Verification of Data Layout Transformations

Ramon Fernández Mir

with Arthur Charguéraud

Inria

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Motivating example

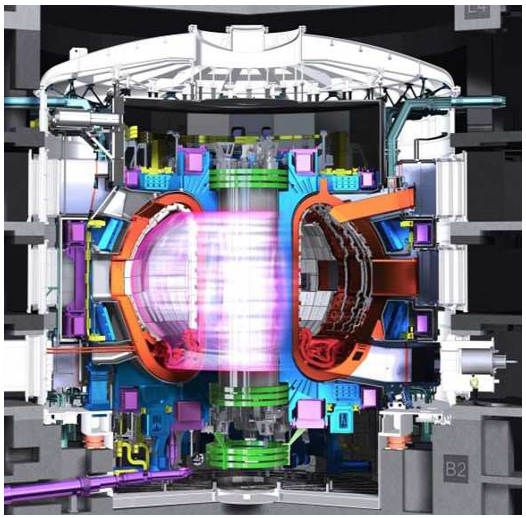


Figure: ITER tokamak

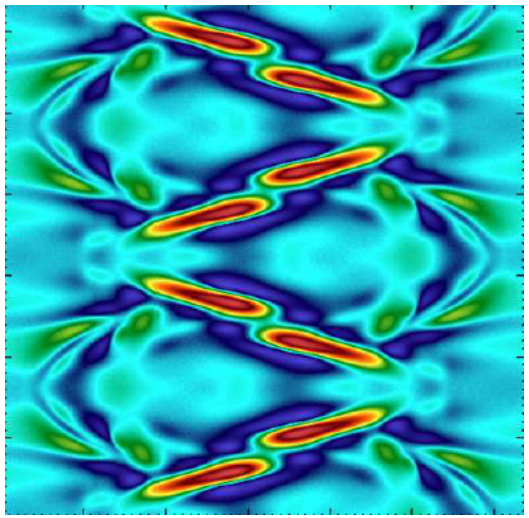


Figure: Plasma physics

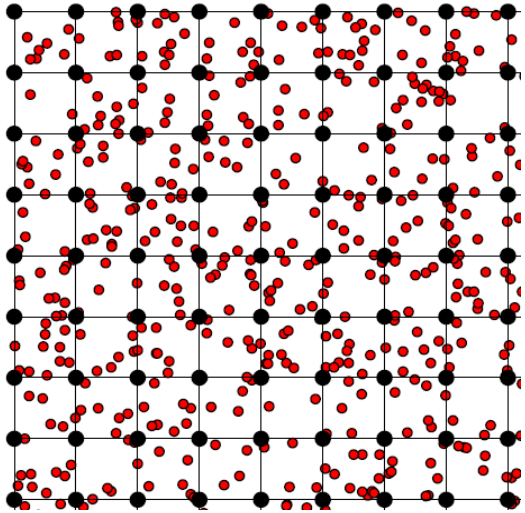


Figure: PIC simulation

Challenges:

- Exploit data-level parallelism.
- Use domain-specific knowledge of the code.
- Do it without introducing any bugs.

Motivating example - initial code

```
typedef struct {
  // Position
  float x, y, z;
  // Other fields
  float vx, vy, vz, c, m, v;
} particle;

particle data[NUM_PARTICLES];

for (int i = 0; i < NUM_PARTICLES; i++) {
  // Some calculation
}
```

Motivating example - splitting

Suppose that the calculation uses mainly the position.

```
typedef struct {
  float vx, vy, vz, c, m, v;
} cold_fields;

typedef struct {
  float x, y, z;
  cold_fields *other;
} particle;

particle data[NUM_PARTICLES];
```

Motivating example - peeling

Further suppose that the intial 'particle' record is not used as part of a dynamic data structure.

```
typedef struct {
  float vx, vy, vz, c, m, v;
} cold_fields;

typedef struct {
  float x, y, z;
} hot_fields;

cold_fields other_data[NUM_PARTICLES];
hot_fields pos_data[NUM_PARTICLES];
```

Motivating example - AoS to SoA

Now, say that we want to take advantage of vector instructions.

```
typedef struct {
  float x[NUM_PARTICLES];
  float y[NUM_PARTICLES];
  float z[NUM_PARTICLES];
} hot_fields;

hot_fields pos_data;
```

Motivating example - AoS to AoSoA

But without reducing too much the locality between accesses to fields of the original struct.

```
typedef struct {
  float x[N];
  float y[N];
  float z[N];
} hot_fields;

hot_fields pos_data[NUM_PARTICLES / N];
```

Motivating example - summary

In short, the transformations we have seen are:

- Splitting.
- Peeling.
- AoS to SoA.
- AoS to AoSoA.

