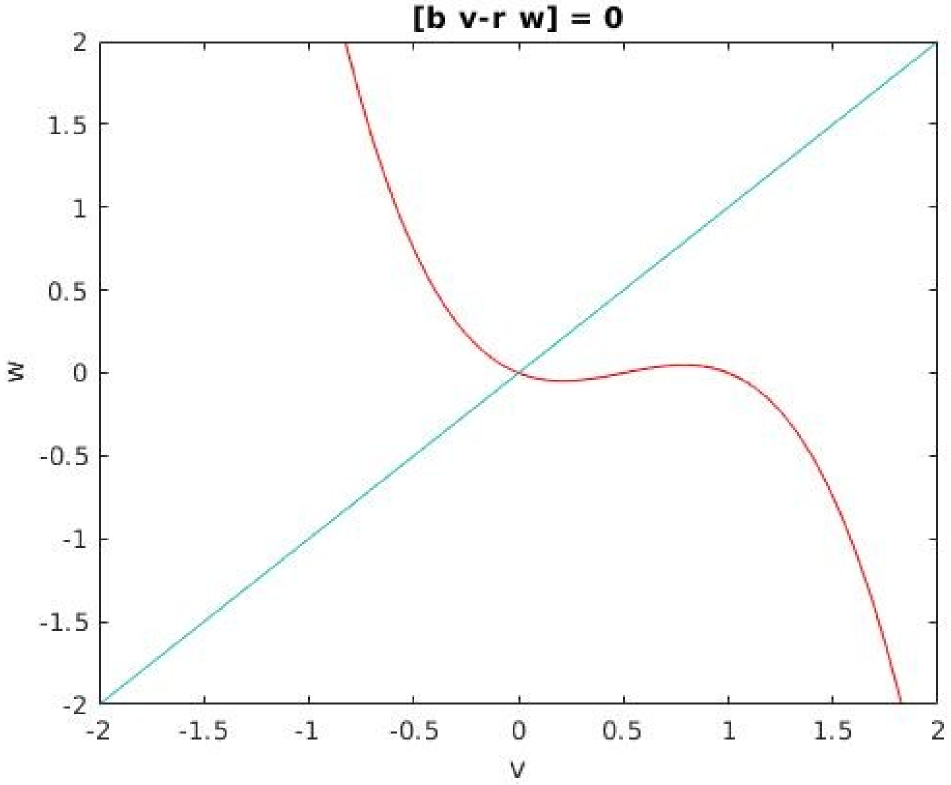
Assignment - Fitz Hugh Nagumo Model

1. For Case 1, when I_{ext} =0; a=0.5, b=r=0.1

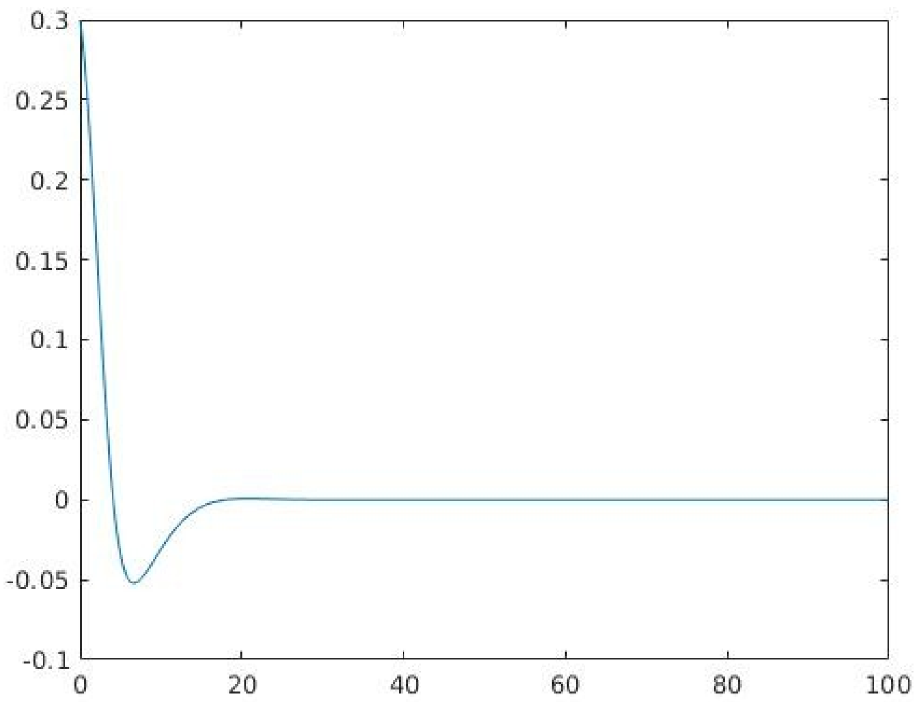
   (a) Phase Plot



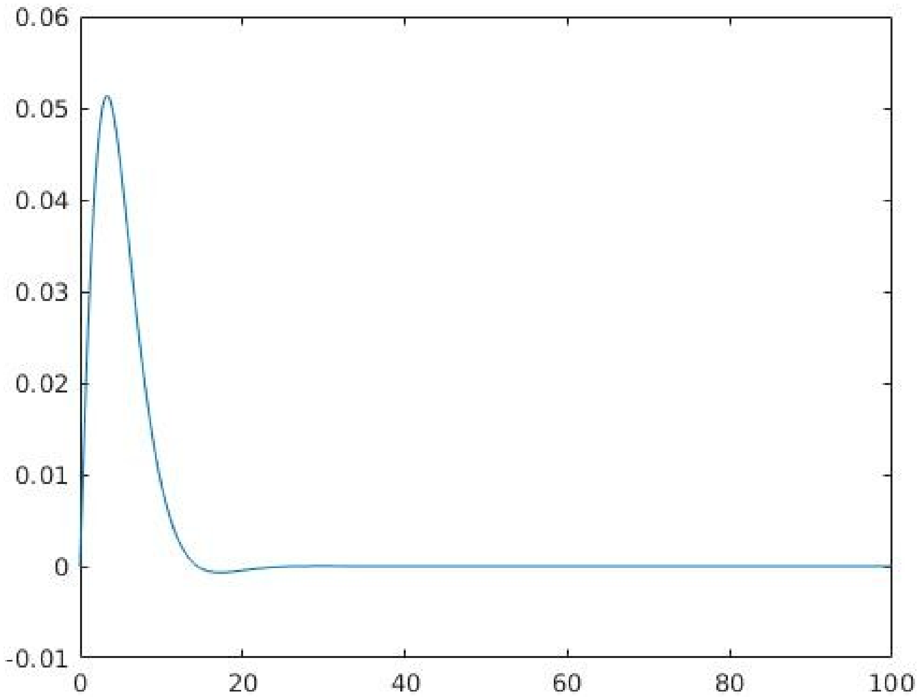
   (b) V(t), W(t), trajectories.

