Perception | **Correlation & Convolution**
Autonomous Mobile Robots

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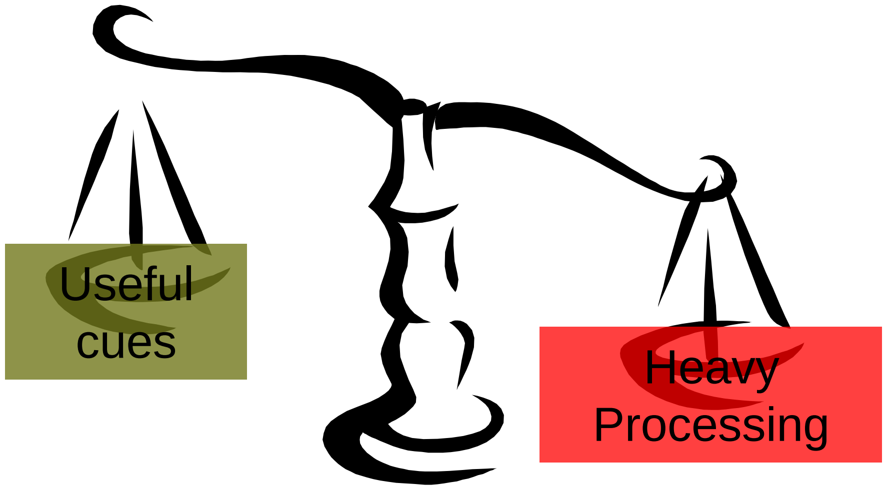
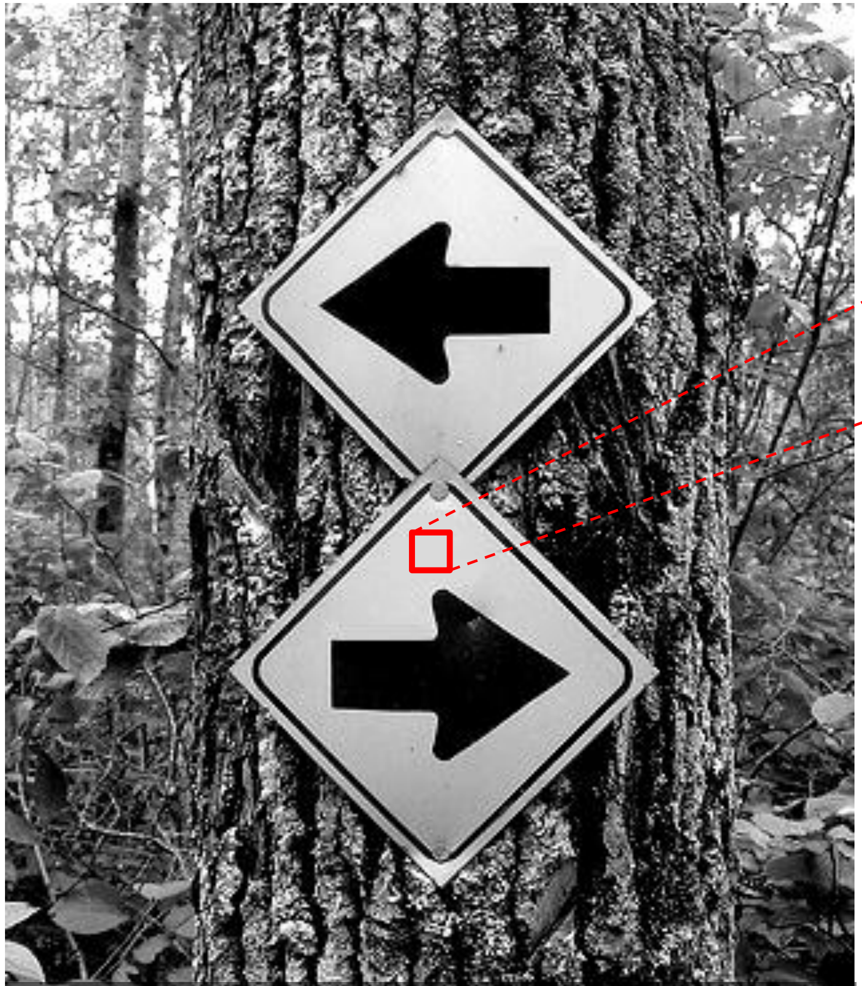
Image Saliency | data reduction

