Balls into bins

K balls, n bins

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

K balls \ n bins	Limit of 1 ball	Unlimited # balls
Same looking	$\binom{n}{k}$	$\binom{n-1+k}{k}$
Unique	$\binom{n}{k}k!$	n^k

$n \times (n-1) \times (n-2) \times \ldots \times (n-k+1)$

$K=n \Rightarrow n!$

n=4
K=2

| 0 | 0 | |

Ball & Separators "Trick"

0||0

00||

|0|0|

11100

10011

11001

n-1 Separators
K balls

Total "characters" = n-1+k

n=4
K=2

$\binom{4}{2} = \frac{4!}{2!2!}$ = 6	$\binom{4-1+2}{2} = \binom{5}{2} = \frac{5!}{3!2!} = \frac{5\cdot4}{2} = 10$
$\binom{4}{2}2! = 12$	$4^2 = 16$

Induction example:

Claim: $\boxed{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}}$ $P(n)$

Proof:

We use induction:

Base $P(1)$: $\sum_{k=1}^{1} k = \frac{1(1+1)}{2}$

$$\begin{array}{ccc} \sum_{k=1}^{1} k & = & \frac{1(1+1)}{2} \\ \| & & \| \\ 1 & = & \frac{1(2)}{2} = 1 \end{array}$$

Hence $P(1)$ holds

Step: Now assume $P(n)$ holds

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Want to prove $\underline{P}(n+1)$:

$$\left(\sum_{k=1}^{n+1} k = \frac{(n+1)(n+1+1)}{2}\right)$$

"Break the sum trick"

$$\sum_{k=1}^{n+1} k = \boxed{\sum_{k=1}^{n} k} + n+1$$

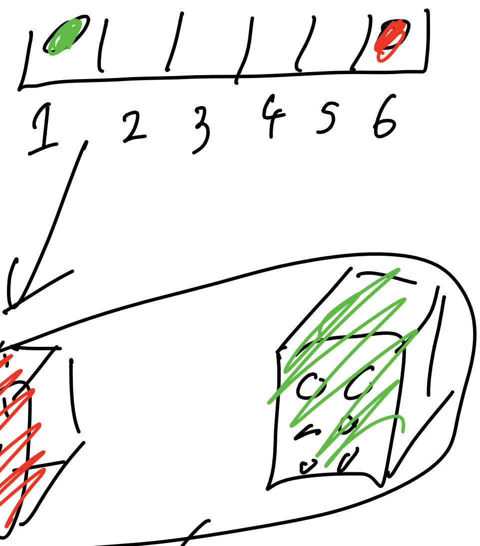
$\downarrow \underline{P}(n)$

$$= \boxed{\frac{(n)(n+1)}{2} + \frac{2(n+1)}{2}}$$

$$= \frac{(n+1)(n+2)}{2}$$ which is $\underline{P}(n+1)$

Balls into bins

n bins / k balls	limit of 1	unlimited
identical		$\binom{n-1+k}{k} = \binom{n-1+k}{n-1}$
unique		36



$n=6$

$k=2$

$$\binom{6-1+2}{2} = \binom{7}{2} = \frac{7!}{(7-2)!\,2!} = \frac{7\cdot 6}{2} = 7\cdot 3 = 21$$

Sampling

pop n=50 / sample size k	Without replacement	With replacement
order does not matter	$\binom{n}{k}$	$\binom{n-1+k}{k}$
order matters	$\binom{n}{k}k!$	n^k