苏泓源, 2019010448 固物 HW2

1.

对一维: $\qquad g_k = \frac{L_x}{2\pi}$

$N(E) = 2 \cdot g_k \cdot 2k = 2 \cdot \frac{L_x}{2\pi} \cdot 2 \cdot \frac{2m^{1/2}}{\hbar} E^{1/2}$

$g_1(E) = dN(E)/dE = \frac{L_x}{\pi\hbar}\left(\frac{2m}{E}\right)^{1/2} = \frac{L}{\pi\hbar}\left(\frac{2m}{E}\right)^{1/2}$

对二维: $g_k = \frac{L_x L_y}{4\pi^2}$

$N(E) = 2 \cdot g_k \cdot \pi k^2 = 2 \cdot \frac{L_x L_y}{4\pi^2} \cdot \pi \frac{2mE}{\hbar^2}$

$g_2(E) = dN/dE = \frac{L_x L_y m}{\pi\hbar^2} = \frac{L^2 m}{\pi\hbar^2}$

2. $g_k = \frac{L_x L_y}{4\pi^2}$

$N(E) = 2 \cdot g_k \cdot \pi k^2 = \frac{L_x L_y m E}{\pi\hbar^2}$,

$dn = dN(E) = \frac{L_x L_y m}{\pi\hbar^2} dE = \frac{L^2 m}{\pi\hbar^2} dE$

$N = \int_0^{E_F^\circ} g_2(E)dE = \frac{S m E_F^\circ}{\pi\hbar^2} = N$

$\therefore E_F^\circ = \frac{\pi\hbar^2 N}{L^2 m} \qquad (S = L_x L_y)$

3. $f(E) = \frac{1}{e^{\frac{E-E_F}{K_B T}}+1}$

a) $\frac{1}{e+1}$ 　　b) $\frac{1}{e^5+1}$ 　　c) $\frac{1}{e^{10}+1}$

4. 证明: $g_k = \frac{V}{8\pi^3}$

$N(E) = 2 \cdot g_k \cdot \frac{4}{3}\pi k^3 = \frac{V}{3\pi^2\hbar^3}(2mE)^{3/2}$

$g(E) = dN/dE = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$

$\bar{u} = \int_0^{E_F^\circ} Eg(E)dE \Big/ \int_0^{E_F^\circ} g(E)dE$

$= \frac{3}{5}E_F^\circ$

5.

$g(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$

$\cdot E^\circ \qquad = \frac{2}{5} \cdot \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{5/2}$

$U_0 = N\bar{u} = \int_0^{E_F^\circ} \frac{3}{5}E_F^\circ \cdot \frac{V}{3\pi^2\hbar^2}(2mE)^{3/2}$

5.

$N = \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{\circ\,3/2}$

$U = N\bar{u} = \frac{3}{5} \cdot \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{\circ\,5/2}$

$= N \cdot \frac{3}{5}E_F^\circ = N \cdot \frac{3}{5} \cdot$

5.

$E_F^\circ = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{\frac{2}{3}}$

$E_F \approx E_F^\circ\left[1 - \frac{\pi^2}{12}\left(\frac{K_B T^2}{E_F^\circ}\right)^2\right]$

$U = \int_0^{E_F} Eg(E) = \frac{2}{5} \cdot \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{5/2} \Big|_{T \to 0}$

$\Big|_{T \to 0}$

$= \frac{2}{5}\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{\circ\,5/2} = U_0$

$= \frac{2}{5} \cdot \frac{1}{2\pi^2} \cdot \left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot \left(\frac{\hbar^2}{2m}(3\pi^2 N)^{\frac{2}{3}}\right)^{\frac{5}{2}} \cdot V^{-\frac{2}{3}}$

$P = -\left(\frac{dU}{dV}\right)_N = \frac{-1}{5\pi^2} \cdot \left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot \left(\frac{\hbar^2}{2m}\right)^{\frac{5}{2}} \cdot (3\pi^2 N)^{\frac{5}{3}} \cdot -\frac{2}{3}V^{-\frac{5}{3}}$

$= \frac{2}{3}\frac{U_0}{V}$