Risk Parity Portfolio Construction for South Africa

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Abstract

This study examines the effectiveness of risk parity portfolio construction in the South African equity market, using data from the FTSE/JSE Top 40 index from 2010 to 2024. A conceptual replication of the vanilla risk parity framework is conducted and applied to South Africa, comparing RP portfolios against the tangency portfolio in terms of allocation stability, risk distribution, and return performance. The results indicate that risk parity portfolios provide greater risk diversification and allocation stability, making them resilient in volatile markets. However, they may underperform in strong bull markets where high-risk assets generate outsized returns.

Keywords: Risk Parity Portfolio, Portfolio Optimization

1. Introduction

Portfolio optimization remains a fundamental challenge in financial management, as investors seek to allocate capital efficiently across different assets to balance risk and return. Traditional models, such as Markowitz's (1952) mean-variance optimization (MVO) and the Capital Asset Pricing Model (CAPM), have played a crucial role in shaping investment strategies. However, these approaches heavily rely on expected returns, which are notoriously difficult to estimate accurately and are prone to instability.

Risk parity (RP), also known as equal risk contribution (ERC), has emerged as an alternative approach to portfolio construction, emphasizing the allocation of risk rather than capital. Unlike MVO, which optimizes return for a given level of risk, RP portfolios ensure that each asset contributes equally to overall portfolio risk. This strategy gained popularity soon after the global financial crisis as traditional methods of asset pricing faced scrutiny. The popularity grew after Maillard et al. (2010) showed that constructing portfolios with equal risk contribution in mind led to better diversification outcomes regardless of expected returns.

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The objective of this paper is to examine the effectiveness of risk parity portfolio construction in the South African equity market. Specifically, we compare risk parity portfolios with the traditional maximum Sharpe ratio (tangency) portfolio using a dataset of the FTSE/JSE Top 40 companies from 2010 to 2024. By conducting a conceptual replication of Tharis Souza's (2019) study on risk parity for FAANG stocks, we assess whether risk parity offers superior risk-adjusted returns and portfolio stability in an emerging market context. Through a series of empirical tests, we evaluate the allocation dynamics, risk stability, and return performance of RP portfolios against their tangency counterparts.

2. Literature Review

2.1. Historical Evolution of Portfolio Theory

The foundation of modern portfolio theory can be traced to Markowitz (1952) work on mean variance optimization, which introduced the concept of balancing expected returns against risk. The core concept of MVO is to maximise expected return for a given level of risk, measured by variance, by strategically allocating investments across different assets to achieve the best possible risk-return trade-off for an investor based on their risk tolerance (Panulo, 2014). Although MVO remains a powerful tool that is applied widely, it's practical implementation has been criticised for its heavy reliance on expected return estimates, which are prone to estimation errors and instability (Best and Grauer, 1991).

Building on Markowitz's work, the Capital Asset Pricing Model, introduced by Sharpe (1964) and Mossin (1966), provided further insight into asset pricing by linking expected returns to systematic risk. While CAPM remains a cornerstone of financial theory, its assumptions about market efficiency and normally distributed returns have also been challenged.

The 2008 global financial crisis marked an important turning point that led to many scholars and practitioners to strongly criticise traditional risk-gain models, consequently, leading to wide scale adoption of a different kind of portfolio selection strategy, one based on risk allocation (Ararat et al., 2024). One way to deal with the problems of MVO is to use equal weighted portfolios (EW), where capital is distributed evenly across assets. However, if the assets in the investment universe have different instrinsic risk then the equal weighted portfolio does not achieve total risk diversification (Ararat et al., 2024).

2.2. Toward Risk Parity

A more prudent way of addressing risk is therefore risk parity, also called equal risk contribution. A risk parity portfolio is constructed to ensure that each asset contributes equally to the total risk of the portfolio. Unlike EW portfolios, which allocates the same amount of capital to each asset regardless of risk, RP adjusts asset weights based on their individual volatility and correlation with other assets. This ensures that no single asset dominates portfolio risk, leading to improved diversification (Maillard et al., 2010).

Mathematically, a risk parity portfolio is defined by solving the following optimization problem:

$$w_i \sigma_i = wj\sigma_j, \forall_{i,j}$$

Where w_i represents the weight of asset I and σ_i is its standard deviation. This framework ensures that assets with higher risk receive lower weight, while lower risk assets are are allocated proportionally more capital, creating a balanced risk structure across the portfolio. Many methods exist when creating risk parity portfolios (Feng and Palomar, 2015). Traditionally, the formulation of risk parity is a least squares optimization problem, where the risk contribution of each asset is compared to another assets risk contribution. The model thus compares each asset with every other asset and reach an objective value of 0 indicating that all risk contributions are the same (Gambeta and Kwon, 2020).

