

# G2S3 Project - UQ group

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# Outline

## 1 Sensitivity analysis

- Motivation
- Methodology
- Results

## 2 Comparison to the observational data

## Different location for the source

# The mathematical model

## The mathematical model

$$y = f(x), \text{ where } x = (x_1, x_2, \dots, x_n) \in U^n = [0, 1]^n \quad (1)$$

is a vector of input parameters with joint p.d.f.  $p(x) = p(x_1, x_2, \dots, x_n)$

- Local sensitivity:  $x = x^* \in U^n, y^* = f(x^*)$
- Global sensitivity  $y = f(x), x \in U^n$

# The mathematical model

Total Sensitivity Index (TSI) of input parameter  $x_i, i \in \{1, \dots, n\}$

$$TSI(x_i) = S_i + \sum_{l_1 \neq i} S_{il_1} + \sum_{l_1, l_2 \neq i, l_1 < l_2} S_{il_1 l_2} + \dots + S_{il_1 \dots l_{n-1}} \quad (2)$$

where  $S_{il_1 \dots l_{j-1}}$  -  $j^{th}$  order sensitivity index of parameter  $x_i$  ( $1 \leq i \leq n$ )

$j = 1$  :  $S_i$  - "the main effect" of  $x_i$

Classification of input parameters:

- -very important:  $0.8 \leq TSI$
- -important:  $0.5 \leq TSI \leq 0.8$
- -unimportant:  $0.3 \leq TSI \leq 0.5$
- -irrelevant:  $TSI \leq 0.3$

# Sobol' Approach

Analysis of Variances:

$$f(x) = f_0 + \sum_{i=1}^n f_i(x_i) + \sum_{1 < i < j < n} f_{ij}(x_i, x_j) \quad (3)$$

where

- $f_0$  - constant
- $\int_0^1 f_{l_1 \dots l_s}(x_1, x_2, \dots, x_{l_s}) dx_{l_k} = 0, 1 \leq k \leq s$  Therefore
- $\int_{U^n} f_{i_1, \dots, i_s} f_{j_1, \dots, j_l} dx = 0, (i_1, \dots, i_s) \neq (j_1, \dots, j_l)$
- $f_0 = \int_{U^n} f(x)$
- $f_{l_1}(x_{l_1}) = \int_{U^n} f(x) \prod_{k \neq l_1} dx_k - f_0, l_1 \in 1, \dots, n$
- $f_{l_1 l_2}(x_{l_1}, x_{l_2}) = \int_{U^n} f(x) \prod_{k \neq l_1, l_2} dx_k - f_0 - f_{l_1}(x_{l_1}) - f_{l_2}(x_{l_2})$

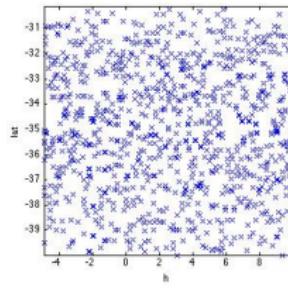
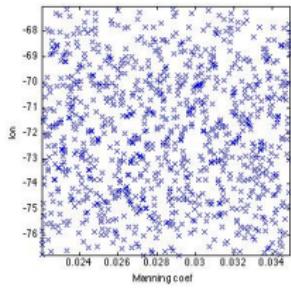
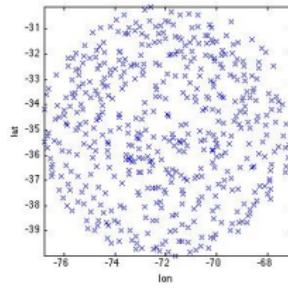
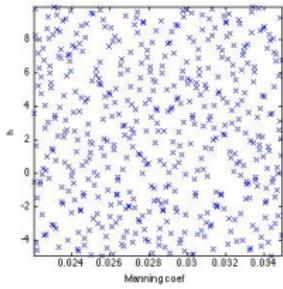
# Sobol' Sensitivity Indices

