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Exercise: 6.1

1. In Fig. 6.13, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.

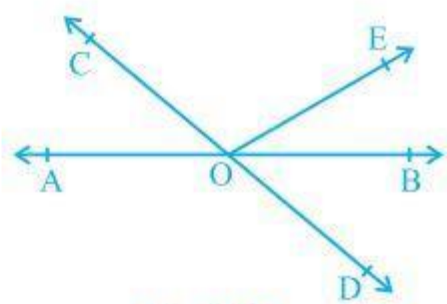

Fig. 6.13

Solution:

From the diagram,

∠AOC + ∠BOE +∠COE and ∠COE +∠BOD + ∠BOE forms a straight line.

So, ∠AOC + ∠BOE +∠COE = ∠COE +∠BOD + ∠BOE = 180°

Now, by putting the values of ∠AOC + ∠BOE = 70° and ∠BOD = 40° we get

∠COE = 110° and

∠BOE = 30°

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2. In Fig. 6.14, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.

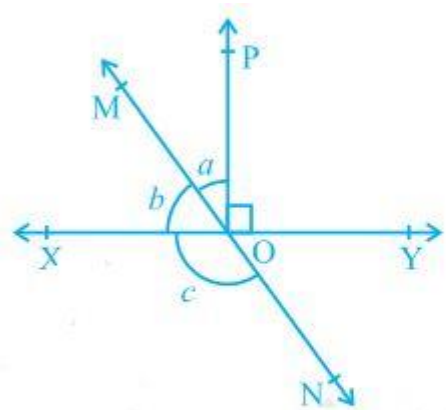


Fig. 6.14

Solution:

We know that the sum of linear pair are always equal to 180°

So,

∠POY + a + b = 180°

Putting the value of ∠POY = 90° (as given in the question) we get,

a + b = 90°

Now, it is given that a : b = 2 : 3 so,

Let a be 2x and b be 3x

∴ 2x + 3x = 90°

Solving this we get

5x = 90°

So, x = 18°

∴ a = 2 × 18° = 36°

Similarly b can be calculated and the value will be

b = 3 × 18° = 54°

From the diagram, b + c also forms a straight angle so,

b + c = 180°

=> c + 54° = 180°

∴ c = 126°

3. In Fig. 6.15, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.

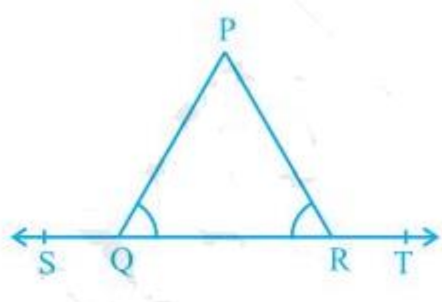


Fig. 6.15

Solution:

Since ST is a straight line so,

∠PQS + ∠PAR = 180° (linear pair) and

∠PRT + ∠PRQ = 180° (linear pair)

Now, ∠PQS + ∠PAR = ∠PRT + ∠PRQ = 180°

Since ∠PQR = ∠PRQ (as
given in the question)
∠PQS = ∠PRT. (Hence proved).

4. In Fig. 6.16, if x + y = w + z, then prove that AOB is a line.

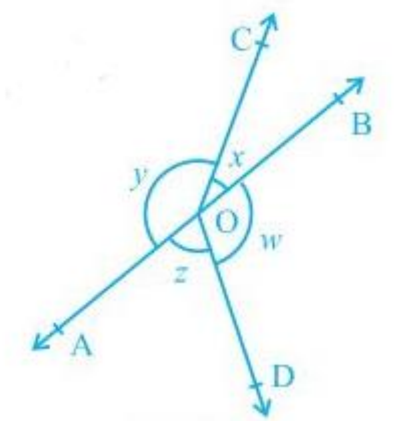


Fig. 6.16

Solution:

For proving AOB is a straight line, we will have to prove x + y is a linear pair

i.e. x + y = 180°

We know that the angles around a point are 360° so,

x + y + w + z = 360°

In the question, it is given that,

x + y = w + z

So, (x + y) + (x + y) = 360°

=> 2(x + y) = 360°

∴ (x + y) = 180° (Hence proved).

5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ROS = 1/2(∠QOS – ∠POS).