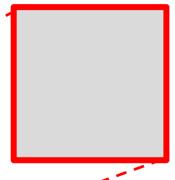
- Monochrome image ⇨ matrix of intensity values
- Typical sizes:
  - 320 x 240 (QVGA)
  - 640 x 480 (VGA)
  - 1280 x 720 (HD)
- Intensities sampled to 256 grey levels ⇨ 8 bits
- Images capture a lot of information



⇨ Reduce the amount of input data:
preserving useful info & discarding redundant info

Image Saliency | what is USEFUL, what is REDUNDANT ?





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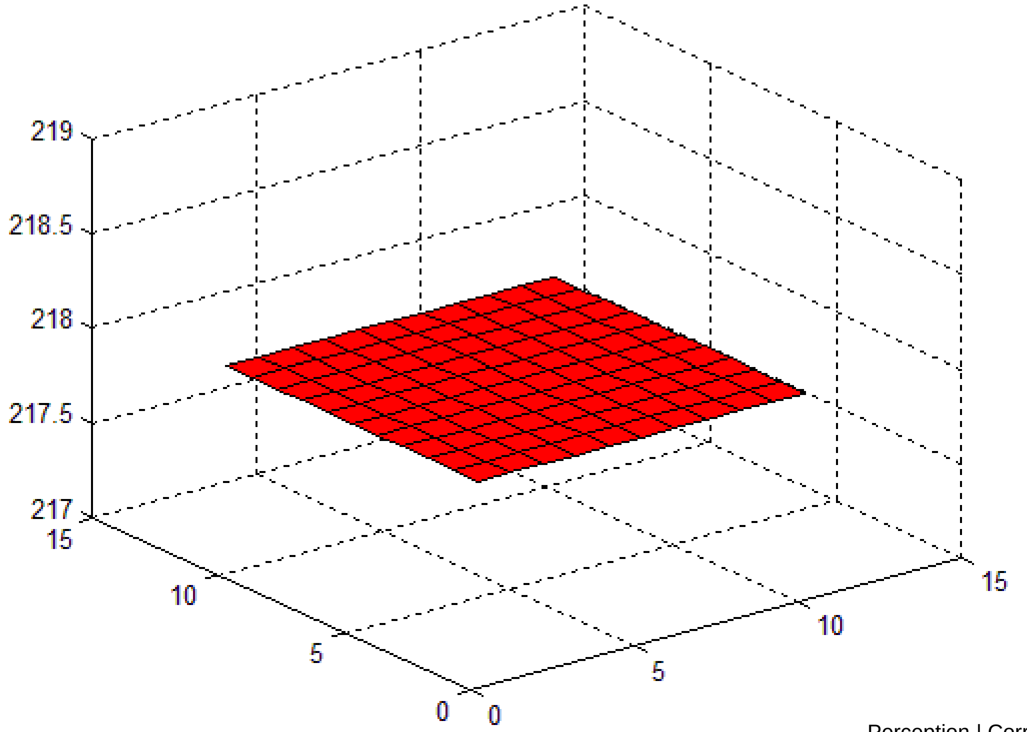

Image Saliency | what is USEFUL, what is REDUNDANT ?

