Δ , H^+ $\qquad Q(z) = \dfrac{z - i}{-iz + 1}$

$H^+ \longrightarrow \Delta$

$G_\Delta^+ = \{ f \in PSL(2, \mathbb{C}) \mid f(\Delta) = \Delta \}$

$G_{H^+}^+ \qquad\qquad H^+ \quad H^+$

<u>Lema</u> $f \in PSL(2,\mathbb{C})$, f circunferencias$^+$ en circunferencias$^+$.

<u>Dem</u> $\quad Ax^2 + Ay^2 + Dx + Ey + F = 0$

ecuación de circunferencia$^+$ $\quad (\underline{A = 0})$

$Re(z) = x = \dfrac{z + \bar{z}}{2} \qquad \overset{Im(z)}{\overset{\shortparallel}{y}} = \dfrac{z - \bar{z}}{2i}$

$$A\left[\frac{(z+\bar{z})^2}{4} - \frac{(z-\bar{z})^2}{4}\right] + D\,\frac{z+\bar{z}}{2} + E\,\frac{z-\bar{z}}{2i} + F = 0$$

$$A z\bar{z} + \underbrace{\frac{D - iE}{2}}_{B} z + \underbrace{\frac{D + iE}{2}}_{\bar{B}} \bar{z} + F = 0$$

$$\boxed{A z\bar{z} + B z + \bar{B}\bar{z} + C = 0 \qquad A, C \in \mathbb{R}}$$

Ecuación circunf $+$ $\quad \mathbb{C}$

$f(z) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} z \\ 1 \end{pmatrix} \qquad ad - cb = 1$

$f : \hat{\mathbb{C}} \longrightarrow \hat{\mathbb{C}} \qquad f[C]$

$$z = f^{-1}(w) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} w \\ 1 \end{pmatrix} \qquad ad-bc=1$$

$$z = \frac{dw-b}{-cw+a} \qquad \bar{z} = \frac{\bar{d}\bar{w}-\bar{b}}{-\bar{c}\bar{w}+\bar{a}}$$

$$\underset{\neq 0}{A} z\bar{z} + Bz + \bar{B}\bar{z} + C = 0$$

$$\Downarrow$$

$$\underset{=0}{A'} w\bar{w} + B'w + \bar{B'}\bar{w} + C' = 0$$

$$A', C' \in \mathbb{R}$$

$\blacksquare$

<u>Corolario</u> 4 puntos en $\mathbb{C}P^1$ estan en una circunferencia + si y solo si su razon cruzada (doble) es real

Fundamental de la geom. proy.

Teo (de 3 en 3) dados 3 puntos z_i arbitrarios en $\mathbb{R}P^1$ y sus imagenes w_i existe una única $T \in Pr(1)$ $\left(PSL(2, \underset{\mathbb{R}}{\mathbb{C}}) \right)$

$$T(z_i) = w_i$$

$\square$

z_1, z_2, z_3 puntos arbitrarios

$w_1, w_2, w_3 \in \mathbb{R}$

$$\mathbb{C} \xrightarrow{T} \mathbb{R}$$

$z \in C(z_1, z_2, z_3)$

$(w_1, w_2; w_3, T(z)) \in \mathbb{R}$

$\shortparallel$

$(z_1, z_2; z_3, z)$

G^+_Δ basta con que se fije la frontera

$G^+_{H^+}$

$G^+_{H^+} = \{ f \in PSL(2, \mathbb{C}) \mid f(\mathbb{R}) = \mathbb{R} \}$

Si $a, b, c, d \in \mathbb{R}$ $f[\mathbb{R}] = \mathbb{R}$

Af Es condición suficiente

$f(z) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} z \\ 1 \end{pmatrix} = \dfrac{az+b}{cz+d}$ ✓

$f(0) = b/d \in \mathbb{R}$ $b = \lambda d$ $\lambda \in \mathbb{R}$

$f(\infty) = a/c \in \mathbb{R}$ $a = \mu c$ $\mu \in \mathbb{R}$

$f(1) = \dfrac{a+b}{c+d} \in \mathbb{R}$ $c + d = k(a+b)$ $k \in \mathbb{R}$

$c = ka + kb - d$

$ka + (k\lambda - 1)d$

$c = k\mu c + (k\lambda - 1) d$

$c = \dfrac{(k\lambda - 1)}{(1 - k\mu)} d$ ✓

$$f(z) = \begin{pmatrix} \mu_1 d & \mu_2 d \\ \mu_3 d & d \end{pmatrix} \begin{pmatrix} z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \mu_1 & \mu_2 \\ \mu_3 & 1 \end{pmatrix}$$

$$\sqrt{\det} \quad \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \quad a'd' - c'b' = 1$$

$$\in PSL(2, \mathbb{R})$$

así tienen la forma

$$\frac{az+b}{cz+d} \qquad \frac{a(-\bar{z})+b}{c(-\bar{z})+d}$$

$$ad - bc = 1$$

$$a, b, c, d \in \mathbb{R}$$

$$G^+_{H^+} = PSL(2, \mathbb{R}) \subset PSL(2, \mathbb{C})$$

G_{H^+}

G_{H^+} componer $\int$ con la reflexión en Im

Para el disco

$$\begin{tikzcd} \Delta \arrow[r, bend left, "Q^{-1}"] & H^{+} \arrow[l, bend left, "Q"] \arrow[loop right] \end{tikzcd}$$

$$Q = \frac{z - i}{-iz + 1}$$

$$Q : H^{+} \longrightarrow \Delta$$

¿ $Q\,T\,Q^{-1}$ forma ? : $\Delta \longrightarrow \Delta$

$G^{+}_{H^{+}}$ $\quad$ $G^{-}_{H^{+}}$

$$G^{+}_{\Delta} \cup G^{-}_{\Delta} = G_{\Delta}$$