Harmonic Mean Problems and Solutions

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1. Ungrouped Data Problems

Problem 1.1

Calculate the harmonic mean of the following values: 4, 5, 8, 10, and 20.

Solution: The harmonic mean (H) is calculated using the formula: H = n / (1/x + 1/x + … + 1/x)

For the given data: 4, 5, 8, 10, and 20: H = 5 / (1/4 + 1/5 + 1/8 + 1/10 + 1/20) H = 5 / (0.25 + 0.2 + 0.125 + 0.1 + 0.05) H = 5 / 0.725 H = 6.90

Therefore, the harmonic mean is 6.90.

Problem 1.2

A car travels 100 km at 40 km/hr and another 100 km at 60 km/hr. Find the average speed for the entire journey using harmonic mean.

Solution: Since equal distances are covered at different speeds, we can use the harmonic mean: H = 2 / (1/40 + 1/60) H = 2 / (3/120 + 2/120) H = 2 / (5/120) H = 2 × 24 H = 48 km/hr

Therefore, the average speed for the entire journey is 48 km/hr.

Problem 1.3

Find the harmonic mean of 2, 4, 6, 8, and 10. Compare it with the arithmetic mean of the same data set.

Solution: Harmonic mean: H = 5 / (1/2 + 1/4 + 1/6 + 1/8 + 1/10) H = 5 / (0.5 + 0.25 + 0.167 + 0.125 + 0.1) H = 5 / 1.142 H = 4.38

Arithmetic mean: A = (2 + 4 + 6 + 8 + 10) / 5 A = 30 / 5 A = 6

The harmonic mean (4.38) is less than the arithmetic mean (6), which follows the AM-HM inequality (A H).

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2. Discrete Frequency Distribution Problems

Problem 2.1

Calculate the harmonic mean of the following discrete frequency distribution:

Value (x)	Frequency (f)
5	3
8	5
10	7
12	4
15	2

Solution: For discrete frequency distribution, the harmonic mean formula is: H = N / Σ(f/x) where N is the total frequency.

Value (x)	Frequency (f)	f/x
5	3	0.6000
8	5	0.6250
10	7	0.7000
12	4	0.3333
15	2	0.1333
Total	N = 21	2.3916

H = 21 / 2.3916 H = 8.78

Therefore, the harmonic mean is 8.78.

Problem 2.2

A retailer sells products at different price points. The following table shows the price per unit and the number of units sold. Calculate the harmonic mean price.

Price per unit ( )	Number of units sold
20	15
25	22
30	18
40	10
50	5

Solution: For discrete frequency distribution, the harmonic mean formula is: H = N / Σ(f/x) where N is the total frequency.

Price (x)	Units sold (f)	f/x
20	15	0.7500
25	22	0.8800
30	18	0.6000
40	10	0.2500
50	5	0.1000
Total	N = 70	2.5800

H = 70 / 2.58 H = 27.13

Therefore, the harmonic mean price is 27.13.

Problem 2.3

For a production line, the following data shows the number of units produced per hour for different machines. Calculate the harmonic mean of production rate.

Units per hour	Number of machines
50	3
60	5
75	8
80	4
100	2

Solution: Using the harmonic mean formula for discrete frequency distribution: H = N / Σ(f/x)

Units per hour (x)	Machines (f)	f/x
50	3	0.0600
60	5	0.0833
75	8	0.1067
80	4	0.0500
100	2	0.0200
Total	N = 22	0.3200

H = 22 / 0.32 H = 68.75

Therefore, the harmonic mean production rate is 68.75 units per hour.

3. Continuous Frequency Distribution Problems (Inclusive Classes)

Problem 3.1

Calculate the harmonic mean for the following continuous frequency distribution with inclusive classes:

Class Interval (Inclusive)	Frequency
10-19	5
20-29	8
30-39	12
40-49	7
50-59	3

Solution: For continuous frequency distribution, we first need to find the midpoint of each class. For inclusive classes, the midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
10-19	14.5	5	0.3448
20-29	24.5	8	0.3265
30-39	34.5	12	0.3478
40-49	44.5	7	0.1573
50-59	54.5	3	0.0550
Total		N = 35	1.2314

H = 35 / 1.2314 H = 28.42

Therefore, the harmonic mean is 28.42.

Problem 3.2

The following table shows the inclusive class intervals of the time (in minutes) taken by workers to complete a task. Calculate the harmonic mean.

