Digital Elevation Models (DEMs) clustering for terrain modeling

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Abstract

We consider the problem of Digital Elevation Models (DEMs) segmentation in homogeneous regions, aiming for identification of plateaux, ridges, small drainages, straight front slopes, valleys, and crests. In the paper we explore and compare different methods for the segmentation that are required / needed when we want to construct a sparse representation of the DEM. Methods that are extensively and successfully used in image segmentation such as Spatial Gaussian Mixture Model and Spectral Clustering are adapted for the case in which each data point has associated range of geomorphometric measures. These are two complex methods which account for the spatial correlation of the elevation points and have the advantage that they can be used for almost any application where relationships between topographic features and other components of landscapes are to be assessed.

1 Introduction

Information about topography is necessary for landscape evaluation, erosion studies, hydrology and geophysical modeling, natural hazard prevention, etc. The classic way to incorporate relief units into a landscape assessment is to delineate them during field survey or using stereo aerial photographs. This approach is relatively time-consuming and the results depend on the subjective decision of the interpreter. Several methods for the creation of landform elements using elevation-derived attributes are described in the literature. Commonly, these techniques developed regions of homogeneity based on common attributes and then classified those regions (or groups of regions) as elements. The most widely used techniques are: watershed segmentation, object-based image analysis, support vector machine, segmentation using heuristic rules and fuzzy logic, fuzzy K-means classification and self organizing map. Many of these techniques have drawbacks, especially when the method relies heavily on hydrological information and requires data-specific knowledge; also these methods don't incorporate autocorrelation between the same attribute at two locations in their models. Digital Elevation Models (DEMs) are digital representations of terrain, and are represented as an array of squared cells (data points/ pixels) with an elevation associated to each data point. They can have different resolutions (5m, 30m, 90m, 120m, etc) and can be obtained from various methods (photometry, radar interferometry, laser altimetry, etc.). Usually the size of a DEM varies from tens to hundreds of kilometers which can lead to thousands to millions of grid points.

In this context, we would like to implement a more complex model segmentation of a DEM of Mammoth Mountain to create non-overlapping groupings of homogeneous regions. Mammoth Mountain (Fig. 1) is a volcano located in California and it was chosen due to the ease of obtaining data sets.

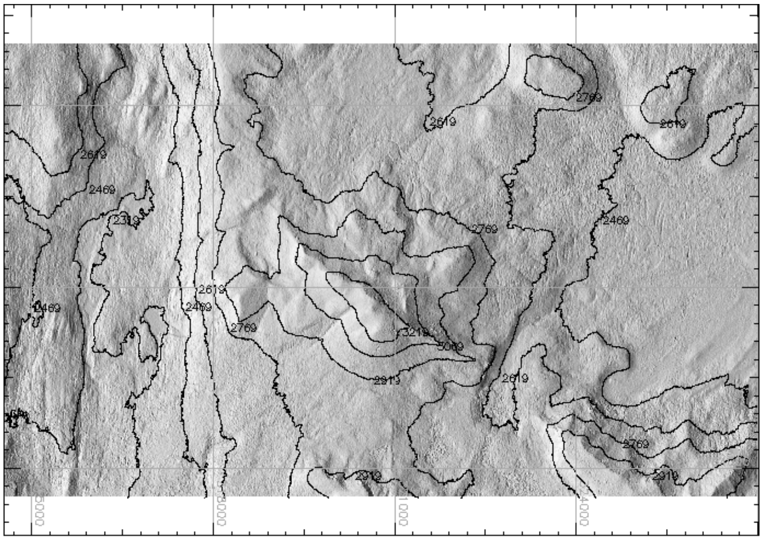


Figure 1: Hillshade plot of the Mammoth Mountain

2 Methodology

Segmentation methods are based on some data point or region similarity in relation to their local neighborhood. A variety of different methods have been proposed for image segmentation such as boundary-based segmentation, region-based segmentation and pixel-labeling. Recently, Expectation-Maximization (EM) algorithm has attracted considerable interest to compute the maximum likelihood estimates, when the observations are unlabeled. The unsupervised clustering techniques do not require training data, but they do require an initial segmentation.

2.1 Spatial Gaussian Mixture and the Expectation-Maximization algorithm

To be able to perform the segmentation of the DEM in homogeneous regions we need to specify a range of geomorphometric measures which can be extracted from the surface. We define a *feature matrix* of DEM attributes, consisting of elevation and first and second derivatives of elevation (slope, profile curvature and tangential curvature). Slope and curvature are easily extracted from a DEM within a Geographical Information System (GIS).

