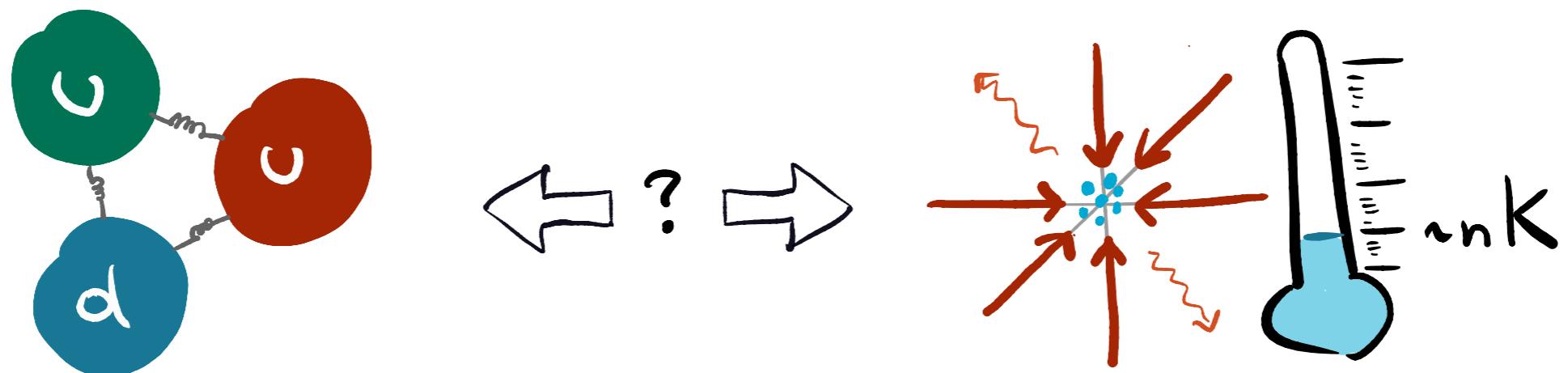
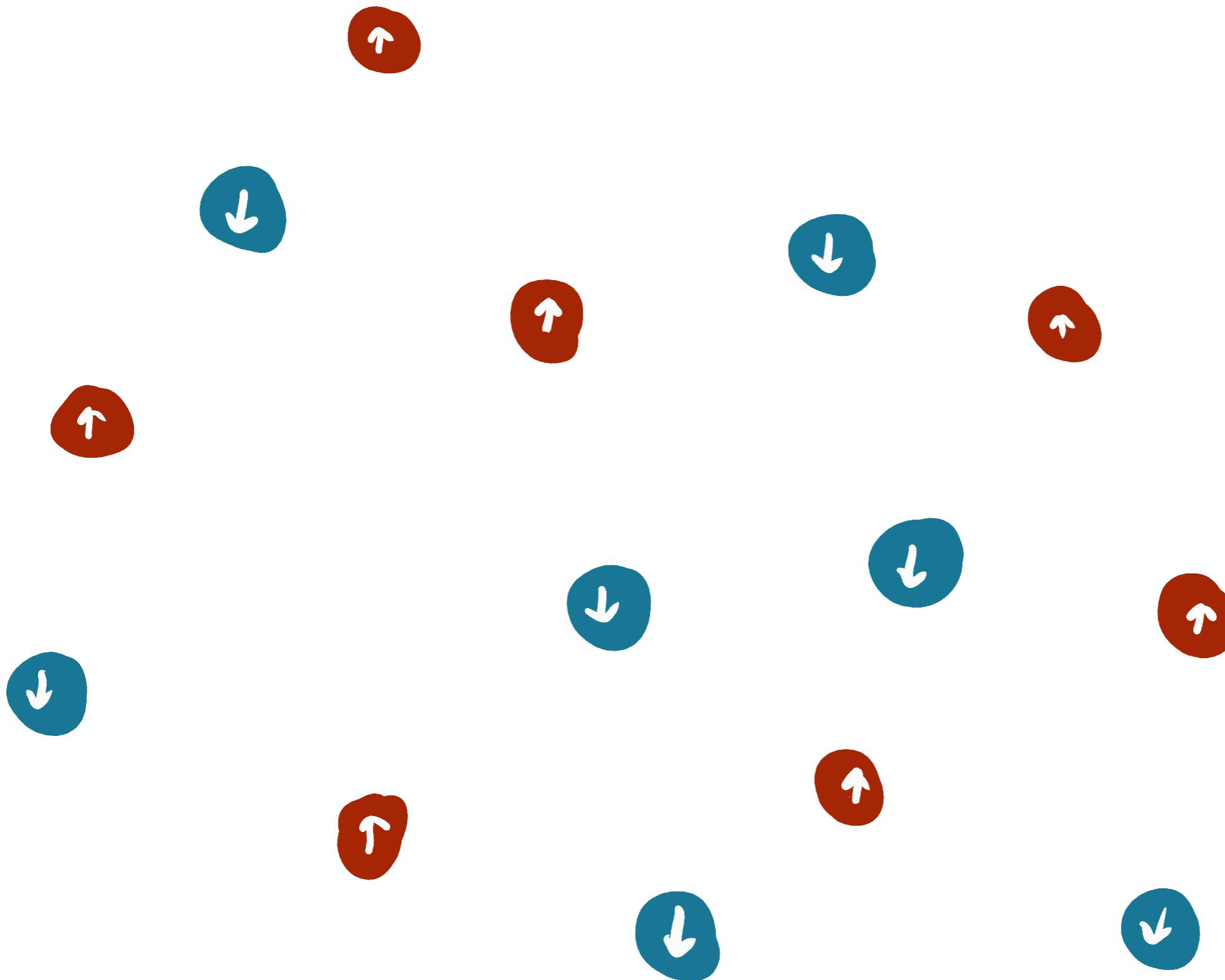


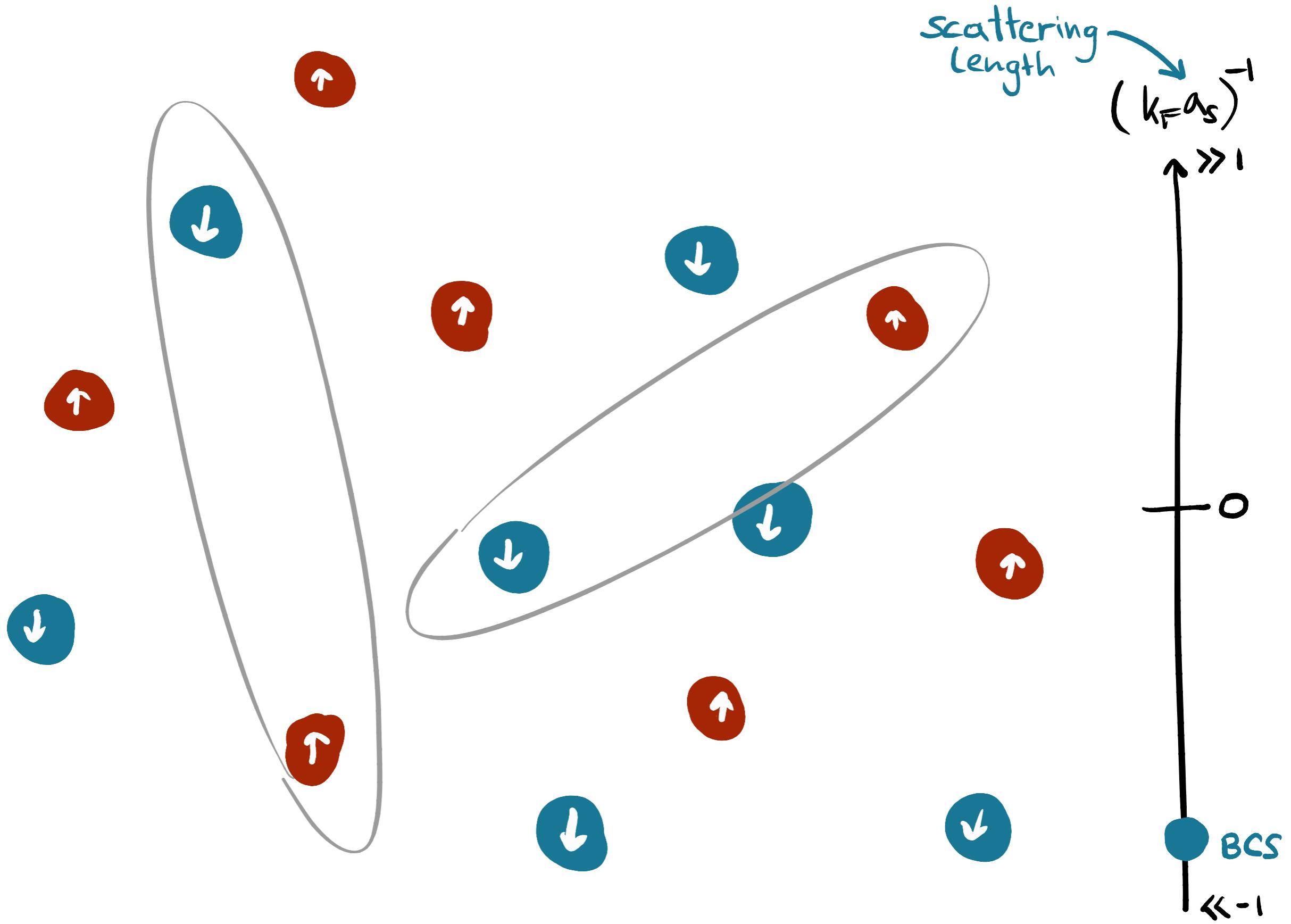
Approaching imbalanced Fermi gases via complex Langevin

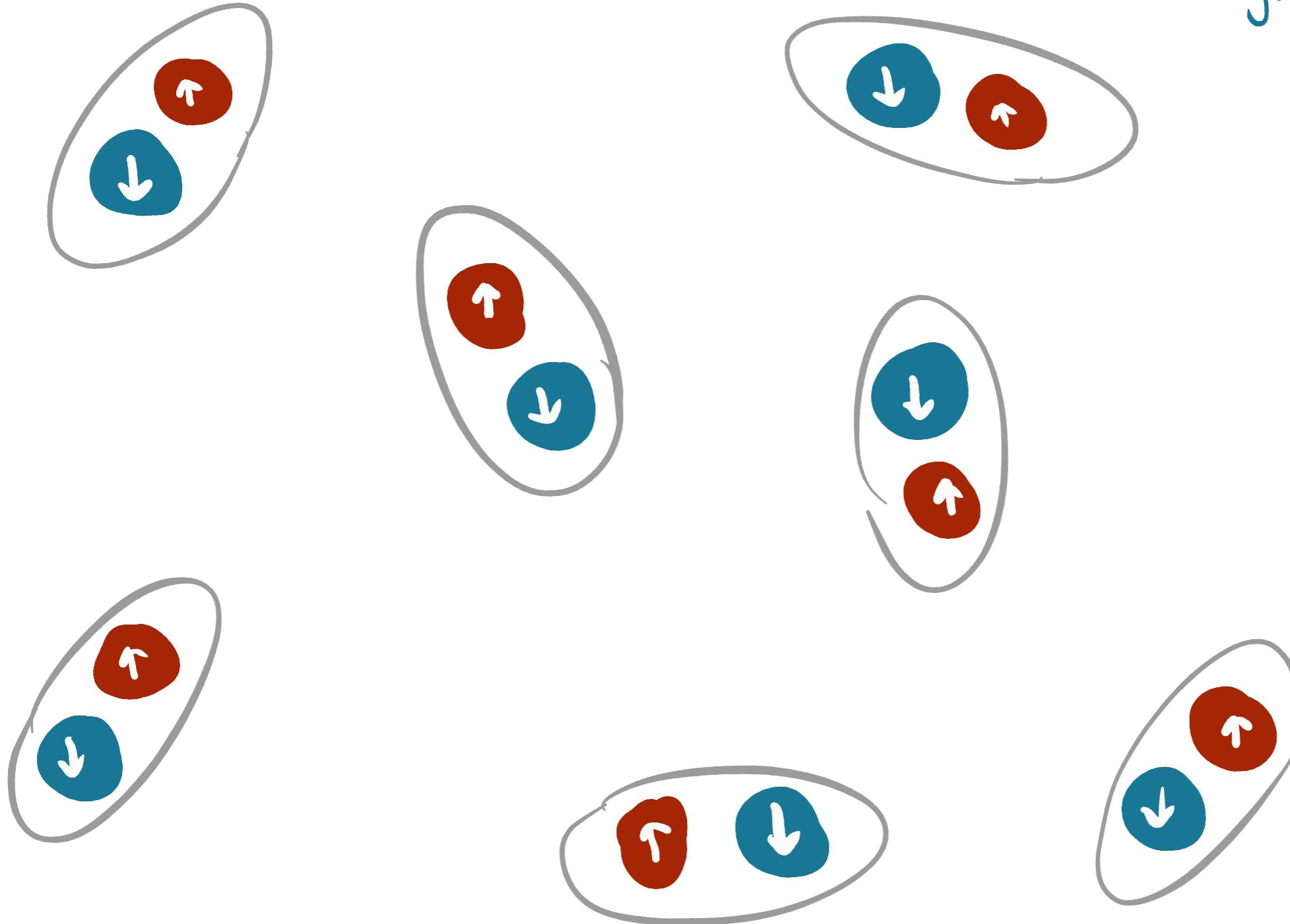


Lukas Rammelmüller, TU Darmstadt

ECT* workshop "High-energy physics at ultra-cold temperatures" - June 10, 2019





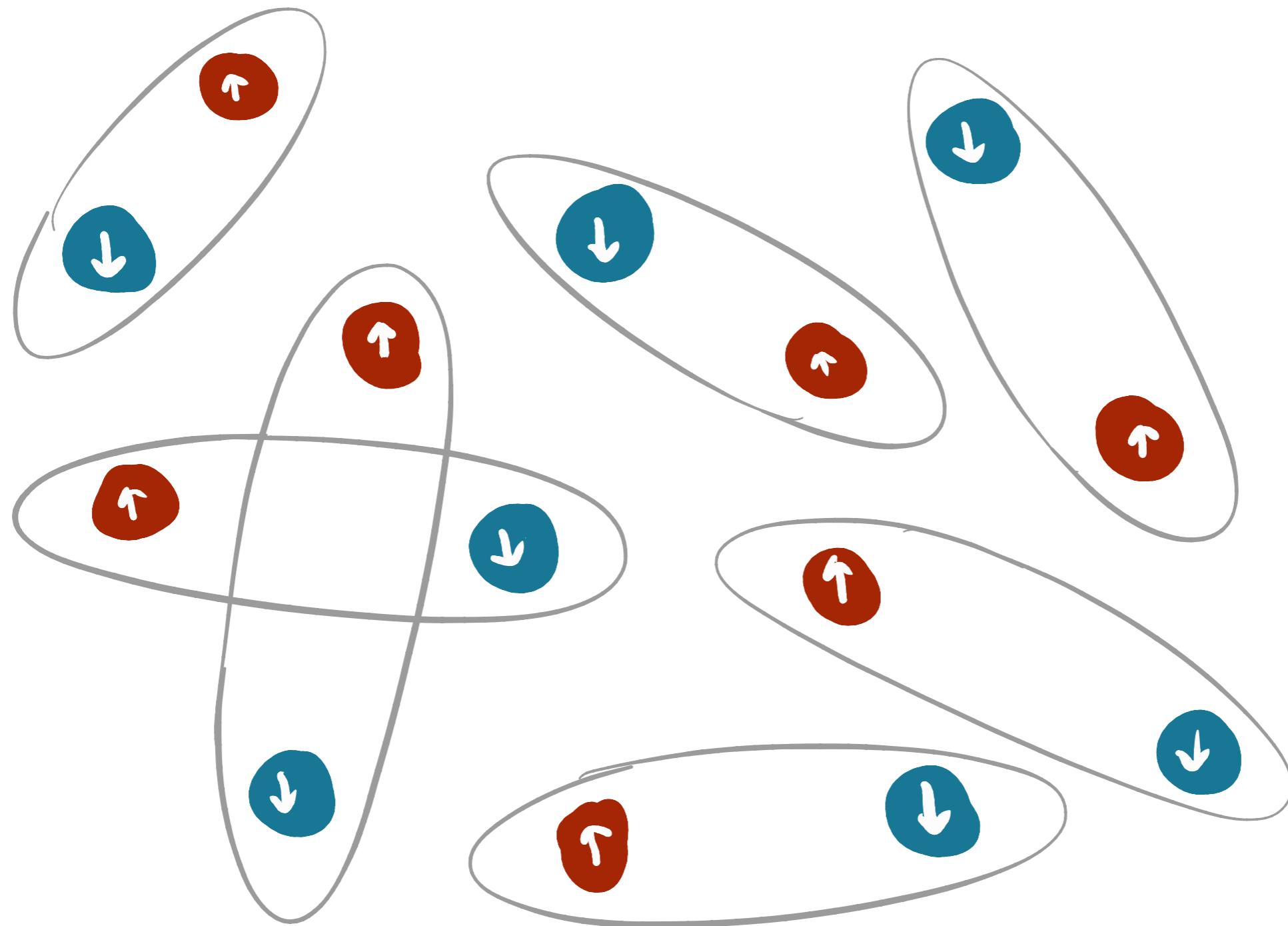


Scattering length
 $(k_F a_S)^{-1}$
 $\uparrow \gg 1$
BEC

0

$\ll -1$

$$a_S \gg n^{-1/3} \gg r_0$$



$$(k_F a_S)^{-1} \gg 1$$

the unitary Fermi gas (UFG)

[reviews: Zwerger '12; Mukaiyama,Ueda '13]

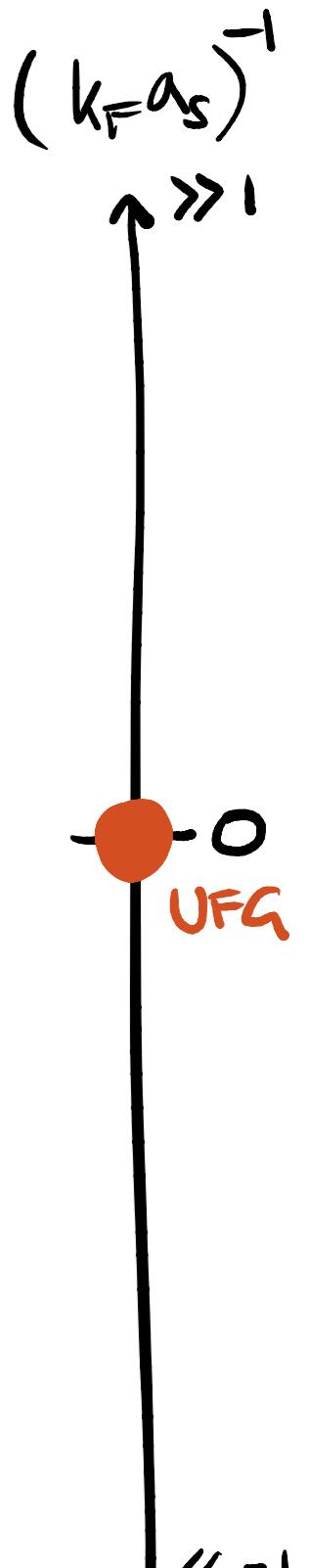
density & temperature are the **only** dimensionful scales in the system

universal scaling functions:

$$E = E_{FG} f_E(\beta\mu)$$

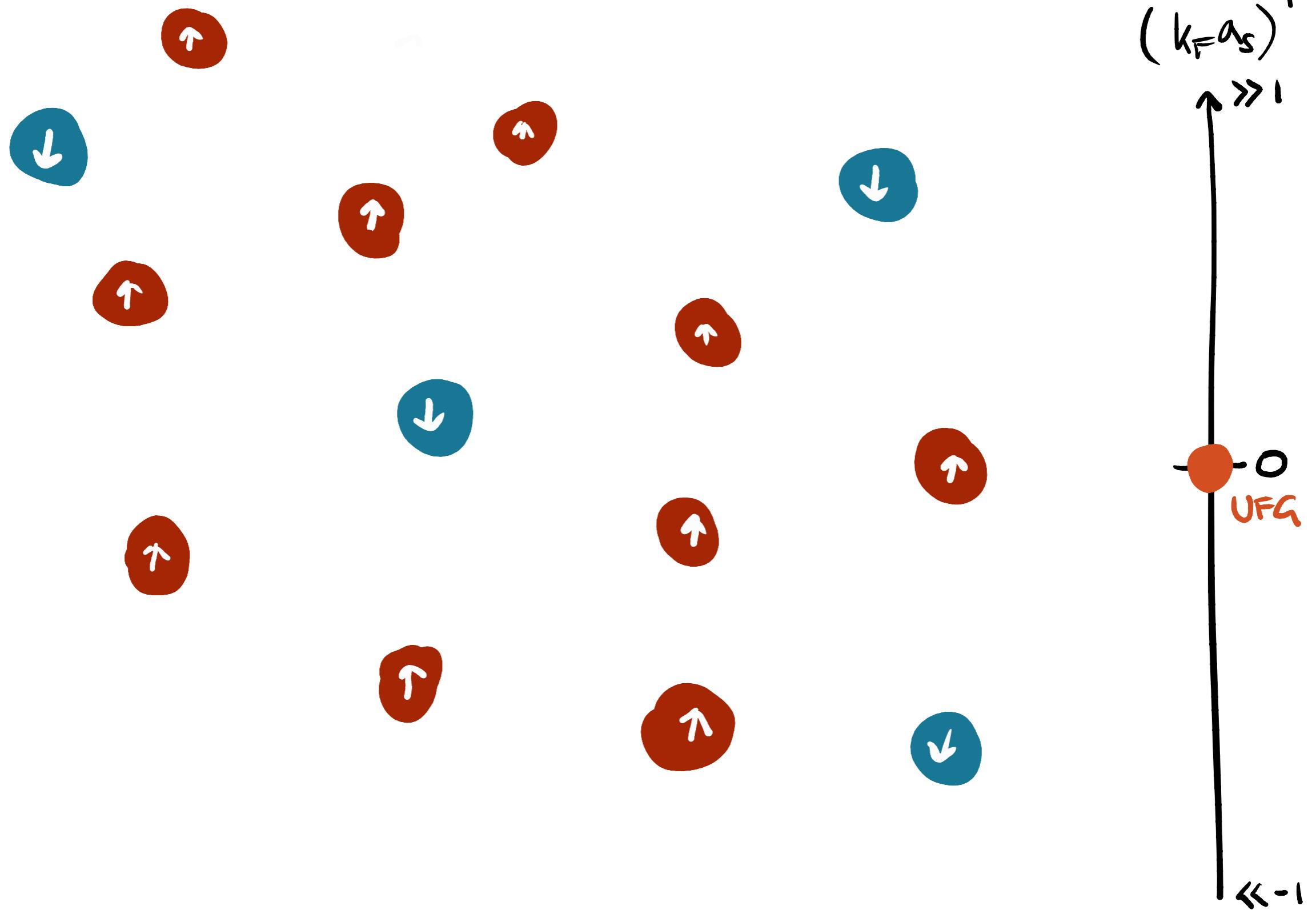
$$P = P_{FG} f_P(\beta\mu)$$

...



numerous experiments:

- first realizations of unitary fermions [Regal,Greiner,Jin '04; Zwierlein et al. '04; Kinast et al. '04]
- universal behavior & thermodynamics [Thomas,Kinast,Turpalov '05; Horikoshi et al. '10]
- temperature vs. polarization phase-diagram [Shin,Schunck,Schirotzek,Ketterle '08]
- measurement of equation of state [Nascimbène et al. '10; van Houcke et al. '12]
- superfluid transition [Ku,Sommer,Cheuck,Zwierlein '12]
- temperature dependence of Tan's contact [Carcy et al. '19; Mukherjee et al. '19]
- and many more...



[reviews: Chevy,Mora '10; Gubbels,Stoof '13]

agenda

part I

quick intro to **stochastic quantization & CL**
(what is it & how can it help us for fermions?)

part II

unitary fermions with finite polarization
(equations of state & thermodynamic response)

part III

correlation functions & pairing at finite polarization
(one-dimensional systems, preliminary)

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-S[\phi]}$$

(partition function)

$$\langle \hat{\mathcal{O}} \rangle = \int \mathcal{D}\phi \mathcal{O}[\phi] P[\phi]$$

(expectation values)

$$P[\phi] = \frac{1}{\mathcal{Z}} e^{-S[\phi]}$$

(probability measure)

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-S[\phi]}$$

(partition function)

$$\langle \hat{\mathcal{O}} \rangle = \int \mathcal{D}\phi \mathcal{O}[\phi] P[\phi]$$

(expectation values)

$$P[\phi] = \frac{1}{\mathcal{Z}} e^{-S[\phi]}$$

(probability measure)

key idea:

probability measure of a **d-dimensional Euclidean path integral** as equilibrium distribution of a **d+1-dimensional random process**

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

random walk governed by

Langevin equation (Brownian motion):

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

random walk governed by

Langevin equation (Brownian motion):

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$

fictitious Langevin time
(not physical)

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

random walk governed by

Langevin equation (Brownian motion):

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + n$$

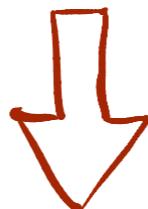
fictitious Langevin time
(not physical)

noise term

$$\langle n \rangle = 0$$
$$\langle n_t n_{t'} \rangle = 2\delta(t - t')$$

the Langevin method

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$



discretization

$$\phi^{(n+1)} = \phi^{(n)} - \left. \frac{\delta S[\phi]}{\delta \phi} \right|_{\phi^{(n)}} \Delta t_L + \sqrt{2 \Delta t_L} \tilde{\eta}$$

(Markov chain)

the Langevin method

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$



discretization

$$\phi^{(n+1)} = \phi^{(n)} - \left. \frac{\delta S[\phi]}{\delta \phi} \right|_{\phi^{(n)}} \Delta t_L + \sqrt{2 \Delta t_L} \tilde{\eta}$$

(Markov chain)

statistical evaluation

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{N} \sum_{i=1}^N \mathcal{O}[\phi_i]$$

$$\sigma \propto \left(\sqrt{\# \text{ of (uncorrelated) samples}} \right)^{-1}$$

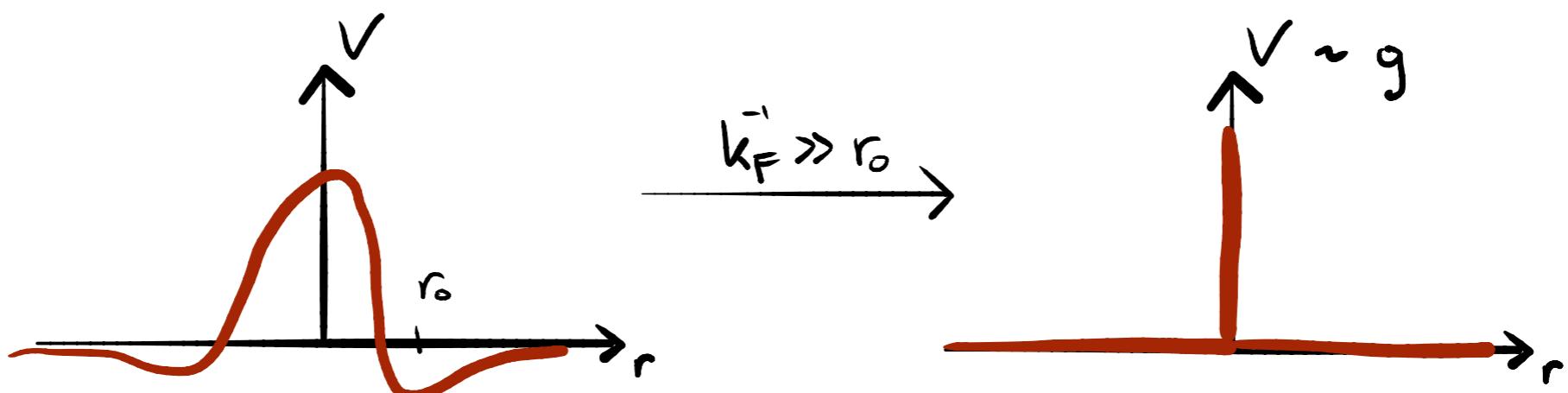
**how can stochastic quantization
help us to study fermions?**

fermions with contact interaction

kinetic part

$$\hat{H} = - \sum_{s=\uparrow,\downarrow} \int d^d x \hat{\psi}_s^\dagger(\vec{x}) \left(\frac{\hbar^2 \vec{\nabla}^2}{2m_s} \right) \hat{\psi}_s(\vec{x})$$
$$+ g \int d^d x \hat{\psi}_\uparrow^\dagger(\vec{x}) \hat{\psi}_\uparrow(\vec{x}) \hat{\psi}_\downarrow^\dagger(\vec{x}) \hat{\psi}_\downarrow(\vec{x})$$

interaction part



what do we need to compute?