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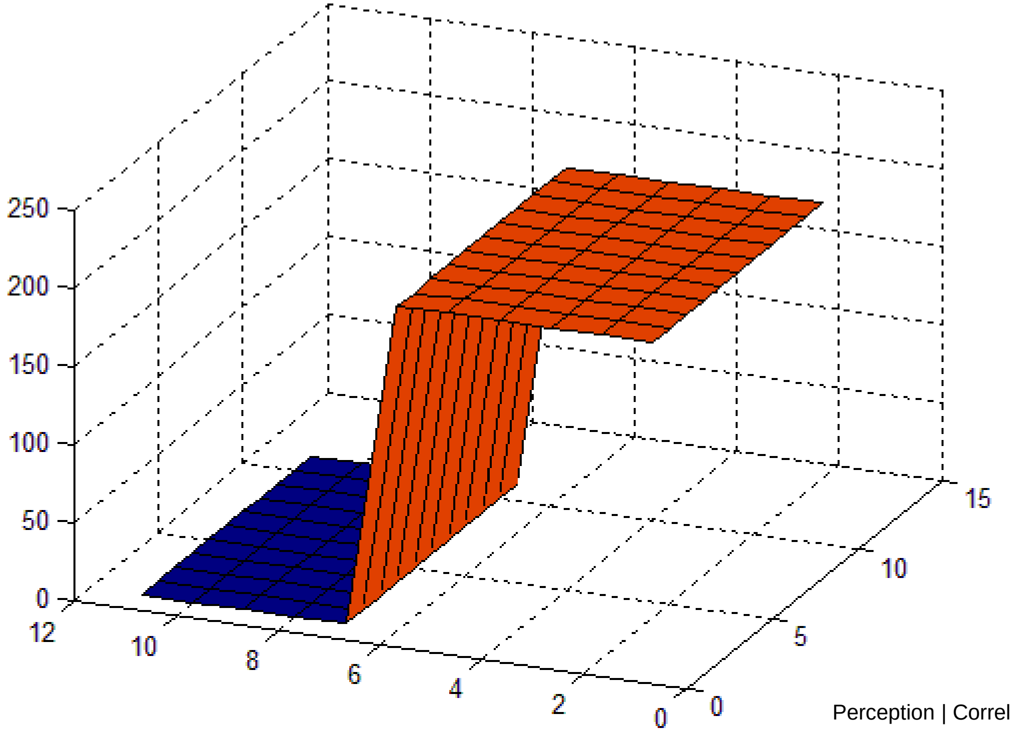
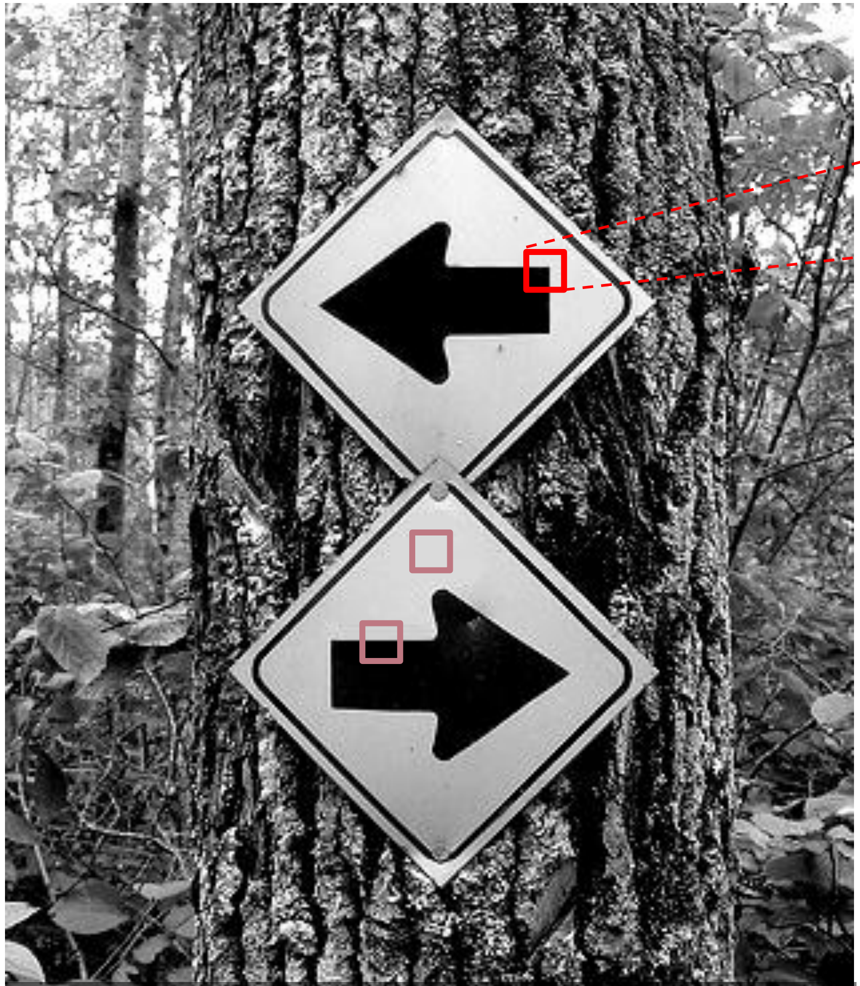


Image Saliency | what is USEFUL, what is REDUNDANT ?



