```
In [57]: import numpy as np
         import matplotlib.pyplot as plt
         import csv
         import pandas as pd
         np.random.seed(2023)
         # Load data
         df = pd.read_csv('ford-prices.csv')
         df = df.replace({',': ''}, regex=True)
         df = df.astype(float)
         print(df.head())
              Year minPrice maxPrice
         0 1992.0 8730.0
                             14840.0
         1 1993.0 8781.0 16535.0
         2 1994.0 9449.0 18328.0
         3 1995.0 10224.0 19571.0
         4 1996.0 10575.0 20295.0
In [58]: # Split data into features (years) and labels (minimum prices and maximum pr
         X = df.iloc[:,0].values.reshape(-1,1)
         y min = df.iloc[:,1].values.reshape(-1,1)
         y_max = df.iloc[:,2].values.reshape(-1,1)
In [59]: # Initialize theta
         theta min = np.random.rand(2,1)
         theta_max = np.random.rand(2,1)
In [60]: # Define the hypothesis function
         def h(X, theta):
             return np.matmul(X, theta)
In [61]: # Define the mean squared error loss function
         def MSE(y, y_hat):
             return np.mean((y-y_hat)**2)
In [62]: def gradient_descent(X, y, theta, alpha, num_iters):
             m = X.shape[0]
             loss = np.zeros((num_iters, 1))
             for i in range(num_iters):
                 y_hat = h(X, theta)
                   print(y_hat.shape)
                 loss[i] = MSE(y, y_hat)
                 #print(loss[i])
                   theta = theta + (alpha)/m * np.matmul(X.T, y- y_hat)
                 error = y_hat - y
                 theta[0] -= alpha * error.mean()
                 theta[1] -= alpha * (error * X[:,1:]).mean()
             return theta, loss
In [63]: # Set hyperparameters
         alpha = 1e-7
```

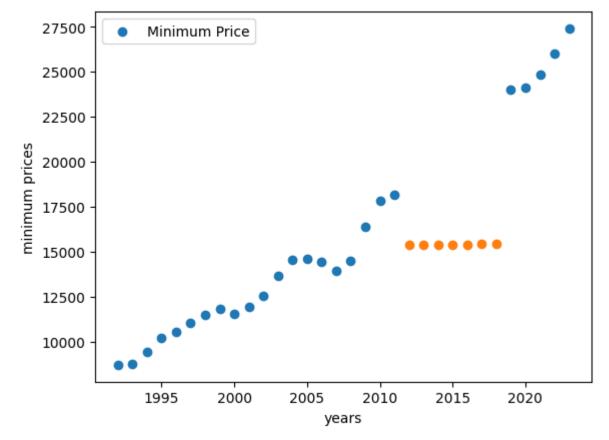
```
num iters = 100
In [64]: # Run gradient descent for minimum prices
         theta min, loss min = gradient descent(np.hstack((np.ones((X.shape[0], 1)),
         # Run gradient descent for maximum prices
         theta_max, loss_max = gradient_descent(np.hstack((np.ones((X.shape[0], 1)),
In [65]: print(f'The values of theta for the minimum prices are : theta0 {theta_min[0
         print(f'The values of theta for the maximum prices are : theta0 {theta_max[@]
         The values of theta for the minimum prices are : theta0 [0.32510465], theta1
         [7.64557759]
         The values of theta for the maximum prices are : theta0 [0.59409396], theta1
         [12.91118457]
In [66]: # Plot the loss curve
         plt.plot(range(num_iters), loss_min, label='Minimum Price')
         plt.plot(range(num_iters), loss_max, label='Maximum Price')
         plt.xlabel('Iteration')
         plt.ylabel('Loss')
         plt.legend()
         plt.show()
               1e8
                                                                Minimum Price
             7
                                                                Maximum Price
             6
             5
             3
             2
             1
             0
                 0
                             20
                                         40
                                                     60
                                                                80
                                                                            100
                                            Iteration
```

predicting the minimum and maximum price of year between 2012 to 2018

```
In [67]: prices_min = []
    prices_max = []
    years = []
    for i in range(2012,2019):
        years.append(i)
        prices_min.append(h(np.array([1,i]), theta_min))
        prices_max.append(h(np.array([1,i]), theta_max))
```

## plotting the Minimum prices of every years

```
In [68]: plt.scatter(X, y_min, label='Minimum Price')
   plt.scatter(np.array(years),np.array(prices_min))
   plt.xlabel('years')
   plt.ylabel('minimum prices')
   plt.legend()
   plt.show()
```



## plotting the Maximum prices of every years

```
In [69]: plt.scatter(X, y_max, label='Maximum Price')
   plt.scatter(np.array(years),np.array(prices_max))
   plt.xlabel('years')
   plt.ylabel('maximum prices')
   plt.legend()
   plt.show()
```

