Introduction to Robotics CSCI/ARTI 4530/6530

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With slides courtesy of Prof. Sebastian Thrun et al.

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Announcements

- Assignment 2 announcement is delayed.
- Mid-term exam will be on Thursday (Oct 4).

Agenda

- A quick recap on EKF Localization based on GPS sensor

- For today
  - Particle Filter localization with an implementation example, if possible.

Bayes Filter

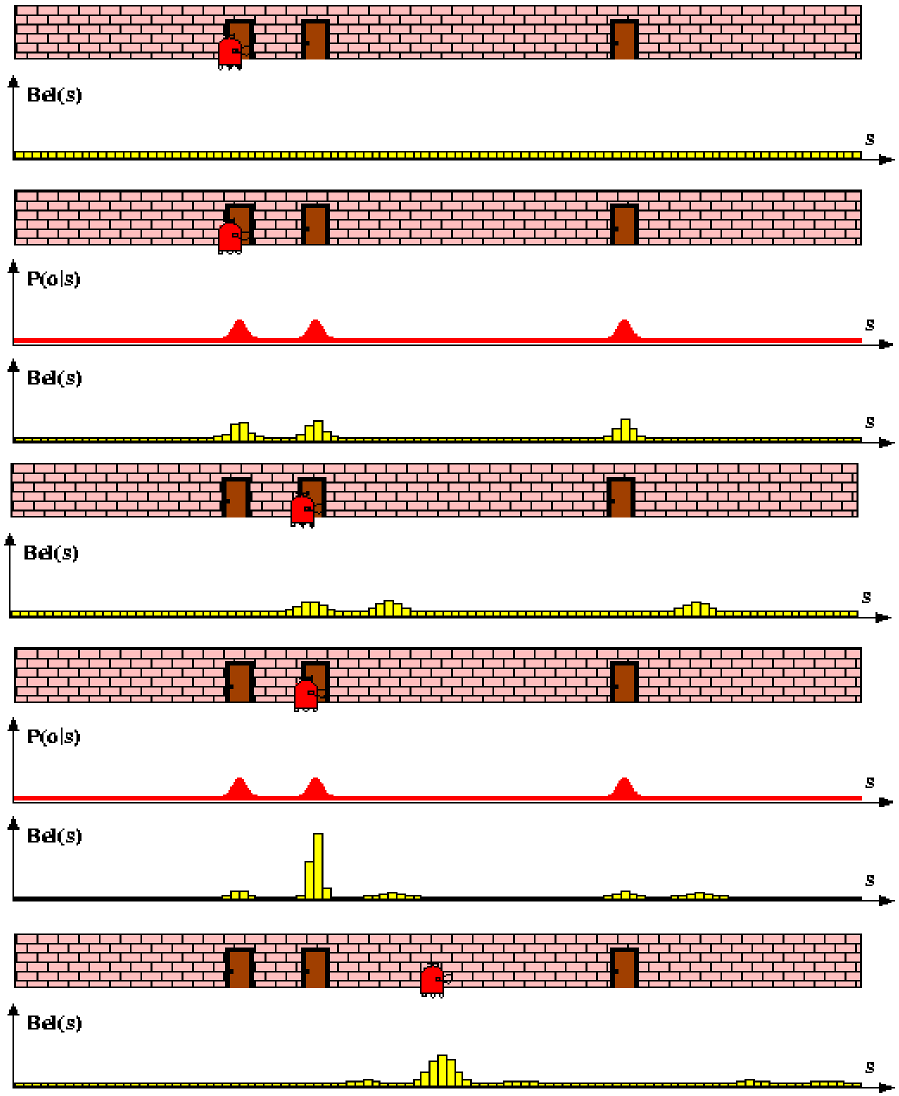
- What is Markov Assumption?
- How Bayesian Filter work?

Bayes Filter

Probabilistic localization

- Previous Belief State
- Motion model
- Prior Estimate
- Observation model
- Posterior Estimate

This process is repeated.



Bayes Filter Algorithm

1. Algorithm **Discrete_Bayes_filter**(*Bel(x),d*):
2. $\eta = 0$
3. If *d* is a perceptual data item *z* then
4. For all *x* do
5. $\boxed{Bel'(x) = P(z \mid x) Bel(x)}$
6. $\boxed{\eta = \eta + Bel'(x)}$

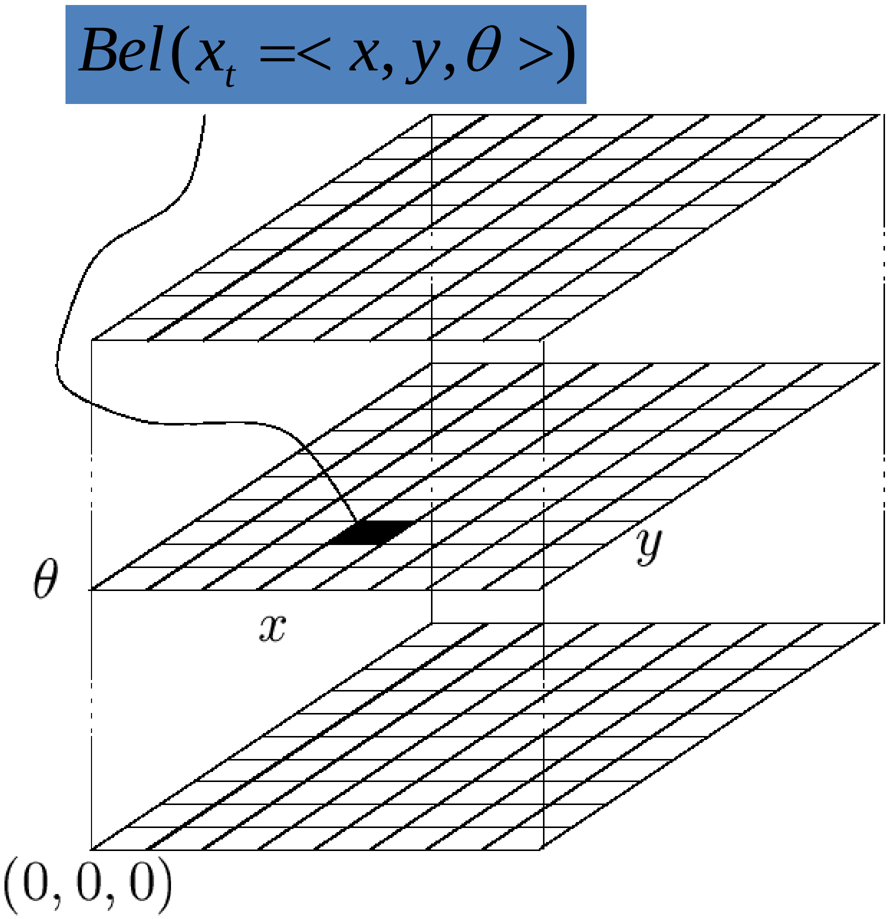
   Bayes Rule
7. For all *x* do
8. $Bel'(x) = \eta^{-1} Bel'(x)$

   Normalization
9. Else if *d* is an action data item *u* then
10. For all *x* do
11. $\boxed{Bel'(x) = \sum_{x'} P(x \mid u, x')\, Bel(x')}$

    Law of Total Probability
12. Return *Bel'(x)*

Grid-map Representation

Piecewise Constant



Bayes Filter Implementation

- To update the belief upon sensory input and to carry out the normalization one has to iterate over all cells of the grid.
- Especially when the belief is peaked (which is generally the case during position tracking), one wants to avoid updating irrelevant aspects of the state space.
- One approach is not to update entire sub-spaces of the state space.
- This, however, requires to monitor whether the robot is de-localized or not.
- **To achieve this, one can consider the likelihood of the observations given the active components of the state space.**

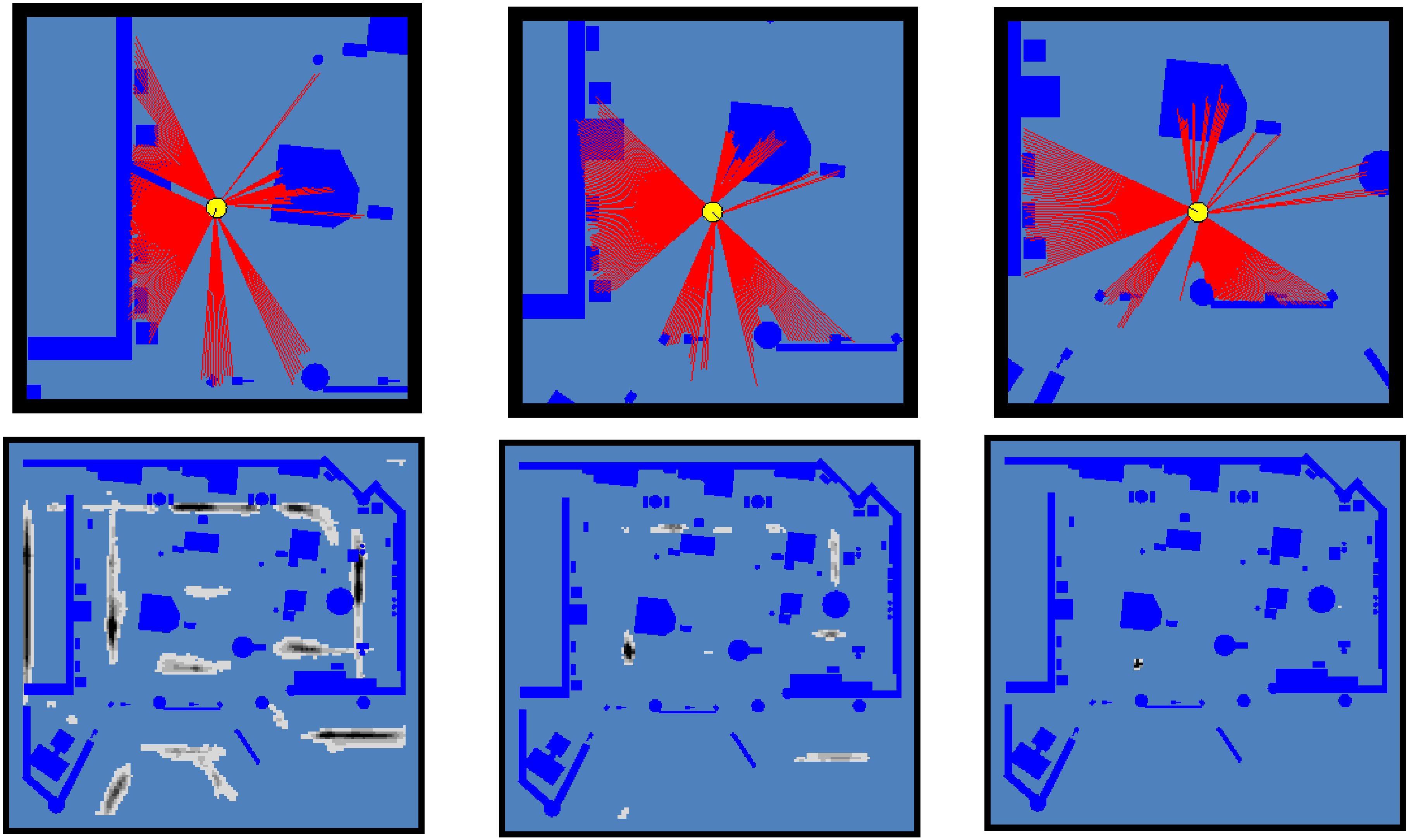
Bayes Filter Implementation

- Assume a bounded Gaussian model for the motion uncertainty.
- This reduces the update cost from $O(n^2)$ to $O(n)$, where n is the number of states.
- The update can also be realized by shifting the data in the grid according to the measured motion.
- In a second step, the grid is then convolved using a separable Gaussian Kernel.
- Two-dimensional example:

$$\begin{array}{|c|c|c|}\hline 1/16 & 1/8 & 1/16 \\ \hline 1/8 & 1/4 & 1/8 \\ \hline 1/16 & 1/8 & 1/16 \\ \hline \end{array} \cong \begin{array}{|c|}\hline 1/4 \\ \hline 1/2 \\ \hline 1/4 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 1/4 & 1/2 & 1/4 \\ \hline \end{array}$$

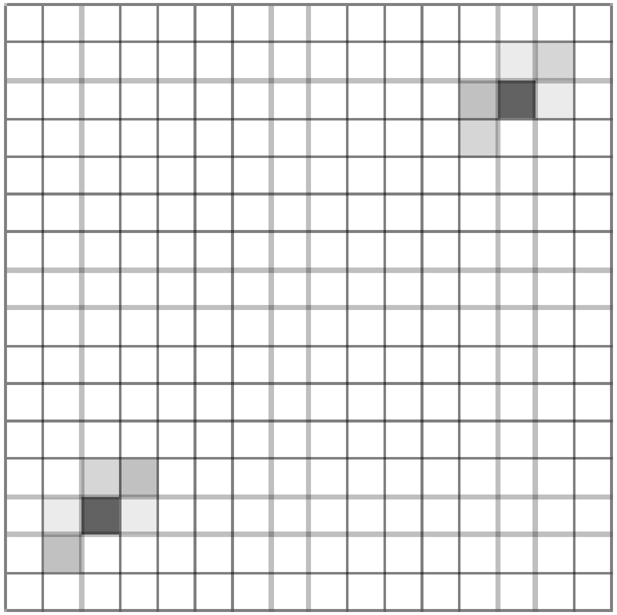
- Fewer arithmetic operations
- Easier to implement

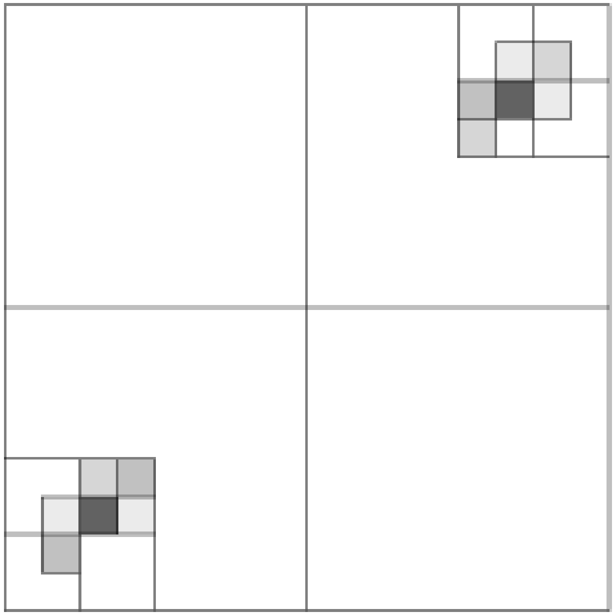
Grid based Localization



Tree-based Representation

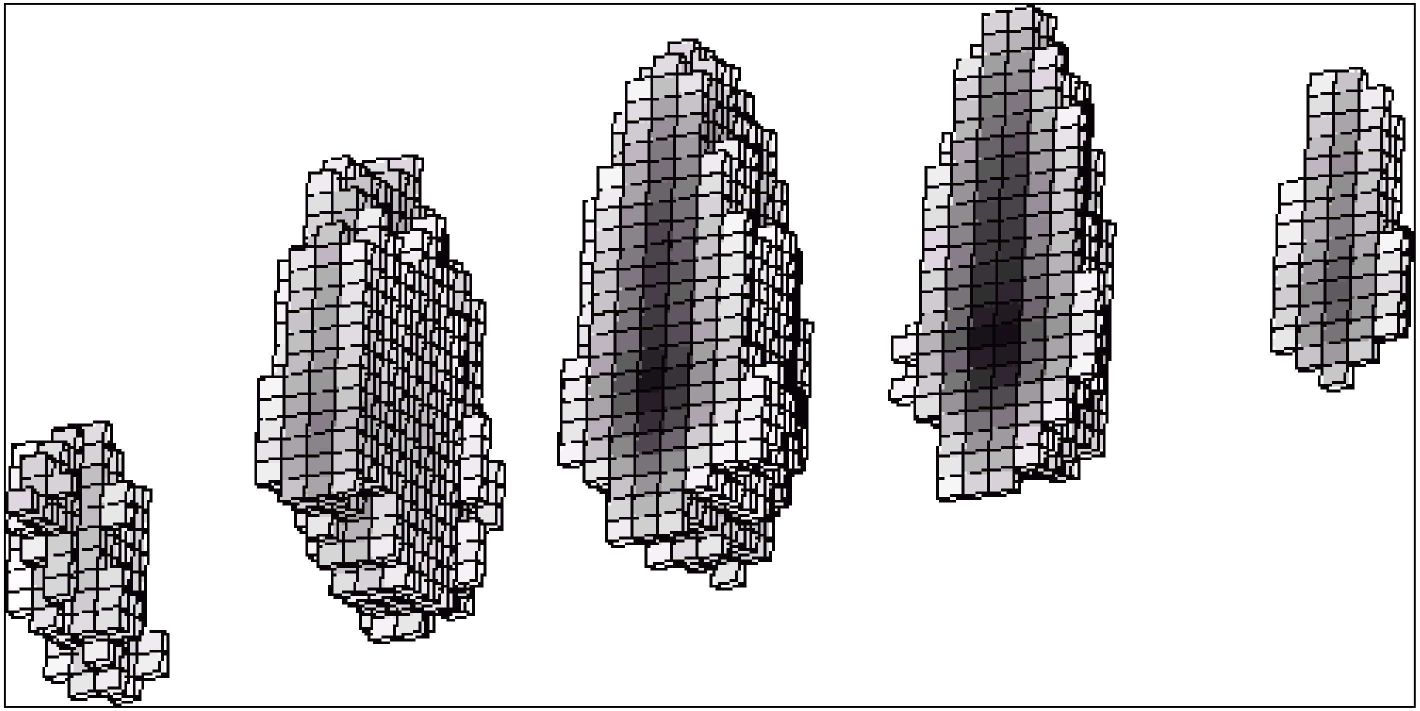
Idea: Represent density using a variant of octrees





Tree-based Representations

- Efficient in space and time
- Multi-resolution



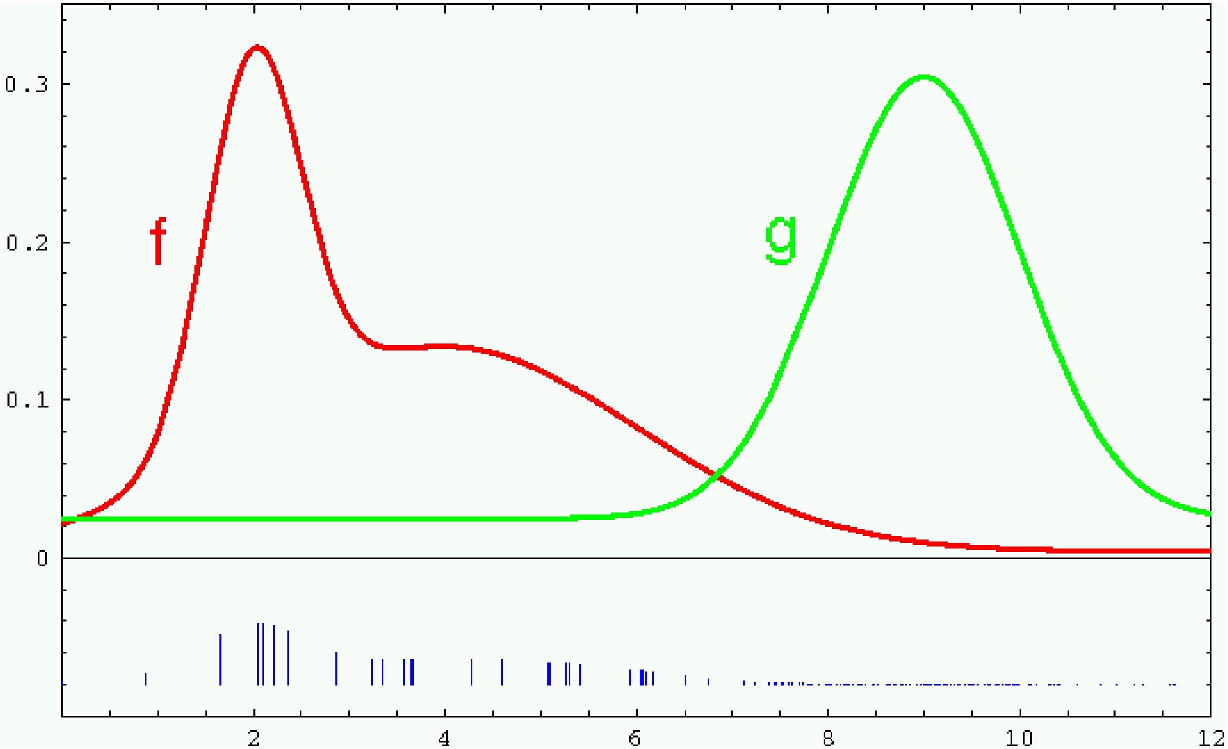
Particle Filters

- Represent belief by random samples
- Estimation of non-Gaussian, nonlinear processes

- Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Particle filter

- Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96]
- Computer vision: [Isard and Blake 96, 98]
- Dynamic Bayesian Networks: [Kanazawa et al., 95]

