Introduction to Robotics CSCI/ARTI 4530/6530

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09/20/2018

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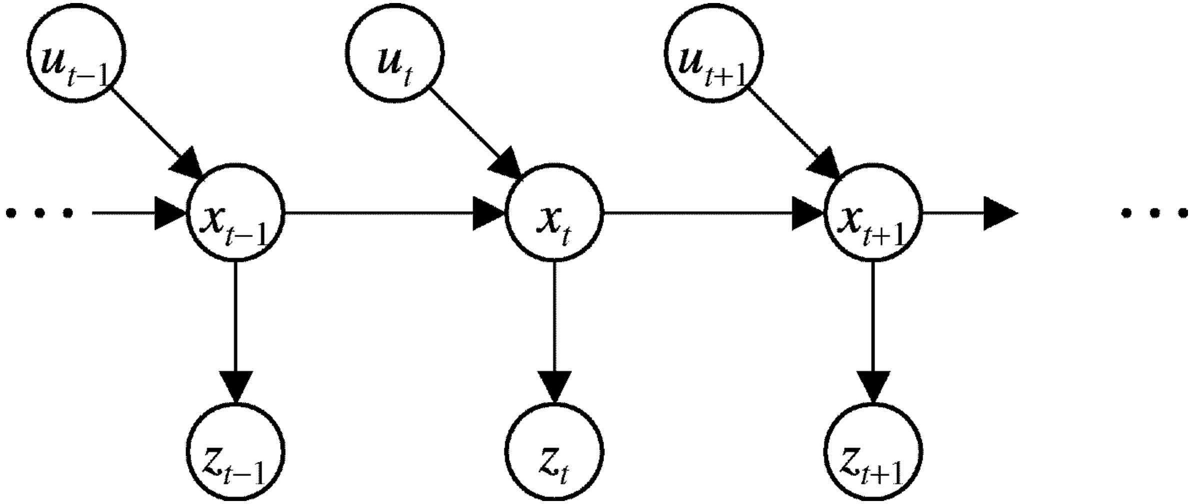
Agenda

- A quick recap on Markov localization

- For today
  - Kalman Filter Localization - continued

Markov Assumption



$$p(z_t \mid x_{0:t}, z_{1:t}, u_{1:t}) = p(z_t \mid x_t)$$

$$p(x_t \mid x_{1:t-1}, z_{1:t}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Underlying Assumptions

- Static world
- Independent noise
- Perfect model, no approximation errors

Bayes Filters

z = observation
u = action
x = state

$$\boxed{Bel(x_t)} = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes
$$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov
$$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob.
$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov
$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov
$$= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \, P(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) \; dx_{t-1}$$

$$\boxed{= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}}$$

Bayes Filter Algorithm

$$Bel(x_t) = \eta\ P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1})\ Bel(x_{t-1})\ dx_{t-1}$$

1. Algorithm **Bayes_filter**(*Bel(x),d*):
2. η=0
3. If *d* is a perceptual data item *z* then
4. For all *x* do
5. $$Bel'(x) = P(z \mid x) Bel(x)$$
6. $$\eta = \eta + Bel'(x)$$
7. For all *x* do
8. $$Bel'(x) = \eta^{-1} Bel'(x)$$
9. Else if *d* is an action data item *u* then
10. For all *x* do
11. $$Bel'(x) = \int P(x \mid u, x')\ Bel(x')\ dx'$$
12. Return *Bel'(x)*

Bayes Filters are Familiar!

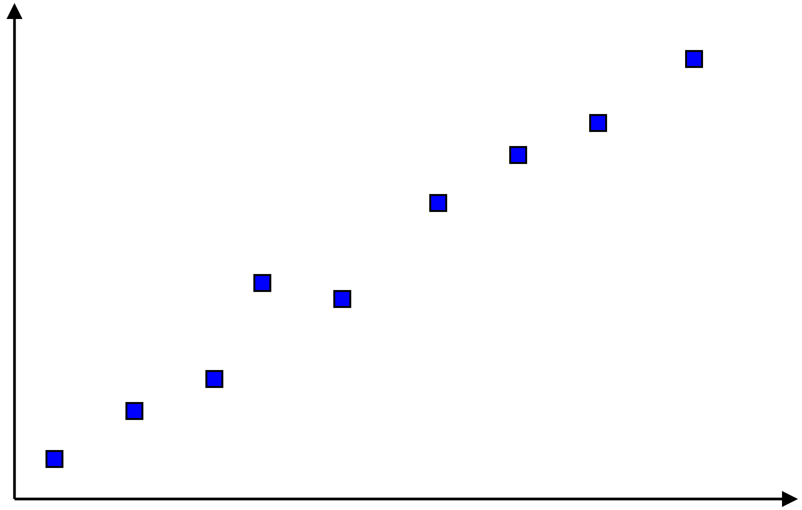
$$Bel(x_t) = \eta \ P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \ Bel(x_{t-1}) \ dx_{t-1}$$

