Principles of Antennas

Lecture #4 Antenna parameters I

Tsinghua University

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General Schedule

课次	日期	主讲教师	教学内容	备注
1	2/24	李越	L1：天线导论	
2	3/3	李越	L2：麦克斯韦方程、均匀平面波	
3	3/10	李越	L3：辅助位函数、辐射解	
4	3/17	李越	L4：天线辐射远场、天线参数I	
5	3/24	李越	L5：天线参数II	
6	3/31	李越	L6：天线链路计算	
7	4/7	李越	L7：天线相关定理	
8	4/15	李越	L8：偶极天线	
9	4/22		期中测验	one-page note
10	4/29	李越	L9：环天线	
11	5/5	李越	停课	
12	5/12	李越	L10：槽天线、口面天线	
13	5/19	李越	L11：天线阵列I	
14	5/26	李越	L12：天线阵列II、行波天线	
15	6/2	李越	L13：微带天线	
16	6/9	李越	L14：反射面天线+总复习	
		期末考试，开卷！		

Indirect method

Electric vector/scalar potential functions

Sources $\vec{J},\rho$ → **APFs** $\vec{A},\phi$ → **Fields** $\vec{E},\vec{H}$

$$\nabla\times\vec{E} = -j\omega\vec{B} \quad\longrightarrow\quad \nabla\times(\vec{E}+j\omega\vec{A})=0$$

$$\boxed{\nabla\times(\nabla f)=0}$$

$$\nabla\cdot\vec{D}=\rho$$

$$\boxed{\vec{E}+j\omega\vec{A}=-\nabla\phi}$$

$$\nabla\times\vec{H}=\vec{J}+j\omega\vec{D}$$

$$\nabla\cdot\vec{B}=0 \quad\longrightarrow\quad \boxed{\vec{B}=\nabla\times\vec{A}}$$

$$\boxed{\nabla\cdot\vec{A}=-j\omega\mu\varepsilon\phi}$$

$$\boxed{\nabla\cdot(\nabla\times\vec{F})=0}$$

Lorentz Gauge

Solve a radiation problem

Sources

$\vec{J}, \rho$

$$\nabla^2 \vec{A} + \omega^2 \mu\varepsilon \vec{A} = -\mu \vec{J}$$

$$\nabla^2 \phi + \omega^2 \mu\varepsilon \phi = -\frac{\rho}{\varepsilon}$$

D'Alembert equations

$$\boxed{g(\vec{r}, \vec{r}\,') = \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|}}$$

Green's function

APFs

$\vec{A}, \phi$

$$\phi(\vec{r}) = \int_{\substack{\text{source}\\ \text{region}}} \frac{\rho}{\varepsilon} g(\vec{r}, \vec{r}\,') dv' \qquad \vec{A}(\vec{r}) = \int_{\substack{\text{source}\\ \text{region}}} \mu \vec{J}(\vec{r}\,') g(\vec{r}, \vec{r}\,') dv'$$

Fields

$\vec{E}, \vec{H}$

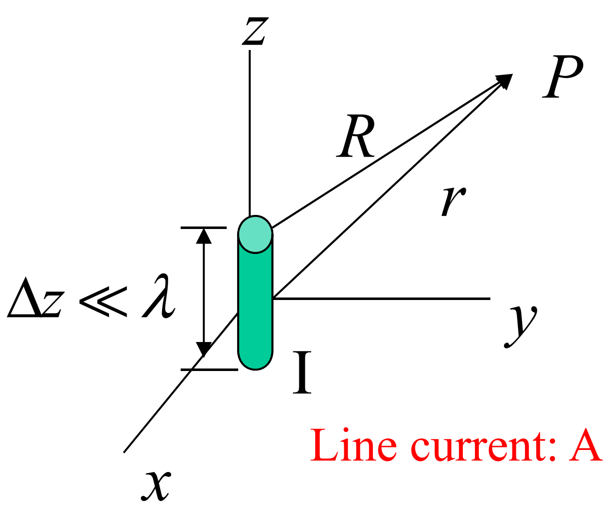
$$\vec{E} = -\nabla\phi - j\omega\vec{A} = -j\omega\vec{A} + \nabla\left(\frac{1}{j\omega\mu\varepsilon}\nabla\cdot\vec{A}\right)$$

$$\vec{H} = \frac{1}{\mu}\nabla\times\vec{A}$$

Simple method: A -> H -> E

Radiation of a Hertz dipole

Infinite short length;
Uniform distribution;
Infinite small radius;



Line current: A

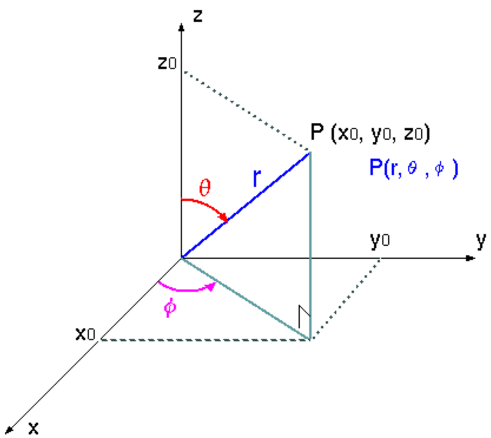
$$\nabla^2 \vec{A} + \omega^2 \mu\varepsilon \vec{A} = -\mu J \hat{z}$$

A -> H -> E

Cartesian Coordinate：for source

$$\vec{A}(x,y,z) = A_z \hat{z} = \hat{z}\int_{-\Delta z/2}^{\Delta z/2} \mu I \frac{e^{-jkr}}{4\pi r} dz = \mu I \Delta z \frac{e^{-jkr}}{4\pi r}\hat{z}$$

Integral region is determined by the source type

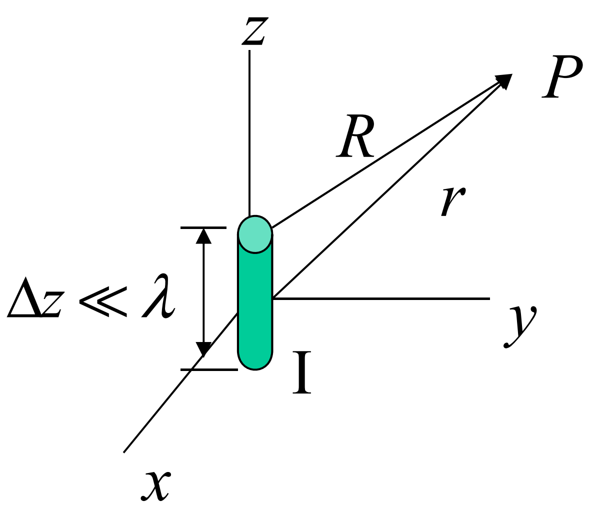


Spherical Coordinate: for radiation field

$$\hat{z} = -\sin\theta\hat{\theta} + \cos\theta\hat{r}$$

$$\vec{A}(\theta,r) = \mu I \Delta z \frac{e^{-jkr}}{4\pi r}(-\sin\theta\hat{\theta} + \cos\theta\hat{r})$$

Radiation of a Hertz dipole (2)



$$\vec{A}(\theta,r)=\mu I\Delta z\frac{e^{-jkr}}{4\pi r}(-\sin\theta\hat{\theta}+\cos\theta\hat{r})$$

