UNIT – II

Theoretical Structure of Quantum Information Systems

What is a Qubit (Spin Qubit)

A **qubit** (quantum bit) is the fundamental unit of information in quantum computing—analogous to the classical **bit**. However, while a classical bit can be **either** 0 **or** 1, a qubit can be in a **superposition** of both 0 and 1 **simultaneously**.

In **spin-based systems**, a qubit is encoded using the **spin of a particle** (like an electron):

- Spin **up** → $|0\rangle$
- Spin **down** → $|1\rangle$

Why "ket"?

The notation $|\cdot\rangle$ is called **Dirac notation**, or **bra–ket notation**, named after physicist **Paul Dirac**.

- $|0\rangle$: called a **"ket"** — it represents a **column vector** (a state in quantum mechanics).
- $\langle 0|$: called a **"bra"** — it represents the **row vector** (the dual of the ket).

Together:

- $\langle 0|0\rangle$ is pronounced **"bra zero ket zero"**, which represents an **inner product** (dot product).

Unlike classical bits (only 0 or 1), a qubit can be in a **superposition**:

$$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$$

Where α and β are complex numbers such that:

$$|\alpha|^2+|\beta|^2=1$$

A Detailed Conceptual Understanding Using Spin and Polarization

1. Spin-½ Particles (like electrons)

What is Spin?

- **Spin** is a fundamental property of particles like electrons.
- Think of it like a tiny magnetic needle that can point **up** or **down**.

Qubit Representation:

- $|0\rangle$= Spin **up** along the z-axis → $|\uparrow z\rangle \rightarrow +\hbar/2$
- $|1\rangle$= Spin **down** along the z-axis → $|\downarrow z\rangle \rightarrow -\hbar/2$

Superposition:

- The spin can also be in a **mix** (superposition) of up and down:

  $|\psi\rangle = \alpha|\uparrow z\rangle + \beta|\downarrow z\rangle$

  where α and β are complex numbers.

Measurement:

- When you measure the spin along the z-axis, the outcome is either **up** or **down** — randomly, based on probabilities $|\alpha|^2$and $|\beta|^2$.

Analogy: Like a coin that's spinning in the air — it's not clearly heads or tails until you catch it.

2. Polarization of Photons

What is Polarization?

- Polarization is the **direction** in which a light wave vibrates.
- Light (photons) can be:
  - **Horizontally polarized** → $|H\rangle$
  - **Vertically polarized** → $|V\rangle$

Qubit Representation:

- $|0\rangle$= Horizontal polarization $|H\rangle$
- $|1\rangle$= Vertical polarization $|V\rangle$

Superposition:

- Light can also be polarized diagonally or circularly:
  - Diagonal: $\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$
  - Right Circular: $\frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$**+** Measurement:
- You use polarizing filters (like sunglasses) to measure the light's polarization.
- The result will be **H** or **V**, again based on probability.

Analogy: Like adjusting window blinds — you can filter light to pass only in certain directions.

Why This Matters

- **Spin** is used in **quantum computers** (trapped ions, electrons).
- **Photon polarization** is used in **quantum communication** (e.g., quantum key distribution like BB84).
- Both are easy to control, measure, and visualize — perfect for **learning and experimentation**.

Comparison: Classical bits vs Quantum bits

1. Basic Definition

Feature	Classical Bit	Quantum Bit (Qubit)
Definition	Smallest unit of classical information	Smallest unit of quantum information
Possible Values	Only **one** value at a time: 0 **or** 1	Can be in a **superposition** of 0 and 1
Mathematical Form	Integer value (0 or 1)	A **qubit** is a two-level quantum system. Its **state** is described by a

		linear combination (superposition) of two basis states: $|0\rangle$and$|1\rangle$ Complex vector: $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$

2. Representation

Feature	Classical Bit	Qubit
Basis	Binary digits: 0, 1	Quantum states: (
Visualization	Line with two points: 0 and 1	**Bloch Sphere**: 3D sphere of all possible states
Example States	0, 1	$|0\rangle= \begin{bmatrix}1\\0\end{bmatrix}$,$|1\rangle=\begin{bmatrix}0\\1\end{bmatrix}$

3. State Change & Manipulation

Feature	Classical Bit	Qubit
State Change	Flipped using logic gates like NOT	Changed using **unitary transformations** (quantum gates)
Gates	AND, OR, NOT, XOR	Hadamard, Pauli-X, Y, Z, CNOT, T, etc.
Reversibility	Not all gates are reversible (e.g., AND)	All quantum gates are **reversible** (unitary)
Operations	Deterministic	Can be **probabilistic** until measured