## Definition

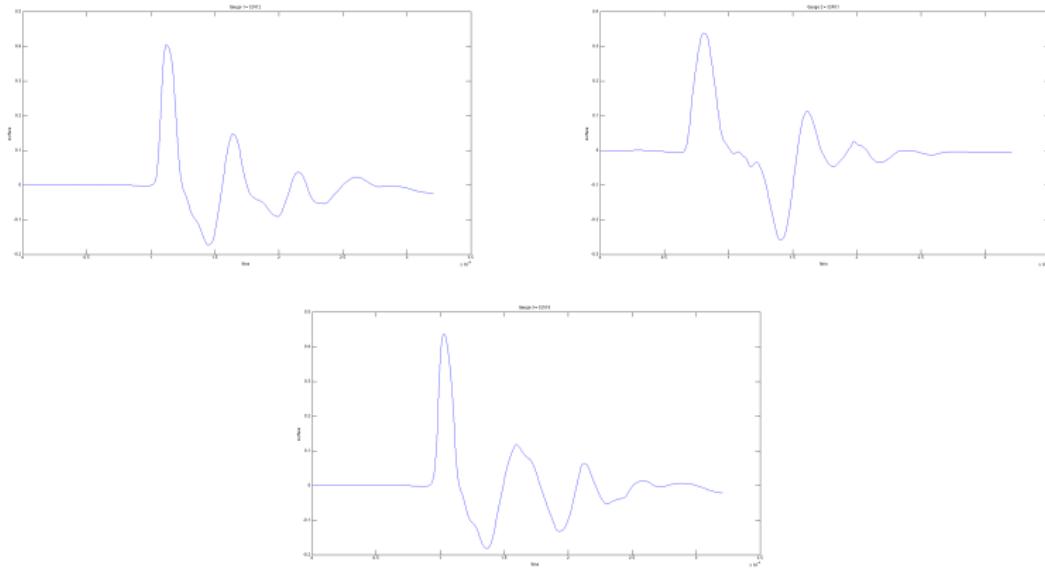
$$S_{l_1 \dots l_s} = \frac{D_{l_1 \dots l_s}}{D}, \quad s \in 1, \dots, n \quad (4)$$

- variances  $D_{l_1 \dots l_s} = \int f_{l_1 \dots l_s}^2 dx_{l_1} \dots dx_{l_s}$
- total variance  $D = \int_{U^k} f^2(x) dx - f_0^2$
- $\sum_{s=1}^k \sum_{l_1 \leq \dots \leq l_s} S_{l_1, \dots, l_s} = 1$