Importance of Sampling



Weight samples: $w = f / g$

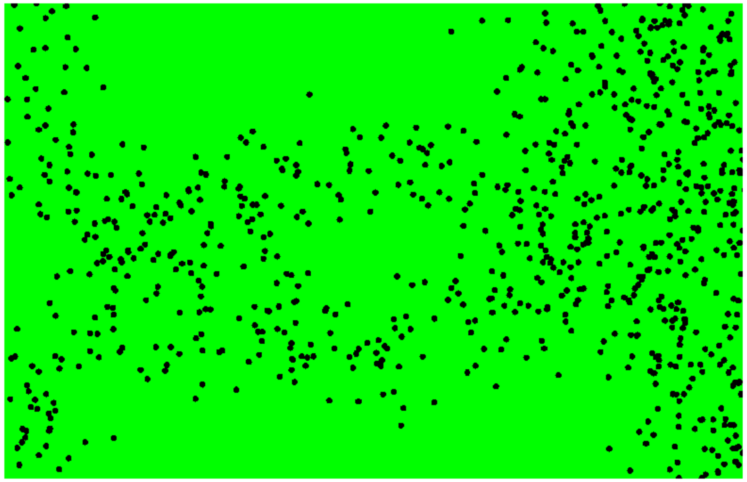
Importance of Sampling with Resampling

$$\text{Target distribution f}: p(x \mid z_1, z_2, \ldots, z_n) = \frac{\prod_k p(z_k \mid x) \quad p(x)}{p(z_1, z_2, \ldots, z_n)}$$

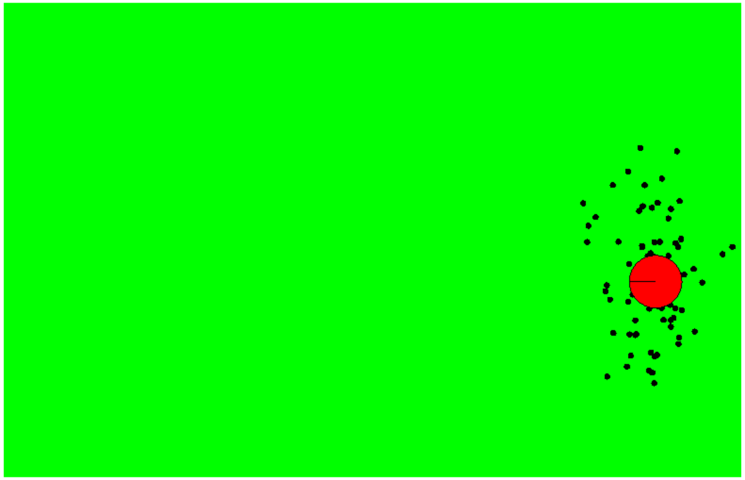
$$\text{Sampling distribution g}: p(x \mid z_l) = \frac{p(z_l \mid x) p(x)}{p(z_l)}$$

$$\text{Importance weights w}: \frac{f}{g} = \frac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \frac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)}$$

Importance of Sampling with Resampling

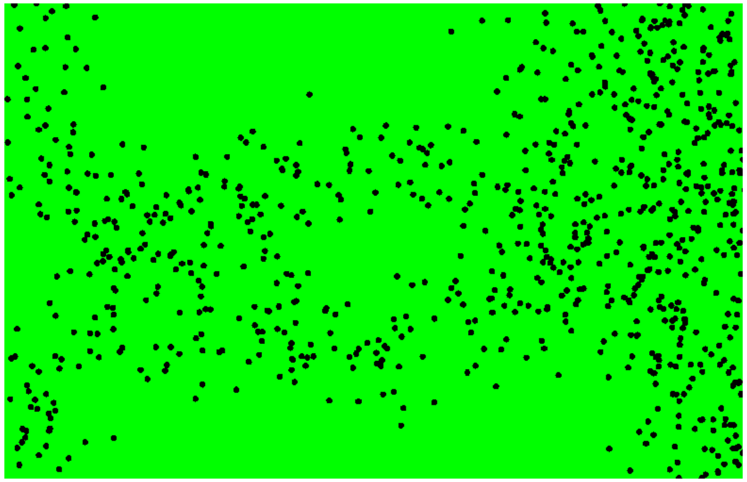


Weighted samples

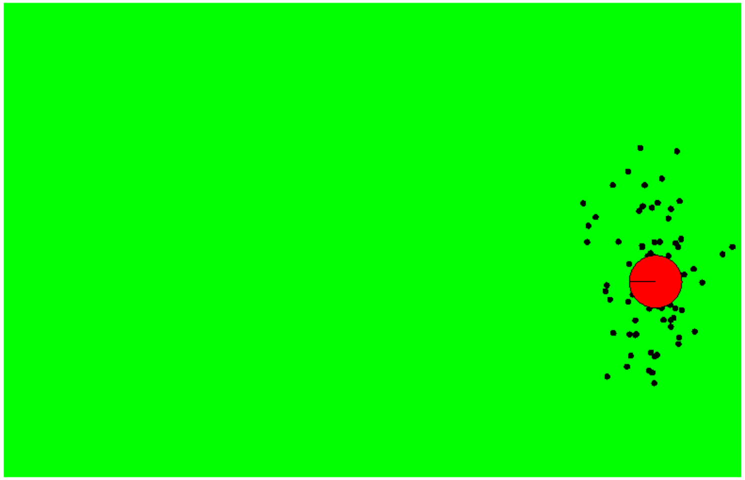


After resampling

Importance of Sampling with Resampling

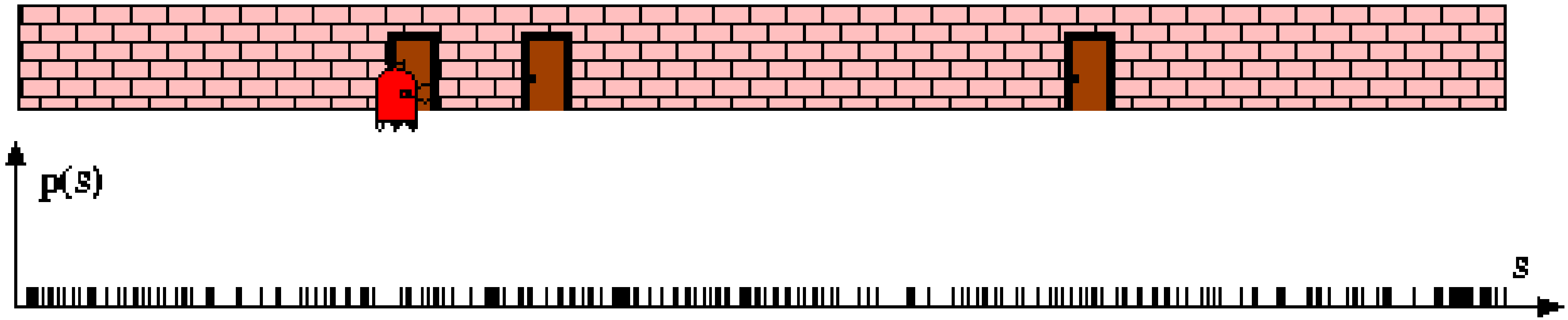


Weighted samples



After resampling

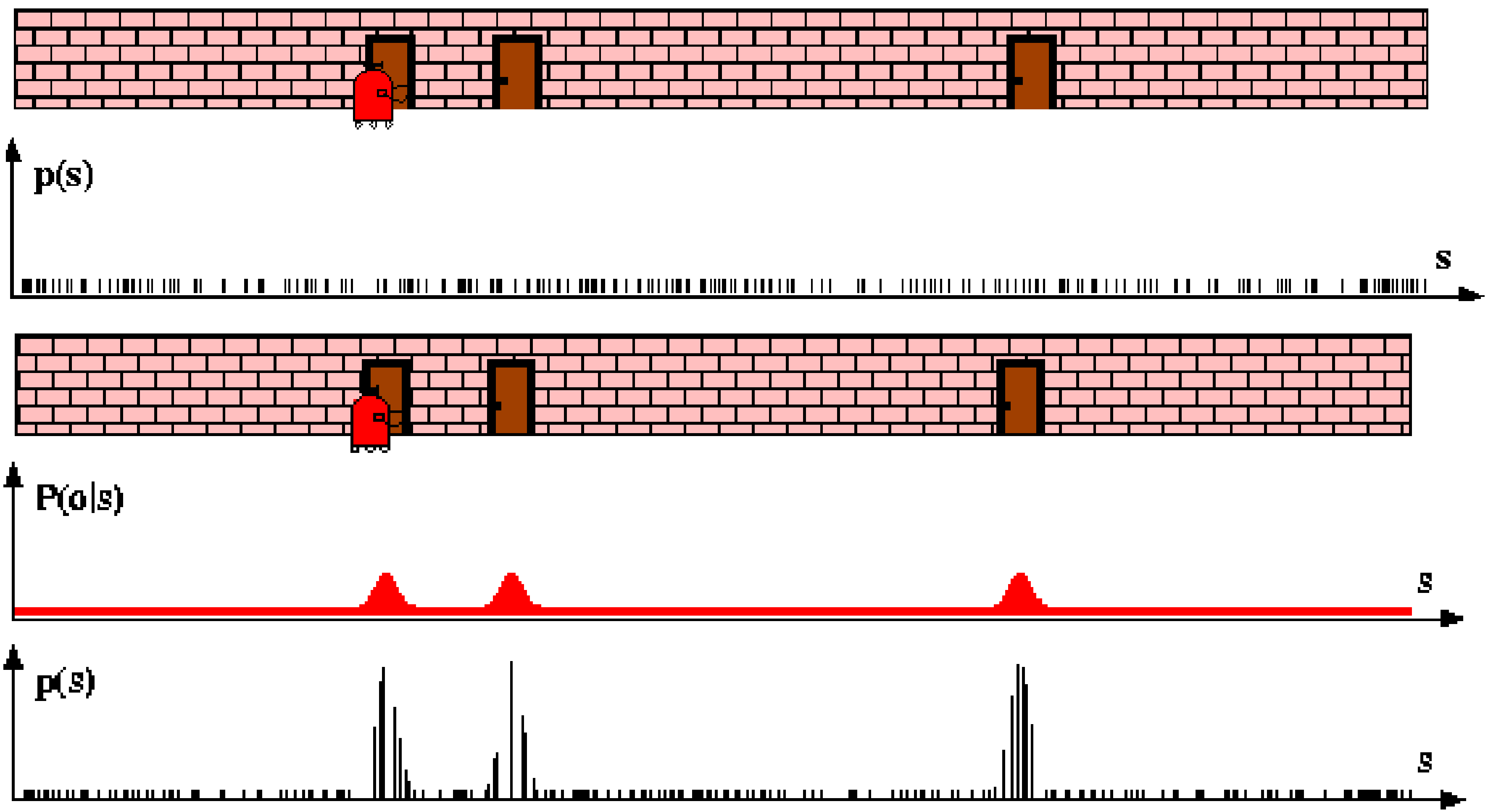
Particle Filters



Sensor Information: Importance Sampling

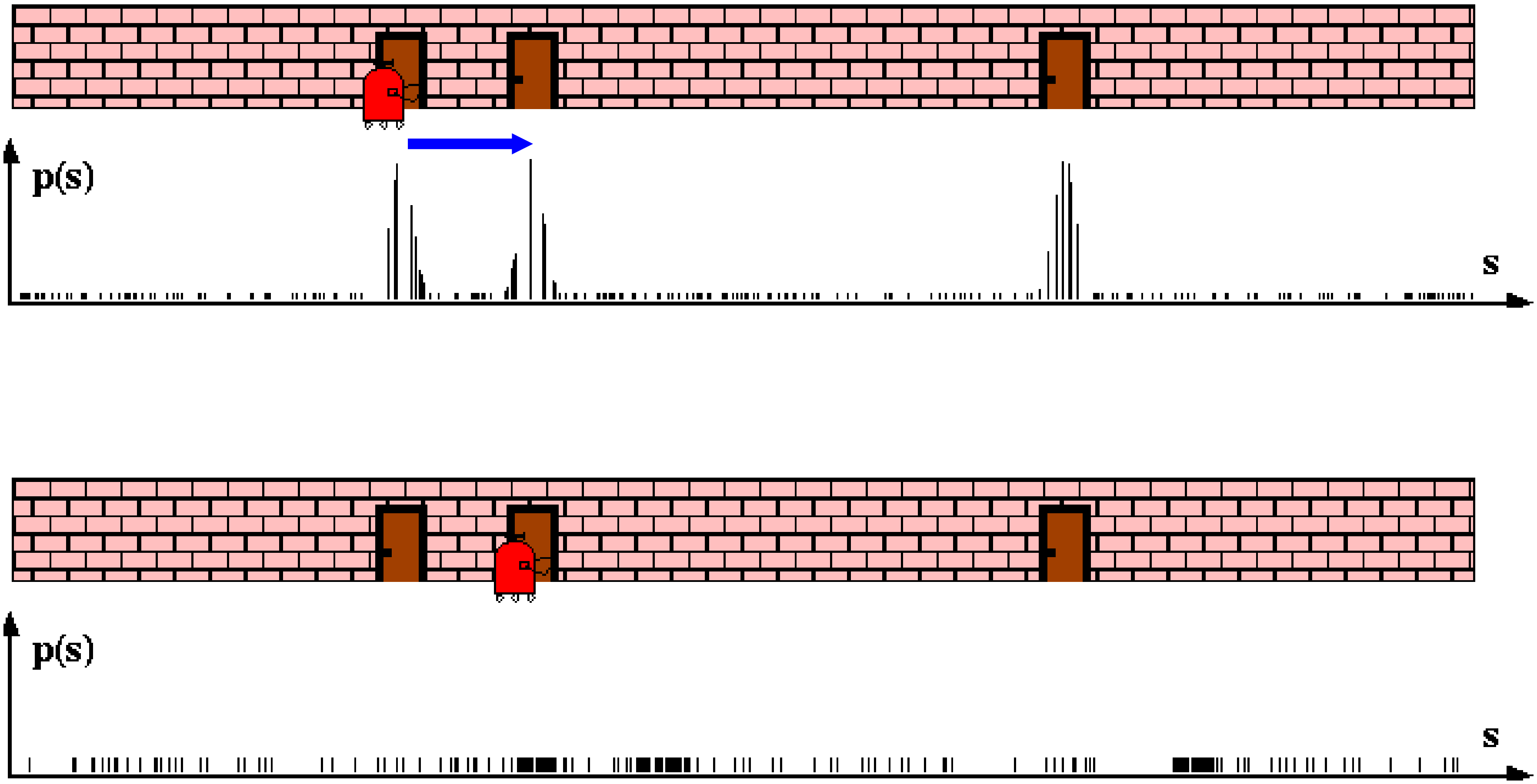
$$Bel(x) \leftarrow \alpha\ p(z\,|\,x)\ Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\ p(z\,|\,x)\ Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z\,|\,x)$$



Robot Motion

$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\, \mathrm{d}x'$$



Sensor Information: Importance Sampling

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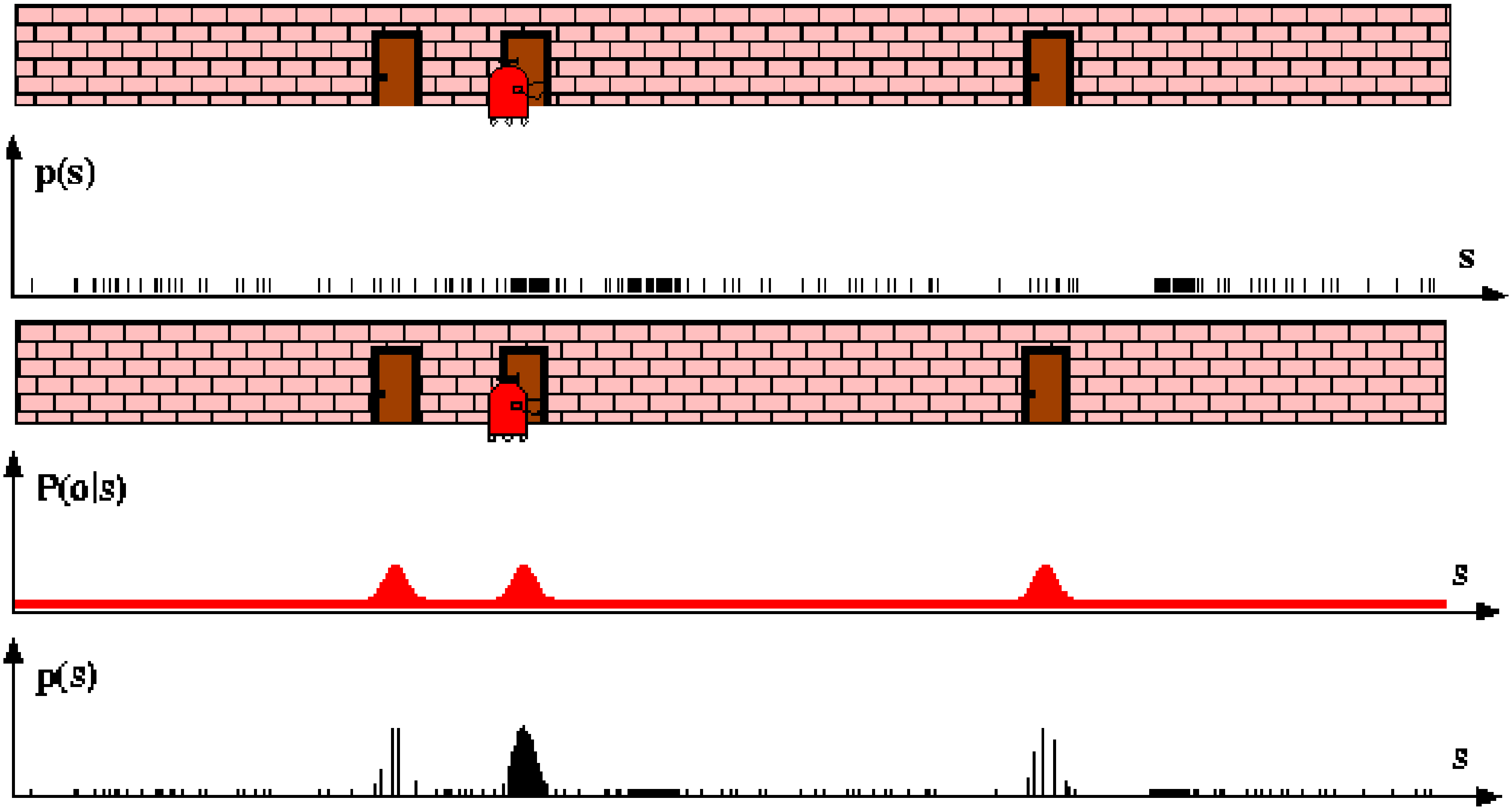

Robot Motion

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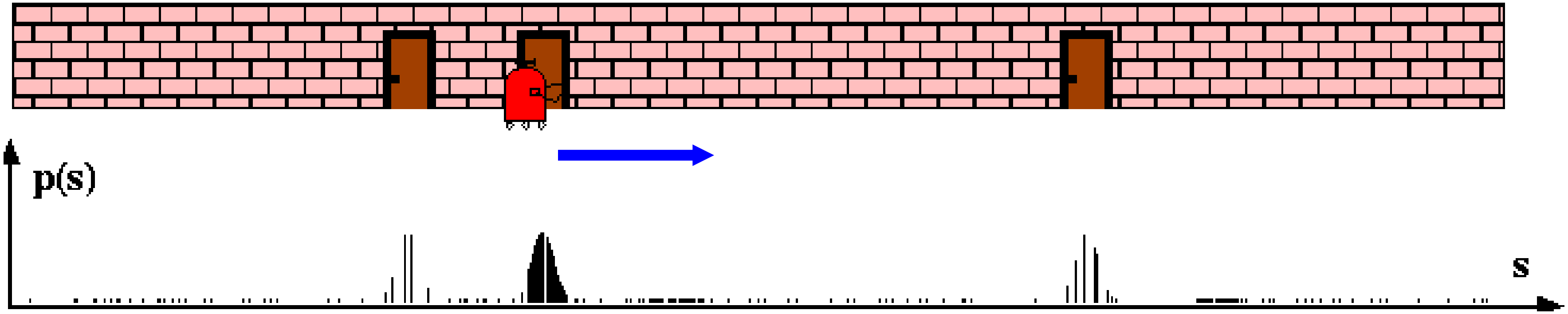
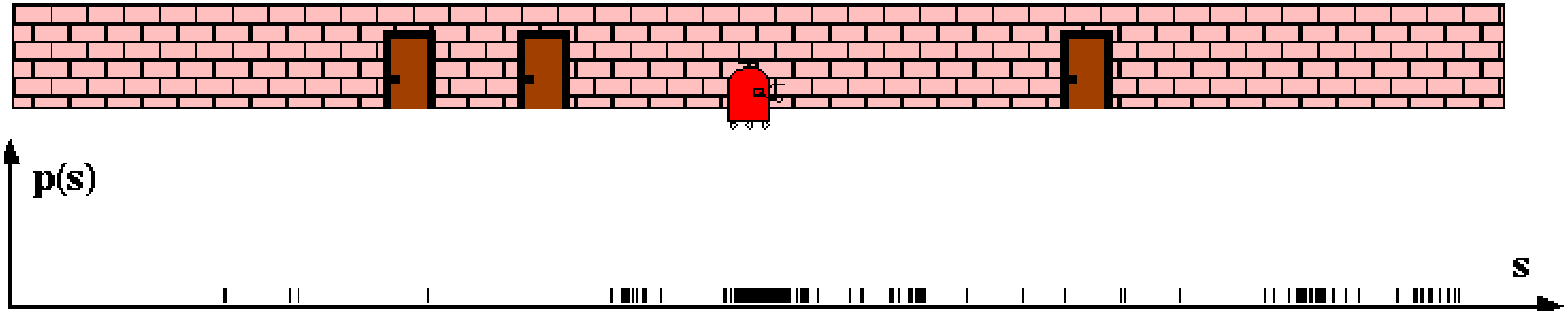




