Advanced Internal Models

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Credit Risk parameters estimation

The distinction between 'model development' and 'calibration'

Estimation of risk parameters: The full modelling process related to the risk parameters including the selection and preparation of data, model development and calibration:

1. **Model development**: The part of the process of the estimation of risk parameters that leads to an appropriate risk differentiation by specifying relevant risk drivers, building statistical or mechanical methods to assign exposures to obligor or facility grades or pools, and estimating intermediate parameters of the model, where relevant
2. **Calibration**: The part of the process of the estimation of risk parameters which leads to appropriate risk quantification by ensuring that when the PD ranking or pooling method is applied to a calibration sample, the resulting PD estimates correspond to the long-run average default rate at the level relevant for the applied method

Topic	**Model Development (Risk differentiation)**	**Calibration (Risk quantification)**
Definition of default	Own choice, ensuring discriminatory power is not hindered under the regulatory definition	CRR, Article 178
Sample depth	Typically short, representative of the application portfolio	Typically long and coincident with the historical observation period
Exclusions	Allowed	Not allowed, usually addressed through risk-drivers
Representativeness tests	Scope of application, definition of default, distribution of risk characteristics, lending standards and recovery policies	In addition, current and foreseeable economic and market conditions
Margin of conservatism	No	Yes

Models should be based on the institutions' internal losses experience:

- PD: at least 5 years
- LGD: at least 5 years (retail) and 7 years (non-retail)
- EAD: at least 5 years (retail) and 7 years (non-retail)

Definition of Default

Before, to measure default, banks used missed instalments. For example if a customer missed 5 instalments in his mortgage. Now a default shall be considered to have occurred with regard to a particular obligor when either or both of the following have taken place:

1. The institution considers that the obligor is unlikely to pay its credit obligations to the institution, the parent undertaking or any of its subsidiaries in full, without recourse by the institution to actions such as realising security

2. The obligor is past due more than 90 days on any material credit obligation to the institution, the parent undertaking or any of its subsidiaries. Competent authorities may replace the 90 days with 180 days for exposures secured by residential or SME commercial real estate in the retail exposure class, as well as exposures to public sector entities). The 180 days shall not apply for the purposes of Article 127

Definition of materiality thresholds

If a borrower is past due in 1€ it is not necessarily in default because the amount is not material. National competent authorities were mandated to establish the thresholds relevant for their jurisdiction, considering as limits:

- Relative threshold: 1 % (both for retail and non-retail)
- Absolute threshold: 100 € (retail) and 500 € (non-retail)

Amount considered is Nominal Value of loan left to pay + accrual interest. (accounting value)

A default is considered to have occurred whenever the materiality thresholds (both the absolute and the relative) are breached for 90 consecutive days. And both threshold have to be met.

For corporate borrowers, if they default in the payment of one loan they are considered in default. For retail clients, they can have the credit card in default but not the mortgage for example.

- The definition of default must be the same across all risk parameters (i.e. if a facility is in default for PD estimation, it is so for EAD and LGD modelling)
- The definition of default must be consistent over time. In case of changes within the historical observation period adequate adjustments must be proposed to make the default definition homogeneous over time. In addition, a margin of conservatism (MoC) must be applied.
- The definition of default used for modelling purposes must be the same as that used in the application of risk parameters to calculate capital requirements.

3.1. Estimation of Risk Parameters

Unlike for retail exposures, where there are millions of defaults, for some corporate exposures there might not be enough data to estimate the LGD

3.1.1. Grade level vs calibration segment level approach

In credit risk parameter estimation, the terms *grade level* and *calibration segment level* refer to different approaches or dimensions in modeling and calibrating risk parameters such as Probability of Default (PD), Loss Given Default (LGD), and Exposure at Default (EAD).

Grade Level

Grade level refers to the classification of exposures based on credit ratings or scores. In this context, borrowers or exposures are grouped into different grades based on their creditworthiness. Each grade typically corresponds to a specific risk level.

- **Credit Ratings**: These grades can be derived from internal rating systems, external rating agencies, or credit scoring models. Each rating grade (e.g., AAA, AA, A, BBB, etc.) indicates a certain level of credit risk.

- **Parameter Estimation**: At the grade level, risk parameters like PD, LGD, and EAD are estimated for each grade. For example, the PD for grade A might be different from the PD for grade BBB.
- **Granularity**: This method provides a granular approach to risk differentiation, allowing for more tailored risk management and capital allocation.

Calibration Segment Level

Calibration segment level involves grouping exposures based on specific characteristics or segmentation criteria that are relevant to risk assessment and parameter calibration. These segments are often determined by factors such as industry, geography, product type, or other relevant risk drivers.

- **Segmentation Criteria**: Exposures are divided into segments based on shared characteristics. For instance, segments could be defined by industry (e.g., retail, manufacturing), geographic location (e.g., country, region), or product type (e.g., mortgage, credit card).
- **Parameter Calibration**: Risk parameters are then calibrated for each segment. For example, the LGD might be calibrated separately for mortgages and credit card debt, reflecting differences in loss patterns and recovery processes.
- **Heterogeneity Management**: This approach helps manage the heterogeneity within the portfolio by acknowledging that different segments might exhibit distinct risk behaviors.

Differences and Usage

- **Purpose**: Grade level calibration is typically used to account for differences in creditworthiness, whereas calibration segment level is used to account for differences in exposure characteristics.
- **Granularity**: Grade level provides a more nuanced view of credit risk based on ratings, whereas calibration segment level provides insights based on other relevant risk characteristics.
- **Application**: In practice, both methods can be used together. For example, within a calibration segment (e.g., retail loans), exposures might still be further divided by grade (e.g., high, medium, low credit scores).

Example in Practice

Imagine a bank estimating PD for its corporate loan portfolio:

- **Grade Level**: The bank might have an internal rating system with grades from 1 to 10. PD is estimated for each grade, with lower grades indicating lower PD and higher grades indicating higher PD.
- **Calibration Segment Level**: The same portfolio might be segmented by industry (e.g., technology, healthcare, manufacturing). PD estimates are adjusted for each industry segment to reflect industry-specific risk factors.

By combining these approaches, the bank can achieve a more comprehensive and accurate estimation of credit risk parameters, ensuring that both the creditworthiness of borrowers and the characteristics of exposures are adequately captured in the risk assessment process.

3.1.2. Probability of Default (PD)

The regulatory PD is a long-run PD, which means taking an average of the default rate over a sufficiently large period of time. The confidence level is 99.99 % and the time horizon is 1-year.

With products with the same scoring to all of them, the PD will vary.

Other features

- It is the result of a time series average, not a default-weighted average (in which each default counts the same).
- At least five years are required to estimate regulatory PDs. And usually banks use between 20-25 years. To be able to compare and use good and bad economic periods.

- Ultimately, it depends on the specific attributes of the facility/obligor (fundamentally the scoring or rating model). Facilities sharing credit risk characteristics are grouped into risk grades to which the same PD is assigned.
- But the PD can be calculated at a more aggregate level, such as portfolio level/calibration segment level, and afterwards distributed across grades.
- And in fact, interpretations surrounding the regulatory long-run PD from a conceptual standpoint remain unsolved... ECB has just published a consultative version to update the ECB Guide

Some technical comments

1. No exclusions are allowed, only 'errors' should be considered (i.e. wrong default assignment or wrong rating model assignment).
2. Any data cleansing activity should be properly justified.
3. Multiple defaults: defaults should only be counted once.
4. Default should be tracked even if the facility/obligor 'leaves' the portfolio, for instance in the case of restructuring events.
5. Care should be taken that the default rate is not biased by events such as portfolio sales or massive (punctual) repayments.

Points 3. and 4. mean that in the denominator you take into account the obligors who are not defaulted at the beginning of the period and had at least one default event during the one-year observation period. The customer does not need to be at default at the end of the period to be counted, just having incurred in default at any time is enough.

Broadly speaking there are two main approaches:

- **Grade level approach**: the long-run average default rates are calculated at risk grade level
- **Portfolio or calibration segment level approach**: the long-run average default rates are calculated at portfolio level (or at calibration segment level).

A calibration segment is defined as similar structural characteristics that define the risk profile (example: LTV for mortgages, industry for SMEs)

Calibration segments should have **time consistency** and **business sense**. A general analysis should be done:

- Percentage of missing values at each reference date.
- Percentage of outliers at each reference date.
- Basic key statistics such as the mean, the median, the standard deviation, or relevant percentiles.

Calibration segments should also have **discriminatory power**.

- After choosing an adequate statistic, the discriminatory power should be measured over time.

Calibration segments: assessment

- Visual inspection of default rates over time, focusing on:
  - Behaviour during downturn periods (which segments are more sensitive to macroeconomic conditions?)
  - Behaviour today (which segments perform worse today?)
  - Separation between default rates (are default rates of the different calibration segments consistently separated over time?)
- Overall discriminatory power over the whole period
- Distribution of exposures over time (percentage of facilities/obligors in each segment)

- Concentration (which may be natural...)

3.1.3. Loss Given Default (LGD)

Economic loss is defined in the GLs on PD and LGD estimation as the following:

$$\textit{Economic Loss} = \textit{Debt at Default} + \textit{Costs - Recoveries} \tag{3.1}$$

LGD has to be estimated using data from the downturn period (if it is more conservative than the long run average). At least 5 years of internal data are required for retail portfolios and 7 years for non-retail portfolios.

3.1.4. Key features

The estimation is performed at facility level, although considering, for non-retail portfolios, all defaults from an obligor perspective (estimation at obligor level allowed by ECB under strict conditions).

Economic loss as opposed to accounting loss: the timing of losses matters via the use of discount factors. In addition, costs, direct and indirect, have to be added.

The estimation generally considers **the existence of collateral**, distinguishing:

- Secured exposures (example: mortgages).
- Unsecured exposures (example: credit cards).

Lifetime view of the parameter: once the facility enters in default costs and recoveries are tracked until the end of the default event (even if the facility is restructured).

The following types of LGD have to be estimated:

- Not in default: long-run LGD and downturn LGD.
- In default: best estimate expected loss (ELBE), long-run LGD and downturn LGD

$$\textit{Realised LGD} = \frac{\textit{Initial debt} - \sum_{t=t_0}^{T}[PV(\textit{Recoveries}_t) - PV(\textit{Costs}_t)]}{\textit{Initial debt}} \tag{3.2}$$

Foundation: Just estimating PD for a portfolio

Advanced approach: Estimating PD and LGD

Some technical comments

- Independent default cycles should be identified. At least 9 months should pass after the probation period to consider that a re-entry into default can be considered as an additional default event
- LGD should be calculated even if the facility 'cures' repaying missed instalments: an 'artificial flow' is added considering the outstanding balance at the point of default exit
- Additional drawings after default should be considered as economic loss:
  - Retail: in LGD or CCF estimation
  - Non-retail: in CCF estimation
- Following the latest regulation the discount rate should be 3-month EURIBOR (or equivalent reference rate) at the point of entry into default + 500 basis points
- Both direct and indirect costs should be considered:
  - Direct costs: directly attributable to each individual default (sale costs, legal costs, etc.).

- Indirect costs: available at pool level such as staff, building, software, etc.

