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Two-degree-of-freedom anti-aliasing technique for wide-band networked control

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Abstract: In this paper we propose a new anti-aliasing technique for wide band networked control schemes. We study the potential equivalence with the usual one-degree-of-freedom anti-aliasing filtering. The benefits of the proposed anti-aliasing approach are illustrated with a simple example that incorporates an MPC controller in a Networked Control framework.

1. INTRODUCTION

Physical systems typically evolve continuously whereas modern controllers and signal processing devices invariably operate in discrete time (see e.g. Goodwin et al. (2010a,b)). Hence, it is necessary to have a device that transforms samples into continuous time signals (hold) and a device that takes samples of the relevant signals in the process to be controlled (see Figure 1). Conversion to sampled data form implicitly involves folding of high frequency system characteristics back into the lower frequency range. This effect is known as *aliasing* and means that different sampled signals may become indistinguishable. Aliasing is particularly problematic when wide-band noise is present. In order to avoid this problem an *Anti aliasing filter* (AAF) is used in analog to digital converters. The purpose of the AAF is to restrict the bandwidth of the signals such that the conditions in the *sampling theorem* are satisfied (Nyquist (1928)).

Recently, in Błachuta and Grygiel (2009a,b, 2010) the impact of the use of an AAF on the control loop performance is studied. It is argued, it is desirable to have an ideal AAF in order to avoid aliasing. However, an AAF also introduces a phase shift in the signal path (Goodwin et al. (2010b, 2001)). Thus, the performance and stability of the control loop are affected by incorporating an AAF.

In this paper, we study a new anti-aliasing filtering technique initially proposed in Goodwin et al. (2009, 2010b) called *Two-Degree-of-Freedom Anti-aliasing filter* (2-DOF-AAF). The proposed scheme is shown in Figure 3. This anti-aliasing filtering technique generalizes the usual One-Degree-of-Freedom Anti-aliasing filter (1-DOF-AAF). Choosing the filter $F_1 = 0$, we obtain the 1-DOF-AAF. Moreover, in the control loop in Figure 3, the controller can be modified such that, in some cases, both schemes are equivalent.

In this paper we show that the 2-DOF-AAF provides better performance than the 1-DOF-AAF. The loop (using 1-DOF-AAF) can even become unstable if a high order filter is used as an AAF due to the phase shift introduced in the loop. On the other hand, if a low order filter is used in the 1-DOF-AAF, then the impact of output disturbances with high frequency content is increased. In the 2-DOF-AAF architecture (Figure 3) one can be utilize an AAF of high order without affecting the closed loop performance. In addition, one can design the feedback law without taking into account the particular AAF utilised.

The layout of the paper is as follows: In Section 2, we present some technical preliminaries. In Section 3, we analyse the signals in the 2-DOF-AAF architecture. In Section 4, we study the potential equivalence between the 2-DOF-AAF and 1-DOF-AAF architectures. Section 5 shows the implication of the results in the context of Networked Control Systems (NCS). In section, 6 we present a simple example. Finally, we present conclusions in Section 7.

2. TECHNICAL PRELIMINARIES

In order to study the closed loop in Figure 3, it is necessary to mix continuous and discrete time signals and systems. This combination of a digital controller and continuous plant is commonly known as *hybrid control* [Goodwin and Salgado (1994); Feuer and Goodwin (1996); Goodwin et al. (2001)].

We introduce the operator $[S(j\omega)]_q$ to denote the folded version of $S(j\omega)$ (Oppenheim and Schafer (1989)), i.e.,

$$[S(j\omega)]_q = \sum_{k=-\infty}^{\infty} S\left(j\omega + j\frac{2\pi}{\Delta}k\right) \qquad (1)$$

where $S(j\omega)$ can be, either, the Fourier transform of a continuous-time signal, or the Fourier transform of the impulse response of a continuous-time linear system.

We next present three well known results that can be used to analyse the closed loop in Figure 3. We assume a zero-order hold for simplicity.