A -> H -> E

$$\vec{H}=\frac{1}{\mu}\nabla\times\vec{A}=\frac{I\Delta z}{4\pi}jk\left(1+\frac{1}{jkr}\right)\frac{e^{-jkr}}{r}\sin\theta\hat{\varphi}$$

$$\vec{E}=\frac{1}{j\omega\varepsilon}\nabla\times\vec{H}$$

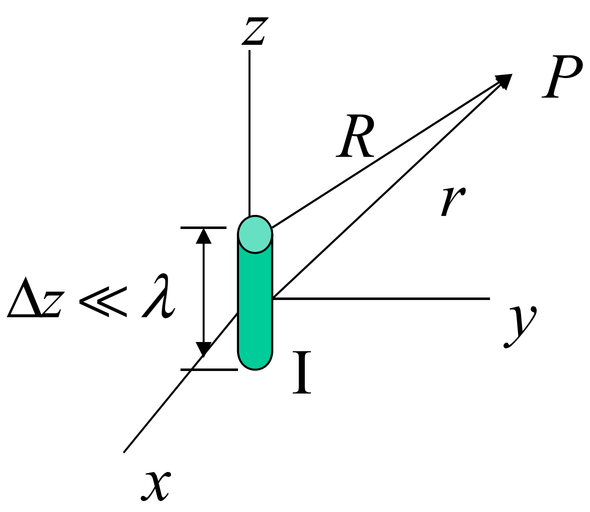
$$=\frac{I\Delta z}{4\pi}j\omega\mu\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]\frac{e^{-jkr}}{r}\sin\theta\hat{\theta}$$

$$+\frac{I\Delta z}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\left[\frac{1}{r}-j\frac{1}{kr^2}\right]\frac{e^{-jkr}}{r}\cos\theta\hat{r}$$

$$\nabla\times\vec{F}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\hat{\varphi}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi}\\ F_r & rF_\theta & r\sin\theta F_\varphi\end{vmatrix}$$

Discussion: (1) polarization component; (2) θ-distribution; (3) decay rate of r’s order.

Radiation of a Hertz dipole (3)



Near field: $kr \ll 1$

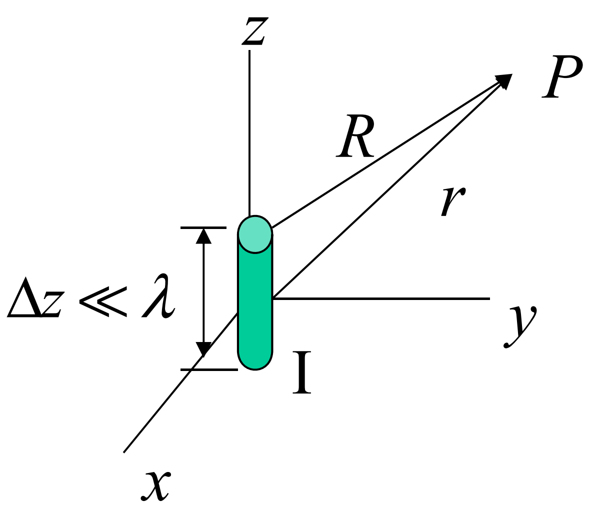
Compare the static-E of a dipole
r^3 decay rate;
two component with 2-factor;
θ-distribution;

$$H_\varphi = \frac{I\Delta z}{4\pi} jk\left(1 + \frac{1}{jkr}\right)\frac{e^{-jkr}}{r}\sin\theta$$

$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]\frac{e^{-jkr}}{r}\sin\theta$$

$$E_r = \frac{I\Delta z}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\left[\frac{1}{r} - j\frac{1}{kr^2}\right]\frac{e^{-jkr}}{r}\cos\theta$$

Far field radiation



Far field: $kr \gg 1$

$$H_\varphi = \frac{I\Delta z}{4\pi} jk\left(1 + \cancel{\frac{1}{jkr}}\right)\frac{e^{-jkr}}{r}\sin\theta$$

$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu\left[1 + \cancel{\frac{1}{jkr}} - \cancel{\frac{1}{(kr)^2}}\right]\frac{e^{-jkr}}{r}\sin\theta$$

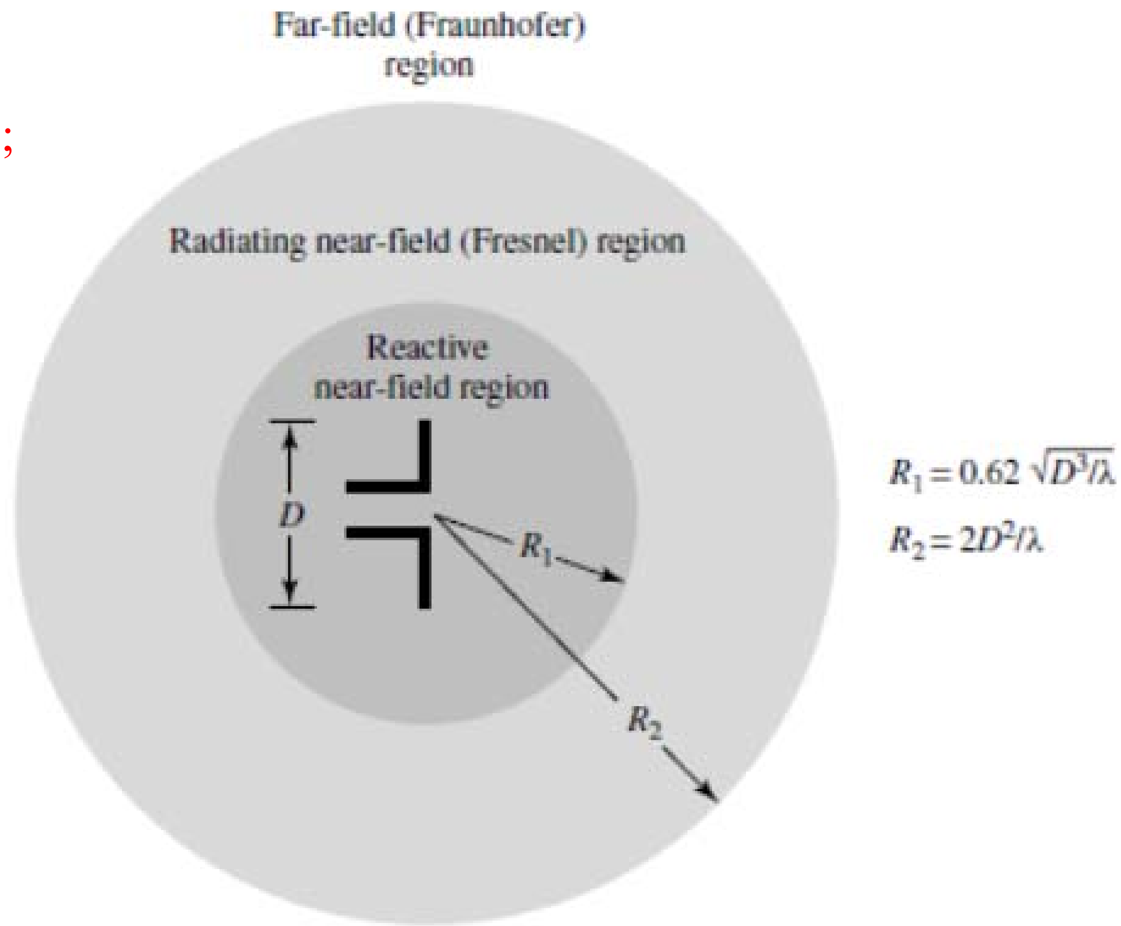
$$E_r = \frac{I\Delta z}{2\pi}\eta\left[\cancel{\frac{1}{r}} - \cancel{j\frac{1}{kr^2}}\right]\frac{e^{-jkr}}{r}\cos\theta \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$H_\varphi = \frac{I\Delta z}{4\pi} jk\frac{e^{-jkr}}{r}\sin\theta \qquad E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu\frac{e^{-jkr}}{r}\sin\theta \qquad \frac{E_\theta}{H_\varphi} = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

Discussion: (1) polarization components; (2) θ-distribution; (3) magnitude and phase; (4) propagation direction (power flow); (5) intrinsic impedance.

