Perception | **Filtering: a worked example**
Autonomous Mobile Robots

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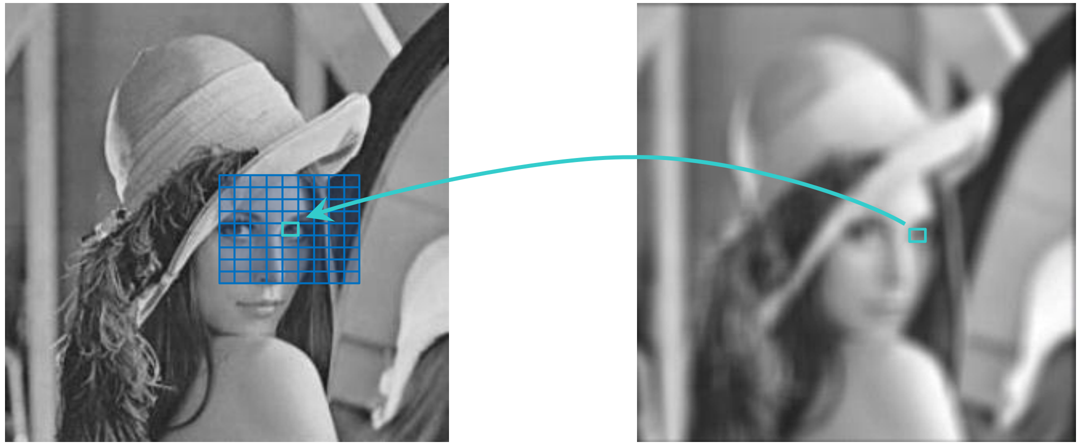
Correlation in 2D

