Pairing in imbalanced 1D Fermi gases

[LR, J.E. Drut, J. Braun, SciPost Phys. 9, 014 (2020)]

Lukas Rammelmüller, LMU Munich

Superfluctuations 2021, recorded talk

slides are available at `rammelmueller.github.io`
for questions please drop me a line at `lukas.rammelmueller@lmu.de`

European Research Council
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outline

part I

brief motivation and **quick reminder on pairing**

(pairing with finite spin imbalance)

technical intermezzo

many-body treament with **complex Langevin**

part II

results: **pairing patterns for spin- and mass-imbalanced 1D fermions**

(pair momentum distributions & noise correlations)

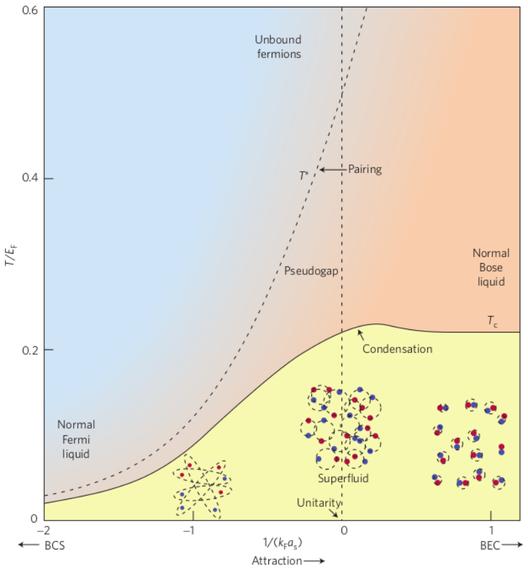
Fermi gases with short-ranged interaction

[Giorgini,Pitaevskii,Stringari '08]

$$\hat{H} = -\sum_{s=\uparrow,\downarrow} \int \mathrm{d}x\, \hat{\psi}_s^\dagger(\vec{x}) \left(\frac{\hbar^2 \vec{\nabla}^2}{2m_s} \right) \hat{\psi}_s(\vec{x}) \quad + \quad g \int \mathrm{d}x\, \hat{\psi}_\uparrow^\dagger(\vec{x})\, \hat{\psi}_\uparrow(\vec{x})\, \hat{\psi}_\downarrow^\dagger(\vec{x})\, \hat{\psi}_\downarrow(\vec{x})$$

(attractive) contact interaction

- well studied problem (experiment & theory): BCS-BEC crossover
  [Zwerger '12]
- **unequal spin populations**: what exactly happens to pair formation?
  [Chevy,Mora '10; Radzihovsky,Sheehy '10; Gubbels,Stoof '13]
- less is known for mixtures of **unequal masses**



[3D BCS-BEC phase diagram (sketch): Randeria '10]

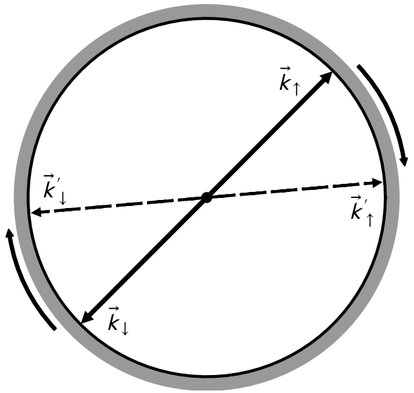
pairing in a nutshell

spin-balanced

unequal spin populations

pairing in a nutshell

spin-balanced

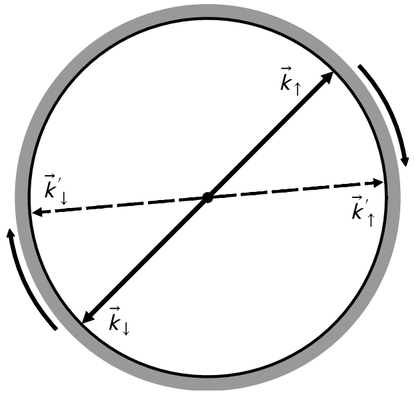


"conventional"
BCS type pairing

unequal spin populations

pairing in a nutshell

spin-balanced



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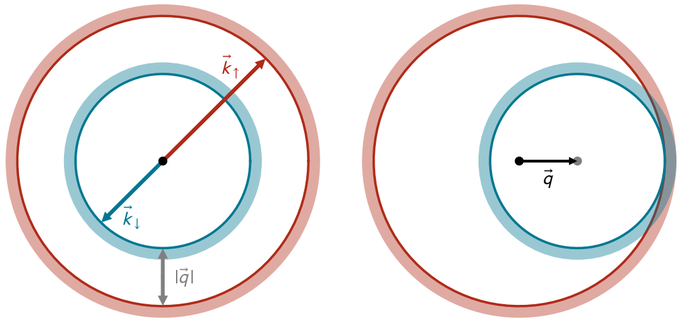

FFLO type pairing
[Fulde,Ferell '64, Larkin,Ovchinnikov '64]

pairing in a nutshell

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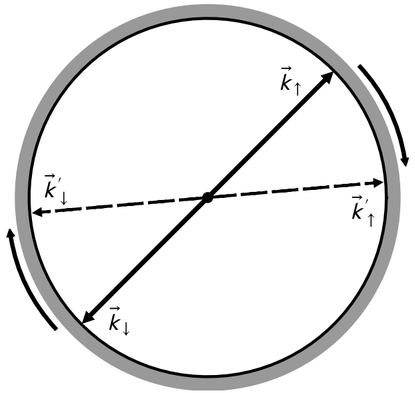