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Representation of the Belief Function

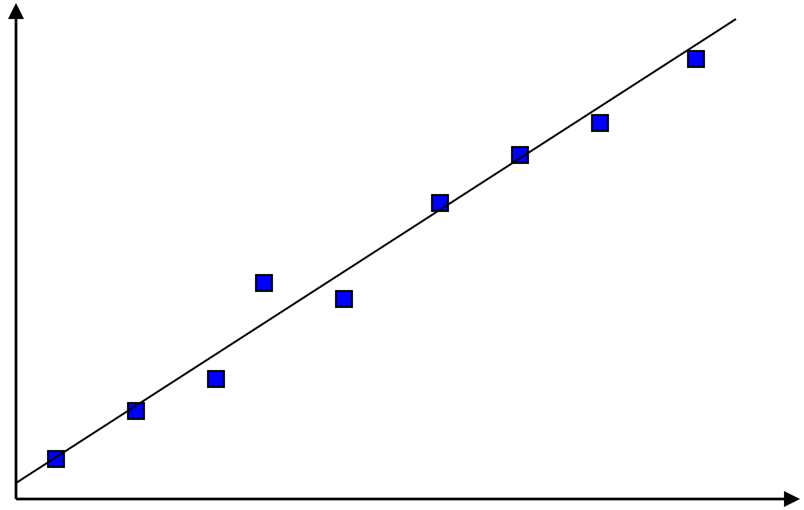
Sample-based representations

e.g. Particle filters



$$(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots (x_n, y_n)$$

Parametric representations

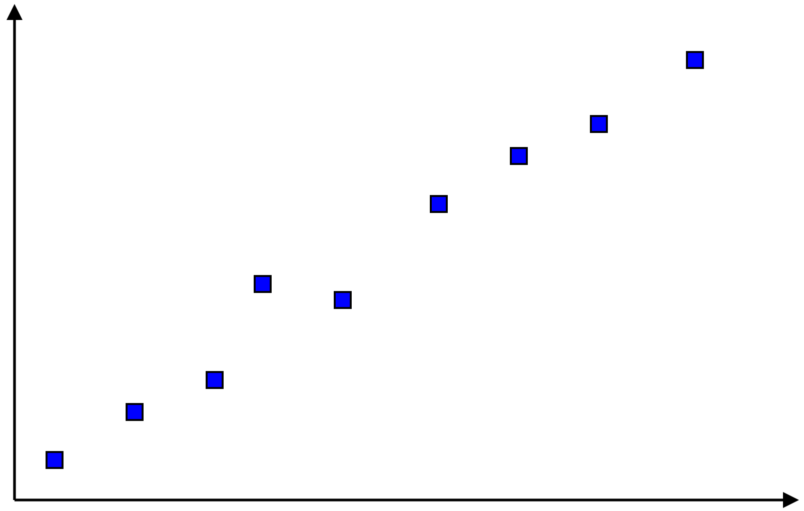


$$y = mx + b$$

Representation of the Belief Function

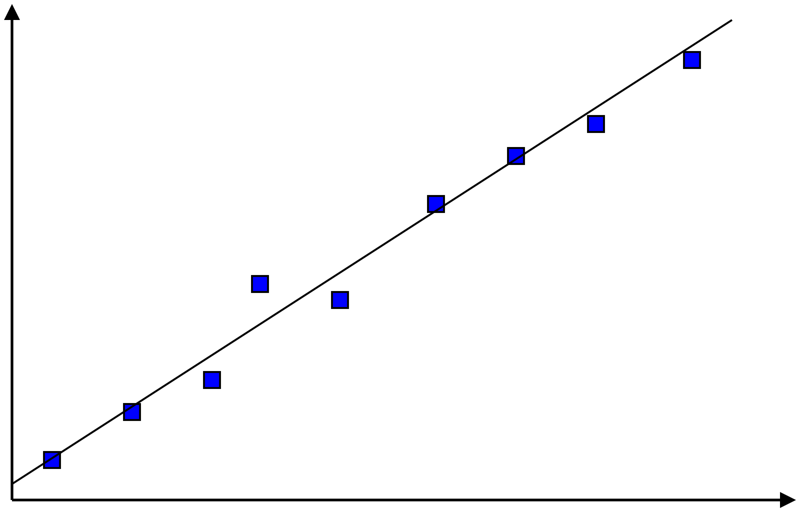
Sample-based representations

e.g. Particle filters