Note that after all these changes, where we wrote:

```
data[i].x
```

Now we have to write:

```
pos_data[i / N].x[i % N]
```

Project goals

- Find the basic transformations that combined give rise to the ones we are interested in.
- Formalize a C-like language with arrays, structs and pointers.
  - On a high-level, to simplify the proofs.
  - On a low-level, to be closer to the semantics of C.
- Define the transformations and prove their correctness.

Basic transformations

1. Field grouping

```
// Before
typedef struct {
  int a, b, c;
} s;

// After
typedef struct {
  int b, c;
} sg;

typedef struct {
  int a; sg fg;
} s';
```

2. Array tiling

```
// Before
int a[N];

// After
int a'[N / B][B];
```

3. Adding indirection

```
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a; int *b;
} s';
```

4. AoS to SoA

```
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a[N]; int b[N];
} s;
```

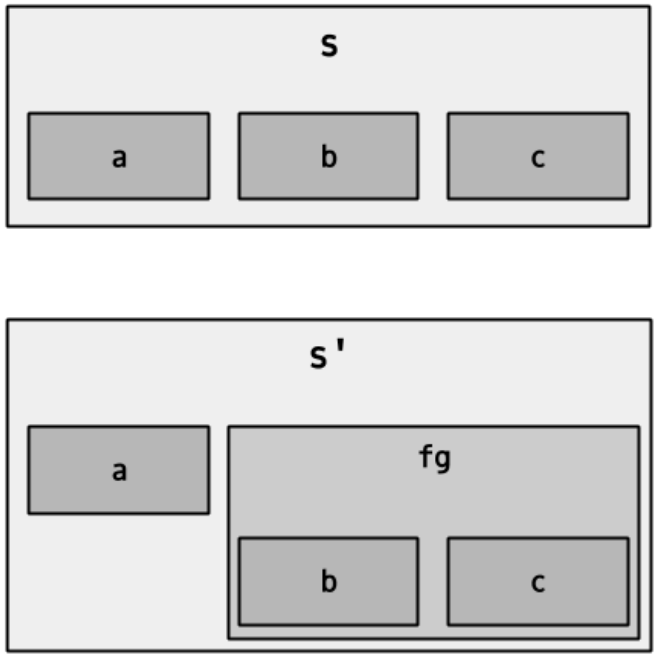
Basic transformations - grouping

1. Field grouping

```
// Before
typedef struct {
  int a, b, c;
} s;

// After
typedef struct {
  int b, c;
} sg;

typedef struct {
  int a; sg fg;
} s';
```