Field zone of antennas

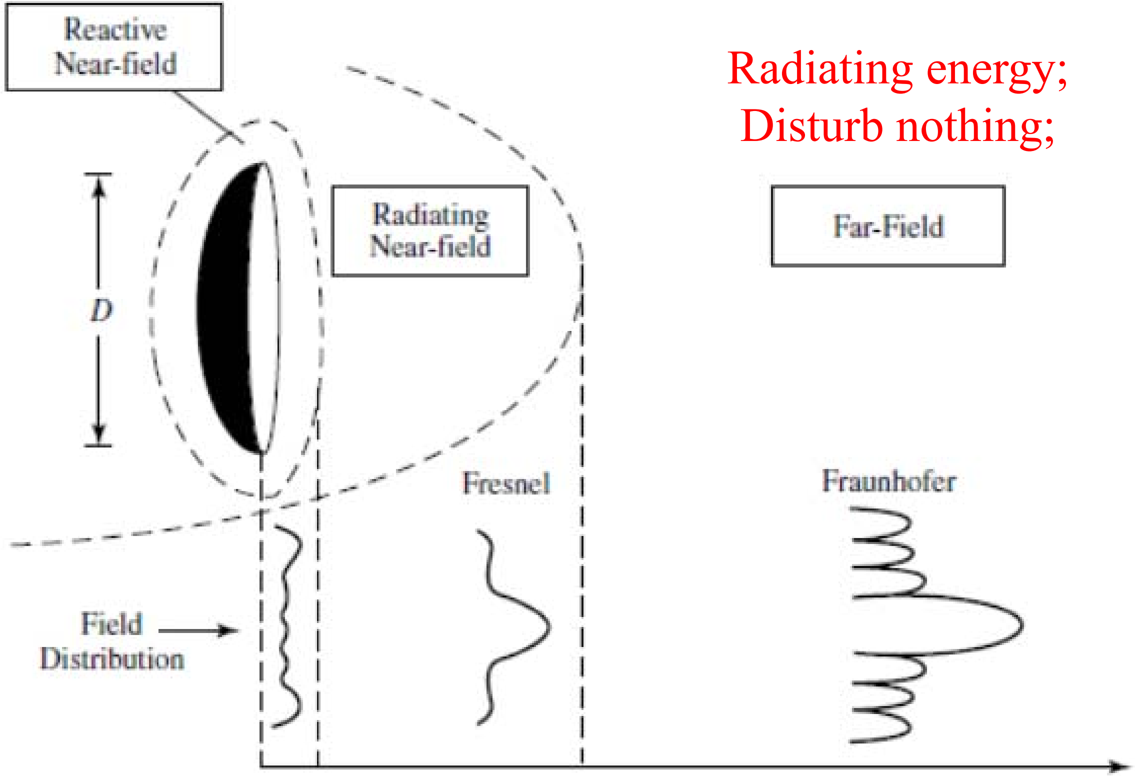
Near field: resonant, field;
Far field: propagation, wave;
Fresnel region: transition;



Field zone of antennas (2)

Reactive field, store energy;
Disturb input impedance and radiation pattern;

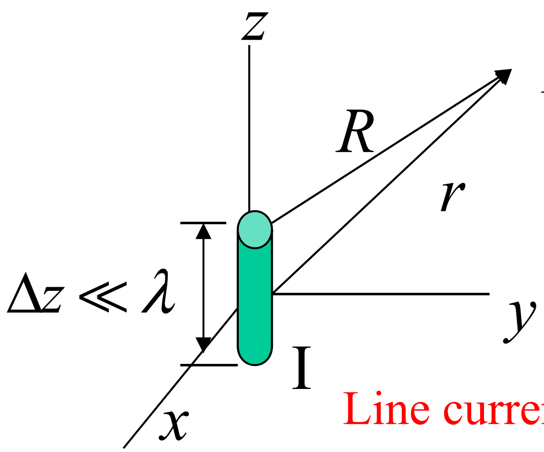
Radiating energy;
Disturb nothing;



Energy transition;
Disturb radiation pattern;

Radiation of a Hertz dipole

Infinite short length;
Uniform distribution;
Infinite small radius;



Line current: A

$$\nabla^2 \vec{A} + \omega^2 \mu\varepsilon \vec{A} = -\mu J\hat{z}$$

$$\vec{A}(x,y,z) = A_z\hat{z} = \hat{z}\int_{-\Delta z/2}^{\Delta z/2} \mu I \frac{e^{-jkr}}{4\pi r} dz = \mu I \Delta z \frac{e^{-jkr}}{4\pi r}\hat{z}$$

$$\vec{A}(\theta,r) = \mu I \Delta z \frac{e^{-jkr}}{4\pi r}(-\sin\theta\hat{\theta} + \cos\theta\hat{r})$$

Far field: $kr \gg 1$

$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]\frac{e^{-jkr}}{r}\sin\theta$$

$$E_r = \frac{I\Delta z}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\left[\frac{1}{r} - j\frac{1}{kr^2}\right]\frac{e^{-jkr}}{r}\cos\theta$$

$$H_\varphi = \frac{I\Delta z}{4\pi} jk\left(1 + \frac{1}{jkr}\right)\frac{e^{-jkr}}{r}\sin\theta$$

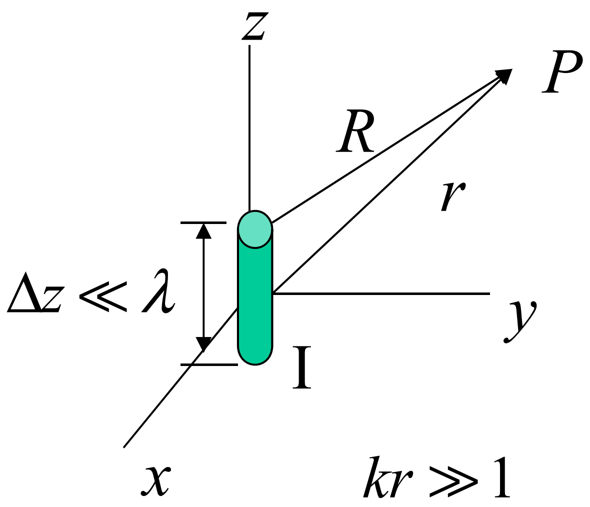
$$\boxed{E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r}\sin\theta}$$

$$\boxed{H_\varphi = \frac{I\Delta z}{4\pi} jk \frac{e^{-jkr}}{r}\sin\theta}$$

$$\frac{E_\theta}{H_\varphi} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta$$

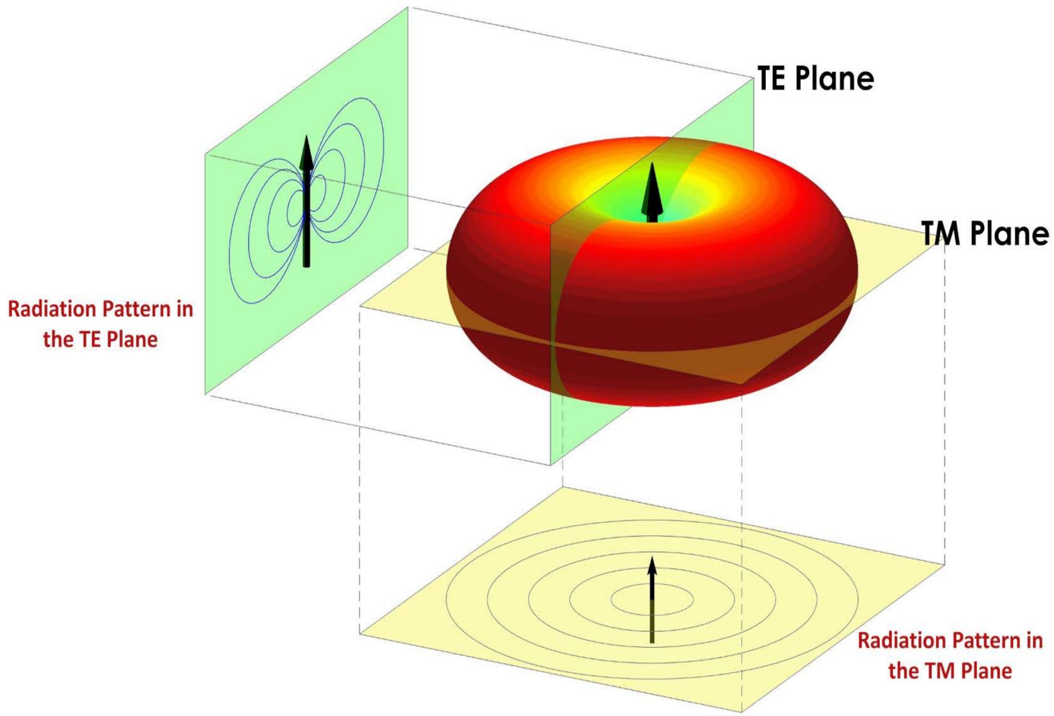
Discussion: (1) components; **(2) θ-distribution;** (3) magnitude and phase; (4) propagation direction; (5) intrinsic impedance.