$$(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots (x_n, y_n)$$

Parametric representations



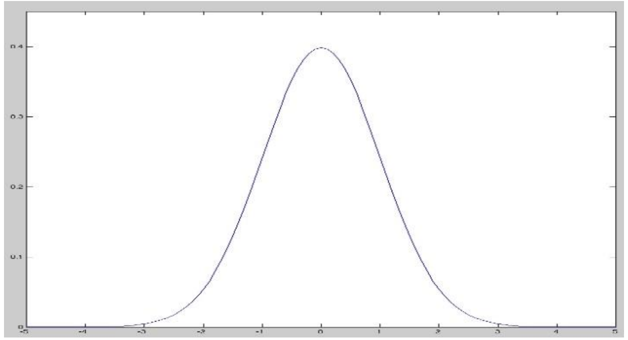
$$y = mx + b$$

Parameterized Bayesian Filter: Kalman Filter

Kalman filters (KF) represent posterior belief by a Gaussian (normal) distribution

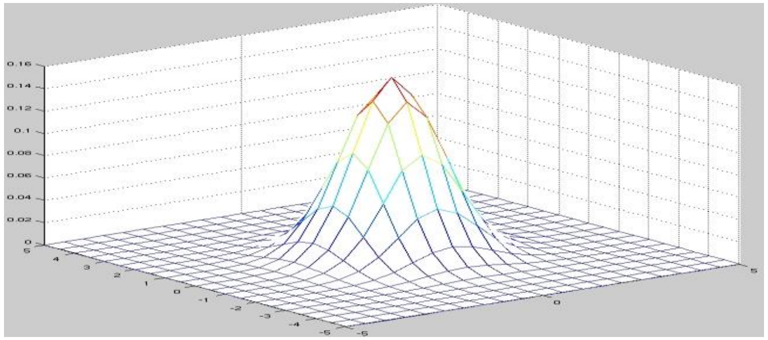
A 1-d Gaussian distribution is given by:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$



An n-d Gaussian distribution is given by:

$$P(x) = \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)}$$



For today – Kalman Filter Basics

Kalman Filter Model

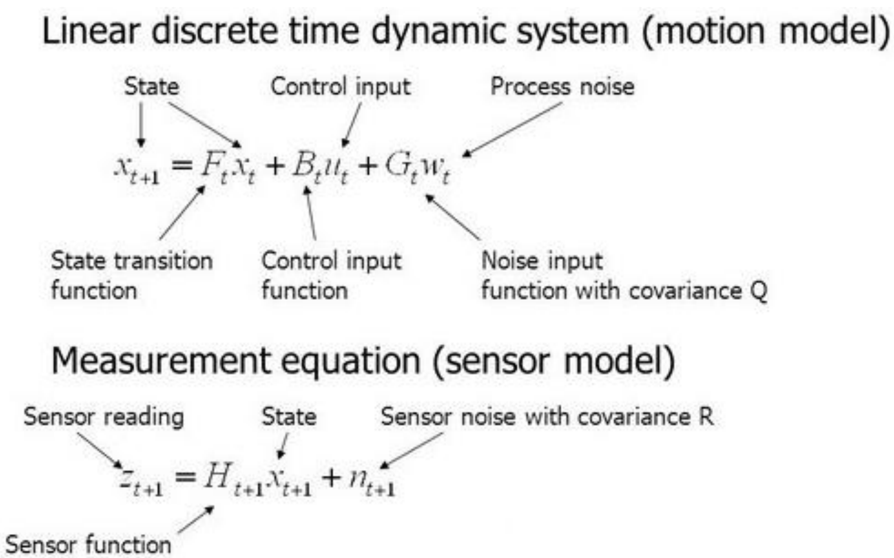


Image courtesy: Dr. Paul E. Rybski

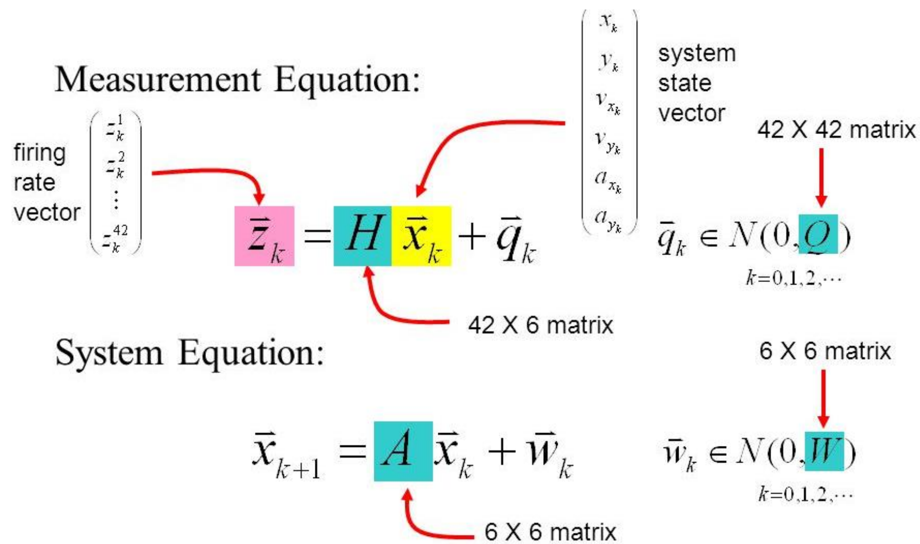
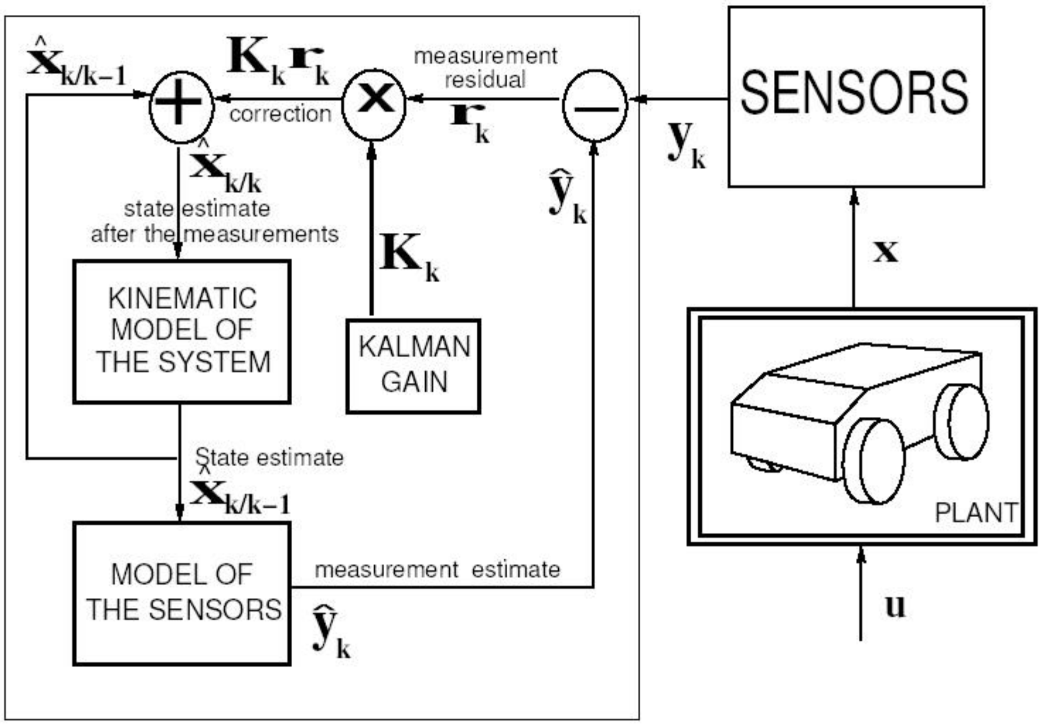