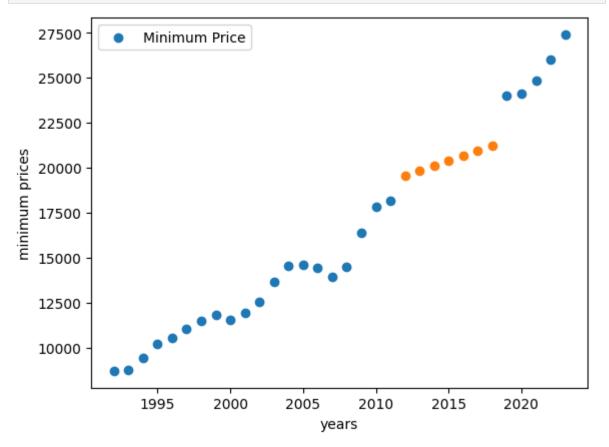


## dynamic learning rate with feature scaling

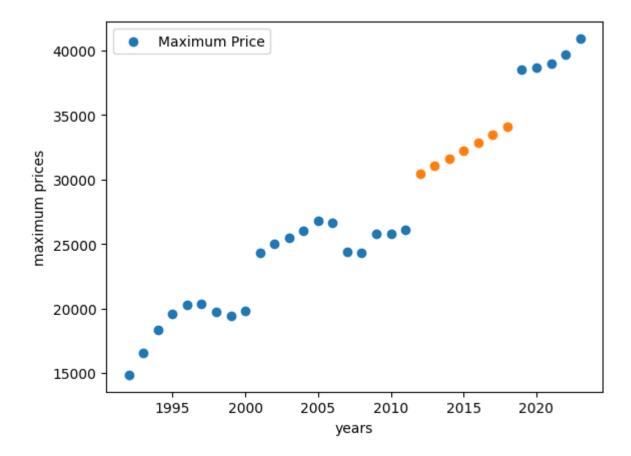
```
In [70]: X scaled = (X - X.mean())/(X.max()-X.min())
         y_min_scaled = (y_min - y_min.mean())/(y_min.max()-y_min.min())
         y_max_scaled = (y_max - y_max.mean())/(y_max.max()-y_max.min())
In [71]: # Initialize theta
         theta_min = np.random.rand(2,1)
         theta_max = np.random.rand(2,1)
         alpha = 1e-3
In [72]: def gradient_descent_dynamic(X, y, theta, alpha, num_iters):
             m = X.shape[0]
             loss = np.zeros((num_iters, 1))
             for i in range(num_iters):
                 y_hat = h(X, theta)
                 loss[i] = MSE(y, y_hat)
                 error = y_hat - y
                 theta[0] -= alpha * error.mean()
                 theta[1] -= (1/(1+i))*alpha * (error * X[:,1:]).mean()
             return theta, loss
In [73]: # Run gradient descent for minimum prices
         theta_min, loss_min = gradient_descent_dynamic(np.hstack((np.ones((X_scaled.
         # Run gradient descent for maximum prices
         theta max, loss max = gradient descent dynamic(np.hstack((np.ones((X scaled.
```

```
print(f'The values of theta for the minimum prices are : theta0 {theta_min[0
         print(f'The values of theta for the maximum prices are : theta0 {theta_max[0
         The values of theta for the minimum prices are : theta0 [0.12788443], theta1
         [0.46813276]
         The values of theta for the maximum prices are : theta0 [0.01998655], theta1
         [0.72736549]
In [75]: # Plot the loss curve
         plt.plot(range(num_iters), loss_min, label='Minimum Price')
         plt.plot(range(num_iters), loss_max, label='Maximum Price')
         plt.xlabel('Iteration')
         plt.ylabel('Loss')
         plt.legend()
         plt.show()
             0.045
             0.040
             0.035
             0.030
                                                                     Minimum Price
                                                                     Maximum Price
             0.025
             0.020
             0.015
             0.010
                      0
                                 20
                                             40
                                                         60
                                                                     80
                                                                                100
                                                Iteration
In [76]: prices_min = []
         prices_max = []
         years = []
         for i in range(2012,2019):
             years.append(i)
             prices_min.append(h(np.array([1,(i-X.mean())/(X.max()-X.min())]), theta_
             prices_max.append(h(np.array([1,(i-X.mean())/(X.max()-X.min())]), theta_
In [77]: plt.scatter(X, y_min, label='Minimum Price')
         plt.scatter(np.array(years),np.array(prices_min))
         plt.xlabel('years')
         plt.ylabel('minimum prices')
```

```
plt.legend()
plt.show()
```



```
In [78]: plt.scatter(X, y_max, label='Maximum Price')
   plt.scatter(np.array(years),np.array(prices_max))
   plt.xlabel('years')
   plt.ylabel('maximum prices')
   plt.legend()
   plt.show()
```



## Summary

```
In [79]: prices_min = h(np.array([1,(2024-X.mean())/(X.max()-X.min())]), theta_min)*(
    prices_max = h(np.array([1,(2024-X.mean())/(X.max()-X.min())]), theta_max)*(
    print(prices_min , prices_max)
```

[22939.18549421] [37773.83463379]

The probable maximum price for ford in 2024 is 37773.83463379 and minimum price would be 22939.18549421 which seems lower than 2023. So buying new card should be benificial.

The new values of theta for the minimum prices are: theta0 0.12788443,theta1 0.46813276 The values of theta for the maximum prices are: theta0 0.01998655,theta1 0.72736549 And form the price prediction plot it's pretty evident that our model is good for the prediction. So we can trust the new values of theta.

There are some hyper parameters like learning rate and number of iteration. We need to tune them to get better result. As we can tell from the loss curve, if we would have run the model for several epochs more, the model would have converged better.

We can also see the difference between the performance of non scaled feature models and scaled feature models. In the first two models where we didn't do feature scaling, they didn't converge properly. We can see that from the prediction results as well. But

when we did the feature scaling the models converged really well. So we can confidently say feature scaling helped in this case.

I will definitely buy the next generation model for sure as they are more cost efficient and the new model should come with new features as well.