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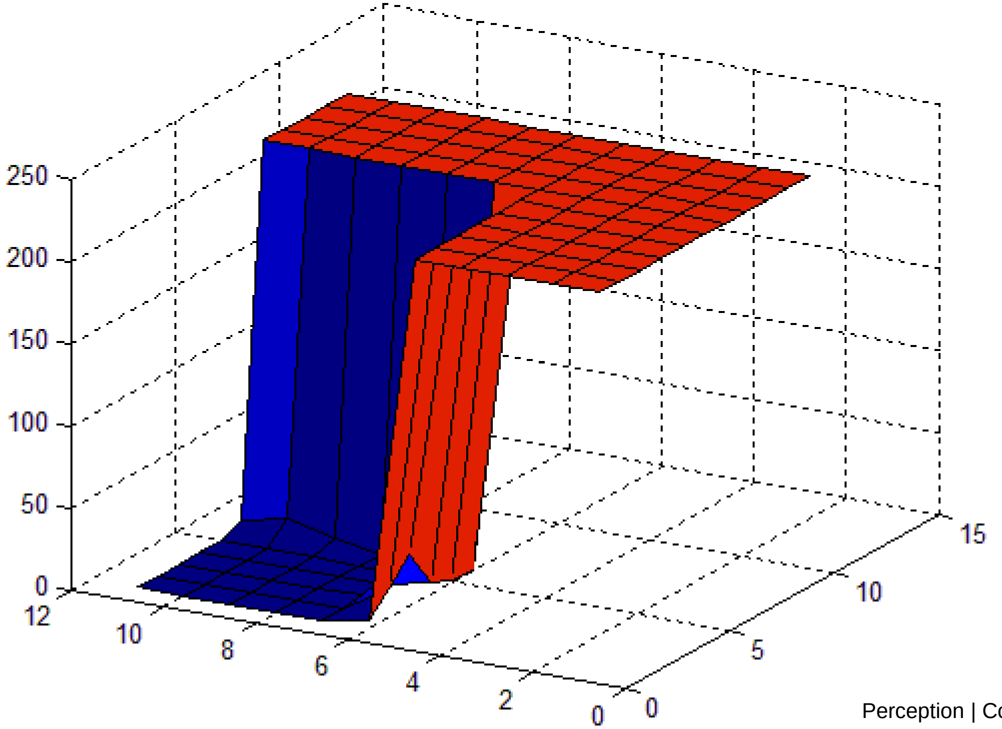


Image filtering | spatial filters

- "filtering": accept / reject certain components
- S_{xy} : neighborhood of pixels around the point (x,y) in an image I
- Spatial filtering operates on S_{xy} to generate a new value for the corresponding pixel at output image J

