Two-degree-of-freedom anti-aliasing technique for wide-band networked control

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Abstract

We propose a new anti-aliasing technique for wide band networked control schemes. We study the potential equivalence with the usual one-degree-of-freedom anti-aliasing filtering. The benefits of the proposed anti-aliasing approach are illustrated with a simple example that incorporates an MPC controller in a Networked Control framework.

Introduction

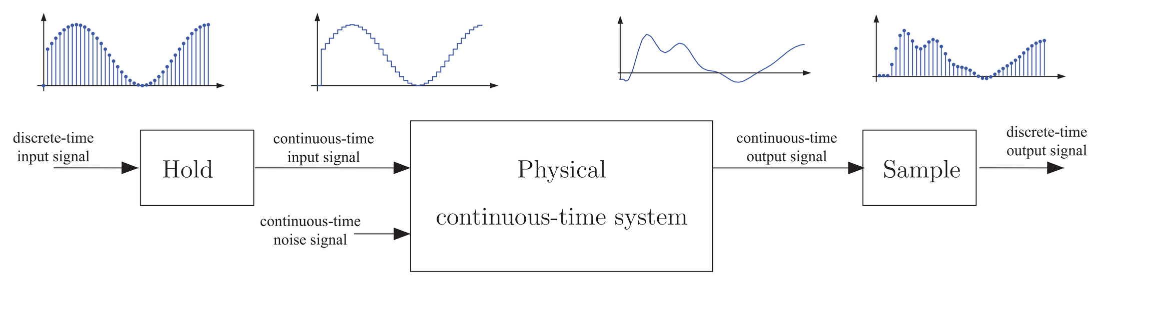


Figure 1: Sampled system.

- Modern controllers and signal processing devices invariably operate in discrete time.
- An *Anti-aliasing filter* is used in analog to digital converters to avoid *aliasing*.
- An Anti-aliasing filter also introduces a phase shift in the signal path. Thus, the performance and stability of the control loop are affected by incorporating an Anti-Aliasing Filter [1].
- We study a new anti-aliasing filtering technique called *Two-Degree-of-Freedom Anti-aliasing filter* (2-DOF-AAF) [2] .
- The 2-DOF-AAF in Figure 2 generalize the usual One-Degree-Of-Freedom Anti-Aliasing Filter (1-DOF-AAF). Note that if $F_1 = 0$ we obtain 1-DOF-AAF.
- The controller can be modified such that, in some cases, both schemes are equivalent.
- The 2-DOF-AAF allow design the feedback law without taking into account the particular Anti-Aliasing Filter utilised.

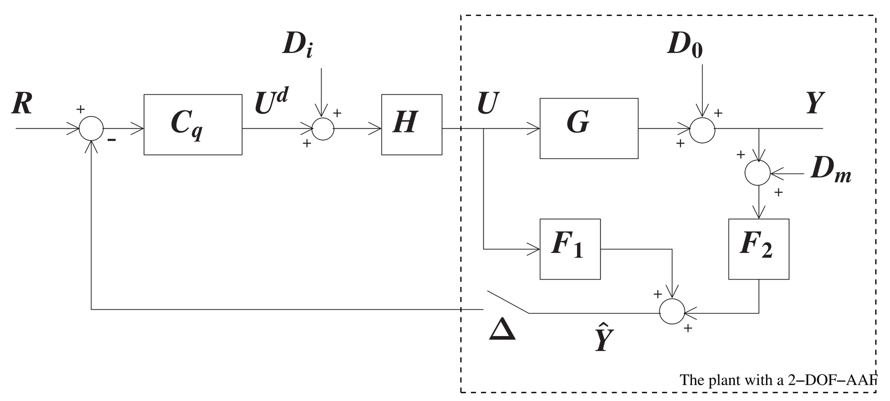


Figure 2: Closed Loop with 2 degree of freedom anti-aliasing filter.

Technical Preliminaries

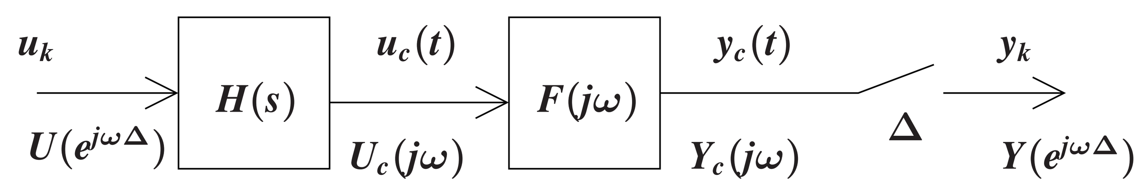


Figure 3: Signals in a sampling process.

