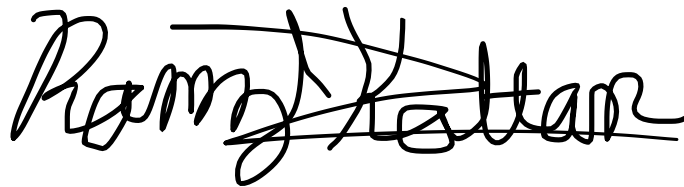


Ramzi Kattan

0032574669

"I pledge on my honor that I have not given or received any unauthorized assistance on this project. As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

A handwritten signature in black ink that reads "Ramzi Kattan". The signature is written in a cursive style with a horizontal line drawn across the middle of the name.

**Purdue University-Credit : Survey Certificate
of Completion**

**Course: wl.202220.AAE.30100.001.10014
: Sig Anly For Aero Engr
Instructor: Dengfeng Sun**

**Submitted: 5/1/2022 12:20 AM
Student: Ramzi Kattan**

Introduction:

This technical report will study the longitudinal dynamics of a Cessna 182 aircraft using Simulink to model 2 separate scenarios under varying trim conditions. Simulink is a powerful app in MATLAB that allows for dynamic modeling and simulations with the use of 'blocks,' each block serving a mathematical or technical objective. Simulink allows you to visualize the steps taken to study complex scenarios. The trim conditions were derived from the AAE301 Spring 2022 "Final Project Assignment" document. This technical report will include the procedure, Simulink model, simulation graphs, and results of the Cessna 182 longitudinal dynamics under the 2 trim conditions.

Procedure:

The procedure for such a study may differ, there are several ways to arrive at the simulation graphs required. Firstly, all known constants are input into a MATLAB script file. The script file is then coded to open a Simulink file so that the two files are linked. Next, all given equations are studied for desired inputs and outputs. Each equation is then given a subsystem, with the previously identified inputs and outputs. Finally, the desired timeseries and variables are exported into the MATLAB workspace. Once in MATLAB workspace, the desired simulations are graphed, this is preferred to the 'scope' block in Simulink because the plot produced by 'scope' cannot be altered and studied the same way as the normal plot function can.

Simulink:

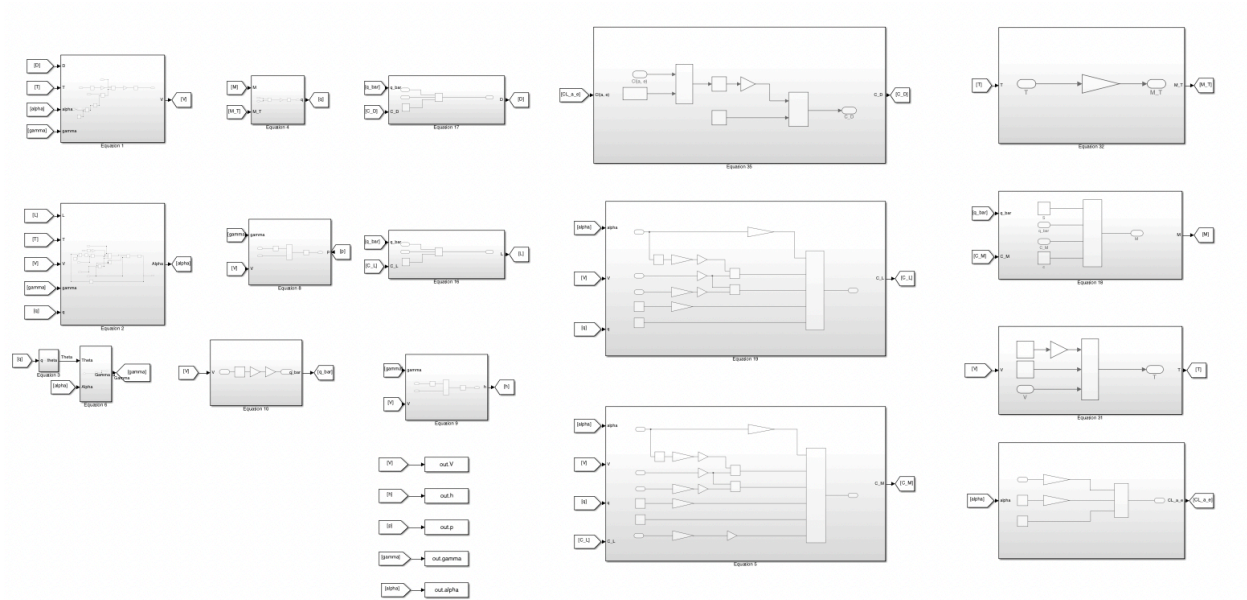


Figure 1: Entire Simulink model

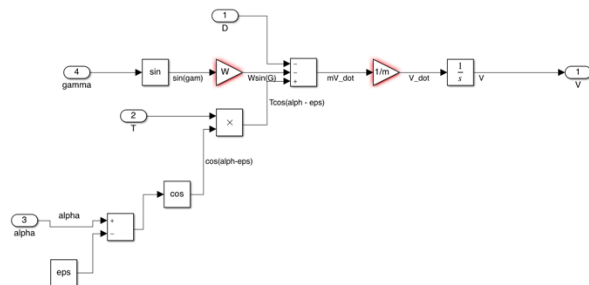


Figure 2: Equation 1 Subsystem

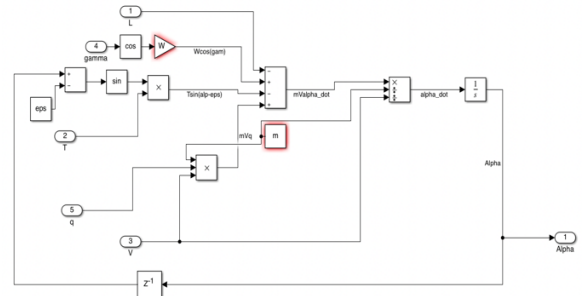


Figure 3: Equation 2 Subsystem

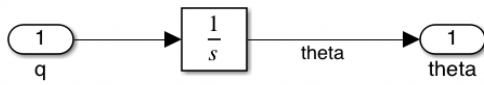


Figure 4: Equation 3 Subsystem

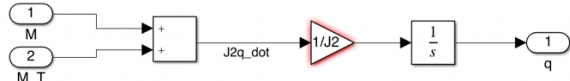


Figure 6: Equation 4 Subsystem



Figure 8: Equation 10 Subsystem

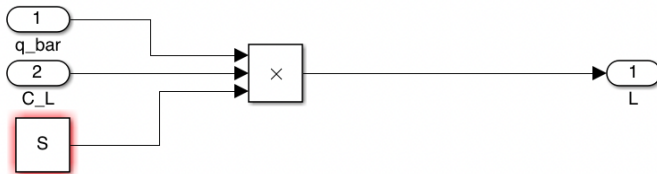


Figure 10: Equation 16 Subsystem

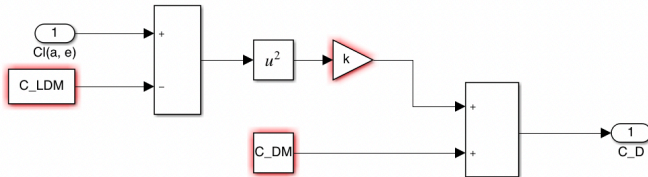


Figure 12: Equation 35

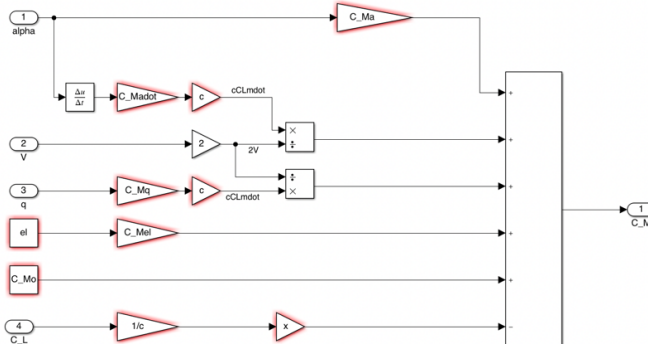


Figure 14: Equation 5 Subsystem

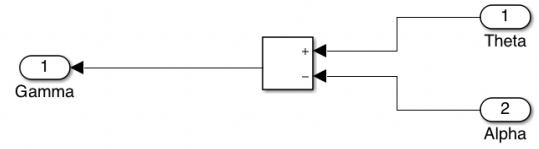


Figure 5: Equation 6 Subsystem

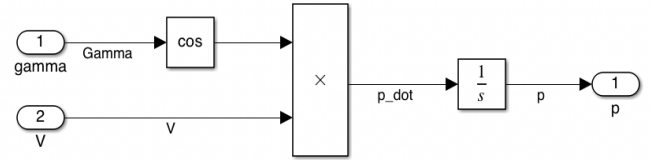


Figure 7: Equation 8 Subsystem

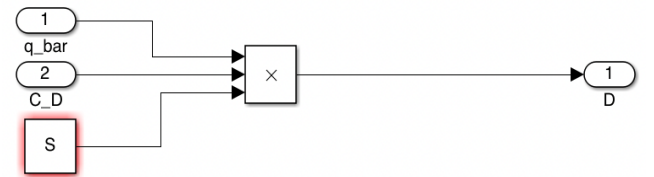


Figure 9: Equation 17 Subsystem

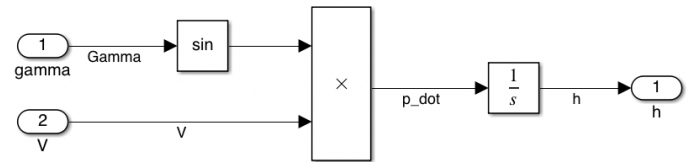


Figure 11: Equation 9 Subsystem

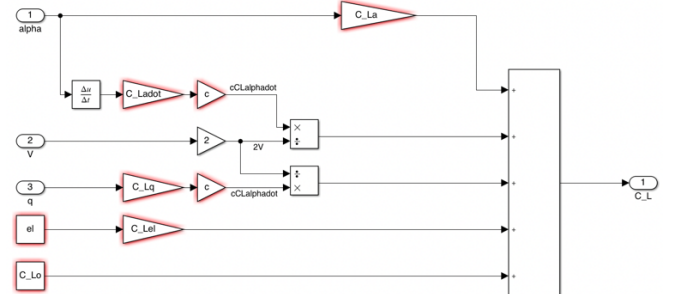


Figure 13: Equation 19

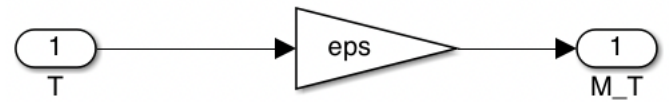


Figure 15: Equation 32 Subsystem

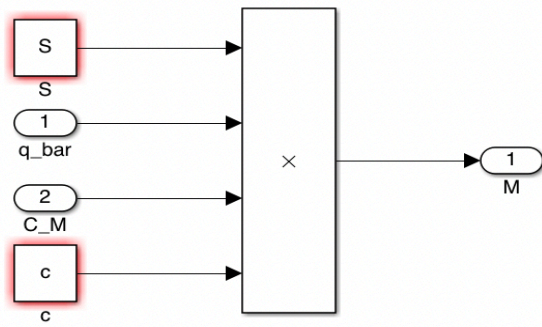


Figure 16: Equation 18

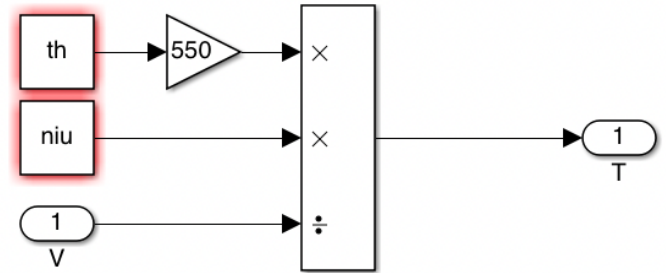


Figure 17: Equation 31 Subsystem

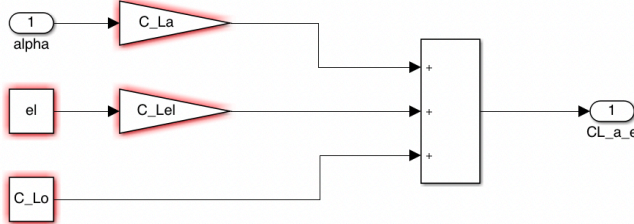


Figure 18: Equation 39 Subsystem

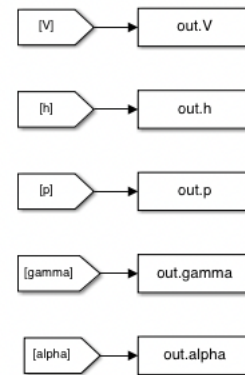


Figure 19: Exporting variables to workspace

Various blocks were used to produce this model. These blocks included constants, gains, addition, subtraction, multiplication, division, inputs, outputs, goto's, from's, to workspace, delays, trigonometric functions, and squares.

Results:

To model the longitudinal dynamics, we must define trim conditions. We will assume steady state flying conditions, this means a constant velocity, V , and constant pitch angle, θ , and therefore a constant angle of attack, α , and a zero-pitch rate, $q = 0$. Due to this, the flight path angle, γ , will also be constant.

Simulation 1:

The first simulation is gliding with no elevator deflection, $el = 0$. In a gliding scenario there is no thrust, $T = 0$. For this to occur, throttling must be zero, $th = 0$. This yields the following simulations for velocity, angle of attack, flight path angle, altitude, and altitude as a function of horizontal range.

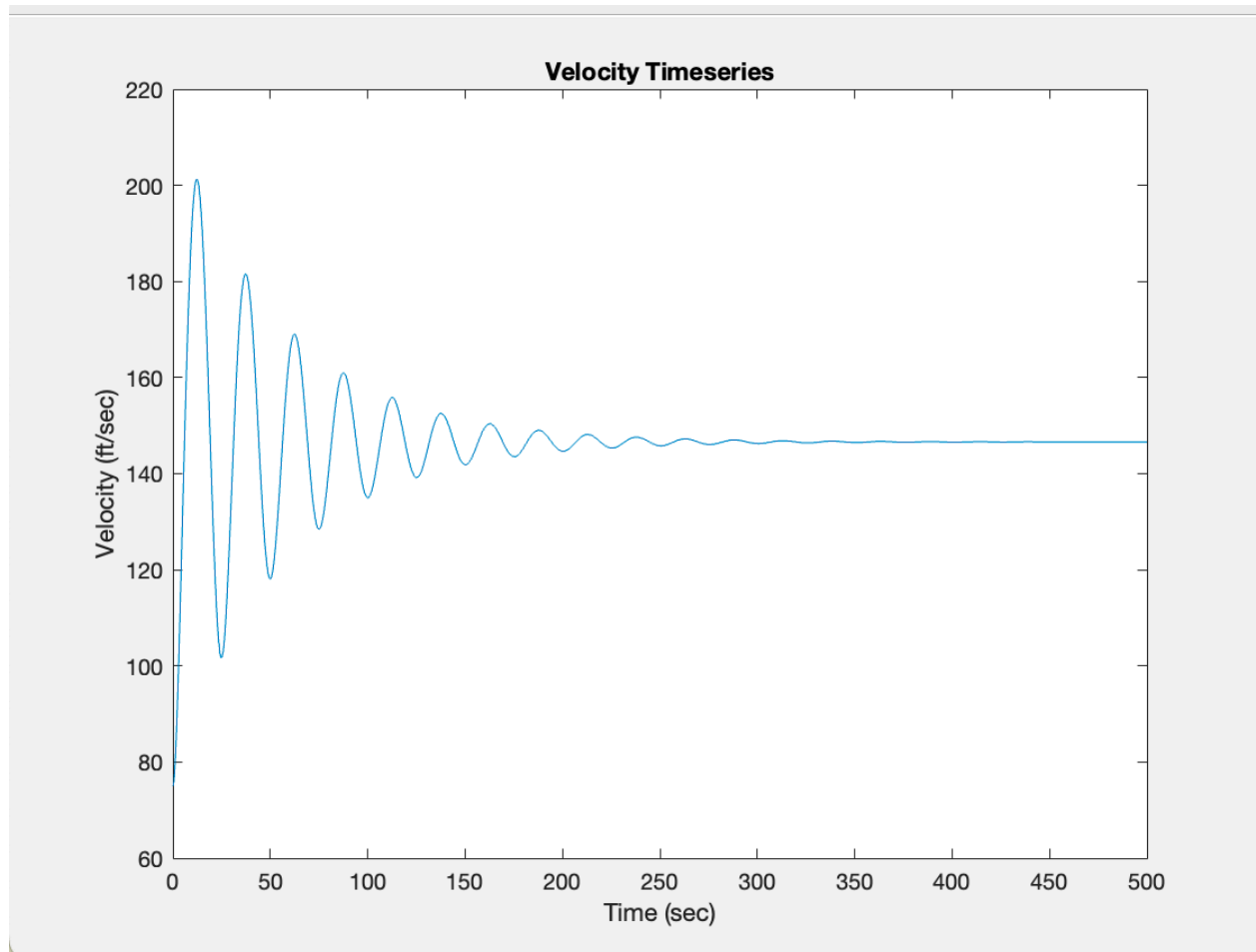


Figure 20: Velocity Timeseries – Sim 1

As expected, the velocity reaches a constant steady state value of $V \approx 146.6 \text{ ft/sec}$. It is expected that the velocity be constant since with no thrust, there is no force to increase the velocity over time.

