Tara Ram Mohan
OPER 527 Homework 2

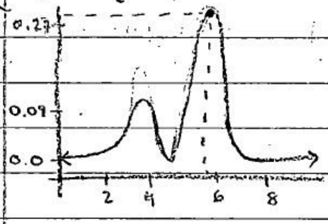
Note that all computer output screenshots are in the order of left to right.

<u>Problem 1</u>

1. $f(x) = (\sin(x)+1)e^{10x - x^2 - 25}, \quad 0 < x < 10$

R code

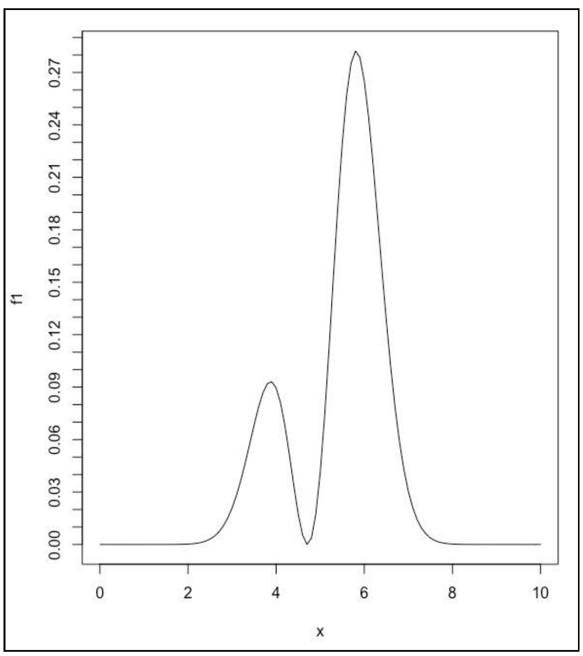
```
f1 <- function (x1) {
    result1 <- (sin(x1) + 1) * exp ((10* x1) - (x1^2) - 25)
    return (result1)
}
plot ( f1, xlim = c(0,10), yaxp = c(0, 0.5, 50))
plot (f1, xlim = c(5.8, 10), ylim = c(0.24, 0.5), yaxp = c(0, 0.5, 50)
```

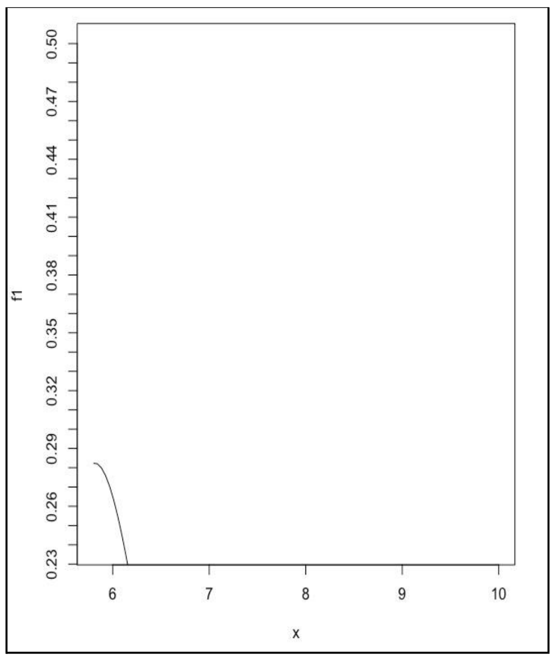


maximizer: $x^* = 5.8$
maximum: $f(x^*) = 0.282$

```
# Problem 1
f1 <- function( x1 ) {
  result1 <- (sin(x1) + 1) * exp((10*x1) - (x1^2) - 25)
  return( result1 )
}

plot( f1 , xlim=c(0,10), yaxp = c(0,0.5,50))
plot( f1 , xlim=c(5.8,10), ylim=c(0.24,0.5) , yaxp = c(0,0.5,50))
```





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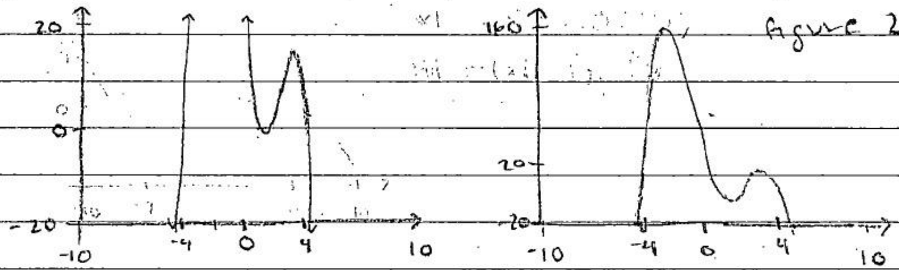
Problem 2a

