Hydrological Frequency Analysis

Frequency Analysis

- First step in designing variety of engineering projects related to water resources problems
  - Design of reservoirs, bridges, highways, drainage systems, soil conservations works, hydro-electric projects etc.
  - In case of soil conservation, we need to determine probability of occurrence of particular extreme rainfall. This information is determined through frequency analysis of point rainfall data.
- Deals with the chance of occurrence of an event equal to or greater than a specified magnitude

Frequency Analysis

- The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions

- The hydrologic data analysed are assumed to be independent and identically distributed, and hydrologic system producing them is considered to be stochastic, space-independent, and time-independent

  - Independent: data is not related with adjacent observation, i.e., randomness

  - Identically distributed: data coming from same population and having same statistical properties (homogeneity). All data reflect same type of hydrologic processes

Frequency Analysis

- Suppose, P is the probability of occurrence of an event (rainfall) whose magnitude is equal to or in excess of a specified magnitude X

- The recurrence interval (return period) is related to P as follows:

$$T = \frac{1}{P} \qquad (1)$$

- This represents the average interval between the occurrence of a rainfall of magnitude equal to or greater than X

Frequency Analysis

- If return period of rainfall of 20 cm in 24 hour is 10 years at a certain station A
  - It implies that on an average rainfall magnitudes equal to or greater than 20 cm in 24 hour occur once in 10 year period, i.e., in long period say 100 years, 10 such events can be expected
- However, this does not mean that such rainfall events will be separated by 10-year intervals
  - It is much more likely that two or more events may occur within one year or month

Frequency Analysis

- In the design of structures generally annual data is used for frequency analysis

- However, analysis is accurate only if length of record is sufficient
  - In the case of soil conservation structures, at least 10 years data are essential
  - Furthermore, estimating the frequencies of expected events greater than twice the length of the record should be avoided

Methods for Frequency Analysis

- There are two methods for performing frequency analysis
  - Empirical or Graphical method
  - Frequency factor method
- Empirical Method PPF
  - The exceedence probability of the event is obtained by the use of empirical formula, known as plotting position formula
  - Several plotting position formulae are developed and some of them are given below:

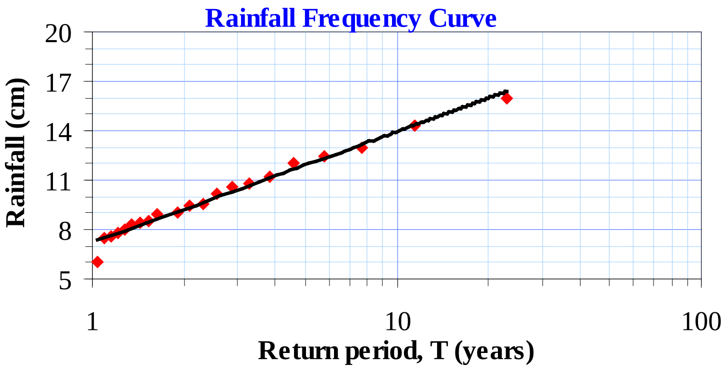
Plotting position formulae

Method	P (probability)	
California	$\frac{m}{N}$	(10)
Hazen	$\frac{m-0.5}{N}$	(11)
Weibull	$\frac{m}{N+1}$	(12)
Chegodayev	$\frac{m-0.3}{N+0.4}$	(13)
Blom	$\frac{m-0.44}{N+0.12}$	(14)

- Where m is rank assigned to the data after arranging them in descending order of magnitude
  - Thus, the maximum value is assigned m =1, the second largest value (m =2), and the lowest value m =N, N being the number of records
  - Weibull formula is the most commonly used plotting position formula

Empirical Method

- Having calculated P and T for all the events in the series, the variation of rainfall magnitude is plotted against the corresponding T on semi-log or log-log paper



- The rainfall magnitude for any recurrence interval can be determined by extrapolating the plot between magnitude and recurrence interval

- Empirical procedures can give good results for small extrapolations but the errors increase with the amount of extrapolation

- For more accurate results, analytical methods using frequency factor are used

Example

- For a station A, the recorded annual 24-h maximum rainfall is given below
  - Estimate the 24-h maximum rainfall with return periods of 13 and 50 years.
  - What would be the probability of a rainfall of magnitude equal to or exceeding 10 cm occurring in 24 h at station A?