## 2D samples



# Outputs 1

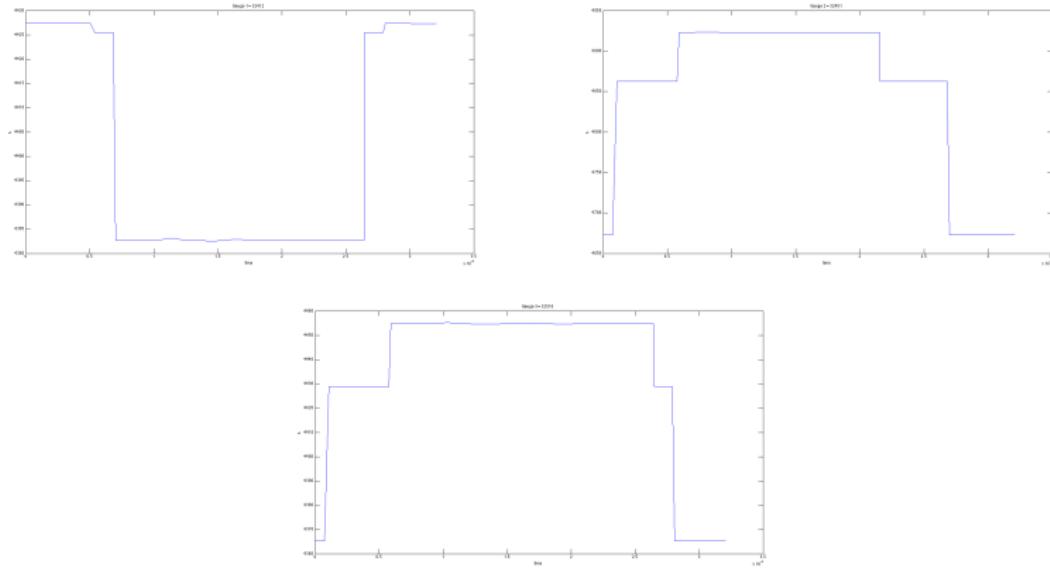


**Figure:** Surface elevation after 130 time steps at: a) gauge 1 b) gauge 2 c) gauge 3

# Results 1

Parameter	Sensitivity [Confidence]	Rank	Parameter	Sensitivity [Confidence]	Rank
1st-Order Sensitivity Indices					
Manning	-0.155750 [ +0.240775 ] 4		Manning	-0.028776 [ +0.081816 ] 3	
R	+0.154082 [ +0.484122 ] 3		R	-0.194375 [ +0.233368 ] 4	
theta	+0.335555 [ +0.592225 ] 1		theta	+0.450594 [ +0.559537 ] 1	
h	+0.163244 [ +0.517045 ] 2		h	+0.333184 [ +0.553066 ] 2	
Total-Order Sensitivity Indices					
Manning	+0.097877 [ +0.349192 ] 4		Manning	+0.068208 [ +0.157423 ] 4	
R	+0.712277 [ +0.550941 ] 1		R	+0.183205 [ +0.310126 ] 3	
theta	+0.425347 [ +0.581164 ] 3		theta	+0.447316 [ +0.549985 ] 2	
h	+0.635118 [ +0.443884 ] 2		h	+0.779188 [ +0.448338 ] 1	
2nd-Order Sensitivity Indices					
Manning * R	+0.223184 [ +0.655293 ] 1		Manning * R	+0.501131 [ +0.539193 ] 5	
Manning * theta	+0.117399 [ +0.414486 ] 2		Manning * theta	+0.561411 [ +0.459554 ] 4	
Manning * h	+0.003365 [ +0.577097 ] 5		Manning * h	+0.661888 [ +0.465414 ] 3	
R * theta	+0.053148 [ +0.554816 ] 3		R * theta	+0.787201 [ +0.494790 ] 1	
R * h	-0.423183 [ +0.712271 ] 6		R * h	+0.773194 [ +0.472911 ] 2	
theta * h	+0.006077 [ +0.930382 ] 4		theta * h	-0.284160 [ +1.778547 ] 6	
Parameter	Sensitivity [Confidence]	Rank	Parameter	Sensitivity [Confidence]	Rank
1st-Order Sensitivity Indices					
Manning	-0.001940 [ +0.053661 ] 3		Manning	-0.053661 [ +0.001940 ] 6	
R	-0.141305 [ +0.229064 ] 4		R	-0.229064 [ -0.141305 ] 5	
theta	+0.357650 [ +0.511374 ] 1		theta	+0.511374 [ -0.357650 ] 2	
h	+0.314340 [ +0.491002 ] 2		h	-0.491002 [ -0.314340 ] 4	
Total-Order Sensitivity Indices					
Manning	+0.030574 [ +0.073128 ] 4		Manning	+0.073128 [ -0.030574 ] 5	
R	+0.151932 [ +0.282981 ] 3		R	-0.282981 [ -0.151932 ] 4	
theta	+0.469673 [ +0.479080 ] 2		theta	-0.479080 [ -0.469673 ] 3	
h	+0.561749 [ +0.403719 ] 1		h	-0.403719 [ -0.561749 ] 2	
2nd-Order Sensitivity Indices					
Manning * R	+0.606387 [ +0.541231 ] 4		Manning * R	-0.541231 [ -0.606387 ] 5	
Manning * theta	+0.536692 [ +0.503986 ] 5		Manning * theta	-0.503986 [ -0.536692 ] 4	
Manning * h	+0.696512 [ +0.420984 ] 3		Manning * h	-0.420984 [ -0.696512 ] 2	
R * theta	+0.829943 [ +0.519430 ] 1		R * theta	-0.519430 [ -0.829943 ] 3	
R * h	+0.759589 [ +0.537556 ] 2		R * h	-0.537556 [ -0.759589 ] 1	
theta * h	+0.034795 [ +1.296518 ] 6		theta * h	-1.296518 [ -0.034795 ] 5	

