A Novel Approach to Model Error Modeling using the Expectation-Maximization Algorithm

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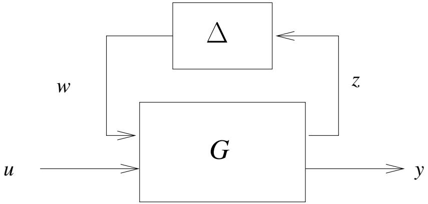
December 2012

Motivation

Robustness is one of the key ideas in control



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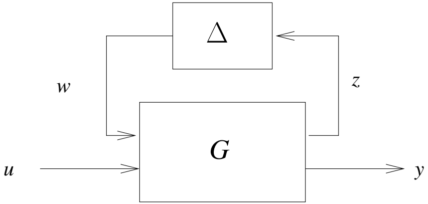

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Where does G and Δ come from?

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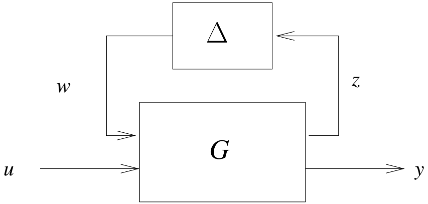

and the basis of robust control theory.

Where does G and Δ come from?
This talk addresses this problem.

Outline

- Explanation of model error description.
- Some previous approaches.
- Presentation of our approach to model error description.
- A numerical example.

Main contribution

Maximum Likelihood Estimate

Provide estimates for the nominal model and model error that correspond to the maximum likelihood estimate.

Model Error Description

In System Identification the aim is to provide a nominal model that tries to describe the dynamics of the true system.

$$y_t = G(q)u_t + v_t \qquad (1)$$

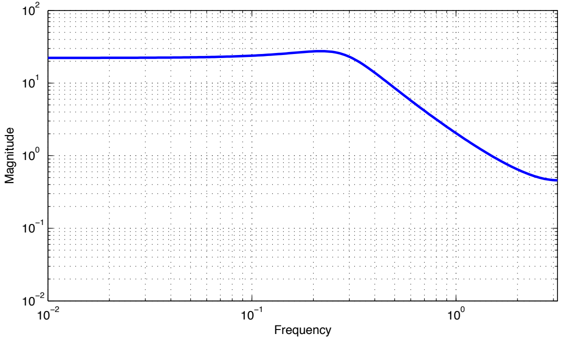

Figure: Magnitude Bode plot.

Model Error Description

However, a systematic error occurs when the true system is more complex than the nominal model.

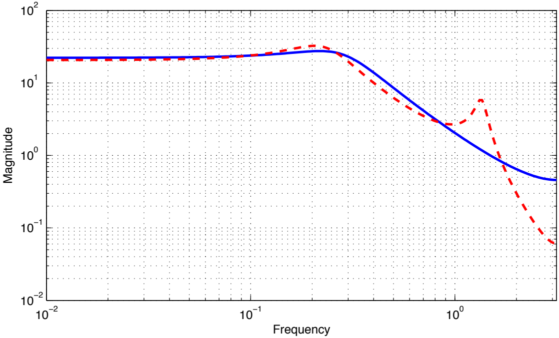

Figure: Magnitude Bode plot.

Model Error Description

Our aim is to provide a model error description corresponding to the nominal model.

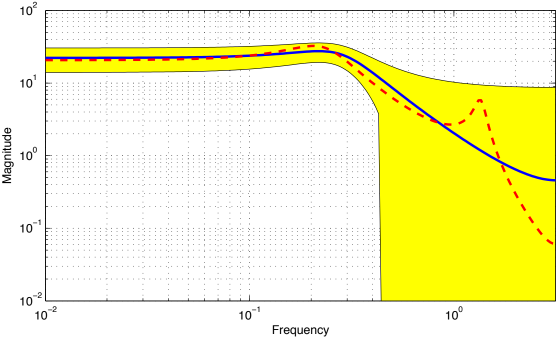

Figure: Magnitude Bode plot.

Some Previous Approaches

- Set Membership.
- Stochastic embedding.
- Model Error Modelling.
- and several others, as the based on finite sample properties, or in resampling, sub-sampling and/or bootstrapping techniques.