4. Measurement

Feature	Classical Bit	Qubit
Measurement	Directly reads 0 or 1	**Collapses** to 0 or 1 with probabilities

Effect of Measurement	Non-destructive	**Destroys** superposition (irreversible collapse)
Determinism	Always gives the same result	May give **different results** unless in eigenstate

5. Superposition & Entanglement

Feature	Classical Bit	Qubit
Superposition	Not possible	Can be in any linear combination of basis states
Entanglement	Not possible	Qubits can be **entangled** → shared state, instant correlations
State Space	2^n configurations (n bits)	2^n-dimensional **complex vector space** (n qubits)

6. Information Processing

Feature	Classical Bit System	Quantum Bit System
Computing Model	Boolean logic + circuits	Quantum circuits + unitary evolution + measurement
Speedup Potential	Limited to hardware efficiency	**Exponential speedup** for some problems (e.g., Shor's, Grover's)
Parallelism	Sequential or parallel hardware needed	Superposition enables **quantum parallelism**

7. Example: Two Bits vs Two Qubits

Feature	Classical 2-bit System	Quantum 2-qubit System
States	One of {00, 01,	$\lvert\psi\rangle=\alpha_{00}\lvert 00\rangle+\alpha_{01}\lvert 01\rangle+\alpha_{10}\lvert 10\rangle+\alpha_{11}$

	10, 11}	$\lvert 11\rangle$ Each α is a complex number (amplitude), $\lvert\alpha_{00}\rvert^2+\lvert\alpha_{01}\rvert^2+\lvert\alpha_{10}\rvert^2+\lvert\alpha_{11}\rvert^2==1$ (for probability),
Entanglement possible?	No	Yes: e.g., Bell state ($\lvert\Phi+\rangle=\frac{1}{\sqrt{2}}(\lvert 00\rangle+\lvert 11\rangle)$)

8. Physical Realizations

Feature	Classical Bit	Qubit
Device	Transistors, capacitors	Electron spins, photon polarization, trapped ions, superconducting circuits
Technology	CMOS, TTL	IBM Q, Google Sycamore, IonQ, photonic and topological qubits

9. Advantages of Qubits Over Classical Bits

Quantum Feature	Advantage
Superposition	Encodes more information than a single bit
Entanglement	Enables **quantum teleportation**, secure communication, and speedups
Reversibility	Energy-efficient operations (theoretically no loss of information)
Quantum Parallelism	Evaluates many possible solutions at once in algorithms

Quantum systems: trapped ions, superconducting circuits, photons (non- engineering view)

1. Trapped Ions

What are they?

- **Single atoms** (like calcium or ytterbium) are stripped of one or more electrons, creating **ions**.
- These ions are **held in place using electromagnetic fields** in a vacuum chamber.

Conceptual View:

- Think of ions as **tiny magnets or compass needles** suspended in space.
- Their internal energy levels (like "spin up" or "spin down") are used to represent **qubit states: |0⟩ and |1⟩**.

How they interact:

- Lasers are used to **manipulate** (rotate) and **measure** their quantum states.
- Qubits can be entangled using shared vibrations (like connecting springs between ions).

Pros:

- Extremely **high fidelity** (very low error rates).
- **Long coherence times** (quantum states last longer).

Cons:

- Slower gate speeds.
- Difficult to scale to many qubits (complex trap design).

2. Superconducting Circuits

What are they?

- Made from tiny loops of metal (e.g., aluminum) that behaves like **artificial atoms** at very cold temperatures.

- Operate at **near absolute zero** in dilution refrigerators.

Conceptual View:

- Imagine a **miniature electrical loop** that can hold a current flowing clockwise = $|0\rangle$, or counterclockwise = $|1\rangle$ — or both at once!
- These current states represent the qubit.

How they interact:

- Controlled using **microwave pulses** to perform quantum gates.
- Qubits can be linked through electromagnetic fields.

Pros:

- **Fast gate speeds** (operations in nanoseconds).
- Easy to **fabricate** using standard chip-making processes.

Cons:

- **Shorter coherence times** (states decay quickly).
- Requires ultra-cold temperatures and precise shielding.

3. Photons

What are they?

- **Particles of light**, typically used in fiber-optic cables or free space.
- Photon qubits are encoded in **polarization, path, or time**.