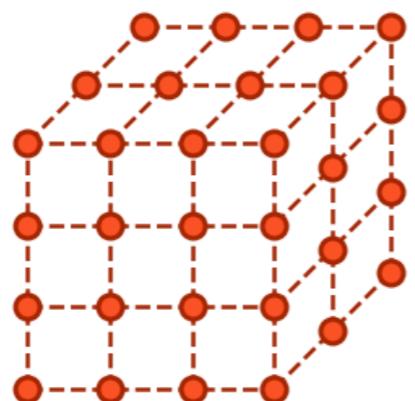
$$\mathcal{Z} = \text{Tr}[\text{e}^{-\beta \hat{H}}] = \text{Tr}[\text{e}^{-\beta(\hat{T} + \hat{V})}]$$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \text{Tr}[\hat{\mathcal{O}} \text{e}^{-\beta \hat{H}}]$$

what do we need to compute?

$$\mathcal{Z} = \text{Tr}[e^{-\beta \hat{H}}] = \text{Tr}[e^{-\beta(\hat{T} + \hat{V})}]$$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \text{Tr}[\hat{\mathcal{O}} e^{-\beta \hat{H}}]$$



+ Trotter decomposition

+ Hubbard-Stratonovich transformation

rewrite the problem as a **path-integral**:

$$\mathcal{Z} = \int \mathcal{D}\phi \det M_\phi^\uparrow \det M_\phi^\downarrow \equiv \int \mathcal{D}\phi e^{-S[\phi]}$$

the path integral

$$\mathcal{Z} = \int \mathcal{D}\phi \det M_\phi^\uparrow \det M_\phi^\downarrow \equiv \int \mathcal{D}\phi e^{-S[\phi]}$$

discrete Langevin equation:

$$\phi^{(n+1)} = \phi^{(n)} + \Delta\phi^{(n)}$$

$$\Delta\phi^{(n)} = -\frac{\delta S[\phi]}{\delta\phi} \Big|_{\phi^{(n)}} \Delta t_L + \sqrt{2\Delta t_L} \eta$$

the path integral & complex actions

$$\mathcal{Z} = \int \mathcal{D}\phi \det M_\phi^\uparrow \det M_\phi^\downarrow \equiv \int \mathcal{D}\phi e^{-S[\phi]}$$

discrete Langevin equation:

$$\phi^{(n+1)} = \phi^{(n)} + \Delta\phi^{(n)}$$

probability measure **not positive (semi-)definite** if any of these conditions applies:

$$\Delta\phi^{(n)} = -\frac{\delta S[\phi]}{\delta\phi} \Big|_{\phi^{(n)}} \Delta t_L + \sqrt{2\Delta t_L} \eta$$

$$\begin{aligned} N_\uparrow &\neq N_\downarrow \\ \mu_\uparrow &\neq \mu_\downarrow \\ m_\uparrow &\neq m_\downarrow \\ g &> 0 \end{aligned}$$

the path integral & complex actions

$$\mathcal{Z} = \int \mathcal{D}\phi \det M_\phi^\uparrow \det M_\phi^\downarrow \equiv \int \mathcal{D}\phi e^{-S[\phi]}$$

discrete Langevin equation:

$$\phi^{(n+1)} = \phi^{(n)} + \Delta\phi^{(n)}$$

probability measure **not** positive (semi-)definite if any of these conditions applies:

$$\Delta\phi_R^{(n)} = -\text{Re} \left[\frac{\delta S[\phi]}{\delta \phi} \right]_{\phi^{(n)}} \Delta t_L + \sqrt{2\Delta t_L} \eta$$

$$\Delta\phi_I^{(n)} = -\text{Im} \left[\frac{\delta S[\phi]}{\delta \phi} \right]_{\phi^{(n)}} \Delta t_L$$

$$\begin{aligned} N_\uparrow &\neq N_\downarrow \\ \mu_\uparrow &\neq \mu_\downarrow \\ m_\uparrow &\neq m_\downarrow \\ g &> 0 \end{aligned}$$

complex action → complex Langevin equation

complex probabilities

$$\int \mathcal{D}\phi \, P[\phi] O[\phi] \rightarrow \int \mathcal{D}\phi_R \mathcal{D}\phi_I \, P[\phi_R + i\phi_I] O[\phi_R + i\phi_I]$$

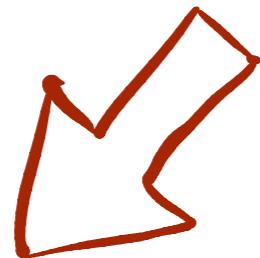
guaranteed convergence if PDF decays
fast enough in imaginary direction and $S[\phi]$ is holomorphic

[Aarts,Seiler,Stamatescu '10; Aarts,James,Seiler,Stamatescu '11]

complex probabilities & possible issues

$$\int \mathcal{D}\phi P[\phi]O[\phi] \rightarrow \int \mathcal{D}\phi_R \mathcal{D}\phi_I P[\phi_R + i\phi_I]O[\phi_R + i\phi_I]$$

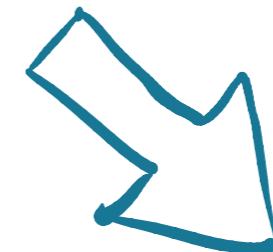
guaranteed convergence if PDF decays
fast enough in imaginary direction and $S[\phi]$ is holomorphic
[Aarts,Seiler,Stamatescu '10; Aarts,James,Seiler,Stamatescu '11]



non-analyticities in the action

- zeros in measure ($\det M = 0$)
- could lead to ergodicity issues (bottlenecks)

[Aarts,Seiler,Sexty,Stamatescu '17]



non-vanishing boundary terms

- convergence to wrong limits possible
- behavior must be monitored

[Scherzer,Seiler,Sexty,Stamatescu '19]

recap: stochastic quantization & CL

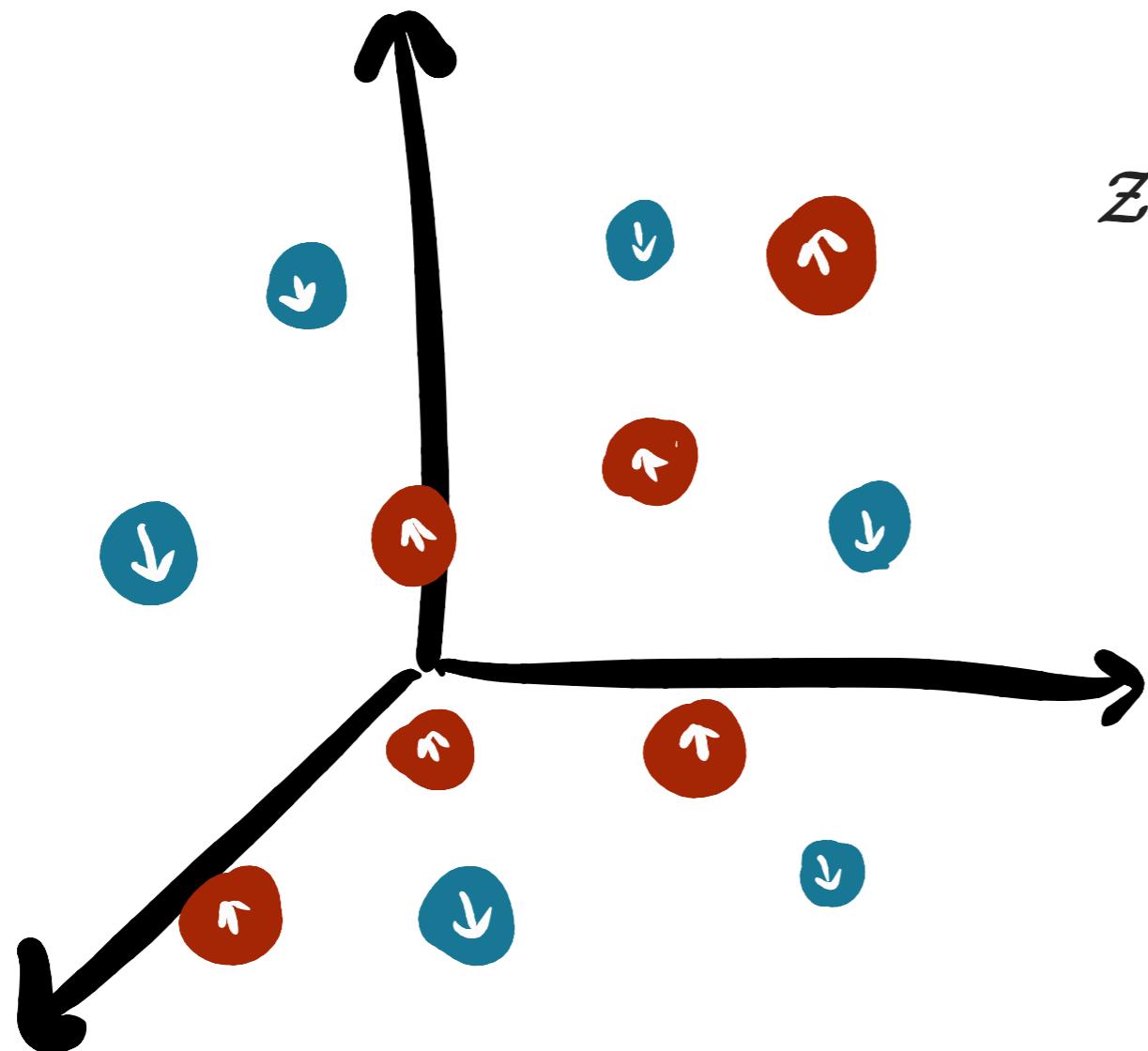
SQ: interpret **Euclidean field theories**
as equilibrium limit of a fictitious **random process**
(allows us to build a **Markov chain**)

complex Langevin provides a way
to **evade sign problems** in some cases

however: not guaranteed to work a-priori
and the **behavior needs to be monitored carefully**

the unitary Fermi gas at finite temperature

[LR, Loheac, Drut, Braun '18]



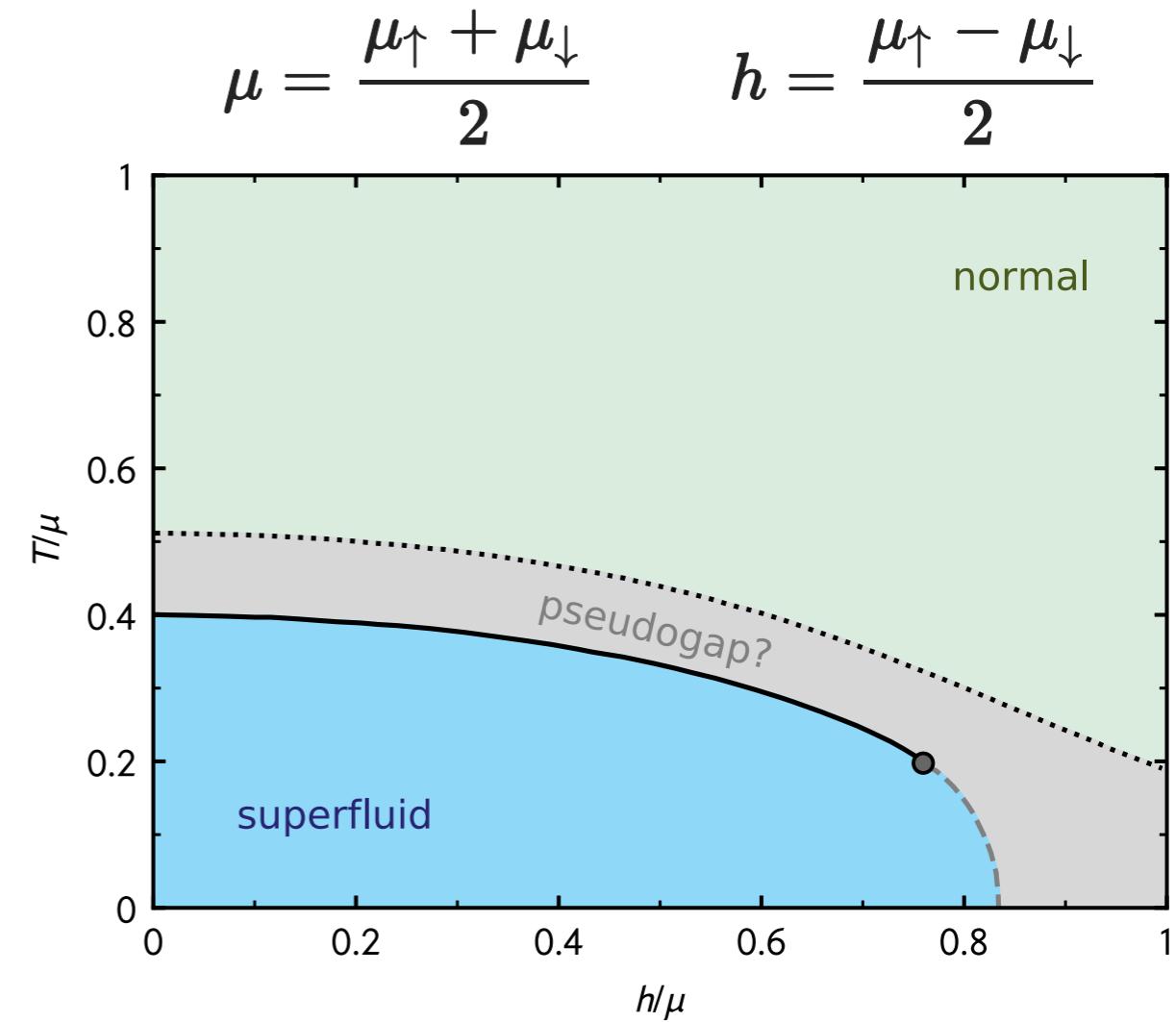
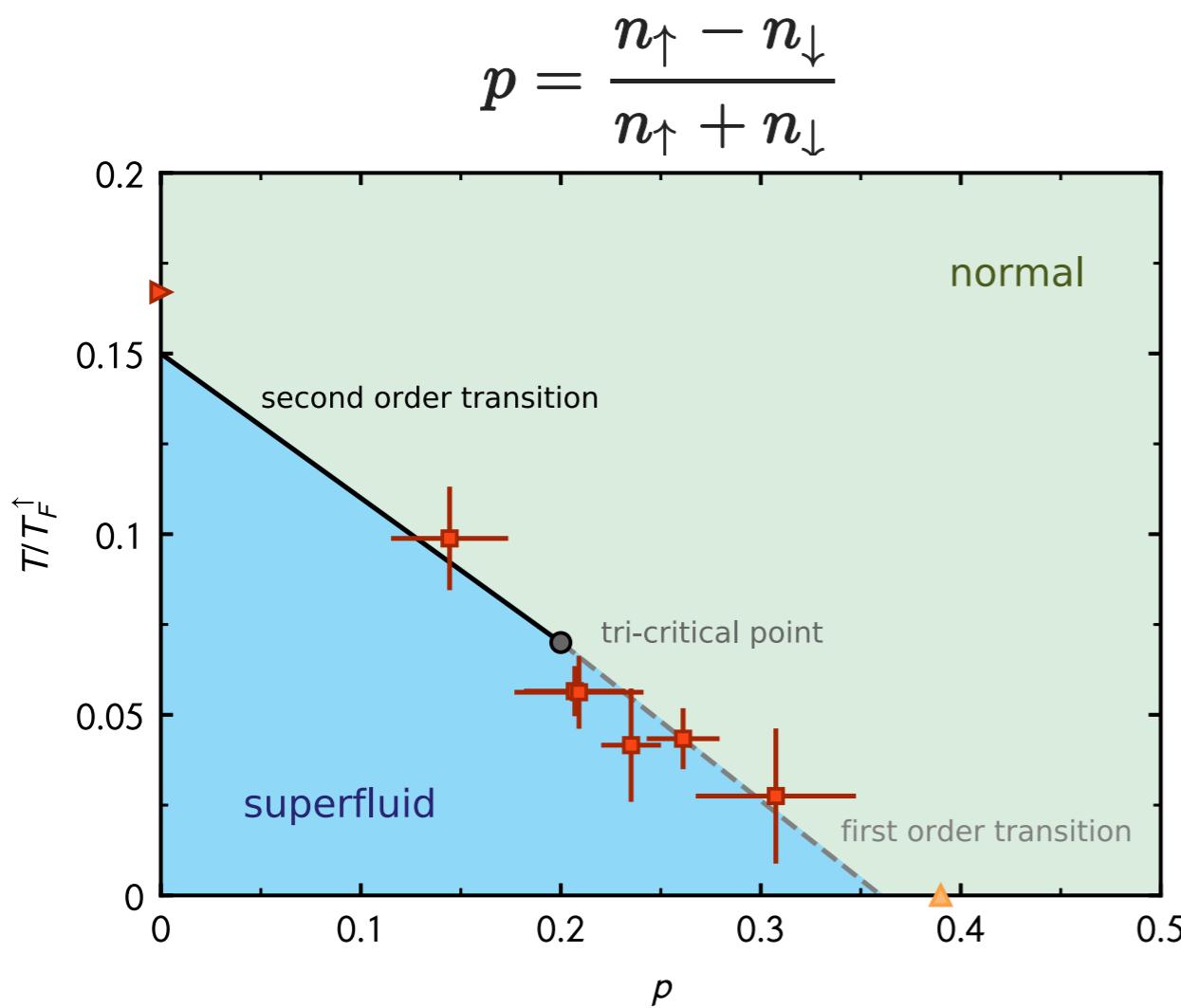
$$\begin{aligned}\mathcal{Z} &= \text{Tr} \left[e^{-\beta(\hat{H}-\mu_{\uparrow}\hat{N}_{\uparrow}-\mu_{\downarrow}\hat{N}_{\downarrow})} \right] \\ &= \text{Tr} \left[e^{-\beta(\hat{H}-\mu\hat{N}-h\hat{M})} \right]\end{aligned}$$

$$\mu = \frac{\mu_{\uparrow} + \mu_{\downarrow}}{2}$$

$$h = \frac{\mu_{\uparrow} - \mu_{\downarrow}}{2}$$

computationally challenging (but feasible)

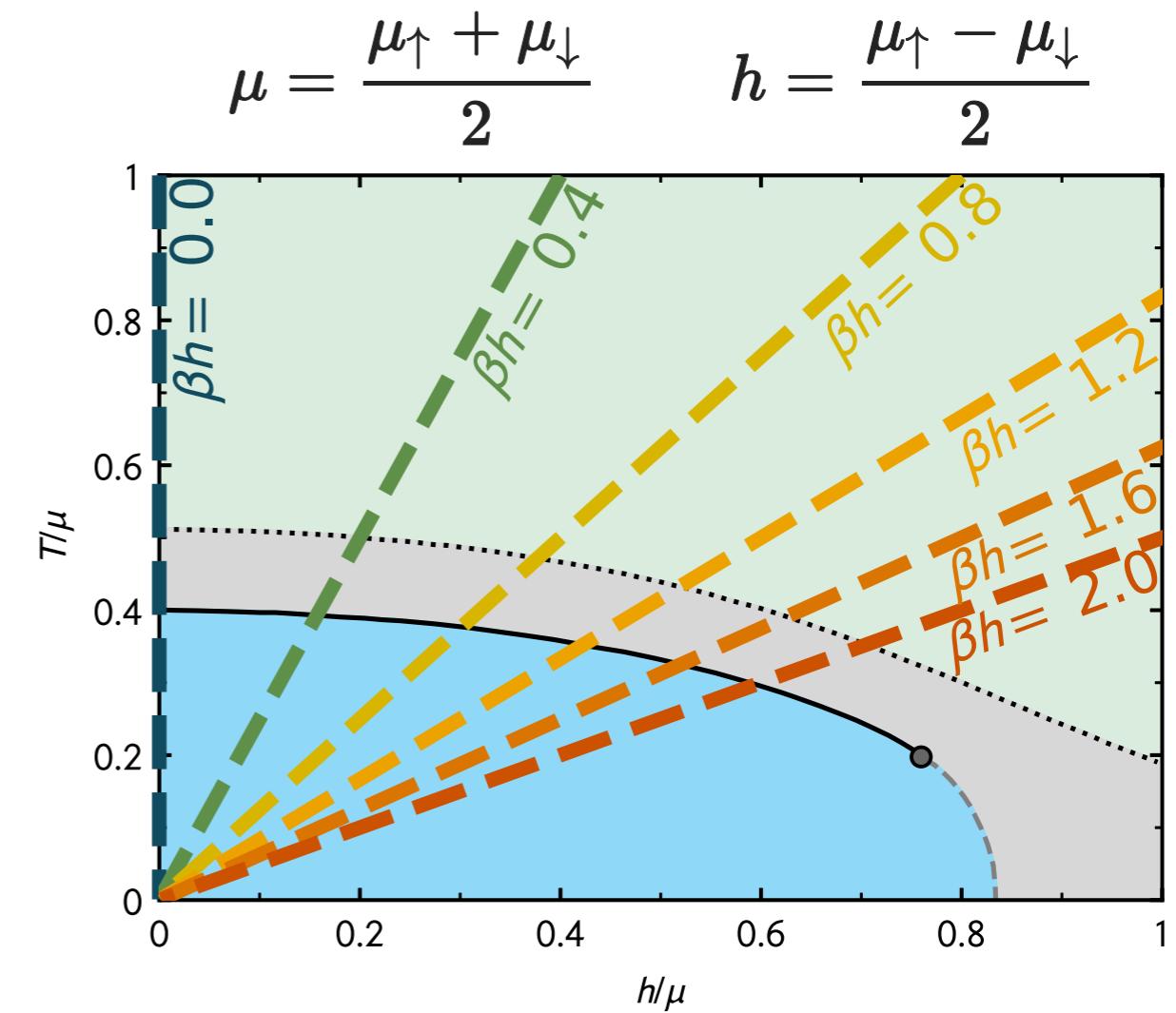
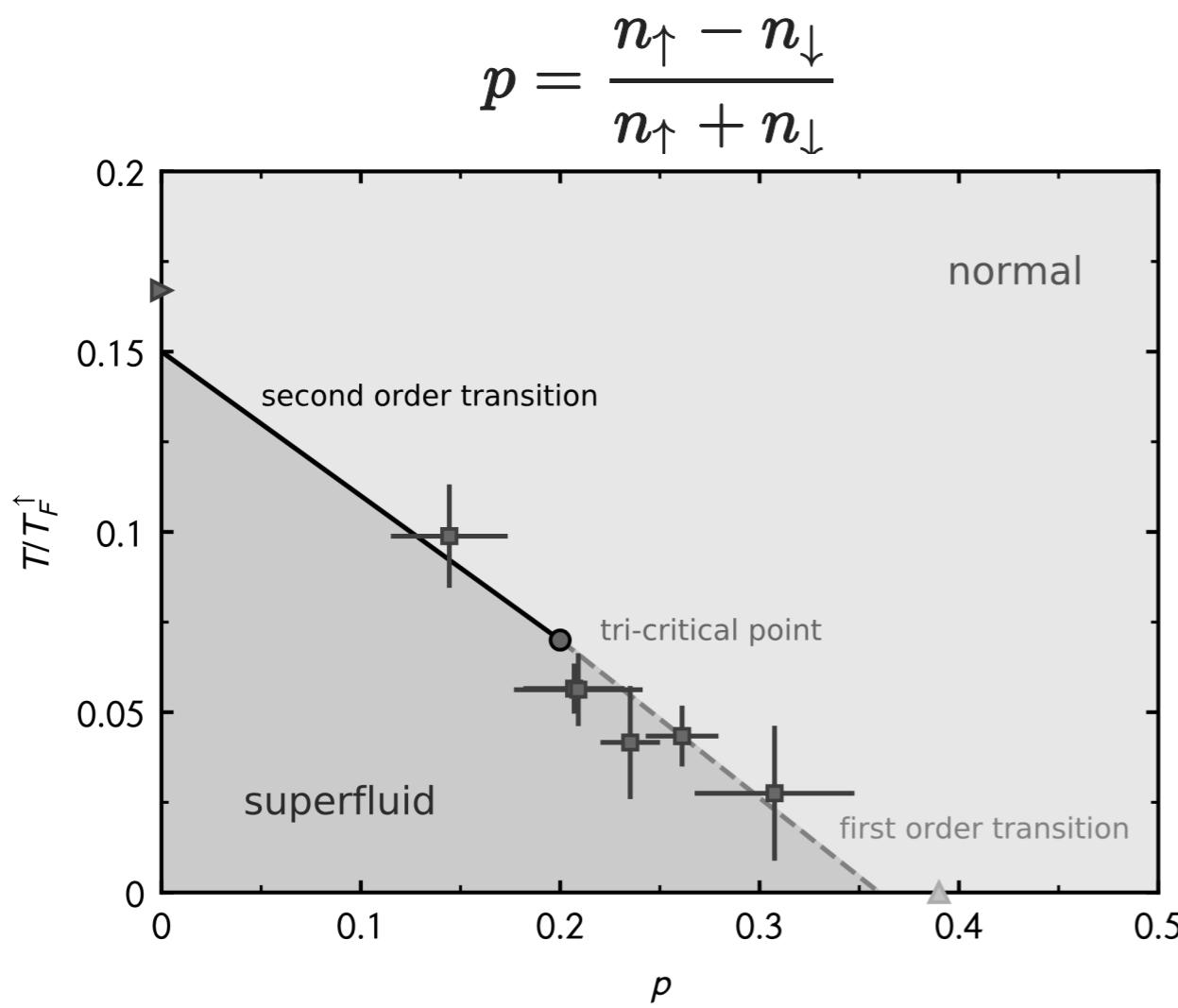
exploring the phase diagram



[experiment: Shin,Schunck,Schirotzek,Ketterle '08]
 [zero-temperature p_c : Lobo,Recati,Giorgini,Stringari '06]
 [balanced T_c : Ku,Sommer,Cheuck,Zwierlein '12]

[fRG: Boettcher et. al '15]

exploring the phase diagram



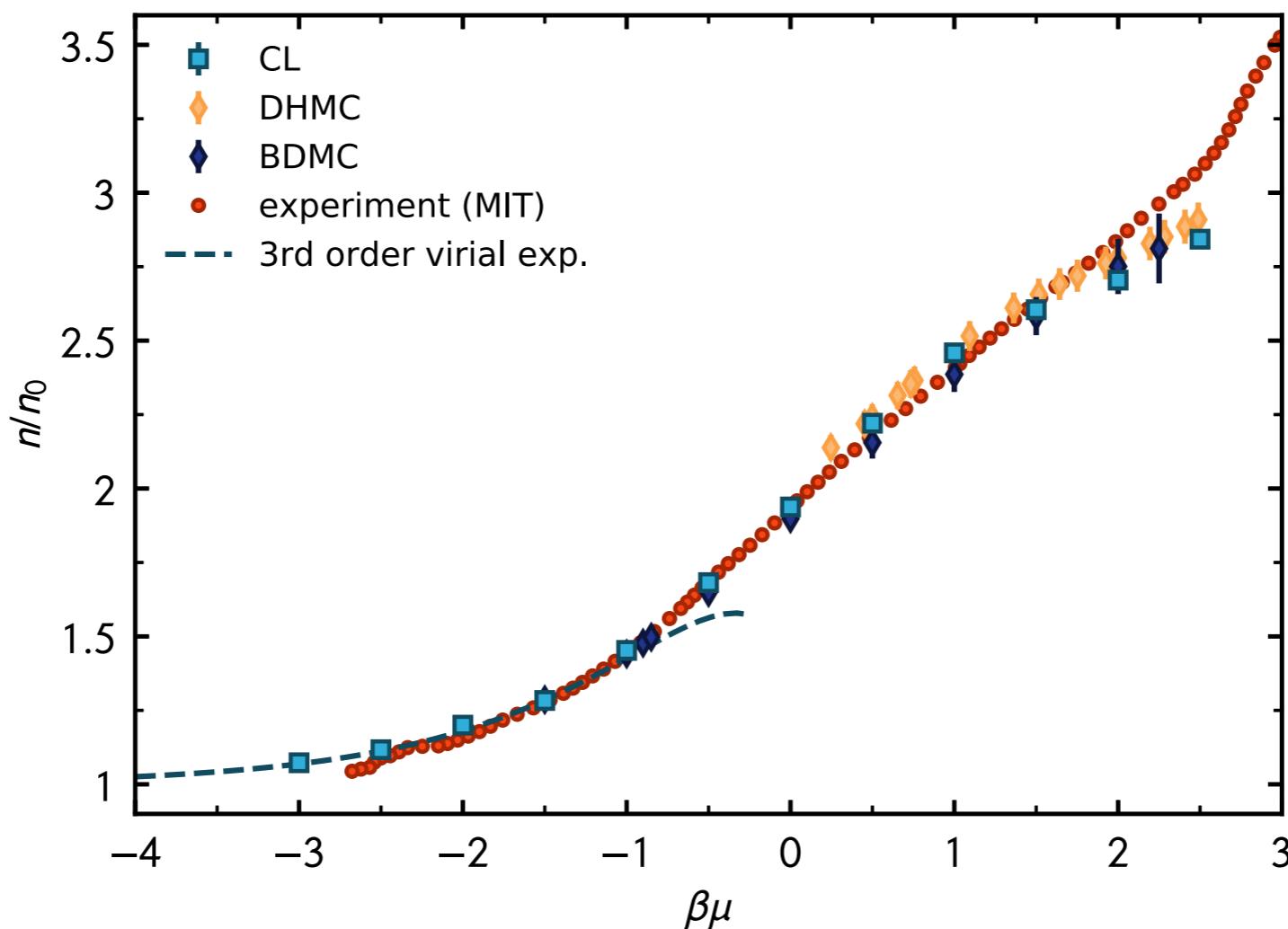
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[fRG: Boettcher et. al '15]

density equation of state

[LR, Loheac, Drut, Braun '18]