Particle Filter Algorithm

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1}, z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \dots n$ ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***
9. **For** $i = 1 \dots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***

Particle Filter Algorithm

$$Bel(x_t) = \eta\ p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\ Bel(x_{t-1})\ dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$\begin{aligned} w_t^i &= \frac{\text{target distributi on}}{\text{proposal distributi on}} \\ &= \frac{\eta\ p(z_t \mid x_t)\ p(x_t \mid x_{t-1}, u_{t-1})\ Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\ Bel(x_{t-1})} \\ &\propto p(z_t \mid x_t) \end{aligned}$$

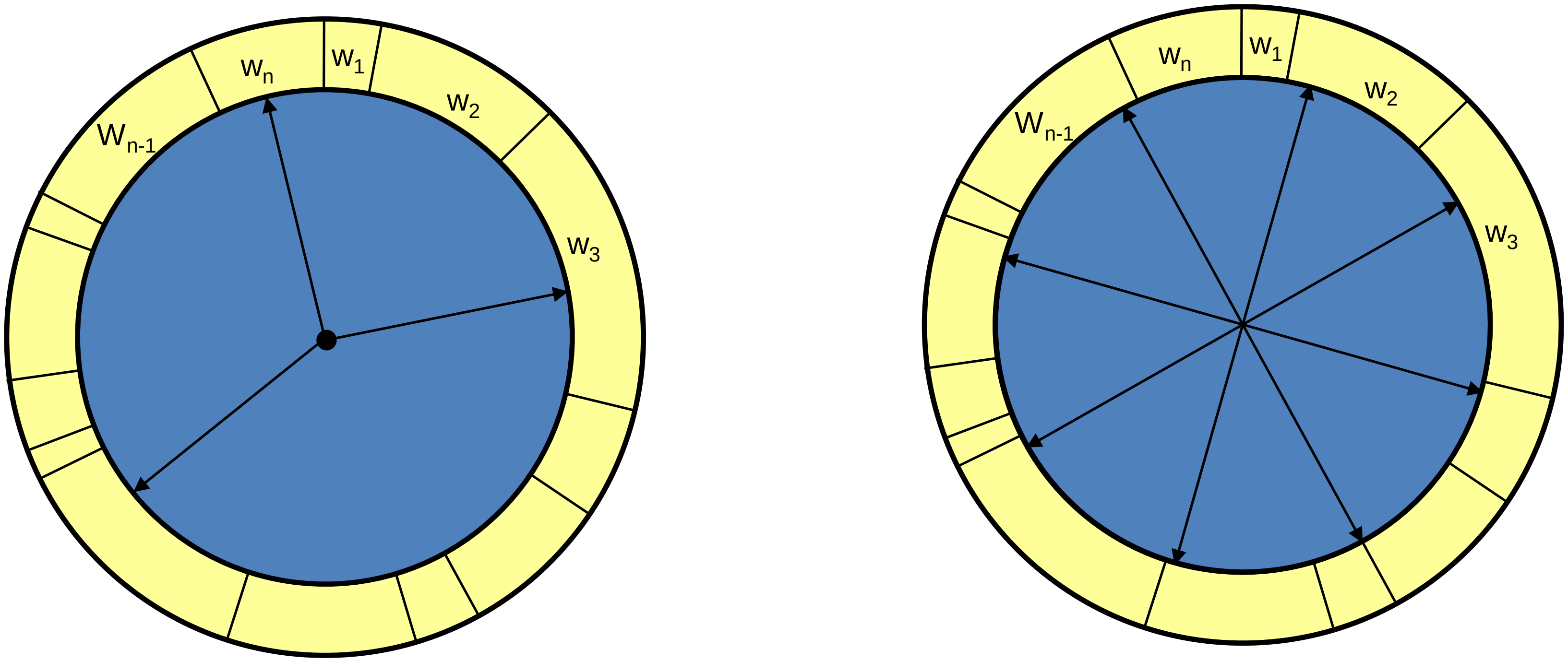
Resampling

- **Given**: Set *S* of weighted samples.

- **Wanted** : Random sample, where the probability of drawing x_i is given by w_i.

- Typically done *n* times with replacement to generate new sample set *S'*.

Resampling



- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

Resampling Algorithm

1. Algorithm **systematic_resampling**(S,n):

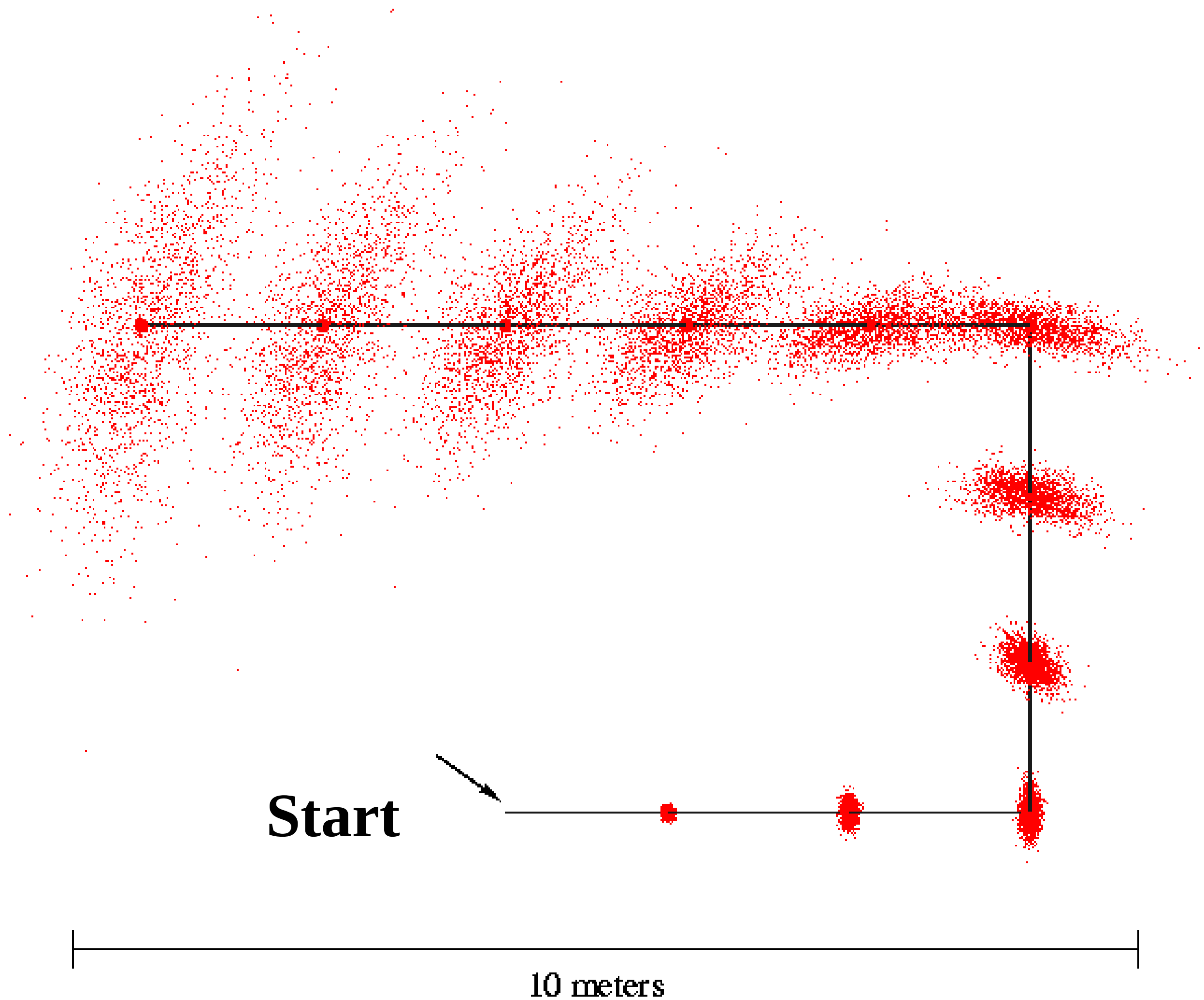
2. $S' = \emptyset, c_1 = w^1$
3. **For** $i = 2 \ldots n$ *Generate cdf*
4. $c_i = c_{i-1} + w^i$
5. $u_1 \sim U]0, n^{-1}], i = 1$ *Initialize threshold*

6. **For** $j = 1 \ldots n$ *Draw samples …*
7. **While** ($u_j > c_i$) *Skip until next threshold reached*
8. $i = i + 1$
9. $S' = S' \cup \left\{ < x^i, n^{-1} > \right\}$ *Insert*
10. $u_{j+1} = u_j + n^{-1}$ *Increment threshold*

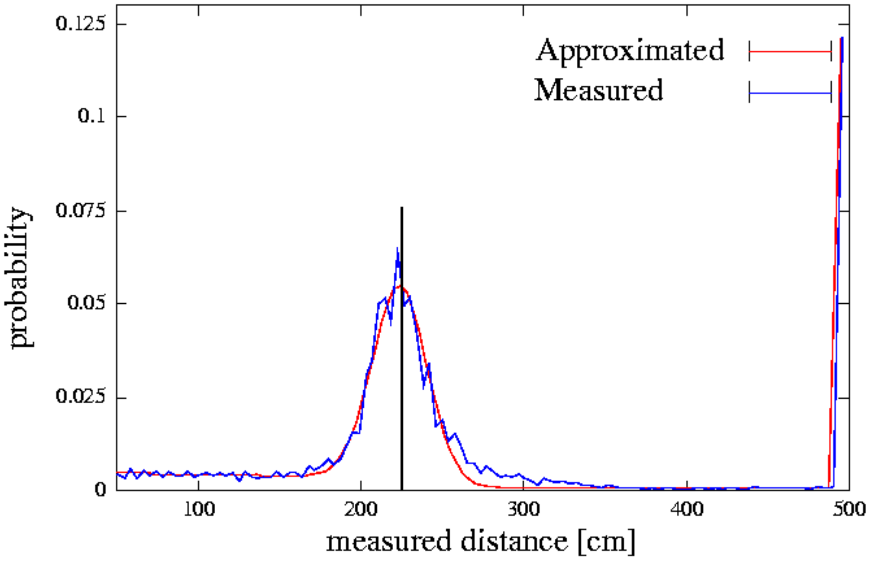
11. **Return** S'