2. a) $f(x) = -x^4 + 3x^3 + 14x^2 - 48x + 32 \quad -10 \le x \le 10$

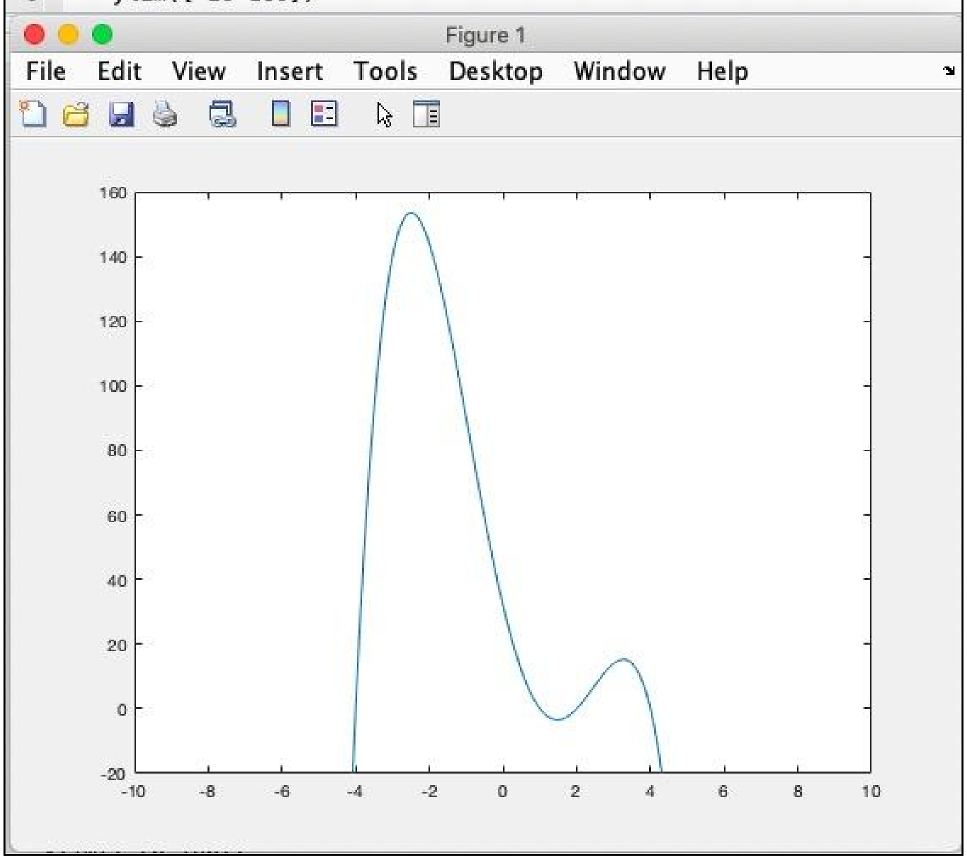


MATLAB code

```
x1 = -10:0.1:10;
f1 = -(x1.^4) + (3* x1.^3) + (14*x1.^2) - (48*x1) + 32;
plot(x1,f1);
xlim([-10 10]);
ylim([-20 20]);   OR   ylim([-20 160]) for figure 2
```

main.m

```
% 2a
x1 = -10:0.1:10;
f1 = -(x1.^4) + (3*x1.^3) + (14*x1.^2) - (48*x1) + 32;
plot(x1,f1);
xlim([-10 10])
ylim([-20 160])
```