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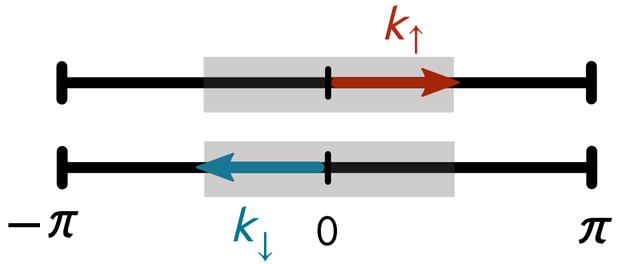
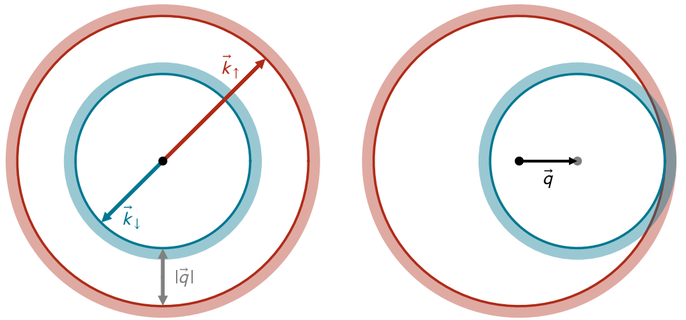


unequal spin populations



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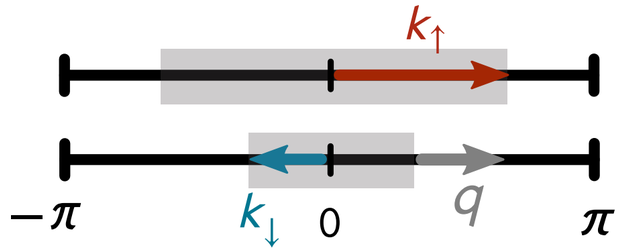


technical digression: complex Langevin

[recent review: Berger et al. '20]

$$\langle \hat{\mathcal{O}} \rangle = \frac{\mathrm{Tr}[\hat{\mathcal{O}}\, \mathrm{e}^{-\beta \hat{H}}]}{\mathrm{Tr}[\mathrm{e}^{-\beta \hat{H}}]} \overset{T \to 0}{=} \frac{\langle \psi_0 | \hat{\mathcal{O}} | \psi_0 \rangle}{\langle \psi_0 | \psi_0 \rangle}$$

- numerical approach: **path-integal sampling** (involves spacetime discretization)

  [lattice methods: Lee '09; Drut,Nicholson '13; Zhang '13]

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\phi \; P[\phi] \mathcal{O}[\phi] = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi \; \mathrm{det} M_{\uparrow}[\phi] \mathrm{det} M_{\downarrow}[\phi] \mathcal{O}[\phi] \equiv \int \mathcal{D}\phi \; \mathcal{O}[\phi] e^{-S_{\mathrm{eff}}[\phi]}$$

- to avoid efficiency issues (sign-problem): **complex stochastic quantization**

  [Parisi,Wu '81; Damgaard,Hüffel '87]

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$

spin-polarized 1D fermions

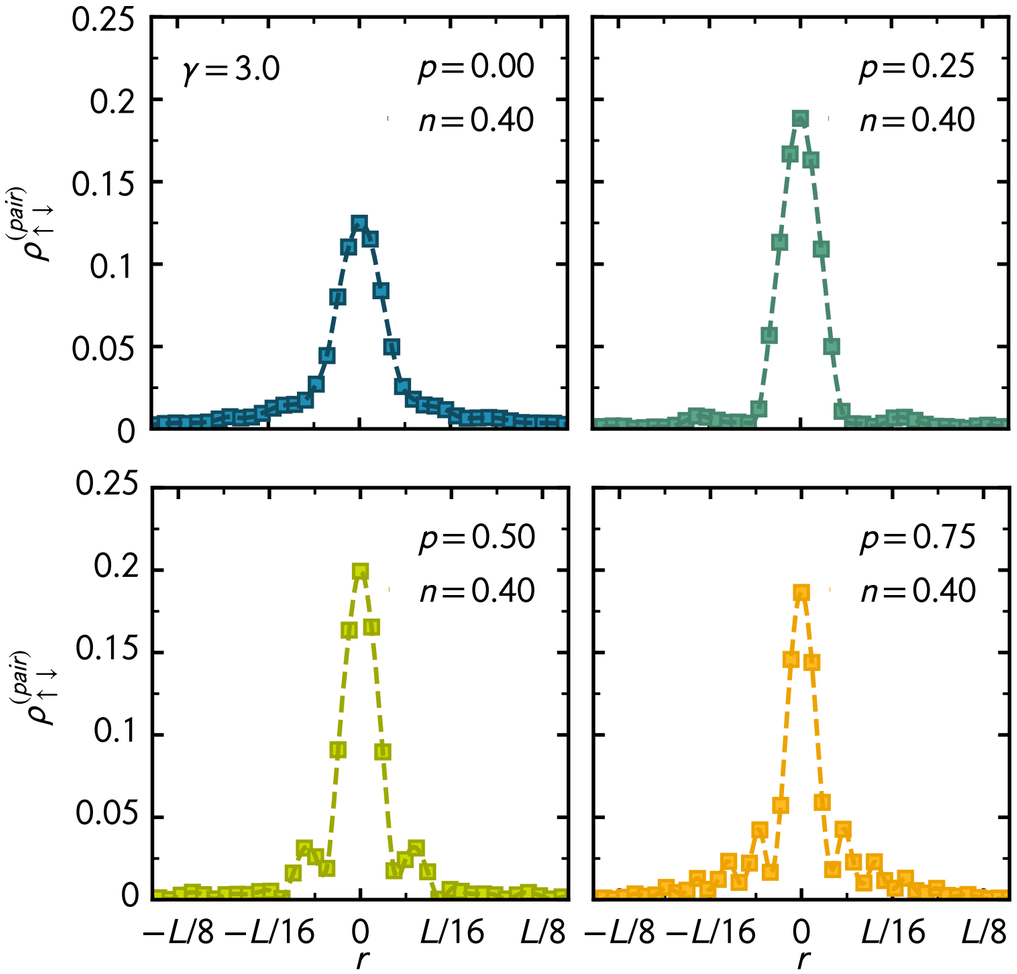
[Feiguin,Heidrich-Meisner,Orso,Zerger '12]