Also called **stochastic universal sampling**

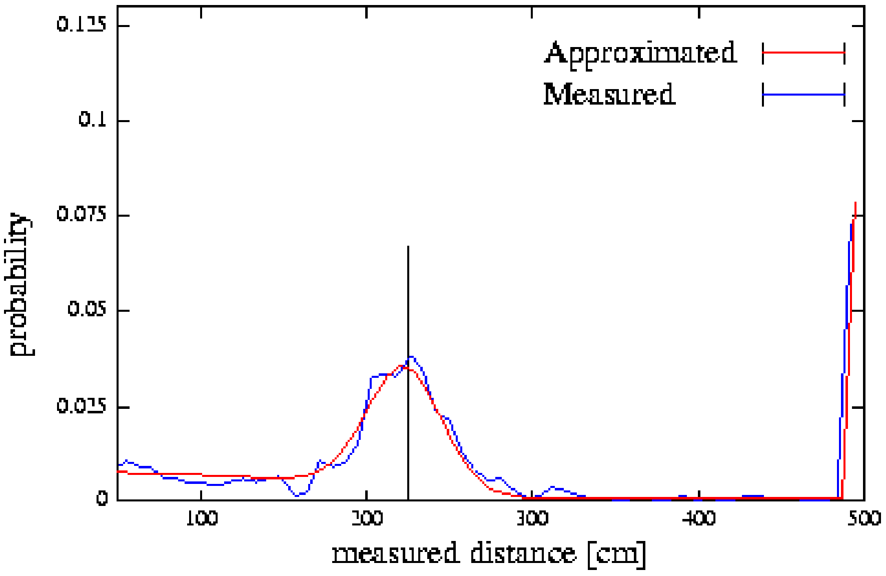
Motion Model Reminder



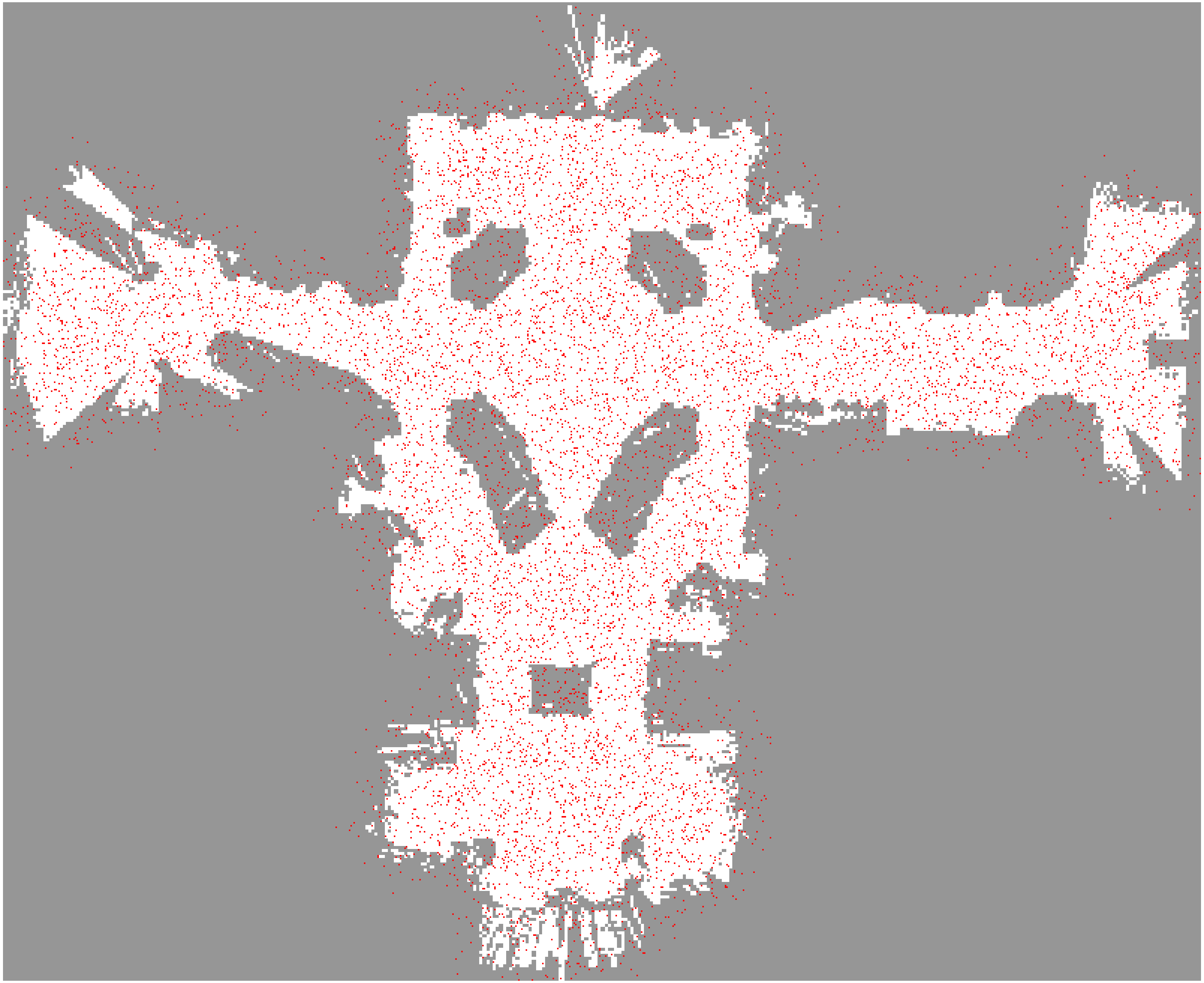
Proximity Sensor Model Reminder

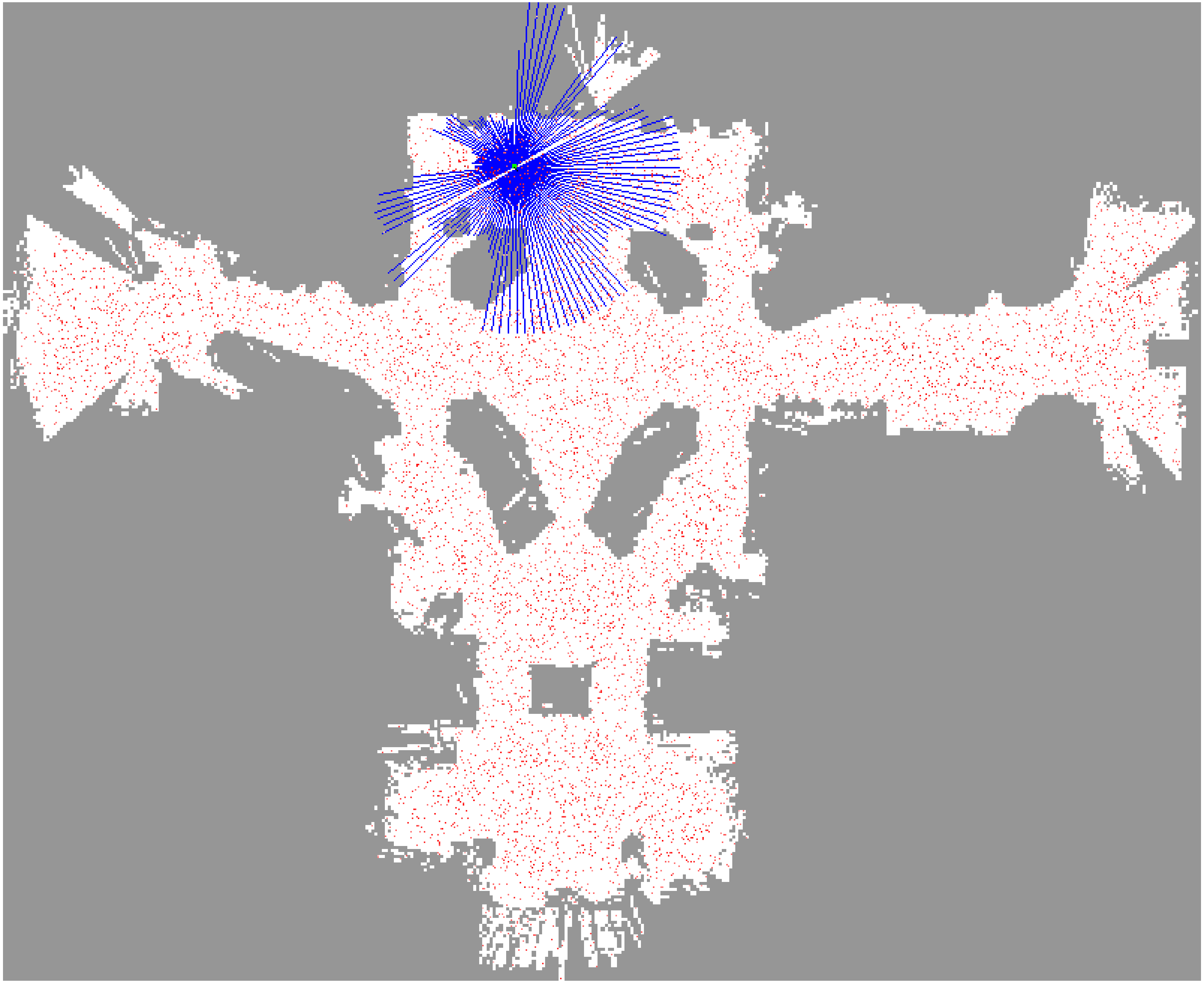


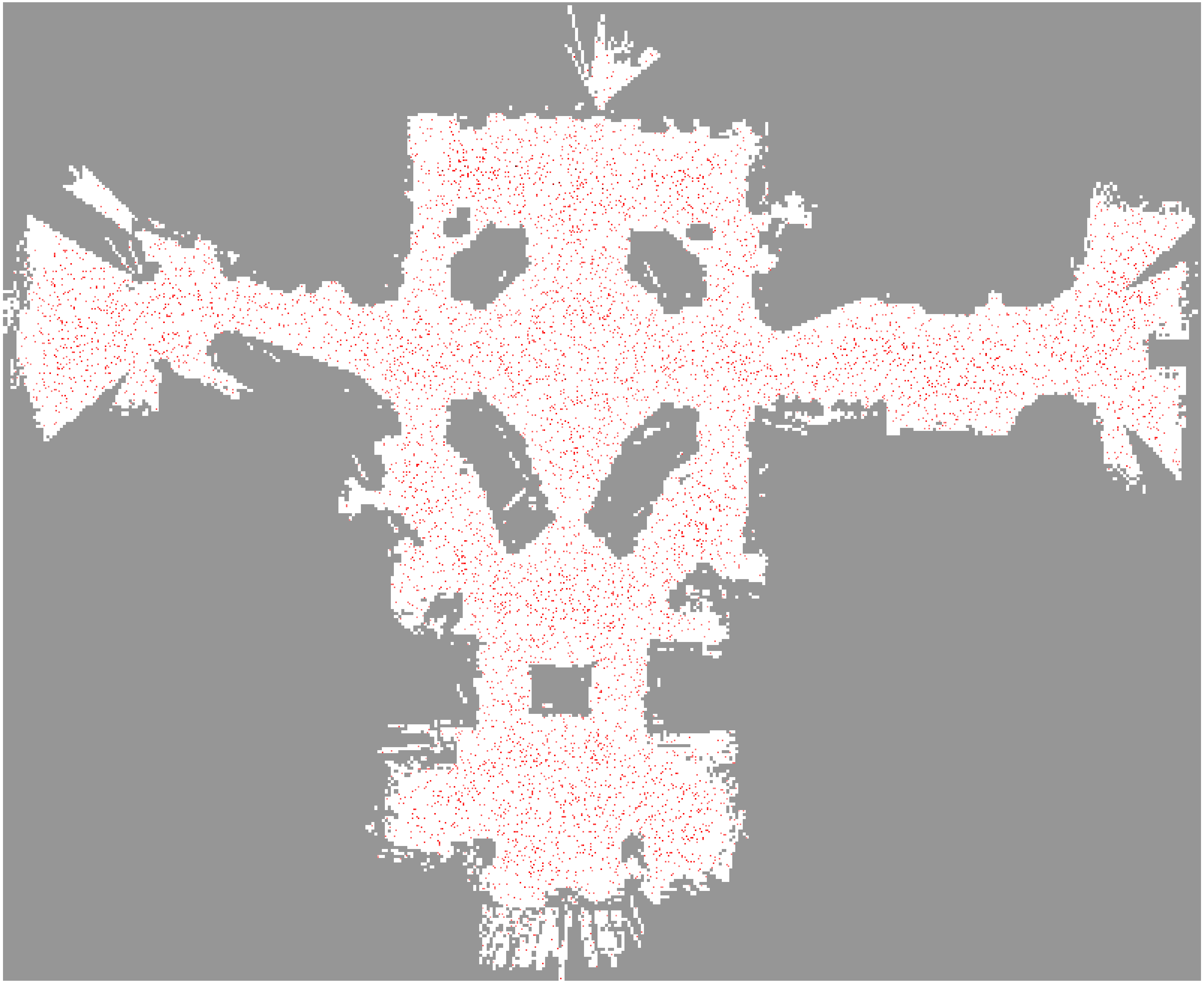
Laser sensor

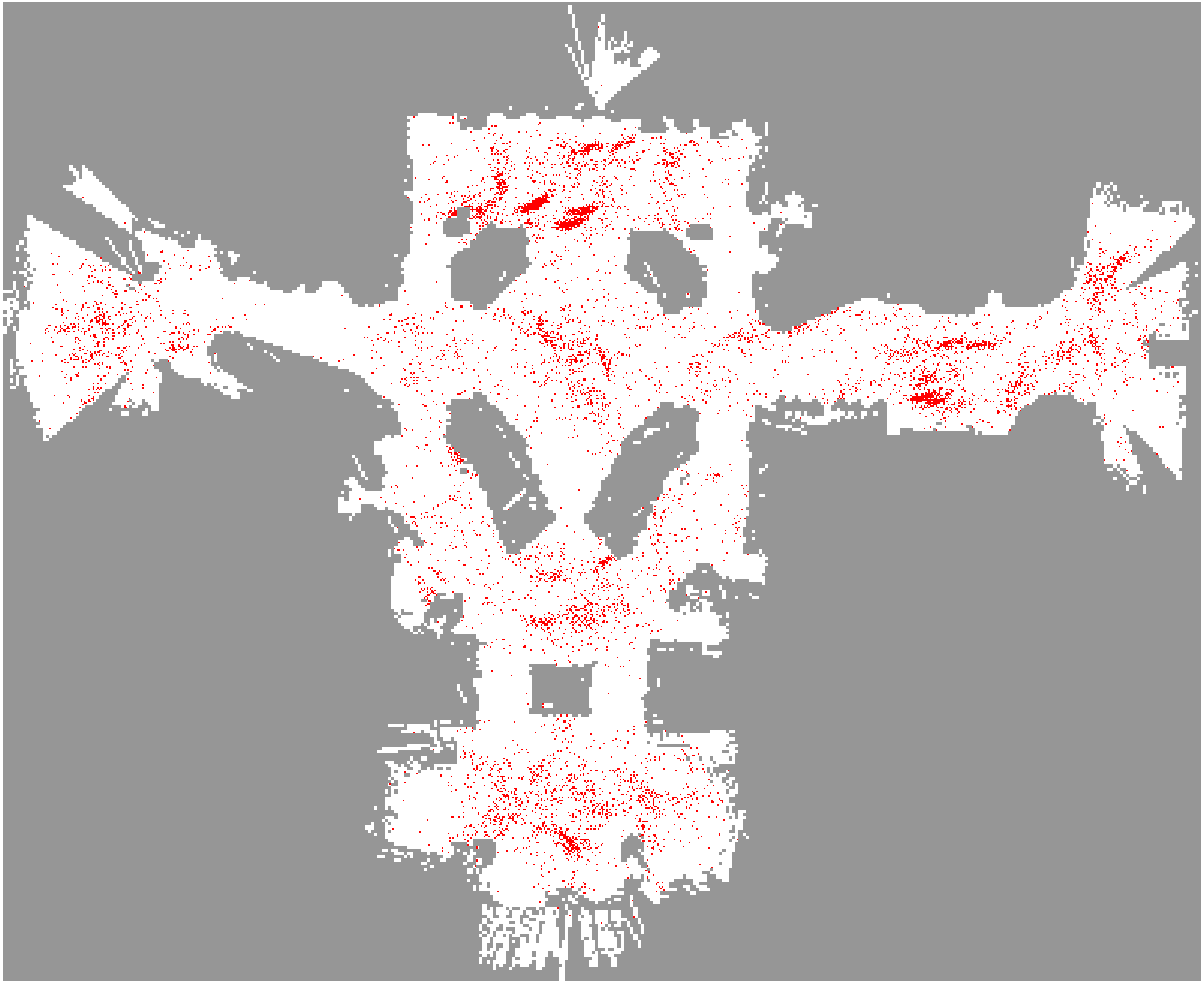