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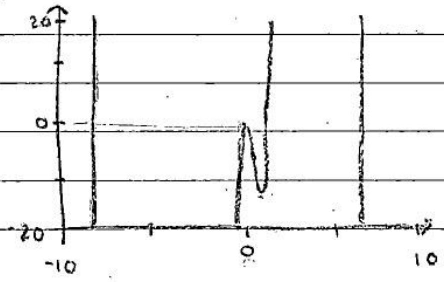
Problem 2b

b) $g(x) = -x^5 + 54x^3 - 40x^2 - 25x \qquad -10 < x < 10$



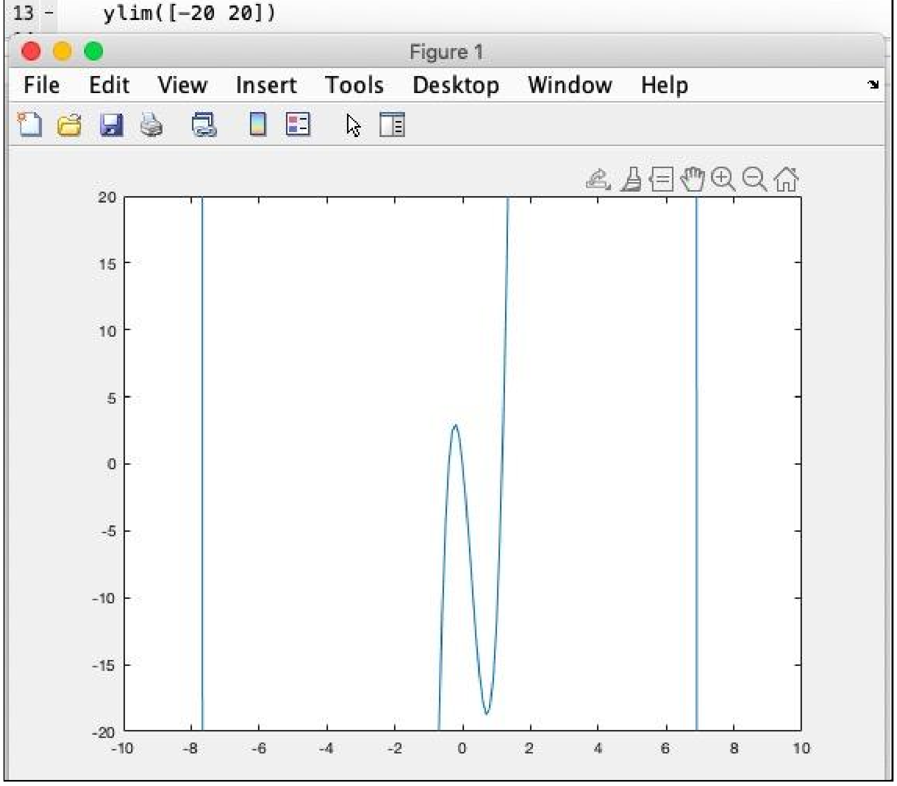
MATLAB code

b)
MATLAB code

```
x2 = -10:0.1:10;
g1 = -(x2.^5) + (54 * x2.^3) - (40 * x2.^2) - (25 * x2);
plot (x2, g1);
xlim ([-10 10]);
ylim([-20 20]);
```

main.m

```
% 2b
x2 = -10:0.1:10;
g1 = -(x2.^5) + (54*x2.^3) - (40*x2.^2) - (25*x2);
plot(x2,g1);
xlim([-10 10])
ylim([-20 20])
```



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Problem 2c

c) A maximum is not guaranteed to exist because the conditions that guarantee a maximum are (1) that f is continuous and (2) that the set is closed, but the set in question, $-10 < x < 10$ or $(-10, 10)$, is open

Problem 2d

d) $f(x) = -x^4 + 3x^3 + 14x^2 - 48x + 32$

$f'(x) = -4x^3 + 9x^2 + 28x - 48$

$f''(x) = -12x^2 + 18x + 28$

Gausse-Newton for Critical Points

$$x_i = x_{i-1} - \frac{f'(x_{i-1})}{f''(x_{i-1})}$$

when $x_0 = 2$, critical point at $\boxed{x = 1.4734}$

when $x_0 = -2$, critical point at $\boxed{x = -2.4918}$

when $x_0 = 0$, critical point at $\boxed{x = 3.2684}$

MATLAB code

flp.m
```
function y1 = flp(x1)
  result1 = -(4*x1^3) + (9*x1^2) + (28*x1) - 48
  y1 = result1;
```

flpp.m
```
function y1 = flpp(x1)
  result2 = -(12*x1^2) + (18*x1) + (28)
  y1 = result2;
```

main.m (lines run one ~~at a time~~ depending on x_0)
```
x1 = 2
x1 = -2
x1 = 0
x1 = x1 - flp(x1)/flpp(x1)
```