[experiment/BDMC: van Houcke et al. '12]
[DPMC: Drut,Lähde,Wlazłowski,Magierski '12]
[diagMC: Rossi,Ohgoe,van Houcke,Werner '18]



classical regime

$k_B T$ dominates

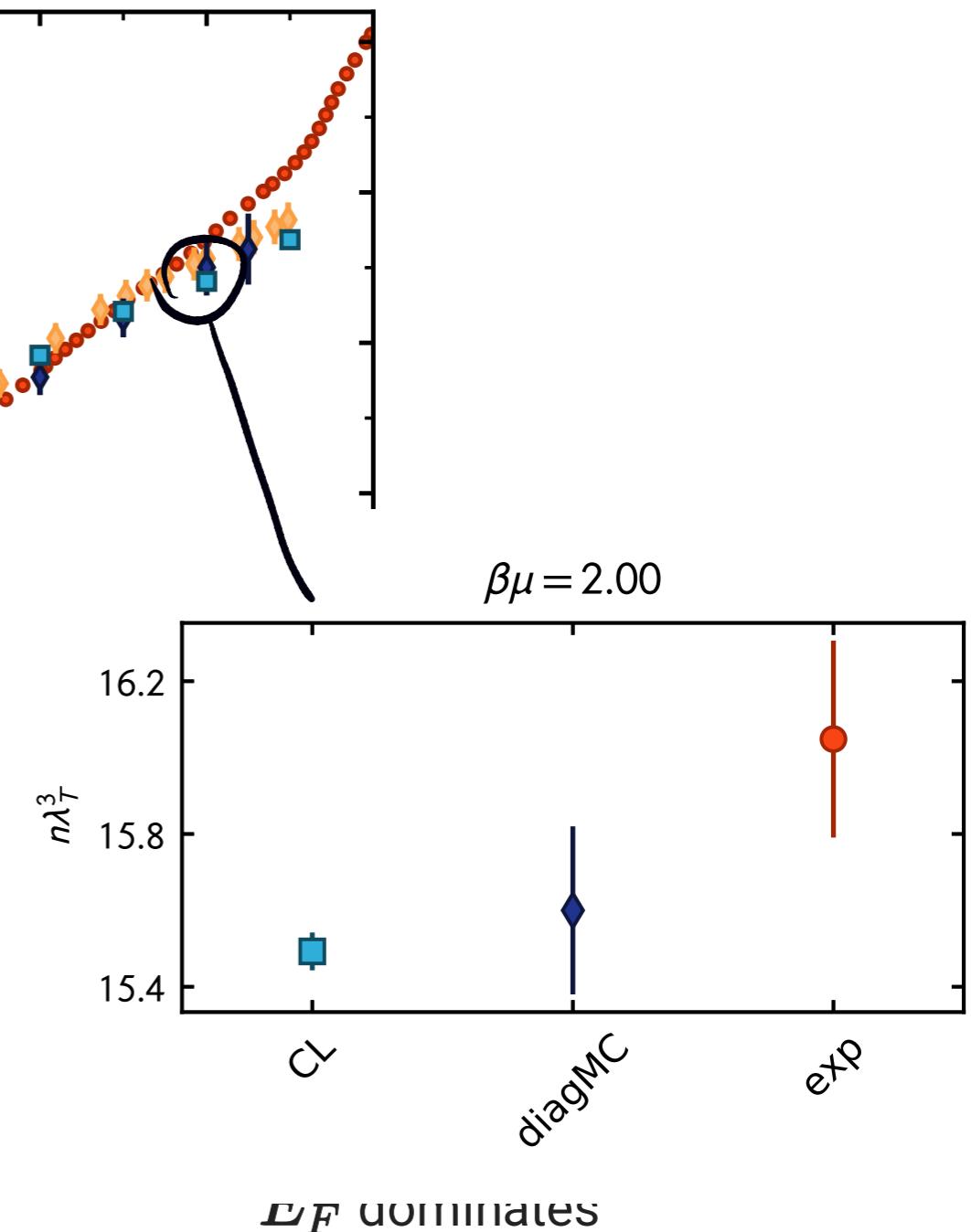
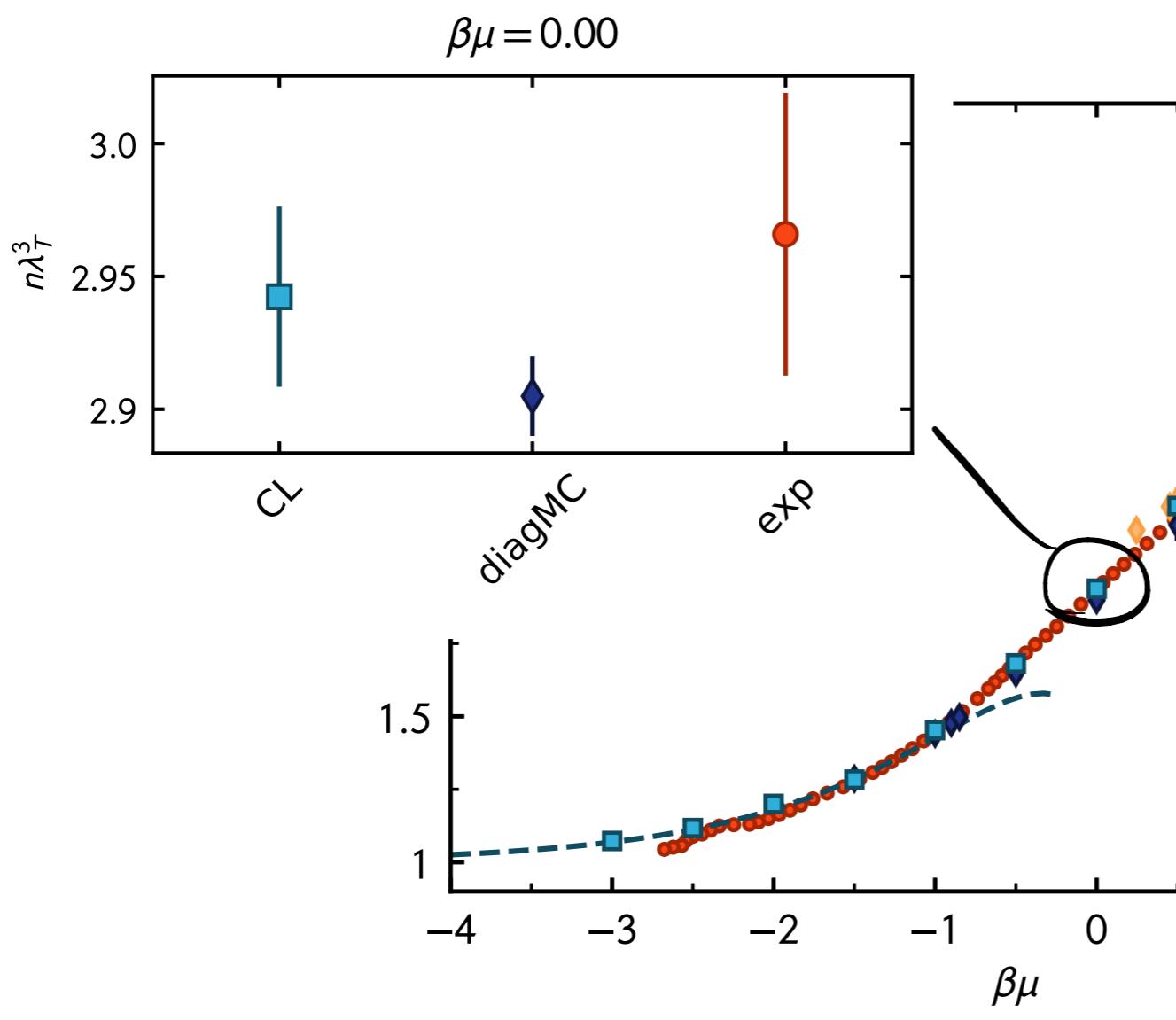
quantum regime

E_F dominates

density equation of state

[LR, Loheac, Drut, Braun '18]

[experiment/BDMC: van Houcke et al. '12]
[DHMC: Drut,Lähde,Wlazłowski,Magierski '12]
[diagMC: Rossi,Ohgoe,van Houcke,Werner '18]



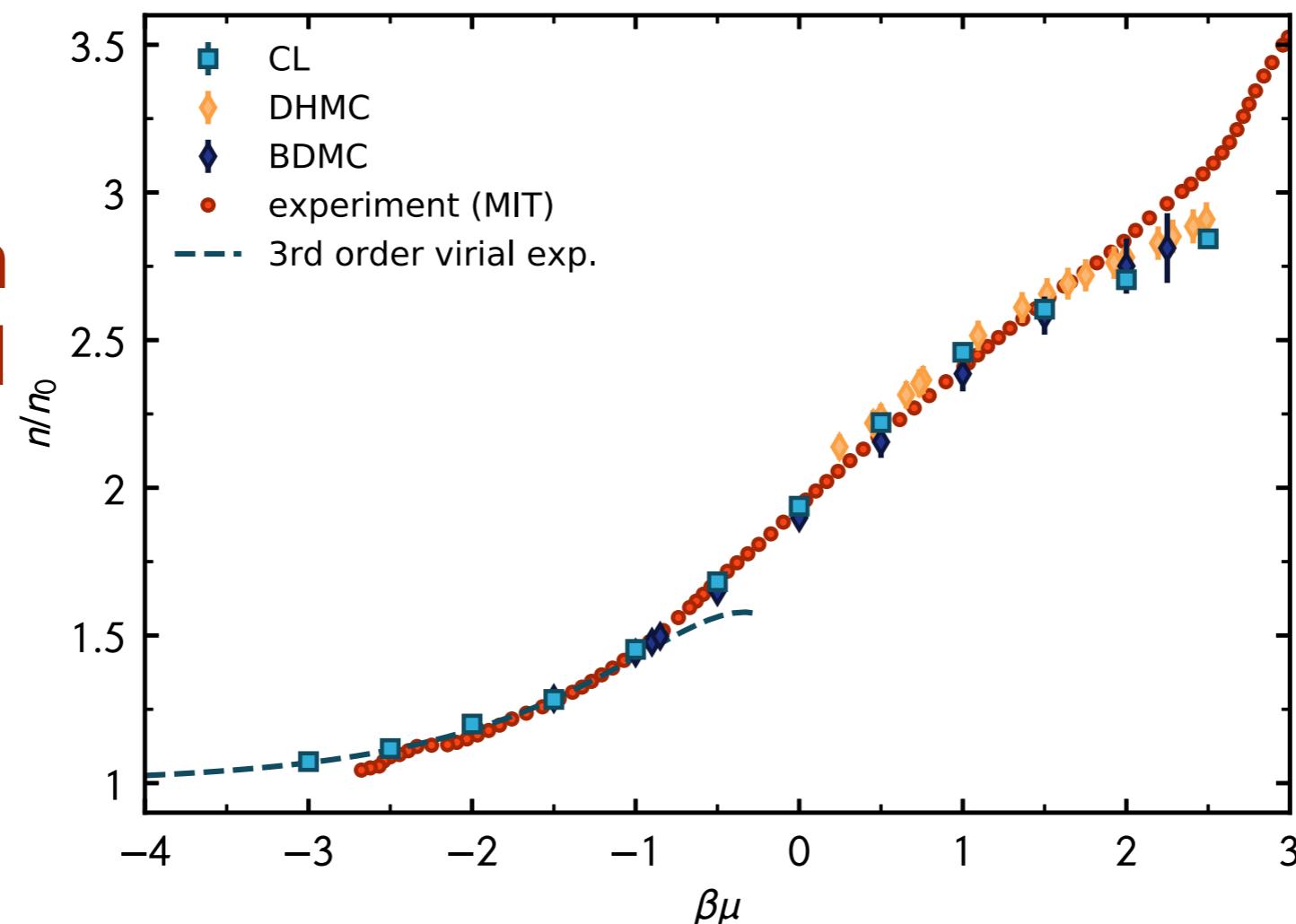
density equation of state

[LR, Loheac, Drut, Braun '18]

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good
agreement with
experiment and
other methods!

CL results:
finite lattice $V = 11^3$



classical regime

$k_B T$ dominates

quantum regime

E_F dominates

density equation of state

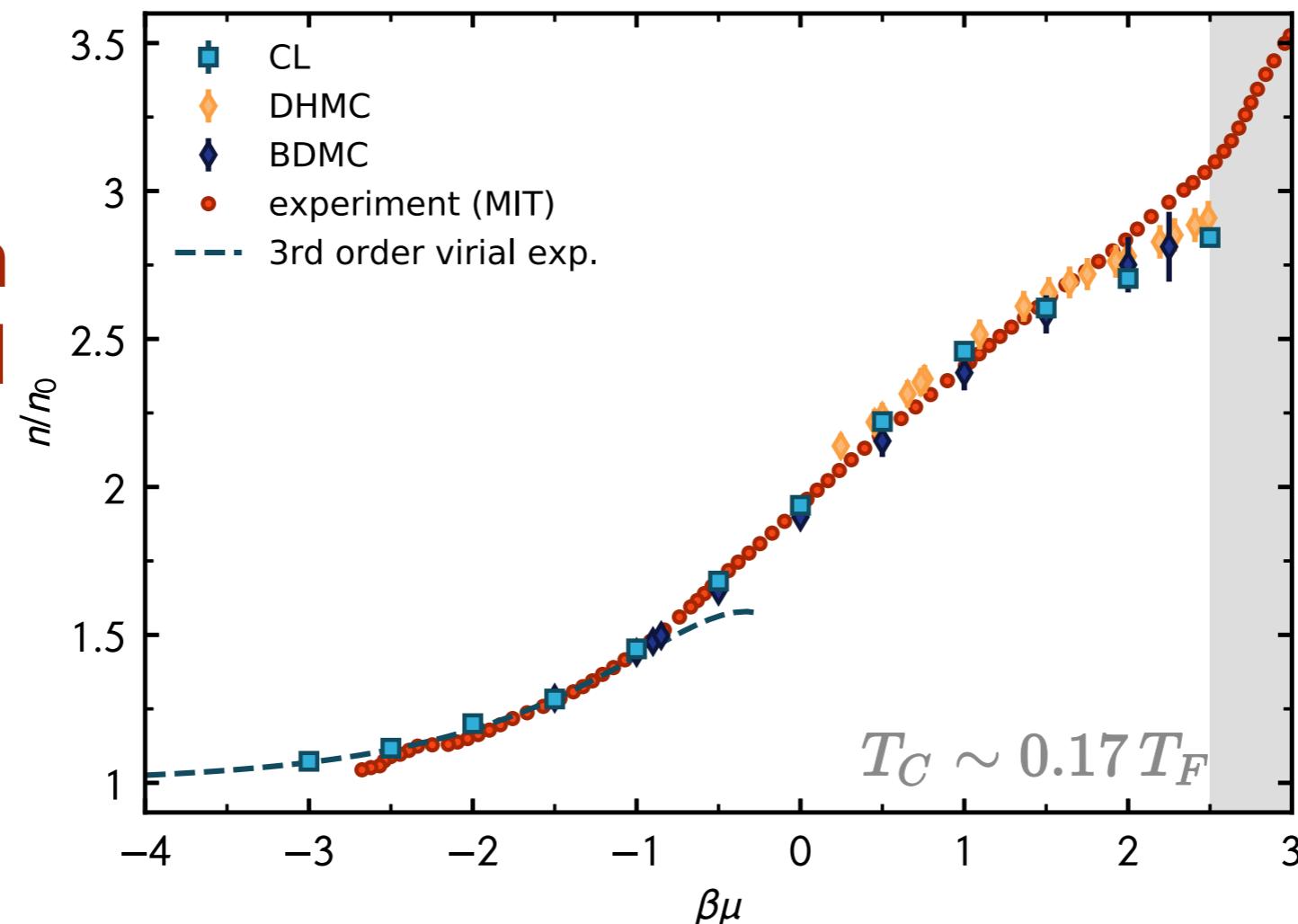
[LR, Loheac, Drut, Braun '18]

[experiment/BDMC: van Houcke et al. '12]
[DPMC: Drut,Lähde,Wlazłowski,Magierski '12]
[diagMC: Rossi,Ohgoe,van Houcke,Werner '18]

good
agreement with
experiment and
other methods!

CL results:

finite lattice $V = 11^3$



classical regime

$k_B T$ dominates

quantum regime

E_F dominates

low temperatures:
 λ_T increases
($\lambda_T \ll V^{1/3}$ must be
fulfilled)

interlude: the virial expansion

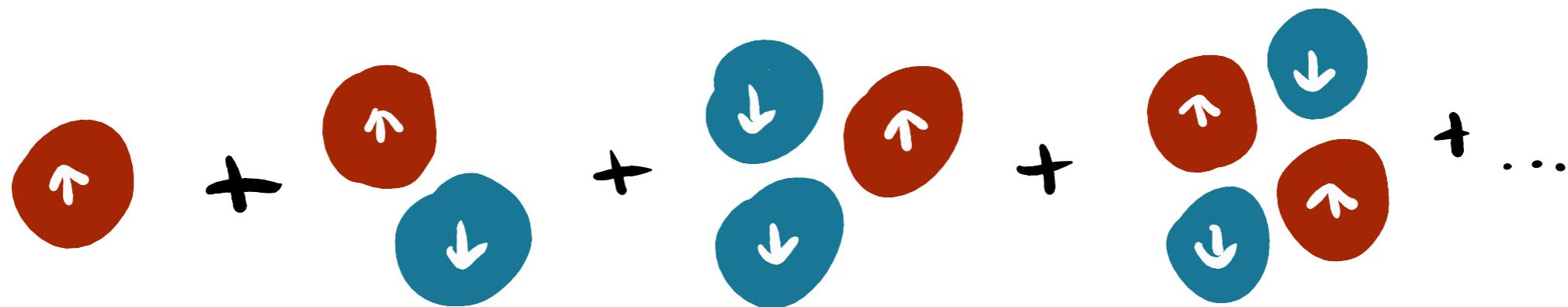
[Liu '13]

dilute gases: few-body correlations dominate

idea: describe the system as **expansion in few-body clusters**

$$z = e^{\beta\mu}$$

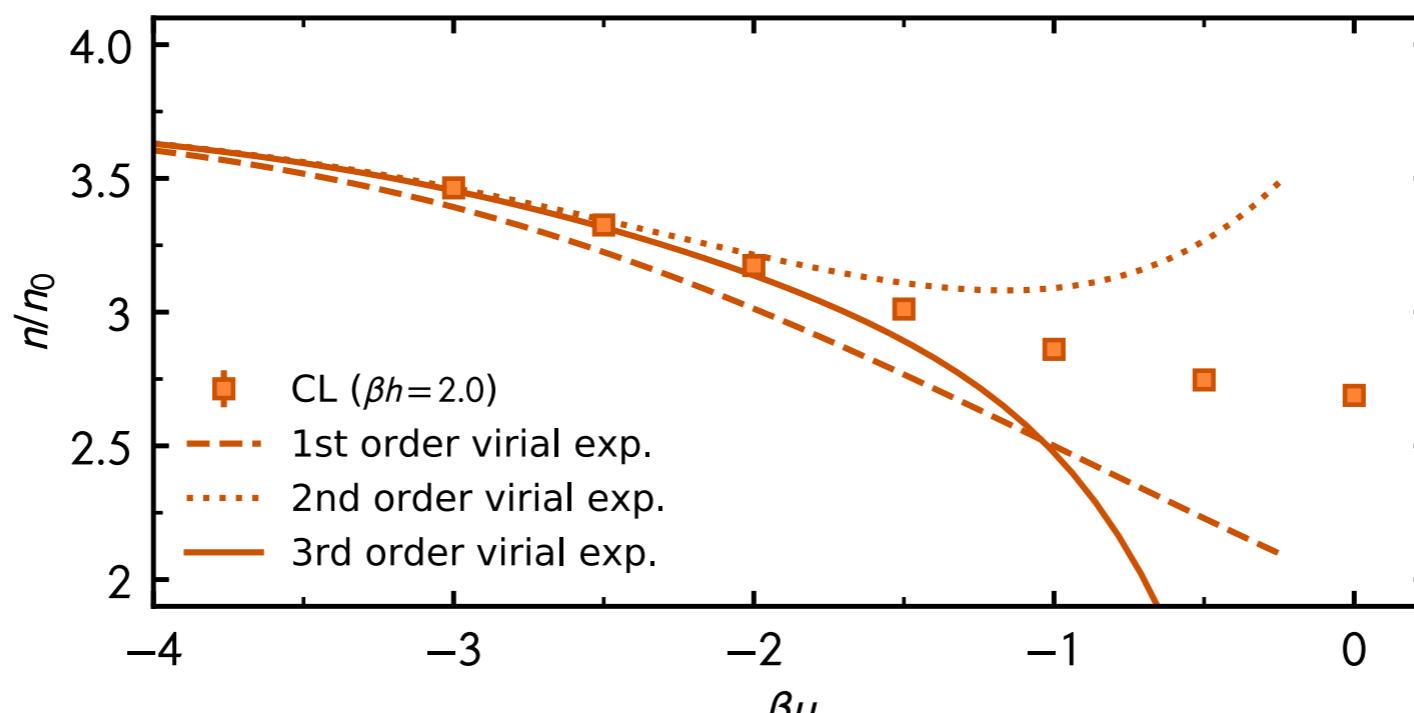
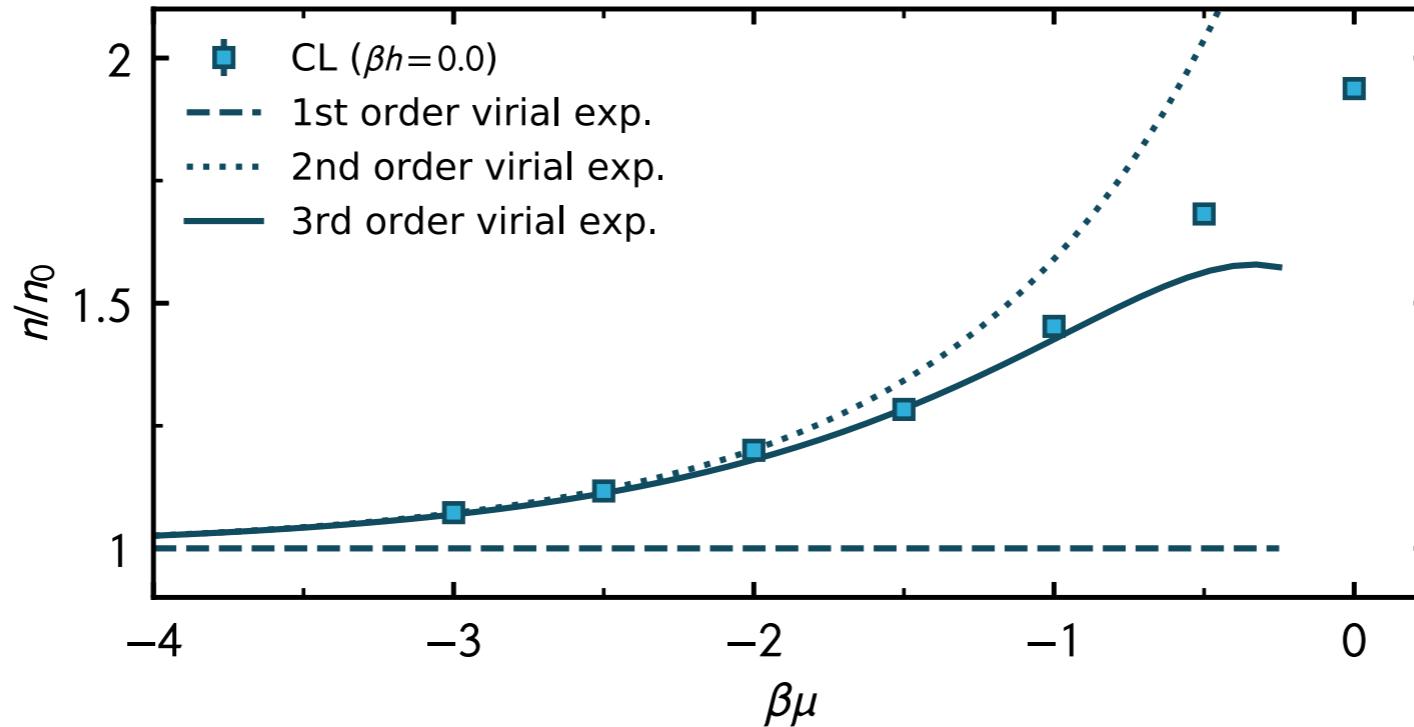
$$\ln \mathcal{Z} = Q_1 \sum_n z^n b_n$$



density EOS (virial regime)

[LR, Loheac, Drut, Braun '18]

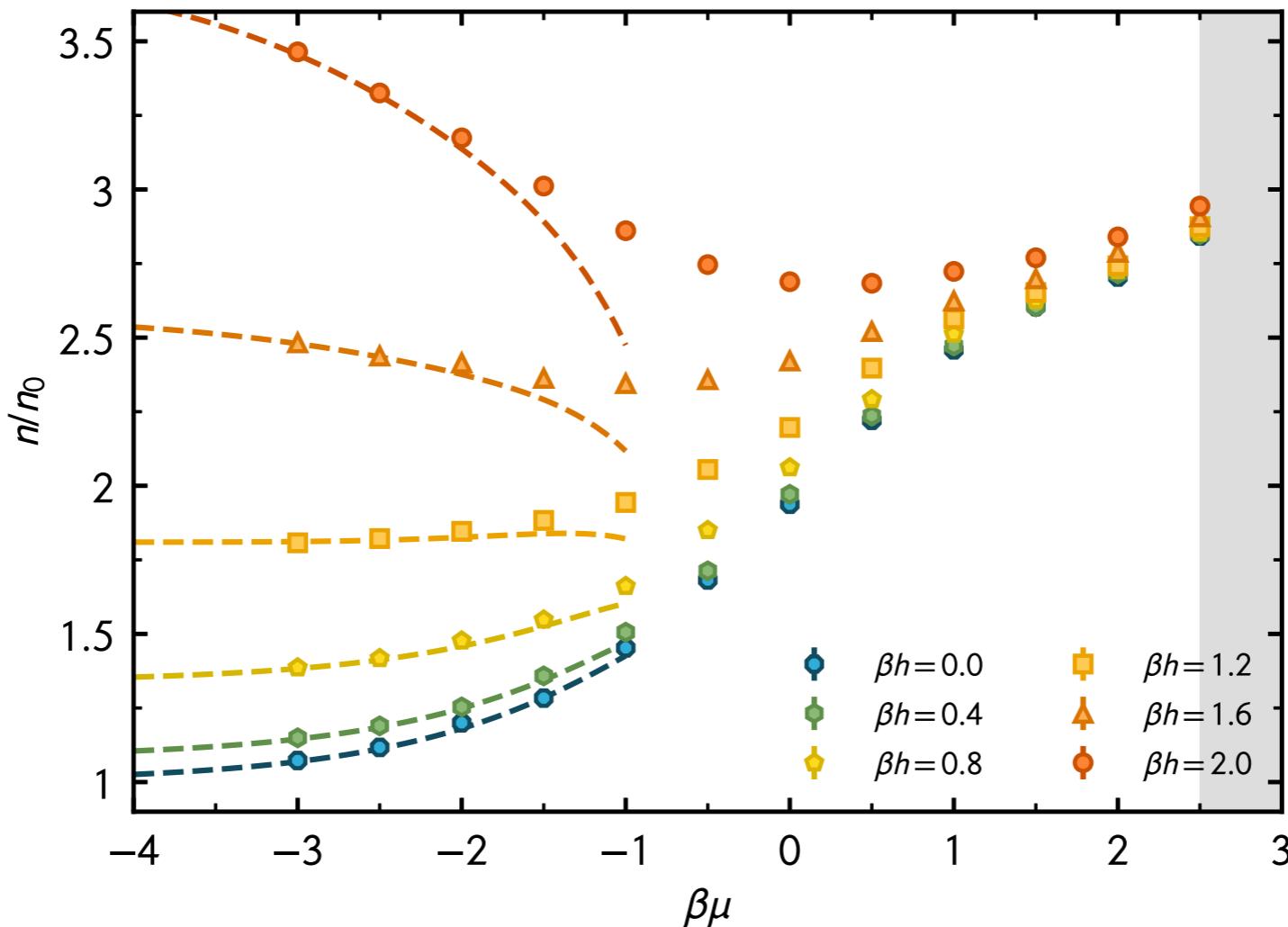
VE approaches
the CL results
order-by-order



