

# Securing Secure Aggregation: Mitigating Multi-Round Privacy Leakage in Federated Learning

Ramy E. Ali

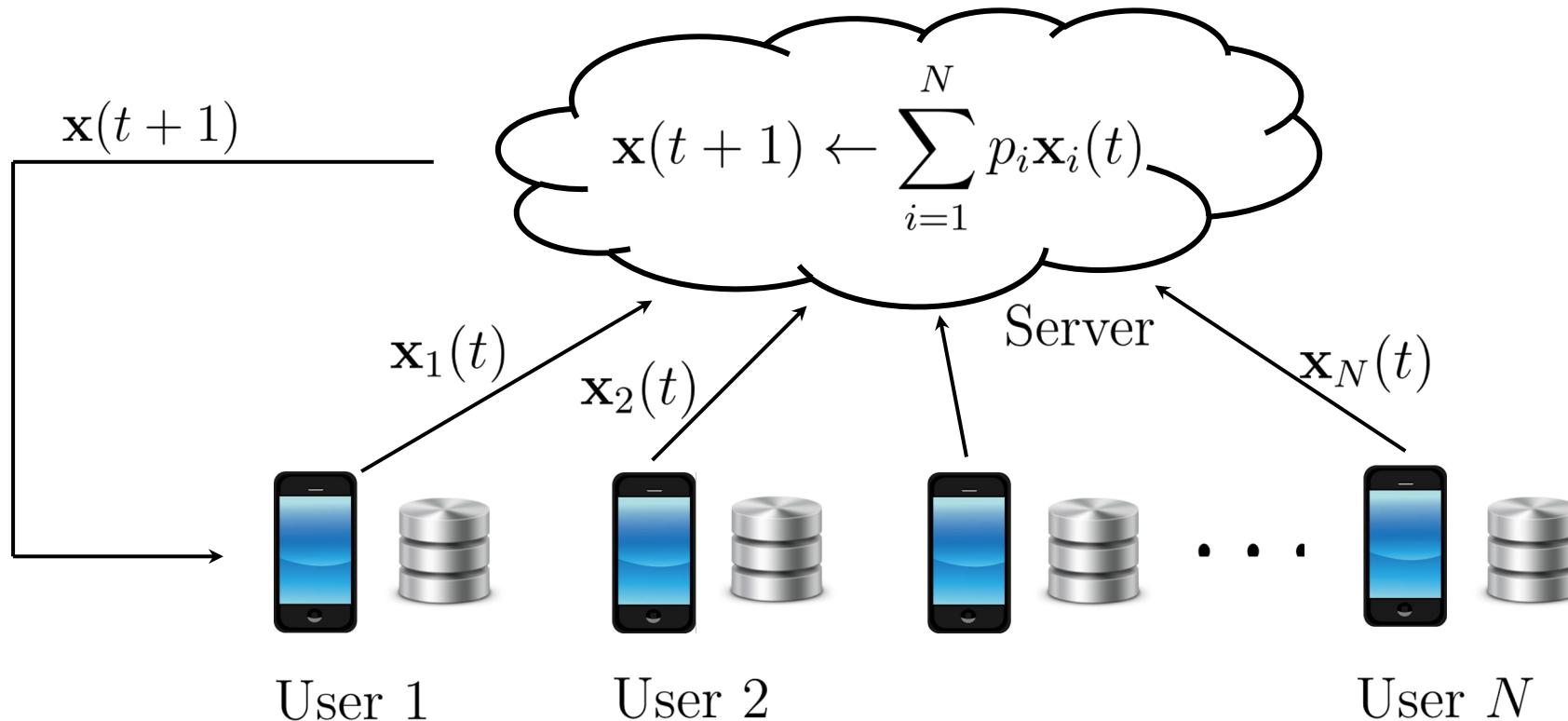


In collaboration with Jinhyun So (USC), Başak Güler (UCR), Salman Avestimehr (USC) and Jiantao Jiao (UC Berkeley)

2021

# The Promise of Federated Learning

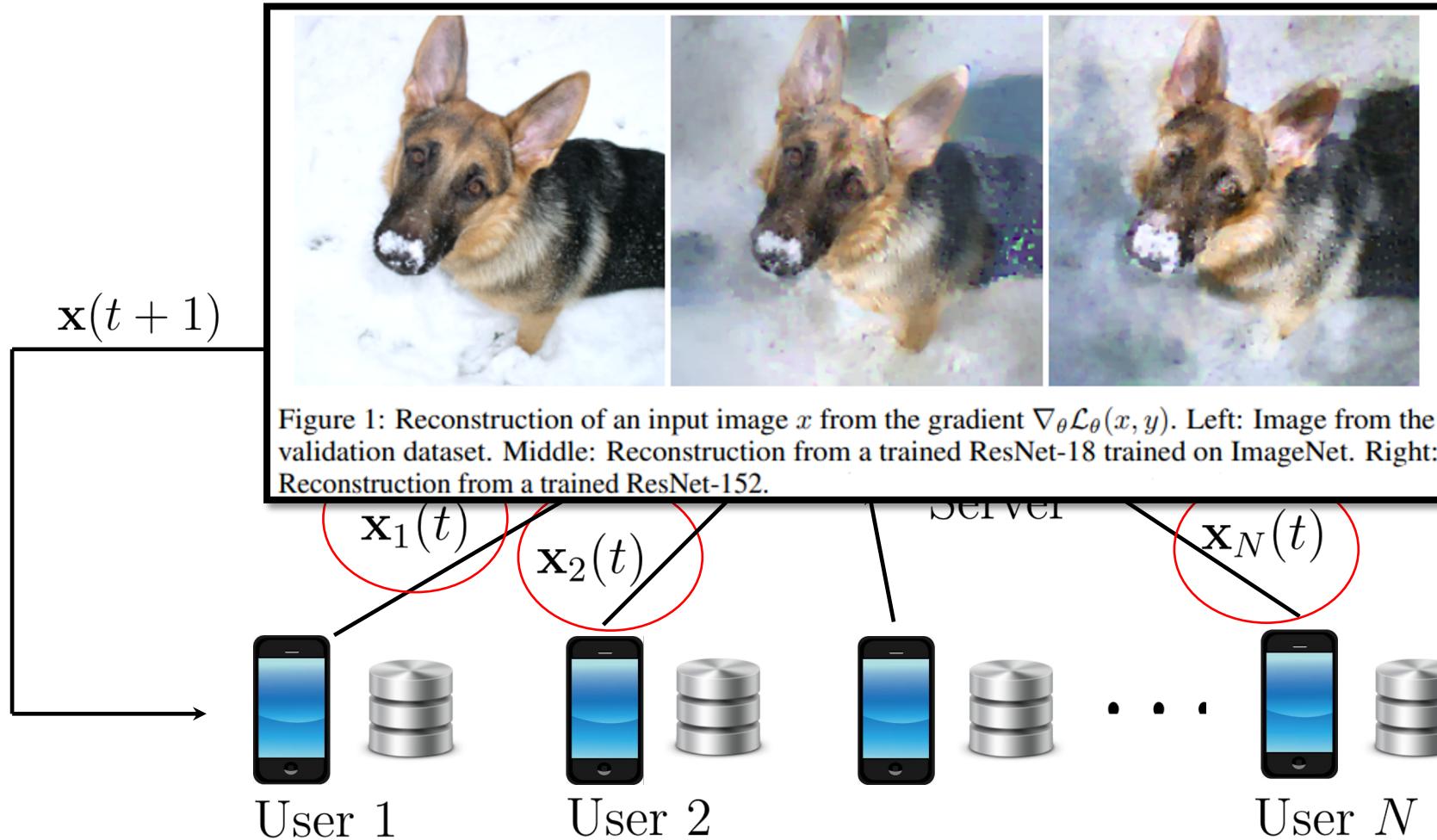
Main principle: train locally - average globally



Ensuring **privacy** by avoiding data sharing?

But ...

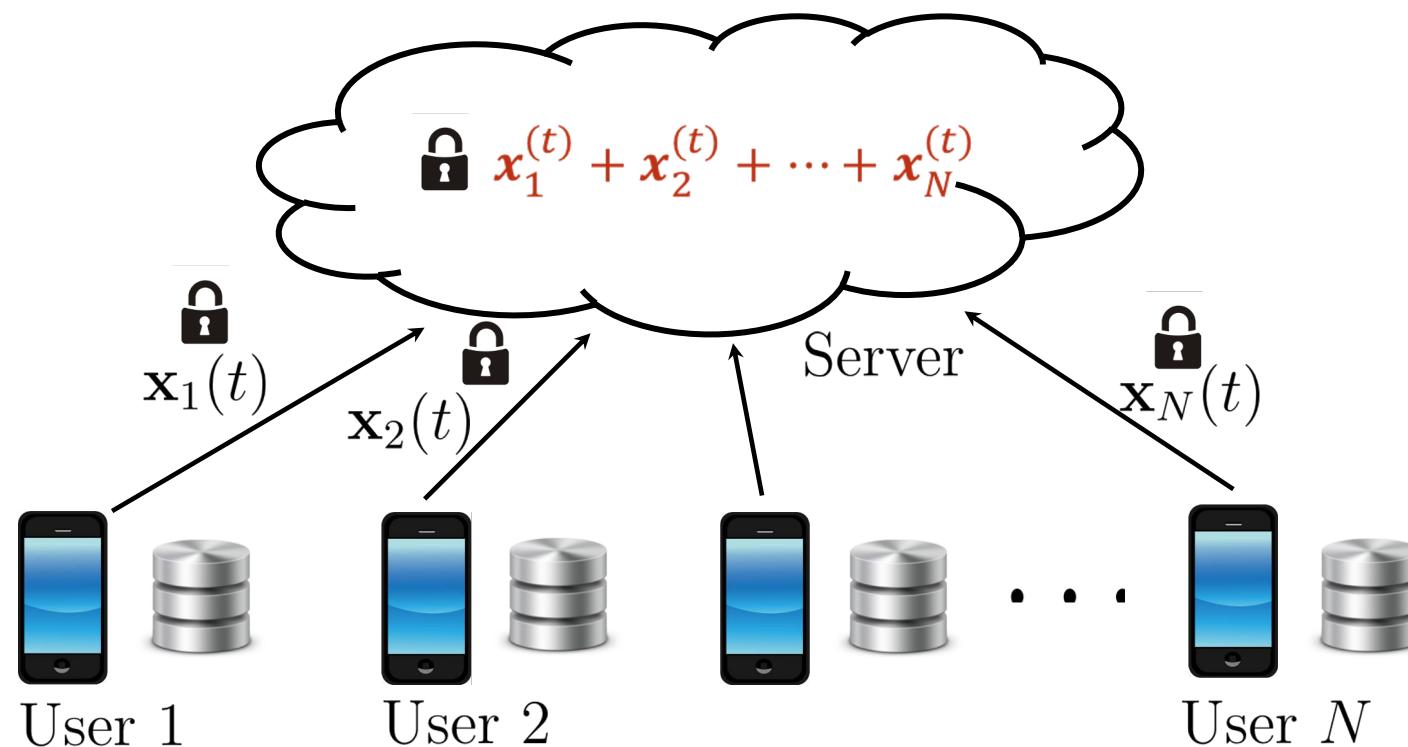
## Model Inversion Attack



**Problem:** Individual model updates can leak sensitive data

# Remedy: Secure Model Aggregation

- Secure aggregation ensures that the server only learns the global model.



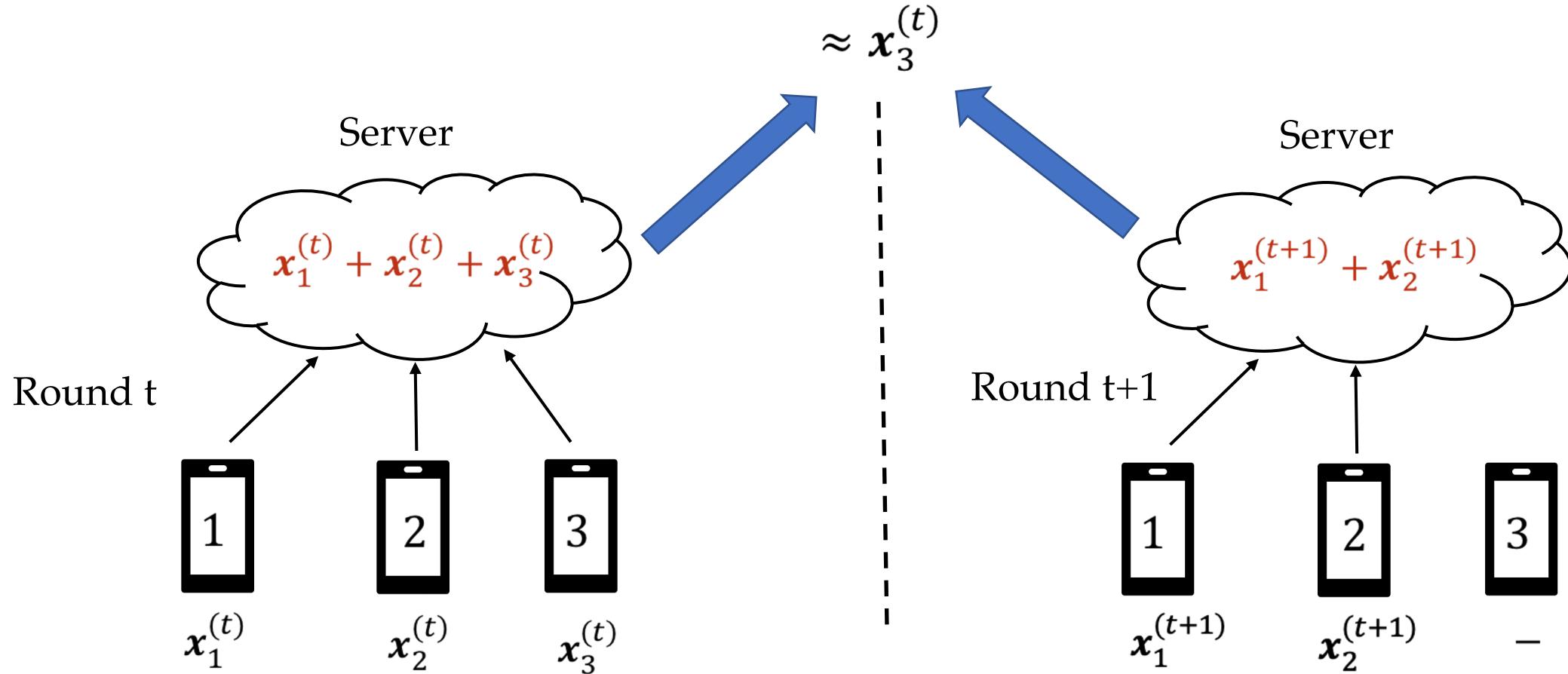
Secure Aggregation is Essentially an MPC problem with [User Dropouts](#)

## Bad news ...

- Secure aggregation, however, is not secure over **multiple** rounds.