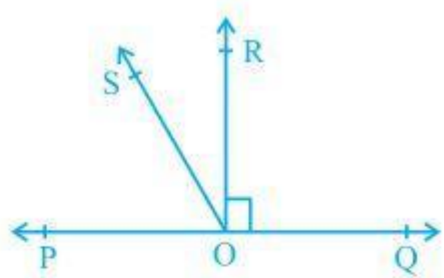


Fig. 6.17

Solution:

In the question, it is given that (OR ⊥ PQ) and ∠POQ = 180°

So, ∠POS + ∠ROS + ∠ROQ = 180°

Now, ∠POS + ∠ROS = 180° - 90° (Since ∠POR = ∠ROQ = 90°)

∴ ∠POS + ∠ROS = 90°

Now, ∠QOS = ∠ROQ + ∠ROS

It is given that ∠ROQ = 90°,

∴ ∠QOS = 90° + ∠ROS

Or, ∠QOS + ∠ROS = 90°

As ∠POS + ∠ROS = 90° and ∠QOS + ∠ROS = 90°, we get

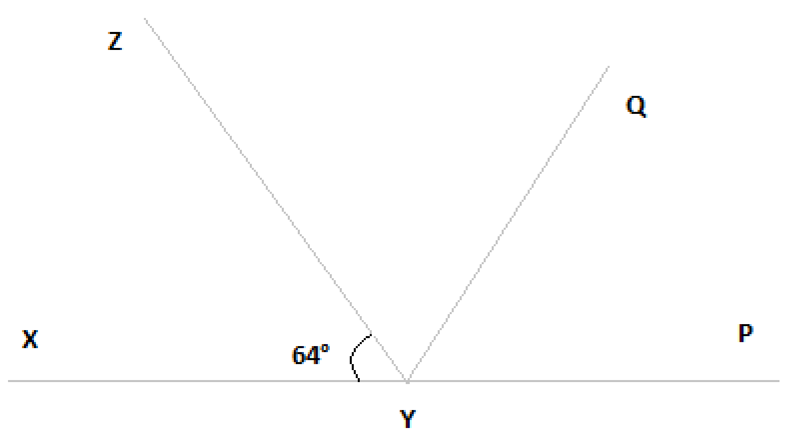
∠POS + ∠ROS = ∠QOS + ∠ROS

=>2 ∠ROS + ∠POS = ∠QOS

Or, ∠ROS = ½ (∠QOS - ∠POS) (Hence proved).

6. It is given that ∠XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.

Solution:



Here, XP is a straight line

So, ∠XYZ +∠ZYP = 180°

Putting the valye of ∠XYZ = 64° we get,

64° +∠ZYP = 180°

∴ ∠ZYP = 116°

From the diagram, we also know that ∠ZYP = ∠ZYQ + ∠QYP

Now, as YQ bisects ∠ZYP,

∠ZYQ = ∠QYP

Or, ∠ZYP = 2∠ZYQ
∴ ∠ZYQ = ∠QYP = 58°

Again, ∠XYQ = ∠XYZ + ∠ZYQ
By putting the value of ∠XYZ = 64° and ∠ZYQ = 58° we get.
∠XYQ = 64° + 58°
Or, ∠XYQ = 122°
Now, reflex ∠QYP = 180° + ∠XYQ
We computed that the value of ∠XYQ = 122°. So,
∠QYP = 180° + 122°
∴ ∠QYP = 302°

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Exercise: 6.2

1. In Fig. 6.28, find the values of x and y and then show that AB || CD.

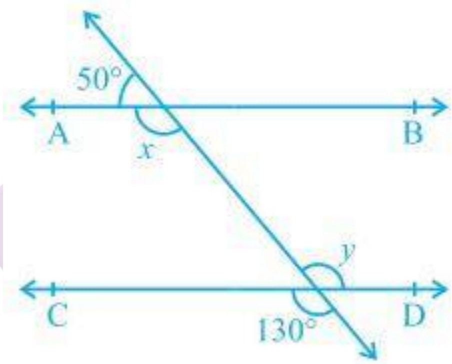

Fig. 6.28

Solution:
We know that a linear pair is equal to 180°.
So, x + 50° = 180°
∴ x = 130°
We also know that vertically opposite angles are equal.
So, y = 130°
In two parallel lines, the alternate interior angles are equal. In this,
x = y = 130°
This proves that alternate interior angles are equal and so, AB || CD.

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2. In Fig. 6.29, if AB || CD, CD || EF and y : z = 3 : 7, find x.