VE deviates earlier
for polarized
systems

density EOS at finite polarization

[LR, Loheac, Drut, Braun '18]



$$h = \frac{\mu_{\uparrow} - \mu_{\downarrow}}{2}$$

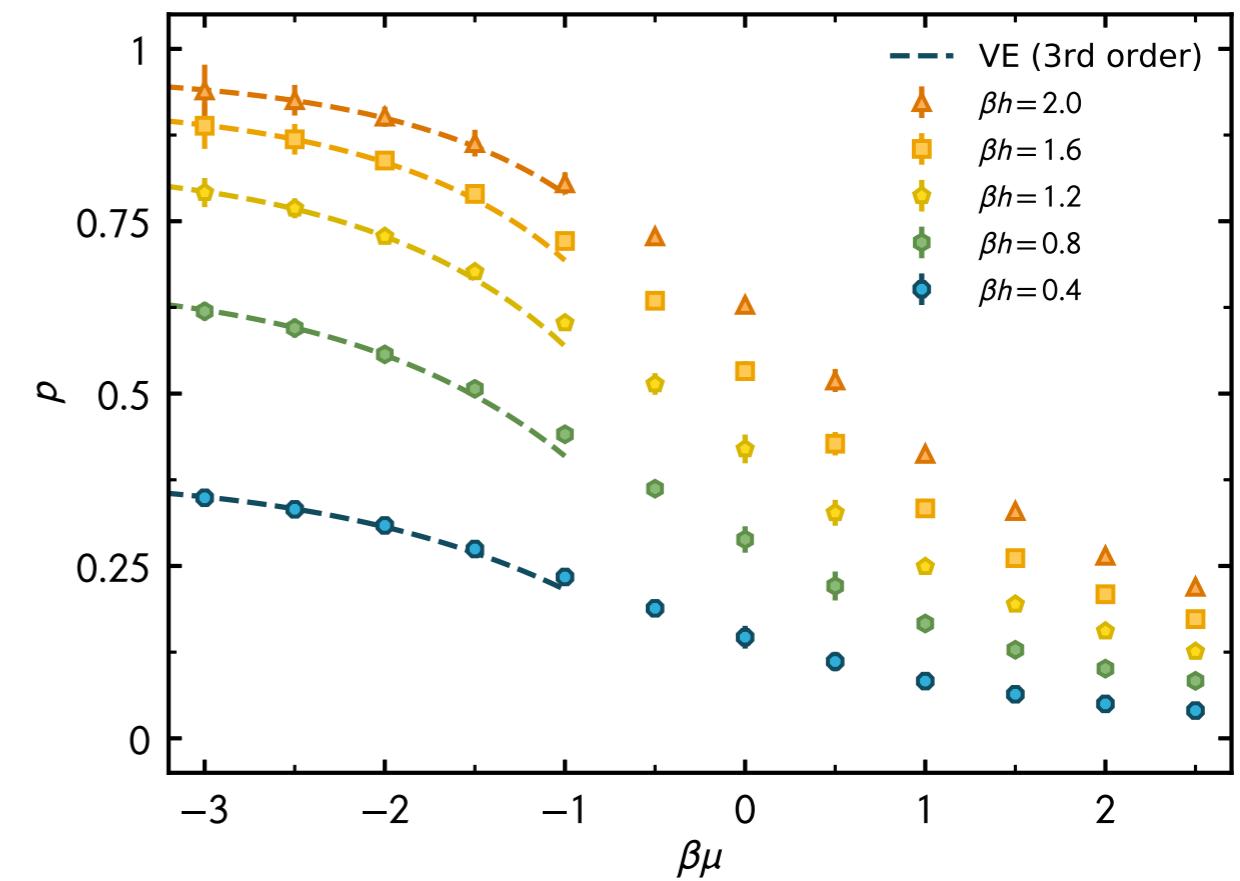
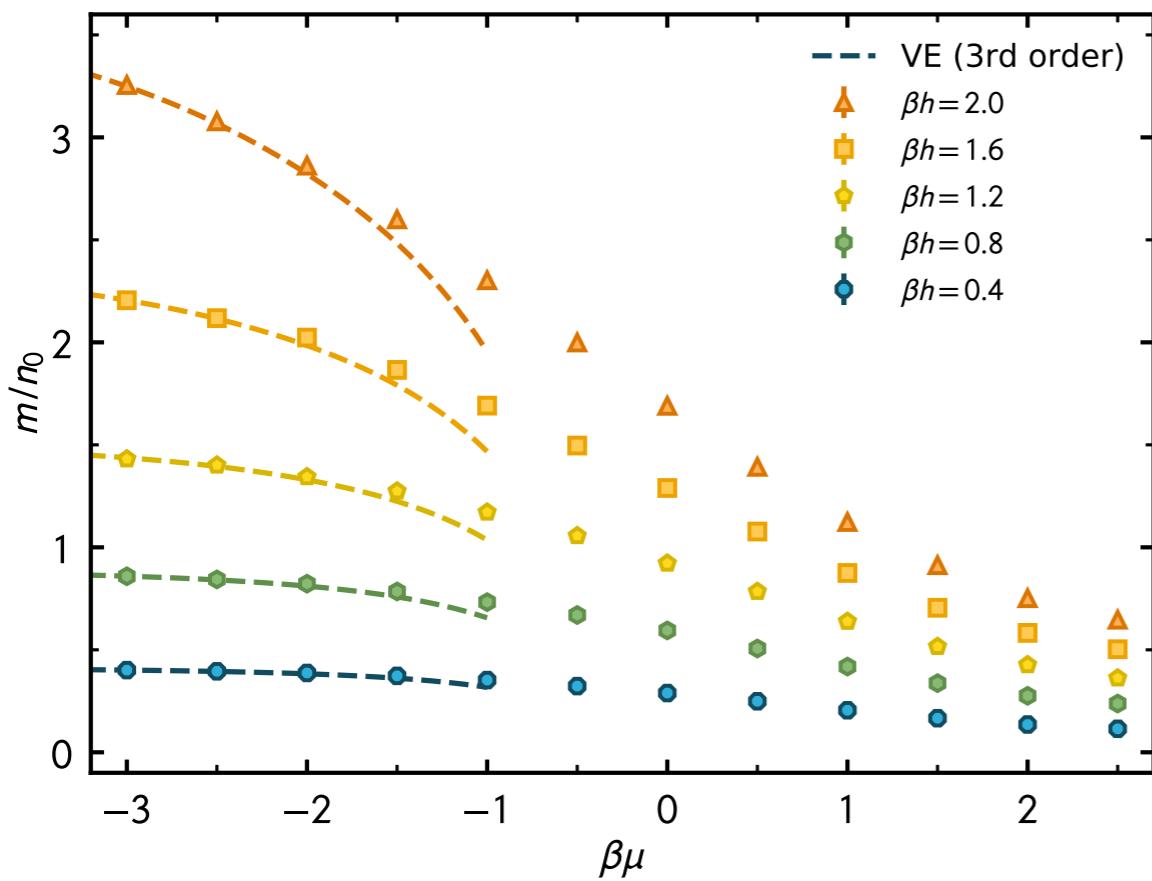
excellent agreement with virial expansion **for all polarizations**
experimentally testable prediction

magnetization & polarization

[LR, Loheac, Drut, Braun '18]

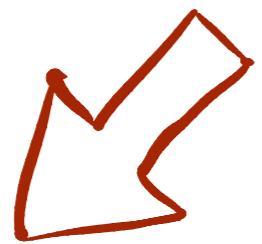
$$m = n_{\uparrow} - n_{\downarrow} = \frac{\partial \ln \mathcal{Z}}{\partial(\beta h)}$$

$$p = \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}} = \frac{m}{n}$$

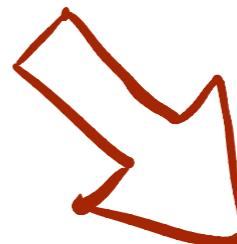


textbook thermodynamics

$$n = \frac{\partial \ln Z}{\partial(\beta\mu)}$$



$$m = \frac{\partial \ln Z}{\partial(\beta h)}$$



pressure & energy

$$P(\beta\mu) = \frac{1}{\beta} \int_{-\infty}^{\beta\mu} n(x) dx$$

$$E = \frac{3}{2} PV$$

thermodynamic response

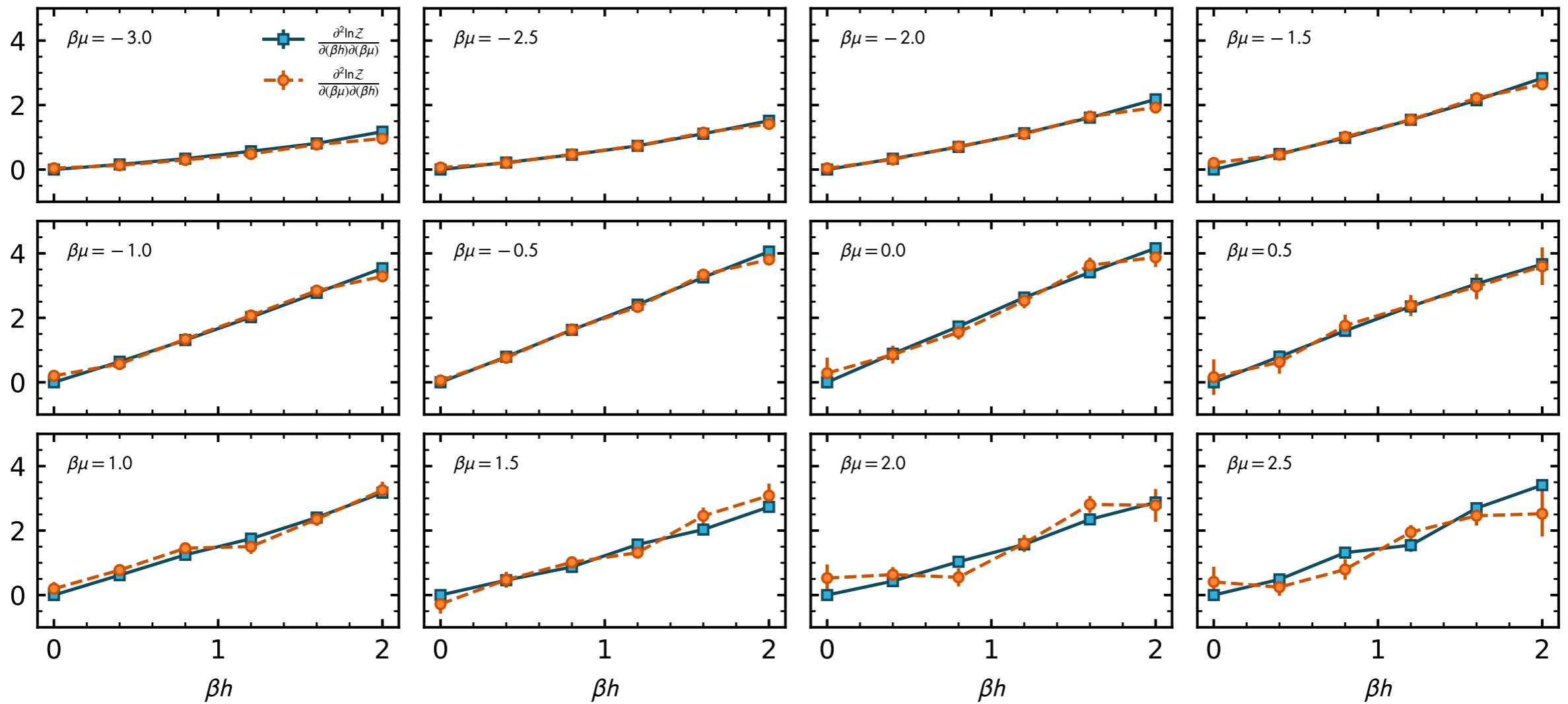
$$\kappa = \frac{1}{n} \left(\frac{\partial n}{\partial P} \right)_{T,V,h}$$

$$\chi = \left(\frac{\partial m}{\partial h} \right)_{T,V,\mu}$$

Maxwell relations: consistency check

[LR, Loheac, Drut, Braun '18]

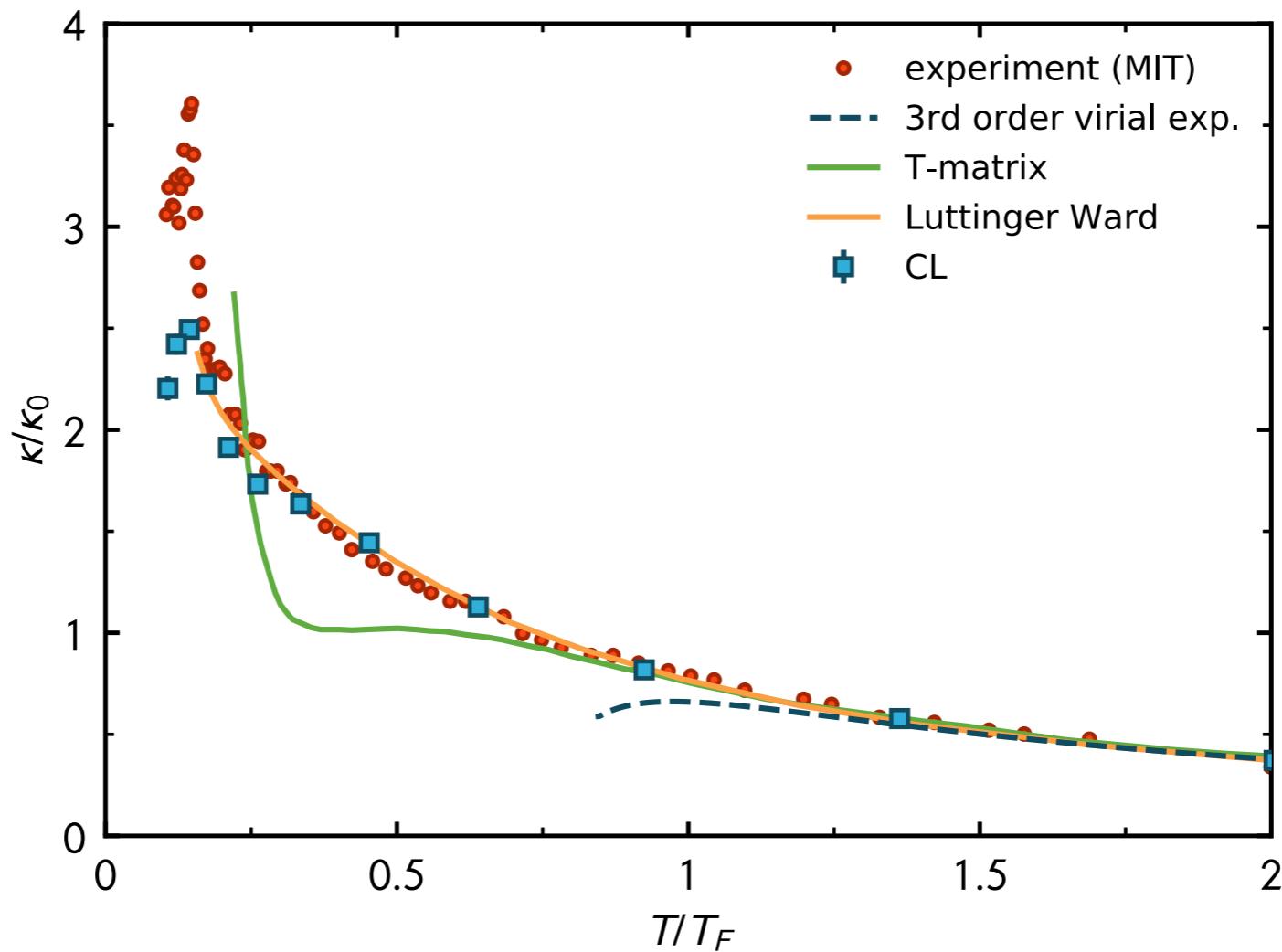
$$\left(\frac{\partial n}{\partial(\beta h)} \right)_{\beta\mu} = ! \left(\frac{\partial m}{\partial(\beta\mu)} \right)_{\beta h}$$



compressibility (unpolarized)

[LR, Loheac, Drut, Braun '18]

$$\kappa = \frac{1}{n} \left(\frac{\partial n}{\partial P} \right)_{T,V,h} = \frac{1}{n^2} \left(\frac{\partial n}{\partial \mu} \right)_{T,V,h}$$



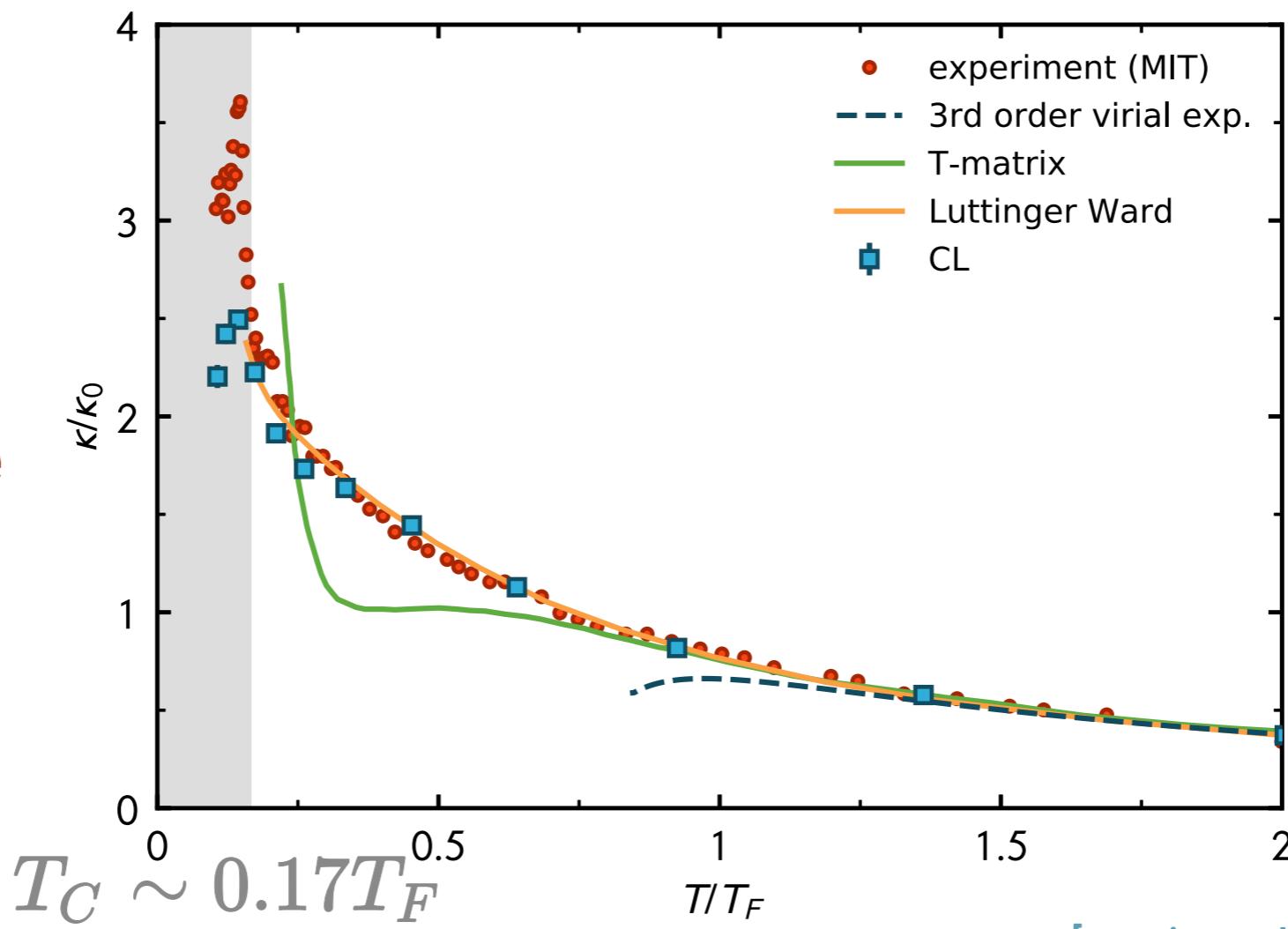
[experiment: Ku,Sommer,Cheuck,Zwierlein '12]
[Luttinger-Ward: Enns,Haussmann '12]

compressibility (unpolarized)

[LR, Loheac, Drut, Braun '18]

$$\kappa = \frac{1}{n} \left(\frac{\partial n}{\partial P} \right)_{T,V,h} = \frac{1}{n^2} \left(\frac{\partial n}{\partial \mu} \right)_{T,V,h}$$

sudden
increase of κ
indicates
superfluid phase
transition



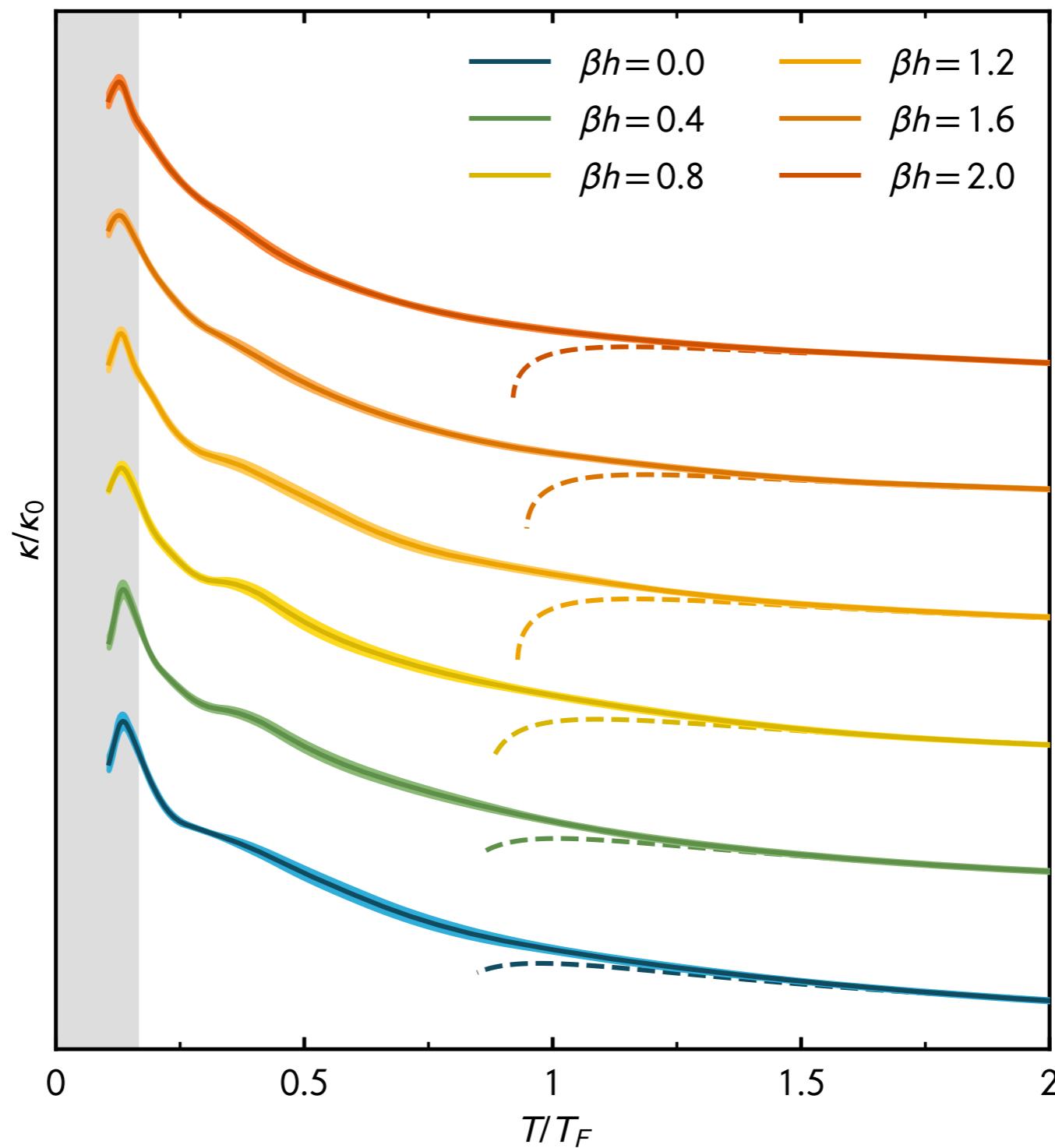
[experiment: Ku,Sommer,Cheuck,Zwierlein '12]
[Luttinger-Ward: Enns,Haussmann '12]

features of curve
recovered with CL

quantitative
disagreement
at low
temperatures

compressibility for polarized systems

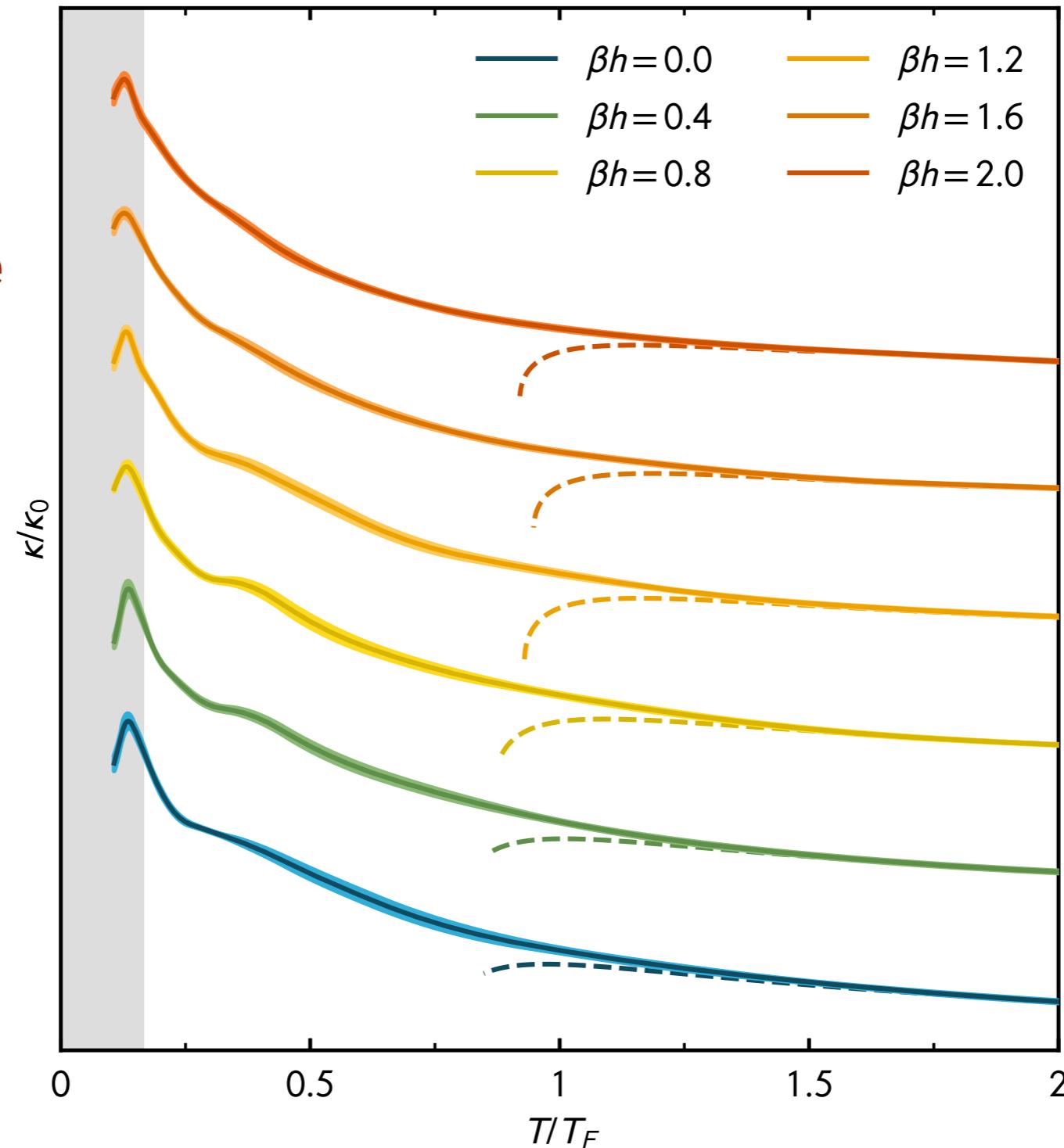
[LR, Loheac, Drut, Braun '18]



compressibility for polarized systems

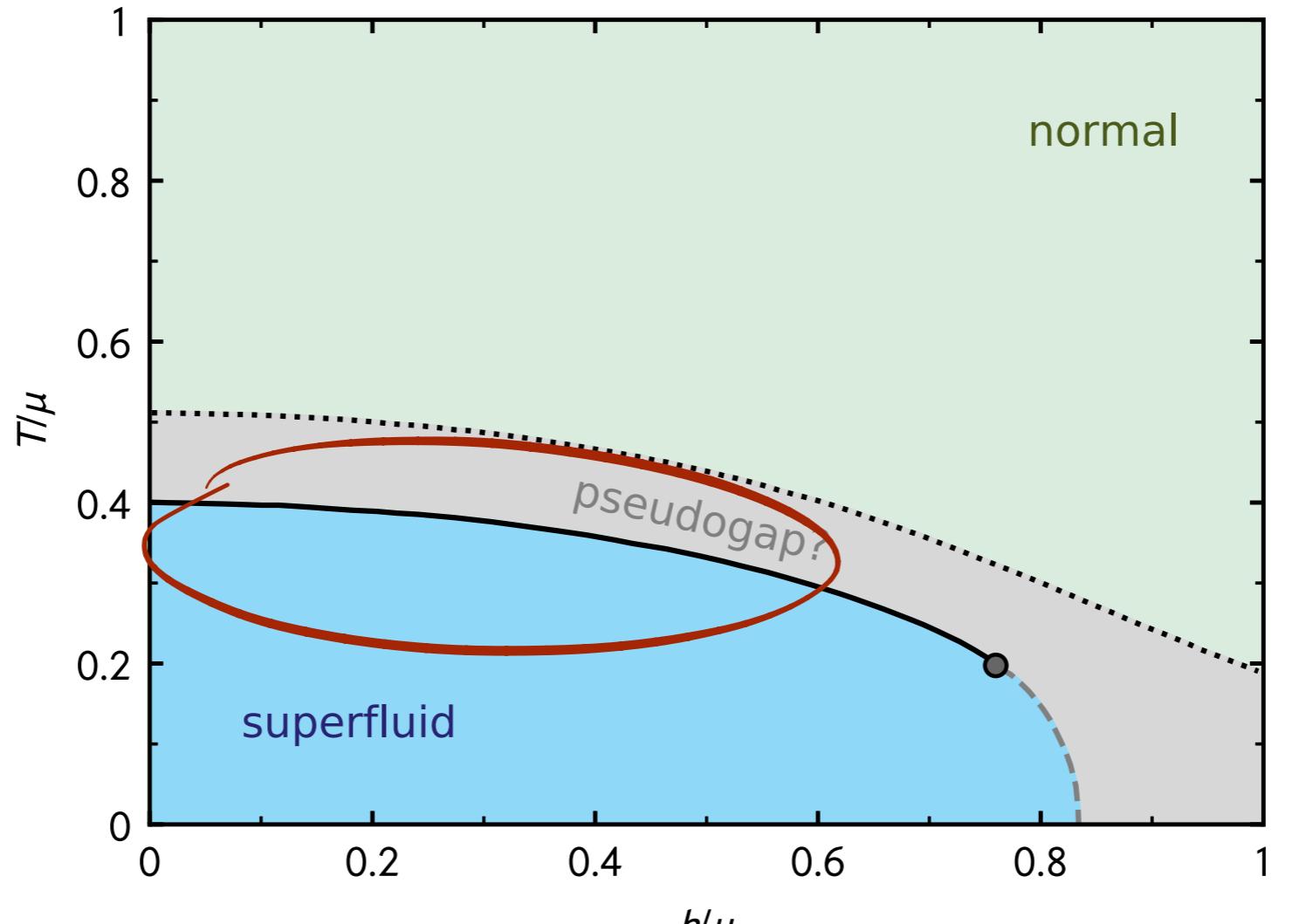
[LR, Loheac, Drut, Braun '18]

