苏泓源 2019010448 统计信号 HW2

1.

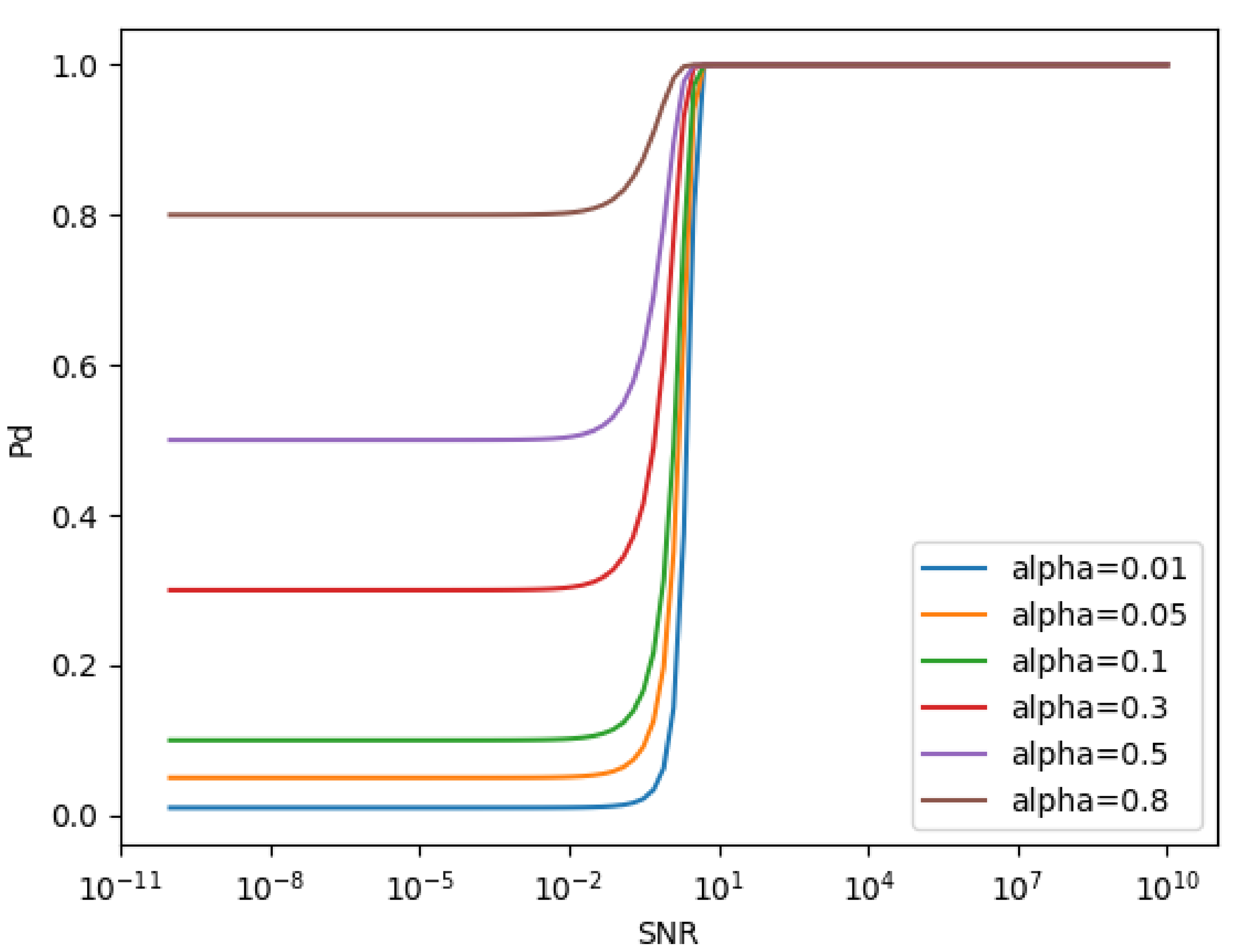
$$\lambda(x)=\frac{p_1(x)}{p_0(x)}=\exp\left\{\frac{2ax-a^2}{2\sigma^2}\right\}\underset{H_0}{\overset{H_1}{\gtrless}}\lambda_0 \Rightarrow x\underset{H_0}{\overset{H_1}{\gtrless}}\frac{a}{2}+\frac{\sigma^2}{a}\ln\lambda_0=V_T$$

$$P_F=\alpha \Rightarrow \int_{V_T}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}dx=Q\left(\frac{V_T}{\sigma}\right)=\alpha \Rightarrow V_T=\sigma Q^{-1}(\alpha)$$

∴判别规则 $x\underset{H_0}{\overset{H_1}{\gtrless}}\sigma Q^{-1}(\alpha)$

$$P_D=\int_{V_T}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-a)^2}{2\sigma^2}}dx=Q\left(\frac{V_T-a}{\sigma}\right)=Q\left(Q^{-1}(\alpha)-\frac{a}{\sigma}\right)$$

图如下所示



2.