Far field radiation of Hertz dipole



$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r} \sin\theta$$

$$H_\varphi = \frac{I\Delta z}{4\pi} jk \frac{e^{-jkr}}{r} \sin\theta$$



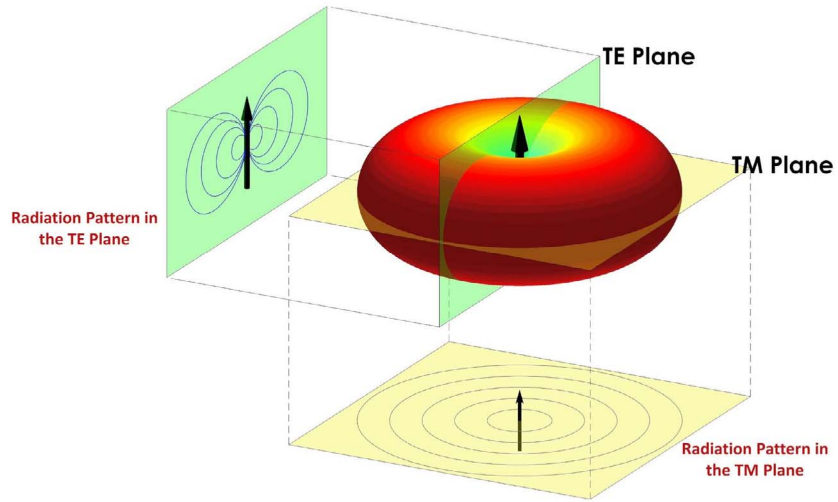
Radiation in different angles.

How to describe?

Antenna parameters

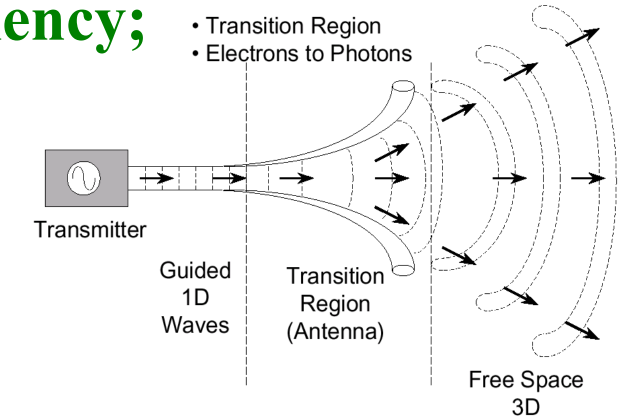
Radiation parameters:

- **Radiation patterns;**
- **Power density;**
- **Radiation intensity;**
- **Directivity and gain;**
- **Polarization;**
- **Effective Aperture and Aperture efficiency;**



Circuit parameters:

- **Input impedance;**
- **Scattering parameters;**



Outline

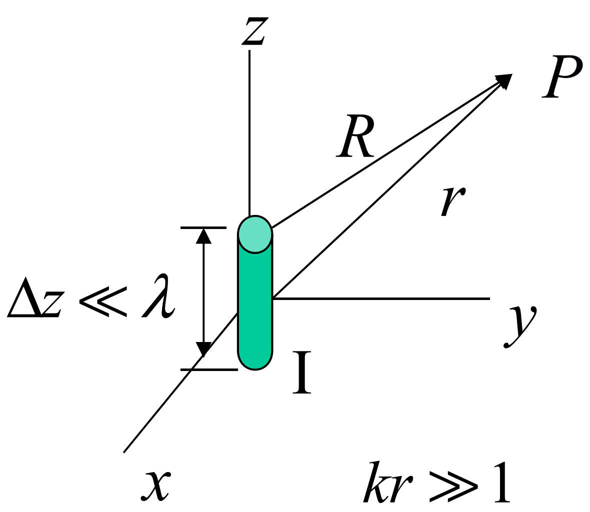
Radiation parameters:

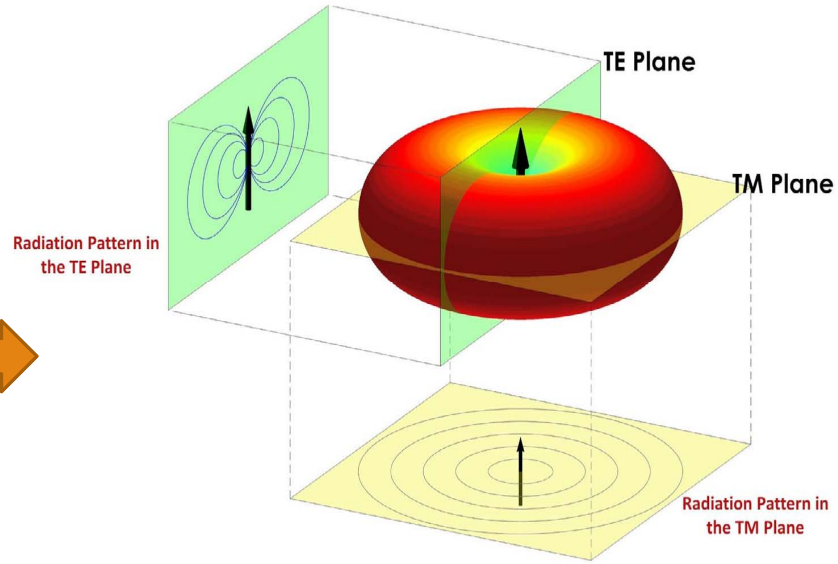
- **Radiation patterns;**
- **Power density;**
- **Radiation intensity;**
- **Directivity and gain;**
- **Polarization;**
- **Effective Aperture and Aperture efficiency;**

Circuit parameters:

- **Input impedance;**
- **Scattering parameters;**

Radiation patterns





$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r} \sin\theta$$

$$H_\varphi = \frac{I\Delta z}{4\pi} jk \frac{e^{-jkr}}{r} \sin\theta$$

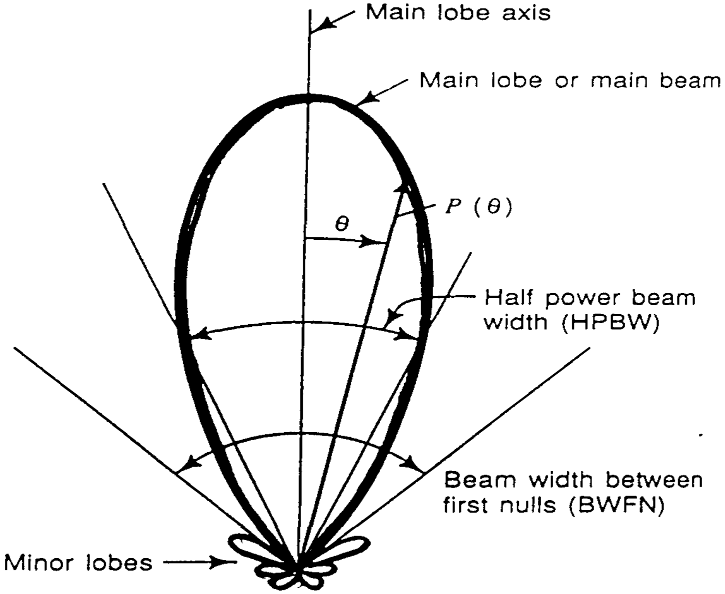
Definition: Angular variation of radiation.

