Astronomy 121: Radio Lab
Lab 1 Report

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August 31, 2016

ABSTRACT

This lab covers the basics of signal measurement and processing from sample rates and Nyquist frequency to mixers and the heterodyne process. We will be testing a variety of signal processing techniques to determine their efficacy and taking note of techniques to avoid. This will serve as a foundation for all further exploration into signal processing and radio astronomy.

1. Introduction

Analog signals are, to put it briefly, fluctuations in voltage. They can be measured, but conventional measuring devices can't really measure continuously; measurements are made discretely at regular time intervals, and the resulting signal is generally representative of the true signal as long as signal sampling conventions are observed. Once signals are measured, they can be manipulated in order to better explore or represent the true signal. One of the most common manipulations involves the Fourier transform, which allows the signal to be mapped from the time domain to the frequency domain—more on this later. Signals of too high a frequency, or signals that carry another signal via amplitude modulation, can be shifted down in frequency by using a mixer and some signal processing. With this basic toolkit of techniques, one can properly measure signals, view and edit their spectral decompositions, and mix them with other signals for a variety of desired results. These are the basics of signal processing, and this lab will demonstrate each one and highlight useful characteristics.

The following sections will provide background information and theory for each of the upcoming topics.

1.1. Nyquist Criterion

The most important of these conventions is the requirement that one must sample above the Nyquist frequency in order to be reliably able to recover the original signal. The Nyquist frequency is twice the original frequency:

$$\nu_{Nyquist} = 2 \times \nu_{signal} \tag{1}$$

That means that one must sample the original signal at *at least* twice the expected signal frequency.

This raises the question, why not sample at something like 100 MHz all the time? There are two main responses to this fair inquiry. First of all, not every device can sample at such a high rate. The picosampler used in the Undergraduate Lab can sample at rates up to 62.5 MHz, which is good enough for many signals. Second, a higher sample rate leads to a larger data file, and if the original signal isn't of a very high frequency anyway, then sampling at 60 MHz for several seconds doesn't make much sense in terms of the amount of data; the signal can be reproduced with a file a fraction of that size.

Part of this lab will sample signals around their Nyquist frequencies to take a look at what potential problems can arise if the criterion is not observed.

1.2. Fourier Spectra and Filtering

The Fourier transform is a mathematical procedure that transforms a signal from a time basis into a frequency basis. Normally, a signal is represented by voltage as a function of time, so signals show up as they are measured by the device. After a Fourier transform, the signals are represented by voltage as a function of frequency; the transform is a change of basis from time domain to frequency domain.

$$E(\nu) = \frac{1}{T}\int_{-T/2}^{T/2} E(t) e^{2\pi i \nu t} dt \tag{2}$$

$$E(t) = \frac{1}{F}\int_{-F/2}^{F/2} E(\nu) e^{-2\pi i \nu t} d\nu \tag{3}$$

The mathematical formula for the Fourier transform is shown above in Equation 2 and the inverse Fourier transform, from frequency back to time is shown in Equation 3.

1.2.1. Fourier Transform

Frequency domain is still an orthonormal basis, but unlike the time domain, where there are two dimensions, and therefore two basis vectors, (time and voltage), frequency domain technically has an infinite number of basis vectors, one for each possible frequency (which is theoretically infinite). A graph of the voltage spectrum after a Fourier transform becomes a graph with each possible frequency on the x axis, and its comparative prevalence in amplitude on the y axis.

While theoretically, an arbitrarily small *dt* could lead to an arbitrarily small *df* and therefore yield a continuous power spectrum, that requires a continuous sample over time– which we already determined is functionally impossible. Realistically, *dt* and *df* are discrete chunks of time or frequency domain, so the integral becomes more of a sum: this is called the Discrete Fourier Transform (DFT). Practical Fourier transforms used in signal processing are likely DFTs (or FFTs–see below). It turns out that our *dt* is the inverse of our sample rate; it is the time *in between* samples. It follows that the *df* will be the space between frequencies that we can detect, which leads to our frequency resolution.

