苏. 冯源 2014010448 固物 HW5

1.

$$U(r) = -\frac{\alpha}{r^m} + \frac{\beta}{r^n}$$

1) 平衡时 $\left.\frac{dU(r)}{dr}\right|_{r=r_0} = +\frac{\alpha m}{r_0^{m+1}} - \frac{n\beta}{r_0^{n+1}} = 0$

$$\Rightarrow r_0 = \left(\frac{\alpha m}{\beta n}\right)^{1/m-n}$$

$$W = -\left(-\frac{\alpha}{r_0^m} + \frac{\beta}{r_0^n}\right) = \frac{\alpha}{\left(\frac{\alpha m}{\beta n}\right)^{\frac{m}{m-n}}} - \frac{\beta}{\left(\frac{\alpha m}{\beta n}\right)^{\frac{n}{m-n}}}$$

2) 引力最大值处 $r_1$ 满足

$$\left.\frac{d^2U(r)}{dr^2}\right|_{r=r_1} = 0$$

$$\Rightarrow \frac{\alpha m(m+1)}{r_1^{m+2}} = \frac{\beta n(n+1)}{r_1^{n+2}}, \quad r_1 = \left[\frac{\alpha m(m+1)}{\beta n(n+1)}\right]^{\frac{1}{m-n}}$$

$$F_{max} = -\left.\frac{\partial U(r)}{\partial r}\right|_{r=r_1} = \frac{-m\alpha}{r_1^{m+1}} + \frac{n\beta}{r_1^{n+1}}$$

其中 $r_1 = \left[\frac{\alpha m(m+1)}{\beta n(n+1)}\right]^{\frac{1}{m-n}}$

3) $m=2, n=10$. $r_0 = \left(\frac{\alpha m}{\beta n}\right)^{\frac{1}{m-n}} = 0.3\,nm$ --①

$$W = \frac{\alpha}{\left(\frac{\alpha m}{\beta n}\right)^{\frac{m}{m-n}}} - \frac{\beta}{\left(\frac{\alpha m}{\beta n}\right)^{\frac{n}{m-n}}} = 4eV \quad \text{--②}$$

由①可得 $\left(\frac{\beta}{\alpha}\right)^{\frac{1}{8}} = 0.3\,nm$，代入②.

解得
$$\begin{cases} \alpha = 7.2\times10^{-38}\ J\cdot m^2 \\ \beta = 9.4\times10^{-115}\ J\cdot m^{10} \end{cases}$$

2. 证明

$$\oplus\ r\ \ominus\ r\ \oplus\ r\ \ominus \cdots$$

$$\frac{\alpha}{r} = 2\left[\frac{1}{r} - \frac{1}{2r} + \frac{1}{3r} - \frac{1}{4r} + \cdots\right]$$

$$\therefore \alpha = 2\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

将 $\ln(1+x)$ 在 1 附近做泰勒展开

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$$

$$\therefore \ln(1+1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

$$\therefore \alpha = 2\ln 2$$

3.

电势能

$$W_1 = -N\cdot\frac{\alpha q^2}{4\pi\varepsilon_0 R} = -N\frac{\ln 2\cdot q^2}{2\pi\varepsilon_0 R}$$

排斥势能（仅考虑相邻）

$$W_2 = 2N\cdot\frac{A}{R^n}$$

总势能

$$W = W_1 + W_2 = 2N\left[\frac{A}{R^n} - \frac{q^2\ln 2}{4\pi\varepsilon_0 R}\right]$$

4.

1) $n = \frac{1}{a^3} = 2.78\times10^{28}\ m^{-3}$

$$\therefore k_F = (3\pi^2 n)^{\frac{1}{3}} = 9.37\times10^9\ m^{-1}$$

$$E_F = \frac{\hbar^2 k_F^2}{2m} = 5.51\times10^{-19}\ J$$

2) $k = \pi/a = 9.52\times10^9\ m^{-1}$

$$\Delta k = k - k_F = 1.5\times10^8\ m^{-1}$$

思考题：

弹性强弱主要由吸引作用和排斥

作用共同决定.

当固体受拉伸，相邻原子距离增大，

吸引力起主导作用；当固体受压缩，

相邻原子距离减小，排斥力起主导作用