Exercise 1

We first load a dataset and examine its dimensions.

In [1]:
```
# If you are running this on Google Colab, uncomment and run the followin
# from google.colab import drive
# drive.mount('/content/drive')
```

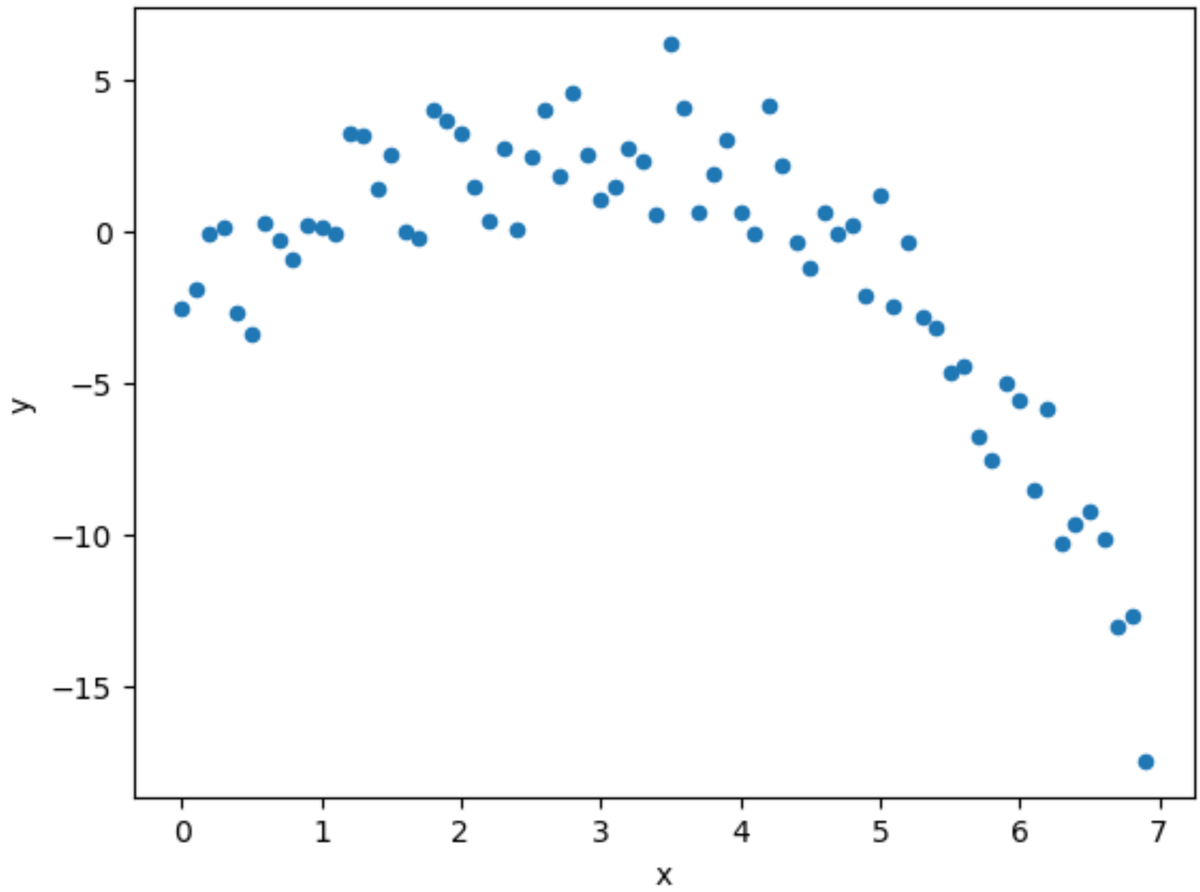
In [2]:
```
import math
import numpy as np

xy_data = np.load('Ex1_polyreg_data.npy')
# If running on Google Colab change path to '/content/drive/MyDrive/IB-Da

np.shape(xy_data)
```

Out[2]:
```
(70, 2)
```

The matrix `xy_data` contains 70 rows, each a data point of the form (x_i, y_i) for $i = 1, \dots, 70$.

1a) Plot the data in a scatterplot.

In [3]:
```
import matplotlib.pyplot as plt
# Your code for scatterplot here

x = xy_data[:,0]
y = xy_data[:,1]

plt.scatter(x, y, s=20)    # s can be used to adjust the size of the dots
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('xy_plot.pdf', bbox_inches = 'tight')
plt.show()
```



1b) Write a function `polyreg` to fit a polynomial of a given order to a dataset.