Fact 1. The continuous and discrete time signals in Figure 2 satisfy the following identity:

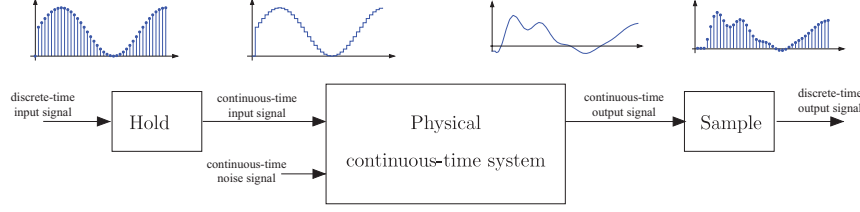$$U_c(j\omega) = H(j\omega)U(e^{j\omega\Delta}) \qquad (2)$$

Fig. 1. Sampled system.

where $H(s) = \frac{1-e^{-s\Delta}}{s\Delta}$ is the transfer function of a zero or-der hold, and $U_c(j\omega) = \int_{-\infty}^{\infty} u_c(t)e^{-j\omega t}dt$ and $U(e^{j\omega\Delta}) = \Delta\sum_{k=-\infty}^{\infty} u_k e^{-j\omega k\Delta}$ are the continuous-time and (scaled) discrete-time Fourier transforms of $u_c(t)$ and u_k respectively.

Proof. See e.g. Feuer and Goodwin (1996). □

Fact 2. The Fourier transform of the discrete time signals in Figure 2 satisfy the following identity:

$$Y(e^{j\omega\Delta}) = [FH]_q U(e^{j\omega\Delta}) \tag{3}$$

where $H(s) = \frac{1-e^{-s\Delta}}{s\Delta}$ is a zero-order hold, and $Y(e^{j\omega\Delta})$, and $U(e^{j\omega\Delta})$ are the (scaled) discrete time Fourier transforms given by $Y(e^{j\omega\Delta}) = \Delta\sum_{k=-\infty}^{\infty} y_k e^{-j\omega k\Delta}$, and

$$U(e^{j\omega\Delta}) = \Delta\sum_{k=-\infty}^{\infty} u_k e^{-j\omega k\Delta}.$$

Proof. See e.g. Goodwin et al. (2010b). □

Fact 3. The continuous and discrete time signals in Figure 2 satisfy the following identity:

$$Y(e^{j\omega\Delta}) = [F(j\omega)U_c(j\omega)]_q \tag{4}$$

where $U_c(j\omega) = \int_{-\infty}^{\infty} u_c(t)e^{-j\omega t}dt$, and $Y(e^{j\omega\Delta}) = \Delta\sum_{k=-\infty}^{\infty} y_k e^{-j\omega k\Delta}$.

Proof. See e.g. Oppenheim and Schafer (1989). □

Remark 1. The above facts can be readily extended to any hold device with Kronecker delta response $H(s)$ (Feuer and Goodwin (1994); Yuz et al. (2004)).▽▽▽
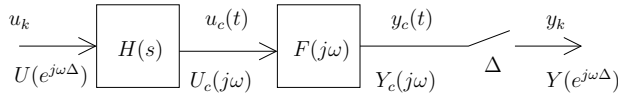

Fig. 2. Signals in a sampling process.

3. A 2-DOF-AAF

We next analyse the signals in the closed loop of Figure 3.

Proposition 4. The Fourier transform of the input and output of the plant in Figure 3 satisfy the following identities:

$$U = \frac{H}{1 + C_q[(F_2G + F_1)H]_q} \cdot \left\{ C_q\big(R_k - [F_2(D_m + D_0)]_q\big) + D_i \right\} \tag{5}$$

$$Y = D_0 + \frac{GH}{1 + C_q[(F_2G + F_1)H]_q} \cdot \left\{ C_q\big(R_k - [F_2(D_m + D_0)]_q\big) + D_i \right\} \tag{6}$$

Proof. Using facts 1 and 2 we have that

$$Y = D_0 + GU \tag{7}$$

$$U = H(D_i + U^d) \tag{8}$$

$$U^d = C_q\left\{ R - \left([F_2(D_m + D_0)]_q \right.\right. \\ \left.\left. + [F_2(GH(D_i + U^d))]_q + [F_1H(D_i + U^d)] \right) \right\} \tag{9}$$