Previous Approach: Set Membership

Deal with the problem of *bounded but unknown errors*. Given a bound for the error, Set Membership delivers a Feasible System Set.

$$\text{Feasible System Set} = \{G \in \mathcal{H} : \|y - Gu\| < \delta\}$$

- Deterministic framework.
- Finding the solution set can be difficult depending on the model structure and the norm used to bound the error.
- The value of δ must be specified a priori.
- Focus into the “worst-case” model.

Previous Approach: Stochastic Embedding

The nominal model structure is embedded into a larger class of model, where the model error is characterised as a realization of a stochastic process.

$$\bar{G} = G \cdot (1 + G_{\Delta}) \tag{2}$$

- Probabilistic framework.
- Solved by using Least Squares.
- The nominal model is not estimated simultaneously with the model error.
- A Random Walk in the frequency domain was proposed as a description for the model error.

Previous Approach: Model Error Modelling

The nominal model and the model error description are obtained in two steps. In the first step the nominal model is obtained by using any standard tool. In the second step, the residues $\varepsilon = y - \hat{y}$ are used to obtain the model error description.

$$y_t = G(q)u_t + v_t \qquad (3)$$

$$\hat{y}_t = \hat{G}(q)u_t \qquad (4)$$

$$\varepsilon_t = y_t - \hat{y}_t \qquad (5)$$

$$\varepsilon_t = F(q)u_t + H(q)w_t \qquad (6)$$

- Probabilistic framework
- Allow the use of standard tools of system identification to estimate the nominal model, as well as the model error.
- The model error should have a complex structure.

Some issues with previous approaches

- In Set membership identification, the upper bound δ needs to be specified a priori. This could be challenging for some specific problems.
- In both probabilistic methods, i.e. Stochastic Embedding and Model Error Modelling, the estimated nominal model is not affected by the model error description.
- In Stochastic Embedding the nominal model complexity is limited by the use of Least Squares.

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Our contribution

Estimate both, the nominal model and the model error description simultaneously. Moreover, this estimate correspond to a Maximum Likelihood Estimate.

Proposed approach (PA)

The nominal model structure is embedded into a larger class of model. The estimation of the nominal model and the model for the error is made simultaneously.

- Probabilistic framework.
- Simultaneous estimation of the nominal model and the model error description.
- The complete estimated model (containing the nominal model and the model error description) correspond to the Maximum Likelihood Estimate.
- Allow more complex structures than Stochastic Embedding for the nominal model. (By using Expectation-Maximization algorithm.)

Model description example

Consider the following description of the model

$$G(j\omega_k) = G_o(j\omega_k)(1 + G_\Delta(j\omega_k)), \tag{7}$$

and the corresponding data generating system

$$Y(j\omega_k) = G(j\omega_k)U(j\omega_k) + V(j\omega_k), \tag{8}$$

where $\omega_k = \frac{2\pi k}{N}$, $Y(j\omega_k)$ and $U(j\omega_k)$ are the Discrete Fourier Transform of the measurement and the input signal, respectively. Moreover, $V(j\omega_k)$ is a realization of a zero mean proper Gaussian process and $G_\Delta(j\omega_k)$ is a realization of a Random Walk process in the frequency domain.

Comparison Example [2] (not in the paper)

Consider the following data generating system

$$y(t) = \frac{q^{-1} + 0.5q^{-2}}{1 - 2.2q^{-1} + 2.42q^{-2} - 1.87q^{-3} + 0.7225q^{-4}} u(t) + w(t) \quad (9)$$

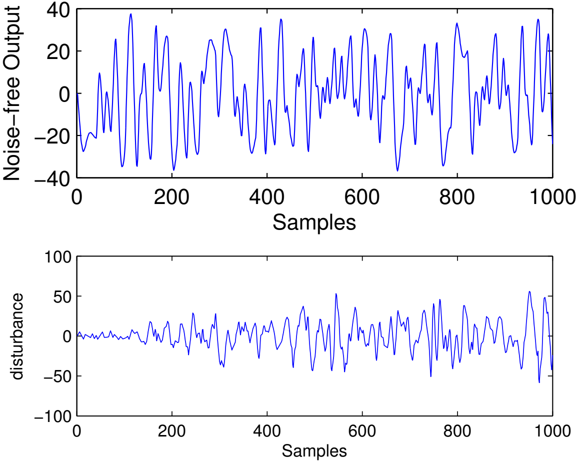
where $u(t)$ is a Pseudo Random Binary Signal (PRBS), and $w(t)$ is chosen to be one of the signal registered during the Loma Prieta earthquake, in October, 1989[1].

