Todays <u>Content</u>:

Modular Arthmetics

Basic And /OR principle

Intro to Subsets vs Subsequences

Subsequence with given sum

Modular Arthimettcs

$$\text{min} \quad \text{man}$$

$$\left.\begin{array}{r}\text{any}\\\text{number}\end{array}\right\} \% \, M = \{0, \quad M-1\}$$

$$\boxed{(a+b) \% m = (a \% m + b \% m) \% m}$$

$$\boxed{(a * b) \% m = (a \% m * b \% m) \% m}$$

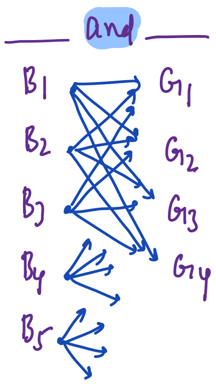
$$\underline{\underline{a \quad b}} \quad m \qquad \boxed{(a-b) \% m = (a \% m - b \% m + m) \% m}$$

$$14 \quad 7 \quad 4 \qquad \frac{(14-7) \% 4}{3} = \frac{(14 \% 4 - 7 \% 4 + 4) \% 4}{(2 - 3 + 4) \% 4 = 3 \% 4 =}$$

$$11 \quad 7 \quad 3 \qquad \frac{(11-7) \% 3}{1} = \frac{(11 \% 3 - 7 \% 3 + 3) \% 3}{(2 - 1 + 3) \% 3 = 4 \% 3 = 1}$$

And/OR Principle

Q1. 5 girs & 4 boys, how many pair : 5*4 = 20 pairs



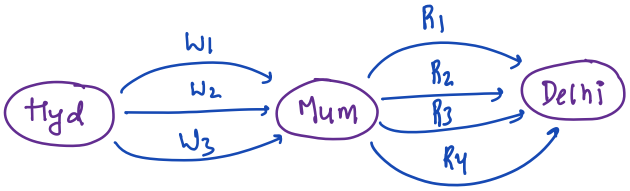
Idea: Pick 1 boy from 5 boys : 5

and → *

Pick 1 girl from 4 girls : 4

20 ways

Q2:



ways to go from Hyd → Delhi via Mumbai

1. Ways Hyd → Mum and ways Mum → Delhi

$\boxed{3 \quad * \quad 4 = 12 \text{ ways}}$

Q3: 3 T/F, how many ways we can answer all of them.

Q1	Q2	Q3
T	T	T
F	F	F
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

ways to Q1 : 2 ways

and *

ways to Q2 : 2 ways

and *

ways to Q3 : 2 ways

= 8 ways

obs

1. and = X

2. OR = +

Subarrays vs Subsequence vs Subsets

Subarray : Continous part of an array.

Subsequence : Sequence obtained by deleting none or more ele from arr[]

1. Data should be arranged based on inc order of index
2. Empty Subsequence is also valid.

Eg:

$$arr[6] = \left\{\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 9 & 6 & 8 \end{array}\right\}$$

$$\{1\ 6\ 8\} \xrightarrow{\text{subseq}} = \{\times\ \times\ 1\ \times\ 6\ 8\}$$

$$\{\overset{3}{9}\ \overset{4}{6}\ \overset{0}{3}\} \xrightarrow{\text{order miss}} = \{3\ \times\ \times\ 9\ 6\ \times\}$$

$$\{\overset{1}{2}\ \overset{3}{9}\ \overset{5}{8}\} \xrightarrow{\text{subseq}} = \{\times\ 2\ \times\ 9\ \times\ 8\}$$

$$\{1\ 9\ 6\ 8\} \xrightarrow{\text{subseq}} = \{\times\ \times\ 1\ 9\ 6\ 8\}$$

$$\{2\ 1\ 9\ 6\} \xrightarrow{\text{subseq}} = \{\times\ 2\ 1\ 9\ 6\ \times\}$$

$$\{\ \} \xrightarrow{\text{subseq}} = \{\times\ \times\ \times\ \times\ \times\ \times\}$$

$$\{3\ 2\ 1\ 9\ 6\ 8\} \xrightarrow{\text{sub}} = \{3\ 2\ 1\ 9\ 6\ 8\}$$

Subarr[] vs Subsequence

$$arr[6] = \left\{\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 9 & 6 & 8 \end{array}\right\}$$

	Subarray:	Subsequence
{3 2 1}	Yes	Yes
{1 9 6 8}	Yes	Yes
{3 1 6 8}	No	Yes
{2 1 9}	Yes	Yes

Obs:

1. All subarrays are Subseqences Yes
2. All subsequen are Subarrays No

Properties of Subsequence

$arr[3] = \{\overset{0}{3}\ \ \overset{1}{1}\ \ \overset{2}{8}\} \xrightarrow{\text{sort[] in inc}} arr[] = \{\overset{0}{1}\ \ \overset{1}{3}\ \ \overset{2}{8}\}$

max	Sum	All Subsequences		All Subsequences	Sum	max
—	0	{ }	↔	{ }	0	—
3	3	{3}	↔	{3}	3	3
1	1	{1}	↔	{1}	1	1
8	8	{8}	↔	{8}	8	8
3	4	{3 1}	Order diff but data Same ↔	{1 3}	4	3
8	11	{3 8}	↔	{3 8}	11	8
8	9	{1 8}	↔	{1 8}	9	8
8	12	{3 1 8}	Order diff but data Same ↔	{1 3 8}	12	8

obs: If we sort arr[]

