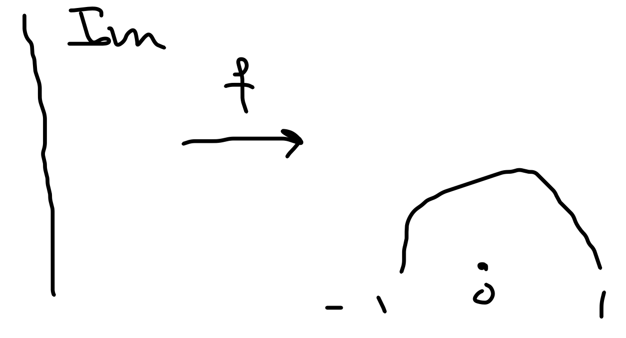
Teorema

$G_{H^+} = Iso(\mathbb{H}, \ )$ ← métrica hiperbólica

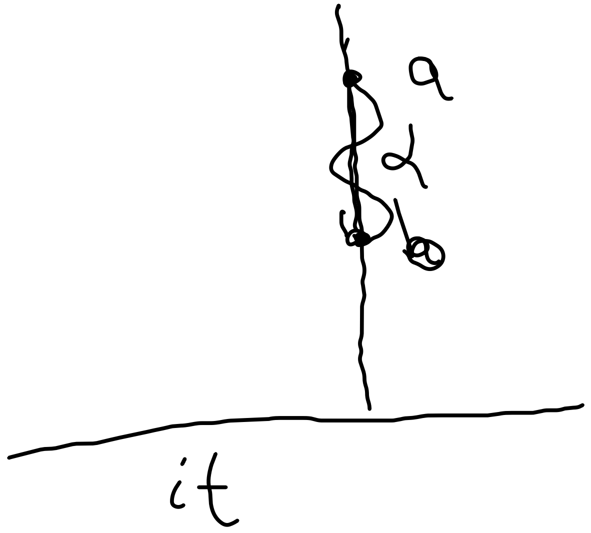


isometrías

traslaciones $a \in \mathbb{R}$

y homotecias

Teo Las rectas hiperbólicas son geodésicas i.e. minimizan la distancia entre sus puntos.



$\alpha : [0,1] \longrightarrow \mathbb{H}^+$

$\alpha(0) = a \quad \alpha(1) = b$

$\alpha(t) = (x(t), y(t))$

$\alpha'(t) = (x'(t), y'(t))$

$$\ell(\alpha) = \int_0^1 \|\alpha'(t)\|_H \, dt$$

$$\|\alpha'(t)\|_H = \|(x'(t), y'(t))\|_H \geq \|(0, y'(t))\|_H$$

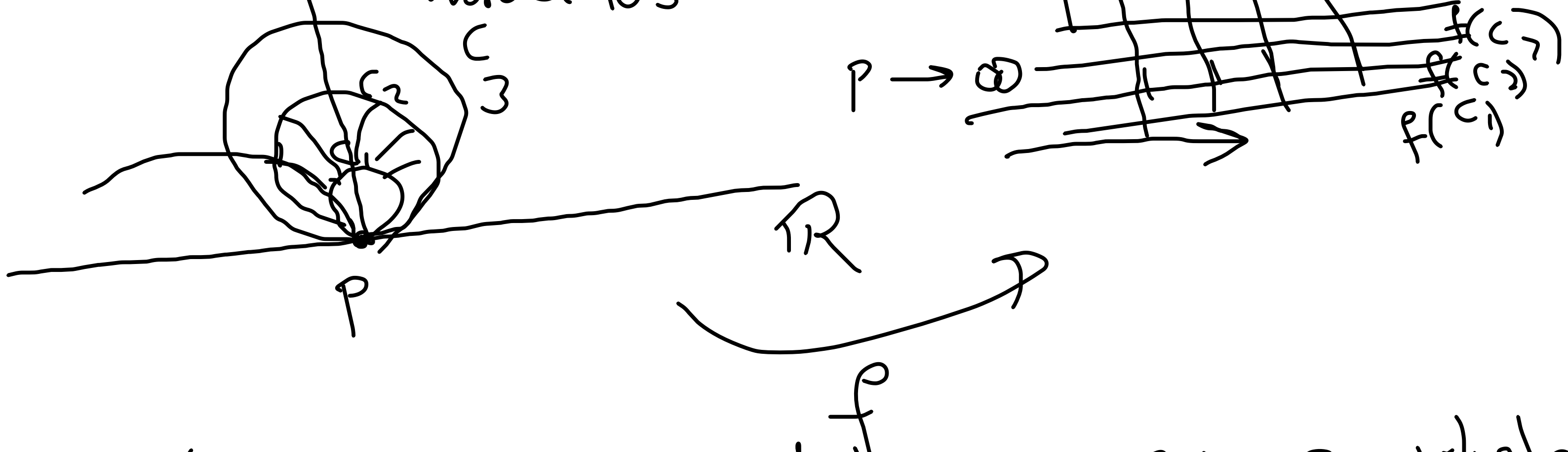
$$\ell(\alpha) \geq \int_0^1 \|(0, y'(t))\|_H \, dt = \int_b^a \frac{1}{t} \, dt$$