Furthermore, the transform (which should[1] be performed symmetrically around 0) will have a limit of $\nu_{sample}/2$ in either direction, which means the lowest frequency detectable is $-\nu_{sample}/2$ (or *df*, depending on how you define "lowest") and the highest frequency detectable is $\nu_{sample}/2$.

Given a data set consisting of N samples, the result of a proper Fourier transform will also comprise N samples. For the original data set, *dt* will naturally be ν_{sample}^{-1}, which is equivalent to T/N, where T is the total time over which samples were taken. Following that, the *df* of the data set transformed to frequency domain will be ν_{sample}/N; one can see the parallelism in how *df* and *dt* are obtained by dividing by N.

IDL has a DFT procedure written into it that calculates the DFT of a data set, which was used throughout the data collection and analysis portion of this lab. When sample sizes are large, the DFT can take a while, so there exists a faster procedure: the aptly-named Fast Fourier Transform (FFT).

1.2.2. Fourier Filtering

Once a signal has been transformed from the time domain to frequency domain, it can be viewed as a collection of frequencies. Through array manipulation, the transform sample can be modified to add or eliminate certain frequencies in order to modify the shape of the wave. One can imagine the algorithm it would take to turn a square wave in to a sine wave in time domain. In frequency domain, it is as easy as zeroing out all frequencies but the fundamentals and transforming back to the time domain.

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[1] This assumes that proper Fourier transform conventions are followed regarding the time and frequency range. These conventions will not be enumerated here; it will be assumed throughout this report that all DFTs are evaluated accordingly.

All techniques mentioned here should be taken as theoretical for now; they will be proven to work in subsequent sections.

1.3. Mixers and the Heterodyne Process

Mixers multiply two signals together. They take advantage of a useful trigonometric identity:

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y) \tag{4}$$

One can see in Equation 4 that two original multiplied signals will appear the same as two overlaid signals[2], one the sum of the original two and the other the difference. This can be used for a variety of useful purposes, but we will limit discussion in further sections of this report to mixing a high frequency signal with a local oscillator to recover the difference between them.

This trigonometric identity makes more sense the other way around:

$$\cos(x+y) + \cos(x-y) = 2\cos(x)\cos(y) \tag{5}$$

Here, one can see that two similar frequencies added together are going to interact somehow. This is the phenomenon of beats. Two similar frequencies interfere in such a way that modulates their volume; one can infer from Equation 5 that another way to model beat frequencies is as a product of the high frequency carrier wave x and the lower frequency amplitude modulation wave y. This can be further clarified as:

$$\cos(f+\delta f) + \cos(f-\delta f) = 2\cos(f)\cos(\delta f) \tag{6}$$

This is incredibly useful, as further sections of this report will prove.

2. Observations and Experiments

Each of the topics discussed in the previous section was tested in the lab; this section will go over how they were tested.

2.1. Nyquist Criterion

The Nyquist criterion was tested for the three obvious cases:

[2]The factor of 2 doesn't matter; it will only be observed in the amplitude.

- $\nu_{sample} > \nu_{Nyquist}$
- $\nu_{sample} < \nu_{Nyquist}$
- $\nu_{sample} = \nu_{Nyquist}$

All data was taken via the `GETPICO` procedure[3] written into our version of IDL. The procedure takes samples remotely from the picosampler connected to a signal generator. Most samples were taken at frequencies that are multiples of 6.25 Hz, due to the way the `GETPICO` procedure works. Small sample sizes were plotted in order to view the individual waveforms; the data that will be shown later used 16 samples.

First, nine sets of data were taken to test what happened right around the Nyquist frequency. The sample rate was kept at 6.25 MHz and the signal frequency was adjusted remotely via an automated procedure using the `SRS1_FRQ` script. Signal frequency was adjusted from $0.1\nu_{sample}$ to $0.9\nu_{sample}$ in coefficient increments of 0.1 $(0.1, 0.2, 0.3, ...0.9)$. This data is useful because it covers proper sampling ($\nu_{signal} = 0.1\nu_{sample}$), sampling at the Nyquist frequency ($\nu_{signal} = 0.5\nu_{sample}$), and undersampling ($\nu_{signal} = 0.9\nu_{sample}$).
Next, two extreme cases were identified and tested. A 6.25 MHz signal was sampled at its own frequency ($\nu_{sample} = \nu_{signal}$) and then a 6.23 MHz signal was significantly undersampled at 62.5 kHz ($\nu_{sample} \ll \nu_{signal}$). The undersampled signal frequency was specifically chosen not to be an integer multiple of the sample frequency to avoid a repetitive sampling; we wanted to demonstrate a normal undersampling result, not a special case of integer multiples.

