

Problem Description

A bus travels to N different stops, and at each stop some amount of people get in and get out. You are given an array A , where integer $A[i]$ gives the amount of people who have got onto the bus (if positive), or the amount of people who have gotten off the bus (if negative) at the stop i .

You are also given a positive integer B , that denotes the capacity of the bus, that is the maximum number of people the bus can hold. The bus can hold 0 to B number of people at any time.

Initially, the bus can have some number of people inside of it, you have to find the total number of possible ways of how many people were initially in the bus before the first stop, such that at any time there are always 0 to B number of people in the bus.

If it is not possible to find any valid number of ways, return 0.

Problem Constraints

$-10^6 \leq A[i] \leq 10^6$
 $1 \leq B \leq 10^9$
 $1 \leq |A| \leq 1000$

min sum of $a_i = -10^9$
 max sum of $a_i = 10^9$ Int range:

Ex: At any moment Bus should contain $[0 \dots B]$

$A = \begin{bmatrix} 2 & 4 & -1 & 3 \end{bmatrix}$ $B = 10$ # Total ways to start = 3
 $p = \begin{bmatrix} 2 & 6 & 5 & 8 \end{bmatrix}$

$\xrightarrow{\text{min}} 2$ $\xrightarrow{\text{max}} 8$
 $\xrightarrow{\text{mins}} 0$ $\xrightarrow{\text{maxs: } B - \text{maxs}} 2 : 10 - 8$: $\frac{\text{maxs} - \text{mins} + 1}{2 - 0 + 1 = 3}$

Ini
 0 : $\begin{bmatrix} 2 & 6 & 5 & 8 \end{bmatrix}$
 1 : $\begin{bmatrix} 3 & 7 & 6 & 9 \end{bmatrix}$
 2 : $\begin{bmatrix} 4 & 8 & 7 & 10 \end{bmatrix}$
 3 : $\begin{bmatrix} 5 & 9 & 8 & 11 \end{bmatrix}$ ← max of $\text{arr}[i] > 10$ Exceeding capacity *

$A = \begin{bmatrix} 2 & -1 & 3 & 3 \end{bmatrix}$ $B = 10$
 $p = \begin{bmatrix} 2 & 1 & 4 & 7 \end{bmatrix}$

$\xrightarrow{\text{min}} 1$ $\xrightarrow{\text{max}} 7$
 $\xrightarrow{\text{mins}} 0$ $\xrightarrow{\text{maxs: } B - \text{maxs}} 3 : 10 - 7$ = $\frac{\text{maxs} - \text{mins} + 1}{3 - 0 + 1 = 4}$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} -2 & -4 & 5 & 3 \end{bmatrix} \end{matrix} \quad B = 10$$

$$P = \begin{bmatrix} -2 & \underline{-6} & -1 & 2 \end{bmatrix} \rightarrow \begin{matrix} \min & \max \\ -6 & 2 \end{matrix}$$

$$\begin{matrix} S \\ 6: \end{matrix} A = \begin{bmatrix} -2 & -4 & 5 & 3 \end{bmatrix} \quad \begin{matrix} \min S: -\min \\ 6 \end{matrix} \quad \begin{matrix} \max S: B - \max \\ 10 - 2 = 8 \end{matrix} = \frac{\max S - \min S + 1}{3}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} -3 & 1 & 4 & -6 \end{bmatrix} \end{matrix} \quad B = 10$$

$$P = \begin{bmatrix} -3 & -2 & 2 & -4 \end{bmatrix} \rightarrow \begin{matrix} \min & \max \\ -4 & 2 \end{matrix}$$

$$\begin{matrix} \min S: -\min \\ 4 \end{matrix} \quad \begin{matrix} \max S: B - \max \\ 8 \end{matrix} \quad \begin{matrix} \max S - \min S + 1 \\ 5 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 2 & 3 & -1 & 2 \end{bmatrix} \end{matrix} \quad B = 10$$

$$P = \begin{bmatrix} 2 & 5 & 4 & 6 \end{bmatrix} \rightarrow \begin{matrix} \min & \max \\ 2 & 6 \end{matrix}$$

$$\begin{matrix} \min S: -\min \\ -2 \rightarrow 0 \end{matrix} \quad \begin{matrix} \max S: B - \max \\ 4 \end{matrix} \quad \begin{matrix} \max S - \min S + 1 \\ 4 - 0 + 1 = 5 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} -3 & 1 & -4 & 2 \end{bmatrix} \end{matrix} \quad B = 10$$