      i. V(0) < a and ω (0)= 0

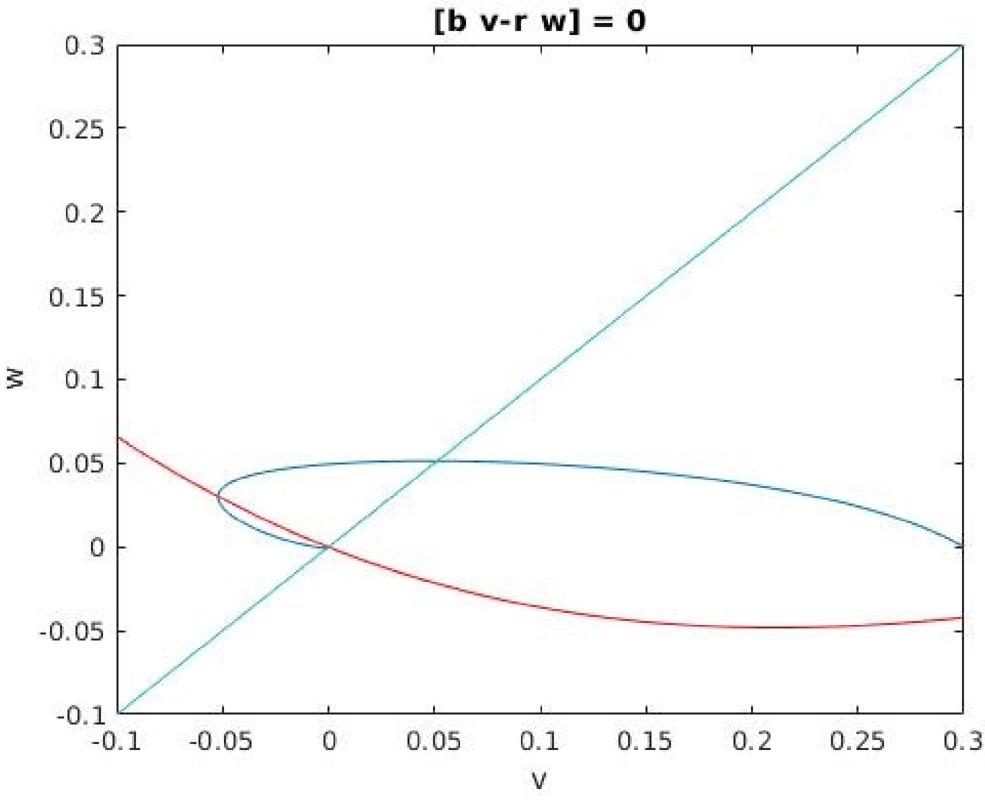
         I. V(t) vs t



II. W(t) vs t

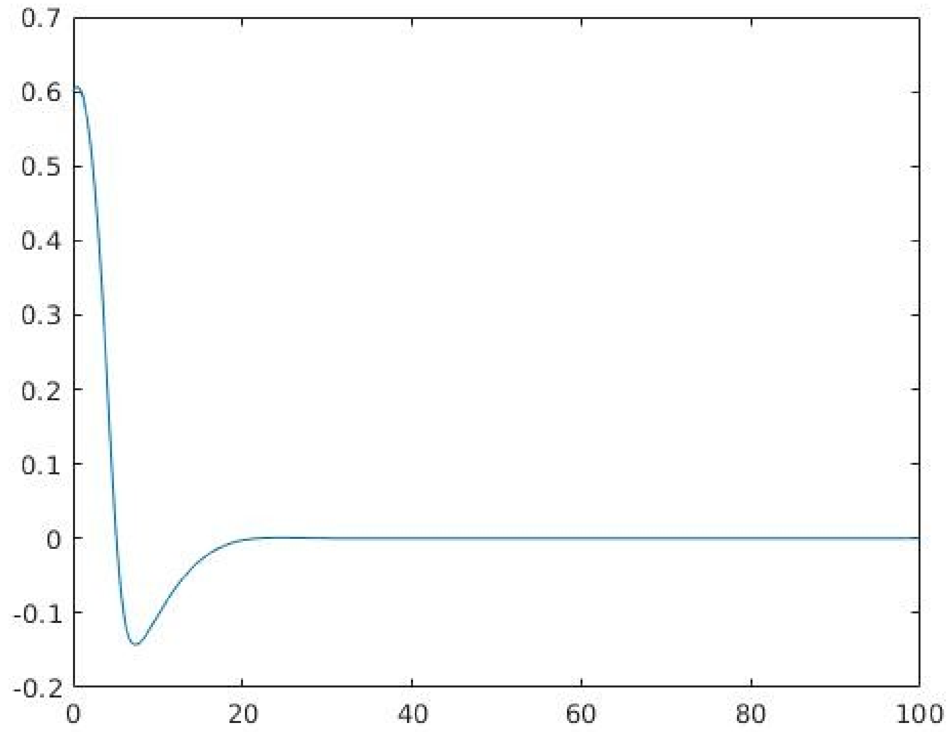


III. Trajectory

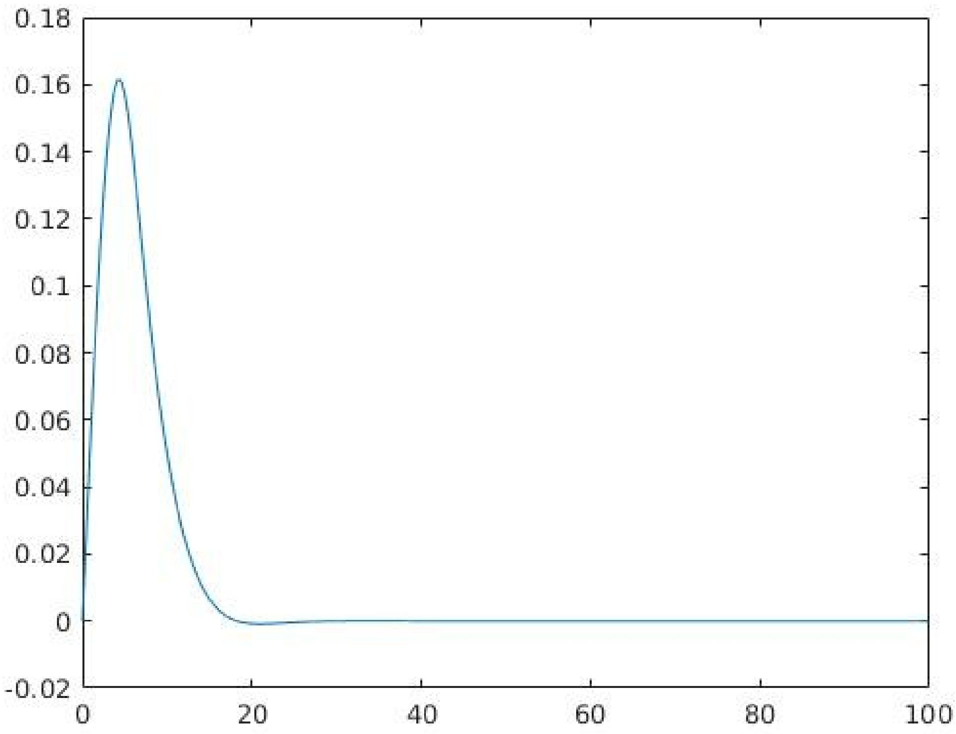


ii. V(0) > a and ω (0)= 0

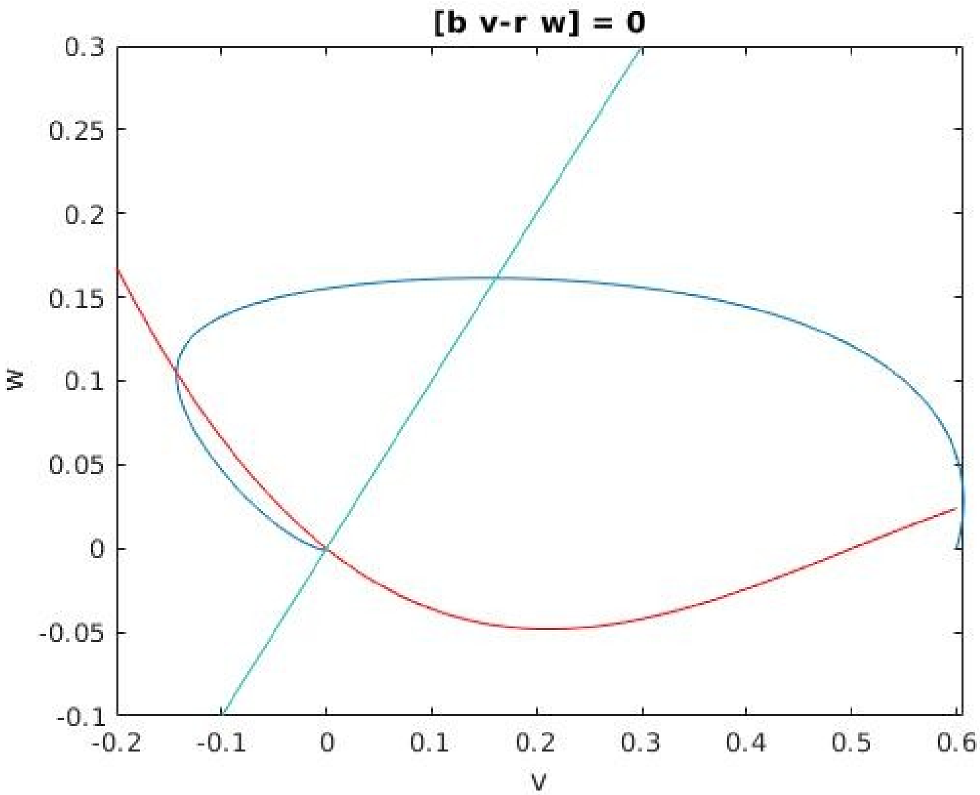
I. V(t) vs t



II. W(t) vs t

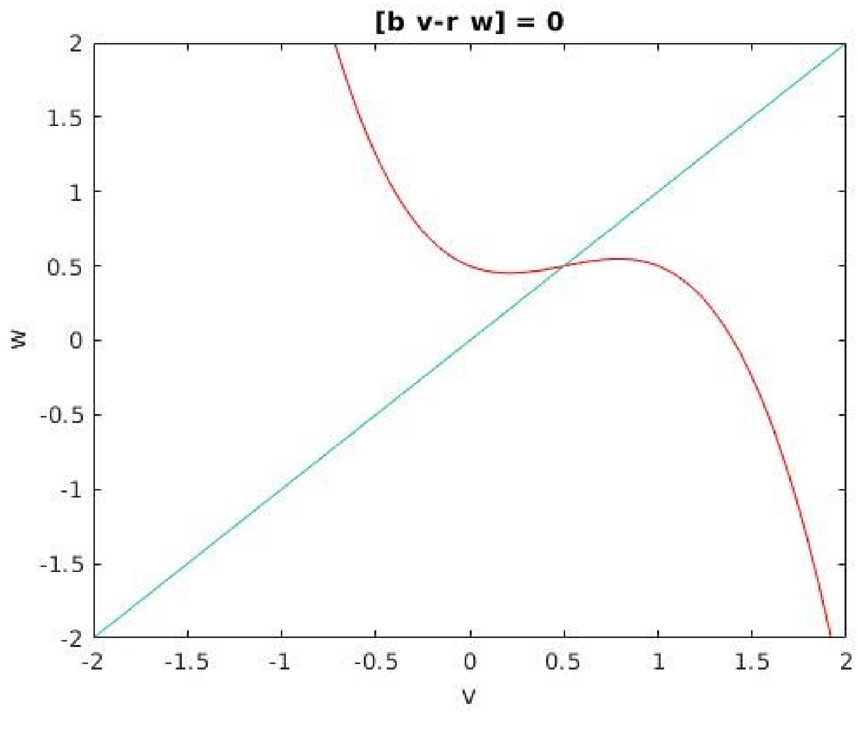


