Verification of Data Layout Transformations

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Motivating example

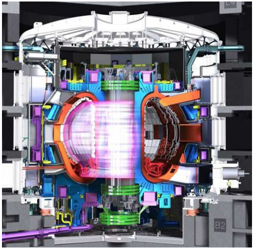


Figure: ITER tokamak

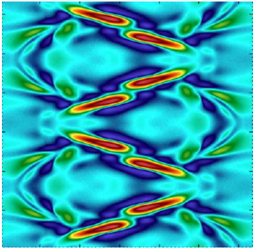


Figure: Plasma physics

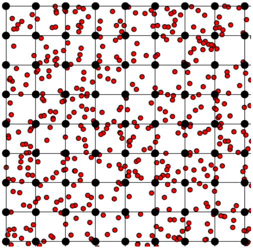


Figure: PIC simulation

Challenges:

- Exploit data-level parallelism.
- Use domain-specific knowledge of the code.
- Do it without introducing any bugs.

Motivating example - initial code

```
typedef struct {
  // Position
  float x, y, z;
  // Other fields
  float vx, vy, vz, c, m, v;
} particle;

particle data[NUM_PARTICLES];

for (int i = 0; i < NUM_PARTICLES; i++) {
  // Some calculation
}
```

Motivating example - splitting

Suppose that the calculation uses mainly the position.

```
typedef struct {
  float vx, vy, vz, c, m, v;
} cold_fields;

typedef struct {
  float x, y, z;
  cold_fields *other;
} particle;

particle data[NUM_PARTICLES];
```

Motivating example - peeling

Further suppose that the intial 'particle' record is not used as part of a dynamic data structure.

```
typedef struct {
  float vx, vy, vz, c, m, v;
} cold_fields;

typedef struct {
  float x, y, z;
} hot_fields;

cold_fields other_data[NUM_PARTICLES];
hot_fields pos_data[NUM_PARTICLES];
```

Motivating example - AoS to SoA

Now, say that we want to take advantage of vector instructions.

```
typedef struct {
  float x[NUM_PARTICLES];
  float y[NUM_PARTICLES];
  float z[NUM_PARTICLES];
} hot_fields;

hot_fields pos_data;
```

Motivating example - AoS to AoSoA

But without reducing too much the locality between accesses to fields of the original struct.

```
typedef struct {
  float x[N];
  float y[N];
  float z[N];
} hot_fields;

hot_fields pos_data[NUM_PARTICLES / N];
```

Motivating example - summary

In short, the transformations we have seen are:

- Splitting.
- Peeling.
- AoS to SoA.
- AoS to AoSoA.

Note that after all these changes, where we wrote:

```
data[i].x
```

Now we have to write:

```
pos_data[i / N].x[i % N]
```

Project goals

- Find the basic transformations that combined give rise to the ones we are interested in.
- Formalize a C-like language with arrays, structs and pointers.
  - On a high-level, to simplify the proofs.
  - On a low-level, to be closer to the semantics of C.
- Define the transformations and prove their correctness.

Basic transformations

1. Field grouping

```c
// Before
typedef struct {
  int a, b, c;
} s;

// After
typedef struct {
  int b, c;
} sg;

typedef struct {
  int a; sg fg;
} s';
```

2. Array tiling

```c
// Before
typedef int a[N];

// After
typedef int a'[N / B][B];
```

3. Adding indirection

```c
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a; int *b;
} s';
```

4. AoS to SoA

```c
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a[N]; int b[N];
} s;
```

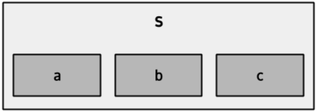
Basic transformations - grouping

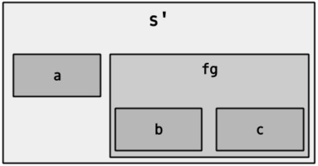
1. **Field grouping**

```c
// Before
typedef struct {
  int a, b, c;
} s;

// After
typedef struct {
  int b, c;
} sg;

typedef struct {
  int a; sg fg;
} s';
```