1) $$\lambda(x)=\exp\left\{\frac{2ax-a^2}{2\sigma^2}\right\}\underset{H_0}{\overset{H_1}{\gtrless}}\frac{\xi(C_{10}-C_{00})}{(1-\xi)(C_{01}-C_{11})}=\frac{1}{11}\frac{\xi}{1-\xi}$$

$$\Rightarrow x\underset{H_0}{\overset{H_1}{\gtrless}}\frac{a}{2}+\frac{\sigma^2}{a}\ln\left(\frac{1}{11}\frac{\xi}{1-\xi}\right)=\frac{a}{2}-\frac{1}{a}\ln\left(11\cdot\frac{1-\xi}{\xi}\right)=V_T$$，其中 $\xi=P(H_0)$

$P_F=Q(V_T)$，$P_D=Q(V_T-a)$

$$\bar{C}_{min}(\xi)=C_{00}P(H_0)(1-P_F)+C_{10}P(H_0)P_F+C_{01}P(H_1)(1-P_D)+C_{11}P(H_1)P_D$$
$$=\xi[1-Q(V_T)]+10\xi Q(V_T)+100(1-\xi)[1-Q(V_T-a)]+(1-\xi)Q(V_T-a$$

极大极小方程：$C(0,x) = C(1,x)$

$$\Rightarrow 100[1-Q(V_T-a)] + Q(V_T-a) = 1-Q(V_T) + 10Q(V_T)$$

$$\Rightarrow 99 - 99Q(V_T-a) - 9Q(V_T) = 0$$

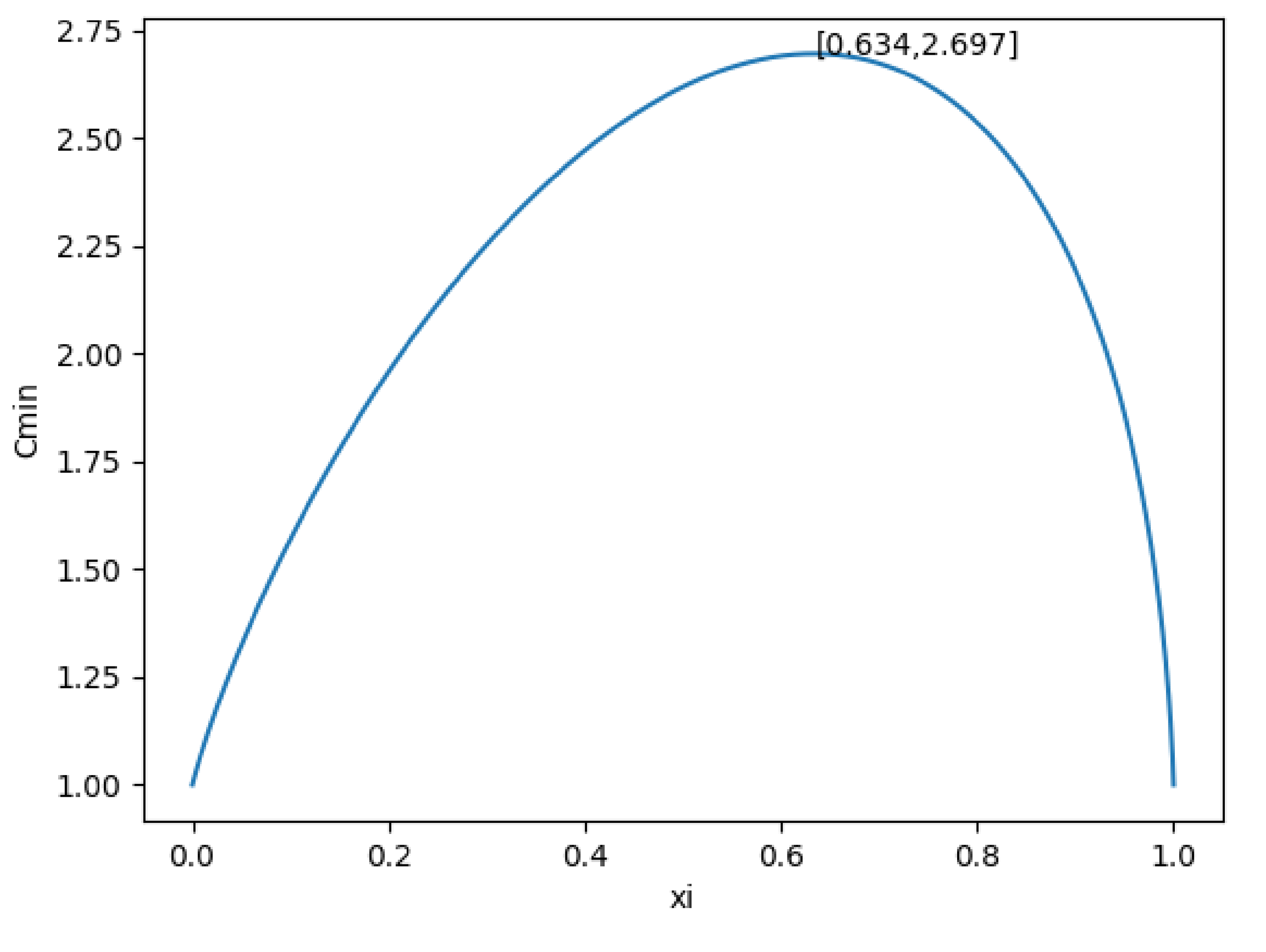
$$\Rightarrow 11Q(V_T-a) + Q(V_T) = 11$$，规则为 $x \underset{H_0}{\overset{H_1}{\gtrless}} V_T$