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Second Derivative Test

2. d) $f''(1.4734) = 28.4703 > 0$ ∴ Minimum

$f''(-2.4918) = -91.3612 < 0$ ∴ Maximum

$f(-2.4918) = 153.3656$

$f''(3.2684) = -41.3581 < 0$ ∴ maximum

$f(3.2684) = 15.2999$

MATLAB code

```
function y1 = f1(x1)
    result1 = -(x1^4)+(3*x1^3)+(14*x1^2)-(48*x1)+32
    y1=result1;
f1pp (1.4734)
f1pp (-2.4918)
f1 (-2.4918)
f1pp (3.2684)
f1(3.2684)
```

∴ there exists a maximum of 153.5656 at $x = -2.4918$ because by Gauss-Newton, a critical point exists at $x = 1.4734, -2.4918, 3.2684$, but the second derivative test reveals that the curve is concave up and a maximum at $x = -2.4918$ and $x = 3.2684$. $f(-2.4918) > f(3.2684)$, so the maximizer is $x = -2.4918$

Computer Input

main.m | f1.m | f1p.m | f1pp.m | +

```
function y1 = f1pp ( x1 )
    result1 = -(12*x1^2) + (18*x1) + (28)
    y1 = result1;
```

main.m | f1.m | f1p.m | f1pp.m | +

```
function y1 = f1p ( x1 )
    result1 = -(4*x1^3) + (9*x1^2) + (28*x1) - (48)
    y1 = result1;
```

main.m | f1.m | f1p.m | f1pp.m | +

```
function y1 = f1 ( x1 )
    result1 = -(x1^4) + (3*x1^3) + (14*x1^2) - (48*x1) + 32
    y1 = result1;
```

main.m | f1.m | f1p.m | f1pp.m

```
% 2d
x1 = 2;
x1 = -2;
x1 = 0;
x1 = x1 - f1p(x1)/f1pp(x1)

f1pp(1.4734)
f1(1.4734)
f1pp(-2.4918)
f1(-2.4918)
f1pp(3.2684)
f1(3.2684)
```

Computer Output

```
>> x1 = 0;
x1 = x1 - f1p(x1)/f1pp(x1)

result1 =

   -48


result1 =

    28
```

```
result1 =

    28


x1 =

    1.7143

>> x1 = x1 - f1p(x1)/f1pp(x1)

result1 =

    6.2974


result1 =

   23.5918


x1 =

    1.4474

>> x1 = x1 - f1p(x1)/f1pp(x1)

result1 =

   -0.7484
```

```
result1 =

   28.9143


x1 =

    1.4732

>> x1 = x1 - f1p(x1)/f1pp(x1)

result1 =

   -0.0057


result1 =

   28.4731


x1 =

    1.4734

>> x1 = x1 - f1p(x1)/f1pp(x1)

result1 =

  -3.4489e-07
```

```
result1 =

  -3.4489e-07


result1 =

   28.4696


x1 =

    1.4734

>> x1 = x1 - f1p(x1)/f1pp(x1)

result1 =

     0


result1 =

   28.4696


x1 =

    1.4734
```

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```
>> f1pp(1.4734)
f1(1.4734)
f1pp(-2.4918)
f1(-2.4918)
f1pp(3.2684)
f1(3.2684)

result1 =

   28.4703


ans =

   28.4703


result1 =

   -3.4475


ans =

   -3.4475
```

```
result1 =

  -91.3612


ans =

  -91.3612


result1 =

  153.5656


ans =

  153.5656


result1 =

  -41.3581
```

Problem 2e

e) Gauss-Newton for zeroes

$g(x) = -x^5 + 54x^3 - 40x^2 - 25x \qquad -10 < x < 10$

$g'(x) = -5x^4 + 162x^2 - 80x - 25$

$x_i = x_{i-1} - \dfrac{g(x_{i-1})}{g'(x_{i-1})}$ where $x_0 = -8, -1, 0, 1, 7$

zeros at $x = -7.6676$ when $x_0 = -8$

$x = -0.4046$ when $x_0 = -1$

$x = 0$ when $x_0 = 0$

$x = 1.1669$ when $x_0 = 1$

$x = 6.9053$ when $x_0 = 7$

2 e)

MATLAB code

g1.m
```
function y1 = g1(x1)
result = -(x1^5) + (54 * x1^3) - (40 * x1^2) - (25 * x1)
y1 = result1;
```

g1p.m
```
function y1 = g1p(x1)
result1 = -(5 * x1^4) + (162 * x1^2) - (80 * x1) - 25
y1 = result1;
```

