Mathematical Firsts—Who Done It?

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In mathematics and other scientific disciplines a common practice is to name a theory, an equation, and other discoveries in honor of the scientist who pioneered the investigation. Some examples of such expressions are Galois theory, Fahrenheit scale, Freudian psychoanalysis, pasteurization, Zorn's lemma, Planck's constant, Linnaean system of botanical classification, Hilbert space, Darwin's theory of evolution, Halley's comet, Keynesian economic theory, Mendelian genetics, and so on.

> Every scientific discovery is named after the last individual too ungenerous to give the credit due his or her predecessors.

Designations of this type are called *eponyms,* and they are so ubiquitous that a volume now exists entitled *Eponyms Dictionaries Index: A Reference Guide to Persons, Both Real and Imaginary, and the Terms Derived from Their Names* (Ruffner 1977).

Robert K. Merton, a noted sociologist, has produced a provocative body of work on the social structure of science in which he views this notion of attaching the name of an individual to a scientific discovery as a feature of the reward system of science (Merton 1957). In writing of "fatherhood" in science, Merton (1965, 102–3) remarked that "on rare occasions the same individual acquires a double immortality, both for what he achieved, and what he failed to achieve, as in the case of Euclidean and non-Euclidean geometries, and Aristotelian and non-Aristotelian logic. Most rarely of all there are eponymies within eponymies, as when Ernest Jones bestows on the Father of Psychoanalysis the title of 'the Darwin of the mind.'"

In the November 1983 issue of the *American Statistician,* an article appeared bearing the puzzling title "Who Discovered Bayes's Theorem?" The author, S. M. Stigler, provides evidence that the expression "Bayes's theorem" may be a misnomer or a pseudo-eponym (Stigler 1983).

The purpose of this article is to present a list of a dozen such mathematical misnomers. For each of these we have included the original eponym together with a brief explanation of the mathematical concept. When possible, the name of the actual originator, the time frame involved, and the reason why the wrong person received credit for the discovery are included. In some cases, of course, identification of the originator is, at best, a highly conjectural matter. Said another way, demonstrating that the wrong person got credit is much easier than identifying the originator. We should mention that when a mathematician's name has become attached to a concept, the individual has likely made some important contribution to the discovery or development of the concept. We have also included brief mention of several other mathematical eponyms that require further investigation.

A Sample of Mathematical Pseudoeponyms

1. The Pythagorean theorem

The Pythagorean theorem states that in a right triangle, the sum of the squares of the legs equals the square of the hypotenuse. It would be difficult to overestimate the importance of this result. It is generally acknowledged that this remarkable theorem was known before the time of Pythagoras of Samos (ca. 582–500 B.C.), the Greek philosopher and mathematician (see, e.g., NCTM [1969], Swetz and Kao [1977], Ang [1978]). Van der Waerden (1983) hypothesized that since the Pythagorean theorem was known in four ancient civilizations—Babylonia, India, Greece, and China—it is probable that a common origin of the whole theory of right triangles exists. Using both written sources and archeological evidence, he constructed an interesting and compelling argument that led him to the following conclusion (van der Waerden 1983, 29): "I am convinced that the excellent neolithic mathematician who discovered the Theorem of Pythagoras had a proof of the theorem." He also remarked that the best account of mathematical science in the Neolithic Age is to be found in Chinese texts.

Euler's theorem on polyhedra should really be Descartes's theorem.

2. Euler's polyhedral theorem

One of the most interesting formulas relating to simple polyhedra is $F + V - E = 2$, where F is the number of faces, V is the number of vertices, and E is the number of edges. The five simple polyhedra are tetrahedron (pyramid), hexahedron (cube), octahedron, dodecahedron, and icosahedron. For the cube, $F = 6$, $V = 8$, and $E = 12$. Although this formula may have been known to Archimedes (ca. 225 B.C.), René Descartes, the French mathematician and philosopher, was the first to state this concept (ca. 1635). Leonard Euler independently discovered the theorem and announced his finding in Petrograd in 1752. Since Descartes's findings were not generally known until his unpublished mathematical works were made available in 1860, the polyhedral formula became known as Euler's theorem rather than Descartes's theorem (Smith 1958, 296).

