After classical logic Nolt turns to extensions of classical logic: modal logics, deontic and tense logics, and second-order logic. Then he presents nonclassical logics: free logics, multi-valued logics, supervaluations, infinite-valued and fuzzy logics, intuitionistic logics, relevance logics, and nonmonotonic logics.

Nolt's motivation of the various logics and their use in analyses of philosophical issues is often good and in the case of the modal logics (including deontic and tense logics) of considerable interest even to those long familiar with the systems. However, in the discussion of modal systems he does not consider the principal criticism of them as involving use-mention confusions, which can be highlighted with an analysis of deduction theorems for them. In the discussion of higher-order logics he misses the main philosophical issue of why we should identify properties and relations with sets and what we mean by all subsets and all predicates.

For the semantics of most of these logics Nolt introduces the predicate interpretation without separately considering the propositional analysis. This is hard to follow, particularly when there are many controversial choices to be made for the quantifiers that obscure the analysis of the propositional connectives. Only for some of the logics that he classifies as nonclassical does he consider just the propositional logic, and those presentations are more accessible.

It is not clear what audience Nolt has in mind for this text. The density of the material and the depth of the philosophical discussions suggest that this book would be suitable only for upper-division undergraduates in philosophy. The complexity of the predicate semantics for modal logics in particular seems at a level accessible only for advanced students of logic.

Overall, this text does not appear to be suitable for a first introduction to logic. But the motivations of many of the logics make it a useful reference for faculty, whether preparing such a course or engaging in research.

RICHARD L. EPSTEIN

Advanced Reasoning Forum, P. O. Box 635, Socorro, NM 87801, USA.
rle@AdvancedReasoningForum.org.

BOB HALE AND CRISPIN WRIGHT. ***The reason's proper study: Essays toward a neo-Fregean philosophy of mathematics.*** Oxford University Press, New York. 2001, 472 pp.

For most of his career Frege believed that (i) arithmetic is a body of truths concerned with independently existing objects and that (ii) arithmetic is derivable from logical principles and definitions alone. This is what Frege meant by the claim that arithmetic is analytic. Moreover, he offered an original defense of this claim. First he argued that ascriptions of number, which for Frege are ascriptions of a number to a concept, are governed by Hume's principle:

> HUME'S PRINCIPLE: The number of F = the number of G if and only if F and G are in one-to-one correspondence.

For all its merits, Hume's principle seems unable to settle mixed identity questions of the form "The number of $F = q$", where q is a singular term purported to refer to an object of an ostensibly different sort. In the face of what has come to be known as the Julius Caesar problem, Frege sought to provide an explicit definition of numerical terms of the form "the number of F":

> The number of F is the extension of the concept *being equinumerous with F*.

This definition takes place against the background of Frege's theory of extensions which would eventually emerge as second-order logic supplemented with Basic Law V:

> BASIC LAW V: The extension of F = the extension of G if and only if F and$G$ are coextensive.

Frege's explicit definition of number had Hume's principle as an immediate consequence. In ***Grundlagen***, Frege outlined a derivation of ordinary arithmetic from his definition of number. Unfortunately, by the time the two volumes of Frege's ***Grundgesetze*** appeared in print with all the technical details of the derivation, Russell had shown Basic Law V to be inconsistent. Frege tried to modify Basic Law V to avoid the contradiction but to no avail. Faced with Russell's discovery, he later abandoned his life's project to ground arithmetic on logical principles and definitions.

Bob Hale and Crispin Wright have led a recent effort to revive Frege's ambition to establish the analyticity of arithmetic. Neo-Fregeanism, as this effort is now known, is a renewed defense of Frege's two main tenets, but with a qualified understanding of the analyticity of arithmetic. Where Hale and Wright differ from Frege is in their assessment of the prospects for such a defense. Frege's mistake, they suggest, had been to seek an explicit definition of number in the face of the Julius Caesar problem. Instead, they propose to make do with Hume's principle alone as a definitional basis for arithmetic. Two technical observations provide some grounds for optimism. The first observation is that Hume's principle is consistent if second-order arithmetic is. Since the consistency of this theory seems beyond serious question, this observation has been taken to show the consistency of Hume's principle itself. The second observation is that, in the context of second-order logic, Hume's principle does indeed suffice for a derivation of all the Dedekind-Peano axioms. This is what, following a suggestion of George Boolos, has come to be known as *Frege's Theorem*.