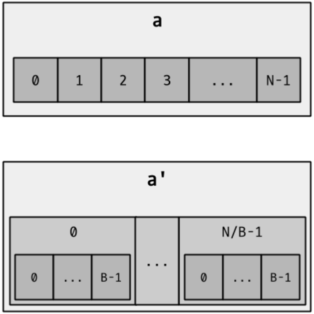


Basic transformations - tiling

2. Array tiling

```
// Before
typedef int a[N];

// After
typedef int a'[N / B][B];
```

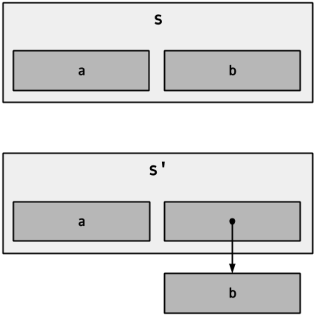


Basic transformations - indirection

3. Adding indirection

```c
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a; int *b;
} s';
```

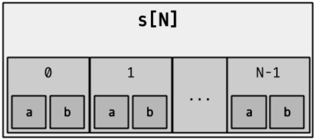


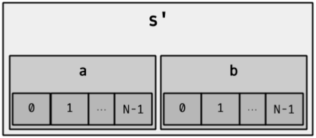
Basic transformations - AoS to SoA

4. AoS to SoA

```
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a[N]; int b[N];
} s;
```





Basic transformations - justification

- **Peeling:** Field grouping twice.
- **Splitting:** Field grouping and then adding indirection on the field holding the group.
- **AoS to SoA:** AoS to SoA.
- **AoS to AoSoA:** Array tiling and then AoS to SoA on the tiles.

Language overview - values and terms

```
Inductive val : Type :=
  | val_error : val
  | val_unit : val
  | val_uninitialized : val
  | val_bool : bool → val
  | val_int : int → val
  | val_double : int → val
  | val_abstract_ptr : loc → accesses → val
  | val_array : typ → list val → val
  | val_struct : typ → map field val → val

Inductive trm : Type :=
  | trm_var : var → trm
  | trm_val : val → trm
  | trm_if : trm → trm → trm → trm
  | trm_let : bind → trm → trm → trm
  | trm_app : prim → list trm → trm
  | trm_while : trm → trm → trm
  | trm_for : var → val → val → trm → trm.
```

Language overview - primitive operations

```
Inductive prim : Type :=
  | prim_binop : binop → prim
  | prim_get : typ → prim
  | prim_set : typ → prim
  | prim_new : typ → prim
  | prim_new_array : typ → prim
  | prim_struct_access : typ → field → prim
  | prim_array_access : typ → prim
  | prim_struct_get : typ → field → prim
  | prim_array_get : typ → prim
```

Examples of the semantics of our language compared to C:

```
get p   : *p                      array_access p i  : p + i
set p v : *p = v                  struct_access p f : &(p->f)
new T   : malloc(sizeof(T))       struct_get s f    : s.f
```

where pointers are represented as pairs:

```
(l, [access_field T f, access_array T' i])
```

which would correspond to the address:

```
l + field_offset(f) + i * sizeof(T')
```

Language overview - semantics

Some crucial definitions:

```
Definition typdefctx := map typvar typ.

Record ll_typdefctx := make_ll_typdefctx {
  typvar_sizes   : map typvar size;
  fields_offsets : map typvar (map field offset);
  fields_order   : map typvar (list field) }.

Definition stack := Ctx.ctx val.

Definition state := map loc val.
```

And the relation that defines the big-step reduction rules:

$$\texttt{red} \subseteq \texttt{typdefctx} \times \texttt{ll\_typdefctx} \times \texttt{stack} \times \texttt{state} \times \texttt{trm} \times \texttt{state} \times \texttt{val}$$

