

# Verification of Data Layout Transformations

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# Motivating example

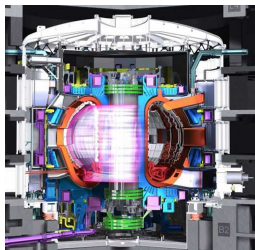


Figure: ITER tokamak

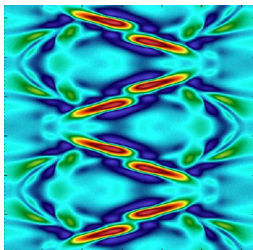


Figure: Plasma physics

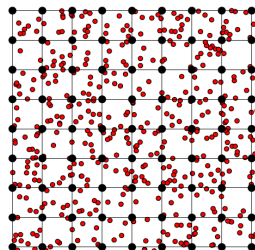


Figure: PIC simulation

## Challenges:

- Exploit data-level parallelism.
- Use domain-specific knowledge of the code.
- Do it without introducing any bugs.

# Motivating example - initial code

```
typedef struct {  
    // Position  
    float x, y, z;  
    // Other fields  
    float vx, vy, vz, c, m, v;  
} particle;  
  
particle data[N];  
  
for (int i = 0; i < N; i++) {  
    // Some calculation  
}
```

## Motivating example - splitting

Suppose that the calculation uses mainly the position.

```
typedef struct {  
    float vx, vy, vz, c, m, v;  
} cold_fields;
```

```
typedef struct {  
    float x, y, z;  
    cold_fields *other;  
} particle;
```

```
particle data[N];
```

## Motivating example - peeling

Further suppose that the initial 'particle' record is not used as part of a dynamic data structure.

```
typedef struct {  
    float vx, vy, vz, c, m, v;  
} cold_fields;
```

```
typedef struct {  
    float x, y, z;  
} hot_fields;
```

```
cold_fields other_data[N];  
hot_fields pos_data[N];
```

# Motivating example - AoS to SoA

Now, say that we want to take advantage of vector instructions.

```
typedef struct {  
    float x[N];  
    float y[N];  
    float z[N];  
} hot_fields;  
  
hot_fields pos_data;
```

## Motivating example - AoS to AoSoA

But without reducing too much the locality between accesses to fields of the original struct.

```
typedef struct {  
    float x[B];  
    float y[B];  
    float z[B];  
} hot_fields;
```

```
hot_fields pos_data[ceil(N/B)];
```

# Motivating example - summary

In short, the transformations we have seen are:

- Splitting.
- Peeling.
- AoS to SoA.
- AoS to AoSoA.



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Note that after all these changes, where we wrote:

```
data[i].x
```

Now we have to write:

```
pos_data[i/N].x[i%N]
```

# Project goals

- Find the basic transformations that combined give rise to the ones we are interested in.

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- Formalize a C-like language with arrays, structs and pointers.
  - Equipped with a high-level semantics, to simplify the proofs.
  - Equipped with a low-level semantics, to be closer to C.

# Project goals

- Find the basic transformations that combined give rise to the ones we are interested in.
- Formalize a C-like language with arrays, structs and pointers.
  - Equipped with a high-level semantics, to simplify the proofs.
  - Equipped with a low-level semantics, to be closer to C.
- Define the transformations and prove their correctness.

# Basic transformations

## 1. Field grouping

```
// Before
typedef struct {
    int a, b, c;
} s;
```

```
// After
typedef struct {
    int b, c;
} sg;
```

```
typedef struct {
    int a; sg fg;
} s';
```

## 2. Array tiling

```
// Before
typedef int a[N];
```

```
// After
typedef int a'[N_/_B][B];
```

## 3. Adding indirection

```
// Before
typedef struct {
    int a, b;
} s;
```

```
// After
typedef struct {
    int a; int *b;
} s';
```

## 4. AoS to SoA

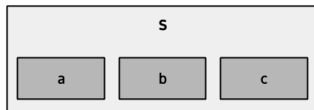
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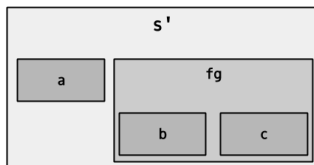
# Basic transformations - grouping

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// Before  
typedef struct {  
    int a, b, c;  
} s;
```