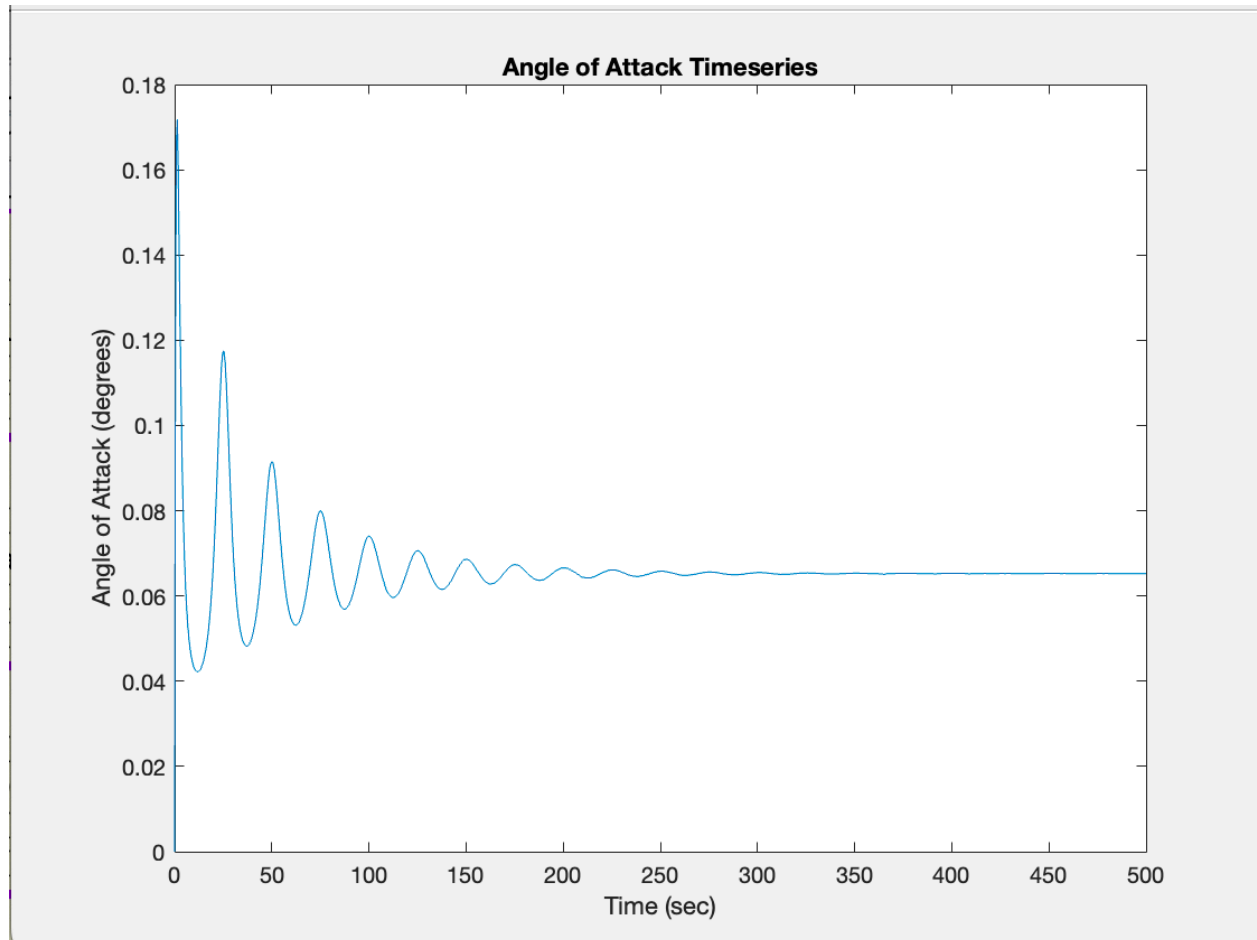


Figure 21: Angle of Attack Timeseries – Sim 1

Again, the steady state of angle of attack shows a constant value of $\alpha \approx 0.0653 \text{ rad} \approx 3.74^\circ$. A small angle of attack such as this is expected for a gliding scenario.

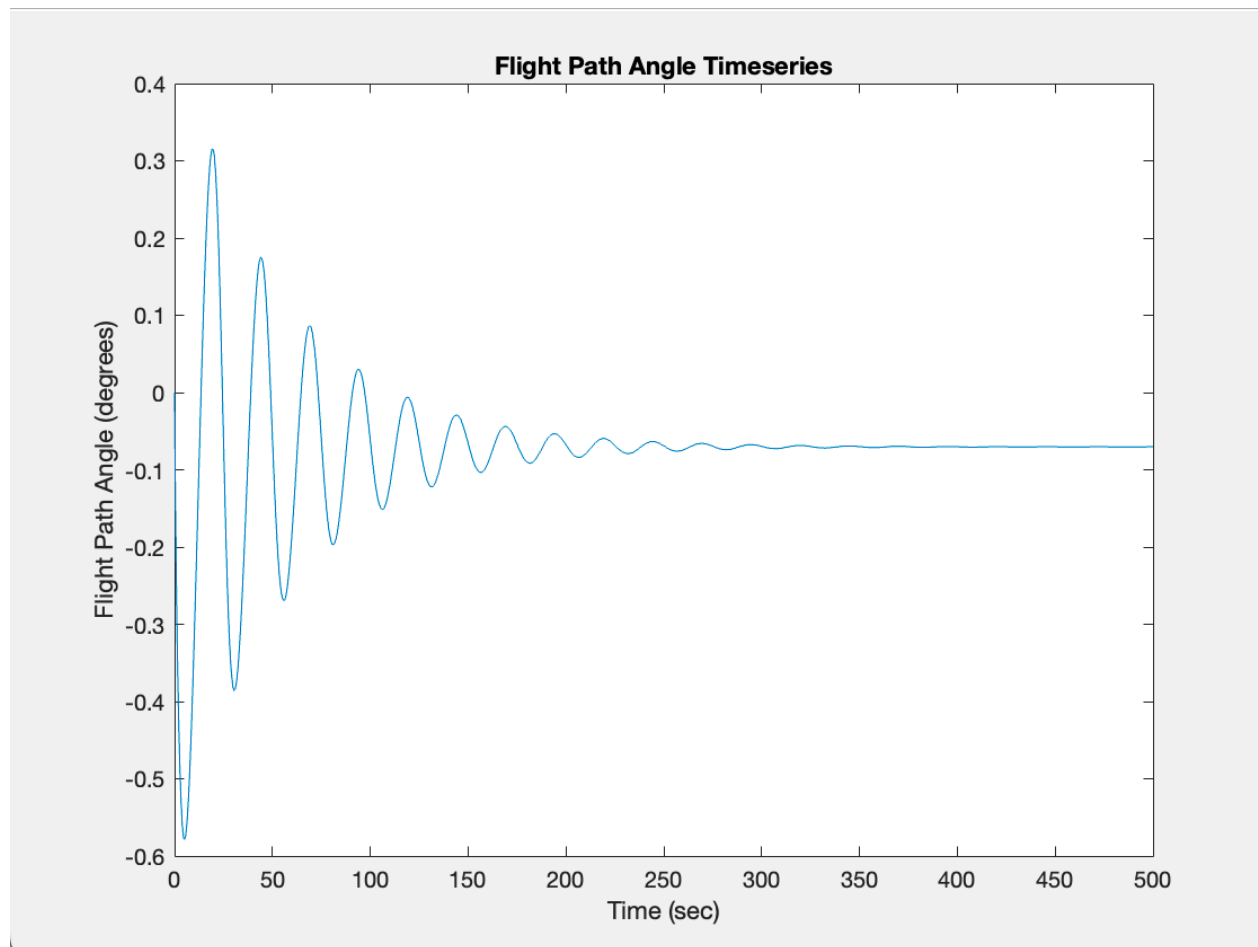


Figure 22: Flight Path Angle Timeseries – Sim 1

The steady state value of flight path angle is $\gamma = -0.0702 \text{ rad} \approx -4.022^\circ$. This means the gliding angle is roughly 4.022° .

