Particle Filter Based Localization of Access Points Using Direction of Arrival on Mobile Robots

Ravi Parashar   Ramviyas Parasuraman

***Abstract*—Localization of autonomous vehicles in unknown and unstructured GPS-denied environments is still a relevant and major research challenge in the field of Robotics. Applications of such research can be found in search and rescue missions and connected vehicles, where multiple robots need an efficient solution for simultaneous localization through multi-sensor integration so that they can effectively cooperate and coordinate tasks amongst themselves. In this paper, we propose a novel method for estimating the position of a WiFi access point in relation to a moving robot. Specifically, we exploit the integration of two sensors: Direction-of-arrival (DOA) of WiFi signals and the robot's odometry and combine them with Gaussian probabilistic sampling in a Particle Filter framework. We evaluate the proposed method in terms of accuracy and computational efficiency through extensive trials on datasets gathered from real-world measurements with mobile robots and compared our method against standard approaches. The results demonstrate superior localization accuracy (up to 3x improvement) and capability for most practical applications.**

***Index Terms*—Localization, Mobile Robots, Direction Of Arrival, WiFi Access Point, Received Signal Strength, Particle Filter**

I. Introduction

Research on efficient and accurate localization, particularly in real-time, in GPS-denied environments, is a very pertinent problem in the field of Robotics [1]. It is well known that GPS signals can be unreliable due to distortions such as walls or weather changes that can result in errors measured in tens of meters [2]. Therefore, GPS data can be quite imprecise when using it as a tool for accurately localizing several robots [3], which makes it unsuitable to be used independently for inter-vehicle localization in real traffic conditions [4].

On the other hand, researchers have focused on alternative technologies such as expensive laser-based ranging systems to obtain accurate localization within a small region. Out of these alternatives, commodity radio and WiFi-based localization have been prominently studied, especially in indoor environments, because of the inexpensive nature of this sensing medium and its extended range [5]. Specifically, consistent improvement of 802.11 WiFi network standards and protocols, including 802.11p for vehicular environments, has led to vast amounts of research on the topic of WiFi signal localization using methods such as machine learning, fingerprinting, and probabilistic frameworks [6], [7], [8]. Efficient solutions to these problems are vital, as there are many potential and new applications of WiFi Access Point (AP) localization [9].

The authors are with the Heterogeneous Robotics Lab, Department of Computer Science, University of Georgia, Athens, GA 30602, USA. Email: {*ravi.parashar25,ramviyas*}*@uga.edu*.

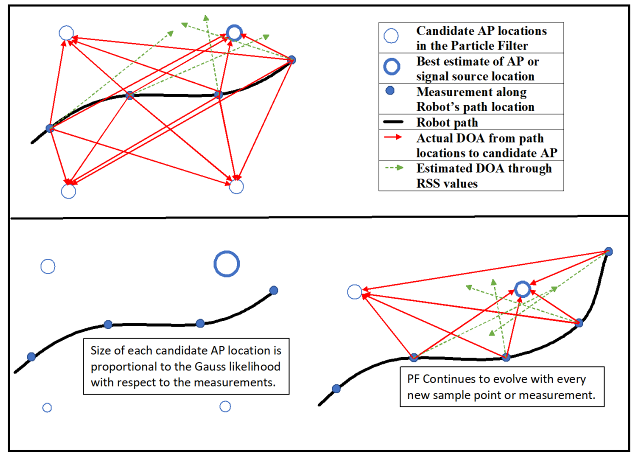


Fig. 1. Illustration of a one-step PF update process using our proposed AP localization method based on Gaussian probability over DOA of RSS.

Many existing solutions either seek accurate offline estimation approaches or real-time self-estimation of a robot's position through multi-sensor integration and the use of AP(s) [10]. Methods that perform online localization of WiFi AP require either dedicated infrastructure with multiple beacons or multiple (sometimes, expensive) sensors spatially distributed in the environment [11]. Both of these assumptions cannot be used in a realistic real-time localization of APs in the V2X scenario, where there is a need for accurate online localization of radio sources, especially in cases of cooperation among connected vehicles or for collision warning systems without dedicated or expensive infrastructure [4].