- Realized LGD estimation should be floored at 0 % at facility level for estimation purposes.
- Calculation can become extremely complex when restructuring processes are involved: need to devise allocation mechanisms, for instance, in N to 1 processes.
- Collateral should be subject to some legal certainty and valuation conditions.
- Collateral should be accounted for at the repossession date, considering the minimum between the debt reduction and the value the collateral is registered within the books.
- A haircut should be considered to account for differences between the collateral value at repossession and the market value, considering the effects of maintenance costs and the discount factor from repossession to sale.

3.1.5. Model Design

Grade level vs Calibration segment level:

- Generally grade level approaches are used (it requires that the drivers are available during the historical period)
- In any case, calibration tests need to be applied focusing especially in whether estimates result in cyclical results (for instance, if using market-based updated value of properties)

Direct estimate vs use of intermediate components (model components approach):

$$LGD = Prob(loss) \cdot LossGivenLoss + Prob(Cure) \cdot LossGivenCure \tag{3.3}$$

Use of structural formula / components (there may be alternative specifications):

$$LGD = 1 + \frac{PV(costs)}{Initial\ Debt} - \frac{min(Initial\ Debt, Property\ value \cdot (1 - Haircut) \cdot Downturn\ factor)}{Initial\ Debt} \tag{3.4}$$

3.1.6. Model development

1. **Identification of candidates:**
   - Retail Mortgages: Loan-to-value (almost mandatory), time on books, balance, % amortised, region, etc.
   - Retail unsecured: type of product, balance, time on books, etc.
   - Non-retail: type of customer, type of collateral and valuation, type of product, etc.
2. **Assessment of the adequacy of the information over time:** Time consistency and business sense during the whole historical period.
3. **Analysis of univariate discriminatory power**:
   - Can we expect consistent discriminatory power over time?
   - If not, which drivers should prevail: those more discriminant today or those more discriminant during the downturn period?
4. Combination of drivers using **multivariate techniques**
5. **Contrast / Assessment / Validation of the results**: generally visual inspection of the results, their behaviour over time and the impact on the volatility of capital requirements
6. **Recoveries may take a long time**. should we consider only ‘closed’ defaults’? All defaults? Defaults with enough performance period even if there are still unresolved cases?

7. Is it a problem if drivers discriminant over the downturn period are no longer that discriminant?

3.1.7. Model calibration

The LGD is a **Default weighed average**: it refers to just the average LGD considering all the defaults in the sample (each default counts once).

Identification of downturn periods

Three key dimensions:

- **Nature**: Identification of the specific economic factors affecting losses in a downturn period
- **Severity**: Most severe value relating to a 12-months period of an economic factor
- **Duration**: A 12-month minimum duration for each downturn period specified. This duration could be enlarged if:
  - There is persistence of the most severe values in a period longer than 12 months
  - Most severe values of different economic factors are attained at different time periods, but refer to the same economic downturn

In case of **several economic downturns**, the one producing more conservative results should be selected.

3.1.8. ELBE and DT LGD in default

ELBE:

- Reflects current economic circumstances
- No MoC
- Should be closely aligned to IFRS 9 LGD in-default

Downturn LGD-in-default:

- Reflects downturn conditions
- Several MoCs associated
- Drivers should be the same as for ELBE

RW for defaulted facilities calculated as:

$$RW = 12{,}5 \cdot max(0, DT\ LGD\ \text{-}\ ELBE) \tag{3.5}$$

3.2. Exposure at Default (EAD)

EAD parameter should be calculated as representative of downturn conditions if the downturn. At least 5 years of internal data are required for retail portfolios and 7 years for non-retail portfolios.

The estimation is performed at facility level, although considering, for non-retail portfolios, all defaults from an obligor perspective.The estimation is restricted to certain types of exposures or products such as:

- Credit lines.
- Credit cards.
- Revolving loans.
- Bank accounts (to the extent off-balance exposures are available) or overdrafts with authorised limits.
- Real estate loans (with off-balance exposures).

At facility level, **CCFs may be negative**, but at an assignment level (i.e. estimated parameter) the EAD cannot be less than the current exposure. This is an aspect subject to discussion under CRR III.

The CCF was not included in the EBA GLs on risk parameters estimation (although included in the ECB Guide). EBA has been mandated to set specific guidelines by 2027.

3.2.1. The realised credit conversion factor

Two main possibilities:

- **Fixed horizon** approach: select the normal or reference conditions 12 months before the default date. Implicitly assumes that default will happen at the 12th month. It may potentially lead to over-conservatism.
- **Variable horizon** approach: select all moments within the previous 12 months to default. Less representative CCFs. Care should be taken by selecting adequate drivers.

Basel III prescribes the use of the fixed horizon (i.e. 12th month previous to default as 'normal' situation).

Potentially unstable CCFs given some circumstances that lead to the appearance of a significant volume of outliers (high percentage of use)

	Value
Drawn	2,990
Undrawn	10
Balance at Default	**3,000**
CCF	**100%**

	Value
Drawn	2,990
Undrawn	10
Balance at Default	**3,010**
CCF	**200%**

	Value
Drawn	2,990
Undrawn	10
Balance at Default	**3,050**
CCF	**600%**

Economically different?

The option is to calculate EAD in such cases with another formula: $EAD = CCF \cdot Drawn\ Balance$

	Value
Drawn	2,990
Balance at Default	**3,000**
CCF	**100.33%**

	Value
Drawn	2,990
Balance at Default	**3,010**
CCF	**100.67%**

	Value
Drawn	2,990
Balance at Default	**3,050**
CCF	**102.01%**

3.2.2. Model design

Generally a **grade level approach** is used, CCFs are directly calculated at risk grade level although the use of a calibration at segment level is not forbidden by the regulation.

The grade level approach is expected to be more risk sensitive, whereas it is not expected to generate undue volatility in capital requirements.

In any case, as in LGD, it is convenient to analyse the degree of cyclicality of the estimates.

The role of calibration segments is not as relevant as in PD, and they may just refer to different products (example: credit cards vs credit lines in an SME portfolio)

3.2.3. Model development

Same as other risk parameters.

1. **Identification of candidates:**
   - Examples: type of product, drawn, undrawn, percentage of use, balance, rating, arrears indicators, etc.
2. **Assessment of the adequacy of the information over time:** Time consistency and business sense during the whole historical period.

3. **Analysis of univariate discriminatory power**:
   - Can we expect consistent discriminatory power over time?
   - If not, which drivers should prevail: those more discriminant today or those more discriminant during the downturn period?
4. Combination of drivers using **multivariate techniques**
5. **Contrast / Assessment / Validation of the results**: generally visual inspection of the results, their behaviour over time and the impact on the volatility of capital requirements

3.2.4. Calibration

CCFs should be estimated by facility grade or pool using the default weighted average resulting from all observed defaults within the data sources.

Two different calibrations should be considered:

- Long-run calibration: considering all defaults in the historical observation period
- Downturn calibration: considering defaults related to the downturn period

Generally, no significant sensitivity to macroeconomic conditions is detected. As opposed to other parameters, the entities can more easily influence the behaviour of clients by restricting limits and CCFs may be lower in a downturn

3.3. Credit Risk Mitigation Techniques (CRMT)

One of the key aspects to consider collateral in the estimates is the **eligibility**:

- **Legal Certainty:** the collateral/credit protection arrangement is legally effective and enforceable in all relevant jurisdictions
- **Collateral Valuation:** the collateral should be regularly valued and satisfy certain requirements (independent valuation, etc.) in order to be used for mitigation purposes
- **Sound risk management practices** (i.e. identification of the source of cash flows in the event of recovery)

Consideration of FCP (i.e. collateral):

- Mitigation should be introduced via LGD modelling according to risk drivers and/or segmentation
- A key aspect is the estimation of haircuts (to what extent the cash flows obtained from the sale of collateral are lower than the market value)

Consideration of UFCP (i.e. guarantor). Two main possibilities:

1. Consideration in LGD modelling in a similar way as FCP

2. Substitution of PD and LGD of direct comparable exposure to the guarantor (i.e. assign the parameters of the guarantor

Capítulo 4

Other Credit Risk Models

Key ideas for the successful implementation of IRB models

- Pursuing a high-quality implementation of IFRS 9 following additional guidance issued by supervisors/regulatorsand other international bodies (EBA, NCAs, BIS, etc.). This implies considering issues such as granularity,idiosyncratic aspects, macroeconomic and forward-looking perspective, etc.
- Ensuring adequate management involvement in all the steps of the modelling process (planning, development andimplementation) to guarantee management integration of IFRS 9 outputs
- Ensuring consistency with existing approaches (IRB, stress test, ratings / scorings, pricing, etc.) within a single,comprehensive and coherent credit risk modelling framework
  - **Rating / Scoring models:** basis for rank-ordering of PD models
  - **IRB parameters:** leverage on data and on several parameters (CCF, ELBE, etc.) appropriately adjusted to reflect IFRS 9 specifities
  - **Stress testing models:** basis for portfolios' macroeconomic sensitivities
- Striking an appropriate balance between compliance with IFRS 9 premises and modelling simplicity (avoidingundue over-complexity) tailored to the characteristics of each portfolio
- Ensuring that risk parameters can be monitored in an easy and transparent way that facilitates taking correctiveactions if deviations are observed

Basel	IFRS 9
One-year horizon	One-year and lifetime horizon
90 days past due	90 days past due (rebuttable presumption)
Long-run and downturn view	Forward-looking view
Static view	Amortising nature
Fundamentally rules-based	Fundamentally principle-based

$$LEL_T = \sum_{t=1}^{T} = \frac{Survivalrate_{t-1} \cdot PD_t \cdot EAD_t \cdot LGT_t}{(1+EIR)^t}$$

Where:

- $\boldsymbol{Survivalrate_t} = \boldsymbol{Survivalrate_{t-1}} \cdot (1 - PD_t - PFR_t)$. The probability that the account will remain in the books not having defaulted before time t.
- $\boldsymbol{PD_t}$: The probability that, conditional to survival, the account will default at time t.
- $\boldsymbol{PFR_t}$: The probability that, conditional to survival, the account will be fully repaid at time t.
- $\boldsymbol{EAD_t}$: The expected exposure the account will have if the account was to enter default at time t.
- $\boldsymbol{LGD_t}$: The loss given default if default occurs at time t.
- $\boldsymbol{EIR}$: Effective interest rate.
- $\boldsymbol{t}$: The future dates after the reporting date in which the account can default.
- $\boldsymbol{T}$: The contractual term of the financial instrument.

EAD for products such as **credit cards or credit lines: conversión factors need to be estimated** (not much difference against Basel if maturity is not longer than one year)

$$EAD = CCF_1 \cdot Drawn\ amount + CCF_2 \cdot Undrawn\ amount$$

Rating	EAD	PD (1Y)	LGD	Stage	Provision	% Provision	Capital	% Capital
A	150,000,000	0.50%	30%	1	225,000	**9.04%**	2,974,718	**47.58%**
B	30,000,000	4.00%	30%	1	360,000	**14.46%**	2,232,460	**35.71%**
C	7,500,000	15.00%	30%	2	779,980	**31.32%**	999,464	**15.99%**
D	3,750,000	100.00%	30%	3	1,125,000	**45.18%**	45,000	**0.72%**
Total	191,250,000	3.57%	30%		2,489,980	**100.00%**	6,251,642	**100.00%**

Figura 4.1: IFRS9. A complementary view on capital. It is common to see this structure where the capital accumulates in the best quality assets. This is because these are the assets with higher **potential unexpected losses.**

Case 1

The general case, the EAD is calculated as the full drawn amount (CCF1=1) plus a fraction of the undrawn amount (0<CCF2<1).

$$EAD = CCF_1 \cdot Drawn\ amount_{t=0} + CCF_2 \cdot Undrawn\ amount_{t=0} \longrightarrow CCF_2 = \frac{EAD - Drawn}{Undrawn} \longrightarrow$$

$$\longrightarrow CCF_2 = \frac{Drawn_{default} - Drawn_{normal}}{Limit - Drawn_{normal}}$$

Case 2

However, when the drawn amount is close to or exceeds the limit, CCF2 may become extremely volatile, losing any economic sense. In these cases, EAD is approximated using a CCF1 different than 1 and a CCF2 equal to zero, since the undrawn amount is very small.

$$EAD = CCF_1 \cdot Drawn\ amount_{t=0} \longrightarrow CCF_1 = \frac{Drawn_{default}}{Drawn_{normal}}$$

In order to assess whether Case 1 or Case 2 applies, the percentage of use is calculated:

$$\%\ of\ use = \frac{Drawn_{noemal}}{Limit}$$

- When the % of use is below a specified threshold, Case 1 applies.
- When the % of use is above a specified threshold, Case 2 applies.