The inputs to the function are a data matrix of dimension $N \times 2$, and $k \geq 0$, the order of the polynomial. The function should compute the coefficients of the polynomial $\beta_0 + \beta_1 x + \ldots + \beta_k x^k$ via least squares regression, and should return the coefficient vector, the fit, and the vector of residuals.

If specified the degree k is greater than or equal to N, then the function must fit an order $(N-1)$ polynomial and set the remaining coefficients to zero.

NOTE: You are *not* allowed to use the built-in function `np.polyfit`.

In [4]:
```
def polyreg(data_matrix, k):
    x_priv = data_matrix[:,0]
    y_priv = data_matrix[:,1]

    all_ones = np.ones(np.shape(x_priv))

    columns = [all_ones]
    for power in range(1, k + 1):
        columns.append(x_priv ** power)
    X = np.column_stack(columns)

    y_values = y_priv

    beta = np.linalg.lstsq(X, y_values, rcond=None)[0]
    fit = X.dot(beta)
    resid = y_values - fit
    return [beta, fit, resid]
```

Use the tests below to check the outputs of the function you have written:

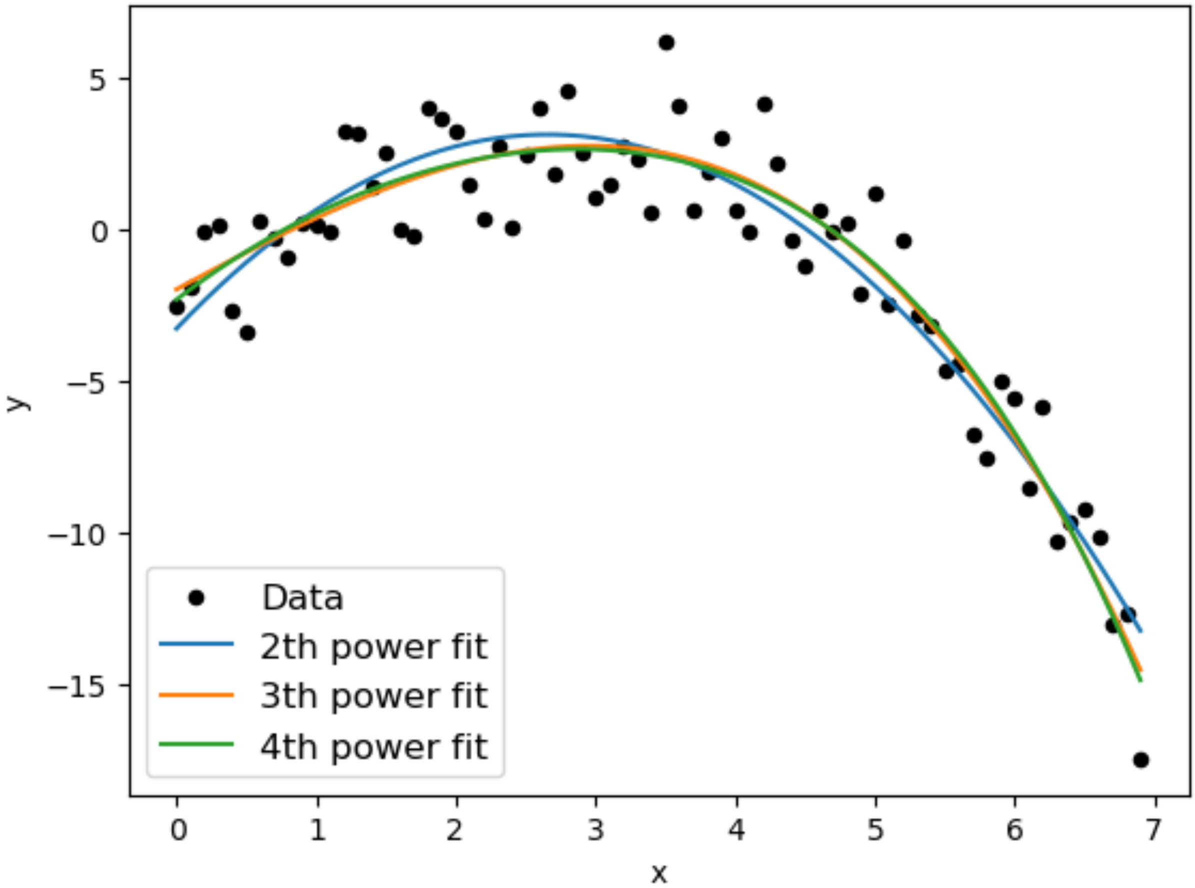
In [5]:
```
# Some tests to make sure your function is working correctly

xcol = np.arange(-1, 1.05, 0.1)
ycol = 2 - 7*xcol + 3*(xcol**2)  # We are generating data accoridng to y
test_matrix = np.transpose(np.vstack((xcol,ycol)))
test_matrix.shape

beta_test = polyreg(test_matrix, k=2)[0]
assert((np.round(beta_test[0], 3) == 2) and (np.round(beta_test[1], 3) ==
# We want to check that using the function with k=2 recovers the coeffici

# Now check the zeroth order fit, i.e., the function gives the correct ou
beta_test = polyreg(test_matrix, k=0)[0]
res_test = polyreg(test_matrix, k=0)[2] #the last output of the function

assert(np.round(beta_test, 3) == 3.1)
assert(np.round(np.linalg.norm(res_test), 3) == 19.937)
```

