Acción $\mathbb{Z}$ en $\mathbb{R}^2$

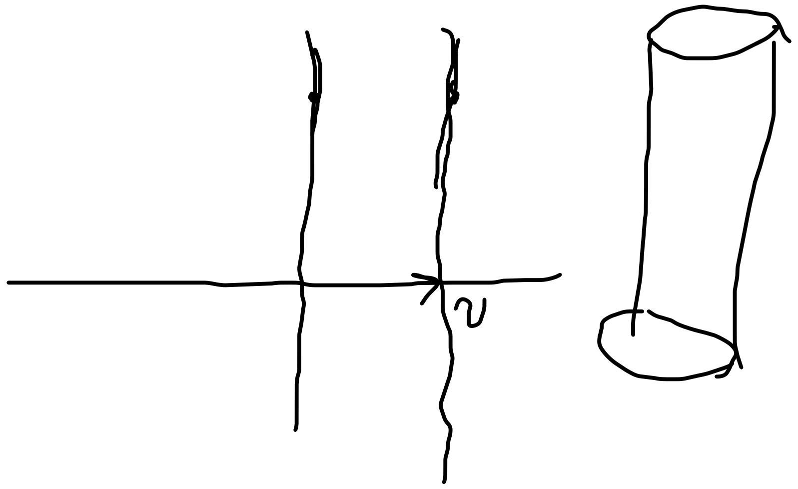
$v \neq 0 \in \mathbb{R}^2$

$T_v(x) = x + v$

$n \longmapsto T^n : \mathbb{R}^2 \to \mathbb{R}^2$

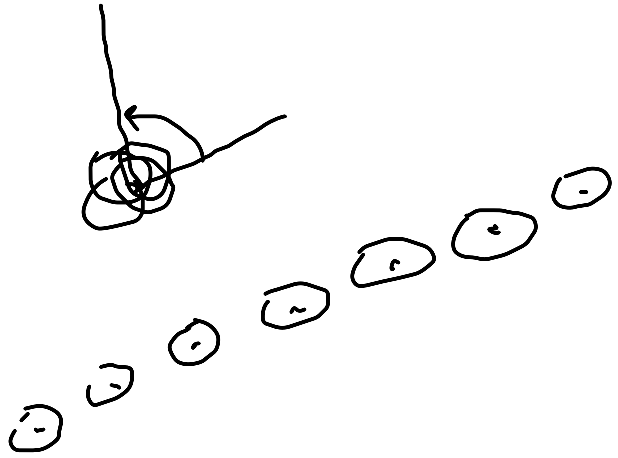
$T : \mathbb{Z} \times \mathbb{R}^2 \to \mathbb{R}^2$