2.2. Fourier Spectra and Filtering

The next part of our experimentation used the `DFT` procedure. In addition to the voltage data from the signal, it requires a time range and a frequency range; for both of these, the proper specifications in the lab manual were observed. The time range for N samples covered $-\frac{N}{2}\frac{1}{\nu_{sample}}$ to $\left(\frac{N}{2}-1\right)\frac{1}{\nu_{sample}}$ in steps of the previously discussed dt. The frequency range for N samples covered $-\frac{\nu_{sample}}{2}$ to $\frac{\nu_{sample}}{2}\left(1-\frac{2}{N}\right)$ in steps of the previously discussed df.

Voltage spectra (real and imaginary) and power spectra (power = voltage × `conj`(voltage))[4] were plotted for several frequencies around 1 MHz, all sampled properly at 6.25 MHz. The 1 MHz signal spectra results will be discussed, since the others were not fundamentally different or surprising.

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[3]The particulars of the GETPICO procedure won't be discussed here. Documentation can be found within IDl.

[4]CONJ is an IDL function that takes the complex conjugate of the argument.

Next, 62.5 kHz square and triangle waves were comfortably sampled at 6.25 MHz. Their power spectra were examined. The square wave was then filtered. Filtering was performed by transforming the signal through the `DFT` procedure into frequency domain. Then, the voltage spectra was examined to find the locations of the fundamental frequencies, and all other frequencies were zeroed out. The modified signal was inverse transformed back to time domain (using the `DFT` procedure's `/inverse` keyword) with the exact same time and frequency ranges so as to retain proper amplitude scaling.

A sample of the filtering code is shown below.

```
; last is the last number in the 2048-element series
; lower is the lower bound of frequency domain that will *not* be zeroed out
; upper is the upper bound of frequency domain that will *not* be zeroed out
; spectrum is the full complex voltage spectrum

last = 2047
lower = 985
upper = last - lower
for i=0,lower do spectrum[i] = 0
for i=upper,last do spectrum[i] = 0
```

The `lower` and `upper` variables were selected by "eyeballing" the power spectrum to find where the fundamentals were. The final filtering procedure included the `DFT`s and plotting, neither of which is remarkable enough to reproduce here.

2.3. Leakage Power and Frequency Resolution

The existence of leakage power was tested by zooming very far in along the y axis[5] using a power spectrum from a well-sampled signal and examining the vertical spread of points to check for a significant number of non-zero elements. If there are a lot of non-zero elements in places where we would expect no frequency presence, then we will consider that leakage power.

Frequency resolution determines how far apart peaks have to be in frequency domain for the observer to identify them as separate peaks. To test this, the number of samples N and the sample rate ν_{sample} were adjusted together to control the df (discussed earlier). The df can be seen as a "bin" in which frequency components are placed. Two or more similar frequency

[5] A normal x range but a very limited y range

components placed in the same bin become indistinguishable to the observer. Frequencies *must* be placed in different bins in order to be distinguishable. In other words, distinguishable frequencies must correspond to different dfs. Our hypothesis and corresponding results will be discussed later.

2.4. Mixers and the Heterodyne Process

In order to run experiments using the mixer, both signal generators were connected to the mixer, whose output was connected to the picosampler. The first experiment did not require any additional equipment. A 1.00 MHz signal (our local oscillator) was mixed with a 1.05 MHz signal (the radio frequency) for the upper sideband case and sampled it at a comfortable 6.25 MHz. After examining the signal and its power spectrum, the voltage spectrum was filtered using the same 'zeroing' method in order to eliminate the sum frequency, which would be 2.05 MHz. The filtered signal was transformed back to the time domain to recover the difference signal of 50 kHz (1.05 MHz -1.0 MHz $=$ 0.5 MHz $=$ 50 kHz). The lower sideband case was also examined to make sure the sum frequency appeared to be 1.95 MHz and the difference was still 50 kHz.