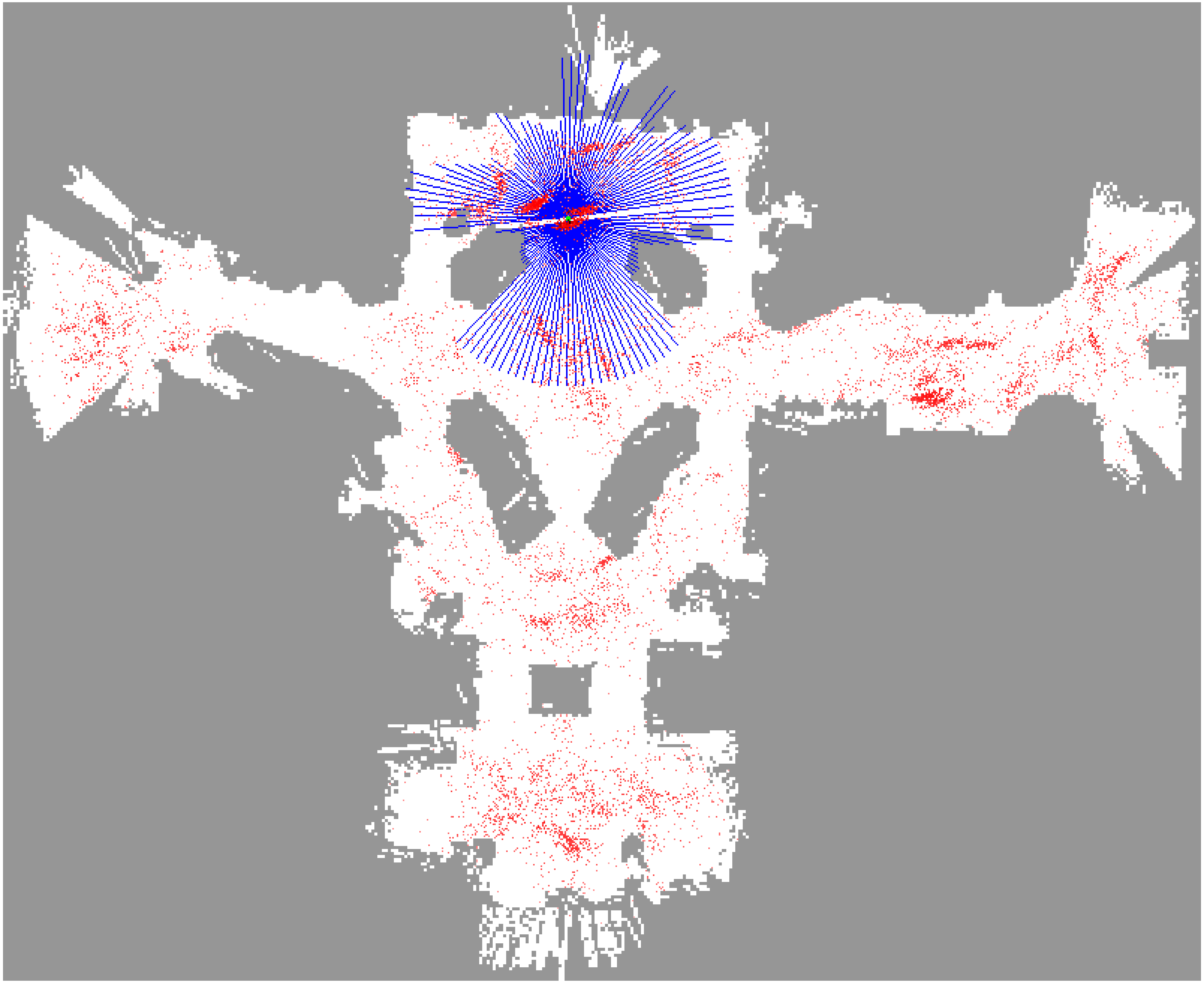
Sonar sensor

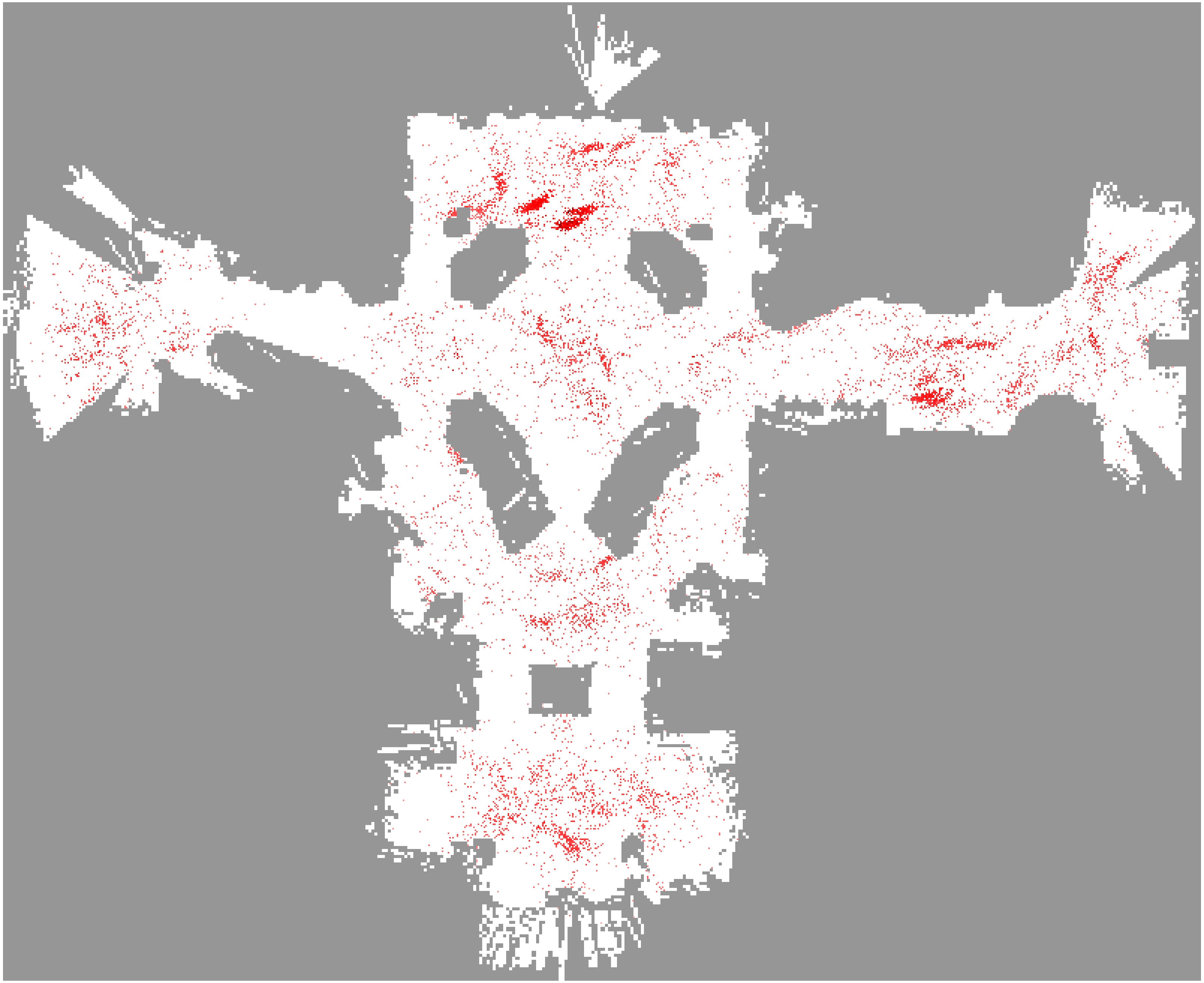


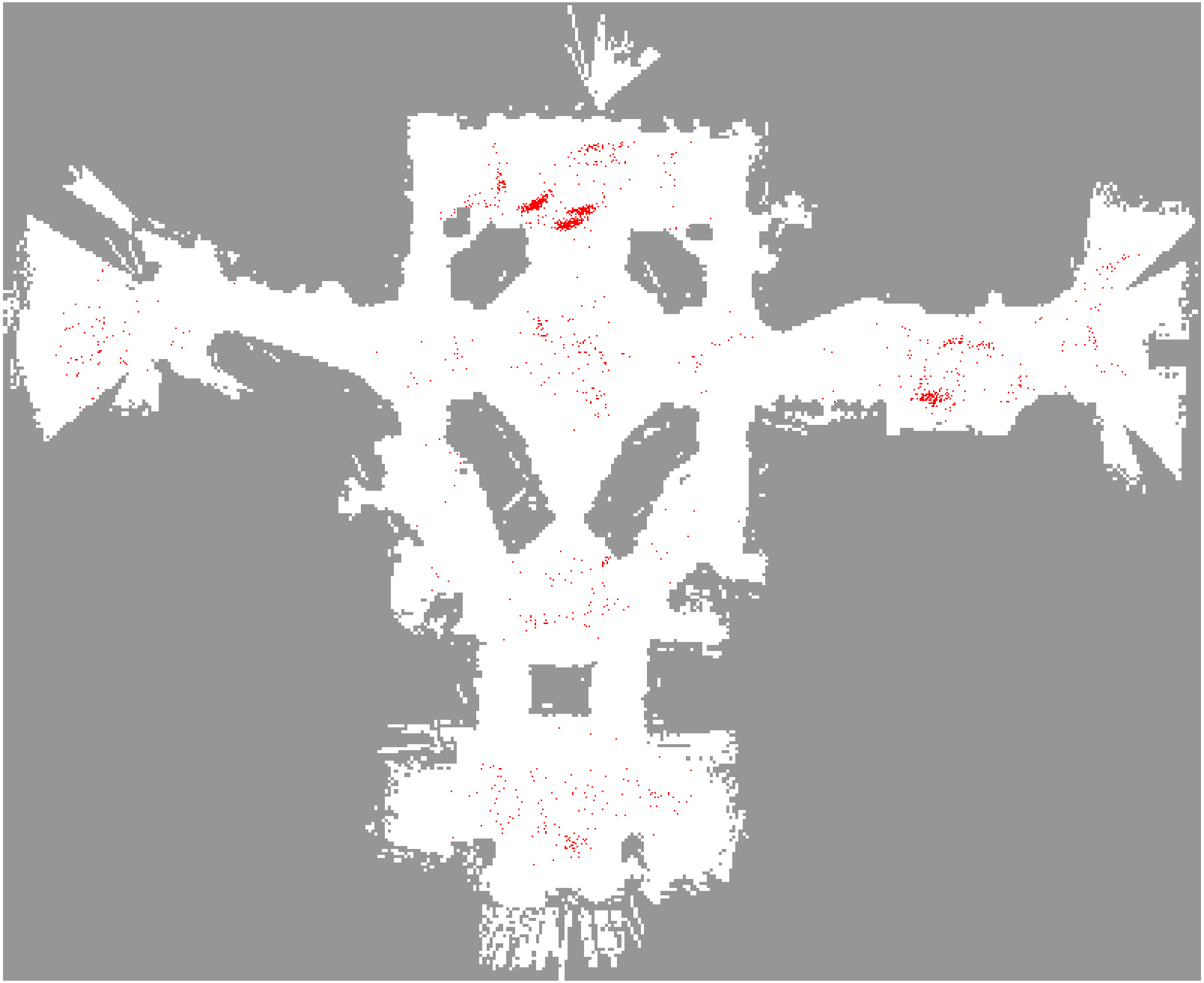


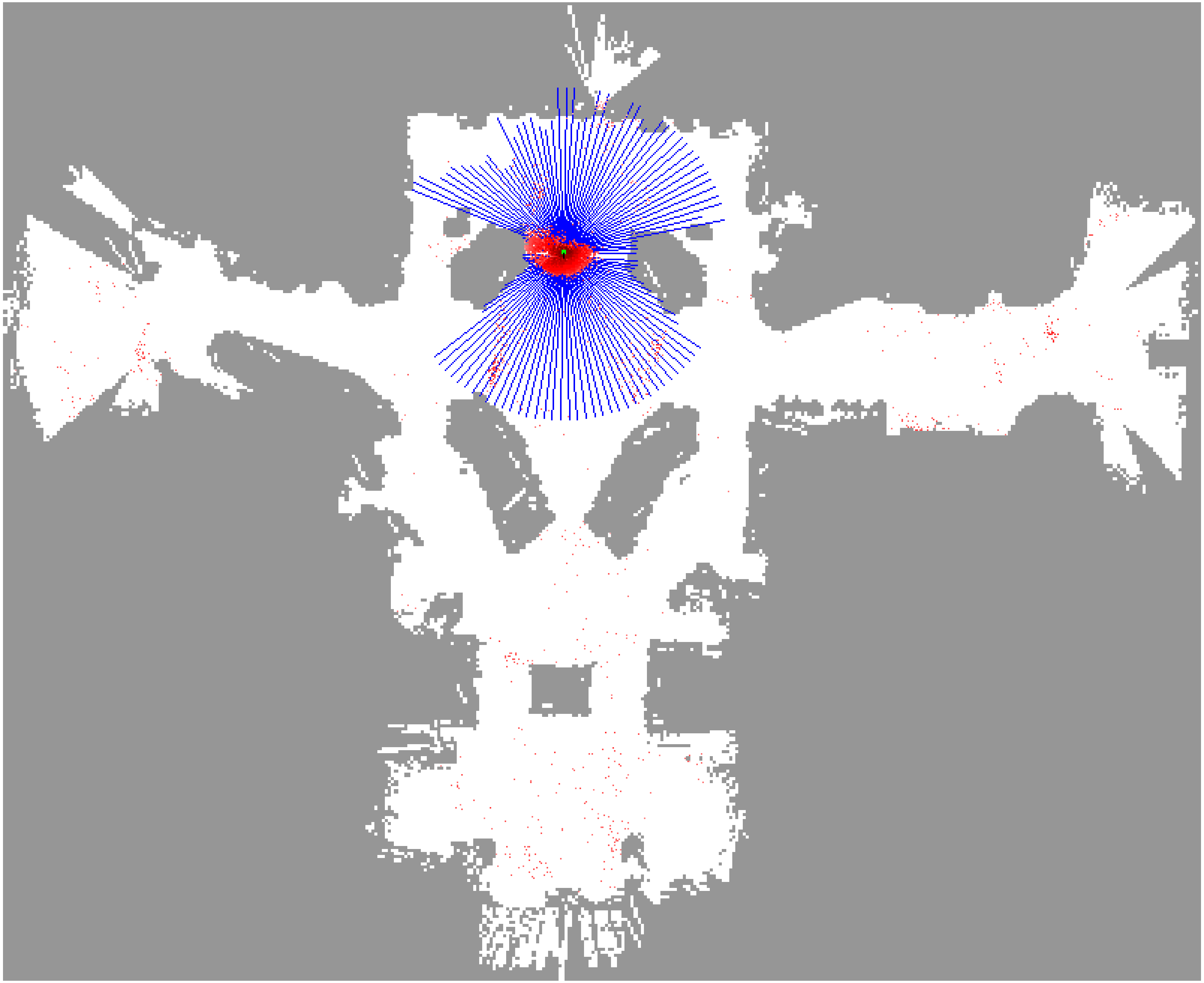


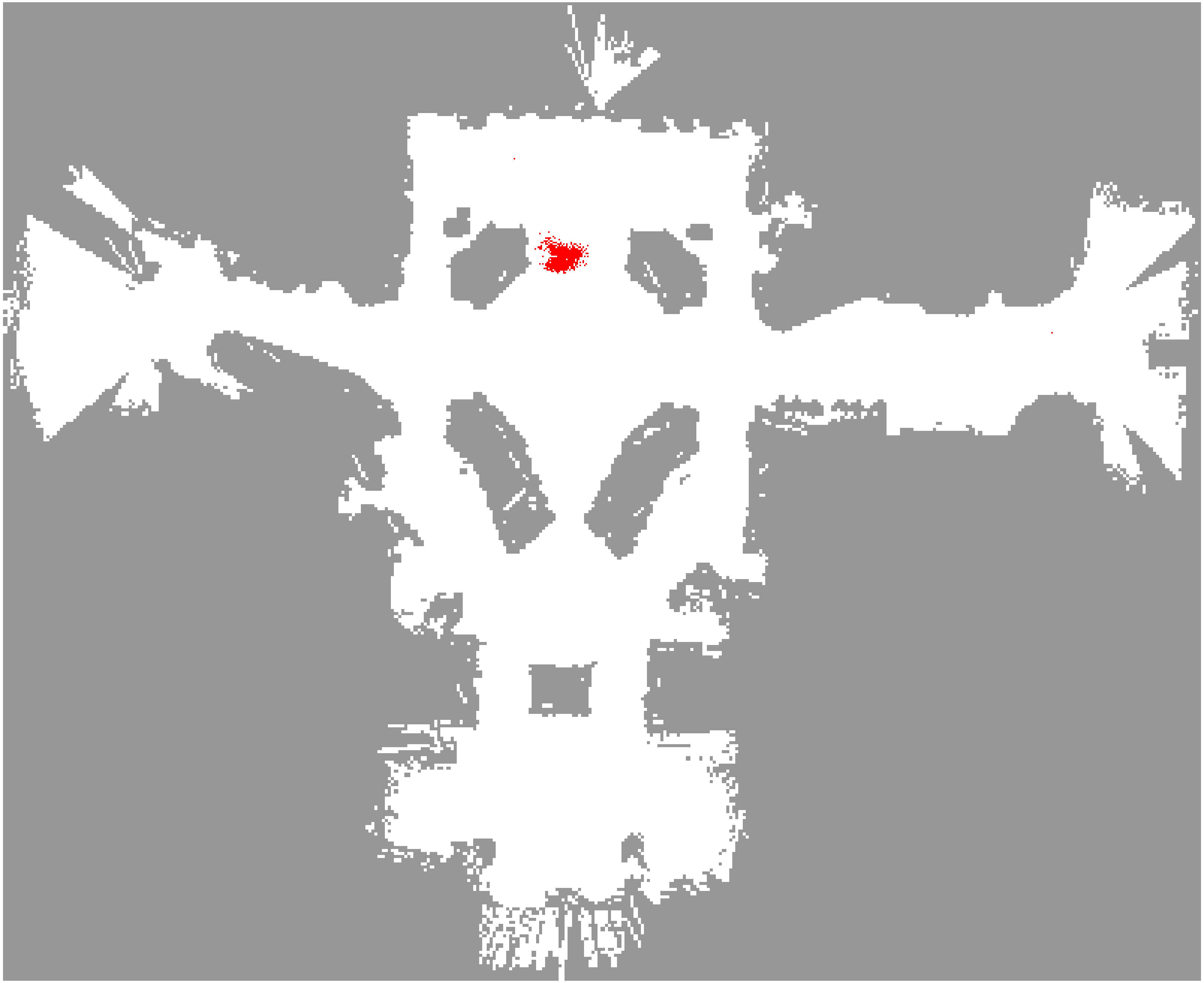


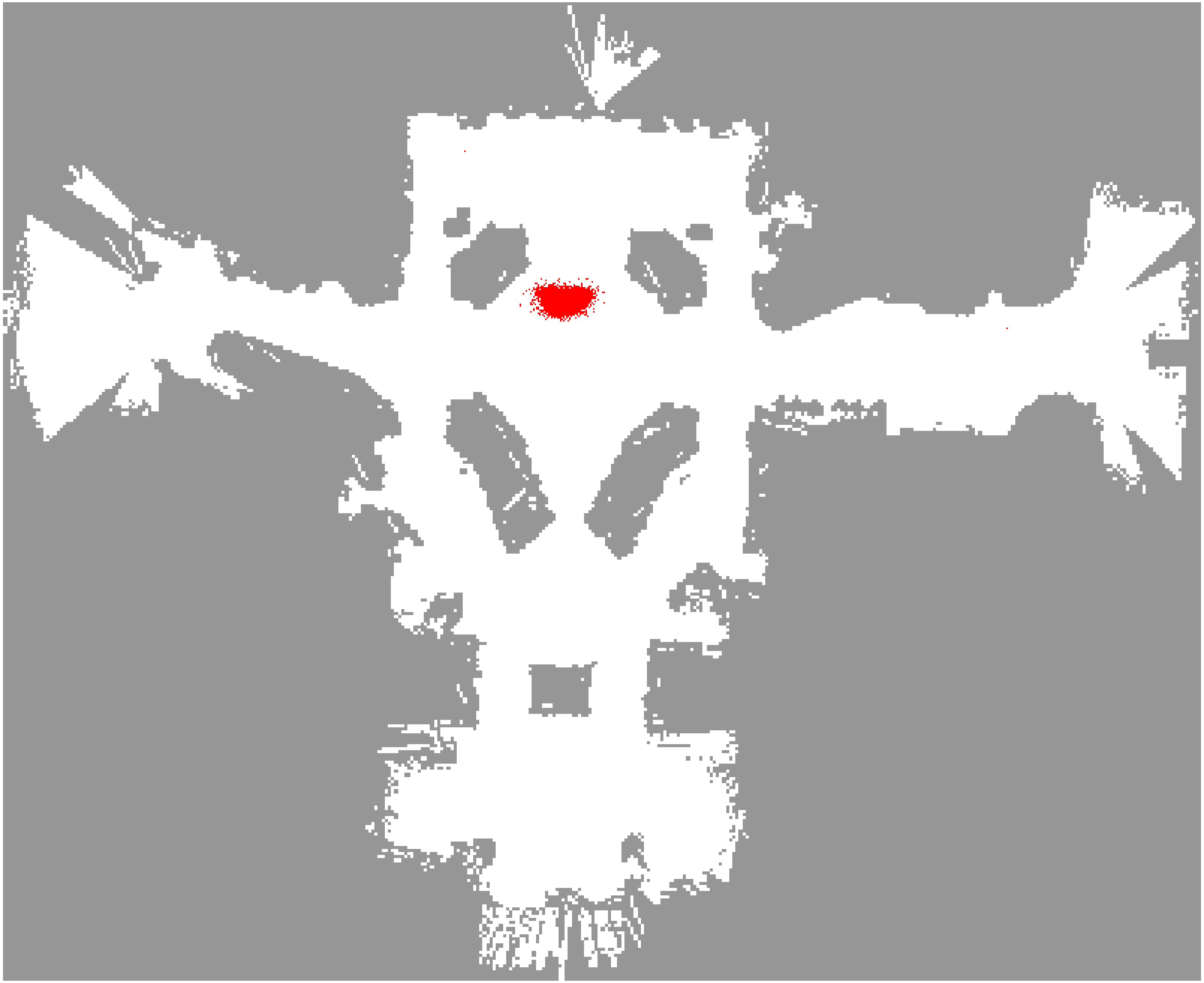


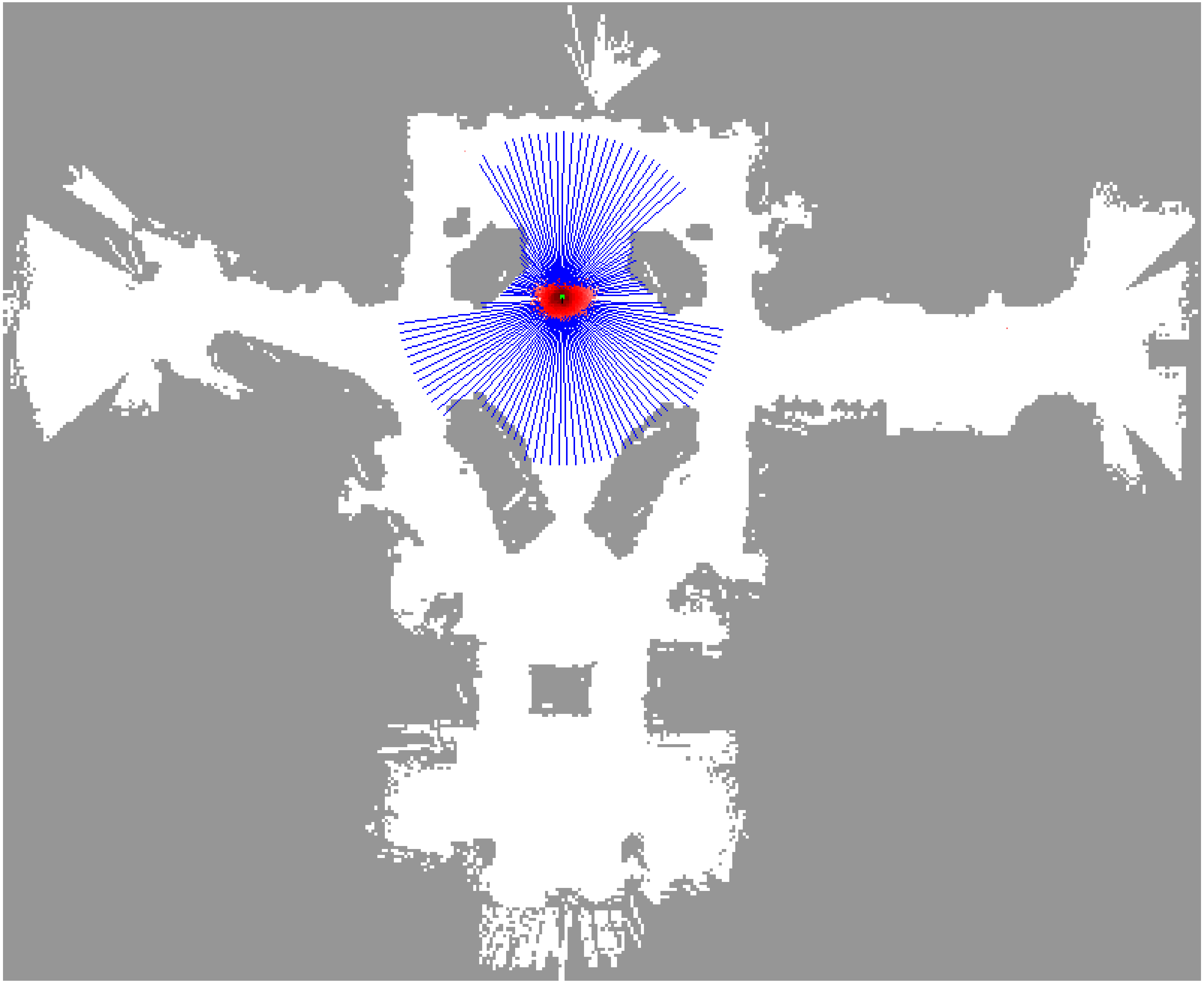


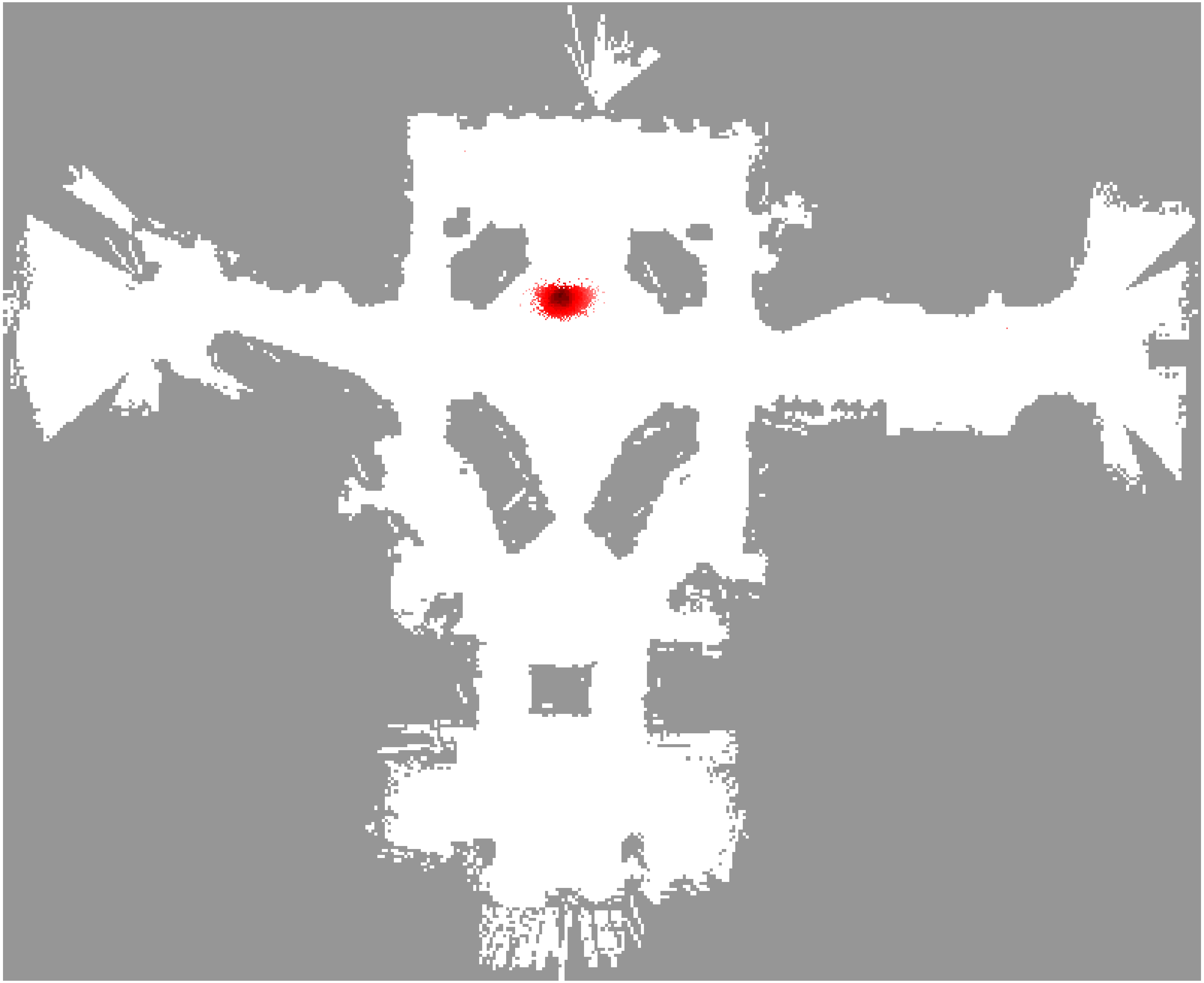