The major drawback of traditional mixture models is lack of spatial correlation in the clustering processing. Sujaritha et. al. proposed an adaptive Spatial Gaussian Mixture Model and EM algorithm which introduces the spatial information into the clustering process. The technique, described below, is what we attempted to implement for our case in which we are dealing with a feature matrix and not a vector in color space.

Gaussian Mixture Model

To define a Gaussian mixture model with $K > 1$ components in $\mathcal{R}^D$ for $D \geq 1$, let x_n be the observation of the nth data point of a DEM. The density function $p(x_n)$ is given by:

$$p(x_n) = \sum_{k=1}^{K} \pi_k \mathcal{N}(x_n|\mu_k, \Sigma_k) \tag{1}$$

where $\pi_1, \pi_2 \cdots \pi_k$ are the mixing coefficients and μ_k, Σ_k are the Gaussian distribution's parameters for each kth component. The mixing coefficients satisfy the following conditions:

$$0 \leq \pi_k \leq 1 \ \ , \ \ \sum_{k=1}^{K} = 1 \tag{2}$$

For the Gaussian mixtures, each component density is a Gaussian probability with μ_k and covariance Σ_k:

$$p(x_n) = \mathcal{N}(x_n|\mu_k, \Sigma_k) = \frac{1}{2\pi^{\frac{D}{2}}} \frac{1}{\det(\Sigma_k)^{\frac{1}{2}}} \exp\{-\frac{(x_n - \mu_k)^T (x_n - \mu_k)}{2\Sigma_k}\} \tag{3}$$

The inherent steps of the EM for the Gaussian mixture approach can be summarized as:

- Initialize the means μ_k, covariance Σ_k, mixing coefficients π_k and the log likelihood.

- **E step** evaluate the responsibilities using the current parameters values:

$$\gamma(z_{nk}) = \frac{\pi_k \mathcal{N}(x_n|\mu_k, \Sigma_k)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(x_n|\mu_j, \Sigma_j)} \tag{4}$$

- **M step** Re-estimate the parameters using the current responsibilities:

$$\mu_k^{new} = \frac{1}{N_k} \sum_{i=1}^{N} \gamma(z_{nk}) x_n \tag{5}$$

$$\Sigma_k^{new} = \frac{1}{N_k} \sum_{i=1}^{N} \gamma(z_{nk})(x_n - \mu_k^{new})(x_n - \mu_k^n ew)^T \tag{6}$$

$$\pi_k = \frac{1}{N} \sum_{n=1}^{N} \gamma(z_{nk}) \tag{7}$$

- Evaluate the log likelihood:

$$\ln p(X|\mu, \Sigma, \pi) = \sum_{n=1}^{N} \ln\{\sum_{k=1}^{K} \pi_k \mathcal{N}(x_n|\mu_k, \Sigma_k)\} \tag{8}$$

Adaptive Spatial Gaussian Mixture Model

In the paper by Sujaritha et.al., a way of incorporating spatial relationships was proposed. Therefore, in the calculation of the density function, the data point x_n will be influenced by its neighbors.

$$\begin{aligned} p(x_n) = \frac{1}{2\pi^{\frac{D}{2}}} \frac{1}{\det(\Sigma_k)^{\frac{1}{2}}} \times [\exp\{-[&\frac{\eta_n^k (x_n - \mu_k)^T (x_n - \mu_k)}{2\Sigma_k} \\ &+ \frac{\eta_n^k}{8} \sum_{X_l \in X_{x_n}} \frac{(x_n - \mu_k)^T (x_n - \mu_k)}{2\Sigma_k}]\}] \end{aligned} \tag{9}$$

where η_n is the parameter that controls the neighbors influence and X_{x_n} is the subset of neighborhood data points of x_n in a 3×3 window. η_n is calculated using the following formula:

$$\eta_i^k = df_{std}^k(i)/x_{std}(i) \tag{10}$$

where

$$df^k_{std}(i) = (\frac{1}{9}[\sum_{X_l \in X_{x_n}} \{(df^k_{x_n} - \mu)^2\} + (df^k_{x_i} - \mu)^2])^{1/2} \tag{11}$$