```
x1 = -8
x1 = -1
x1 = 0
x1 = 1
x1 = 7
x1 = x1 - g1(x1)/g1p(x1)
```
main.m (lines run one at a time depending on x_0)

$(-7.6676, -0.4046) \rightarrow g(-1) = -68 < 0$

$(-0.4046, 0) \rightarrow g(-0.2) = 2.9683 > 0$

$(0, 1.1669) \rightarrow g(1) = -12 < 0$

$(1.1669, 6.9053) \rightarrow g(2) = 190 > 0$

MATLAB code
```
g1(-1)
g1(-0.2)
g(1)
g(2)
```

$\therefore g(x) \geq 0$ in the intervals: $(-0.4046, 0) \cup (1.1669, 6.9053)$

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Computer Input

```
main.m × | g1.m × | g1p.m × | main.m × | +

% 2e
x1 = -8;
x1 = -1;
x1 = 0;
x1 = 1;
x1 = 7;
x1 = x1 - g1(x1)/g1p(x1)
g1(-1)
g1(-0.2)
g1(1)
g1(2)
```

```
main.m × | g1.m × | g1p.m × | main.m × | +

function y1 = g1p ( x1 )
    result1 = -(5*x1^4) + (162*x1^2) - (80*x1) - (25)
    y1 = result1;
```

```
main.m × | g1.m × | g1p.m × | main.m × | +

function y1 = g1 ( x1 )
  result1 = -(x1^5) + (54*x1^3) - (40*x1^2) - (25*x1)
  y1 = result1;
```

Computer Output

```
>> x1 = 7;
x1 = x1 - g1(x1)/g1p(x1)

result1 =

  -420


result1 =

      -4652


x1 =

    6.9097

>> x1 = x1 - g1(x1)/g1p(x1)

result1 =

  -18.7226
```

```
result1 =

  -18.7226


result1 =

  -4.2408e+03


x1 =

    6.9053
```

```
>> g1(-1)
g1(-0.2)
g1(1)
g1(2)

result1 =

   -68


ans =

   -68


result1 =

    2.9683
```

```
ans =

    2.9683


result1 =

   -12


ans =

   -12


result1 =

   190
```

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Problem 2f

From part e:

2. f) $g(x) \geq 0$ when x is in $(-0.405, 0) \cup (1.1699, 6.9053)$

From part d:

Gauss Newton: critical points exist at $x_1 = 1.4734$

$x_2 = -2.4918$

$x_3 = 3.2684$

Only $x_1 = 1.4734$ and $x_2 = 3.2684$ in range where $g(x) \geq 0$

since $f''(1.4734) > 0$ and $f''(3.2684) < 0$, there exists a maximum at $x = 3.2684$ and a minimum at $x = 1.4734$ by the second derivative test (see part d)

$\therefore$ maximum: $\boxed{f(x^*) = 15.2999}$

maximizer: $\boxed{x^* = 3.2684}$

Problem 3a

3. a) $x_0 = 5.8$ would be a good starting point for the Gauss-Newton method because based on our plot graphed in question 1, we estimated that the max was at $x = 5.8$, which means less steps in the Gauss-Newton method to arrive at the critical point.

Problem 3b

b) $f(x) = (\sin(x) + 1)e^{10x - x^2 - 25}$ $\quad 0 < x < 10$

$f'(x) = (-2x + 10)e^{10x - x^2 - 25}(\sin x + 1) + (\cos x)e^{10x - x^2 - 25}$

$f'(x) = (-e^{10x - x^2 - 25})((2x-10)\sin x - \cos x + 2x - 10)$

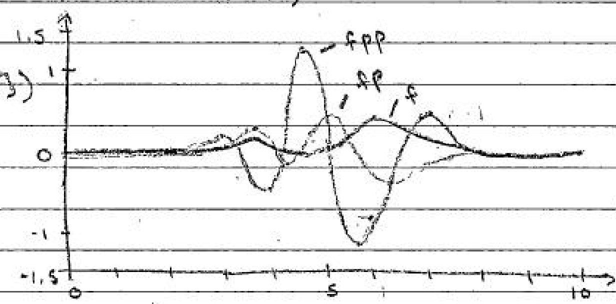
$f''(x) = (2x-10)(-e^{10x-x^2-25})((2x-10)\sin x - \cos x + 2x - 10)) + [(2)\sin x + (2x-10)\cos x + \sin x + 2](-e^{10x-x^2-25})$

$= (2x-10)(-e^{10x-x^2-25})((2x-10)\sin x - \cos x + 2x - 10) + (3\sin x + (2x-10)\cos x + 2)(-e^{10x-x^2-25})$

3. b)

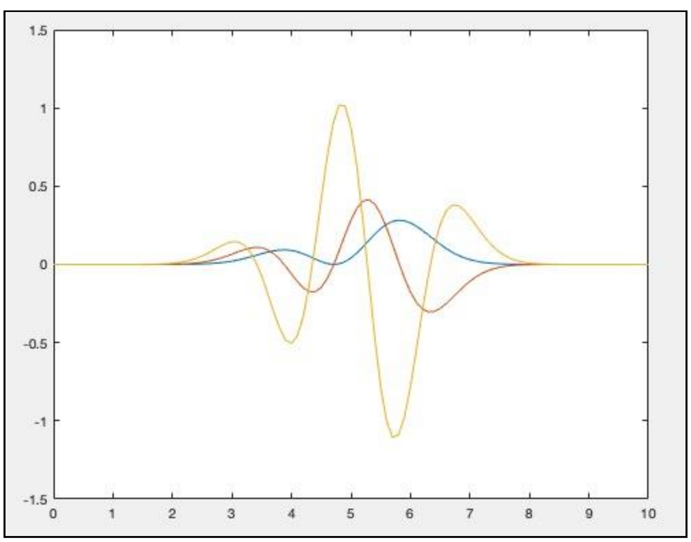
MATLAB code

