CS600 ALGORITHMS AND DATA STRUCTURES

HOMEWORK-2 UIN:01161389

Use the definition of big-O to prove that $1\cdot2+2\cdot3+3\cdot4+\cdots+(n-1)\cdot n$ is $O(n^3)$.

Using Mathematical Induction, w.k.T

$1\cdot2+2\cdot3+3\cdot4+\cdots+n(n+1)$ is $\frac{n(n+1)(n+2)}{3} \rightarrow (1)$

Given,

$1\cdot2+2\cdot3+3\cdot4+\cdots+(n-1)n+n(n+1)$

$1\cdot2+2\cdot3+3\cdot4+\cdots+n(n+1)+(n-1)n$

From equation (1)

$$\frac{n(n+1)(n+2)}{3}+(n-1)n$$

$$\frac{n(n^2+3n+2)}{3}+n^2-n$$

$$\frac{n^3+3n^2+2n+3n^2-3n}{3}$$

$$\frac{n^3+6n^2-n}{3}$$

So, $f(n)=\frac{n^3+6n^2-n}{3}$ is $O(n^3)$

Assuming $n>1$, then

$$\frac{f(n)}{g(n)} = \frac{\frac{1}{3}n^3+2n^2-\frac{1}{3}n}{n^3} < \frac{\cancel{\frac{1}{3}n^3}+2n^3-\cancel{\frac{1}{3}n^3}}{n^3}$$

$$=2$$

Choose $C=2$, Note that $2n^2<2n^3$ and $\frac{1}{3}n<\frac{1}{3}n^3$

Thus, $\frac{n^3+6n^2-n}{3}$ is $O(n^3)$ because.

$\frac{n^3+6n^2-n}{3} \le n^3$ whenever $n>1$.