3. L'Hôpital's rule

The first textbook on the calculus was published in Paris in 1696 (Struik 1967, 113). Its author, the Marquis Guillaume François Antoine de L'Hôpital, included in the text a method for finding the limiting value of a fraction whose numerator and denominator simultaneously approach zero as a limit. This method is now known as L'Hôpital's rule, even though it was discovered by Johann Bernoulli. Apparently L'Hôpital paid Bernoulli a regular salary, and under their pact Bernoulli was obliged to send L'Hôpital his mathematical discoveries (Struik 1963).

4. Leibniz's method of determinants

The concept of a determinant first appeared in the Western world in 1693 in a series of letters to L'Hôpital from Gottfried Leibniz. On this basis, Leibniz is credited with inventing the method of determinants (Boyer 1968, 443–44). In 1683, however, Seki Kōwa, the greatest of the seventeenth-century Japanese mathematicians, produced a mathematical treatise that contained the concept of determinants (Struik 1967, 154; Smith 1958, 476). A determinant is a function that assigns a numerical value to a square array of symbols. Determinants are useful in solving systems of simultaneous equations.

5. Cardan's formula

The formula for the roots of a cubic equation that appears in textbooks dealing with the theory of equations is called Cardan's formula because it first appeared in print in his *Ars Magna* [The great art] in 1545. Girolamo Cardano (i.e., Jerome Cardan), who was a gambler, a doctor, and a mathematics teacher, wheedled the solution of the cubic from Niccolò Tartaglia under solemn oath to the latter that he would not reveal the secret. Evidence also indicates that Scipione del Ferro discovered the solution to the cubic even earlier (ca. 1515) than Tartaglia, but he failed to publish his findings (Miller 1932; Smith 1958, 459–61; Feldman 1961).

6. Bernoulli's system of polar coordinates

Although Jakob Bernoulli is usually credited with the discovery of polar coordinates, Boyer (1949) has presented convincing evidence that indicates that Sir Isaac Newton was actually the originator of this geometric concept. Polar coordinates, like rectangular coordinates, are used to locate the position of a point in a plane. With rectangular coordinates the point is located by specifying its distance from two perpendicular axes, whereas with polar coordinates the point is specified by its distance and direction from a fixed reference point relative to a given reference line. The point is called the *pole* and the line is called the *polar axis*.

7. Mascheroni's geometric constructions

In 1797 Lorenzo Mascheroni, the Italian geometer and poet, published his *Geometria del Compasso* [Geometry of compasses]. It contained the rather surprising result that all pointwise Euclidean geometric constructions that can be made with Euclidean tools—(i.e., with compasses and unmarked straightedges) can also be made with compasses alone. It was not until 1928 that it became known that Georg Mohr, an obscure Danish mathematician, had published a book containing essentially the same results, with proof, 125 years before Mascheroni's publication. This fact was accidently discovered when a mathematics student, browsing in a Copenhagen bookstore, found a copy of Mohr's book (Hallerberg 1960; Eves 1969, 97–98).

8. Gauss's number plane

The first published account of the graphical representation of complex numbers (i.e., numbers of the form $a + bi$, where a and b are real numbers and i is imaginary) appeared in 1798 in the *Transactions of the Royal Danish Academy* and was written by Caspar Wessel, a Norwegian surveyor. Although Carl Friedrich Gauss did not publish his research on this concept until 1831, the complex number plane is now referred to as Gauss's number plane rather than Wessel's number plane. Apparently Wessel's work went virtually unnoticed (Miller 1932; Boyer 1968, 548). The concept is an important one, for it made it possible for mathematicians to visualize imaginary numbers.

9. Playfair's axiom

Through a given point can be drawn only one line parallel to a given line. This alternative to Euclid's celebrated fifth postulate was used by John Playfair (1748–1819), the Scottish physicist and mathematician. Since he was responsible for making it well known in modern times, it is now called Playfair's axiom (Smith 1958, 283). It had actually been stated, however, around 460 by the Neoplatonic philosopher Proclus (Eves 1969, 124). It was necessary for mathematicians to consider instead, in developing the non-Euclidean geometries (Wolfe 1945), the possibility that no line, or more than one line, can be drawn through a given point parallel to a given line.

L'Hôpital's rule is really Johann Bernoulli's rule.

