Assessing Extreme Event Risk for Geophysical Flows

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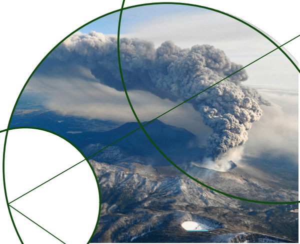


Outline

1. Volcanic events

2. Hazard maps

3. Numerical models
   - TITAN2D
   - `Puff`

4. Extreme events
   - Alaska volcanoes

Volcanic events

When modeling volcanic eruptions we deal with two types of flows:

- on the ground (i.e pyroclastic flows)
- above ground (i.e ash dispersion)

Figure : Volcanic eruptions[1]

The objective: HAZARD MAPS

[1] pictures from public domain

Hazard maps

What do we have

Expert belief and intuition

Models of the physics of individual flows (PDE based)

Data on past events

Methodology for quantifying uncertainty

High end computing and data services

What do we need

- Given a location x and time t – what is the hazard of a catastrophic event? e.g. $P(flow > 1m \; in \; t) \approx 0.000001$?
- Given an area, what is the hazard of an event in the next t time period of all locations?
- Given locations A and B, should we evacuate from A to B at any cost?

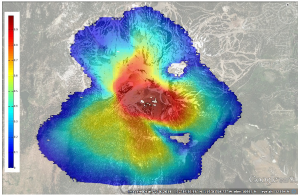


Figure : Hazard map – Mammoth Mountain, Ca

Challenges

- C1. **Expensive simulators** [a] - we deal with complex physics in transport, dispersion, aggregation ...
- C2. **Uncertainties in model parameters and inputs** governs observed flow/transport characteristics
- C3. **Field variables** - uncertainty in the topography/wind (full field variable used in input) is difficult to characterize.
- C4. **Extreme events**, sparse observations.

[a] To obtain three-digit accuracy in the expected value would require $O(10^6)$ simulations

Approach

- Sparse observations – insufficient to build a forecast
  $\Rightarrow$ Use models to supplement.
- Model (Uncertain parameters, Source, Initial and Boundary Conditions)
- Sample uncertain source conditions and uncertain terrain/windfield

$$\Downarrow$$

$$\boxed{\text{Model Ensemble}}$$

$$\Downarrow$$

$$\boxed{\text{Surrogate/ Emulator}}$$

$$\Downarrow$$

$$\mathbf{P}(\text{Flow/Ash }(\mathbf{x}, t))$$

Hazard maps - volcanic flows

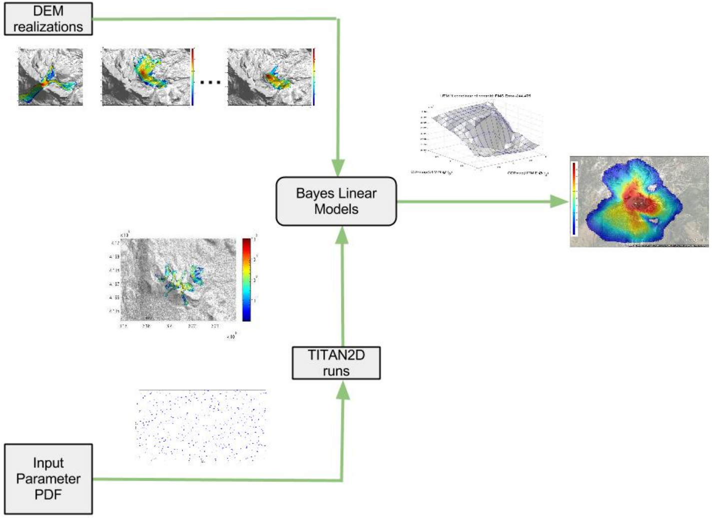


Figure : Hazard maps workflow volcanic flow

C1. The geophysical mass flow model TITAN2D

$$\text{Mass conservation:} \quad \frac{\partial h}{\partial t} + \frac{\partial(V_x \cdot h)}{\partial x} + \frac{\partial(V_y \cdot h)}{\partial y} = 0$$

$$\text{Momentum} \quad \frac{\partial hV_x}{\partial t} + \frac{\partial(V_x \cdot hV_x + 0.5k_{ap}g_z h^2)}{\partial x} + \frac{\partial(V_y \cdot hV_x)}{\partial y} = S_x$$

$$\text{conservation:} \quad \frac{\partial hV_y}{\partial t} + \frac{\partial(V_x \cdot hV_y)}{\partial x} + \frac{\partial(V_y \cdot hV_y + 0.5k_{ap}g_z h^2)}{\partial y} = S_y$$

