TIME-FREQUENCY ANALYSIS OF GRAVITATIONAL-WAVE SIGNALS

DSP Project 2021 EE386

Guide: Prof Krishnan CMK

Mentee: P.Ramyashri

National Institute of Technology Karnataka, Surathkal,191EE138

Link to codes and project log: https://github.com/rum1887/TimeFreq-Analysis

1. Introduction

Need to analyze gravitational-wave signals

The gravitational waves that are detected by interferometers are caused by some of the most energetic events in the Universe - colliding black holes, merging neutron stars, exploding stars, and possibly even the birth of the Universe itself. Detecting and analyzing the information carried by gravitational waves is allowing us to observe the Universe in a way never before possible, providing astronomers and other scientists with their first glimpses of literally un-seeable wonders. Historically, scientists have relied almost exclusively on electromagnetic (EM) radiation (visible light, X-rays, radio waves, microwaves, etc.) to study the Universe. Gravitational waves, however, are completely unrelated to EM radiation. They are as distinct from light as hearing is from vision. Every massive object that accelerates produces gravitational waves. Gravitational-wave data is recorded as a collection of time series from a network of detectors.

Sources and Types of Gravitational Waves

Every massive object that accelerates produces gravitational waves. This includes humans, cars, airplanes, etc., but the masses and accelerations of objects on Earth are far too small to make gravitational waves big enough to detect with our instruments. To find big enough gravitational waves, we have to look far outside of our own solar system.

LIGO scientists have defined four categories of gravitational waves based on what generates them: Continuous, Compact Binary Inspiral, Stochastic, and Burst. Each category of the object generates a unique or characteristic set of signals that LIGO's interferometers can sense, and that researchers can look for in LIGO's data.[7]

Need for Time-frequency analysis

Data from gravitational wave detectors are recorded as time series that include contributions from myriad noise sources in addition to any gravitational wave signals. When regularly sampled data are available, such as for ground-based and future space-based interferometers, analyses are typically performed in the frequency domain, where stationary (time-invariant) noise processes can be modeled very efficiently. In reality, detector noise is not stationary due to a combination of short-duration noise transients and longer-duration drifts in the power spectrum. This non-stationarity produces correlations across samples at different frequencies, obviating the main advantage of a frequency domain analysis.
Here an alternative time-frequency approach to gravitational-wave data analysis is proposed that uses discrete, orthogonal wavelet wavepackets

2. Background and literature review

Review of most used algorithms for analysis of gravitation signal.

a) Time series analysis:
Not used much, except for the following cases.
   i) Pulsar timing array [1]
   ii) Time-domain Low latency LIGO/Virgo searches. [2]
   iii) Analysis of black hole ringdowns [3]

b) Frequency analysis:
The majority of the analysis is performed in the frequency domain.
The primacy of frequency domain analyses is due to the advantages it confers for modeling detector noise, under the assumption that the noise properties are at least approximately stationary.

c) Time-frequency analysis:
The wavelet domain provides a time-frequency representation of that data that is well suited for modeling non-stationary noise processes. For the most commonly encountered type of gravitational wave signal - produced by the merger of compact binary stars on quasi-circular orbits - it turns out that the waveforms can be computed much more efficiently in the wavelet domain than in the time or frequency domains. [6]

   A. The Wilson-DaubechiesMeyer (WDM) wavelet basis [4]

3. Methodology

Phase 1 (Learn): Extensive research and implementation of the following Time-series formulations.

A. Short-time Fourier transform ✔
B. Wigner distribution, bilinear time-frequency distribution ❌
C. Modified Wigner distribution ❌
D. Wavelet transform; discrete-time (Haar) and continuous-time (Morlet).❌

Phase 2 (Implement): Research specific to gravitational wave data.

A. Data collection and Understanding the data
   https://www.gw-openscience.org/about/
B. Study and implement WDM Transform as mentioned in the base paper.

4. Detailed discussions

Time-frequency analysis: Analysing signal in both time and frequency domain simultaneously.

STFT is a form of time-frequency analysis.

Need for STFT?

We know the frequency components present in the signal for the entire length, but we don't exactly know when these frequency components occur.
Audio and gravitational wave data are dynamic, we want to know how frequencies evolve.

How Do we do this?

We do this by considering chunks. And applying fft() locally.

How to chunk?

Applying a windowing function. The windowing function used is Hann.

Frame size used : 2048 samples (No of samples on which DFT is applied)
Window size used : 2048 (Size of the window function)
Hop size used : 512 samples (No of samples hopped between two consecutive frames)

Why do we need overlapping frames?

To avoid spectral leakage.

Mathematics behind STFT

$$\mathbf{STFT}\{x[n]\}(m,\omega) \equiv X(m,\omega) = \sum_{n=-\infty}^{\infty} x[n]w[n-m]e^{-j\omega m}$$

How is this different from the DFT equation?