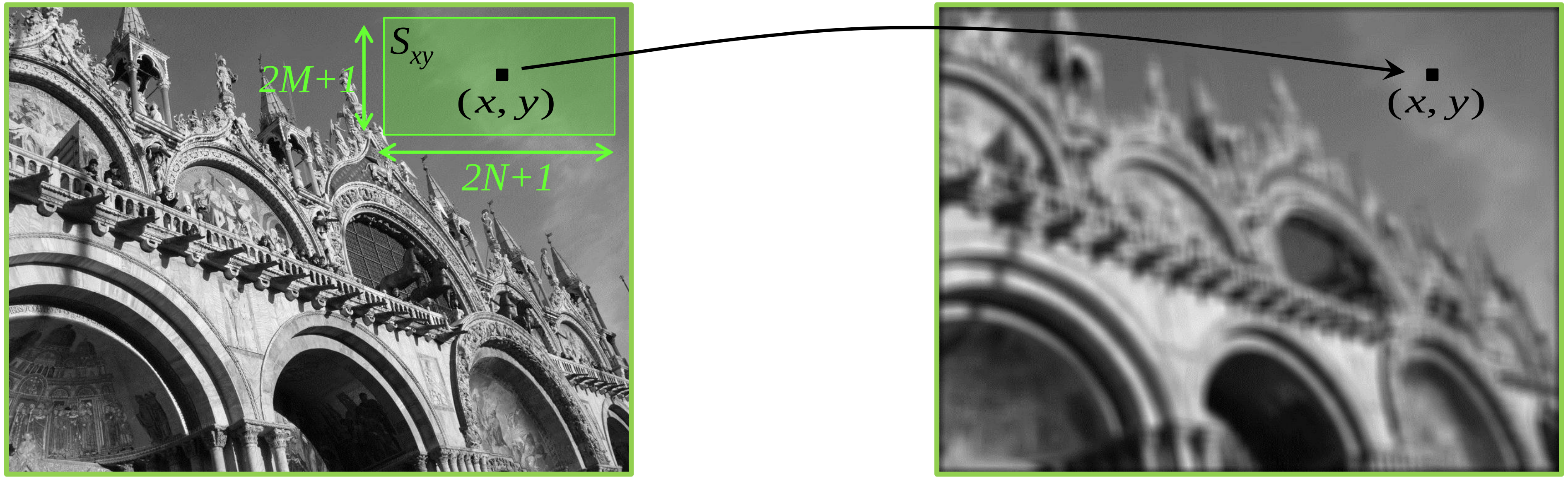


Image I

Filtered Image $J = F(I)$

- For example, an averaging filter is: $J(x,y) = \dfrac{\sum_{(r,c)\in S_{xy}} I(r,c)}{(2M+1)(2N+1)}$

Image filtering | linear, shift-invariant filters

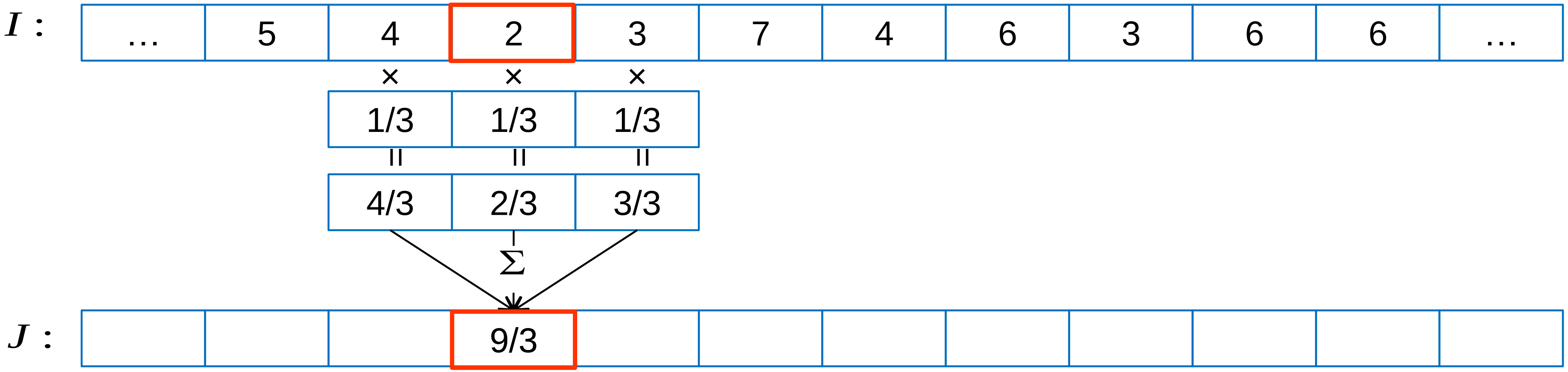
- **Linear**: every pixel is replaced by a linear combination of its neighbors

- **Shift-invariant**: the same operation is performed on every point on the image
- Basic & very useful filtering operations:
  - Correlation
  - Convolution