$$P = \begin{bmatrix} -3 & -2 & -6 & -4 \end{bmatrix} \rightarrow \begin{matrix} \min & \max \\ -6 & -2 \end{matrix}$$

$$\begin{matrix} \min S: -\min \\ 6 \end{matrix} \quad \begin{matrix} \max S: B - \max \\ 10 - (-2) = 12 > \text{limit} = 10 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{bmatrix} -4 & -2 & 3 & -1 \end{bmatrix} & B = 10 \end{matrix}$$

$$P = \begin{bmatrix} -4 & -6 & -3 & -4 \end{bmatrix} \rightarrow \begin{matrix} \min & \max \\ -6 & -3 \end{matrix}$$

$$\rightarrow \begin{matrix} \mins: -\min & \maxs: B - \max \\ 6 & 10 - (-3) = 13 > \text{Limit} = 10 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{bmatrix} -2 & 3 & -6 & 9 \end{bmatrix} & B = 4 \end{matrix}$$

$$P = \begin{bmatrix} -2 & 1 & -5 & 4 \end{bmatrix} \rightarrow \begin{matrix} \min & \max \\ -5 & 4 \end{matrix}$$

$$\rightarrow \begin{matrix} \mins: -\mins & \maxs: B - \max \\ 5 & 0 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 \\ \begin{bmatrix} 2 & 3 & 5 & 1 \end{bmatrix} & B = 6 \end{matrix}$$

$$P = \begin{bmatrix} 2 & 5 & 10 & 11 \end{bmatrix} \rightarrow \begin{matrix} \min & \max \\ 2 & 11 \end{matrix}$$

$$\rightarrow \begin{matrix} \mins & \maxs: B - \max : \\ 0 & -5 & 0 \end{matrix}$$

```
int Bus (int ar[]) {
```

```
    int n = ar.length;
```

```
    int pf[n];
```

```
    pf[0] = ar[0]
```

```
    int min = pf[0], max = pf[0];
```

```
    for (i = 1; i < n; i++) {
```

```
        pf[i] = pf[i-1] + ar[i]
```

```
        if (min > pf[i]) {
```

```
            min = pf[i]
```

```
        } if (max < pf[i]) {
```

```
            max = pf[i]
```

```
        }
```

```
    int mins = -1 * min; } 2 Cases
```

```
    if (mins < 0) {
```

```
        mins = 0
```

```
    }
```

```
    int mans = B - max; } 2 Cases
```

```
    if (mans > B) {
```

```
        mans = B
```

```
    }
```

```
    if (mins > B || mans < 0) // We cannot start
```

```
        return 0
```

```
    }
```

```
    return mans - mins + 1
```

```
}
```

Max Submatrix Sum

Given a row-wise column wise sorted matrix return max Submatrix Sum

TL BR

Ex1:

	0	1	2	3
0	-20	-16	-4	8
1	-10	-8	12	14
2	-1	6	21	30
3	5	7	28	42

$$\rightarrow -8 + 12 + 14 + 6 + 21 + 30 + 7 + 28 + 42 = \underline{\underline{152}}$$

Ex2:

	0	1	2	3
0	-20	-16	-4	-1
1	-10	-8	-2	5
2	-4	2	4	8

$$\rightarrow -2 + 5 + 4 + 8 = \underline{\underline{15}}$$

Ex3:

	0	1	2
0	-50	-40	-30
1	-35	-20	-15
2	-19	-14	-3

$$\rightarrow \underline{\underline{-3}}$$

Idea: Say $mat[N][M]$

obs1: BR of max Submatrix is $\underline{N-1} \underline{M-1}$

obs2: TL of max Submatrix can be anywhere

so, try every cell as TL.

```
int maxSum(int mat[N][M]) { TC: O(N*M) SC: O(N*M)
```

```
int ans = INT_MIN
```

```
Construct pfmat[N][M]. —————> TC: O(N*M)
```

```
i = 0; i < M; i++) { —————> TC: O(N*M) * 1
```

```
    j = 0; j < N; j++) {
```

```
        // Top left: [i, j] Bottom right: [N-1, M-1]
```

```
        // For above submatrix get sum.
```

```
        { Using pfmat[N][M] get the sum }
```

```
    }
```

```
}
```