```
// After  
typedef struct {  
    int b, c;  
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typedef struct {  
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```



# Basic transformations - tiling

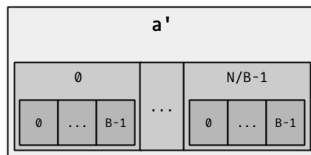
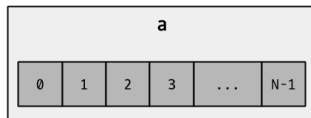
## 2. Array tiling

```
// Before
```

```
typedef int a[N];
```

```
// After
```

```
typedef int a'[N/_B][_B];
```

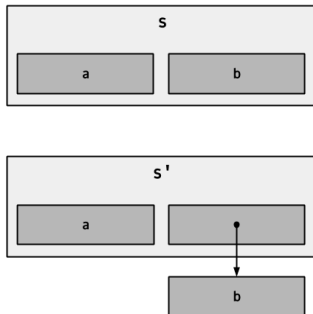


# Basic transformations - indirection

## 3. Adding indirection

```
// Before  
typedef struct {  
    int a, b;  
} s;
```

```
// After  
typedef struct {  
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```

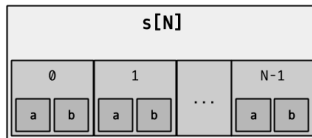




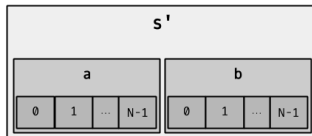
# Basic transformations - AoS to SoA

## 4. AoS to SoA

```
// Before  
typedef struct {  
    int a, b;  
} s;
```



```
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typedef struct {  
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# Basic transformations - justification

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# Basic transformations - justification

- **Splitting:** Field grouping and then adding indirection on the field holding the group.
- **Peeling:** Field grouping twice.
- **AoS to SoA:** AoS to SoA.
- **AoS to AoSoA:** Array tiling and then AoS to SoA on the tiles.

# Language overview - values and terms

Inductive val : Type :=

- | val\_error : val
- | val\_unit : val
- | val\_uninitialized : val
- | val\_bool : bool → val
- | val\_int : int → val
- | val\_double : int → val
- | val\_abstract\_ptr : loc → accesses → val
- | val\_array : typ → list val → val
- | val\_struct : typ → map field val → val

Inductive trm : Type :=

- | trm\_var : var → trm
- | trm\_val : val → trm
- | trm\_if : trm → trm → trm → trm
- | trm\_let : bind → trm → trm → trm
- | trm\_app : prim → list trm → trm
- | trm\_while : trm → trm → trm
- | trm\_for : var → val → val → trm → trm.

# Language overview - primitive operations

```
Inductive prim : Type :=  
| prim_binop : binop → prim  
| prim_get : typ → prim  
| prim_set : typ → prim  
| prim_new : typ → prim  
| prim_new_array : typ → prim  
| prim_struct_access : typ → field → prim  
| prim_array_access : typ → prim  
| prim_struct_get : typ → field → prim  
| prim_array_get : typ → prim
```

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| prim_array_access : typ → prim  
| prim_struct_get : typ → field → prim  
| prim_array_get : typ → prim
```

Examples of the semantics of our language compared to C:

get ptr : *ptr	array_access ptr i : ptr + i
set ptr v : *ptr = v	struct_access ptr f : &(ptr->f)
new T : malloc(sizeof(T))	struct_get s f : s.f

where pointers are represented as pairs:

$(l, [\text{access\_field } T \ f, \text{access\_array } T' \ i])$

which would correspond to the address:

$l + \text{field\_offset}(f) + i * \text{sizeof}(T')$



# Language overview - semantics

Some crucial definitions:

**Definition** `typdefctx := map typvar typ.`

`Record ll_typdefctx := make_ll_typdefctx {  
 typvar_sizes : map typvar size;  
 fields_offsets : map typvar (map field offset);  
 fields_order : map typvar (list field) }.`

**Definition** `stack := Ctx.ctx val.`

**Definition** `state := map loc val.`

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```

**Definition** `stack` := Ctx.ctx val.

**Definition** `state` := map loc val.

And the relation that defines the big-step reduction rules:

**Inductive** `red` : typdefctx → stack → state → trm → state → val → Prop

# Language overview - typing

The allowed types are:

```
Inductive typ : Type :=  
| typ_unit : typ  
| typ_int : typ  
| typ_double : typ  
| typ_bool : typ  
| typ_ptr : typ → typ  
| typ_array : typ → option size → typ  
| typ_struct : map field typ → typ  
| typ_var : typvar → typ.
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With their corresponding definitions (analogous to stack and state):

**Definition** gamma : Ctx.ctx typ.

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  | typ_ptr : typ → typ  
  | typ_array : typ → option size → typ  
  | typ_struct : map field typ → typ  
  | typ_var : typvar → typ.
```

With their corresponding definitions (analogous to stack and state):

```
Definition gamma : Ctx.ctx typ.
```

```
Definition phi : map loc typ.
```

Typing is defined as the following relation:

```
Inductive typing : typdefctx → gamma → phi → trm → typ → Prop
```

# Language overview - properties

For memory accesses, we know the type of the data being manipulated:

```
Inductive typing_val (C:typdefctx) (f:phi) : val → typ → Prop :=  
  | typing_val_abstract_ptr : ∀ l p T,  
    read_phi C f l p T →  
    typing_val C f (val_abstract_ptr l p) (typ_ptr T)
```

```
Inductive typing (C:typdefctx) : gamma → phi → trm → typ → Prop :=  
  | typing_get : ∀ G f T t1,  
    typing C G f t1 (typ_ptr T) →  
    typing C G f (trm_app (prim_get T) (t1::nil)) T
```

Typing result for full execution:

```
Theorem type_soundness : ∀ C LLC m t v T,  
  red C LLC empty_stack empty_state t m v →  
  typing C empty_gamma empty_phi t T →  
  ~is_error v →  
  ∃ f, typing_val C f v T  
  ∧ state_typing C f m.
```

# Field grouping

The arguments of the transformation are:

- The struct name Ts.
- The fields b and c (fs).
- The new struct name Tg.
- The new field fg.

```
// Before  
typedef struct {  
    int a, b, c;  
} Ts;
```

```
// After  
typedef struct {  
    int b, c;  
} Tg;
```

```
typedef struct {  
    int a; Tg fg;  
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} Tg;
```

```
typedef struct {
    int a; Tg fg;
} Ts;
```

These are used to define a transformation for:

- |           |             |              |
|-----------|-------------|--------------|
| • terms,  | • accesses, | • states and |
| • values, | • contexts, | • stacks.    |



# Field grouping - terms

We start with the transformation of the source code. In particular, we look at the struct access case:

```
Inductive tr_trm (gt:group_tr) : trm → trm → Prop :=
| tr_trm_struct_access_group : ∀fs Ts fg Tg f op0 t1 op2 op1 t1',
  gt = make_group_tr Ts fs Tg fg →
  f ∈ fs →
  (* The access s.f *)
  op0 = prim_struct_access (typ_var Ts) f →
  (* The access s'.fg.f *)
  op1 = prim_struct_access (typ_var Ts) fg →
  op2 = prim_struct_access (typ_var Tg) f →
  tr_trm gt t1 t1' →
  tr_trm gt (trm_app op0 (t1::nil)) (trm_app op2 ((trm_app op1 (t1'::nil))::nil))
```

## Field grouping - values

Values need to be changed in the source code. For instance, if we look at the interesting case:

```
Inductive tr_val (gt:group_tr) : val → val → Prop :=
| tr_val_struct_group : ∀Ts Tg s s' fg fs sg,
  gt = make_group_tr Ts fs Tg fg →
  fs ⊆ dom s →
  fg ∉ dom s →
  dom s' = (dom s \ - fs) ∪ {fg} →
  dom sg = fs →
  (* Contents of the grouped fields. *)
  s'[fg] = val_struct (typ_var Tg) sg →
  (∀ f ∈ dom sg, tr_val gt s[f] sg[f]) →
  (* Contents of the rest of the fields. *)
  (∀ f ∈ dom s \ fs, tr_val gt s[f] s'[f]) →
  tr_val gt (val_struct (typ_var Ts) s) (val_struct (typ_var Ts) s')
```

And in the stack and the memory so, from `tr_val`, we naturally define `tr_stack` and `tr_state`.

# Field grouping - accesses

For accesses, if we look at the interesting case:

```
Inductive tr_accesses (gt:group_tr) : accesses → accesses → Prop :=  
  | tr_accesses_field_group : ∀Ts fs fg Tg f a0 p a1 a2 p',  
    gt = make_group_tr Ts fs Tg fg →  
    f ∈ fs →  
    (* The access s.f *)  
    a0 = access_field (typ_var Ts) f →  
    (* Becomes s'.fg.f *)  
    a1 = access_field (typ_var Ts) fg →  
    a2 = access_field (typ_var Tg) f →  
    tr_accesses gt p p' →  
    tr_accesses gt (a0::p) (a1::a2::p')
```

This is used in:

```
Inductive tr_val (gt:group_tr) : val → val → Prop :=  
  | tr_val_abstract_ptr : ∀l p p',  
    tr_accesses gt p p' →  
    tr_val gt (val_abstract_ptr l p) (val_abstract_ptr l p')
```

# Field grouping - typdefctx

We ‘update’ the type definitions context as follows:

```
Inductive tr_typdefctx (gt:group_tr) : typdefctx → typdefctx → Prop :=  
  | tr_typdefctx_intro : ∀Tfs Tfs' Tfsg Ts fs Tg fg C C',  
    gt = make_group_tr Ts fs Tg fg →  
    Ts ∈ dom C →  
    dom C' = dom C ∪ {Tg} →  
    (* The original map from fields to types. *)  
    C[Ts] = typ_struct Tfs →  
    (* The map for the new struct and for the grouped fields. *)  
    tr_struct_map gt Tfs Tfs' Tfsg →  
    C'[Ts] = typ_struct Tfs' →  
    C'[Tg] = typ_struct Tfsg →  
    (* The other type variables stay the same. *)  
    (∀ T ∈ dom C \ {Ts}, C'[T] = C[T]) →  
    tr_typdefctx gt C C'.
```

# Field grouping - sanity checks

We need a way of checking that the transformation is well-defined.

```
Inductive group_tr_ok : group_tr → typdefctx → Prop :=  
| group_tr_ok_intros : ∀Tfs Ts fs fg Tg gt C,  
  gt = make_group_tr Ts fs Tg fg →  
  Ts ∈ dom C →  
  (* The struct Ts can be transformed. *)  
  C[Ts] = typ_struct Tfs →  
  Tg ∉ dom C →  
  fs ⊆ dom Tfs →  
  fg ∉ dom Tfs →  
  (* Ts doesn't appear anywhere else in the typdefctx. *)  
  (∀ Tv ∈ dom C, ~free_typvar C Tt C[Tv]) →  
  group_tr_ok gt C.
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Regardless of `group_tr`, we need to check that everything is well-formed:

- The `typdefctx` is well-formed if the type definitions are productive.