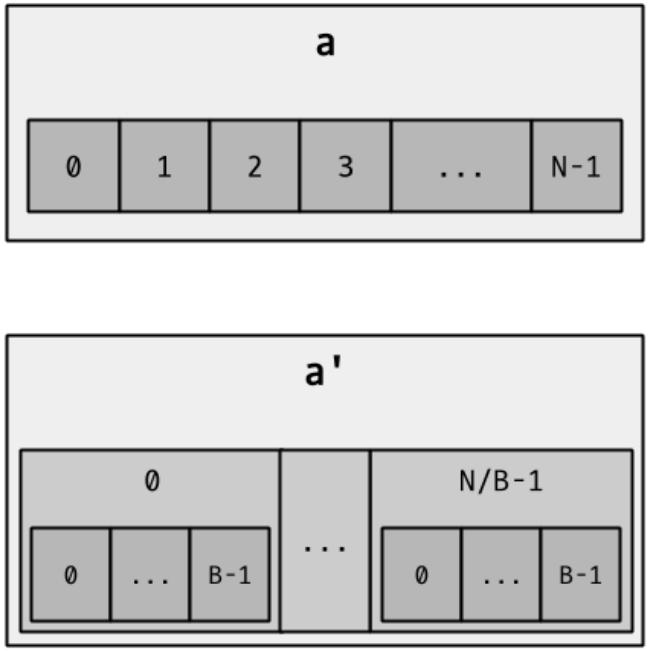


Basic transformations - tiling

2. Array tiling

```
// Before
int a[N];

// After
int a'[N/B][B];
```

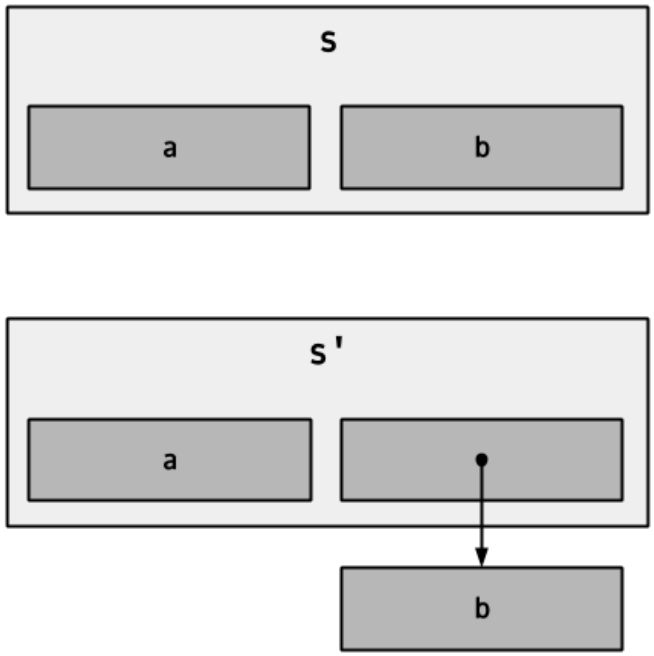


Basic transformations - indirection

3. Adding indirection

```
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a; int *b;
} s';
```

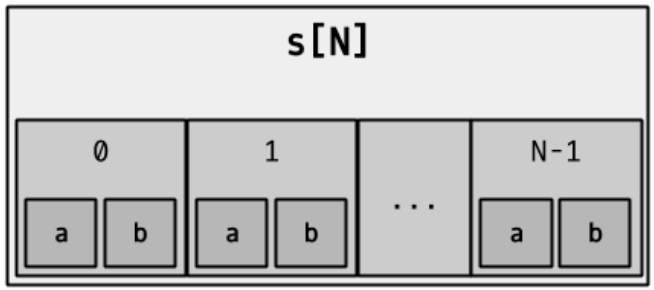


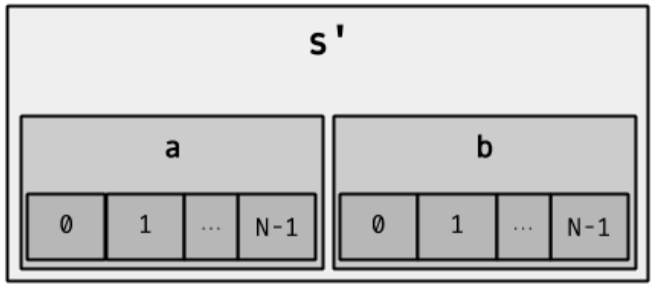
Basic transformations - AoS to SoA

4. AoS to SoA

```
// Before
typedef struct {
  int a, b;
} s;

// After
typedef struct {
  int a[N]; int b[N];
} s;
```





Basic transformations - justification

- **Peeling:** Field grouping twice.
- **Splitting:** Field grouping and then adding indirection on the field holding the group.
- **AoS to SoA:** AoS to SoA.
- **AoS to AoSoA:** Array tiling and then AoS to SoA on the tiles.

Language overview - values and terms

```
Inductive val : Type :=
  | val_error : val
  | val_unit : val
  | val_uninitialized : val
  | val_bool : bool → val
  | val_int : int → val
  | val_double : int → val
  | val_abstract_ptr : loc → accesses → val
  | val_array : typ → list val → val
  | val_struct : typ → map field val → val

Inductive trm : Type :=
  | trm_var : var → trm
  | trm_val : val → trm
  | trm_if : trm → trm → trm → trm
  | trm_let : bind → trm → trm → trm
  | trm_app : prim → list trm → trm
  | trm_while : trm → trm → trm
  | trm_for : var → val → val → trm → trm.
```

Language overview - primitive operations

```
Inductive prim : Type :=
  | prim_binop : binop → prim
  | prim_get : typ → prim
  | prim_set : typ → prim
  | prim_new : typ → prim
  | prim_new_array : typ → prim
  | prim_struct_access : typ → field → prim
  | prim_array_access : typ → prim
  | prim_struct_get : typ → field → prim
  | prim_array_get : typ → prim
```