$$= \ln\left(a/b\right)$$

↗ minimiza

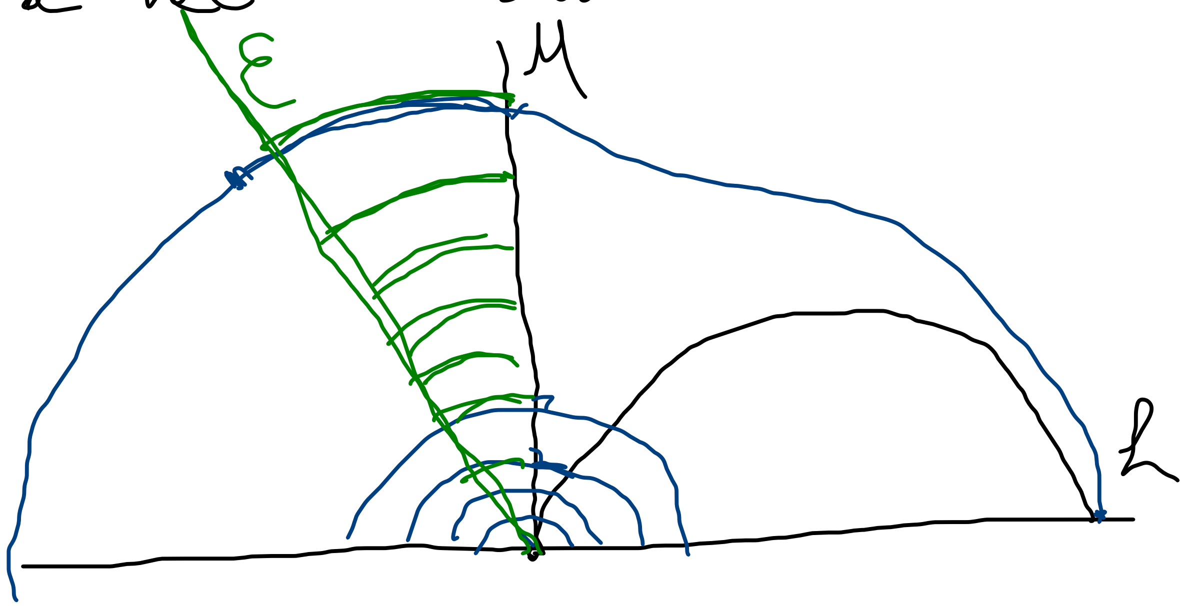
Proposición Los horociclos tangentes en un punto al infinito son equidistantes

horociclos



$\infty \in \mathbb{R}$

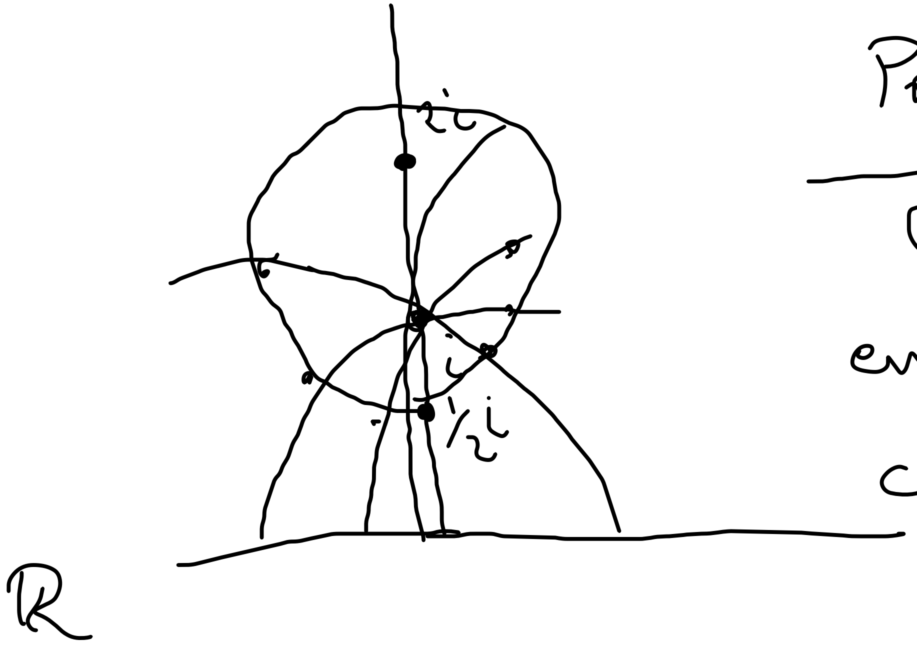
Afirmación Rectas paralelas no son equidistantes de hecho su distancia crece tanto como se quiera