1c) Use `polyreg` to fit polynomial models for the data in `xy_data` for $k = 2, 3, 4$:

- Plot the fits for the three cases on the same plot together with the scatterplot of the data. The plots should be labelled and a legend included.
- Compute and print the SSE and R^2 coefficient for each of the three cases.
- Which of the three models you would choose? Briefly justify your choice.

In [6]:
```
def plot_fit_and_data(data, k_list):
    x = data[:,0]
    y = data[:,1]

    plt.xlabel("x")
    plt.ylabel("y")
    _, fit_0, _ = polyreg(data, 0)

    plt.rcParams['figure.figsize'] = [12, 7]
    plt.scatter(x, y, s=20, color = "black", label = "Data")
    for i in k_list:
        beta_priv, fit_priv, resid_priv = polyreg(data, i)
        plt.plot(x, fit_priv, label = f"{i}th power fit")
        SSE = np.linalg.norm(y - fit_priv)**2
        SSE_0 = np.linalg.norm(y - fit_0)**2
        R_square = np.round_(1- SSE/SSE_0, decimals = 4)
        print(f"{i}th power fit, SSE is {SSE}, R_square is {R_square}.")

    plt.legend(fontsize = "large")
    plt.savefig('k_fit.pdf', bbox_inches = 'tight')
    plt.show()


plot_fit_and_data(xy_data, [2, 3, 4])
```

```
2th power fit, SSE is 172.18102528988547, R_square is 0.8876.
3th power fit, SSE is 152.40580488915805, R_square is 0.9005.
4th power fit, SSE is 151.22778969027124, R_square is 0.9013.
```



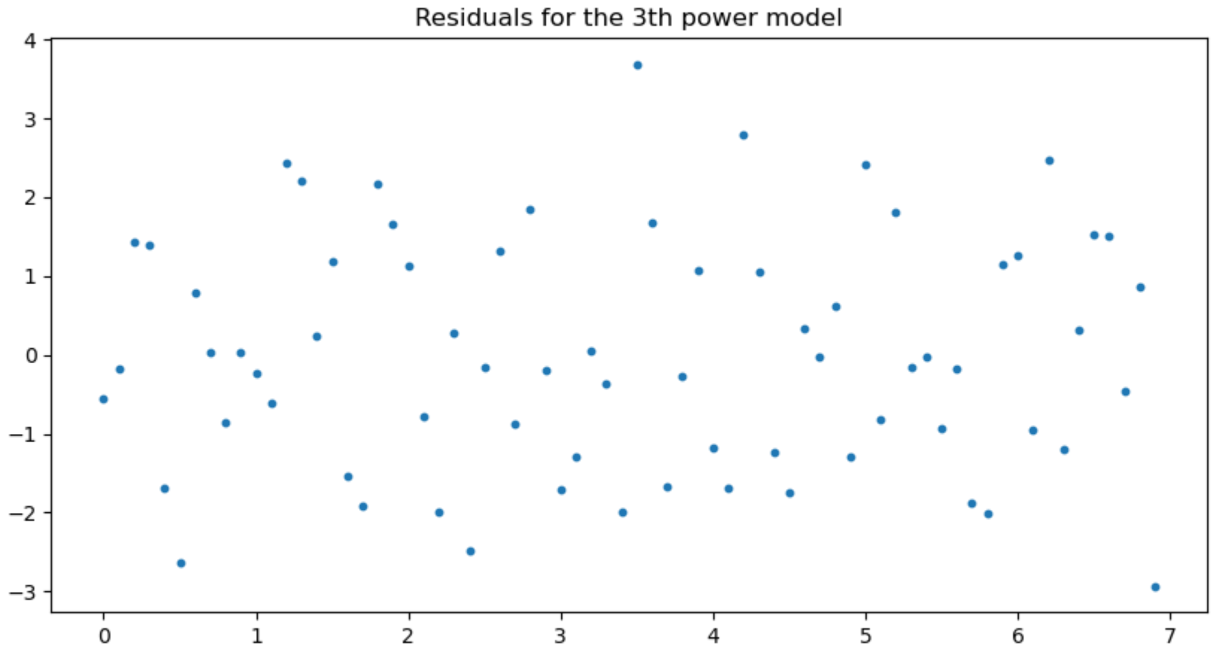
State which model you choose and briefly justify your choice.

