Perception | **Line Extraction**
Autonomous Mobile Robots

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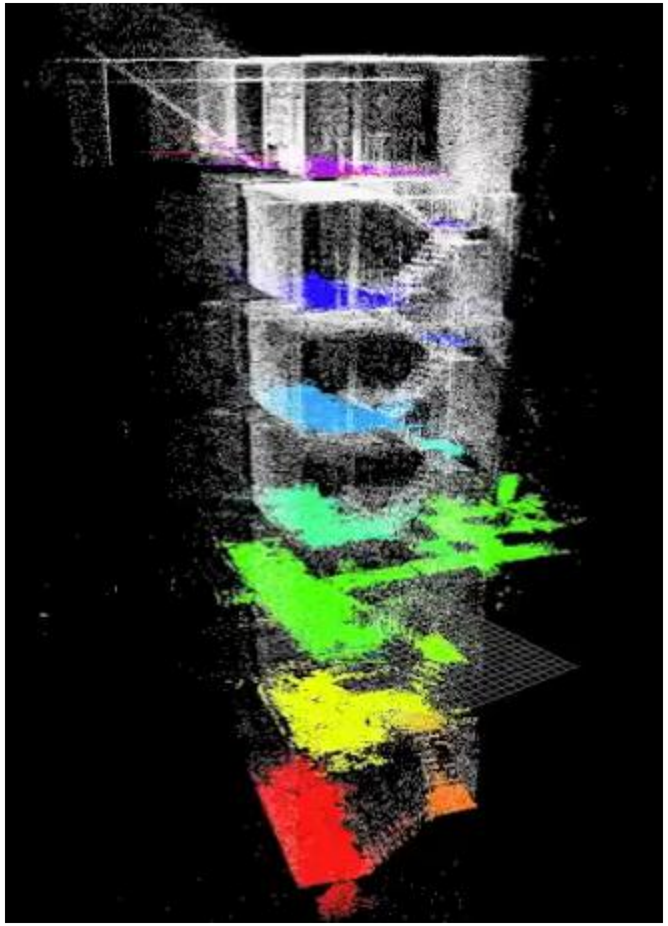
Line Extraction from a point cloud

Extract lines from a point cloud (e.g. range scan)

- Three main problems:
  - How many lines are there?
  - **Segmentation**: Which points belong to which line ?
  - **Line Fitting/Extraction**: Given points that belong to a line, how to estimate the line parameters ?

- Algorithms we will see:
  1. Split-and-merge
  2. RANSAC
  3. Hough-Transform

Courtesy of F. Pomerleau and F. Colas



Line Extraction | split-and-merge (standard)

- Originates from Computer Vision.
- A recursive procedure of fitting and splitting.
- A slightly different version, called Iterative end-point-fit, simply connects the end points for line fitting.

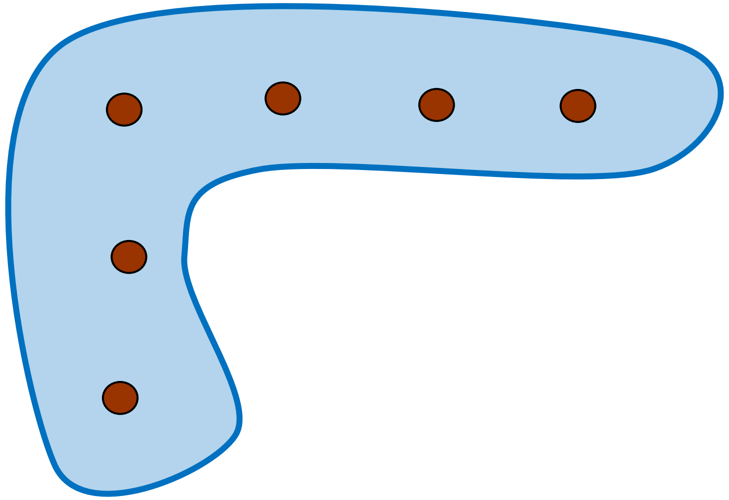
Initialise set **S** to contain all points

Split

- Fit a line to points in current set **S**
- Find the most distant point to the line
- If distance > threshold ⇨ split set & repeat with left & right sets

Merge

- If two consecutive segments are close/collinear enough, obtain the common line and find the most distant point
- If distance <= threshold, merge both segments



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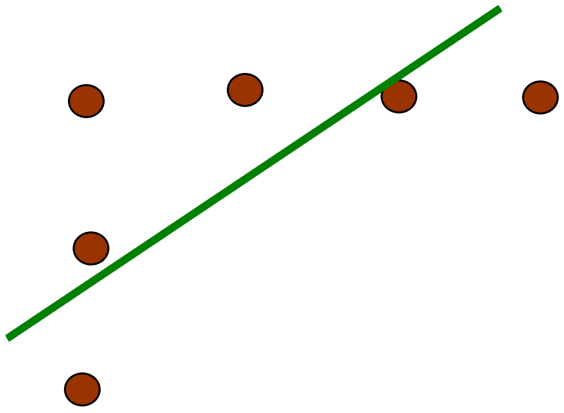

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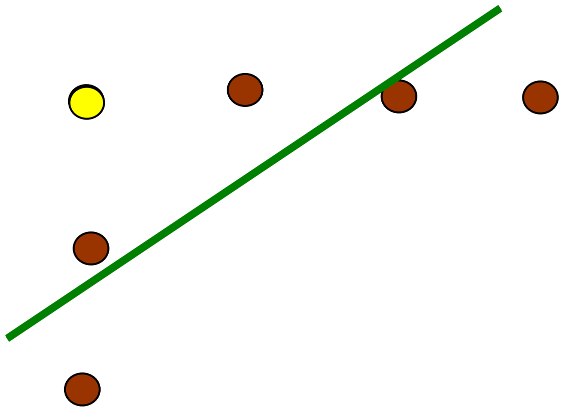

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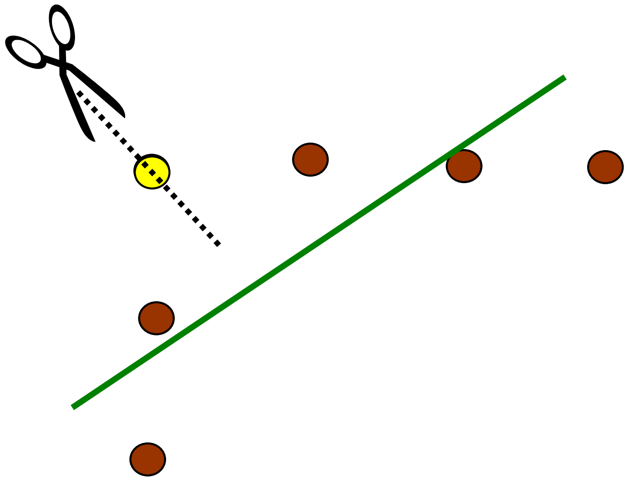