Language overview - typing

The allowed types are:

```
Inductive typ : Type :=
  | typ_unit : typ
  | typ_int : typ
  | typ_double : typ
  | typ_bool : typ
  | typ_ptr : typ → typ
  | typ_array : typ → option size → typ
  | typ_struct : map field typ → typ
  | typ_var : typvar → typ.
```

With their corresponding definitions (analogous to stack and state):

```
Definition gamma : Ctx.ctx typ.

Definition phi : map loc typ.
```

Typing is defined as the following relation:

$$\texttt{typing} \subseteq \texttt{typdefctx} \times \texttt{gamma} \times \texttt{phi} \times \texttt{trm} \times \texttt{typ}$$

Language overview - properties

For memory accesses, we know the type of the data being manipulated:

```
Inductive typing_val (C:typdefctx) (f:phi) : val → typ → Prop :=
  | typing_val_abstract_ptr : ∀l p T,
      read_phi C f l p T →
      typing_val C f (val_abstract_ptr l p) (typ_ptr T)

Inductive typing (C:typdefctx) : gamma → phi → trm → typ → Prop :=
  | typing_get : ∀G f T t1,
      typing C G f t1 (typ_ptr T) →
      typing C G f (trm_app (prim_get T) (t1::nil)) T
```

Typing result for full execution:

```
Theorem type_soundness : ∀C LLC m t v T,
  red C LLC empty_stack empty_state t m v →
  typing C empty_gamma empty_phi t T →
  ~is_error v →
  ∃f, typing_val C f v T
   ∧ state_typing C f m.
```

Field grouping

The arguments of the transformation are:

- The struct name s.
- The fields b and c.
- The new struct name sg.
- The new field fg.

```c
// Before
typedef struct {
  int a, b, c;
} s;

// After
typedef struct {
  int b, c;
} sg;

typedef struct {
  int a; sg fg;
} s';
```

These are used to define a transformation for:

- terms,
- values,
- accesses,
- contexts,
- states and
- stacks.

Field grouping - terms

We start with the transformation of the source code. In particular, we look at the struct access case:

```
Inductive tr_trm (gt:group_tr) : trm → trm → Prop :=
  | tr_trm_struct_access_group : ∀fs Tt fg Tg f op0 t1 op2 op1 t1',
      gt = make_group_tr Tt fs Tg fg →
      f ∈ fs →
      (* The access s.f *)
      op0 = prim_struct_access (typ_var Tt) f →
      (* The access s'.fg.f *)
      op1 = prim_struct_access (typ_var Tt) fg →
      op2 = prim_struct_access (typ_var Tg) f →
      tr_trm gt t1 t1' →
      tr_trm gt (trm_app op0 (t1::nil)) (trm_app op2 ((trm_app op1 (t1'::nil))::nil))
```

Field grouping - values

Values need to be changed in the source code. For instance, if we look at the interesting case:

```
Inductive tr_val (gt:group_tr) : val → val → Prop :=
  | tr_val_struct_group : ∀Tt Tg s s' fg fs sg,
      gt = make_group_tr Tt fs Tg fg →
      fs ⊆dom s →
      fg ∉ dom s →
      dom s' = (dom s \− fs) ∪ {fg} →
      dom sg = fs →
      (* Contents of the grouped fields. *)
      s'[fg] = val_struct (typ_var Tg) sg →
      (∀ f ∈dom sg, tr_val gt s[f] sg[f]) →
      (* Contents of the rest of the fields. *)
      (∀ f ∈dom s \ fs, tr_val gt s[f] s'[f]) →
      tr_val gt (val_struct (typ_var Tt) s) (val_struct (typ_var Tt) s')
```

And in the stack and the memory so, from `tr_val`, we naturally define `tr_stack` and `tr_state`.