The question remains whether Frege's Theorem may be reasonably taken to underwrite a vindication of the analyticity of arithmetic. The present volume collects together 15 papers by Bob Hale and Crispin Wright which, at the time of publication, represented their best effort in that direction. That effort had taken place in a span of several years and all but one of the papers in the collection had appeared in print before the publication of this volume. The book includes an introduction and a postcript written especially for the occasion. The introduction surveys the targets of the neo-Fregean program and explains how the different papers in the volume contribute to the fulfillment of its philosophical obligations. The postcript, in contrast, provides a useful summary of 18 problems still facing the neo-Fregean program and sketches directions for further research. The topics addressed in the papers reflect the breadth of ontological and epistemological questions raised by the neo-Fregean philosophy of arithmetic. The papers are divided into five main parts.

Part I (Ontology and Abstraction Principles) consists of five papers largely centered on two challenges. One has to do with the neo-Fregean conception of object as just what a singular term refers to. The central challenge is to be able to identify expressions as singular terms independently of the assumption that they refer or purport to refer to objects. Another challenge for the neo-Fregean is related to Frege's remark in ***Grundlagen*** §64 that the two sides of instances of abstraction principles in general and Hume's principle in particular *carve* a single content in two different ways. A quick glance at an instance of Hume's principle suggests that the left and right hand side of the biconditional should diverge in their truth conditions in view of the fact that one side is a statement of numerical identity while the other is a statement of one-one correspondence. The challenge is to motivate an understanding of *truth condition* on which the two sides of an instance of Hume's principle may reasonably be said to have identical truth conditions.

Part II (Responses to Critics) consists of four papers written mostly in response to criticisms by Hartry Field and Michael Dummett. One theme in common with the first part is Hartry's Field objection based in the apparent divergence in truth conditions between the two sides of instances of Hume's principle. Field had urged the rejection of that principle in favor of a conditionalized version of the form: "If numbers exist, then the number of F = the number of G if and only if they are in one-to-one correspondence." Other concerns are more general and not specific to the neo-Fregean program. One of the papers by the first author discusses Field's generalization of Benacerraf's classic epistemological challenge for platonism. Two more papers react to Michael Dummett's criticisms of Frege's philosophy of arithmetic and the more recent neo-Fregean revival. Some of Dummett's objections center on the viability of a neo-Fregean vindication of Frege's platonism and have to do with what he thinks is the inability of Hume's principle to secure a robust reference for the numerical terms or to counter ontological reductionism. A related objection concerns the viability of a neo-Fregean solution of the Julius Caesar problem. Discussion of what is probably Dummett's most serious objection is postponed to Part III, where Wright considers the question of whether the neo-Fregean project is marred by the impredicativity of Hume's principle.

Part III (On Hume's Principle) consists of four papers written by Crispin Wright on Hume's principle. This part is largely centered on two themes. One is the impredicativity of abstraction of Hume's principle, which Michael Dummett has found objectionable as a sort of vicious circularity that threatens the prospects of both Frege's philosophy of arithmetic and its neo-Fregean revival. Wright argues for the legitimacy of impredicative abstraction principles in general and denies that impredicativity places a serious obstacle for the neo-Fregean defense of the analyticity of arithmetic. The other theme of this part has to do with a critical obligation for the neo-Fregean, who must provide a reasonable understanding of analyticity on which Hume's principle may appropriately be called analytic. The neo-Fregean answer is that Hume's principle serves as an implicit definition of the concept of number. The task of course is to elaborate and defend the neo-Fregean account of *implicit definition* on which Hume's principle indeed qualifies as an implicit definition.

Part IV (On the Differentiation of Abstracts) consists of a single joint effort to survey what they seem to the authors the best lines of response to the Julius Caesar problem, which is what led Frege to seek an explicit definition of number in the first place. If Hume's principle is to succeed as a complete explanation of the concept of number, then one might expect it to settle the truth conditions of mixed identity statements of the form "the number of $F = q$", where "q" is a term explicitly purported to denote an object of an ostensibly different sort. Hale and Wright hope that the criterion of identity associated with the sortal concept *number* as introduced by Hume's principle imposes some restrictions on the extension of the concept of number that suffice, for example, to exclude the possibility that a number be identical with an object that falls under a sortal concept governed by a different criterion of identity.

Part V (Beyond Number Theory) consists of an attempt by Bob Hale to extend the neo-Fregean program beyond arithmetic to cover real analysis. The neo-Fregean ambition is to identify further abstraction principles with a claim to provide a foundation for other branches of ordinary mathematics and perhaps even set theory. In the specific case of analysis, Hale identifies and studies an abstraction principle which generates real numbers as ratios of quantities.