Suggested values for the threshold are 90 % or 95 %

Sometimes it can be very hard to estimate some things like the **lifetime of credit cards**

4.1. EAD with amortization of installments

$$EAD_t = EAD_t^{contr} \cdot \prod_{k=1}^{t-1}(1 - OF_k)$$

Where:

- EAD_t^{contr} is the contractual EAD at time t
- OF_k is the overpayment factor of the account at each time k before t

It is essential to understand the contractual characteristics of the products along with the incentives obligors have to reduce their debt.

Potential drivers:

- The interest rate paid for the loanç
- Payment benefits during the first years
- Time to maturity / Time on books
- Macroeconomic factors
- The existence of penalties for early redemptions

The **PFR, probability of full redemption or full prepayment,** enters into the IFRS 9 expected loss framework as part of the survival rate term and is the probability of early cancellation of a contract at time t, conditional to having survived until t-1:

$$SR_t = SR_{t-1} \cdot (1 - PD_t - PFR_t)$$

Where:

- SR_t is the survival rate at time t

- PFR_t is the probability of full prepayment at time t

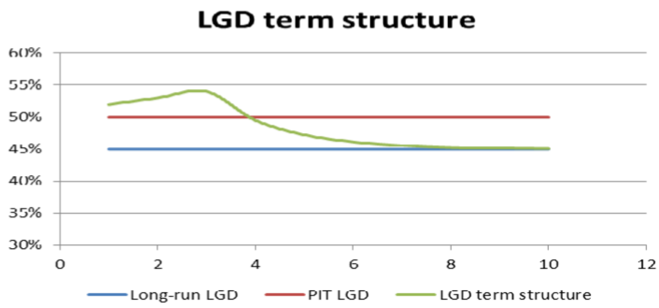
And the PFR has the following characteristics:

- Defined at facility level
- May be estimated in a similar way as PDs
- Similar drivers as in the case of partial prepayments
- The greater the prepayments the lower the ECLs

4.2. LGD - term structure

LGD: the usual work-out approach

$$LGD = \frac{Initial\ Debt - \sum_{t=t_0}^{T}[PV(R_t) - PV(C_t)]}{Initial\ Debt}$$



But...

- The effective interest rate is used to discount cash flows
- No indirect costs are considered
- Estimates may be adjusted to account for macroeconomic conditions
- And the **term structure** must be considered (characteristics may evolve over time (i.e.differences between mortgages and usecured loans)

The use of structural approaches is common, especially when expert parameters are required

1. Loss Given Default:

$$LGD_t = \frac{min(EAD_t^s, EAD_t)}{EAD_t} \cdot LGD_s + \frac{max(EAD_t - EAD_t^s, 0)}{EAD_t} \cdot LGD_u$$

Where:

- LGD_t is the LGD value applicable to the exposure atdefault correspondent to time period t
- EAD_t^s is the adjusted secured portion of the exposure at default
- EAD_t is the total exposure at default
- LGD_s is the LGD value applicable to the secured portion of the exposure. This parameter may be equal to 0 %
- LGD_u is the unsecured LGD estimate

2. Collateral value modelling

$$EAD_s = (1 - Haircut_t) \cdot CV_{t0}$$

Where:

- $Haircut_t$ is the adjustment to the collateral value (modelled or expert)

- CV_{t0} is the collateral value at reporting date

4.3. Key modelling approaches

4.3.1. Top-down: based on aggregated data

$$LN(\frac{PD_t}{1-PD_t}) = \beta_0 + \beta_1 \cdot x_1 + ... + \beta_n \cdot x_n \Longrightarrow PD_t = \frac{1}{1+e^{-(\beta_0+\beta_1 \cdot x_1+...+\beta_n \cdot x_n)}}$$

- Generally specified through a logit transformation to ensure that results are constrained between 0 -1
- The "modelled" variable is an aggregate, for instance, the default rate of the portfolio
- Time series econometric theory is applied to estimate the parameters and general aspects should be satisfied:
  - Proper specification of the model. Linearity?
  - Lack of autocorrelation. Omission of relevant variables?
  - Stationarity of variables. Spurious regression?
  - Consistency of the time series. Structural breaks?

4.3.2. Bottom up: same idea, but based on granular data at facility / obligor level

$$PD = \frac{1}{1+e^{-(\alpha+\beta_1 \cdot Individual\ Variables+\beta_2 \cdot Policy\ Variables+\beta_3 \cdot Macro\ Variables)}}$$

Each facility / obligor is a single observation of the model (in TD, just one observation per period)

Some advantages

- Predictions can be generated at an individual level
- The model adapts more easily to changes in the structure of portfolios
- Interactions between macroeconomic and idiosyncratic factors can be modelled more easily

But...

- Greater computational cost
- Greater modelling complexity

4.3.3. Bottom up in an expanded way: transition matrices

	Rating (t +1)			
Rating (t)	**Low**	**Mid**	**High**	**Default**
Low	75%	9%	6%	10%
Mid	11%	80%	5%	4%
High	3%	6%	90%	1%

- In addition to the probability of default, the models permit to predict distributions conditioned to scenarios

- Several ways to address the estimation, one of the most commons at "row level"
- But again, greater modelling complexity

4.4. Machine Learning Models

The EBA is very conservative on the use of ML models. However there are more and more developments in this field.

Clear benefits are perceived:

- Efficiency delivering financial services: cost reduction
- Inclusiveness: access to credit services by non-usual population
- Consumers may access to more personalised products and services

Whereas some risks are identified:

- Explainability and interpretability
- Fairness and avoidance of bias
- Traceability and auditability
- Data protection
- Consumer rights protection

The most promising use cases across the Banking industry: fraud detection, anti-money laundering, collection strategies.

Current IRB regulatory framework does not seem very prone to these techniques. ML may find its regulatory way through indirect channels: e.g. specific variables within the scoring.

4.5. Climate Risk

Climate Risks manifest through two types of risk:

- **Physical risks**: risks from extreme weather events (acute) and long-term gradual shifts of the climate (chronic)
- **Transition risks**: risks related to the process of transitioning towards a low-carbon economy

How are IRB Models affected?.The ECB has set some expectations:

- Risk drivers: Where climate-related and environmental risks drivers are found to be relevant and material, institutions should include such risk drivers in their internal models
- Margins of conservatism: For deficiencies stemming from missing or inaccurate information

In the short-term:

- E&S risks can be taken into account in the risk differentiation, risk quantification, or application of the estimates to the extent that they satisfy existing regulatory requirements (do not imply a reduction of discriminatory power, are based on sufficient and reliable information, only apply to a well-justified number of cases, etc.).
- Institutions should account for relevant environmental risks in the prudent valuation of immovable property collateral.

In the medium-to-long-term:

- Specific E&S drivers may be added to the list of drivers to be analyzed when developing IRB Models.

- As the impact of E&S risk is reflected on internal data, institutions should reflect them in PD and LGD estimates via a redevelopment of the models.

At this stage, the EBA considers it premature to modify the RW supervisory formula and does not recommend introducing environment-related adjustment factors. Instead, it recommends considering E&S risks in stress testing programs.

Capítulo 5

Credit Conversion Factors

5.1. EAD

The exposure at default (EAD) is the value of the rights for a given counterparty at the default date. In many cases this is assumed/approximated to be the outstanding amount, however this only happened in some cases and in an approximate way. Generally speaking the exposure at default is unknown, it is a random variable.

Some examples:

- For loans the EAD is approximated as the outstanding amount. But the unpaid installments have to be added to the EAD, plus the future discounted installments (the contract interest rate may be different from the current interest rates)
- For derivatives like an interest rate swap (IRS), the EAD will depend on the time of default and the level of the interest rates at this time
- For a credit card, the EAD will be close to the total limit. Before default counterparties then to use all the available limit.
- For a guarantee the EAD usually is the guaranteed amount for retail clients

The **exposure** as we have seen depends on the **time of default** and on the value of the **market factors** at this time, it is a random variable with a mean and volatility, and this generates the so called exposure risk profiles for a given confidence level.