- Brief study of these filters in the simplest case of 1D images (i.e. a row of pixels) & their extension to 2D

Image filtering | correlation

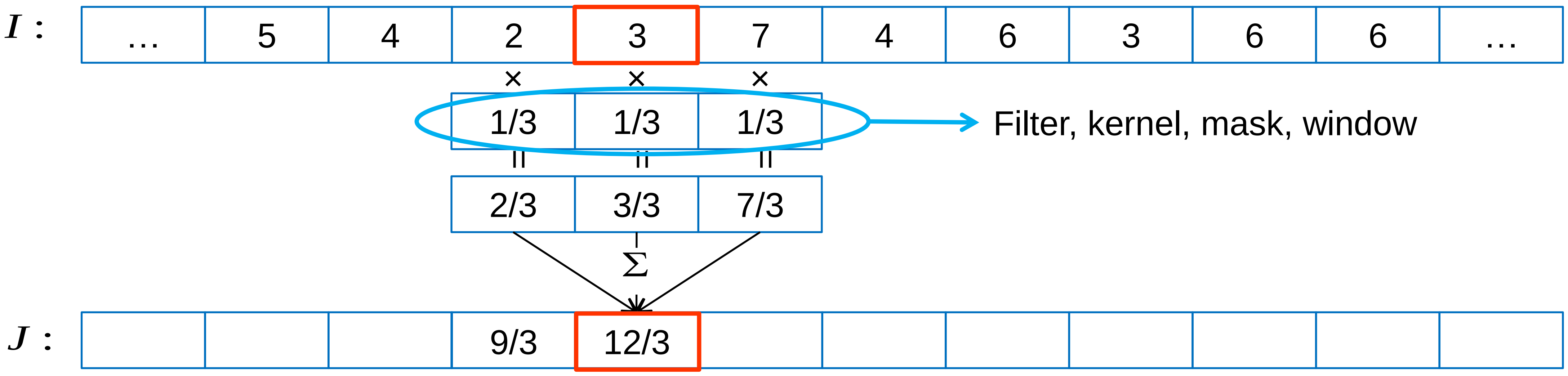
- An averaging filter



- How to handle boundaries?
  - Ignore filtered values at boundaries
  - Pad image with zeros
  - Pad image with first/last image values

Image filtering | correlation

- An averaging filter



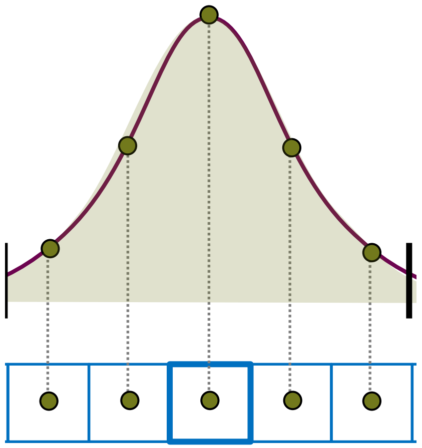
- Formally, Correlation is $J(x) = F \circ I(x) = \sum_{i=-N}^{N} F(i) I(x+i)$

- In this smoothing example $F(i) = \begin{cases} 1/3, & i \in [-1,1] \\ 0, & i \notin [-1,1] \end{cases}$

Other examples of smoothing filters?

