RF LOADING EFFECTS OF FIELD EMISSION CURRENT IN THE SRF CAVITIES

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Abstract

The field emission is an electron discharge which occurs at a very high electric field due to the surface contaminants that are attached to the surface of superconducting radio-frequency (SRF) cavities. The power absorbed by the field emission current loads the RF cavity, ultimately reducing the cavity voltage. In the present work, we have simulated RF loading effects caused by the field emission current in the elliptic SRF cavity which are regularly tested at the Vertical Test Stand (VTS) in RRCAT. This is done by incorporating the field emitted current, modelled as a current source, into the equivalent parallel LCR circuit that represents the RF cavity. The magnitude of the field emitted current is calculated using the Fowler-Nordheim equation. We simulated the RF cavity behaviour for an RF pulse. The results show that the cavity voltage decreases and becomes limited due to the power absorbed by the field emitted electrons. When the RF power is switched off, the cavity voltage decreases more rapidly than the natural decay rate until the field emission threshold is reached. Our simulations are able to reproduce the experimental (qualitative) observations noted by H. Piel and those routinely observed at VTS.

INTRODUCTION

The superconducting radio-frequency (SRF) cavities are natural choice for charge particle acceleration in high-power and high current accelerators. During RF testing of these cavities, many performance-limiting phenomena are encountered, which hinder the realization of a high acceleration gradient in the cavity. Among these phenomena, Multipacting and Field emission are major processes, which are caused by the electronic discharge within the cavity. These phenomena are particularly detrimental to the SRF cavities as the RF energy absorbed by the discharged electrons is dissipated as heat on the cavity surface. This additional surface heat not only quenches superconductivity, but also places extra stress on the cryogenic system, which must maintain the superconducting temperature of 2K. Additionally, a significant amount of X-radiation and sudden decline in the quality factor is also observed when these phenomena occur [1]. In both cases, a significant amount of RF energy is absorbed by the electrons, leading to the RF loading of the cavity.

In the present work, the RF loading effect because of the field emission has been studied in detail. Field emission is a quantum mechanical tunneling of electrons from the surface of the cavity material when it is exposed to a very high electric field (~10^4 MV/m). Such a high electric field is ordinarily not possible in the cavity, except at the irregularly shaped surface contaminants where the field gets enhanced multiple times (~100-200 times) [1]. These contaminants get attached to the surface during the cavity manufacturing and processing. Field emission mostly occurs from the iris region of the elliptic cavity.

During the experiments, the multipacting, which originates differently, and the field emission exhibit similar symptoms which can be confusing in their identification. These two phenomena can be distinguished mainly by observing the RF signatures of the cavity. For instance, the cavity voltage stagnates when a multipacting barrier appears and cannot be increased further even if the input RF power is increased. The extra input RF power is spent on increasing the multipacting current and remaining is reflected back to the source. On the contrary, one can still increase the cavity voltage (albeit slowly) beyond a threshold where field emission appears.

The quality factor (Q) of the cavity drops suddenly by an order of magnitude during the multipacting. On the other hand, Q decreases gradually as a function of cavity voltage in the field emission. Similar behavior is observed in the radiation pattern as a function of cavity voltage. A subtle difference can also be observed in the cavity voltage waveform during the multipacting, which could show oscillations about the lower limit of the multipacting barrier. Additionally, the field emission current increases exponentially with the cavity voltage (beyond a threshold), whereas the multipacting current increases exponentially in time.

In this paper, we have simulated the RF loading effects of the field emission current using the LCR-circuit representation of the elliptic SRF cavity. These are 5-cell, 650 MHz, $\beta_g = 0.92$ cavities which will be used for particle acceleration in the high energy section of the H^-accelerator at Fermilab [2]. A typical RF parameter which has been used in our study are presented in the Table 1.

Table 1: Typical RF parameters of the elliptic SRF cavity tested at VTS in RRCAT.

Quality Factor (Q_0)	4.95×10^{10}
External Quality Factor (Q_{ext})	2.24×10^{10}
R_{sh}/Q	587.55
Cavity Length (L) (m)	1.0437
Resonance Frequency (MHz)	650.00095

The field emission current is modelled as an additional driving current source to the circuit. The resulting waveforms of the cavity voltage and the reflected power are presented when the cavity is excited by another RF power source in the pulsed mode. The obtained waveforms match with those presented in the literature [3] and were regularly observed during the experiments at the Vertical Test Stand (VTS) at RRCAT.

THE CIRCUIT MODEL

The RF cavity can be modelled using a driven parallel LCR circuit [4]. As shown in the schematic diagram in the Figure 1, the field emission current (I_{FE}) has been introduced as an additional current source, other than the RF current source (I_{RF}). In this figure, L and C represent equivalent inductance and capacitance of the RF cavity. The shunt impedance is shown by R_0, and R_{ext} is the shunt impedance of the external circuit transformed to the cavity side. The total shunt impedance (or loaded shunt impedance R_L) is defined as $(1/R_0\ + 1/R_{ext}\ )^{-1}$. Here, ω_0 $(= 1/\sqrt{LC})$ is the resonance frequency (~650 MHz) corresponding to the fundamental accelerating mode TM_{010} of the cavity. The loaded quality factor of the cavity Q_L can be represented by $\omega_0 R_L C$.