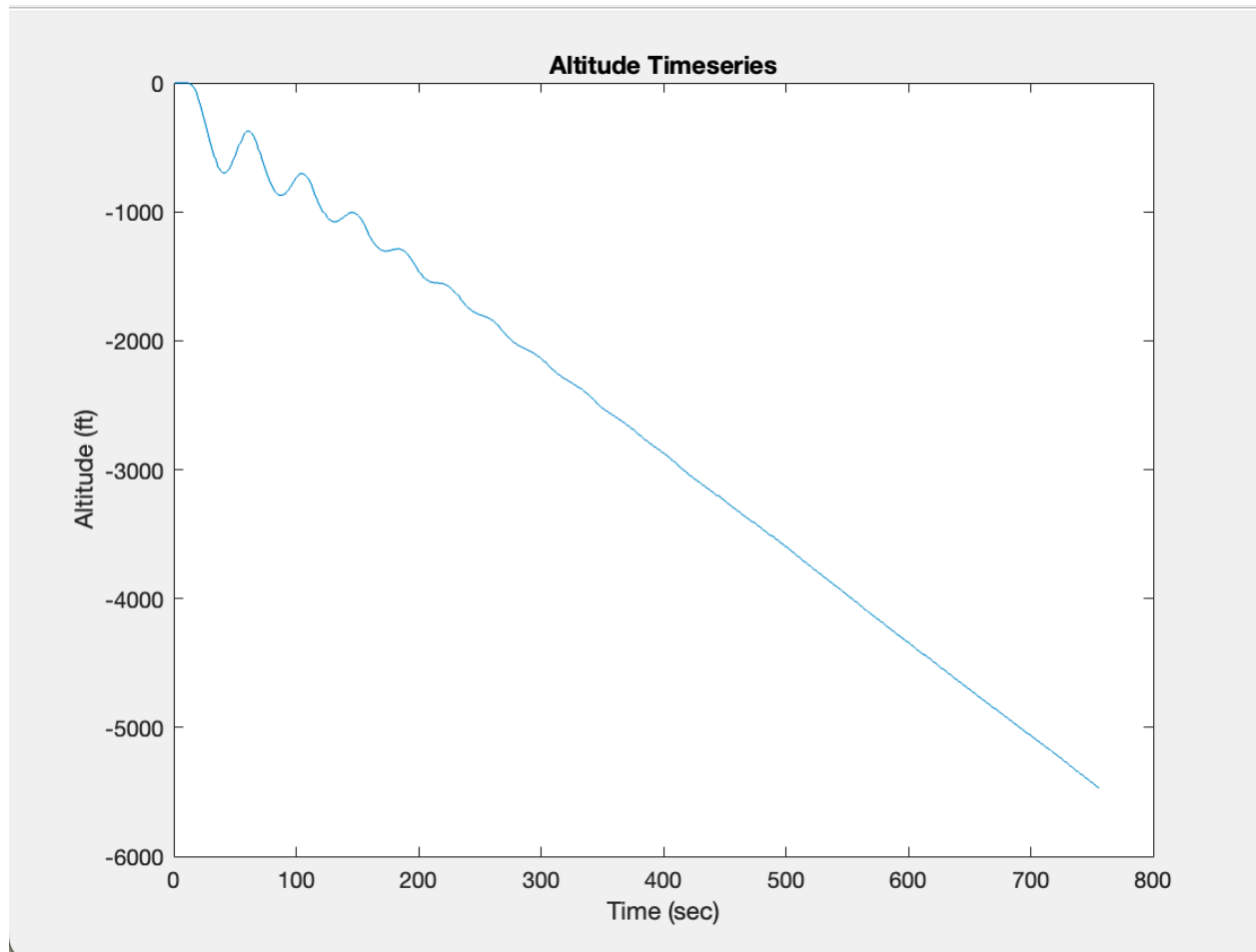


Figure 23: Altitude Timeseries – Sim 1

As expected, the steady state of altitude demonstrates the nature of flight when there is no thrust and a constant velocity, aircrafts lose altitude over time as they glide. The graph demonstrates the aircraft loses altitude linearly as a function of time, a result of gliding.

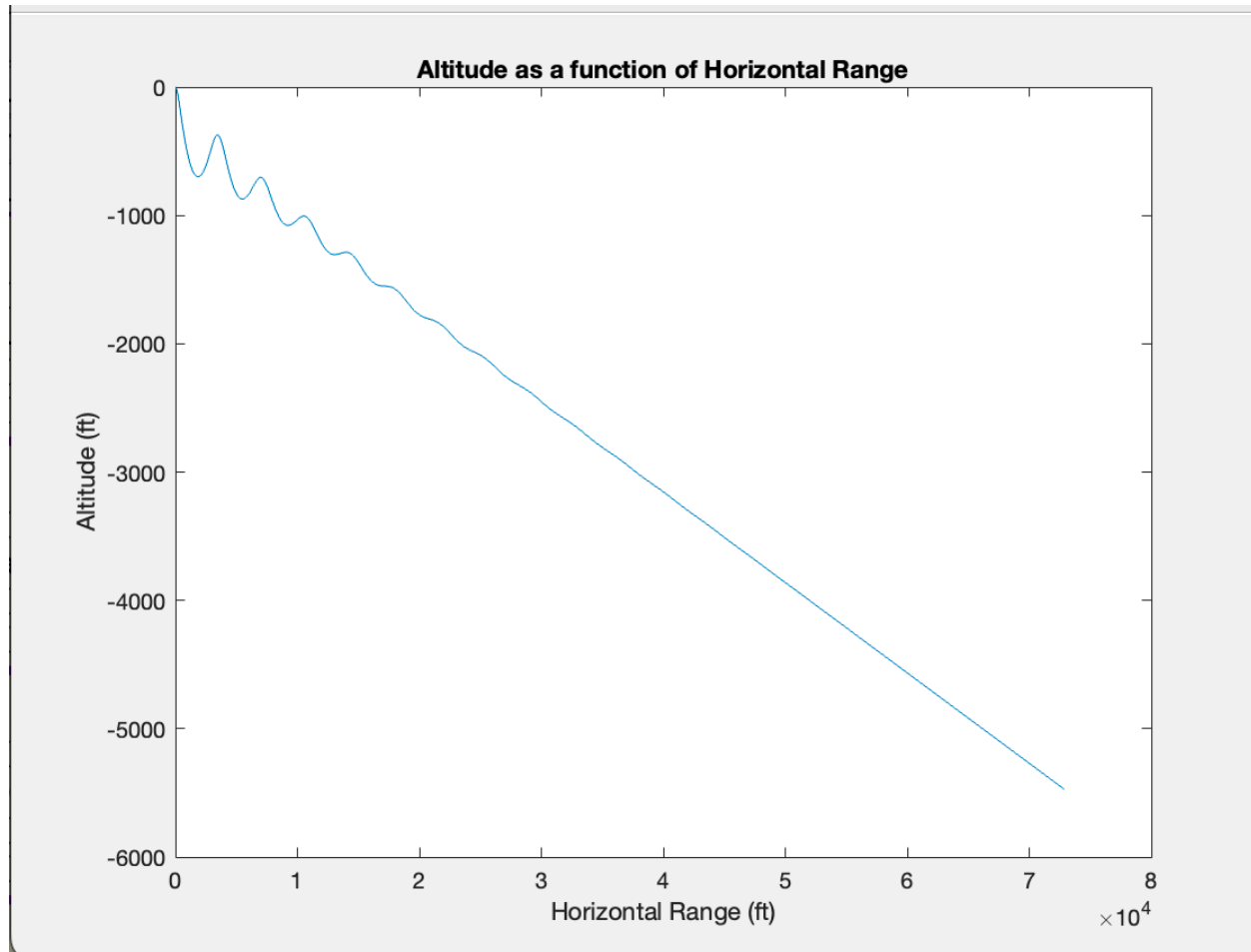


Figure 24: Altitude as a function of Horizontal Distance – Sim 1

Again, aircrafts lose altitude as they glide, this is true with respect to time and with respect to the horizontal range of the aircraft, therefore figure 24 demonstrates expected results. The graph shows that the aircraft loses altitude linearly with respect to horizontal range, a result of gliding.

To verify these results, equation (47) from the “Final Project Assignment” document will be used:

$$\begin{aligned}
 (V)^2 &= -\frac{2W\sin(\gamma)}{\rho S C_D(\alpha)} \\
 &= -\frac{2(2650)\sin(-0.0702)}{(2.377 \times 10^{-3})(174)(0.0223 + 0.0554(0.307 + 4.41(0.0653))^2)} \\
 &= 21377.88 \text{ ft/sec}^2 \\
 V &= 146.212 \approx 146.6.
 \end{aligned}$$

This validates the values of V , α , & γ therefore the values of altitude and horizontal range are also validated. It is clear the model provided an accurate and precise simulation for simulation 1.

Simulation 2:

The second simulation is horizontal level flight with 100 hp of throttle, $hp = 100$. For a horizontal level flight, the flight path angle is 0, $\gamma = 0$. In order to simulate this properly, the correct value of el must be found, this is possible by substitution different values into el until $\gamma = 0$. A hint was provided that the correct value for el was between 0.0270 and 0.0280. The values from 0.0270 to 0.0280 were tested until the graph of γ leveled to 0. Select test cases are shown below.