A few years have now elapsed since the publication of Hale and Wright's volume and much work has been done to address some of the problems they raise in their postcript. The problems listed in the author's postcript are centered on three main fronts: (a) Abstraction principles and their credentials to serve as a foundation for ordinary mathematics, (b) the legitimacy of higher-order logic, and (c) the prospects for an extension of the neo-Fregean program for the rest of mathematics. Two problems of the first sort still strike this reviewer as particularly challenging. One is the question of what justifies the special epistemological status of philosophically virtuous abstraction principles as opposed to others. What confers on Hume's principle, for example, a privileged epistemological and ontological status as a foundation of arithmetic? Not only do we seem to lack an explanation of why some abstraction principles enjoy a more privileged epistemological status than others, we seem to have little reason to expect a principled and informative distinction between philosophically virtuous abstraction principles and all of the rest. Michael Dummett once raised the objection that next to what are supposed to be philosophically virtuous abstraction principles like

Hume's principle lie inconsistent principles like Basic Law V. Unfortunately, a series of refinements of Dummett's *bad company objection* would seem to suggest that there is little hope to set philosophical virtuous abstraction principles apart from less virtuous principles.

More progress has been achieved on some of the other fronts. Hale's approach to real analysis has been recently supplemented by an alternative neo-Fregean account of the real numbers based on Dedekind's construction of the real numbers as Dedekind cuts. Crispin Wright and Stewart Shapiro, for example, have explored this route in which successive abstraction principles eventually take one from cardinals to Dedekind cuts which are then identified with real numbers. The prospects for a neo-Fregean account of set theory are a different matter entirely. Very briefly, the challenge is to identify an acceptable abstraction principle that might serve as a basis for a well-motivated neo-Fregean set theory from which to recover a sufficient amount of set theory to deserve the name.

GABRIEL UZQUIANO

Department of Philosophy, Ohio State University, 230 N. Oval Mall, Columbus, OH 43210, USA. uzquiano-cruz.1@osu.edu.

GRAHAM PRIEST. ***An introduction to non-classical logic.*** Cambridge University Press, 2001, xxi + 242 pp.

The book offers a systematic presentation of non-classical *propositional* logics ("classical" logic understood just as the standard propositional logic as commonly taught). For most of the logics presented, the possible world semantics is used and the proof systems are based on tableaux. Each chapter dealing with a logic ends with a section with proofs of theorems, a historical survey and suggestions for further reading. The notion of a formula is standard, with one remarkable feature that the author never uses the name "implication" and instead calls the connective "conditional". In each chapter there is a shorter or longer discussion on how the connective of conditional corresponds (and in fact does not correspond) to the intuitive notion of the conditional "if – then" of the natural language, understood as relating some proposition (the consequent) to some other proposition (the antecedent) on which, in some sense, it depends (p. 9). This seems to be a reasonable philosophical question (as a part of logical analysis of natural language), but one can ask if this is a *logical* question. The author explicitly says (p. 1) that "the point of logic is to give an account of the notion of validity: what follows from what". But from this point of view, implication is, in each reasonable logic, a well defined connective and modus ponens just expresses its behavior w.r.t. validity. The *paradoxes* of implication just show that implication does not formalize the intuitive notion of conditional, which is interesting (more or less) but is not a fault.

The chapters are as follows: *Chapter* 1 – *Classical logic and material conditional.* Formulas, trees and tableau rules for classical logic are defined; the discussion on paradoxes starts. *Chapter* 2 – *Basic modal logic.* Possible world semantics is defined and the system K (after "Kripke") is introduced (no restriction to the accessibility relation). Philosophical discussion on the meaning of possible worlds—three approaches discussed (realism, actualism, meinongianism). And of course tableau rules, completeness proof, as in all chapters (not stressed below). *Chapter* 3 – *Normal modal logics.* These are K and stronger logics putting some restrictions to the accessibility relation, in particular the famous S5 is presented. There is a discussion on the notion of necessity. *Chapter* 4 – *Non-normal worlds, strict conditionals.* Each structure of possible worlds contains a subset of "normal" possible worlds; in a non-normal world, each formula is possible and no formula is necessary. Lewis's S2 and S3 are introduced and Lewis's notion of strict conditional (defined as $\Box(A \supset B)$) is discussed. *Chapter* 5 – *Conditional logics.* There is a new connective $>$ and the possible world structure has a system $\{R_A \mid A \text{ a formula}\}$ of relations, i.e., each formula has its accessibility relation. $A > B$ is true in a world w if B is true in each world R_A-accessible from w. *Chapter* 6 – *Intuitionistic logic.* Usual possible world semantics for this logic presented (R reflexive and transitive, evaluation