Verification of Data Layout Transformations

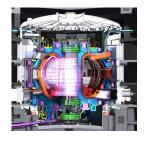
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with Arthur Charguéraud

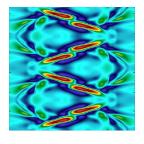
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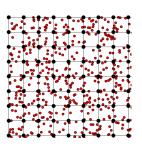
Motivating example







Plasma physics



PIC simulation

Challenges:

- Exploit data-level parallelism.
- Use domain-specific knowledge of the code.
- Do it without introducing any bugs.

Motivating example - initial code

```
typedef struct {
  // Position
  float x, y, z;
  // Other fields
  float vx, vy, vz, c, m, v;
} particle;
particle data[N];
for (int i = 0; i < N; i++) {
  // Some calculation involving data[i]
```

Motivating example - splitting

Suppose that the calculation uses mainly the position.

```
typedef struct {
  float vx, vy, vz, c, m, v;
} cold_fields;

typedef struct {
  float x, y, z;
  cold_fields *other;
} particle;

particle data[N];
```

Motivating example - peeling

Further suppose that the intial 'particle' record is not used as part of a dynamic data structure.

```
typedef struct {
  float vx, vy, vz, c, m, v;
} cold_fields;

typedef struct {
  float x, y, z;
} hot_fields;

cold_fields other_data[N];
hot_fields pos_data[N];
```

Motivating example - AoS to SoA

Now, say that we want to take advantage of vector instructions.

```
typedef struct {
  float x[N];
  float y[N];
  float z[N];
} hot_fields;
hot_fields pos_data;
```

Motivating example - AoS to AoSoA

But without reducing too much the locality between accesses to fields of the original struct.

```
typedef struct {
  float x[B];
  float y[B];
  float z[B];
} hot_fields;

hot_fields pos_data[ceil(N/B)];
```

Motivating example - summary

In short, the transformations we have seen are:

- Splitting.
- Peeling.
- AoS to SoA.
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Note that after all these changes, where we wrote:

Now we have to write:

Project goals

• Find the basic transformations that combined give rise to the ones we are interested in.

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- Formalize a C-like language with arrays, structs and pointers.
 - Equipped with a high-level semantics, to simplify the proofs.
 - Equipped with a low-level semantics, to be closer to C.

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- Formalize a C-like language with arrays, structs and pointers.
 - Equipped with a high-level semantics, to simplify the proofs.
 - Equipped with a low-level semantics, to be closer to C.
- Define the transformations and prove their correctness.

Basic transformations

1. Field grouping

```
// Before
typedef struct {
  int a, b, c;
} s;

// After
typedef struct {
  int b, c;
} sg;

typedef struct {
  int a; sg fg;
} s';
```

2. Array tiling

```
// Before
typedef int a[N];
// After
typedef int a'[N/B][B];
```

3. AoS to SoA

```
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a[N]; int b[N];
} s';
```

4. Adding indirection

```
// Before
typedef struct {
  int a; T b;
} s;

// After
typedef struct {
  int a; T *b;
} s';
```

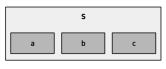
Basic transformations - grouping

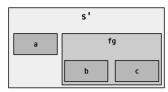
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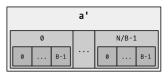
Basic transformations - tiling

2. Array tiling

```
// Before
typedef int a[N];

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```



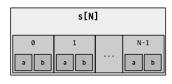


Basic transformations - AoS to SoA

3. AoS to SoA

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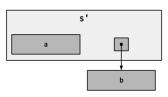
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typedef struct {
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- **Splitting:** Field grouping and then adding indirection on the field holding the group.
- Peeling: Field grouping twice.
- AoS to SoA: AoS to SoA.
- AoS to AoSoA: Array tiling and then AoS to SoA on the tiles.

Language overview - values and terms

```
Inductive val: Type:=
    val_error : val
    val unit: val
    val_uninitialized : val
    val bool: bool → val
    val int: int \rightarrow val
    val\_double : int \rightarrow val
    val_abstract_ptr : loc \rightarrow accesses \rightarrow val
    val_array : typ \rightarrow list val \rightarrow val
    val\_struct : typ \rightarrow map field val \rightarrow val
Inductive trm : Type :=
    trm_var : var → trm
    trm val : val → trm
    trm if : trm \rightarrow trm \rightarrow trm \rightarrow trm
    trm let : bind \rightarrow trm \rightarrow trm \rightarrow trm
    trm\_app : prim \rightarrow list trm \rightarrow trm
    trm while : trm \rightarrow trm \rightarrow trm
    trm for : var \rightarrow val \rightarrow val \rightarrow trm \rightarrow trm
```

Language overview - primitive operations