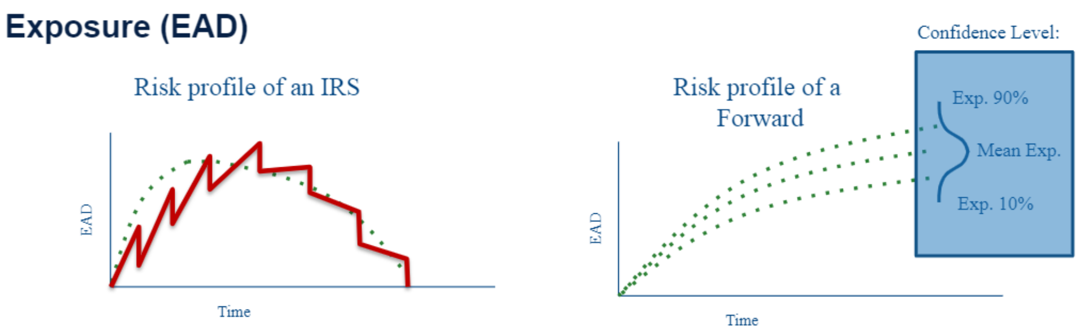


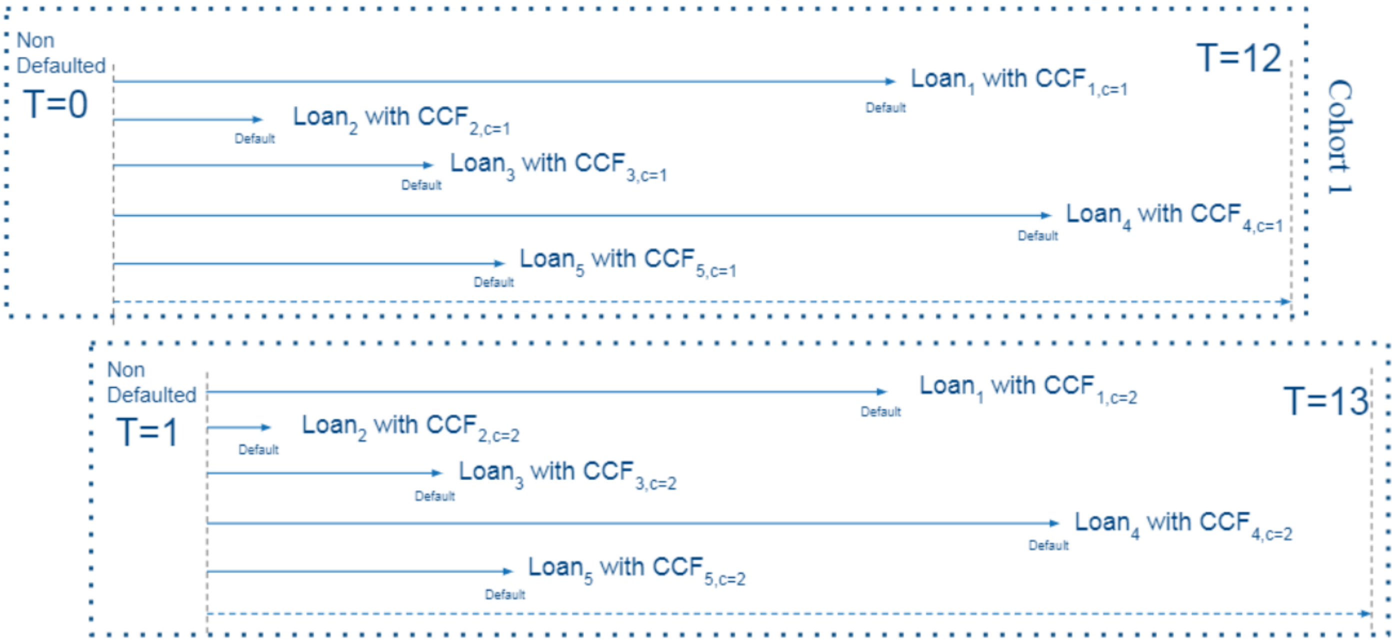
Figura 5.1: Generally the exposure has been considered to be the same as the outstanding amount, however inthe case of the derivatives this is not the case (as there is not a outstanding amount).

5.2. CCF

Credit Conversion Factors (CCFs), for assets with a drawn and a undrawn part (credit cards, credits,guarantees) the EAD is defined as:

$$EAD_D = Drawn_0 + CCF * Undrawn_0 \tag{5.1}$$

When using cohorts (months), same defaults are considered several times:



The CCF of a given cohort can be calculated in several ways:

$$\overline{CCF_c} = \frac{\sum_{i=1}^{N} CCF_i}{N} \tag{5.2}$$

$$\overline{CCF_c} = \frac{\sum_{i=1}^{N} Undrawn_{0,i} \cdot CCF_i}{\sum_{i=1}^{N} Undrawn_{0,i}} = \frac{\sum_{i=1}^{N} EAD_i - Drawn_i 0, i}{\sum_{i=1}^{N} Undrawn_{0,i}} \tag{5.3}$$

$$\overline{CCF_c} = Median\{CCF_1, CCF_2, ..., CCF_N\} \tag{5.4}$$

CCF cohorts: lets check the independent identically distributed (i.i.d.) assumption

- Overlapping cohorts, "repeated" (dependent) information
- Non-Overlapping, independent information

The CCF should be estimated **considering drivers/segments** such that:

1. The CCF of contracts on a given segment are as homogeneous as possible
2. The CCF of contracts on a different segments are as heterogeneous as possible

Statistical test have to be used, such as:

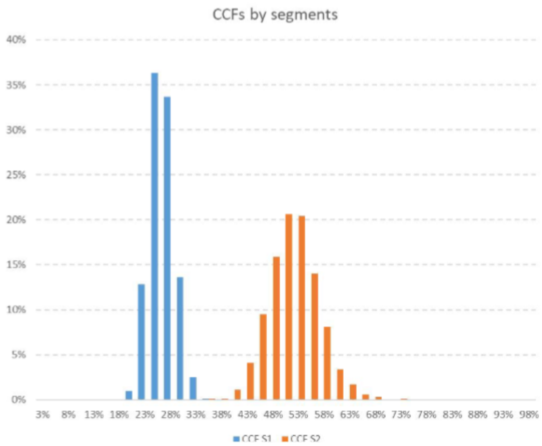
- Equal Mean/Median tests t-Student
- Equal Distribution test Kolmogorov-Smirnoff

Possible drivers are, product:

- Retail: credit cards, credit
- Wholesale: credit cards, credits lines, guarantees

How does the regulation estimates CCFs:

- CCF as simple average (count weighted)
- Downturn CCF :
  - Empirically do not depend of the economic cycle
  - No reg. guidelines on downturn CCF as there are for DTLGD
- Additional drawing after default (CCF vs LGD)
- Initial 5y $\rightarrow$ 7y of data for non retail
- Initial 2y $\rightarrow$ 5y years of data for retail
- EBA standards, not fully available for CCF



Open issues regarding CCFs:

- What to do with negative CCF defaults i.e. the EAD is smaller than current drawn (mortgages)
- What to do with additional disposals during default:
  - EAD1, exposure at the default date and additional disposals are introduced in the LGD
  - EAD2, exposure at default date plus additional disposals
- Stressed CCFs, the LGD has a Downturn LGD equivalent. This is very relevant if there exists PD-EAD correlation. "It seems that there is not a relation between the CCF and the economic cycle, but this does not mean that using a single average is a good idea if we want to account for the possible variability of losses"
- Backtesting the CCF or the sum of EAD

5.3. Real estate haircuts

A component of the LGD and DLGD but also for accountancy purposes. Usually the available NPL databases do not have the linkage between the defaulted loan and the sale of the collateral.

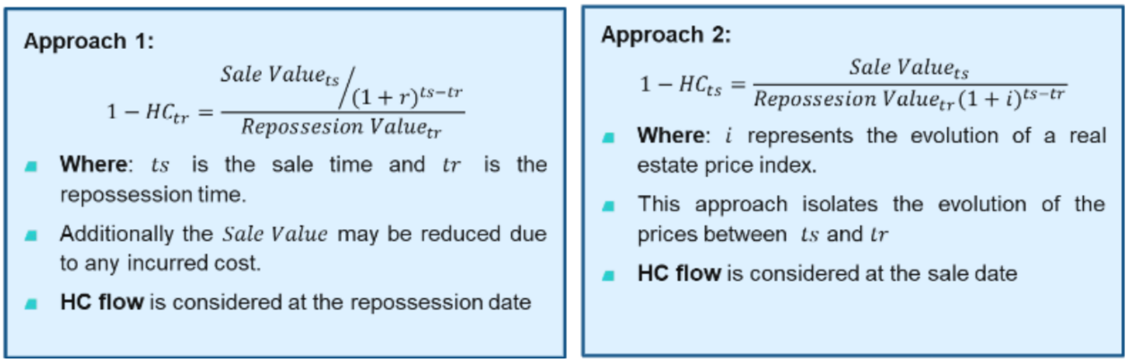
Haircut: enable to transform a repossession value into a sale value. Usually the repossession value is based on an appraisal value. Additionally they should consider costs and discount factor.

Costs have to be considered, repossession, taxes, maintenance, sale... HC application has to be consistent with the estimation approach taken.

We can calculate HC_t for each asset sold and estimate an average HC by:

- **Sale year:** for all the sold assets. *Is there any bias?*
- **Repossession year:** only for those assets already sold. *Is there any bias?*
- **Risk drivers:** Houses, garages, storage rooms, land... or even repossession value and months since repossession.

Figura 5.2: Two main approaches for HC calculation



The estimation is very similar to the CCF. For a given segment with N observations we have that:

$$\bar{HC}_c = \frac{\sum_{i=1}^{N} HC_i}{N} \tag{5.5}$$

When backtesting we have to check both the estimated $\bar{HC}_c$ and $Sale\ price = (1 - \bar{HC}_c) \cdot Reposs\ value$

5.3.1. EBA guidelines

Repossession value, is the min:

- Debt reduction
- Accounting asset value

What is the effect? Can it be zero?

The flow should we recognized at the **date of repossession**, not the sale date.

5.4. Margin of Conservatism (MOCs)

The EBA guidelines says that institutions should add to the estimated parameters (PD, LGD, CCF) a margin of conservatism (MoC) that is related to the expected range of estimation errors.

Two types of deficiencies are identified:

- **Category A**: Identified data and methodological deficiencies.
- **Category B**: Relevant changes to underwriting standards, risk appetite, collection and recovery policies, and any other source of additional uncertainty.

Two types of adjustments should be done to the parameter estimates:

- Appropriate adjustment to overcome any bias, for example, changes in default definition.
- Margin of conservatism to reflect the uncertainty in the estimation:
  - **Category A**: MoC related to data and methodological deficiencies identified under Category A.
  - **Category B**: MoC related to relevant changes to underwriting standards, risk appetite, collection and recovery policies, and any other source of additional uncertainty identified under Category B.
  - **Category C**: The general estimation error, statistical uncertainty.

$Total\ MOC = MOC_A + MOC_B + MOC_C$

Many issues remain open:

- "The general estimation error" is a very broad definition. What sources of uncertainty should be considered? The DT period for the DLGD? Or the period for the average long-run PD?
- For a given model, how to add the intra-category MoCs?
- How to account for the diversification between models?
- What is the best-allowed level to estimate-apply the MoCs? Model, portfolio...

5.5. Credit Risk Backtest

Backtest is the process of showing that the models and parameters employed by the institution are adequate according to the observed experience

We can backtest:

- Whole model: whole portfolio and subsegments discrimination and predictions
- Variables of the model: their stability (distribution) and discrimination

Models and parameters should be **monitored frequently, at least annually.**

5.5.1. GINI coefficient

In the context of backtesting credit risk models, the Gini coefficient is used to evaluate the discriminatory power of a model in distinguishing between good and bad credits. Here's how it typically works:

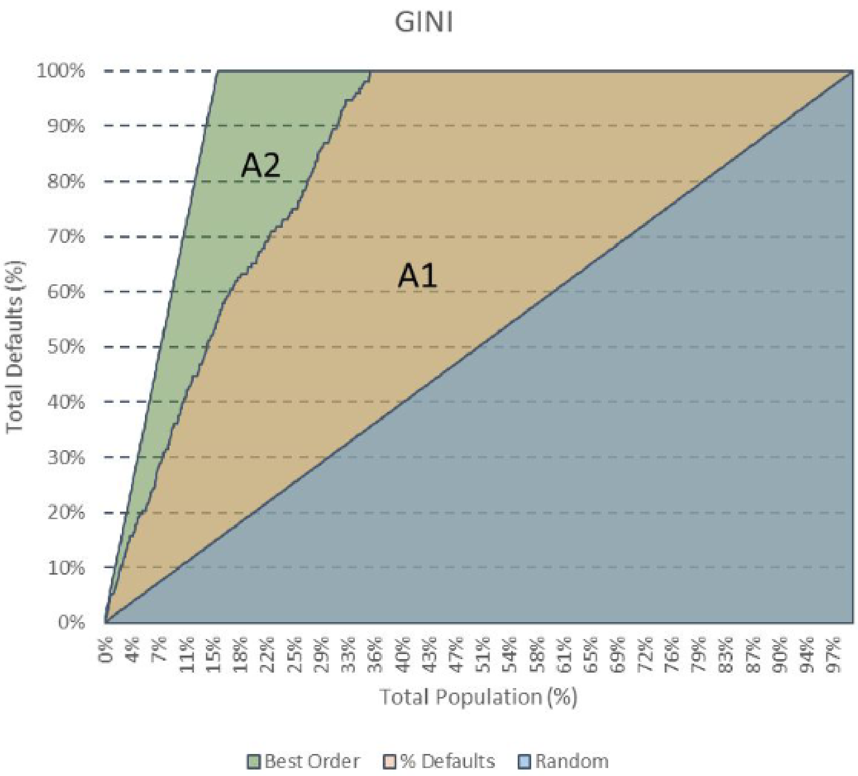
1. **Ranking**: The credit risk model ranks individuals or entities based on their estimated credit risk. This could be based on various factors such as credit scores, financial ratios, and historical behavior.
2. **Segregation**: Individuals or entities are divided into groups, often deciles or quintiles, based on their rank. The highest-ranked group represents the lowest credit risk, while the lowest-ranked group represents the highest credit risk.
3. **Observed Outcomes**: The actual outcomes for each group are observed over a given period. For instance, if the period under scrutiny is one year, the outcomes could include default rates, delinquency rates, or other measures of credit performance.
4. **Calculation**: The Gini coefficient is calculated based on the cumulative distribution of the observed outcomes across the ranked groups. It essentially quantifies how well the model discriminates between good and bad credits. A higher Gini coefficient indicates better discriminatory power. The formula for calculating the Gini coefficient is:

$$G = \frac{\sum(X_i \times Y_{i+1} - X_{i+1} \times Y_i)}{\sum X_i \times Y_i}$$

