



# **Markowitz Mean Variance Portfolio Selection**

**DSA 5303**

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# Presentation Outline



STUDY SCOPE



SOLUTION  
APPROACH



THEORETICAL  
FRAMEWORK



INPUT STOCK  
DATA

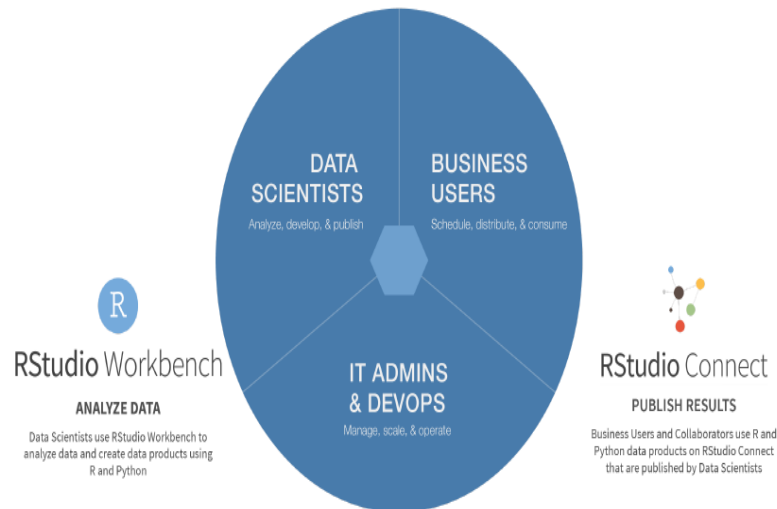


ANALYSIS AND  
RESULTS



CONCLUSIONS

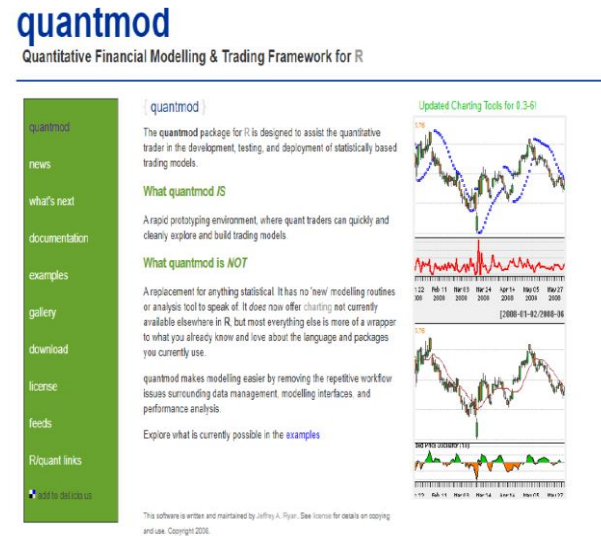
# STUDY SCOPE



- Stock Data from Publicly Held Companies
  - Small Cap
  - Mid Cap
  - Large Cap
- Application of R Studio Packages
- Theoretical Framework of Markowitz's Mean Variance Model
- Analysis and Results
  - Efficient Frontier
  - Portfolio Weights
  - Portfolio Return and Covariance
- Dashboard Development
- Conclusions

# SOLUTION APPROACH

- Quant Mod Package
  - Stock Data
- Time Series Package
  - Conversion of Data Frame to Time Series Object
- fPortfolio Package
  - Portfolio Analysis
- Shiny Package
  - Dashboard Development



```
# long portfolio with target return at 0.15
```

```
spec1<-portfolioSpec()
setRiskFreeRate(spec1)<-0.05
setTargetReturn(spec1) <- 0.15
```

```
scaplong<-efficientPortfolio(scap, spec = spec1, constraints = "LongOnly")
```

```
scaplongfrontier<-portfolioFrontier(scap, spec1)
```

```
tailoredFrontierPlot(object = scaplongfrontier, mText = "MV Portfolio -  
LongOnly Constraints",  
risk = "Cov")
```



# THEORETICAL FRAMEWORK

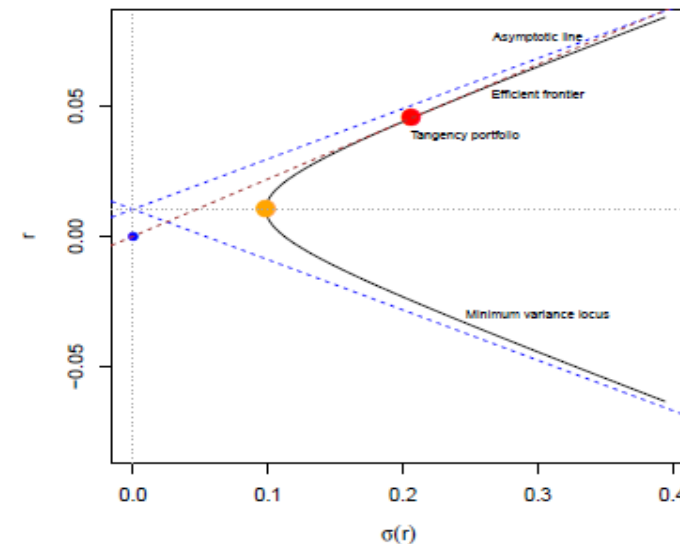
- Markowitz's Mean Variance Model , circa 1952
- Efficient Frontier – Locus of Minimum Variance Points Above Minimum Variance Point
- Covariance Used as Target Risk and Not VaR and CVaR.

$$\min_w w^T \hat{\Sigma} w$$

s.t.

$$w^T \hat{\mu} = \bar{r}$$
$$w^T \mathbf{1} = 1$$

The formula expresses that we minimize the variance-covariance risk  $\bar{\sigma}^2 = w^T \hat{\Sigma} w$ , where the matrix  $\hat{\Sigma}$  is an estimate of the covariance of the assets. The vector  $w$  denotes the individual investments subject to the condition  $w^T \mathbf{1} = 1$  that the available capital is fully invested. The expected or target return  $\bar{r}$  is expressed by the condition  $w^T \hat{\mu} = \bar{r}$ , where the  $p$ -dimensional vector  $\hat{\mu}$  estimates the expected mean of the assets. Markowitz' portfolio model<sup>1</sup> has a unique solution:



# INPUT STOCK DATA

- Quant Mod Used
- Stocks Traded on NYSE
- Small Cap
  - Under a Billion to ~10 Billion
- Mid Cap
  - <50 Billion
- Large Cap
  - Up to and Over 1 Trillion

Table 1: Large Cap Stock Mix with Market Cap

Stock Symbol	Market Cap (\$billion)
AAPL	2,410
MSFT	2,150
GOOG	1,800
IBM	126.35
BP	81.58
XOM	243.72
CSCO	233.34
DUK	80.85
JNJ	453.31
INTLC	217.94

Table 2: Mid Cap Stock Mix with Market Cap

Stock Symbol	Market Cap (\$billion)
GATX	3.27
LEN	32.84
JBLU	4.68
MLM	22.66
NFLX	229.07
RS	9.98
RJF	17.79
SLAB	6.67
ROST	43.82
WRB	12.98

Table 3: Small Cap Stock Mix with Market Cap

Stock Symbol	Market Cap (\$billion)
AOS	11.28
DX	0.599
NNBR	0.295
BLDR	9.22
GME	11.57
SAVA	2.78
AZPN	9.95
FDS	13.50
PVH	7.47
VMI	5.03



# INPUT STOCK DATA

## ANNUAL RETURN RATES, LARGE CAP

- Stock Price Fluctuations – Bar Charts
- Stocks – Volume Traded
- Periodic Returns
  - Annual Rate of Returns Calculated

Date	AAPL	MSFT	IBM	GOOG	BP	XOM	CSCO	DUK	JNJ	INTC
12/31/2007	129.55%	19.02%	11.24%	48.39%	8.77%	22.86%	-1.42%	0.85%	0.86%	30.37%
12/31/2008	-56.91%	-45.39%	-22.15%	-55.51%	-36.12%	-14.79%	-39.79%	-25.58%	-10.30%	-45.01%
12/31/2009	146.90%	56.79%	55.54%	101.52%	24.03%	-14.58%	46.87%	14.66%	7.66%	39.15%
12/31/2010	53.07%	-8.43%	12.12%	-4.20%	-23.81%	7.23%	-15.50%	3.49%	-3.97%	3.09%
12/30/2011	25.56%	-6.99%	25.29%	8.74%	-3.24%	15.92%	-10.63%	23.53%	6.03%	15.31%
12/31/2012	31.40%	2.89%	4.17%	9.52%	-2.57%	2.11%	8.68%	-3.33%	6.89%	-14.97%
12/31/2013	5.42%	40.06%	-2.08%	58.43%	16.74%	16.93%	14.15%	8.17%	30.66%	25.90%
12/31/2014	37.72%	24.16%	-14.46%	-5.97%	-21.58%	-8.65%	24.03%	21.05%	14.17%	39.79%
12/31/2015	-4.64%	19.44%	-14.22%	44.56%	-18.00%	-15.68%	-2.37%	-14.54%	-1.77%	-5.07%
12/30/2016	10.03%	12.00%	20.61%	1.71%	19.58%	15.79%	11.27%	8.73%	12.16%	5.28%
12/29/2017	46.11%	37.66%	-7.57%	35.58%	12.44%	-7.33%	26.74%	8.36%	21.27%	27.27%
12/31/2018	-6.79%	18.74%	-25.91%	-1.03%	-9.78%	-18.47%	13.13%	2.60%	-7.64%	1.67%
12/31/2019	86.16%	55.26%	17.92%	29.10%	-0.47%	2.33%	10.69%	5.69%	13.03%	27.53%
12/31/2020	80.75%	41.04%	-6.09%	31.03%	-45.63%	-40.93%	-6.69%	0.38%	7.89%	-16.76%
8/4/2021	10.75%	25.58%	11.19%	49.67%	12.62%	35.76%	24.45%	16.57%	10.15%	8.19%



# ANALYSIS AND RESULTS

## LARGE CAP

- 10 Stock Collection Analyzed
- Long Only Strategy Evaluated
- Desired Return 15 pct
- Covariance(Risk) Determined for Target Return
- Portfolio Weights Determined

### TITLE: LARGE CAP, LONG ONLY

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRquadprog  
 Optimize: minRisk  
 Constraints: LongOnly

Portfolio Weights:

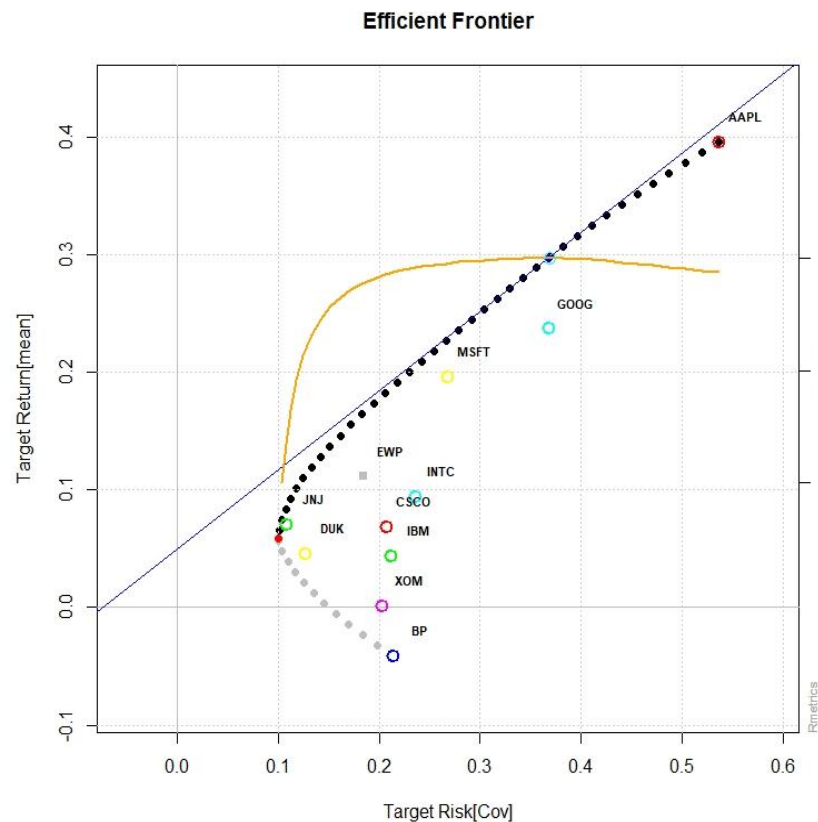
AAPL MSFT IBM GOOG BP XOM CSCO DUK JNJ INTC  
 0.2180 0.0654 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.7166 0.0000