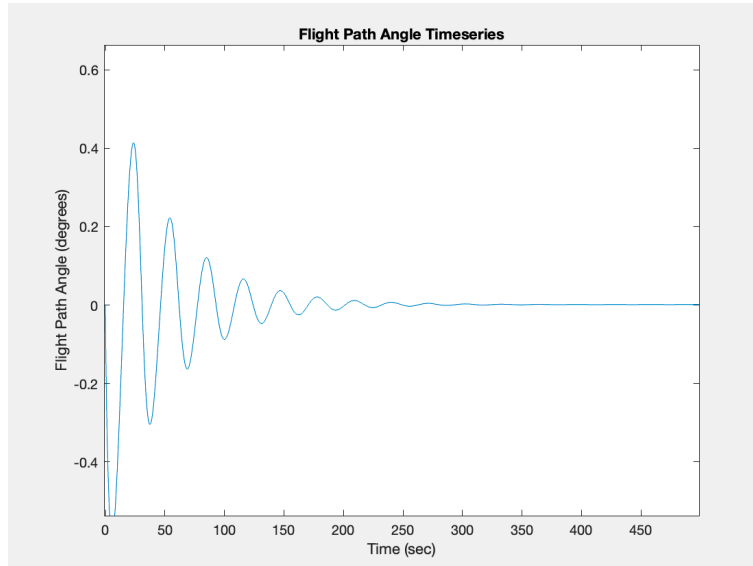


Figure 25: Test 1: $el = 0.0270$, $\gamma \approx 0.00102$

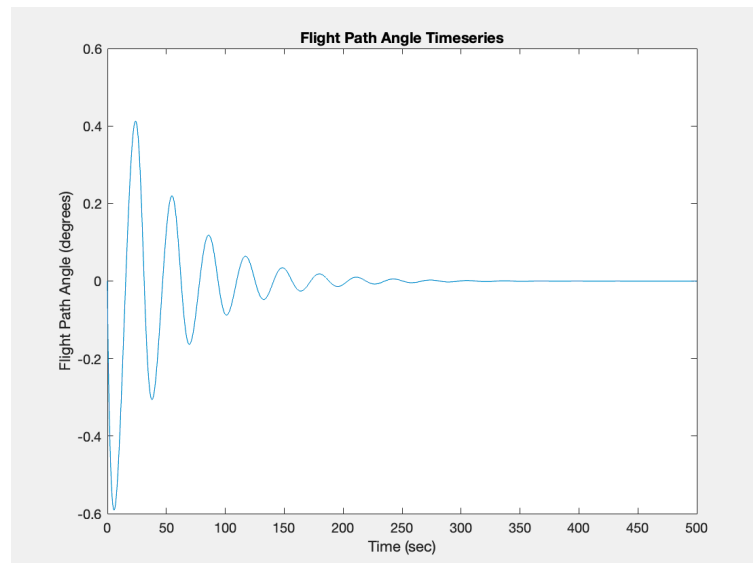


Figure 26: Test 2: $el = 0.0280$, $\gamma \approx -0.0004$

Clearly, the correct value of el is much closer to 0.0280.

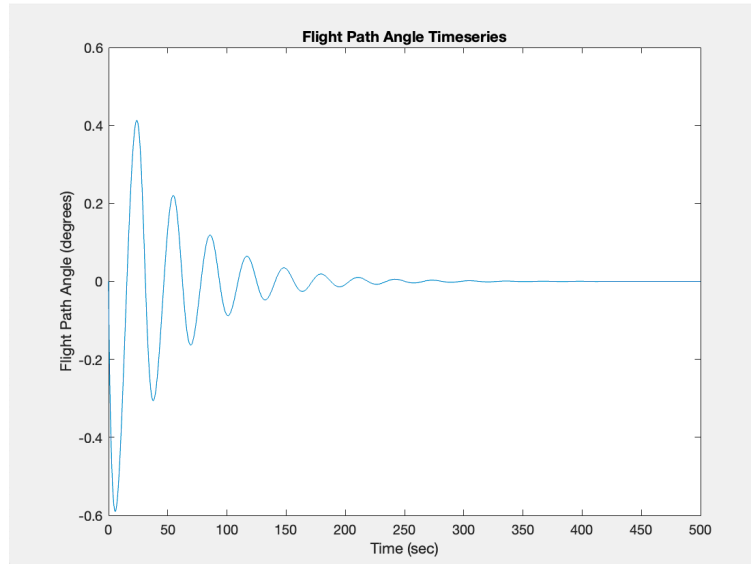


Figure 27: $el = 0.0278$, $\gamma \approx 4.24 \times 10^{-5}$

This value of $el = 0.0278$ provides a flight path angle closest to 0 compared to the other values of el in this range, therefore $el = 0.0278$ will be the value used.

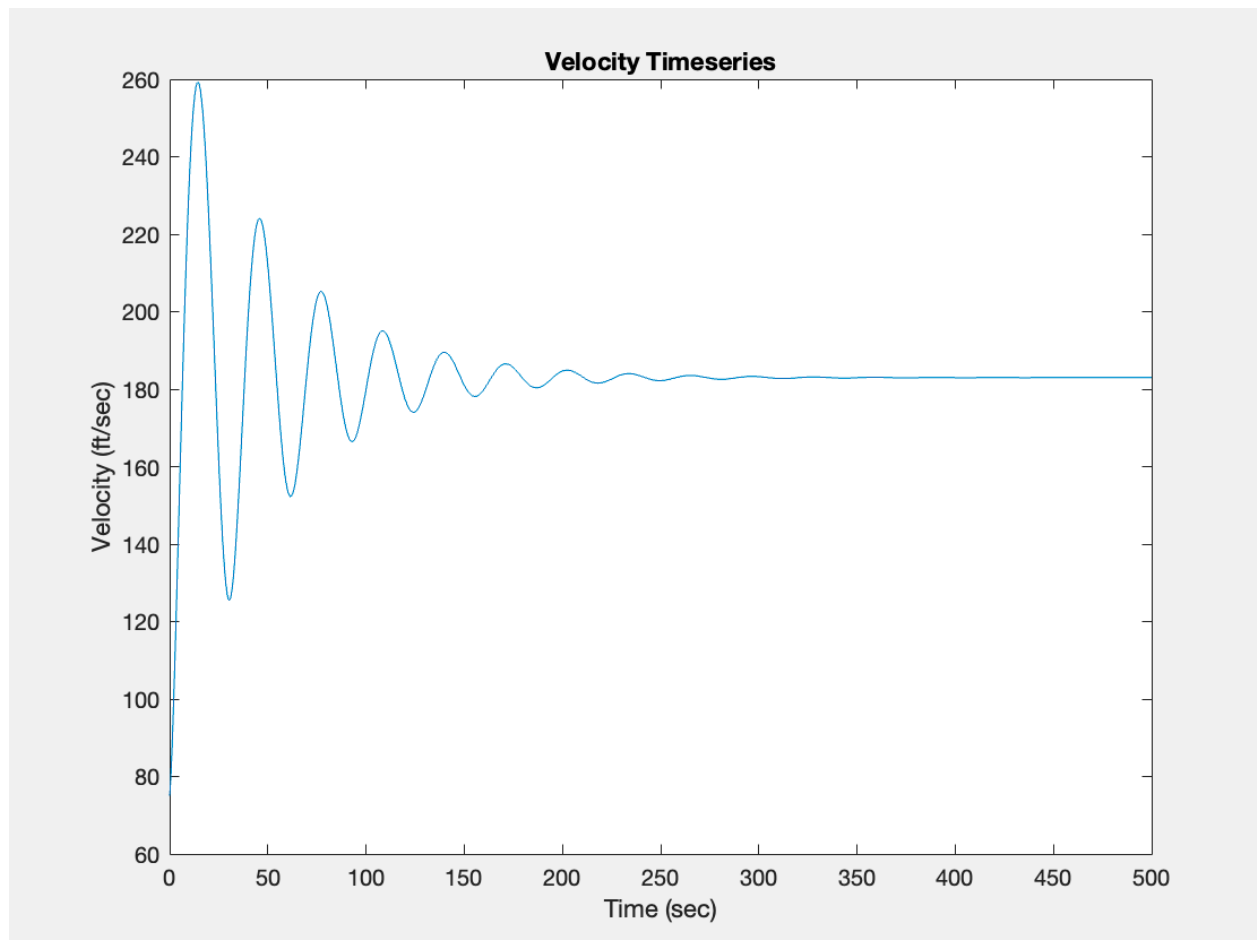


Figure 28: Velocity Timeseries – Sim 2

It can be seen from this simulation that the steady state velocity is higher than that of simulation 1, this is expected since this model has a thrust value. The velocity for this simulation is $V \approx 183 \text{ ft/sec}$.