Then, solving for U^d in (9), and using (7) and (8) we obtain the result. □

In Goodwin et al. (2009) the filters F_1, and F_2 are chosen such that the following condition holds:

$$F_1 + F_2G = G \tag{10}$$

The latter condition on the filters F_1 and F_2 appears in the literature as a tool to design *unbiased estimates* (Serón et al. (1997)). A general procedure to design F_1 and F_2 for a given plant G is presented in the Appendix.

Remark 2. Notice that, when one uses a 2-DOF technique where the filters satisfy (10), it is not necessary to consider the effect of anti-aliasing filters to design the controller.▽▽▽

Remark 3. Since anti-aliasing filters are not ideal, aliasing occurs (specially close to the Nyquist frequency). Moreover, the tradeoff between avoiding folding and affecting the signal path suggests the use of a low order anti-aliasing filter. On the other hand, it is possible to use a higher order anti-aliasing filter in the 2-DOF AAF architecture without affecting the signal path. This is potentially of importance in wide-band control.▽▽▽

4. EQUIVALENCE BETWEEN 1-DOF-AAF AND 2-DOF-AAF ARCHITECTURES

In this section we analyse the potential equivalence between the 2-DOF-AAF and the usual 1-DOF-AAF when the system is operating in closed loop.

We first define when two control loop architectures are considered equivalent.
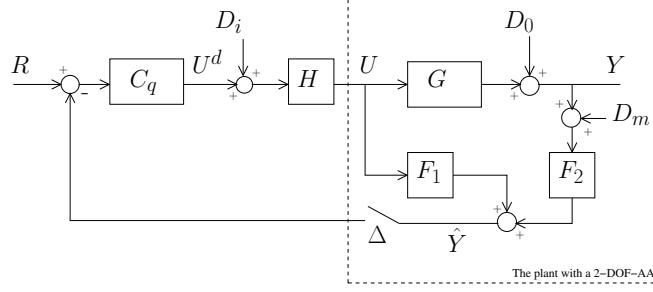
Fig. 3. Closed Loop with 2 degree of freedom anti-aliasing filter.

Definition 1. Two control architectures are equivalent if and only if, for given disturbances D_i, D_0, D_m, and any reference signal R (see Figure 3), the Fourier transform of the continuous time input and output of the plant are the same for all frequencies.$\triangledown\triangledown\triangledown$

We then have

Theorem 5. For the control architecture (shown in Figure 3) using a 2-DOF-AAF, where the controller is linear, we have that:

(1) If the input disturbance D_i is identically zero ($D_i(e^{j\omega\Delta}) = 0, \forall \omega \in [-\frac{\pi}{\Delta}, \frac{\pi}{\Delta}]$), then there exists a 1-DOF-AAF control architecture ($F_1 = 0$) such that both the 1-DOF and 2-DOF AAF are equivalent. A particular case satisfying the above condition occurs when the controller in the 1-DOF-AAF control is given by:

$$C'_q = \frac{C_q}{1 + C_q[F_1 H]_q} \quad (11)$$

and the AAF in the 1-DOF is equal to the filter F_2 used in the 2-DOF-AAF control architecture.

(2) If the input disturbance D_i is not identically zero (i.e. there exists a $\omega_0 \in [-\frac{\pi}{\Delta}, \frac{\pi}{\Delta}]$ such that $D_i(e^{j\omega_0\Delta}) \neq 0$), then there does not exist a 1-DOF-AAF control architecture that is equivalent to the 2-DOF-AAF architecture.

Proof.

(1) If D_i is identically equal to zero, and the controller is linear, then using Proposition 4 we have that the plant input is given by:

$$U = \frac{HC_q}{1 + C_q[H(F_1 + F_2 G)]_q} Z \quad (12)$$

$$Z = R - [F_2(D_m + D_0)]_q \quad (13)$$

then, considering the relationship (11), we have that

$$\frac{HC_q}{1 + C_q[(F_2 G + F_1)H]_q} = \frac{HC'_q}{1 + C'_q[F_2 GH]_q} \quad (14)$$

Then, noting that Z, in (13), only depends on the filter F_2, we obtain the result.