We introduce the operator $[S(j\omega)]_q$ to denote the folded version of $S(j\omega)$, i.e.,

$$[S(j\omega)]_q = \sum_{k=-\infty}^{\infty} S\left(j\omega + j\frac{2\pi}{\Delta}k\right) \qquad (1)$$

where $S(j\omega)$ can be, either, the Fourier transform of a continuous-time signal, or the Fourier transform of the impulse response of a continuous-time linear system.
The continuous and discrete time signals in Figure 3 satisfy:

$$U_c(j\omega) = H(j\omega)U(e^{j\omega\Delta}) \qquad (2)$$
$$Y(e^{j\omega\Delta}) = [FH]_q U(e^{j\omega\Delta}) \qquad (3)$$
$$Y(e^{j\omega\Delta}) = [F(j\omega)U_c(j\omega)]_q \qquad (4)$$

A 2-DOF-AAF

The Fourier transform of the input and output of the plant in Figure 2 satisfy the following identities:

$$U = \frac{H}{1+C_q[(F_2G+F_1)H]_q} \cdot \left\{ C_q(R - [F_2(D_m + D_0)]_q) + D_i \right\} \qquad (5)$$

$$Y = D_0 + \frac{GH}{1+C_q[(F_2G+F_1)H]_q} \cdot \left\{ C_q(R - [F_2(D_m + D_0)]_q) + D_i \right\} \qquad (6)$$

$$Y = D_0 + GU \qquad (7)$$
$$U = H(D_i + U^d) \qquad (8)$$

$$U^d = C_q\left\{ R - \left([F_2(D_m + D_0)]_q + [F_2(GH(D_i + U^d))]_q + [F_1 H(D_i + U^d)] \right) \right\} \qquad (9)$$

In [2] the filters F_1, and F_2 are chosen such that the following condition holds:

$$F_1 + F_2 G = G \qquad (10)$$

The latter condition on the filters F_1 and F_2, allow design the feedback law without taking into account the particular Anti-Aliasing Filter utilised.

Equivalence between 1-DOF-AAF and 2-DOF-AAF architectures

For the control architecture (shown in Figure 2) using a 2-DOF-AAF, where the controller is linear, we have that:

1. If the input disturbance D_i is identically zero ($D_i(e^{j\omega\Delta}) = 0, \forall \omega \in [-\frac{\pi}{\Delta}, \frac{\pi}{\Delta}]$), then there exists a 1-DOF-AAF control architecture ($F_1 = 0$) such that both the 1-DOF and 2-DOF AAF are equivalent. A particular case satisfying the above condition occurs when the controller in the 1-DOF-AAF control is given by:
$$C_q' = \frac{C_q}{1 + C_q[F_1 H]_q} \qquad (11)$$
and the AAF in the 1-DOF is equal to the filter F_2 used in the 2-DOF-AAF control architecture.
2. If the input disturbance D_i is not identically zero (i.e. there exists a $\omega_0 \in [-\frac{\pi}{\Delta}, \frac{\pi}{\Delta}]$ such that $D_i(e^{j\omega_0\Delta}) \neq 0$), then there does not exist a 1-DOF-AAF control architecture that is equivalent to the 2-DOF-AAF architecture.

Implications for Networked Control Systems

Advances in communication technology have motivated the use of general purpose communication networks in control (see e.g. [3]). Figure 4 shows a Networked Control System (NCS).

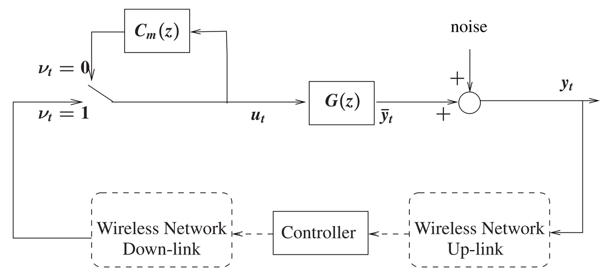


Figure 4: Networked Control System architecture.

In [3], a simple approach has been studied where the wireless network is replaced by an additive noise source where the associated variance appears as a degree of freedom in the design.

