(3) **mutually exclusive events**

A set of events is said to be mutually exclusive if the occurrence of any one of them excludes the occurrence of the others.

Two events A & B are mutually exclusive if A occurs and B does not occur and vice versa.

In other words, A & B cannot occur simultaneously

i.e $A \cap B = \phi$

$P(A \cap B) = P(\phi) = 0$

$\boxed{P(A \cap B) = 0}$

$\boxed{P(A \cup B) = P(A) + P(B)}$

(4) **Independent Events**

A set of events is said to be independent if the occurrence of any one of them does not depend on the occurrence or non-occurrence of the others.

When two events A & B are independent

then $\boxed{P(B|A) = P(B)}$, $\boxed{P(A|B) = P(A)}$

If the events A & B are independent, the multiplication (product) theorem takes the form

$$\boxed{P(A \cap B) = P(A)\,P(B)}$$

Conversely if $P(A \cap B) = P(A)P(B)$ the events A & B are said to be independent (pairwise independent)

$\therefore$ It can be extended to any number of independent events

If $A_1, A_2, A_3 \dots A_n$ are $n$ independent events

$$P(A_1 \cap A_2 \cap A_3 \dots \cap A_n) = P(A_1)P(A_2)P(A_3)\dots P(A_n)$$

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problems

(1) When A and B are two mutually exclusive events such that $P(A) = 1/2$ and $P(B) = 1/3$ find $P(A \cup B)$ and $P(A \cap B)$.

Sol: here A & B are mutually exclusive events

$$P(A \cup B) = P(A) + P(B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

&

$$P(A \cap B) = 0$$

(2) If $P(A) = 0.65$, $P(B) = 0.4$ and $P(A \cap B) = 0.24$

can A and B be independent events ?

Sol: We know that if A & B are

independent events then

$$P(A \cap B) = P(A) \cdot P(B)$$

Given $P(A \cap B) = 0.24$ &

$$P(A) \cdot P(B) = (0.65)(0.4) = 0.26$$

$$\therefore P(A \cap B) \neq P(A) \cdot P(B)$$

$\therefore$ A & B are not independent events

(3) Given $P(A) = 1/2$, $P(B) = 1/3$, & $P(AB) = 1/4$

find the value $P(A+B)$.

Sol: Given $P(A) = 1/2$, $P(B) = 1/3$ & $P(AB) = P(A \cap B) = 1/4$

By using additional theorem

$$P(A+B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{5}{6} - \frac{1}{4} = \frac{20-6}{24}$$

$$= \frac{14}{24} = \frac{7}{12}$$

$$P(A+B) = 7/12$$

(4) let A and B be two events with $P(A) = 1/2$, $P(B) = 1/3$ and $P(A \cap B) = 1/4$ find $P(A|B)$, $P(A \cup B)$, $P(A'|B')$

Sol: Given $P(A) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$

By conditional prob.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = 3/4$$

$$\boxed{P(A|B) = 3/4}$$

By additional theorem

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{5}{6} - \frac{1}{4} = \frac{20-6}{24}$$
$$= \frac{14}{24} = 7/12$$

$$\boxed{P(A \cup B) = 7/12}$$

&

By conditional prob, $P(A'|B') = \dfrac{P(A' \cap B')}{P(B')}$

Now By Demorgan's Law

$$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$
$$= 1 - \frac{7}{12} = \frac{5}{12} \quad (\because P(A') = 1 - P(A))$$

$$P(B') = 1 - P(B) = 1 - \frac{1}{3} = 2/3$$

$$\therefore P(A'|B') = \frac{P(A' \cap B')}{P(B')} = \frac{5/12}{2/3} = \frac{5}{8} \quad //$$

(5) In a random experiment, $P(A) = 1/12$, $P(B) = 5/12$ and

$P(B|A) = 1/15$. find $P(A \cup B)$.

Sol: By conditional prob. $\left( P(B|A) = \frac{P(A \cap B)}{P(A)} \right)$

$$P(A \cap B) = P(A) \cdot P(B|A) = \frac{1}{12} \times \frac{1}{15} = \frac{1}{180}$$

By additional theorem

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{12} + \frac{5}{12} - \frac{1}{180} = \frac{\cancel{72}^{\,6\,1}}{\cancel{144}\ \cancel{12}_{\,2}} - \frac{1}{180} = \frac{178}{2 \times 180}$$

$$P(A \cup B) = \frac{89}{180}$$