# Outputs 2

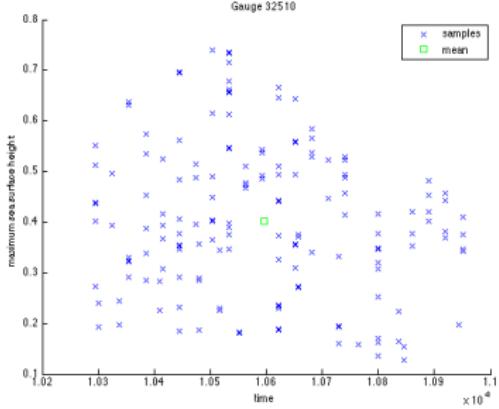
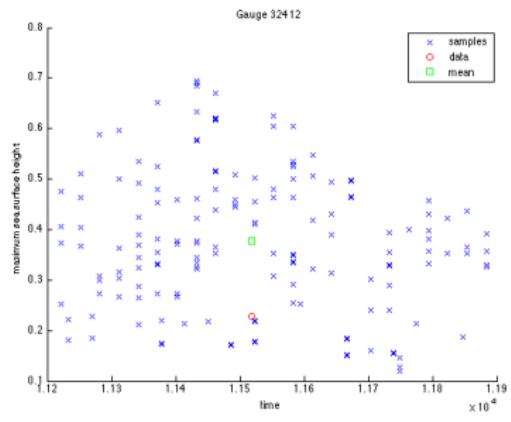
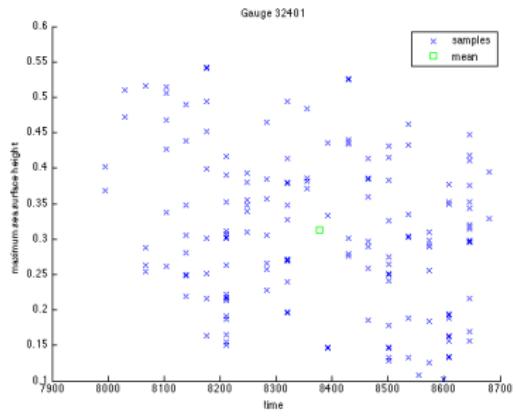