```
Inductive prim : Type :=
    | prim_binop : binop → prim
    | prim_get : typ → prim
    | prim_set : typ → prim
    | prim_new : typ → prim
    | prim_new_array : typ → prim
    | prim_struct_access : typ → field → prim
    | prim_array_access : typ → prim
    | prim_struct_get : typ → field → prim
    | prim_array_get : typ → prim
```

Examples of the semantics of our language compared to C:

where pointers are represented as pairs:

```
(1, [access_field T f, access_array T' i])
```

which would correspond to the address:

```
1 + field_offset(f) + i * sizeof(T')
```

Language overview - semantics

Some crucial definitions:

```
Definition typdefctx := map typvar typ.
Record ll_typdefctx := make_ll_typdefctx {
   typvar_sizes : map typvar size;
   fields_offsets : map typvar (map field offset);
   fields_order : map typvar (list field) }.
Definition stack := Ctx.ctx val.
Definition state := map loc val.
```

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Definition stack := Ctx.ctx val.
Definition state := map loc val.
```

And the relation that defines the big-step reduction rules:

```
\texttt{Inductive red} : \texttt{typdefctx} \to \texttt{stack} \to \texttt{state} \to \texttt{trm} \to \texttt{state} \to \texttt{val} \to \texttt{Prop}
```

Language overview - typing

The allowed types are:

```
Inductive typ: Type:=
  | typ_unit: typ
  | typ_int: typ
  | typ_double: typ
  | typ_bool: typ
  | typ_ptr: typ → typ
  | typ_array: typ → option size → typ
  | typ_struct: map field typ → typ
  | typ_var: typvar → typ.
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```

With their corresponding definitions (analogous to stack and state):

```
Definition gamma := Ctx.ctx typ.

Definition phi := map loc typ.
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Language overview - typing

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    | typ_struct : map field typ → typ
    | typ_var : typvar → typ.
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With their corresponding definitions (analogous to stack and state):

```
Definition gamma := Ctx.ctx typ.

Definition phi := map loc typ.
```

Typing is defined as the following relation:

```
Inductive typing: typdefctx \rightarrow gamma \rightarrow phi \rightarrow trm \rightarrow typ \rightarrow Prop
```

Language overview - properties

For memory accesses, we know the type of the data being manipulated:

```
Inductive typing_val (C:typdefctx) (f:phi) : val → typ → Prop :=
  | typing_val_abstract_ptr : ∀l p T,
      read_phi C f l p T →
      typing_val C f (val_abstract_ptr l p) (typ_ptr T)

Inductive typing (C:typdefctx) : gamma → phi → trm → typ → Prop :=
  | typing_get : ∀G f T t1,
      typing C G f t1 (typ_ptr T) →
      typing C G f (trm_app (prim_get T) (t1::nil)) T
```

Typing result for full execution:

```
Theorem type_soundness: ∀C LLC m t v T,
red C LLC empty_stack empty_state t m v →
typing C empty_gamma empty_phi t T →
~is_error v →
∃f, typing_val C f v T
∧ state_typing C f m.
```

Field grouping

The arguments of the transformation are:

- The struct name Ts.
- The fields b and c (fs).
- The new struct name Tg.
- The new field fg.

```
// Before
typedef struct {
  int a, b, c;
} Ts;

// After
typedef struct {
  int b, c;
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typedef struct {
  int a; Tg fg;
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```

These are used to define a transformation for:

terms,

accesses,

states and

values,

contexts,

stacks.

Field grouping - terms

We start with the transformation of the source code. In particular, we look at the struct access case:

```
Inductive tr_trm (gt:group_tr): trm → trm → Prop :=
  | tr_trm_struct_access_group : ∀fs Ts fg Tg f op0 t1 op2 op1 t1',
    gt = make_group_tr Ts fs Tg fg →
    f ∈ fs →
        (* The access s.f *)
    op0 = prim_struct_access (typ_var Ts) f →
        (* The access s'.fg.f *)
    op1 = prim_struct_access (typ_var Ts) fg →
        op2 = prim_struct_access (typ_var Tg) f →
    tr_trm gt t1 t1' →
    tr_trm gt (trm_app op0 (t1::nil)) (trm_app op2 ((trm_app op1 (t1'::nil))::nil))
```

Field grouping - values

Values need to be changed in the source code. For instance, if we look at the interesting case:

```
Inductive tr_val (gt:group_tr): val \rightarrow val \rightarrow Prop := 
 | tr_val_struct_group: \forallTs Tg s s' fg fs sg, 
 gt = make_group_tr Ts fs Tg fg \rightarrow 
 fs \subseteqdom s \rightarrow 
 fg \notin dom s \rightarrow 
 dom s' = (dom s \setminus- fs) \cup {fg} \rightarrow 
 dom sg = fs \rightarrow 
 (* Contents of the grouped fields. *) 
 s'[fg] = val_struct (typ_var Tg) sg \rightarrow 
 (\forall f \in dom sg, tr_val gt s[f] sg[f]) \rightarrow 
 (* Contents of the rest of the fields. *) 
 (\forall f \in dom s \setminus fs, tr_val gt s[f] s'[f]) \rightarrow 
 tr_val gt (val_struct (typ_var Ts) s) (val_struct (typ_var Ts) s')
```

And in the stack and the memory so, from tr_val, we naturally define tr_stack and tr_state.

Field grouping - accesses

For accesses, if we look at the interesting case:

```
Inductive tr_accesses (gt:group_tr): accesses \rightarrow accesses \rightarrow Prop := | tr_accesses_field_group: \forall Ts \ fs \ fg \ Tg \ fa \ 0 \ pa \ 1 \ a2 \ p', gt = make_group_tr Ts fs Tg fg \rightarrow f \in fs \rightarrow (* The access s.f *) a0 = access_field (typ_var Ts) f \rightarrow (* Becomes s'.fg.f *) a1 = access_field (typ_var Ts) fg \rightarrow a2 = access_field (typ_var Tg) f \rightarrow tr_accesses gt p' \rightarrow tr_accesses gt (a0::p) (a1::a2::p')
```

This is used in:

```
\begin{split} & \text{Inductive tr\_val (gt:group\_tr)} : \text{val} \rightarrow \text{val} \rightarrow \text{Prop} := \\ & | \text{tr\_val\_abstract\_ptr} : \forall 1 \text{ p p'}, \\ & \text{tr\_accesses gt p p'} \rightarrow \\ & \text{tr\_val gt (val\_abstract\_ptr 1 p) (val\_abstract\_ptr 1 p')} \end{split}
```

Field grouping - typdefctx

We 'update' the type definitions context as follows:

Field grouping - sanity checks

We need a way of checking that the transformation is well-defined.

```
Inductive group_tr_ok : group_tr → typdefctx → Prop :=
  | group_tr_ok_intros : ∀Tfs Ts fs fg Tg gt C,
    gt = make_group_tr Ts fs Tg fg →
    Ts ∈ dom C →
    (* The struct Ts can be transformed. *)
    C[Ts] = typ_struct Tfs →
    Tg ∉ dom C →
    fs ⊆dom Tfs →
    fg ∉ dom Tfs →
    (* Ts doesn't appear anywhere else in the typdefctx. *)
    (∀ Tv ∈ dom C, ~free_typvar C Tt C[Tv]) →
    group_tr_ok gt C.
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Regardless of group_tr, we need to check that everything is well-formed:

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Regardless of group_tr, we need to check that everything is well-formed:

- The typdefctx is well-formed if the type definitions are productive.
- Terms, values, stacks and states are well-formed if all the types that appear in them exist.

Field grouping - theorem

In the end the theorem that we prove is:

```
Theorem red_tr: \forall gt LLC C C' t t' v m,
  red C LLC empty_stack empty_state t m v →
 ~is error v →
  group_tr_ok gt C →
  tr_typdefctx gt C C' →
  tr_trm gt t t' →
  wf_typdefctx C \rightarrow
  wf_trm C t →
  ∃v'm', tr_val gt v v'
       ∧ tr_state gt m m'
       ∧ red C' LLC empty_stack empty_state t' m' v'.
```

Field grouping - induction

To make the proof work we strengthen it as follows:

```
Theorem red_tr_ind: \( \forall gt LLC C C' t t' v S S' m1 m1' m2, \)
  red C LLC S m1 t m2 v \rightarrow
 ~is_error v →
  group_tr_ok gt C →
  tr_typdefctx gt C C' →
  tr_trm gt t t' →
  tr_stack gt S S' →
  tr_state gt m1 m1' →
  wf_typdefctx C →
  wf_trm C t →
  wf stack CS \rightarrow
  wf state C m1 \rightarrow
  ∃v'm2', tr_val gt v v'
         ∧ tr_state gt m2 m2'
         ∧ red C' LLC S' m1' t' m2' v'.
```

Array tiling

We need to know:

- The name of the array being changed (Ta).
- The new name for the tiles (Tt).
- The size of the tiles (K).