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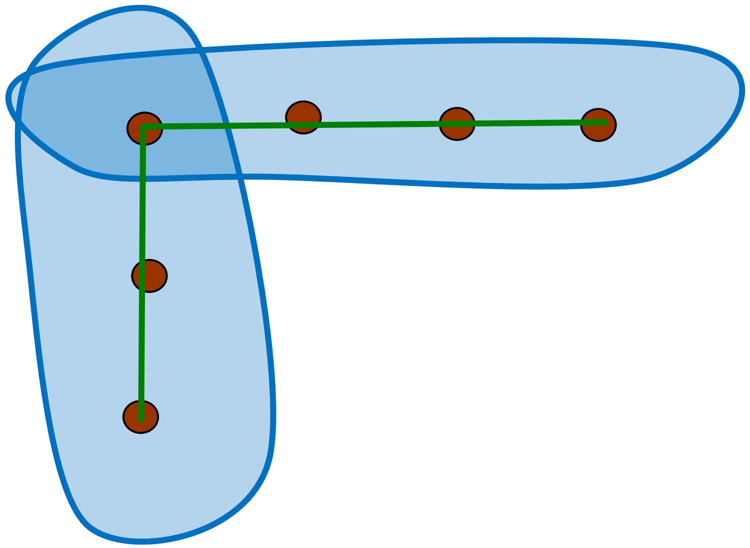
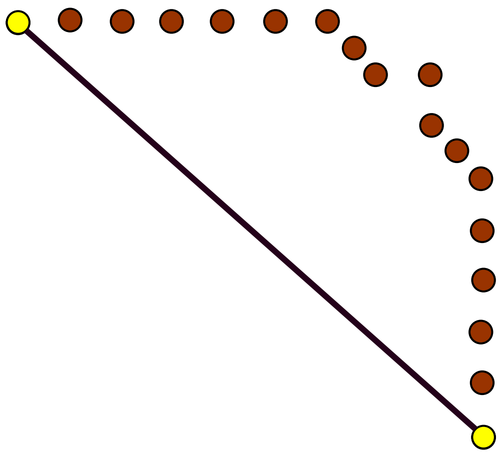


Split-and-Merge | iterative end-point-fit

- Iterative end-point-fit: simply connects the end points for line fitting.



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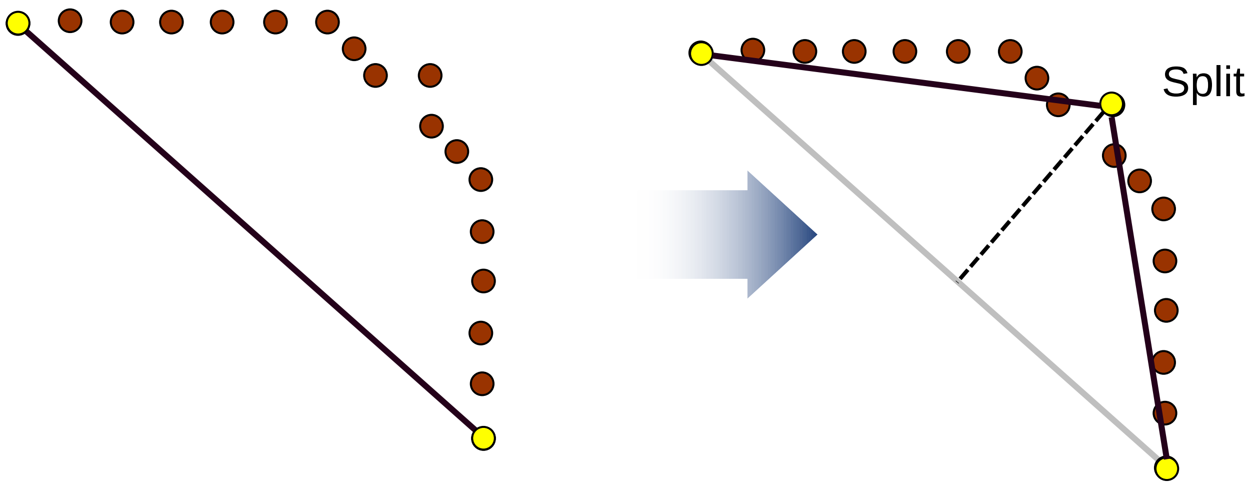

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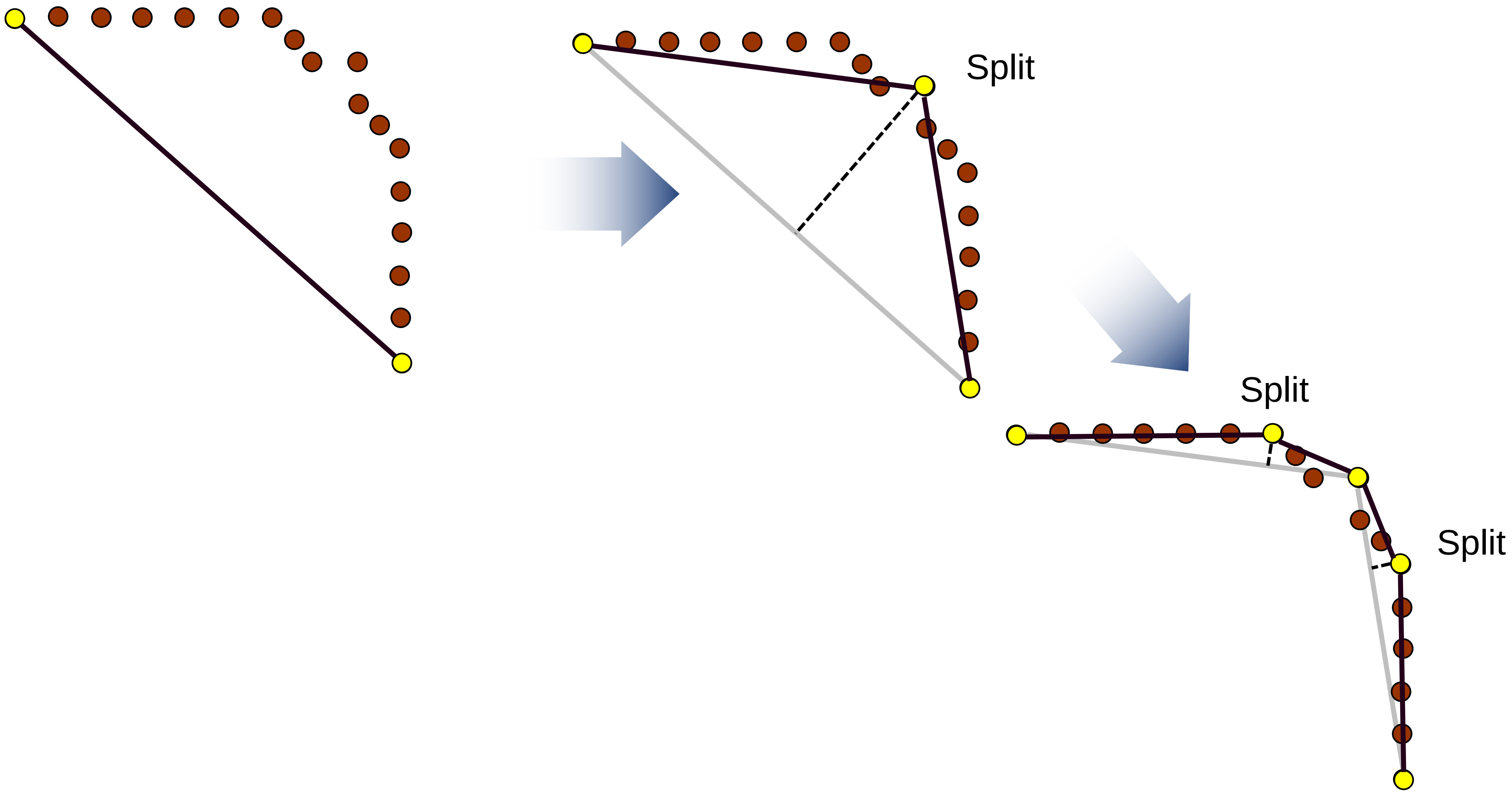