(2) In this case we have that the input signal is given by

$$U = \frac{H}{1 + C_q[H(F_1 + F_2 G)]_q} [C_q Z + D_i] \quad (15)$$

$$Z = R - [F_2(D_m + D_0)]_q \quad (16)$$

We use contradiction. We consider a 2-DOF-AAF architecture with $F_1 \neq 0$, $F_2 \neq 0$, and 1-DOF-AAF given by F'_2, and the corresponding controller C'_q. We

have that the two control architectures are equivalent if and only if

$$\frac{HC_q Z}{1 + C_q[H(F_1 + F_2 G)]_q} = \frac{HC'_q Z'}{1 + C_q[H(F'_2 G)]_q} \quad (17)$$

$$\frac{H}{1 + C_q[H(F_1 + F_2 G)]_q} = \frac{H}{1 + C_q[H(F'_2 G)]_q} \quad (18)$$

Then, using (18) in (17) we have that $ZC_q = C'_q Z'$ where $Z' = R - [F'_2(D_m + D_0)]_q$. Then, considering the part of Z and Z' that depends only on the reference signal, we have that $C_q = C'_q$. Then, considering only the part of Z and Z' that depends on the disturbance signals, we have that $[F_2(D_m + D_0)]_q = [F'_2(D_m + D_0)]_q$. However, then using (18) we have that $F_1 = 0$. This contradicts the initial assumption, and completes the proof.$\triangledown\triangledown\triangledown$

5. IMPLICATIONS FOR NETWORKED CONTROL SYSTEMS

Standard control theory (see e.g. Goodwin et al. (2001)) assumes perfect communication between plant and controllers. However, advances in communication technology have motivated the use of general purpose communication networks in control (see e.g. Nair et al. (2007); Schenato et al. (2007); Hespanha et al. (2007); Goodwin et al. (2008); Silva et al. (2010)). Figure 4 shows a Networked Control System (NCS) where the input commands and the measurements used by the controller are transmitted through a network.
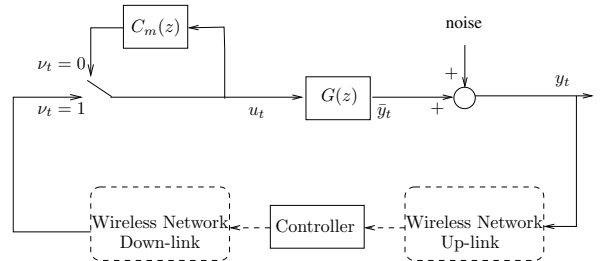

Fig. 4. Networked Control System architecture.

In an NCS framework, new challenges arise. For example, typical communication links are subject to data-rate limits, are prone to data loss, and may experience random delays. In Silva et al. (2010), a simple approach has been studied where the wireless network is replaced by an additive noise source where the associated variance appears as a degree of freedom in the design.

Hence, in the case that one is designing a controller for a continuous-time plant in an NCS framework it is relevant to consider the case $D_i \neq 0$. In this situation, the proposed 2-DOF-AAF cannot be transformed to the usual 1-DOF-AAF architecture (see Theorem 5).

6. NUMERICAL EXAMPLE

In this section we present a simple example to illustrate the advantages of using the 2-DOF-AAF presented in this paper. Consider the following plant:

$$G(s) = \frac{1}{s+1} \quad (19)$$

We use the control architecture of Figure 3, with a zero-order-hold (ZOH). The sampling frequency is chosen as $\omega_s = 20[rad/s]$ which corresponds to a Nyquist frequency $\omega_N = 10[rad/s]$. We study the following scenarios:

Scenario I The controller is linear and all disturbances and noise are zero, i.e. $d_i(k\Delta) = 0$, $d_0(t) = 0$, and $d_m(t) = 0$.