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- Secure aggregation, however, is not secure over **multiple** rounds.
- **Intuition:** partial user participation leads to privacy leakage

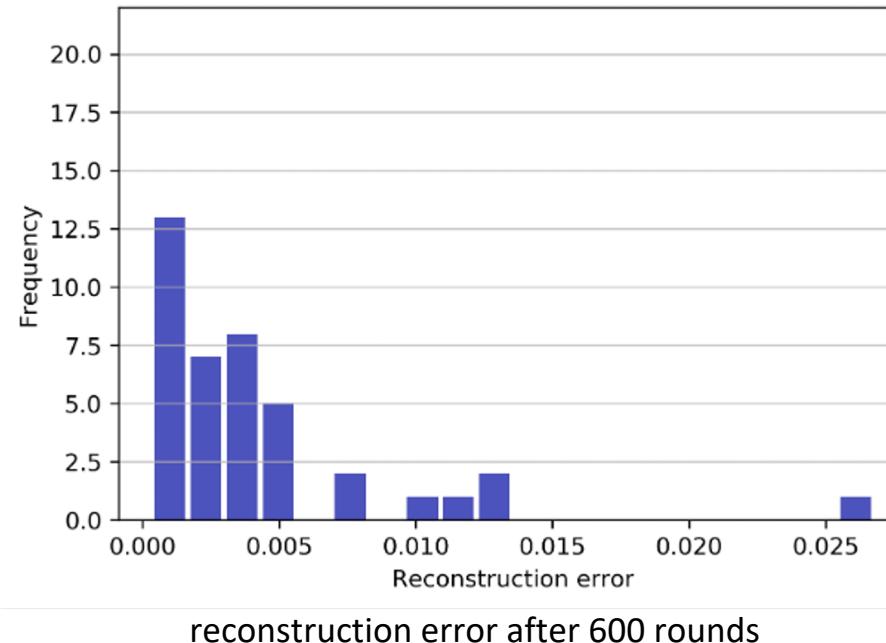


# This is a serious issue!

- Random selection may reveal all individual models.
- **Experiment**
  - N=40 users
  - MNIST dataset with non-IID distribution
  - K=8 users are selected at random at each round
  - The server estimates the individual gradients through least-squares

Reconstruction error of  
user  $i$  at time  $t$

$$\frac{\|\mathbf{x}_i^{(t)} - \hat{\mathbf{x}}_i^{(t)}\|^2}{\|\mathbf{x}_i^{(t)}\|^2}$$



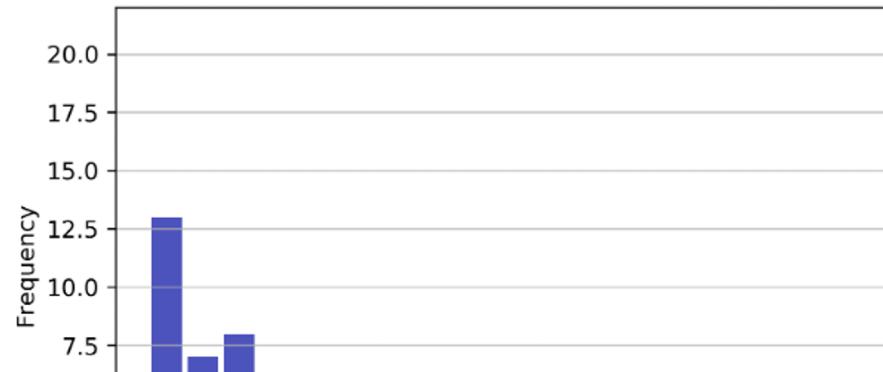
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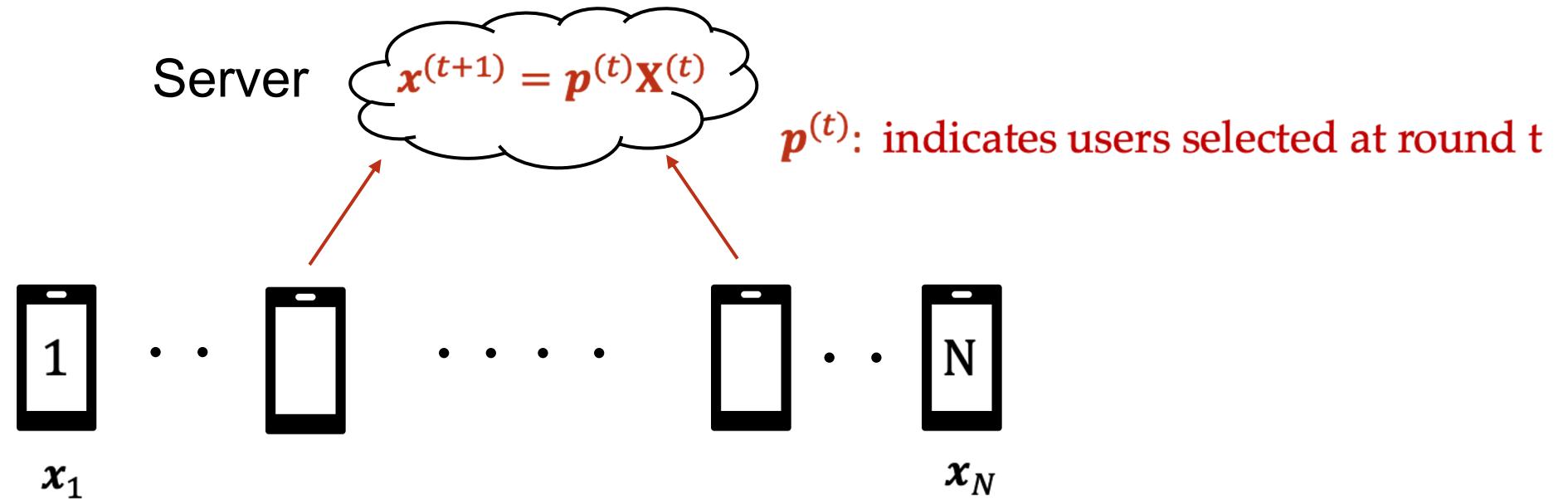
Similar concerns have also been reported in some recent works:

- Pejó et al. "Quality Inference in Federated Learning with Secure Aggregation.", 2020.
- M. Lam et al. "Gradient Disaggregation: Breaking Privacy in Federated Learning by Reconstructing the User Participant Matrix.", ICML 2021.

# This talk

- Introduce a notion/metric for multi-round privacy
- Propose Multi-RoundSecAgg, which ensures multi-round privacy
  - It further optimizes the average number of participating users (convergence rate) and fairness in user selection
  - It also introduces a trade-off between “privacy” and “convergence rate”

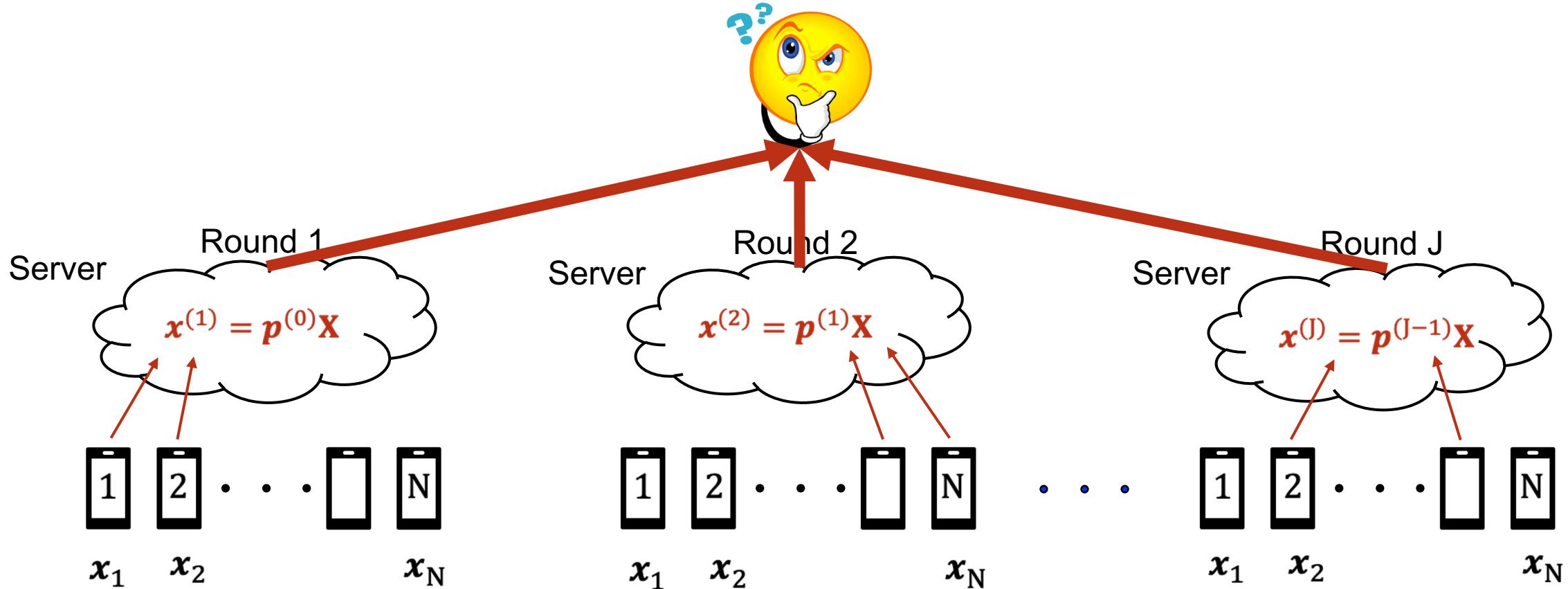
# Federated Averaging with Partial User Participation



- Participation matrix  $\mathbf{P}^{(J)} = \begin{pmatrix} \mathbf{p}^{(0)} \\ \vdots \\ \mathbf{p}^{(J-1)} \end{pmatrix} \in \{0,1\}^{J \times N}$ ,  $J$ : number of rounds

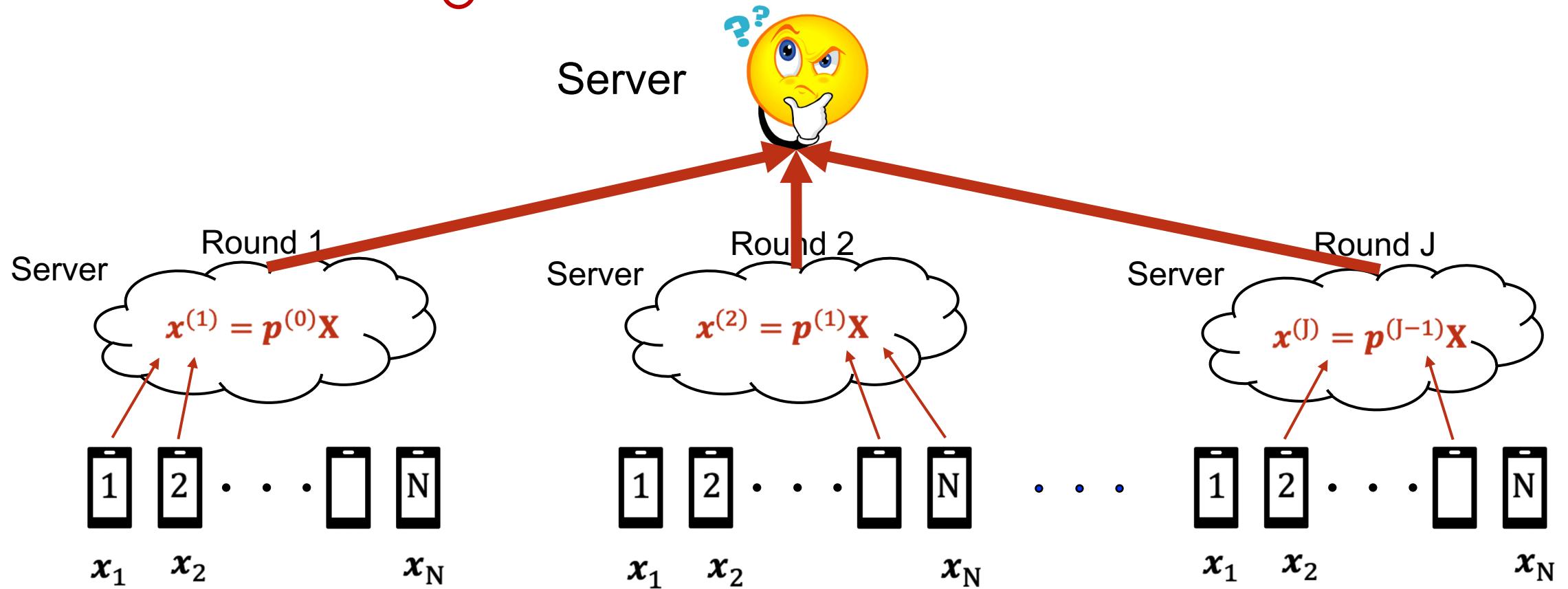
How should we choose  $\mathbf{P}^{(J)}$  to ensure long-term privacy?

# Metric 1: Multi-round Privacy (T)



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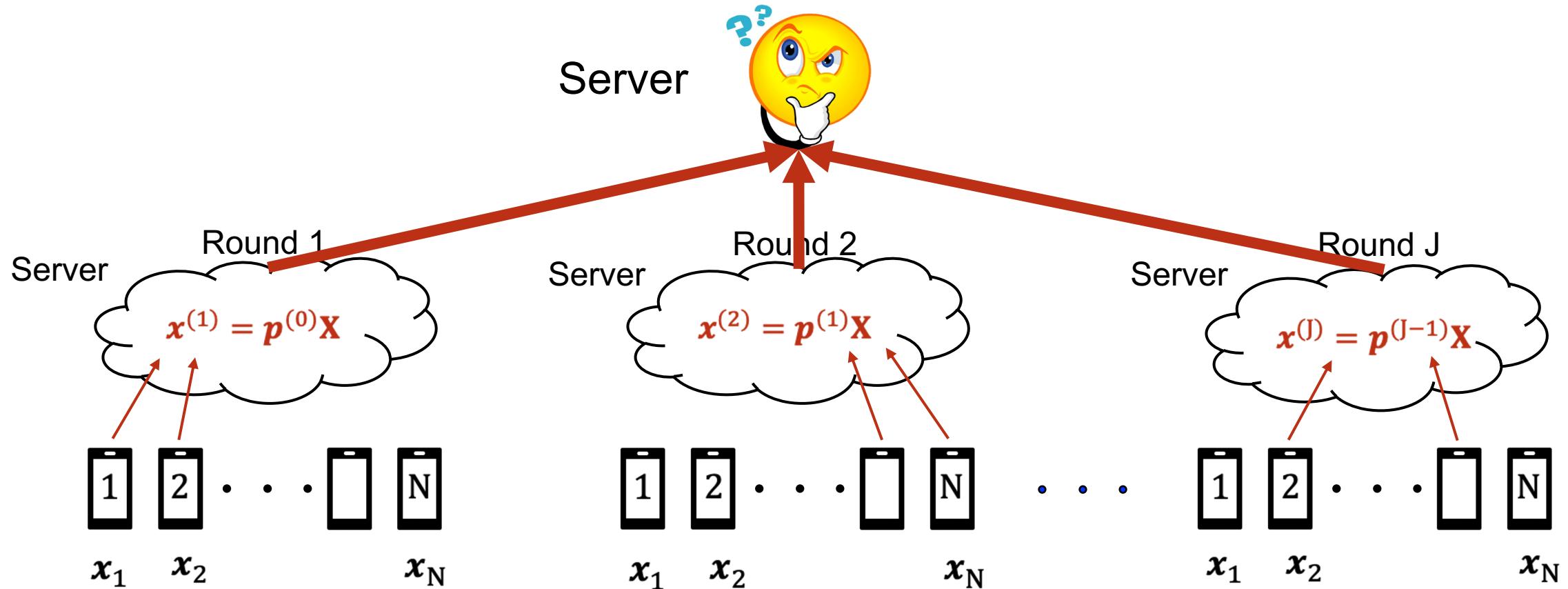
The server must not learn any aggregate model of less than T users.