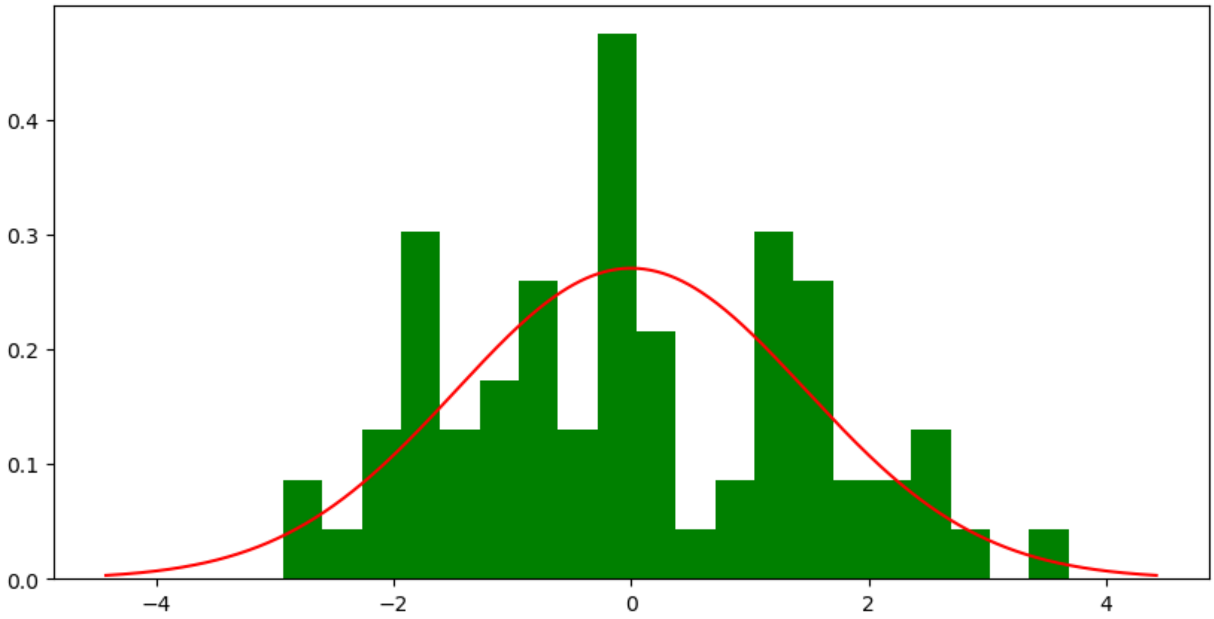
I will choose 3th power fit, as it has a similar R_square value compared with 4th power fit, and the SSE is also smaller than that of 2th power fit.

1d) For the model you have chosen in the previous part (either $k = 2/3/4$):

- Plot the residuals in a scatter plot.
- Plot a histogram of the residuals along with a Gaussian pdf with zero mean and the same standard deviation as the residuals.

In [7]:
```
from scipy.stats import norm

x = xy_data[:,0]
y = xy_data[:,1]

beta_priv, fit_priv, resid_priv = polyreg(xy_data, 3)

resid_priv = y - fit_priv

plt.rcParams['figure.figsize'] = [10, 5]
plt.scatter(x, resid_priv, s=10)
plt.title('Residuals for the 3th power model')
plt.savefig('quad_resid_3th_power_ex1.png', bbox_inches = 'tight')
plt.show()

# Plot normed histogram of the residuals
n, bins, patches = plt.hist(resid_priv, bins=20, density=True, facecolor=

# Plot Gaussian pdf with same mean and variance as the residuals
res_priv_stdev = np.std(resid_priv)  #standard deviation of residuals
xvals = np.linspace(-3*res_priv_stdev,3*res_priv_stdev,1000)
plt.plot(xvals, norm.pdf(xvals, loc=0, scale=res_priv_stdev), 'r')
plt.show()
```


Residuals for the 3th power model
4
3
2
1
0
−1
−2
−3
0
1
2
3
4
5
6
7


0.4
0.3
0.2
0.1
0.0
−4
−2
0
2
4