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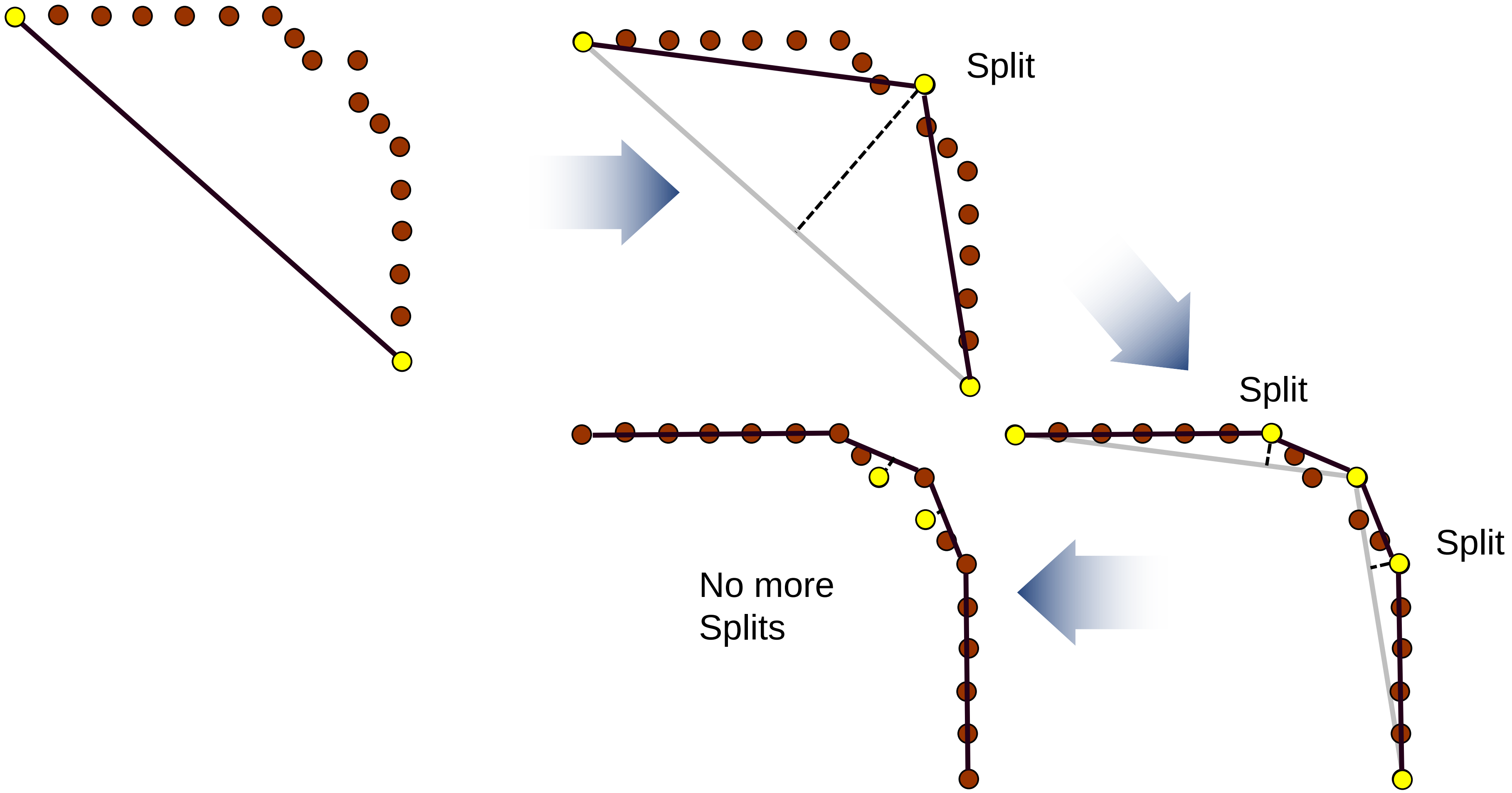

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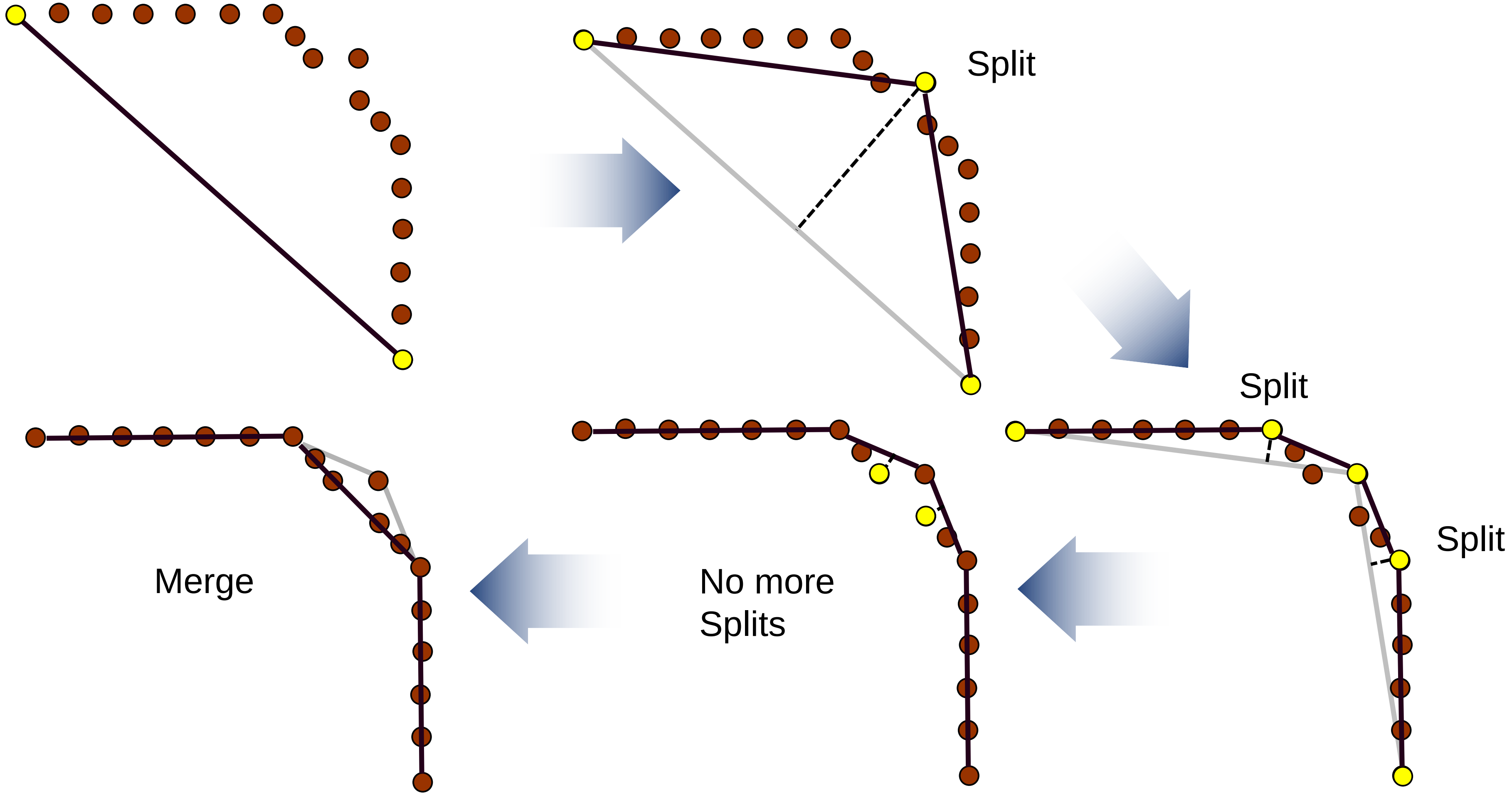
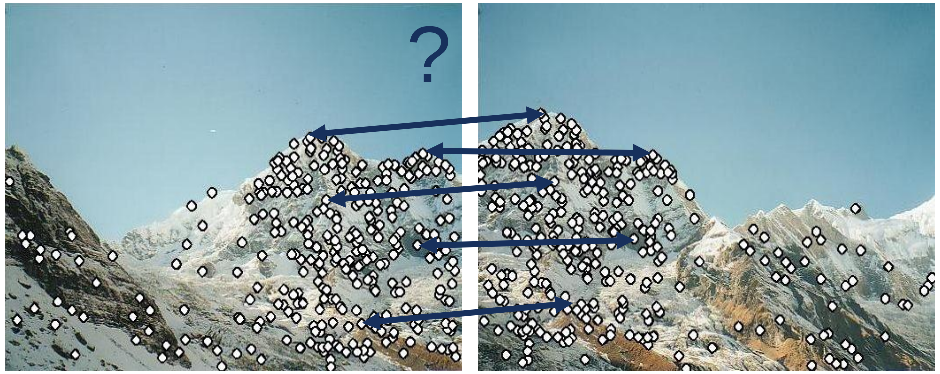


Line Extraction | RANSAC

- **RANSAC** = **RAN**dom **SA**mple **C**onsensus.
- A generic & robust fitting algorithm of models in the presence of outliers (i.e. points which do not satisfy a model)
- Applicable to any problem where the goal is to **identify the inliers which satisfy a predefined model**.
- Typical applications in robotics are: line/plane extraction, feature matching, structure from motion, …



- RANSAC is **iterative** and **non-deterministic** ⇨ the probability to find a set free of outliers increases as more iterations are used
- Drawback: A non-deterministic method, results are different between runs.

