

苏. 汪. 汪. 2019010448 天线 HW4

1.

$$\text{err} < 10\% : r/\lambda > \underline{-4.7\text{dB}}$$

$$< 1\% : r/\lambda > \underline{0.5\text{dB}}$$

2.

$$\begin{aligned}\vec{S}(x, y, z; t) &= \vec{E}(x, y, z; t) \times \vec{H}(x, y, z; t) \\ &= \text{Re}[\tilde{\vec{E}}(x, y, z)e^{j\omega t}] \times \text{Re}[\tilde{\vec{H}}(x, y, z)e^{j\omega t}]\end{aligned}$$

$$\begin{aligned}\text{Re}[\tilde{\vec{H}}(x, y, z)e^{j\omega t}] &= \frac{1}{2}[\tilde{\vec{H}}(x, y, z)e^{j\omega t} \\ &\quad + \frac{1}{2}[\tilde{\vec{H}}^*(x, y, z)e^{-j\omega t}]\end{aligned}$$

$$\therefore \vec{S}(x, y, z; t) = \frac{1}{2}\text{Re}[\tilde{\vec{E}} \times \tilde{\vec{H}}^*] + \frac{1}{2}\text{Re}[\tilde{\vec{E}} \times \tilde{\vec{H}}e^{j\omega t}]$$

$$\begin{aligned}\vec{S}_{\text{av}}(x, y, z) &= \frac{1}{T} \int_0^T \vec{S}(x, y, z; t) dt \\ &= \frac{1}{2}\text{Re}[\tilde{\vec{E}} \times \tilde{\vec{H}}^*] + 0 \\ &= \frac{1}{2}\text{Re}[\tilde{\vec{E}} \times \tilde{\vec{H}}^*]\end{aligned}$$

3.

a) 35dB

b) $10^{3.5} \approx 3162.3$

3.2

$$U = \sin\theta \sin\phi$$

$$\begin{aligned}U_{\text{av}} &= \frac{\int_{\Omega} U d\Omega}{\int_{\Omega} d\Omega} = \frac{\int_{\Omega} \sin\theta \sin\phi \sin\theta d\theta d\phi}{4\pi} \\ &= \frac{\int_0^\pi \sin\phi d\phi \int_0^\pi \sin^2\theta d\theta}{4\pi} = \frac{1}{8}\pi\end{aligned}$$

$$D = \frac{U_{\text{max}}}{U_{\text{av}}} = 8\pi \quad (\theta = \phi = 90^\circ)$$

b) $\theta = 90^\circ$ 时. $\sin\phi = \frac{1}{2} \Rightarrow \phi = 30^\circ$

$$\text{beamwidth} = 2 \cdot 30^\circ \cdot \lambda/180^\circ = \frac{1}{3}\lambda$$

$\phi = 90^\circ$ 时同理

$$\text{beamwidth} = \frac{1}{3}\lambda$$