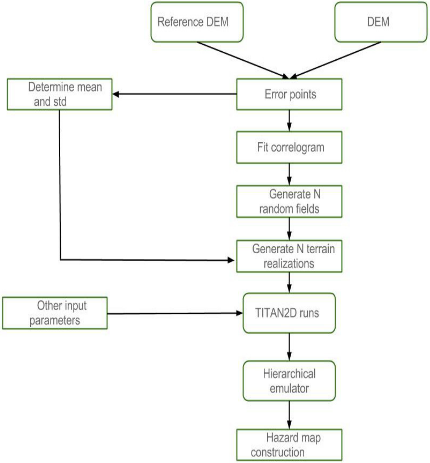
Inputs to TITAN2D are initial volume, initiation location, basal and internal friction angles and Digital Elevation Models (DEMs)

$$k_{ap} = 2\frac{1 \mp \sqrt{1 - cos^2(\phi_{int})(1 + \tan^2(\phi_{bed}))}}{\cos^2} - 1 \qquad (1)$$

$$S_x = g_x h - \frac{V_x}{\sqrt{V_x^2 + V_y^2}} \max\left(g_z + \frac{V_x^2}{r_x}, 0\right) h \tan((\phi_{bed}) -$$

$$-sgn\left(\frac{\partial V_x}{\partial y}\right) hk_{ap} \frac{\partial(g_x h)}{\partial y} \sin((\phi_{int})$$

C2. DEM uncertainty- spatial autocorrelation



- DEM uncertainty is correlated to the feature of the terrain (Ehlschlaeger and Shortridge '96).
- Strong relationship between elevation error and terrain, and that is influenced by the spatial autocorrelation of the error (as the difference of two realizations) (Wechsler et. al. '06, Hebeler and Purves '08).
- The Gaussian model is commonly used as an error model [a]

[a] Stefanescu et al., DEM uncertainty and hazard analysis. Proc. of Royal Society A, 2012.

DEM realizations approach

The model assumes that the total error is the sum of a large number of random, additive effects

$$R(\mathbf{u}) = m(\mathbf{u}) + m(m(T)) + (m(s^2(T)) \cdot \varepsilon) \cdot Z(\mathbf{u}) \tag{2}$$

where R(**u**) is a possible DEM and Z(**u**) is a random field which captures the autocorrelative effect between points. The following equation is used to generate the random field:

$$Z(\mathcal{U}) = \frac{\sum_v w_{u,v}\varepsilon_v}{\sqrt{\sum_v w_{u,v}^2}}, \quad u \in \mathcal{U},\ v \in \mathcal{V} \tag{3}$$

$$w_{u,v} = \begin{cases} 1 & : d_{u,v} \leq F \\ \left(1 - \frac{d_{u,v}-F}{D-F}\right)^E & F < d_{u,v} \leq D,\ u \in \mathcal{U},\ v \in \mathcal{V} \\ 0 & : d_{u,v} > D \end{cases} \tag{4}$$

$w_{u,v}$ is the spatial autocorrelative effect between points $u \in \mathcal{U}$ and $v \in \mathcal{V}$

Uncertainty in DEM.

Hazard map

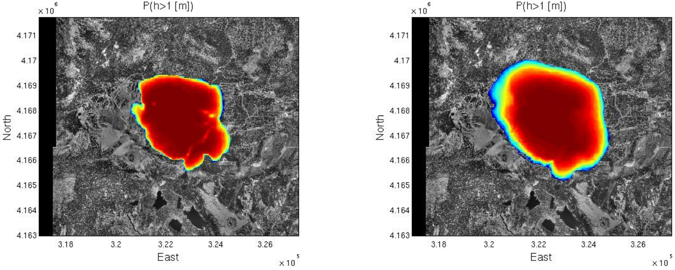


Figure : Probability that a flow will exceed 1 m in depth as a function of position on Mammoth Mountain, CA, given the uncertainties in DEM and input parameters (a) 4 uncertain parameters (east and north location, basal friction, height)– the arrow indicates the center of the town (b) 8 uncertain parameters (including DEM)