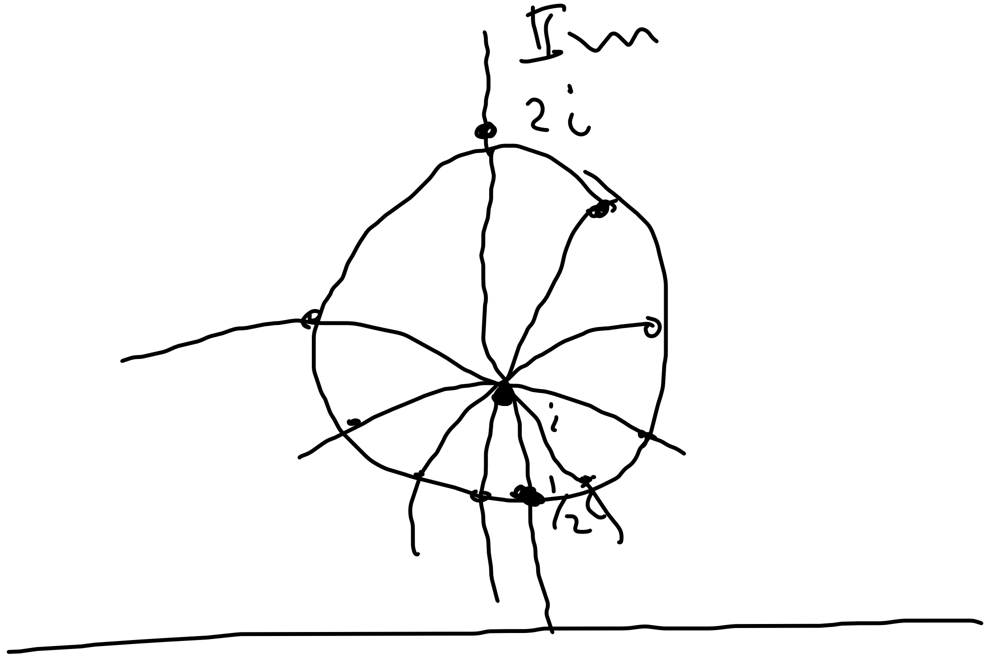


E es equidistante a M

Proposición

Una circunferencia hiperbólica en una circunferencia euclidiana con el centro desplazado hacia Re