Time (minutes)	Number of Workers
5-9	8
10-14	15
15-19	20
20-24	12
25-29	5

Solution: Using the midpoint formula for inclusive classes: Midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
5-9	7	8	1.1429
10-14	12	15	1.2500
15-19	17	20	1.1765
20-24	22	12	0.5455
25-29	27	5	0.1852
Total		N = 60	4.3001

H = 60 / 4.3001 H = 13.95

Therefore, the harmonic mean time is 13.95 minutes.

Problem 3.3

A survey collected data on the monthly expenditure (in $) of families in a neighborhood. Calculate the harmonic mean of the expenditure.

Monthly Expenditure ($)	Number of Families
500-699	12
700-899	18
900-1099	25
1100-1299	15
1300-1499	10

Solution: Using the midpoint formula for inclusive classes: Midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
500-699	599.5	12	0.020016
700-899	799.5	18	0.022514
900-1099	999.5	25	0.025013
1100-1299	1199.5	15	0.012505
1300-1499	1399.5	10	0.007145
Total		N = 80	0.087193

H = 80 / 0.087193 H = 917.50

Therefore, the harmonic mean of monthly expenditure is $917.50.

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4. Continuous Frequency Distribution Problems (Exclusive Classes)

Problem 4.1

Calculate the harmonic mean for the following continuous frequency distribution with exclusive classes:

Class Interval (Exclusive)	Frequency
10-20	6
20-30	10
30-40	15
40-50	9
50-60	5

Solution: For exclusive classes, the midpoint (x) = (lower limit + upper limit) / 2 Note that in exclusive classes, the upper limit of one class is the lower limit of the next class.

Class Interval	Midpoint (x)	Frequency (f)	f/x
10-20	15	6	0.4000
20-30	25	10	0.4000
30-40	35	15	0.4286
40-50	45	9	0.2000
50-60	55	5	0.0909
Total		N = 45	1.5195

H = 45 / 1.5195 H = 29.61

Therefore, the harmonic mean is 29.61.

Problem 4.2

The following table shows the exclusive class intervals of the height (in cm) of plants in a garden. Calculate the harmonic mean.

Height (cm)	Number of Plants
15-25	8
25-35	12
35-45	20
45-55	10
55-65	5

Solution: Using the midpoint formula for exclusive classes: Midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
15-25	20	8	0.4000
25-35	30	12	0.4000
35-45	40	20	0.5000
45-55	50	10	0.2000
55-65	60	5	0.0833
Total		N = 55	1.5833

H = 55 / 1.5833 H = 34.74

Therefore, the harmonic mean height is 34.74 cm.

Problem 4.3

A dataset of the weight (in kg) of individuals is given below with exclusive class intervals. Calculate the harmonic mean.

Weight (kg)	Frequency
40-50	7
50-60	15
60-70	25
70-80	18
80-90	10
90-100	5

Solution: Using the midpoint formula for exclusive classes: Midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
40-50	45	7	0.1556
50-60	55	15	0.2727
60-70	65	25	0.3846
70-80	75	18	0.2400
80-90	85	10	0.1176
90-100	95	5	0.0526
Total		N = 80	1.2231

H = 80 / 1.2231 H = 65.41

Therefore, the harmonic mean weight is 65.41 kg.

5. Unequal Class Intervals Problems

Problem 5.1

Calculate the harmonic mean for the following frequency distribution with unequal class intervals:

Class Interval	Frequency
0-10	5
10-25	12
25-45	18
45-70	10
70-100	5

Solution: For unequal class intervals, we still use the midpoint of each class, but we need to be careful with the calculation.

Class Interval	Midpoint (x)	Frequency (f)	f/x
0-10	5	5	1.0000
10-25	17.5	12	0.6857
25-45	35	18	0.5143
45-70	57.5	10	0.1739
70-100	85	5	0.0588
Total		N = 50	2.4327

H = 50 / 2.4327 H = 20.55

Therefore, the harmonic mean is 20.55.

Problem 5.2

A survey on annual income (in $1000s) yielded the following data with unequal class intervals. Calculate the harmonic mean income.

Income ($1000s)	Number of People
20-30	15
30-45	25
45-70	35
70-100	20
100-150	5

Solution: Using the midpoint formula for each class interval: Midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
20-30	25	15	0.6000
30-45	37.5	25	0.6667
45-70	57.5	35	0.6087
70-100	85	20	0.2353
100-150	125	5	0.0400
Total		N = 100	2.1507

H = 100 / 2.1507 H = 46.50

Therefore, the harmonic mean income is $46,500.