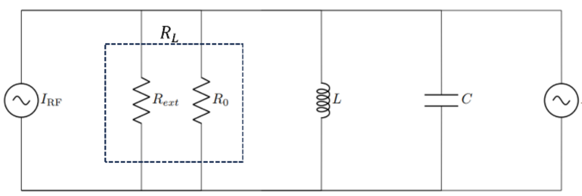

Figure 1: The circuit diagram of the resonantly driven RF cavity represented by a parallel LCR circuit, driven by two different current sources belonging to the RF power source (I_{RF}) and the field emission current (I_{FE}).

The evolution of the cavity voltage (V_{cav}) can be described by the following second order ordinary differential equation (ODE) [5],

$$\ddot{V}_{cav} + \frac{\dot{V}_{cav}}{R_L C} + \omega_0^2\, V_{cav} = \frac{\dot{I}_{RF}}{C} + \kappa\,\frac{\dot{I}_{FE}}{C}. \tag{1}$$

Here, parameter κ is an important scaling factor which is a ratio between the average energy gain by the field emitted electrons and the energy gain by the particles along the cavity axis. The cavity voltage from the above-mentioned equation oscillates sinusoidally in time (time period ~ 1.5 ns), whereas the change in the amplitude is much slower (in seconds) for the SRF cavities which are tested at the VTS. This clear distinction between the two-time scales allows us to strip off the fast-varying terms and form an envelope equation for the V_{cav}.

We can substitute equation (1) by $V_{cav} = V_0(t)\, e^{i\omega t}$, $I_{RF} = I_0(t)\, e^{i(\omega t+\phi)}$ and $I_{FE} = I_{FE0}(V_0)\, e^{i(\omega t+\psi)}$, where ω is the RF frequency which is same as ω_0 when the RF cavity is driven at resonance. The ϕ (= 0 because $\omega = \omega_0$) and ψ are the phase difference of RF and field emission current with the cavity voltage. Note that the field emission current source oscillates at the same frequency as the RF source, but in opposite phase *i.e.* $\psi = \pi$. This indicates the that the field emission current loads the cavity resistively. A detailed description of this methodology is given in our earlier work [6]. After some algebraic manipulations on equation (1), the envelope equation (in complex form) can be written as following,

$$\left[i + \frac{1}{2Q_L}\right] \dot{V}_0 + \left[\frac{i\omega_0}{2Q_L}\right] V_0 = \frac{R_L}{2Q_L}\left[\dot{I_0}\, e^{i\phi} + \kappa \dot{I}_{FE0} e^{i\psi}\right] + \frac{i\omega_0 R_L}{Q_L}\left[I_0 e^{i\phi} + \kappa I_{FE0} e^{i\psi}\right]. \tag{2}$$

The above-mentioned equation requires knowledge of the field emission current I_{FE0}. The I_{FE0} can be estimated from the dark current (I_{DC}) by dividing with the factor η, which represents the fraction of the field emitted particles exiting through the either side of the cavity through beam pipe. Normally, most of the field-emitted electrons strike the cavity surface and get buried there, with only a small fraction < 5% ($\eta \approx 0.05$) escaping through the beam pipes. We had performed particle tracking calculations in [7] and shown η as a function of cavity voltage in the Figure 2. The same figure also plots the average energy gain by the field emitted electrons at any cavity voltage. This information is particularly useful to estimate κ and the total power absorbed by the field emission current.

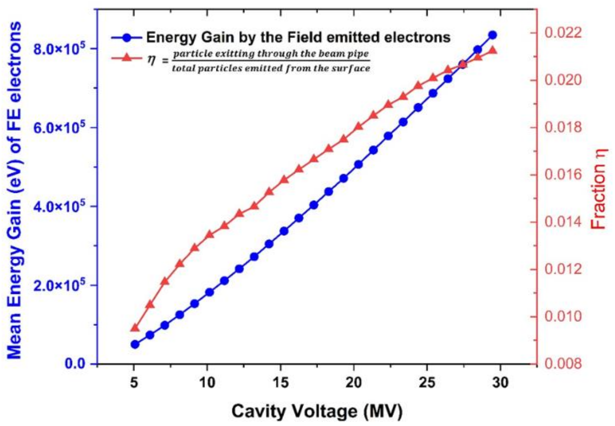

Figure 2: The mean energy gain by the field emitted electron and the fraction (η) of the field emitted electrons leaving the RF cavity through beam pipes.