In this paper, we propose a new method to perform AP (radio source) localization using a mobile robot by exploiting the integration of a mobile robot's odometer sensor and Received Signal Strength Indicator (RSSI) data of WiFi signals (acting as another sensor). Specific contributions include fusing the robot's trajectory information from the odometry and the Direction of Arrival (DOA) information from RSSI to estimate the probability of candidate AP locations through a Gaussian probabilistic approach for resampling. This is then coupled with a novel Particle Filter (PF) based framework to estimate AP location (to which the mobile robot or a connected vehicle is connected) for achieving a balance between real-time efficiency and superior accuracy of online localization. An illustration of the proposed PF method is shown in Fig. 1. Our method is compared against standard approaches on open real-world datasets of indoor RSS measurements[1] [12], [13].

[1] Datasets link: https://github.com/herolab-uga/indoor-rssi-mobile-robot.

II. Related Work

Literature in localization using radio signals is diverse. Some work deals with localizing the AP (radio sources), while most work focuses on the localization of the robots or vehicles (self-localization). Here, we specifically look at locating a single robot in relation to one or more WiFi AP(s) or vice versa. Given that the indoor environment is more challenging than the outdoor line of sight scenarios, we concentrate our attention on indoor localization methods, which, once proven, can be easily extended to outdoor situations. Most methods exploiting radio signals use the RSSI metric, which produces localization errors in the order of a few meters [14], [15], [16].

Most works use Bayesian estimation techniques such as Kalman Filters or PF. In [10], the robot localization is achieved using a fusion of GPS, Odometry, and RSSI values. Several works use a combination of online and offline approaches. For instance, in [6], the authors propose to build a WiFi mapping of the environment using Gaussian Processes with RSS from a single AP and then localize the robot within the WiFi map. They also show that WiFi localization performed consistently better than typical LIDAR and Camera-based data demonstrating the superior strengths of WiFi signals as localization means. In [11], localization of a robot is performed using RSS data from multiple APs and a combined approach of offline learning and online PF. This method achieved a few meters of localization accuracy. [7] employs a similar approach. In general, methods like random forests and fingerprinting based offline machine learning methods are more appropriate in environments with multiple WiFi APs [17], [18].

A robot could also utilize DOA estimations of the RSS signal [19]. One needs at least two DOA estimations to approximate the location of the WiFi signal source. Approaches that exploit DOA (or gradient directions) of RSS data have been demonstrated in large environments with the ability to control the robot's trajectory [20] or applying spatial correlation techniques to generalize the DOA information within a discrete space grid of the sampled trajectory [21], [22]. They provide DOA estimation accuracy in the order of 25-35 degrees and can work well in outdoor or line of sight conditions.

The literature review suggests that an accuracy close to 1 m or sub-cm level is required for reasonable WiFi localization systems [1], [23], [4]. Previous approaches typically use combinations of filtering, fingerprinting, maximum likelihood, and least-squares methods to obtain location estimates. We depart from previous approaches by calculating Gaussian probability-based likelihood based on the errors in RSS DOA measured along robot's path and the AP location is dynamically estimated using a PF, which are multi-hypothesis, multi-modal, global estimation algorithms, to achieve a balance between both accuracy requirements and efficiency for online operation. Therefore, when added with motion models of an AP, we can accommodate the localization of a moving AP as well.

We compare our method with standard approaches such as Weighted Centroid Localization (WCL) [24] and Maximum Likelihood (ML) over DOA [25], [15] in a fair setting.

III. Proposed Approach

Problem Statement Consider a mobile robot connected to a fixed WiFi AP. The robot records its path along with DOA of RSS measurements as the tuple: $m_l = \{x_l, y_l, DOA_l\}$, $l = l, l-1, \ldots l-M$, where (x_l, y_l) is the location of the robot at location l and M is the number of previous samples considered along the completed path trajectory so far. The problem is to find the best estimate of the AP location $(x^*_{AP}, y^*_{AP}) \in \mathbb{R}^2$ which maximizes the probability of observing the measurement tuples when AP is at the estimated location $P(x_{AP}, y_{AP} | m_l, m_{l-1}, \ldots, m_{l-M})$, given that we employ an arbitrary method to estimate DOA from RSS values.

