Solutions of Mathematica based Questions for Assignment 1, MATH7501, Sem 1, 2021

Question 2 c

In[1]:=
```
Sum[k^2, {k, 1, n}]
```
Out[1]=
$$\frac{1}{6} n (1+n) (1+2n)$$

In[2]:=
```
2 n^3 + 3 n (n + 1) - 2 n // Simplify
```
Out[2]=
$$n \left(1 + 3n + 2n^2\right)$$

Question 2 d

In[3]:=
```
expr = 2 (i - j) n (n + 1)/2 - 4 n (n + 1) (2 n + 1)/6 + n i j;
FullSimplify[expr]
```
Out[4]=
$$i j n + (i-j) n (1+n) - \frac{2}{3} n (1+n) (1+2n)$$

In[5]:=
```
check =
  Table[2 (i - j) n (n + 1)/2 - 4 n (n + 1) (2 n + 1)/6 + n i j /. n → 2, {i, 1, 3}, {j, 1, 4}];
check // MatrixForm
```
Out[6]//MatrixForm=
$$\begin{pmatrix} -18 & -22 & -26 & -30 \\ -10 & -12 & -14 & -16 \\ -2 & -2 & -2 & -2 \end{pmatrix}$$

Question 2 c

In[7]:=
```
SeedRandom[1]
randM[r_] := Table[RandomInteger[{1, r}], {3}, {3}]
```

In[9]:=
```
randCommuteTrial[r_] := Module[{},
  A = randM[r];
  B = randM[r];
  If[A.B == B.A, 1, 0]
 ]
```

In[10]:=
```
n = 10^5;
est[r_] := N[Total[Table[randCommuteTrial[r], {n}]] / n]
```

In[12]:=
```
Table[{r, est[r]}, {r, 2, 5}] // TableForm
```

Out[12]//TableForm=

2	0.00333
3	0.00007
4	0.00002
5	0.

Question 5 a

In[26]:=
```
xVals = {2.4, 4.7, 4.9, 2.9, 8.1};
y = {3.1, 2.7, 4.8, 7.6, 5.4};
A = Table[If[j == 1, 1, xVals[[i]]], {i, 1, 5}, {j, 1, 2}];
A // MatrixForm
```

Out[29]//MatrixForm=

$$\begin{pmatrix} 1 & 2.4 \\ 1 & 4.7 \\ 1 & 4.9 \\ 1 & 2.9 \\ 1 & 8.1 \end{pmatrix}$$

In[101]:=
```
β = {0, 0};
bestNorm = ∞;
Do[
 βtry = {RandomReal[{0, 5}], RandomReal[{0, 5}]};
 norm = Norm[A.βtry - y];
 If[norm < bestNorm,
  β = βtry;
  bestNorm = norm;
  Print["New best norm: ", bestNorm];
 ];
 , 10^6]
```

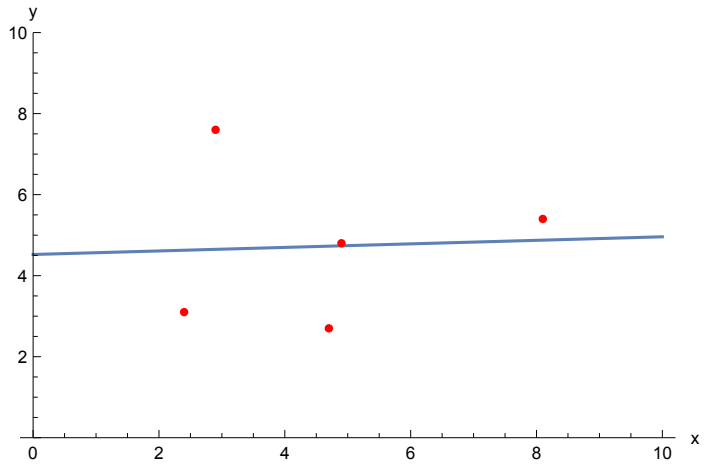
```
New best norm: 28.1809
New best norm: 5.6241
New best norm: 5.07316
New best norm: 3.94038
New best norm: 3.93268
New best norm: 3.93114
New best norm: 3.92946
New best norm: 3.9283
New best norm: 3.92825
New best norm: 3.92817
New best norm: 3.92816
New best norm: 3.92815
```

In[104]:=
```
β
```
Out[104]=
```
{4.52278, 0.0436704}
```

In[105]:=
```
f[x_] := β[[1]] + β[[2]] x
```

In[135]:=
```
plt1 = Plot[f[x], {x, 0, 10},
   AxesLabel → {"x", "y"}, AxesOrigin → {0, 0}, PlotRange → {0, 10}];
plt2 = ListPlot[Transpose[{xVals, y}], PlotStyle → Red, PlotRange → All];
Show[plt1, plt2]
```

Out[137]=



Question 5 b

In[139]:=
```
βformula = Inverse[Transpose[A].A].Transpose[A].y
```
Out[139]=
```
{4.5207, 0.0433267}
```

```
(*This is similar to the Monte Carlo simulation result*)
```

Question 6 c

In[140]:=
```
CC[r_] := Range[1, r]
```

In[142]:=
```
isInA[x_] := Mod[x, 3] == 0
```

In[147]:=
```
CCinterSectionA[r_] := Select[CC[r], isInA]
```

In[148]:=
```
Length[CCinterSectionA[10]]
```
Out[148]=
```
3
```

In[150]:=
```
Length[CCinterSectionA[20]]
```
Out[150]=
```
6
```

In[151]:=
```
Length[CCinterSectionA[100]]
```
Out[151]=
```
33
```