Bayesian Synthetic Likelihood for Calibration of Stochastic Radio Channel Model

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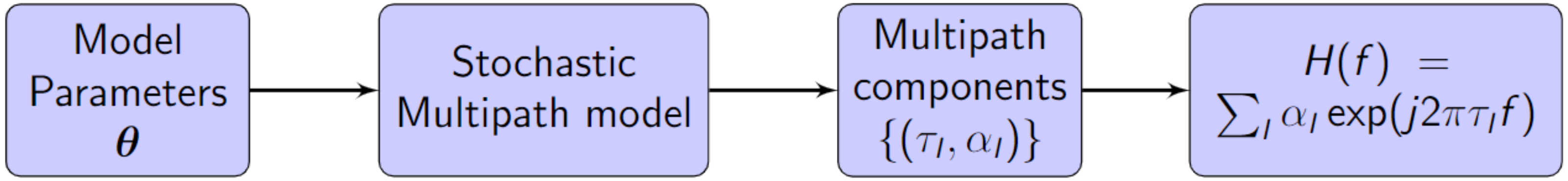
Stochastic Radio Channel Models

Stochastic models of the radio channel are useful simulation tools for designing communication systems

Simulating transfer function from a stochastic multipath model:



Model parameters $\boldsymbol{\theta}$ can be adjusted to mimic different environments.

Hence, their parameters need to be calibrated for the model to be useful.

Calibration: Set model parameters from

- physical knowledge
- measurements

Calibration from measurements

Aim: Given the data Y, estimate parameters θ such that the model fits to the data.

Problem: For most stochastic channel models, the likelihood function $f(Y|\theta)$ is intractable and cannot be evaluated numerically.

Therefore, classical estimation techniques such as

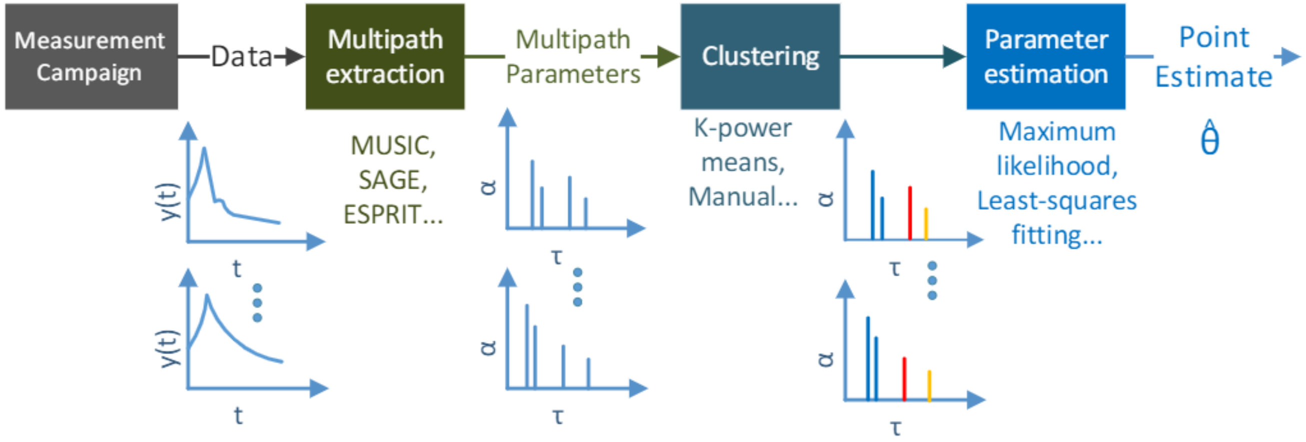
Maximum a Posteriori (MAP) estimate: $\hat{\theta}_{MAP} = \underset{\theta}{\operatorname{argmax}} f(\boldsymbol{Y}|\theta)p(\theta)$

Maximum Likelihood (ML) estimate: $\hat{\theta}_{ML} = \underset{\theta}{\operatorname{argmax}} f(\boldsymbol{Y}|\theta)$

are unrealizable.

State-of-the-art Calibration Method

Multi-step approaches are common:



Drawbacks:

- Requires sophisticated algorithms (multipath extraction, clustering) — cumbersome to use due to several heuristic choices
- Prone to errors (e.g., estimation artifacts, censoring effects)
- Overall performance of these algorithms are hard to investigate
- Calibration methods are specific to the models
  - redesign for new models
  - hard to compare models due to lack of a common calibration method

Calibration of Radio Channel Models is a Synthetic Likelihood Problem

Main observation: Calibration of stochastic channel models is a Bayesian Synthetic Likelihood (BSL) problem:

- The likelihood function is **intractable** and cannot be evaluated numerically.
- The model is **generative** in nature, i.e., it is easy to simulate from.