   Where:

   - G is the Gini coefficient.
   - X_i represents the cumulative proportion of individuals or entities (e.g., cumulative percentage of population).
   - Y_i represents the cumulative proportion of outcomes (e.g., cumulative percentage of defaults).
   - i ranges from 1 to the total number of groups.
5. **Interpretation**: A Gini coefficient of 0 implies the model has no discriminatory power (i.e., it's no better than random chance), while a coefficient of 1 indicates perfect discrimination (i.e., the model perfectly ranks individuals from lowest to highest risk).

6. **Comparison**: The Gini coefficient can be used to compare different versions of a credit risk model or different models altogether. It helps in assessing which model performs better in terms of ranking credit risk.



Similar to the GINI is the **AUC (Area Under the Curve)**. The AUC graphs percentage of defaults detected or true positive (Y axis) vs false alarms or false positive (X axis) .

When backtesting a model we should consider many factors:

- Gini evolution over time for the whole model
- Gini evolution over time for the relevant segments of the model (e.g., Client/Non-client)
- GINI confidence intervals
- Perimeter and default definition have to be homogeneous over time (New Def. of Default)
- Ranking (score) has to be clean of analysts' interventions (adjustments).
- Rating philosophy, the backtest can help understanding the rating philosophy, tools:
  - Migration matrices
  - Backtest over longer default horizons

LGD Models: models to rank/predict LGD can also be backtested:

- Alternative 1: check ordering and use a GINI curve, for example, change an LGD of 60 % by 60 observations with default = 1 and 40 observations with default = 0. Same as using weights.
- Alternative 2: similar to 1 but check if the worst-ranked loans account for the most of the real portfolio loss (EAD * LGD).
- Alternative 3: backtest predicted average LGD * EAD and portfolio accumulated LGD * EAD.

5.5.2. Stability of the model

It is important to check the stability of the models. For example when using Gini, it is important to check the Gini evolution over time of each variable, as well as the distribution of each continuous variable (visually or with statistical tests like Kolmogorov-Smirnov).

For discrete variables there is a simpler approach, the **Stability Index**.

$$SSI = \sum_{i=1}^{Buckets} (Pop\_Ini_i - Pop\_Fin_i) \cdot ln(\frac{Pop\_Ini_i}{Pop\_Fin_i}) \tag{5.6}$$

Where Pop_Ini_i is the percentage of observations in the bucket i for the initial portfolio. If:

- SSI $<$ 10 % no shift
- 10 % $<$ SSI $<$ 25 % minor shift
- SSI $>$ 25 % major shift

Another test for discrete variables is the **Chi-Square Test**

$$X^2_{M-1} = \sum_{i=1}^{Buckets} \frac{(Pop\_Ini_i - Pop\_Fin_i)^2}{Pop\_Ini_i} \tag{5.7}$$

It measures the relative error, but there might be a huge relative error in buckets with a very small population, which is a major drawback.

5.5.3. Backtest error types

Two error Types can be defined:

- **Error Type I**: reject a good model because of bad luck (false negative)
- **Error Type II**: approve a bad model because of good luck. Regulator point of view (false positive)

5.5.4. Predictions

Backtesting PiT parameters is "easier" than TtC parameters. We can compare the portfolio estimates at time t with the observed behaviour between t and t+12 months.

PiT PD: the number of defaults follows a binomial distribution.

PiT NPL additions: the PD and CCF may be correctly estimated, but we should also compare: *NPL* $Additions_{t,t+12}$ *(MM€)*

- There are no statistical tests available to test NPL additions.
- PiT NPL additions try to account for correlation between PD and EAD.
- If the PD and the EAD are correctly calculated, is PD * EAD always going to be equal to the NPL additions? Not necessarily because of PD and EAD correlation.

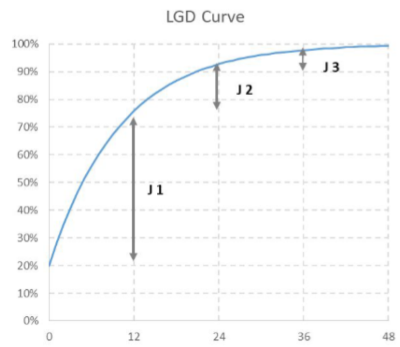
Two possible approaches for LGD:

Average approach: errors are compensated

$$Loss\ shortfall = 1 - \frac{\sum_{i=1}^{N} PredictedLGD_i \cdot EAD_i}{\sum_{i=1}^{N} ObservedLGD_i \cdot EAD_i}$$

One-By-One approach: Errors are NOT compensated

$$MAD = \frac{\sum_{i=1}^{N} |OLGD_i - LGD_i| \cdot EAD_i}{\sum_{i=1}^{N} EAD_i}$$



PiT LGD: the LGD takes several years to be fully observed; it requires more than 12 months to be observed. However, we can backtest the increments of the LGD curve used.

- J1: the recoveries of the loans with 0 months in default over 12 months.
- J2: the recoveries of the loans with 12 months in default over 12 months.
- J3: the recoveries of the loans with 24 months in default over 12 months.

For the PiT Expected Loss, as with the PiT NPL additions it is good to compare:

$$NPL\ additions_{t,t+12} \cdot Last\ Mature\ LGD$$

Basel - CRR requires TtC PD and Downturn LGD for capital requirements calculations

5.5.5. Migrations

Migration analysis has two purposes:

- Understanding the migration dynamic of the scores and/or PD parameters.
- Monitoring if the dynamic changes.

5.6. Credit Risk Loss Distribution

We can differentiate between two loss distributions:

1. Short term loss distribution, the one for the next year
2. Long term loss distributions, the average over many years

Depending on the objective we will use PiT or TtC parameters.

Under independence we have that:

$$EL = PD \cdot EAD \cdot LGD$$

But **are defaults independent between clients?**. Not really, clients have connections mainly due to macroeconomy.

5.6.1. The Vasicek model

The **Vasicek (Basel)** model captures the correlation. The wealth (V_i) of a client can be modeled as:

$$V_i = \sqrt{\rho_i} \cdot Z + \sqrt{1 - \rho_i} \cdot \epsilon_i$$

Where:

- Z is a single common macroeconomic factor with *N(0,1)* distribution

- ϵ_i is an idiosyncratic factor with *N(0,1)* distribution
- ρ_i is a coefficient that measures the linkage of all the clients

In the Vasicek model **default happens** if $V_i < K_i$, and $K_i = \Phi^{-1}(PD_i)$. But this PD has to be an estimate of the long run default rate (TtC). Using PiT PD is not statistically correct.

Under some assumptions, the Vasicek model has a closed formula. The asumption is **ASRF**, asymptotic single risk factor portfolio, i.e infinite identical clients exposed to the same macroeconomic factor

As a function of the **macroeconomy level**

$$\textit{Default Rate (Z)} \; = \Phi(\frac{\Phi^{-1}(PD) - \sqrt{\rho}Z}{\sqrt{1-\rho}}) \tag{5.8}$$

As a function of the **probability of the macroeconomy**

$$\textit{Default Rate (p)} \; = \Phi(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\Phi^{-1}(p)}{\sqrt{1-\rho}}) \tag{5.9}$$

All the idiosyncratic risk (ϵ_i) disappears.

As correlation increases the distribution gets more asymmetric but the mean does not change. The ASRF works quite well for 100 loans.

There is a **relation between the correlation and the volatility of the time series**, as we have seen the shape of the distribution depends on the correlation parameters.

The **exat relation is**:

$$\rho = \frac{var(\Phi^{-1}(\textit{Default Rate}))}{1 + var(\Phi^{-1}(\textit{Default Rate}))} \tag{5.10}$$

What is the effect of the default definition on the Expected loss? And on the loss distribution?. Losses should be more or less the same independently of the default definition:

If we define 30 days arrears as default PD increases but LGD decreases:

- No effect on the expected loss
- Effect on the los distribution if the correlations are not adjusted

But default definition may affect the LGD by the union of different year defaults. This is, make the defaults longer, will have little impact on PD but great on LGD.

5.6.2. Extensions of the Vasicek model

We could consider several macroeconomic factors: For example if we have 50 loans exposed to macroeconomic factor Z_1 and 50 other loans exposed to macroeconomic factor Z_2 and we consider $C = corr(Z_1, Z_2); C \in [-1, 1]$ we have that as C becomes more negative, extreme losses are reduced.

Random recoveries (PD-LGD), introduces an additional risk factor, not just the default but also the recoveries.

Recoveries have a constant std and a negative correlation with default rates.

Simple ASRF based model for random recoveries:

$$\textit{Default Rate } (Z_{PD}) = \Phi(\frac{\Phi^{-1}(PD) - \sqrt{\rho_{PD}}Z_{PD}}{\sqrt{1-\rho_{PD}}}) \tag{5.11}$$

$$Default\ Rate\ (Z_{LGD}) = \Phi(\frac{\Phi^{-1}(LGD) - \sqrt{\rho_{LGD}}Z_{LGD}}{\sqrt{1-\rho_{LGD}}}) \tag{5.12}$$

What is the importance of the parameter ρ_{LGD}?

- Reduces the chance of medium losses but increases the chance of high losses

What is the importance of the parameter $corr(Z_{PD}, Z_{LGD})$?

- Positive correlation always increases the risk

Rating migration model

So far we only had two states Default, Non-Defaut, but we may have multiple states with multiple possible losses (values). We can use the Vasicek model, if the value drops to a given level there is a rating migration. Under a multiple states models:

- Loss distribution is smoother compared with a default model.
- The portfolio may suffer higher losses. i.e. some loans may default and others migrate to lower values.

Moody's KMV model

Two approaches depending on the available data:

- Equity, simulate the value of a company and compare it with the short term debt plus 50 % of the long term debt.
- Parameters, Vasicek model where Moody's G-Corr provides the correlation between the macrofactors (geo/sector).

Credit Metrics, multiple state model to valuate bonds, the correlation between different companies is extracted from stock market returns.

Credit Risk + /IFRS9 type, models the relation between the defaults and losses with macroeconomic variables. Then we can simulate macrovariables

5.7. Economic and Regulatory Credit Risk Capital Requirements

From a very simplified point of view:

- The provisions account for expected loses (PiT vs TtC, 1y vs lifetime). IASB-FASB.
- The capital accounts for possible variation of losses around the expected value. Basel-EU.

