

We have the nonlinear state space representation,

$$\dot{X}(t) = A(t)X(t) + B(t)u(t) + E(t)$$

where,

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} \text{ and } X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \text{ where our input } u = [u_1] = [F]$$

In other words, we have,

$$\dot{X} = f(X, u)$$

In other words, we have,

$$\dot{x}_1 = f_1(x_1, x_2, x_3, x_4, x_5, x_6, F)$$

$$\dot{x}_2 = f_2(x_1, x_2, x_3, x_4, x_5, x_6, F)$$

$$\dot{x}_3 = f_3(x_1, x_2, x_3, x_4, x_5, x_6, F)$$

$$\dot{x}_4 = f_4(x_1, x_2, x_3, x_4, x_5, x_6, F)$$

$$\dot{x}_5 = f_5(x_1, x_2, x_3, x_4, x_5, x_6, F)$$

$$\dot{x}_6 = f_6(x_1, x_2, x_3, x_4, x_5, x_6, F)$$

where all these functions are nonlinear functions of time. Remembering the multivariate version of the Taylor Series Expansion, for example for a simple two variable case, linearizing about  $\bar{a}$  and  $\bar{b}$  for the first order Taylor Polynomial,

$$f(x, y) \approx f(\bar{a}, \bar{b}) + \left. \frac{\partial f}{\partial x} \right|_{x=\bar{a}, y=\bar{b}} * (x - \bar{a}) + \left. \frac{\partial f}{\partial y} \right|_{x=\bar{a}, y=\bar{b}} * (y - \bar{b})$$

We want to linearize about the equilibrium point,

$$X_e = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \\ \bar{x}_5 \\ \bar{x}_6 \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{\dot{x}} \\ \bar{\theta}_1 \\ \dot{\bar{\theta}}_1 \\ \bar{\theta}_2 \\ \dot{\bar{\theta}}_2 \end{bmatrix} = \begin{bmatrix} x_e \\ \dot{x}_e \\ \theta_{1e} \\ \dot{\theta}_{1e} \\ \theta_{2e} \\ \dot{\theta}_{2e} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and } u_e = [\bar{u}_1] = [F_e] = 0$$

For our situation, taking a multivariate first order Taylor Series approximation (but with six variables, not just two as shown above), we get:

$$\begin{aligned}
\dot{x}_1 &\approx f_1(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6, F) + \frac{\partial f_1}{\partial x_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_1 - \bar{x}_1) + \frac{\partial f_1}{\partial x_2} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_2 - \bar{x}_2) + \dots + \frac{\partial f_1}{\partial x_6} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_6 - \bar{x}_6) + \frac{\partial f_1}{\partial u_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (u_1 - \bar{u}_1) \\
\dot{x}_2 &\approx f_2(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6, F) + \frac{\partial f_2}{\partial x_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_1 - \bar{x}_1) + \frac{\partial f_2}{\partial x_2} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_2 - \bar{x}_2) + \dots + \frac{\partial f_2}{\partial x_6} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_6 - \bar{x}_6) + \frac{\partial f_2}{\partial u_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (u_1 - \bar{u}_1) \\
\dot{x}_3 &\approx f_3(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6, F) + \frac{\partial f_3}{\partial x_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_1 - \bar{x}_1) + \frac{\partial f_3}{\partial x_2} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_2 - \bar{x}_2) + \dots + \frac{\partial f_3}{\partial x_6} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_6 - \bar{x}_6) + \frac{\partial f_3}{\partial u_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (u_1 - \bar{u}_1) \\
\dot{x}_4 &\approx f_4(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6, F) + \frac{\partial f_4}{\partial x_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_1 - \bar{x}_1) + \frac{\partial f_4}{\partial x_2} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_2 - \bar{x}_2) + \dots + \frac{\partial f_4}{\partial x_6} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_6 - \bar{x}_6) + \frac{\partial f_4}{\partial u_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (u_1 - \bar{u}_1) \\
\dot{x}_5 &\approx f_5(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6, F) + \frac{\partial f_5}{\partial x_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_1 - \bar{x}_1) + \frac{\partial f_5}{\partial x_2} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_2 - \bar{x}_2) + \dots + \frac{\partial f_5}{\partial x_6} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_6 - \bar{x}_6) + \frac{\partial f_5}{\partial u_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (u_1 - \bar{u}_1) \\
\dot{x}_6 &\approx f_6(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6, F) + \frac{\partial f_6}{\partial x_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_1 - \bar{x}_1) + \frac{\partial f_6}{\partial x_2} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_2 - \bar{x}_2) + \dots + \frac{\partial f_6}{\partial x_6} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (x_6 - \bar{x}_6) + \frac{\partial f_6}{\partial u_1} \Big|_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u=\bar{u}_1}} * (u_1 - \bar{u}_1)
\end{aligned}$$