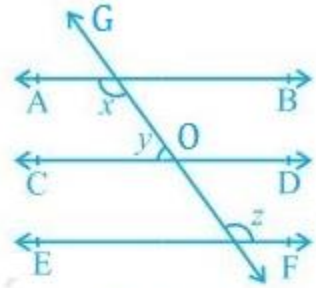


Fig. 6.29

Solution:

It is known that AB ∥ CD and CD ∥ EF

As the angles on the same side of a transversal line sums up to 180°,

x + y = 180° -----(i)

Also,

∠O = z (Since they are corresponding angles)

and, y + ∠O = 180° (Since they are a linear pair)

So, y + z = 180°

Now, let y = 3w and hence, z = 7w (As y : z = 3 : 7)

∴ 3w + 7w = 180°

Or, 10 w = 180°

So, w = 18°

Now, y = 3 × 18° = 54°

and, z = 7 × 18° = 126°

Now, angle x can be calculated from equation (i)

x + y = 180°

Or, x + 54° = 180°

∴ x = 126°

3. In Fig. 6.30, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.

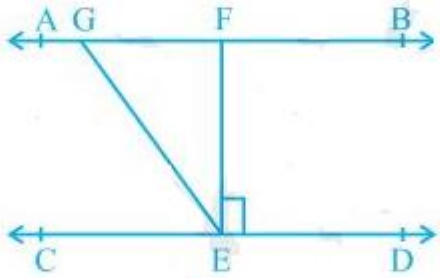


Fig. 6.30

Solution:

Since AB || CD GE is a transversal.

It is given that ∠GED = 126°

So, ∠GED = ∠AGE = 126°   (As they are alternate interior angles)

Also,

∠GED = ∠GEF + ∠FED

As

EF ⊥CD, ∠FED = 90°

∴ ∠GED = ∠GEF + 90°

Or, ∠GEF = 126 – 90° = 36°

Again, ∠FGE + ∠GED = 180° (Transversal)

Putting the value of ∠GED = 126° we get,

∠FGE = 54°

So,

∠AGE = 126°

∠GEF = 36° and

∠FGE = 54°

4. In Fig. 6.31, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.
[Hint : Draw a line parallel to ST through point R.]

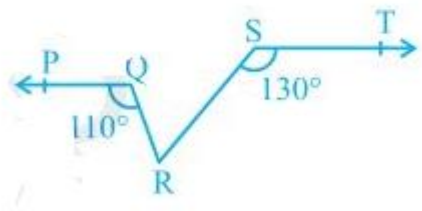
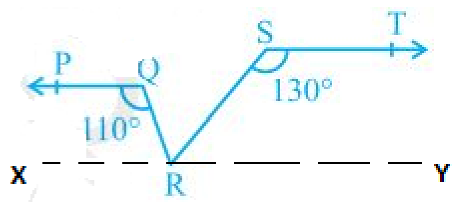


Fig. 6.31

Solution:

First, construct a line XY parallel to PQ.



We know that the angles on the same side of transversal is equal to 180°.
So, ∠PQR + ∠QRX = 180°
Or,∠QRX = 180° - 110°
∴ ∠QRX = 70°
Similarly,
∠RST + ∠SRY = 180°
Or, ∠SRY = 180° - 130°
∴ ∠SRY = 50°
Now, for the linear pairs on the line XY-
∠QRX + ∠QRS + ∠SRY = 180°
Putting their respective values we get,
∠QRS = 180° - 70° - 50°
Or, ∠QRS = 60°

5. In Fig. 6.32, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.

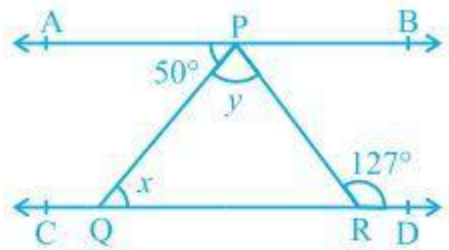


Fig. 6.32

Solution:

From the diagram,

∠APQ = ∠PQR (Alternate interior angles)

Now, putting the value of ∠APQ = 50° and ∠PQR = x we get,

x = 50°

Also,

∠APR = ∠PRD (Alternate interior angles)

Or, ∠APR = 127° (As it is given that ∠PRD = 127°)

We know that

∠APR = ∠APQ + ∠QPR

Now, putting values of ∠QPR = y and ∠APR = 127° we get,

127° = 50° + y

Or, y = 77°

Thus, the values of x and y are calculated as:

x = 50° and

y = 77°

6. In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.

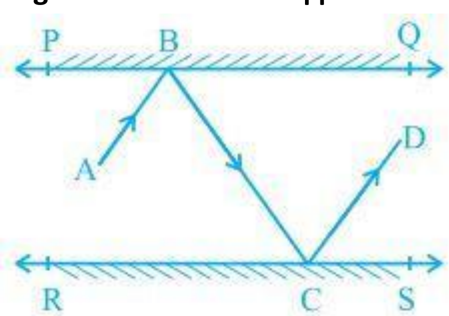


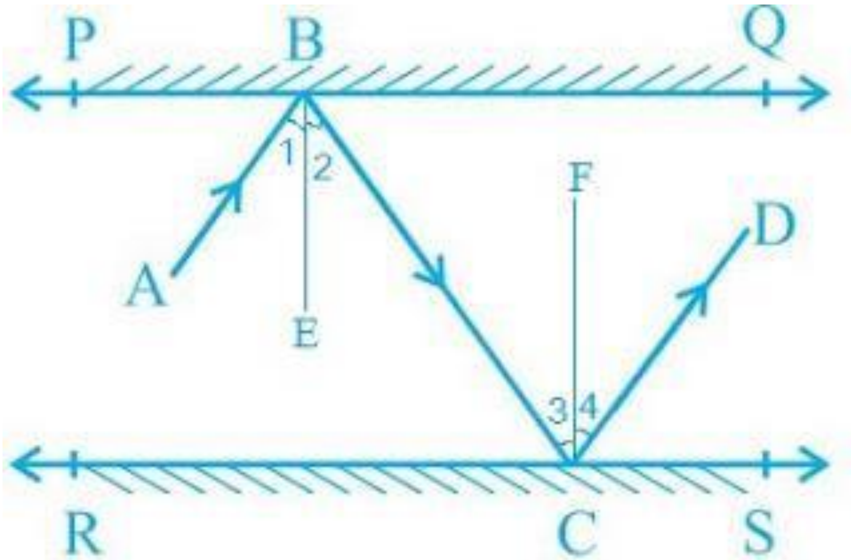
Fig. 6.33

Solution:

First, draw two lines BE and CF such that BE ⊥PQ and CF ⊥RS.

Now, since PQ || RS,

So, BE || CF



We know that,

Angle of incidence = Angle of reflection (By the law of reflection)

So,

∠1 = ∠2 and

∠3 = ∠4

We also know that alternate interior angles are equal. Here, BE ⊥ CF and the transversal line BC cuts them at B and C

So, ∠2 = ∠3 (As they are alternate interior angles)

Now, ∠1 + ∠2 = ∠3 + ∠4

Or, ∠ABC = ∠DCB

So, AB ∥ CD (alternate interior angles are equal)

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Exercise: 6.3

1. In Fig. 6.39, sides QP and RQ of ΔPQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.

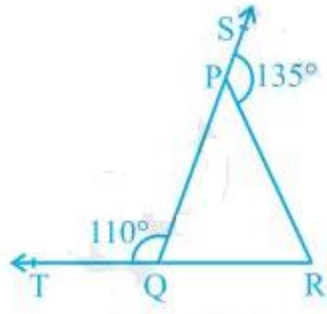

Fig. 6.39

Solution:

It is given the TQR is a straight line and so, the linear pairs (i.e. ∠TQP and ∠PQR) will add up to 180°

So, ∠TQP + ∠PQR = 180°

Now, putting the value of ∠TQP = 110° we get,

∠PQR = 70°

Consider the ΔPQR,

Here, the side QP is extended to S and so, ∠SPR forms the exterior angle.

Thus, ∠SPR (∠SPR = 135°) is equal to the sum of interior opposite angles. (triangle property)

Or, ∠PQR + ∠PRQ = 135°

Now, putting the value of ∠PQR = 70° we get,

∠PRQ = 135° - 70°

Or, ∠PRQ = 65°

2. In Fig. 6.40, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of Δ XYZ, find ∠OZY and ∠YOZ.

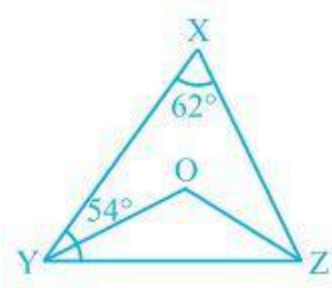

Fig. 6.40

Solution:

We know that the sum of the interior angles of the triangle.

So, ∠X +∠XYZ + ∠XZY =
180°
Putting the values as given in the question we get,
62° + 54° + ∠XZY = 180°
Or, ∠XZY = 64°
Now, we know that ZO is the bisector so,
∠OZY = ½ ∠XZY
∴ ∠OZY = 32°
Similarly, YO is a bisector and so,
∠OYZ = ½ ∠XYZ
Or, ∠OYZ = 27° (As ∠XYZ = 54°)
Now, as the sum of the interior angles of the triangle,
∠OZY +∠OYZ + ∠O = 180°
Putting their respective values we get,
∠O = 180° - 32° - 27°
Or, ∠O = 121°