# Metric 1: Multi-round Privacy ( $T$ )

- A multi-round privacy  $T$  requires that any non-zero partial sum that the server can reconstruct to be of the form

$$a_1 \sum_{j \in \mathcal{S}_1} x_j + a_2 \sum_{j \in \mathcal{S}_2} x_j + \cdots + a_n \sum_{j \in \mathcal{S}_n} x_j, \text{ where } |\mathcal{S}_i| \geq T.$$



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- **Example ( $T=2$ ):** the best the server can do is reconstructing  $\mathbf{x}_i + \mathbf{x}_j$  (for some  $i$  and  $j$ )

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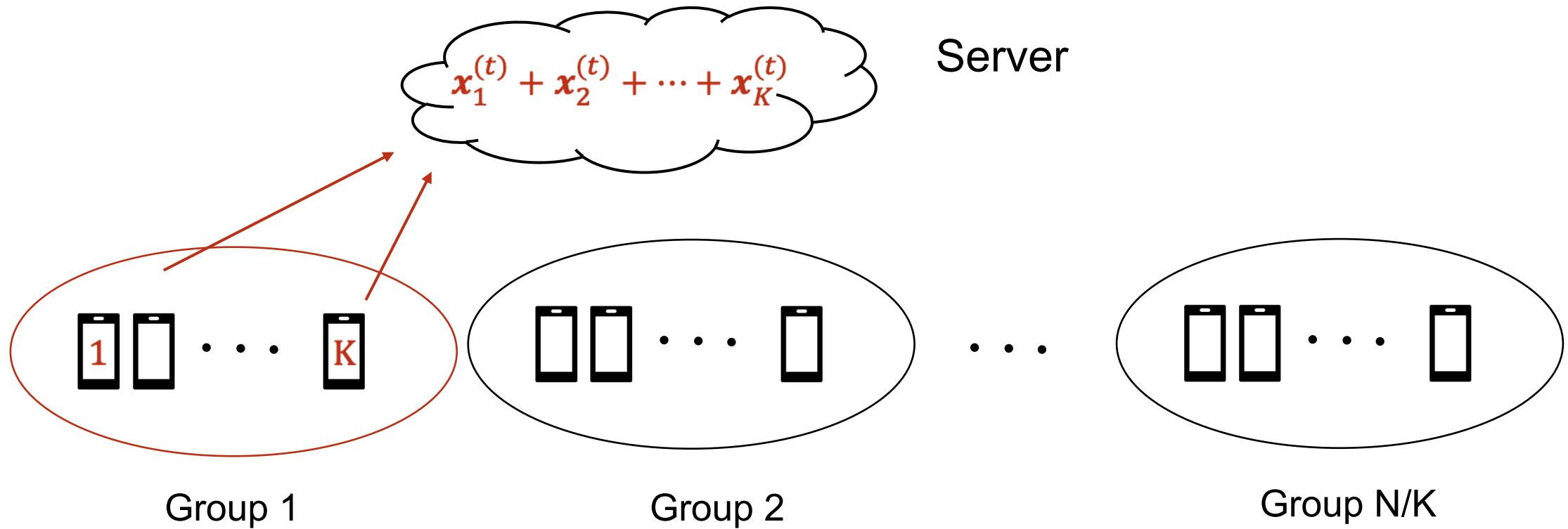
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- **Example ( $T=2$ ):** the best the server can do is reconstructing  $\mathbf{x}_i + \mathbf{x}_j$  (for some  $i$  and  $j$ )
- Worst-case (strong) assumption 1: the model coefficients in each group are the same
- Worst-case (strong) assumption 2: user's model stays the same across different rounds

# Baselines

## 1. User Partitioning

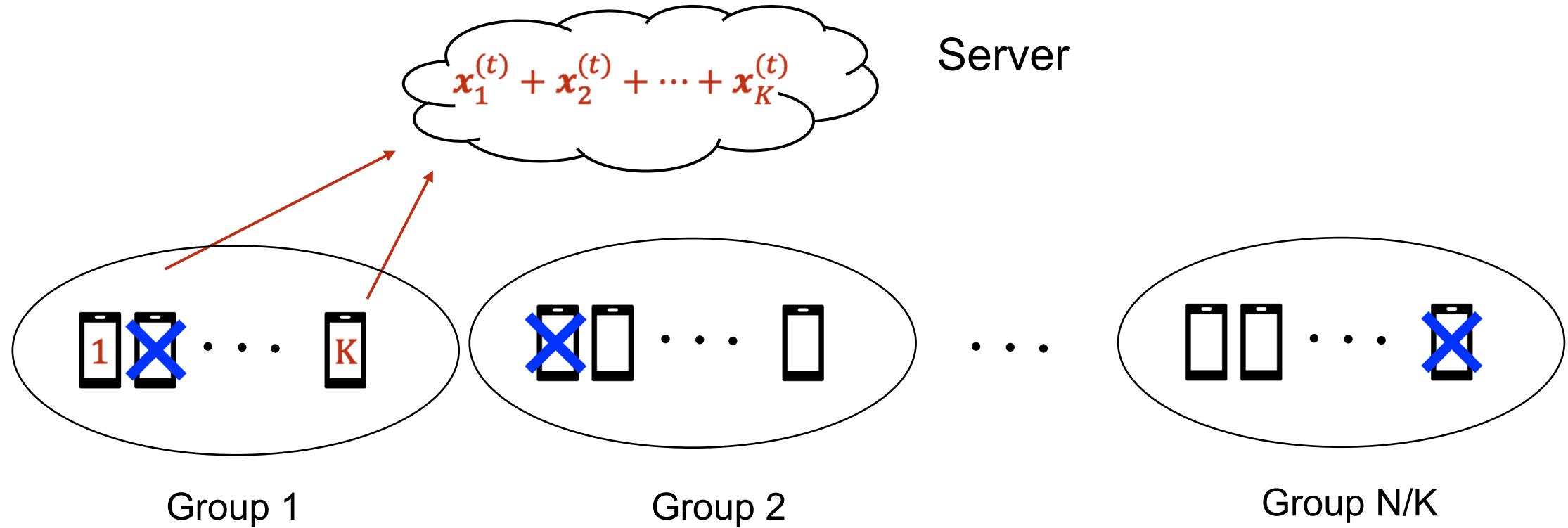
- Large multi-round privacy  $T = \text{group size}$  



# Baselines

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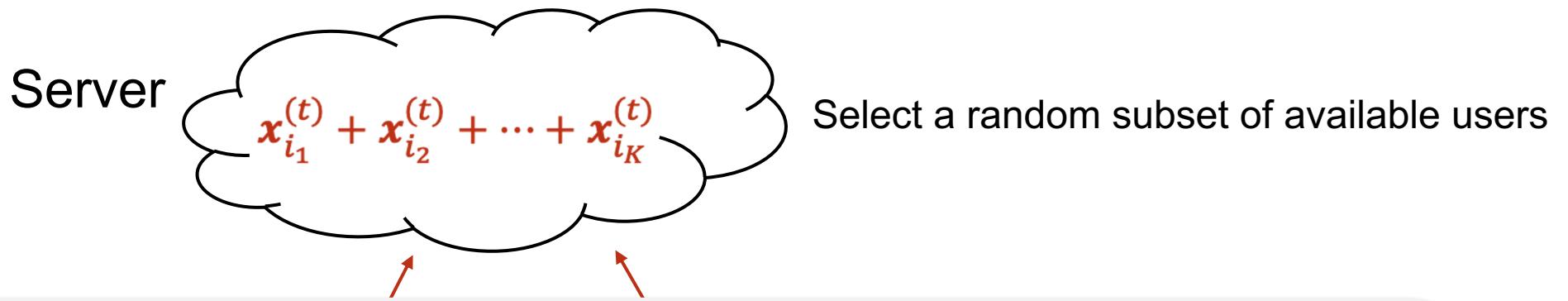
- Large multi-round privacy  $T = \text{group size}$
- In many rounds, however, no groups are available



# Baselines

## 2. Random Selection

- Small multi-round privacy  $T = 1$  



**Theorem:** In Random Selection, the server can reconstruct all individual models of the  $N$  users after  $N$  rounds with probability at least

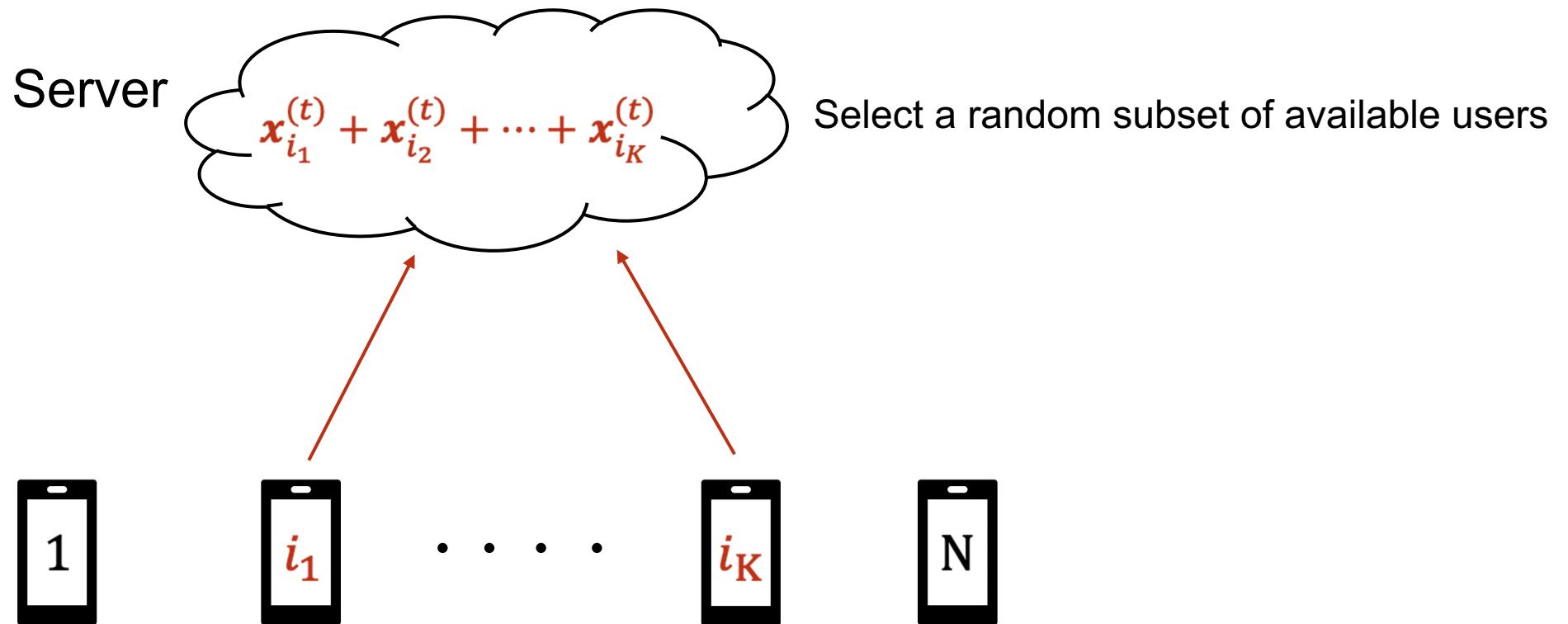
$$1 - \exp(-cN),$$

where  $c$  is a constant.

# Baselines

## 2. Random Selection

- Small multi-round privacy  $T = 1$
- Any subset of available users can be selected in any round