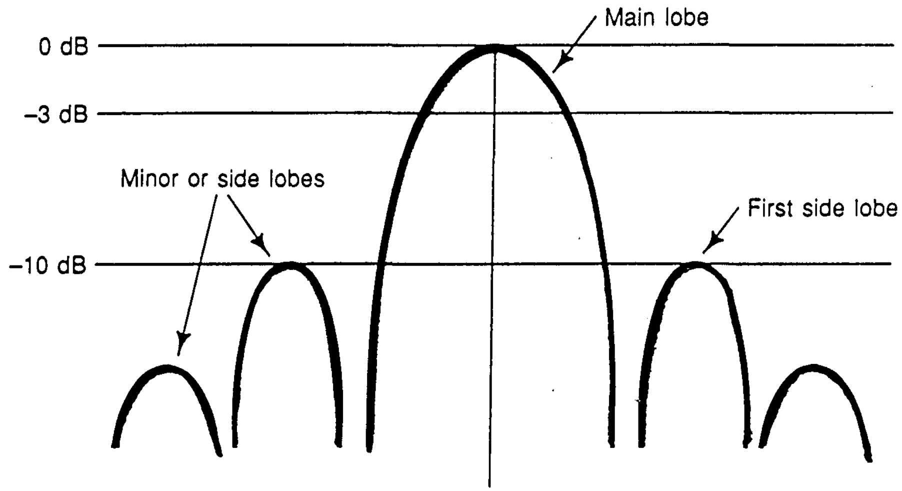
- Angular distribution; variables (θ, φ);
- Far field; no relation with r;
- Power density/intensity, magnitude;
- 3-D, 2-D (E-plane, H-plane);

Pattern parameters

2-D plot:

- Polar (r-mag, θ-angle)
- Planar (y-mag, x-angle)

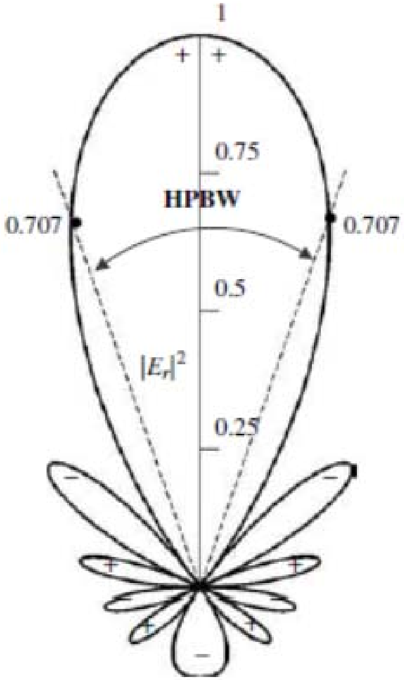




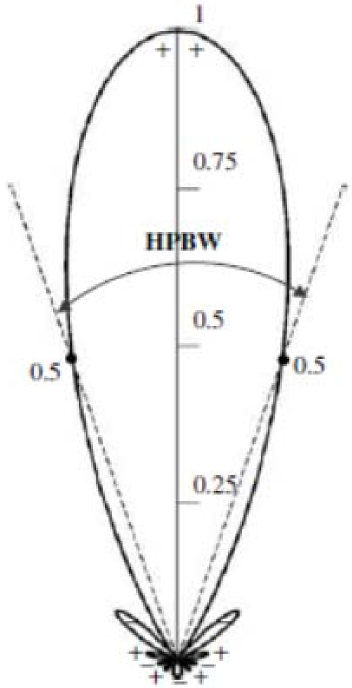
- Main lobe: maximum radiation;
- Minor/side lobes, first side lobe (1 or 2);
- Null, 2 first nulls;
- Grating lobe, for array;

- Half power beam width: HPBW, 3-dB BW;
- Beam width between first nulls, BWFN;
- Side lobe level suppression: main lobe and first side lobe;

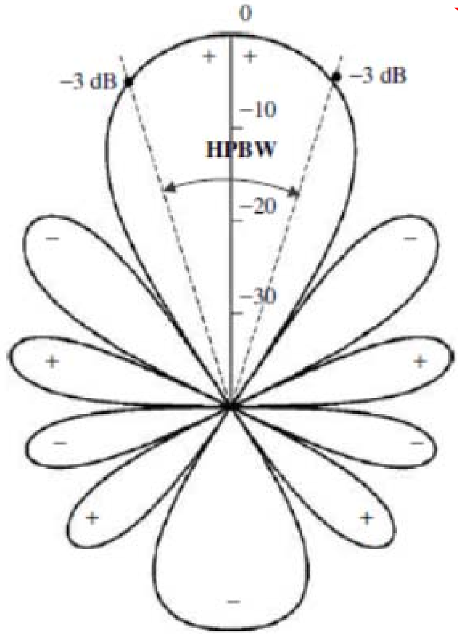
Pattern parameters (2)



(a) Field pattern (in linear scale)



(b) Power pattern (in linear scale)

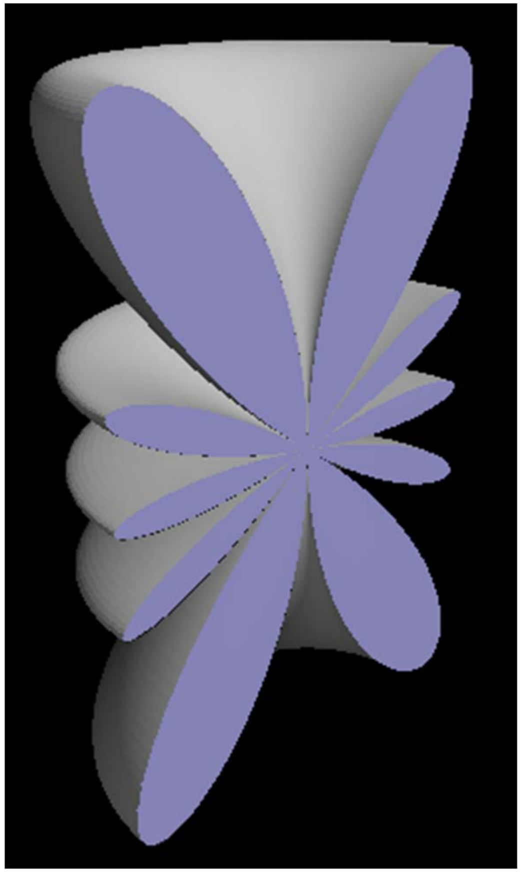


(c) Power pattern (in dB)

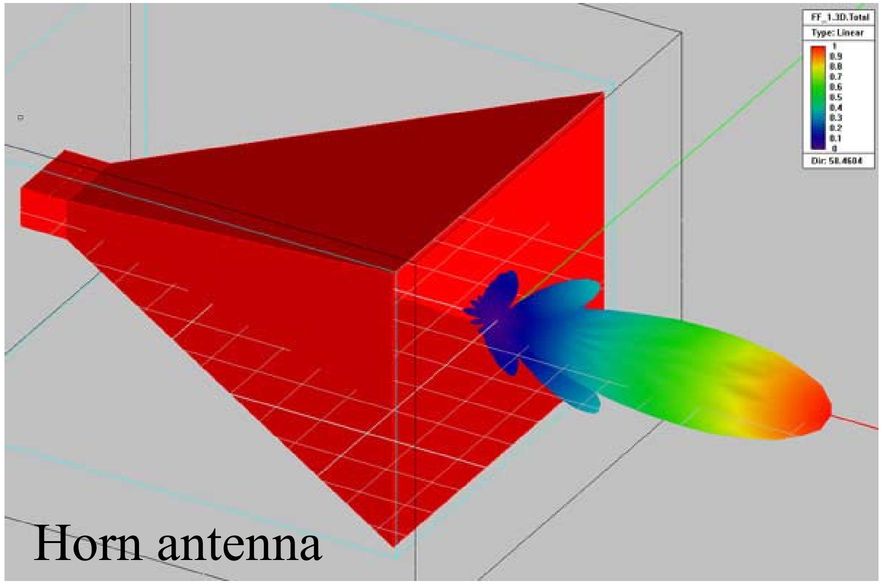
- Normalized radiation pattern;
- Field pattern and power pattern;
- HPBW in linear scale, 0.707 or 0.5;

- Log scale, dB, only for power pattern;
- From linear to log, ×10;
- For field pattern, log ×20;
- 3-dB beam width.