Numerical Example

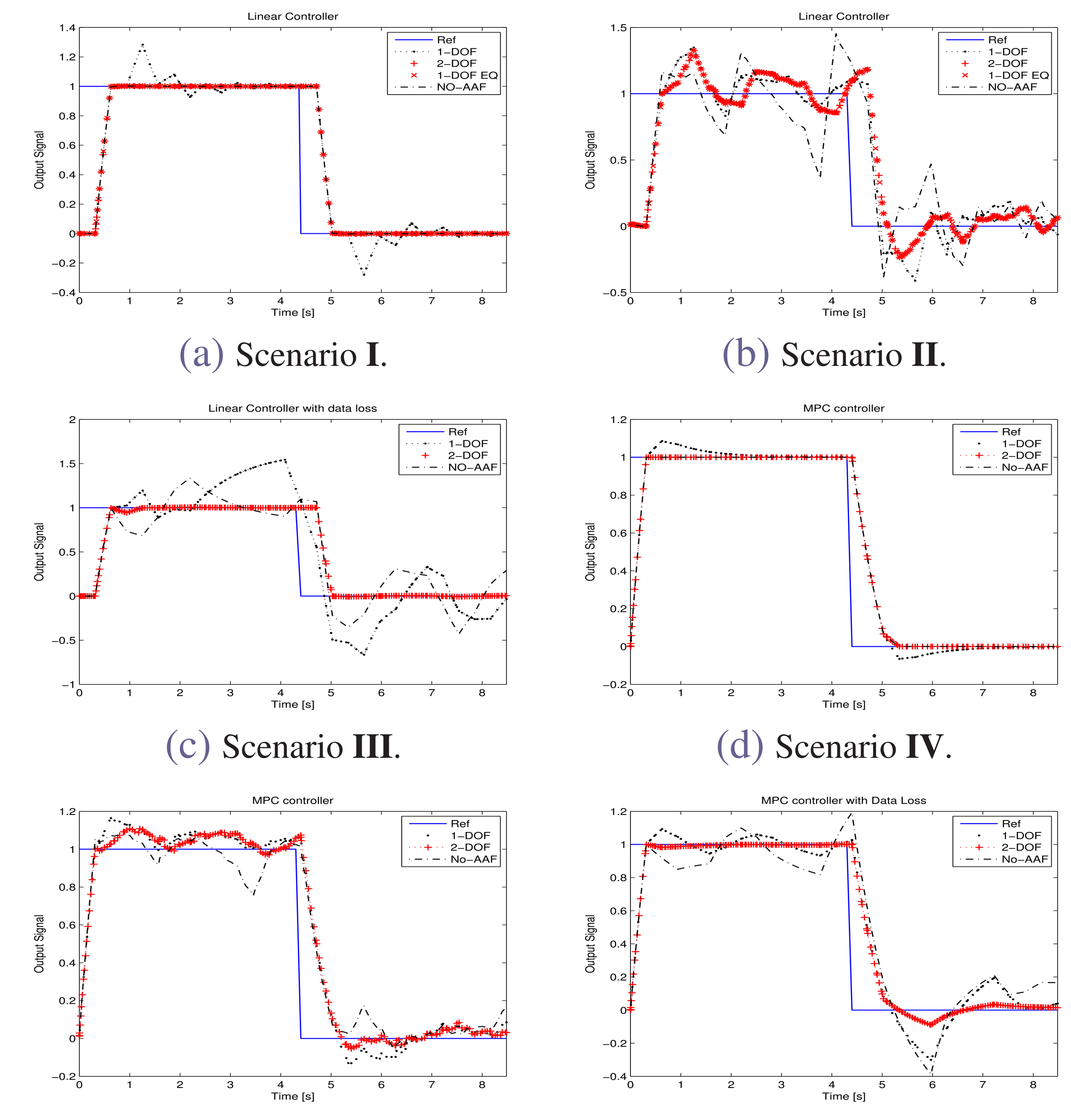
Considering the following plant:

$$G(s) = \frac{1}{s+1} \qquad (12)$$

We study the following scenarios:
I Linear controller, Noise free, and without data loss.
II Linear controller, with noise, and without data loss.
III Linear controller, only a sinusoidal disturbance, and with data loss.
IV MPC controller, Noise free, and without data loss.
V MPC controller, with noise, and without data loss.
VI MPC controller, only a sinusoidal disturbance, and with data loss.

	1-DOF	2-DOF	1-DOF EQ	NO-AAF
I	1.3157	1	1	1
II	1.1313	1	1	1.3033
III	1.3199	1	-	1.4696
IV	1.2060	1	-	1
V	1.0846	1	-	1.2032
VI	1.1171	1	-	1.2009

Table: Integral of the absolute tracking error over the simulation time ratio, $\frac{\rho}{\rho_{2DOF}}$.



(a) Scenario **I**. (b) Scenario **II**.
(c) Scenario **III**. (d) Scenario **IV**.
(e) Scenario **V**. (f) Scenario **VI**.

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