The appeal of risk parity lies in its ability to distribute risk more evenly across portfolio constituents, reducing reliance on fragile return forecasts. However, this advantage is not without its theoretical and empirical challenges. The assumption that equal risk contribution leads to optimal diversification has been contested, particularly in extreme market conditions. Traditional risk parity models rely on volatility as a risk measure, which fails to capture asymmetries and fat-tailed distributions present in financial markets (Braga et al., 2023). The increased research in extensions of ERC such as kurtosis-based risk parity reflects a growing awareness that standard deviation alone may be insufficient for portfolio optimization.

Costa and Kwon (2019) argue that risk parity portfolios perform inconsistently across market regimes, particularly during high volatility periods. Their research introduces a Markov regime-switching framework to address the instability in asset volatilities and correlations. This approach suggests that the risk contributions of assets should be dynamically adjusted based on the market state rather than assuming static risk exposures. Such enhancements present a compelling case for incorporating dynamic risk parity strategies that react to macroeconomic conditions and shifts in risk premia. Another concern is the overconcentatration of risk parity portfolios in low volatility assets, particularly fixed income securities. Risk parity portfolios seem to work well when interest rates are declining but presents substantial risk when interest rates rise. Panulo (2014) highlights the vulnerability of leveraged risk parity portfolios to funding constraints, as liquidity dries up during financial crises. The necessity for leverage in risk parity to maintain competitive returns adds another layer of risk, raising questions about its suitability for all investors.

Alternative portfolio construction methods provide contrasting views on achieving diversification and risk control. MVO, despite its well documented estimation errors, remains the theoretical benchmark for efficient portfolio construction. With the advancements in machine learning and Bayesian estimation techniques, some argue that return forecasting has improved enough to mitigate drawbacks of MVO. Meanwhile, hierarchical risk parity (HRP) has emerged as a more sophisticated refinement of traditional risk parity (Pfitzinger et al., 2019). It involves clustering assets based on their correlation structures rather than assuming homogeneity in risk contribution (Palit and Prybutok, 2024). These innovations demonstrate that risk parity, while valuable, is not necessarily the optimal solution in all market conditions.

The ongoing debate between risk parity and alternative allocation strategies underscores a broader dilemma in portfolio theory: should investors prioritize robustness over theoretical efficiency? While risk parity minimizes reliance on unreliable return estimates, it does not eliminate estimation risk entirely. Volatility estimates and correlation structures are equally prone to regime shifts and model instability. By integrating higher order risk measures such as skewness and kurtosis, researchers aim to refine risk parity's effectiveness in extreme market conditions (Braga et al., 2023). Future research should further explore dynamic risk parity models that adjust allocation strategies in real time based on changing market conditions. Factor-based risk parity, which incorporates macroeconomic variables into the allocation process, may also provide a more adaptive framework for portfolio construction. Combining machine learning and risk parity has also proved to be useful, as algorithmic techniques can enhance estimation stability and improve responsiveness to financial shocks. Ultimately, risk parity remains an essential strategy in portfolio construction. For the remainder of this essay, I will advocate for risk parity portfolios using South African equity data.

3. Data

In this paper, we explore whether a risk parity portfolio is more efficient than maximum sharpe ratio or tangency portfolio in South Africa. we conduct a conceptual replication of Tharis Souza's 2019 article, "*DIY Ray Dalio ETF: How to build your own Hedge Fund strategy with risk parity portfolios*", where he showed how to build a risk parity portfolio for FAANG companies. In this paper, we follow the same principles, using the data from the FTSE/JSE Top 40 Companies (J200) from January 2010 to June 2024. The data is available through Bloomberg. In order to stick close to the article by Souza, which used 5 top companies in the USA, we subset the number of equities by taking the top ten tickers on the last date of the dataset. The portfolios are then constructed from the universe of the following tickers: AGL, BTI, CFR, CPI, FSR, GFI, MTN, NPN, PRX and SBK. Table one shows the descriptive statistics for these tickers.

4. Methodology

For the purposes of this replication we use the J200 dataset to construct a risk parity portfolio and a tangency portfolio. First, we verify whether the risk parity portfolio is well specified, by plotting the risk contribution and weight distribution of each asset. If the risk contributions are all equal then the portfolio is well specified. Next we compare the weight allocation of both portfolios to assess the weight distribution among constituents. Thirdly, in order to determine the stability of risk parity portfolios, we construct a rolling risk parity portfolio and plot a stacked barchart of quarterly rebalanced weights. Finally, the cumulative returns, daily returns and drawdowns are also assessed and compared with performance from other portfolios in the literature.