Image filtering | constructing filter from a continuous f^n

- Common practice for image smoothing:
  use a Gaussian



Normalize filter so that values always add up to 1

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

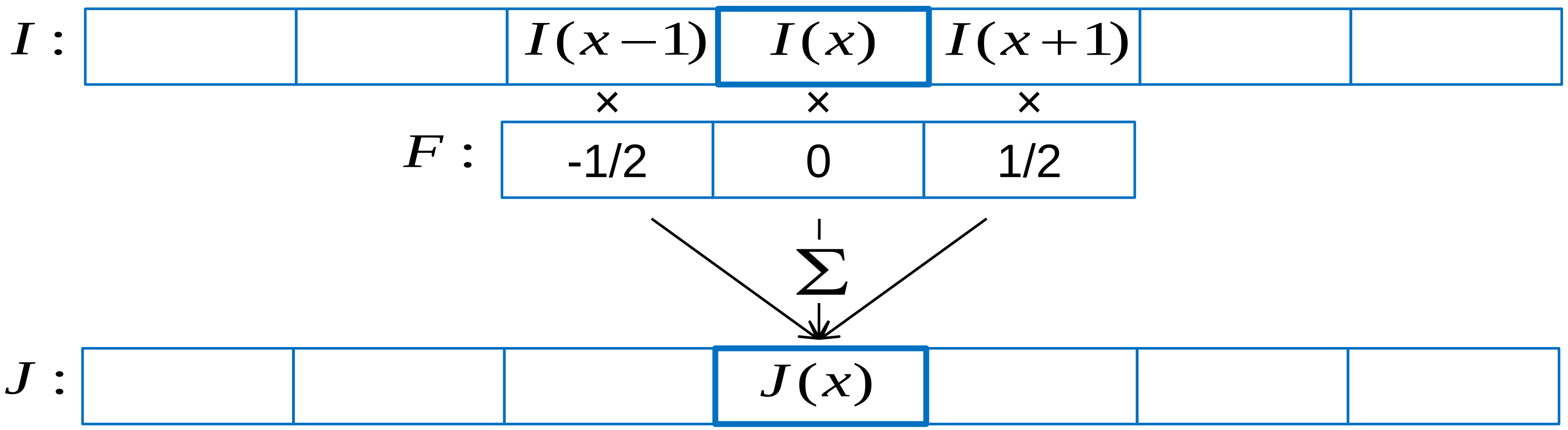
$$\mu = 0$$

σ : controls the amount of smoothing

- Near-by pixels have a bigger influence on the averaged value rather than more distant ones

Image filtering | taking derivatives with correlation

- Derivative of an image: quantifies how quickly intensities change (along the direction of the derivative)
- Approximate a derivative operator:



$$J(x) = \frac{I(x+1) - I(x-1)}{2}$$

Image filtering | matching using correlation

- Find locations in an image that are similar to a ***template***
- Filter = template

3	8	3

➪ test it against all image locations

I :

3	2	4	1	3	8	4	0	3	8	7	7

- Similarity measure: Sum of Squared Differences (**SSD**) – minimize

$$\sum_{i=-N}^{N}(F(i)-I(x+i))^2=\sum_{i=-N}^{i=N}(F(i))^2+\sum_{i=-N}^{i=N}(I(x+i))^2-2\sum_{i=-N}^{i=N}(F(i)I(x+i))$$

J :

26	37	21	50	54	1	50	65	59	16	42	17

Image filtering | matching using correlation

- Find locations in an image that are similar to a ***template***
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I :

3	2	4	1	3	8	4	0	3	8	7	7

- Similarity measure: Sum of Squared Differences (**SSD**) – minimize

$$\sum_{i=-N}^{N}\left(F(i)-I(x+i)\right)^2=\sum_{i=-N}^{i=N}\left(F(i)\right)^2+\sum_{i=-N}^{i=N}\left(I(x+i)\right)^2-2\underbrace{\boxed{\sum_{i=-N}^{i=N}\left(F(i)I(x+i)\right)}}_{\text{Correlation}}$$

J :

26	37	21	50	54	1	50	65	59	16	42	17

- Similarity measure: Correlation? – maximize

J :

30	37	41	29	51	85	56	21	48	86	101	77

Image filtering | NCC: Normalized Cross Correlation

- Find locations in an image that are similar to a ***template***
- Filter = template

3	8	3

  ➪ test it against all image locations

I :

3	2	4	1	3	8	4	0	3	8	7	7

- Correlation value is affected by the magnitude of intensities
- Similarity measure: Normalized Cross Correlation (**NCC**) – maximize

$$\frac{\sum_{i=-N}^{i=N}\left(F(i)I(x+i)\right)}{\sqrt{\sum_{i=-N}^{i=N}\left(F(i)\right)^2}\sqrt{\sum_{i=-N}^{i=N}\left(I(x+i)\right)^2}}$$

J :