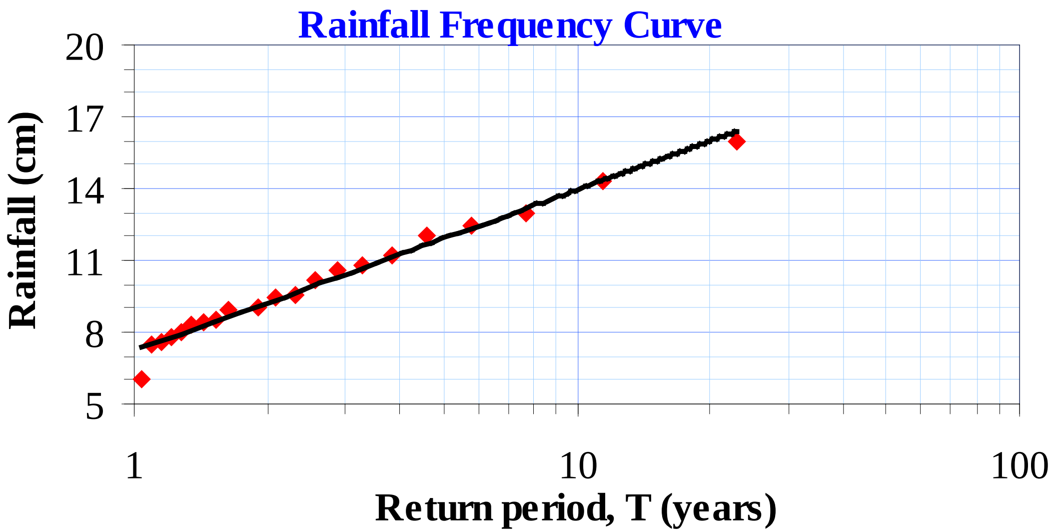
Year	**Rainfall (cm)**	**Year**	**Rainfall (cm)**	**Year**	**Rainfall (cm)**
1950	13.0	1957	12.5	1964	8.5
1951	12.0	1958	11.2	1965	7.5
1952	7.6	1959	8.9	1966	6.0
1953	14.3	1960	8.9	1967	8.4
1954	16.0	1961	7.8	1968	10.8
1955	9.6	1962	9.0	1969	10.6
1956	8.0	1963	10.2	1970	8.3
				1971	9.5

Solution

- Selecting Weibull Formula for Plotting Position

m	Rainfall, cm	P =m/n+1	T=1/P	m	Rainfall, cm	P =m/n+1	T=1/P
1	16	0.043	23.00	12	9	0.522	1.92
2	14.3	0.087	11.50	13	8.9	0.565	1.77
3	13	0.130	7.67	14	8.9	0.609	1.64
4	12.5	0.174	5.75	15	8.5	0.652	1.53
5	12	0.217	4.60	16	8.4	0.696	1.44
6	11.2	0.261	3.83	17	8.3	0.739	1.35
7	10.8	0.304	3.29	18	8	0.783	1.28
8	10.6	0.348	2.88	19	7.8	0.826	1.21
9	10.2	0.391	2.56	20	7.6	0.870	1.15
10	9.6	0.435	2.30	21	7.5	0.913	1.10
11	9.5	0.478	2.09	22	6	0.957	1.05

Solution



- After interpolating and extrapolating the above graph, we can determine rainfall magnitude for 13 and 50-year return period respectively