Hazard maps - Ash transport

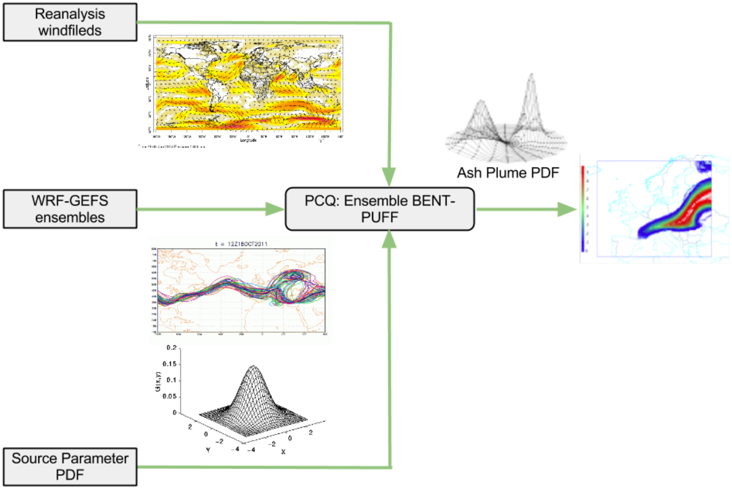
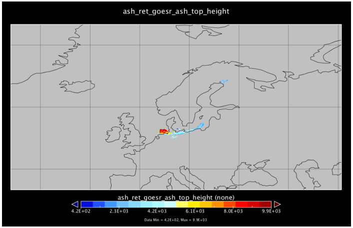


Figure : Hazard maps workflow ash transport

C1. `Puff` Lagrangian VATD model

Tracks $O(10^6)$ Lagrangian point particles at location $R_i(t_k)$ is propagated via an advection/diffusion equation [2].

$$R_i(t_{k+1}) = R_i(t_k) + W(t_k)\Delta t + Z(t_k)\Delta t + S_i(t_k)\Delta t \tag{5}$$



Inputs: Windfield from NWP, Mass Eruption Source parameters
Output: Particle concentration, Maximum height

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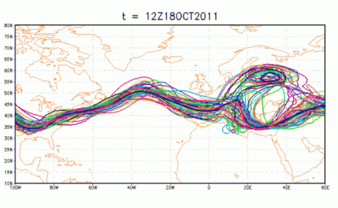
[2] Searcy, C. et al. (1998) PUFF: A high-resolution volcanic ash tracking model. J. Volc. and Geo. Res., 80, 1-16

C2. GEFS ensemble

The NCEP GEFS ensembles consists of 21 members and is run 4 times daily (0000, 0600, 1200, and 1800 UTC) out to 384-h (16 day) lead time. The underlying model for the GEFS is the NCEP Global Forecasting System (GFS), a high-resolution spectral atmospheric model run 4 times daily at the Environmental Modeling Center.



Ensemble spaghetti plot courtesy of http://www.pandowae.de/en/projects/block

We are using the ensemble members produced four times daily, at 0000, 0600, 1200, and 1800 UTC, starting 0000 UTC April 14 2010 to 0000 UTC April 18 2010 at 1°latitude by 1°longitude grid.