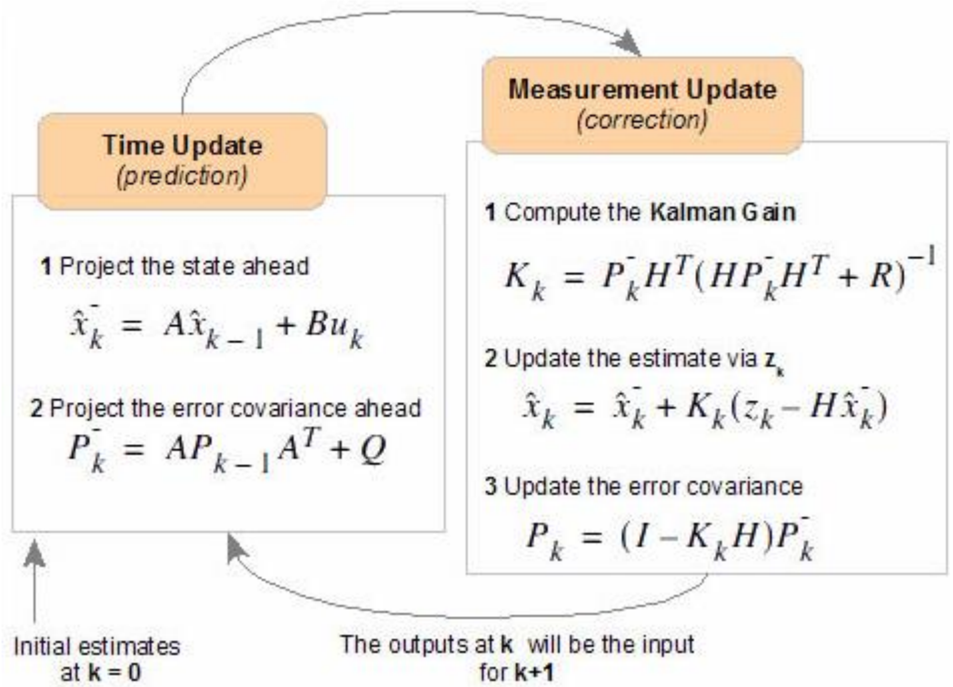
Image courtesy: Brown University

For today – Kalman Filter Basics

Kalman Filter Model



For today – Kalman Filter Basics



Kalman Gain (K) = Error in Estimate / (Error in Estimate + Measurement Error)

New Estimate = Old Estimate + K (Measurement - Old Estimate)

New Error in Estimate = (1 – K) (Old Error in Estimate)

Image courtesy: Bilgin Esme, Kalman Filter for Dummies

For today – Kalman Filter Basics

Propagation (motion model):

$$\hat{x}_{t+1/t} = F_t \hat{x}_{t/t} + B_t u_t$$

- State estimate is updated from system dynamics

$$P_{t+1/t} = F_t P_{t/t} F_t^{\,T} + G_t Q_t G_t^{\,T}$$

- Uncertainty estimate *GROWS*

Update (sensor model):

$$\hat{z}_{t+1} = H_{t+1} \hat{x}_{t+1/t}$$

- Compute expected value of sensor reading

$$r_{t+1} = z_{t+1} - \hat{z}_{t+1}$$

- Compute the difference between expected and “true”

$$S_{t+1} = H_{t+1} P_{t+1/t} H_{t+1}^{\,T} + R_{t+1}$$

- Compute covariance of sensor reading

$$K_{t+1} = P_{t+1/t} H_{t+1}^{\,T} S_{t+1}^{\,-1}$$

- Compute the Kalman Gain (how much to correct est.)

$$\hat{x}_{t+1/t+1} = \hat{x}_{t+1/t} + K_{t+1} r_{t+1}$$

- Multiply residual times gain to correct state estimate

$$P_{t+1/t+1} = P_{t+1/t} - P_{t+1/t} H_{t+1}^{\,T} S_{t+1}^{\,-1} H_{t+1} P_{t+1/t}$$

- Uncertainty estimate SHRINKS

For today – KF Localization

See the attached EdX slides