Question1:

1) Decision Variables:
   B1 = Number of Collegiate backpacks to produce per week.
   B2 = Number of Mini backpacks to produce per week.

2) Objective Function:
   The objective is to maximize the total profit(P). The profit for each Collegiate backpack is $32, and for each Mini backpack is $24. So, the objective function is:

   Maximize P = 32*B1 + 24*B2

3) Constraints:
   a. Total square footage of nylon fabric used for Collegiate and Mini backpacks should not exceed the 5000 square feet available each week
      i. 3*B1 + 2*B2 <=5000
   b. From the question, the sales forecasts indicate that at most 1000 collegiates and 1200 minis can be sold per week. Hence, we have:
      i. B1<=1000 and B2<=1200
   c. Back savers company has 35 laborers, each providing 40 hours of labor per week. Each Collegiate backpack requires 45 minutes (which is 0.75 hours) of labor to produce, and each Mini backpack requires 40 minutes (which is 0.667 hours) of labor to produce. So, the constraint would be:
      i. 0.75*B1+0.667*B2 ≤ 35*40 ➔ 0.75*B1+0.667*B2 = 1400
   d. Since we cannot produce a negative number of backpacks, so B1 and B2 should be greater than 0:
      i. B1>=0 and B2>=0

4) Full mathematical Formulation of the Linear programming problem:

   Objective Function: Maximize P = 32*B1 + 24*B2

   Subject to the following constraints:

   $$3 \cdot B1 + 2 \cdot B2 \leq 5000$$
   $$B1 \leq 1000$$
   $$B2 \leq 1200$$
   $$0.75 \cdot B1 + 0.667 \cdot B2 \leq 1400$$
   $$B1 \geq 0$$
   $$B2 \geq 0$$

This linear programming problem can be solved using LP software or methods to find the values of B1 and B2 that maximize the total profit P

Question 2:

1) Decision Variables:
   a) As per the question, we need to define decision variables to represent how much of each size (large, medium, and small) should be produced by each plant(1,2,3). So, the decision variables are as follows:
      i. X_{L1} – Number of large units produced at plant 1 per day
      ii. X_{M1} – Number of medium units produced at plant 1 per day
      iii. X_{S1} – Number of small units produced at plant 1 per day
      iv. X_{L2} – Number of large units produced at plant 2 per day
      v. X_{M2} – Number of medium units produced at plant 2 per day
      vi. X_{S2} – Number of small units produced at plant 2 per day
      vii. X_{L3}– Number of large units produced at plant 3 per day
      viii. X_{M3} – Number of medium units produced at plant 3 per day
      ix. X_{S3} – Number of small units produced at plant 3 per day
2) Formulate a linear programming model for this problem:
   The objective is to maximize the total profit(P). The profit for each unit size is given as follows:
   a) Large: $420 profit per unit
   b) Medium: $360 profit per unit
   c) Small: $300 profit per unit

   Hence, the Objective function would be to Maximize Total Profit:

$P = 420^* X_{L1} + 420^* X_{L2} + 420^* X_{L3} + 360^* X_{M1} + 360^* X_{M2} + 360^* X_{M3} + 300^* X_{S1} + 300^* X_{S2} + 300^* X_{S3}$

   Constraints:
   a) Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. So the constraints are:
      i. Plant 1: $X_{L1} + X_{M1} + X_{S1} <= 750$
      ii. Plant 2: $X_{L2} + X_{M2} + X_{S2} <= 900$
      iii. Plant 3: $X_{L3} + X_{M3} + X_{S3} <= 450$
   b) Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Additionally, each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. So the constraints are:
      i. Plant 1: $20^*X_{L1} + 15^*X_{M1} + 12^*X_{S1} <= 13000$
      ii. Plant 2: $20^*X_{L2} + 15^*X_{M2} + 12^*X_{S2} <= 12000$
      iii. Plant 3: $20^*X_{L3} + 15^*X_{M3} + 12^*X_{S3} <= 5000$
   c) As per the question: Sales forecasts indicate the maximum number of units to be sold for each size or also called Demand forecast. So the demand constraints are:
      i. Plant 1: $X_{L1} + X_{L2} + X_{L3} <= 900$

ii. Plant 2: $X_{M1} + X_{M2} + X_{M3} <= 1200$
iii. Plant 3: $X_{S1} + X_{S2} + X_{S3} <= 750$

d) To avoid layoffs, each plant should use the same percentage of its excess capacity. Let **percent_excess** represent this percentage and the constraints would be as follows:
   i. Plant 1: $X_{L1} + X_{M1} + X_{S1} <= 750$*percent_excess
   ii. Plant 2: $X_{L2} + X_{M2} + X_{S2} <= 900$*percent_excess
   iii. Plant 3: $X_{L3} + X_{M3} + X_{S3} <= 450$*percent_excess

e) Lastly, we have non-negative constraints: Since we cannot produce negative number of units. The constraints are:
   i. $X_{L1} >=0$, $X_{M1}>=0$, $X_{S1} >=0$, $X_{L2}>=0$, $X_{M2} >=0$, $X_{S2}>=0$, $X_{L3} >=0$, $X_{M3}>=0$, $X_{S3}>=0$

This linear programming problem can be solved using Excel to find the values of these decision variables that maximize the total profit(P) while satisfying all the above constraints.