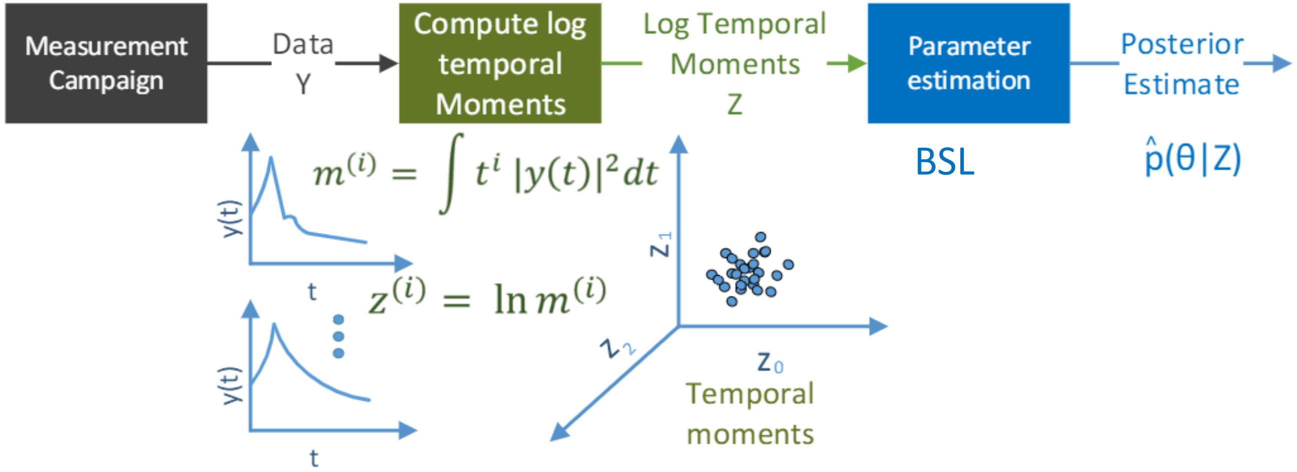
Radio channel models hold both the properties.

- Therefore, BSL can potentially be used to calibrate the models.
  - Similar approaches based on ABC have been proposed [1,2]

1. Bharti and Pedersen, 2019, Calibration of stochastic radio channel models using ABC
2. Bharti et al., 2020, Learning parameters of stochastic radio channel models using summaries

Proposed Calibration Method [1,2]

Use easy-to-compute generic summaries:



Advantages:

- General calibration method applicable to different channel models
- Simpler processing chain
- Fit is based on explicit choice of summaries
- Information on posterior is obtained (not only point estimates)

1. Bharti and Pedersen, 2019, Calibration of stochastic radio channel models using ABC
2. Bharti et al., 2020, Learning parameters of stochastic radio channel models using summaries

Likelihood-Free Inference Methods

We are interested in sampling from the posterior:

$$p(\theta|y) \ \alpha \ p(y|\theta)p(\theta)$$

But $p(y|\theta)$ is intractable!!

Likelihood-free methods can be used when simulation of data from the model is feasible.

- True for most stochastic channel models
- It is typical to reduce the dataset to a lower-dimensional set of summary statistics, $\boldsymbol{s}_y$

Bayesian Synthetic Likelihood (BSL)

Sythetic likelihood [Wood 2010] uses a multi-variate normal approximation

$$p(\boldsymbol{s}_y|\boldsymbol{\theta}) \approx \aleph(\boldsymbol{s}_y : \mu(\boldsymbol{\theta}), \Sigma(\boldsymbol{\theta}))$$

BSL:

- Bayesian version of SL
- Used for calibration in this paper

Algorithm 1: MCMC BSL for Calibration of Stochastic Channel Models.

Input: Prior distribution $p(\boldsymbol{\theta})$, observed summary statistics $\mathbf{s_y}$, number of model realisations for summary statistic computation, N_r, number of summary statistics vectors used per likelihood, L, number of MCMC steps, K

Output: Approximate posterior distribution of $\boldsymbol{\theta}$

Draw $\boldsymbol{\theta}^0$ from the prior distribution;

for $j = 1$ **to** L **do**

  Draw N_r model realisations using $\boldsymbol{\theta}^0$;

  Calculate a summary statistics vector $\mathbf{s}_j^*$;

end

Calculate log-likelihood, $p(\mathbf{s_y}|\boldsymbol{\theta}^0)$ using (3);

for $i = 1$ **to** $K - 1$ **do**

  Draw $\boldsymbol{\theta}^*$ from proposal distribution $\mathcal{N}(\boldsymbol{\theta}^{i-1}, \Sigma_{\theta})$;

  for $j = 1$ **to** L **do**

    Draw N_r model realisations using $\boldsymbol{\theta}^*$;