- (some) exact solutions available via **Bethe ansatz**
  [Guan,Batchelor,Lee '13]
- **FFLO type pairing** stable in a wide parameter range
  [Lüscher,Noack,Läuchli '08]
- no true long-range order in 1D, **polynomial decay of correlation functions:**

$$|C(r)| \propto r^{-\Delta}$$

pair density matrix

[LR, Drut, Braun '20]



$$\rho_{\uparrow\downarrow}(x,x') \equiv \langle \hat{\psi}^\dagger\uparrow(x')\,\hat{\psi}^\dagger\downarrow(x')\;\hat{\psi}\downarrow(x)\,\hat{\psi}\uparrow(x)\rangle$$

~ (instantaneous) propagation of an on-site pair

- polynomial decay **+ spatial modulation**

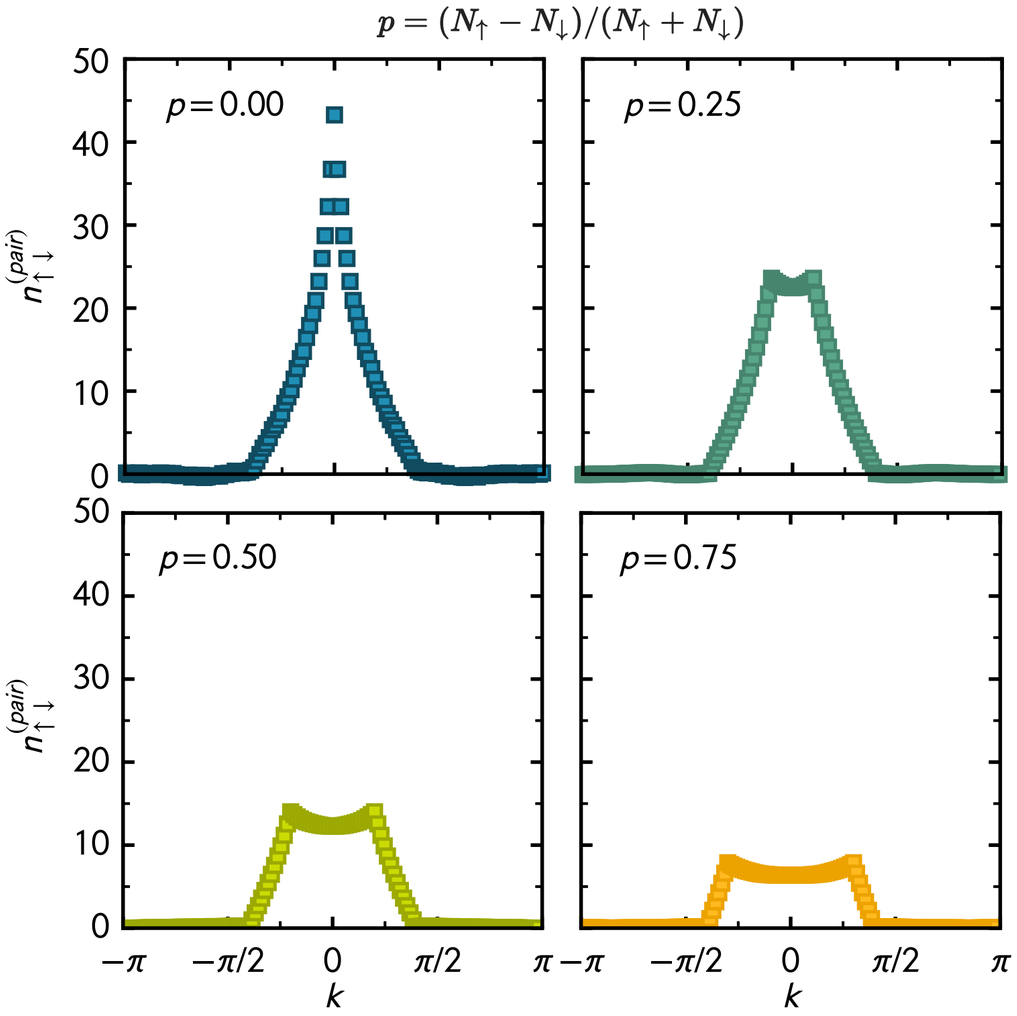
$$\rho_{\uparrow\downarrow}(x,x') \propto \frac{|\cos(q|x-x'|)|}{|x-x'|^{\Delta}}$$