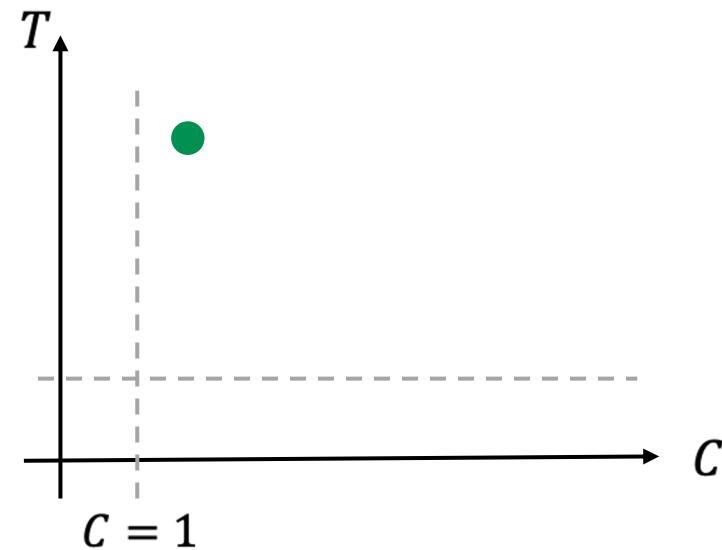
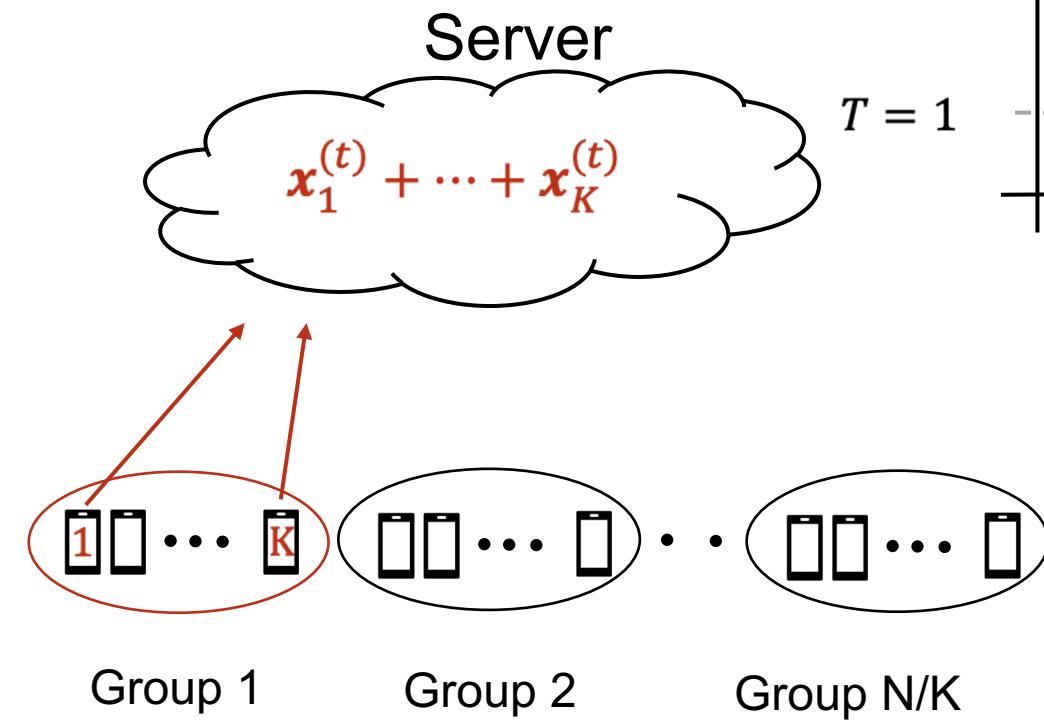
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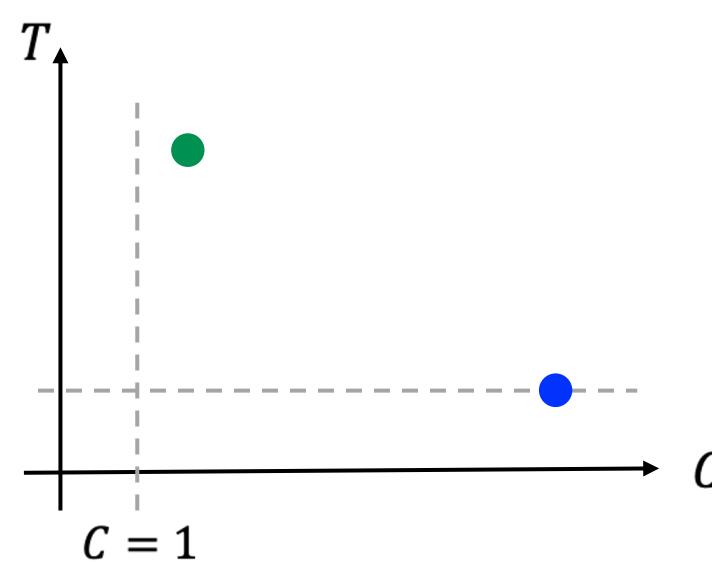
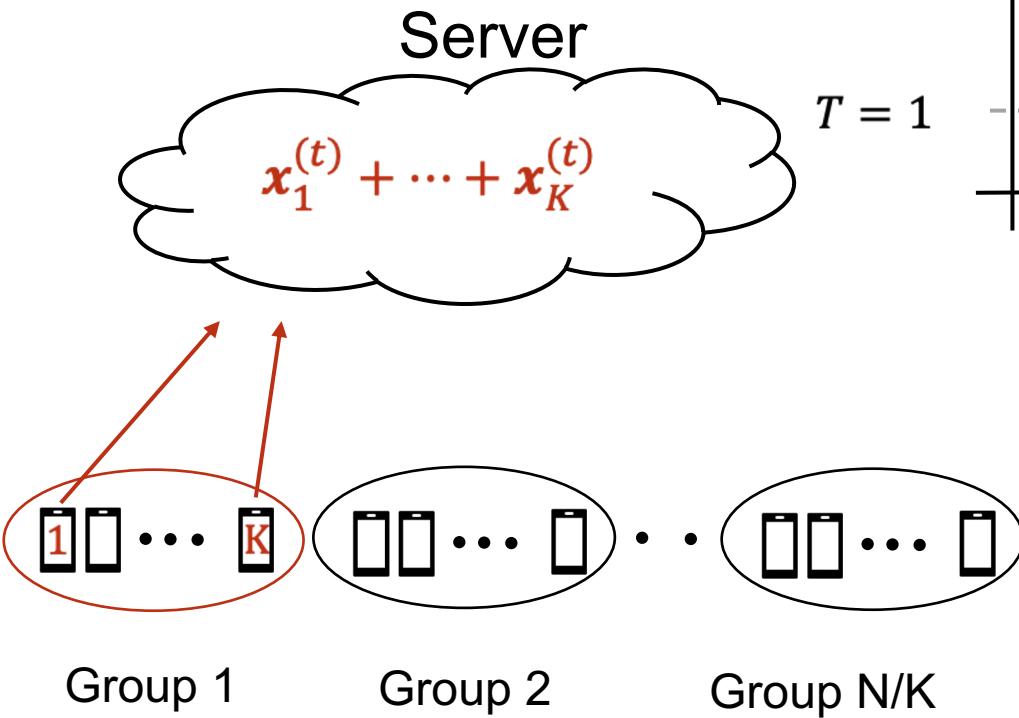
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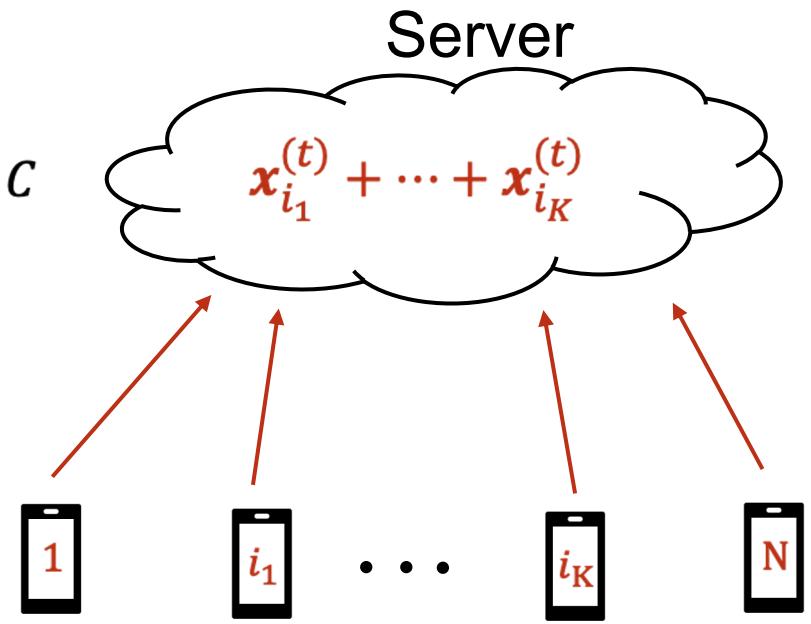
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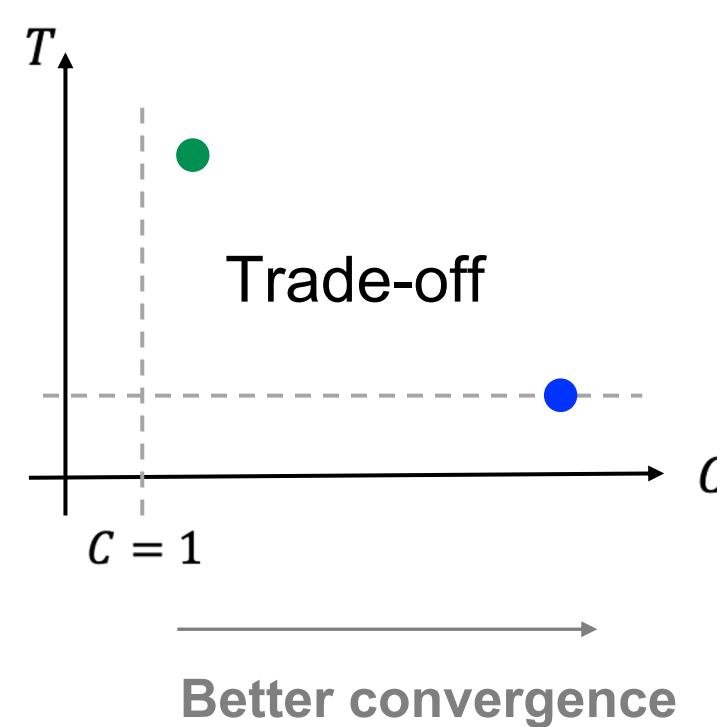
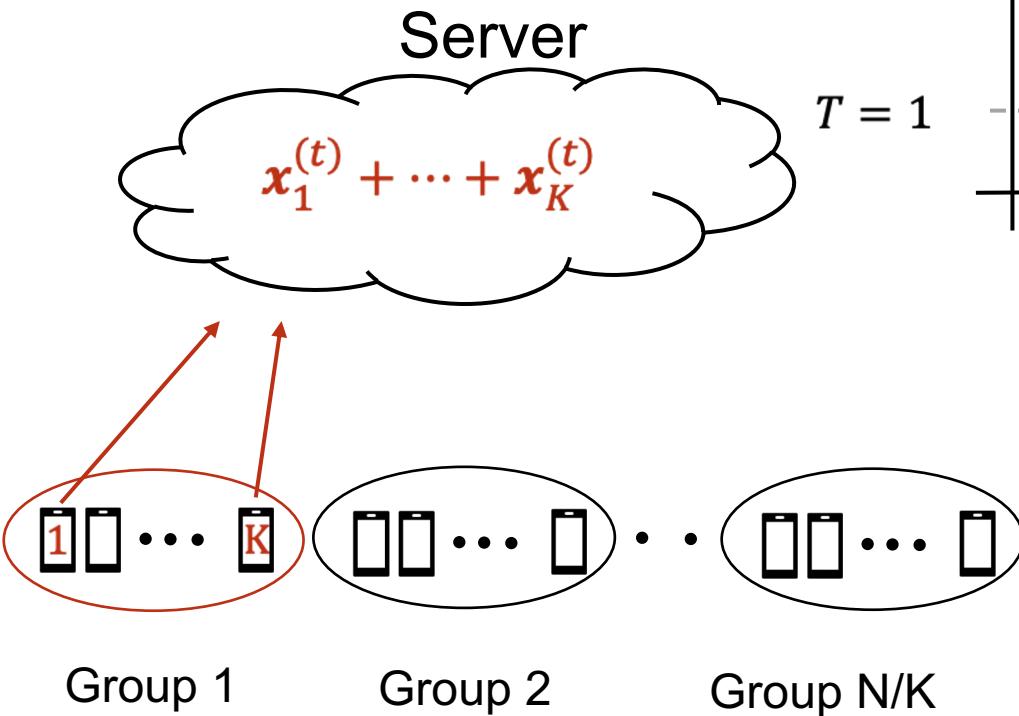
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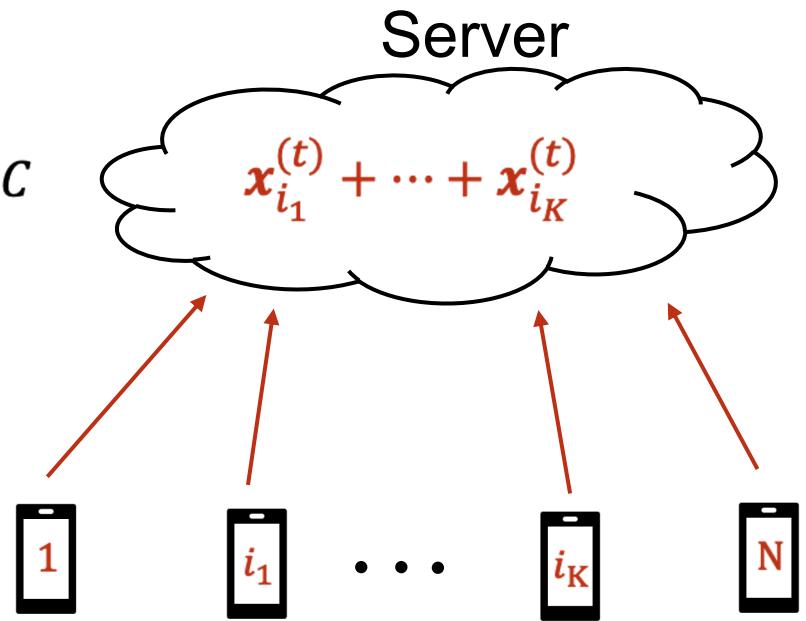
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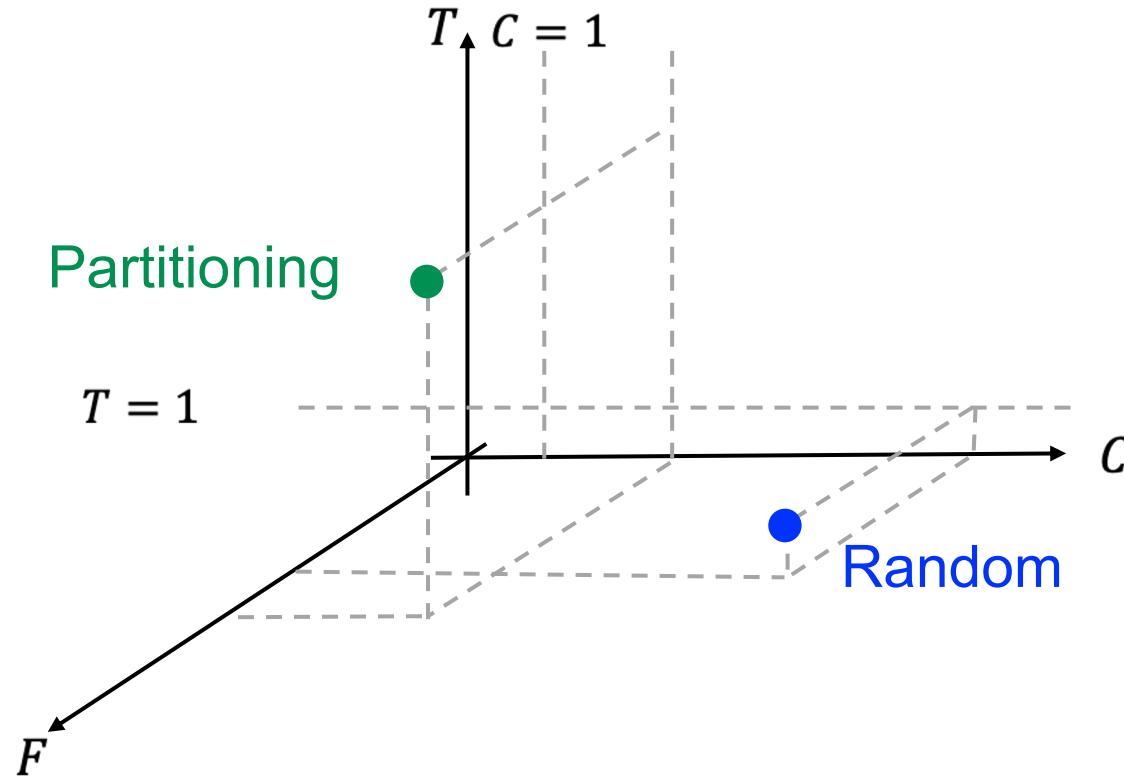
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## Metric 3: Aggregation Fairness Gap (F)

- Aggregation Fairness Gap  $F$

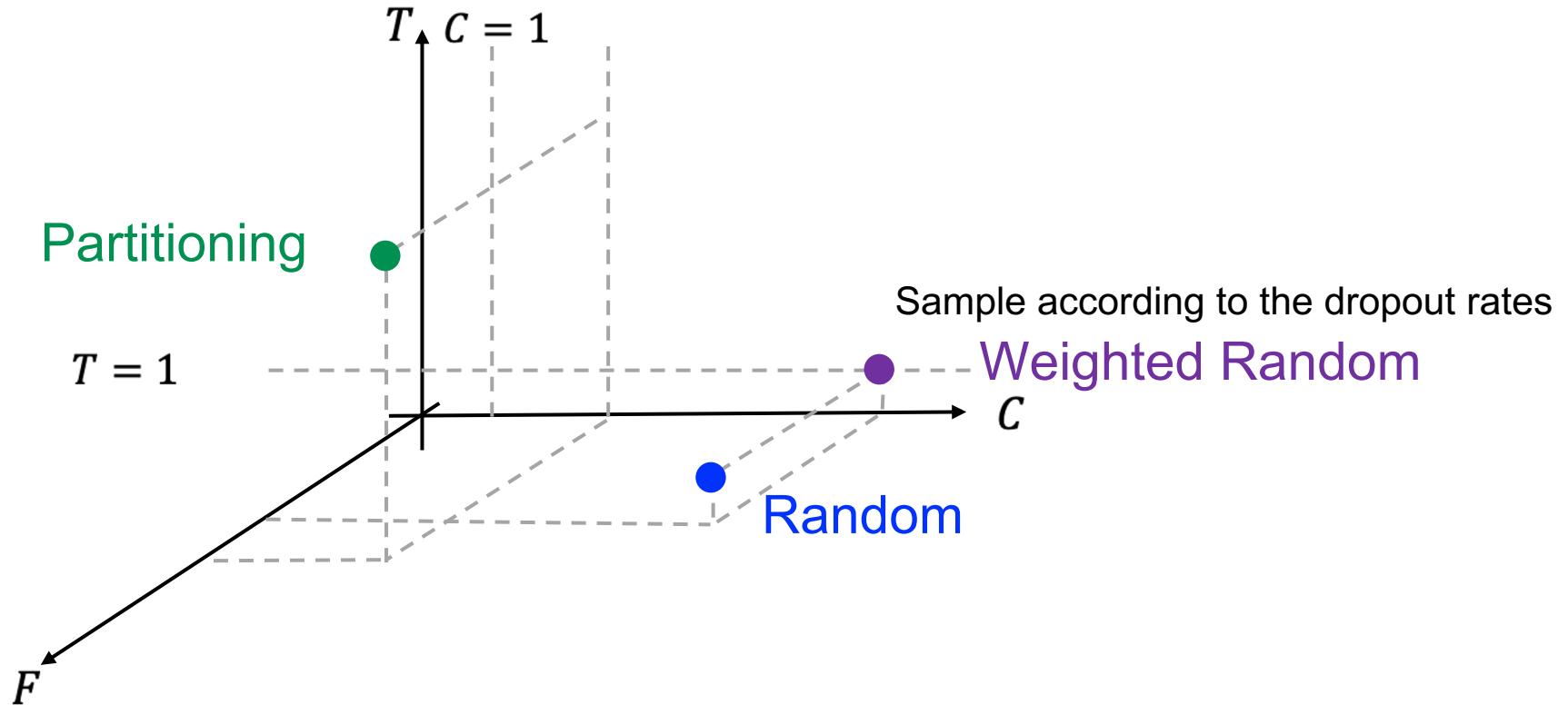
$F = \text{max. average participation frequency} - \text{min. average participation frequency}$



