Computer Science 130B
Winter 2014
Homework #2

Due: Jan 31, Friday

Problem 1 Consider the coin change problem. You are at a check out register, and have an unlimited supply of quarters, dimes, nickels, and pennies. You have to make change for a customer. Design a greedy algorithm to make out an amount of x cents that uses the smallest number of coins. Prove the optimality of your algorithm.

Problem 2 You are pairing couples for a very conservative formal ball. There are n men and m women, and you know the height and gender of each person there. Each dancing couple must be a man and a woman, and the man must be at least as tall as, but no more than 3 inches taller than, his partner. You wish to maximize the number of dancing couples given this constraint. Give a greedy algorithm to accomplish this and analyze the complex of your algorithm. Can your greedy strategy guarantees the optimal solution? If not, given a counter example. You do not need to prove your strategy is optimal.

Problem 3 You are given n intervals on the x-axis, each specified by a starting and an end x-coordinate, or internal i goes from s_i to e_i inclusively and $s_i < e_i$. We want to select a maximum set of mutually non-overlapping intervals (here, maximum means that the selected set contains the largest number of the input intervals).

There are many greedy strategies that can be used to select intervals from the given set. For example, some reasonable strategies are selecting intervals based on (1) their starting x locations (smaller ones first), (2) their end x locations (smaller ones first), (3) their lengths (shorter ones first), and (4) the number of other intervals that they overlap with (fewer overlaps first).. It is not clear if all these different strategies will generate the same results and/or if any of them will always generate the optimal results (the most number of intervals included). For each strategy, give an example if you believe that such a strategy will not generate the optimal results. You do not need to prove a particular strategy is optimal.

Problem 4 Assume n programs of length $l_1, l_2, \cdots, l_n$ are to be stored on a tape. Program i is to be retrieved with frequency f_i. If the program are stored in the order of $i_1, i_2, \cdots, i_n$, the expected retrieval time (ERT) is

$$\frac{\sum_j (f_{i_j} \sum_{k=1}^{j} l_{i_k})}{\sum_j f_{i_j}}$$

a. Show that storing programs in nondecreasing order of l_i does not necessarily minimize ERT.
b. Show that storing programs in non-increasing order of f_i does not necessarily minimize ERT.
c. Show that storing programs in non-increasing order of f_i/l_i does minimize ERT. **Note:** You can assume that the tape is long enough to hold all the programs. So $\sum_j f_{i_j} = 1$.