$$F \circ I(x,y) = \sum_{j=-M}^{M} \sum_{i=-N}^{N} F(i,j) I(x+i, y+j)$$

- Example:
  Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

This example was generated with a 21x21 mask



Filtering | correlation in 2D

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  Constant averaging filter

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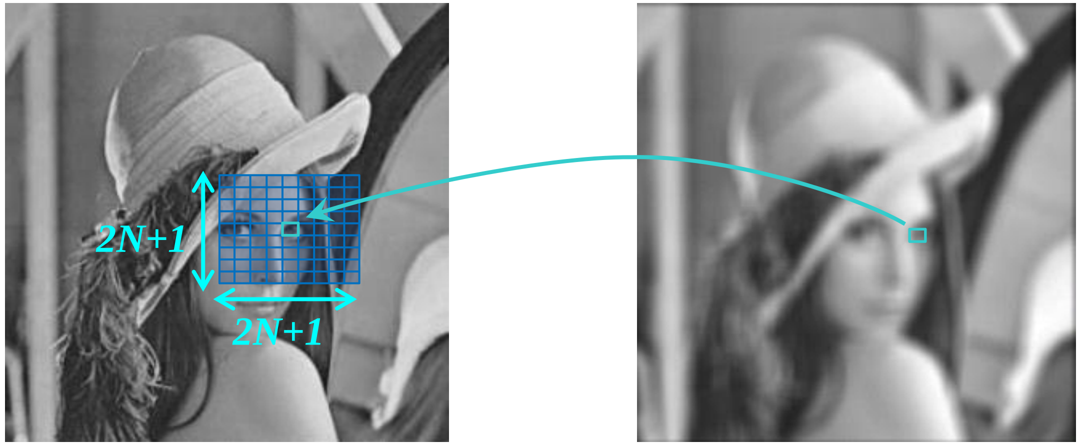
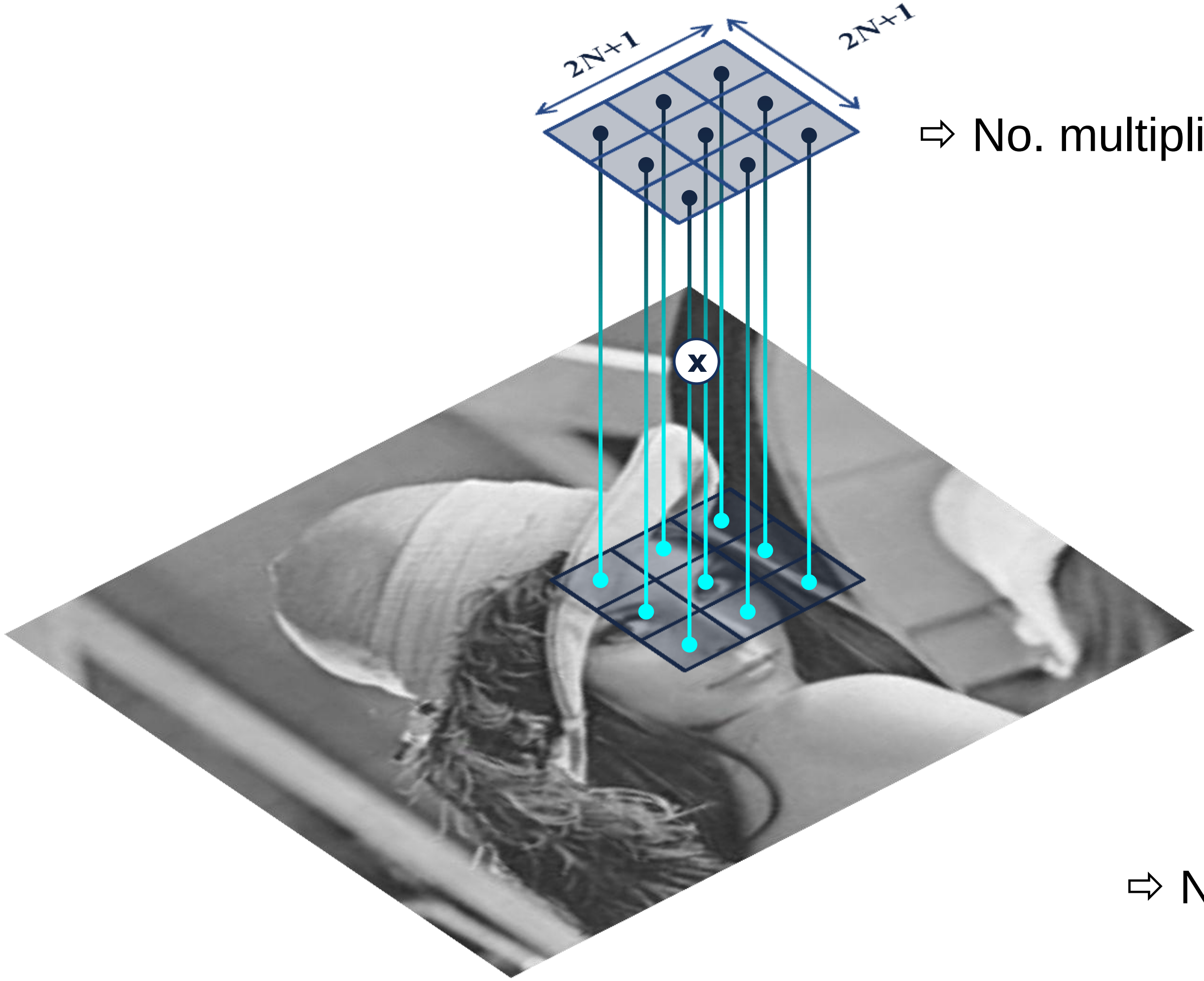
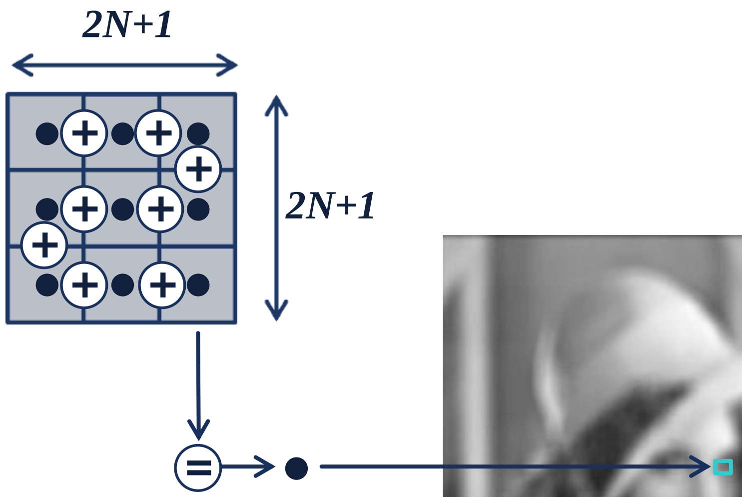