III. Trajectory

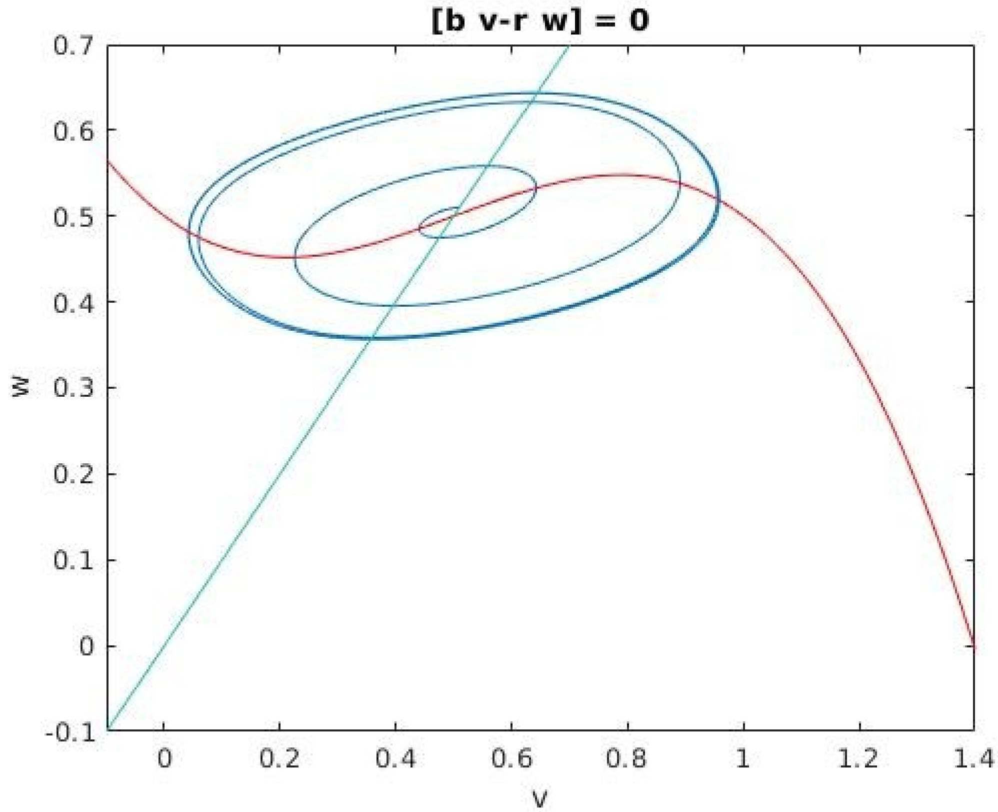


2. Case 2: When the w-nullcline intesects the v-nullcine in the middle branch, where the slope of the function is positive, it is known that a limit cycle is formed in the phase plane. The values of I_{ext} lie between the values I_1 and I_2 for which the w-nullcline intersects the v-nullcline at the critical/stationary points, on either side of the branch. Thus the appraoch taken in MATLAB is solving the equation F'(x)=0, to get the roots, and then solving for intersection at those values to get I_1 , I_2. See the code, for more.
   I_2 =0.7406; I_1 =0.2594; We take I_{ext} =0.5, satisfying the required condition.

