

Δ, H^+

$$Q(z) = \frac{z - i}{-iz + 1}$$

$$H^+ \longrightarrow \Delta$$

$$G_{\Delta}^+ = \left\{ f \in \text{PSL}(2, \mathbb{C}) \mid f(\Delta) = \Delta \right\}$$

$$G_{H^+}^+$$

$$H^+ \quad H^+$$

Lema $f \in \text{PSL}(2, \mathbb{C})$, f
en circunferencias⁺.

circunferencias⁺

Der $Ax^2 + Ay^2 + Dx + Ey + F = 0$
ecuación de circunferencia⁺ ($A = 0$)

$$R_0(z) = X = \frac{z + \bar{z}}{2} \quad \text{Im}(z) = \frac{z - \bar{z}}{2i}$$

$$A \left[\frac{(z + \bar{z})^2}{4} - \frac{(z - \bar{z})^2}{4} \right] + D \frac{z + \bar{z}}{2} + E \frac{z - \bar{z}}{2i} + F = 0$$

$$A z \bar{z} + \underbrace{\frac{D - iE}{2}}_B z + \underbrace{\frac{D + iE}{2}}_{\bar{B}} \bar{z} + F = 0$$

$$A z \bar{z} + B z + \bar{B} \bar{z} + C = 0 \quad A, C \in \mathbb{R}$$

Ecuación circunferencia⁺ \mathbb{C}

$$f(z) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} z \\ 1 \end{pmatrix} \quad ad - bc = 1$$

$$f: \hat{\mathbb{C}} \longrightarrow \hat{\mathbb{C}} \quad f[\hat{C}]$$

$$z = f^{-1}(w) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} w \\ 1 \end{pmatrix} \quad ad - bc = 1$$

$$z = \frac{dw - b}{-cw + a} \quad \bar{z} = \frac{\bar{d}\bar{w} - \bar{b}}{-\bar{c}\bar{w} + \bar{a}}$$

$$\neq 0 \quad A z \bar{z} + B z + \bar{B} \bar{z} + C = 0$$

↓

$$\stackrel{=0}{=} A' w \bar{w} + B' w + \bar{B}' \bar{w} + C' = 0$$

$A', C' \in \mathbb{R}$ □

Corolario 4 puntos en \mathbb{CP}^1
están en una circunferencia⁺
si y solo si su razón cruzada
(doble) es real

← Fundamental de la
geom. proy.

Teo (de 3 en 3) dados 3 puntos z_i
arbitrarios en \mathbb{RP}^1 y sus imágenes w_i
existe una única $T \in \text{Pr}(1)$ $\left(\text{PSL}(2, \frac{\mathbb{C}}{\mathbb{R}}) \right)$
 $T(z_i) = w_i$ □

z_1, z_2, z_3 puntos arbitrarios
 \xrightarrow{T}
 $w_1, w_2, w_3 \in \mathbb{R}$

$$C \xrightarrow{T} \mathbb{R}$$

$$z \in C(z_1, z_2, z_3)$$

$$(w_1, w_2; w_3, T(z)) \in \mathbb{R}$$

"

$$(z_1, z_2; z_3, z)$$

$$G^+$$

basta con que se
fije la frontera

$$G_{H^+}^+$$

$$G_{H^+}^+ = \{ f \in \text{PSL}(2, \mathbb{C}) \mid f(\mathbb{R}) = \mathbb{R} \}$$

$$s. \quad a, b, c, d \in \mathbb{R} \quad f[\mathbb{R}] = \mathbb{R}$$

Ap Es condición suficiente

$$f(z) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} z \\ 1 \end{pmatrix} = \frac{az+b}{cz+d}$$

$$f(0) = b/d \in \mathbb{R}$$

$$f(\infty) = a/c \in \mathbb{R}$$

$$f(1) = \frac{a+b}{c+d} \in \mathbb{R}$$

$$b = \lambda d \quad \lambda \in \mathbb{R}$$

$$a = \mu c \quad \mu \in \mathbb{R}$$

$$c+d = k(a+b) \quad k \in \mathbb{R}$$

$$c = ka + kb - d$$

$$ka + (k\lambda - 1)d$$

$$c = k\mu c + (k\lambda - 1)d$$

$$c = \frac{(k\lambda - 1)}{(1 - k\mu)} d \quad \checkmark$$

$$f(z) = \begin{pmatrix} \mu_1 d & \mu_2 d \\ \mu_3 d & d \end{pmatrix} \begin{pmatrix} z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \mu_1 & \mu_2 \\ \mu_3 & 1 \end{pmatrix}$$

$$\sqrt{\det} \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \quad a'd' - c'b' = 1$$

$$\in \text{PSL}(2, \mathbb{R})$$

$$G_{H^+}^+ = \text{PSL}(2, \mathbb{R}) \subset \text{PSL}(2, \mathbb{C})$$

G_{H^+} componer \uparrow con la reflexión
en $\mathbb{I}m$

así tener la forma

$$\frac{az+b}{cz+d}$$

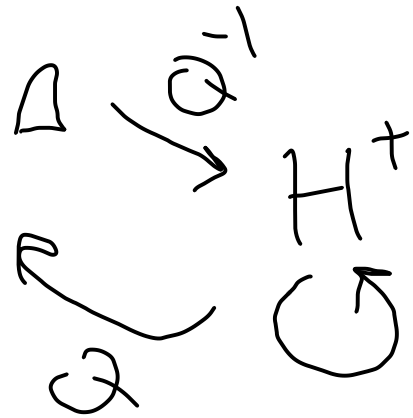
$$\frac{a(-\bar{z})+b}{c(-\bar{z})+d}$$

$$ad - bc = 1$$

$$a, b, c, d \in \mathbb{R}$$

$$G_{H^+}$$

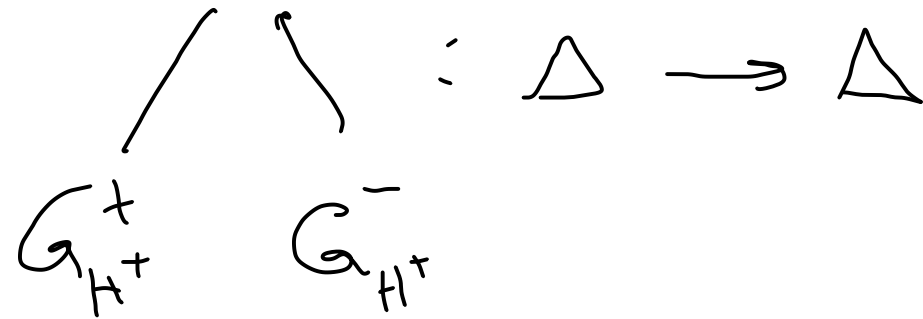
Para el disco



$$Q = \frac{z - i}{-iz + 1}$$

$$Q: H^+ \longrightarrow \Delta$$

$i \quad Q \tau Q^{-1}$ forma?



$$G_{\Delta}^+ \cup G_{\Delta}^- = G_{\Delta}$$