- If $size(F) = (2N+1)^2$ i.e. this is a square filter
- 2D Correlation ⇨ no. multiplications per pixel $=$ **?**
  no. additions per pixel $=$ **?**

Filtering | correlation in 2D



⇨ No. multiplications per pixel = $(2N+1)^2$



⇨ No. additions per pixel = $(2N+1)^2 - 1$

Filtering | correlation in 2D

$$F \circ I(x,y) = \sum_{j=-M}^{M} \sum_{i=-N}^{N} F(i,j) I(x+i, y+j)$$

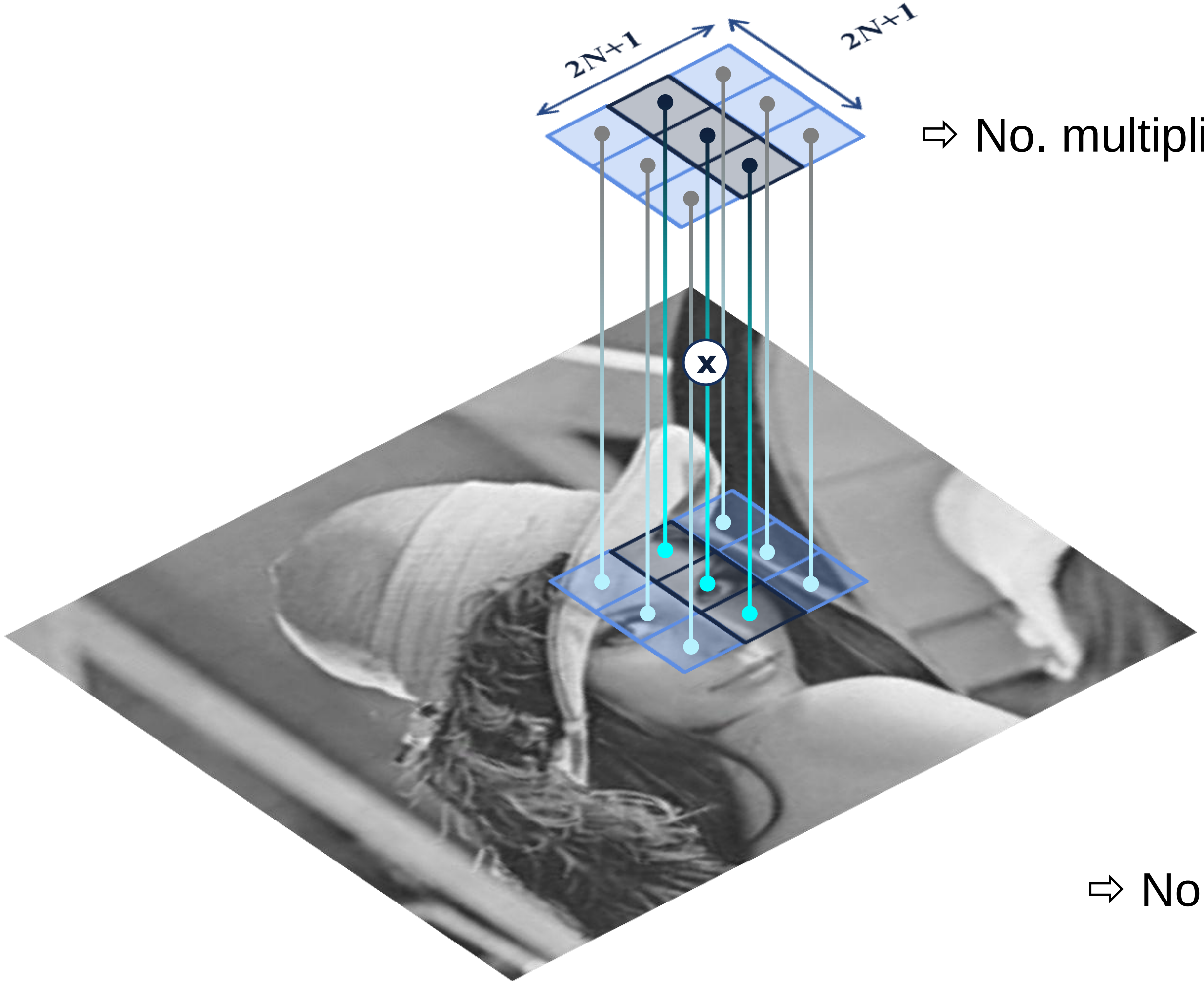
- Example:
  Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

