苏泓源 2019010448 天线 HW1

1、

$$\iiint_V (\nabla\cdot\vec{D})\,dV \equiv \iiint_V \rho\,dV \qquad \iint \vec{B}\cdot d\vec{S} = 0$$

$$\oint_l \vec{E}\cdot d\vec{l} = \iint_S \left(-\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{S}$$

$$\oint_l \vec{H}\cdot d\vec{l} = \mu_0 I$$

2、

1) $\nabla\cdot(3\hat{x}+2\hat{y}+\hat{z}) = 0$

$\nabla\times(3\hat{x}+2\hat{y}+\hat{z}) = 0$

2) $\nabla\cdot(3\hat{\rho}+2\hat{\phi}+\hat{z}) = 3/\rho$

$\nabla\times(3\hat{\rho}+2\hat{\phi}+\hat{z}) = \frac{2}{\rho}\hat{z}$

3) $\nabla\cdot(3\hat{r}+2\hat{\theta}+\hat{\phi}) = \frac{2}{\rho}(3+\cot\theta)$

$\nabla\times(3\hat{r}+2\hat{\theta}+\hat{\phi})$

$= \frac{1}{r}\cot\theta\,\hat{r} - \frac{1}{r}\hat{\theta} + \frac{2}{r}\hat{\phi}$

3. 在直角坐标下

$$\nabla\times(\nabla f) = \nabla\times\left(\frac{\partial f}{\partial x}\hat{x}+\frac{\partial f}{\partial y}\hat{y}+\frac{\partial f}{\partial z}\hat{z}\right)$$

$$=\left(\frac{\partial^2 f}{\partial z\partial y}-\frac{\partial^2 f}{\partial y\partial z}\right)\hat{x}+\left(\frac{\partial^2 f}{\partial x\partial z}-\frac{\partial^2 f}{\partial z\partial x}\right)\hat{y}+\left(\frac{\partial^2 f}{\partial y\partial x}-\frac{\partial^2 f}{\partial x\partial y}\right)\hat{z} = 0$$

$$\nabla\cdot(\nabla\times\vec{F}) = \nabla\cdot\left[\left(\frac{\partial f_z}{\partial y}-\frac{\partial f_y}{\partial z}\right)\hat{x}+\left(\frac{\partial f_x}{\partial z}-\frac{\partial f_z}{\partial x}\right)\hat{y}+\left(\frac{\partial f_y}{\partial x}+\frac{\partial f_x}{\partial y}\right)\hat{z}\right]$$

$$=\frac{\partial^2 f_z}{\partial y\partial x}-\frac{\partial^2 f_y}{\partial z\partial x}+\frac{\partial^2 f_x}{\partial z\partial y}-\frac{\partial^2 f_z}{\partial x\partial y}+\frac{\partial^2 f_y}{\partial x\partial z}-\frac{\partial^2 f_x}{\partial y\partial z} = 0$$

$$\nabla(\nabla\cdot\vec{F}) = \left(\frac{\partial^2 f_x}{\partial x^2}+\frac{\partial^2 f_x}{\partial y\partial x}+\frac{\partial^2 f_z}{\partial z\partial x}\right)\hat{x}+\left(\frac{\partial^2 f_x}{\partial x\partial y}+\frac{\partial^2 f_y}{\partial y^2}+\frac{\partial^2 f_z}{\partial z\partial y}\right)\hat{y}+\left(\frac{\partial^2 f_x}{\partial x\partial z}+\frac{\partial^2 f_y}{\partial y\partial z}+\frac{\partial^2 f_z}{\partial z^2}\right)\hat{z}$$

$$\nabla\times(\nabla\times\vec{F}) = \nabla\times\left[\left(\frac{\partial f_z}{\partial y}-\frac{\partial f_y}{\partial z}\right)\hat{x}+\left(\frac{\partial f_x}{\partial z}-\frac{\partial f_z}{\partial x}\right)\hat{y}+\left(\frac{\partial f_y}{\partial x}-\frac{\partial f_x}{\partial y}\right)\hat{z}\right]$$

$$=\left(\frac{\partial^2 f_y}{\partial x\partial y}+\frac{\partial^2 f_z}{\partial x\partial z}-\frac{\partial^2 f_x}{\partial y^2}-\frac{\partial^2 f_x}{\partial z^2}\right)\hat{x}+\left(\frac{\partial^2 f_z}{\partial y\partial z}+\frac{\partial^2 f_x}{\partial y\partial x}-\frac{\partial^2 f_y}{\partial z^2}-\frac{\partial^2 f_y}{\partial x^2}\right)\hat{y}$$

$$+\left(\frac{\partial^2 f_x}{\partial z\partial x}+\frac{\partial^2 f_y}{\partial z\partial y}-\frac{\partial^2 f_z}{\partial x^2}-\frac{\partial^2 f_z}{\partial y^2}\right)\hat{x}$$

$\therefore \nabla(\nabla\cdot\vec{F}) - \nabla\times(\nabla\times\vec{F})$

$= \left(\frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2} + \frac{\partial^2 f_z}{\partial z^2}\right)(\hat{x}+\hat{y}+\hat{z}) = \nabla^2\vec{F}$

4.

$\vec{\tilde{E}}(x) = E_0 e^{-j(k_x x + \frac{\lambda}{2})}\hat{z}$, $\vec{E}(x,t) = E_0 \cos(\omega t - k_x x - \frac{\lambda}{2})\hat{z}$

$\vec{\tilde{H}}(x) = \frac{E_0}{\sqrt{\mu_0/\varepsilon_0}} e^{-j(k_x x - \frac{\lambda}{2})}\hat{y}$, $\vec{H}(x,t) = \frac{E_0}{\sqrt{\mu_0/\varepsilon_0}} \cos(\omega t - k_x x + \frac{\lambda}{2})\hat{y}$