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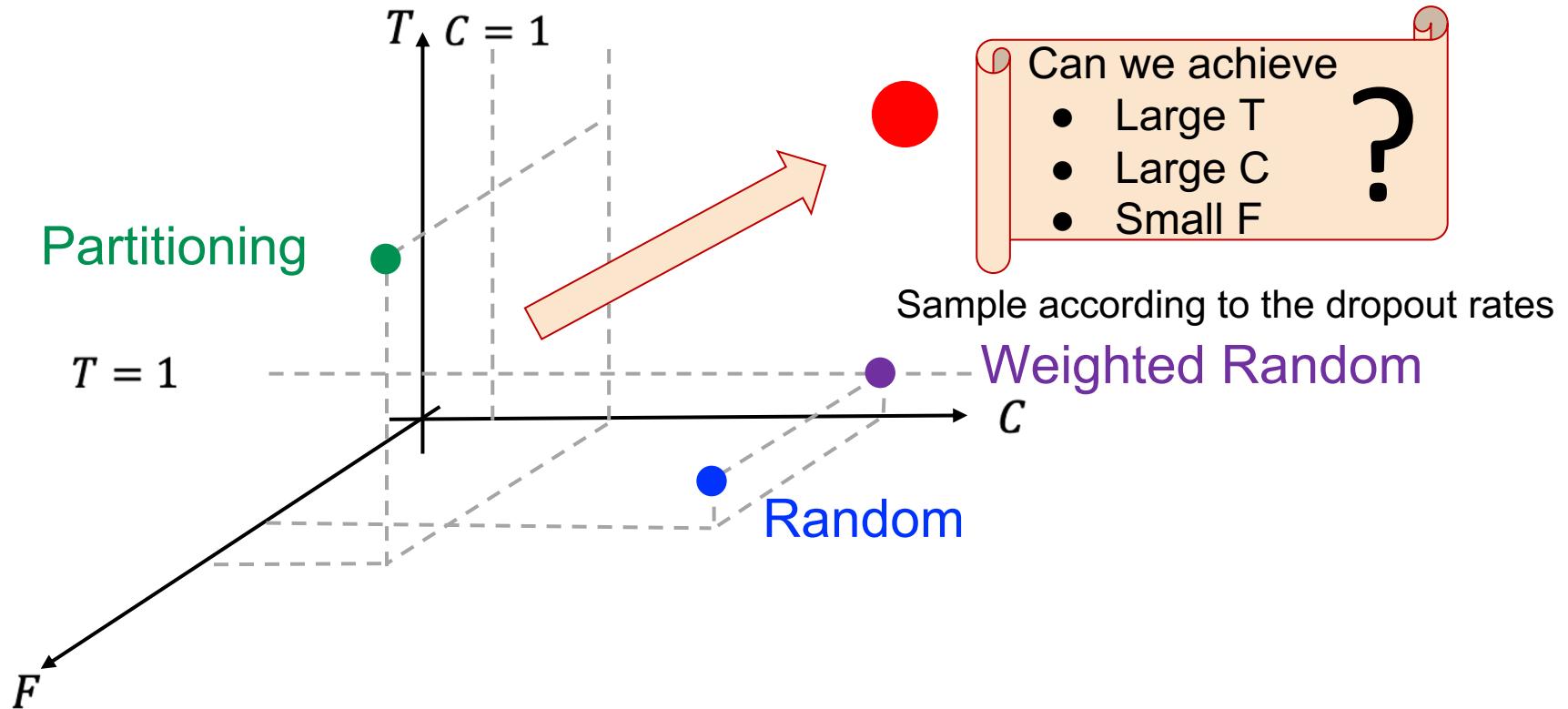
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# Proposed Approach: Multi-RoundSecAgg

## 1) Batch Partitioning

- Input:  $N, K \leq N, 1 \leq T \leq K$
- Output: A family of  $K$ -user sets satisfying the multi-round privacy  $T$ .  
This family is represented by a matrix  $\mathbf{B}$ .

## 2) Available batch selection to guarantee fairness

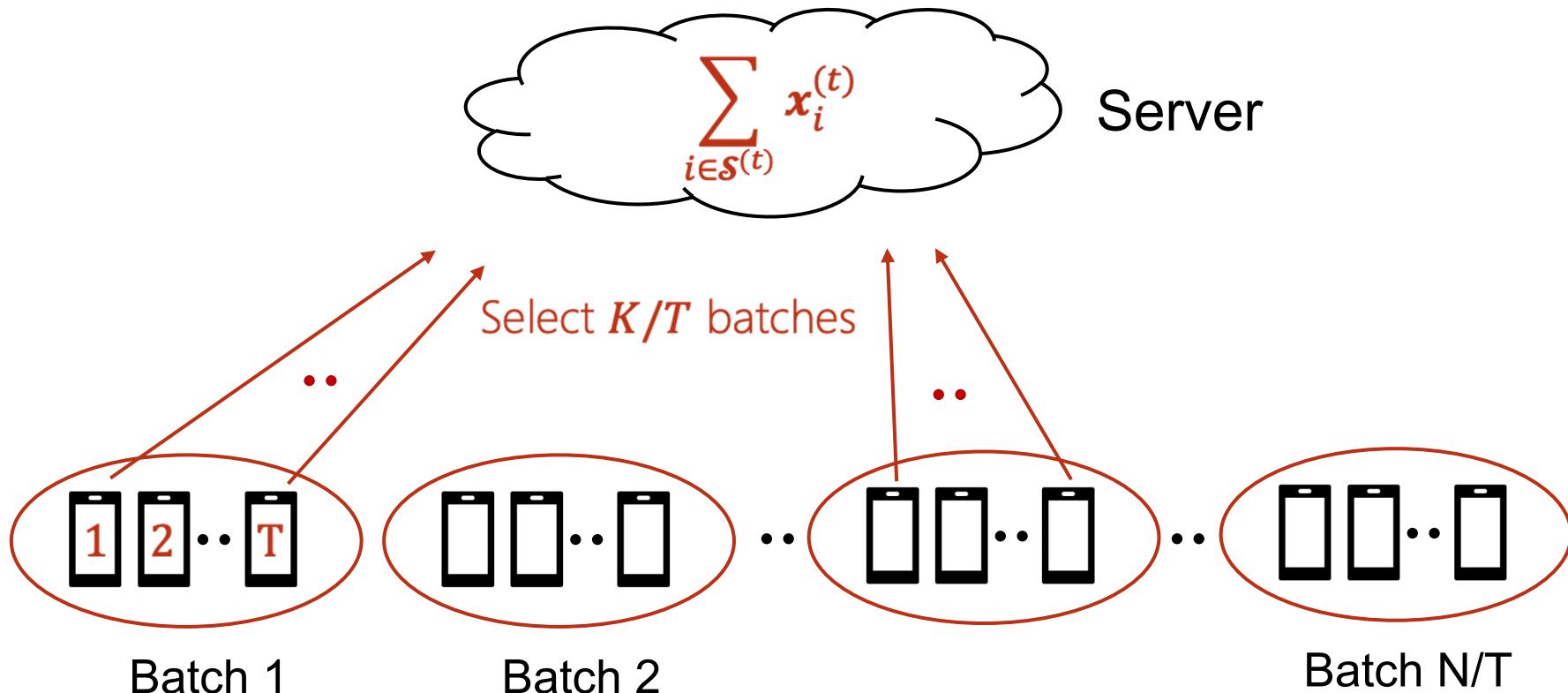
- Input: Set of available users at round  $t$  and  $\mathbf{B}$ .
- Output: Set of users that will participate at round  $t$ .

# Multi-RoundSecAgg

## 1) Batch Partitioning

Idea: Partition users into  $T$ -user batches; allow selection of any  $K/T$  available batches

This results in a family of  $R_{BP} = \binom{N/T}{K/T}$  sets.



# Multi-RoundSecAgg

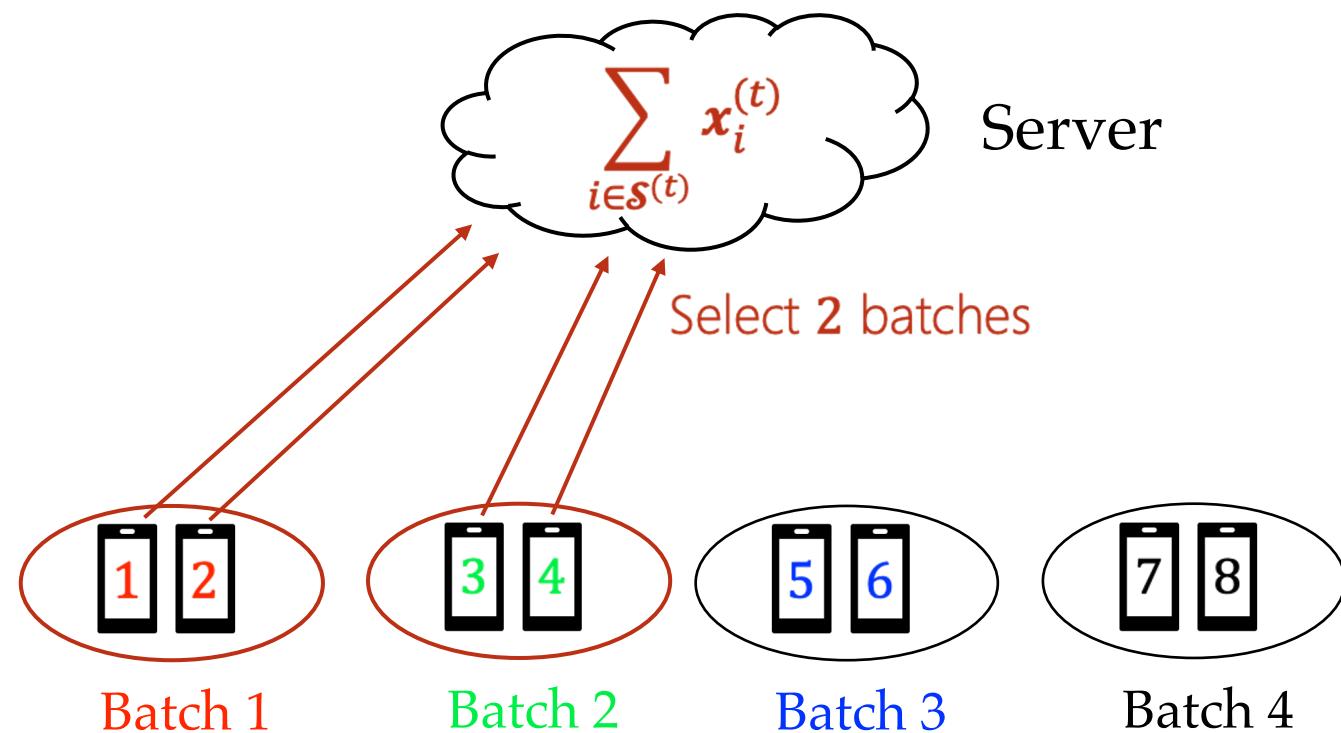
## 1) Batch Partitioning

Example ( $N = 8, K = 4$  &  $T = 2$ )

$$R_{BP} = \binom{N/T}{K/T} = 6 \text{ sets}$$

$$\mathbf{B} = \left( \begin{array}{|c|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 0 & 0 \\ \hline 1 & 1 & 0 & 0 & 1 & 1 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 1 \\ \hline \end{array} \right)$$

Batch 1   Batch 2   Batch 3   Batch 4



# Multi-RoundSecAgg

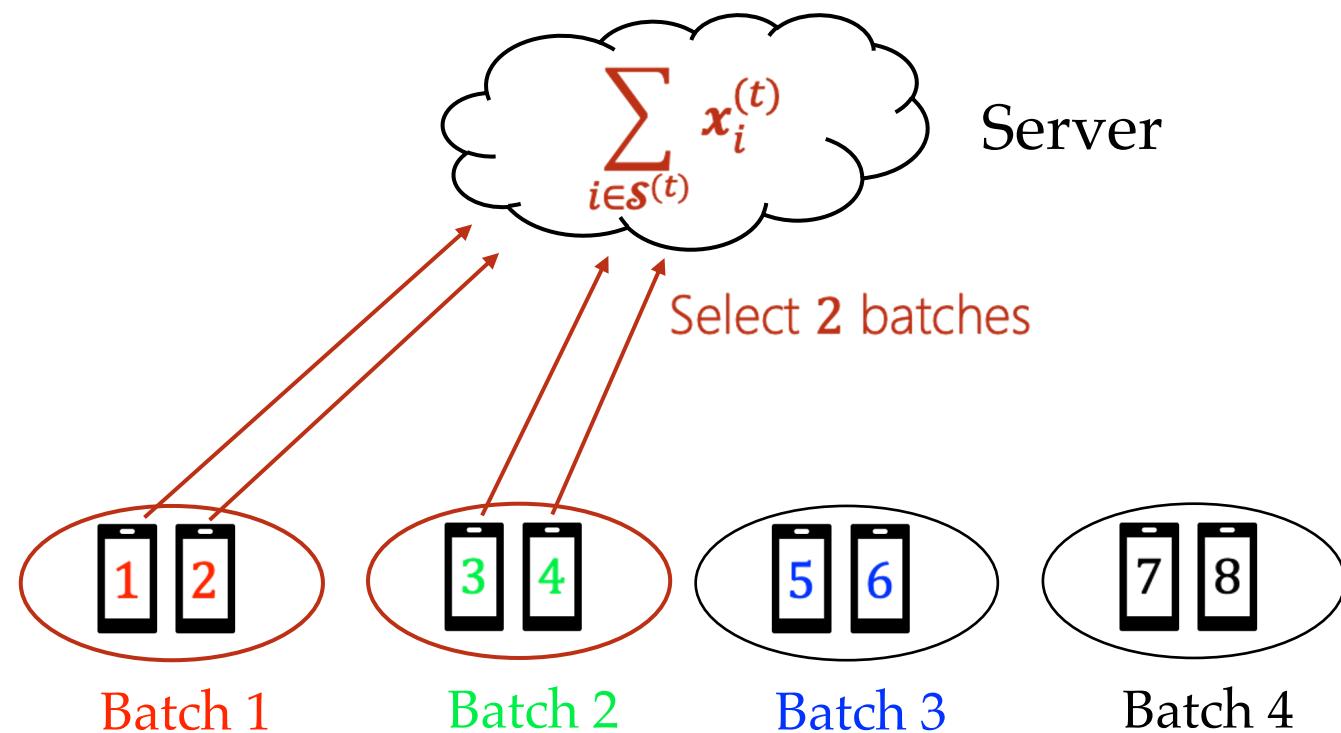
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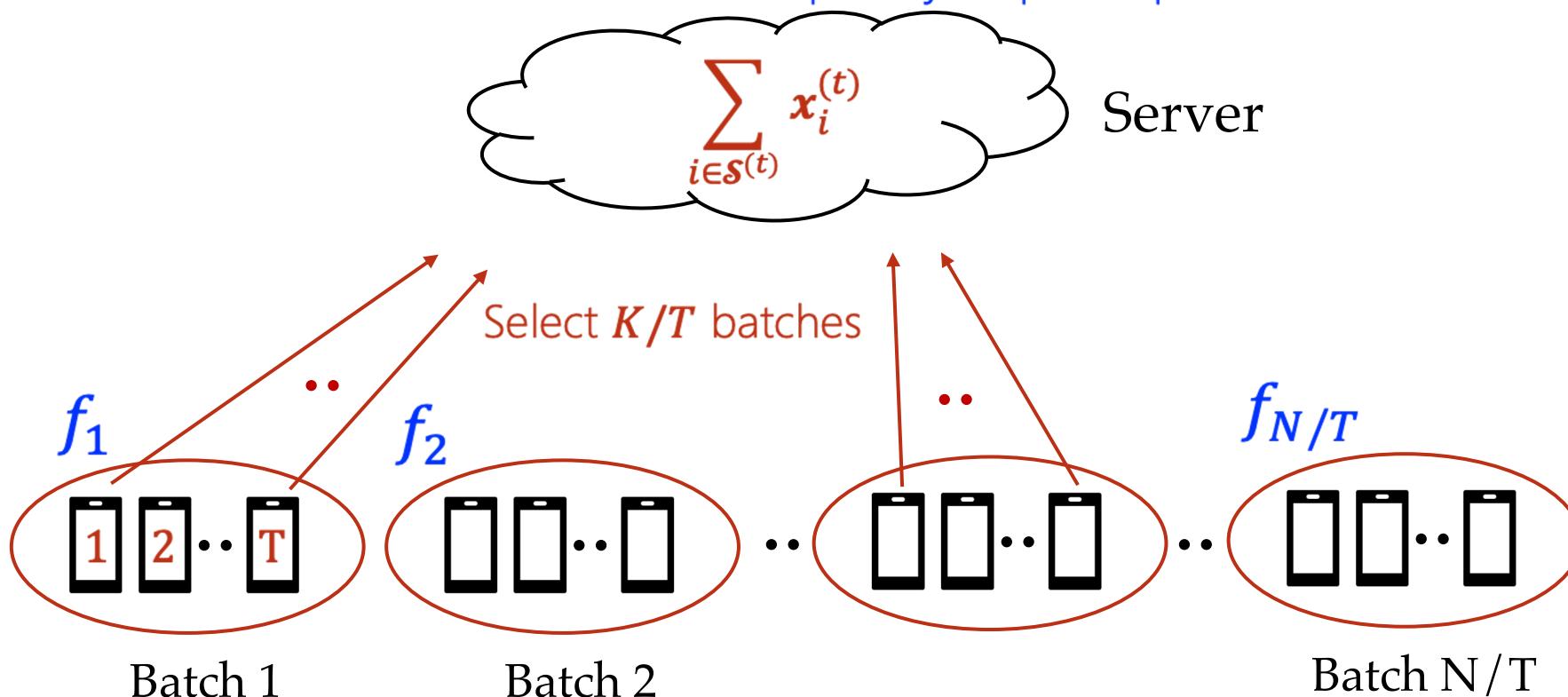


**Theorem:** aggregated models in the same batch can't be separated across different rounds even through non-linear mixtures of received aggregates.