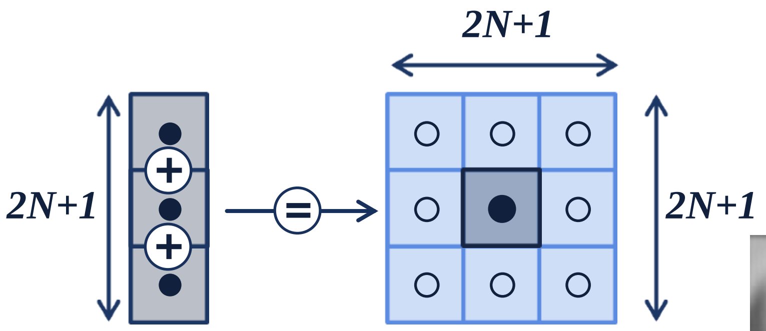
"separable" filter

- If $size(F) = (2N+1)^2$ i.e. this is a square filter
- 2D Correlation ⇨ no. multiplications per pixel $= (2N+1)^2$
  no. additions per pixel $= (2N+1)^2 - 1$
- 2 × 1D Correlation ⇨ no. multiplications per pixel = **?**
  no. additions per pixel = **?**

Filtering | correlation in 2D

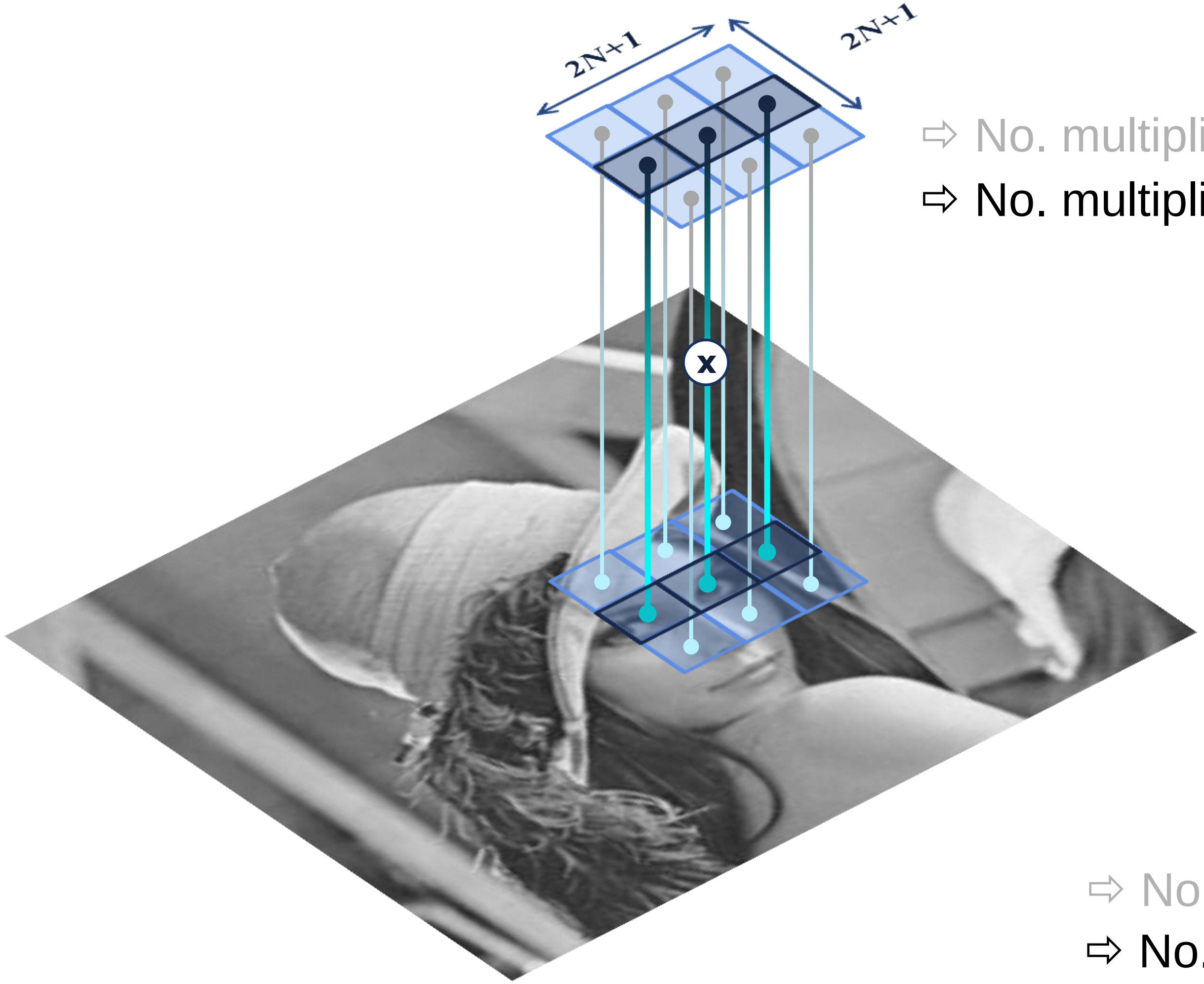


⇨ No. multiplications per pixel so far = $2N+1$



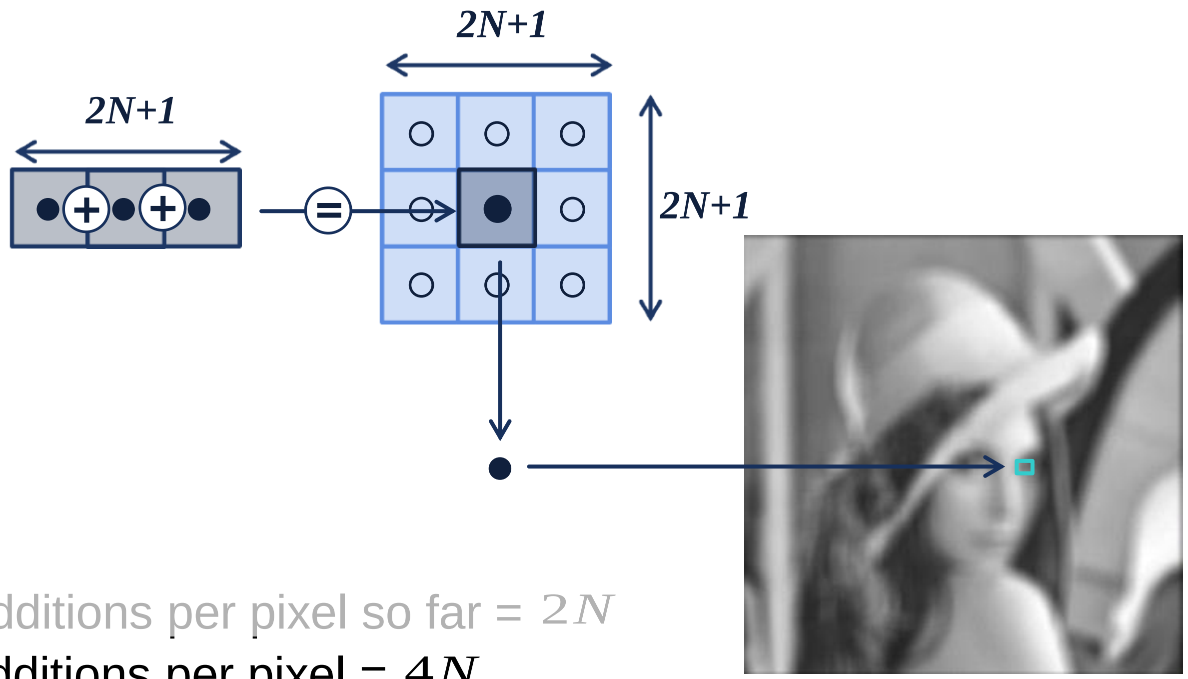
⇨ No. additions per pixel so far = $2N$

Filtering | efficient correlation in 2D