  13 year RI Rainfall = 14.55 cm

  50 year RI Rainfall = 18.00 cm

- For Rainfall = 10 cm, T =2.4 years and P = 0.417

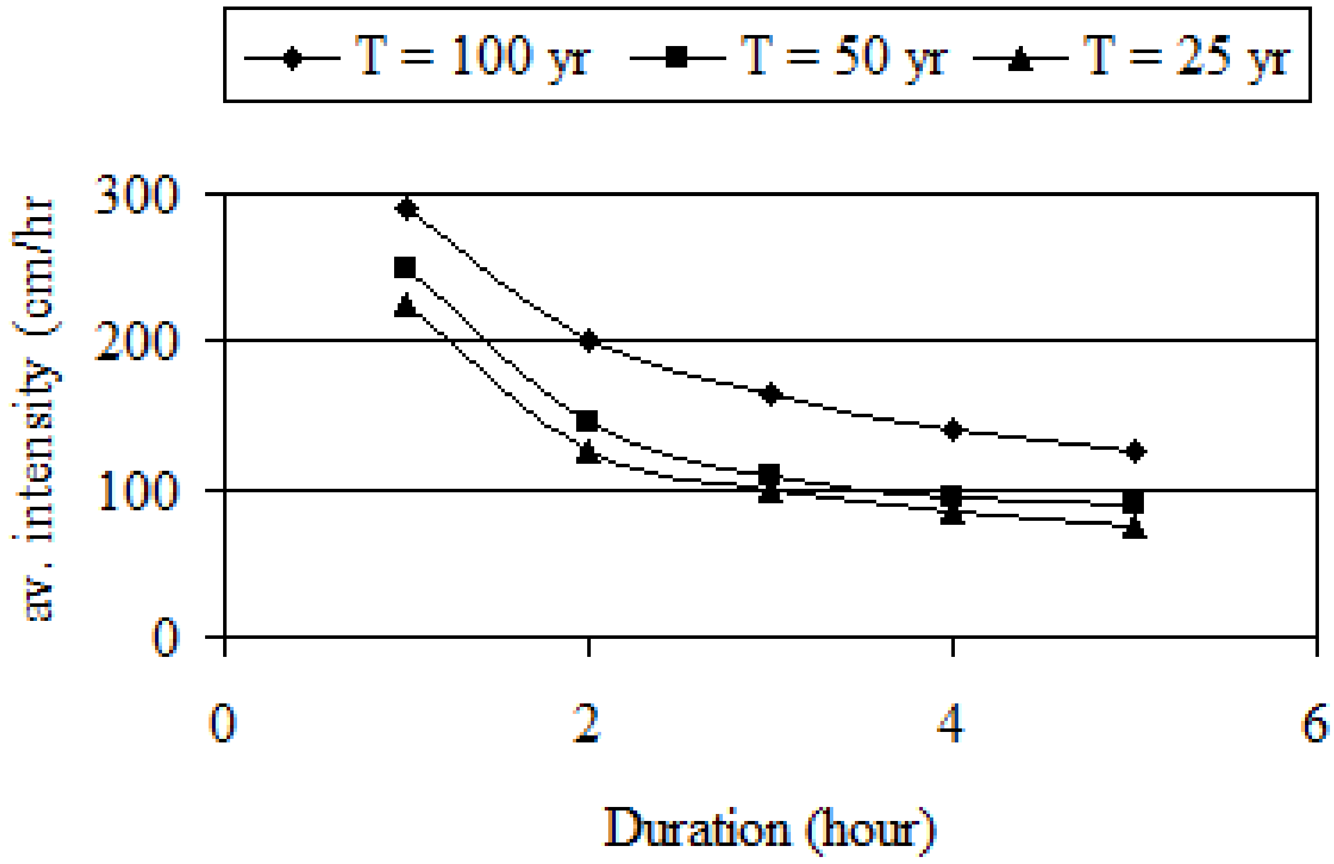
Intensity Duration Frequency Relationship

- Many studies require rainfall intensities of different durations and return periods.
  - watershed management, runoff disposal and erosion control,
- The relationship between intensity (i, cm/hr), duration (D, hr) and return period (T, years) can be expressed as follows:

$$i = \frac{KT^x}{(D+a)^n}$$

Where K, x, a and n are constants for a given catchment.

IDF Curve



T = 100 yr
T = 50 yr
T = 25 yr
300
200
100
0
av. intensity (cm/hr
0
2
4
6
Duration (hour)

Values of constants in IDF eqn – (Source: CSWCRTI- Dehradun)

City	K	x	a	n
Bhopal	6.93	0.189	0.50	0.878
Nagpur	11.45	0.156	1.25	1.032
Chandigarh	5.82	0.160	0.40	0.750
Bellary	6.16	0.694	0.50	0.972
Raipur	4.68	0.139	0.15	0.928

Example

- Compute 10 year, 1 h design rainfall intensity for Bhopal and Nagpur

Solution:

For Bhopal

$$i = \frac{6.93(10)^{0.189}}{(1+0.50)^{0.878}} = \frac{10.708}{1.427} = 7.50 \text{ cm/hr}$$

For Nagpur

$$i = \frac{11.45(10)^{0.156}}{(1+1.25)^{1.032}} = \frac{16.398}{2.309} = 7.10 \text{ cm/hr}$$

End of Lecture