```
x1 = -10 : 0.1 : 10;
f = (sin(x1) + 1) .* exp ((10.* x1) - (x1.^2)-25);
fp = ((-2.*x1)+10) .* exp ((10.*x1)-(x1.^2)-25)
     .*(sin(x1)+1) + (cos(x1) .* exp((10.*x1)-(x1.^2)-25));
fpp = (2.*x1-10).*(exp((10.*x1)-(x1.^2)-25))
     .*((2.*x1-10).*sin(x1) - cos(x1 + 2.*x1 -10)
     -(exp((10.*x1 - (x1.^2)-25)).*(3.*sin(x1)
     +(2.*x1-10).*cos(x1)+2)
plot (x1,f)
xlim ([0 10])
ylim ([-1.5 1.5])
hold on
plot (x1,fp)
hold on
plot(x1,fpp)
```



main.m

```
% 3b
x1 = -20:0.1:20;
f = (sin(x1) + 1) .* exp((10.* x1)-(x1.^2)-25);
fp = ((-2.*x1)+10) .* exp((10.*x1)-(x1.^2)-25) .* (sin(x1) + 1) + (cos(x1) .* exp((10.*x1)-(x1.^2)-25));
fpp = (2.*x1-10).*(exp((10.* x1)-(x1.^2)-25)).*((2.*x1-10).*sin(x1) - cos(x1) + 2.*x1 - 10)
fpp = fpp  - (exp((10.* x1)-(x1.^2)-25)).*(3.*sin(x1)+(2.*x1 - 10).*cos(x1) + 2)
plot(x1,f)
xlim([0 10])
ylim([-1.5 1.5])
hold on
plot(x1,fp)
hold on
plot(x1,fpp)
```



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Problem 3c

c) Gauss - Newton for critical points

$x_i = x_{i-1} - \frac{f'(x_{i-1})}{f''(x_{i-1})}$ where we choose $x_0 = 3.5, 4.5, 5, 6$

critical point at $x = 3.8756$ when $x_0 = 4$

$x = 4.7724$ when $x_0 = 5$

$x = 5.8140$ when $x_0 = 6$

$f''(3.8756) = -0.4684 < 0$ ∴ Max

$f''(4.7724) = 0.9206 > 0$ ∴ Min

$f''(5.8140) = -1.0803 < 0$ ∴ Max

$f(3.8756) = 0.0932$ $\quad f(3.8756) < f(5.8140)$

$f(5.8140) = 0.2824$

$\boxed{\begin{array}{l} \therefore \text{maximizer}: x^* = 5.8140 \\ \text{maximum}: f(x^*) = 0.2824 \end{array}}$

MATLAB code →

3. c)

MATLAB code

f2.m
```
function y1 = f2(x1)
    result1 = (sin(x1) + 1) * exp((10 * x1)-(x1^2)-25)
    y1 = result1;
```

f2p.m
```
function y1 = f2p(x1)
    result1 = ((-2.*x1)+10).* exp((10.*x1)-(x1.^2)-25)
        .*((sin(x1)+1)+(cos(x1).* exp((10.*x1-
        (x1.^2)-25)))
    y1=result1;
```

f2pp.m
```
function y1 = f2pp(x1)
    result1 = (2.*x1-10).*(exp((10.*x1)-(x1.^2)-25))
        .*((2.*x1-10).*sin(x1) - cos(x1)+2.*x1-10)
        -exp((10.*x1-(x1.^2)-25)).*(3.*sin(x1)+
        (2.*x1-10).*cos(x1)+2)
    y1=result1;
```

main.m
```
i = 0
x2 = 4      3 iterations   }  lines run one
x2 = 5      4 iterations   }  at a time depending
x2 = 6      4 iterations   }  on x0
i = i+1
x2 = x2 - f2p(x2)/f2pp(x2)
f2pp(3.8756)
f2pp(4.7724)
f2pp(5.8140)
f2(3.8756)
f2(5.8140)
```