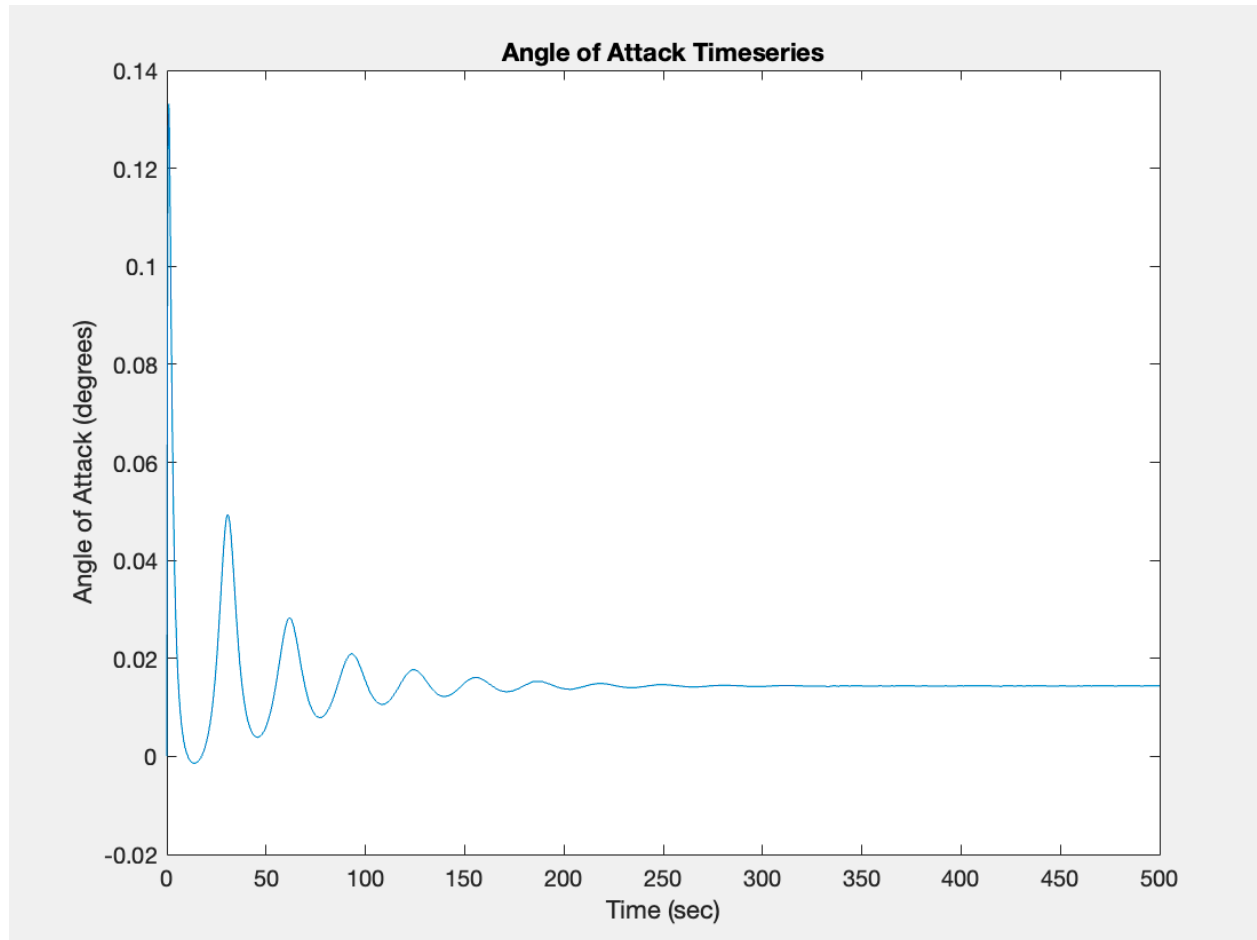


Figure 29: Angle of Attack Timeseries – Sim 2

The resultant angle of attack reaches a steady state value of $\alpha = 0.0144 \text{ rad} \approx 0.825^\circ$. This is expected for a horizontal level flight because when thrust is present, if $\alpha > 0$ the aircraft would no longer be flying horizontally level.

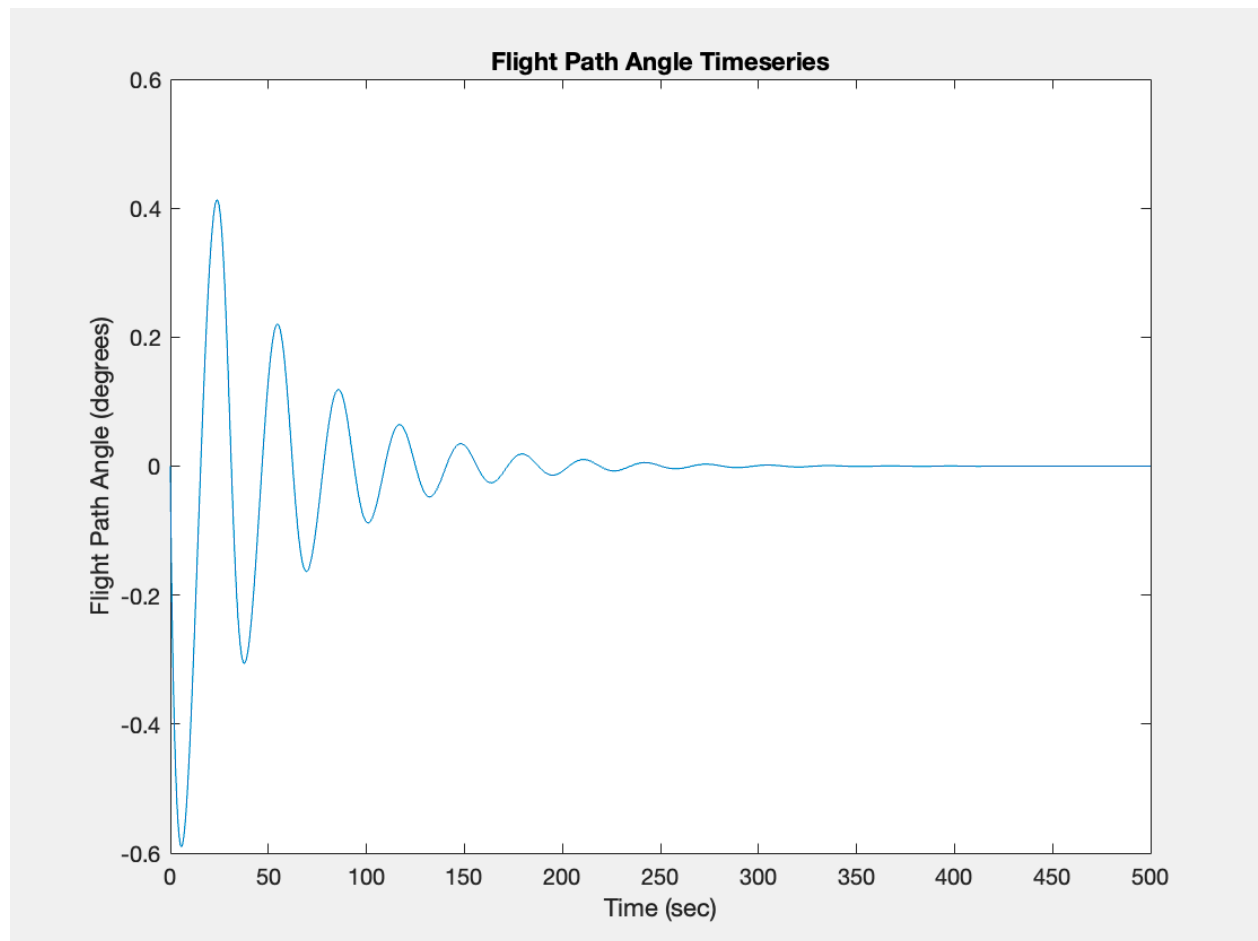


Figure 30: Flight Path Angle Timeseries – Sim 2

The flight path angle was made to be equal to zero since this is the desired simulation, a flight path angle of 0 is required for horizontal level flight, therefore this simulation is expected.