- loose interpretation: **spatially fluctuating "order parameter"**, excess majority particles reside at nodes

numerical results obtained with complex Langevin method (statistical error $\lesssim$ symbol size)

pair momentum distribution

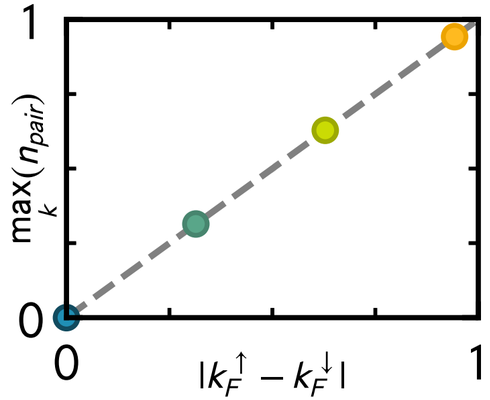
[LR, Drut, Braun '20]



$$n_{\uparrow\downarrow}(k) = \int \mathrm{d}p\, \mathrm{d}q\, \langle \hat{\psi}^\dagger_{-p-k,\uparrow}\, \hat{\psi}^\dagger_{p,\downarrow}\, \hat{\psi}_{k-q,\downarrow}\, \hat{\psi}_{q,\uparrow} \rangle$$

~ likelihood to find a pair with momentum k

off-center peak:
hallmark of **FFLO type pairing**



numerical results obtained with complex Langevin method (statistical error $\lesssim$ symbol size)

mass-imbalanced systems

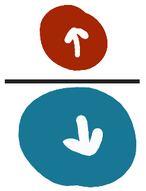
[LR, Drut, Braun '20]

heavy majority

heavy minority

- imbalance characterized through mass-ratio $\kappa = \frac{m_\uparrow}{m_\downarrow}$

$\kappa =$  ≥ 1

$\kappa =$  ≤ 1

- experimentally accessible mixtures: ^{6}Li - ^{40}K, ^{6}Li - ^{53}Cr, ^{40}K - ^{161}Dy, ...

[Taglieber et al. '08; Wille et al. '08, Voigt et al. '09; Naik et al. '11; Ravensbergen et al. '18; '20; Neriet al. '20]

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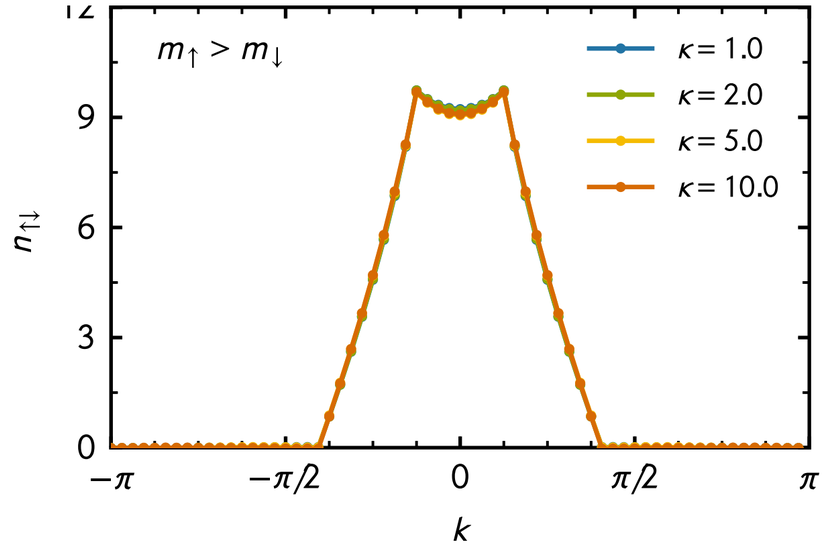
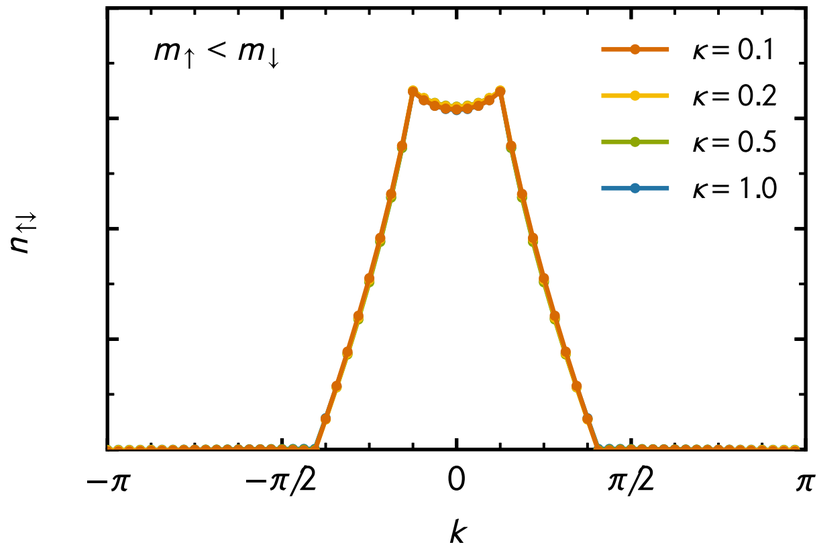