1. Order before & after sorting change
   But data remains same.

2. Sum of Subsequences
   Max of Subsequences
   Min of Subsequences

   } Are not effected by sorting

Subsets: Subsets obtained by deleting none or more ele from arr[]

1. Empty Subset is also valid.

$ar[3] = \{3 \quad 1 \quad 8\} \xrightarrow{sort[]} ar[] = \{1 \quad 3 \quad 8\}$

All Subsets		All Subsets
{ }	⟷	{ }
{3} {1} {8}	⟷	{1} {3} {8}
{3 1}	Order: Not Matter Data: Same	{1 3}
{1 8}	⟷	{1 8}
{3 8}	⟷	{3 8}
{3 1 8}	Order: Not Matter Data: Same	{1 3 8}

Count of Subsets & Subsequences

$$ar[3] = \{\overset{0}{6} \quad \overset{1}{7} \quad \overset{2}{4}\}$$

F T F T F T

Subseq / Subset = { 2 * 2 * 2 }

make choice at index 0 = 2
and *
make choice at index 1 = 2
and *
make choice at index 2 = 2 } = 8

{ } {6} {7} {4} {6 7} {7 4} {6 4} {6 7 4}

$ar[4] = \{7 \quad 2 \quad 9 \quad 5\} =$

$Seq/set = \{2 * 2 * 2 * 2\} = 2^4 = 16$

$ar[n] = \{a_1 \quad a_2 \quad a_3 \quad \ldots \quad a_n\}$

$\boxed{Seq/sub = \{2 * 2 * 2 * \ldots 2\} = 2^n}$

Q1) Given an ar[N], check if There enists a subsequence with sum = k

Constraints

$1 <= N <= 20$

$1 <= ar[i] <= 10^6$

En: ar[5] = { 2 5 3 11 7 }   (indices 0 1 2 3 4)

K = 20 : { 2 11 7 } True

k = 8 : { 5 3 } True

k = 9 : { 2 7 } True

K = 19 : { 5 3 11 } True

K = 50 : False

Idea: For all subsequence get sum & compare == k.

If There enists such sequence : Return True

If No such sequence : False

Generate all SubSeq:

→ delet an ele : 0

↳ select an ele : 1

A = [7 5 3]   (indices 0 1 2)

decimal	Bi 2	1	0		All sub
0	0	0	0	→	{ }
1	0	0	1	=	{ 7 }
2	0	1	0	=	{ 5 }
3	0	1	1	→	{ 7 5 }
4	1	0	0	=	{ 3 }
5	1	0	1	=	{ 7, 3 }
6	1	1	0	→	{ 5, 3 }
7	1	1	1	=	{ 7 5 3 }

$$A = \begin{array}{cccc} 0 & 1 & 2 & 3 \\ [2 & 6 & 4 & 5] \end{array}$$

decimal	3	2	1	0	subseq
0 –	0	0	0	0	{ }
1	0	0	0	1	{2}
2	0	0	1	0	{6}
.	0	0	1	1	{2 6}
.	0	1	0	0	{4}
	0	1	0	1	{2 4}
	0	1	1	0	{6 4}
	0	1	1	1	{2 6 4}
	1	0	0	0	{5}
	1	0	0	1	{2 5}
10	3 1 ar[3]	2 0	1 1 ar[1]	0 0	{6 5}
	1	0	1	1	{2 6 5}
	1	1	0	0	{4 5}
	1	1	0	1	{2 4 5}
15 -	1	1	1	1	{2 6 4 5}

Obs:

1) N = 3 : We have 8 sub = [0, 7], check 3 bits

2) N = 4 : We have 16 sub = [0, 15], check 4 bits

→ $\boxed{N : \text{Subseq} = 2^N = [0, 2^N - 1]}$ $\boxed{\text{Check } n \text{ bits}}$

```
boolean subseq(int arr[], int N, int k){   TC: O(2^N * N) = O(N*2^N)   SC: O(1)
    for(num = 0; num < (1<<N); num++){ // (1<<N) = 2^N
        int sum = 0;
        // For a num get requried subsequence.
        for( i = 0; i < N; i++){ // i is indicating bit pos
            if( (num >> i) & 1 == True){ // i^th bit pos in num is 1 or 0.
                sum = sum + arr[i]
            }
        }
        if( sum == k){ return True}
    }
    return False;
}
```

man N = 20.

$20 \times 2^{20} <= 10^8$: Yes

$$A = \begin{array}{ccc} 0 & 1 & 2 \\ [7 & 5 & 3] \end{array}$$: num = $\{0, 2^3 - 1\} = \{0\ 7\}$, i = [0, 2]

num:	S	2	1	0 ... i
0	S = 0	0	0	0
1	S = 7	0	0	1 {7}
2	S = 5	0	1 {5}	0
3	S = 12	0	1 {5}	1 {7}
4	S = 3	1 {3}	0	0
5	S = 10	1 {3}	0	1 {7}
6	S = 8	1 {3}	1 {5}	0
7	S = 15	1 {3}	1 {5}	1 {7}