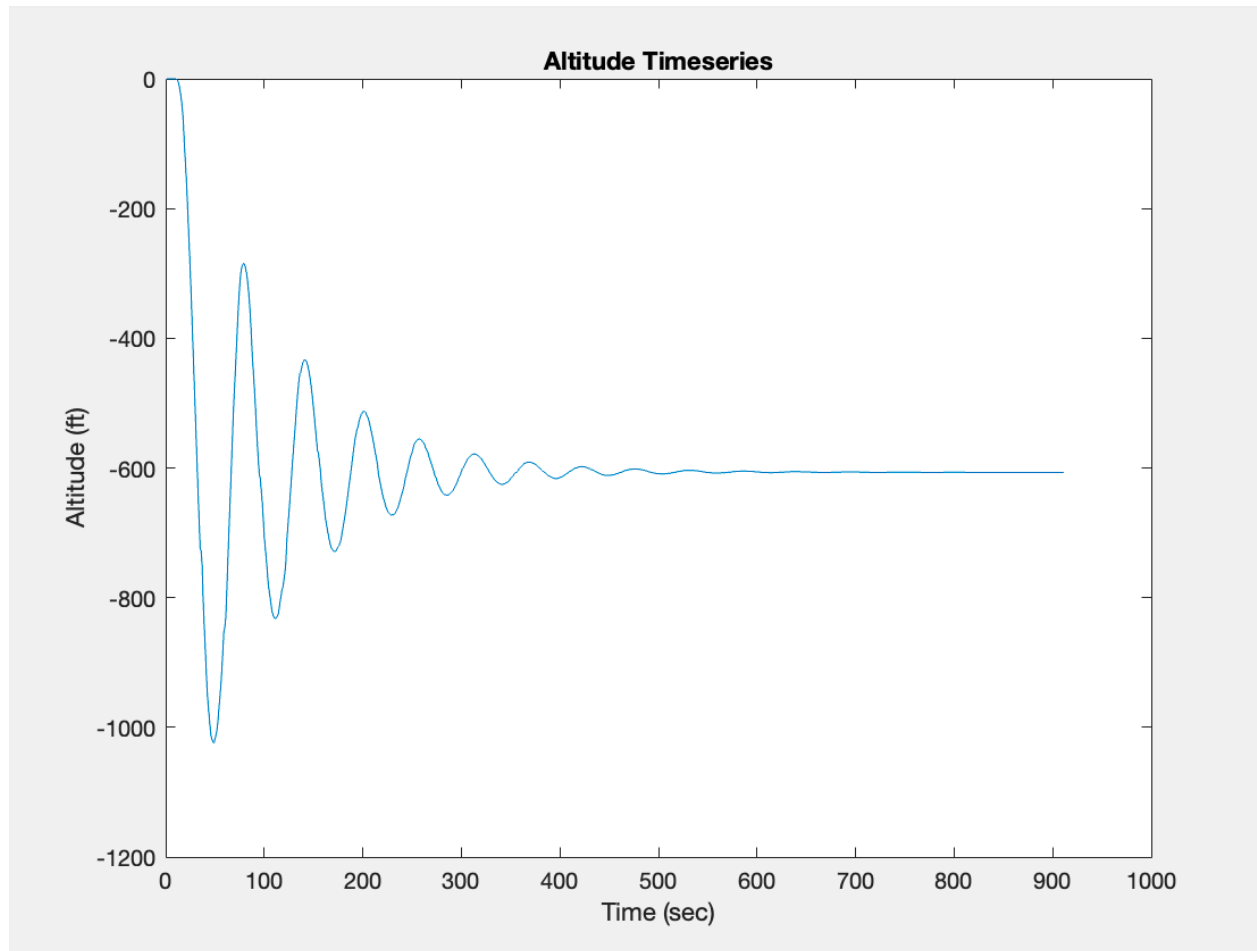


Figure 31: Altitude Timeseries – Sim 2

This simulation demonstrates the constant altitude level the flight is flying at during its horizontal level flight. A constant altitude is expected for such a flight.

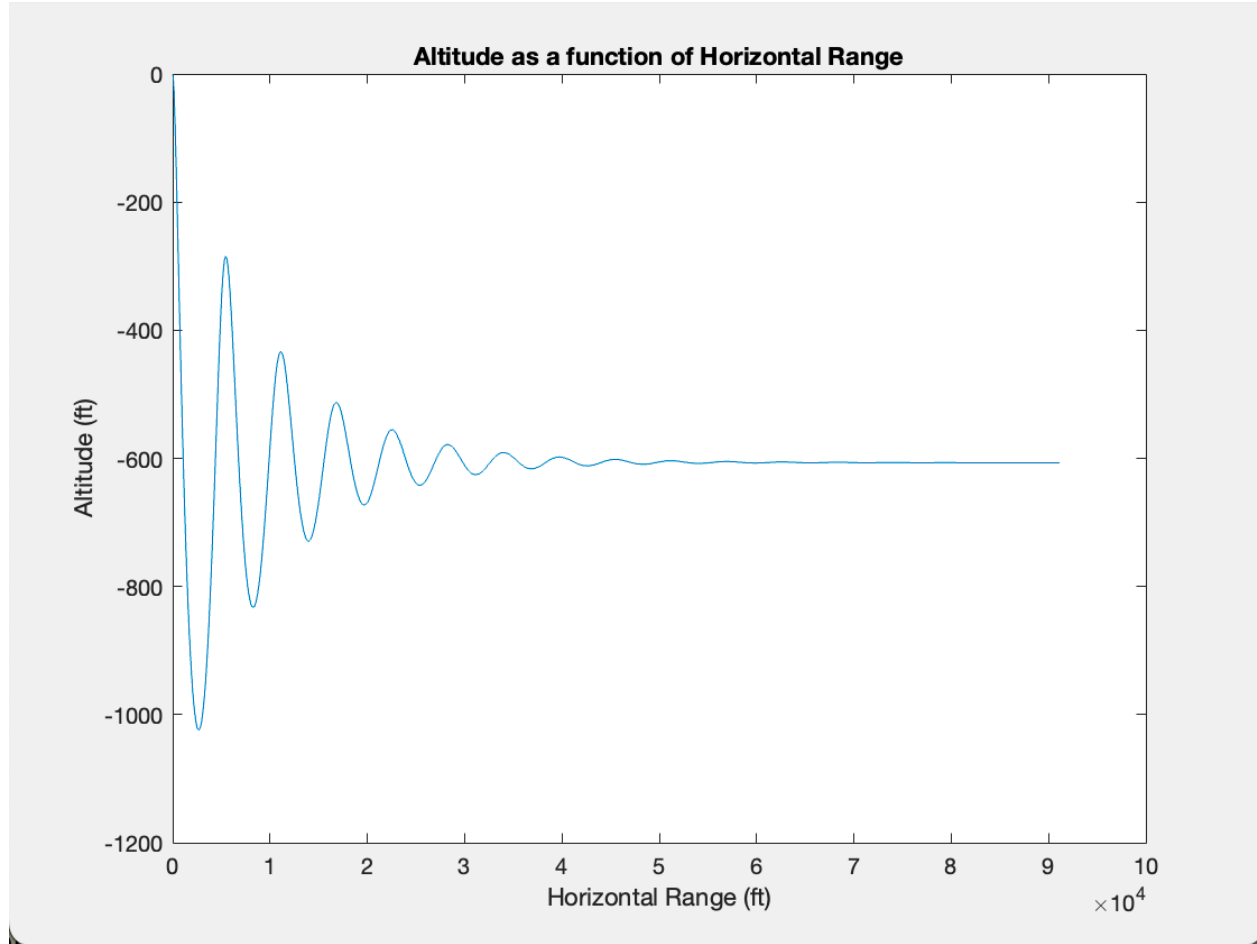


Figure 32: Altitude as a function of Horizontal Range – Sim 2

The altitude as a function of horizontal range is very similar to the that of the altitude timeseries, as expected. Also, the simulation demonstrates that the plane flies at a constant altitude, equal to that of the height from the altitude timeseries.

To verify these results, equation (53) will be used.

$$\frac{C_L}{C_D} \approx \frac{W}{T}$$

$$\frac{C_L}{C_D} = \frac{0.307 + 4.41(0.0144) + 0.43(0.0278)}{0.0233 + 0.0554(0.307 + 4.41(0.0144) + 0.43(0.0278))^2} = 12.179$$

$$\frac{W}{T} = \frac{2650}{550 * 100 * \frac{0.7}{183}} = 12.596$$

Therefore, the simulations are accurate and precise as equation (53) is proven. This validates the values of V , α , & γ therefore the values of altitude and horizontal range are also validated.

Conclusion:

The Simulink model outlined in figures 1-19 provided an accurate model for both sets of trim conditions for each simulation, 1 and 2.

Simulation 1 was a gliding scenario with no thrust and therefore the aerodynamic forces exactly balance those of the aircraft and the Cessna glides with a constant velocity as it linearly loses altitude over time and over its horizontal range with a constant flight angle and angle of attack.

Simulation 2 was a scenario with 100 hp of throttle providing thrust flying horizontally where the weight is equal to the lift and the thrust is equal to the drag. For this flight, the aircraft is still in equilibrium and therefore the aircraft flies with constant velocity. In addition, in order to remain on the same level, the aircraft has minimal angle

of attack, the flight path is again constant, and the aircraft remains at a constant altitude with respect to time and horizontal distance.