RANSAC | how it works

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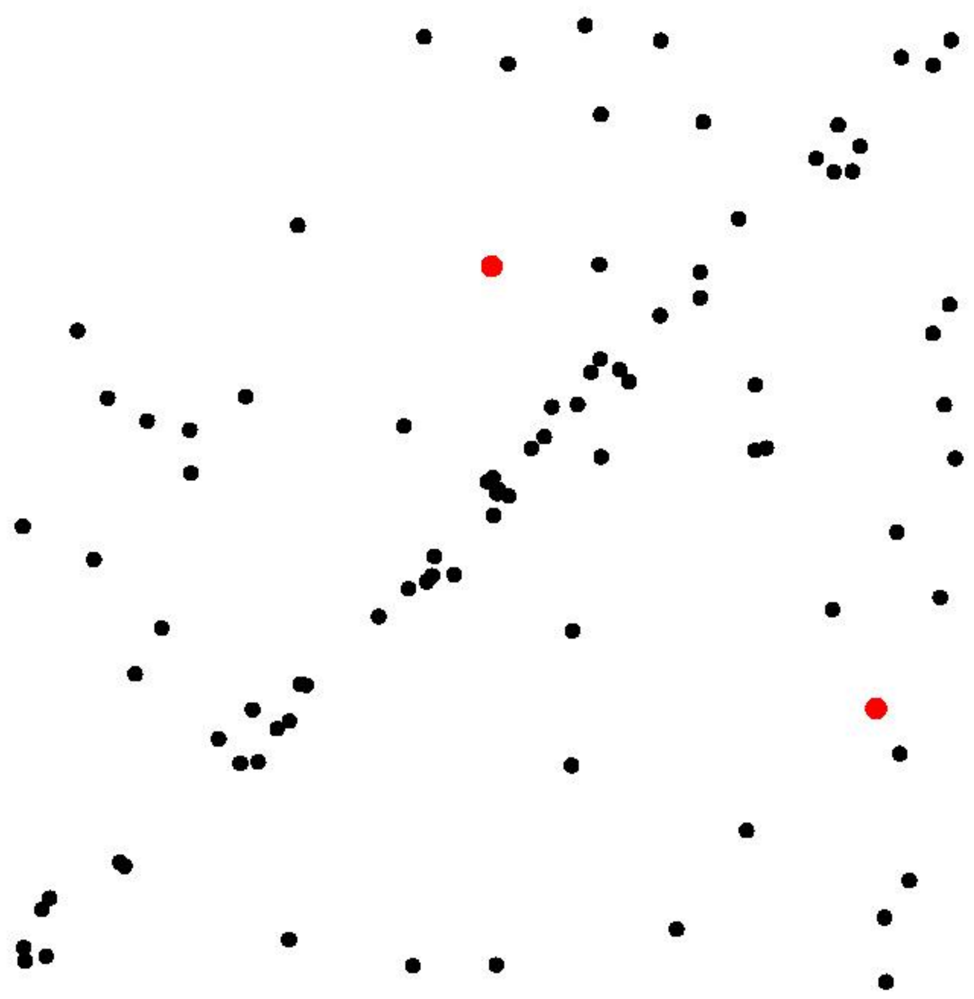
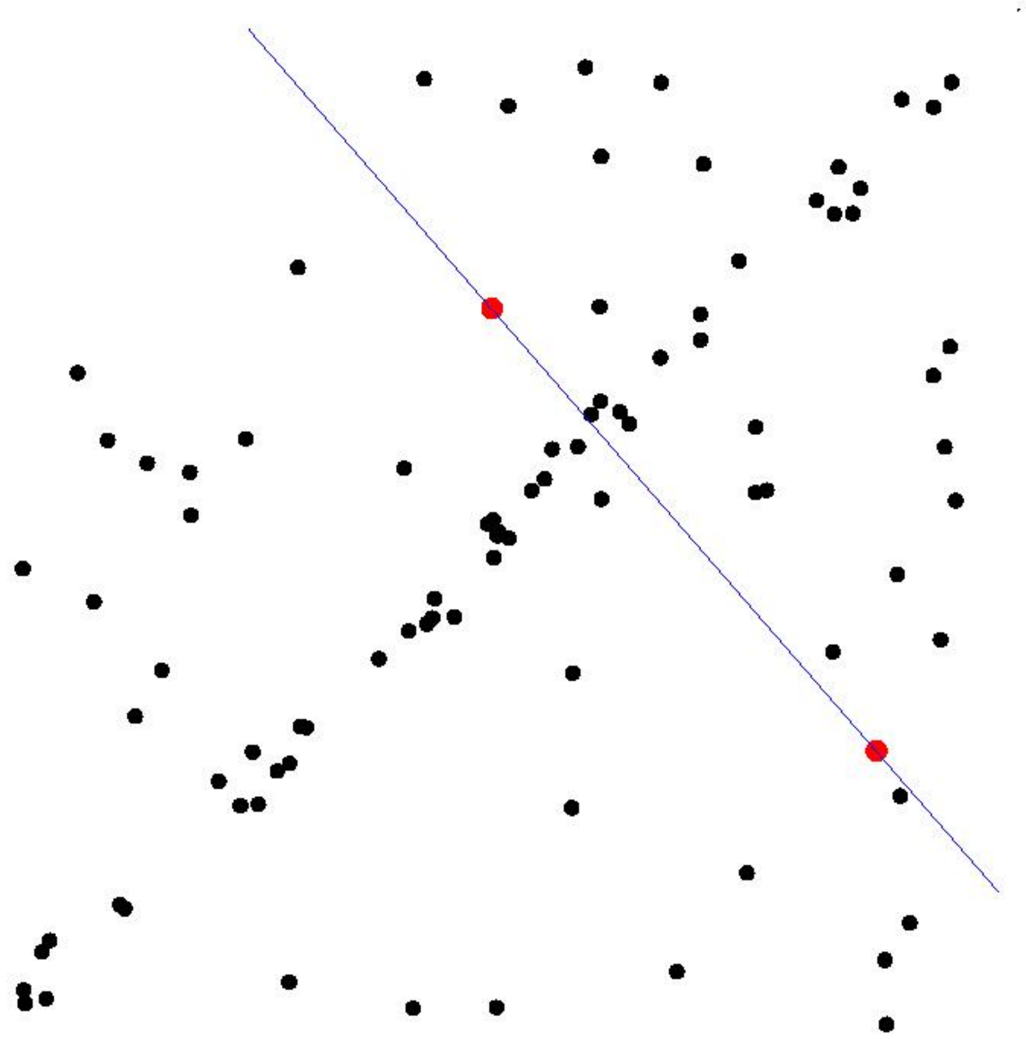