3. In Fig. 6.41, if AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.

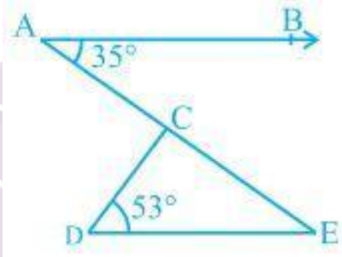

Fig. 6.41

Solution:
We know that AE is a transversal since AB ∥ DE
Here ∠BAC and ∠AED are alternate interior angles.
Hence, ∠BAC = ∠AED
It is given that ∠BAC = 35°
=> ∠AED = 35°
Now consider the triangle CDE. We know that the sum of the interior angles of a triangle is 180°.
∴ ∠DCE +∠CED + ∠CDE = 180°
Putting the values we get
∠DCE + 35° + 53° = 180°

Or, ∠DCE = 92°

4. In Fig. 6.42, if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.

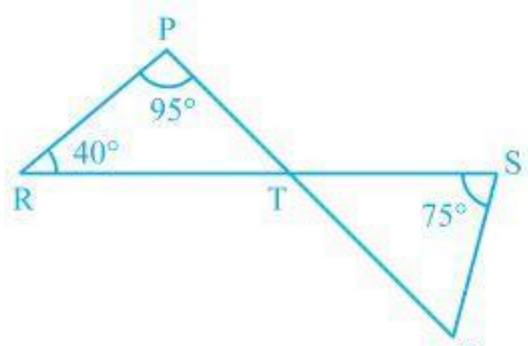

Fig. 6.42

Solution:

Consider triangle PRT.
∠PRT +∠RPT + ∠PTR = 180°
So, ∠PTR = 45°
Now ∠PTR will be equal to ∠STQ as they are vertically opposite angles.
So, ∠PTR = ∠STQ = 45°
Again in triangle STQ,
∠TSQ +∠PTR + ∠SQT = 180°
Solving this we get,
∠SQT = 60°

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5. In Fig. 6.43, if PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.

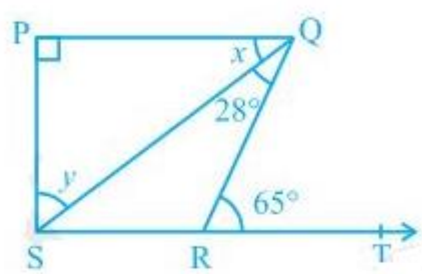

Fig. 6.43

Solution:

x +∠SQR = ∠QRT (As they are alternate angles since QR is transversal)
So, x + 28° = 65°
∴ x = 37°
It is also known that alternate interior angles are same and so,
∠QSR = x = 37°
also,
Now,
∠QRS +∠QRT = 180° (As they are a Linear pair)
Or, ∠QRS + 65° = 180°
So, ∠QRS = 115°
Now, we know that the sum of the angles in a quadrilateral is 360°. So,
∠P + ∠Q+ ∠R +∠S = 360°
Putting their respective values we get,
∠S = 360° - 90° - 65° - 115°
Or, ∠QSR + y = 360°
=>y = 360° - 90° - 65° - 115° - 37°
Or, y = 53°

6. In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR = 1/2∠QPR.

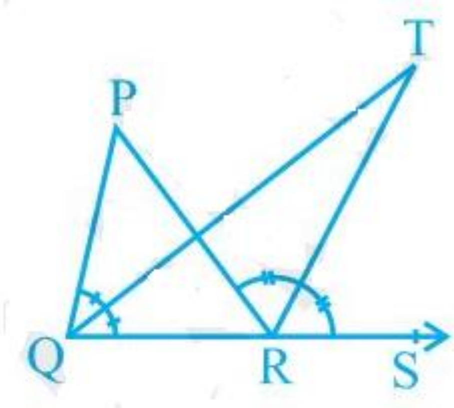

Fig. 6.44

Solution:

Consider the ΔPQR. ∠PRS is the exterior angle and ∠QPR and ∠PQR are interior angles.

So, ∠PRS = ∠QPR + ∠PQR (According to triangle property)

Or, ∠PRS - ∠PQR = ∠QPR -----------(i)

Now, consider the ΔQRT,

∠TRS = ∠TQR + ∠QTR

Or, ∠QTR = ∠TRS - ∠TQR

We know that QT and RT bisect ∠PQR and ∠PRS respectively.

So, ∠PRS = 2 ∠TRS and ∠PQR = 2 ∠TQR

Now, ∠QTR = ½ ∠PRS - ½ ∠PQR

Or, ∠QTR = ½ (∠PRS - ∠PQR)

From (i) we know that ∠PRS - ∠PQR = ∠QPR

So, ∠QTR = ½ ∠QPR (hence proved).