2) $a=3$ 时

$$11Q(V_T-3) + Q(V_T) = 11, \quad V_T = 0.883 = \frac{a}{2} - \frac{1}{a}\ln\left(11 \cdot \frac{1-\xi}{\xi}\right)$$

$\therefore \xi = \xi_0 \approx 0.634$

$\overline{C}_{min}(\xi)$ 图像如下



3.

$$p_1(\vec{r}) = \prod_{i=1}^{m} p_1(r_i) = \left(\frac{1}{\sqrt{2}\sigma_n}\right)^m \exp\left\{-\frac{\sqrt{2}\sum_{i=1}^{m}|r_i-A|}{\sigma_n}\right\}$$

$$p_0(\vec{r}) = \prod_{i=1}^{m} p_0(r_i) = \left(\frac{1}{\sqrt{2}\sigma_m}\right)^n \exp\left\{-\frac{\sqrt{2}\sum_{i=1}^{m}|r_i|}{\sigma_n}\right\}$$

$$\ln(\lambda(\vec{r})) = \ln\left(\frac{p_1(\vec{r})}{p_0(\vec{r})}\right) = \frac{\sqrt{2}}{\sigma_n}\sum_{i=1}^{m}(|r_i| - |r_i - A|)$$

$\therefore$ 判决规则 $$\frac{\sqrt{2}}{\sigma_n}\sum_{i=1}^{m}(|r_i| - |r_i+10|) \underset{H_0}{\overset{H_1}{\gtrless}} \ln\lambda_0.$$

进一步地，设有 n 个 r_i 满足 $r_i \in (-10, 0)$，k 个满足 $r_i < -10$

$$\sum_{i=1}^{m}(|r_i| - |r_i+10|) = \sum_{A<r_i<0}(A-2r_i) - kA + (m-n-k)A$$
$$= mA - 2kA - 2\sum_{A<r_i<0} r_i$$

$$\Rightarrow \sum_{i=1}^{m} r_i \mathbb{1}\{A < r_i < 0\} \underset{H_1}{\overset{H_0}{\gtrless}} \frac{m}{2}A - \sum_{i=1}^{m} A \mathbb{1}\{r_i < A\} - \frac{\sigma_n}{2\sqrt{2}} \ln \lambda_0 .$$

$$\Rightarrow \sum_{i=1}^{m} \left[r_i \mathbb{1}\{-10 < r_i < 0\} + A \mathbb{1}\{r_i < -10\} \right] \underset{H_1}{\overset{H_0}{\gtrless}} \frac{m}{2}A - \frac{\sigma_n}{2\sqrt{2}} \ln \lambda_0$$