Scenario II The controller is linear and the disturbances and noise are Gaussian sequences having zero mean and variance 0.0001 for both $d_i(k\Delta)$, and $d_0(t)$, and variance 0.05 for $d_m(t)$.

Scenario III The controller is linear and we consider that data loss occurs. We consider a sinusoidal output disturbance with frequency $2.3*\omega_N$ and amplitude $1/3$. We use the data loss process described in Figure 4 where the data loss process in the measurement, ν_t, satisfies a Bernoulli distribution with probability $p = 0.1171$.

Scenario IV We use a non-linear MPC controller where all disturbances and noise are zero, i.e. $d_i(k\Delta) = 0$, $d_0(t) = 0$, and $d_m(t) = 0$.

Scenario V We use a non-linear MPC controller where the disturbances and noise are Gaussian sequences having zero mean and variance 0.0001 for both $d_i(k\Delta)$, and $d_0(t)$, and variance 0.05 for $d_m(t)$.

Scenario VI We use a nonlinear MPC controller and the output disturbance is a sinusoidal signal with frequency $2.3*\omega_N$ and amplitude $1/3$. We use the data loss process in Figure 4 where the data loss process, ν_t, satisfies a Bernoulli distribution with probability $p = 0.1171$.

The actuator uses a hold-input strategy to compensate the data loss, i.e., in Figure 4 $C_m(z) = z^{-1}$ (Schenato et al. (2007)).

For scenarios **I**, **II**, and **III**, we design a linear controller for the discretized plant $G_d(z)$ using the command `lqg` of MATLAB, with weighting matrices $Q_{xu} = 10^{-5}I_2$, $Q_{wv} = diag(0.0001, 0.0001, 0.05)$, $Q_i = 1$. This controller is denoted as $C_q(z)$.

For scenarios **IV**, **V**, and **VI**, we design an MPC controller by using the command `mpc(Gd,h)` where h is the sampling rate. The weights that we use are: 10^{-5} for the control signal rate, and the default values of the command for the rest, i.e., 0 for the magnitude of the control signal, and 1 for the output. We use the following constraint in the control signal: $-1 < u_k$.

For the 1-DOF-AAF we use a first order Butterworth filter, $B_1(s)$, with cut-off frequency $\omega_c = \omega_N$. We notice that the loop becomes unstable if higher order AAFs are used.
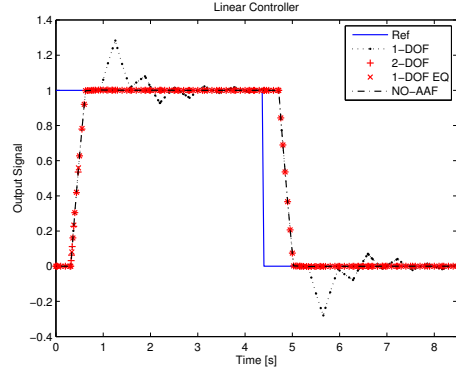

Fig. 5. Simulation for scenario **I** (Linear controller, Noise free, and without data loss).

For the 2-DOF-AAF we use a fifth order Butterworth filter, $B_5(s)$, with cut-off frequency $\omega_c = \omega_N$ for F_2 and we choose F_1 to satisfy (10). For every scenario we show the impact of the following AAF configurations:

NO-AAF: No AAF is used, i.e. $F_1(s) = 0$, and $F_2(s) = 1$.
1-DOF: $F_1(s) = 0$, and $F_2(s) = B_1(s)$.
2-DOF: We consider $F_1(s)$ such that $F_1(s) + G(s)F_2(s) = G(s)$, and $F_2(s) = B_5(s)$.
1-DOF EQ: $F_1(s) = 0$, $F_2(s) = B_5(s)$, and we use a controller C_q' that satisfies (11).