Max Subsequence Sum

Given $arr[N]$, calculate sum of max of every subsequence

$$A[] = \begin{array}{ccc} 0 & 1 & 2 \\ \hline 2 & 5 & 3 \end{array}$$

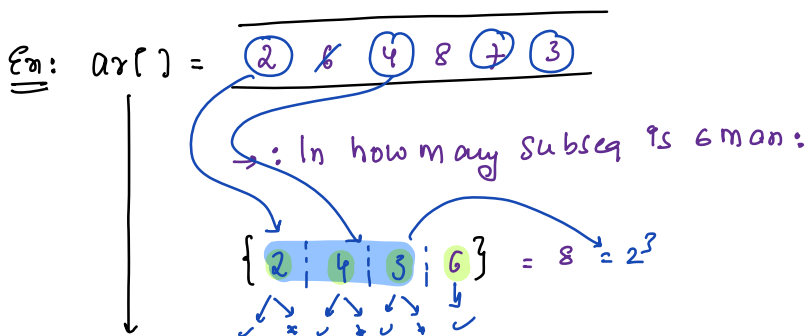
All Sub: \rightarrow max

{ }	0	Sum of all sub = 28.
{ 2 }	2	In how many subseq 2 is max = 1 = $2 \times 1 = 2$
{ 5 }	5	In how many subseq 5 is max = 2 = $3 \times 2 = 6$
{ 3 }	3	In how many subseq 3 is max = 4 = $5 \times 4 = 20$
{ 2 5 }	5	
{ 5 3 }	5	
{ 2 3 }	3	
{ 2 5 3 }	5	

Idea:

For every number get its contribution & add in final.

$$= arr[i] \times \{ \text{count of subseqs in which } arr[i] \text{ is max} \}$$



Ex: $arr[] = [2, 6, 4, 3, 5, 7, 8]$

\rightarrow In how many subseq is 6 max

{ 2, 4, 3, 5, 6 }

4 elements \rightarrow sub: $2^4 = 16$

Ex: $arr[] =$

-3 6 7 2 10

\Rightarrow In how many subseq is 6 max

{ -3 2 6 }

2 ele \rightarrow Sub: $2^2 = 4$

Idea:

For every number get its contribution & add in final.

$= arr[i] \times \{ \text{count of Subseqs in which } arr[i] \text{ is max} \}$

{get count of ele $< arr[i]$ in $arr[] = x$ }

{count of Subseq in which $arr[i]$ max = 2^x }

Solve: For every ele get count of ele $< arr[i]$?

\downarrow

$arr[5] = \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ \{ & 10 & 3 & 5 & -1 & 6 & 3 \end{matrix}$

$arr.sort() = \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ \{ & -1 & 3 & 5 & 6 & 10 \end{matrix}$
in inc

ele $< = \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix}$

$arr[5] = \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ \{ & 6 & 11 & 2 & 4 & 12 \end{matrix}$

$arr.sort() = \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ \{ & 2 & 4 & 6 & 11 & 12 \end{matrix}$

ele $< = \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix}$


```
int manSub(int arr[])
```

```
Arrays.sort(arr) // sort given arr[] in inc order
```

```
int ans = 0;
```

```
for (i = 0; i < n; i++) {
```

```
    // ele = arr[i]
```

```
    int less = i; // ele less than arr[i] = i
```

```
    int count = less * 2^i // In how many sub arr[i] is max
```

```
    int contri = count * arr[i]
```

```
    ans = ans + contri
```

```
}  
return ans;
```

Dry run:

A[] =

0	1	2
2	3	2

According to Code: $n = 3$

{ }	0
{ 2 }	2
{ 3 }	3
{ 2 }	2
{ 2 3 }	3
{ 3 2 }	3
{ 2 2 }	2
{ 2 3 2 }	3

2 2 3

ans = 0

for (i = 0; i < 3; i++) ans = ans + 2ⁱ * arr[i]

i = 0: ans = ans + 2⁰ * 2 = ans = 2

i = 1: ans = ans + 2¹ * 3 = ans = 6

i = 2: ans = ans + 2² * 2 = ans = 18

man : 18

✓ Sum of max of every Sub: ✓

✓ Sum of min of every Sub: TODD ✓

Sum of (max - min) in Sub: ✓ Sum of max of Subseq -
Sum of min of Subseq?