Conceptual View:

- Think of photons like **arrows of light** that can point in different directions:
  - Horizontal = $|0\rangle$
  - Vertical = $|1\rangle$
  - Diagonal = superposition
- They fly through circuits or air like **tiny flying messengers**.

How they interact:

- Manipulated using **beam splitters, mirrors, and polarizers**.

- Difficult to make them interact directly → usually use **interference and detection**.

Pros:

- **Excellent for communication** (long-distance transmission).
- **Low noise** and no need for cooling.

Cons:

- Hard to implement logic gates (weak interaction between photons).
- Photonic quantum computing is still **emerging**.

Hilbert Spaces – The Mathematical Stage

Definition:

A **Hilbert space** is a **complete vector space** equipped with an **inner product**.

It provides the **mathematical framework** where **quantum states live**.

Think of a Hilbert space as the **quantum version of 3D space**, but with **infinite or finite dimensions**, and complex numbers instead of real ones.

Properties of Hilbert Spaces:

Property	Description
Vector Space	Quantum states are vectors
Inner Product	Allows calculation of angles, lengths, and probabilities
Completeness	Any convergent sequence of vectors stays inside the space
Orthonormal Basis	States like

Examples:

- A **single qubit** lives in a **2D Hilbert space**:
  H2=span{|0⟩,|1⟩}0⟩,|1⟩
- A **2-qubit system** lives in a **4D space**:
  $H_2 \otimes H_2 = H_4$
- Infinite-dimensional spaces (like for the position of a particle) also exist in continuous quantum mechanics.

Quantum States – The Vectors in Hilbert Space

Definition:

A **quantum state** is a **unit vector** (length = 1) in a Hilbert space. It represents the **complete description of a quantum system**.

Operators – Abstract Interpretation in Quantum Mechanics

- In quantum theory, **operators** are **mathematical tools** that represent **physical processes**, **measurements**, and **transformations** of quantum states.
- They act on **quantum states**, which are vectors in a **Hilbert space**, and produce new vectors or extract meaningful values like measurement outcomes.

What is an Operator?

An **operator** is a rule that **acts on a state** (like a function acts on a number or vector).
If |ψ⟩|\psi\rangle|ψ⟩ is a quantum state and *O* is an operator,
then:
$\hat{O}\,|\psi\rangle=|\phi\rangle$
This means the operator **transforms** the state |ψ⟩| into a new state |ϕ⟩|

Types of Operators

Operator Type	Role in Quantum Mechanics	Symbol & Example
Identity Operator	Leaves the state unchanged	$\hat{I}$
Hermitian Operator	Represents a **physical observable** (e.g., position, spin)	$\hat{A}†=\hat{A}$
Unitary Operator	Describes **time evolution** or **quantum gates**	$\hat{U}†=†\hat{U}=\hat{I}$
Projection Operator	Projects state onto a subspace (e.g., measurement)	$\hat{p}\phi=\|\phi\rangle$
Hamiltonian	Total energy operator → governs dynamics	Appears in Schrödinger's equation

Abstract Interpretations

1. **Measurement as Operator**
    - Observable quantities (position, momentum, spin, energy) are represented by **Hermitian operators**.
    - The **eigenvalues** of the operator are the possible **outcomes** of measurement.
    - The **state collapses** to the corresponding **eigenvector** upon measurement.
2. **Time Evolution as Operator**
    - A closed quantum system evolves according to a **unitary operator**:

      $|\psi(t)\rangle= \hat{U}†\,(t,t_0)|\psi(t_0)\rangle$

- This operator preserves the **length** (norm) of the quantum state — maintaining probabilities.

3. Operators and Observables

- Every observable in quantum mechanics corresponds to a **mathematical operator**.
- You don't "touch" spin or energy directly — you **apply an operator** and calculate what outcomes are possible and how likely they are.

Example (Abstract)

Suppose is an observable (Hermitian operator), and |ψ⟩ is the quantum state.

- If $\hat{U}$ |ψ⟩=a|ψ⟩, then:
  - a is a **measurable value** (eigenvalue).
  - |ψ⟩ is in a **definite state** for observable $\hat{A}$.

If not, the measurement may yield different values probabilistically, based on the **expansion** of |ψ⟩ in terms of eigenstates of $\hat{A}$.

1. Identity Operator

- **Meaning**: "Do nothing" operation.
- Acts as a neutral element.
- In any equation or transformation, it **leaves the state untouched**.