Types of Radiation pattern



Wire antenna, L = 5 λ



Horn antenna

- **Directional: uni-, bi-directional;**
- **Omnidirectional: for 2D;**
- **Isotropic: for 3D.**

Outline

Radiation parameters:

- **Radiation patterns;**
- **Power density;**
- **Radiation intensity;**
- **Directivity and gain;**
- **Polarization;**
- **Effective Aperture and Aperture efficiency;**

Circuit parameters:

- **Input impedance;**
- **Scattering parameters;**

Radiation Power Density

Instantaneous Poynting vector $\vec{S}(x, y, z, t)$

$$\vec{S}(x, y, z, t) = \vec{E}(x, y, z, t) \times \vec{H}(x, y, z, t)$$

- Unit and name;

$\vec{S}$ = instantaneous Poynting Vector (W/m^2)

$\vec{E}$ = instantaneous electric field intensity (V/m)

$\vec{H}$ = instantaneous magnetic field intensity (A/m)

$$P = \oiint_s \vec{S} \cdot d\vec{s} = \oiint_s \vec{S} \cdot \hat{n}\ ds$$

- Flux and integral surface;

P = instantaneous total power crossing a closed surface (W)

$\hat{n}$ = unit vector normal to the surface

ds = infinitesimal area of the closed surface (m^2)

Radiation Power Density (2)

Instantaneous Poynting vector $\vec{S}(x,y,z,t)$

Time average Poynting vector $\vec{S}_{av}(x,y,z)$

- Remove the time factor

$$\vec{S}_{av}(x,y,z) = \frac{1}{T}\int_0^T \vec{S}(x,y,z,t)dt$$

T=intgral time period

Time harmonic field

$$\vec{E}(x,y,z;t) = \mathrm{Re}\left[\tilde{\vec{E}}(x,y,z)e^{j\omega t}\right] \qquad \vec{H}(x,y,z;t) = \mathrm{Re}\left[\tilde{\vec{H}}(x,y,z)e^{j\omega t}\right]$$

$$\vec{S}_{av}(x,y,z) = \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}}(x,y,z)\times\tilde{\vec{H}}^*(x,y,z)\right]$$

- Compare with circuits
- E&H analogy to V/I

Radiation Power Density (con.)

Prove $\vec{S}_{av}(x,y,z)=\frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}}(x,y,z)\times\tilde{\vec{H}}^*(x,y,z)\right]$

$$\vec{S}(x,y,z;t)=\vec{E}(x,y,z;t)\times\vec{H}(x,y,z;t)=\mathrm{Re}\left[\tilde{\vec{E}}(x,y,z)e^{j\omega t}\right]\times\mathrm{Re}\left[\tilde{\vec{H}}(x,y,z)e^{j\omega t}\right]$$

$$\mathrm{Re}\left[\tilde{\vec{H}}(x,y,z)e^{j\omega t}\right]=\frac{1}{2}\left[\tilde{\vec{H}}(x,y,z)e^{j\omega t}\right]+\frac{1}{2}\left[\tilde{\vec{H}}^*(x,y,z)e^{-j\omega t}\right]$$

$$\vec{S}(x,y,z;t)=\mathrm{Re}\left[\tilde{\vec{E}}(x,y,z)e^{j\omega t}\right]\times\left[\frac{1}{2}\left[\tilde{\vec{H}}(x,y,z)e^{j\omega t}\right]+\frac{1}{2}\left[\tilde{\vec{H}}^*(x,y,z)e^{-j\omega t}\right]\right]$$

$$=\frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}}\times\tilde{\vec{H}}^*\right]+\frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}}\times\tilde{\vec{H}}e^{j2\omega t}\right]$$

With time factor, period

No time factor

$$\vec{S}_{av}(x,y,z)=\frac{1}{T}\int_0^T\vec{S}(x,y,z,t)dt \qquad \vec{S}_{av}(x,y,z)=\frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}}\times\tilde{\vec{H}}^*\right]$$

Summary for Time harmonic field

Radiation Power Density $\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right]$

Total Radiation Power $P_{rad} = \oiint_s \vec{S}_{av} \cdot d\vec{s} = \oiint_s \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right] \cdot d\vec{s}$

Outline

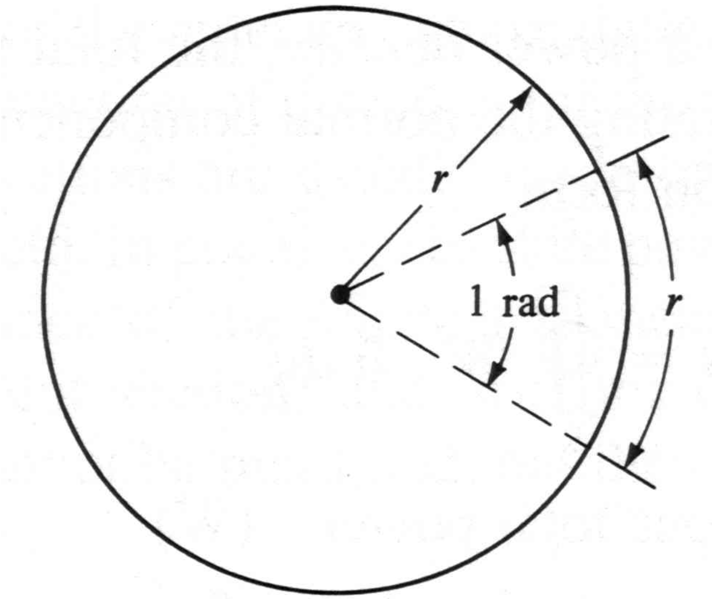
Radiation parameters:

- **Radiation patterns;**
- **Power density;**
- **Radiation intensity;**
- **Directivity and gain;**
- **Polarization;**
- **Effective Aperture and Aperture efficiency;**

Circuit parameters:

- **Input impedance;**
- **Scattering parameters;**

Radian and Steradian

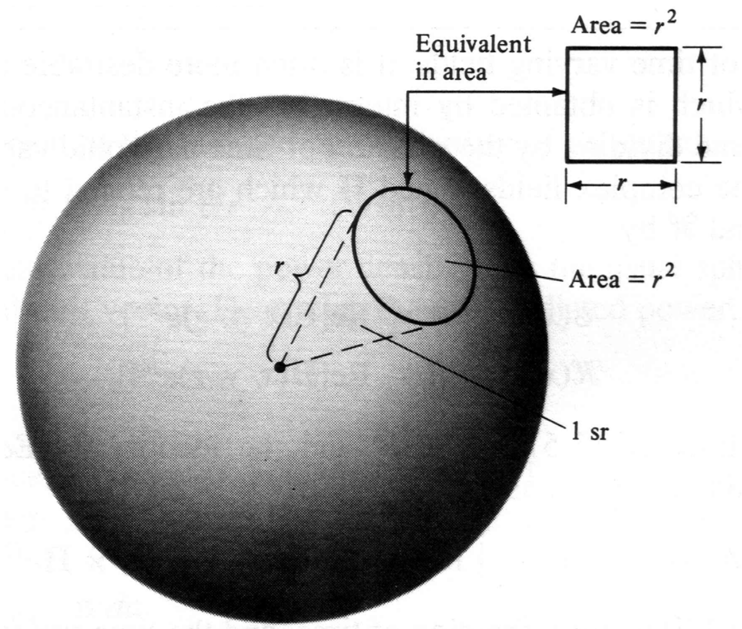


(a) Radian

Radian is the measure of **a plane angle (2D)**. There are 2π radian in a full circle.

