1. Introduction to Harmonic Mean

The **Harmonic Mean (HM)** is a measure of central tendency that is useful when dealing with **rates, ratios, and time-based problems**. It is calculated as the reciprocal of the arithmetic mean of reciprocals of a given set of numbers.

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2. Formula for Harmonic Mean

For **n** values $x_1, x_2, x_3, \ldots, x_n$, the **harmonic mean (HM)** is given by:

$$HM = \frac{n}{\sum \frac{1}{x_i}}$$

Where:

- n = number of observations
- x_i = individual values

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3. Calculation of Harmonic Mean

Case 1: Ungrouped Data (Individual Values)

For a dataset $x_1, x_2, \ldots, x_n$:

$$HM = \frac{n}{\sum \frac{1}{x_i}}$$

Example 1: Find the HM of **4, 5, and 6**.

$$HM = \frac{3}{(\frac{1}{4} + \frac{1}{5} + \frac{1}{6})}$$

$$= \frac{3}{(0.25 + 0.2 + 0.1667)}$$

$$= \frac{3}{0.6167} = 4.87$$

Thus, **HM ≈ 4.87**.

Case 2: Grouped Data (Frequency Distribution)

For a frequency distribution:

$$HM = \frac{\sum f_i}{\sum \frac{f_i}{x_i}}$$

Where:

- f_i = frequency of class
- x_i = midpoint of class interval

Example 2: Frequency Distribution

Class Interval	Midpoint (x)	Frequency (f)
10 - 20	15	3
20 - 30	25	5
30 - 40	35	4
40 - 50	45	2

Steps to Calculate HM:

1. Compute $\frac{f}{x}$ for each class.
2. Sum $\sum \frac{f}{x}$.
3. Compute $HM = \frac{\sum f}{\sum \frac{f}{x}}$.

x	f	$\frac{f}{x}$
15	3	0.200
25	5	0.200
35	4	0.114
45	2	0.044
Σ f = 14		**Σ $\frac{f}{x}$ = 0.558**

$$HM = \frac{14}{0.558} = 25.09$$

Thus, **HM ≈ 25.09**.

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4. Properties of Harmonic Mean

- **Always Positive**: HM is always a positive value.
- **Influenced by Small Values**: HM is **highly affected by small values** in the dataset.
- **Reciprocal Relationship**: The HM of a dataset is the **reciprocal of the arithmetic mean of the reciprocals** of the dataset.
- **Suitable for Rates & Ratios**: HM is the best mean for cases involving **speed, efficiency, and density**.

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5. Merits of Harmonic Mean

☑ **Best for Rates & Ratios**: HM is widely used in problems involving speed, time, and work.
☑ **All Values are Considered**: It considers every value in the dataset, unlike the mode or median.
☑ **Less Impact of Large Values**: Unlike the arithmetic mean, HM **reduces the effect of large values**.

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6. Demerits of Harmonic Mean

✖ **Cannot Handle Zero or Negative Values**: Since it involves reciprocals, HM is **undefined if any value is zero**.
✖ **Difficult to Compute**: Compared to arithmetic mean, HM **requires more complex calculations**.
✖ **Highly Affected by Small Values**: Even a single small value in the dataset **significantly lowers HM**.

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7. Applications of Harmonic Mean

📌 **Speed & Distance Problems –** Used in **average speed calculations** when distances are constant.
📌 **Finance & Investment –** Used in **financial ratios, interest rates, and stock price indices**.
📌 **Work & Time Problems –** Applied in problems involving **work efficiency and productivity**.
📌 **Physics & Engineering –** Used in **resistance in parallel circuits** and **fluid mechanics**.

8. More Example Problems on Harmonic Mean

Example 3: Speed Calculation

A person travels **60 km at 30 km/h** and another **60 km at 40 km/h**. Find the **average speed**.

Solution:

$$HM = \frac{2}{(\frac{1}{30} + \frac{1}{40})}$$

$$= \frac{2}{(0.0333 + 0.025)}$$

$$= \frac{2}{0.0583} = 34.29$$

Thus, **average speed ≈ 34.29 km/h**.

Example 4: Work Efficiency Calculation

Two workers A and B can complete a job in **10 hours and 15 hours** respectively. What is their **average work rate**?

Solution:

$$HM = \frac{2}{(\frac{1}{10} + \frac{1}{15})}$$

$$= \frac{2}{(0.1 + 0.0667)}$$

$$= \frac{2}{0.1667} = 12$$

Thus, the **average work efficiency is 12 hours**.

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9. Conclusion

- **Harmonic Mean** is useful when dealing with **rates and ratios**.
- It is best suited for **speed, time, and work problems**.
- It is **highly affected by small values** but **reduces the impact of large values**.
- **Not defined for zero or negative values**.
- **Commonly used in physics, finance, and statistics**.