Covariance Risk Budgets:

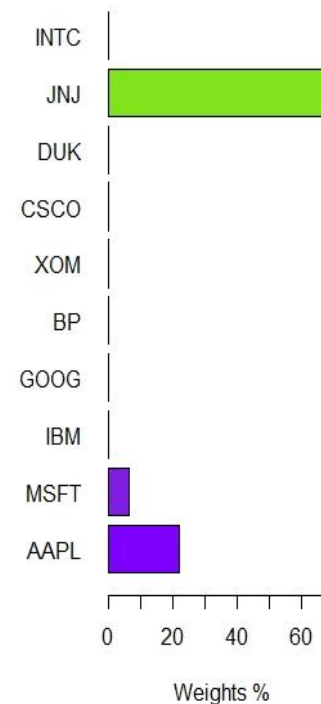
AAPL MSFT IBM GOOG BP XOM CSCO DUK JNJ INTC  
 0.6039 0.0872 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3090 0.0000

Target Returns and Risks:

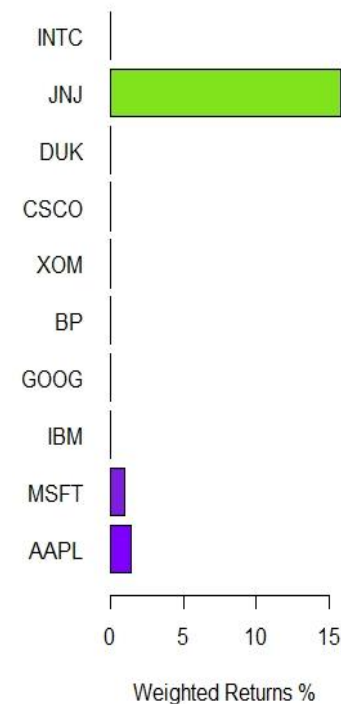
mean Cov  
 0.1500 0.1657



Portfolio Weights



Weighted Portfolio Returns



## RESULTS

- ✓ JNJ, MSFT, AAPL Best Performing



# ANALYSIS AND RESULTS

## LARGE CAP

- 10 Stock Collection Analyzed
- Long/Short Strategy
- Desired Return 15 pct
- Covariance(Risk) Determined for Target Return
- Portfolio Weights Determined

### TITLE: LARGE CAP, SHORT SELL INCLUDED

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRshortExact  
 Optimize: minRisk  
 Constraints: Short

Portfolio Weights:

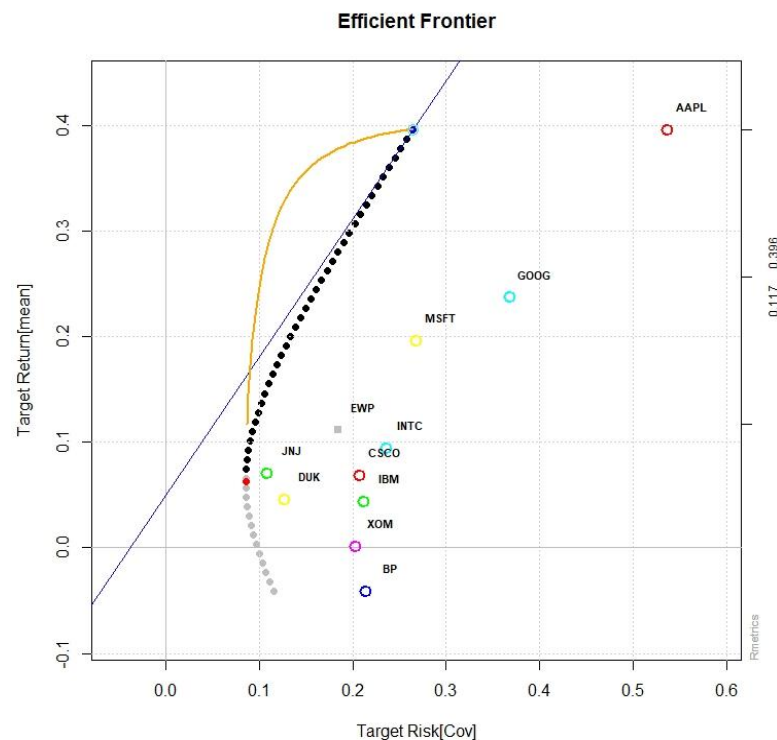
AAPL MSFT IBM GOOG BP XOM CSCO DUK JNJ INTC  
 0.1053 -0.1042 0.0894 0.0708 -0.7834 0.4955 0.4891 -0.0765 0.9621 -0.2481

Covariance Risk Budgets:

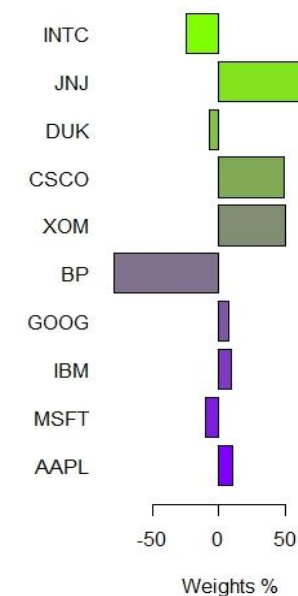
AAPL MSFT IBM GOOG BP XOM CSCO DUK JNJ INTC  
 0.2150 -0.1246 0.0494 0.0970 -0.1497 0.1844 0.3207 -0.0428 0.6402 -0.1896

Target Returns and Risks:

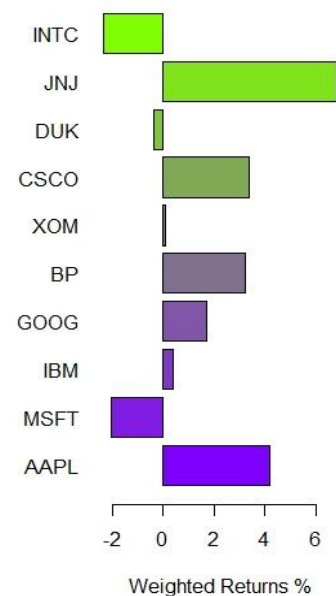
mean Cov  
 0.1500 0.1076



Portfolio Weights



Weighted Portfolio Returns



## RESULTS

- ✓ Long Primarily on JNJ, CSCO, and XOM
- ✓ Short Heavily on Underperforming Stock - BP