weak dependence
of the critical
temperature on
polarization
indicated



challenging to
extract precise T_C

UFG phase diagram



$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2}$$

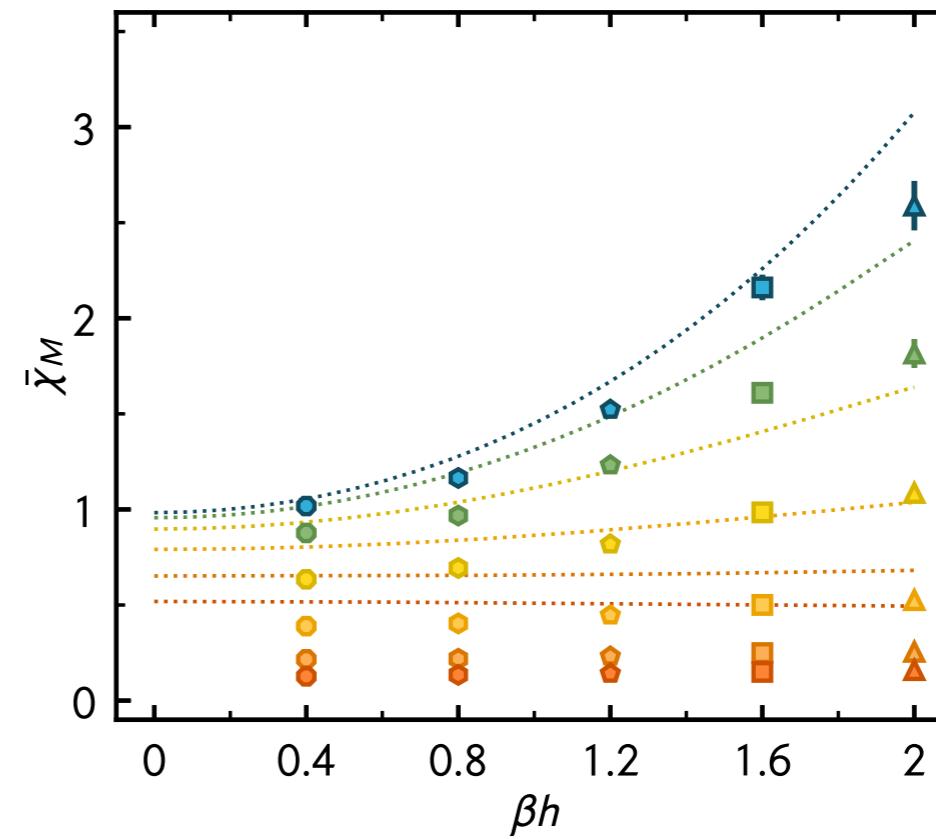
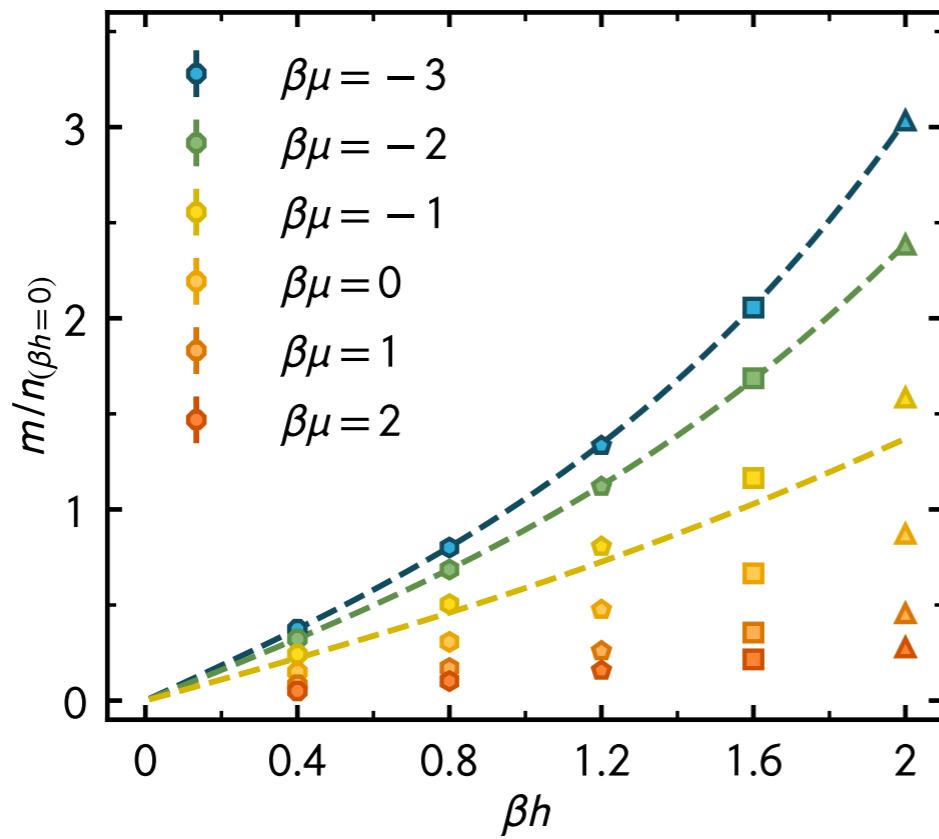
$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$

[fRG: Boettcher et. al '15]

spin susceptibility

[LR, Loheac, Drut, Braun '18]

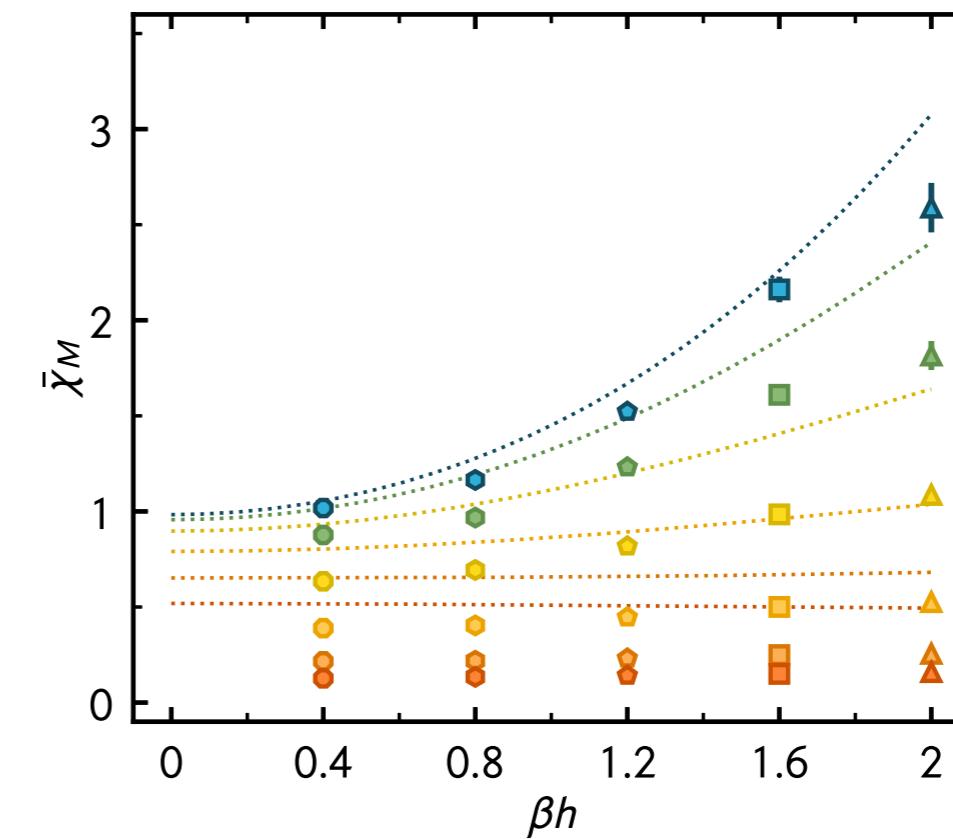
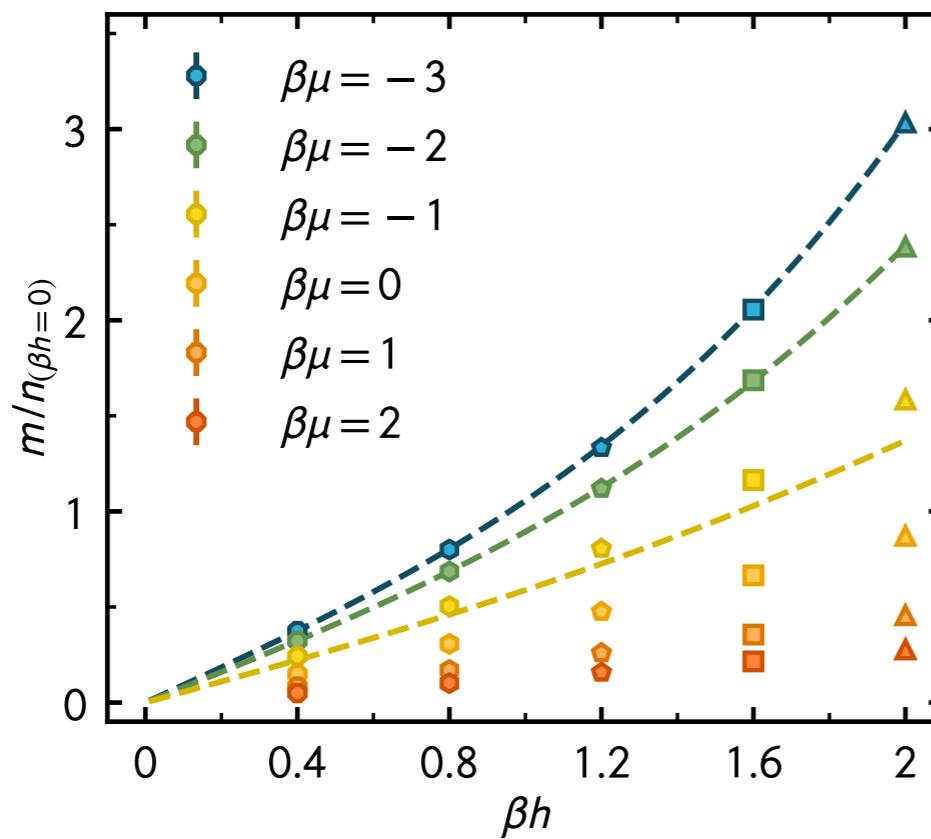
$$\chi = \left(\frac{\partial m}{\partial h} \right)_{T,V,\mu}$$



spin susceptibility

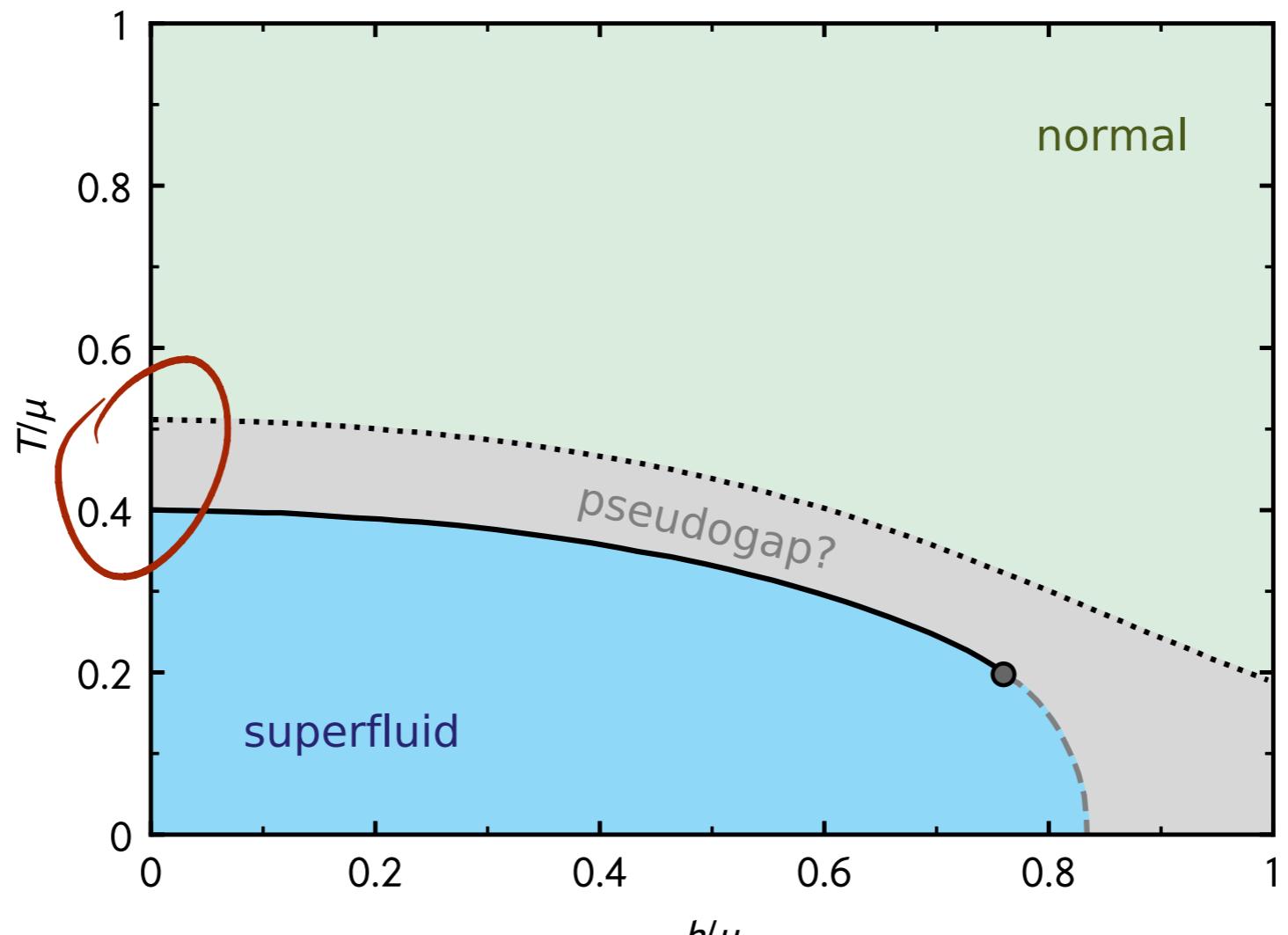
[LR, Loheac, Drut, Braun '18]

$$\chi = \left(\frac{\partial m}{\partial h} \right)_{T,V,\mu}$$



Pauli susceptibility field independent at low field and temperature
UFG: dependence on βh very similar to FG, but rescaled

UFG phase diagram (sketch)



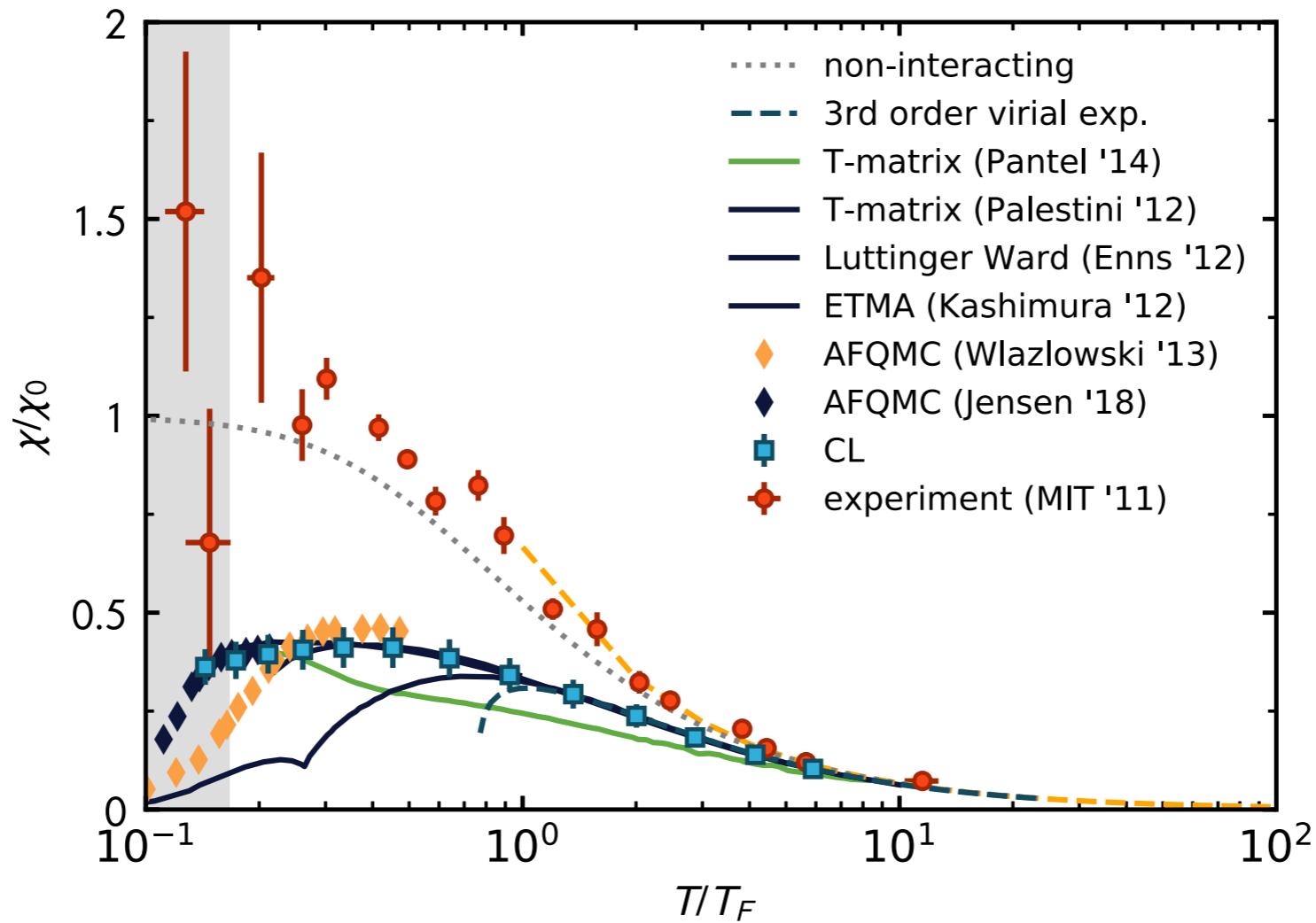
$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2}$$

$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$

[fRG: Boettcher et. al '15]

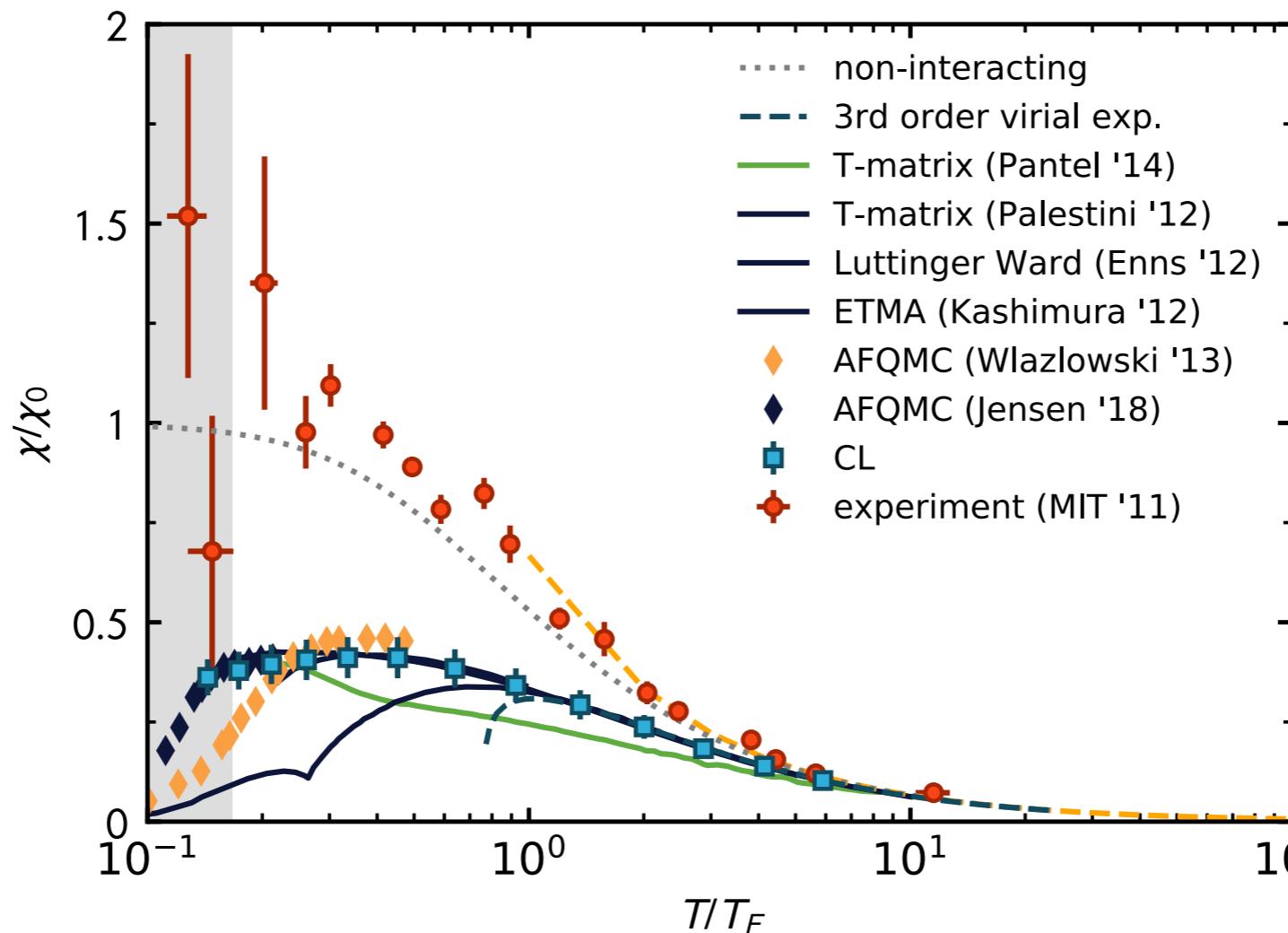
magnetic susceptibility (unpolarized)

[LR, Loheac, Drut, Braun in preparation]



magnetic susceptibility (unpolarized)

[LR, Loheac, Drut, Braun in preparation]



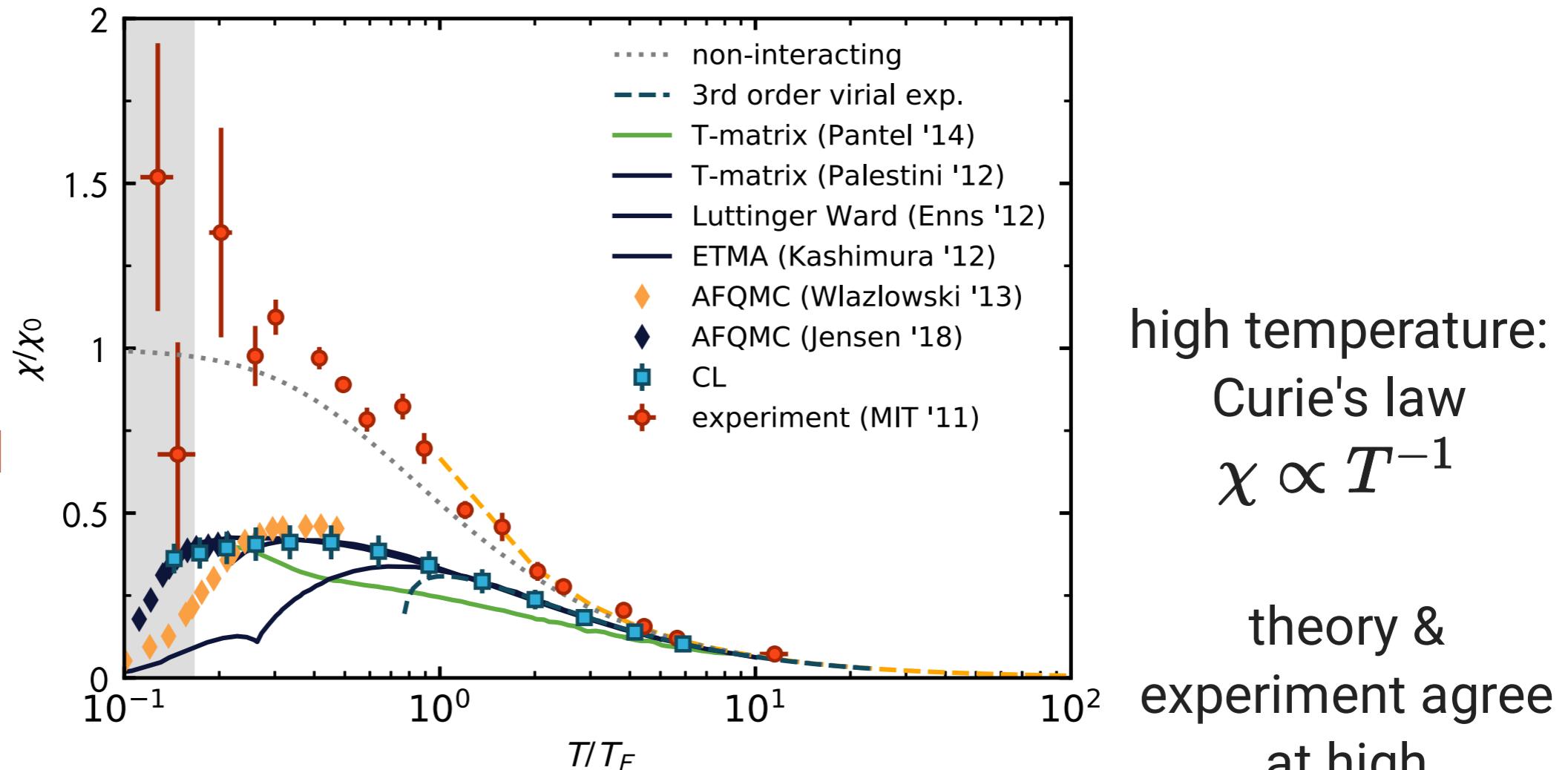
high temperature:
Curie's law
 $\chi \propto T^{-1}$

theory &
experiment agree
at high
temperatures

magnetic susceptibility (unpolarized)

[LR, Loheac, Drut, Braun in preparation]

low
temperature:
discrepancy
between
experiment and
theory



Pseudogap:
suppression of χ at $T > T_C$

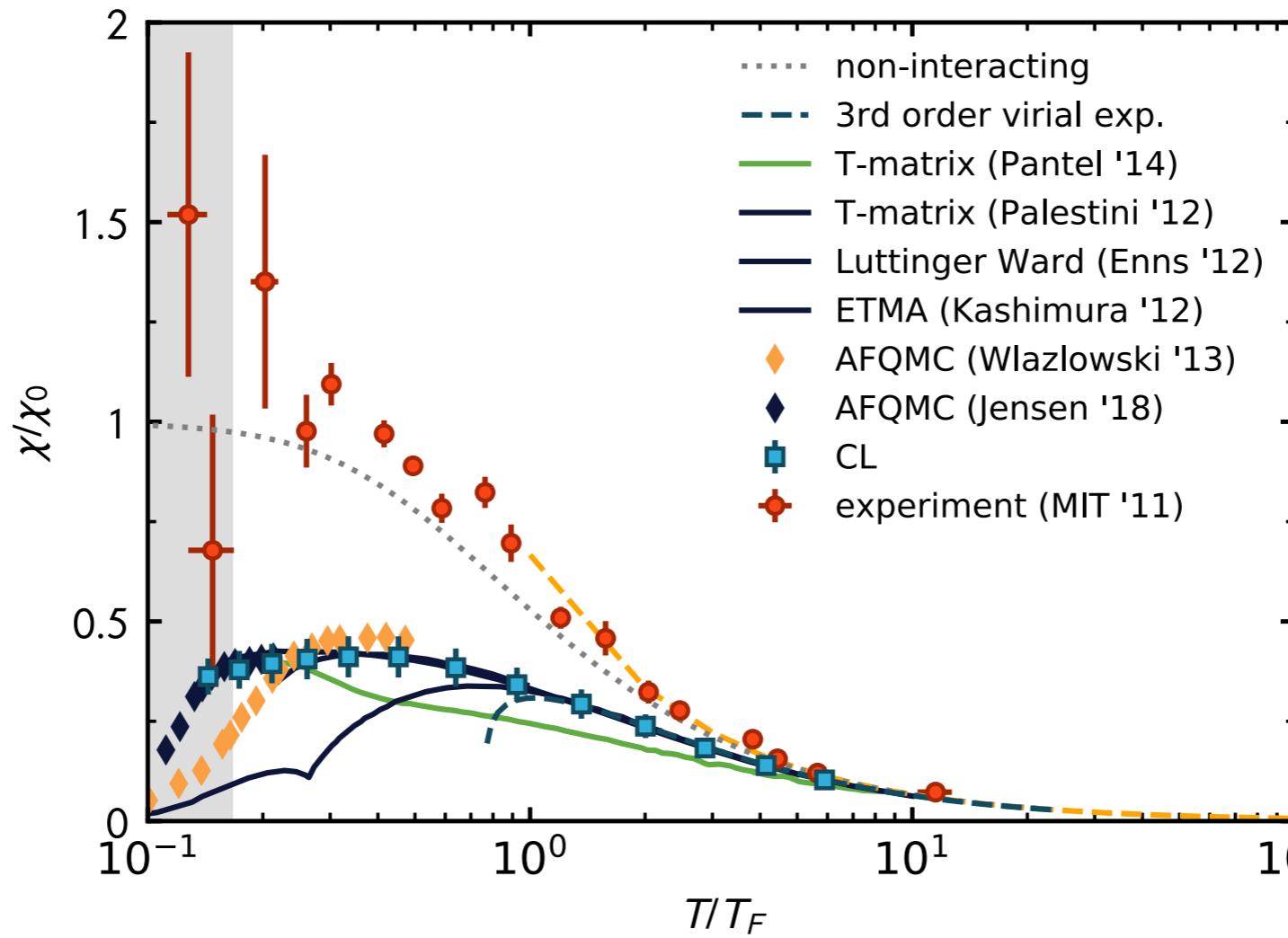
theory &
experiment agree
at high
temperatures

[recent review: Jensen et al. '18]

magnetic susceptibility (unpolarized)