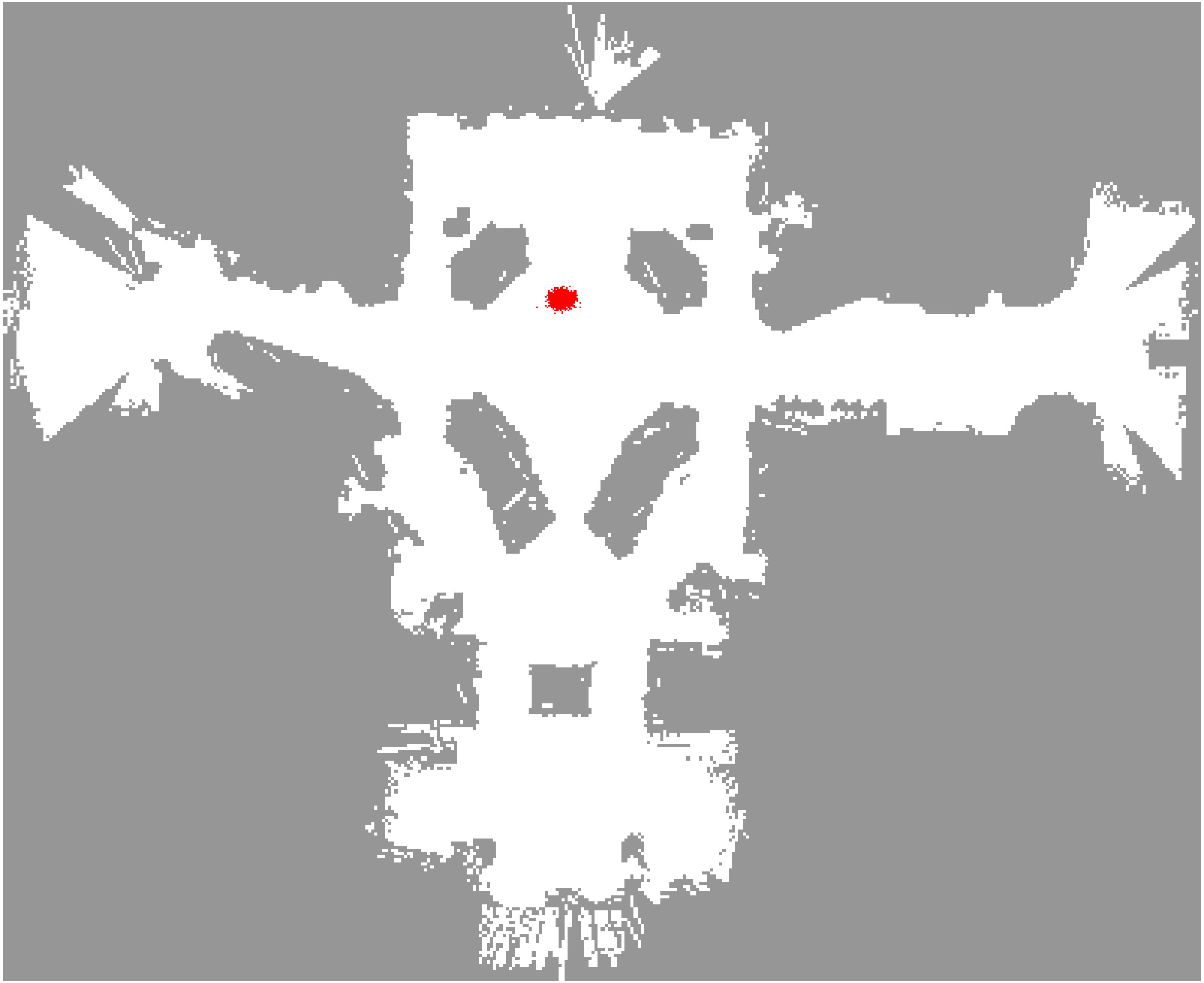


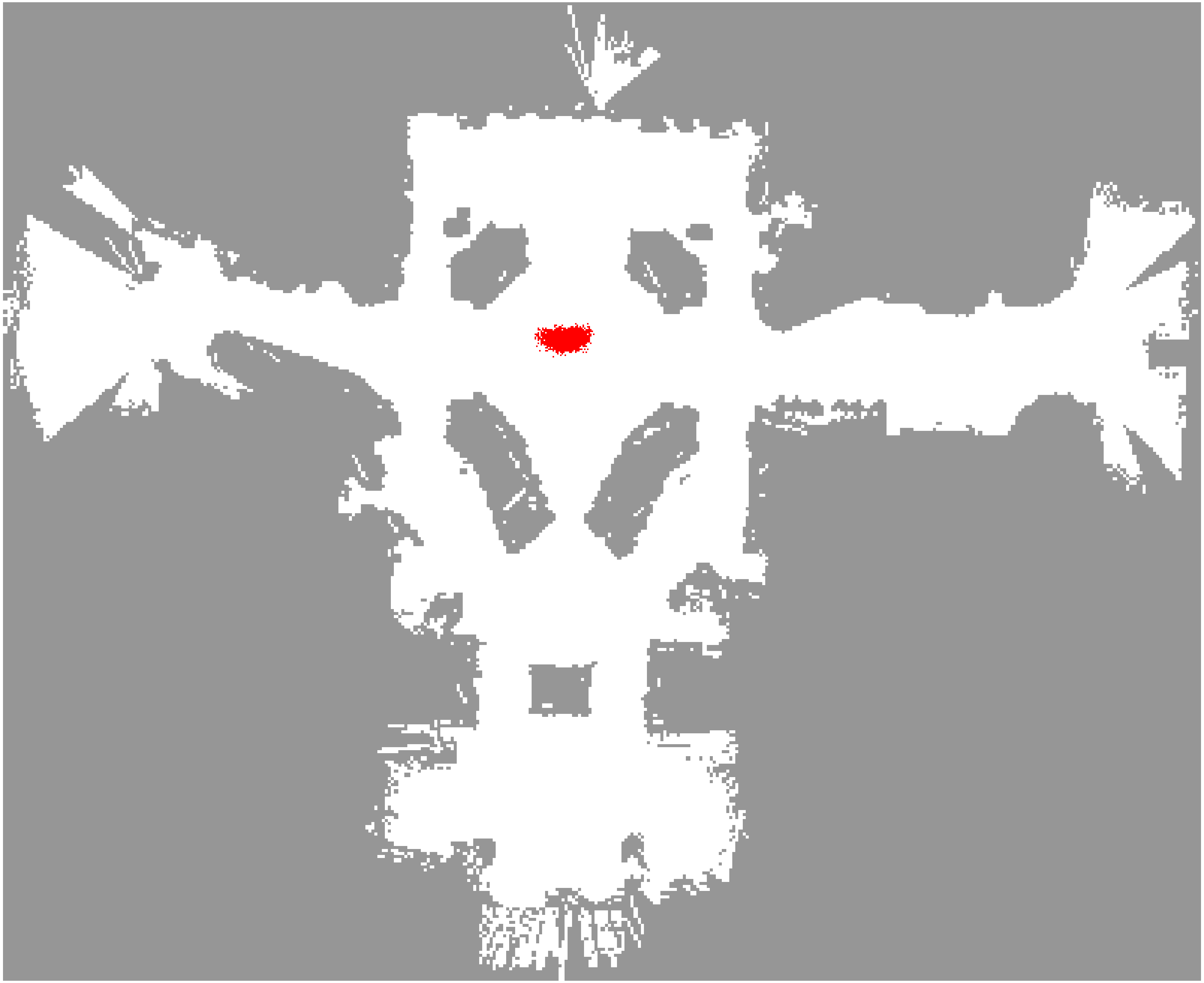


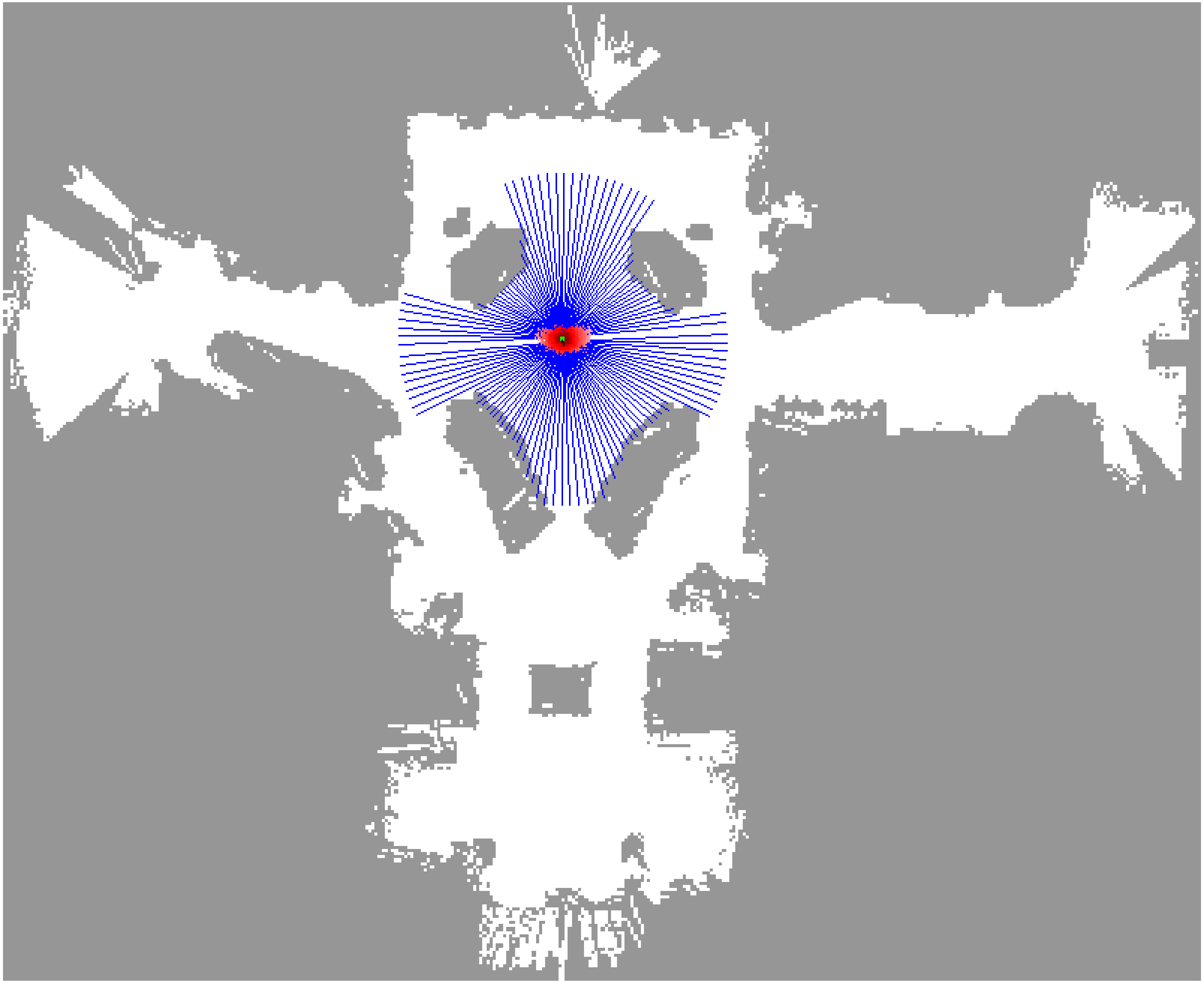


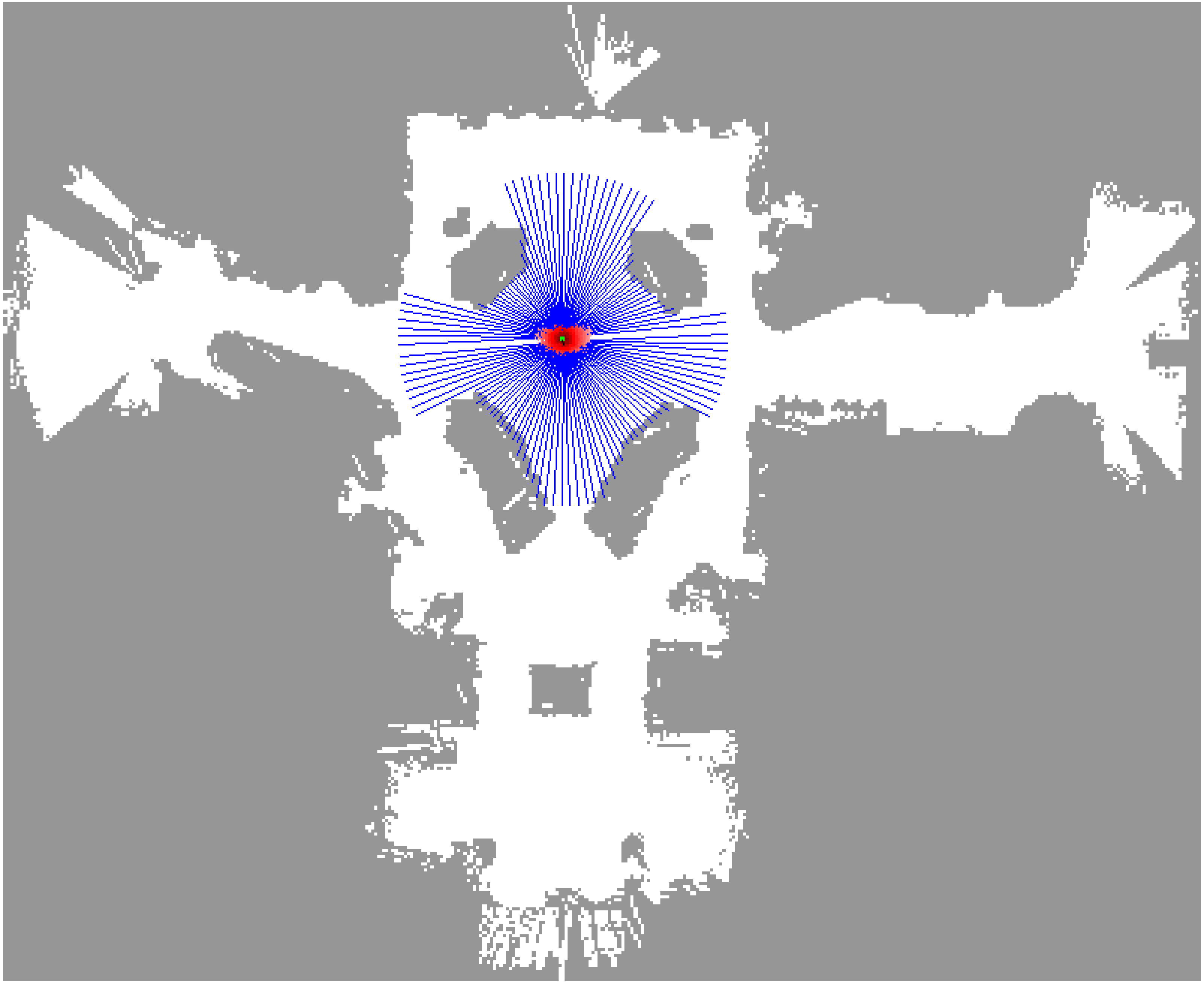




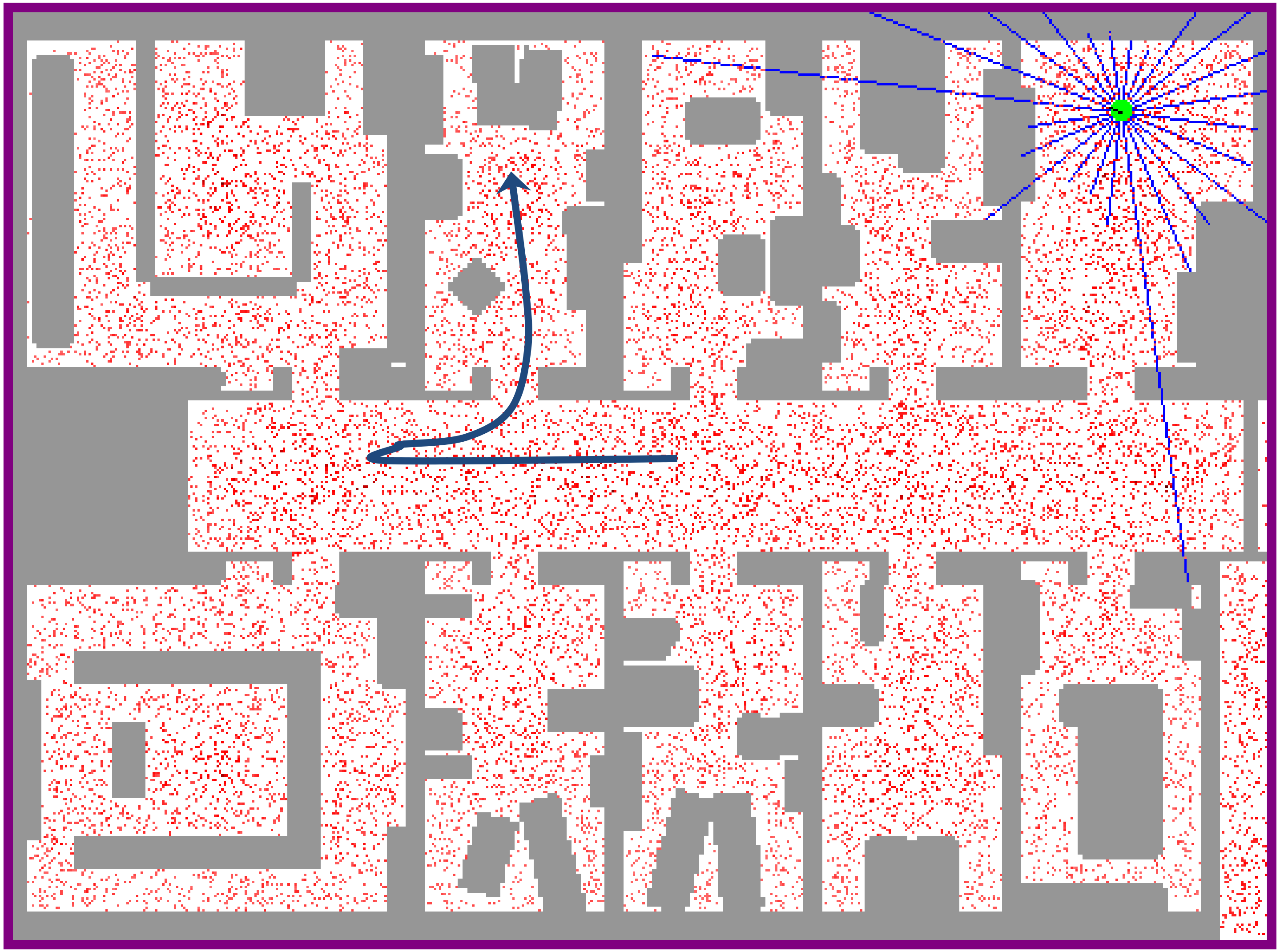




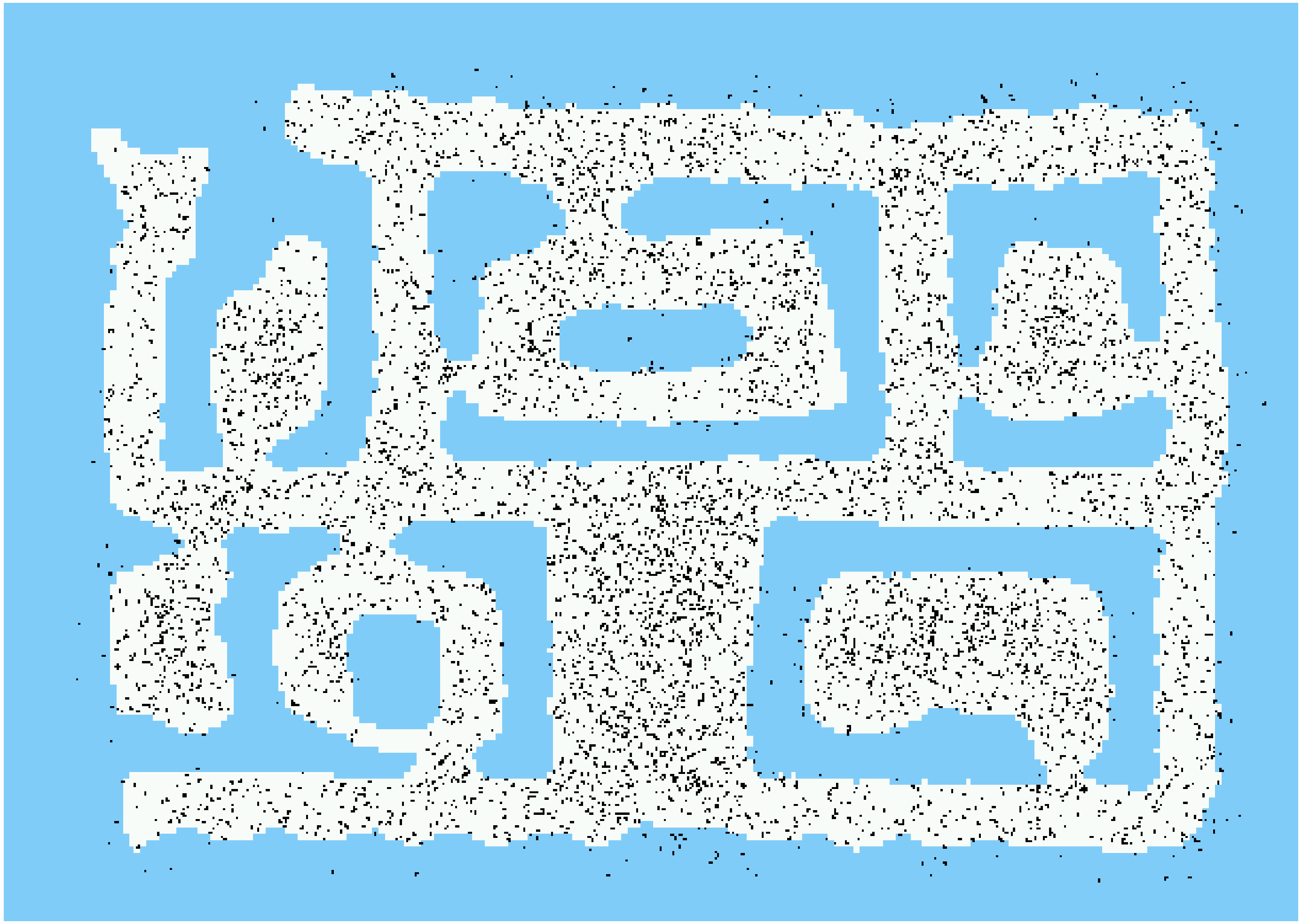




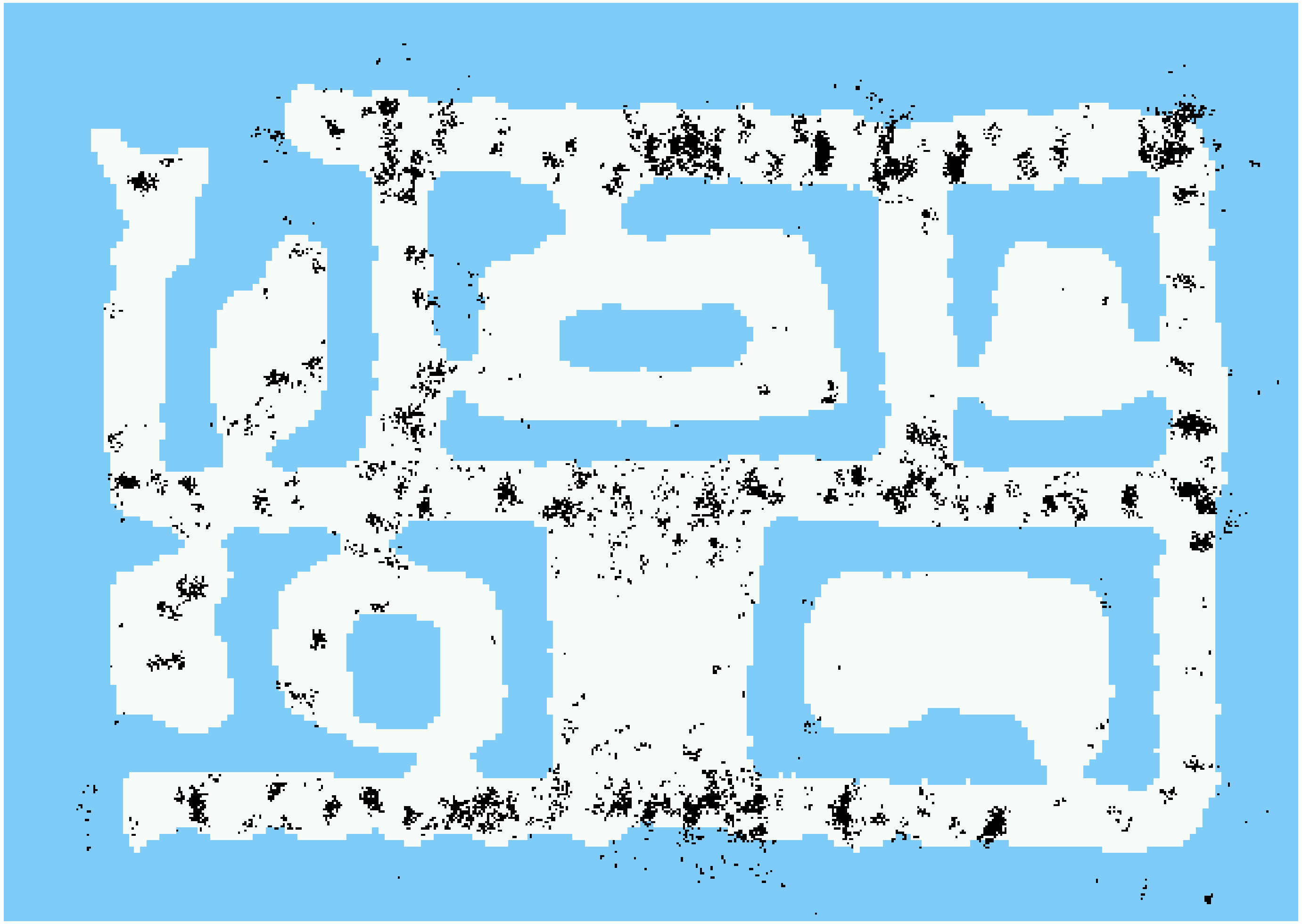
Sample-based Localization (sonar)