Remark 4. Notice that MPC is typically used in NCS framework (see, e.g. Yu et al. (2004); Liu et al. (2006); Jing et al. (2007); Chen et al. (2010); Pin et al. (2009)). In all the above papers, no anti-aliasing filter is considered in the design of the control system. ▽▽▽

Table 1 shows the value of the ratio $\frac{\rho}{\rho_{2DOF}}$, where $\rho = \frac{1}{T}\int_0^T |z(t) - y(t)|dt$, ρ_{2DOF} is the value of ρ for the **2-DOF** case, $z(t)$ is the reference signal passed through a ZOH, and $T = 2638[s]$ is the simulation time.

Figure 5 shows the results for all AAF configurations for scenario **I**. We see that **2-DOF**, **1-DOF EQ**, and **NO-AAF** cases provide good performance. However, for the **1-DOF** case, the control loop shows poor performance.

The simulation results for scenario **II** for all the AAF cases are shown in Figure 6. The **2-DOF** and **1-DOF EQ** have the same performance (see Table 1). The **NO-AAF** case has poor performance compared to the **2-DOF** and **1-DOF** case. The **2-DOF** case shows the best performance.

Figure 7 shows the results for scenario **III**. We see that the **2-DOF** case shows the best performance. In Table 1 we notice that, for scenario **III**, the **NO-AAF** shows the worst performance.

Figure 8 shows the simulation results for all AAF configurations (except for **1-DOF EQ**), considering scenario **IV**. The **2-DOF** and **NO-AAF** cases have the same behaviour (see Table 1). The performance of the **2-DOF** case is better than the performance for the **1-DOF** case.

Simulation results for scenario **V** are shown in Figure 9. The performance of the **1-DOF** case is worse than the performance for the **NO-AAF**. The **2-DOF** gives the best performance.

	1-DOF	2-DOF	1-DOF EQ	NO-AAF
I (Linear controller, Noise free, and without data loss)	1.3157	1	1	1
II (Linear controller, with noise, and without data loss)	1.1313	1	1	1.3033
III (Linear controller, only a sinusoidal disturbance, and with data loss)	1.3199	1	-	1.4696
IV (MPC controller, Noise free, and without data loss)	1.2060	1	-	1
V (MPC controller, with noise, and without data loss)	1.0846	1	-	1.2032
VI (MPC controller, only a sinusoidal disturbance, and with data loss)	1.1171	1	-	1.2009

Table 1. Integral of the absolute tracking error over the simulation time ratio, $\frac{\rho}{\rho_{2DOF}}$, for all the scenarios and AAF configurations.
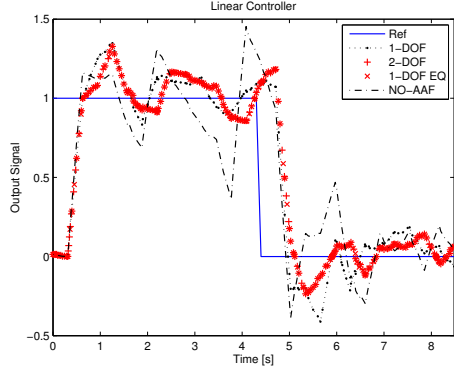

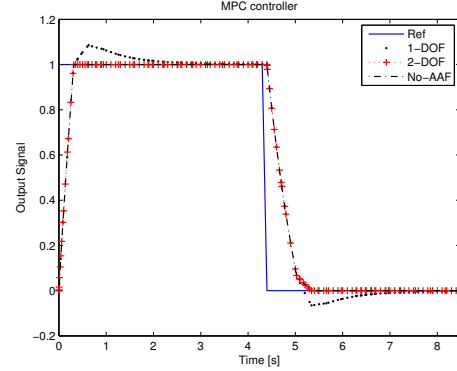Fig. 6. Simulation for scenario **II** (Linear controller, with noise, and without data loss).

Fig. 7. Simulation for scenario **III** (Linear controller, only sinusoidal disturbance, and with data loss).

Figure 10 shows the simulation results considering scenario **VI**. The **2-DOF** gives the best performance. Table 1 shows that for scenario **VI**, the **NO-AAF** gives the worst performance.

7. CONCLUSIONS

In this paper we have studied a new anti-aliasing technique with a two-degree-of-freedom architecture. The new AAF technique reduces the effect of the phase shift without affecting the signal path. This means that the control design procedure becomes independent of sampling artifacts.