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  (* The struct Ts can be transformed. *)  
  C[Ts] = typ_struct Tfs →  
  Tg ∉ dom C →  
  fs ⊆ dom Tfs →  
  fg ∉ dom Tfs →  
  (* Ts doesn't appear anywhere else in the typdefctx. *)  
  (∀ Tv ∈ dom C, ~free_typvar C Tt C[Tv]) →  
  group_tr_ok gt C.
```

Regardless of `group_tr`, we need to check that everything is well-formed:

- The `typdefctx` is well-formed if the type definitions are productive.
- Terms, values, stacks and states are well-formed if all the types that appear in them exist.

# Field grouping - theorem

In the end the theorem that we prove is:

**Theorem** `red_tr`:  $\forall \text{gt LLC } C \ C' \ t \ t' \ v \ m,$   
    `red C LLC empty_stack empty_state t m v`  $\rightarrow$   
    `~is_error v`  $\rightarrow$   
    `group_tr_ok gt C`  $\rightarrow$   
    `tr_typdefctx gt C C'`  $\rightarrow$   
    `tr_trm gt t t'`  $\rightarrow$   
    `wf_typdefctx C`  $\rightarrow$   
    `wf_trm C t`  $\rightarrow$   
     $\exists v' \ m', \text{ tr\_val gt } v \ v'$   
         $\wedge \text{tr\_state gt } m \ m'$   
         $\wedge \text{red } C' \text{ LLC empty\_stack empty\_state } t' \ m' \ v'.$



## Field grouping - induction

To make the proof work we strengthen it as follows:

**Theorem** `red_tr_ind`:  $\forall \text{gt LLC C C' t t' v S S' m1 m1' m2,}$   
`red C LLC S m1 t m2 v`  $\rightarrow$   
`~is_error v`  $\rightarrow$   
`group_tr_ok gt C`  $\rightarrow$   
`tr_typdefctx gt C C'`  $\rightarrow$   
`tr_trm gt t t'`  $\rightarrow$   
`tr_stack gt S S'`  $\rightarrow$   
`tr_state gt m1 m1'`  $\rightarrow$   
`wf_typdefctx C`  $\rightarrow$   
`wf_trm C t`  $\rightarrow$   
`wf_stack C S`  $\rightarrow$   
`wf_state C m1`  $\rightarrow$   
 $\exists v' m2',$  `tr_val gt v v'`  
 $\wedge$  `tr_state gt m2 m2'`  
 $\wedge$  `red C' LLC S' m1' t' m2' v'.`

# Array tiling

We need to know:

- The name of the array being changed ( $T_a$ ).
- The new name for the tiles ( $T_t$ ).
- The size of the tiles ( $K$ ).

# Array tiling

We need to know:

- The name of the array being changed ( $Ta$ ).
- The new name for the tiles ( $Tt$ ).
- The size of the tiles ( $K$ ).

Similarly, we also define:

- `tiling_tr_ok`,
- `tr_typdefctx`,
- `tr_accesses`,
- `tr_val`,
- `tr_stack`,
- `tr_state` and
- `tr_trm`.

In this case, we change all the instances of `t[i]` to `t[i/K][i%K]` where `t` has type `typ_var Ta`.

# Array tiling - some specifics

We use:

- $I$  for the length of the original array,
- $J$  for the length of the array of tiles and
- $K$  for the length of the tile.

These are related by the definitions:

**Definition**  $\text{nb\_tiles } (K \ I \ J:\text{int}) : \text{Prop} :=$   
 $J = I / K + \text{If } (I \bmod K = 0) \text{ then } 0 \text{ else } 1.$

**Definition**  $\text{tiled\_indices } (I \ J \ K \ i \ j \ k:\text{int}) : \text{Prop} :=$   
 $i = j * K + k$   
 $\wedge \text{index } I \ i$   
 $\wedge \text{index } J \ j$   
 $\wedge \text{index } K \ k.$

# Array tiling - key components

The crucial case of `tr_val` from the array `aI` to `aJ` is captured by:

$$\begin{aligned} \forall i\ j\ k\ aK, \text{ tiled\_indices } I\ J\ K\ i\ j\ k \rightarrow \\ aJ[j] = (\text{val\_array } (\text{typ\_var } Tt) aK) \rightarrow \\ \text{tr\_val } tt\ aI[i]\ aK[k] \end{aligned}$$

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For the translation accesses and primitive operations, the aim is for all the accesses

$$l1 \ ++ \ (\text{access\_array } (\text{typ\_var } Ta) \ i) :: l2$$

to be transformed to

$$l1 \ ++ \ (\text{access\_array } (\text{typ\_var } Ta) \ (i/K)) :: (\text{access\_array } (\text{typ\_var } Tt) \ (i \bmod K)) :: l2.$$

# AoS to SoA

For this transformation, we need to know:

- The name of the array being changed ( $T_a$ ).
- The fields names and types of the struct being changed ( $Tfs$ ).
- The size of the array ( $K$ ).

# AoS to SoA

For this transformation, we need to know:

- The name of the array being changed ( $Ta$ ).
- The fields names and types of the struct being changed ( $Tfs$ ).
- The size of the array ( $K$ ).

This transformation is similar to array tiling in many ways. One key difference is that the accesses

```
l1 ++ (access_array Ta i)::(access_field (typ_struct Tfs) f)::l2
```

are transformed to

```
l1 ++ (access_field Ta f)::(access_field (typ_array Tfs[f] K) i)::l2.
```



# High-level transformations - summary

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**Problem:** This could all be just a hack if we don't link it with a more concrete, CompCert-style semantics...

# High-level to low-level transformation

The grammar is extended with:

```
Inductive val : Type :=  
| val_concrete_ptr : loc → val  
| val_words : list word → val.
```

```
Inductive prim : Type :=  
| prim_ll_get : typ → prim  
| prim_ll_set : typ → prim  
| prim_ll_new : typ → prim  
| prim_ll_access : typ → prim.
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```

There are two sides of this transformation:

- The memory.
- The programs.