Examples of the semantics of our language compared to C:

```
get p   : *p                        array_access p i  : p + i
set p v : *p = v                    struct_access p f : &(p->f)
new T   : malloc(sizeof(T))         struct_get s f    : s.f
```

where pointers are represented as pairs:

```
(l, [access_field T f, access_array T' i])
```

which would correspond to the address:

```
l + field_offset(f) + i * sizeof(T')
```

Language overview - semantics

Some crucial definitions:

```
Definition typdefctx := map typvar typ.

Record ll_typdefctx := make_ll_typdefctx {
  typvar_sizes   : map typvar size;
  fields_offsets : map typvar (map field offset);
  fields_order   : map typvar (list field) }.

Definition stack := Ctx.ctx val.

Definition state := map loc val.
```

And the relation that defines the big-step reduction rules:

```
red ⊆ typdefctx × ll_typdefctx × stack × state × trm × state × val
```

Language overview - typing

The allowed types are:

```
Inductive typ : Type :=
  | typ_unit : typ
  | typ_int : typ
  | typ_double : typ
  | typ_bool : typ
  | typ_ptr : typ → typ
  | typ_array : typ → option size → typ
  | typ_struct : map field typ → typ
  | typ_var : typvar → typ.
```

With their corresponding definitions (analogous to stack and state):

```
Definition gamma : Ctx.ctx typ.

Definition phi : map loc typ.
```

Typing is defined as the following relation:

$$\texttt{typing} \subseteq \texttt{typdefctx} \times \texttt{gamma} \times \texttt{phi} \times \texttt{trm} \times \texttt{typ}$$

Language overview - properties

Need to think of something...
An approximation to type safety:

```
Theorem type_soundness : ∀C LLC m t v T,
  red C LLC nil empty t m v →
  typing C nil empty t T →
  ∃f, typing_val C f v T
   ∧ state_typing C f m.
```

Field grouping

The arguments of the transformation are:

- The name of the struct being changed.
- The fields being grouped.
- The name of the new struct that will contain said fields.
- The new field holding the new struct.

These are used to define a transformation for:

- type definitions contexts,
- accesses,
- values,
- terms,
- states and
- stacks.

Field grouping - typdefctx

We 'update' the type definitions context as follows:

```
Inductive tr_typdefctx (gt:group_tr) : typdefctx → typdefctx → Prop :=
  | tr_typdefctx_intro : ∀Tfs Tfs' Tfsg Tt fs Tg fg C C',
      gt = make_group_tr Tt fs Tg fg →
      dom C' = dom C ∪ {Tg} →
      (* The original map from fields to types. *)
      C[Tt] = typ_struct Tfs →
      (* The map for the new struct and for the grouped fields. *)
      tr_struct_map gt Tfs Tfs' Tfsg →
      C'[Tt] = typ_struct Tfs' →
      C'[Tg] = typ_struct Tfsg →
      (* The other type variables stay the same. *)
      (∀ T,
        T ∈ dom C →
        T ≠Tt →
        C'[T] = C[T]) →
      tr_typdefctx gt C C'.
```

Field grouping - accesses

For accesses, if we look at the interesting case:

```
Inductive tr_accesses (gt:group_tr) : accesses → accesses → Prop :=
  | tr_accesses_field_group : ∀Tt fs fg Tg f a0 p a1 a2 p',
      gt = make_group_tr Tt fs Tg fg →
      f ∈ fs →
      (* The access s.f *)
      a0 = access_field (typ_var Tt) f →
      (* Becomes s'.fg.f *)
      a1 = access_field (typ_var Tt) fg →
      a2 = access_field (typ_var Tg) f →
      tr_accesses gt p p' →
      tr_accesses gt (a0::p) (a1::a2::p')
```

This is used in:

```
Inductive tr_val (gt:group_tr) : val → val → Prop :=
  | tr_val_abstract_ptr : ∀l p p',
      tr_accesses gt p p' →
      tr_val gt (val_abstract_ptr l p) (val_abstract_ptr l p')
```

Transformations - tiling

tiling

Transformations - AoS to SoA

AoS to SoA

Transformations - proof

statement and proof

High-to-low level transformation

A few slides on this.

Project extent

what has been done and what hasn't quite and statistics

Future work

for instance functions etc, combining them. Code realisations...

Conclusion

conclusion