Problem 5.3

The following table shows the time (in minutes) taken to complete a test with unequal class intervals. Calculate the harmonic mean.

Time (minutes)	Number of Students
10-15	8
15-25	15
25-40	25
40-60	12
60-90	5

Solution: Using the midpoint formula for each class interval: Midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
10-15	12.5	8	0.6400
15-25	20	15	0.7500
25-40	32.5	25	0.7692
40-60	50	12	0.2400
60-90	75	5	0.0667
Total		N = 65	2.4659

H = 65 / 2.4659 H = 26.36

Therefore, the harmonic mean time is 26.36 minutes.

Problem 5.4

The marks obtained by students in an examination are given below with unequal class intervals. Calculate the harmonic mean.

Marks	Number of Students
0-20	5
20-35	12
35-55	20
55-80	15
80-100	8

Solution: Using the midpoint formula for each class interval: Midpoint (x) = (lower limit + upper limit) / 2

Class Interval	Midpoint (x)	Frequency (f)	f/x
0-20	10	5	0.5000
20-35	27.5	12	0.4364
35-55	45	20	0.4444
55-80	67.5	15	0.2222
80-100	90	8	0.0889
Total		N = 60	1.6919

H = 60 / 1.6919 H = 35.46

Therefore, the harmonic mean mark is 35.46.

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6. Merits, Demerits, and Applications of Harmonic Mean

Merits of Harmonic Mean

1. **Appropriate for Averaging Rates and Ratios:** The harmonic mean is the most appropriate average when dealing with rates, ratios, speeds, or prices where the denominator remains constant.
2. **Less Influenced by Large Values:** Unlike the arithmetic mean, the harmonic mean gives more weight to smaller values and is less affected by extremely large values in the dataset.
3. **Accurate for Speed Calculations:** When calculating average speed over fixed distances, the harmonic mean provides the mathematically correct result.
4. **Preserves Reciprocal Relationships:** The harmonic mean preserves the reciprocal relationship between variables, making it valuable in specific scientific and engineering applications.
5. **Useful in Financial Analysis:** For financial ratios like price-to-earnings (P/E) ratios, the harmonic mean provides more appropriate averages than arithmetic means.

Demerits of Harmonic Mean

1. **Limited to Positive Values:** The harmonic mean can only be calculated for positive values, making it unusable for datasets containing zero or negative values.
2. **Computational Complexity:** Calculating the harmonic mean requires more computational steps than the arithmetic mean, making it less intuitive and more prone to calculation errors.
3. **Highly Sensitive to Small Values:** While less influenced by large values, the harmonic mean is extremely sensitive to small values, which can disproportionately affect the result.
4. **Difficult to Interpret:** For non-specialists, the harmonic mean can be difficult to understand and interpret compared to more commonly used measures like the arithmetic mean.
5. **Limited General Applicability:** The harmonic mean is applicable only in specific contexts and is not as versatile as the arithmetic mean for general statistical purposes.

Applications of Harmonic Mean

1. **Average Speed Calculation:** When traveling the same distance at different speeds, the harmonic mean provides the correct average speed.
2. **Electrical Engineering:** For calculating the effective resistance of resistors connected in parallel or the effective capacitance of capacitors in series.
3. **Financial Analysis:**
   - Computing average price-to-earnings ratios
   - Portfolio performance evaluation
   - Average cost per share when investing different amounts at different prices
4. **Statistical Analysis:**
   - F1-score in classification (harmonic mean of precision and recall)
   - Combining samples of different sizes
5. **Physical Sciences:**
   - Computing average density
   - Calculating mean conductivity
   - Determining effective focal length in optical systems
6. **Computer Performance Benchmarking:** Used in measuring average performance rates in computing systems, particularly in SPEC benchmarks.

7. **Demographic Studies:** Used in calculating average rates of population growth, mortality rates, or other demographic ratios.
8. **Production and Manufacturing:**
    - Average productivity rates
    - Machine efficiency rates
    - Production cost analysis
9. **Transportation Studies:** Computing average travel times, speeds, or flow rates in traffic management.
10. **Educational Assessment:** Analyzing average rates of student performance across different test metrics or schools.