Teo $Iso^{+}(\mathbb{H}) = PSL(2,\mathbb{R})$

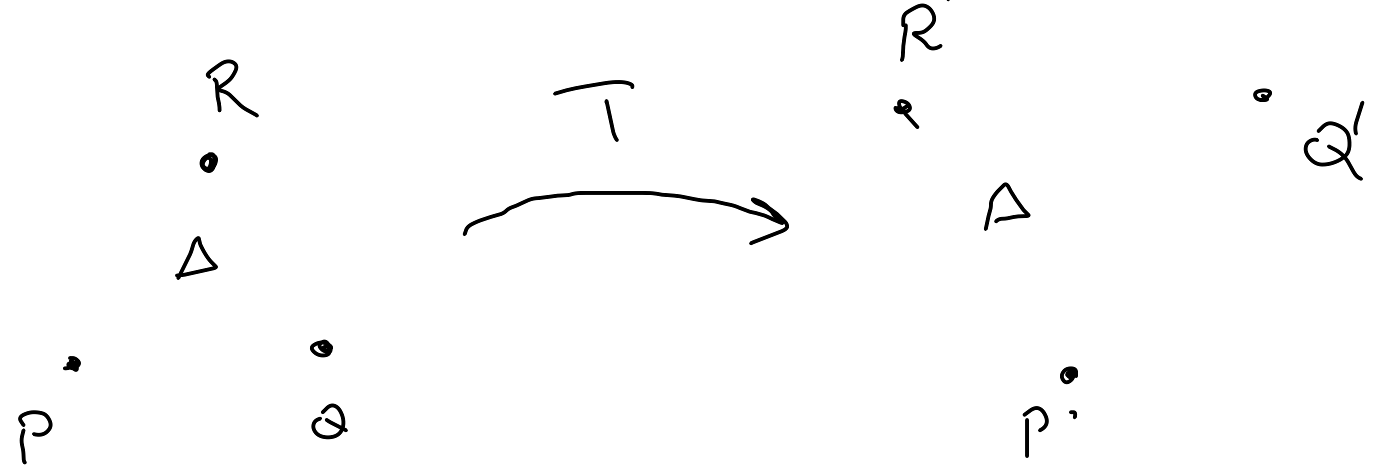
$S \in \mathbb{H}^{+}$

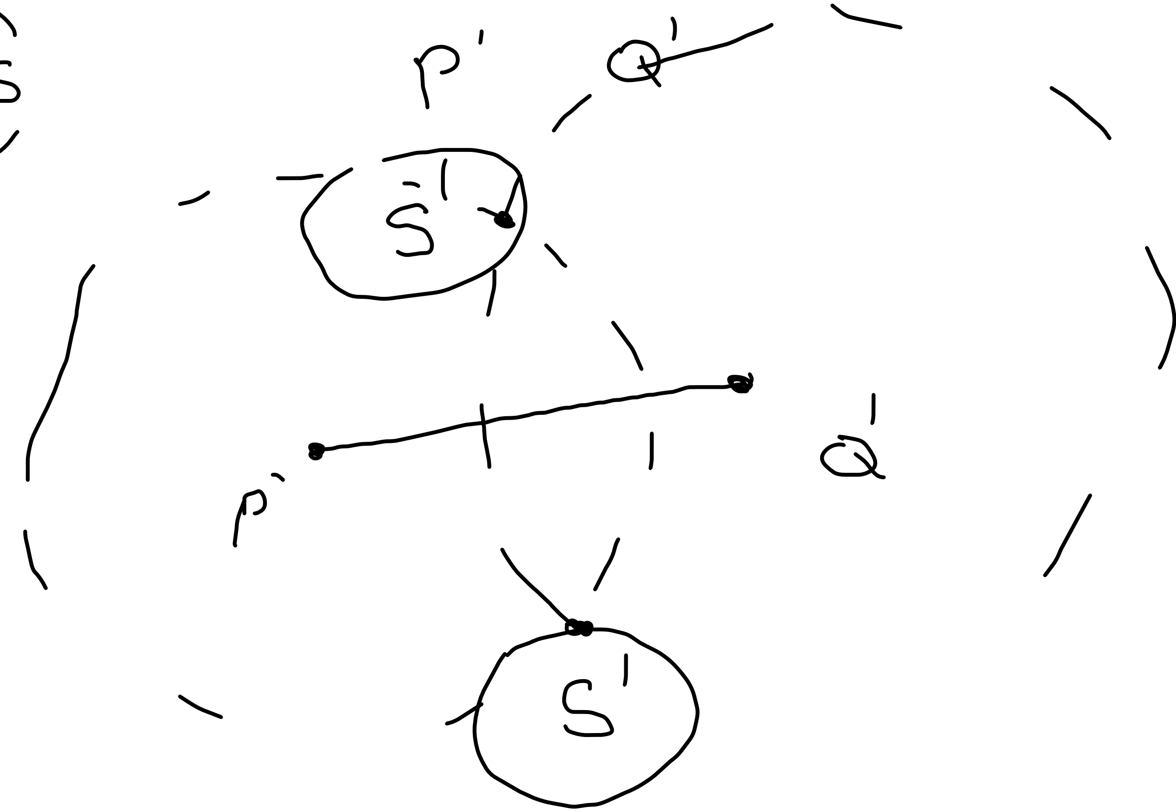
Ya vimos $\supset$ falta $\subset$

preserve la orientación

Sea T una isometría de $\mathbb{H}^{+}$

$\Delta\, PQS$





$T : x \longrightarrow x'$

Tomo S' tal que

$$(P,Q;R,S) = (P',Q';R',T(S))$$

$$f : \begin{matrix} P \\ Q \\ R \end{matrix} \longrightarrow \begin{matrix} P' \\ Q' \\ S' \end{matrix}$$