The final experiment split the local oscillator frequency using a power splitter and mixed each half with the radio frequency. The cables from the local oscillator to each mixer were of approximately equal length. The upper sideband case was tested first: the local oscillator frequency was 10.5 MHz and the radio frequency was 11.025 MHz, so a sum frequency of 20.5 MHz and an amplitude modulation frequency of 500 kHz were expected, and so the signal was sampled at the picosampler's limit of 62.5 MHz in order to stay well above the Nyquist frequency. Both mixed waves were sampled by the picosampler; one was treated as the real part of the signal and the other was treated as the imaginary component. The complex wave was constructed by picking apart the `GETPICO` output to separate the A and B inputs. N elements were taken from each (we used 1024 elements) and combined as such:

```
; real is an array containing N elements from the A input, which will be treated as the
; imaginary is an array containing N elements from the B input, which will be treated as

complex_signal = complex(real, imaginary)
```

The result, `complex_signal` is a complex array of N elements. The `DFT` of that array was then taken and held for comparison. The experiment was repeated with all the same values except for a modified radio frequency of 9.975 MHz in order to test the lower sideband case.

Next, the upper sideband case was repeated with one modification. The cable from the radio frequency to the mixer that led to the picosampler's B input was replaced with two of the marked "21 MHz $\frac{\lambda}{4}$" cables in order to achieve a $\frac{\lambda}{4}$ (90°)shift at 10.5 MHz. This long cable experiment was repeated for the lower sideband and all spectra examined.

3. Data Analysis

Here we will discuss our findings.

3.1. Nyquist Criterion

The tests on the Nyquist criterion proved that the signal must be sampled at at least twice the Nyquist criterion, and preferably well above it. As one can see in the above

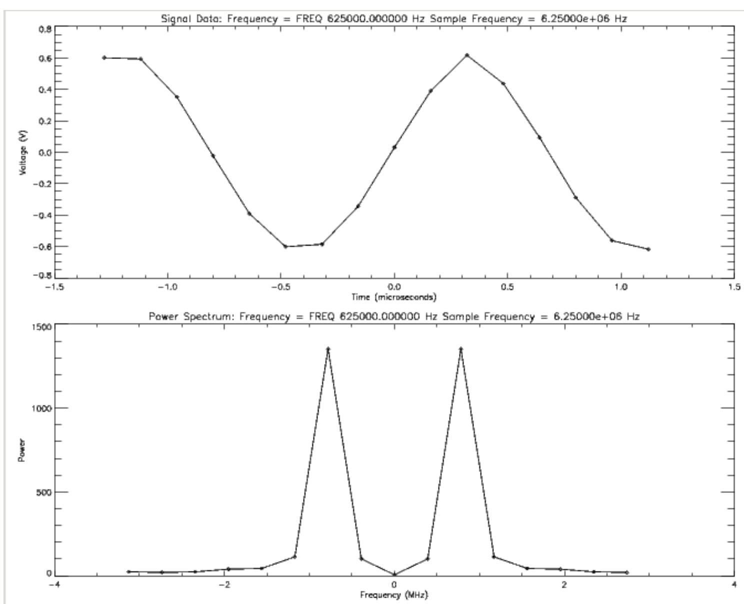

Fig. 1.— Signal was 0.1 times sample rate

figures, these two signals appear to have the same frequency even though the second signal had a frequency 9 times that of the first one. Aliasing makes them seem alike, which can be misleading. The first signal is properly sampled and the recreation is accurate. The second one is a faulty measurement.