   (a) Phase Plot



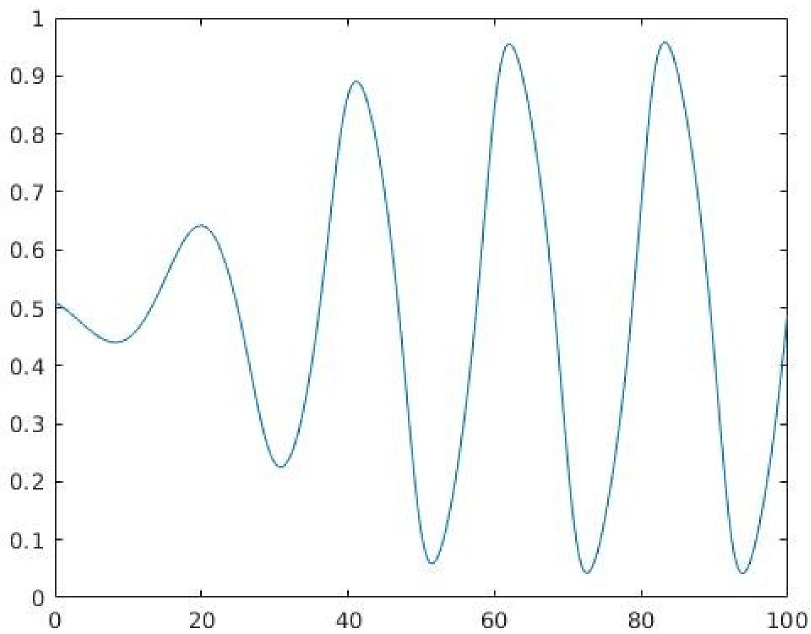
(b) First we need to solve for the fixed point. Including our value of I_{ext} in the equation, we solve the equations of the v and w nullclines to get the following solutions:
(0.5, 0.5), (1/2 – (3^(1/2)*1i)/2, 1/2 – (3^(1/2)*1i)/2), ((3^(1/2)*1i)/2 + 1/2 , (3^(1/2)*1i)/2 + 1/2). The only real solution of these is (0.5, 0.5), and that is the fixed point.
To show that the fixed point is unstable, for a small perturbation, (0.001, 0.001) about the fixed point, we need to show that the trajectory does not return to the fixed point. To show this, the trajectory of (v,w) from (0.501, 0.501) is shown below.



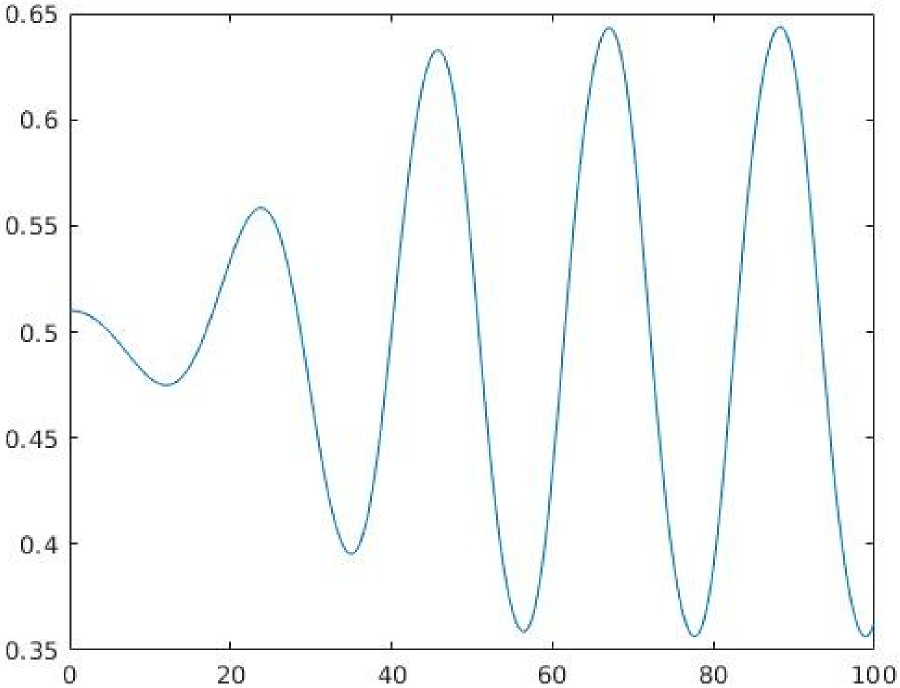
It can be seen that from the point mentioned, the trajectory does not return to the fixed point; it spirals to form a limit cycle. Thus the fixed point is unstable.

(c) V(t), W(t)