$\mathbb{R}^2 / \mathbb{Z}$ = Cilindro



$\mathbb{R}^2$

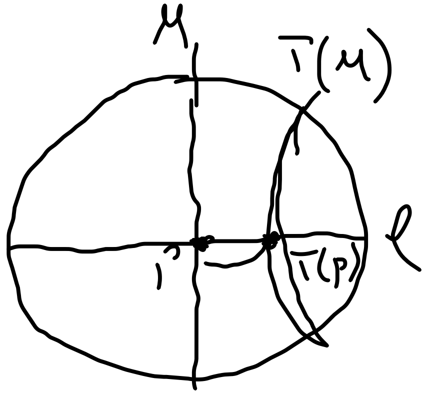
$\mathbb{H}^2$



$T : \mathbb{H}^2 \to \mathbb{H}^2$

$\mathbb{H}^2$ sin puntos fijos

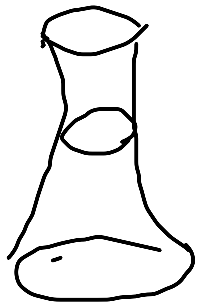
$T(z) = kz$



$0, \infty$

ℓ eje imag es invariante

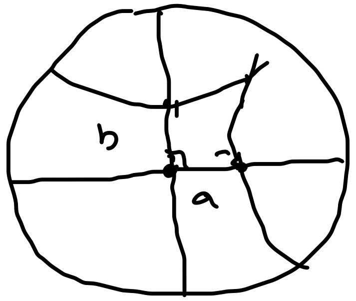
$d(p, T(p)) = a = \ln(k)$

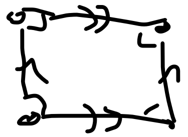


cilindro

$\mathbb{H}^2$

Superficies compactas

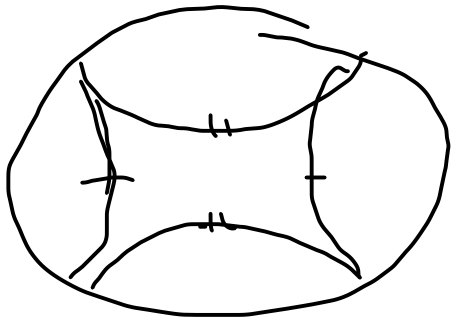




diferenciable

$\mathbb{R}^2$ →

Teorema (Clasificación) Toda superficie compacta y orientada es una esfera con asas



$g=1$

toro ponchado

Def Característica de Euler $\chi = 2-2g$

Obs Es un invariante topológico.

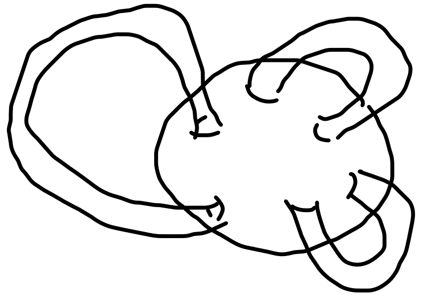
6 8 - - - . $\sum k = 2\pi$

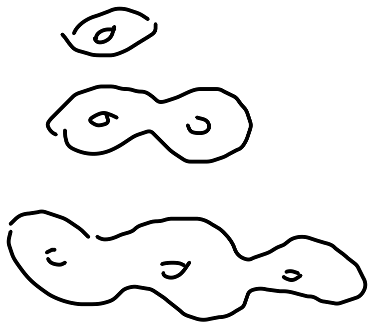
$\mathbb{C}$

Teorema (Uniformización) Sup. Riemann puede dotarse de una estructura $\begin{cases}\text{elíptica}\\ \text{parabólica}\\ \text{hiperbólica}\end{cases}$ según sea conformemente equivalente al conj. de clases de equivalencia de $\begin{cases}\hat{\mathbb{C}}\\ \mathbb{C}\\ \Delta\end{cases}$ bajo un subgrupo discontinuo de $PSL(2,\mathbb{C})$.

Polígono 4g lados → Toro con g asas

$g>2$

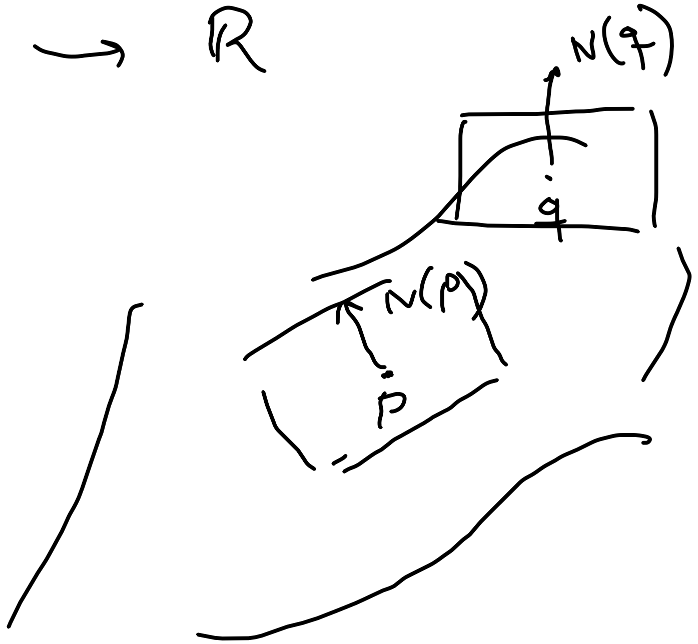




Teorema (Gauss - Bonnet)