The extreme cases also showed what we predicted. The case in which the signal was sampled at its own frequency led to straight lines, which makes sense because the sample would occur at the same point in the signal's period every time. The case in which the signal

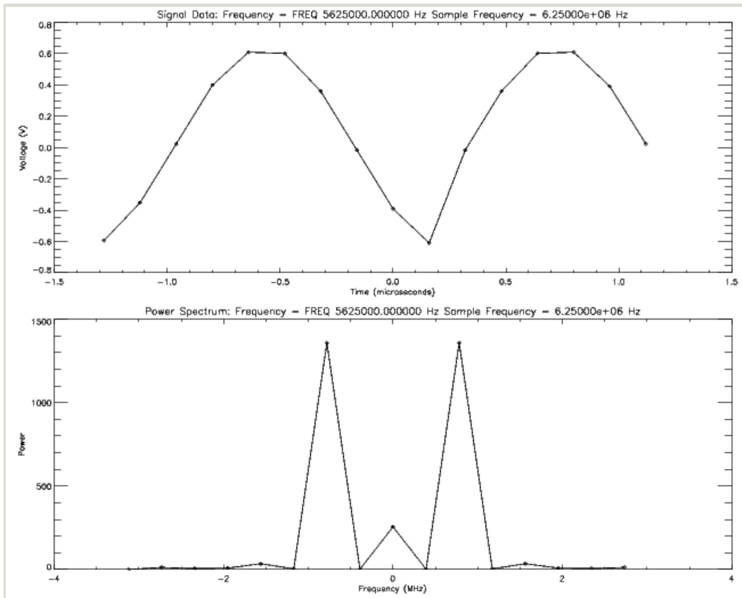

Fig. 2.— Signal was 0.9 times sample rate

was significantly undersampled yielded what appeared to be a low frequency signal, which was entirely due to aliasing. The recreation of this signal was not at all accurate since many periods were missed in between each sample.

3.2. Fourier Spectra and Analysis

The Fourier transforms of several samples were taken and examined. The voltage spectrum was complex, with positive and negative as well as both real and imaginary components. The voltage spectra appeared differently for different measurements of the same signal, which we attribute to differences in the perceived phase of the signal by the sampling. The power spectrum was all positive and real, since it is an absolute value. Power spectra were symmetric around $t = 0$; frequencies left of 0 were negative.

We filtered a square wave using the aforementioned zeroing technique. The filtering resulted in a very nice sine wave since the identification of the fundamental frequency in the power spectrum was quite accurate.