- Select sample of 2 points at random

RANSAC | how it works



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample

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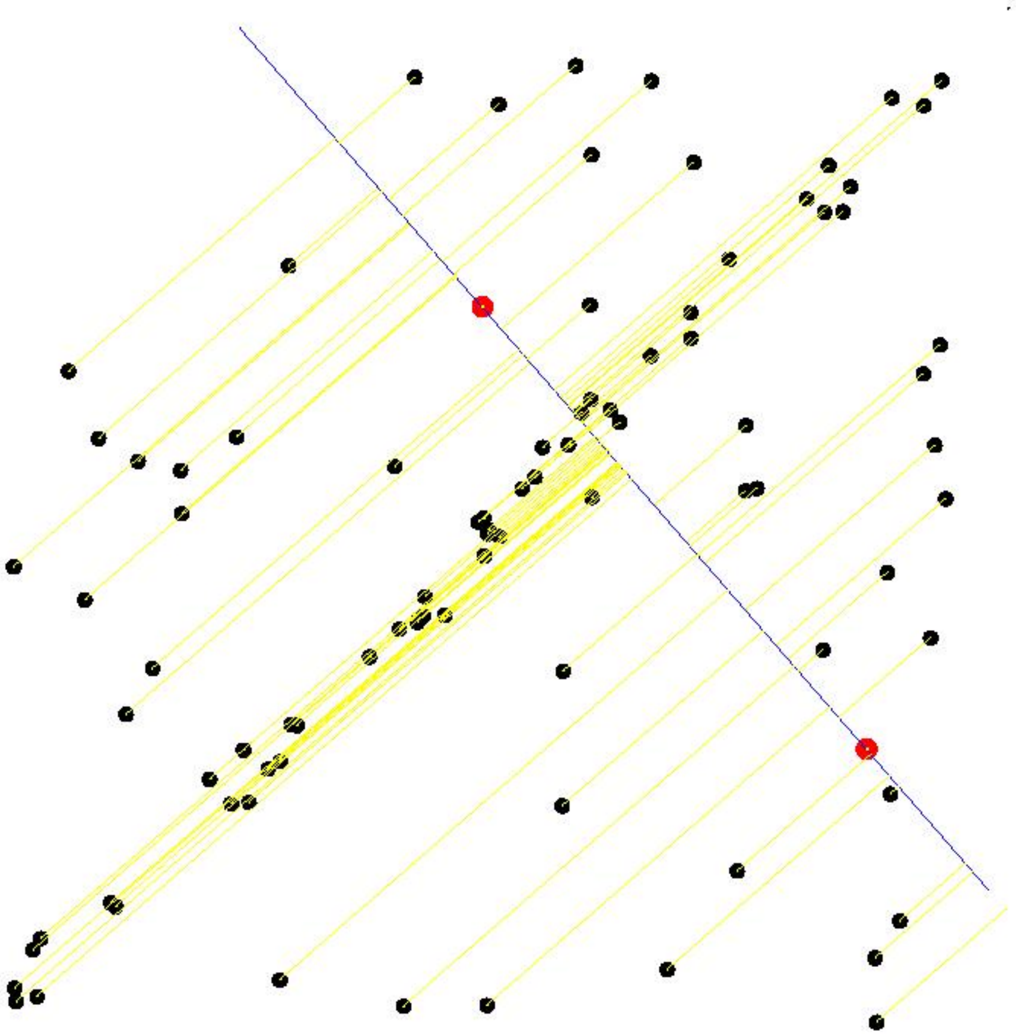

- Select sample of 2 points at random
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- Calculate error function for each data point

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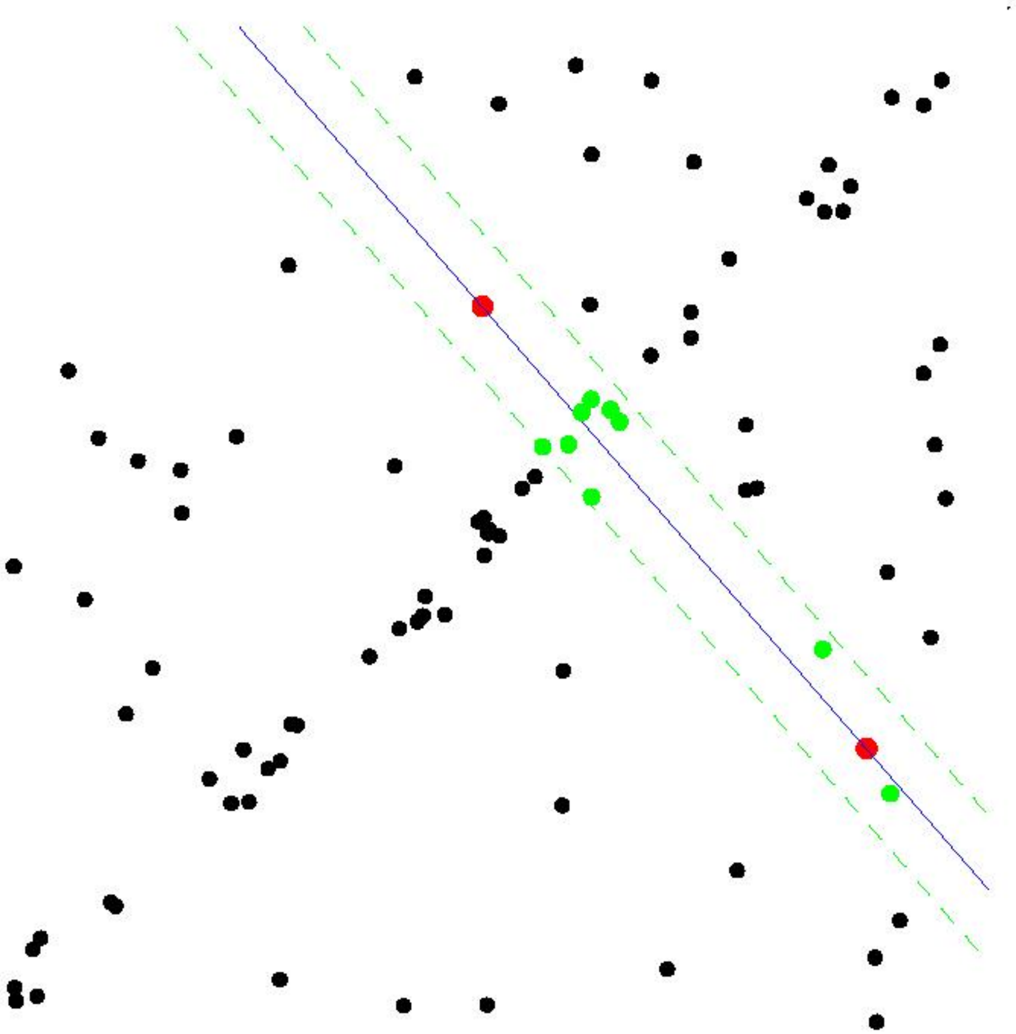
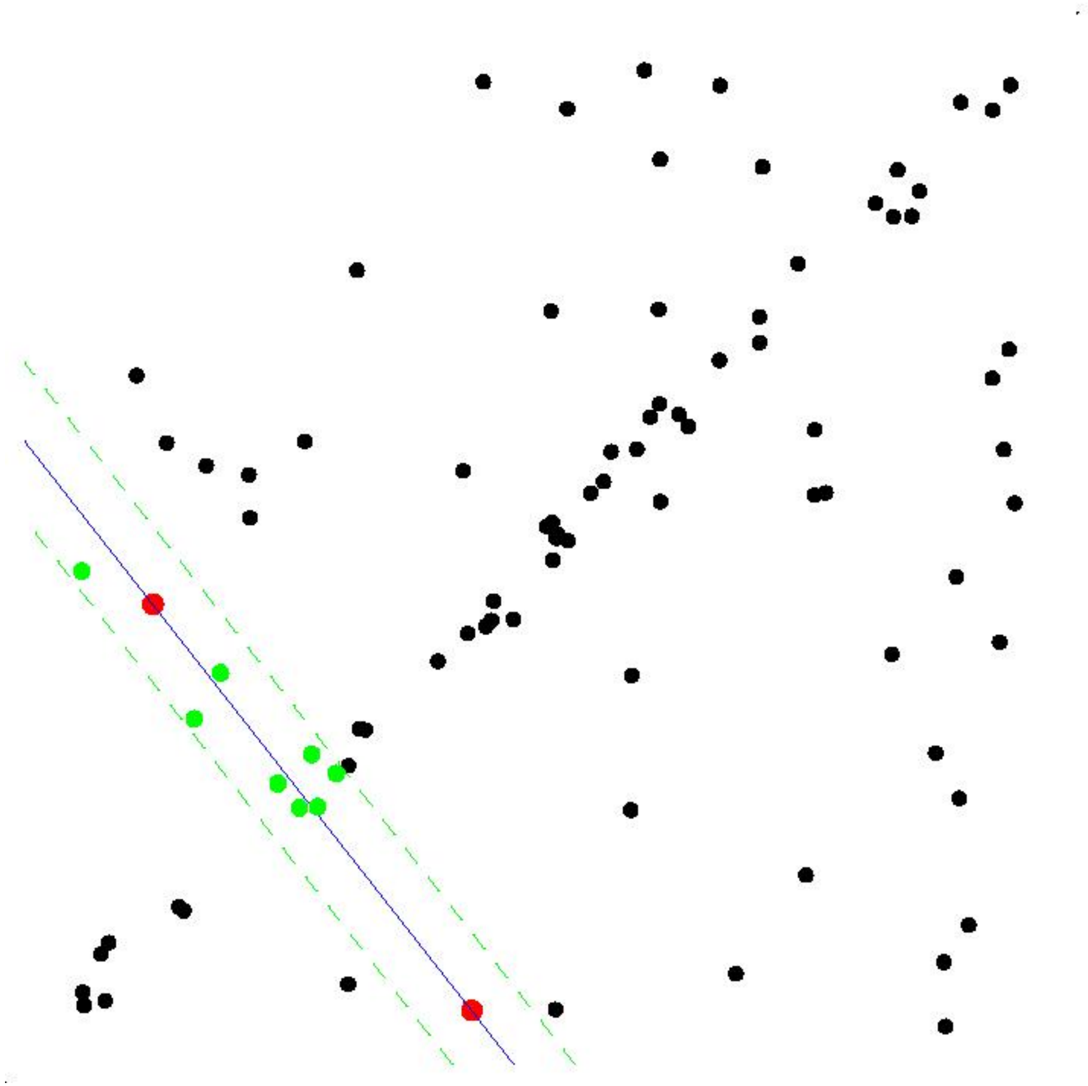


- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that supports current hypothesis

RANSAC | how it works



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
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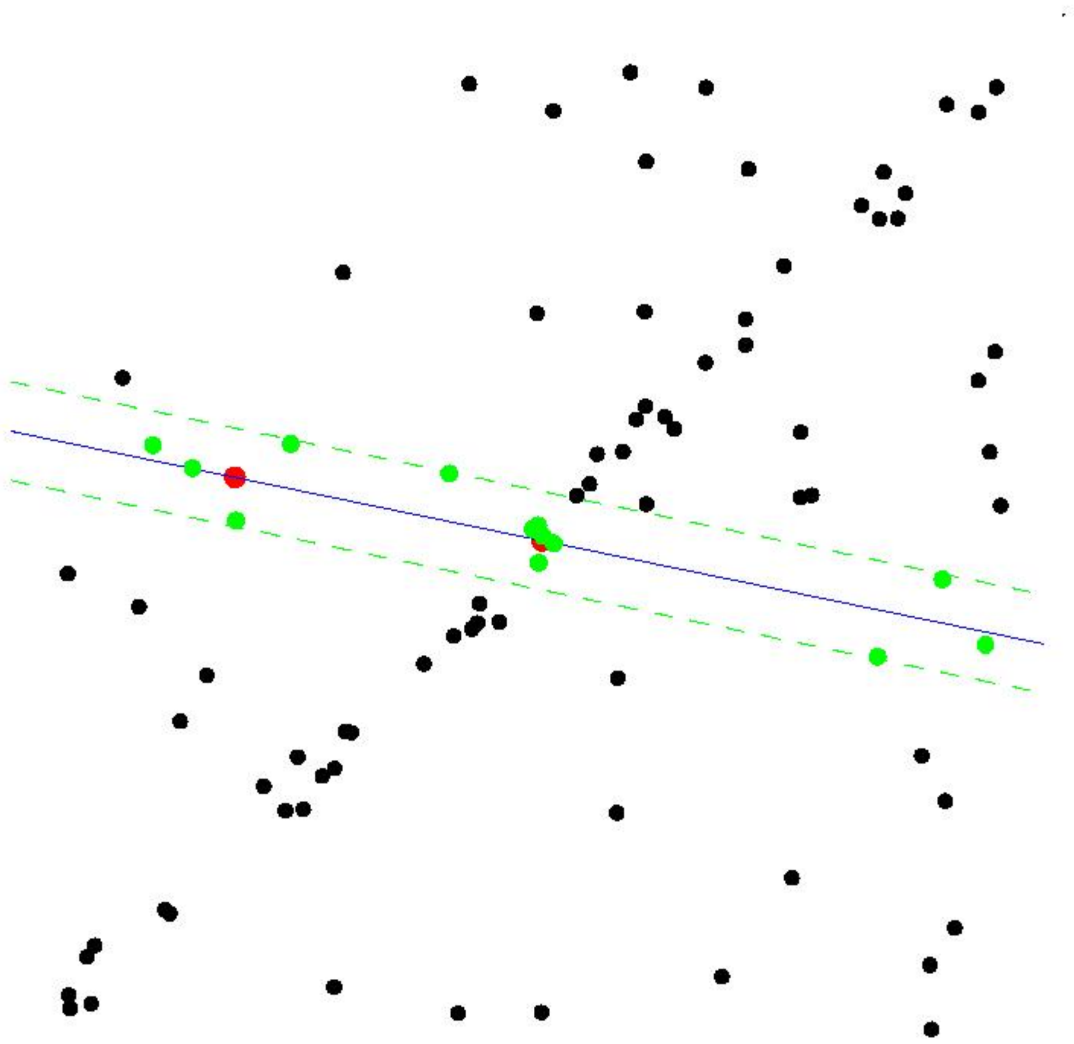
- Select data that supports current hypothesis
- Repeat sampling

