Verification of Data Layout Transformations

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Motivation

- To have an efficient data structure for stacks, queues, deques, sequences, catenable sequences, random access sequences, strings
- Both ephemeral and persistent versions with full persistence
- Convert between ephemeral and persistent versions in constant time

Demo Coq

```
Lemma foo : ∀x, ∃y, x = y.
Proof using.
  foo_solved.
Qed.
```

```
Lemma bar : ∀x, ∃y, x = y.
Proof using.
  bar_solved.
Qed.
```

Related Work

Structure	**Memory**	**Time**	**Limitations**
Arrays	$1\times$	$1\times$	concat/split/resize: $O(n)$
Vectors	$2-4\times$	$2\times$	concat/split: $O(n)$
Lists	$3\times$	$3\times$	concat/split/random access $O(n)$
Finger trees	$> 3\times$	$> 3\times$	Not transient
Ropes	?	?	More complex access to ends, not automatically balanced
Chunked Seq	$< 1.2\times$	$< 2\times$	

Interface

Chunks: fixed capacity arrays in which elements are stored
K = size of chunks

Operation	**Ephemeral**	**Persistent**
push/pop/front/back	$O(1 + \frac{1}{K} \log_K n)$	$O(K + \frac{1}{K} \log_K n)$
usual case	$O(1)$	$O(1)$
concat/split/get/set	$O(K \log_K n)$	$O(K \log_K n)$
iter/fold/...	$O(n)$	$O(n)$
Ephemeral $\rightarrow$Persistent		
destructive	$O(1)$	
nondestructive	$O(K)$	
Persistent $\rightarrow$Ephemeral	$O(K)$	

Chunked Sequence - key idea

```
type 'a chunk = {
  data : 'a array;
  mutable head : head;
  mutable size : int; }

type 'a seq =
| Empty
| Struct of 'a chunk * ('a chunk) seq * 'a chunk
```

Tree Structure

[image]

Sequence Representation - persistent

Pchunk:
Fixed capacity persistent sequence
Implemented using a view on a shared "support" chunk

```
type 'a pchunk = {
  support : 'a chunk;
  mutable view : segment; }

type segment = int * int
```

The shared chunk is reusable when popping or when pushing past its bounds. Other push cases need copy-on-write.

Pops are always O(1), pushes are in amortized O(1) if iterated.

```
type 'a pseq =
| Empty
| Struct of 'a pchunk * ('a pchunk) seq * 'a pchunk
```

Versions

Goal: Work on pchunks with in-place updates in ephemeral sequences

Solution: Maintain whether a chunk is shared or uniquely possessed in ephemeral sequences

Invariants:
Persistent: all chunks are marked false
Ephemeral: some are false and shared, some are true and were created in this sequence

Ephemeral $\rightarrow$persistent $=$ mark all chunks back to false
Version number trick enables this to be done in constant time.

```
type 'a pchunk = {
 version : version;
 support : 'a chunk;
 mutable view : segment; }
```

Transient Sequences - Types

```
type 'a seq = {
  mutable version : version;
  mutable front : 'a chunk;
  mutable middle : ('a pchunk) pseq;
  mutable back : 'a chunk;
}

type 'a pseq =
| Empty of 'a
| Struct of 'a pchunk * ('a pchunk) pseq * 'a pchunk
```

Note: Persistent sequence version number is stored in back chunk.

Summary and Additional Fields

```
type 'a chunk = {
  mutable head : int;
  mutable size : int;
  mutable data : 'a array;
  default : 'a; }
```

```
type 'a pchunk = {
  version : version;
  support : 'a chunk;
  mutable view : segment;
  mutable weight : weight; }
```

```
type 'a seq = {
  mutable version : version;
  mutable front : 'a chunk;
  mutable free_front : ('a chunk) option;
  mutable middle : ('a pchunk) PWSeq.t;
  mutable free_back : ('a chunk) option;
  mutable back : 'a chunk;
}
```

```
type 'a pseq =
    | Empty of 'a
    | Struct of weight * 'a pchunk * ('a pchunk) t * 'a pchunk
```