# ANALYSIS AND RESULTS

## MID CAP

- 10 Stock Collection Analyzed
- Long Only Strategy
- Desired Return 15 pct
- Covariance(Risk) Determined for Target Return
- Portfolio Weights Determined

### TITLE: MID CAP, LONG ONLY

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRquadprog  
 Optimize: minRisk  
 Constraints: LongOnly

### Portfolio Weights:

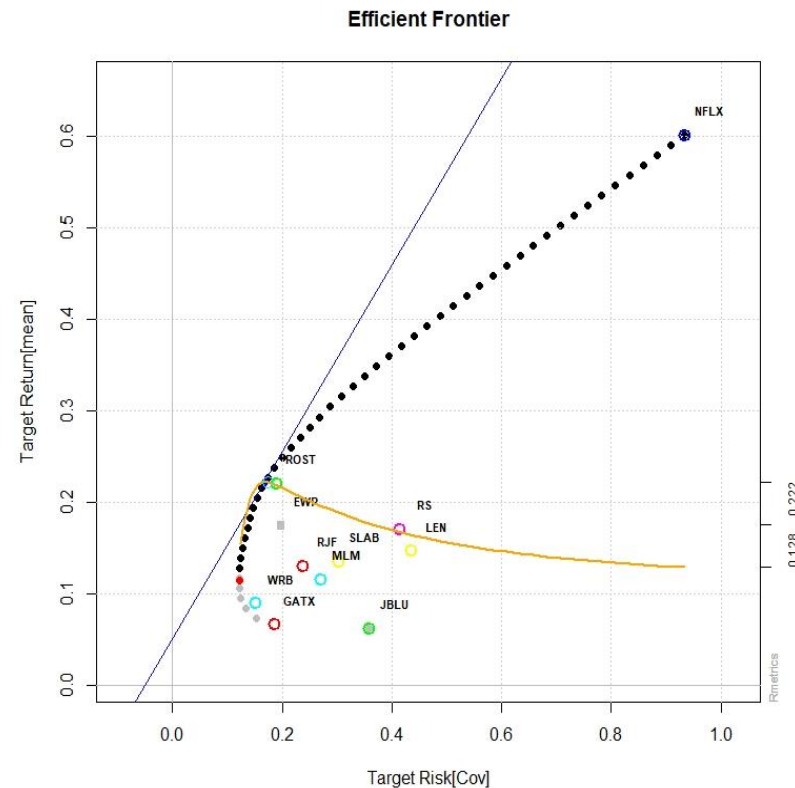
GATX	LEN	JBLU	MLM	NFLX	RS	RJF	SLAB	ROST	WRB
0.0000	0.0000	0.0000	0.0588	0.0334	0.0000	0.0159	0.1386	0.2647	0.4885

### Covariance Risk Budgets:

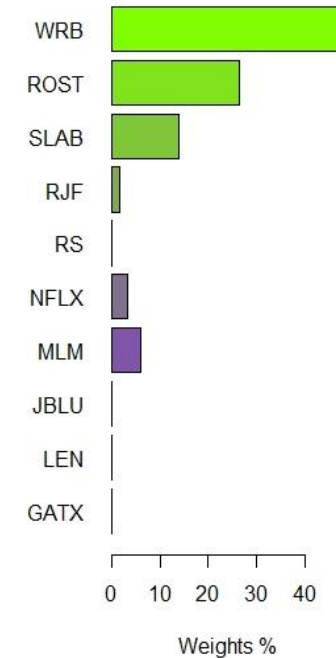
GATX	LEN	JBLU	MLM	NFLX	RS	RJF	SLAB	ROST	WRB
0.0000	0.0000	0.0000	0.0535	0.0731	0.0000	0.0151	0.1332	0.3140	0.4110

### Target Returns and Risks:

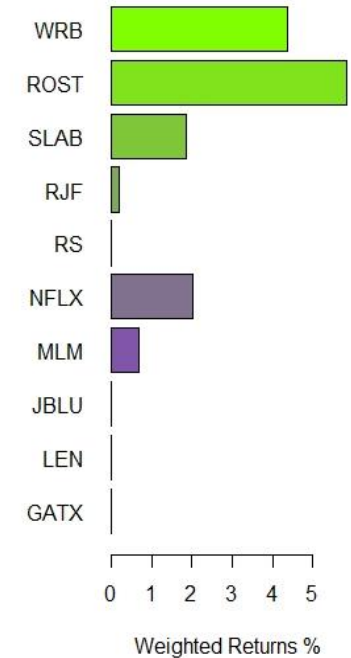
mean	Cov
0.1500	0.1284



### Portfolio Weights



### Weighted Portfolio Returns



## RESULTS

- ✓ Best Performing Stocks - WRB, ROST, SLAB
- ✓ NFLX Included Despite High Volatility

# ANALYSIS AND RESULTS

## MID CAP

- 10 Stock Collection Analyzed
- Long/Short Strategy
- Desired Return 15 pct
- Covariance(Risk) Determined for Target Return
- Portfolio Weights Determined

### TITLE: MID CAP, SHORT SELL INCLUDED

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRshortExact  
 Optimize: minRisk  
 Constraints: Short

Portfolio Weights:

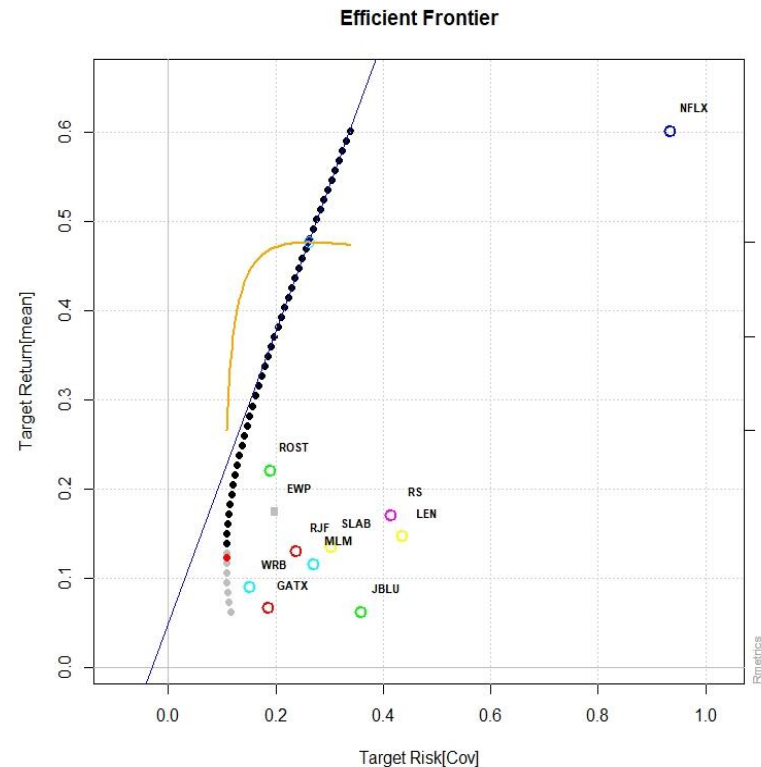
GATX	LEN	JBLU	MLM	NFLX	RS	RJF	SLAB	ROST	WRB
-0.2912	-0.1323	-0.1812	0.0477	0.0177	-0.3305	0.5075	0.3211	0.2824	0.7589

Covariance Risk Budgets:

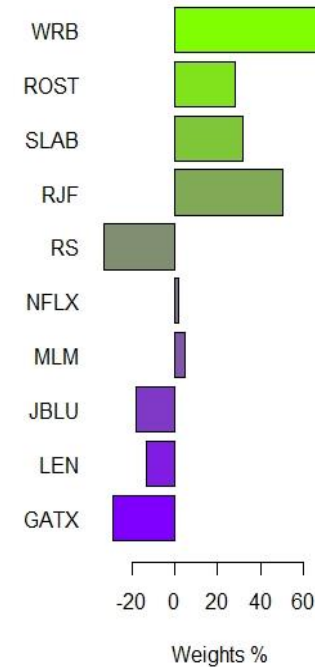
GATX	LEN	JBLU	MLM	NFLX	RS	RJF	SLAB	ROST	WRB
-0.2668	-0.1319	-0.1652	0.0461	0.0257	-0.3373	0.4977	0.3163	0.3024	0.7130

Target Returns and Risks:

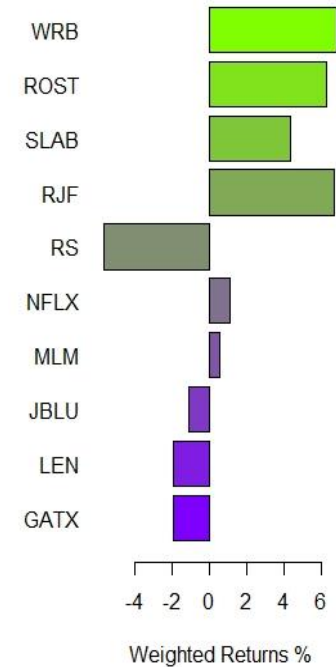
mean	Cov
0.1500	0.1094



Portfolio Weights



Weighted Portfolio Returns



## RESULTS

- ✓ Long Primarily on WRB, ROST, SLAB, RJF
- ✓ Short on GATX, JBLU - Lower Performing Stocks

# ANALYSIS AND RESULTS

## SMALL CAP

- 10 Stock Collection Analyzed
- Long Only Strategy Evaluated
- Desired Return 15 pct
- Covariance(Risk) Determined for Target Return
- Portfolio Weights Determined

### TITLE: SMALL CAP, LONG ONLY

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRquadprog  
 Optimize: minRisk  
 Constraints: LongOnly

Portfolio Weights:

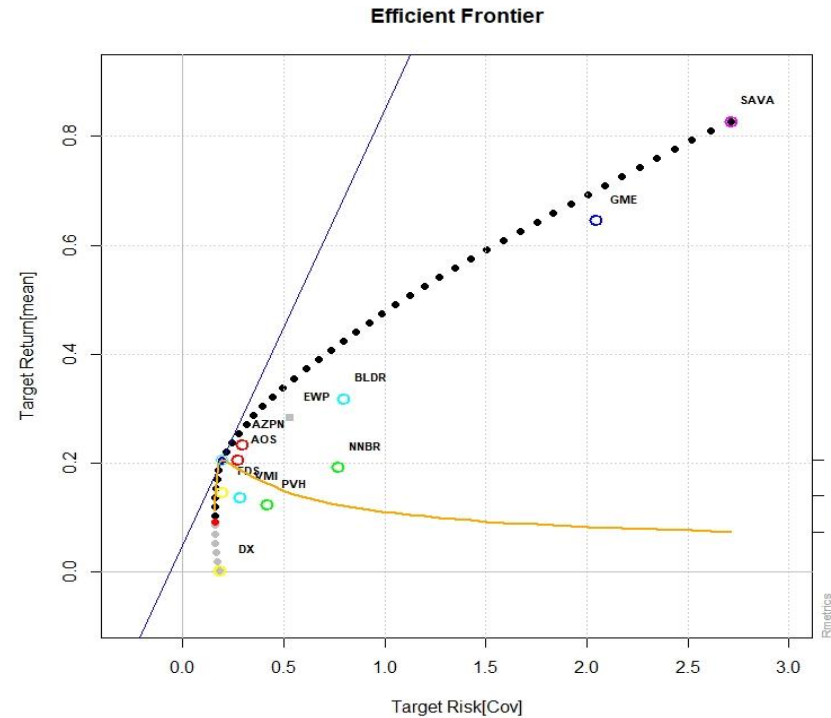
AOS	DX	NNBR	BLDR	GME	SAVA	AZPN	FDS	PVH	VMI
0.1001	0.1661	0.0000	0.0000	0.0112	0.0000	0.1895	0.5188	0.0000	0.0144

Covariance Risk Budgets:

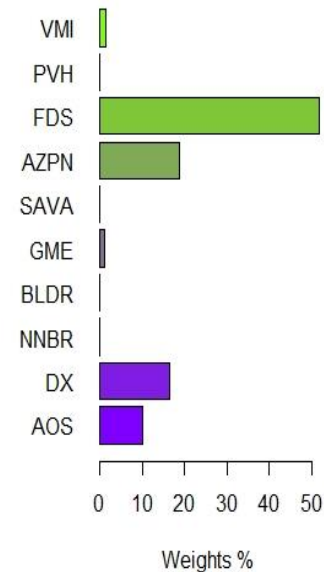
AOS	DX	NNBR	BLDR	GME	SAVA	AZPN	FDS	PVH	VMI
0.1085	0.1285	0.0000	0.0000	0.0197	0.0000	0.2134	0.5158	0.0000	0.0141

Target Returns and Risks:

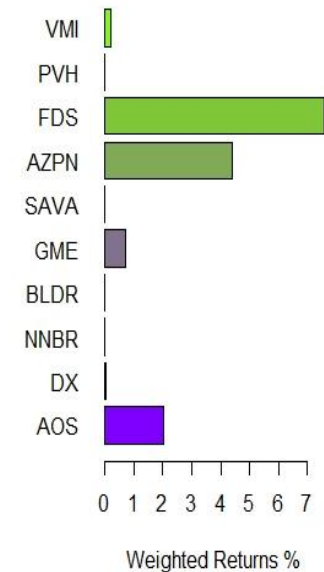
mean	Cov
0.1500	0.1653



Portfolio Weights



Weighted Portfolio Returns



## RESULTS

- ✓ Best Performing Stocks - FDS, AZPN, AOS

# ANALYSIS AND RESULTS

## SMALL CAP

- 10 Stock Collection Analyzed
- Long/Short Strategy
- Desired Return 15 pct
- Covariance(Risk) Determined for Target Return
- Portfolio Weights Determined

### TITLE: SMALL CAP, SHORT SELL INCLUDED

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRshortExact  
 Optimize: minRisk  
 Constraints: Short

### Portfolio Weights:

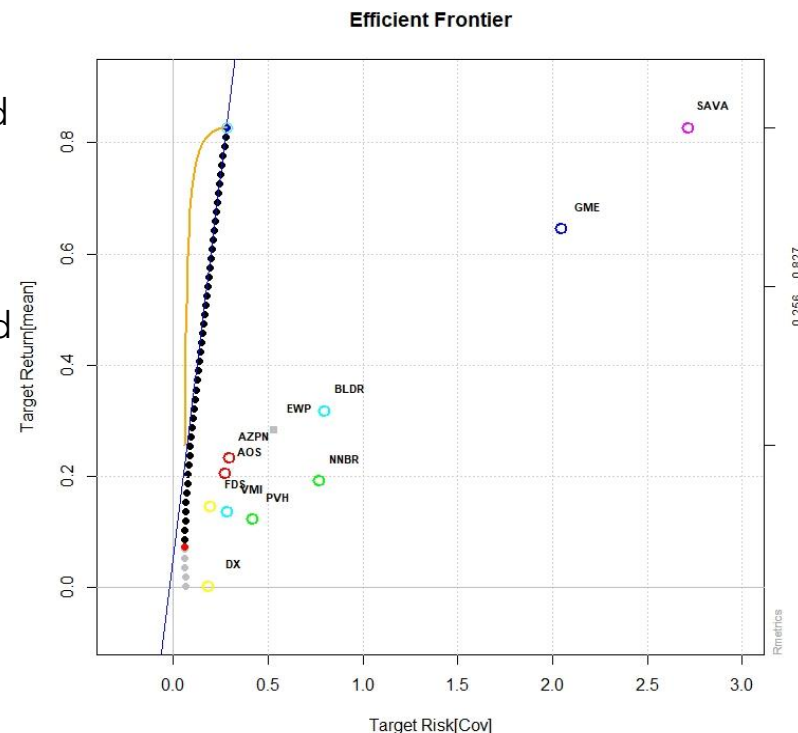
AOS	DX	NNBR	BLDR	GME	SAVA	AZPN	FDS	PVH	VMI
0.5401	-0.3132	-0.2847	-0.2332	-0.0464	0.0170	-0.0094	0.7670	0.1852	0.3777

### Covariance Risk Budgets:

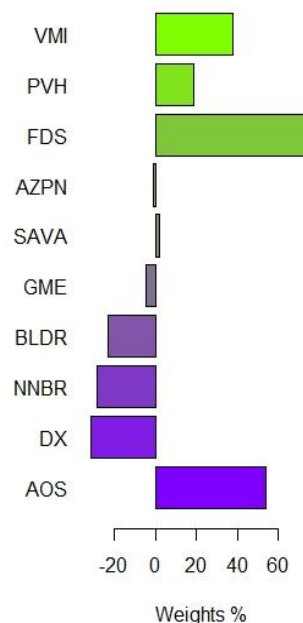
AOS	DX	NNBR	BLDR	GME	SAVA	AZPN	FDS	PVH	VMI
0.6073	-0.2081	-0.3115	-0.3218	-0.0986	0.0431	-0.0112	0.7604	0.1739	0.3664

### Target Returns and Risks:

mean	Cov
0.1500	0.0671



### Portfolio Weights



### Weighted Portfolio Returns



## RESULTS

- ✓ Long Primarily on FDS and AOS
- ✓ Short on BLDR and NNBR – Lower Performing Stocks and Higher Volatility



# ANALYSIS AND RESULTS MIXED CAP

- 14 Stock Collection Analyzed
- Long Only Strategy
- Desired Return 15 pct
- Best Performing from Previous Analysis
- Lower Performing (BP) Included - Short Sell Potential

## TITLE: MIXED CAP, LONG ONLY

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRquadprog  
 Optimize: minRisk  
 Constraints: LongOnly