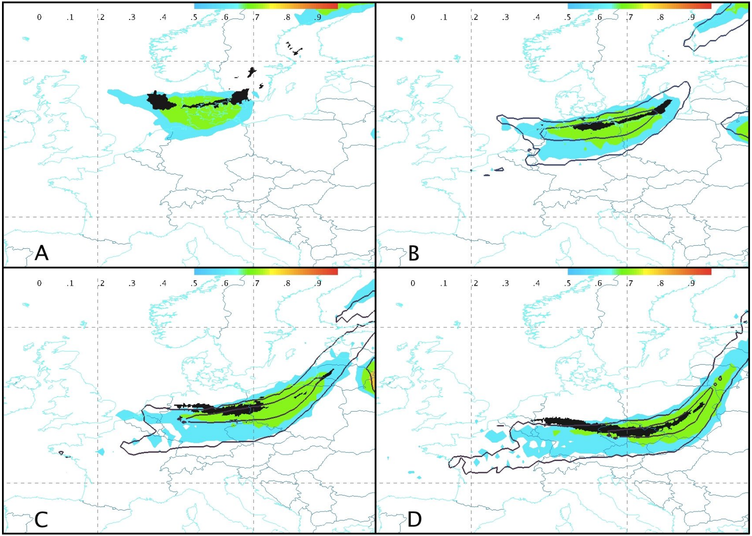
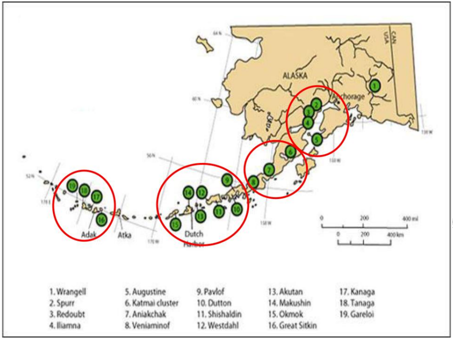
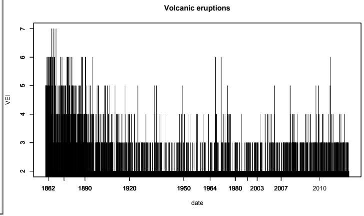


Figure : Probability of having airborne ash when accounting for source parameters only (solid color), source parameters and windfield variability (probability con- tour) and corresponding satellite image at different times (black color), (a) 0000 UTC April 16 2010, (b) 0006 UTC April 16 2010, (c) 0012 UTC April 16 2010, and (d) 0018 UTC April 16 2010

Alaska volcanoes

Cluster volcanoes based on location/ frequency/ magnitude (VEI – Volcanic Explosivity Index) to determine the time intervals between events above some threshold *Q* and the correlation of the return intervals.





Difficulties implementing the method:

- Sparse data spanned over a large period of time
- The assumption of a discrete time stochastic process
- The process needs to be stationary, although stationarity is a property which almost never applies to realistic processes.
- Applying concepts from stationary processes to data which might originate from a non-stationary process could result in reduced performance of our prediction algorithms.
- The method of determining the precursor is especially accurate when the PDF of the process has one clearly defined maximum.

Peak over threshold – a point process model

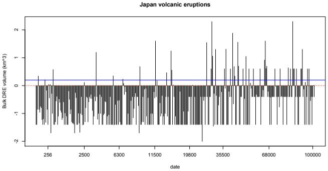
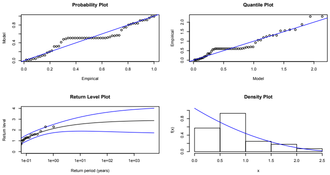
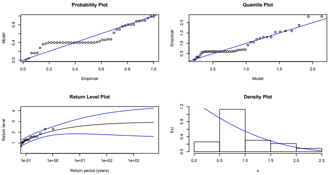


Figure : The threshold is set for 0 (red) and 0.2 (blue)



(a)



(b)

Figure : Peak over threshold approach a) threshold equals 0 b) threshold equals 0.2

The Mathematics of Risk

A "hazard function" ($W_Q(t;\Delta t)$) is defined as the probability that within the next Δt units of time at least one event/ eruption above Q occurs, if the last Q-exceeding event occurs t time units ago.

$$W_Q(t,\Delta t) = \frac{\int_t^{t+\Delta t} P_Q(r)dr}{\int_t^{\infty} P_Q(r)dr}. \tag{6}$$

where P_Q is the probability distribution function (PDF) of the return intervals. For uncorrelated records:

$$\ln P_Q(r) \sim -r/R_Q \tag{7}$$

where R_Q is the mean return interval.
CDF: $C_Q(t) = \int_{-\infty}^{t} P_Q(r)dr$ results:

$$W_Q(t,\Delta t) = [C_Q(t+\Delta t) - C_Q(t)]/[1 - C_Q(t)]. \tag{8}$$

where C_Q can be find out (if data permits) using precursor methods, fractal and multi fractal analysis, etc.

Conclusions

- We account for variability in the input parameters, field parameters.
- Developed methods to generate DEM ensembles.
- Using WRF wind ensemble we are trying to capture the dependency on initial conditions that can lead to large differences in the forecasts.
- Attempt to estimate what future extreme levels of a process might be expected based on a historical series of observations.