$\hat{I}|\psi\rangle=|\psi\rangle$

2. Hermitian Operator

- **Meaning**: Anything that can be **measured** in quantum mechanics is represented by a **Hermitian operator**.
- Example observables: position x^, momentum $\hat{p}$spin $\hat{S}$, energy $\hat{H}$

$$\hat{A}|^{\dagger}=\hat{A}$$

- **Eigenvalues are real** → physically measurable outcomes.
- **Eigenstates form a complete basis** → any quantum state can be expressed in this basis.

3. Unitary Operator

- **Meaning**: Describes **quantum evolution** and **quantum gates**.
- Reversible: no information is lost.
- Preserves probabilities:

$$\hat{U}|^{\dagger}=\hat{U}\hat{U}=\hat{I}$$

- Used to **transform** or **rotate** states without destroying coherence.

4. Projection Operator

- **Meaning**: Extracts part of a state (e.g., "measure if the system is in state $|\phi\rangle|$").
- Collapses the quantum state into a **subspace**:

$$\hat{p}_{\phi}=|\phi\rangle$$

- Used in measurement formalism and filtering.
- Properties:
    - Hermitian: $\hat{p}^{\dagger}=\hat{p}$
    - Idempotent: $\hat{p}^{2}=\hat{p}$

5. Hamiltonian Operator

- **Meaning**: Represents the **total energy** (kinetic + potential) of the system.
- Governs **how quantum states evolve in time**:
- The **solution** to this equation gives the **unitary operator** $\hat{p}(t)$ for time evolution.
- Always **Hermitian**, ensuring **energy is real** and time evolution is unitary.

Entanglement and Non-Locality in Quantum Systems

1. What is Entanglement?

- **Entanglement** is a quantum phenomenon where the states of two or more particles become **correlated** in such a way that **they cannot be described independently**, even when separated by large distances.

$$\text{Bell State: } |\Phi+\rangle=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

This is a **2-qubit entangled state** where:

- The system is in a **superposition** of both qubits being 0 (**$|00\rangle$**) and both being 1 (**$|11\rangle$**).
- It means **neither qubit has a definite value on its own**, but they are **perfectly correlated**.
- The factor $\frac{1}{\sqrt{2}}$ ensures the total probability is 1. This means: if one qubit is measured and found to be 0, the other is instantly 0; if it is 1, the other is also 1.

2. What is Non-Locality?

- **Non-locality** refers to the fact that **entangled particles affect each other instantly**, even when they are **far apart**, without any signal passing between them.
- It **defies classical notions of locality**, but does **not violate relativity**, because it does **not transmit information faster than light**.
- Verified by experiments testing **Bell's inequalities** — quantum predictions differ from classical ones, and **nature behaves non-locally**.

The Role of Entanglement and Non-Locality

A. Foundational Role in Quantum Theory

- Entanglement reveals that **quantum systems are holistic**: you cannot fully describe parts without knowing the whole.
- It challenges **classical realism** and **local hidden variable theories**.
- It confirms that quantum mechanics is **probabilistic and fundamentally non-local** in structure.

B. Practical Role in Quantum Technologies

Application	Role of Entanglement & Non-locality
Quantum Teleportation	Transfers quantum information using shared entangled pairs
Quantum Cryptography (QKD)	Detects eavesdropping by checking for entanglement loss
Superdense Coding	Doubles classical capacity using entanglement
Quantum Computing	Entanglement allows qubits to represent and process **exponentially many states** simultaneously
Bell Test Experiments	Verifies non-locality, ruling out classical explanations

C. Entanglement in Composite Systems

- A composite system of qubits can exhibit **entangled states**, which allow:
  - **Correlated measurement outcomes**
  - **Interference across multiple branches of computation**

- **Non-classical parallelism**, beyond what classical computers can simulate efficiently

Comparison between quantum information and classical information

1. What Is *Classical Information*?

- **Definition**: Information encoded using **bits** — each bit is either 0 or 1.
- **Processing**: Uses classical logic gates (AND, OR, NOT).
- **Transmission**: Through classical channels (electrical signals, radio waves).
- **Storage**: In transistors, magnetic disks, etc.
- **Measurement**: Reveals the value without affecting the system.

2. What Is *Quantum Information*?

- **Definition**: Information stored in **qubits** — quantum systems that can be in a **superposition** of 0 and 1.
- **Processing**: Uses **quantum gates** (unitary operations).
- **Transmission**: Can use **entangled particles** or **quantum states of photons**.
- **Storage**: In spin systems, superconducting circuits, trapped ions, etc.
- **Measurement**: Collapses the quantum state — changes or destroys the original state.