### Portfolio Weights:

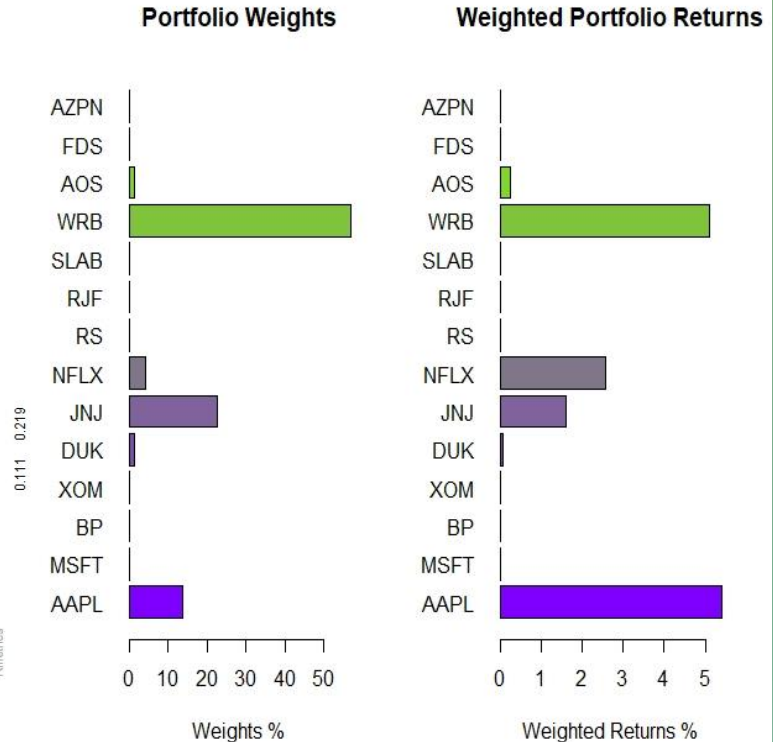
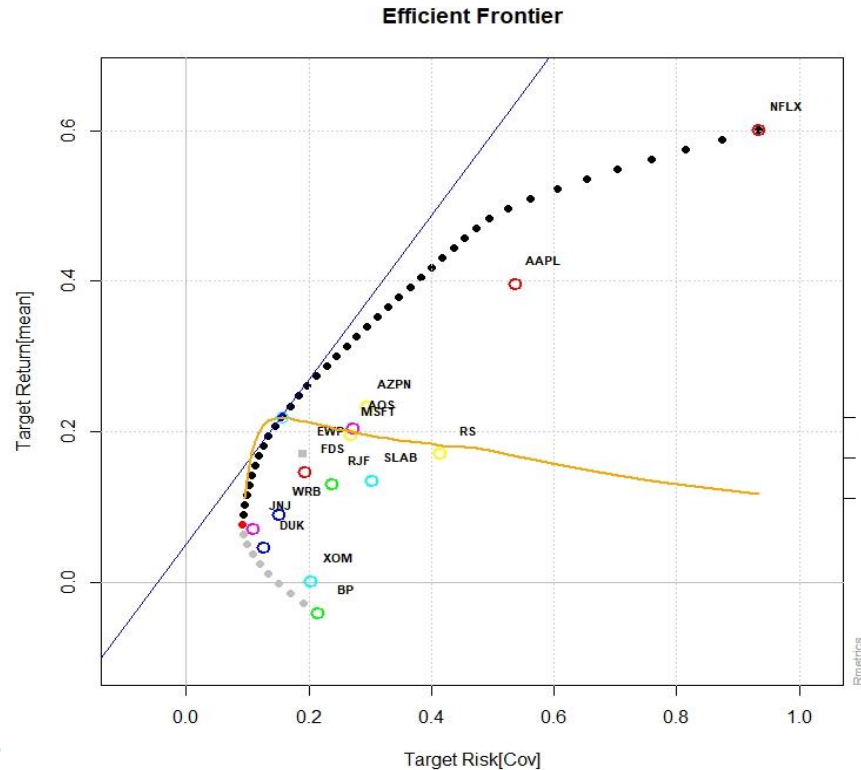
AAPL	MSFT	BP	XOM	DUK	JNJ	NFLX	RS	RJF	SLAB
0.1366	0.0000	0.0000	0.0000	0.0119	0.2263	0.0427	0.0000	0.0000	0.0000
0.5707	0.0118	0.0000	0.0000						

### Covariance Risk Budgets:

AAPL	MSFT	BP	XOM	DUK	JNJ	NFLX	RS	RJF	SLAB
0.2689	0.0000	0.0000	0.0000	0.0070	0.1559	0.1185	0.0000	0.0000	0.0000
0.4353	0.0144	0.0000	0.0000						

### Target Returns and Risks:

mean	Cov
0.1500	0.1103



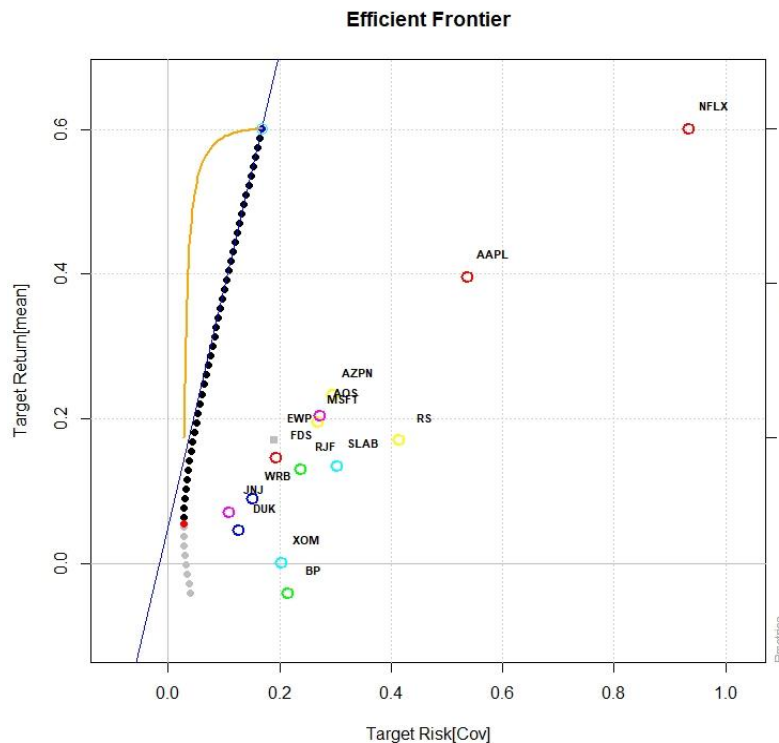
## RESULTS

- ✓ Best Performing Stocks - WRB, JNJ, AAPL
- ✓ NFLX Included Despite Higher Volatility