The nominal model to estimate is a second order Output Error model.

[1] This signal has been made available by The MathWorks Inc.

[2] Wolfgang Reinelt. i4c: Identification for control package (version 1.1b5). Linköping University, Linköping, Sweden, Sept. 2000.
`http://www.wolfgang-reinelt.de/i4c/`

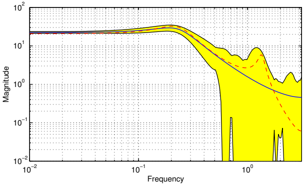
Comparison Example
Output composition



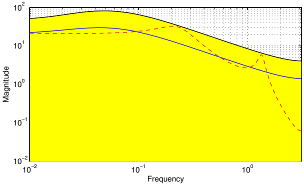
$SNR = 1$

Comparison Example

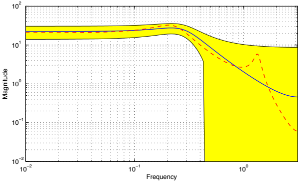
Results for 99% confidence



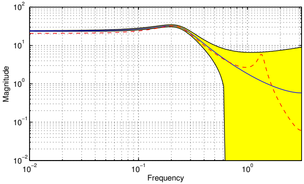
(a) Model Error Modeling



(b) Set Membership



(c) PA with constant variance undermodelling, with $N_{exp} = 1$.



(d) PA RW undermodelling with $N_{exp} = 25$.

Summary

- We develop a new method to provide a model error description.
- The developed method allow to estimate the nominal model and the model error description, simultaneously.
- The complete model (containing the nominal model and the model error description) correspond to the Maximum Likelihood Estimate.

Thanks for your attention!

Any questions?

Wolfgang Reinelt.
i4c: Identification for control package (version 1.1b5).
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Numerical example

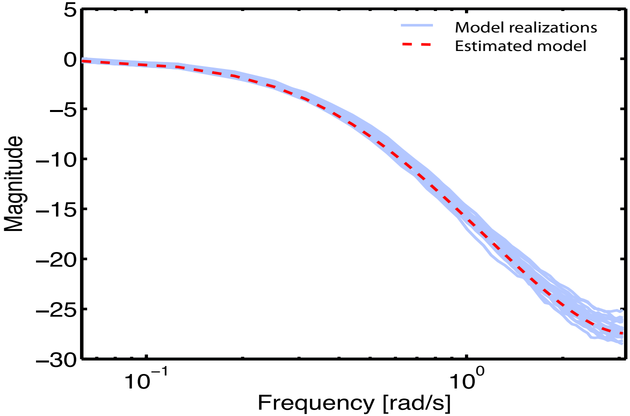
Consider the nominal model

$$G_o(z) = \frac{0.12}{z^2 - 1.3z + 0.42}, \tag{10}$$

and $G_\Delta(j\omega_k)$ having variance $\sigma_w^2 = 4 \cdot 10^{-4}$, and $V(j\omega_k)$ having variance $\sigma_v^2 = 0.1$.

We run $N_{exp} = 20$ different experiments each with $L = 50$ samples, with the same input signal $\{U_k\}_{k=0}^{L-1}$, generated as a proper Gaussian sequence with zero mean and variance $\sigma_u^2 = 4$.

On each experiment we have different realizations of the noise and the uncertainty.



The estimated model is given by
$G(\hat{\beta}, z) = 0.116/(z^2 - 1.307z + 0.423)$, $\hat{\sigma}_w^2 = 0.0033$, and $\hat{\sigma}_v^2 = 0.0997$.

Expectation-Maximization Algorithm

The complete data is composed by the measured data $\mathcal{Y}$ and also an unmeasured data $\mathcal{X}$, known as the *hidden variables*.

E-step

$$\mathcal{Q}(\theta, \hat{\theta}_i) = \mathbf{E}\{\log p(\mathcal{Y}, \mathcal{X}|\theta)|\mathcal{Y}, \hat{\theta}_i\}. \tag{11}$$

M-step

$$\hat{\theta}_{i+1} = \arg\max_{\theta\in\Omega} \mathcal{Q}(\theta, \hat{\theta}_i). \tag{12}$$

Random walk realizations