# Multi-RoundSecAgg

2) Available batch selection to guarantee fairness

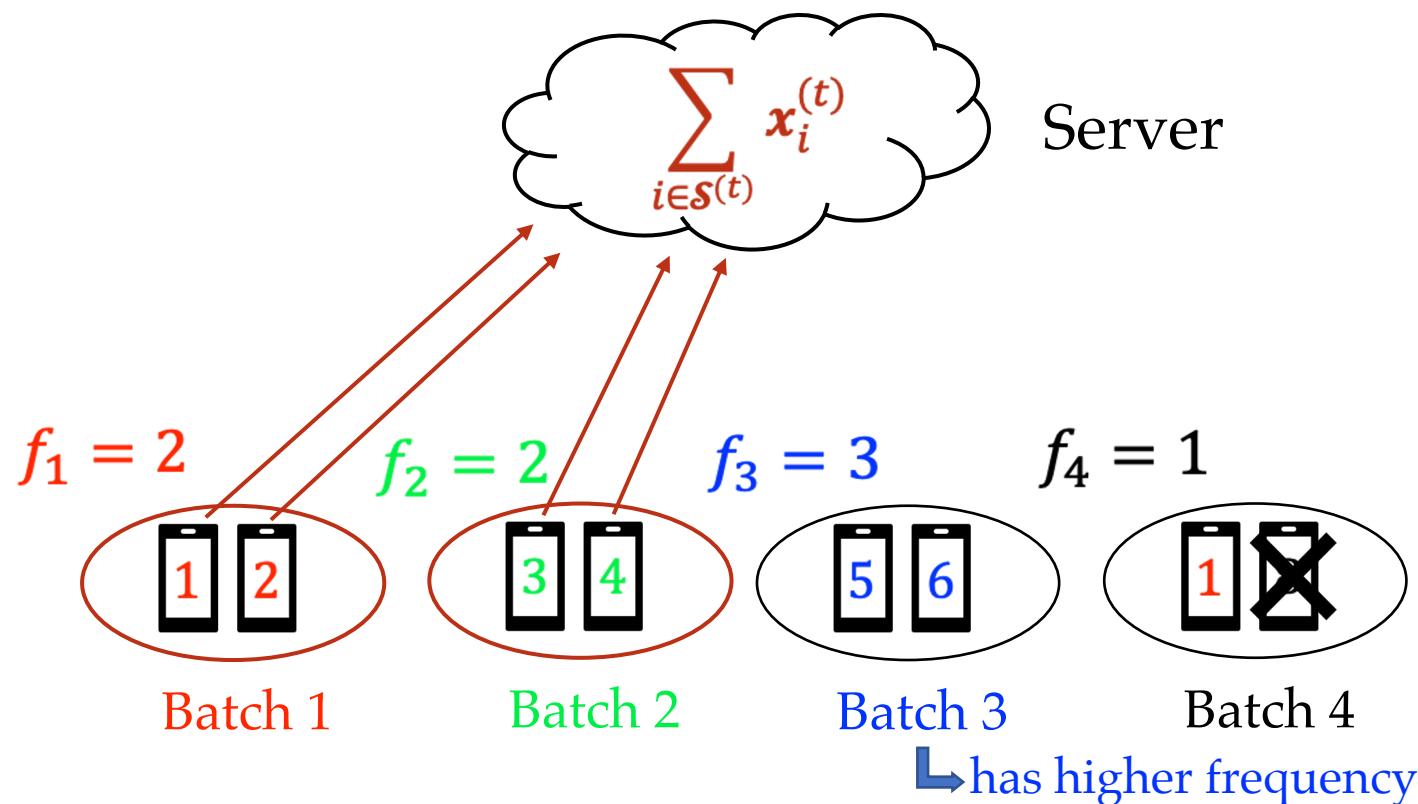
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- Idea: Select based on the minimum frequency of participation.



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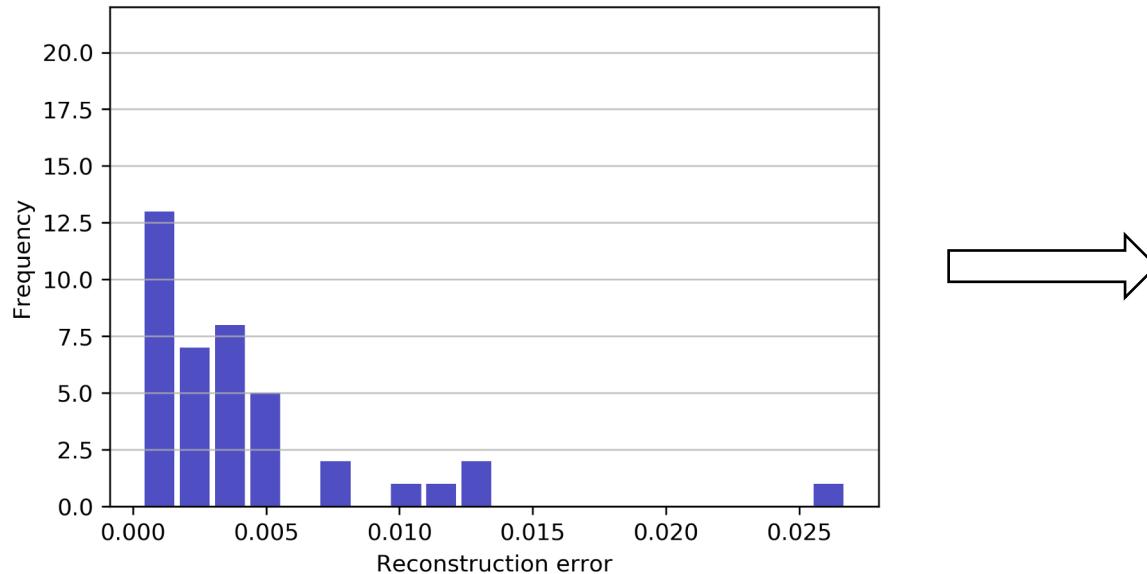
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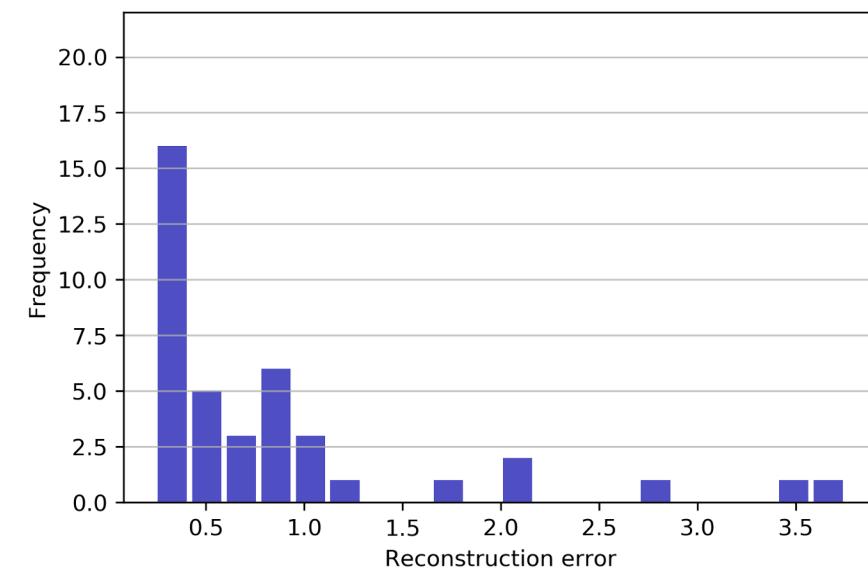


# An Illustrative Experiment

- Experiment (N=40 users)
  - MNIST Dataset & Non-IID Setting.
  - K=8 users are selected at random at each round.
  - Dropout probability  $p_i \sim \{0.1, 0.2, 0.3, 0.4, 0.5\}$
  - The server estimates the individual gradients through least squares.



- **Random Selection**  
Reconstruction Error < 0.005 for many users



- **Multi-RoundSecAgg (T=2)**  
Reconstruction Error > 0.25 for all users

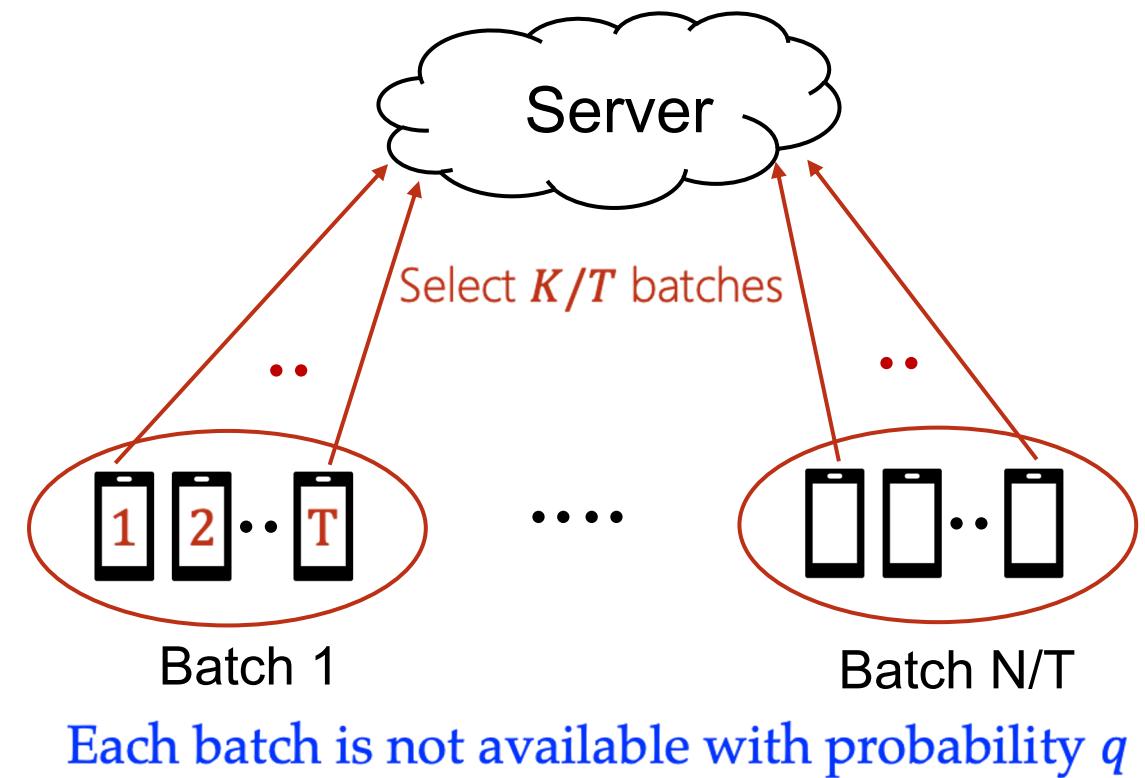
# Multi-RoundSecAgg Theoretical Guarantees

Theorem: Multi-RoundSecAgg with parameters  $N, K \leq N$  and  $1 \leq T \leq K$  ensures

1. a multi-round privacy  $1 \leq T \leq K$ ,
2. an aggregation fairness gap  $F = 0$ , and
3. an average aggregation cardinality  $C$

$$C(T) = K \left( 1 - \sum_{i=N/T-K/T+1}^{N/T} \binom{N/T}{i} q^i (1-q)^{N/T-i} \right),$$

$q = 1 - (1-p)^T$ ,  $p$ : dropout probability



# Multi-RoundSecAgg Theoretical Guarantees

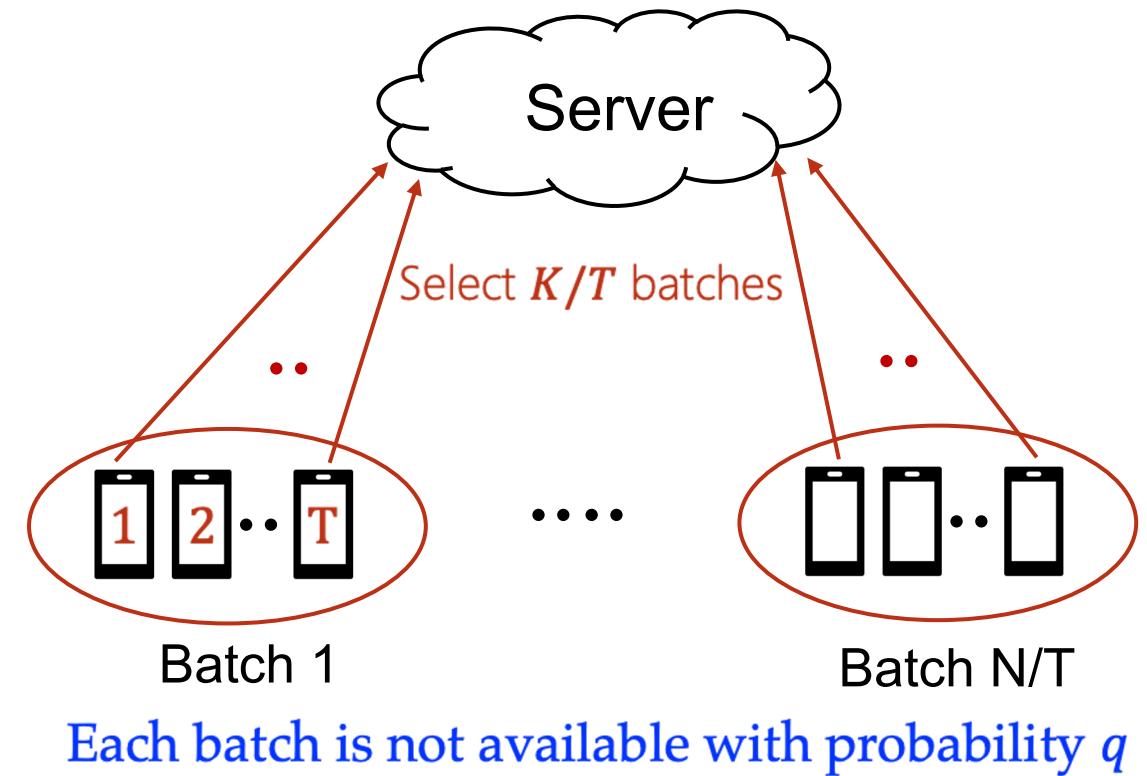
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**Example** ( $N = 120, K = N/10 = 12, p = 0.2$ )  
for  $T = N/20 = 6$ , we have  $C=11.77 \approx N/10$