This approximation holds well only when the difference between the state and equilibrium point is small, for example,  $(x_1 - \bar{x}_1)$  is small, because we have discarded all Higher Order Terms (H.O.T.). The HOT containing, for example say,  $(x_1 - \bar{x}_1)^2$ , or  $(x_1 - \bar{x}_1)^5$ , are all very small if  $(x_1 - \bar{x}_1)$  is very small, hence the approximation that the HOT are zero holds well only if this is the case. Note the \* symbol above simply denotes multiplication, and the “bar” symbol denotes each partial should be evaluated at the equilibrium state. We can write this more compactly using matrices, (note we moved the constant term in the front of the equation further back):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} \approx \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} & \frac{\partial f_2}{\partial x_6} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} & \frac{\partial f_3}{\partial x_6} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} & \frac{\partial f_4}{\partial x_5} & \frac{\partial f_4}{\partial x_6} \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_3} & \frac{\partial f_5}{\partial x_4} & \frac{\partial f_5}{\partial x_5} & \frac{\partial f_5}{\partial x_6} \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_4} & \frac{\partial f_6}{\partial x_5} & \frac{\partial f_6}{\partial x_6} \end{bmatrix}_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u_1=\bar{u}_1}} \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \\ x_3 - \bar{x}_3 \\ x_4 - \bar{x}_4 \\ x_5 - \bar{x}_5 \\ x_6 - \bar{x}_6 \end{bmatrix} + \begin{bmatrix} f_1(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6) \\ f_2(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6) \\ f_3(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6) \\ f_4(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6) \\ f_5(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6) \\ f_6(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u_1} \\ \frac{\partial f_2}{\partial u_1} \\ \frac{\partial f_3}{\partial u_1} \\ \frac{\partial f_4}{\partial u_1} \\ \frac{\partial f_5}{\partial u_1} \\ \frac{\partial f_6}{\partial u_1} \end{bmatrix}_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u_1=\bar{u}_1}} [u_1 - \bar{u}_1]$$

In the above equations we already know that for example  $f_1(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_6)$  equals the derivative of our state evaluated at the equilibrium point, or  $\dot{x}_1$ . Making this substitution, we get:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} \approx \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} & \frac{\partial f_2}{\partial x_6} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} & \frac{\partial f_3}{\partial x_6} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} & \frac{\partial f_4}{\partial x_5} & \frac{\partial f_4}{\partial x_6} \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_3} & \frac{\partial f_5}{\partial x_4} & \frac{\partial f_5}{\partial x_5} & \frac{\partial f_5}{\partial x_6} \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_4} & \frac{\partial f_6}{\partial x_5} & \frac{\partial f_6}{\partial x_6} \end{bmatrix}_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u_1=\bar{u}_1}} \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \\ x_3 - \bar{x}_3 \\ x_4 - \bar{x}_4 \\ x_5 - \bar{x}_5 \\ x_6 - \bar{x}_6 \end{bmatrix} + \begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \\ \dot{\bar{x}}_4 \\ \dot{\bar{x}}_5 \\ \dot{\bar{x}}_6 \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u_1} \\ \frac{\partial f_2}{\partial u_1} \\ \frac{\partial f_3}{\partial u_1} \\ \frac{\partial f_4}{\partial u_1} \\ \frac{\partial f_5}{\partial u_1} \\ \frac{\partial f_6}{\partial u_1} \end{bmatrix}_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u_1=\bar{u}_1}} [u_1 - \bar{u}_1]$$