Field grouping - accesses

For accesses, if we look at the interesting case:

```
Inductive tr_accesses (gt:group_tr) : accesses → accesses → Prop :=
  | tr_accesses_field_group : ∀Tt fs fg Tg f a0 p a1 a2 p',
      gt = make_group_tr Tt fs Tg fg →
      f ∈ fs →
      (* The access s.f *)
      a0 = access_field (typ_var Tt) f →
      (* Becomes s'.fg.f *)
      a1 = access_field (typ_var Tt) fg →
      a2 = access_field (typ_var Tg) f →
      tr_accesses gt p p' →
      tr_accesses gt (a0::p) (a1::a2::p')
```

This is used in:

```
Inductive tr_val (gt:group_tr) : val → val → Prop :=
  | tr_val_abstract_ptr : ∀l p p',
      tr_accesses gt p p' →
      tr_val gt (val_abstract_ptr l p) (val_abstract_ptr l p')
```

Field grouping - OK

We need a way of checking that the transformation is well-defined.

```
Inductive group_tr_ok : group_tr → typdefctx → Prop :=
  | group_tr_ok_intros : ∀Tfs Tt fs fg Tg gt C,
      gt = make_group_tr Tt fs Tg fg →
      Tt ∈ dom C →
      (* The struct Tt can be transformed. *)
      C[Tt] = typ_struct Tfs →
      Tg ∉ dom C →
      fs ⊆dom Tfs →
      fg ∉ dom Tfs →
      (* Tt doesn't appear anywhere else in the typdefctx. *)
      (∀ Tv ∈ dom C, ~free_typvar C Tt C[Tv]) →
      group_tr_ok gt C.
```

Field grouping - typdefctx

We 'update' the type definitions context as follows:

```
Inductive tr_typdefctx (gt:group_tr) : typdefctx → typdefctx → Prop :=
  | tr_typdefctx_intro : ∀Tfs Tfs' Tfsg Tt fs Tg fg C C',
      gt = make_group_tr Tt fs Tg fg →
      dom C' = dom C ∪ {Tg} →
      (* The original map from fields to types. *)
      C[Tt] = typ_struct Tfs →
      (* The map for the new struct and for the grouped fields. *)
      tr_struct_map gt Tfs Tfs' Tfsg →
      C'[Tt] = typ_struct Tfs' →
      C'[Tg] = typ_struct Tfsg →
      (* The other type variables stay the same. *)
      (∀ T ∈ dom C \ {Tt}, C'[T] = C[T]) →
      tr_typdefctx gt C C'.
```

Field grouping - main theorem

In the end the theorem that we prove is:

```
Theorem red_tr: ∀gt LLC C C' t t' v m2,
  red C LLC empty_stack empty_state t m2 v →
  group_tr_ok gt C →
  tr_typdefctx gt C C' →
  tr_trm gt t t' →
  wf_typdefctx C →
  wf_trm C t →
  ~is_error v →
  ∃v' m2',   tr_val gt v v'
          ∧  tr_state gt m2 m2'
          ∧  red C' LLC S' m1' t' m2' v'.
```

Field grouping - induction

To make the proof work we strengthen it as follows:

```
Theorem red_tr_ind: ∀gt LLC C C' t t' v S S' m1 m1' m2,
  red C LLC S m1 t m2 v →
  group_tr_ok gt C →
  tr_typdefctx gt C C' →
  tr_trm gt t t' →
  tr_stack gt S S' →
  tr_state gt m1 m1' →
  wf_typdefctx C →
  wf_trm C t →
  wf_stack C S →
  wf_state C m1 →
  ~is_error v →
  ∃v' m2',    tr_val gt v v'
          ∧  tr_state gt m2 m2'
          ∧  red C' LLC S' m1' t' m2' v'.
```

Array tiling

We need to know:

- The name of the array being changed (`Ta`).
- The new name for the tiles (`Tt`).
- The size of the tiles (`K`).

Similarly, we also define:

- `tiling_tr_ok`,
- `tr_typdefctx`,
- `tr_accesses`,
- `tr_val`,
- `tr_stack`,
- `tr_state` and
- `tr_trm`.

In this case, we change all the instances of `t[i]` to `t[i/K][i%K]` where `t` has type `typ_var Ta`.