Our solution to the localization of a WiFi AP in relation to a moving robot employs Gaussian probability calculations using DOA estimates, which are used to weight the particles in a particle filter and ultimately produce an accurate AP location estimate. Firstly, let us look at how DOA will be calculated at a particular location. There are a number of ways to calculate DOA; one particular approach we employ from [12], [26] uses the central finite difference method on RSS measurements from WiFi antennas placed at the corners of a mobile robot. RSS can be modeled as a vector with two components and the gradient with respect to the center of the robot can be represented as $\vec{g} = [g_x, g_y]$. Using the central finite difference method [27], the gradient can be calculated as follows:

$$g_x = \frac{R_{UR} - R_{UL}}{2\Delta_{SX}} + \frac{R_{LR} - R_{LL}}{2\Delta_{SX}}, \tag{1}$$

$$g_y = \frac{R_{UR} - R_{LR}}{2\Delta_{SY}} + \frac{R_{UL} - R_{LL}}{2\Delta_{SY}}. \tag{2}$$

Here, Δ_{SX} is the distance between an antenna and the center of the robot on the x-axis, Δ_{SY} is the distance between an antenna and the center of the robot on the y-axis, and R_{UR}, R_{LR}, R_{UL}, and R_{LL} are the RSS values at the upper right, lower right, upper left, and lower left receivers, respectively on the mobile robot, measured at the current path location. These values must be adjusted to take into account the current orientation of the robot θ_l.

$$DOA_l = \arctan(\frac{g_y}{g_x}) + \theta_l \tag{3}$$

This gives us the DOA of the WiFi signal at path location l using the RSS gradients. There are several other ways to estimate the DOA from RSS measurements, mainly based on spatial correlation [20], [21], [22].

To reduce the noise that comes with this estimated DOA calculation because of inherent noise and fluctuations in the RSS, we use an exponential weighted moving average (EWMA), which uses a fixed size window of previous path locations where data was collected. Therefore, the Filtered DOA at a path location l with $K-1$ previous path locations included in the window can be written as

$$\widetilde{DOA}_l = \frac{1}{0.99^0 + 0.99^1 + \ldots + 0.99^{K-1}} * \sum_{n=0}^{K-1} 0.99^n * DOA_{l-n} \tag{4}$$

The actual DOA (ADOA) from a potential candidate AP location, with coordinates (x_c, y_c), to the robot's current location, with coordinates (x_r, y_r), is calculated as follows:

$$ADOA_l = arctan(\frac{y_r - y_c}{x_r - x_c}). \quad (5)$$

We can then obtain the error between the estimated and actual DOAs for a candidate AP location at (x_c, y_c).

$$err_l^c = ADOA_l - \widetilde{DOA}_l. \quad (6)$$

Now we can use a Gaussian probability formula (similar to [28]) to calculate the probability of the i^{th} candidate AP location in the particle $q_i = (x_i, y_i, w_i), i \in (1, ..N)$, where N is the number of particles in the PF and w_i is the weight of each particle calculated over a set of previous path samples as

$$w_i \propto P_l(q_i) = \prod_{k=0}^{M-1} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(err_{t-k}^c)^2}{2\sigma^2}} \right]. \quad (7)$$

Each estimated DOA measure has an intrinsic angular error. The variance of this error is represented as σ, which is assumed to be known as we know the accuracy of the method being employed to estimate the DOA. We take the product of the Gauss probability of the error of DOA over M - 1 previous path locations of the robot (imitating spatially distributed samples) so that the filtered DOA estimates from previous path locations can be used in an equivalent manner as readings from multiple sensors. This product of the DOA probability is used as the weights of the particles in the PF, which is then used in the resampling procedure in the next iteration of the PF step.

The particle filter generates initial hypotheses with a uniform sampling of candidate AP locations across the environment using a bound around the current robot location. The Gaussian probability is calculated for each unique particle being the signal source. The particles are then assigned weights equivalent to their probability, and the weights w_i are normalized as

$$w_i^*(q_i) = \frac{w_i(q_i)}{\sum_{i=0}^{N-1} w_i(q_i)}. \quad (8)$$

The probability that a particle from this set of particles is regenerated is based on this normalized weight. The particle with the highest weight is the new best estimate of the WiFi AP. This process is continued until either there is only one unique particle remaining or there are no new samples. Note the PF is iterated for every new measurement tuple. The pseudo-code of the process is depicted in Alg. 1.

