5. degree to rad

$$29^\circ \times \frac{\pi}{180^\circ} = \frac{29\pi}{180}$$

$x = \frac{29\pi}{180}$

$f(x) = \cos(x)$ estimate the value (linear approximation)

$$f(x_0 + \Delta x) = f(x_0) + f'(x_0)\Delta x$$

$$\frac{29\pi}{180} = 0.5061 \text{ rad} = x_0$$

$$f\left(\frac{29\pi}{180}\right) = \cos\left(\frac{29\pi}{180}\right)$$

$$f(x) = \cos(x)$$

$$f'(x) = -\sin(x)$$

$$f(x_0) = \cos(0.5) = 0.8776$$

$$f'(x_0) = -\sin(0.5) = -0.4794$$

$$\cos(29^\circ) = \cos\left(\frac{29\pi}{180}\right) = 0.8776 + [-0.4794(0.0061)]$$

$$\approx 0.8747$$

$$f(x_0 + \Delta x) = f(x_0) + f'(x_0)\Delta x$$

Week 5

1. $y = x^2y^3 + x^3$

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2y^3) + \frac{d}{dx}(x^3)$$

$$\frac{d}{dx}(y) = 1 \cdot \frac{dy}{dx}$$

$$\frac{d}{dx}(x^2y^3) = \frac{d}{dx}(x^2) \cdot \frac{d}{dx}(y^3)$$

Chain rule : $vu' + uv'$

$v = x^2$ $\qquad$ $u = y^3$

$v' = 2x$ $\qquad$ $u' = 3y^2 \cdot \frac{dy}{dx}$

$$3y^2 \cdot \frac{dy}{dx}(x^2) + 2x(y^3)$$

$$3y^2x^2\frac{dy}{dx} + 2xy^3$$

$$\frac{dy}{dx} = 3y^2x^2\frac{dy}{dx} + 2xy^3$$

$$\frac{dy}{dx} = 3y^2x^2\frac{dy}{dx} + 2xy^3$$

$$\frac{dy}{dx} - 3y^2x^2\frac{dy}{dx} = 2xy^3$$

$$\frac{dy}{dx}\left(1 - 3y^2x^2\right) = 2xy^3$$

$$\frac{dy}{dx} = \frac{2xy^3}{1-3y^2x^2}$$

2. $e^{xy} = e^{4x} - e^{5y}$

$$\frac{d}{dx}(e^{xy}) = \frac{d}{dx}(e^{4x}) - \frac{d}{dx}(e^{5y})$$

e^{xy} : chain rule : $vu' + uv'$

let $u = xy \Rightarrow e^u$

$$\frac{dy}{dx} = vu' + uv'$$

$v = x \qquad u = y$

$v' = 1 \qquad u' = 1 \cdot \frac{dy}{dx}$

$$\frac{dy}{dx} = x\left(\frac{dy}{dx}\right) + y(1)$$

$$= x \cdot \frac{dy}{dx} + y$$

$$\frac{dy}{dx} = e^{xy}\left(x\frac{dy}{dx} + y\right)$$

e^{4x} : let $u = 4x \Rightarrow e^u$

$$\frac{du}{dx} = 4$$

$$\frac{dy}{dx} = e^{4x} \cdot 4$$

$$= 4e^{4x}$$

e^{5y} : let $u = 5y \Rightarrow e^u$

$$\frac{du}{dx} = 5 \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = 5 \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = 5e^{5y}\frac{dy}{dx}$$

$$e^{xy}\left(x\frac{dy}{dx} + y\right) = 4e^{4x} - e^{5y}\left(5 \cdot \frac{dy}{dx}\right)$$

$$e^{xy}\left(x\frac{dy}{dx}+y\right)=4e^{4x}-e^{5y}\left(5\cdot\frac{dy}{dx}\right)$$