    Calculate a summary statistics vector $\mathbf{s}_j^*$;

  end

  Calculate log-likelihood, $p(\mathbf{s_y}|\boldsymbol{\theta}^*)$ using (3);

  Compute $r = \exp\left(p(\mathbf{s_y}|\boldsymbol{\theta}^*) - p(\mathbf{s_y}|\boldsymbol{\theta}^{i-1})\right)$;

  if $\mathcal{U}(0,1) < r$ **then**

    $\boldsymbol{\theta}^i = \boldsymbol{\theta}^*$;

  else

    $\boldsymbol{\theta}^i = \boldsymbol{\theta}^{i-1}$;

  end

end

Summary Statistics – Temporal Moments

Temporal moments have been used since 1970s in wireless communications.

$$m_i = \int t^i |y(t)|^2, \qquad i = 0,1, \dots, (l-1)$$

They are used to compute widely used statistics such as the instantaneous received power, P_0, mean delay, $\bar{\tau}$, and rms delay spread, τ_{rms}:

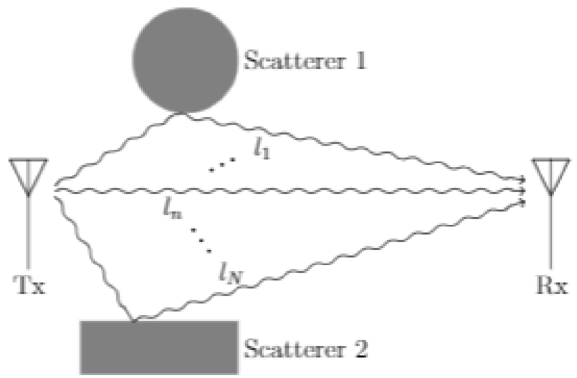
$$P_0 = m_0, \qquad \bar{\tau} = \frac{m_1}{m_0}, \qquad \text{and} \qquad \tau_{rms} = \sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2}$$

They have been found to be well-modeled by a multivariate log-normal distribution in various environments [3]

Mean and covariances used for calibration in this paper.

Example 1: Turin Model

A widely used stochastic radio channel model



$$H(f) = \sum_{l=0}^{\infty} \alpha_l e^{-i2\pi\tau_l f}$$

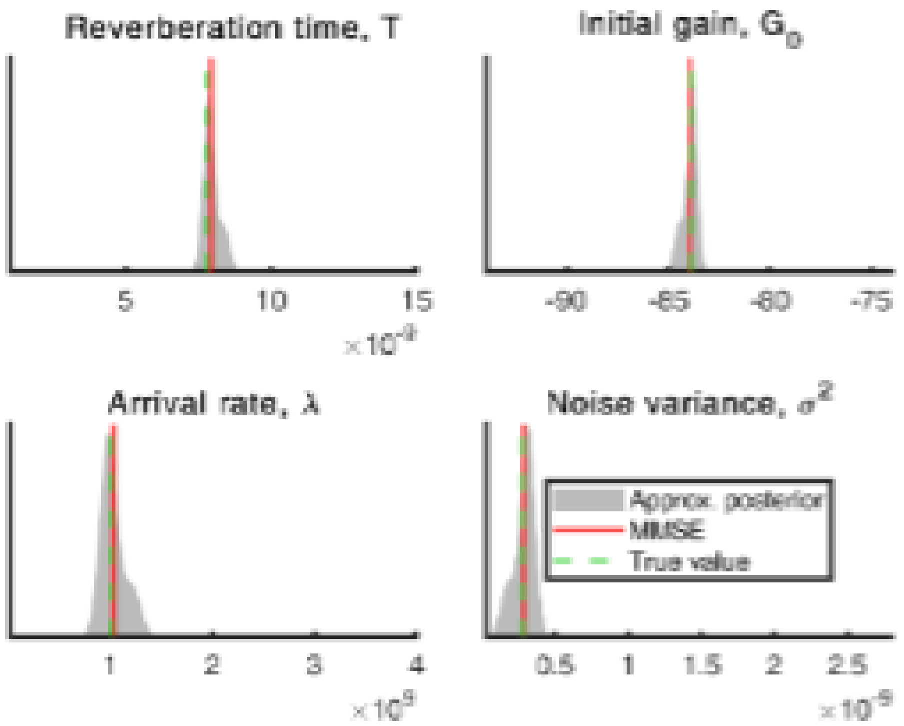
τ -> PPP and α -> complex normal conditioned on τ.