The Vasicek model is a good starting point to measure the risk of a portfolio. The process to measure Regulatory Capital requirements would be:

- Use the Vasicek model ASRF and TtC parameters Long term loss distribution.
- Considers random LGD, through the DLGD.
- Capital = ASRF Model(99.9 %) − Stress Expected Loss. 1.06 multiplier to be removed.
- Regulatory correlations by portfolio.
- Maturity Adjustments on the EAD, "to account for possible rating migration"

1. Retail exposures, mortgages, revolving and others:

$$Capital = EAD \cdot DLGD \cdot [\Phi(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\Phi^{-1}(99{,}9\,\%)}{\sqrt{1-\rho}}) - PD] \tag{5.13}$$

Note that we subtract $EAD \cdot PD \cdot DLGD$. This is a expected loss under stressed LGD.

The correlations are:

$$\rho_{others} = 3\,\% \cdot \frac{1 - e^{-35 \cdot PD}}{1 - e^{-35}} + 16\,\% \cdot (1 - \frac{1 - e^{-35 \cdot PD}}{1 - e^{-35}}) \tag{5.14}$$

2. Wholesale exposures

$$Capital = EAD \cdot DLGD \cdot [\Phi(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\Phi^{-1}(99{,}9\,\%)}{\sqrt{1-\rho}}) - PD] \cdot \frac{1 + (M - 2{,}5) \cdot b(PD)}{1 - 1{,}5 \cdot b(PD)} \tag{5.15}$$

Where the term $\frac{1+(M-2{,}5) \cdot b(PD)}{1-1{,}5 \cdot b(PD)}$ is the maturity adjustment.

The correlations are:

$$\rho_{others} = 12\,\% \cdot \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} + 24\,\% \cdot (1 - \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}}) - 4\,\% \cdot (\frac{S-5}{45}) \tag{5.16}$$

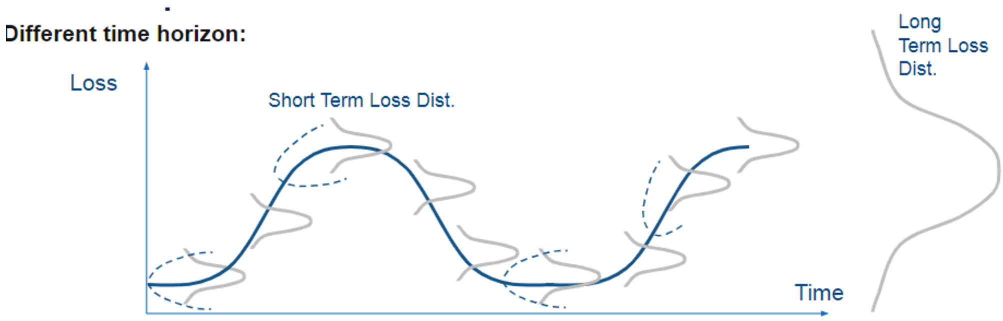
Where S are the Sales in MM€ (this last term is only for SMEs)

What is the effect of the PD on the correlation?:

- As the PD increases the correlation decreases

What is the effect of the maturity adjustment?:

- As the PD decreases the mat. adjustment increases



In the graph the short term approach does not introduce cyclical cap requirements, however when loss distributions are asymmetric/non-linear the capital requirements get cyclical

More issues on Economic Capital:

- Maturity adjustment, Should it be considered? Or a migration matrix?
- Subtract PD * LGD or PD * DLGD
- Internal data correlations based on the variance of the internal series
- We can combine all the models from the previous section:
  - Geographic diversification
  - Random and correlated recoveries

5.7.1. The IRB Repair Programme

The IRB Repair Programme was launched by EU institutions after an intense debate on the role of internal models to calculate regulatory capital. The main objectives are:

1. Preserve the risk sensitivity of capital requirements
2. But... restoring the reliability in internal models to appropriately calculate own funds requirements
3. And... harmonising practices across entities and jurisdictions (level playing field)

A comprehensive new set of regulation by the EBA:

1. Guidance to supervisors to assess IRB models compliance
2. Change of the definition of default
3. Harmonisation of risk modelling practices
4. Review of the credit risk mitigation framework

Accompanied by interpretations from supervisors: ECB Guide to Internal Models, PRA supervisory statements, etc.

Along with a new (intrusive) style of supervision (IMI, TRIM, TRIMI, TRIMIX, etc.) Accompanied by a strong regulatory and supervisory scrutiny:

- One-step (both at the same time) versus two-step approach (one by one)
- Dual deadline:
  - Jan'21 Classification with new definition of default
  - Jan'22 Whole set of methodological changes and recalibration with new DoD

Guidelines on PD estimation, LGD estimation and treatment default exposures (EBA)

Institutions should choose a calibration method that is appropriate for their LGD estimation methodology from the following approaches:

(a) **the calibration of LGD estimates to the long-run average LGD calculated for each grade or pool**, in which case they should provide additional calibration tests at the level of the relevant calibration segment

(b) **the calibration of LGD estimates to the long-run average LGD calculated at the level of calibration segment**. In this case institutions should at least compare this long-run average LGD with the average LGD estimate applied to the same set of observations as those used for calculating the long-run average LGD and, where necessary, correct the individual LGD estimates for the application portfolio accordingly, for instance by using a scaling factor. Where realised values are higher than estimated values at the level of calibration segment, institutions should correct the estimates upwards or readjust their estimation in order to reflect their loss experience.

Capítulo 6

Internal Model Methods for Counterparty Credit Risk

The Asian crisis in 1997, Russia default in 1998 and failure of LTCM the same year triggered increased attention to counterparty credit risk. In early 2000 it has seen defaults of large corporates (Enron, WorldCom, Parmalat). In 2007-2009 the global financial crisis emerged: counterparty credit risk took a prominent role in risk management practices, CVA has become a permanent component of derivative pricing and other components started to emerge i.e. funding and capital costs.

6.1. Credit Risk, Counterparty Risk and Credit Valuation Adjustment

6.2. Regulatory Capital for Counterparty Risk

Counterparty risk is strictly linked to derivative contracts and markets. Derivatives are traded on exchanges or via Over-The-Counter (OTC).

The credit risk of derivatives is traditionally called counterparty risk; however counterparty risk is intrinsically linked to other financial risks that are faced when trading derivatives.

Counterparty risk has a very important component of operational risk attached to it. When we are measuring counterparty risk we have to leverage in very complex processes, regulations and models which can cause operational failures.

Counterparty risk is the risk that one or more counterparties are not able to meet their obligations. This sounds a lot like credit risk but here it is not related to bonds and originations but more to derivatives.

6.2.1. Bond pricing and credit risk

A corporate bond entitles the bondholder to a series of cashflows in the future – the coupons NC_T at coupon dates T_i and principal N at maturity T_m (where C is coupon and T is accrual fraction).

This is conditional on the company not defaulting by the time the payment is due. Of course if the counterparty defaults we do not receive the remaining cashflows nor the notional.

- Upon default, we can get a fraction of the value – the recovery rate R.
- If D(0, T_i) for different T_i represents the discount curve built from risk-free rates, and λ represents the default intensity (i.e. probability of survival after time t is $e^{-\lambda t}$, then the value of the corporate bond is

The price of the bond is a weigthed sum of the cashflows multiplied by the probability or defaulting:

$$Price = NC_T \sum_{i=1}^{m-1} D(0,T_i)(e^{-\lambda T_i} + R(1-e^{-\lambda T_i})) + N \cdot D(0,T_m)e^{-\lambda T_m} + R(1-e^{-\lambda T_m}) \tag{6.1}$$

More practically, the bond is traded on a spread off the risk-free rate, so that the value of the bond is given by

$$Price = NC_T \sum D(0,T_i)e^{-sT_i} + N \cdot D(0,T_m)e^{-sT_m} \tag{6.2}$$

where the spread s is whatever value is needed to match the quoted bond price P. The credit risk of the bond is captured by the spread s or the default intensity λ.

Default Intensity and Survival Probability

Let $h(t) = e^{-\int_0^t \lambda(u)du}\lambda(t)$ be the probability density function of the default intensity. Then the probability of survival up to time T is given by $\int_0^t h(t)dt = e^{-\int_0^t \lambda(u)du}$

Extension to derivatives

The derivative is different from the bond for many reasons:

- The is a exchange of payments
- The derivative payoff depends on the value of some underlying variables, e.g. changes in levels of interest rates or spot value of a stock.
- What changes the value of the derivative is the change in the underlying, it is not dependant on itself like the bond

One of the most common derivatives is the Interest Rate Swap (IRS), usually traded on OTC markets, where one party A pays regular coupons at time T_i (typically annually) based on a fixed rate C, and the other party B pays regular coupons at time T_i^* (typically semi-annually) based on the floating reference interest rate (e.g. Euribor). So party A pays cashflows $N \cdot C \cdot \tau_i$ whilst party B pays cashflows $Nf(T_{i-1}^*, T_i^*) \cdot \tau_i^*$, where N is notional, τ_i and τ_i^* are accrual fractions and $f(T_{i-1}^*, T_i^*)$ is the Euribor rate that sets at time T_{i-1}^* and pays at time T_i^*.

A typical IRS is based on a fixed rate C chosen at time 0, so that the value of the swap is zero i.e. no cashflows are initially exchanged; but the future value of the swap depends on whether the overall levels of the reference interest rate rise or drop.

How can we extend the concept of credit risk to derivatives?

One key difference with the bonds is that in an IRS the cashflows can be negative whereas the coupons from the bond are always positive. In order to quantify the counterparty risk we need to differientiate between

$$\textit{Expexted Positive Exposure (EPE)} = EPE(t) = E[max(V(t),0)]$$

$$\textit{Expexted Negative Exposure (ENE)} = ENE(t) = E[min(V(t),0)]$$

The real loss here is when I am expecting to get money in the future but I don't receive it because the counterparty defaults.

6.3. Concept of CVA

The CVA tries to price the risk of the default of the counterparty. We need to be able to quantify the likelihood of the default of the counterparty. We also need to quantify how much we will lose in case of the default and how much we will recover after the default.

This is very similar to the capital formula for expected losses where we have a probability of default PD and a loss fiven defaul LGD.

Looking at a derivative in general, the CVA is given by:

$$-(1-R)\int_0^T E[\frac{e^{-\int_0^t \lambda(u)du}\lambda(t)\cdot max(V(t),0)}{B(t)}]dt \tag{6.3}$$

- 1-R ... The recovery is where most uncertainty lies.

There are many simplifications

The evolution of the credit spreads (i.e default intensity) are independent from the market. This seems innocent but it is actually quite important. This assumption is equivalent to assuming that our counterparty it is not large enough to move the market with its default.

Since the expression has no analytical solution, we discretize it

How does this formula differ from the practice?

The biggest difference is that we have done this for one transaction but a typical institution does many thousands of deals with a large counterparty, and even hundreds with smaller ones. How do we deal with this? Can we sum everything up?.

Suppose upon default of your counterparty, you owe €50m on deal A and it owes you €30m on deal B, then shouldn't your net exposure be €20m to your counterparty? That depends on whether these çontracts are nettable", e.g. what happens if you owe the US subsidiary of your counterparty and its Japanese subsidiary owes you? This highlights the concept of a netting set - i.e. the set of all positions that could be aggregated for purposes of determining exposure upon default, as a result of legal agreements.