```
main.m × f2p.m × f2.m × f2pp.m × +
function y1 = f2 ( x1 )
    result1 = (sin(x1) + 1) * exp((10*x1)-(x1^2)-25)
    y1 = result1;
```

```
main.m × f2p.m × f2.m × f2pp.m × +
function y1 = f2p ( x1 )
    result1 = ((-2.*x1)+10) .* exp((10.*x1)-(x1.^2)-25) .* (sin(x1) + 1) + (cos(x1) .* exp((10.*x1)-(x1.^2)-25));
    y1 = result1;
```

```
main.m × f2p.m × f2.m × f2pp.m × +
function y1 = f2pp ( x1 )
    result1 = (2.*x1-10).*(exp((10.* x1)-(x1.^2)-25)).*((2.*x1-10).*sin(x1) - cos(x1) + 2.*x1 - 10)
    result1 = result1 - (exp((10.* x1)-(x1.^2)-25)).*(3.*sin(x1)+(2.*x1 - 10).*cos(x1) + 2)
    y1 = result1;
```

```
main.m* × f2p.m × f2.m × f2pp.
22      % 3d
23 -    i = 0;
24 -    x2 = 4;
25 -    x2 = 5;
26 -    x2 = 6;
27 -    x2 = x2 - f2p(x2)/f2pp(x2);
28 -    i = i + 1;
29
30 -    f2pp(3.8756)
31 -    f2pp(4.7124)
32 -    f2pp(5.8140)
33 -    f2(3.8756)
34 -    f2(5.8140)
```

```
>> % 3d
i = 0;
x2 = 4;
x2 = 5;
x2 = 6;
x2 = x2 - f2p(x2)/f2pp(x2);
i = i + 1;

result1 =

    0.3539
```

```
result1 =

    0.3539


result1 =

   -0.7799

>> x2 = x2 - f2p(x2)/f2pp(x2);
i = i + 1;

result1 =

   -0.0692


result1 =

   -1.1060

>> x2 = x2 - f2p(x2)/f2pp(x2);
i = i + 1;

result1 =

  -7.3766e-04


result1 =

   -1.0807
```

```
>> x2 = x2 - f2p(x2)/f2pp(x2);
i = i + 1;

result1 =

  -1.1981e-07


result1 =

   -1.0803

>> x2 = x2 - f2p(x2)/f2pp(x2);
i = i + 1;

result1 =

  -2.9816e-15


result1 =

   -1.0803

>> x2 = x2 - f2p(x2)/f2pp(x2);
i = i + 1;

result1 =

     0


result1 =

   -1.0803
```

```
>> f2pp(3.8756)
f2pp(4.7124)
f2pp(5.8140)
f2(3.8756)
f2(5.8140)