Algorithm 1: PF Localization of AP using RSS DOA

```
Pr = [] % Initial List of Particles in the PF ;
for n=1 to N do
    sample x_0 from prior distribution P(X_0);
    add x_0 to Pr;
end
while data stream tuple m_t does not end do
    sample x_t from the transition model P(X_{t+1}|x_t);
    X_{t+1} = X_t % motion model of fixed AP ;
    W = [] % initialize weights;
    for x_{t+1} in Pr do
        add P(e_{t+1}|x_{t+1}) to W % normalized weights
          in Eq. (8);
    end
    Rr = [] % particles re-sampling;
    for n=1 to N do
        Choose p = Pr[m] with probability W[m]
          where 1 < m < N;
        add p to R;
    end
    Pr = Rr;
    x_l^* = x_i ∈ Pr where w_i(x_i) is max(w_i) ;
    % choosing the max weighted particle as the best
      AP location estimate.
end
```

IV. Experimental Validation

Validation of our solution was done using seven datasets containing temporal information, positional information, and RSS values measured at five antennae, four of which we have mentioned previously and a central antenna. All of the datasets have the WiFi AP placed at the same location (9,0), in relation to the robot's starting position (0,0). They contain readings from a robot traversing the same indoor environment in different trajectories. These datasets are obtained from data gathered during our previous research [26], [12], and parts of this dataset are publicly available as part of [13] (Dataset 1 in this paper). The rest of the datasets are also released in Github (see Sec. I).

The environment is a 20-meter x 26-meter indoor office hall with multiple rooms. The frequency of sampling on the robot is around 5Hz. Fig.2 shows the robot trajectories in these seven datasets. In the last two datasets (Dataset 6 and Dataset 7), the robot does not move in the x-y plane, but instead, the robot spins on its location for a few minutes.

We implement comparison algorithms such as the Maximum Likelihood (ML), which uses the DOA error for the expectation minimization of the MSE and the Weighted Centroid Localization (WCL), which uses the RSS measurements in their weight calculation of weighted centroid estimation $w_l = \sqrt{(10^{RSS_l/10})^g}$ with $g = 2$. The w_l for each path location l is then normalized and used to estimate AP location as $[x_{AP}^{wcl}, y_{AP}^{wcl}]^T = \sum_{l=1}^{M} w_l [x_l, y_l]^T$. For WCL, M covers all samples (run over the entire dataset containing all path locations because of the nature of this algorithm). Except for WCL (since it is deterministic), we run 100 trials per algorithm - ours (PF) and ML for each dataset. From this, we obtain AP estimates for each probabilistic method per trial. We then calculate Root Mean Squared Error (RMSE) and standard deviation (std) calculated across the trials.

Before comparing the localization accuracy, we first set the important parameters of our PF method. These parameters are the number of particles N in the PF, the number of previous

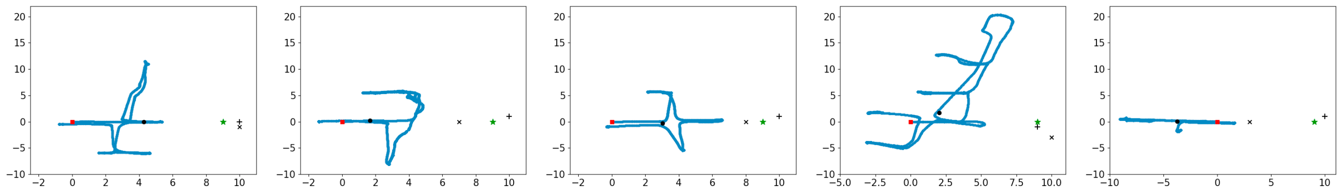


Fig. 2. The robot path trajectories of the first five datasets used in our experiments (Dataset 1 to 5). ⋆ indicates the true AP location and ■ indicates the robot start position. +, ∘, and x indicate the AP location estimate from our PF, WCL, and ML, respectively on a sample trial.

path locations K to use for filtering the DOA values, and the number of previous path locations M to use for the Gaussian probability calculation in Eq. (7).

First, we attempt to obtain an optimal[2] value for K by obtaining the number which yields the least average circular error between estimated DOA and actual DOA over all path locations for dataset 1. The circular average error for a dataset with N timesteps is then calculated as follows:

$$error_{circ} = arctan\left(\frac{\frac{1}{N}\sum_{n=1}^{N} sin(error)}{\frac{1}{N}\sum_{n=1}^{N} cos(error)}\right) \quad (9)$$

For each timestep, the actual DOA, the angle from the actual WiFi AP location to the current path location, is calculated. The error is merely the absolute value of the difference between the actual DOA and the estimated DOA EWMA. Fig. 3(a) presents the impact of K on the DOA estimation accuracy on Dataset 1. The best value for $K = 100$ (beyond which there is no improvement in DOA error), which is chosen to be used in the rest of the evaluations.