μ is the mean value of df in the 3 × 3 window:

$$df^k_{x_n} = (\frac{(x_n - \mu_k)^T (x_n - \mu_k)}{2\Sigma_k} \tag{12}$$

In order to eliminate the unbalanced effect on the weighting functions between smooth and sharp edges, the df is divided by the standard deviation of all the data points in the 3 × 3 window

$$x_{std}(i) = \{(\frac{1}{9}\sum_{X_l \in X_{x_n}} (x_l - \hat{x})^2 + (x_n - \hat{x})^2)\}^{1/2} \tag{13}$$

2.2 K-means

Since EM algorithm convergences to local maxima, initialization is very important. The initialization of the Gaussian mixture parameters is done using K-means. K-means is essentially a sorting and binning procedure where the user determines the number of bins / clusters to be used and the rules for sorting. The K-means procedure sorts a set of data points into a specified number of clusters based on the feature vector for each data point [3]. Comparison of the K-means algorithm with the EM algorithm for Gaussian mixtures shows that they are very similar [6]. The K-means algorithm uniquely assigns each data point within a cluster whereas theEM algorithm makes the assignment based on the posterior probabilities. The data set available is a 5m DEM, covering an area of $\approx 94km^2$ which results in $\approx$ 3 millions data points. To speed up the computational time, the majority of the analysis was performed on a decimated DEM having a 120m resolution and 6550 data points. The following steps are implemented for the K-means algorithm:

- We choose $k = 4$, 6 and 10 clusters from the DEM. There are $\mu_1, \mu_2, \cdots, \mu_k$ means that we use for initialization.

- For each elevation point the nearest mean is calculated.

$$c_i = \underset{\mu_j \in \{\mu_1, \mu_1, \cdots, \mu_k\}}{\operatorname{argmax}} (x_i - \mu_j)^2 \tag{14}$$

- The means values were updated based on the elevation points assigned to them.

- The above steps were repeated until the locations of the means were no longer changing by a significant amount.

2.3 Spectral Clustering

A digital representation of a terrain surface is an approximation of reality and is often subject to significant error. The error is usually not known in terms of both magnitude and spatial distribution. There are, in fact large uncertainties associated with the construction of DEMs. DEM vendors generally provide users with a measure of vertical accuracy in the form of the root mean squared error (RMSE) statistic. One key feature of the DEM grid points, which are spatial data, is the autocorrelation of observations in space. Generally, spatial autocorrelation refers to the correlation between the same attribute at two locations. Observations in close spatial proximity tend to be more related than observations at larger distances or separation. Based on this assumption our next clustering method will be Spectral Clustering [2]-[4].

Spectral Clustering has been used with success in the field of computer vision for data clustering. Compared with traditional clustering algorithms, Spectral Clustering has some advantages: it can recognize the clusters of unusual shapes and obtain the globally optimal solutions in a continuous domain by eigendecomposition [5]. This property makes Spectral

Clustering more suitable for many applications, such as speech separation, image segmentation, large scale integration design, and so on.

The main idea of Spectral Clustering is to build a weighted graph $G(V, E)$, where V represents vertices and E, edges. We represent each elevation point as a node in the graph G and the links between the adjacent data points will form the edges of the graph. Like other clustering algorithms, Spectral Clustering attempts to partition data points into groups such that the members of a group are similar to each other and dissimilar to data points outside of the group. Spectral clustering has a simple formulation and can be solved by standard linear algebra techniques, however, it typically produces better results than traditional clustering methods such as K-means and mixture models [6]-[7] Given data points, an affinity matrix can be represented by a weighted adjacency matrix W, where w_{ij} is a measure of the similarity between x_i and x_j. The affinity matrix is used to preserve the local structure of the patterns. It expresses the degree of similarity between points, and it must have the following properties: i) non-negative; ii) symmetric; iii) invertible. We have chosen the heat kernel for calculating the affinity matrix, as:

$$\mathbf{W}_{ij} = \begin{cases} \exp\frac{-\|F(i)-F(j)\|}{\sigma_F^2} * exp\frac{-\|x(i)-x(j)\|}{\sigma_x^2}, & if \parallel x(i) - x(j) \parallel \leq r \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

where $F(i)$ represents the DEM feature vector for node i, and $x(i)$ represents the coordinate location of i^{th} node. where $\sigma > 0$ is a tuning parameter that controls the decay of the affinity [8]. The degree d_i of node i is the sum of all edge weights incident on x_i:

$$d_i = \sum_{j=1}^{n} w_{ij} \tag{16}$$

The graph Laplacian matrix is defined as:

$$L = D - W \tag{17}$$

L satisfies the following properties:

- For every $f \in \mathcal{R}^n$ we have :

$$f'Lf = \frac{1}{2}\sum_{i,j=1}^{n} w_{ij}(f_i - f_j)^2. \tag{18}$$

- L is symmetric and positive semi-definite.
- The smallest eigenvalue of L is 0 and the corresponding eigenvector is the constant unity vector 1.
- L has n non-negative, real-valued eigenvalues $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$.