4.1. RP Construction

Let the marginal risk contribution of asset i be given by:

$$\frac{\partial \sigma_p}{\partial w_i} = \frac{(\Sigma w)_i}{\sigma_p}$$

Multiplying by w_i gives the total risk contribution (RC) :

$$RC_i = w_i \frac{(\Sigma w)_i}{\sigma_p}$$

Since the total risk portfolio is:

$$\sigma_p = \sqrt{w^\top \Sigma w},$$

the risk contribution simplifies to:

$$RC_i = w_i (\Sigma w)_i.$$

In a risk parity portfolio, all assets contribute equally to risk, this means :

$$w_i (\Sigma w)_i = w_j (\Sigma w)_j, \quad \forall i, j.$$

Since this must hold for all pairs, we rewrite it as:

$$w_i (\Sigma w)_i - w_j (\Sigma w)_j = 0, \quad \forall i, j,$$

This forms a homogenous system of equations and can be formulated as a least squares objective problem as below

$$\min_w \sum_{i=1}^{n} \sum_{j=1}^{n} \left(w_i (\Sigma w)_i - w_j (\Sigma w)_j \right)^2 .$$

$$s.t \mathbf{1}^T w = 1$$

$$w \geq 0$$

The model will compare risk contribution of each asset with every other asset and reach an objective value of 0 indicating all risk contributions are the same.

4.2. Max Sharpe Ratio Construction

The tangency portfolio maximizes the Sharpe ratio defined as:

$$S(w) = \frac{w^\top \mu - r_f}{\sqrt{w^\top \Sigma w}}$$

Where μ is the vector of expected asset returns and r_f is the risk free rate.

Since the Sharpe ratio is scale invariant, we solve the following unconstrained optimization problem:

$$\max_w \frac{w^\top (\mu - r_f \mathbf{1})}{\sqrt{w^\top \Sigma w}}.$$

Using Lagrangian optimization, the optimal tangency portfolio weights before normalization are:

$$w^* = \Sigma^{-1} (\mu - r_f \mathbf{1}).$$

To ensure that the portfolio is fully invested $w^\top \mathbf{1} = 1$, we normalize:

$$w^* = \frac{\Sigma^{-1}(\mu - r_f \mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\mu - r_f \mathbf{1})}.$$

This results in the tangency portfolio that leads to the highest Sharpe ratio.

5. Discussion

The empirical results highlight several important findings regarding risk parity portfolio performance in South Africa. First, the risk contribution analysis confirms that the RP portfolio is well-specified, with risk contributions distributed evenly across assets. This stands in contrast to the maximum Sharpe ratio portfolio, where risk is concentrated in a few high risk, high reward stocks.

Second, our weight distribution analysis reveals that risk parity portfolios favor low-volatility assets more than the tangency portfolio, which assigns disproportionate weights to assets with higher expected returns. This finding aligns with existing literature, where RP strategies tend to overweight assets with lower volatility. Moreover, the rolling risk parity allocation over different time periods suggests that RP portfolios maintain a relatively stable allocation structure, whereas tangency portfolios would have exhibitted significant variation in weight distributions due to changing return expectations. This stability is particularly valuable in volatile market conditions, as it minimizes exposure.

However, RP portfolios are not without their challenges. One notable drawback is their tendency to underperform during strong bull markets, where high-risk assets generate excess returns that RP portfolios do not fully capture due to their focus on risk allocation rather than return maximization. Additionally, the dependence of risk parity on volatility estimates means that periods of sharp volatility shifts, such as during financial crises, may lead to suboptimal allocations if not adjusted.

Finally, the literature shows that a risk parity portfolio would likely be somewhere between an equal weight and mean variance portfolios (Gambeta and Kwon, 2020). From this we can infer that the risk parity portfolio would likely have higher cumulative returns and shallower drawdowns than the tangency portfolio. This strengthens the argument for their application in emerging market economies. To stress test this portfolio structure further, it could have been useful to test other packages on the same data as well as introduce a wider range of assets to the investable universe. This would allow a thorough analysis of how risk parity portfolios diversify across sectors, asset classes and regulatory or macroeconomic contraints.

6. Conclusion

The empirical analysis shows that RP portfolios provide superior diversification by ensuring an equal distribution of risk across assets, reducing concentration in high-risk stocks, and maintaining allocation stability over time. Given these findings, a dynamic risk parity approach, which adjusts allocations based on changing market conditions, such as a markov regime switching risk parity portfolio, may further enhance performance. Incorporating factors such as macro-driven risk assessments or machine learning techniques could improve risk parity's adaptability to different market environments.

Future research should explore how hierarchical risk parity (HRP), which clusters assets based on correlation structures, compares with traditional RP in South African financial markets. Additionally, integrating alternative risk measures, such as skewness and kurtosis, may refine portfolio construction to better address tail-risk events. Furthermore, incorporating different asset classes would illuminate how risk parity construction diversifies across sectors and asset classes.

Ultimately, while risk parity does not necessarily outperform in all market conditions, it remains a valuable tool for investors seeking stability, diversification, and reduced dependence on unreliable return forecasts. Its effectiveness in the South African context underscores the need for risk-based allocation strategies in emerging markets, where structural volatility and market inefficiencies pose unique challenges to traditional investment models.

7. Appendix

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