[LR, Loheac, Drut, Braun in preparation]

low
temperature:
discrepancy
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Pseudogap:
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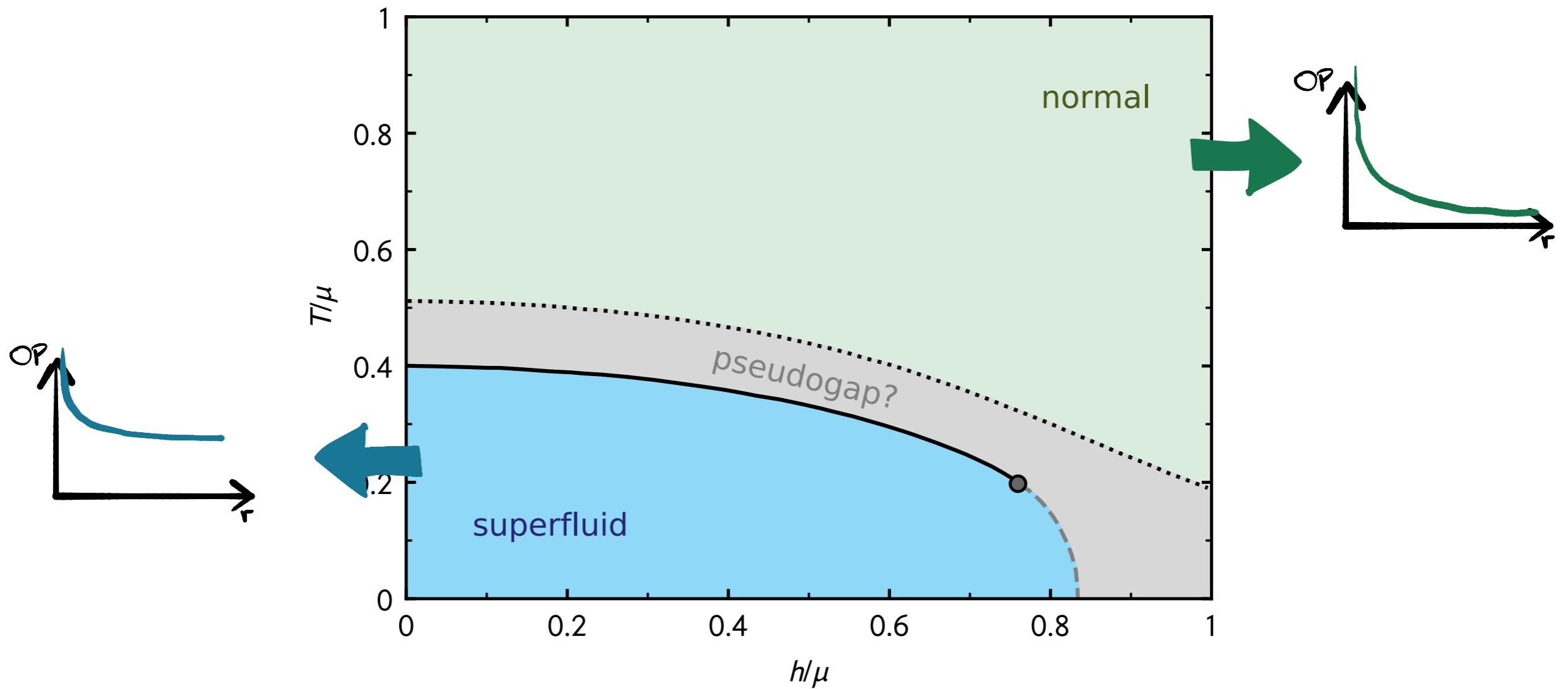
CL: pseudogap possible
 T^* and T_C seem to be very close

recap: unitary fermions

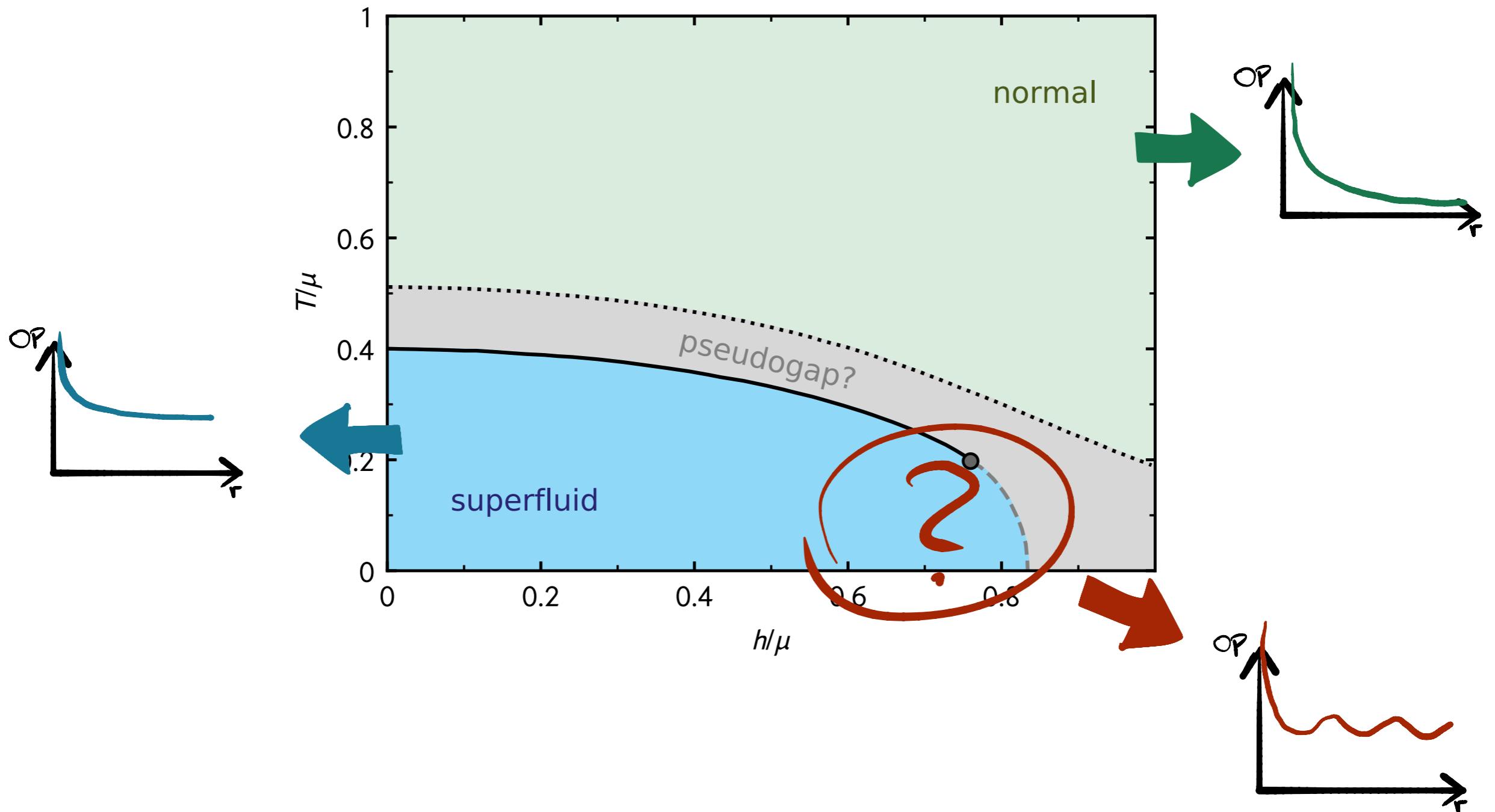
EOS, magnetic properties & response accessible
for the unitary Fermi gas at finite temperature and
polarization

CL **matches state-of-the art results**
from other methods and experiments
wherever available

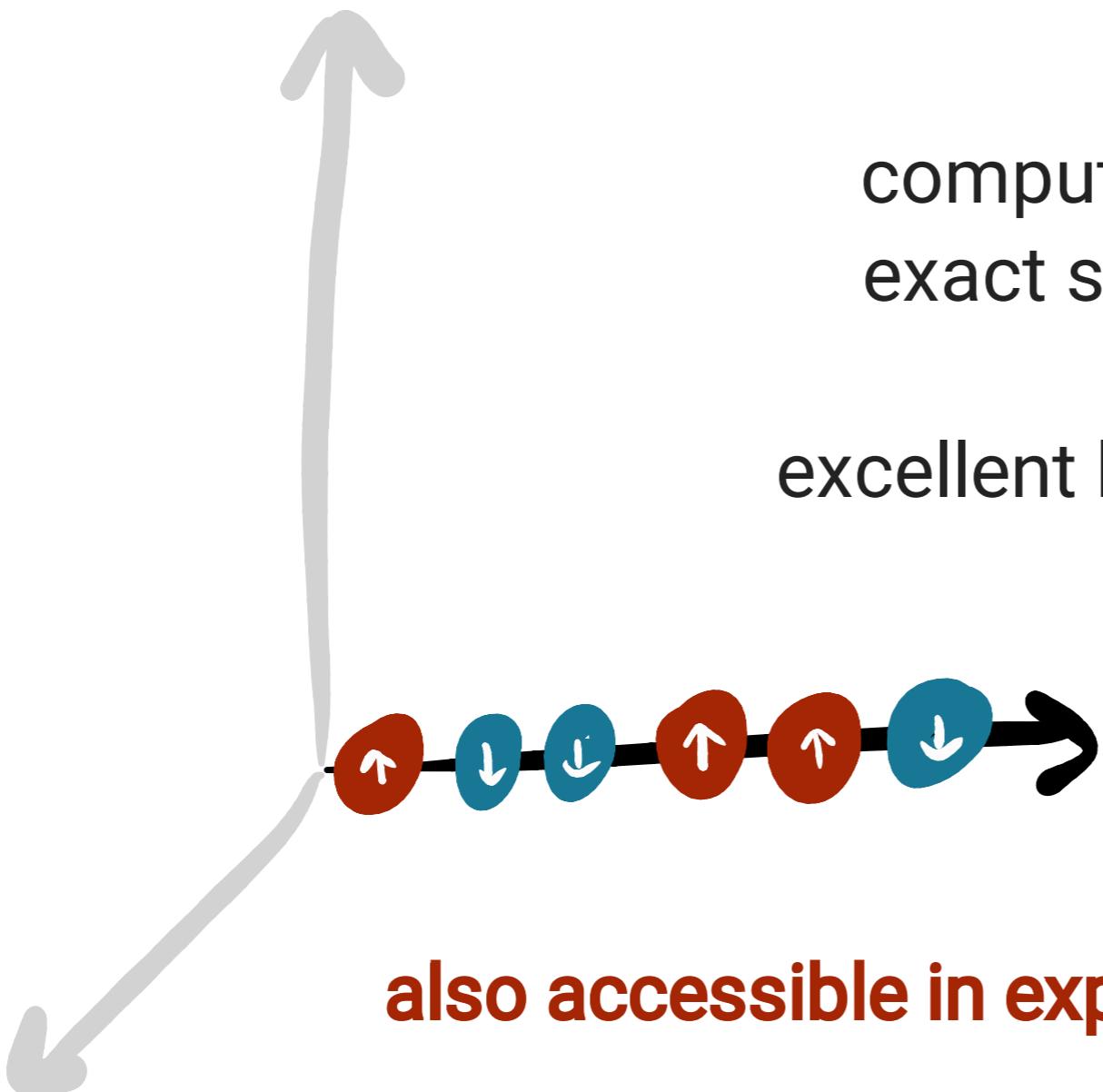
UFG phase diagram (sketch)



UFG phase diagram (sketch)



one-dimensional systems



computationally cheap &
exact solutions available

=

excellent benchmark systems

also accessible in experiment

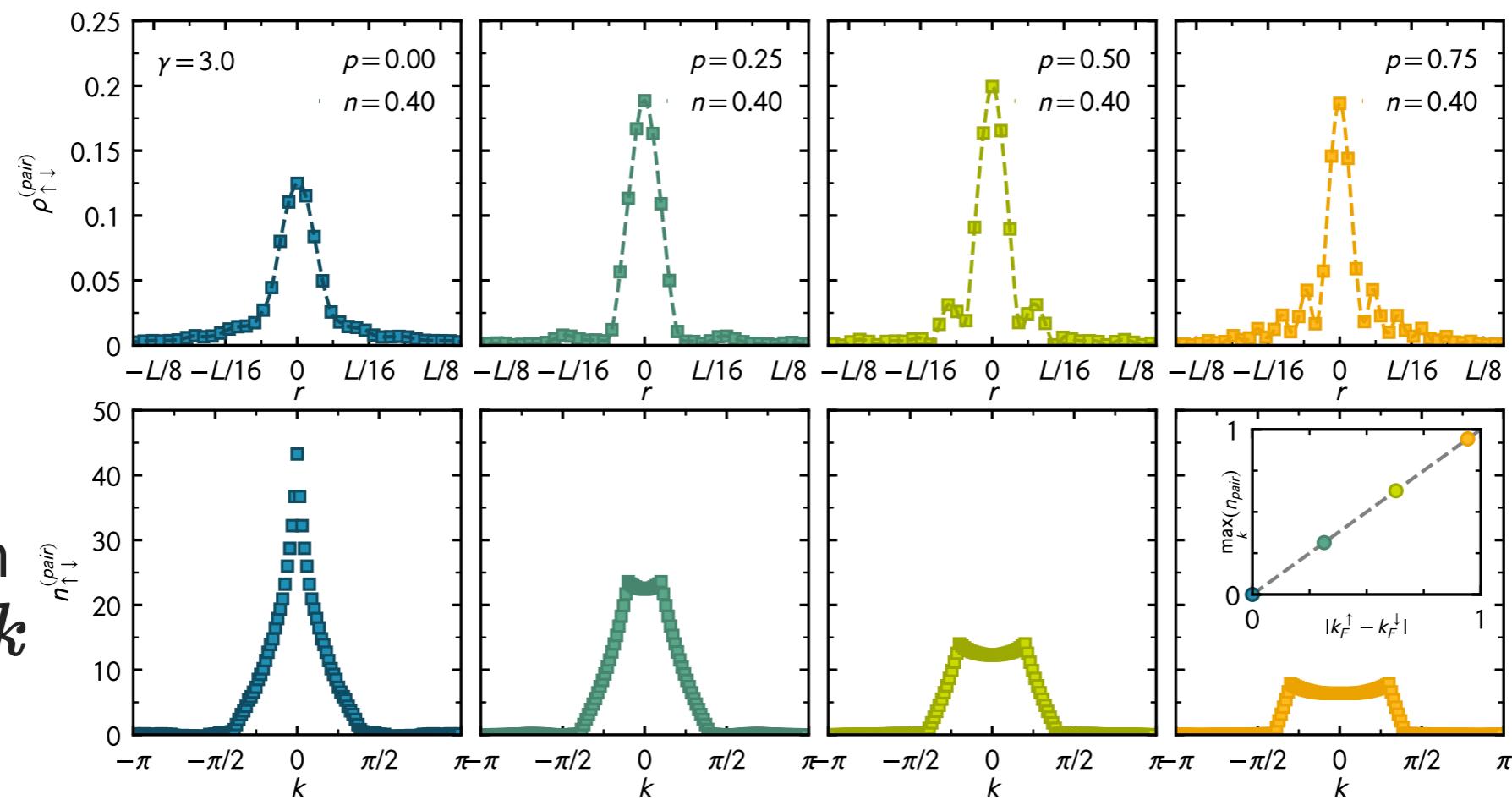
pair correlations

[LR, Drut, Braun in preparation]

$$\rho_{pair}(|x - x'|) = \langle \hat{\psi}_\uparrow^\dagger(x) \hat{\psi}_\downarrow^\dagger(x) \hat{\psi}_\downarrow(x') \hat{\psi}_\uparrow(x') \rangle$$

$$p = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

\sim likelihood
of a pair with
momentum k



spatially
fluctuating
order-
parameter

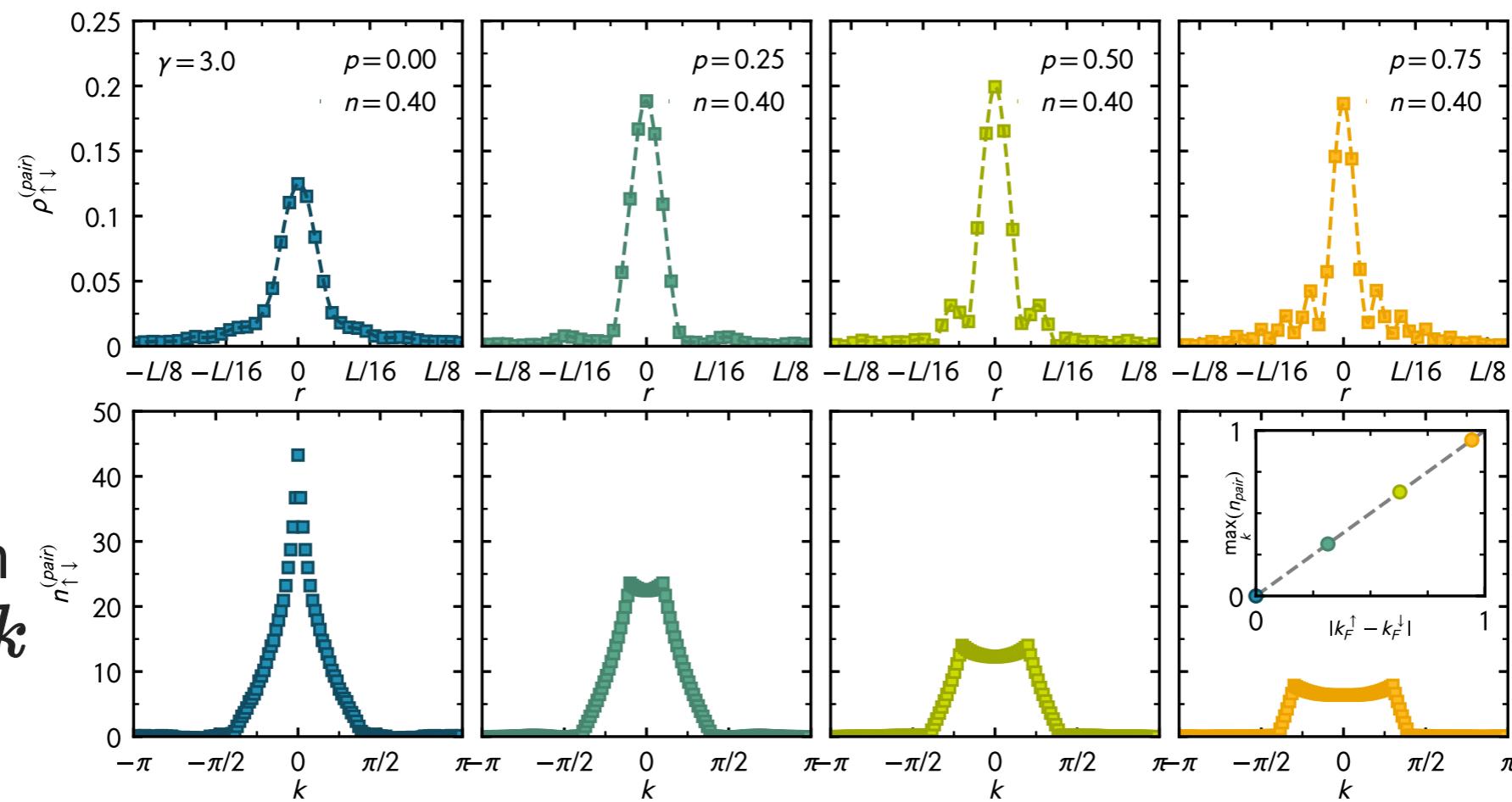
pair correlations

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$$\rho_{pair}(|x - x'|) = \langle \hat{\psi}_\uparrow^\dagger(x) \hat{\psi}_\downarrow^\dagger(x) \hat{\psi}_\downarrow(x') \hat{\psi}_\uparrow(x') \rangle$$

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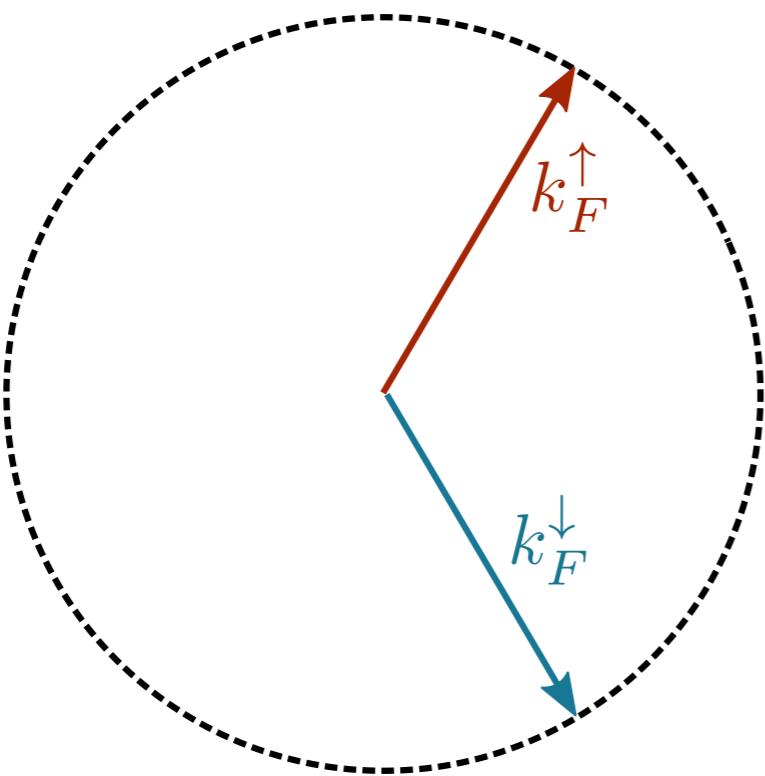
\sim likelihood
of a pair with
momentum k



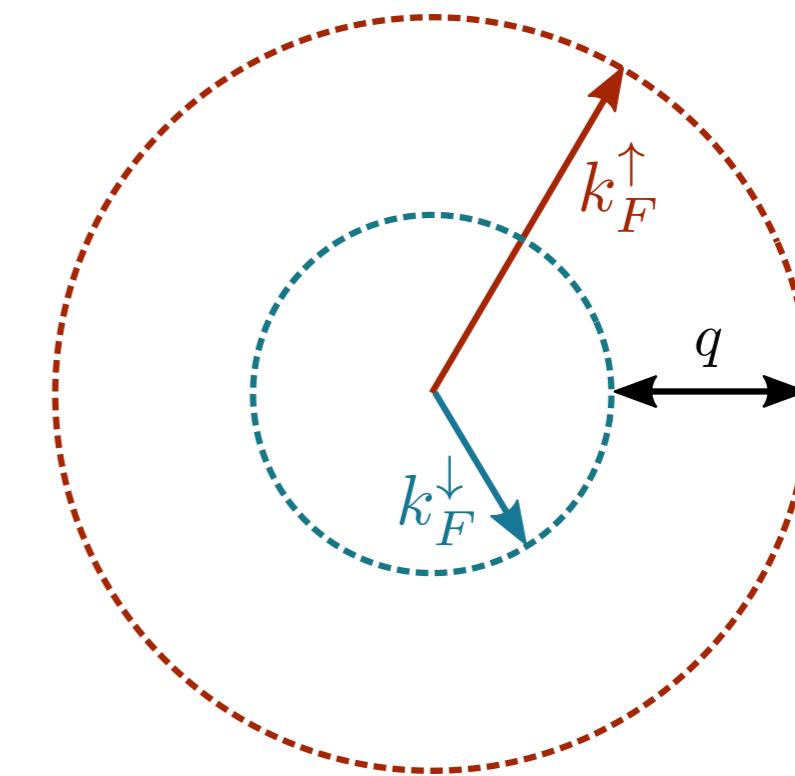
spatially
fluctuating
order-
parameter

off-center peak: hallmark of **FFLO type pairing**

pairing (schematically)



$$\vec{q} \equiv \vec{k}_F^\uparrow - \vec{k}_F^\downarrow = 0$$

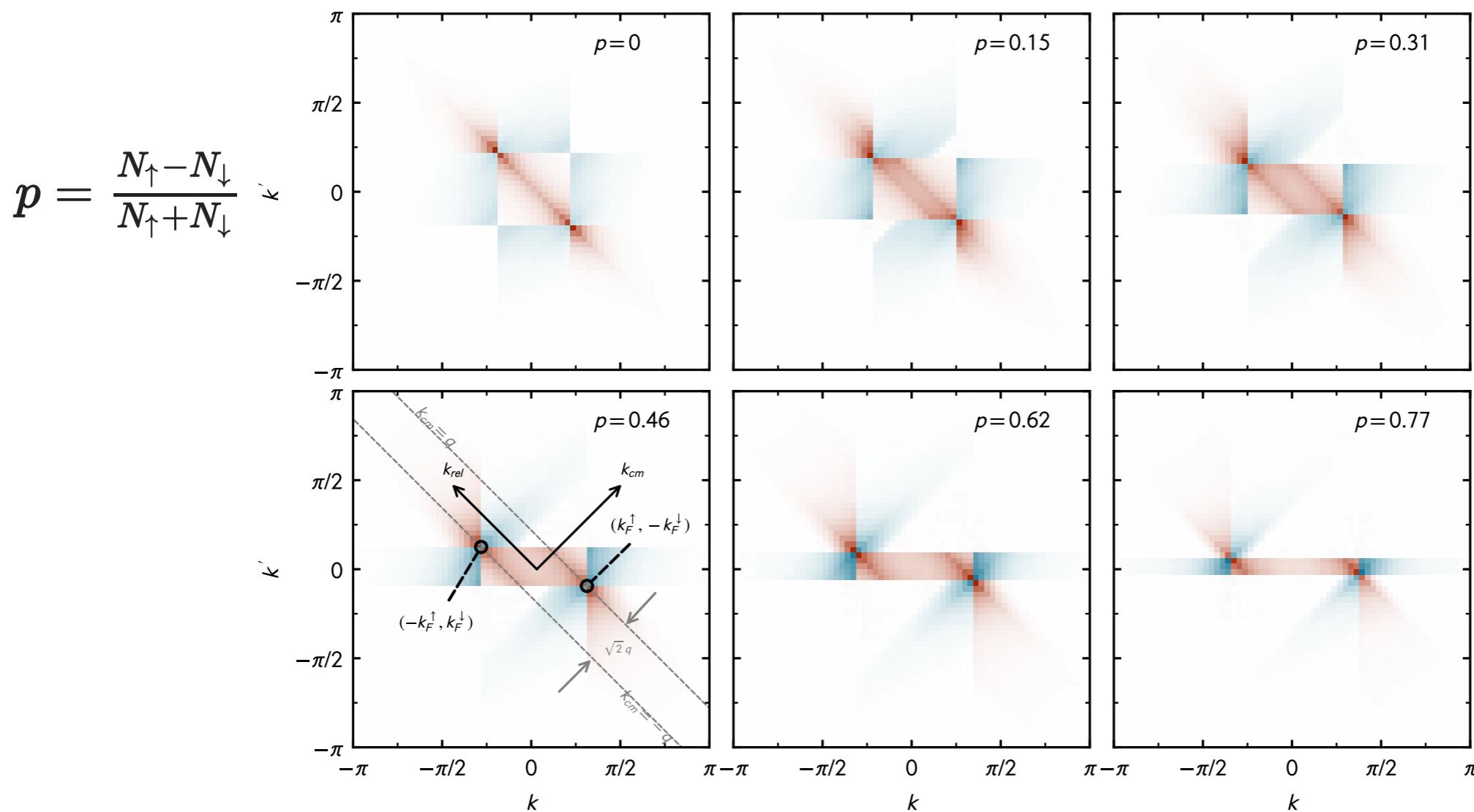


$$\vec{q} \neq 0$$

density-density correlation (shot noise)

[LR, Drut, Braun in preparation]

$$n_{\uparrow\downarrow}(k, k') = \langle \hat{\psi}_{k\uparrow}^\dagger \hat{\psi}_{k\uparrow} \hat{\psi}_{k'\downarrow}^\dagger \hat{\psi}_{k'\downarrow} \rangle - \langle \hat{\psi}_{k\uparrow}^\dagger \hat{\psi}_{k\uparrow} \rangle \langle \hat{\psi}_{k'\downarrow}^\dagger \hat{\psi}_{k'\downarrow} \rangle$$