Well, we know that by definition the equilibrium point should not change once it's reached there, in other words, the rate of change at the equilibrium point should be zero. Therefore, the entire term goes to zero.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} \approx \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} & \frac{\partial f_2}{\partial x_6} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} & \frac{\partial f_3}{\partial x_6} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} & \frac{\partial f_4}{\partial x_5} & \frac{\partial f_4}{\partial x_6} \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_3} & \frac{\partial f_5}{\partial x_4} & \frac{\partial f_5}{\partial x_5} & \frac{\partial f_5}{\partial x_6} \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_4} & \frac{\partial f_6}{\partial x_5} & \frac{\partial f_6}{\partial x_6} \end{bmatrix}_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u_1=\bar{u}_1}} \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \\ x_3 - \bar{x}_3 \\ x_4 - \bar{x}_4 \\ x_5 - \bar{x}_5 \\ x_6 - \bar{x}_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u_1} \\ \frac{\partial f_2}{\partial u_1} \\ \frac{\partial f_3}{\partial u_1} \\ \frac{\partial f_4}{\partial u_1} \\ \frac{\partial f_5}{\partial u_1} \\ \frac{\partial f_6}{\partial u_1} \end{bmatrix}_{\substack{x_1=\bar{x}_1 \\ x_2=\bar{x}_2 \\ \vdots \\ x_6=\bar{x}_6 \\ u_1=\bar{u}_1}} [u_1 - \bar{u}_1]$$

Next, in our specific case, all the states at the equilibrium point we have selected are zero, as shown in our equation for  $X_e$  earlier. Making this substitution:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} \approx \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} & \frac{\partial f_2}{\partial x_6} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} & \frac{\partial f_3}{\partial x_6} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} & \frac{\partial f_4}{\partial x_5} & \frac{\partial f_4}{\partial x_6} \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_3} & \frac{\partial f_5}{\partial x_4} & \frac{\partial f_5}{\partial x_5} & \frac{\partial f_5}{\partial x_6} \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_4} & \frac{\partial f_6}{\partial x_5} & \frac{\partial f_6}{\partial x_6} \end{bmatrix} \begin{bmatrix} x_1 - 0 \\ x_2 - 0 \\ x_3 - 0 \\ x_4 - 0 \\ x_5 - 0 \\ x_6 - 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u_1} \\ \frac{\partial f_2}{\partial u_1} \\ \frac{\partial f_3}{\partial u_1} \\ \frac{\partial f_4}{\partial u_1} \\ \frac{\partial f_5}{\partial u_1} \\ \frac{\partial f_6}{\partial u_1} \end{bmatrix} [u_1 - 0]$$

$\begin{matrix} x_1 = \bar{x}_1 \\ x_2 = \bar{x}_2 \\ \vdots \\ x_6 = \bar{x}_6 \\ u_1 = \bar{u}_1 \end{matrix} \qquad \begin{matrix} x_1 = \bar{x}_1 \\ x_2 = \bar{x}_2 \\ \vdots \\ x_6 = \bar{x}_6 \\ u_1 = \bar{u}_1 \end{matrix}$

Finally, we can call these matrices, which are the Jacobian evaluated at our equilibrium point, the A and B matrices. These are constant matrices because we evaluate them at the equilibrium point. In general, linearizing about a specific equilibrium point results in constant A and B matrices and hence a Linear Time Invariant (LTI) system, whereas linearizing about a trajectory results in a Linear Time Varying (LTV) system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} \approx A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + B [u_1]$$

We have linearized the nonlinear state space representation to obtain the linear state space representation. As mentioned earlier this linear approximation holds for small differences between the state and the equilibrium point.