así $T = f \in PSL(2,\mathbb{R})$

$Q: H^+ \to \Delta$

$z_0 \longrightarrow z$ $\quad u,v$

Como se ve la métrica en Δ

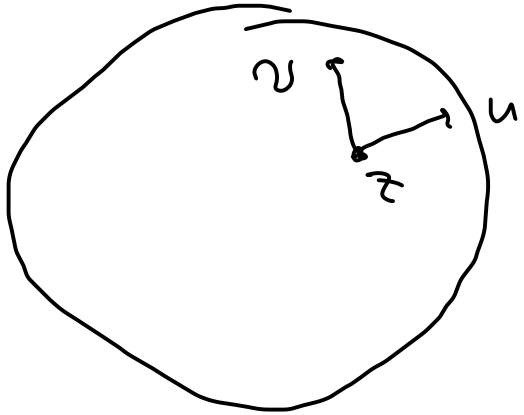
$Q: H^+ \longrightarrow \Delta$ $\qquad \dfrac{z-i}{-iz+1} = f(z)$

$\cdot_H \qquad (JQ)^{-1}$

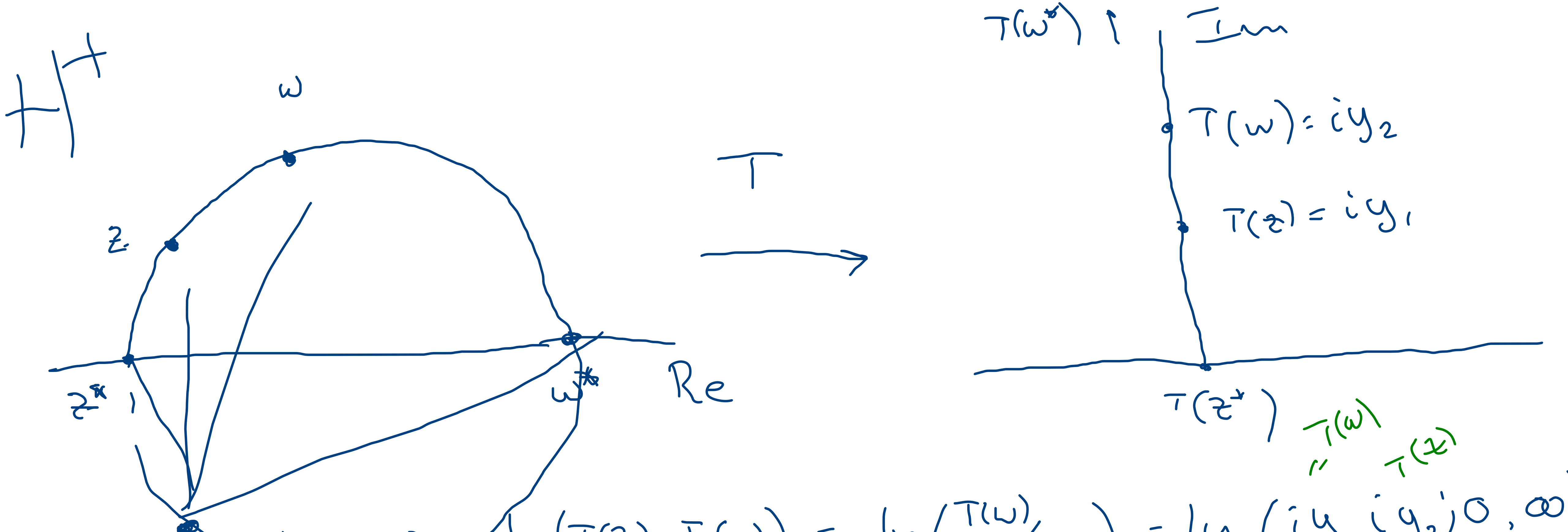
tenemos

$$\boxed{u \cdot_\Delta v = 4\,\frac{u \cdot v}{(1-r^2)^2}}$$

$u, v \in T_{re^{i\theta}}\Delta$



$z = re^{i\theta}$



$$d_H(z,w) = d_H(T(z),T(w)) = \ln\left(T(w)/T(z)\right) = \ln\left(\overset{T(w)}{iy_1}, \overset{T(z)}{iy_2}; 0, \infty\right)$$

$$\boxed{d_H(z,w) = \ln\left(z, w, z^*, w^*\right)}$$

usando φ vale en Δ