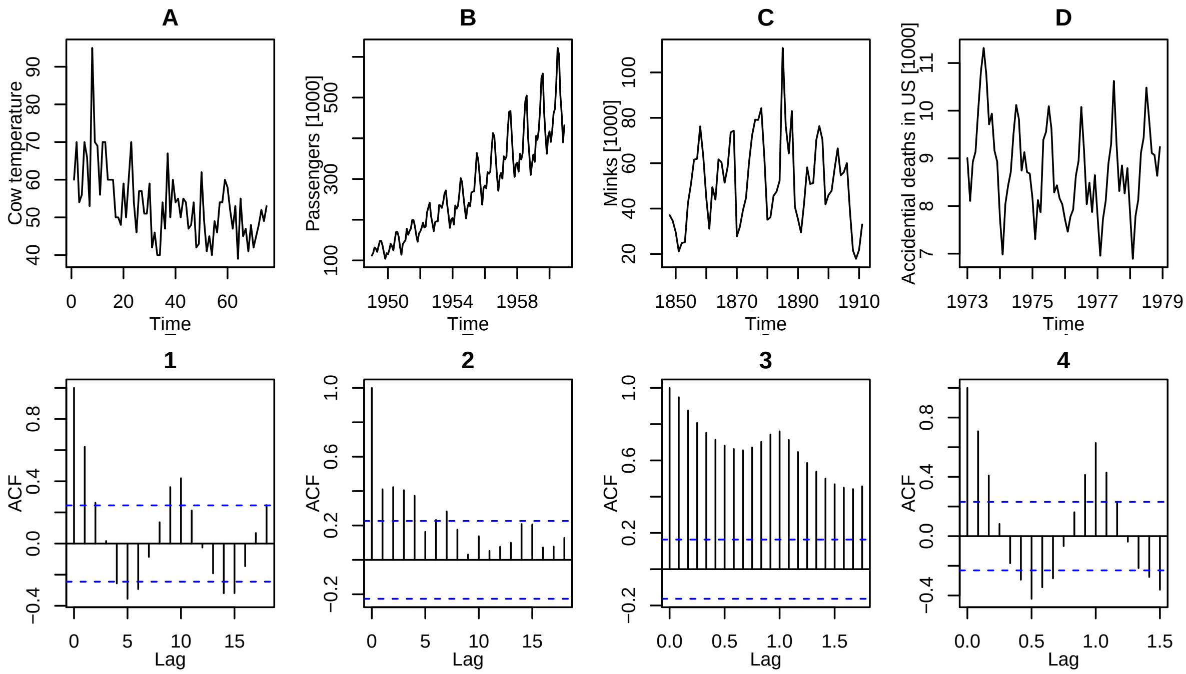
RTP Exercise Sheet

Series 2

Exercise 2.1

Below you find the plots and the correlograms of four datasets. The correlograms have been permutated. Please find for each data sets (A-D) the appropriate corellogram (1 - 4).



R-hints:

```r
library(fma)
# Data set cow temperature
cowtemp
# Data set air passengers
AirPassengers
# Data set mink trappings
mink
# Data set accidential deaths in the US
usdeaths
```

Exercise 2.2

In this exercise, we will calculate the lagged scatter plot and the plug-in estimator without employing the internal R function.

a) Write a function to calculate the lagged scatter plot estimator for the autocorrelation. For this, you may extend the code given in the lecture notes.

b) Develop a function to calculate the plug-in estimator for the autocorrelation.

c) Calculate the two estimates for the `beer` and the `chicken` dataset.

Exercise 2.3

In this exercise, we would like to investigate the properties of an AR(1) process.

a) Simulate a realisation of the process

$$X_t = 0.8 \cdot X_{t-1} + e_t$$

with e_t an innovation process of length 1000.

R-hints:

```
arima.sim(list(ar = c(0.5, 0.2)), n = 100)
```

simulates a realisation of the process $X_t = 0.5 \cdot X_{t-1} + 0.2 \cdot X_{t-2} + e_t$ with length 100.

b) Calculate the theoretical autocorrelation function and the plug-in estimator of the autocorrelation of the simulation results in a) and plot both curves for lags from 0 to 100.