Array tiling - some specifics

We use:

- I for the length of the original array,
- J for the length of the array of tiles and
- K for the length of the tile.

These are related by the definitions:

```
Definition nb_tiles (K I J:int) : Prop :=
  J = I / K + If (I mod K = 0) then 0 else 1.

Definition tiled_indices (I J K i j k:int) : Prop :=
      i = j * K + k
   ∧  index I i
   ∧  index J j
   ∧  index K k.
```

Array tiling - key components

The crucial case of tr_val from the array aI to aJ is captured by:

$$\forall \texttt{i j k aK},\ \texttt{tiled\_indices I J K i j k} \rightarrow$$
$$\texttt{aJ[j]} = (\texttt{val\_array (typ\_var Tt) aK}) \rightarrow$$
$$\texttt{tr\_val tt aI[i] aK[k]}$$

For the translation accesses and primitive operations, the aim is for all the accesses

$\texttt{l1} \mathbin{++} (\texttt{access\_array (typ\_var Ta) i})::\texttt{l2}$

to be transformed to

$\texttt{l1} \mathbin{++} (\texttt{access\_array (typ\_var Ta) (i/K)})::(\texttt{access\_array Tt (i mod K)})::\texttt{l2}.$

High-level transformations - summary

So far we have presented:

- Field grouping.
- Array tiling.
- AoS to SoA.

The correctness of these is proved (up to a couple axioms).

Problem: This could all be just a hack if we don't link it with a more concrete, CompCert-style semantics...

High-to-low level transformation

The grammar is extended with:

```
Inductive val : Type :=
  | val_concrete_ptr : loc → offset → val
  | val_words : list word → val.

Inductive prim : Type :=
  | prim_ll_get : typ → prim
  | prim_ll_set : typ → prim
  | prim_ll_new : typ → prim
  | prim_ll_access : typ → prim.
```

There are two sides of this transformation:

- The memory.
- The programs.

High-to-low level transformation - OK

We need to ensure consistency between the type definition context (`C`) and the low-level context (`LLC`). In particular:

- The type variable sizes in `LLC` math with the types in `C`.
- The field offsets match with the order of the fields and the sizes of each of their types.

High-to-low level transformation - memory

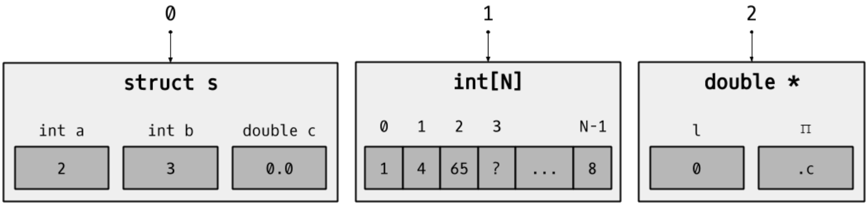


Figure: High-level memory.

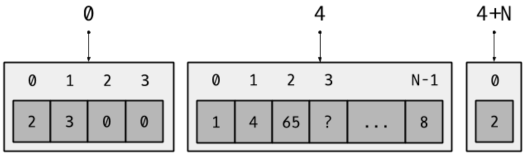


Figure: Low-level memory.

High-to-low level transformation - program

TODO: Details on the transformation of terms.

High-to-low level transformation - proof

TODO: Main theorem.

Project extent

Accomplished goals:

- Defined a high-level language convenient to argue about data-layout transformations.
- Found a way to connect it to realistic low-level semantics.
- Basically proved the correctness of:
  - Field grouping.
  - Array tiling.
  - AoS to SoA.

Some statistics:

lines of spec	lines of proof	lines of comments
2637	2623	606

Future work

Next steps:

- The transformation ‘adding indirection’.
- Realizations of the transformations as functions.
- Some arithmetic results in the tiling and low-level transformations.
- Work on loops and add loop transformations.
- Connect the low-level language with CompCert.

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