苏泓源 2019010448 贝叶斯 HW3

4.4

$n \to \infty$ 时，θ 会收敛到 θ_0（方差 $\to 0$）

将 $\phi = f(\theta)$ 在 θ_0 处作泰勒展开，由于

$\theta - \theta_0 \to 0$，故 $\phi = f(\theta) \approx f(\theta_0) + \frac{f'(\theta_0)}{1!}(\theta - \theta_0)$

$+ o((\theta-\theta_0)^2)$ 可以认为是线性的。

2.

$f(\log\sigma) \cdot d(\log\sigma) = f(\sigma^2) d\sigma^2$

$\therefore f(\sigma^2) = f(\log\sigma) \cdot \frac{1}{2\sigma^2}$

$\therefore \log p(\mu, \sigma^2 | Y)$

$= C - (n+2)\log\sigma - \frac{1}{2\sigma^2}[(n-1)S^2 + n(\bar{y}-\mu)^2]$

$d\log p(\mu, \sigma^2|Y) / d\mu = -\frac{n}{\sigma^2}(\mu - \bar{y})$

$d\log p(\mu, \sigma^2|Y) / d\sigma^2 = -\frac{n+2}{2\sigma^2} + \frac{(n-1)S^2}{2\sigma^4}$

$\therefore \hat{\mu} = \bar{y}, \quad \hat{\sigma}^2 = \frac{n-1}{n+2} S^2$

$d^2\log p(\mu, \sigma^2|Y) / d\mu^2 = -\frac{n}{\sigma^2}$

$d^2\log p(\mu, \sigma^2|Y) / d\mu d\sigma^2 = 0$

$d^2\log p(\mu, \sigma^2|Y) / d(\sigma^2)^2 = \frac{n+2}{2\sigma^4} - \frac{(n-1)S^2}{\sigma^6}$

$= -\frac{n+2}{2\hat{\sigma}^4}$

$\therefore p(\mu, \sigma^2|Y) \approx N\left[\begin{pmatrix} \bar{y} \\ \hat{\sigma}^2 \end{pmatrix}, \begin{pmatrix} \frac{\hat{\sigma}^2}{n} & 0 \\ 0 & \frac{2(\hat{\sigma}^2)^2}{n+2} \end{pmatrix}\right]$

$\hat{\sigma}^2 = \frac{n-1}{n+2} S^2$