R-hints:

```
ARMAacf(ar = c(0.5, 0.2), lag.max = 10)
```

calculates the theoretical autocorrelation function of the AR(2) process $X_t = 0.5 \cdot X_{t-1} + 0.2 \cdot X_{t-2} + e_t$ for lags up to 10.

c) What is the functional dependence of the theoretical autocorrelation function on the lag k and $\alpha_1 = 0.8$?

d) Now compare the theoretical partical autocorrelation function with the estimated version for the simulated process. Which particularity do you observe for the two representations?

Exercise 2.4

An analytical device measures the creatine concentration of human muscular tissue. In this exercise, we would like to check whether it is operating correctly, i.e. the measured values does not depend on the measuring instance.

A sample with known concentration is split into 157 samples and measured by the device one after the other. You can find them in the data under

`http://stat.ethz.ch/Teaching/Datasets/WBL/kreatin.dat`.

In this exercise, we focus only on the variable "gehalt"(content) in the data.

a) Which stochastic model should this series of data follow if the machine is working correctly?

b) Use the time series plot, the autocorrelations (and the partial autocorrelations) to determine whether these data fit the ideal model found in Part a) or not.

Exercise 2.5

Let us now consider the electricity production of Australia in GWh in the period from January 1958 to December 1990. You may download the data from

`http://stat.ethz.ch/Teaching/Datasets/WBL/cbe.dat`.

The aim of this exercise is to compare the effect of different algorithms to decompose a time series representation in trend, seasonality and remainder by means of their (partial) autocorrelation function.

a) Start by considering the plot of the time series. Why is not meaningful to interpret the correlogram of this time series?

   Explain in a few sentences.

b) Decompose the time series into trend, seasonal component and remainder using the R function `decompose()`, which performs the decomposition with moving averages. Plot the remainder and its correlogram and interpret the plots in a few sentences.

   R-Hints:

```r
# example for decompose function
decomp <- decompose(tselec, type = "multiplicative")

# example to calculate the plugin estimator of the
# autocorrelation function
acf(..., na.action = na.pass, plot = TRUE)
```

The function employs a filter to estimate the trend; therefore, the first and the last few entries of the decomposition are not defined, i.e. the have the value `NA` in R. To prevent issues of R, the parameter `na.action = na.pass` (asking R to ignore `NA` entries) has to be employed.

c) Decompose the log-transformed time series using the R function `stl()`. Estimate the seasonal effect once by averaging over all years (parameter `s.window = "periodic"`) and once by choosing an appropriate smoothing window (parameter `s.window = ...`). Recall that the window length has to be odd. An appropriate smoothing window may be determined by the R-function `monthplot()`. For both estimation approaches (averaging and smoothing window), plot the remainder and its correlogram, and comment on the plots.

   R-hint:

```
elec.stl <- stl(log(tselec), s.window = ...)
```

d) Explain why you used the parameter `type = "multiplicative"` in Task b), and why you log-transformed the time series before performing an `stl()` decomposition in Task c).

e) As a last algorithm consider the differencing approach. Choose a lag of 1 and 12 (months) to eliminate a trend and periodic structures. Plot the resulting time series and autocorrelation function. Compare the results to the previous methods.

Exercise 2.6

In this exercise, we consider two time series `ts1` and `ts2`, which putatively were created by an AR process. You may download the data from

`http://stat.ethz.ch/Teaching/Datasets/WBL/ts_S3_A2.dat`

a) Visualise both time series. Are both time series stationary? What is their mean?

b) Consider the (partial) autocorrelation function and decide whether the two time series can be generated by an AR process. If yes, what is the order of the respective AR process?

   Hint: The partial auto correlation function of an AR(p) process displays a sudden drop for lags larger than p.

Exercise 2.7

Let us consider the AR(3) model with coefficients $\alpha_1 = 0.6, \alpha_2 = -0.5$ and $\alpha_3 = 0.4$:

$$X_t = 0.6 \cdot X_{t-1} - 0.5 \cdot X_{t-2} + 0.4 \cdot X_{t-3}$$

a) Simulate one realisation of length 50 of the time series and plot it. Would you assume that this time series is stationary?

b) Calculate the estimated (partial) autocorrelation function and compare it to the theoretical function. Hint: Compare exercise 2.3 for hints.

c) Preview to week 3: Calculate the roots of the polynomial $\Phi(z) = 1 - \alpha_1 \cdot z - \alpha_2 \cdot z^2 - \alpha_3 \cdot z^3$ with the R function `polyroot`. What do you observe for the absolute value of the roots?

Disclaimer: Parts of the exercises are adopted from 'Applied Time Series Analysis' course at ETHZ by Marcel Dettling.