heavy minority

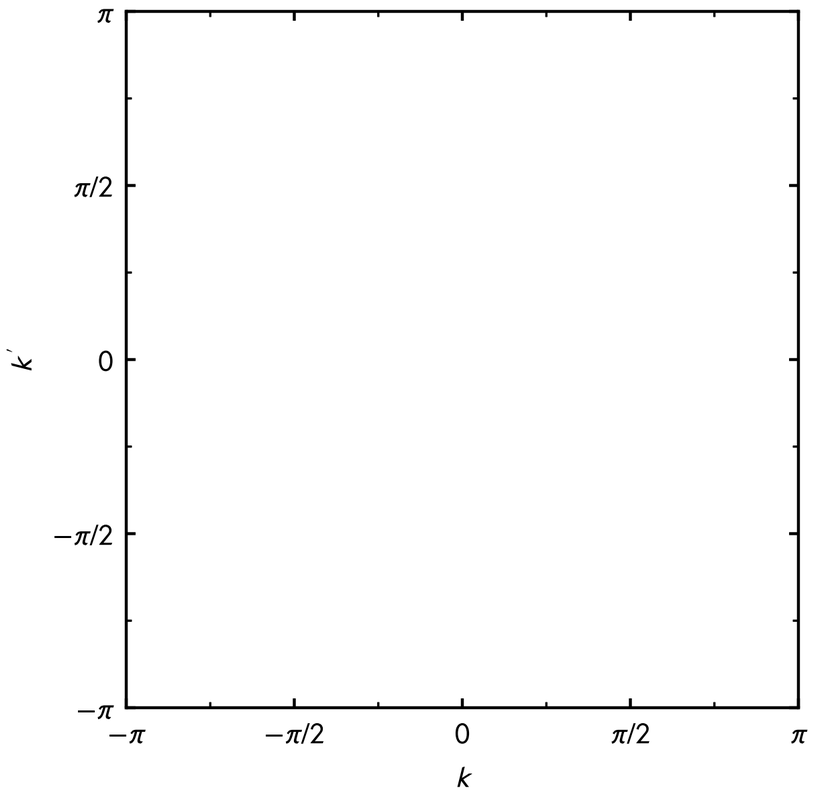


(essentially) no change up to large mass imbalance

better probes: noise correlations

[Mathey '04; Luescher,Laeuchli,Noack '07; LR, Drut, Braun '20]

$$G_{\uparrow\downarrow}(k,k') = \langle \delta\hat{n}_{k\uparrow}\delta\hat{n}_{k'\downarrow} \rangle = \langle \hat{n}_{k\uparrow}\hat{n}_{k'\downarrow} \rangle - \langle \hat{n}_{k\uparrow}\rangle\langle \hat{n}_{k'\downarrow} \rangle$$



- interpretation: covariance matrix of momentum-densities
- exhibits clear signatures of pairing fuctuations

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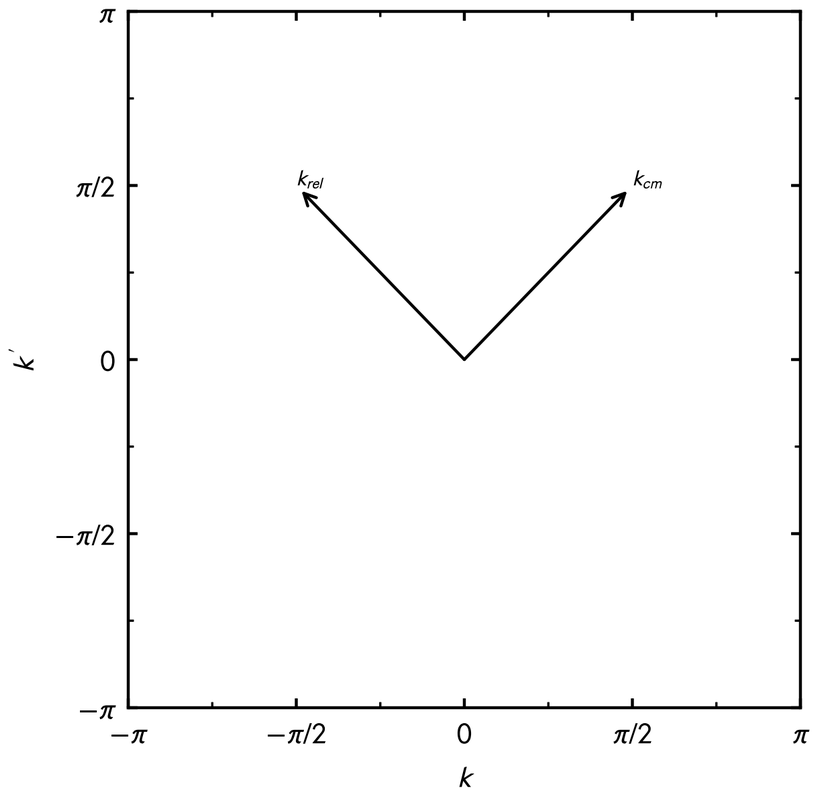

