant despite increasing mass imbalance, density-density correlator shows difference in structure




## 3D unitary fermions with finite spin asymmetry

[LR, Loheac, Drut, Braun '18; LR, Drut, Braun *in preparation*]

$$\mathcal{Z} = \mathrm{Tr}[\mathrm{e}^{-\beta(\hat{H}-\mu_\uparrow\hat{N}_\uparrow-\mu_\downarrow\hat{N}_\downarrow)}] = \mathrm{Tr}[\mathrm{e}^{-\beta(\hat{H}-\mu\hat{N}-h\hat{M})}]$$




### density equation of state & polarization

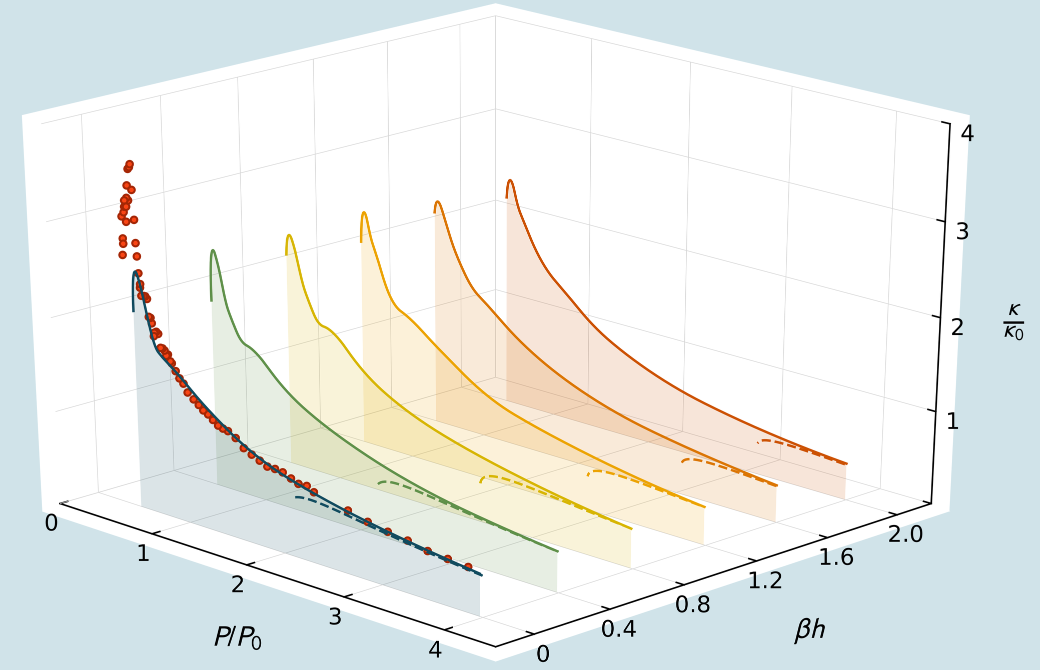
- balanced gas: excellent agreement with experimental results and state-of-the-art numerical data [experiment/BDMC: Van Houcke et al. '12; DHMC: Drut, Lähde, Wlazlowski, Magierski '12]
- low temperatures: thermal wavelength $\lambda_T$ increases, finite volume effects visible (currently $V = 11^3$)
- virial expansion (VE): valid at high temperature $(\beta\mu < 0)$, provides an important benchmark for polarized systems
- CL shows excellent agreement with 3rd order VE for all polarizations studied

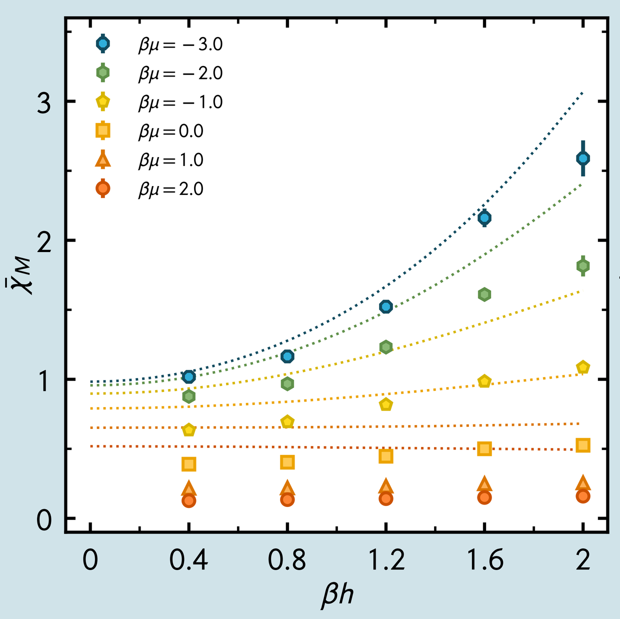
### isothermal compressibility $\kappa = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_{T,V,h}$

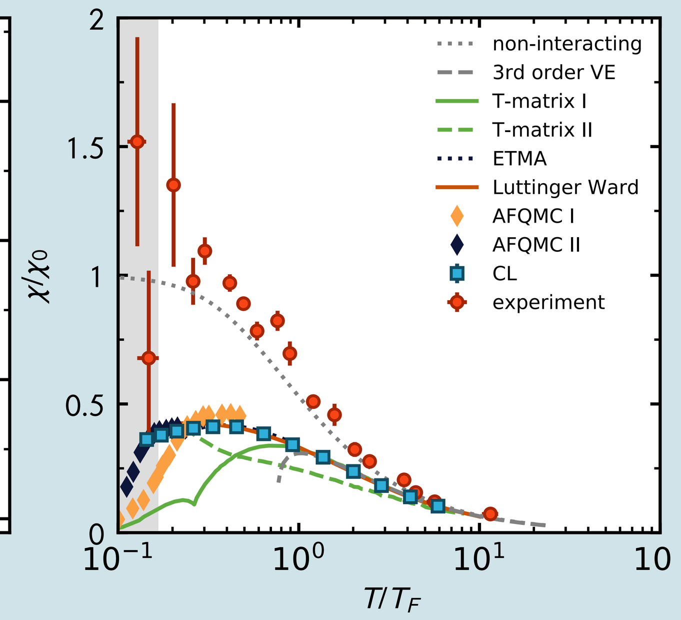
- agrees well with experiment & VE in the balanced case [experiment: Ku et al. '12]
- peak shows weak dependence on polarization, suggests a flat phase-boundary near the balanced limit

### spin susceptibility $\chi = \left(\frac{\partial m}{\partial h}\right)_{T,V,\mu}$

- functional form similar to non-interacting Fermi gas (Pauli susceptibility) but rescaled
- zero polarization: theoretical methods agree at high-temperature, experimental values disagree [experiment: Sommer et al. '11; T-matrix: Pantel et al. '14, Palestini et al. '12; ETMA: Kashimura et al. '12; Luttinger-Ward: Enns et al. '12; AFQMC: Wlazlowski et al. '13, Jensen et al. '18]
- low temperature: CL suggests absence of pseudogap, however, accuracy needs to be increased




HIC for FAIR Helmholtz International Center
DFG Deutsche Forschungsgemeinschaft German Research Foundation
HGS-HIRe for FAIR
TECHNISCHE UNIVERSITÄT DARMSTADT