The steps involved in segmentation using Spectral Clustering are summarized in Figure 2. The most interesting aspect of Spectral Clustering is the mapping of the data points into a new space with k-dimensions by means of eigenvector decomposition. These eigenvectors induce an embedding of the data points in a low-dimensional subspace wherein a partitioning based on the Normalized Cut (NCut) can be used. If we let A and B represent a bipartition

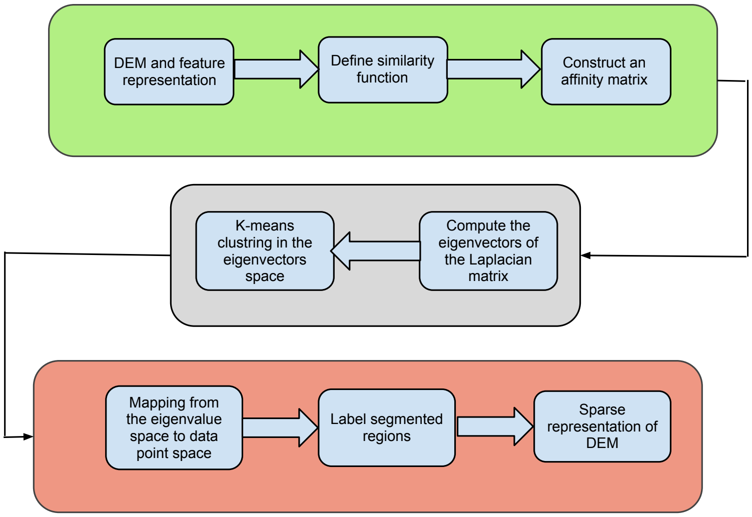

DEM and feature representation
Define similarity function
Construct an affinity matrix
K-means clustring in the eigenvectors space
Compute the eigenvectors of the Laplacian matrix
Mapping from the eigenvalue space to data point space
Label segmented regions
Sparse representation of DEM

Figure 2: Spectral Clustering workflow

of V, ($A \cup B = V$ and $A \cap B = \emptyset$). It can be defined $cut(A,B) = \sum_{i \in A, j \in B} W_{ij}$ and $assoc(A,V) = \sum_{i \in A, j \in V} W_{ij}$. The normalized cut between sets A and B is then given by:

