# LLOYD-MAX QUANTIZER

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**Calculation of quantification level $\gamma$, where $f(x)$ is probability density function.**

$$\gamma = \frac{\int_{m_k}^{m_{k-1}} xf(x)dx}{\int_{m_k}^{m_{k-1}} f(x)dx} \tag{1}$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}} \tag{2}$$

**Numerator:**

$$\int_{m_k}^{m_{k-1}} xf(x)dx = \sqrt{\frac{\sigma^2}{2\pi}} [e^{-\frac{m_k^2}{2\sigma^2}} - e^{-\frac{m_{k-1}^2}{2\sigma^2}}] \tag{3}$$

Maths:

$$\int_{m_k}^{m_{k-1}} xf(x)dx = \int_{m_k}^{m_{k-1}} \frac{x}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx \tag{4}$$

$$g(x) = e^{-\frac{x^2}{2\sigma^2}} \tag{5}$$

$$\frac{dg(x)}{dx} = -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \tag{6}$$

$$\int_{m_k}^{m_{k-1}} xf(x)dx = -\sqrt{\frac{\sigma^2}{2\pi}} \int_{m_k}^{m_{k-1}} -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx = -\sqrt{\frac{\sigma^2}{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \Big|_{m_k}^{m_{k-1}} \tag{7}$$

**Denominator:**

$$\int_{m_k}^{m_{k-1}} f(x)dx = qfunc(\frac{m_k}{\sigma}) - qfunc(\frac{m_{k-1}}{\sigma}) \tag{8}$$

Maths:

$$\int_{m_k}^{m_{k-1}} f(x)dx = \int_{m_k}^{m_{k-1}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx = qfunc(\frac{m_k}{\sigma}) - qfunc(\frac{m_{k-1}}{\sigma}) \tag{9}$$

**Results:** $\gamma_k = \frac{\sqrt{\frac{\sigma^2}{2\pi}} [e^{-\frac{m_{k-1}^2}{2\sigma^2}} - e^{-\frac{m_k^2}{2\sigma^2}}]}{qfunc(\frac{m_k}{\sigma}) - qfunc(\frac{m_{k-1}}{\sigma})}$