$$\Phi \doteq \frac{C}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}.$$

Radian and Steradian (2)



(b) Steradian

Steradian is the measure of **a solid angle (3D)**. There are 4π steradian in a closed sphere.

$$\Omega \doteq \frac{S}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi \; sr$$

Unit solid angle

ster

Note: $ds = r^2 \sin\theta d\theta d\phi$

$$\boxed{d\Omega = \frac{ds}{r^2} = \sin\theta d\theta d\phi}$$

differential solid angle of sphere

Radiation Intensity

Radiation intensity: $U(\Omega)=U(\theta,\varphi)$

For far field, the radiated power per solid angle in a given direction.

$$U(\theta,\varphi)=r^2 S(r,\theta,\varphi)$$

$U(\theta,\varphi)=$ radiation intensity (W/unit solid angle)

$S(r,\theta,\varphi)=$ power density (W/m^2)

$$P_{rad}=\int_0^{2\pi}\int_0^{\pi}U d\Omega=\int_0^{2\pi}\int_0^{\pi}U(\theta,\varphi)\sin\theta\, d\theta d\varphi$$

Another way to calculate total radiation power.

Isotropic radiation $\quad P_{rad}=\int_0^{2\pi}\int_0^{\pi}U d\Omega=4\pi U \qquad U=\dfrac{P_{rad}}{4\pi}$

Summary for Time harmonic field

Radiation Power Density
$$\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right]$$

Total Radiation Power
$$P_{rad} = \oiint_s \vec{S}_{av} \cdot d\vec{s} = \oiint_s \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right] \cdot d\vec{s}$$

Radiation Intensity
$$U(\theta,\varphi) = r^2 S_{av}(r,\theta,\varphi)$$

Total Radiation Power
$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U d\Omega = \int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\sin\theta\, d\theta d\varphi$$

Outline

Radiation parameters:

- **Radiation patterns;**
- **Power density;**
- **Radiation intensity;**
- **Directivity and gain;**
- **Polarization;**
- **Effective Aperture and Aperture efficiency;**

Circuit parameters:

- **Input impedance;**
- **Scattering parameters;**

Directivity

Directivity: the ratio of the radiation intensity in a certain direction to the average radiation intensity (isotropic intensity).

$$D(\theta,\phi)=\frac{U(\theta,\phi)}{U_{av}} \qquad U_{av}=\frac{P_{rad}}{4\pi} \qquad D(\theta,\varphi)=\frac{U(\theta,\varphi)}{P_{rad}/4\pi}$$

Directivity of an antenna: maximum value $D=\dfrac{U_{\max}}{U_{av}}=\dfrac{U_{\max}}{P_{rad}/4\pi}$

(Effective) Solid angle

- Directivity of an angle;
- Directivity of an antenna;
- Physics understanding: directional or omnidirectional antenna;
- Effective solid angle, large or small.

$$\Omega_A=\frac{P_{rad}}{U_{\max}} \qquad D=\frac{U_{\max}}{U_{av}}=\frac{4\pi}{\Omega_A}$$

$$P_{rad}=U_{\max}\Omega_A=U_{av}4\pi$$

Solid angle in engineering approximation

$$P_{rad} = U_{\max}\Omega_A = U_{av} 4\pi = \oiint_s U(\theta,\varphi)\sin\theta d\theta d\varphi \qquad \Omega_A = \frac{P_{rad}}{U_{\max}}$$

$$\boxed{\Omega_A \simeq \Theta_{1r}\Theta_{2r}}$$

Θ_{1r} = half-power beamwidth in one plane (rad)
Θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)

$$\boxed{D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}}}$$

Summary for Time harmonic field

Radiation Power Density

$$\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right]$$

Total Radiation Power

$$P_{rad} = \oiint_s \vec{S}_{av} \cdot d\vec{s} = \oiint_s \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right] \cdot d\vec{s}$$

Radiation Intensity

$$U(\theta,\varphi) = r^2 S_{av}(r,\theta,\varphi)$$

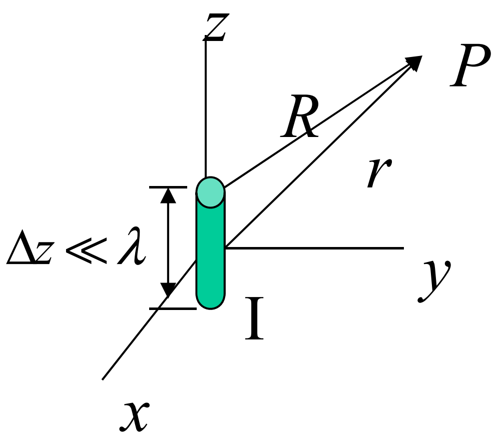
Total Radiation Power

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U d\Omega = \int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\sin\theta\, d\theta d\varphi$$

Directivity

$$D = \frac{U_{\max}}{U_{av}} = \frac{U_{\max}}{P_{rad}/4\pi} \qquad D = 4\pi/\Omega_A$$

Directivity of Hertz dipole



$$\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right] = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 k\omega\mu \frac{\sin^2\theta}{r^2}\hat{r}$$

$$= \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 k^2\eta \frac{\sin^2\theta}{r^2}\hat{r}$$

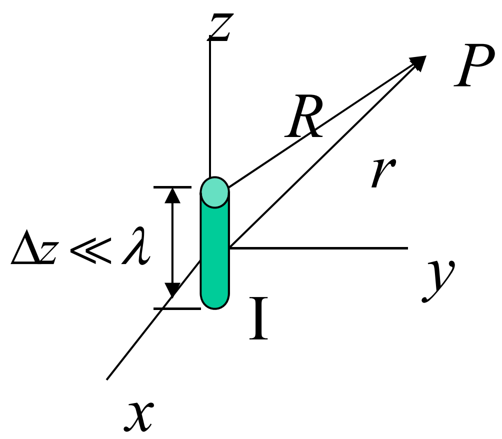
$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r}\sin\theta$$

$$H_\varphi = \frac{I\Delta z}{4\pi} jk \frac{e^{-jkr}}{r}\sin\theta$$

$$P_{rad} = \oiint_s \vec{S}_{av} \cdot d\vec{s} = \int_0^{2\pi}\int_0^{\pi} \vec{S}_{av} \cdot \hat{r}\, r^2 \sin\theta \, d\theta d\varphi$$

$$= \frac{4\pi}{3}\left(\frac{I\Delta z}{4\pi}\right)^2 k^2\eta$$

Directivity of Hertz dipole (2)



$$U = r^2 S_{av} = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 k^2 \eta \sin^2\theta$$

$$U_{\max} = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 k^2 \eta$$

$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r}\sin\theta$$

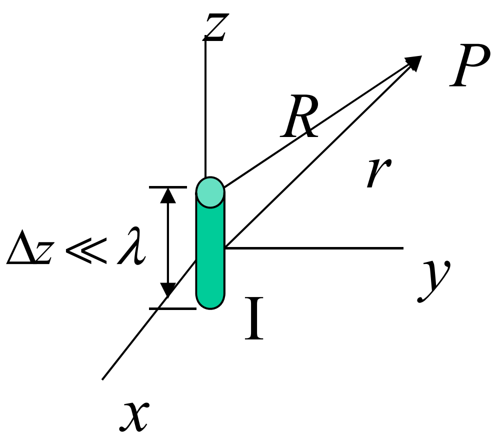
$$H_\varphi = \frac{I\Delta z}{4\pi} jk \frac{e^{-jkr}}{r}\sin\theta$$

$$\Omega_A = \frac{P_{rad}}{U_{\max}} = \frac{\frac{4\pi}{3}\left(\frac{I\Delta z}{4\pi}\right)^2 k^2\eta}{\frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 k^2\eta} = \frac{8\pi}{3}$$

$$\boxed{D = 4\pi/(8\pi/3) = 1.5}$$

$$\boxed{D_{dB} = 10\log\frac{3}{2} = 1.76\ \text{dB}}$$

Directivity of Hertz dipole (3)