i. V(t) vs t



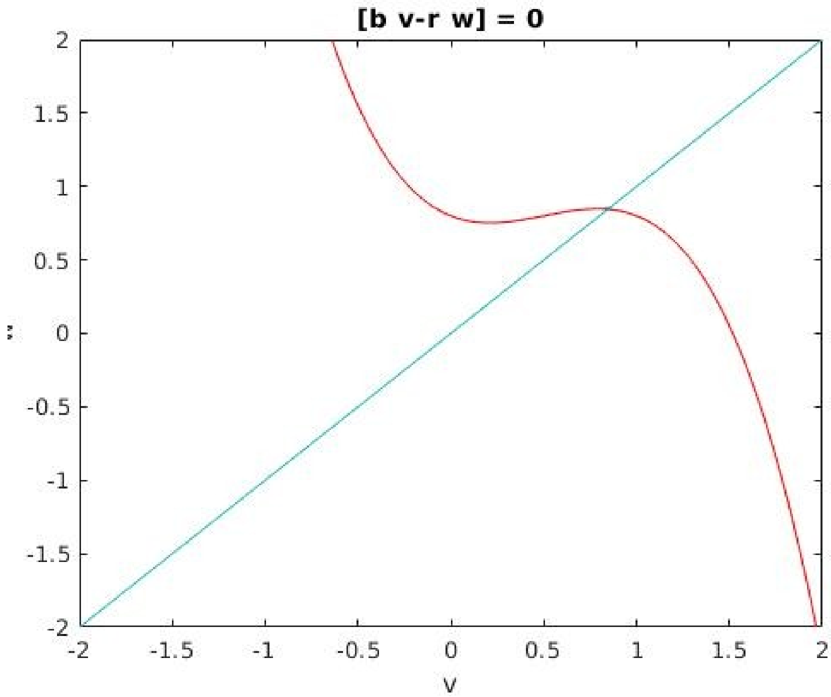
ii. W(t) vs t



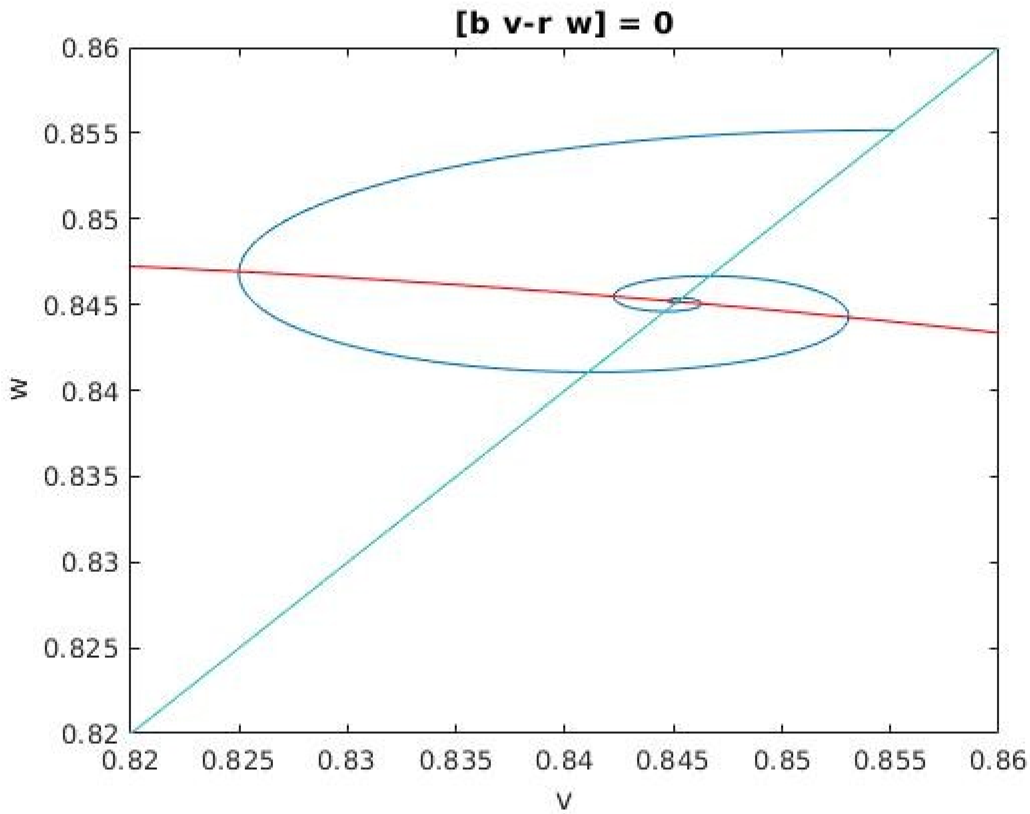
It can be seen that both V(t) and W(t) show periodic oscillations, corresponding to the limit cycle.

3. Case 3: We take the value for I_{ext}=0.8 >I_2 (found in 2).

(a) Phase Plot



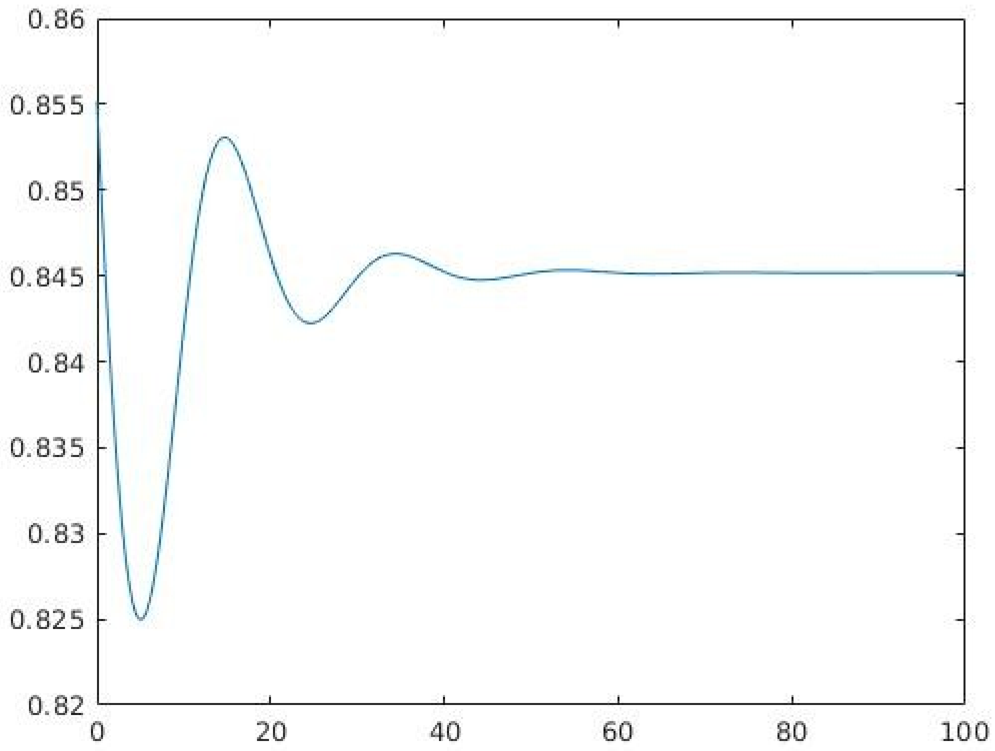
(b) First, we find the fixed point for the value of I_{ext}, solving the equations of the w and v nullclines with that value to find the intersection point. Doing this using MATLAB, we find that there are three solutions, of which (0.845, 0.845) is the only real solution. We then plot a trajectory starting from a small perturbation (0.01, 0.01) away.