10. Diophantine equations

The term *Diophantine equations,* which appears in books dealing with number theory, refers to indeterminate linear equations (Struik 1967, 70–71). This term usually implies that only integral solutions are of interest. Although several Hindu mathematicians (e.g., Brahmagupta, ca. 625) were interested in problems of this type, Diophantus of Alexandria (ca. 250) was not, and the term is, therefore, a misnomer (Miller 1932). Diophantus usually worked with quadratic equations, typically gave just one solution to the problems he considered, and was interested only in positive rational solutions (van der Waerden 1983).

11. Cramer's rule

Gabriel Cramer published his *Introduction a l'Analyse des Lignes Courbes Algebriques* [Introduction to the analysis of algebraic curves] in 1750. In the appendix he gives a method for solving systems of linear equations using determinants, which is now known as Cramer's rule. In 1748, however, the rule appeared in a posthumous publication by Colin Maclaurin, entitled *A Treatise of Algebra*. Although the rule was first stated by Maclaurin, Cramer's superior mathematical notation was probably instrumental in popularizing the method;

Cramer's rule is really Maclaurin's rule.

thus, it has been suggested that the procedure be referred to as the Maclaurin-Cramer rule (Boyer 1966).

12. Pascal's triangle

Although Blaise Pascal's *Traite du Triangle Arithmetique* [Treatise on the arithmetic triangle] was published posthumously in 1665, he was using the arithmetic triangle that now bears his name as early as 1653 (Eves 1969, 261–62). This mathematical concept was known to the Arabs in the eleventh century. It is believed to have been imported into China and recorded by Chu Shih-Chieh, the greatest of the Chinese algebraists of his time, in 1303 (Boyer 1968, 227–28). David (1962) indicated that Pascal's arithmetic triangle was published earlier by his teacher, Hèrigone, and Hogben (1951) claimed that the arithmetic triangle was discovered by Omar Khayyám, the Persian poet and astronomer, almost six centuries before Pascal. Since Pascal developed many properties of the triangle and applied them to probability, it is now known as Pascal's triangle. Perhaps the most commonly encountered property of the arithmetic triangle is that its rows contain the coefficients in the binomial expansion.

In addition to these twelve misnomers, the following are some other eponyms that are worthy of further investigation: Heron's formula, Horner's root-extraction algorithm, Descartes's rule of signs, Gaussian distribution, Mercator's series, Mercator's projection, Cartesian coordinates, Oughtred's circular slide rule, and Clairaut's equation. Readers interested in investigating eponyms, pseudoeponyms, and related notions in the history of mathematics will find the Thirty-first Yearbook of the National Council of Teachers of Mathematics, *Historical Topics for the Mathematics Classroom* (NCTM 1969), to be a valuable source of information.

Concluding Comments

Apparently pseudoeponyms of the type we have been discussing are more common than one might believe. In fact, S. M. Stigler, convinced that these misnomers are the rule rather than the exception, has formulated Stigler's law of eponymy (1980, 147), which asserts that "no scientific discovery is named after its original discoverer." In support of this self-eponymization, he has indicated that "evidence in favor of [Stigler's law of eponymy] is readily available in any field whose history has been subjected to serious scrutiny. Thus in ... mathematical statistics it can be found that Laplace employed Fourier transforms in print before Fourier published on the topic, that Lagrange presented Laplace transforms before Laplace began his scientific career, that Poisson published the Cauchy distribution in 1824, 29 years before Cauchy touched on it in an incidental manner, and that Bienaymé stated and proved the Chebychev inequality a decade before and in greater generality than Chebychev's first work on the topic" (Stigler 1980, 148).

We conclude by noting that Stigler's law of eponymy (1980) was adumbrated by—

1. an aphorism attributable to Whitehead (Merton 1968, 1): "Everything of importance has been said before by someone who did not discover it";
2. the following quotation from the economist G. J. Stigler (1966, 77): "If we should ever encounter a case where the theory is named for the correct man, we will note it";
3. the following statement by an unknown historian of science: "Every scientific discovery is named after the last individ-

ual too ungenerous to give due credit to his predecessors"; and

4. the mathematical historian C. B. Boyer (1968, 460), who said, "Clio, the muse of history, often is fickle in the matter of attaching names to theorems."

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