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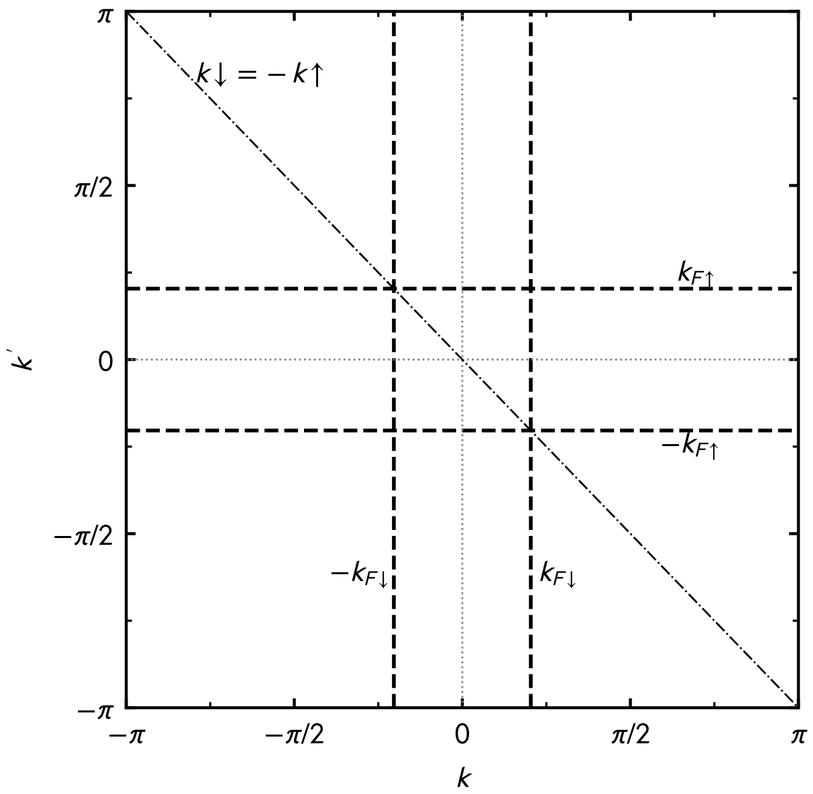

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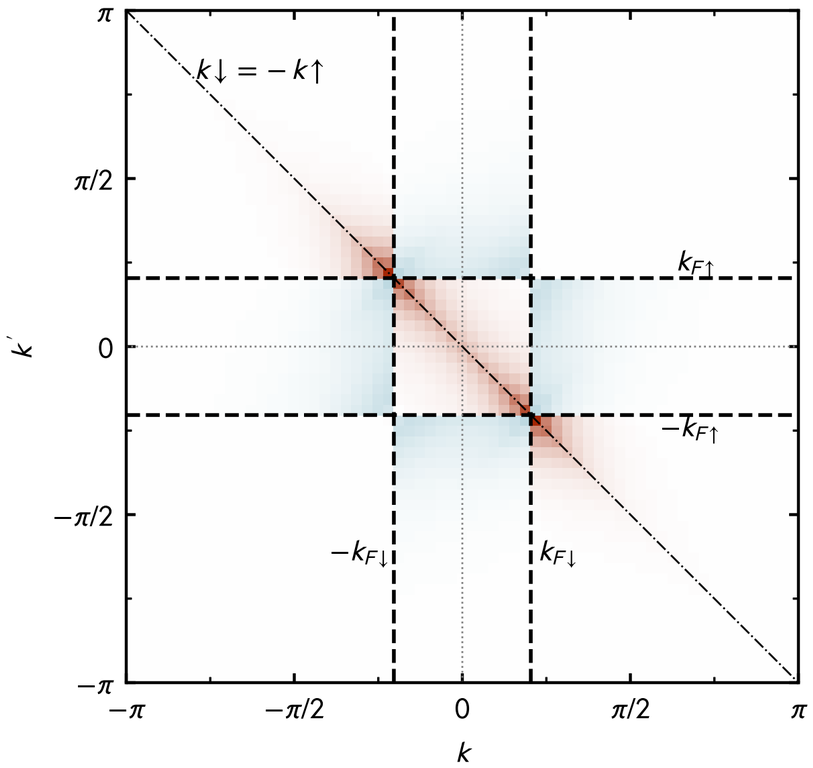

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positive correlations: particle-particle
negative correlations: particle-hole

shot noise for imbalanced systems (heavy majoriy)

[LR, Drut, Braun '20]

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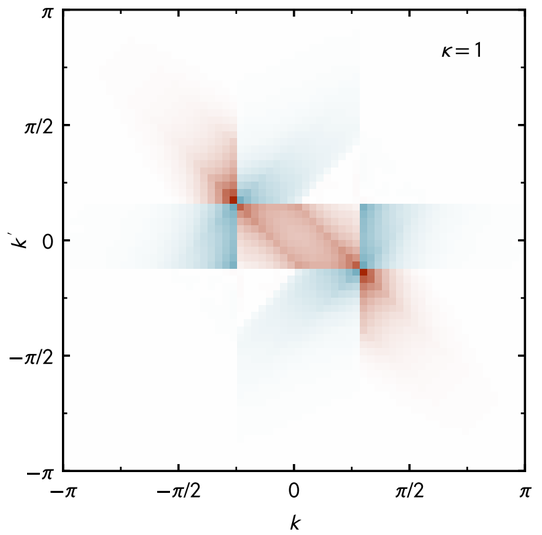

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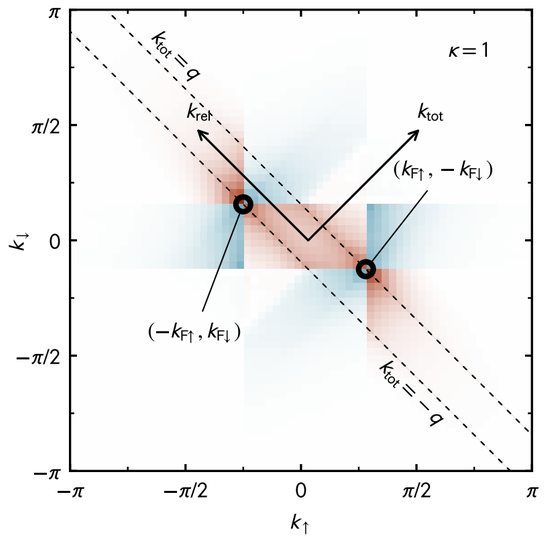