# High-level to low-level transformation - memory

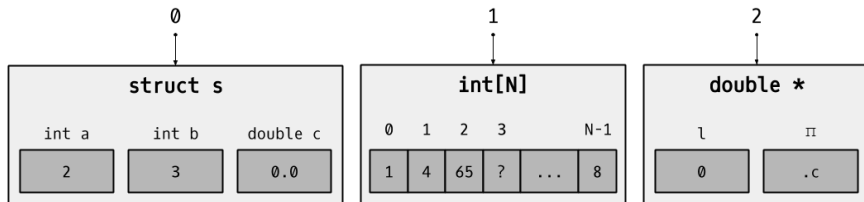


Figure: High-level memory.

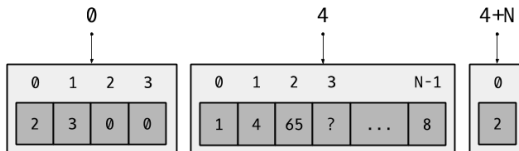


Figure: Low-level memory.

# High-level to low-level transformation - program

The values in the source code are all kept the same except for pointers:

**Inductive**  $\text{tr\_val} (C:\text{typedefctx}) (LLC:\text{ll\_typedefctx}) (a:\text{alpha}) : \text{val} \rightarrow \text{val} \rightarrow \text{Prop} :=$   
|  $\text{tr\_val\_abstract\_ptr} : \forall p \text{ l o},$   
|  $\text{tr\_ll\_accesses } C \text{ LLC } p \text{ o} \rightarrow$   
|  $\text{tr\_val } C \text{ LLC } a (\text{val\_abstract\_ptr } l \text{ p}) (\text{val\_concrete\_ptr } (a[l] + o)).$



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    `tr_ll_accesses C LLC p o` →  
    `tr_val C LLC a (val_abstract_ptr l p) (val_concrete_ptr (a[l] + o)).`

For terms, as an example,

`trm_app (prim_struct_access T f) (t::nil)`

becomes

`trm_app (prim_ll_access T[f]) (t'::(field_offset T f)::nil).`

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The semantics of `prim_ll_access` is, in fact, that of addition.

# High-level to low-level transformation - LLC

We need to ensure coherency between the type definition context ( $C$ ) and the low-level context (LLC). In particular:

- The type variable sizes in LLC match with the types in  $C$ .
- The field offsets match with the order of the fields and the sizes of each of their types.

# High-level to low-level transformation - theorem

The goal is to prove:

**Theorem** `red_tr_warmup` :  $\forall C \text{ LLC } T \text{ m a v t' m' v' ,}$   
    `red C LLC empty_stack empty_state t m v`  $\rightarrow$   
    `typing C empty_gamma empty_phi t T`  $\rightarrow$   
    `~is_error v`  $\rightarrow$   
    `ll_typdefctx_ok C LLC`  $\rightarrow$   
    `tr_trm C LLC a t t'`  $\rightarrow$   
    `wf_typdefctx C`  $\rightarrow$   
    `wf_trm C t`  $\rightarrow$   
    `wf_typ C T`  $\rightarrow$   
     $\exists v' m' , \text{ tr\_state } C \text{ LLC a m m'}$   
         $\wedge \text{ tr\_val } C \text{ LLC a v v'}$   
         $\wedge \text{ red } C \text{ LLC empty\_stack empty\_state t' m' v' .$

# Project extent

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Some statistics:

lines of spec	lines of proof	lines of comments
2723	3103	668



# Future work

Next steps:

- The transformation ‘adding indirection’.

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- Connect the low-level language with CompCert.

# Verification of Data Layout Transformations

**Ramon Fernández Mir**

with Arthur Charguéraud

Inria

17/09/2018