$$NCut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)} \tag{19}$$

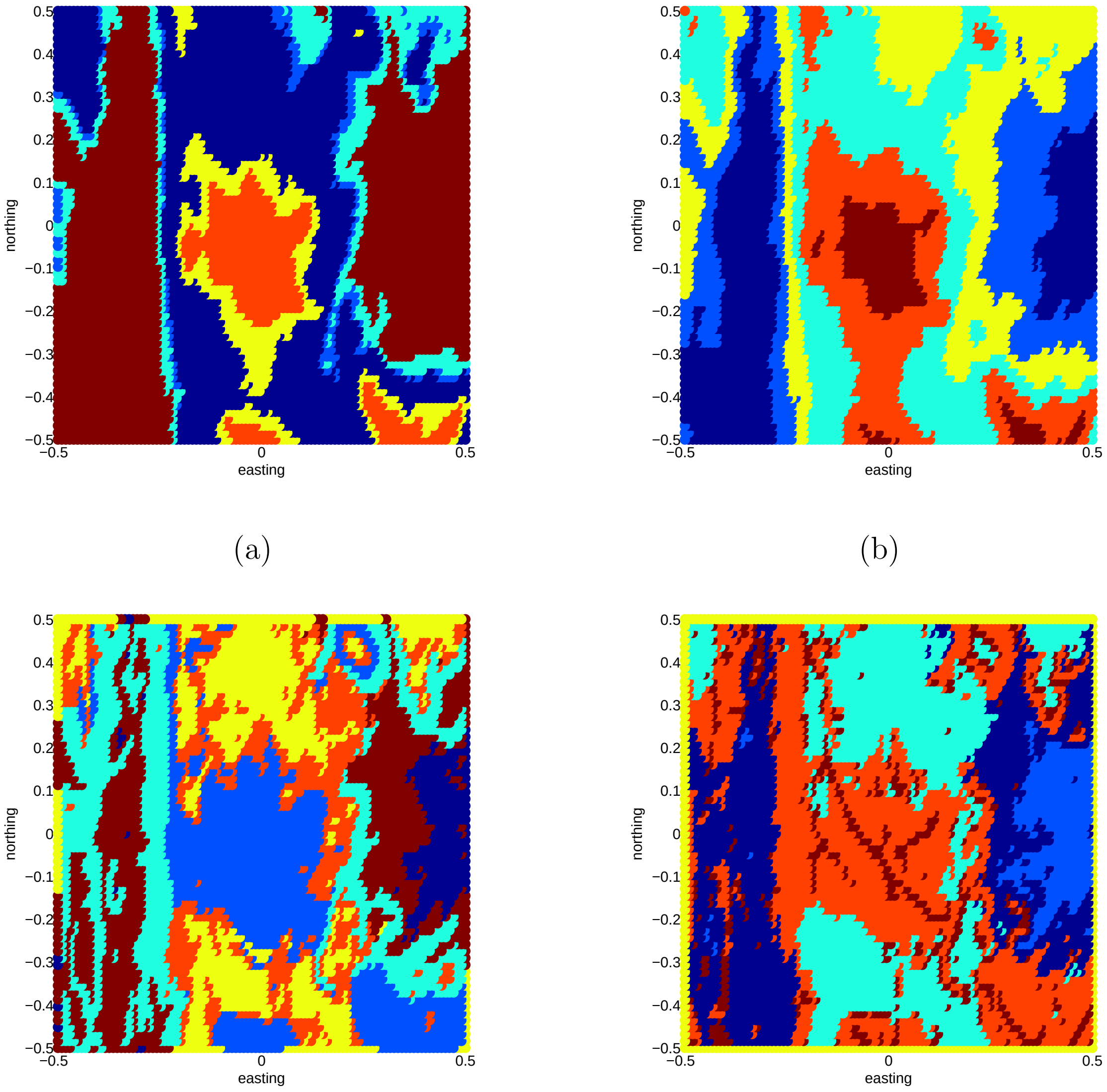
The problem is to find A and B such that $NCut(A,B)$ is minimized. The solution of this problem can be obtained from the Fiedler eigenvector [9].

3 Results and Conclusions

The data set available is a 5m DEM, covering an area of $\approx 94km^2$ which results in ≈ 3 millions data points. To speed up the computational time, the majority of the analysis was performed on a decimated DEM having a 120m resolution and 6550 data points.

The implementation of equation 11 it is still a work in progress. We succeeded in performing the classic EM Gaussian Mixture (EMGM) on the *feature matrix* $X = \{x_k | 1 \leq k \leq N\}$ with dimensionality d, where x_k is a $d \times 1$ matrix. In this case $d = 4$ and $N = 6550$. The feature were previously scaled such that their means were equal to zero. The initial parameters were chosen from the results of K-means results. The results are presented in Figure 3 (b) and are compared to the Matlab build-in function (Fig. 3 (a)). The results are quite different and we assume that using K-means for parameter initialization makes a significant difference in the clustering process. Also, adding more geomorphometric features results in differing assignments of data points to clusters.

Spectral Clustering was performed for $K = 6$ and $K = 20$ (Fig. 4 (a), (b)). We observe that classical EMGM clustering (Fig. 3 (b)) and Spectral Clustering (Fig. 4 (a)) give similar results which give us confidence of the *goodness of fit* for those methods, considering that we had not used any performance measures. Spectral Clustering requires setting a set of parameters, namely, the scaling parameters σ_F and σ_x. These parameters are very important



(a)

(b)

(c)

(d)

Figure 3: EM Gaussian Model clustering for a) K=6 using Matlab build in function for one feature (elevation) *gmdistribution* b) K=6 using an improved EMGM – one feature (elevation) c) K=6 using 2 features (elevation and slope) d) K=6 using 4 feature (elevation, slope, transversal, and profile curvature)

since they add more weigh to the features that we consider that are more important than the others. Appropriate setting of σ is crucial for obtaining good segmentation results in Spectral Clustering. Unfortunately, it is difficult to choose the appropriate σ value, and it is always set manually. The incorrect value of σ can degrade the performance as the Spectral Clustering is highly sensitive to σ, and different values of σ may lead to drastically different results (Fig. 4 (c), (d)). The proper setting of the scaling parameter still remains an open issue, and there is no known effective method.