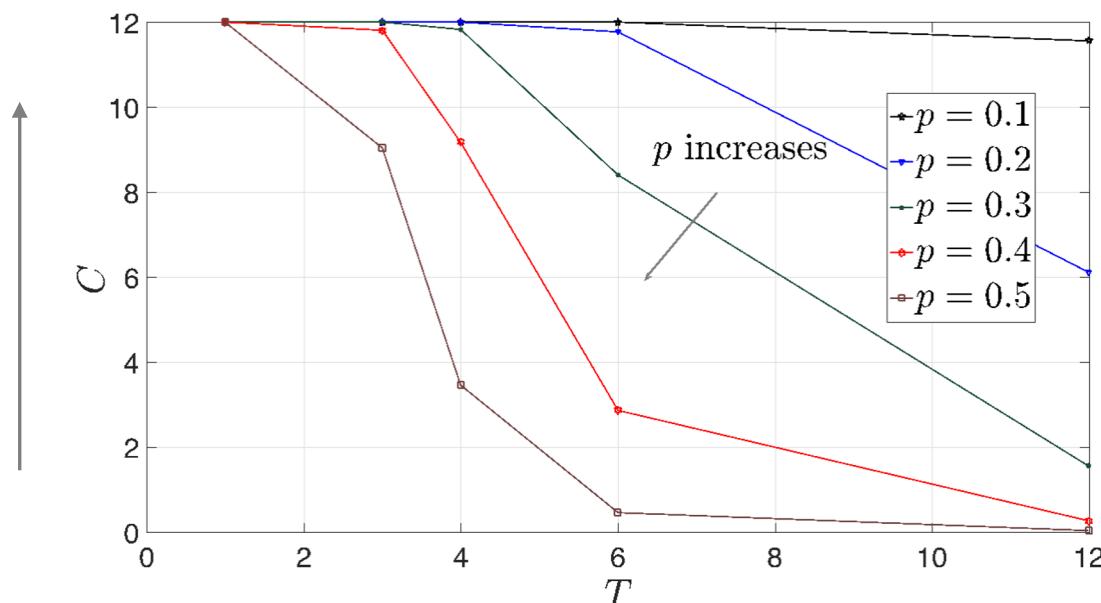


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Better convergence



$N = 120$  and  $K = 12$

Trade-off between “Multi-round Privacy Guarantee” & “Average Aggregation Cardinality”

# Multi-RoundSecAgg Convergence Guarantees

## Assumptions

1. The loss functions  $L_1, L_2, \dots, L_N$  are  $\rho$ -smooth.
2. The loss functions  $L_1, L_2, \dots, L_N$  are  $\mu$ -strongly convex.
3. The variance of the stochastic gradients at user  $i$  is bounded by  $\sigma_i^2$ .
4. The expected squared norm of the stochastic gradients is uniformly bounded by  $G^2$ .

$$E[L(\mathbf{x}^{(J)})] - L^* \leq \frac{\rho}{\gamma + \frac{c}{K}JE - 1} \left( \frac{2(\alpha+\beta)}{\mu^2} + \frac{\gamma}{2} E[\|\mathbf{x}^{(0)} - \mathbf{x}^*\|] \right) ,$$

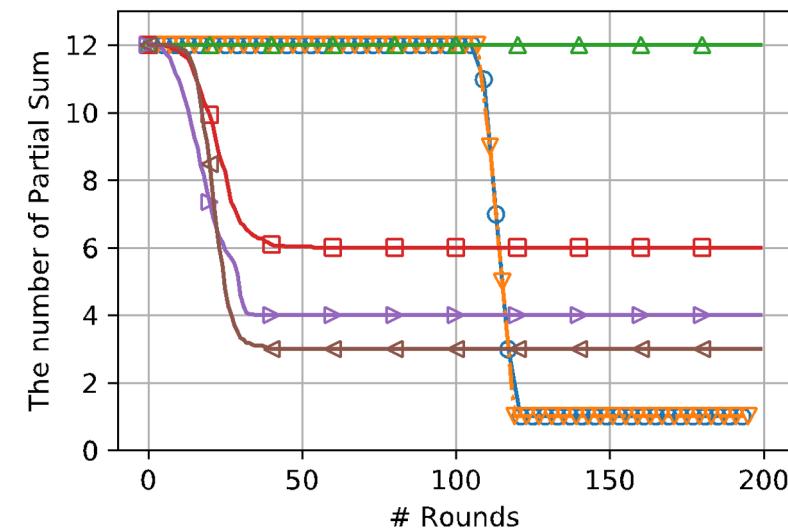
$c$  controls the convergence rate

# Experiments

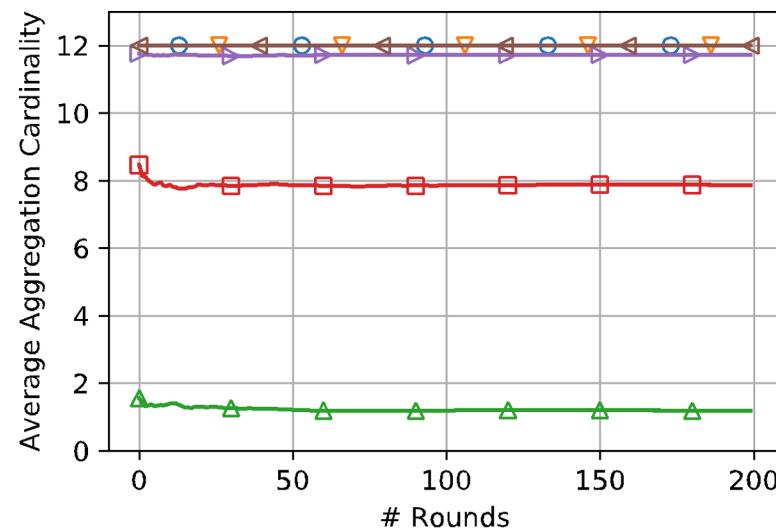
## Setup

- $N = 120$  users,  $K = 12$  users.
- Dataset: CIFAR-10
- Architecture: LeNet
- Dropout probability,  $p_i \sim \{0.1, 0.2, 0.3, 0.4, 0.5\}$

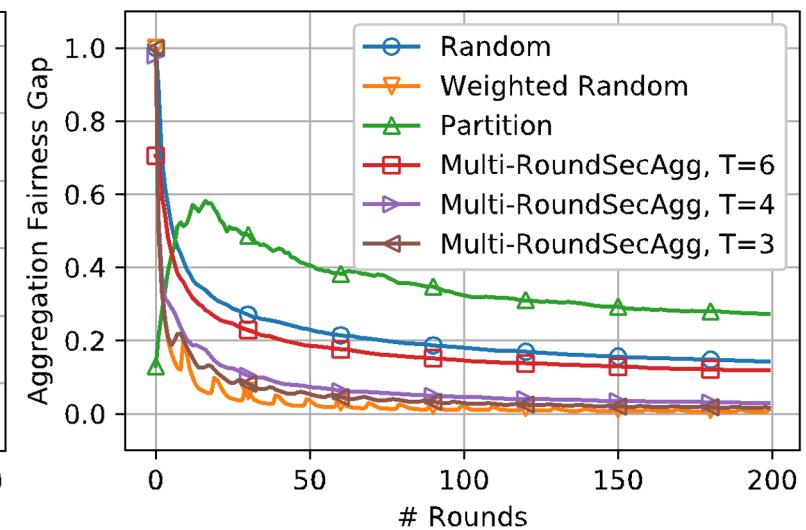
## Key Metrics



Multi-round privacy guarantee ( $T$ )



Average aggregation cardinality ( $C$ )

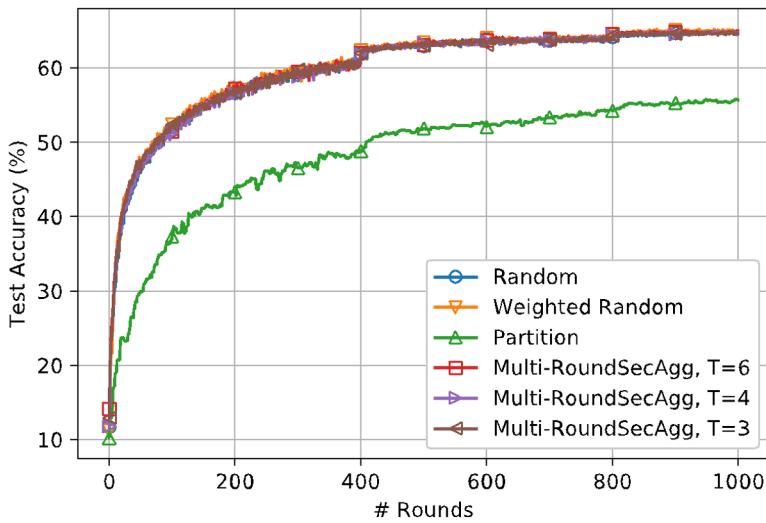


Aggregation fairness gap ( $F$ )

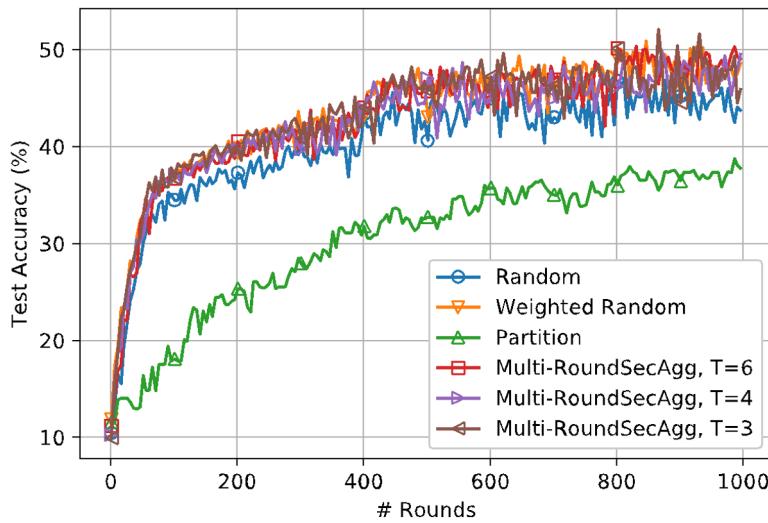
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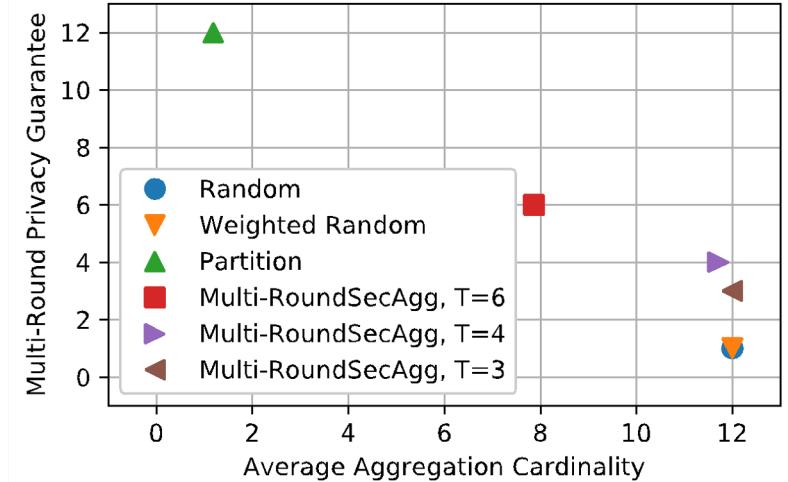
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(a) I.I.D Data Distribution



(b) Non I.I.D Data Distribution



(c) Trade-off between multi-round privacy guarantee & average aggregation cardinality

# Concluding Remarks

- Random user selection in FL can lead to serious privacy leakage
- MultiRoundSecAgg is the first scheme for mitigating this challenge
- MultiRoundSecAgg reveals an interesting tradeoff between “privacy” and “convergence rate” in FL

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- Random user selection in FL can lead to serious privacy leakage
- MultiRoundSecAgg is the first scheme for mitigating this challenge
- MultiRoundSecAgg reveals an interesting tradeoff between “privacy” and “convergence rate” in FL
- Potential future directions
  - Our metric for multi-round privacy is very strong. Careful relaxations may lead to substantial improvements (e.g., in aggregation cardinality)
  - Formalizing a fundamental trade-off between privacy and convergence-rate in FL?
  - Our protocol guarantees that only an aggregate of models can be learned. How to bound privacy leakage from aggregate models?

$$\text{🔒 } x_1^{(t)} + x_2^{(t)} + \dots + x_N^{(t)}$$



Questions?  
Thank you

# Additional Slides

# Optimality of Multi-RoundSecAgg

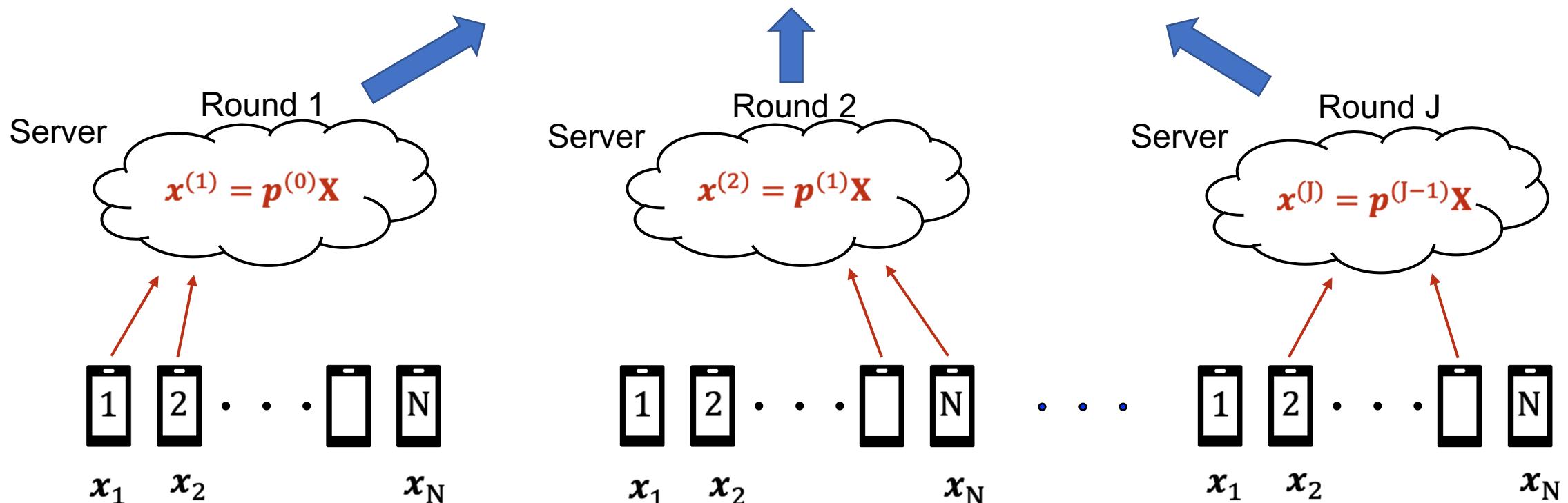
- Any strategy satisfying the multi-round privacy guarantee must have a batch partitioning structure.
- For given  $N, K \leq N/2$  and  $T$ , any strategy satisfying a multi-round privacy  $\mathbf{T}$  can have at most  $R_{\max}$  user sets

$$R_{\max} \leq \binom{N/T}{K/T} = R_{\text{BP}} \quad \text{number of sets in BP}$$

# Our Multi-round Privacy is Strong

- A multi-round privacy  $T$  requires that any non-zero partial sum that the server can reconstruct to be of the form