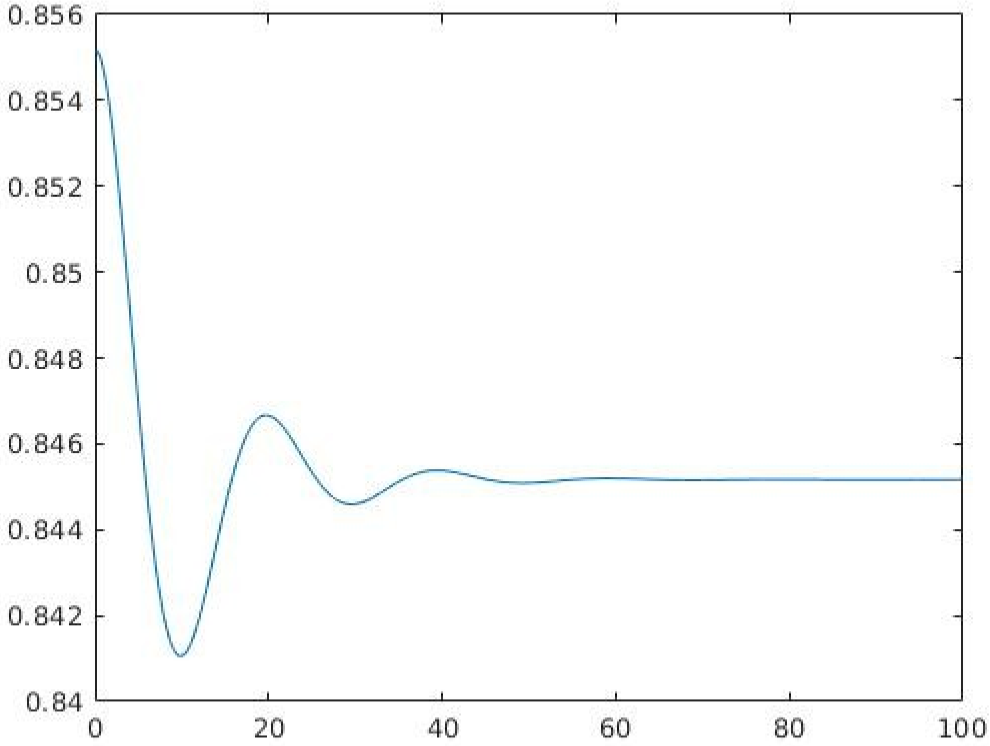
The trajectory spirals back to the fixed point, thus showing that it is a stable fixed point.

(c) V(t), W(t):

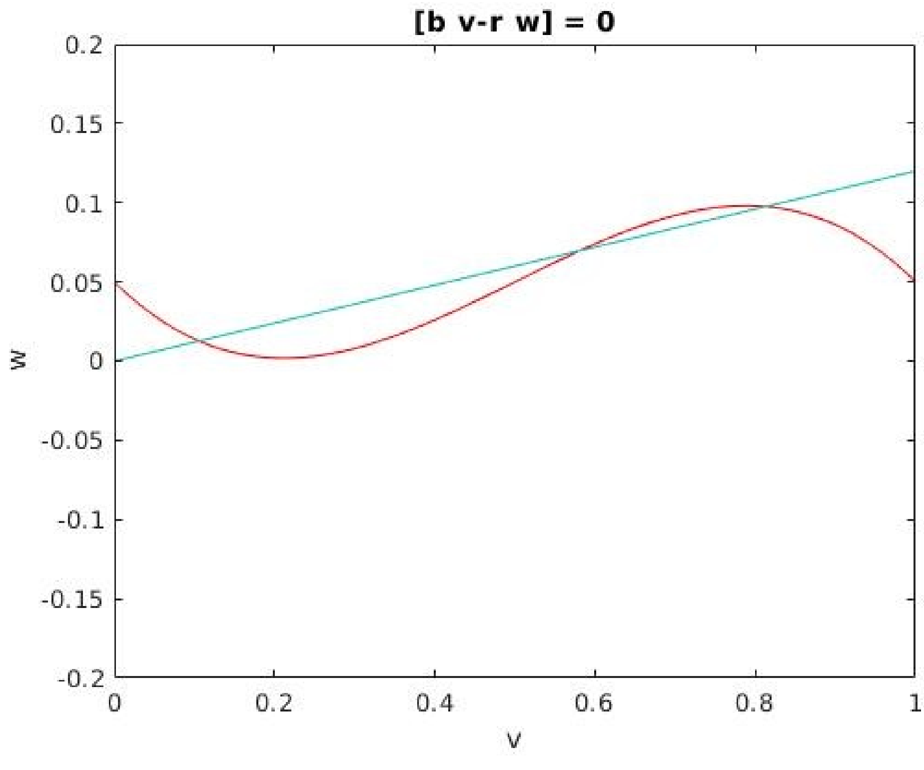
i. V(t)



ii. W(t)



4. Case 4: We already calculated the coordinates of the two stationary points; in order to get some handle on the values of b, r and I_{ext}, we surmise that the slope b/r has to be less than the slope of the two stationary points (1,2); essentially:
   $$\frac{y_2 - y_1}{x_2 - x_1} < \frac{b}{r}$$
   Thus, we calculate this value in the MATLAB; the condition we get is 0< b/r < 0.1667. We thus settle upon the value of b/r=0.12. By varying the parameters and observing the values, we get the values as follows: Im=0.05, b=0.012, r=0.1.
   (a) Phase Plot:



(b) We find the values of three points to be P1=(0.106, 0.013), P2=(0.581, 0.007), P3=(0.813, 0.097).

i. At P1:
Δ= 0.021610934380017096513764468456288>0
τ= -0.21610934380017096513764468456288<0
stable point.