$$xe^{xy}\frac{dy}{dx}+ye^{xy}=4e^{4x}-5e^{5y}\frac{dy}{dx}$$

$$xe^{xy}\frac{dy}{dx}+5e^{5y}\frac{dy}{dx}=4e^{4x}-ye^{xy}$$

$$\frac{dy}{dx}\left(xe^{xy}+5e^{5y}\right)=4e^{4x}-ye^{xy}$$

$$\frac{dy}{dx}=\frac{4e^{4x}-ye^{xy}}{xe^{xy}+5e^{5y}}$$

3. $\cos^2 x + \cos^2 y = \cos(2x + 2y)$

$$\frac{d}{dx}(\cos^2 x) + \frac{d}{dx}(\cos^2 y) = \frac{d}{dx}(\cos 2x + 2y)$$

$$\frac{d}{dx}(\cos x)^2$$

$u = \cos x$

$$\frac{d}{dx}(u^2)$$

$$\frac{dy}{du} = u^2$$

$$= 2u$$

$$= 2\cos x$$

$$\frac{du}{dx} = \cos x$$

$$= 1 \cdot -\sin x$$

$$= -\sin x$$

$$\frac{d}{dx}(\cos y)^2$$

let $u = \cos y$

$$\frac{d}{dx}(u^2)$$

$$\frac{dy}{du} = 2u$$

$$= 2\cos y$$

$$\frac{d}{dx}(\cos y)$$

$$= -\sin y \cdot \frac{dy}{dx}$$

$$= (2\cos y)\left(-\frac{dy}{dx}\sin y\right)$$

$$\frac{d}{dx}(\cos(2x + 2y))$$

$u = 2x + 2y$

$$u' = 2 + 2 \cdot \frac{dy}{dx}$$

$$\frac{d}{dx} = \cos u$$

$$= -\sin u$$

$$= -\sin(2x + 2y)$$

$$= \left(2 + 2\frac{dy}{dx}\right)\left(-\sin(2x + 2y)\right)$$

$$= \boxed{-2\sin(2x + 2y) - 2\sin(2x + 2y)\frac{dy}{dx}}$$

$$= 2u$$

$$= 2\cos x$$

$$\frac{du}{dx} = \cos x$$

$$= 1 \cdot -\sin x$$

$$= -\sin x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 2\cos x \times (-\sin x)$$

$$\boxed{= -2\sin x \cos x}$$

$$\frac{d}{dx}(\cos y)$$

$$= -\sin y \cdot \frac{dy}{dx}$$

$$= (2\cos y)\left(-\frac{dy}{dx}\sin y\right)$$

$$= \boxed{-2\sin y \cos y \frac{dy}{dx}}$$

$$= -\sin(2x+2y)$$

$$= \left(2 + 2\frac{dy}{dx}\right)\left(-\sin(2x+2y)\right)$$

$$\boxed{= -2\sin(2x+2y) - 2\sin(2x+2y)\frac{dy}{dx}}$$

$$-2\sin x\cos x - 2\sin y\cos y\frac{dy}{dx} = -2\sin(2x+2y) - 2\sin(2x+2y)\frac{dy}{dx}$$

$$-2\sin y\cos y\frac{dy}{dx} + 2\sin(2x+2y)\frac{dy}{dx} = -2\sin(2x+2y) + 2\sin x\cos x$$

$$\frac{dy}{dx}\left(-2\sin y\cos y + 2\sin(2x+2y)\right) = -2\sin(2x+2y) + 2\sin x\cos x$$

$$\frac{dy}{dx} = \frac{-2\sin(2x+2y) + 2\sin x\cos x}{-2\sin y\cos y + 2\sin(2x+2y)}$$

4. $x = 3 + \sqrt{x^2 + y^2}$

$$\frac{d}{dx}(x) = \frac{d}{dx}(3) + \frac{d}{dx}(x^2 + y^2)^{1/2}$$

$$\frac{d}{dx} = (x^2 + y^2)^{1/2} \qquad \frac{d}{dx} = x^2 + y^2$$

let $u = (x^2 + y^2)$ $\qquad = 2x + 2y \cdot \frac{dy}{dx}$

$$\frac{du}{dx} = u^{1/2}$$

$$= \frac{1}{2} u^{-1/2}$$

$$= \frac{1}{2}(x^2 + y^2)^{-1/2}\left(2x + 2y \frac{dy}{dx}\right)$$

$$1 = \frac{1}{2\sqrt{x^2 + y^2}}\left(2x + 2y \frac{dy}{dx}\right)$$

$$1 = \frac{1}{2} \cdot \frac{1}{\sqrt{x^2+y^2}}\left(2x + 2y \frac{dy}{dx}\right)$$

$$1 = \frac{2x + 2y\frac{dy}{dx}}{2\sqrt{x^2+y^2}}$$

$$2\sqrt{x^2+y^2} = 2x + 2y\frac{dy}{dx}$$

$$\frac{2\sqrt{x^2+y^2}}{2x+2y} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2\sqrt{x^2+y^2}}{2x+2y}$$

5. $$\frac{x-y^3}{y+x^2} = x+2$$

$$\frac{d}{dx}\left(\frac{x-y^3}{y+x^2}\right) = \frac{d}{dx}(x+2)$$

5. $$\frac{x - y^3}{y + x^2} = x + 2$$

$$\frac{d}{dx}\left(\frac{x - y^3}{y + x^2}\right) = \frac{d}{dx}(x + 2)$$

Quotient rule : $$\frac{vu' - uv'}{v^2}$$

$u = x - y^3$ $\qquad$ $v = y + x^2$

$u' = \frac{d}{dx}(x) - \frac{d}{dx}(y^3)$ $\qquad$ $v' = \frac{d}{dx}(y) + \frac{d}{dx}(x^2)$