We have analysed the potential equivalence between the new AAF and the usual one-degree-of-freedom anti-aliasing scheme. We have illustrated the potential of the proposed technique with a simple example using a linear controller and an MPC strategy in an NCS framework.

Fig. 8. Simulation for scenario **IV** (MPC controller, Noise free, and without data loss).

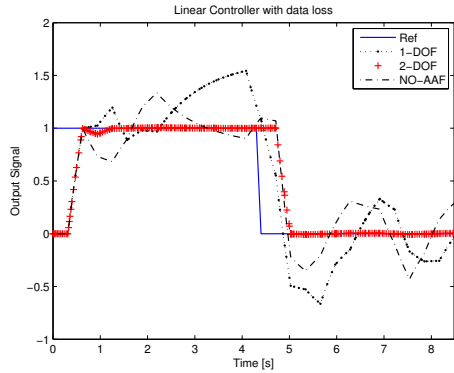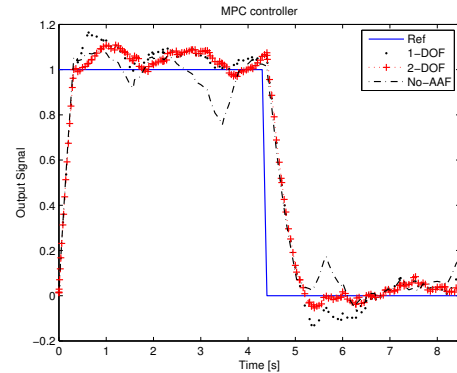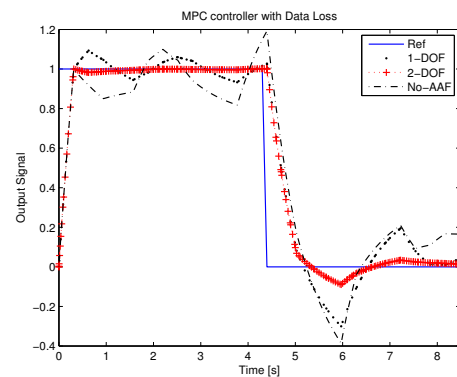Fig. 9. Simulation for scenario **V** (MPC controller, with noise, and without data loss).

Fig. 10. Simulation for scenario **VI** (MPC controller, only a sinusoidal disturbance, and with data loss).

Appendix A. UNBIASED ESTIMATES

There exists a general class of 2-DOF filters which yield the following key property (see Figure 3): The transfer function from $u(t)$ to $\hat{y}(t)$ is the same as that from $u(t)$ to $y(t)$. This class of filters can be analyzed, in the Fourier transform domain, as follows:

$$\hat{Y}(j\omega) = F_1(j\omega)U(j\omega) + F_2(j\omega)Y(j\omega) \qquad (A.1)$$

Clearly, the transfer function from $u(t)$ to $\hat{y}(t)$ is given by:

$$\frac{\hat{Y}(j\omega)}{U(j\omega)} = F_1(j\omega) + F_2(j\omega)G(j\omega) \qquad (A.2)$$

where $G(j\omega)$ is the plant transfer function. We say that the 2-DOF filter is **unbiased** if this transfer function is the same as the plant transfer function. Hence, the condition for unbiasedness is:

$$F_1(j\omega) + F_2(j\omega)G(j\omega) = G(j\omega) \qquad (A.3)$$

All linear time-invariant observers have the above property as we show below:

Lemma 6. Consider a plant represented by the state-space model:

$$\dot{x} = Ax + Bu \qquad (A.4)$$
$$y = Cx \qquad (A.5)$$

and the class of (full order) observers given by:

$$\dot{\hat{x}} = A\hat{x} + Bu + J(y - \hat{y}) \qquad (A.6)$$
$$\hat{y} = C\hat{x} \qquad (A.7)$$

Then, all such observers are unbiased

Proof. The proof can be found, for example, in Goodwin et al. (2001); Serón et al. (1997)

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