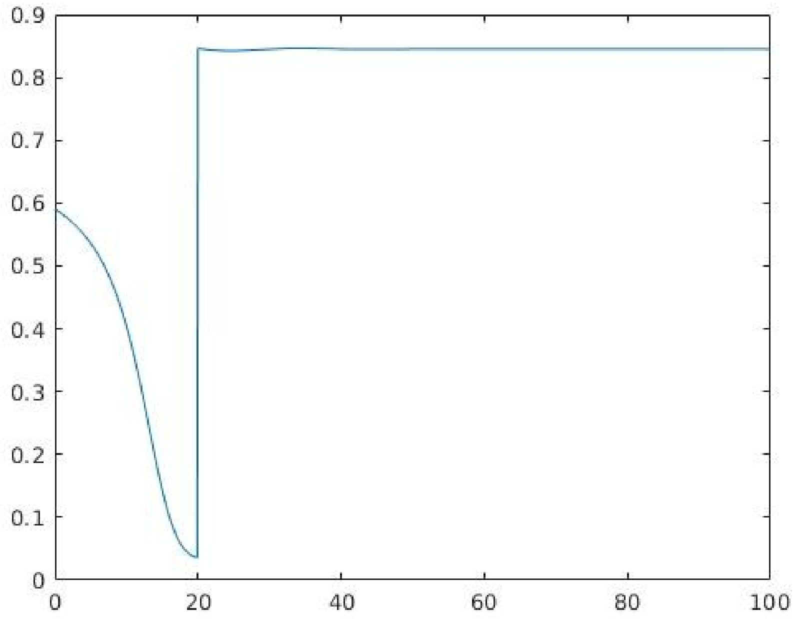
ii. At P2:
Δ= -0.023031065100007658711481382572219 <0
τ= 0.23031065100007658711481382572219>0
saddle node.

iii. At P3:
Δ=0.0044201307199905621977169141159314 >0
τ=-0.044201307199905621977169141159314 <0
stable point.

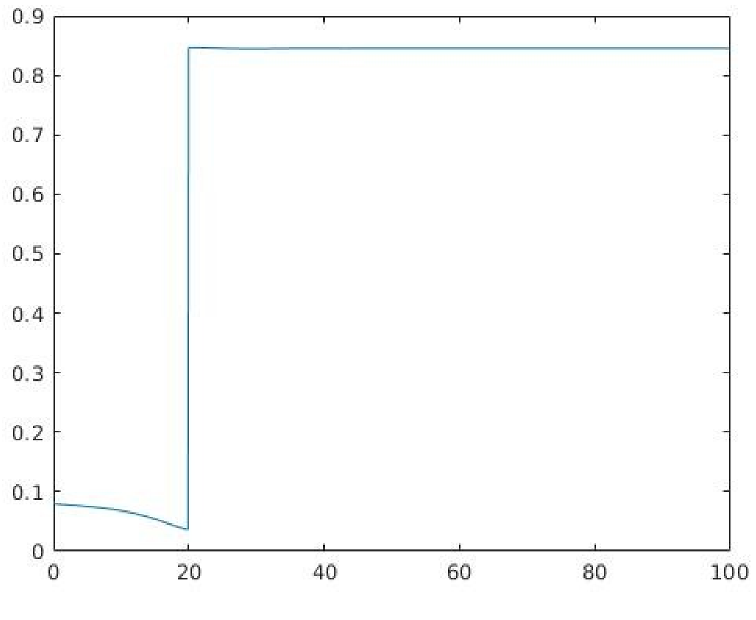
(c) V(t), W(t):

i. At P2=(0.581, 0.007)

I. V(t)

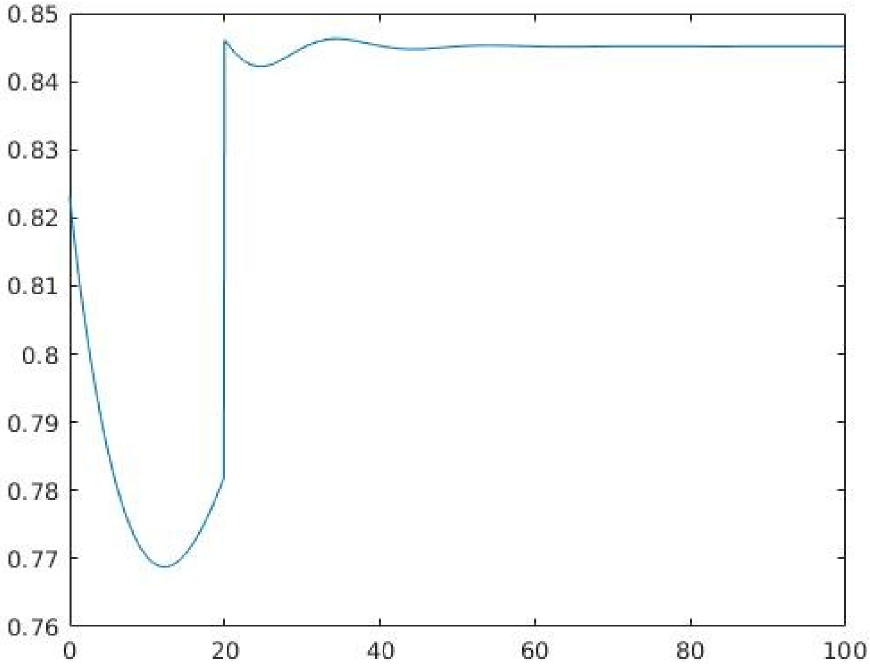


II. W(t)



ii. At P3= (0.813, 0.097)

I. V(t)



II. W(t)