⇨ No. multiplications per pixel so far = $2N+1$

⇨ No. multiplications per pixel = $2(2N+1)$



⇨ No. additions per pixel so far = $2N$

⇨ No. additions per pixel = $4N$

Filtering | efficient correlation in 2D

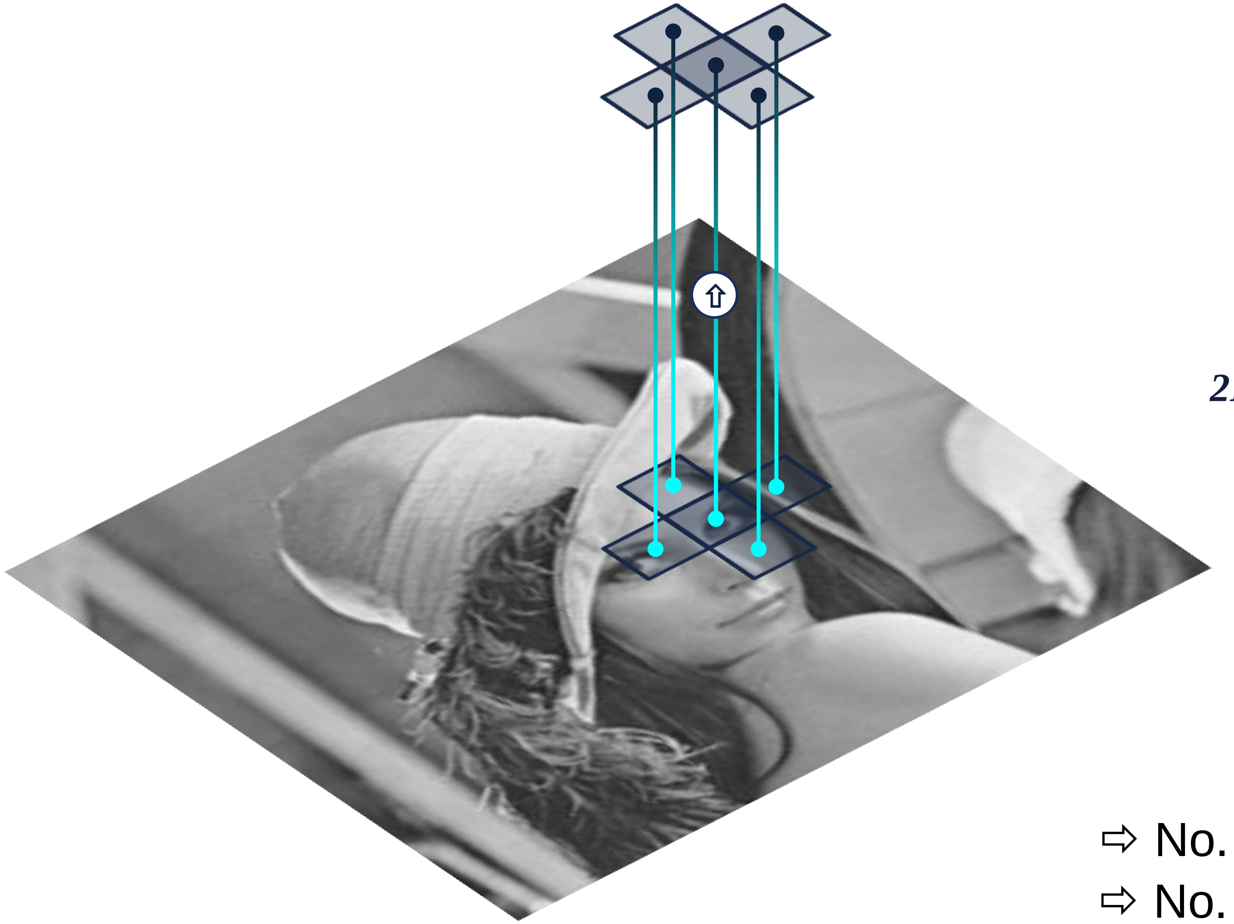
- Example:
  Constant averaging filter

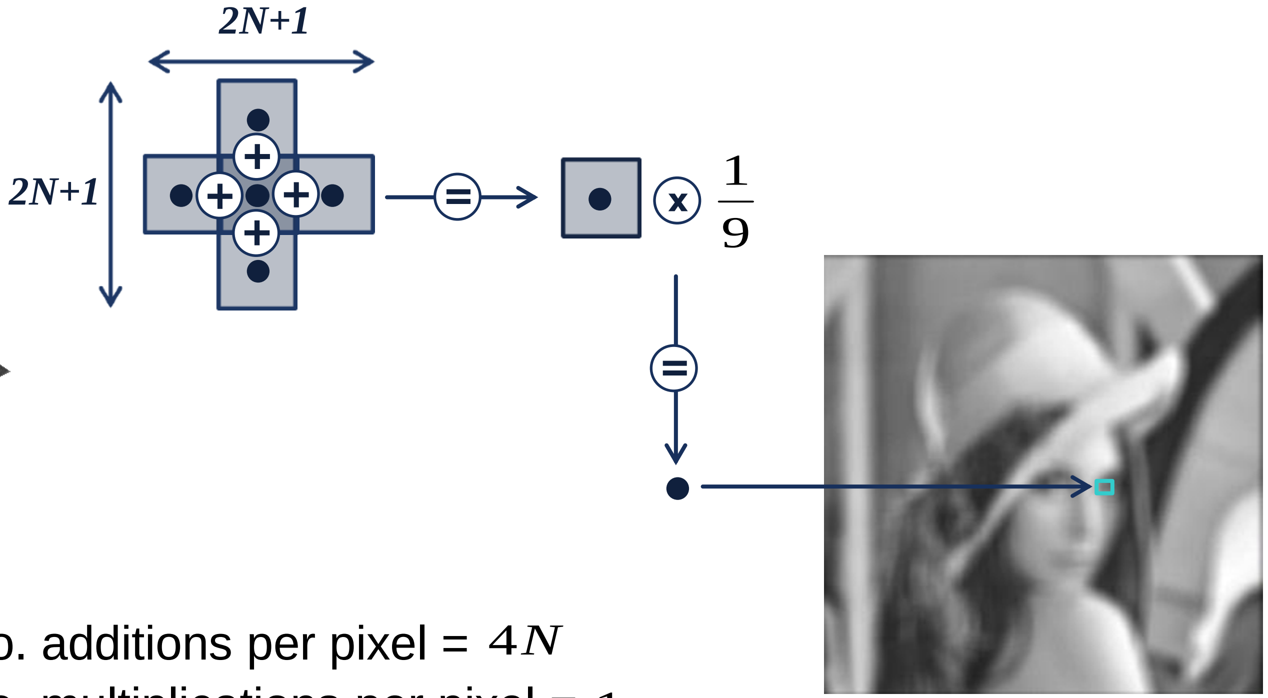
$$F=\begin{bmatrix}\frac{1}{9}&\frac{1}{9}&\frac{1}{9}\\\frac{1}{9}&\frac{1}{9}&\frac{1}{9}\\\frac{1}{9}&\frac{1}{9}&\frac{1}{9}\end{bmatrix}=\begin{bmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{1}{3}\end{bmatrix}\cdot\begin{bmatrix}\frac{