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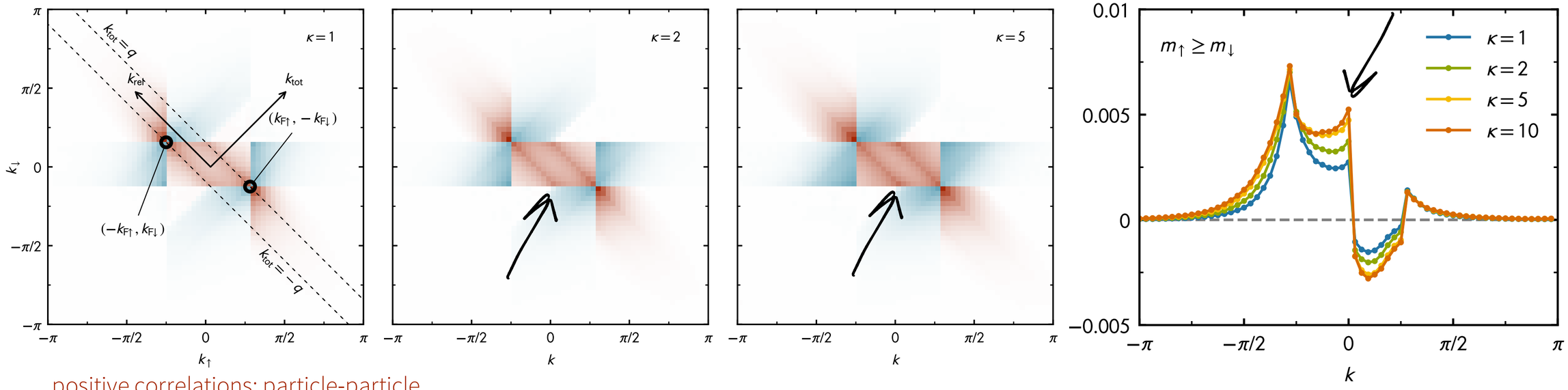

positive correlations: particle-particle

negative correlations: particle-hole

secondary peaks emerge with growing mass ratio

shot noise for imbalanced systems (heavy majoriy)

[LR, Drut, Braun '20]

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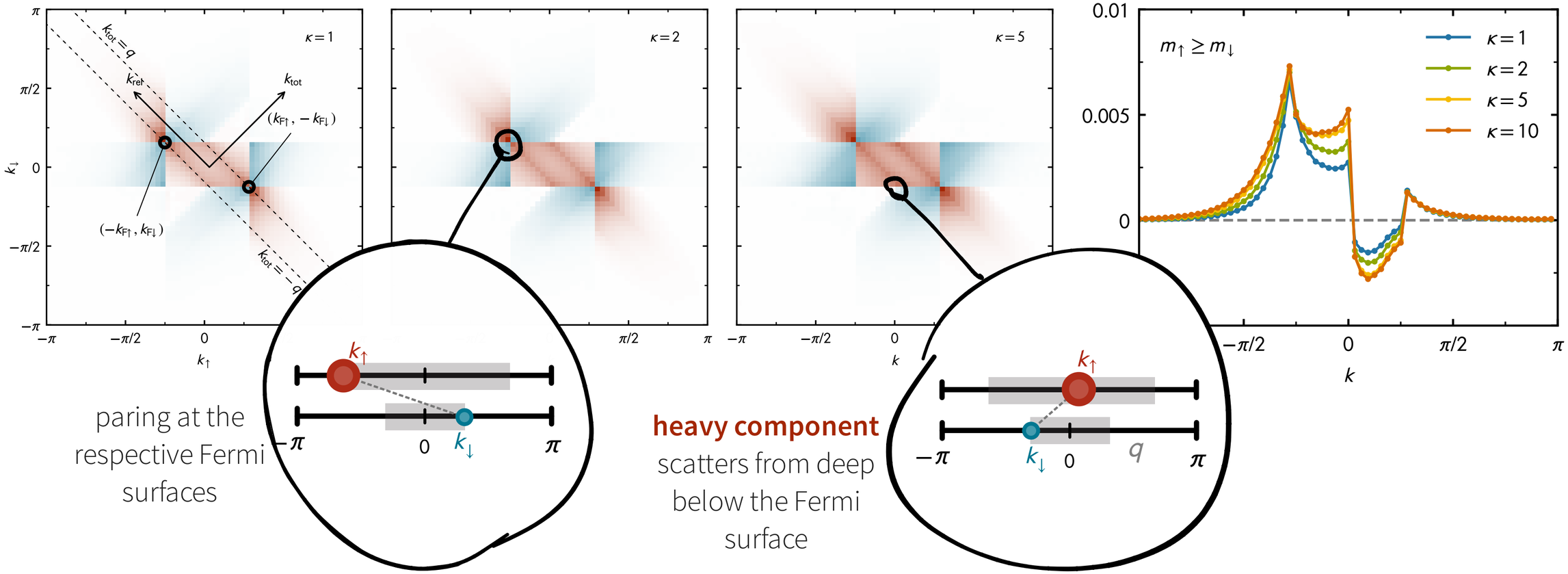
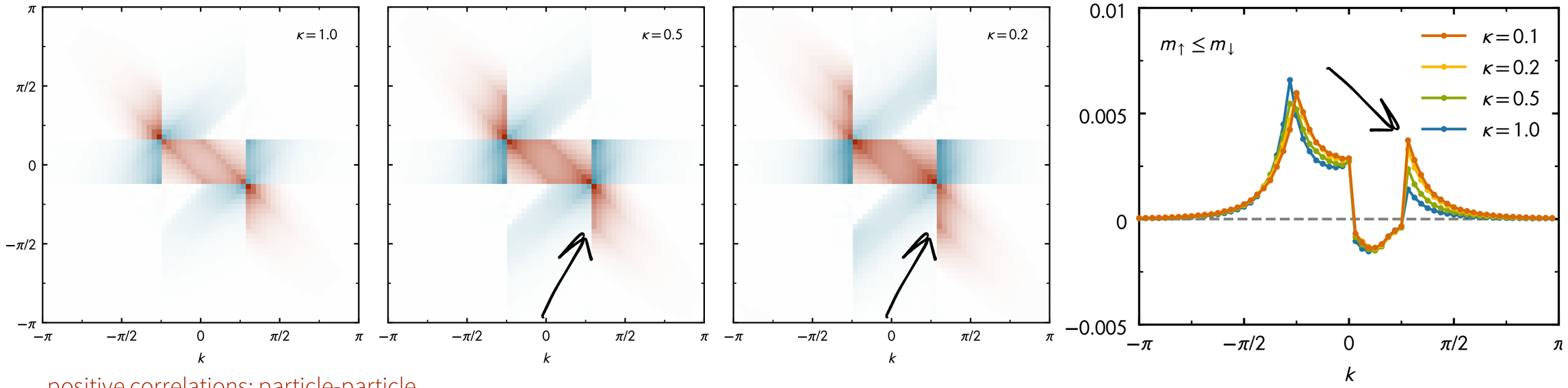