Second, we determine N - the number of particles in the PF. Over 100 runs, the average absolute localization error of the WiFi AP estimation from the ground truth WiFi AP location is calculated. Fig. 3(b) shows that the lowest average localization error is obtained when the number of particles $N = 400$. This is the value set for all future evaluations.

Finally, we move on to determining a good number of previous samples to use for Gaussian probability calculation in Eq. 7. Setting the other parameter values, we calculate the average error of the Wi-Fi AP estimation from the actual Wi-Fi AP location over 100 runs. Fig. 3(c) shows that the lowest localization error is achieved when the number of previous samples $M = 20$. We will use these parameter values to evaluate the program performance on different datasets and to compare to standard methods.

V. Results and Discussion

Table I presents the results of the comparison of our approach with others in terms of localization accuracy. It is important to remember that the parameter values for PF were "optimized" only on dataset 1. We now interpret these results obtained from running the programs on several datasets to assess how well our program performs with other trajectories (paths of the robot in the environment) and against other localization methods. Note that all of these trajectories are still in the same environment.

TABLE I
RMSE and standard deviation of errors for 7 datasets and average of the datasets using PF, ML, and WCL methods.

		PF (ours)		ML		WCL	
Dataset	Total Samples	RMSE	std	RMSE	std	Error	std
1	1681	0.933	0.465	1.300	0.259	4.733	N/A
2	6640	1.261	0.271	2.170	0.340	7.348	N/A
3	1561	1.118	0.446	0.916	0.411	5.973	N/A
4	3228	1.640	0.478	3.023	0.507	7.175	N/A
5	2722	1.744	0.728	5.629	0.910	12.718	N/A
6	351	1.442	0.505	7.602	0.700	8.995	N/A
7	371	1.446	0.418	7.489	0.946	9.000	N/A
All	16554	1.369	0.473	4.019	0.582	7.992	N/A

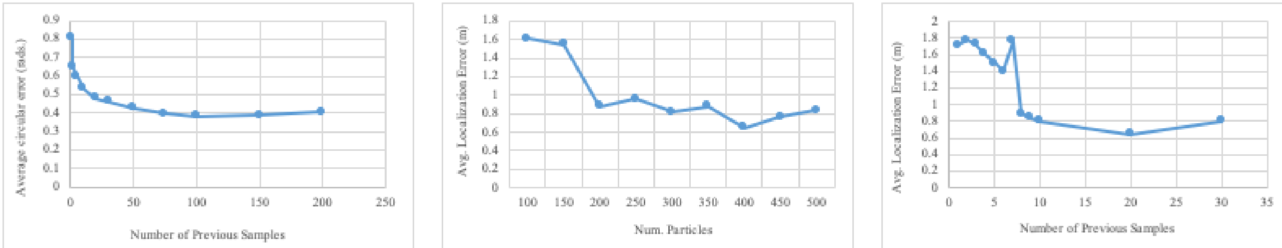
Once again, RMSE for localization error (m) and standard deviation of errors over 100 runs are used to evaluate the performance of the algorithms, with the exception of WCL, which is deterministic. Datasets 1-7 in Table I are representative of the errors of actual datasets. "All" shows the average results of the algorithms for combined datasets 1-7.

In general, our PF approach provides superior meter-level localization performance and produces significant accuracy improvement up to 3x to that of other compared methods. For all datasets other than 1, the localization RMSE for the PF approach is significantly higher than for the Dataset 1. This makes sense intuitively as the parameters were optimized for only that dataset. However, we can see that the localization RMSE of 1.369 m on combined datasets obtained by our PF approach is competitive with the de-facto benchmark of around 1 m localization accuracy. It is also important to remember that although this program is being run on recorded datasets currently, it could easily be adapted for real-time localization.

If multiprocessing is added in order to calculate Gaussian probability in parallel for all particles, this program could be easily adapted to work in real-time. Currently, without multiprocessing, the program achieves real-time speed for completing an iteration (0.2 seconds) in 4-5 timesteps. To adapt the current approach for real-time needs, a queue could be created for timesteps that need to be processed, and within around 20 timesteps, the queue would be clear, and the program would be caught up with the current timestep.