**Figure:** Height after 160 time steps at: a) gauge 1 b) gauge 2 c) gauge 3

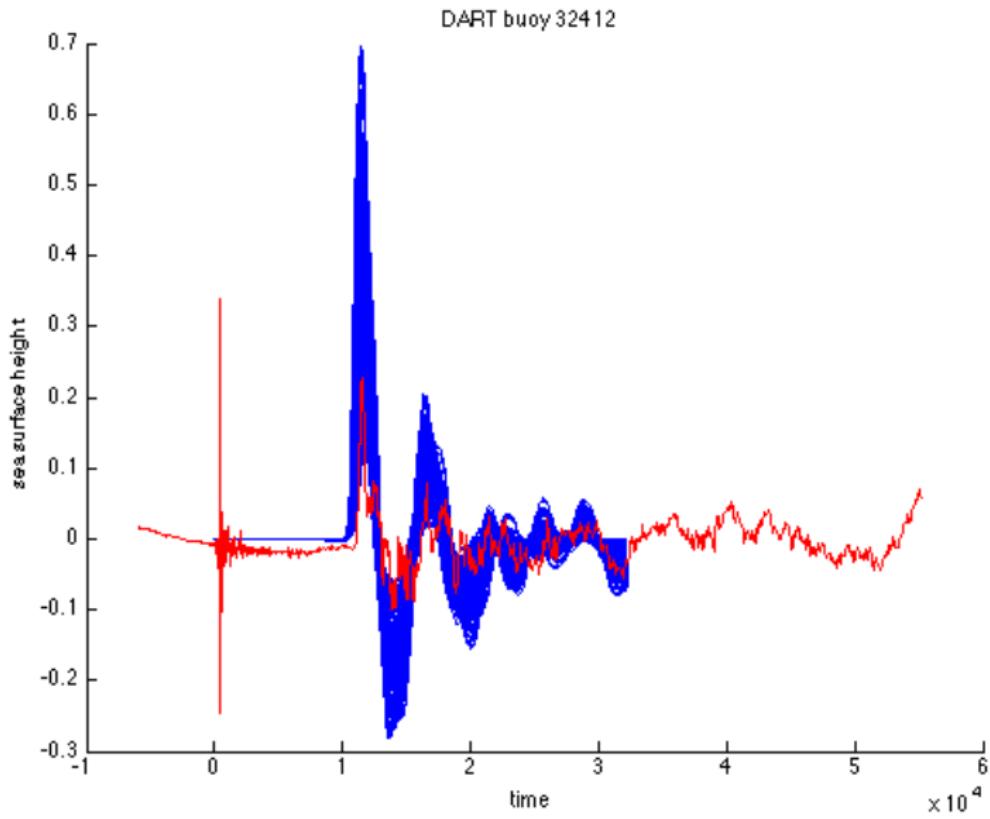
# Results 2

Parameter	Sensitivity [Confidence]	Rank	Parameter	Sensitivity [Confidence]	Rank
1st-Order Sensitivity Indices					
Manning	-0.001940 [ +0.057550 ] 3		Manning	-0.001940 [ +0.052150 ] 3	
R	-0.141305 [ +0.210916 ] 4		R	-0.141305 [ +0.222653 ] 4	
theta	+0.357650 [ +0.541096 ] 1		theta	+0.357650 [ +0.558873 ] 1	
h	+0.314340 [ +0.518185 ] 2		h	+0.314340 [ +0.501342 ] 2	
Total-Order Sensitivity Indices					
Manning	+0.030574 [ +0.078288 ] 4		Manning	+0.030574 [ +0.071014 ] 4	
R	+0.151932 [ +0.279152 ] 3		R	+0.151932 [ +0.274437 ] 3	
theta	+0.469673 [ +0.476666 ] 2		theta	+0.469673 [ +0.472958 ] 2	
h	+0.561749 [ +0.407516 ] 1		h	+0.561749 [ +0.410131 ] 1	
2nd-Order Sensitivity Indices					
Manning * R	+0.606387 [ +0.546098 ] 4		Manning * R	+0.606387 [ +0.549519 ] 4	
Manning * theta	+0.536692 [ +0.487736 ] 5		Manning * theta	+0.536692 [ +0.463229 ] 5	
Manning * h	+0.696512 [ +0.425572 ] 3		Manning * h	+0.696512 [ +0.423411 ] 3	
R * theta	+0.829943 [ +0.522535 ] 1		R * theta	+0.829943 [ +0.528170 ] 1	
R * h	+0.759589 [ +0.567075 ] 2		R * h	+0.759589 [ +0.528212 ] 2	
theta * h	+0.034795 [ +1.363929 ] 6		theta * h	+0.034795 [ +1.242868 ] 6	
Parameter	Sensitivity [Confidence]	Rank	Parameter	Sensitivity [Confidence]	Rank
1st-Order Sensitivity Indices					
Manning	-0.001940 [ +0.050712 ] 3		Manning	-0.001940 [ +0.050712 ] 3	
R	-0.141305 [ +0.203323 ] 4		R	-0.141305 [ +0.203323 ] 4	
theta	+0.357650 [ +0.571649 ] 1		theta	+0.357650 [ +0.571649 ] 1	
h	+0.314340 [ +0.509073 ] 2		h	+0.314340 [ +0.509073 ] 2	
Total-Order Sensitivity Indices					
Manning	+0.030574 [ +0.069622 ] 4		Manning	+0.030574 [ +0.069622 ] 4	
R	+0.151932 [ +0.282230 ] 3		R	+0.151932 [ +0.282230 ] 3	
theta	+0.469673 [ +0.468142 ] 2		theta	+0.469673 [ +0.468142 ] 2	
h	+0.561749 [ +0.412431 ] 1		h	+0.561749 [ +0.412431 ] 1	
2nd-Order Sensitivity Indices					
Manning * R	+0.606387 [ +0.522936 ] 4		Manning * R	+0.606387 [ +0.522936 ] 4	
Manning * theta	+0.536692 [ +0.500382 ] 5		Manning * theta	+0.536692 [ +0.500382 ] 5	
Manning * h	+0.696512 [ +0.424605 ] 3		Manning * h	+0.696512 [ +0.424605 ] 3	
R * theta	+0.829943 [ +0.524892 ] 1		R * theta	+0.829943 [ +0.524892 ] 1	
R * h	+0.759589 [ +0.522950 ] 2		R * h	+0.759589 [ +0.522950 ] 2	
theta * h	+0.034795 [ +1.370904 ] 6		theta * h	+0.034795 [ +1.370904 ] 6	

# Comparison to the observational data



## sample comparison at DART buoy



So long and thanks for all the fish!