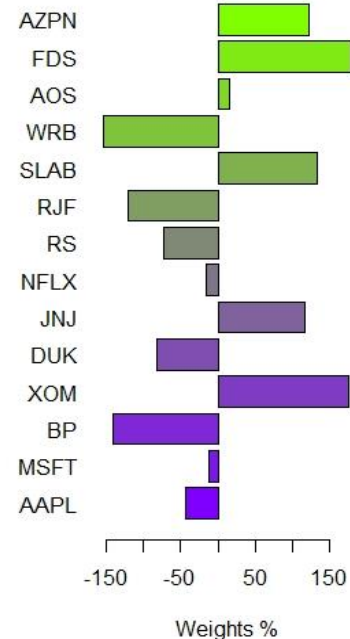


# ANALYSIS AND RESULTS MIXED CAP

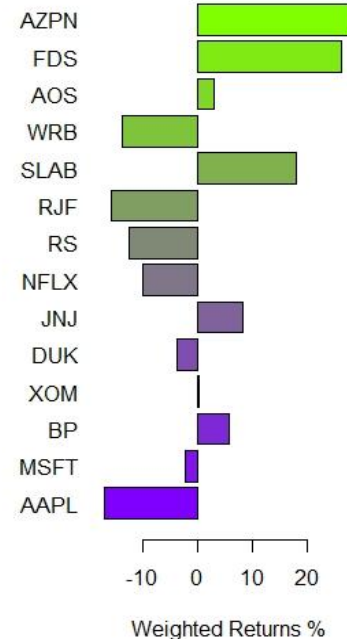
- 14 Stock Collection Analyzed
- Desired Return 15 pct
- Long/Short Strategy
- Lower Performing (BP) Included - Short Sell Potential



Portfolio Weights



Weighted Portfolio Returns



## TITLE:MIXED CAP, SHORT SELL INCLUDED

MV Efficient Portfolio

Estimator: covEstimator  
 Solver: solveRshortExact  
 Optimize: minRisk  
 Constraints: Short

Portfolio Weights:

AAPL	MSFT	BP	XOM	DUK	JNJ	NFLX	RS	RJF	SLAB
-0.4278	-0.1195	-1.4145	1.7567	-0.8183	1.1691	-0.1663	-0.7378	-1.2105	1.3377
-1.5334	0.1534	1.7928	1.2185						

Covariance Risk Budgets:

AAPL	MSFT	BP	XOM	DUK	JNJ	NFLX	RS	RJF	SLAB
-1.0067	-0.1498	0.0720	0.3238	-0.3501	0.6609	-0.5790	-0.8214	-1.0819	1.2294
1.0252	0.1997	1.7555	1.7730						

Target Returns and Risks:

mean	Cov
0.1500	0.0399

## RESULTS

- ✓ Long Primarily on AZPN, FDS, SLAB, JNJ, and XOM
- ✓ Short on BP, DUK, RS, WRB- Lower Performing Stocks, Higher Volatility Relative to Other Stocks

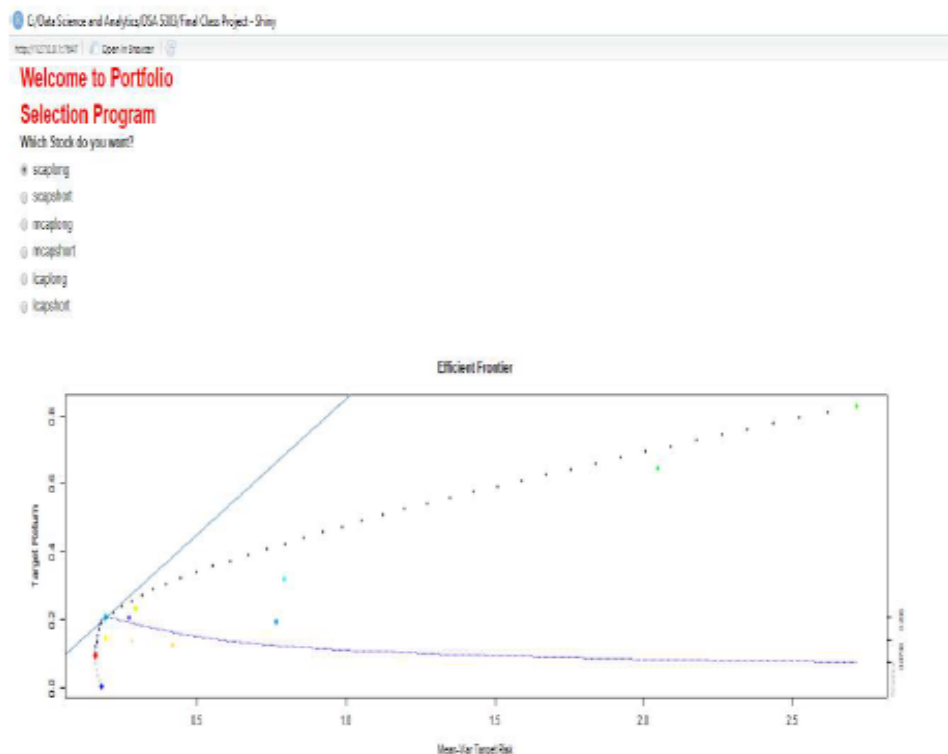
# SHINY DASHBOARD

## ➤ 2 Apps Developed

- App1 : Plots Efficient Portfolio for Portfolio Selected by User

## ➤ Periodic Returns

- App2: Prints Output with Portfolio Weights/Risk Budgets and Target Returns/Risk for Portfolio Selected by User



# CONCLUSIONS

- R Packages - Robust Application for Stocks Analysis and Portfolio Selection
- Diversified Portfolio Can be Selected using Markowitz's Model
- User Can:
  - Determine His/Her Risk Tolerance and Maximize Return Based on Individual Tolerance
  - Determine Risk for His/Her Desired Return
- Allocation of Weights for Large, Mid, and Small Cap Developed
- Mixed Cap Portfolio Developed Using Best Performing Stocks from Individual Analysis - Lowest Covariance (Risk) Obtained for this Portfolio Amongst All Portfolios Analyzed

## Asset Allocation Weights/Returns, Frontier Plot, Mixed Cap Long