$= 1 - 3y^2\frac{dy}{dx}$ $\qquad$ $= \frac{dy}{dx} + 2x$

$= 1 - 3y^2 \frac{dy}{dx}$ $\qquad$ $= \frac{dy}{dx} + 2x$

$$= \frac{(y+x^2)\left(1 - 3y^2\frac{dy}{dx}\right) - (x-y^3)\left(2x + \frac{dy}{dx}\right)}{(y+x^2)^2}$$

$$= \frac{y - 3y^3\frac{dy}{dx} + x^2 - 3y^2x^2\frac{dy}{dx} - 2x^2 + x\frac{dy}{dx} - 2xy^3 - y^3\frac{dy}{dx}}{y^2 + 2x^2y + x^4}$$

$$y - y^2 - 2x^2y - x^4 + x^2 - 2x^2 + 2xy^3 = 3y^3\frac{dy}{dx} + y^3\frac{dy}{dx} + 3x^2y^2\frac{dy}{dx} + x$$

$$y - y^2 - 2x^2y - x^4 + x^2 - 2x^2 + 2xy^3 = \frac{dy}{dx}\left(3y^3 + y^3 + 3x^2y^2 + x\right)$$

$$y - y^2 - 2x^2y - x^4 + x^2 - 2x^2 + 2xy^3 = 3y^3\frac{dy}{dx} + y^3\frac{dy}{dx} + 3x^2y^2\frac{dy}{dx} + x$$

$$y - y^2 - 2x^2y - x^4 + x^2 - 2x^2 + 2xy^3 = \frac{dy}{dx}\left(3y^3 + y^3 + 3x^2y^2 + x\right)$$

$$x^2 - 2x^2 + y - y^2 - x^4 - 2x^2y + 2xy^3 = \frac{dy}{dx}\left(3y^3 + y^3 + 3x^2y^2 + x\right)$$

$$\frac{-x^2 + y - y^2 - x^4 - 2x^2y + 2xy^3}{\left(3y^3 + y^3 + 3x^2y^2 + x\right)} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-x^2 + y - y^2 - x^4 - 2x^2y + 2xy^3}{\left(3y^3 + y^3 + 3x^2y^2 + x\right)}$$

6. $$\frac{y}{x^3} + \frac{x}{y^3} = x^2 y^4$$

$$\frac{d}{dx}\left(\frac{y}{x^3}\right) + \frac{d}{dx}\left(\frac{x}{y^3}\right) = \frac{d}{dx}\left(x^2 y^4\right)$$

$$\frac{d}{dx}\left(\frac{y}{x^3}\right) : \quad \frac{vu' - uv'}{v^2}$$

$u = y \qquad v = x^3$

$u' = \frac{dy}{dx} \qquad v' = 3x^2$

$$= \frac{(x^3)\left(\frac{dy}{dx}\right) - (y)(3x^2)}{(x^3)^2}$$

$$= \frac{(x^3)\left(\frac{dy}{dx}\right) - (y)(3x^2)}{(x^3)^2}$$

$$\boxed{= \frac{x^3 \frac{dy}{dx} - 3x^2 y}{x^6}}$$

$\frac{d}{dx}\left(\frac{x}{y^3}\right)$: $u = x$ $\quad V = y^3$

$u' = 1$ $\quad v' = 3y^2 \frac{dy}{dx}$

$$\boxed{= \frac{y^3 - 3y^2 x \frac{dy}{dx}}{y^6}}$$

$\frac{d}{dx}(x^2 y^4)$: Product rule : $vu' + uv'$

$u = x^2$ $\quad v = y^4$

$u' = 2x$ $\quad V' = 4y^3 \frac{dy}{dx}$

$$u' = 2x \qquad v' = \ldots \frac{}{dx}$$

$$= (y^4)(2x) + (x^2)\left(4y^3 \frac{dy}{dx}\right)$$

$$\boxed{= 2xy^4 + 4x^2y^3 \frac{dy}{dx}}$$

$$\frac{\times y^6 \quad x^3 \frac{dy}{dx} - 3x^2y}{\times y^6 \quad x^6} + \frac{y^3 - 3y^2x \frac{dy}{dx} \quad \times x^6}{y^6 \quad \times x^6} = 2xy^4 + 4x^2y^3 \frac{dy}{dx}$$

$$\frac{x^3y^6 \frac{dy}{dx} - 3x^2y^7}{x^6y^6} + \frac{x^6y^3 - 3y^2x^7 \frac{dy}{dx}}{x^6y^6} = 2xy^4 + 4x^2y^3 \frac{dy}{dx}$$