Netting provisions are part of the IDSA Master Agreement and are usually defined with respect to:

- Payment netting: net cashflows occurring on the same day
- Close-out netting: termination of all contracts with an insolvent counterparty and offsetting of all transaction values

6.3.1. Regulation

CCR RWA and CVA RWA

The regulatory framework for capital is based on risk-weighted assets (RWAs) requirements i.e. the required capital to hold is a proportion of the bank's asset and exposures appropriately risk-weighted (the total amount of RWAs).

RWA converts uncertain exposure profiles (e.g. in a derivative like a swap) into "equivalent" units of corresponding standard assets (e.g. bonds)

The Counterparty Credit Risk from a capital point of view is strictly connected with market and credit risk.

What makes Basel 3.1 different from Basel 3 and Basel 2 is that the way to compute Counterparty Credit Risk was heavily structured. The current CRR allows the firms to use exposures from internal models to calculate their capital for CVA. But due to the huge variability it was impossible to compare among banks. Also banks with similar risk profile could produce very different capital requirements. Basel 3.1 aims to make the CVA comparable among banks and also risk sensitive.

Chapter 6 of CRR deals specifically with the capital requirement for Counterparty Credit Risk. The intention is to translate the uncertainty of CVA to pricing of instruments that everybody can understand.

Banks are allowed to use a combination of internal models and standarized approach for some instruments that banks may not regularly trade.

CVA affects the banks profits and ultimately the compensation of traders. RWA affects dividends.

As we have seen in the first part, the CVA calculation involves 3 parts: the probability of default, the loss given default and the exposure at default. The regulation prescribes PDs for specific risk types.

Regulators assume that exposures to a counterparty cannot be quickly neutralised (e.g. unwinding a swap will damage client relations) - i.e. a 1 year horizon is necessary. Further it is assumed that short dated repeat business will roll over at the end of each period - i.e. exposures will be non-decreasing over the 1 year horizon. This is called *Effective Expected Exposure (EEE)*. Define $EAD = \alpha \times EEPE$ where α (usually 1.4) is the supervisory multiplier for conservatism.

()()() rest of slide 28 and 29

EEPE is the area bounded by the EEE borders (in red)

Effective Maturity

- For trades under 1y, effective maturity = actual maturity
- For trades over 1y, effective maturity = total area weighted by discount factors (red plus blue beyond 1y) / area under EEE curve weighted by discount factors (in red)

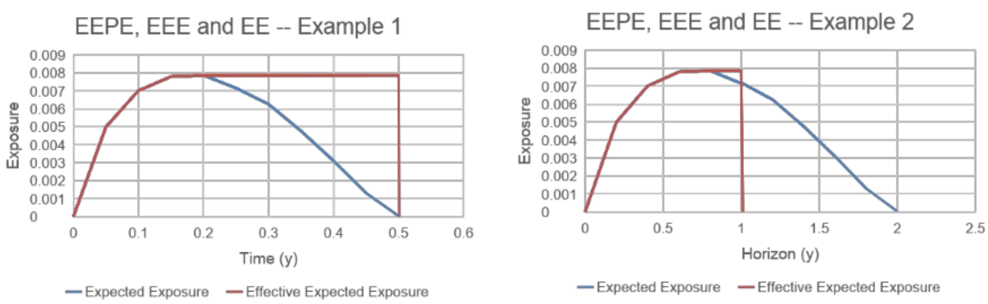


Figura 6.1: As long as the blue line continues to grow, the red line coincides with it.

Mathematically, the EEE is the area under the red curve.

6.4. Modelling counterparty credit risk

What are we modelling?. The starting point is to understand a typical institution's portfolio

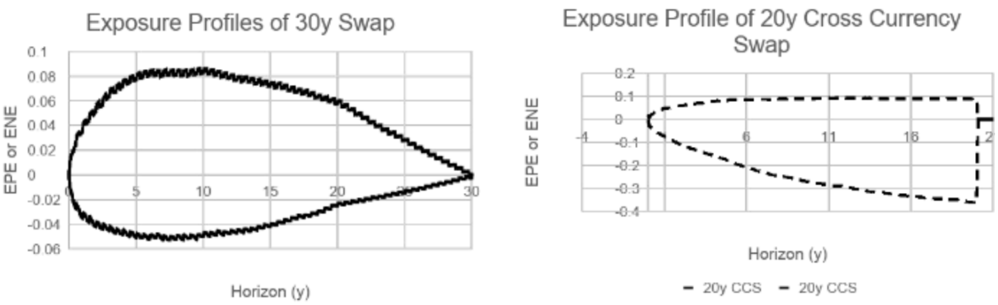
- By far, a typical bank's portfolio is dominated by interest rate swaps, cross currency swaps (CCS), and FX forwards.
- Banks also have a lot of credit default swaps in the books to hedge credit risk and CVA.
- Options constitute a relatively minor population, and exotics are generally even smaller; credit derivatives also do not contribute much.
- Conceptually, typical exposure of an option at future time is not much more than an at-the-money forward; after all, exposure is highest when an instrument is in the money – see profiles in subsequent slides.
- Not all instruments have counterparty risk – instruments that are assets/securities (e.g., securitized products, bonds, or other assets) as opposed to bilateral deals have market risk only.

Category	Instrument	Low	High
Linear IR & FX	Swaps, CCS & FX Forwards	50 %	70 %
IR	Non-Standard Swaps (incl Basis but not CMS)	5 %	10 %
IR	IR Options (Swaptions, Caps, Bermudans)	2 %	7 %
IR	IR Exotics (CMS, Spread, Ranges)	0 %	3 %
Inflation	Inflation ZC and YoY Swaps	1 %	5 %
Inflation	Inflation Options	0 %	2 %
Inflation	Inflation Exotics (incl LPI)	0 %	2 %
FX	FX Options	1 %	3 %
FX	FX Exotics (Barriers, Multi-FX, PRDC)	0 %	2 %
Credit	Credit Deriv (CDS, CDS Index, Tranches, Opt)	5 %	15 %
Equities	Equities Linear (incl TRS)	2 %	7 %
Equities	Equities Options	1 %	5 %
Equities	Equities Exotics	0 %	3 %
Commodities	Commodities (Swaps, Forwards, Options)	0 %	5 %

Tabla 6.1: Ranges of risk weights for different categories of financial instruments.

- Maturity plays a critical role when computing CCR RWA and for CVA. Interest rate and inflation instruments have the longest maturities (typically up to 30 years). These long maturities are used by pension funds and other actors that want to hedge their long-term exposure.

This composition will drive the decisions of banks when they need to allocate resources to develop models for the CVA.



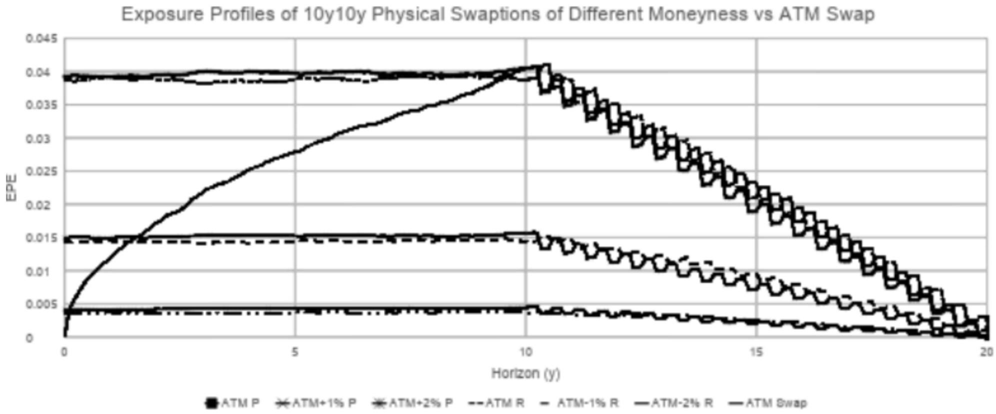
The top part of the shape represents the expected positive exposure, which gives you the average price of the swap in the future where the swap is positive.

Why do we see such a weird shape?. Initially the likelyhood of observing scenarios where the price of the swap is positive grows, as scenarios where variable is higher than fixed increase. As time passes, more and more cashflows expire and the Expected Positive Exposure starts decreasing until it reaches 0 at maturity.

Cross currency swap

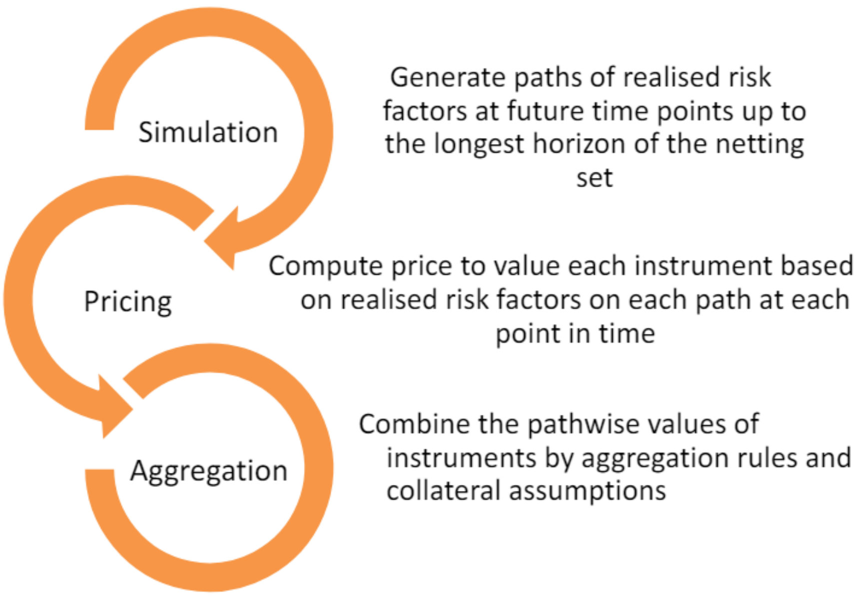
The CCS, in addition to the exchange of currencies we take the notionals and exchange them again at the end, thus we carry the counterparty risk until the end of the maturity, driving the exposure up. For example if we enter into a cross currency swap where we exchange 1m euros for 1.2m dollars, what will happen is that that money will be invested in risk free rates and we will be responsible for paying the interest on the swapped amounts in their respective currencies throughout the duration of the swap.

Swaptions



- The figure shows the shape of exposure profiles of physically settled swaptions – effectively, they become a swap after exercise, and so exposure only drops to zero upon swap maturity.
- Observe how the profiles of swaptions of different moneyness compare vis a vis each other, and the at-the-money swap.
- For cash settled swaptions, the profile is similar up to expiry and thereafter the exposure drops to zero.
- For FX and equity options, the profile is also similar up to expiry and then drops to zero thereafter.

The framework for modelling counterparty exposure involves 3 major components: simulation, pricing and aggregation



The probabilities of the paths are implied in the pricing. For example if we use Black-Scholes, we are generating points that are log-normal. Maybe these points themselves don't distribute as a lognormal. Wen we have all the prices of the derivatives that were generated randomly, we can aggregate them into percentiles for example.

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- A typical institution will have exposure to multiple asset classes (Rates, FX – needed to convert to base currency anyway, Equities, Credit, and maybe Commodities), and in each asset class to multiple underlyings.