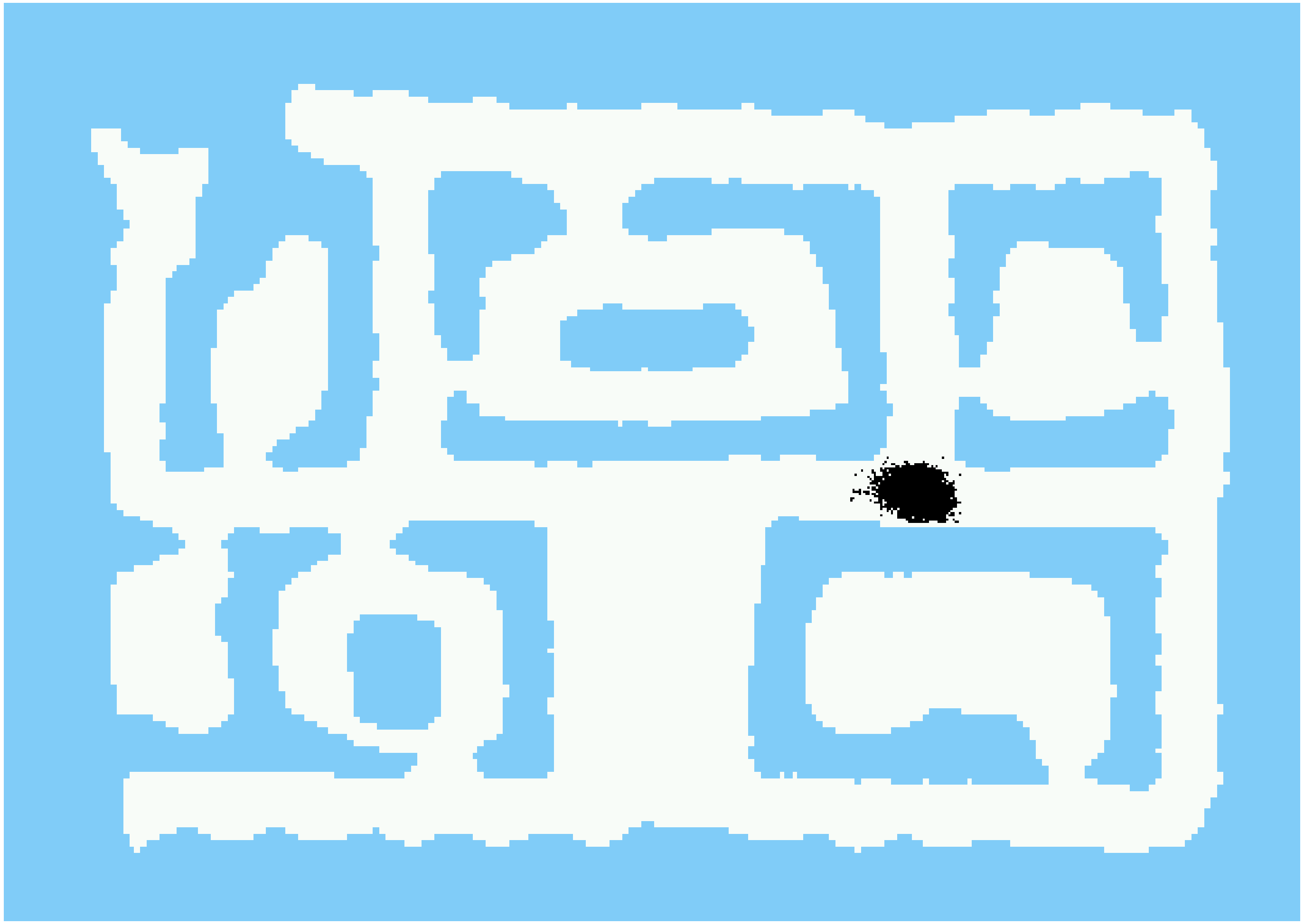
Initial Distribution



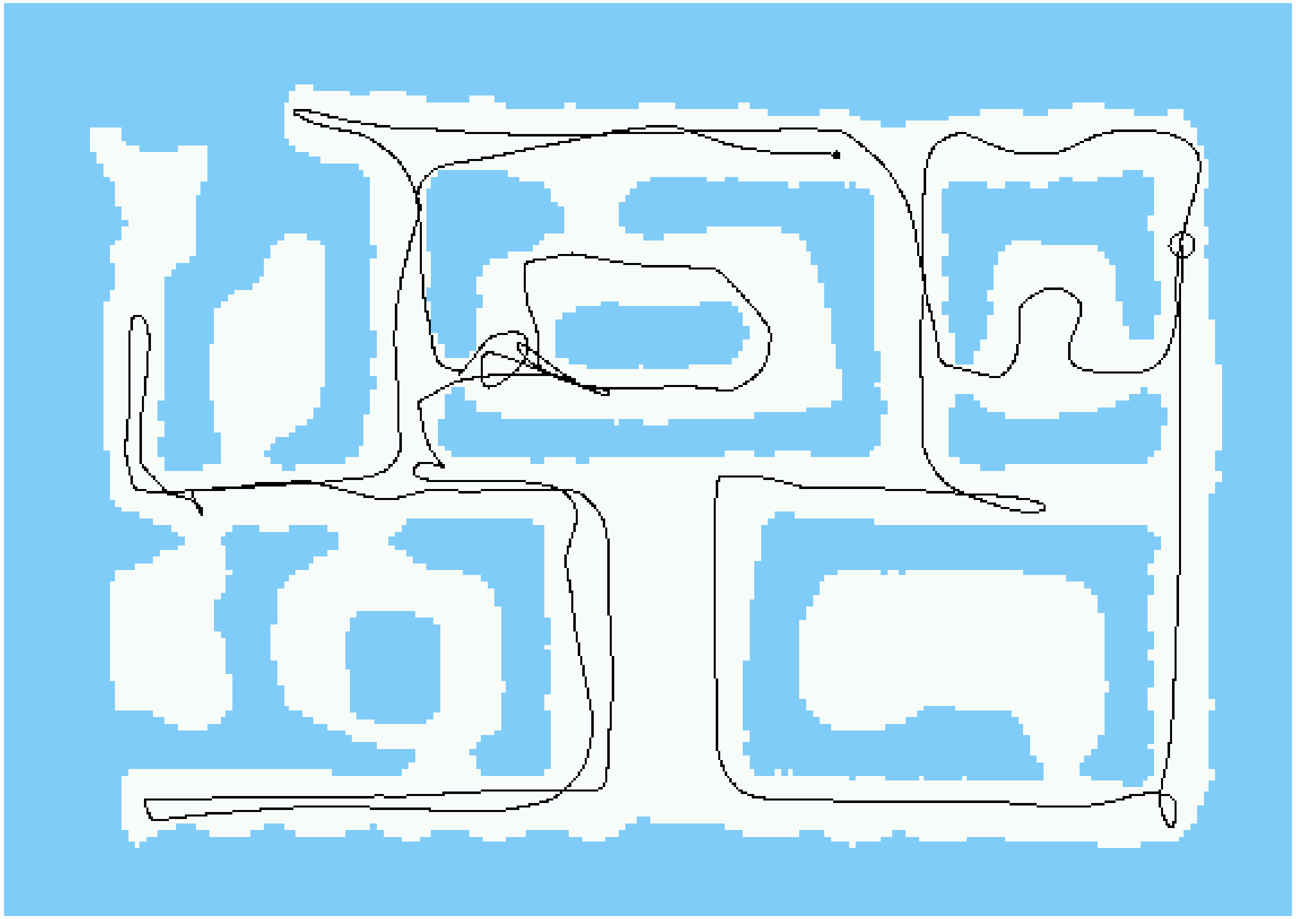
After Incorporating Ten Ultrasound Scans



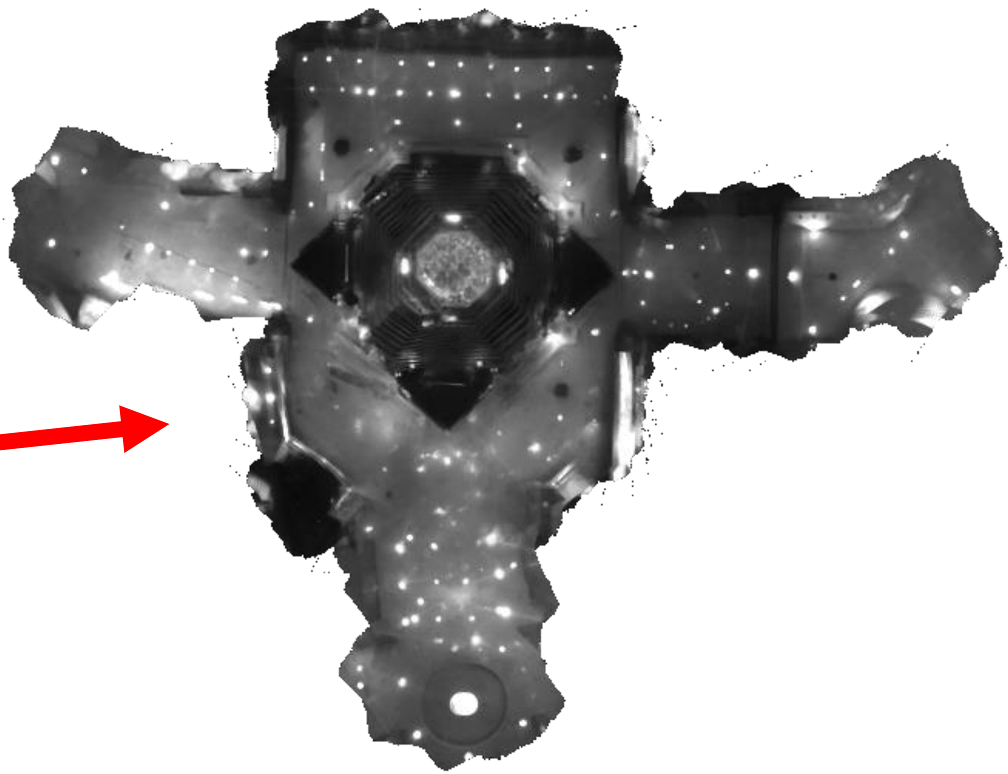
After Incorporating 65 Ultrasound Scans



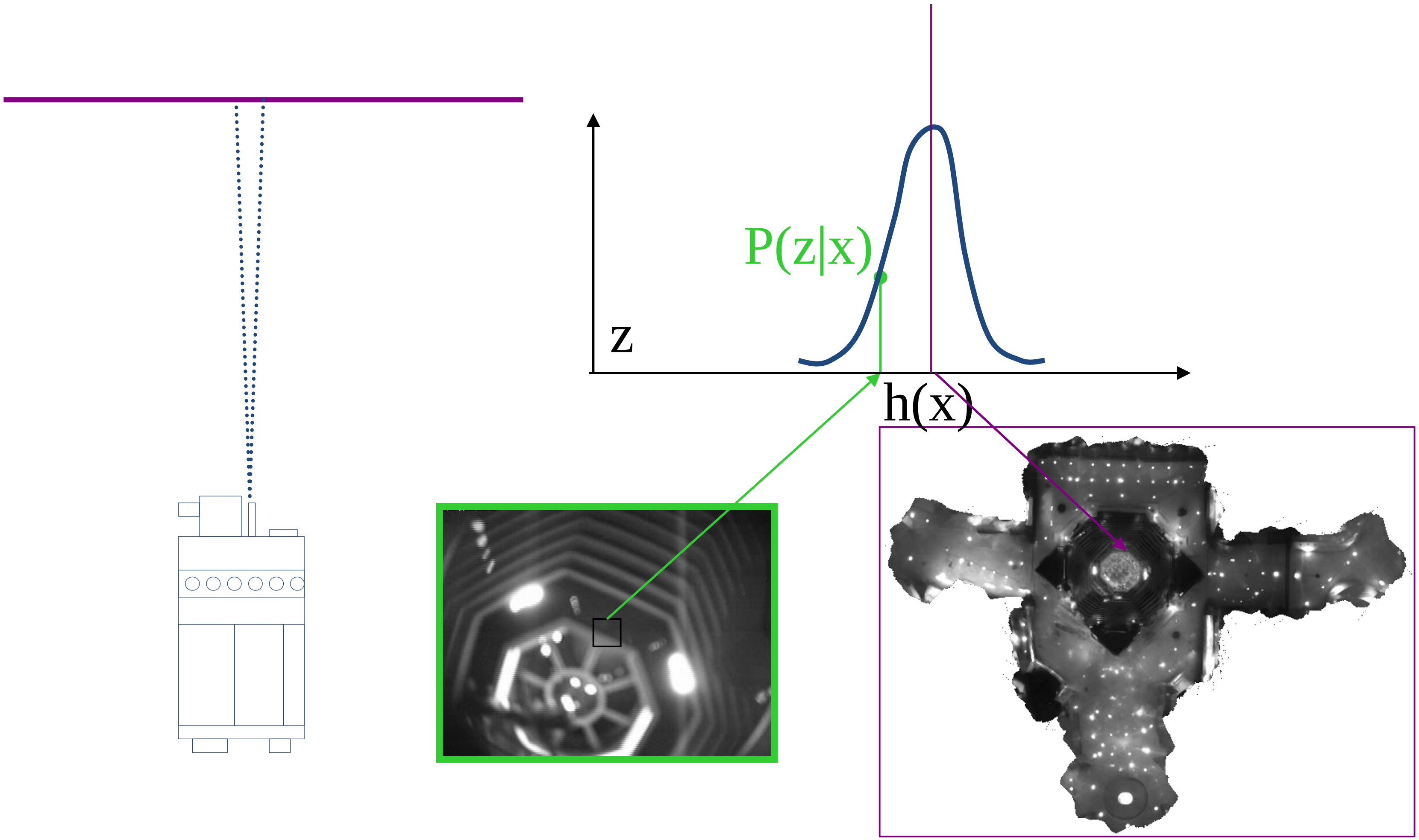
Estimated Path



Using Ceiling Maps for Localization

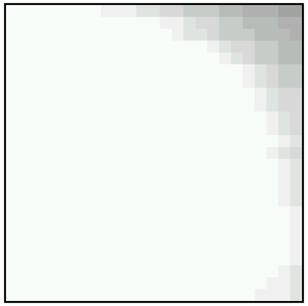


Vision-based Localization

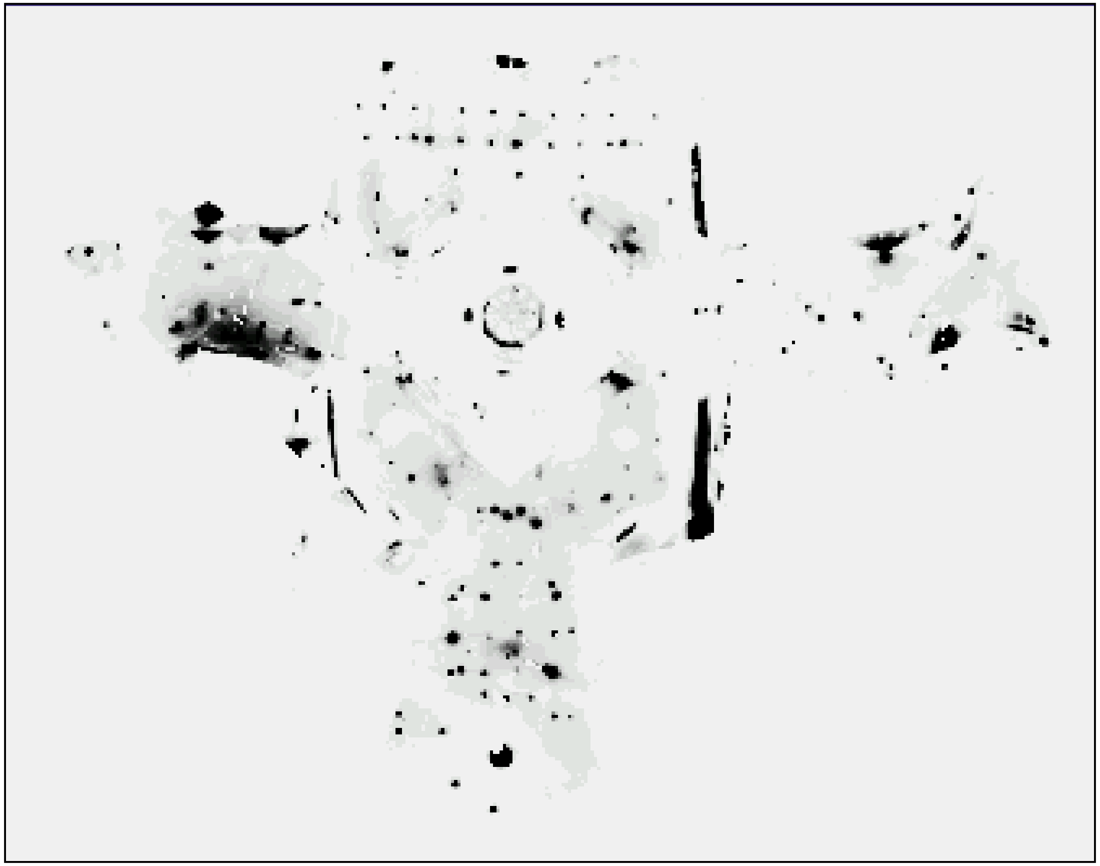


Under a Light

Measurement z:

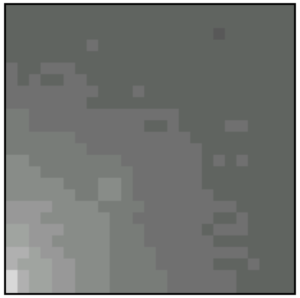


***P(z|x)*:**

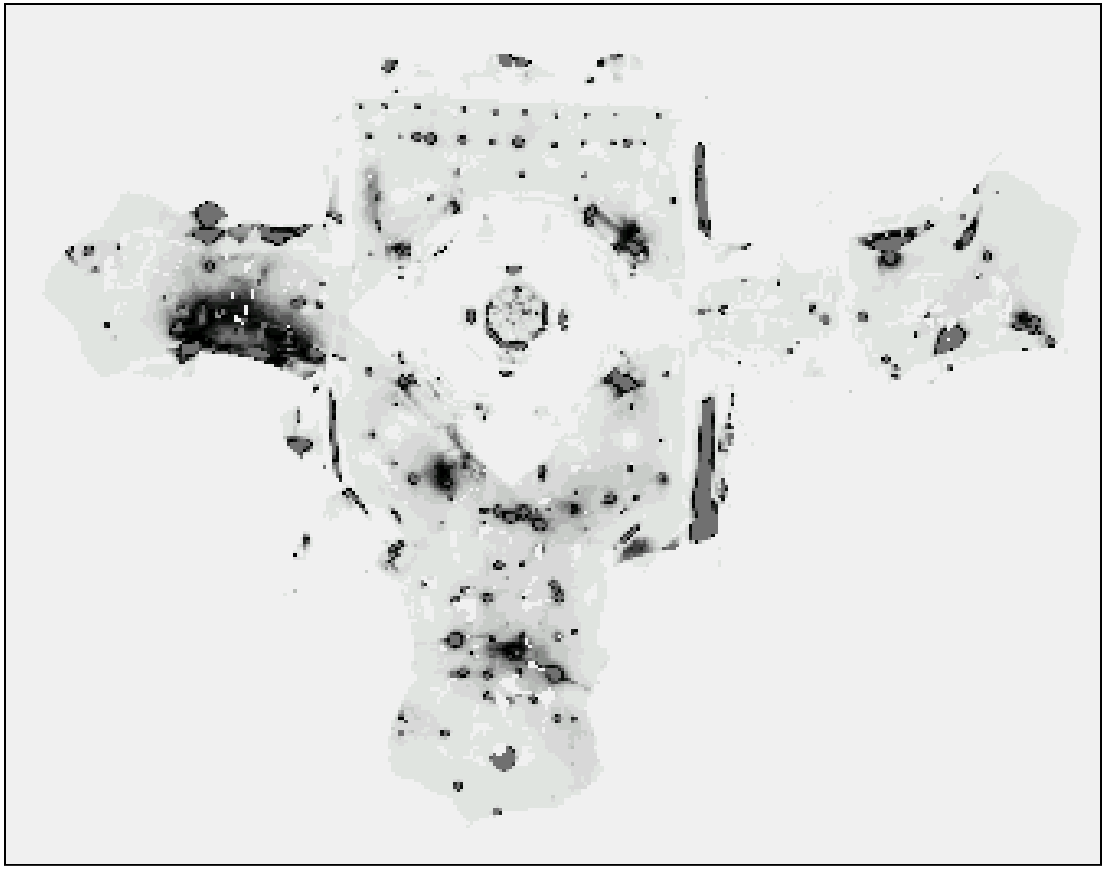


Next to a Light

Measurement z:



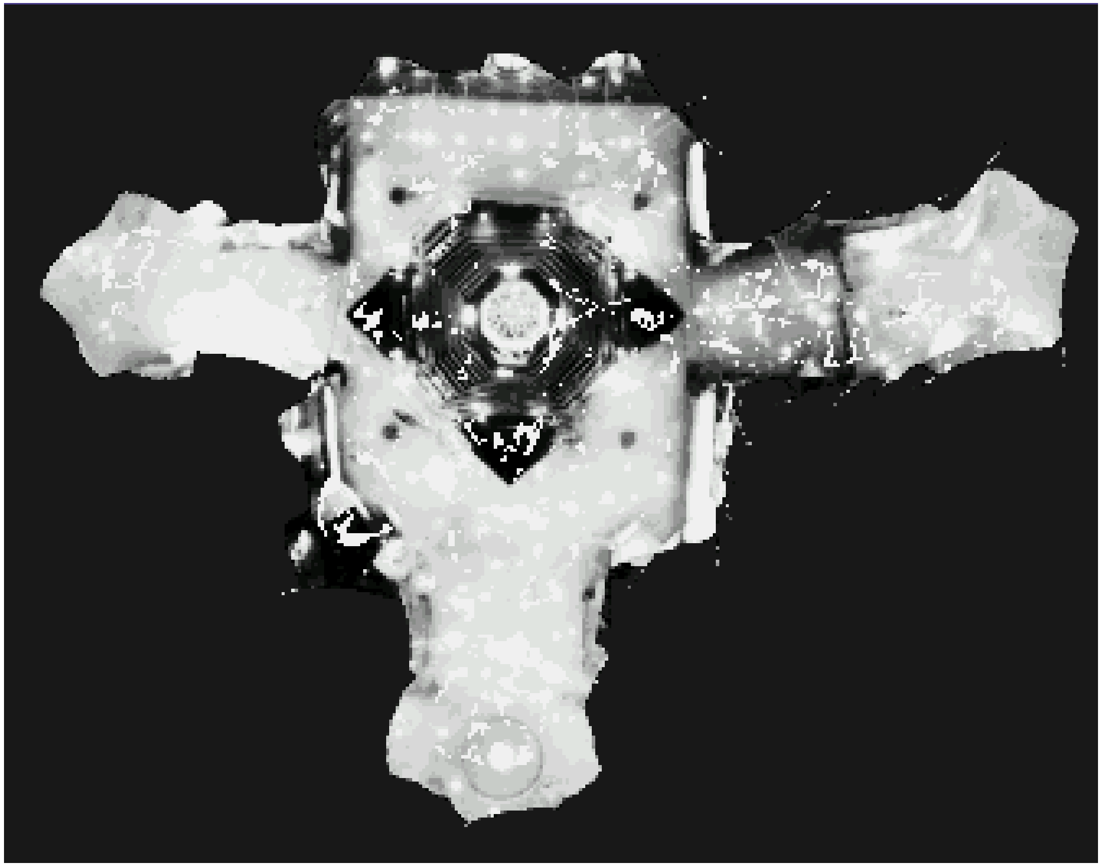
***P(z|x)*:**



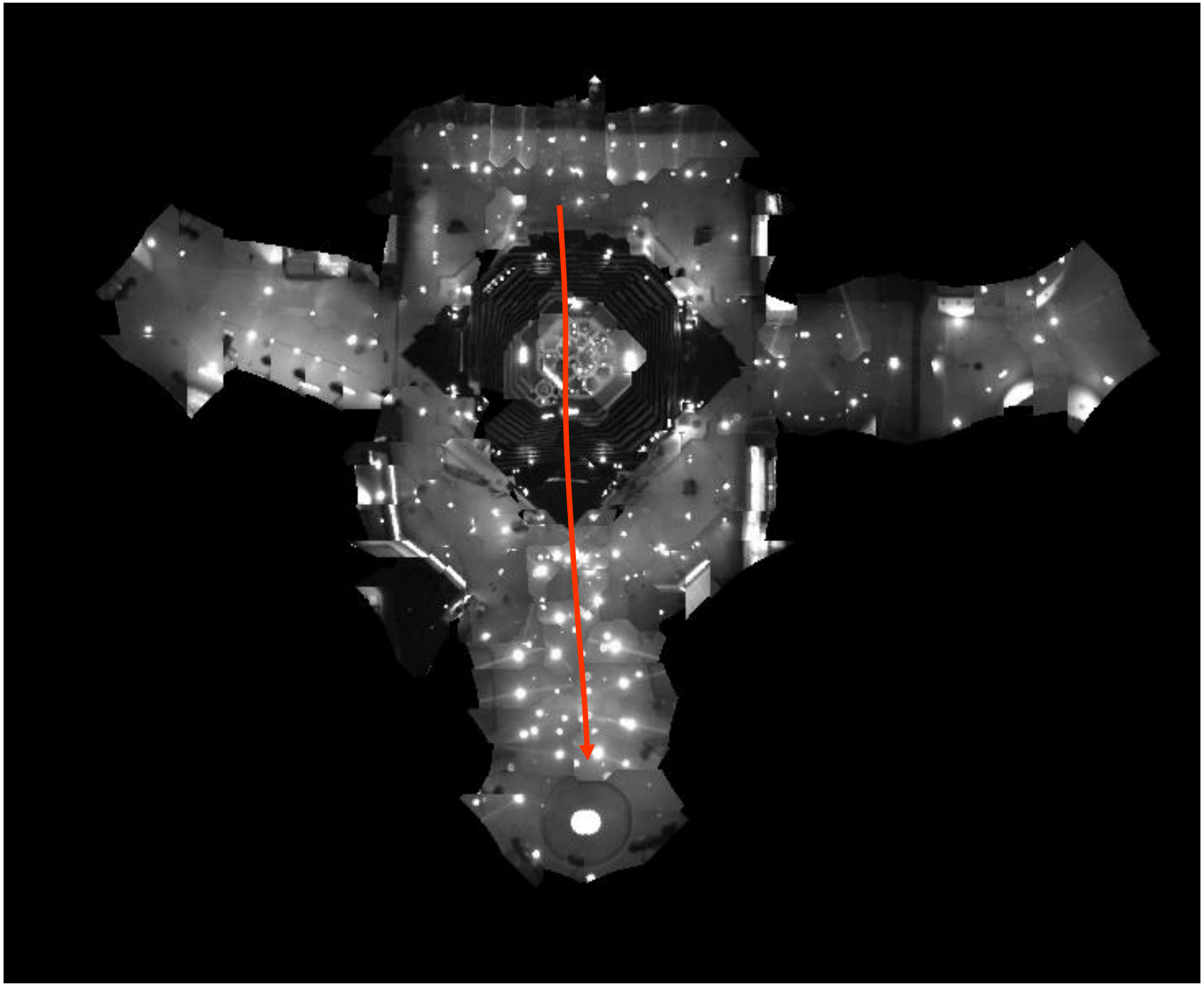
Elsewhere

Measurement z:

***P(z|x)*:**



Global Localization Using Vision



Limitations

- The approach described so far is able to
  - track the pose of a mobile robot and to
  - globally localize the robot.

- How can we deal with localization errors (i.e., the kidnapped robot problem)?

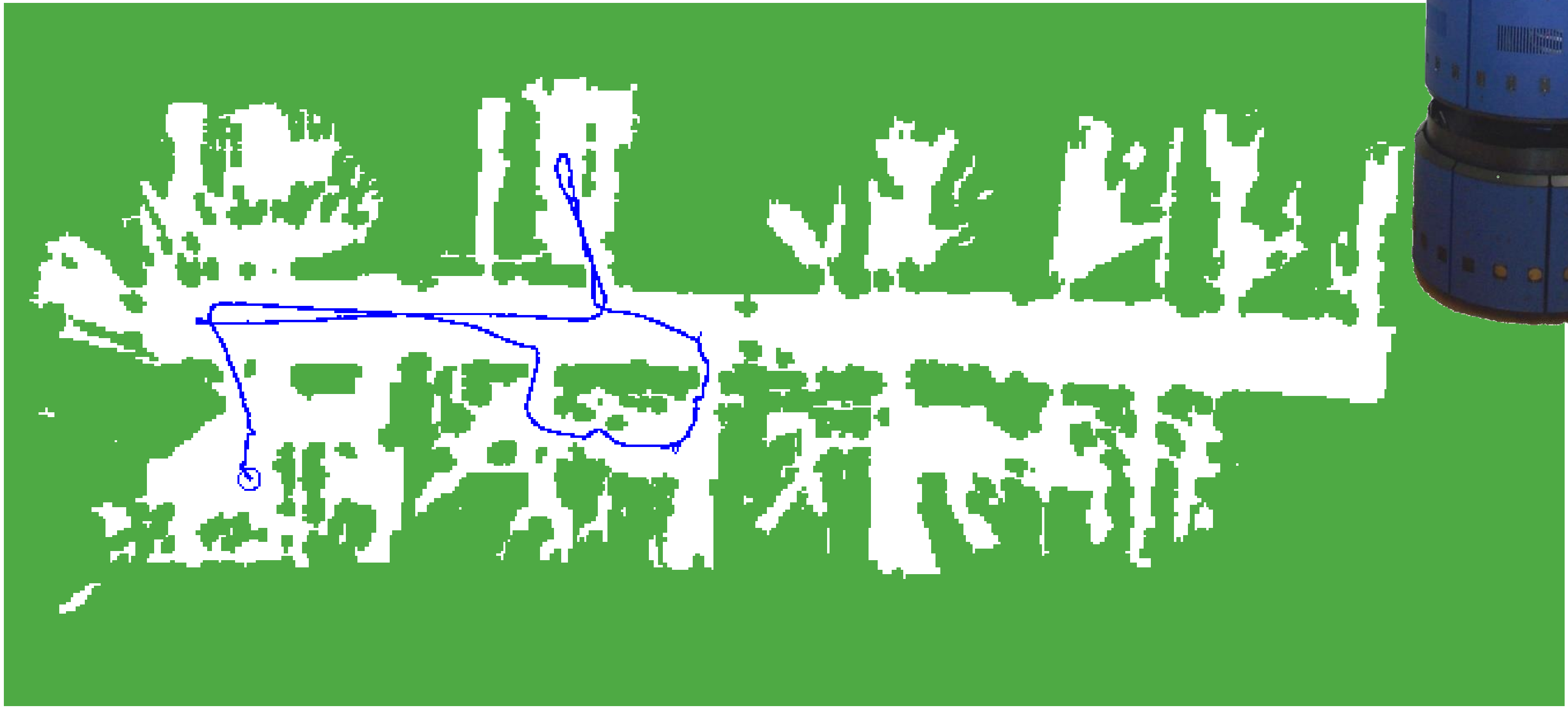
Approaches

- Randomly insert samples (the robot can be teleported at any point in time).
- Insert random samples proportional to the average likelihood of the particles (the robot has been teleported with higher probability when the likelihood of its observations drops).

Random Samples: Vision-Based Localization

936 Images, 4MB, .6secs/image

Trajectory of the robot:

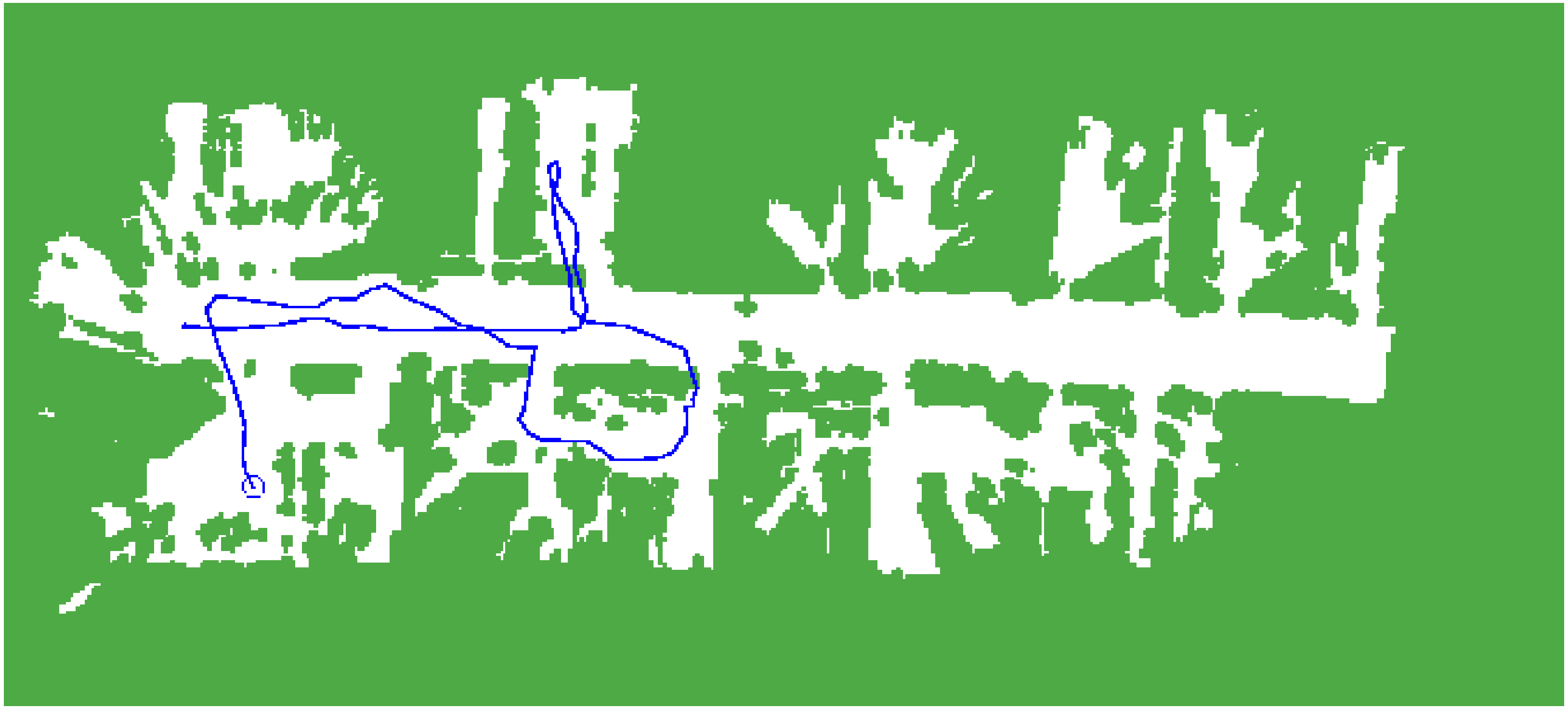


Odometry Information

Image Sequence

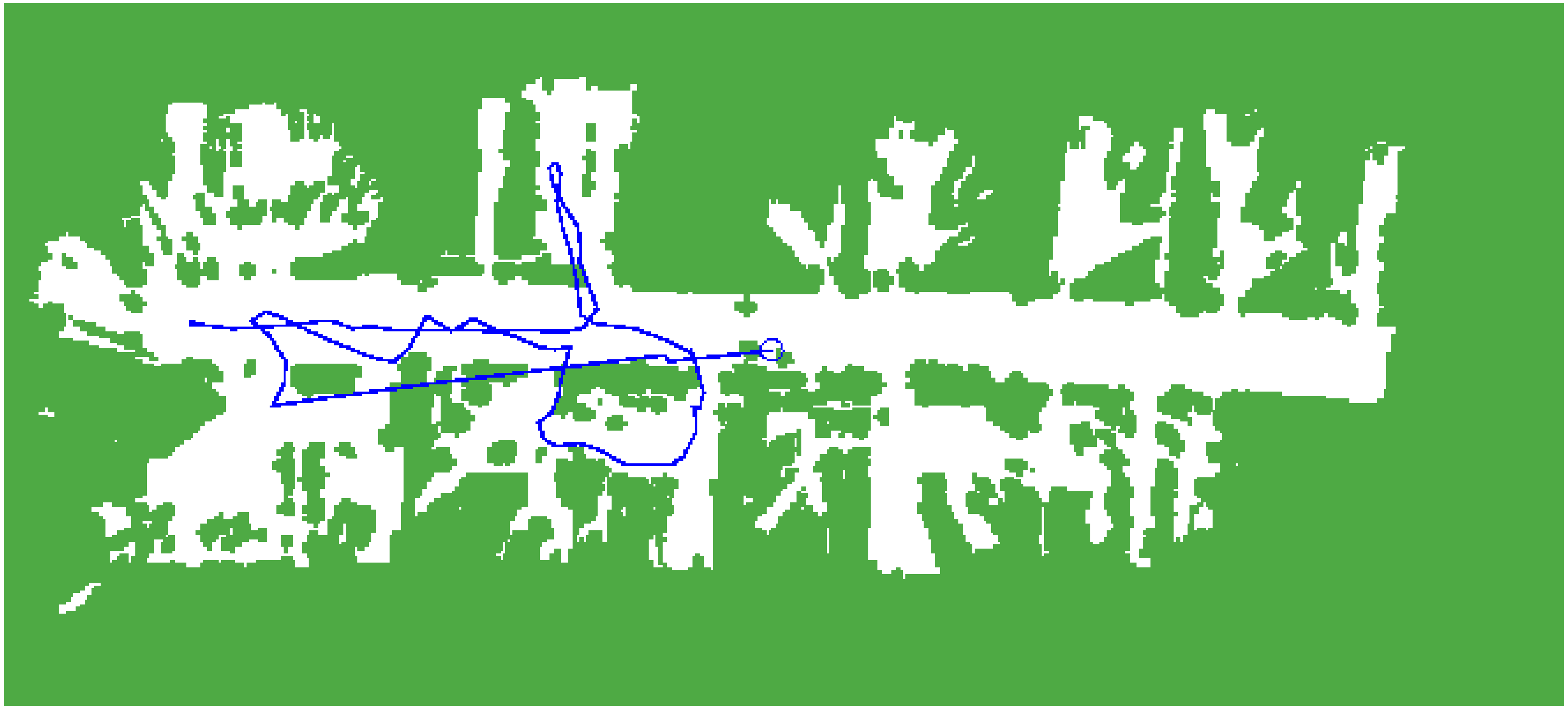
Resulting Trajectories

Position tracking:

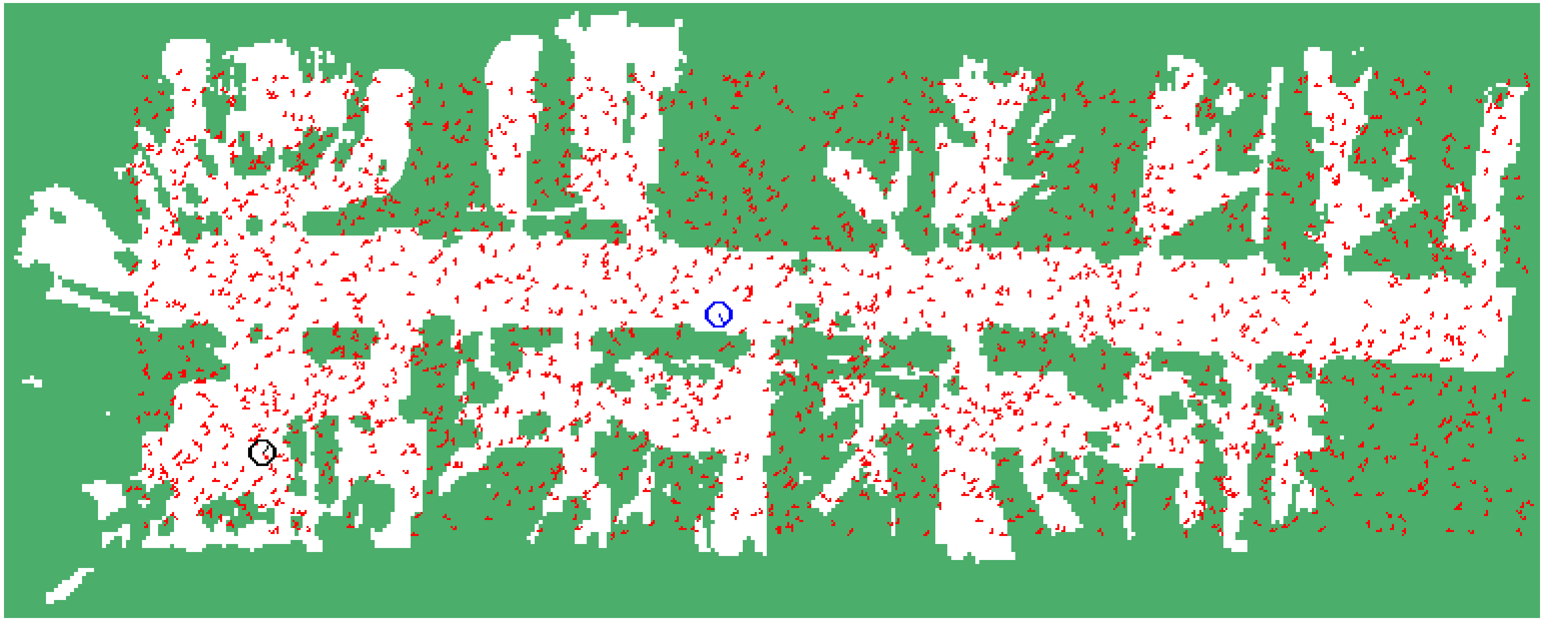


Resulting Trajectories

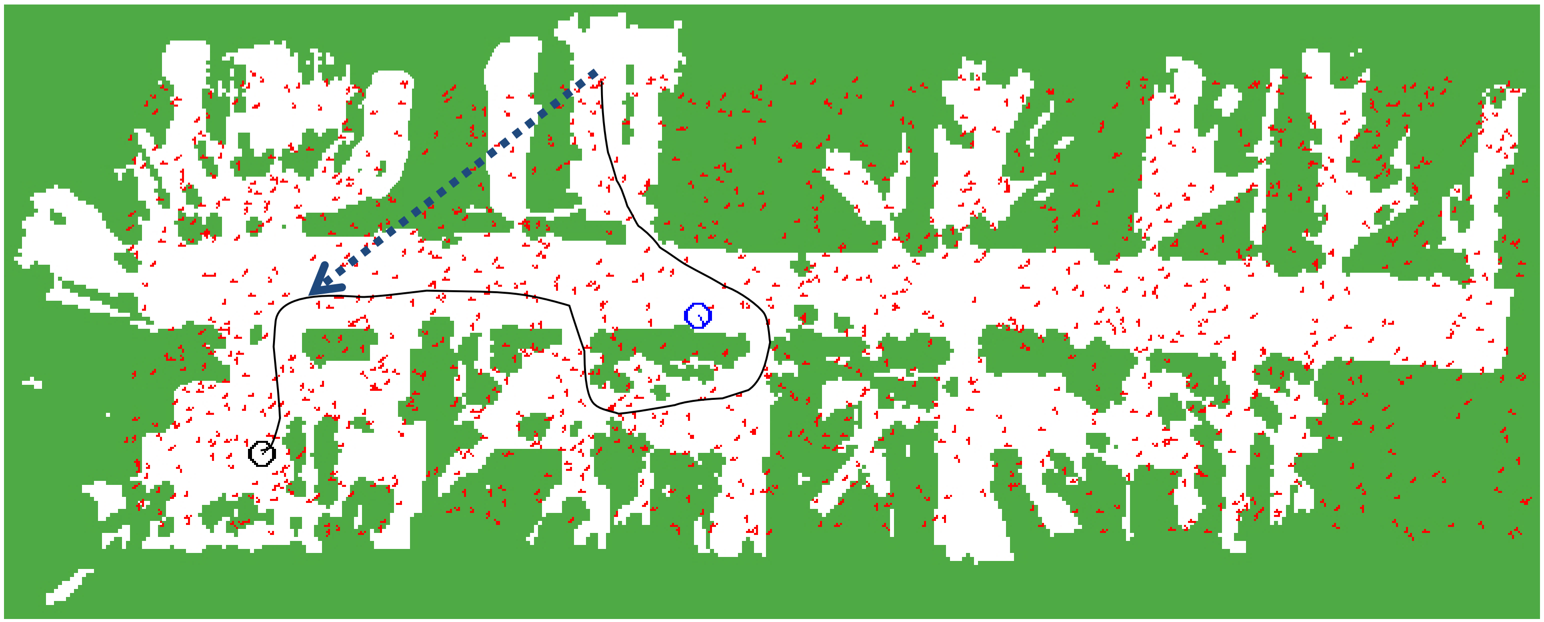
Global localization:



Global Localization



Kidnapping the Robot



Summary

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples.
- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.