$$\int_S K \, dA = 2\pi \underline{\chi(S)}$$

$K : S \to \mathbb{R}$

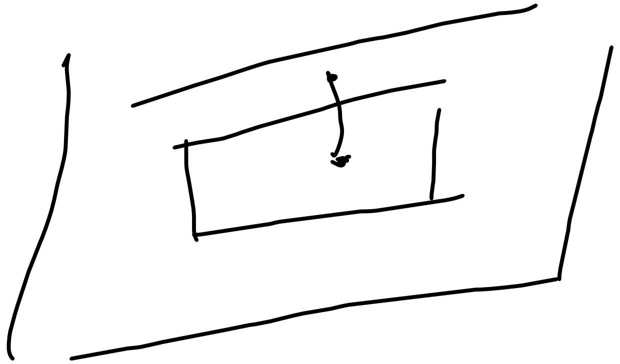


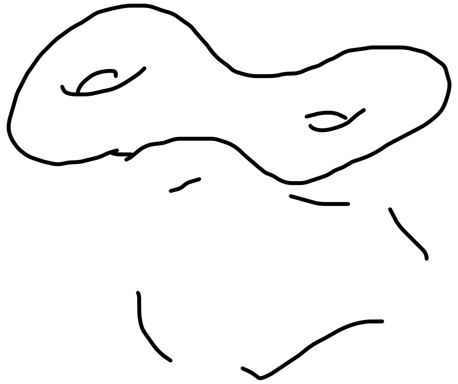
$\underline{S} \subset \mathbb{R}^3 \xrightarrow{N} \mathbb{S}^2$

$\det(dN) \in \mathbb{R}$

K :

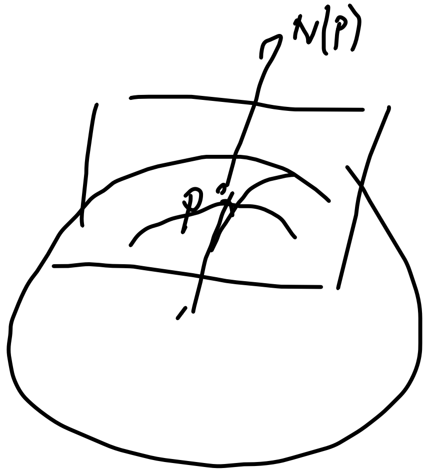
$K = 0$



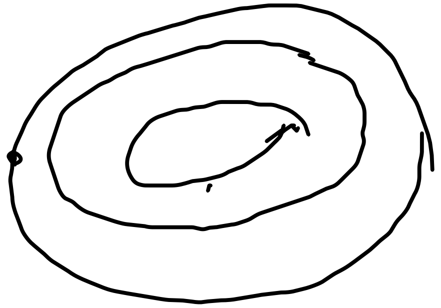


$K = 1$

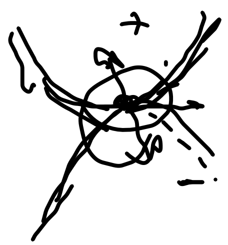
$\mathbb{S}^2$



$N = Id$



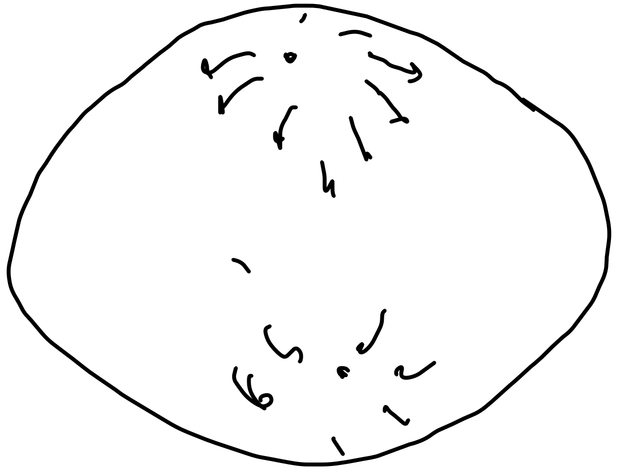
< 0

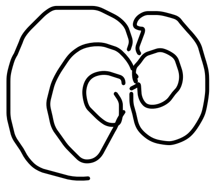


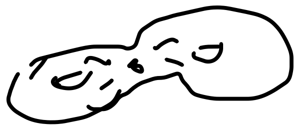
Teorema Poincaré - Hopf

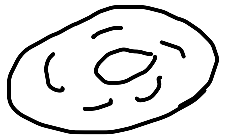
Dado un campo vectorial en S la suma de los índices de los puntos singulares aislados es igual a $\chi(S)$.

$\chi(S) = 2$









$\chi = 0$