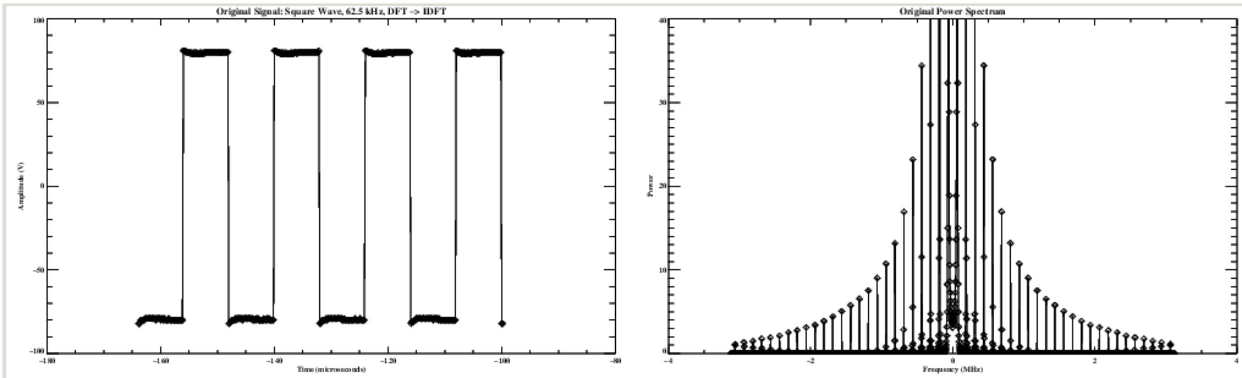

Fig. 3.— Unfiltered Square and Spectrum

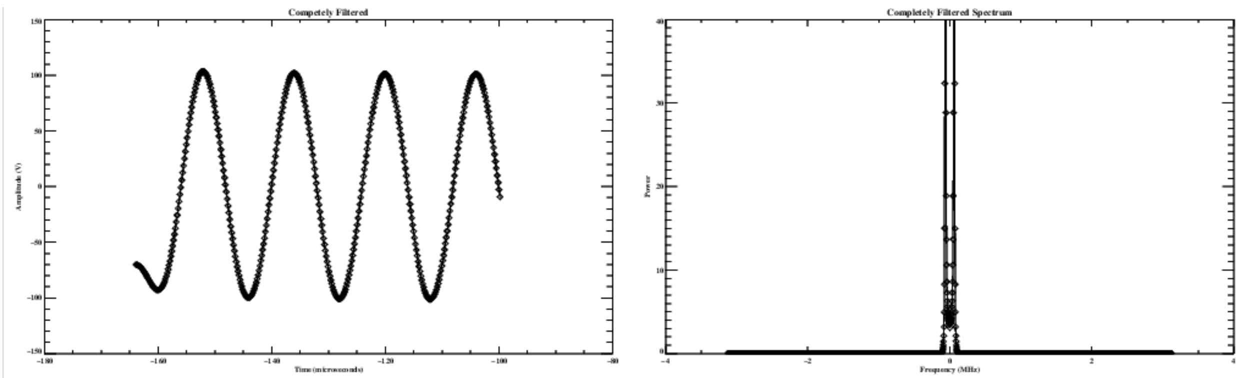

Fig. 4.— Filtered Square and Spectrum

3.3. Leakage Power and Frequency Resolution

When we investigated for leakage power, we found a multitude of non-zero elements to the power spectrum. Most of these had very small magnitude (around 0.5 compared to the fundamental at 1000), but they prove that the spectrum isn't zero everywhere besides fundamental frequencies.

We found that, in order to resolve two separate peaks in a spectrum, the peaks needed to be at least $2df$ away from each other. This makes sense because one peak must be in one bin and the other in another in order for them to be distinguishable, but there must be a bin in between them with much less of a peak, or else the two peaks will look like one wide peak. The central bin is essential to distinguishing the two peaks from each other.

3.4. Mixers and the Heterodyne Process

After mixing the 1 MHz signal with the 1.05 MHz signal, we found that we could filter the wave and recover the 50 kHz wave. In the original mixed signal, the 50 kHz wave looked like an amplitude modulation, and we were easily able to Fourier filter out the 2.05 MHz carrier wave just as we filtered the square wave into a sine.

Our mixing of the 10.5 MHz waves to get both real and complex input to the `DFT` was quite interesting. The original mixing, with no intentional phase shift, led to what we would expect from normal mixing, as with the 1 MHz signal, for both the upper and lower sideband cases. As one can see in Figure 5, corresponding sets of positive and negative peaks