In contradistinction to ML, PF and WCL can produce results in real-time. ML localizes after the stream of data has ended, so it cannot be done in real-time. ML performs comparably to PF for the first three datasets, but significantly worse for

[2]Note that these may not be truly optimal values because not every possible combination is tried and they may vary across the datasets.



(a) Avg. circular error of DOA estimation vs. num of samples K in EWMA Filter (Eq. (4)).

(b) Avg. AP localization error vs. num. of particles N in the Particle Filter.

(c) Avg. AP localization error vs. num. of samples M in Gauss Probability (Eq. (7)).

Fig. 3. Estimation of the parameters in the proposed PF localization approach. Best values are K = 100, N = 400, and M = 20. These parameter values are optimized based on the analysis of Dataset 1.

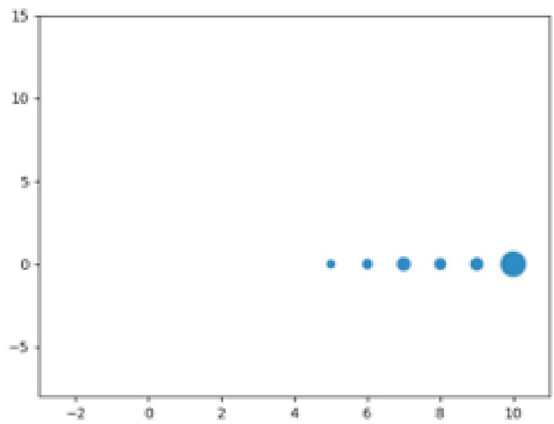


Fig. 4. Consistency of Wi-Fi AP estimates using our PF approach over the iterations (one iteration per every new sample arrived) averaged over ten runs on Dataset 1. The intensity of the estimated location signifies the frequency of that estimation. Note, the ground truth location of the AP is (9,0).

the others. Both algorithms (ML and PF) use DOA error, but the latter has the advantage of using PF based on Gaussian probability. This allows PF to iteratively generate particles from an initial set with likelihood proportional to probabilities.

WCL performs the worst out of the three approaches. This is because the algorithm is heavily biased by the path of the robot. The robot does not venture near the AP often in the datasets, and thus, the WCL algorithm performs very poorly.

The PF algorithm using the Gaussian probability of DOA error clearly enjoys the benefit of not being biased by robot path. The PF method was able to localize as soon as the robot covered a few meters of area in the environment, whereas other methods would need extensive sampling of the environment. For instance, in Dataset 1, the PF approach achieved the unique particle (converged to single AP location candidate) within 100 to 200 samples, which correspond to the first four meters of the robot's path. Fig. 4 shows the consistency of PF estimation.

Further, consider the Datasets 6 and 7, where the robot does not explore the environment (no movement in x-y plane) but only spins on its origin location. Obviously, the ML and WCL algorithms failed to localize the AP due to insufficient sampling. But, the PF algorithm consistently resulted in good localization accuracy (around 1.45m) because of the use of the reliable DOA measurements and iterative PF updates.

Additional advantages offered by our PF approach include the ability to localize an AP without needing to know the wireless channel parameters. Furthermore, the multi-hypothesis PF approach allow us to achieve global localization, thereby supporting moving AP cases (where the AP itself is on an another robot for example) and in kidnapped robot problems.

One may ask how this approach will yield fruitful advances in the localization of a robot or multiple robots themselves, as the paper's scope is limited to the localization of a WiFi AP. Upon localization of the WiFi AP w.r.t the robot, the robot can self-localize if the AP location is known in a global coordinate frame. In a multi-robot context, applications of WiFi DOA estimation and localization include building connectivity graph and consensus control [29], [13], [30].

Further testing could be done for our PF approach to determine if it could benefit from an alteration of the distribution of particle probabilities such that a small portion of the particles with lower probability may have a higher chance of propagation (in resampling) than they would based solely on normalized probability. That is, to add random injections of particles with lower weights to add diversity to the particles and avoid bias in the results.

In our implementation, we currently set a resolution of 1 m grid in the environment to search for candidate AP locations. This grid spacing could be made finer; perhaps this could reduce the localization error. To improve the estimates of optimal parameters for both numbers of previous timesteps, this should be done with experimentation across a variety of trajectories in multiple environments. There could be an interesting pattern between certain aspects of an environment and the optimal values for these parameters. We plan to exploit these avenues for further work.