RANSAC | how it works



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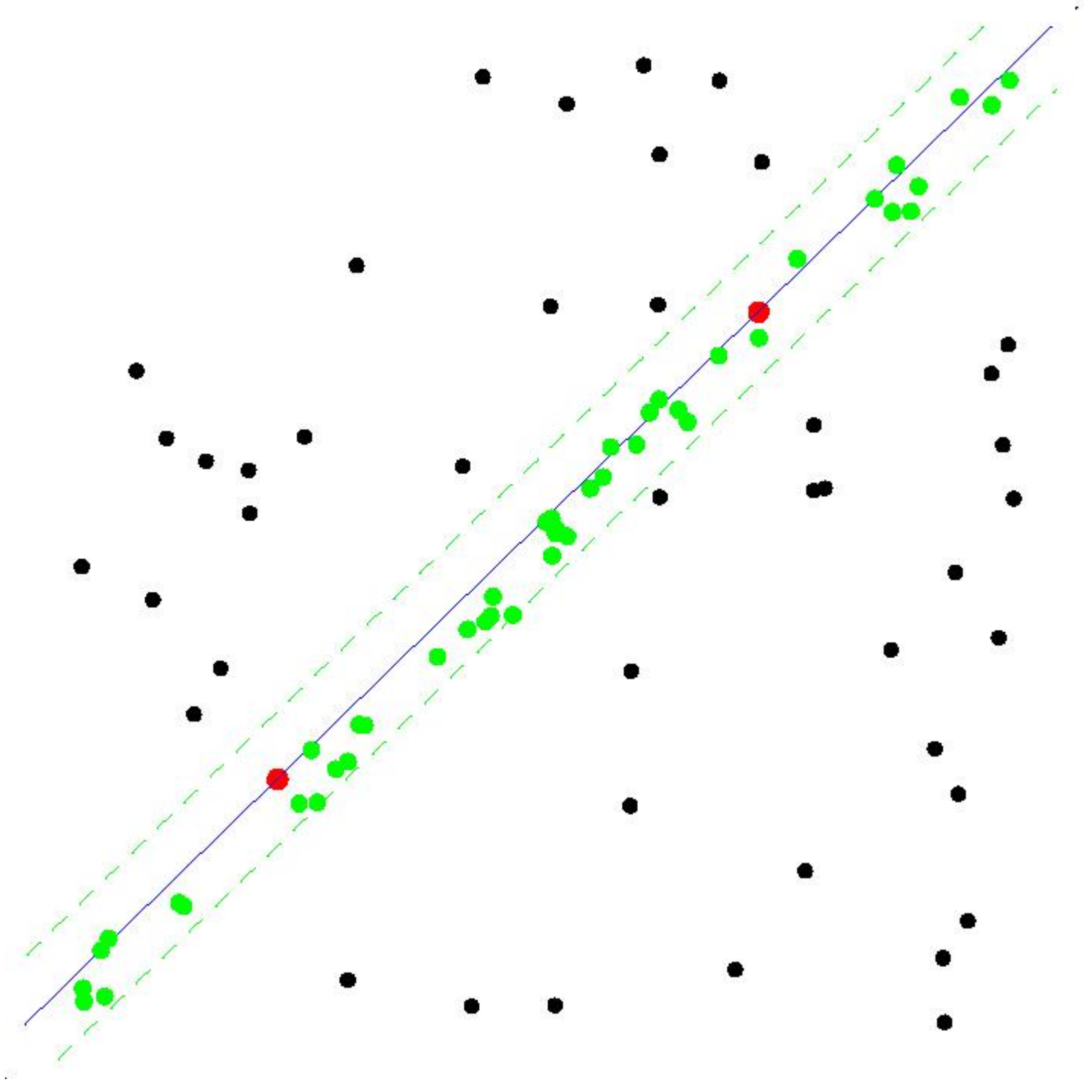

Set with the maximum number of inliers obtained within *k* iterations

RANSAC | how many iterations?

- We cannot know in advance if the observed set contains the max. no. inliers
  ➪ ideally: check all possible combinations of **2** points in a dataset of **N** points.
- No. all pairwise combinations: **N(N-1)/2**
  ➪ computationally infeasible if **N** is too large.
  example: laser scan of **360** points ➪ need to check all 360*359/2= **64’620** possibilities!
- Do we really need to check all possibilities or can we stop RANSAC after iterations?
  Checking a **subset** of combinations is enough if we have a **rough** estimate of the percentage of inliers in our dataset
- This can be done in a probabilistic way

RANSAC | how many iterations?

- ***w*** := number of inliers ***/ N***
  N := tot. no. data points
  ⇨ ***w*** : fraction of inliers in the dataset ⇨ ***w* =** P(selecting an inlier-point from the dataset)

- Let ***p*** := P(selecting a minimal set of points free of outliers)

- Assumption: the 2 points necessary to estimate a line are selected independently
  ⇨ **?** = P(both selected points are inliers)
  ⇨ **?** = P(at least one of these two points is an outlier)

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  ⇨ $\boldsymbol{w^2}$ = P(both selected points are inliers)
  ⇨ $\boldsymbol{1-w^2}$ = P(at least one of these two points is an outlier)

- Let ***k*** := no. RANSAC iterations executed so far
  ⇨ **?** = P(RANSAC never selects two points that are both inliers)

RANSAC | how many iterations?

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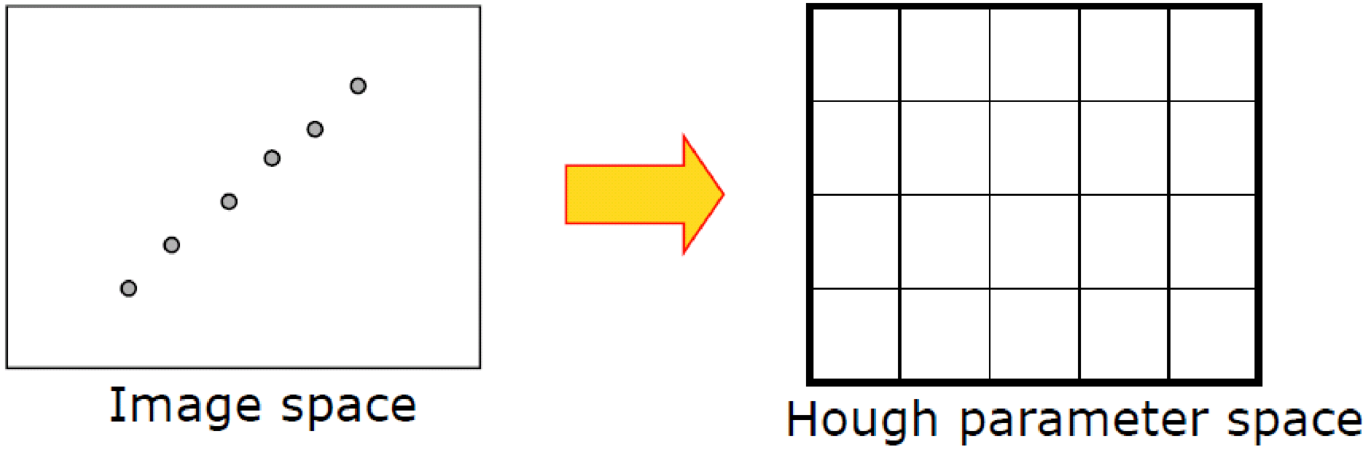
- Let ***k*** := no. RANSAC iterations executed so far
  ⇨ $(1-w^2)^k$ = P(RANSAC never selects two points that are both inliers)
  ⇨ $1-p = (1-w^2)^k$ and therefore:

$$k = \frac{\log(1-p)}{\log(1-w^2)}$$

- In practice we need only a rough estimate of ***w***.
  More advanced variants of RANSAC estimate the fraction of inliers & adaptively set it on every iteration.

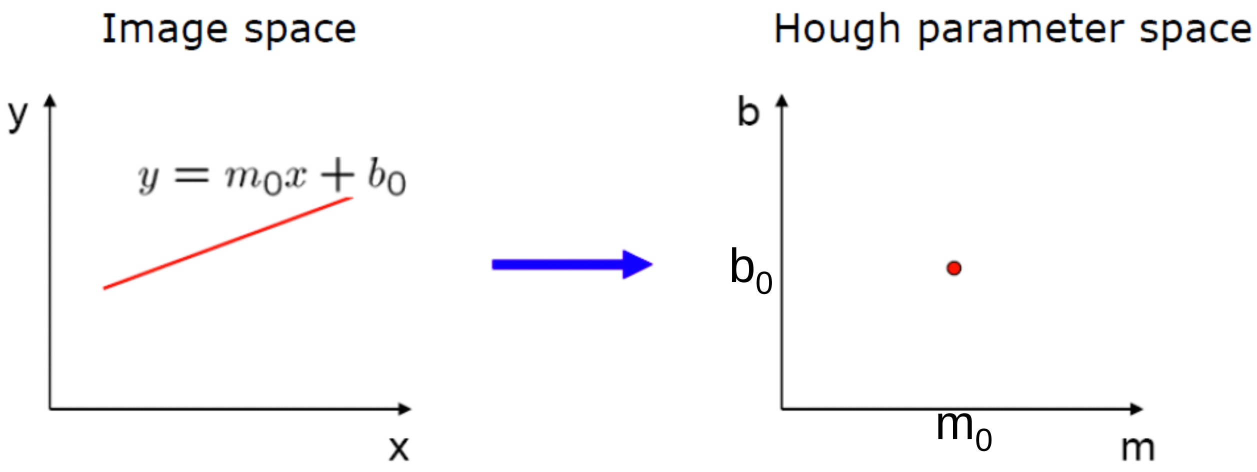
Line Extraction | Hough-transform

- Edges **vote** for plausible line locations
- Map image space into Hough parameter space
- Hough space parameterizes coordinate space w.r.t line characteristics
- In practice, it’s a discretized accumulator array (comprising of voting bins)



Hough-Transform | Hough space

- A line in the image corresponds to a point in Hough space



- What does a point (x_0, y_0) in the image space map to in the Hough space?