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Similarly, we also define:

- tiling_tr_ok,
 - tr_typdefctx,
 - tr_accesses,
 - tr_val,

- tr_stack,
- tr_state and
- tr_trm.

In this case, we change all the instances of t[i] to t[i/K] [i%K] where t has type typ_var Ta.

Array tiling - some specifics

We use:

- I for the length of the original array,
- J for the length of the array of tiles and
- K for the length of the tile.

These are related by the definitions:

Array tiling - key components

The crucial case of tr_val from the array aI to aJ is captured by:

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The crucial case of tr_val from the array aI to aJ is captured by:

```
\foralli j k aK, tiled_indices I J K i j k \rightarrow
             aJ[j] = (val_array (typ_var Tt) aK) →
             tr_val tt aI[i] aK[k]
```

For the translation accesses and primitive operations, the aim is for all the accesses

```
11 ++ (access_array (typ_var Ta) i)::12
```

to be transformed to

```
11 ++ (access_array (typ_var Ta) (i/K))::(access_array (typ_var Tt) (i mod K))::12.
```

AoS to SoA

For this transformation, we need to know:

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This transformation is similar to array tiling in many ways. One key difference is that the accesses

```
11 ++ (access_array Ta i)::(access_field (typ_struct Tfs) f)::12
```

are transformed to

```
\label{eq:local_constraint} 11 \; ++ \; (\texttt{access\_field} \; \texttt{Ta} \; \texttt{f}) :: (\texttt{access\_field} \; (\texttt{typ\_array} \; \texttt{Tfs[f]} \; \texttt{K}) \; \texttt{i}) :: 12.
```

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Problem: This could all be just a hack if we don't link it with a more concrete, CompCert-style semantics...

High-level to low-level transformation

The grammar is extended with:

```
Inductive val : Type :=
  | val_concrete_ptr : loc → val
  | val_words : list word → val.

Inductive prim : Type :=
  | prim_ll_get : typ → prim
  | prim_ll_set : typ → prim
  | prim_ll_new : typ → prim
  | prim_ll_access : typ → prim.
```

High-level to low-level transformation

The grammar is extended with:

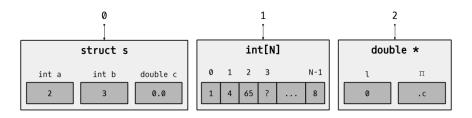
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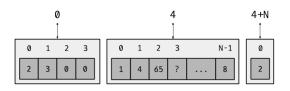
There are two sides of this transformation:

- The memory.
- The programs.

High-level to low-level transformation - memory



High-level memory.



Low-level memory.

High-level to low-level transformation - program

The values in the source code are all kept the same except for pointers:

```
Inductive tr_val (C:typdefctx) (LLC:ll_typdefctx) (a:alpha) : val \rightarrow val \rightarrow Prop := | tr_val_abstract_ptr : \forallp l o, tr_ll_accesses C LLC p o \rightarrow tr_val C LLC a (val_abstract_ptr l p) (val_concrete_ptr (a[l] + o)).
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```
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   | tr_val_abstract_ptr : ∀p 1 o,
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          tr_val \ C \ LLC \ a \ (val_abstract_ptr \ l \ p) \ (val_concrete_ptr \ (a[1] + o)).
```

For terms, as an example,

```
trm_app (prim_struct_access T f) (t::nil)
```

becomes

```
trm_app (prim_ll_access T[f]) (t'::(field_offset T f)::nil).
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```

The semantics of prim_ll_access is, in fact, that of addition.

High-level to low-level transformation - LLC

We need to ensure coherency between the type definition context (C) and the low-level context (LLC). In particular:

- The type variable sizes in LLC match with the types in C.
- The field offsets match with the order of the fields and the sizes of each of their types.

High-level to low-level transformation - theorem

The goal is to prove:

```
Theorem red_tr_warmup : \forall C \ LLC \ T \ m \ a \ v \ t' \ m' \ v',
  red C LLC empty_stack empty_state t m v →
  typing C empty_gamma empty_phi t T →
 ~is error v →
  ll_typdefctx_ok C LLC →
  tr trm C LLC a t t' →
  wf_typdefctx C \rightarrow
  wf_trm C t →
  wf_typ C T \rightarrow
  ∃v'm', tr_state C LLC a m m'
       ∧ tr val C LLC a v v'
       ∧ red C LLC empty_stack empty_state t' m' v'.
```

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Some statistics:

lines of spec lines of proof lines of comments 2723 3103 668

Next steps:

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Verification of Data Layout Transformations

Ramon Fernández Mir

with Arthur Charguéraud

Inria

17/09/2018