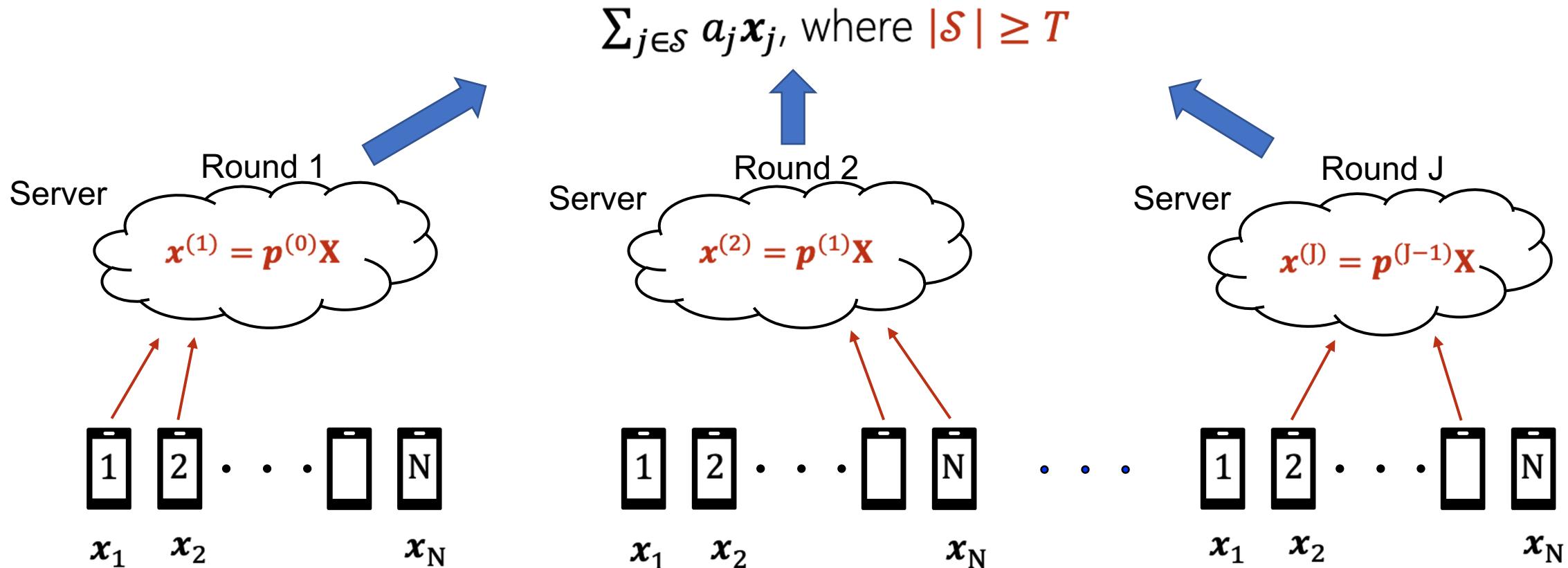
$$a_1 \sum_{j \in S_1} x_j + a_2 \sum_{j \in S_2} x_j + \dots + a_n \sum_{j \in S_n} x_j, \text{ where } |S_i| \geq T.$$



Each group has the same coefficient.

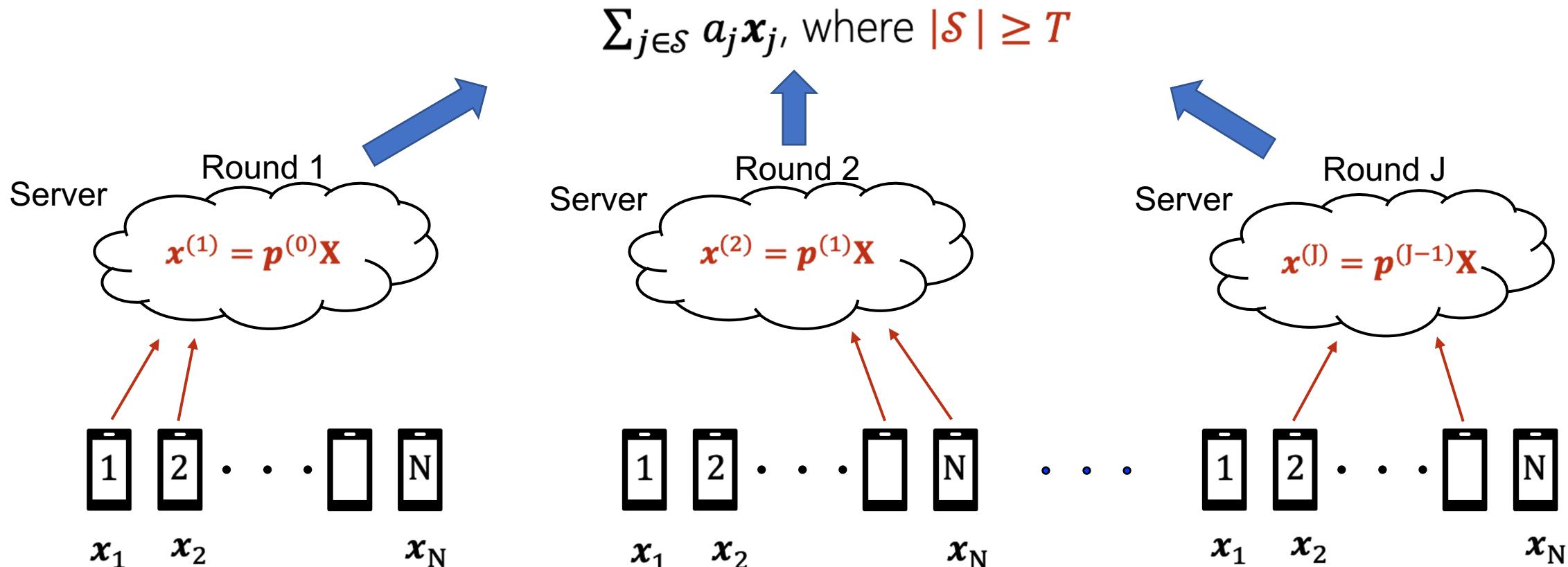
# Relaxed Multi-round Privacy

- The relaxed multi-round privacy  $T$  requires that any non-zero partial sum that the server can reconstruct to be of the form



# Relaxed Multi-round Privacy

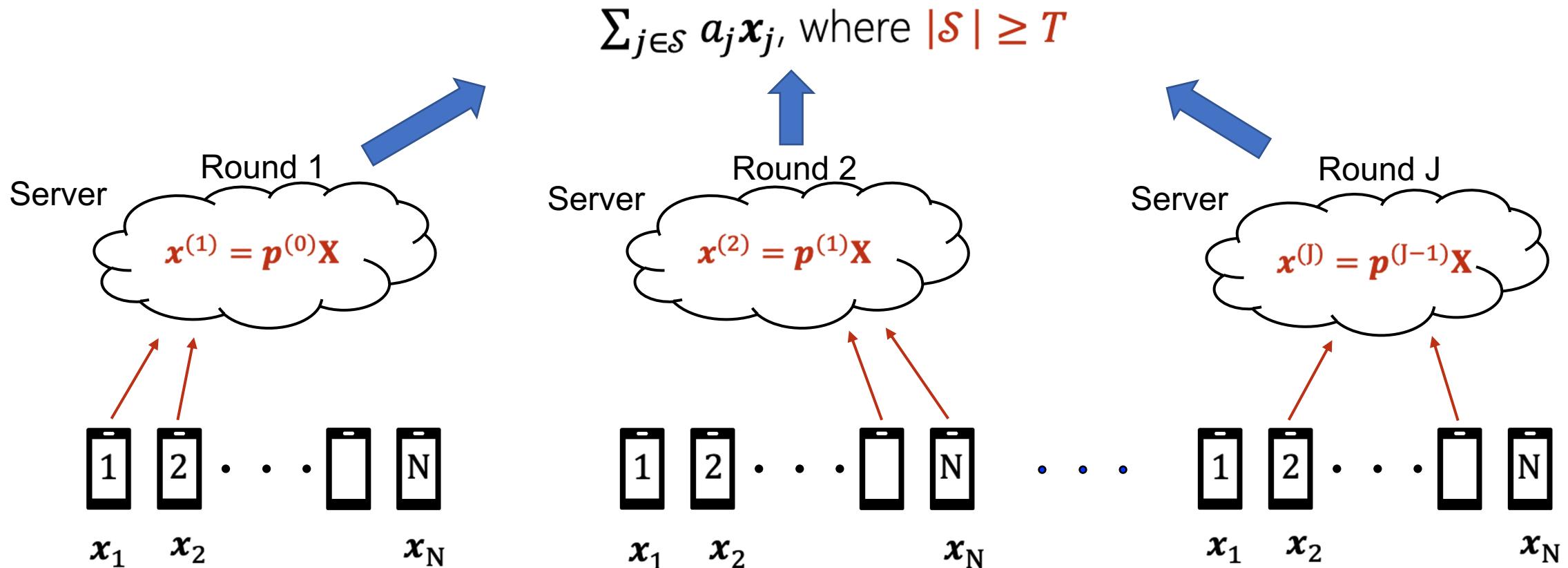
- The relaxed multi-round privacy  $T$  requires that any non-zero partial sum that the server can reconstruct to be of the form



When  $T = 2$ , this allows reconstructing  $a_i \mathbf{x}_i + a_j \mathbf{x}_j$  which can reveal  $\mathbf{x}_i$  if  $a_i \gg a_j$ .

# Relaxed Multi-round Privacy

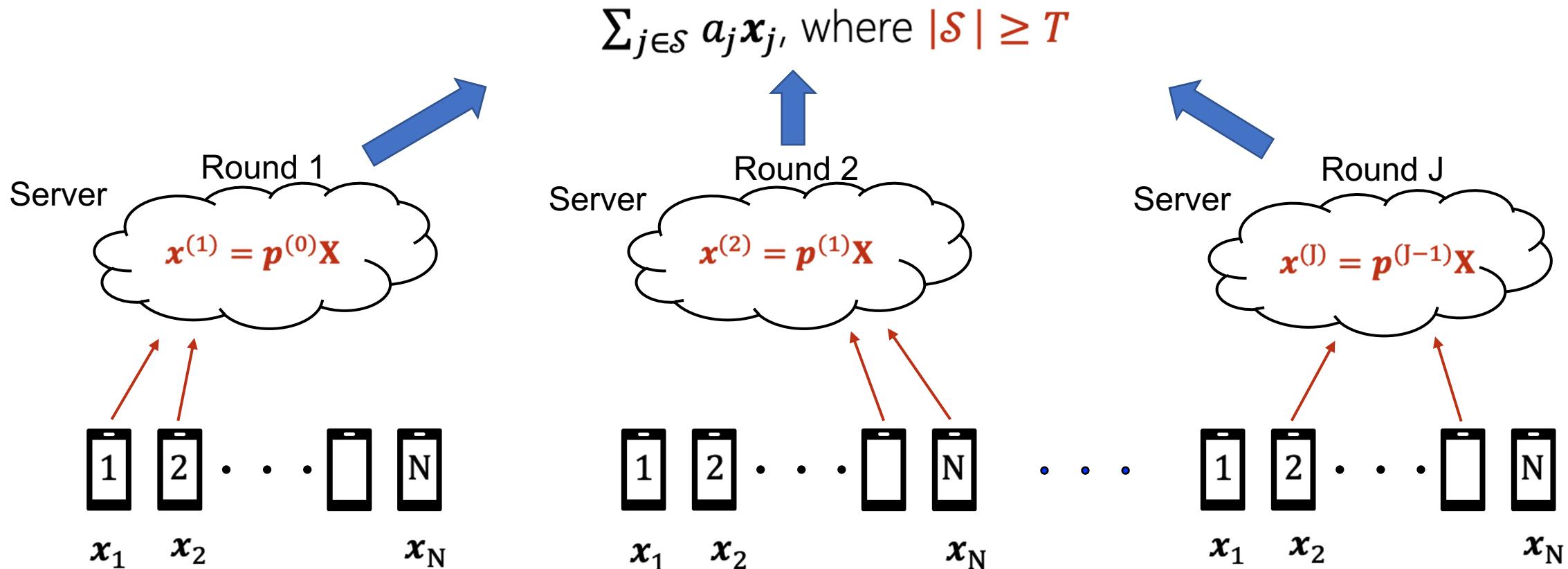
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Batch partitioning is not necessary

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What's the optimal strategy?

## Relaxed Multi-round Privacy

- The relaxed multi-round privacy  $T$  requires that any non-zero partial sum that the server can reconstruct to be of the form

$$\sum_{j \in S} a_j \mathbf{x}_j, \text{ where } |S| \geq T$$

Example (N=6, K=4, T=2)

$$\mathbf{B}_{old} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \xrightarrow{2 \times} \quad \mathbf{B}_{new} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

# Aggregation Fairness Gap F & Average Aggregation Cardinality C

Aggregation Fairness Gap F

$$F = \max_{i \in [N]} \limsup_{J \rightarrow \infty} \frac{1}{J} E \left[ \sum_{t=0}^{J-1} \{\boldsymbol{p}^{(t)}\}_i \right] - \min_{i \in [N]} \liminf_{J \rightarrow \infty} \frac{1}{J} E \left[ \sum_{t=0}^{J-1} \{\boldsymbol{p}^{(t)}\}_i \right]$$


maximum average participation frequency      minimum average participation frequency

Average Aggregation Cardinality C (Average number of participating users)

$$C = \liminf_{J \rightarrow \infty} \frac{1}{J} E \left[ \sum_{t=0}^{J-1} \|\boldsymbol{p}^{(t)}\|_0 \right]$$

# Multi-RoundSecAgg Convergence Guarantees

## Assumptions

1. The loss functions  $L_1, L_2, \dots, L_N$  are  $\rho$ -smooth.
2. The loss functions  $L_1, L_2, \dots, L_N$  are  $\mu$ -strongly convex.
3. The variance of the stochastic gradients at user  $i$  is bounded by  $\sigma_i^2$ .
4. The expected squared norm of the stochastic gradients is uniformly bounded by  $G^2$ .

$$E[L(\mathbf{x}^{(J)})] - L^* \leq \frac{\rho}{\gamma + \frac{c}{K}JE - 1} \left( \frac{2(\alpha+\beta)}{\mu^2} + \frac{\gamma}{2} E[\|\mathbf{x}^{(0)} - \mathbf{x}^*\|] \right) ,$$

where  $E$  is the number of local SGD steps,  $\alpha = \frac{1}{N} \sum_{i=1}^N \sigma_i^2 + 6\rho\Gamma + 8(E-1)^2G^2$ ,  
 $\beta = \frac{4(N-K)E^2G^2}{K(N-1)}$ ,  $\Gamma = L^* - \sum_{i=1}^N L_i$  and  $\gamma = \max\{\frac{8\rho}{\mu}, E\}$

# Necessity of Batch Partitioning (BP)

- Any strategy satisfying the multi-round privacy guarantee must have a batch partitioning structure.

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## Proof Idea

Consider any scheme with a matrix  $\mathbf{V}_{R \times N}$  and denote the linear combination used by the server by  $\mathbf{z}_{1 \times R}, z_i \sim U[0, 1], i \in [R]$ .

For the scheme to have privacy  $T$ , at least  $T$  elements of  $\mathbf{zV}$  have to be equal.

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For the scheme to have privacy  $T$ , at least  $T$  elements of  $\mathbf{z}\mathbf{V}$  have to be equal.

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→ Batch partitioning is necessary

- For given  $N, K \leq N/2$  and  $T$ , any strategy satisfying a multi-round privacy  $T$  can have at most  $R_{\max}$  user sets

$$R_{\max} \leq \binom{N/T}{K/T} = R_{\text{BP}} \text{ number of sets in BP}$$

# References

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- [2] Balázs Pejó, and Gergely Biczók. "**Quality Inference in Federated Learning with Secure Aggregation.**" arXiv:2007.06236 (2020).
- [3] M. Lam et al. "**Gradient Disaggregation: Breaking Privacy in Federated Learning by Reconstructing the User Participant Matrix.**" arXiv:2106.06089 (2021).