$$\frac{x^3y^6 \frac{dy}{dx} - 3x^2y^7 + x^6y^3 - 3y^2x^7 \frac{dy}{dx}}{x^6y^6} = 2xy^4 + 4x^2y^3 \frac{dy}{dx}$$

$$x^6y^6$$

$$x^3y^6\frac{dy}{dx} - 3x^2y^7 + x^6y^3 - 3y^2x^7\frac{dy}{dx} = 2x^7y^{10} + 4x^8y^9\frac{dy}{dx}$$

$$x^3y^6\frac{dy}{dx} - 3y^2x^7\frac{dy}{dx} - 4x^8y^9\frac{dy}{dx} = 2x^7y^{10} + 3x^2y^7 - x^6y^3$$

$$\frac{dy}{dx}\left(x^3y^6 - 3y^2x^7 - 4x^8y^9\right) = 2x^7y^{10} + 3x^2y^7 - x^6y^3$$

$$\frac{dy}{dx} = \frac{2x^{\cancel{7}\,5}y^{\cancel{10}\,8} + 3\cancel{x^2}y^{\cancel{7}\,5} - x^{\cancel{6}\,4}y^{\cancel{3}}}{x^{\cancel{3}}y^{\cancel{6}\,4} - 3\cancel{y^2}x^{\cancel{7}\,5} - 4x^{\cancel{8}\,6}y^{\cancel{9}\,7}}$$

$$\frac{dy}{dx} = \frac{2x^5y^8 + 3y^5 - x^4y}{xy^4 - 3x^5 - 4x^6y^7}$$

Past Year Question

1. $$x^3 = \left(\sqrt{64 \cdot 04}\right)^3$$

$$x = \sqrt{64 \cdot 04}$$

$$x_0 \approx 8$$ estimation value of x

$$f(x) = x^3$$

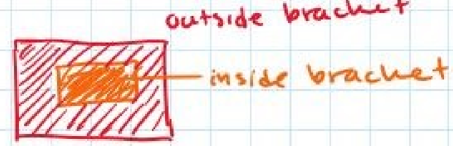
$$f'(x) = 3x^2$$

$$f'(x_0) = f'(8) = 3(8)^2 = 192$$

$$f(x) = f(8) = 8^3 = 512$$

$$\left(\sqrt{64 \cdot 04}\right)^3 - \frac{2}{\sqrt{64 \cdot 04}} \approx \left[512 + 192\left(\sqrt{64 \cdot 04}\right)\right] - \frac{2}{\sqrt{64 \cdot 04}}$$

$$\approx 512 \cdot 25$$



2. $f(x) = \sqrt{3x-2}$ $\quad x = 2$ to $x = 2.03$

$= (3x-2)^{\frac{1}{2}}$

let $u = 3x-2$

$f(x) = u^{\frac{1}{2}}$

$= \frac{1}{2} u^{-\frac{1}{2}}$

$= \frac{1}{2}(3x-2)^{-\frac{1}{2}} \cdot 3$

$= \frac{3}{2}(3x-2)^{-\frac{1}{2}}$

$= \dfrac{3}{2(3x-2)^{\frac{1}{2}}}$

$f'(x) = \dfrac{3}{2\sqrt{3x-2}}$

$x_0 = 2$

$f'(x_0) = \dfrac{3}{2\sqrt{3(2)-2}} = \dfrac{3}{4}$

$f(x_0) = \sqrt{3(2)-2} = 2$ $\quad \Delta x$

$$f'(x_0) = \frac{3}{2\sqrt{3(2)-2}} = \frac{3}{4}$$

$$f(x_0) = \sqrt{3(2)-2} = 2$$

Δx

$f(x)$ $\quad x_0 \quad f'(x_0)(x_1 - x_0)$

$$\sqrt{3x-2} \approx 2 + \frac{3}{4}(2.03 - 2)$$

$$\approx 2.0225$$

$x = \frac{29\pi}{180}$

5. degree to rad $\quad f(x) = \cos(x)$ estimate the value (linear approximation)

$$29° \times \frac{\pi}{180°} = \frac{29\pi}{180}$$

$$f(x_0 + \Delta x) = f(x_0) + f'(x_0)\Delta x$$

$$\frac{29\pi}{180} = 0.5061 \text{ rad} = x_0$$

$$f\left(\frac{29\pi}{180}\right) = \cos\left(\frac{29\pi}{180}\right)$$

$$f(x) = \cos(x)$$

$$f'(x) = -\sin(x)$$

$$f(x_0) = \cos(0.5) = 0.8776$$

$$f'(x_0) = -\sin(0.5) = -0.4794$$

$$\cos(29°) = \cos\left(\frac{29\pi}{180}\right) = 0.8776 + [-0.4794(0.0061)]$$

$$\approx 0.8747$$