Ultimately, the goal of this research is to construct a localization algorithm for the simultaneous, real-time localization of multiple robots. We find that the results we obtained in this paper are fairly promising, and the localization method is already competitive with the benchmark. With further experimentation and improvements as suggested, the localization accuracy should only improve and be able to be applied to more realistic scenarios and multi-robot systems.

VI. SUMMARY

In this paper, we have proposed a novel solution for the localization of a single WiFi AP in relation to a moving robot. The solution uses a particle filter that generates several points/hypotheses over the environment. For each of the particles that represents a candidate AP location, the Gaussian probability of the point being the source is calculated by using the error between the estimated DOA, obtained through RSS measurements, and the actual DOA. The particle with the highest probability is assigned to be the current estimate of the WiFi AP location. When the program reaches the end of the input data, or there is only one unique particle left, the program returns the current estimate of the WiFi AP location and terminates. Our results show an average error of 1.369 m in the indoor environment based on real-world datasets from our previous studies. Our PF algorithm demonstrates significantly better accuracy than the weighted centroid localization and maximum likelihood algorithms.

Further experimentation in both simulated and real environments is essential to make more substantial claims about the performance of the solution. Furthermore, we plan to improve the real-time performance of our method by using multi-processing in calculating the probability for each particle or by creating a queue for iterations. Simulations will allow for better estimates of optimal parameters for the program in a given environment. Hierarchical particle filtering, by iteratively increasing fineness of grid lines, also seems to offer improvements in localization accuracy. Overall, these results show that the proposed PF approach using Gaussian probability for DOA errors should be explored further, and the results we obtain are promising for its application in multi-robot localization and connected vehicle scenarios.

REFERENCES

[1] F. Zafari, A. Gkelias, and K. K. Leung, "A survey of indoor localization systems and technologies," *IEEE Communications Surveys Tutorials*, vol. 21, no. 3, pp. 2568–2599, 2019.

[2] M. B. Kjærgaard, H. Blunck, T. Godsk, T. Toftkjær, D. L. Christensen, and K. Grønbæk, "Indoor positioning using gps revisited," in *International conference on pervasive computing*. Springer, 2010, pp. 38–56.

[3] "Gps accuracy." [Online]. Available: https://www.gps.gov/systems/gps/performance/accuracy/

[4] A. Boukerche, H. A. Oliveira, E. F. Nakamura, and A. A. Loureiro, "Vehicular ad hoc networks: A new challenge for localization-based systems," *Computer communications*, vol. 31, no. 12, pp. 2838–2849, 2008.

[5] D. Song, C.-Y. Kim, and J. Yi, "Simultaneous localization of multiple unknown and transient radio sources using a mobile robot," *IEEE Transactions on Robotics*, vol. 28, no. 3, pp. 668–680, 2012.

[6] J. Biswas and M. Veloso, "Multi-sensor mobile robot localization for diverse environments," in *Robot Soccer World Cup*. Springer, 2013, pp. 468–479.

[7] B.-F. Wu and C.-L. Jen, "Particle-filter-based radio localization for mobile robots in the environments with low-density wlan aps," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 12, pp. 6860–6870, 2014.

[8] D. Vasisht, S. Kumar, and D. Katabi, "Decimeter-level localization with a single wifi access point," in *13th {USENIX} Symposium on Networked Systems Design and Implementation ({NSDI} 16)*, 2016, pp. 165–178.

[9] Y. Oh, R. Parasuraman, T. McGraw, and B.-C. Min, "360 vr based robot teleoperation interface for virtual tour," in *1st International Workshop on Virtual, Augmented, and Mixed Reality for HRI (VAM-HRI)*, 2018.

[10] R. Maidana, A. Amory, and A. Salton, "Outdoor localization system with augmented state extended kalman filter and radio-frequency received signal strength," in *2019 19th International Conference on Advanced Robotics (ICAR)*. IEEE, 2019, pp. 604–609.

[11] V. Seshadri, G. V. Zaruba, and M. Huber, "A bayesian sampling approach to in-door localization of wireless devices using received signal strength indication," in *Third IEEE international conference on pervasive computing and communications*. IEEE, 2005, pp. 75–84.

[12] S. Caccamo, R. Parasuraman, F. Båberg, and P. Ögren, "Extending a ugv teleoperation flc interface with wireless network connectivity information," in *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2015, pp. 4305–4312.