0.919	0.759	0.988	0.628	0.655	0.994	0.691	0.464	0.620	0.860	0.876	0.859

Image filtering | ZNCC: Zero-mean Normalized Cross Correlation

- Find locations in an image that are similar to a ***template***
- Filter = template

3	8	3

➪ test it against all image locations

I :

3	2	4	1	3	8	4	0	3	8	7	7

- For better invariance to intensity changes, try to eliminate all absolute effects on intensities

Similarity measure: Zero-mean Normalized Cross Correlation (**ZNCC**) – maximize

$$\frac{\sum_{i=-N}^{i=N}\left(F(i)-\mu_F\right)\left(I(x+i)-\mu_{I_x}\right)}{\sqrt{\sum_{i=-N}^{i=N}\left(F(i)-\mu_F\right)^2}\sqrt{\sum_{i=-N}^{i=N}\left(I(x+i)-\mu_{I_x}\right)^2}}$$

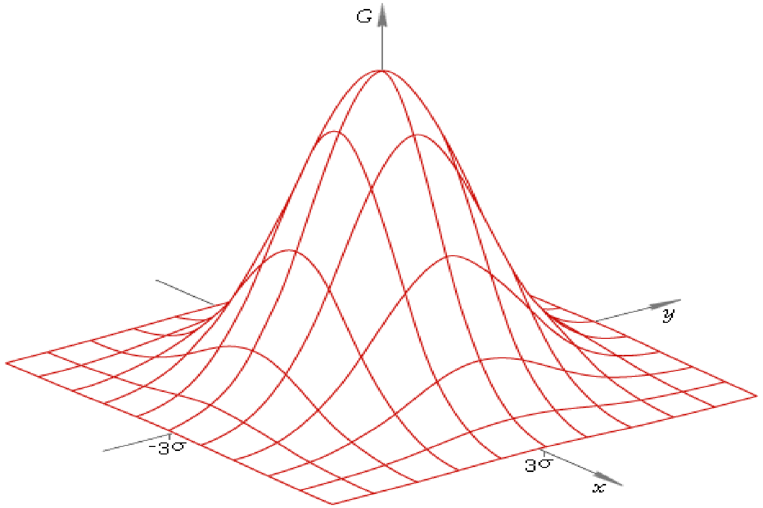
where

$$\begin{cases} \mu_F = \dfrac{\sum_{i=-N}^{N} F(i)}{2N+1} \\ \mu_{I_x} = \dfrac{\sum_{i=-N}^{N} I(x+i)}{2N+1} \end{cases}$$

Image filtering | 2D Gaussian smoothing

- Correlation in 2D: $F \circ I(x,y) = \sum_{j=-M}^{M} \sum_{i=-N}^{N} F(i,j) I(x+i, y+j)$
- A general, 2D Gaussian $G(x,y) = \frac{1}{2\pi |S|^{1/2}} e^{-\frac{1}{2}\begin{pmatrix} x \\ y \end{pmatrix} S^{-1} (x \quad y)}$
- We usually want to smooth by the same amount in both *x* and *y* directions $S = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$
- So this simplifies to:

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \underbrace{\boxed{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}}}_{G_\sigma(x)} \cdot \underbrace{\boxed{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}}}}_{G_\sigma(y)}$$



- A “separable” filter

Image filtering | convolution

- Convolution is ***equivalent*** to Correlation with a flipped filter before correlating
- CONVOLUTION: $J(x) = F * I(x) = \sum_{i=-N}^{N} F(i)I(x-i)$
- CORRELATION: $J(x) = F \circ I(x) = \sum_{i=-N}^{N} F(i)I(x+i)$

So if $F = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
$F' = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$
Then, $J(x) = J'(x)$

- Likewise, in 2D we flip the filter both horizontally & vertically

$$J(x,y) = F * I(x,y) = \sum_{j=-M}^{M} \sum_{i=-N}^{N} F(i,j)I(x-i,y-j)$$

- Key difference with correlation: **convolution is associative**: $F * (G * I) = (F * G) * I$
- Very useful!
- Example: smooth an image & take its derivative ⇨ convolve the Derivative filter with the Gaussian filter & convolve the resulting filter with the Image