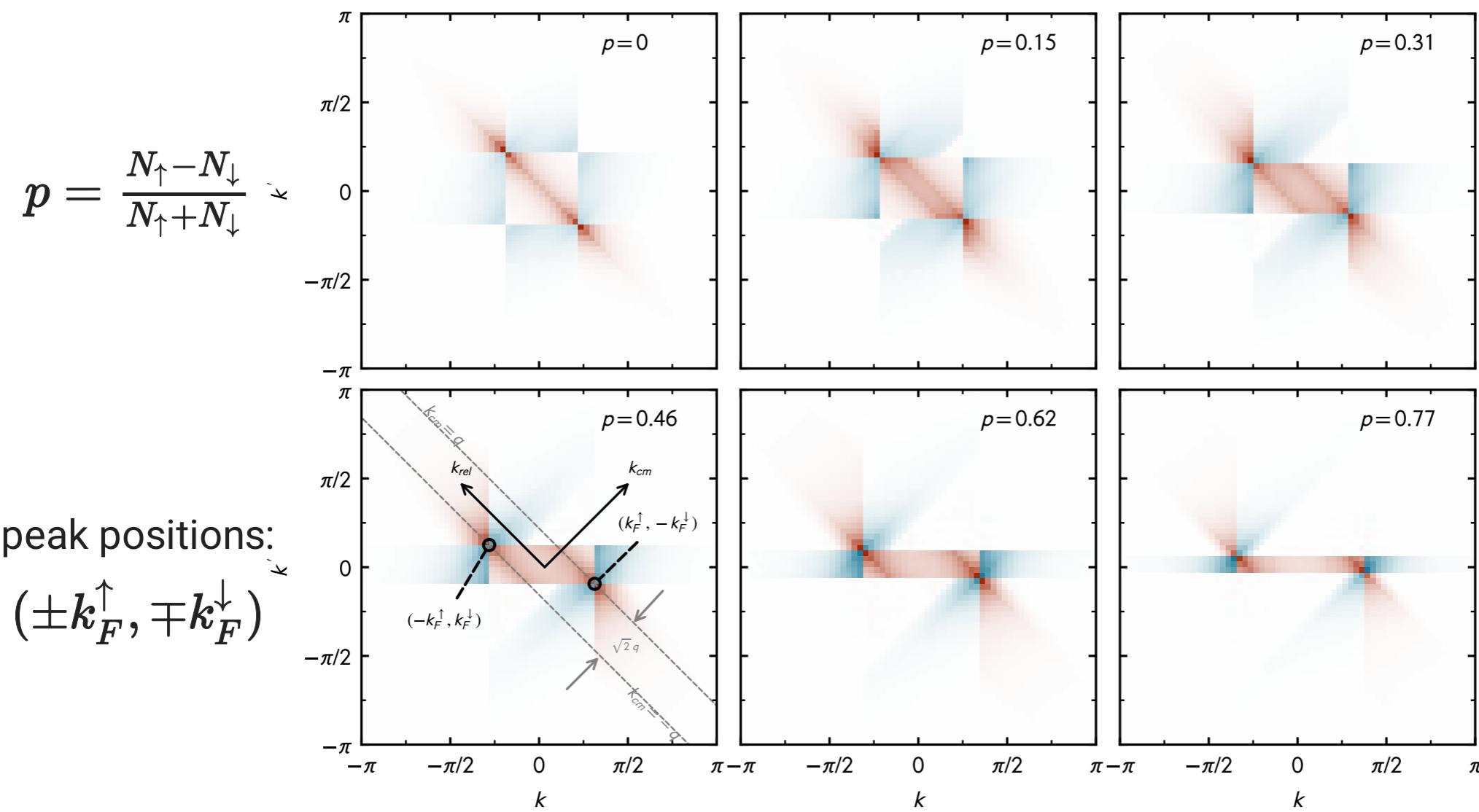


resolution of **internal structure** of fermionic pairs

density-density correlation (shot noise)

[LR, Drut, Braun in preparation]

$$n_{\uparrow\downarrow}(k, k') = \langle \hat{\psi}_{k\uparrow}^\dagger \hat{\psi}_{k\uparrow} \hat{\psi}_{k'\downarrow}^\dagger \hat{\psi}_{k'\downarrow} \rangle - \langle \hat{\psi}_{k\uparrow}^\dagger \hat{\psi}_{k\uparrow} \rangle \langle \hat{\psi}_{k'\downarrow}^\dagger \hat{\psi}_{k'\downarrow} \rangle$$



resolution of **internal structure** of fermionic pairs

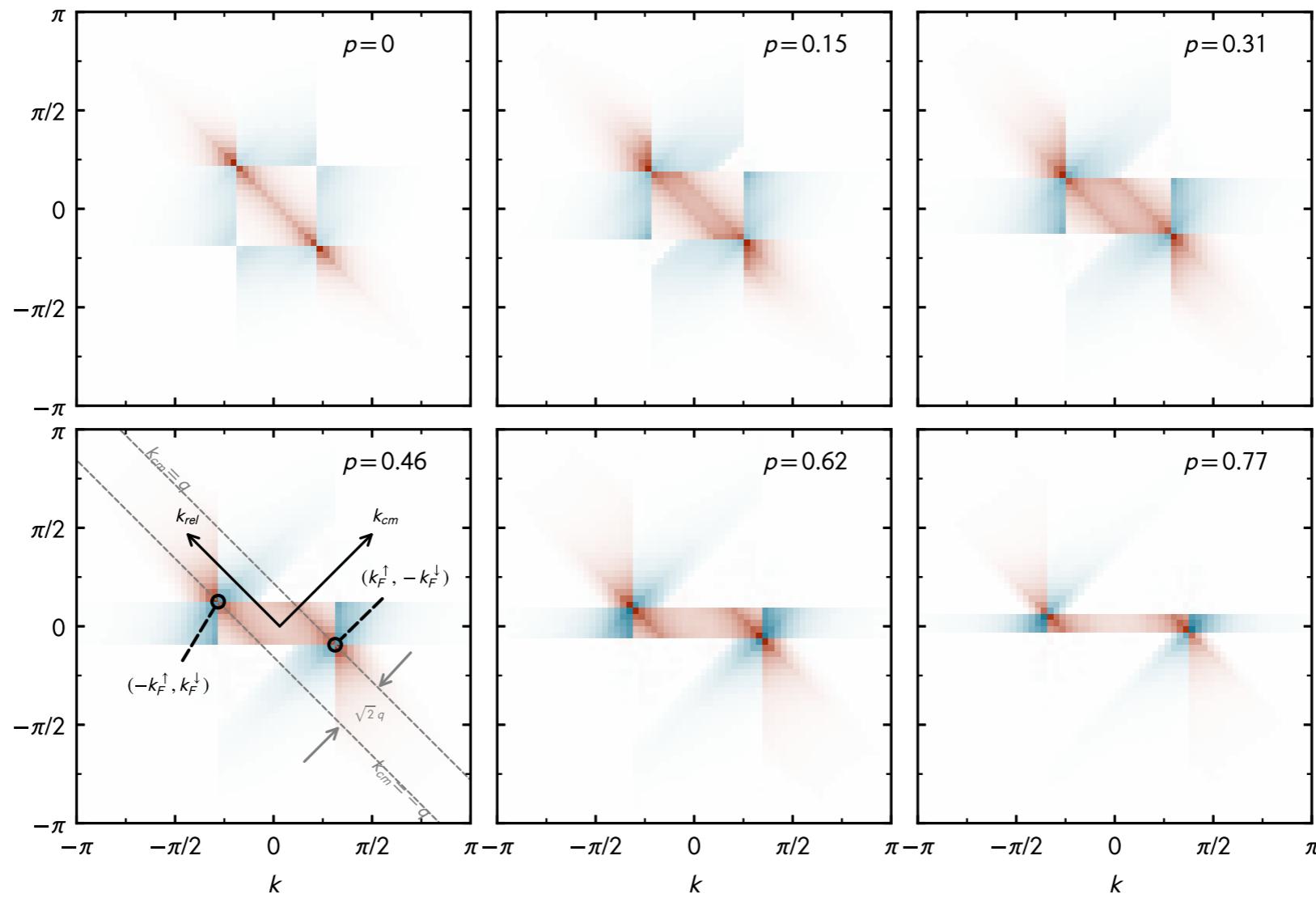
density-density correlation (shot noise)

[LR, Drut, Braun in preparation]

$$n_{\uparrow\downarrow}(k, k') = \langle \hat{\psi}_{k\uparrow}^\dagger \hat{\psi}_{k\uparrow} \hat{\psi}_{k'\downarrow}^\dagger \hat{\psi}_{k'\downarrow} \rangle - \langle \hat{\psi}_{k\uparrow}^\dagger \hat{\psi}_{k\uparrow} \rangle \langle \hat{\psi}_{k'\downarrow}^\dagger \hat{\psi}_{k'\downarrow} \rangle$$

$$p = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

peak positions:
 $(\pm k_F^\uparrow, \mp k_F^\downarrow)$



positive
correlations:
particle-
particle

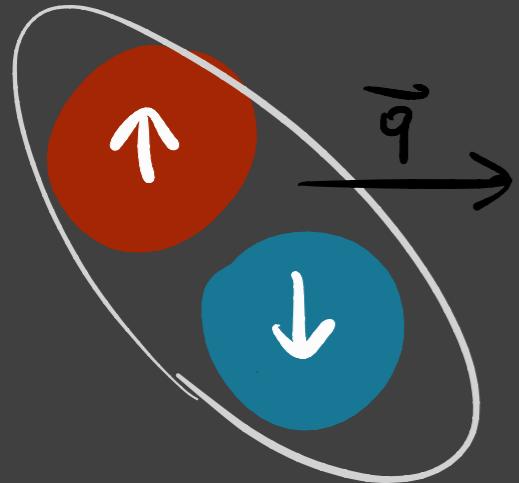
negative
correlations:
particle-hole

resolution of internal structure of fermionic pairs

recap

**complex Langevin is a valuable tool
to study ultracold Fermi gases
(it works quite well)**

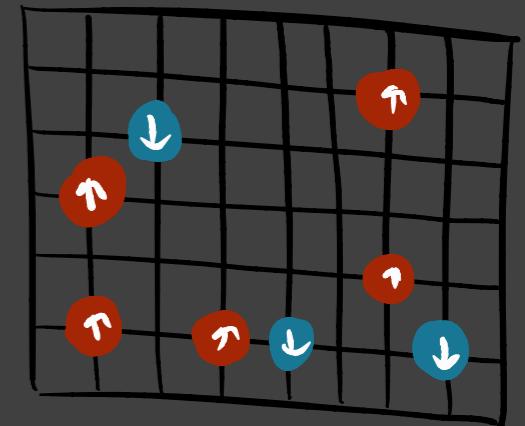
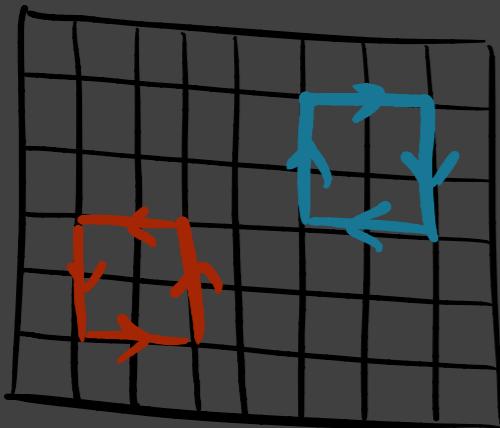
stay tuned!



looking for inhomogeneous phases in the UFG

thermodynamics of 2D fermions at finite polarization

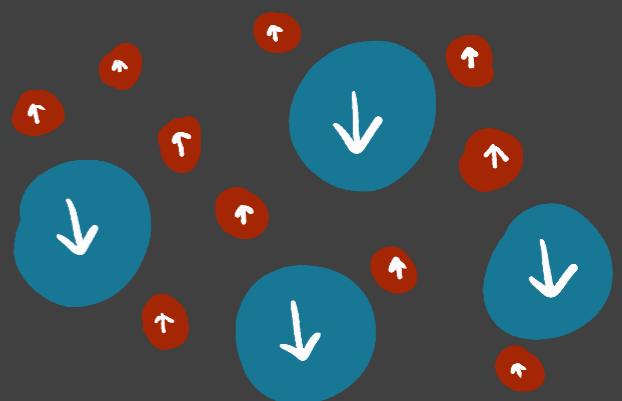
[with Josh McKenney, Andrew Loheac & Joaquin Drut, UNC Chapel Hill]



vortex formation in 2D rotating bosons

[Casey Berger & Joaquin Drut, UNC Chapel Hill]

effect of mass-imbalance on fermion pair formation



the team



Jens Braun



Florian Ehmann



Joaquin Drut



Andrew Loheac



Casey Berger



TU Darmstadt



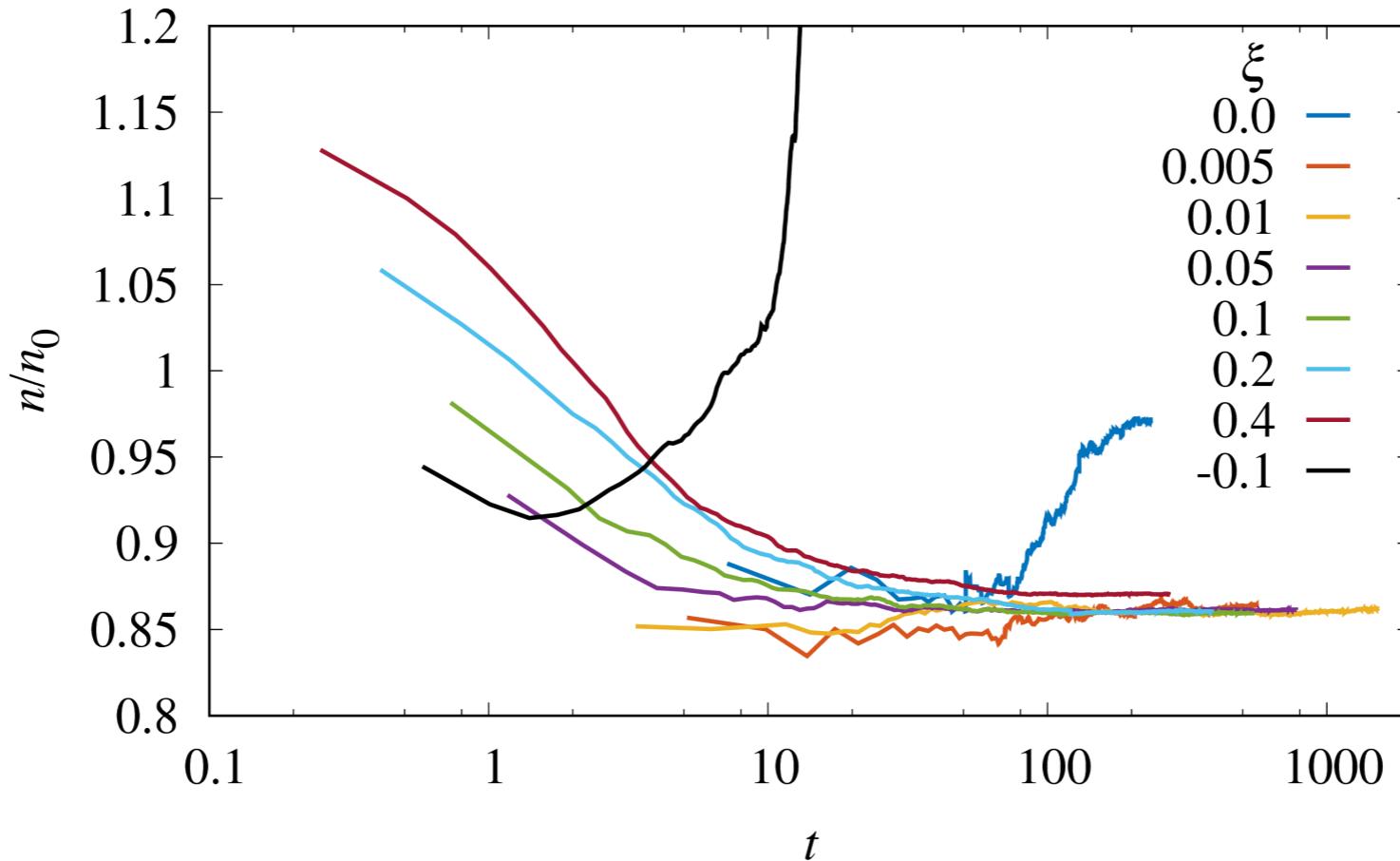
UNC Chapel Hill

APPENDIX

regulator to stabilize numerics

[Loheac,Drut '17]

$$\phi^{(n+1)} = \phi^{(n)} - \frac{\delta S[\phi]}{\delta \phi} \Big|_{\phi^{(n)}} \Delta t_L - 2\xi \phi^{(n)} + \sqrt{2\Delta t_L} \eta$$

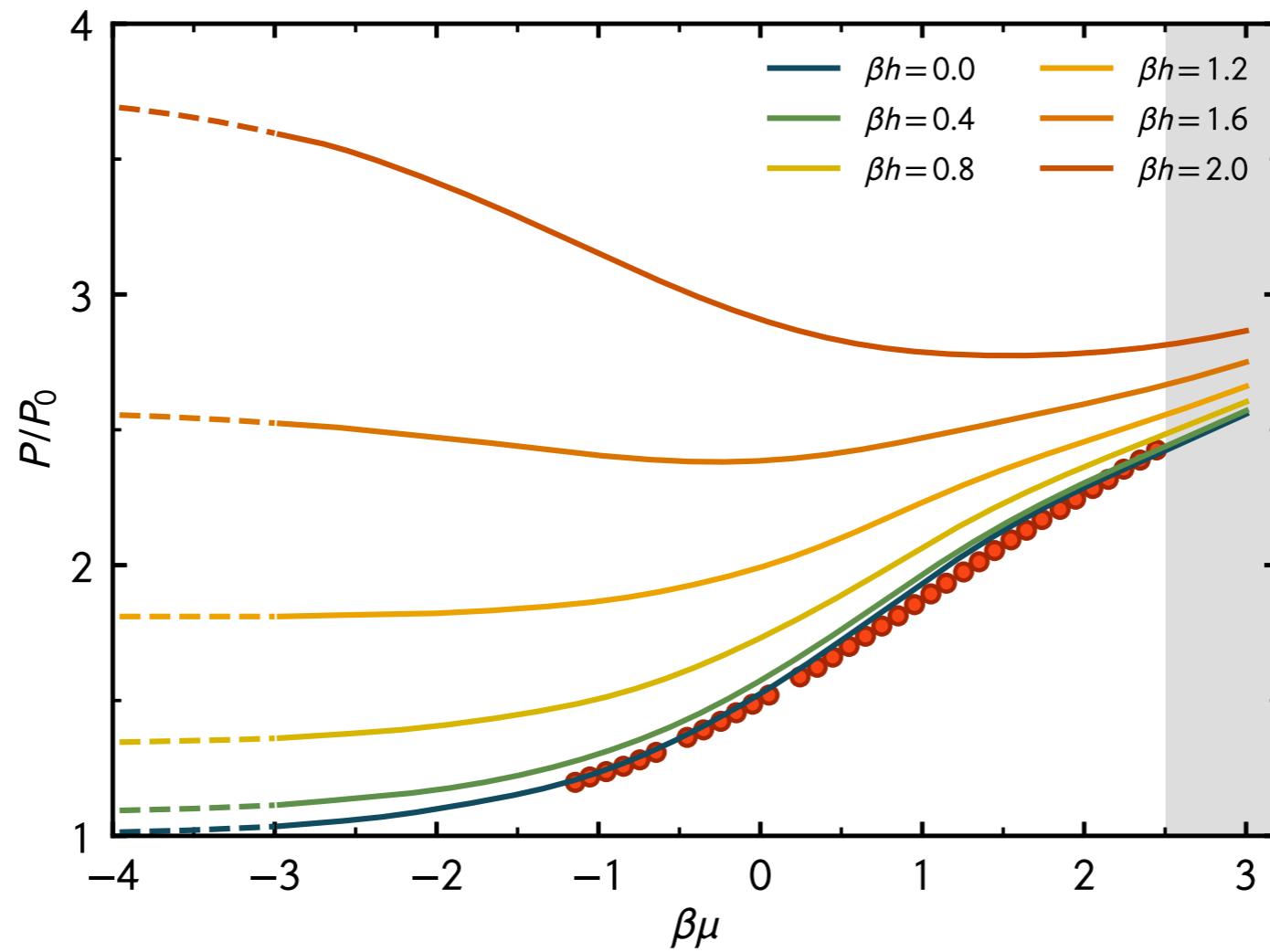


unregulated runs tend to fail: ξ stabilizes CL trajectories

pressure equation of state

[LR, Loheac, Drut, Braun '18]

$$P(\beta\mu) = \frac{1}{\beta} \int_{-\infty}^{\beta\mu} n(x) dx$$

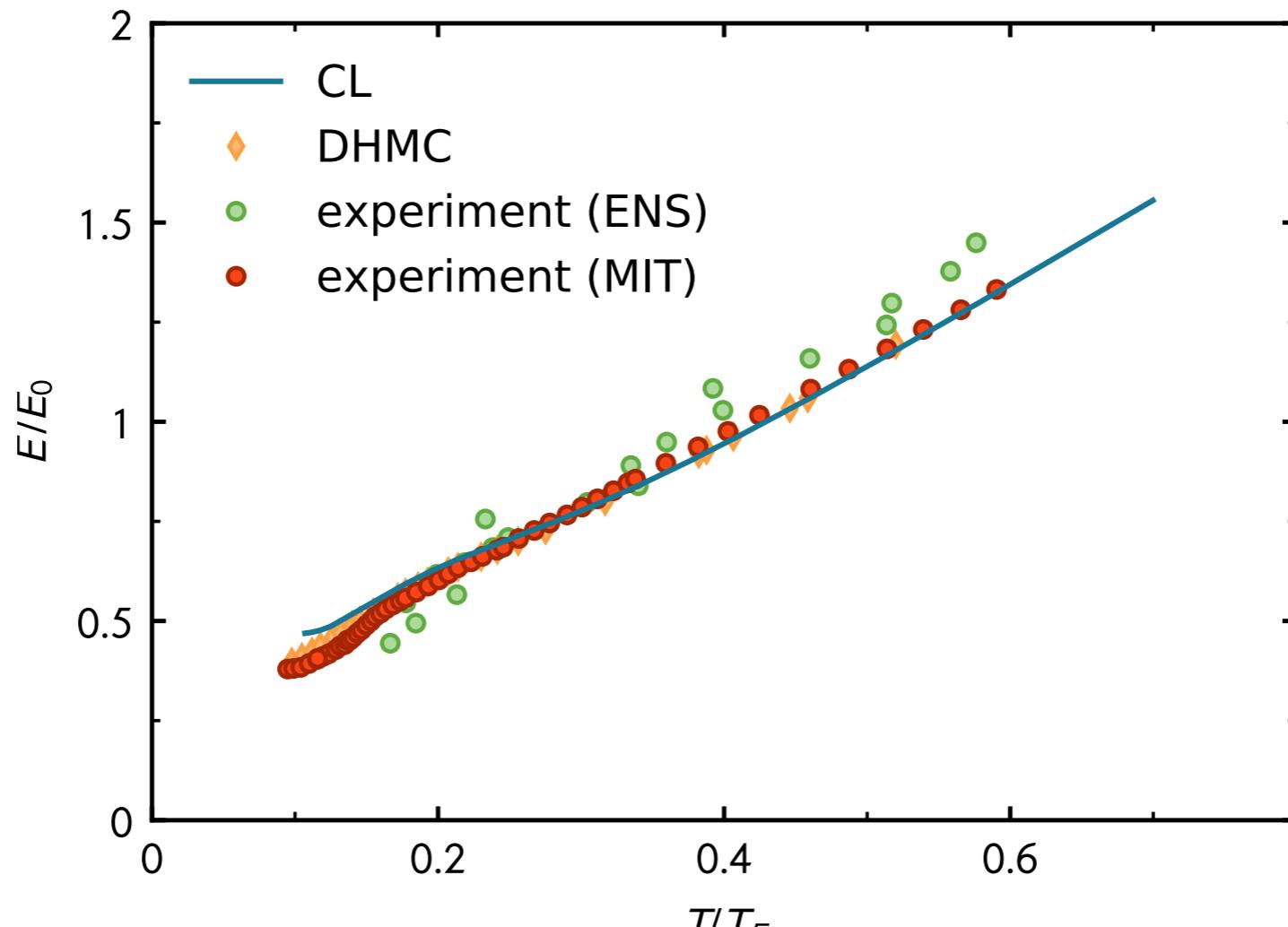


[experiment: Ku,Sommer,Cheuck,Zwierlein '12]

energy equation of state

[LR, Loheac, Drut, Braun in preparation]

$$E = \frac{3}{2} PV$$



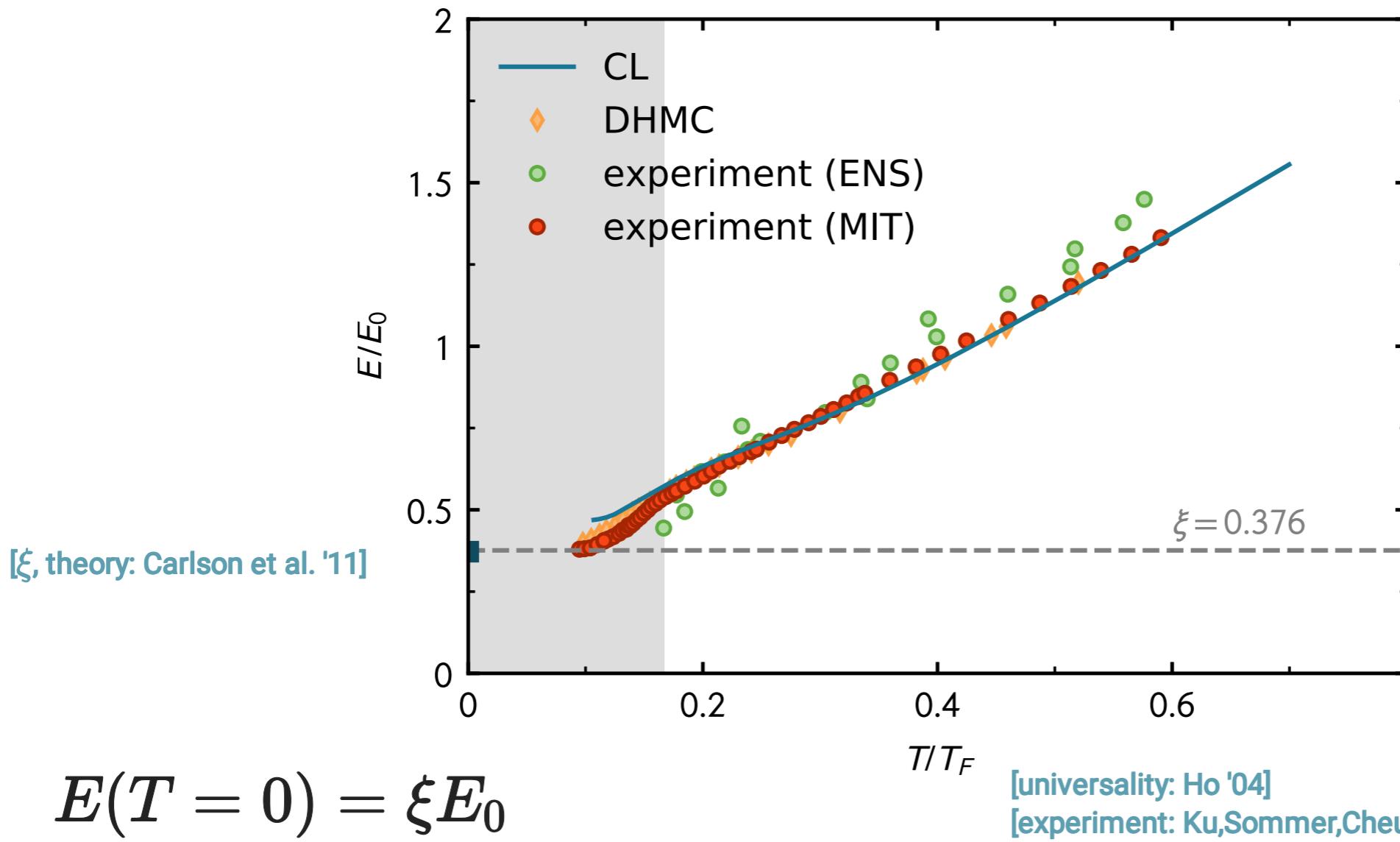
[universality: Ho '04]

[experiment: Ku,Sommer,Cheuck,Zwierlein '12; Nascimbène et al. '10]

energy equation of state

[LR, Loheac, Drut, Braun in preparation]

