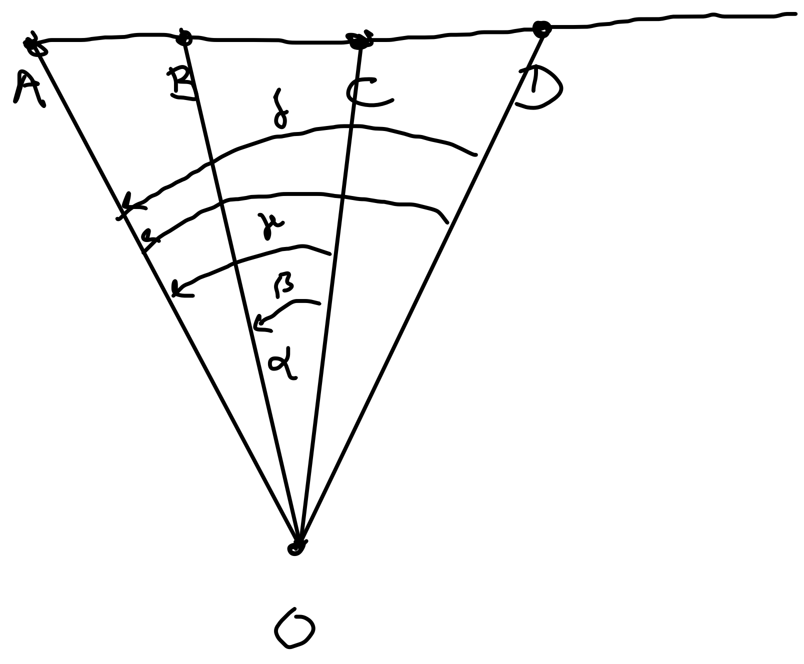
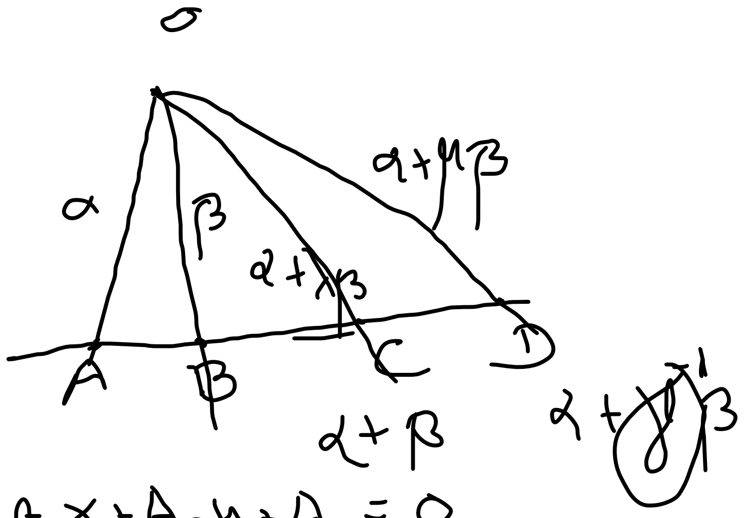
Razón cruzada





$\alpha \quad A_1x+A_2y+A_3=0$

β

$$(A,B;C,D) = \frac{\lambda}{\mu}$$

$A = (a_1 : a_2)$ $\quad =1$

$$(A,B;C,D) = \frac{|\quad|\,|\quad|}{|\quad|\,|\quad|}$$

<u>Teo</u> A,B,C,D colineales

$T \in PGL(3,\mathbb{R})$

$\gamma = (A,B;C,D) = (T(A),T(B);T(C),T(D))$

via los haces $\alpha, \beta, \alpha+\beta, \alpha+\gamma^{-1}\beta$ y su <u>transformado</u>

Conicas en $\mathbb{P}^2$

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0$$

homogeneizarla

$$Ax^2+2Bxy+Cy^2+2Dxz+2Eyz+Fz^2=0$$

$$(x\ y\ z)\begin{pmatrix} A & B & D \\ B & C & E \\ D & E & F \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\begin{pmatrix} \pm 1 \lambda_1 & & \\ & \pm 1 \lambda_2 & \\ & & \pm \lambda_3 \end{pmatrix} = C$$

Rango 3, 2, 1

No singulares rango 3

signatura $(\# > 0 - \# < 0)$ $\quad x^2+y^2+z^2=0 = \emptyset$

3

signatura 1

$x^2+y^2-z^2=0$ conica no singular

singulares rango 2

signatura 2

$x^2+y^2=0$ $\quad (0:0:z)$ punto proyectivo

signatura 0

$x^2-y^2=0$ $\quad \left.\begin{matrix}(x:x:z)\\(x:-x:z)\end{matrix}\right\}$ rectas proy.

rango 1

$x^2=0$ $\quad (0:y:z)$ ← recta doble.

$$C = \begin{pmatrix} A & B & D \\ B & C & E \\ D & E & F \end{pmatrix}$$ matriz simétrica 6 variables $\in \mathbb{P}^5(\mathbb{R})$

λ

$\mathbb{R}^*$

$(A:B:C:D:E:F) \in \mathbb{P}^5(\mathbb{R})$

representa una cónica no singular si

$\det(C) \neq 0$ espacio de las no singulares es dim 5

$ACF + 2BED - CD^2 - AE^2 - FB^2 = 0 \leftarrow$ hipersuperficie de $\mathbb{P}^5$

dim 4 (cónicas singulares) (pares de rectas)

Cónicas de rango 1 dim 2 (rectas)

$f[A] = \{f(a) \mid a \in A\}$

$C = \{(x:y:z) \in \mathbb{P}^2 \mid x^t C x = 0\}$

$\mathbb{R} \longrightarrow \mathbb{C}$

$x^2 + y^2 + z^2 = 0 \quad = \emptyset$

Def $\nu : \mathbb{P}^2 \longrightarrow \mathbb{P}^5$

$(x:y:z) \longmapsto (x^2 : y^2 : z^2 : 2xy : 2xy : 2yz)$

la Veronese, inyectiva, diferenciable

su imagen $\subset \mathbb{P}^5$ es la superficie Veronese

$\Pi_c = (A:B:C:D:E:F)^\perp$ hiperplano

$Ax_0 + Bx_1 + Cx_2 + Dx_3 + Ex_4 + Fx_4 = 0$

$C = (A:B:C:D:E:F)$

$\nu[C] \subset \Pi_c \cap$ Superf Veronese

$C \longmapsto \Pi_c \in (\mathbb{P}^5)^*$