CSCI/ARTI 4530/6530 Introduction to Robotics - Fall 2018
Assignment 3: Robot Localization and Navigation

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General Information

Deadline: 11:59 pm, Nov 20 (Tuesday)
Worth: 75 pts (undergraduate); 100 pts (graduate)

This assignment is to make yourself familiar with the robotics localization, mapping, navigation algorithms. Please be as specific as possible while writing the answers because your grade will be based not only on the correctness of the solutions but also the approaches you take.

Important note: This is a two-part assignment. This assignment is NOT a group assignment and everybody should work on it INDIVIDUALLY. I encourage you to discuss the problems in this assignment but you are expected to solve and write the solutions individually.

In order to complete this assignment successfully, you must first carefully read your own notes and associated slides on an robot localization and navigation (path planning).

Problems

[Part I] (35/60 points) Paper and Pencil

1. (15 pts) We noted that EKF linearization is an approximation. Let a mobile robot operate in a planar environment. Its state is (x, y, θ). Suppose we know x and y with high certainty, but the orientation θ is unknown. This is reflected by our initial estimate:

$$\mu = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 10000 \end{pmatrix}$$

   (a) Draw, graphically, your best model of the posterior over the robot pose after the robot moves $d = 1$ unit forward. Assume that the robot moves noise free. For your drawing, ignore θ and draw the posterior over (x, y) only.

   (b) Now, develop this motion into a prediction step for the EKF. For this, define a state transition function and linearize it. Then, generate a new Gaussian estimate of the robot pose using the linearized model. Be sure to give the exact mathematical equations for each step and state the Gaussian that results.

2. (20 pts) Monte Carlo localization is biased for any finite sample size – i.e., the expected value of the location computed by the algorithm differs from the true expected value. In this question, you are asked to quantify this bias.

To simplify, consider a world with four possible robot locations, $X = \{x_1, x_2, x_3, x_4\}$. Initially, we draw $N \geq 1$ samples uniformly from among those locations. As usual, it is perfectly acceptable if more than one sample is generated for any of the locations in X. Let Z be a Boolean sensor variable characterized by the following conditional probabilities:

$$
\begin{array}{ll}
p(z|x_1) = 0.8 & p(\neg z|x_1) = 0.2 \\
p(z|x_2) = 0.4 & p(\neg z|x_2) = 0.6 \\
p(z|x_3) = 0.1 & p(\neg z|x_3) = 0.9 \\
p(z|x_4) = 0.1 & p(\neg z|x_4) = 0.9
\end{array}
$$

MCL uses these probabilities to generate particle weights, which are subsequently normalized and used in the resampling process. For simplicity, let us assume that we only generate one new sample in the resampling process, regardless of N. This sample might correspond to any of the four locations in X. Thus, the sampling process defines a probability distribution over X.

(a) Let $N = 1, 2, \ldots 5$ and $N = \infty$. What is the resulting probability distribution over X (i.e., posterior probability) for this new sample for the different values of N when the observation is z?

(b) The difference between two probability distributions p and q can be measured by the *KL divergence*, which is defined as,

$$KL(p, q) = \sum_i p(i)\ log \frac{p(i)}{q(i)}.$$

What are the KL divergences between distributions in (a) and the true posterior? (Set q as the true posterior in the KL divergence.)

3. (25 pts; **Graduate students only**) Consider a drone living in a 3D world and equipped with a perfect compass (it always knows its orientation). Let the robot move independently in all three Cartesian dimensions (i.e., x, y and z) by setting velocities along the three dimensions. Its motion noise is Gaussian and independent for all directions.

   The robot is surrounded by a number of beacons that emit radio signals. The emission time of each signal is known, and the robot can determine from each signal the identity of the emitting beacon (there is no correspondence problem). The robot also knows the locations of all beacons, and it is given an accurate clock to measure the arrival time of each signal. However, the robot cannot sense the direction from which it received a signal.

   Mathematically derive the motion and measurement models, along with the Taylor approximation. Use these models to write down the final EKF localization process assuming known correspondence. You *may* also include the algorithm.

[Part II] (40 points) ROS Practicals

For an Occupancy Grid Map output from a SLAM algorithm (e.g., Hector SLAM), implement either RRT or A* path planning algorithm for a specified goal position as a service provider (provide the goal position as input and obtain the path to goal as response). Use the laser scan data provided to you (in *Slack* for generating the map with the SLAM node in ROS.

What and how to hand it in

Hand in the *typed* answers to Part I in eLC A3 submission folder by the deadline. Please be sure to indicate the question numbers alongside the answers. The document should include your name, student id, and all the answers. You'll submit Part II of your assignment in a *single zipped file* by the deadline via eLC (same submission folder). The zip file should contain your *ros_package* called *yourlastname_a3*. (Please use all lower case letters in the package name). In a separate README file (within the Zip file of Part II), describe how to compile and run your programs, and mention the output that you obtained in your evaluations.

Assignments that are **late** but within a day of the deadline will be penalized 25% of the total number of points. Two days of delay will be penalized 50%. Assignments submitted later than two days will not be accepted.