Appendix: MATLAB Code

```
%% AAE301 Final Project

%% Given Constants
g = 32.2; % gravity (ft/sec^2)
rho = 0.002377; % air density (slug/ft^3)
W = 2650; % weight (lb)
m = W/g; % mass (slug)
S = 174; % wing parameter (ft^2)
J2 = 1346; % moment of inertia (slug ft^2)
c = 4.9; % wing parameter 2 (ft)
C_Lo = 0.307; %
C_La = 4.41; % (rad^-1)
C_Lel = 0.43; % (rad^-1)
C_Ladot = 1.7; % (rad^-1)
C_Lq = 3.9; % (rad^-1)
C_Mo = 0.04; %
C_Ma = -0.613; % (rad^-1)
C_Mel = -1.122; % (rad^-1)
C_Madot = -7.27; % (rad^-1)
C_Mq = -12.4; % (rad^-1)
niu = 0.7; % coefficient
eps = 0; %
e_T = 0; %
C_DM = 0.0223; %
k = 0.0554; %
C_LDM = 0; %
q = 0;
x = 0;
el = 0.0278;
th = 100;
tspan = 500;

V = out.V;
h = out.h;
p = out.p;
alpha = out.alpha;
gamma = out.gamma;

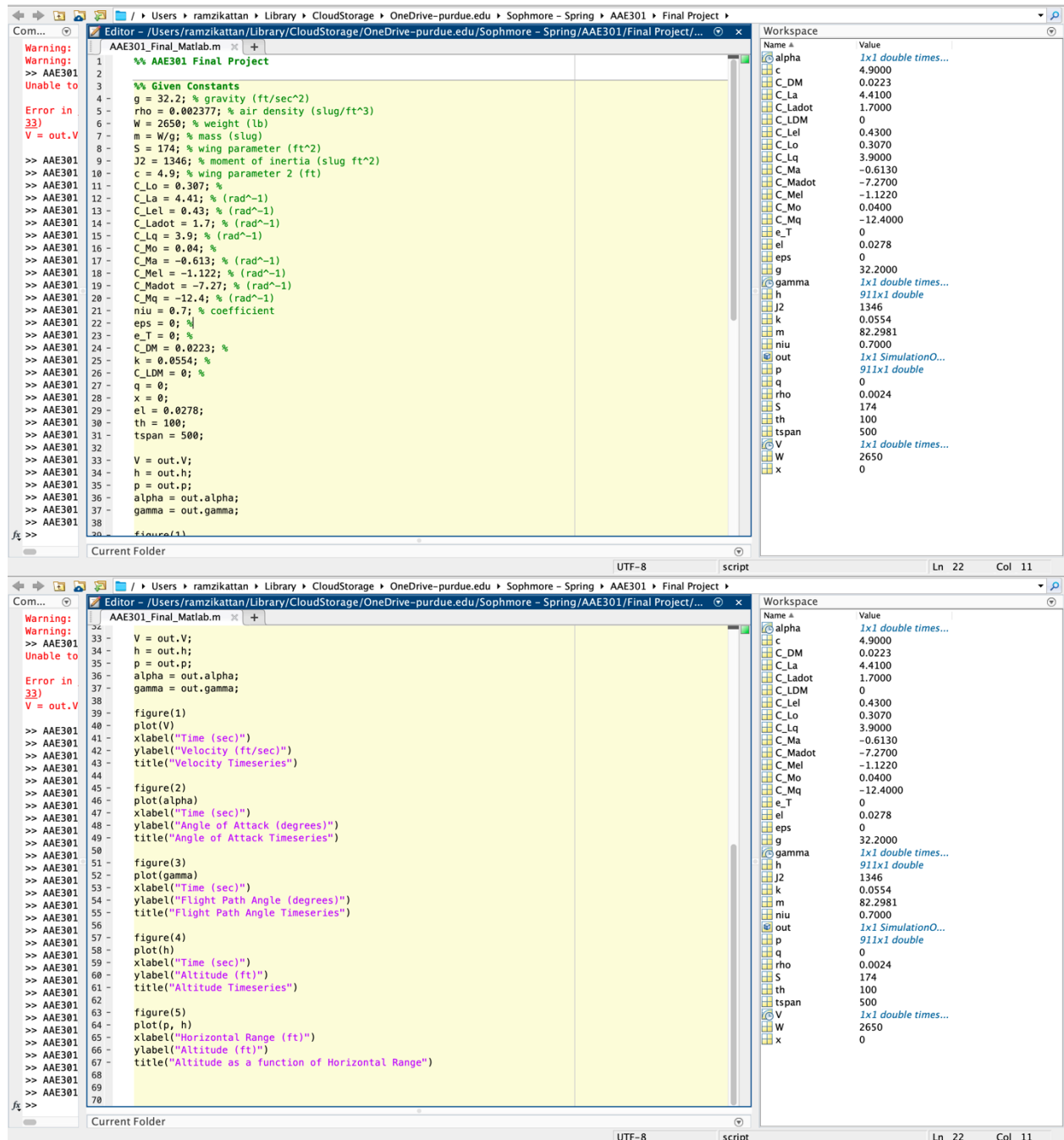
figure(1)
plot(V)
xlabel("Time (sec)")
ylabel("Velocity (ft/sec)")
title("Velocity Timeseries")

figure(2)
plot(alpha)
xlabel("Time (sec)")
ylabel("Angle of Attack (degrees)")
title("Angle of Attack Timeseries")
```

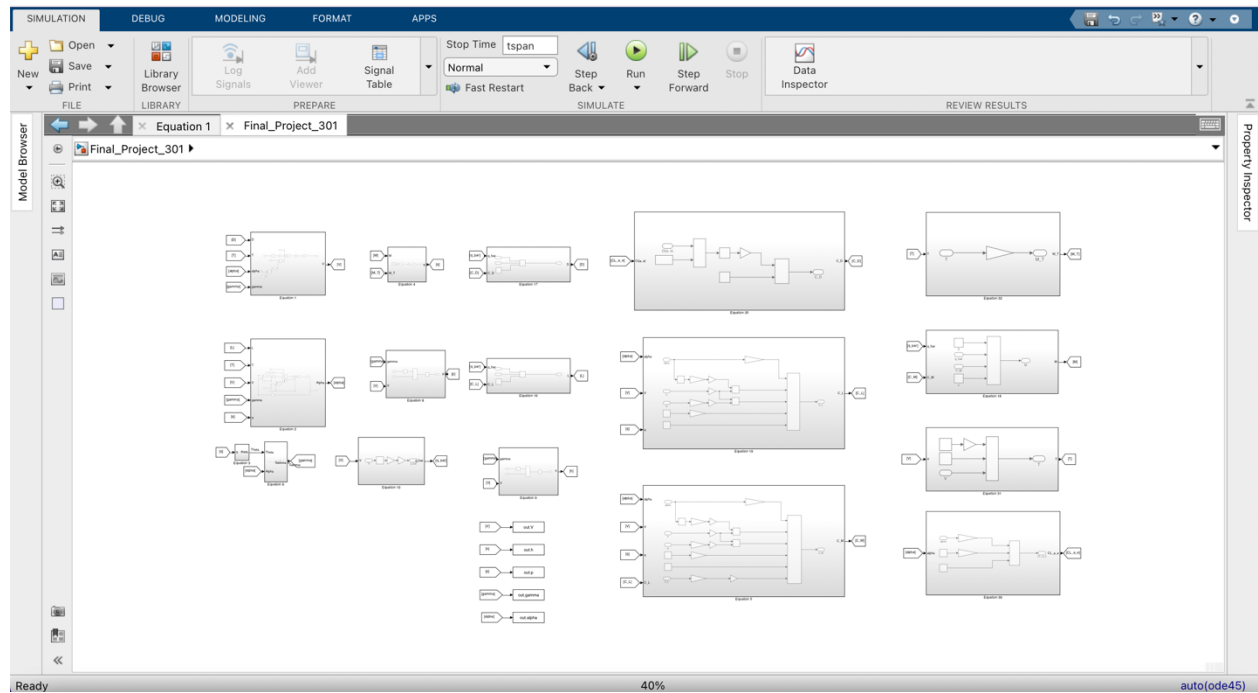
```
figure(3)
plot(gamma)
xlabel("Time (sec)")
ylabel("Flight Path Angle (degrees)")
title("Flight Path Angle Timeseries")
```

```
figure(4)
plot(h)
xlabel("Time (sec)")
ylabel("Altitude (ft)")
title("Altitude Timeseries")
```

```
figure(5)
plot(p, h)
xlabel("Horizontal Range (ft)")
ylabel("Altitude (ft)")
title("Altitude as a function of Horizontal Range")
```



Simulink Screenshot



Links

[Simulink File](#)
[MATLAB Code](#)