The dark current I_{DC} was estimated in [7] for B92A-RRCAT-505 SRF cavity which was tested earlier at VTS. The I_{DC} can be obtained from the modified form of the Fowler-Nordheim equation, which can be written as follows,

$$I_{FE0}\ [\text{in } A] = \frac{I_{DC}}{\eta} = \left(\frac{5700\times 10^{\frac{4.52}{\sqrt{W_f}}}}{\eta\ W_f^{1.75}} \left[\text{in } \frac{A}{m^2\left(\frac{MV}{m}\right)^{2.5}}\right]\right) A(\beta E_s)^{2.5} e^{\frac{65300\times W_f^{1.5}}{\beta E_s}}. \tag{3}$$

Here, A is the aggregate area ($\sim 3.09 \times 10^{-16}\ m^2$) and β is the aggregate field enhancement factor (~105) of all the emitters in the cavity. These parameters were determined by fitting the experimental data from VTS. the work function of niobium is represented by W_f (in eV) and E_s (in MV/m) is the maximum surface electric field available in the cavity. The E_s can be obtained from the cavity voltage (V_0) by dividing it with 2.14.

One can numerically solve the ODE in Equation (2) in conjunction with the equation (3) using an ODE solver. This gives evolution of the cavity voltage in time.

RESULTS

Using the methodology described in the previous section, we have simulated the RF loading effects due to field emission current. We have plotted cavity voltage as a function of time in the Figure 3. For these calculations, the RF power was ON (at P_{for} = 100 W) for initial 100 seconds and turned OFF thereafter. In the beginning, both the waveforms corresponding to the field emission and the no field emission case, follow each other as the field emission current is extremely small. Slowly the field emission current increases and the cavity voltage deviates from the no field emission case. For instance, at 2.8 s, the field emission current is 0.08 μA and difference in the cavity voltages is ~ 0.01%. Soon (at 7.5 seconds) a steady state is reached where a constant cavity voltage of 26.5 MV is achieved for the given P_{for} = 100 W. At this time, the field emission current is ~94.5μA, the reflected power is ~ 7.3 W and power consumed by field emission current is 68.6 W. The RF power lost on the cavity surface is ~ 24.1 W.

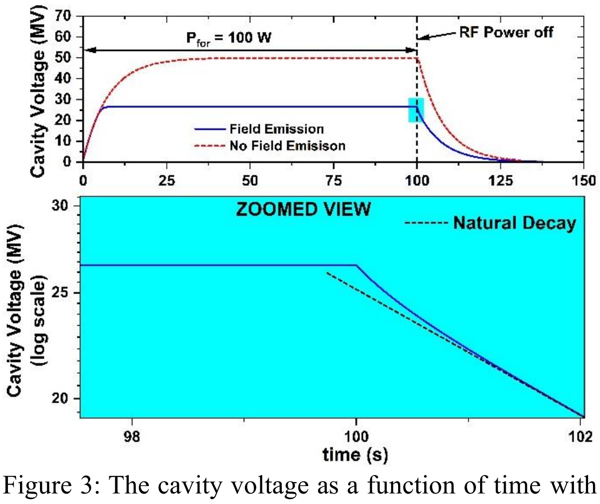

Figure 3: The cavity voltage as a function of time with field emission current. The case when field emission is absent is plotted for reference. A zoomed view of portion when RF power is turned OFF is also shown.

This can be seen from the Figure 4, which shows the field emission current (I_{FE0}), reflected power (P_{ref}) and the power consumed by the field emitted electrons (P_{FE}) as a function of time. Further, in the absence of the RF generator power (after 100 seconds), the electromagnetic energy contained in the cavity not only dissipates on the cavity surface and leaks through the RF coupler ports, but also powers up the field emission process. As a result, the cavity voltage decreases more rapidly compared to the case without field emission. This is clearly illustrated in the zoomed-in view of the cavity voltage vs. time plot in Figure 3. This characteristic behaviour of the cavity voltage has been reported in [3] and regularly observed at VTS in RRCAT.

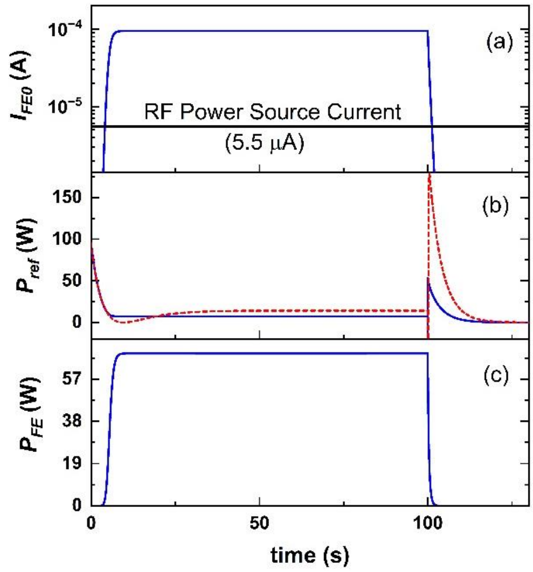

Figure 4: The time evolution of (a) field emission current, (b) reflected power and (c) the power consumed by the field emitted electrons.

In conclusion, we have modelled the RF loading effects of the field emission current by incorporating an additional current source in the LCR circuit representation of the RF cavity. Through our simulations, we have reproduced experimentally observed cavity voltage waveform when field emission occurs.

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