1. Outputs a 2D heatmap, independent variables being time and frequency.
2. Presence of windowing function.
3. Summation variable N in DFT spans the entire duration of time, whilst in STFT N equals the frame rate which is, in turn, the number of samples.

Comparison between DFT and STFT

STFT # represents “Number of”	**DFT** # represnets “Number of”
Output size STFT = (#frequency bins, #time)	OUTPUT SIZE DFT = (#frequency bins)
2D spectral matrix with complex coefficients wrt time and freq	Spectral vector with N fourier coefficients
Frequency bins in STFT = framesize/2 +1 (symmetry in DFT)	Frequency bins in DFT = No of samples
Time bins = (signal size - frame size / hop size) + 1	Concept of time doesn't exist in DFT

Time-frequency tradeoff?

Increasing the frequency resolution means increasing the frame size which in turn means increasing the including bigger chunks so lesser time resolution.
Increasing the time resolution means taking smaller chunks, and hence lesser frequency resolution.

5. Results

Studied in detail and implemented STFT on 4 Audio tracks [8] (Link to the implementation).

Block diagram

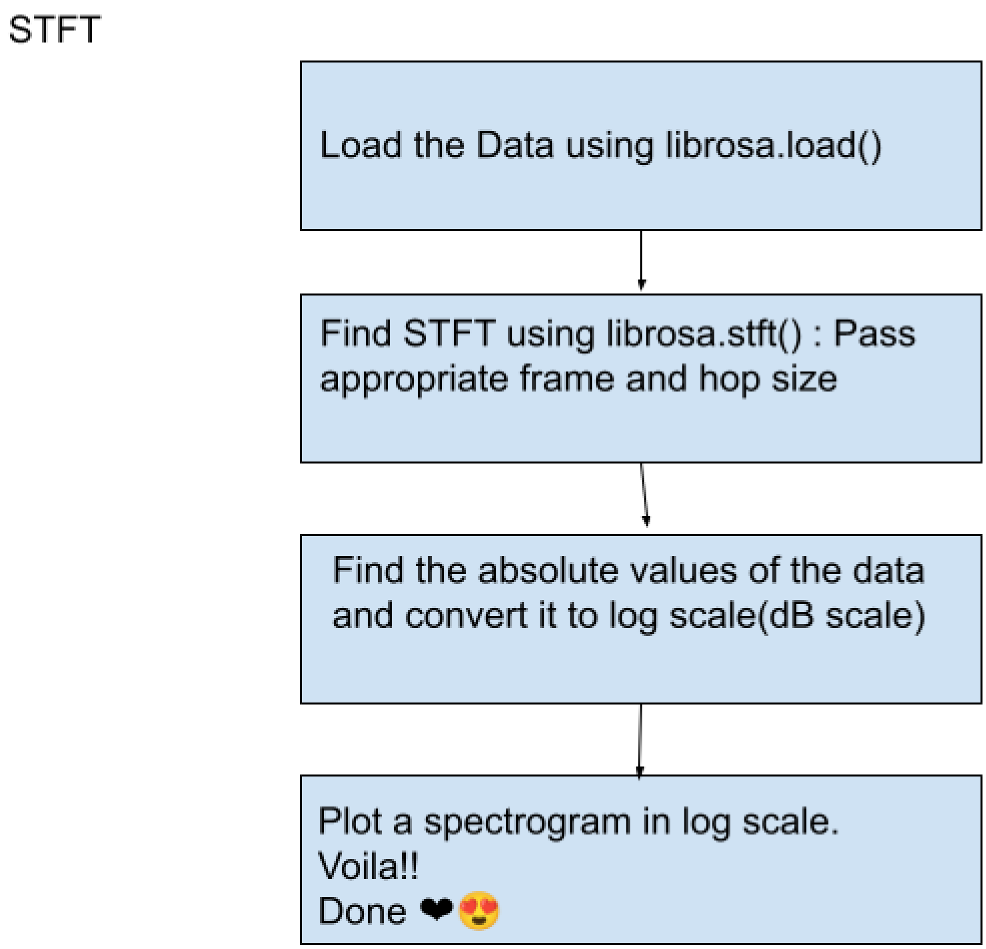
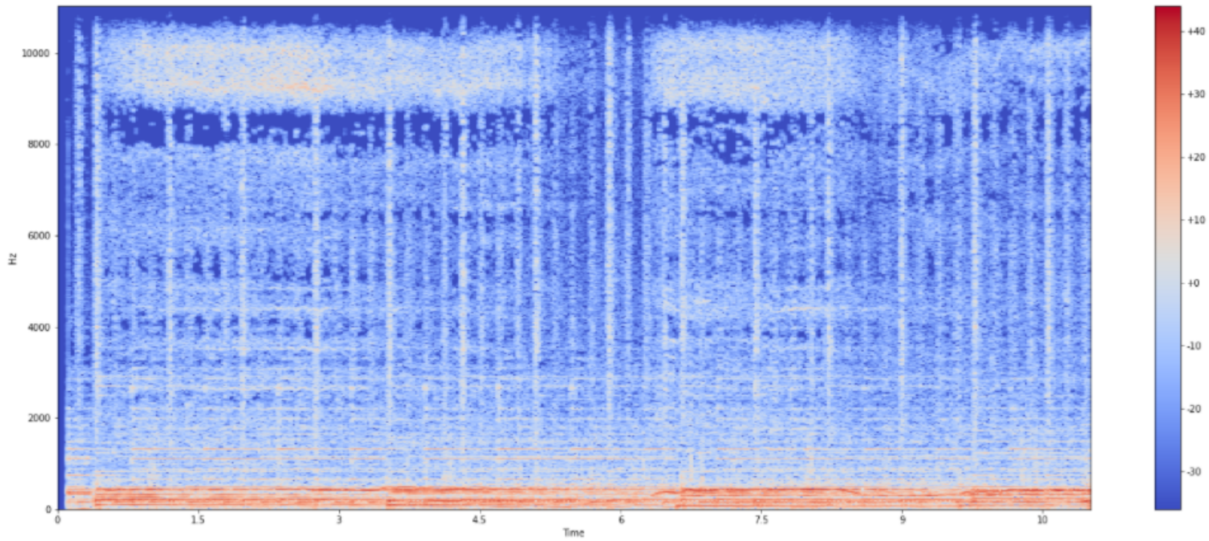
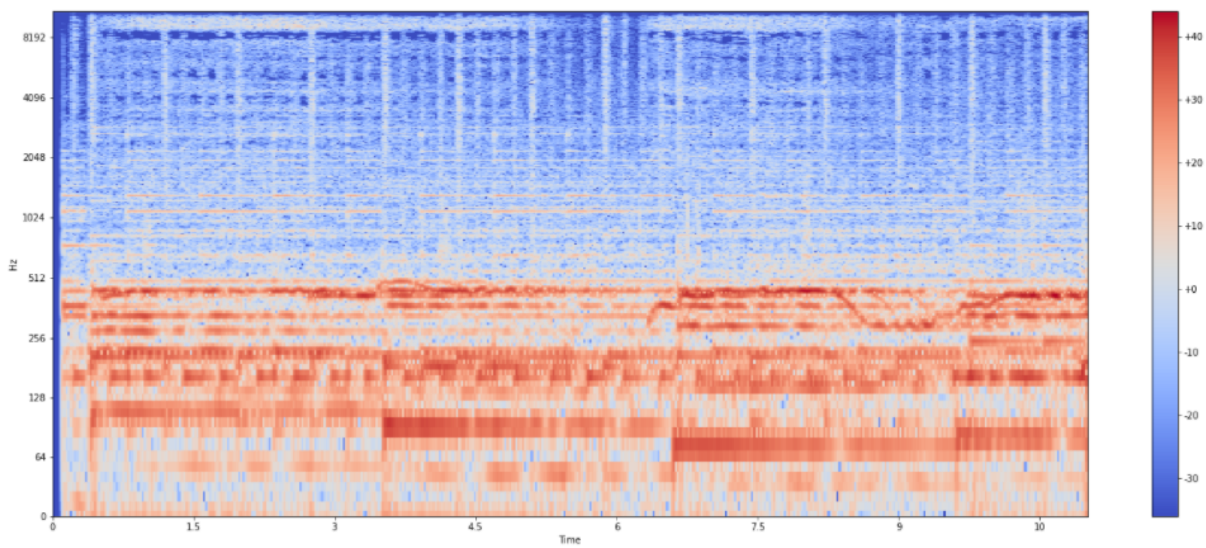


Image: Track 1 Spectrogram: The magnitude (STFT outputs complex coefficients, so taking magnitude makes it real) of the coefficients obtained from STFT is the plot as a heatmap.



Log Spectrogram: Visually more appealing



In theory, STFT calculates the Fourier transform of the signal in a short time truncated by the window. Although the STFT can characterize the time-varying features of a non-stationary signal, the TF representation (TFR) smears in a large region since the window cannot compactly support both the time and frequency domain. This must lead to the energy dispersion problem.

6. Conclusions

A time-frequency approach to gravitational-wave data analysis using discrete wavelet transforms offers many advantages over the traditional time or frequency domain approaches.

Wavelet-based analyses are well suited for modeling non-stationary instrument noise. Wavelet domain models of binary merger signals are often significantly faster to compute than time or frequency domain models.
Together these advantages make a strong case for moving gravitational wave data analysis to the wavelet domain.

7. Plan for the next month

a. Implement STFT on gravitational wave data.
b. Study and implement the other classes of time-frequency formulations (Time 15 days)
c. Complete phases 2 (Time required 15 days)

Bibliography

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[8] https://github.com/rum1887/TimeFreq-Analysis