Hough-Transform | Hough space

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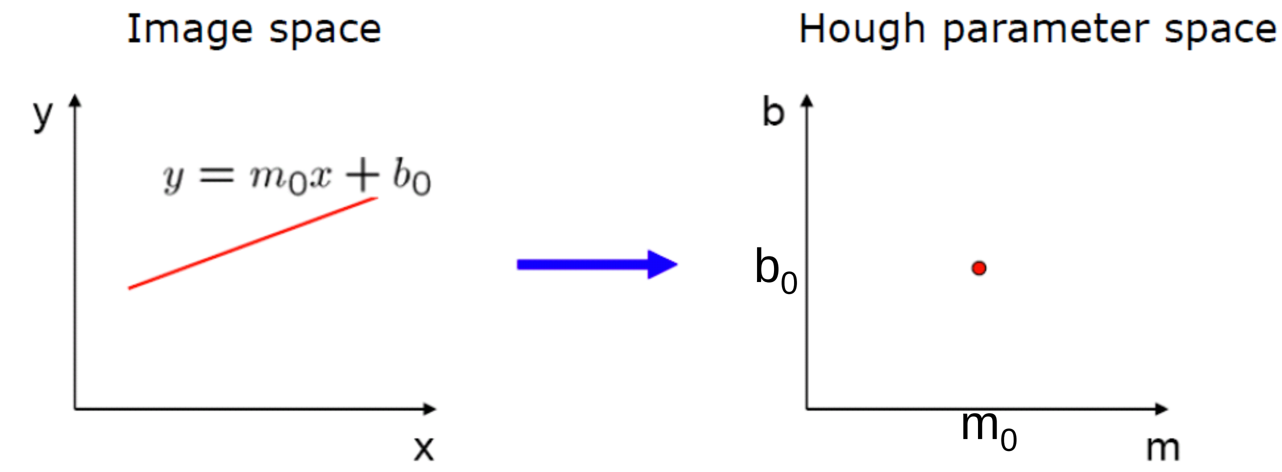
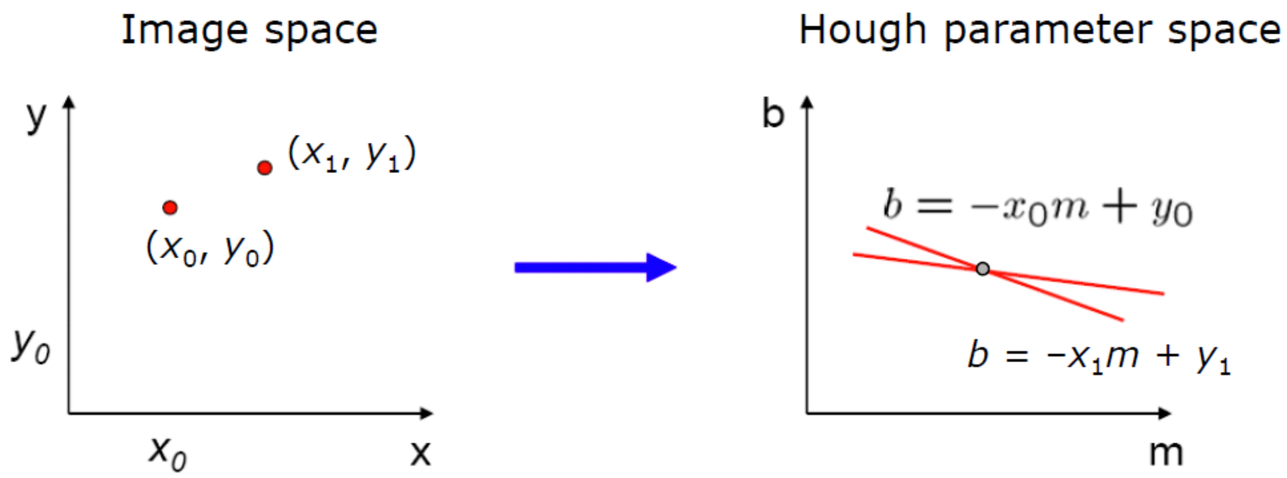
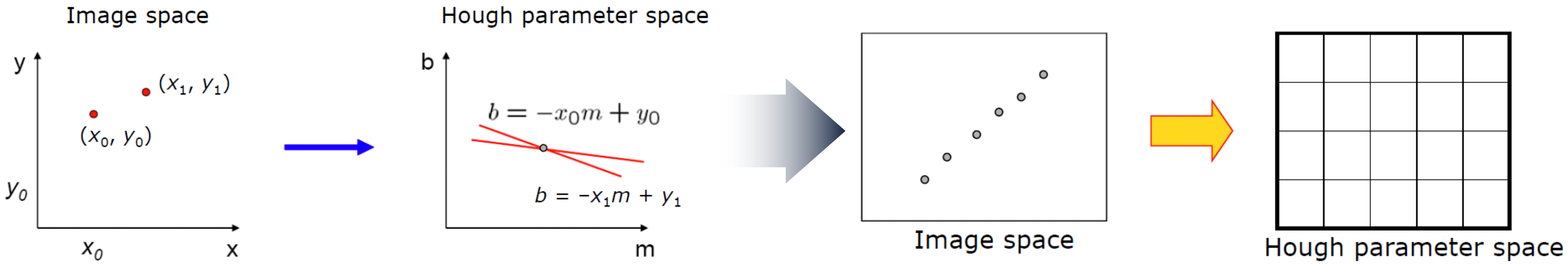


- What does a point (x_0, y_0) in the image space map to in the Hough space?
- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?
  - At the intersection of: $b = -x_0 m + y_0$ and $b = -x_1 m + y_1$



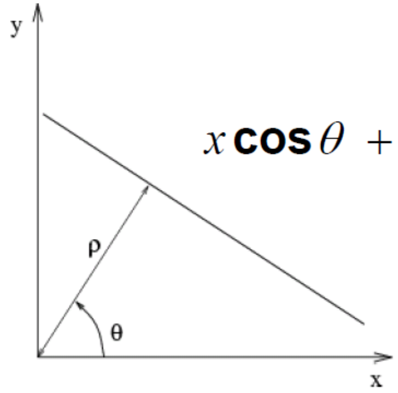
Hough-Transform | how it works

- Each point in image space, votes for line-parameters in Hough parameter space



- Problems with the *(m,b)* space:
  - Unbounded parameter domain
  - Vertical lines require infinite *m*

- Alternative: polar representation



$$x\cos\theta + y\sin\theta = \rho$$

Each point in image space will map to a sinusoid in the (ρ,θ) parameter space

Line Extraction | relative merits

- Split-and-merge: fastest
  - Deterministic & makes use of the sequential ordering of raw scan points
(: points captured according to the rotation direction of the laser beam)

- If applied on randomly captured points only RANSAC and Hough-Transform would segment all lines.

- RANSAC and Hough-Transform:
more robust to outliers

Courtesy of F. Pomerleau and F. Colas