- The typical exposure framework comprises a Monte Carlo engine to generate the simulated paths of correlated risk factors at future times.
- The approach would be to:
  1. Select the distribution for each underlying risk factor based on certain assumptions on drift and volatility (e.g., calibrated to market quantities or historical returns).
  2. Define the correlation assumptions between risk factors and the appropriate correlation matrix.
  3. Generate (pseudo) random numbers based on the chosen distributions to compute the realized paths of risk factors at future horizons.

6.4.1. Simulation

Inputs to the simulation

CRR allows for the use of market implied or historical parameters for the underlying distributions for CCR RWA; for CVA RWA, implied credit spreads must be used

Market implied: We have prices that are being quoted in the market. We can infer the volatility or the interest rate curve or any other risk factors. These are the implied volatilities derived from the markets that we can see in sites like bloomberg. We have to be very careful because if we imply this volatility with the Black-Scholes formula from the stock price, it needs to be consistent with the simulation models afterwards (we will have to assume lognormal fot the simulations in this case)

- Market implied (also known as risk-neutral) is where the underlying risk factors are calibrated to market observables, namely the forwards (interest rate curve, inflation curve, FX spot, equity spot, credit curve, commodity forward curve) and the vols (caplet/swaption vol, YoY/ZC vols, FX vols, equity vols, CDS Index option vols, commodity option vols).
- Historical is where the underlying risk factor vols are calibrated based on returns on risk factors over a period of time (e.g., 1 year to 3 years), and sometimes the drifts are calibrated in the same way as well (or otherwise set to 0).
- In reality, a mix of market implied and historical parameters tend to be used, e.g., it is usually typical to try and recover market forwards whilst keeping historical volatility; and it is almost always the case that correlations are based on historical estimates rather than implied.

The argument for market implied is that it is closer to what is done in CVA (and hence CVA RWA), which is part of P&L, and where traders attempt to hedge away CVA risk – e.g., the asymmetric nature of CVA converts an FX forward into an FX option, so it is natural to consider the cost of hedging the FX risk by buying an FX option.

- Pragmatically, one can ask however if it is actually possible to hedge away CVA risk – after all, CVA involves the exposure contingent on default of the counterparty and really you can only hedge each component separately (e.g., hedge IR risk assuming default probabilities) if at all – e.g., for many counterparties, there is no liquid CDS hedge, so you can only hedge the counterparty credit risk by proxying with a CDS index.

The argument for historical is that if unhedged, then the realized distribution may not correspond to what is predicted by forwards but may be better reflected by historical returns – this assumes that the past is a good indicator of the future, and also that our choice of period to calculate the returns represents the past.

- Ultimately, regulators seem to permit both approaches, subject to the approach being reasonable, and importantly being validated by backtesting.

Observations:

- The large number of underlyings and long horizons require, unlike in pricing, a degree of pragmatism on reducing complexity and a solution sufficiently realistic and robust to work for all risk factors until longest required horizon without breaking

- This is also justified since the bulk of exposure is to linear derivatives like interest rate swaps, cross currency swaps and FX forwards; for options, the largest contribution is for deep in-the-money, therefore it is more critical to generate plausible future paths than modelling more accurately future volatilities
- Finally, irrespective to how future distributions are generated for all correlated risk factors, backtesting is relied upon to give confidence that the model is overall conservative

6.4.2. Pricing

See table 6.2

6.5. Capital for CVA

Regulatory capital for CVA is covered in part 3 of CRR. It is composed of 3 articles.

The regulation specifies the definition of CVA, the scope and defines 2 approaches to calculate CVA:

- **Standarized Approach**: It is expexted that banks follow this approach as sonn as they get permission for it
- **Basic Approach:** It is simpler that the standarized approach. However it is more punitive (it requires higher capital) so there is an incentive to swith to the standarized.

In addition there is also a **simplified approach**, designed for very small banks that do not have advanced capabilities or that have a very small amount of derivative transactions.

Difference between counterparty and reference credit spreads.

To calculate own funds requirements for CVA in Standardized Approach, Article 383 introduces risk classes:

- Interest Rates
- Counterparty Credit Spreads
- Reference Credit Spreads
- Equity Risk
- Commodity Risk
- Foreign Exchange Risk

and two types of risk:

- Delta Risk, which captures CVA changes due to movements in non-volatility related risk factors
- Vega Risk, which captures CVA changes due to movements in volatility related risk factors

Own Funds Requirements for CVA Risk are determined by calculating own funds requirements for Delta Risk and Vega Risk separately in each risk class and aggregating them together.

For each factor k *weighted sensitivities* for the aggregate CVA and of the market value of all eligible hedges are used:

$$WS_k^{CVA} = RW_k \cdot S_k^{CVA}$$

$$WS_k^{Hedges} = RW_k \cdot S_k^{Hedges}$$

Where:

- k is the index that denotes the risk factor k
- RW_k is the risk weight that applies to risk factor k
- WS_k^{CVA} is the weighted sensitivity of the aggregate CVA
- S_k^{CVA} is the net sensitivity of the aggregate CVA with respect to risk factor k

- WS_k^{Hedges} is the weighted sensitivity of the market value of all the eligible hedges in the CVA portfolio with respect to risk factor k
- S_k^{Hedges} is the net sensitivity of the market value of all the eligible hedges in the CVA portfolio with respect to risk factor k

Once the different buckets are defined, the net weighted sensitivities in the same risk bucket b are aggregated giving rise to bucket specific sensitivity K_b:

$$k_b = \sqrt{\sum_k WS_k^2 + \sum_{k\in b}\sum_{\in b, k\neq l} \rho_{k,l} WS_k WS_l + R \cdot \sum_{k\in b}^{max}((WS_k^{hedges})^2)}$$

This formula assumes that all of your sensitivities are correlated. The first two terms are the expression for the expected value of two correlated random variables. The last term is placed to represent that hedges are not perfect as a conservatism term.

Risk-class specific own funds

$$formulahere$$

The Debit Valuation Adjustment is the calculation of CVA done by the counterparty of the bank. This DVA is an indicator of how much they would not pay if they go bankrupt and stop paying for the counterparty.

Regulatory capital is heavily related to other uses of Counterparty models, namely XVA and Credit Limits (Peak Exposure). XVA (Cross Valuation Adjustments) developed significantly after 2008 when it became clear that banks are no longer risk free, funding at Libor is far from reality, there is greater push for exchange clearing, and capital has become increasingly expensive. Prior to 2008, CVA was already well established to reflect the losses from counterparty default or credit spread changes.

- The natural mirror image of CVA is Debt Valuation Adjustment (DVA), i.e. how much your counterparty can lose as a result of your default. Post 2008, when it is clear banks are not risk free, this becomes more important to the counterparty.
- Funding Valuation Adjustment (FVA) reflects the cost of funding needed to sustain one's derivative positions – e.g. if you have a swap where you anticipate paying more early in the life and receiving more later on, there are funding implications if uncollateralised; or if you have a collateral arrangement to post USD collateral vs EUR collateral, there are funding implications.
- Margin Valuation Adjustment (MVA) is the cost of maintaining positions due to initial margins required for exchange traded or cleared derivatives, or as part of Standardised Initial Margin Model.
- Capital Valuation Adjustment (KVA) is the economic cost of tying up capital to support the business.

Potential future exposure: It is the concept of projecting the distribution of the future exposure of the bank and taking the 95th or 99th percentile.

6.6. Fallacies of Counterparty Risk and Worthwhile Highlights

From the counterparty perspective the timing of the cashflows matters a lot for risk quantification, not just the PV of the cashflows. Also the correlation in the long term correlation matters a lot.

6.6.1. Aggregation

We have to think about capital calculations in two steps. Firstly we need to simulate a lot of scenarios. We generate 3 dimensional arrays. The first dimension is type, the second is the market state and the third one is a vector of thousands of instruments prices.

The hard part is generating accurate and correct netting set exposures in the future timeline.

Pricer Type	Approach	Modelling Assumptions	Scope of Use
Linear Pricers	Only interpolation and extrapolation of market data	Only interpolation and extrapolation of market data	Extensively used for interest rate and cross currency swaps, FX, equity, commodity and inflation forwards, Credit Default Swaps
Analytic Option Pricers	Closed form pricers based on Black-Scholes model for FX and equity vanilla options and the normal model for interest rate swaptions	Typically lognormal (Black-Scholes) or normal dynamics for underlying together with interpolation/extrapolation of vols; can involve complex dynamics if they lead to simple approximations for vol (e.g. SABR for interest rates)	European-exercise swaptions, caps, equity options, FX options, commodity options, inflation zero-coupon and year-on-year caps
Quasi Analytic Methods	Simple analytic formula for convexity or quanto adjustments or replication by a portfolio of vanilla options	Simple convexity and quanto adjustments tend to assume simple dynamics (e.g. normal or lognormal) for underlying; replication requires prices of vanilla options for all strikes, effectively vol extrapolation	Simple convexity adjustments are used for futures, inflation year-on-year swaps; quanto adjustments are used for quantoes (i.e. payoffs made in non-natural currency – e.g. Apple stock payable in EUR); replication is used for Libor-in-Arrears swaps and caps, Consta
Copula Methods	Involves getting implied distributions of risk factors, and using two dimensional integration given correlation assumptions (e.g. CMS spread options), or Monte Carlo if more than two factors are involved	Implied risk factor distributions require prices of vanilla options for all strikes, effectively vol extrapolation; alternatively, can assume simple distribution (e.g. lognormal or normal) for underlyings	CMS Spread Options, Quanto Payoffs
1 Factor Partial Differential Equations (PDEs)	Solution by stepping back from maturity based on terminal payoff at different grid points to current time on a 1-factor PDE	Can support local vol FX/equities, interest rate term structure models involving 1-state variable (e.g. 1-factor Hull-White)	Used to price options where it is necessary to compare the value at two different points in time, and where there is one underlying without stochastic volatility (e.g. early exercise of American option in FX/equities, or Bermudan swaptions in 1 factor short rate model, or barrier option in FX)
2 Factor PDEs	Solution by stepping back from maturity based on terminal payoff at different grid points to current time in 2-factor PDE	Can support stochastic vol models in FX/equities or where two underlyings are involved, and models involving stochastic rates and equities, or rate models with 2 state variables	As for 1-factor model, but where there are 2 underlyings or a rates model with 2 state variables (e.g. 1-factor Cheyette) or a stochastic volatility model for FX/equities, or a model with stochastic rates and equities (e.g. to price a payoff dependent on CMS and stock price)
3 Factor PDEs	Solution by stepping back from maturity based on terminal payoff at different grid points to current time in 2-factor PDE	Can support models in FX/equities where three underlyings are involved, and models involving stochastic rates and FX, or rate models with 3 state variables	As for 1-factor model, but where there are 3 underlyings or a rates model with 3 state variables (e.g. 1-factor Cheyette with stochastic vol), or most typically a model with stochastic rates and FX (e.g. long-dated FX involving Power Reverse Dual Currencies in Japan)
Monte Carlo Methods	Generate a large number of paths of realisations of risk factors, evaluate discounted payoffs along these paths, and finally take average to get derivative value	Can support any risk factor dynamics	Used for strongly path dependent payoffs, or those involving many underlyings, or where Libor Market Model involved; Longstaff-Schwartz algorithm often used for early exercise – i.e. generate a bunch of state variables, determine if optimal to exercise along those paths; then regress future values against state variables to create a function to determine exercise based on state variables

Tabla 6.2: Summary of Pricer Types