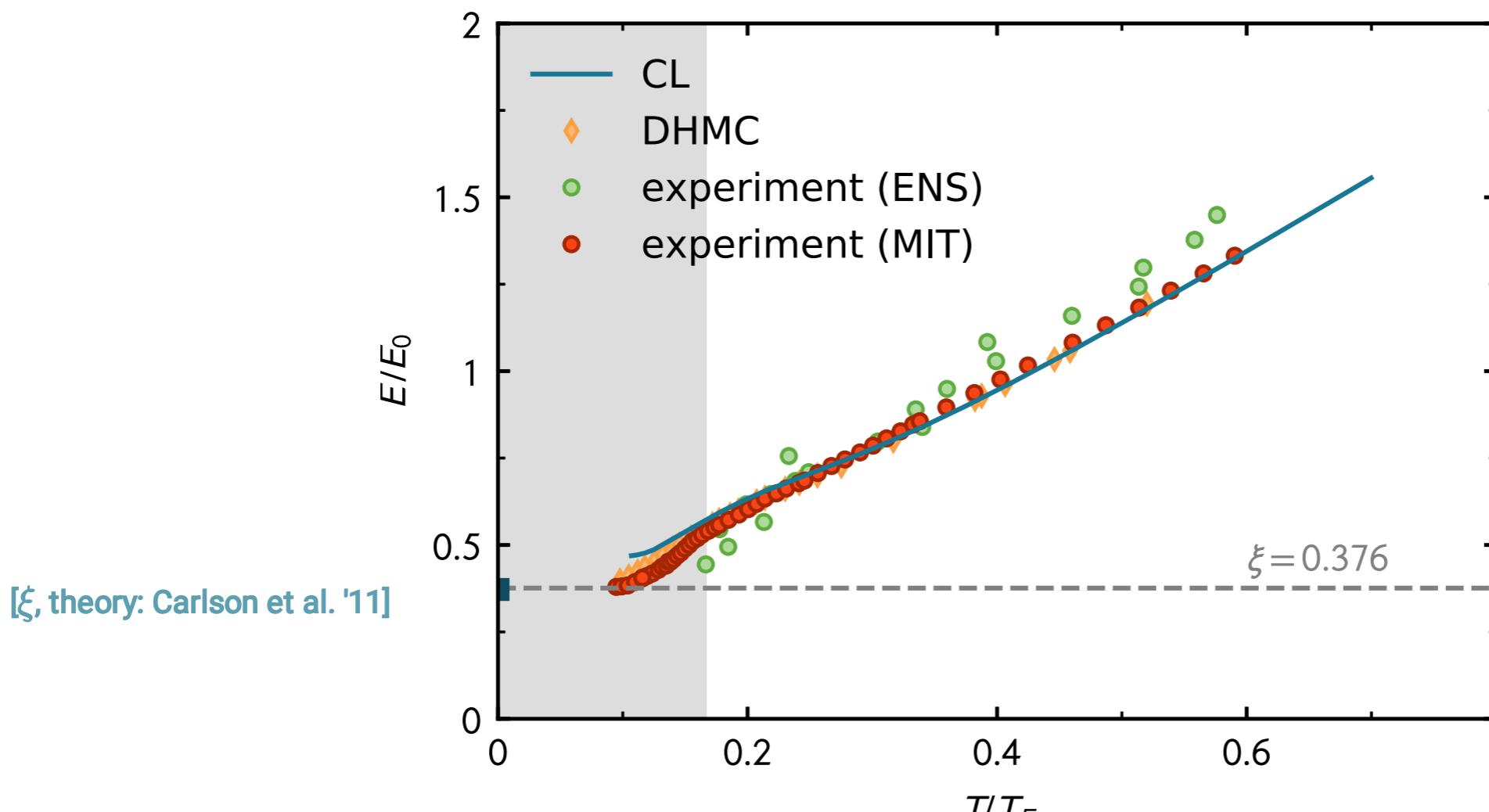
$$E = \frac{3}{2} PV$$



energy equation of state

[LR, Loheac, Drut, Braun in preparation]

$$E = \frac{3}{2} PV$$



$$E(T = 0) = \xi E_0$$

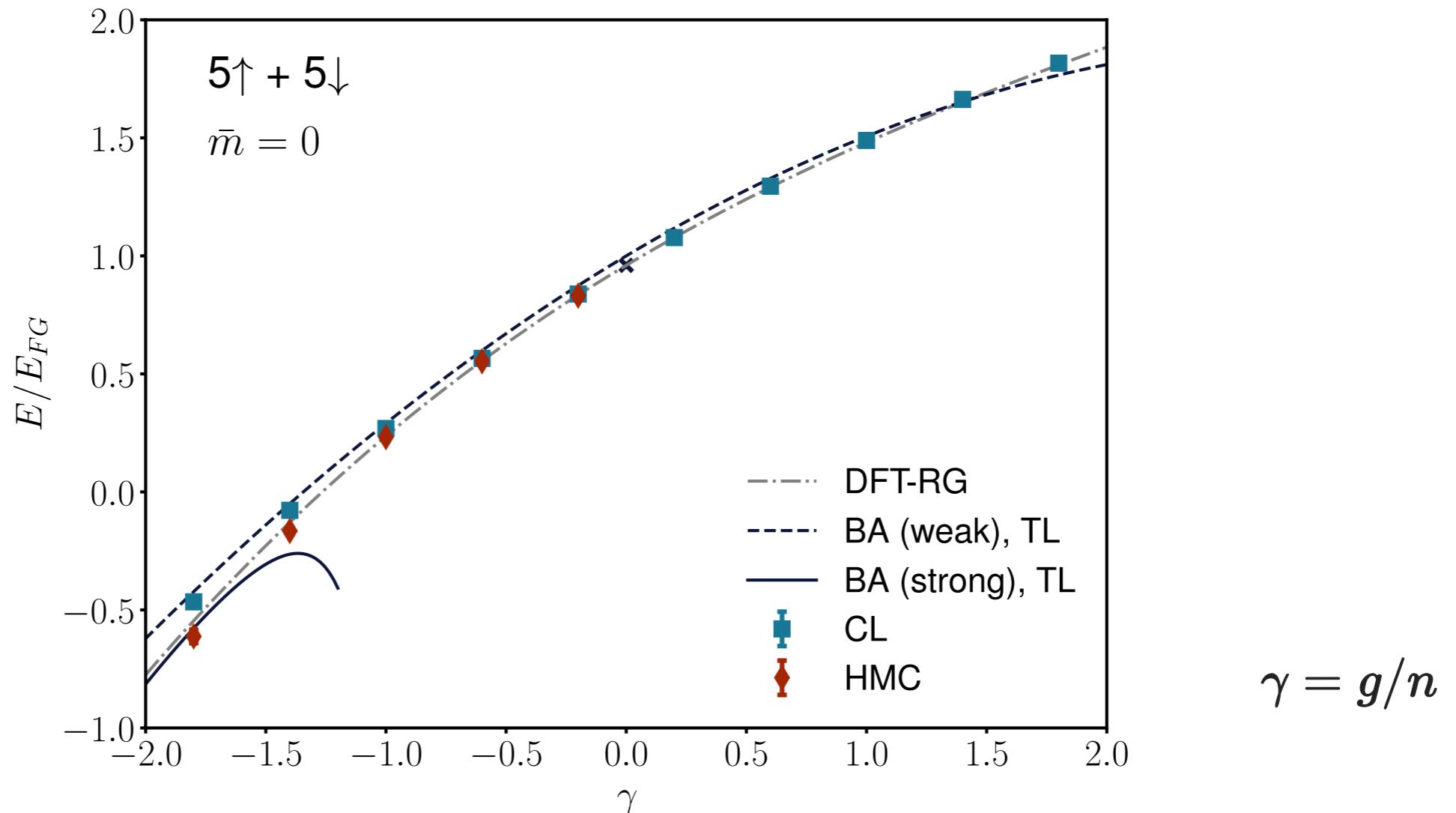
[universality: Ho '04]
[experiment: Ku,Sommer,Cheuck,Zwierlein '12; Nascimbène et al. '10]

at low
temperature:
larger lattices,
improved
operators

[Endres et al. '11; Drut '12]

first step: compare to other methods

[LR,Porter,Drut,Braun '17]



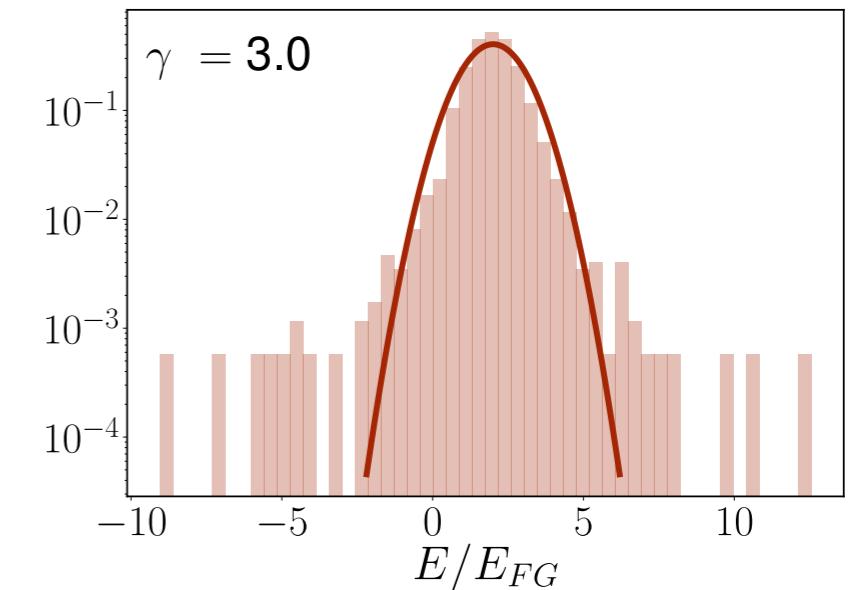
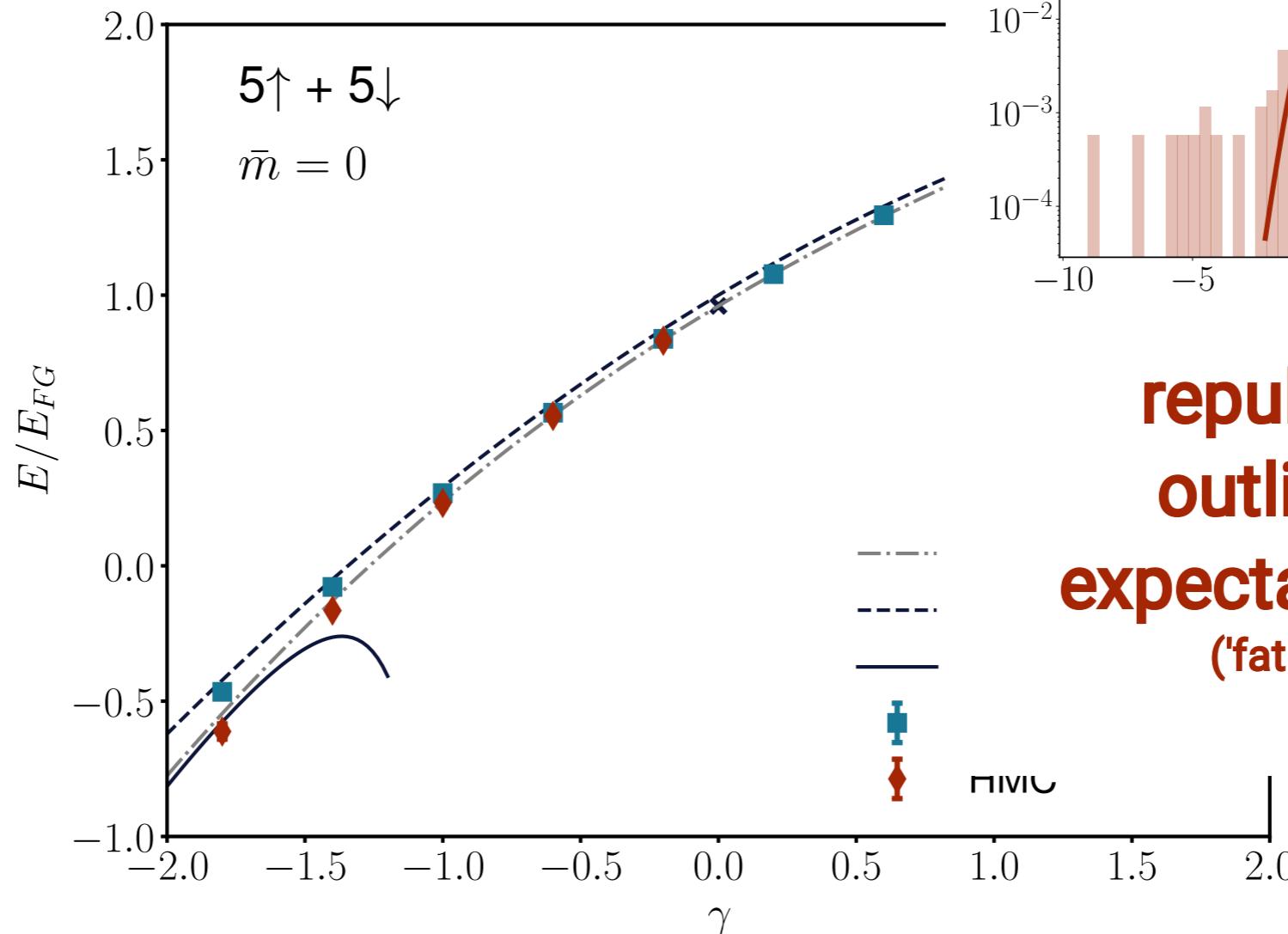
[BA: Iida, Wadati '07; Tracy, Widom '16]

[DFT-RG: Kemler, Pospiech, Braun '17; Kemler '17]

[HMC: LR, Porter, Loheac, Drut '15]

first step: compare to other methods

[LR,Porter,Drut,Braun '17]



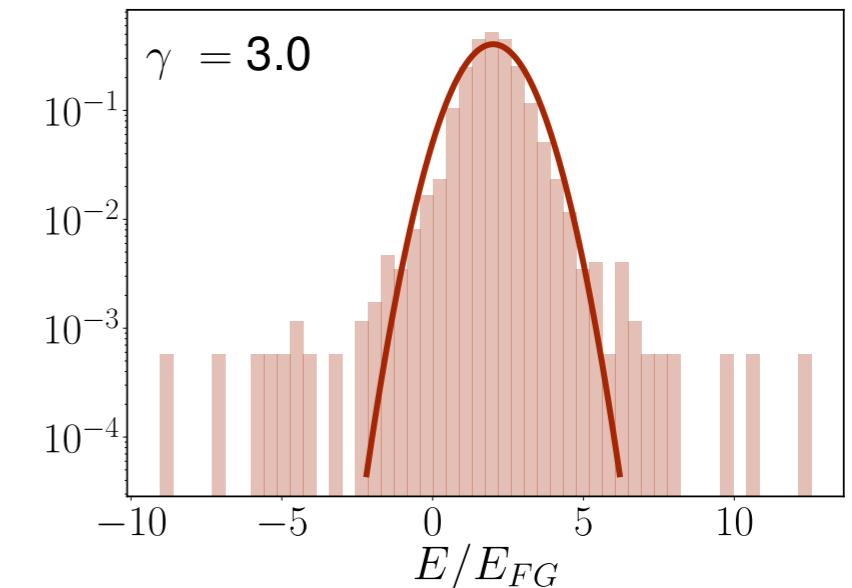
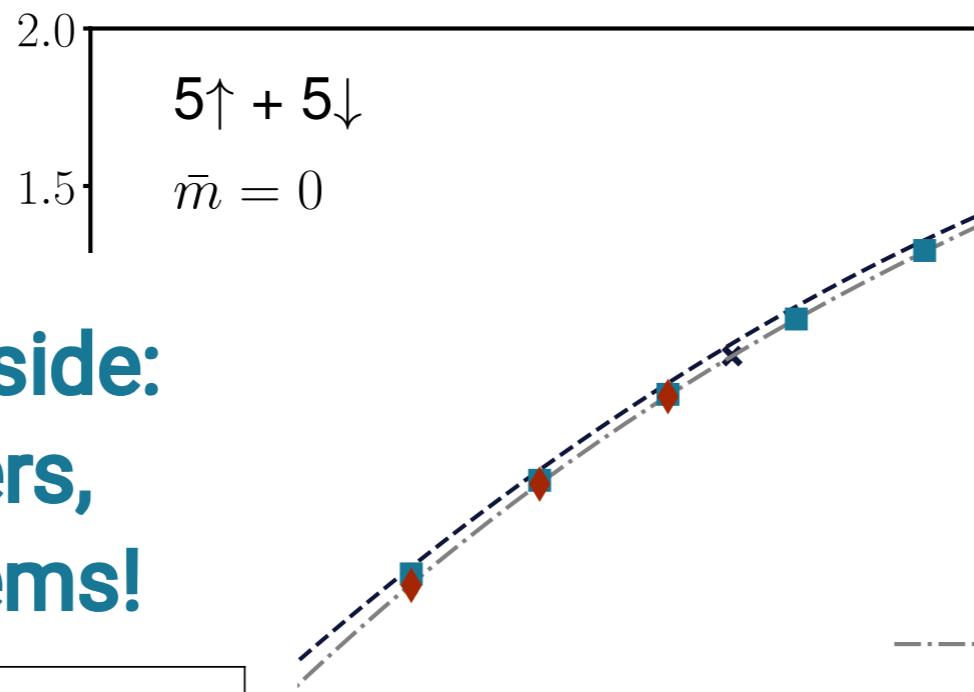
**repulsive side:
outliers skew
expectation values!
('fat tail' problem)**

[BA: Iida, Wadati '07; Tracy, Widom '16]
[DFT-RG: Kemler, Pospiech, Braun '17; Kemler '17]
[HMC: LR, Porter, Loheac, Drut '15]

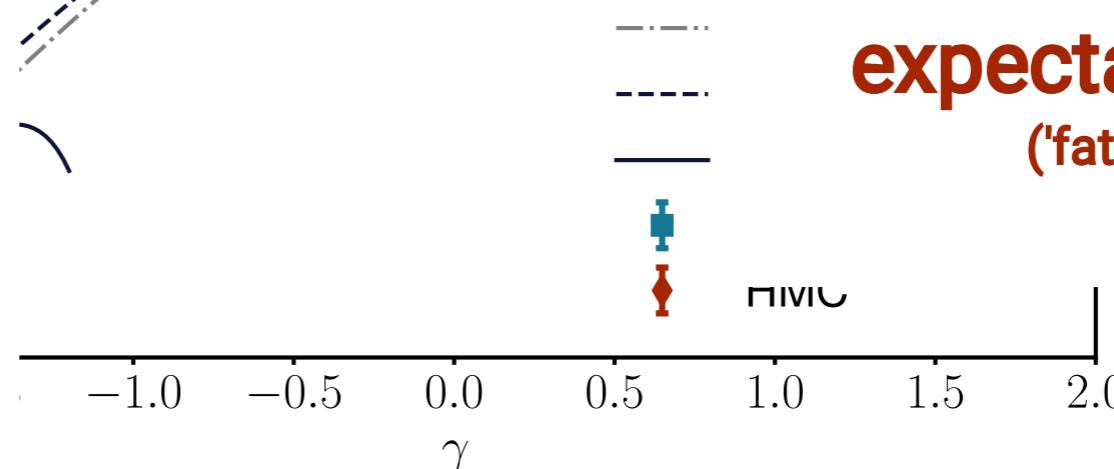
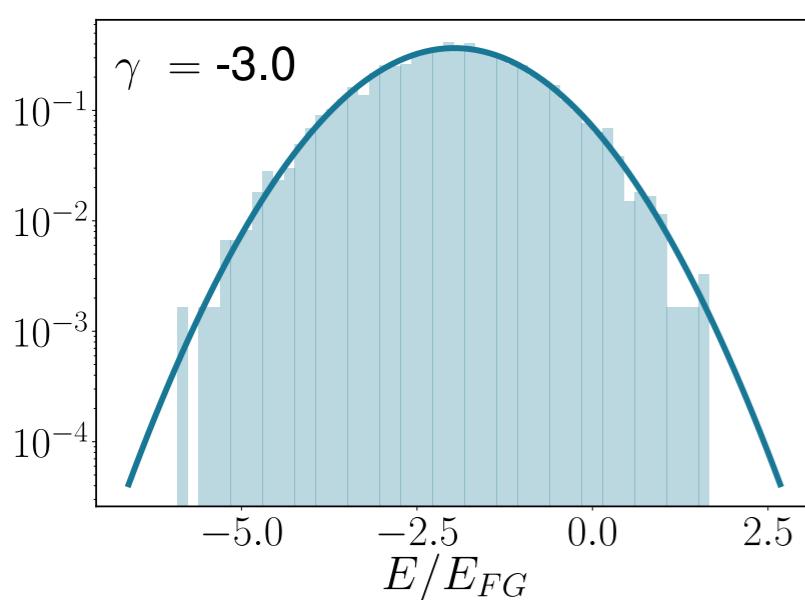
first step: compare to other methods

[LR,Porter,Drut,Braun '17]

attractive side:
no outliers,
no problems!



repulsive side:
outliers skew
expectation values!
('fat tail' problem)



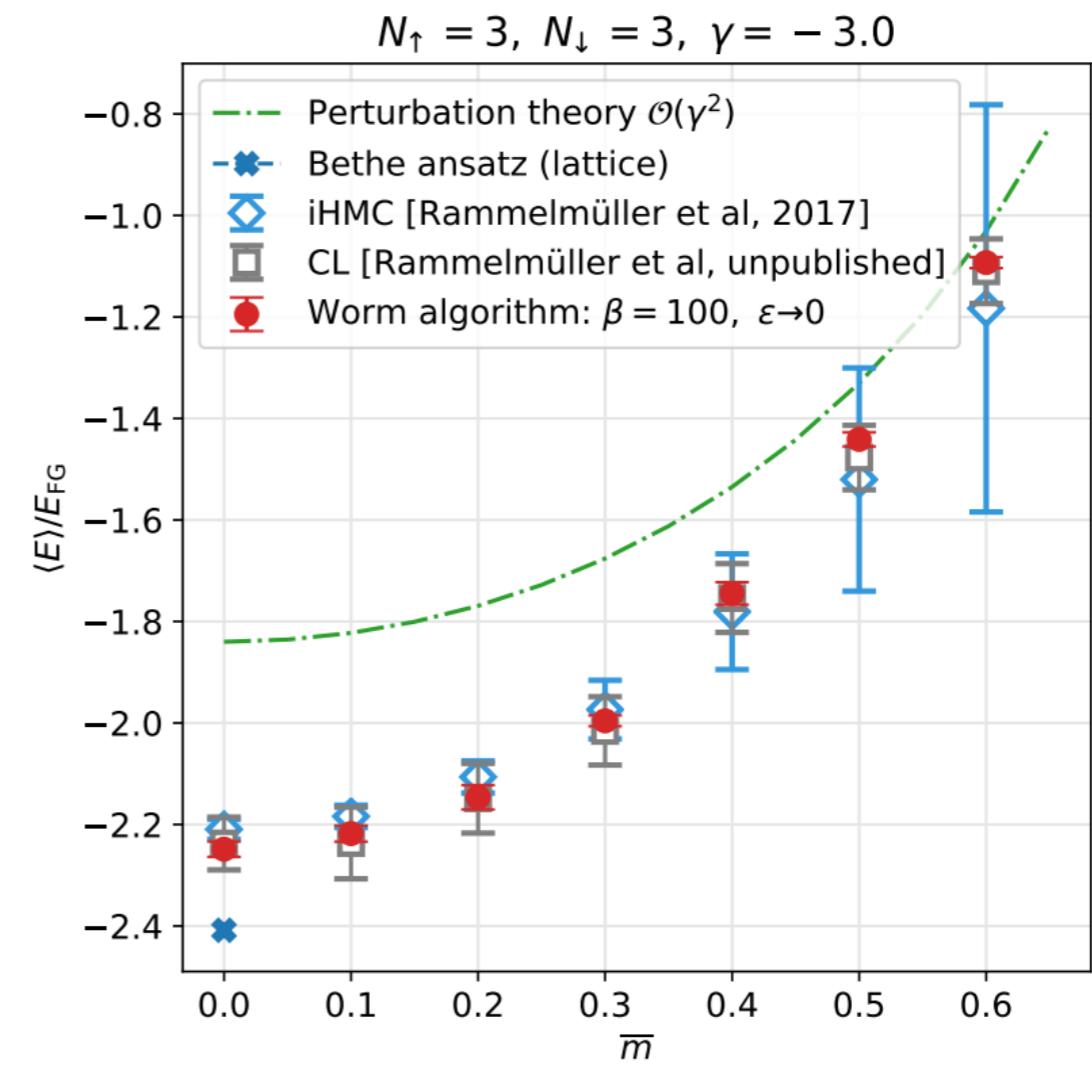
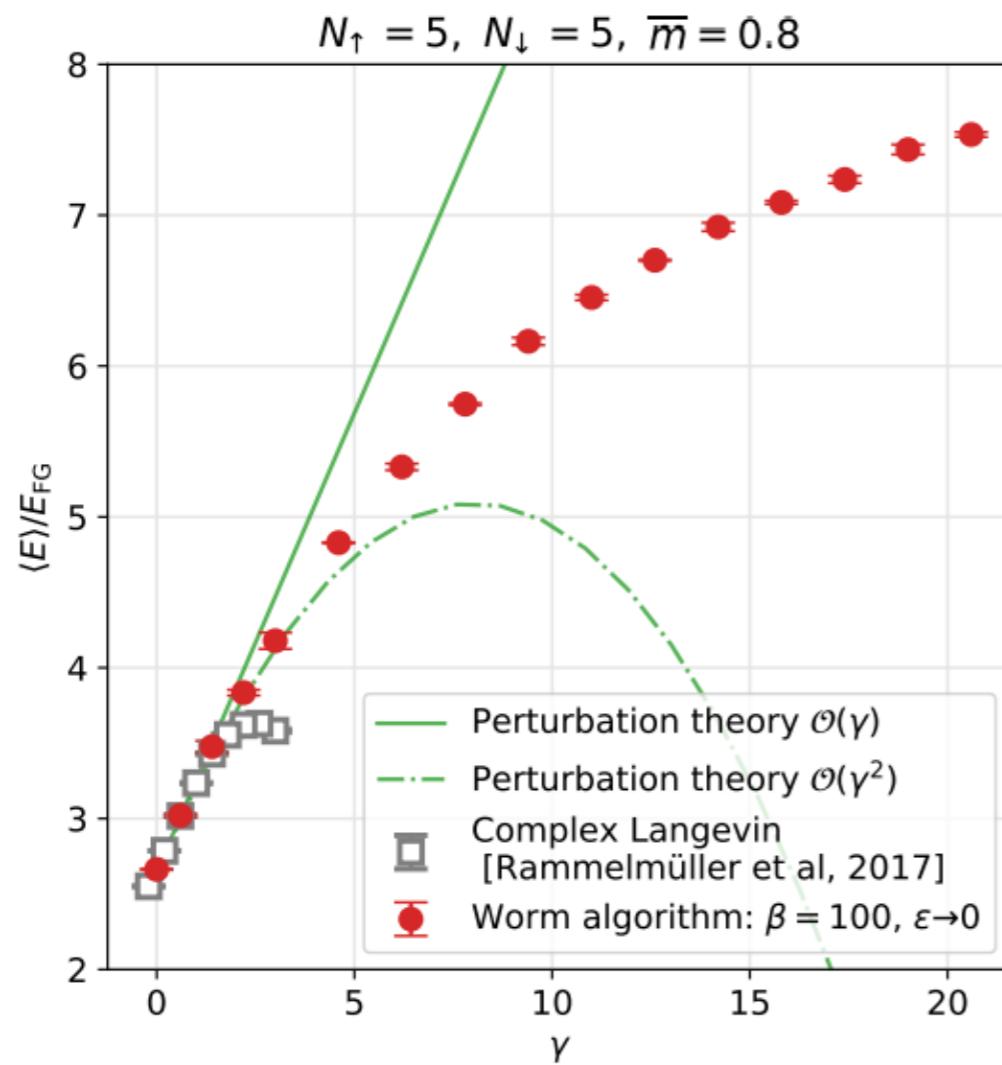
[BA: Iida, Wadati '07; Tracy, Widom '16]
[DFT-RG: Kemler, Pospiech, Braun '17; Kemler '17]
[HMC: LR, Porter, Loheac, Drut '15]

more comparisons: mass-imbalance

[worldline: Singh,Chandrasekharan '18]

[CL/iHMC: LR,Porter,Drut,Braun '17]

$$\bar{m} = \frac{m_\uparrow - m_\downarrow}{m_\uparrow + m_\downarrow}$$

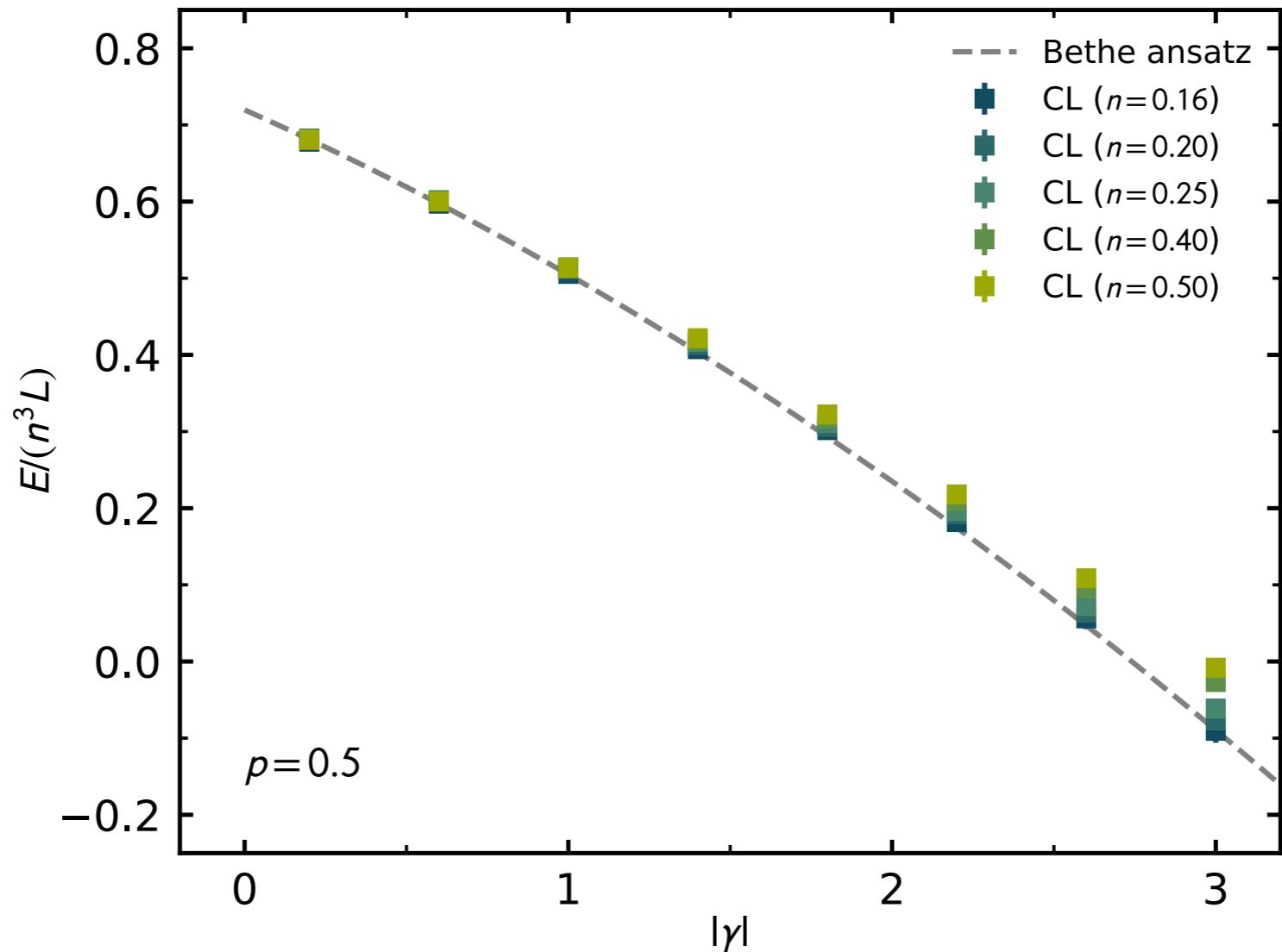


discrepancy with worldline methods for **repulsive interaction**

polarized 1D fermions: equation of state

[LR, Drut, Braun in preparation]

excellent
agreement at
low densities
(zero-range
limit)



$$\gamma = g/n$$
$$p = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

[Iida, Wadati '07; Tracy, Widom '16]