result1 =

  -3.5396e-05


result1 =

   -0.4689


ans =

   -0.4689


result1 =

   5.8353e-06


result1 =

    0.9206


ans =

    0.9206
```

```
ans =

    0.9206


result1 =

  -4.4546e-05


result1 =

   -1.0803


ans =

   -1.0803


result1 =

    0.0932


ans =

    0.0932


result1 =

    0.2824
```

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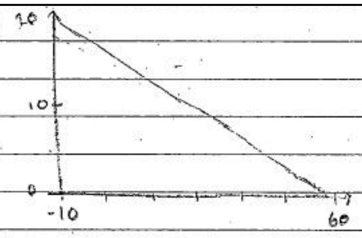
Problem 3d

d) It took 11 iterations to find the maximum

See iteration notes in 3c for more info.

Problem 4

4. a)



x4 = -70:0.1:70;
f4 = 17 - (0.3.*x4);
plot(x4, f4);
xlim([-10 60]);
ylim([0 20]);

MATLAB code

b) $3x \geq 0$

$x \geq 0$ feasible region: where $x \in [0,20]$

$\frac{1}{2}x \leq 19$

$x \leq 38$

$17x \leq 340$

$x \leq 20$

c) Since $f(x)$ is a line, it is both concave and convex, so the maximum and minimum exist at end points

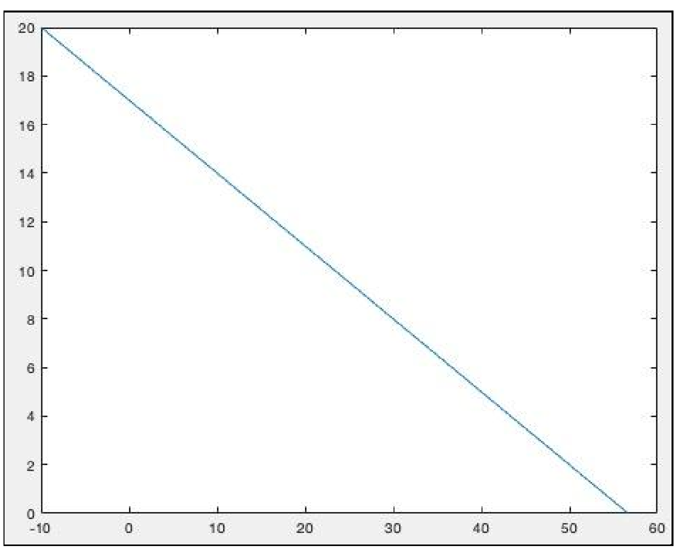
maximizer: $x^* = 0$

maximum: $f(x^*) = 17 - 0.3(0) = 17$

d) minimizer: $z^* = 20$

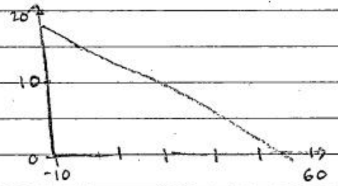
minimum: $f(z^*) = 17 - 0.3(20) = 11$

```
% 4a
x4 = -70:0.1:70;
f4 = 17 - (0.3 .* x4);
plot(x4,f4);
xlim([-10 60]);
ylim([0 20]);
```



Problem 5

a)




```
xs = -70:0.1:70;
fs = 15 + cos(13/7).*xs;
plot (xs, fs);
xlim ([-10 60]);
ylim ([0 20]);
```

MATLAB code

b) $14x \geq 0$

$x \geq 0$

$\tan\left(\frac{14}{3}\right)x \leq 22$

$x \leq \frac{22}{\tan(14/3)} \approx 1.007$

$\sin\left(\frac{71}{13}\right)x \leq -7$

$x \leq \frac{-7}{\sin(71/13)} \approx 9.559$

$\cos\left(4/7\right)x \leq -4$

$x \leq \frac{-4}{\cos(4/7)} \approx 6376.665$

feasible region where $x \in \left[0, \frac{22}{\tan(14/3)}\right]$

c) Since $f(x)$ is a line, it is both concave and convex, and the maximum and minimum exist on the endpoints

maximizer : $x^* = 0$

minimizer : $f(x^*) = 15 + \cos\left(\frac{13}{7}\right)(0) = 15$

d) minimizer : $x^* = \frac{22}{\tan(14/3)} \approx 1.007$

minimum : $f(x^*) = 15 + \cos\left(\frac{13}{7}\right)\left(\frac{22}{\tan(14/3)}\right)$

≈ 14.7157

```
% 5a
x5 = -70:0.1:70;
f5 = 15 + cos(13/7).*x5;
plot(x5,f5);
xlim([-10 60])
ylim([0 20])
```