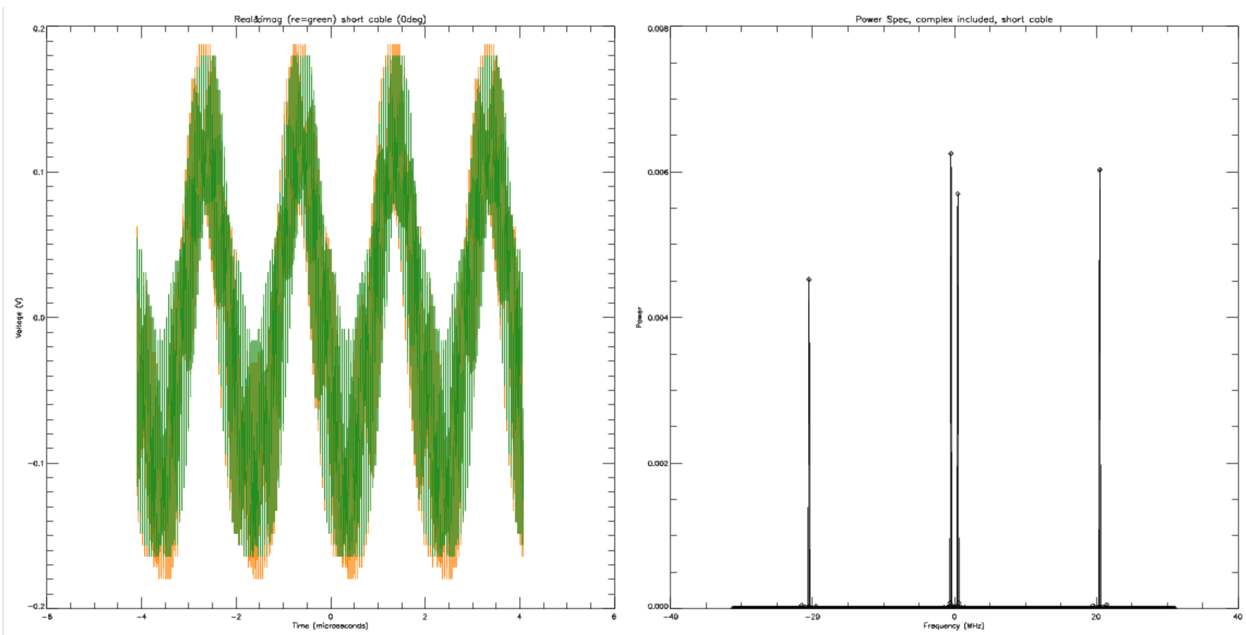

Fig. 5.— Upper sideband case with no phase shift

are about the same size. However, adding the phase shift made a big difference. Figure 6 and 7 show the upper and lower sideband cases with phase shifts. The power spectra now have distinct positive and negative peaks and the symmetry is broken. This means that we have found a way to distinguish the upper sideband from the lower.

Interestingly, the upper sideband case shows the right sum frequency but a more prevalent negative than positive difference frequency, which is the opposite of what we expected. The lower sideband case shows the inverse of this, so at least there is parallelism in what we do not understand. This remained an unsolved problem throughout the lab.

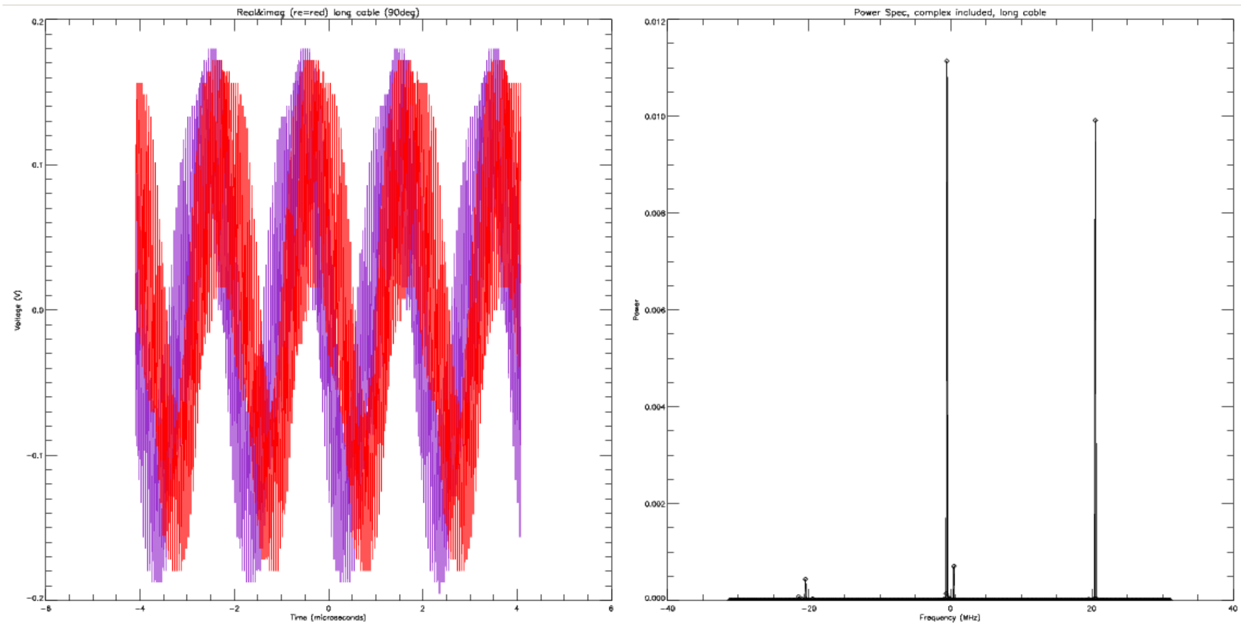

Fig. 6.— Upper sideband case with phase shift

4. Interpretation and Conclusion

We have determined the functionality of several basic signal processing techniques. We tested the Nyquist frequency and sampled around it to see what happened. We were not surprised when sampling below the Nyquist frequency results in an improper measurement of the wave.