1}{3}&\frac{1}{3}&\frac{1}{3}\end{bmatrix}=\frac{1}{9}\begin{bmatrix}1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}1&1&1\end{bmatrix}$$

"separable" filter

- If $size(F)=(2N+1)^2$ i.e. this is a square filter
- 2D Correlation ⇨ no. multiplications per pixel $=(2N+1)^2$
  no. additions per pixel $=(2N+1)^2-1$
- 2 × 1D Correlation ⇨ no. multiplications per pixel $=2(2N+1)$
  no. additions per pixel $=4N$
- 2 × 1D Correlation (with const. factor) ⇨ no. multiplications per pixel = ?
  no. additions per pixel = ?

Filtering | more efficient correlation in 2D





⇨ No. additions per pixel = $4N$

⇨ No. multiplications per pixel = 1

Filtering | more efficient correlation in 2D

- Example:
  Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

"separable" filter

- If $size(F) = (2N+1)^2$ i.e. this is a square filter
- 2D Correlation ⇨ no. multiplications per pixel $= (2N+1)^2$
  no. additions per pixel $= (2N+1)^2 - 1$
- 2 × 1D Correlation ⇨ no. multiplications per pixel $= 2(2N+1)$
  no. additions per pixel $= 4N$
- 2 × 1D Correlation ⇨ no. multiplications per pixel $= 1$
  (with const. factor) no. additions per pixel $= 4N$