Key Differences: Classical Vs Quantum Information

Feature	Classical Information	Quantum Information
Unit	Bit (0 or 1)	Qubit (superposition of 0 and 1)
Storage State	One state at a time	Infinite combinations in

		superposition
Copying (Cloning)	Bits can be copied freely	**No-cloning theorem**: Qubits can't be copied exactly
Measurement	Doesn't disturb the system	**Collapses** the state (irreversible change)
Operations	Logical gates (deterministic)	Quantum gates (reversible, probabilistic outcomes)
Entanglement	Not possible	Possible — creates non-local correlations
Communication	Standard digital methods	Can use **quantum teleportation**, **quantum cryptography**
Security	Vulnerable to interception	**Inherently secure** (e.g., quantum key distribution)
Parallelism	Requires multiple processors	Achieved with superposition in one quantum system

Important Principles Unique to Quantum Information

1. **Superposition**: A qubit can be both 0 and 1 at the same time.
2. **Entanglement**: Qubits can be linked so that measuring one affects the other — instantly.
3. **No-Cloning Theorem**: Quantum states cannot be copied exactly.
4. **Quantum Interference**: Used to boost correct outcomes in algorithms (like Grover's).
5. **Measurement Collapse**: Observing a quantum system changes it irreversibly.

Philosophical implications: Randomness in Quantum Mechanics

In quantum mechanics, **randomness is built-in** — it's not because we don't know enough, it's because nature behaves that way.

In Classical Physics:

- If you roll a die and know **everything** about how it's thrown (angle, speed, air, etc.), you could **predict** the result.
- That's **classical randomness** — it only seems random because we lack full information.

In Quantum Mechanics:

- Even if we know **everything** about a system (its wavef
- unction), the outcome of some measurements is still **truly random**.
- Example: An electron is in a **superposition** (a mix) of spin-up and spin-down:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow z\rangle + |\downarrow z\rangle)$$

Now consider two aspects: When you measure its **spin along the z-axis**, you get:

- $|\uparrow z\rangle$ with **50% probability**
- $|\downarrow z\rangle$ with **50% probability**
- This means the electron is **equally likely** to be spin-up or spin-down along the **z-axis**.

Key Points:

- You **can't predict** which result you'll get — even if the experiment is done **exactly the same** every time.
- There is **no hidden rule or variable** deciding the outcome behind the scenes.

  In quantum mechanics, **nature doesn't always follow predictable rules**. Sometimes, it genuinely **makes a random**

choice — and this is not due to our ignorance but is a **fundamental property** of how the quantum world works.This randomness is **intrinsic** — it's a **natural feature** of quantum systems.

Philosophical implication: Reality isn't deterministic at the smallest scales — **nature chooses randomly**.

2. Determinism – Evolution Vs. Measurement

In quantum mechanics, a system behaves in **two different ways**:

Type	**What Happens**	**Is It Predictable?**
Unitary Evolution	The system changes smoothly over time (like rotation on the Bloch sphere) using a rule called **Schrödinger's equation**	✅ Yes, fully predictable
Measurement (Collapse)	When we **measure**, the system suddenly jumps to one specific outcome (like spin-up or spin-down)	❌ No, outcome is random

- The system **evolves smoothly and predictably** until we **measure it**.
- Measurement causes a sudden, random **change in the system's state** — this is called **collapse**.
- This creates a puzzle called the **measurement problem**:
  Why does the wavefunction collapse during measurement, and how does this happen? So, **quantum mechanics is partly predictable (before measurement) and partly random (when measured).**

Philosophical implication: Quantum mechanics combines deterministic evolution with **non-deterministic collapse**.

3. The Observer – A Unique Role in Quantum Mechanics

1. **Active Role**:
   Measurement **changes** the system — it's not just observation, but interaction.
2. **Choice of Measurement Matters**:
   The **basis chosen** by the observer (e.g., z-axis vs. x-axis) **determines the possible outcomes** (e.g., spin-up vs. spin-down along that axis).
3. **Observer-Dependent Reality (QBism)**:
   The **wavefunction** reflects the observer's **belief** about the system, not an objective state of nature.
4. **Conceptual Entanglement**:
   The **observer and system are not separable** — they form a single quantum description during measurement.
5. **State Collapse**:
   The act of measurement causes the state to **collapse** to one definite outcome ((e.g.$|\uparrow z\rangle$))

Philosophical implication: The **observer's measurement defines the observed reality** — contrasting with classical physics where measurement merely reveals an existing property.