We Fourier transformed waves using the `DFT` procedure and examined their spectra. Using the voltage spectra in frequency domain, we modified the signal's spectral content and were able to retrieve a different wave upon the inverse Fourier transform. This allowed us to Fourier filter a square wave in to a sine wave as well as filter out the sum frequencies from mixed signals in order to retrieve only the amplitude modulation frequencies.

We found that we need to have a *df* of no larger than half the size of the smallest difference we want to be able to see in peaks. This is our resolution and will determine what frequencies we can distinguish and which are lost in obscurity in the same bins.

Lastly, we used mixers to test our ability to separate a lower frequency wave from its high frequency carrier wave; this is how AM radio works, and also how radio astronomers reduce the frequency of signals they want to sample. We tested the effectiveness of a Sideband Separating Mixer (SSB) in which we used complex input to the `DFT` procedure to differentiate positive from negative frequencies and, by the same token, differentiate between the upper and lower sideband. While we did not get exactly the result we expected in this last

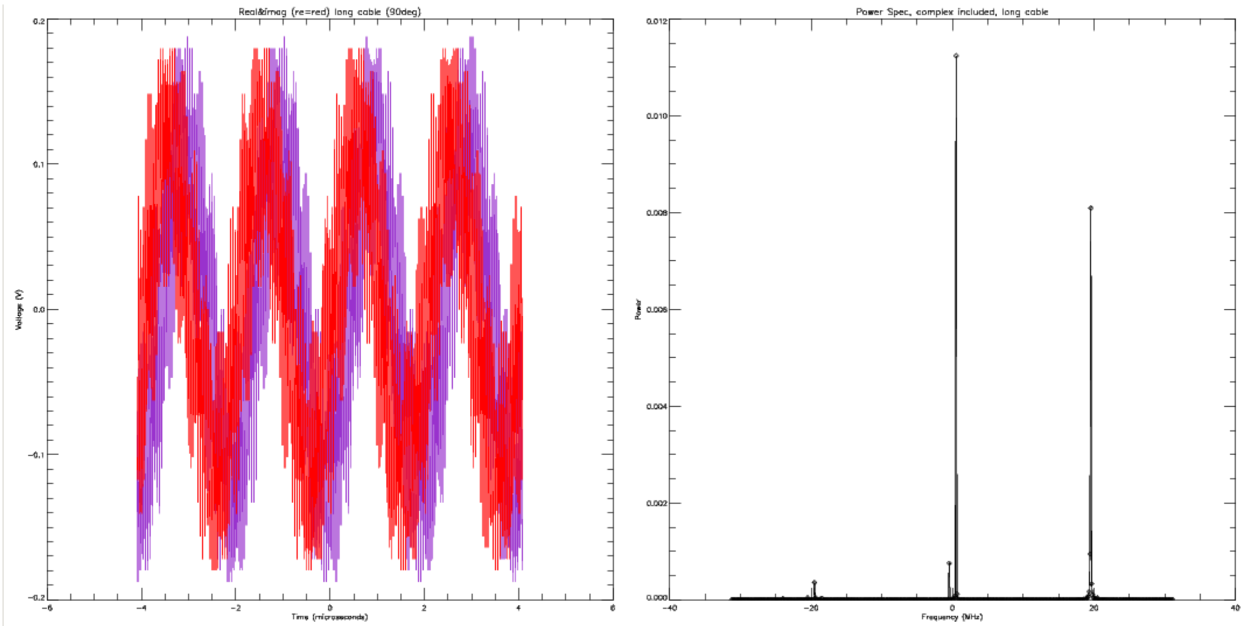

Fig. 7.— Lower sideband case with phase shift

experiment, it was the only significant anomaly of the entire lab.

We have covered the basics of signal processing. By now, we are confident in these abilities and are ready to put them to the test in more astronomical situations.