[13] R. Parasuraman, S. Caccamo, F. Baberg, and P. Ogren, "Crawdad dataset kth/rss (v. 2016-01-05)," 2016.

[14] R. Parker and S. Valaee, "Vehicular node localization using received-signal-strength indicator," *IEEE Transactions on Vehicular Technology*, vol. 56, no. 6, pp. 3371–3380, 2007.

[15] M. Pajovic, P. Orlik, T. Koike-Akino, K. J. Kim, H. Aikawa, and T. Hori, "An unsupervised indoor localization method based on received signal strength (rss) measurements," in *2015 IEEE Global Communications Conference (GLOBECOM)*, 2015, pp. 1–6.

[16] M. Robinson and I. Psaromiligkos, "Received signal strength based location estimation of a wireless lan client," in *IEEE Wireless Communications and Networking Conference, 2005*, vol. 4. IEEE, 2005, pp. 2350–2354.

[17] E. Jedari, Zheng Wu, R. Rashidzadeh, and M. Saif, "Wi-fi based indoor location positioning employing random forest classifier," in *2015 International Conference on Indoor Positioning and Indoor Navigation (IPIN)*, 2015, pp. 1–5.

[18] S. Lee and J. Kim, "Random forest and wifi fingerprint-based indoor location recognition system using smart watch," *Human-centric Computing and Information Sciences*, vol. 9, p. 6, 02 2019.

[19] A. P. Subramanian, P. Deshpande, J. Gao, and S. R. Das, "Drive-by localization of roadside wifi networks," in *IEEE INFOCOM - The 27th Conference on Computer Communications*. IEEE, 2008, pp. 718–725.

[20] J. N. Twigg, J. R. Fink, L. Y. Paul, and B. M. Sadler, "Rss gradient-assisted frontier exploration and radio source localization," in *2012 IEEE International Conference on Robotics and Automation*. IEEE, 2012, pp. 889–895.

[21] D. Han, D. G. Andersen, M. Kaminsky, K. Papagiannaki, and S. Seshan, "Access point localization using local signal strength gradient," in *International Conference on Passive and active network measurement*. Springer, 2009, pp. 99–108.

[22] G. Verma, F. Dagefu, B. M. Sadler, J. Twigg, and J. Fink, "Doa estimation for autonomous systems in complex propagation environments," in *2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*. IEEE, 2018, pp. 1–5.

[23] A. Kokkinis, L. Kanaris, A. Liotta, and S. Stavrou, "Rss indoor localization based on a single access point," *Sensors*, vol. 19, p. 3711, 08 2019.

[24] Q. Dong and X. Xu, "A novel weighted centroid localization algorithm based on rssi for an outdoor environment," *JCM*, vol. 9, pp. 279–285, 2014.

[25] C. Wong, R. Klukas, and G. G. Messier, "Using wlan infrastructure for angle-of-arrival indoor user location," in *2008 IEEE 68th Vehicular Technology Conference*. IEEE, 2008, pp. 1–5.

[26] R. Parasuraman, S. Caccamo, F. Baberg, P. Ogren, and M. Neerincx, "A new ugv teleoperation interface for improved awareness of network connectivity and physical surroundings," *Journal of Human-Robot Interaction*, vol. 6, p. 48–70, 12 2017.

[27] R. Parasuraman, T. Fabry, K. Kershaw, and M. Ferre, "Spatial sampling methods for improved communication for wireless relay robots," in *2013 International Conference on Connected Vehicles and Expo (ICCVE)*. IEEE, 2013, pp. 874–880.

[28] R. Levorato and E. Pagello, "Probabilistic 2d acoustic source localization using direction of arrivals in robot sensor networks," in *International Conference on Simulation, Modeling, and Programming for Autonomous Robots*. Springer, 2014, pp. 474–485.

[29] S. Luo, J. Kim, R. Parasuraman, J. H. Bae, E. T. Matson, and B.-C. Min, "Multi-robot rendezvous based on bearing-aided hierarchical tracking of network topology," *Ad Hoc Networks*, vol. 86, pp. 131–143, 2019.

[30] R. Parasuraman and B.-C. Min, "Consensus control of distributed robots using direction of arrival of wireless signals," in *Distributed Autonomous Robotic Systems*. Springer, 2019, pp. 17–34.