Another simple way to calculate Directivity

$$U = r^2 S_{av} = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 k^2 \eta \sin^2\theta$$

Normalized radiation intensity

$$\bar{U} = \sin^2\theta$$

$$\bar{U}_{\max} = 1$$

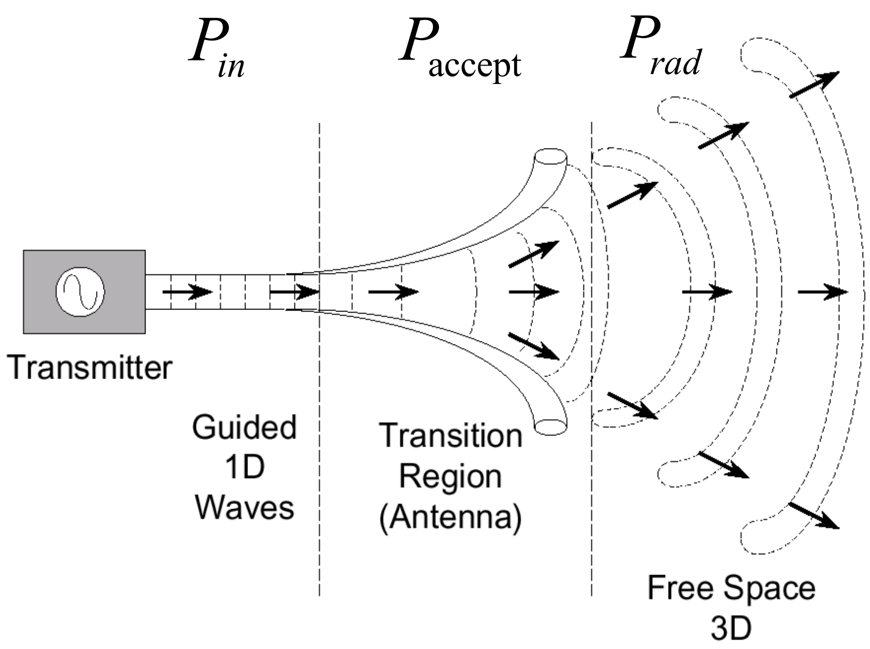
$$E_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r}\sin\theta$$

$$H_\varphi = \frac{I\Delta z}{4\pi} jk \frac{e^{-jkr}}{r}\sin\theta$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \bar{U}\sin\theta\, d\theta d\varphi = \frac{8\pi}{3}$$

$$D = 4\pi/(8\pi/3) = 1.5 \qquad D_{dB} = 10\log\frac{3}{2} = 1.76 \text{ dB}$$

Radiation Efficiency



Reflection efficiency $e_1 = \frac{P_{accept}}{P_{in}}$

Radiation efficiency $e_2 = \frac{P_{rad}}{P_{accept}}$

Total efficiency

$$e = e_1 e_2 = \frac{P_{accept}}{P_{in}} \frac{P_{rad}}{P_{accept}} = \frac{P_{rad}}{P_{in}}$$

Return loss

Dielectric and metallic loss

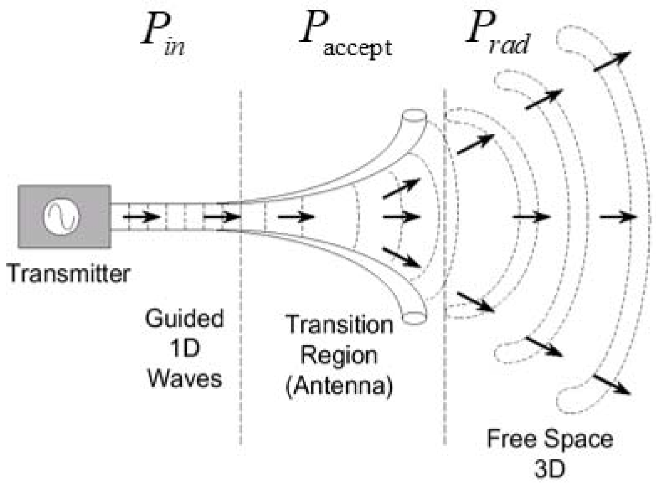
Maximum efficiency is 100%.

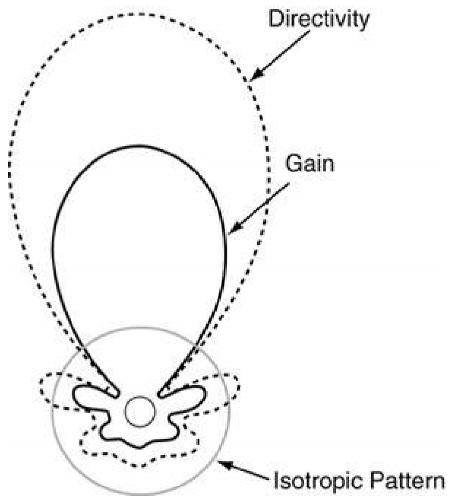
Efficiency of an antenna is total efficiency, not radiation efficiency.

Gain

Antenna gain is defined as: $G = \dfrac{U_{\max}}{P_{in}/4\pi}$ $\qquad G(\theta,\varphi) = \dfrac{U(\theta,\varphi)}{P_{in}/4\pi}$

Directivity: $D = \dfrac{U_{\max}}{U_{av}}$ $\qquad G = \dfrac{U_{\max}}{P_{in}/4\pi} = \dfrac{U_{\max}}{U_{av}} \dfrac{U_{av}}{P_{in}/4\pi} = \dfrac{U_{\max}}{U_{av}} \dfrac{P_{rad}}{P_{in}} = D \cdot e$





$$\boxed{\begin{array}{l} \textit{Directivity} \\ \textit{Efficiency} \rightarrow G = D \cdot e \\ \quad \textit{Gain} \end{array}}$$

$$G_{dB} = 10 \log G$$

Homework #4

- Download from 网络学堂；
- Due to 2023-3-24.

General Schedule

课次	日期	主讲教师	教学内容	备注
1	2/24	李越	L1：天线导论	
2	3/3	李越	L2：麦克斯韦方程、均匀平面波	
3	3/10	李越	L3：辅助位函数、辐射解	
4	3/17	李越	L4：天线辐射远场、天线参数I	
5	3/24	李越	L5：天线参数II	
6	3/31	李越	L6：天线链路计算	
7	4/7	李越	L7：天线相关定理	
8	4/15	李越	L8：偶极天线	
9	4/22		期中测验	one-page note
10	4/29	李越	L9：环天线	
11	5/5	李越	停课	
12	5/12	李越	L10：槽天线、口面天线	
13	5/19	李越	L11：天线阵列I	
14	5/26	李越	L12：天线阵列II、行波天线	
15	6/2	李越	L13：微带天线	
16	6/9	李越	L14：反射面天线+总复习	
		期末考试，开卷！		