paring at the respective Fermi surfaces

heavy component scatters from deep below the Fermi surface

shot noise for imbalanced systems (heavy minority)

[LR, Drut, Braun '20]



positive correlations: particle-particle

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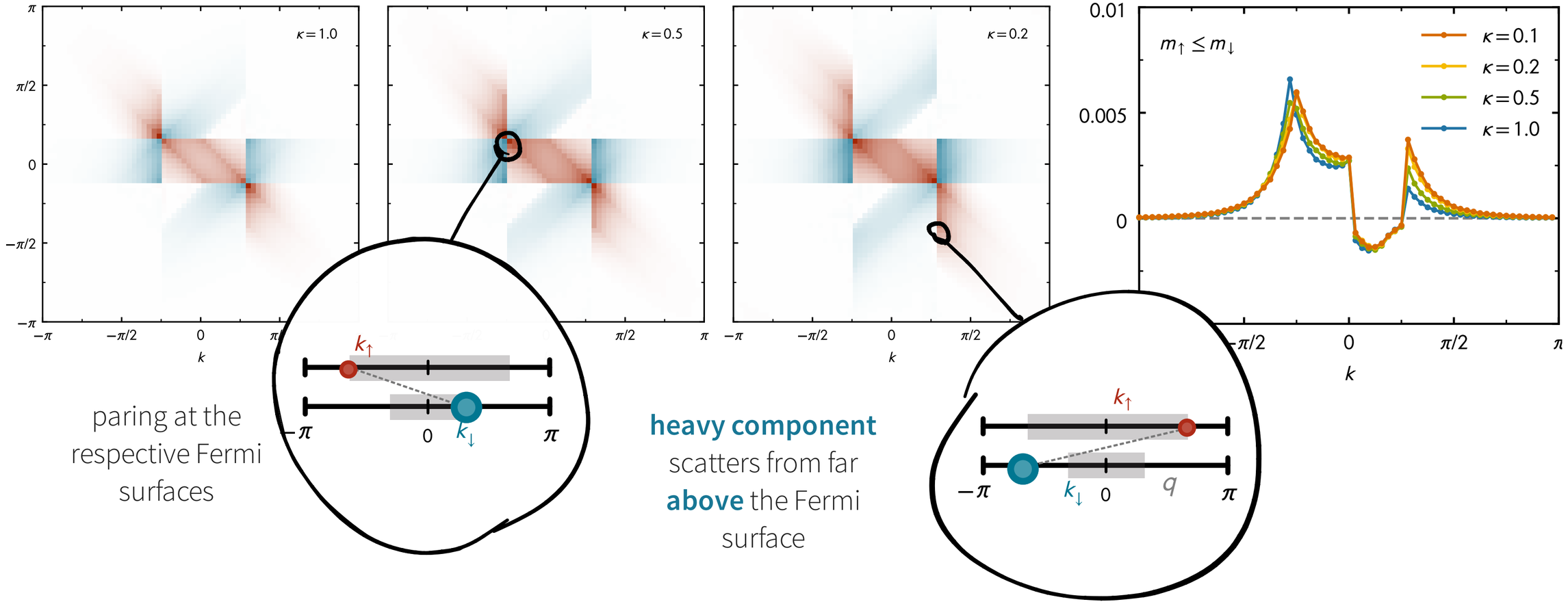
reversed situation for heavy-minority

shot noise for imbalanced systems (heavy minority)

[LR, Drut, Braun '20]



paring at the respective Fermi surfaces

heavy component scatters from far **above** the Fermi surface

recap & future directions

analysis of noise correlations showed
novel, sub-leading type of pairing that develops
for spin-polarized **1D fermions with growing mass ratio**

analysis of noise correlations useful **beyond simple**
systems and noise correlations in 2D/3D could potentially help to identify
FFLO type pairing in experiments