- Popular reverberation model used
- Parameters to estimate:

$$\theta = [T, G_0, \lambda, \sigma_n]$$

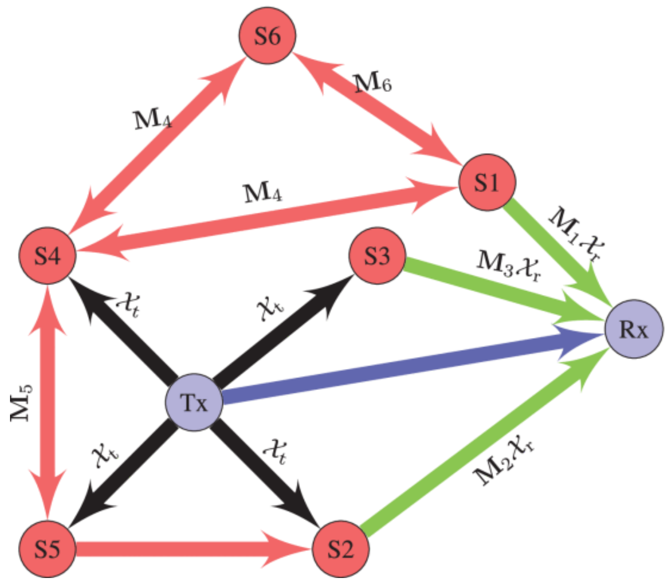
Settings:

- Flat prior
- $T = 7.8 \times 10^{-9}\text{s}; \quad \text{G}_0 = -83.9 \text{ dB}$
- $\lambda = 1 \times 10^9 \, Hz; \qquad \sigma_n = 2.8 \times 10^{-10}$



Example 2: Polarized Propagation Graph [4]

An elegant model based on simple direction graph $G = (V, E)$



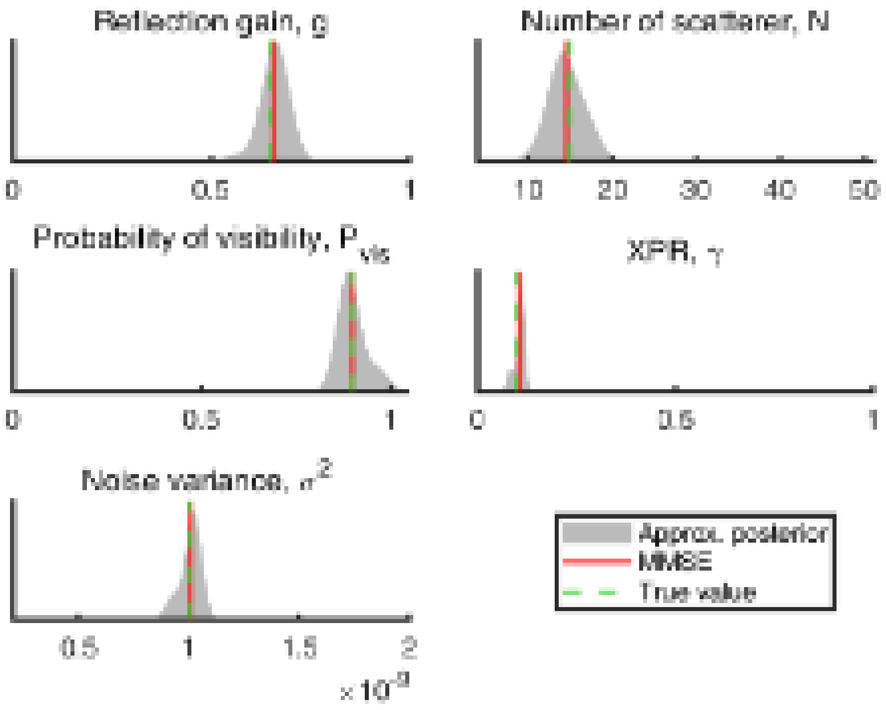
Transfer function:

$$H(f) = D(f) + R(f)[I - B(f)]^{-1}T(f)$$

- Stochastic PPG model used
  - Parameters : $\theta = [g, N_s, P_{vis}, \gamma, \sigma^2]$

Settings: Flat prior

- $g = 0.65$; $N_s = 15$; $P_{vis} = 0.9$
- $\gamma = 0.1$, $\sigma_n^2 = 10^{-9}$



4. Adeogun, et. al, 2019, Polarimetric Wireless Indoor Channel Modeling Based on Propagation Graph

Conclusions

- Calibration of stochastic channel models is a likelihood-free inference problem.
- The proposed BSL method relies on temporal moments of the received signal.
  - **No multipath extraction or clustering is circumvented.**
- The method is automatic as no pre- or post-processing of the data and estimates or additional input from the user are required.
- Priors can be set according to the information available.
  - We set range of flat priors according to physical or system constraints
- The proposed method can accurately calibrate channel models of very different mathematical structure enabling model comparison.
  - Performance obtained are like those from ABC based calibration methods