Acknowledgment

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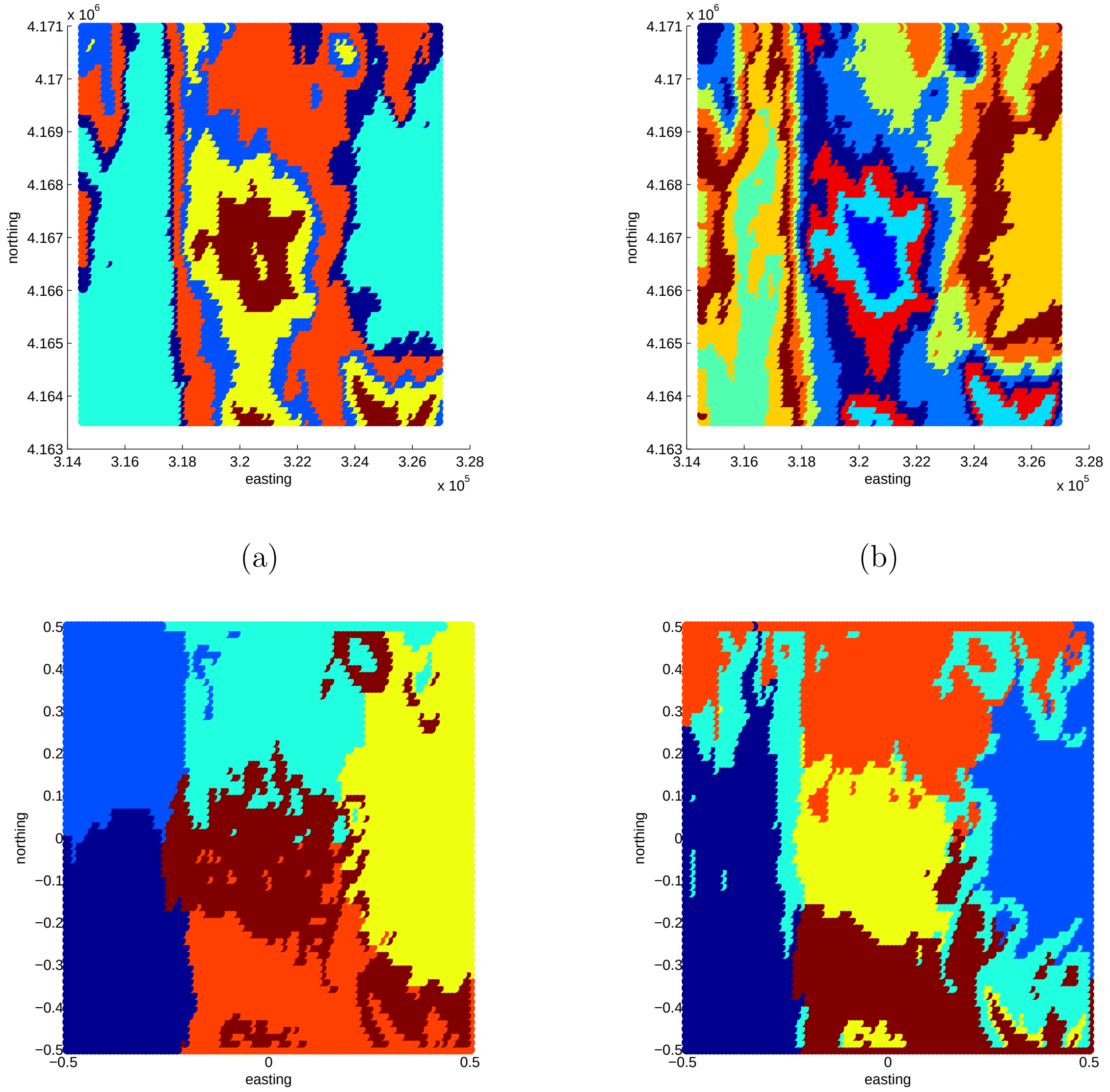

(a)

(b)

(c)

(d)

Figure 4: Spectral Clustering for a) K=6 – one feature b) K=20 – one feature c) K=6 where $\sigma_F = 0.8$ and $\sigma_x = 1$ – 4 feature d) K=6 $\sigma_F = 0.8$ and $\sigma_x = 0.8$ – 4 feature

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