

# বিদ্যাসাগর বিশ্ববিদ্যালয়

### VIDYASAGAR UNIVERSITY

### **BCA**

## 1st Semester Examination 2021

### DISCRETE MATHEMATICS

**PAPER-1103** 

Full Marks: 70

Time: 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

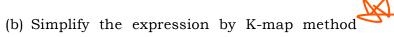
Illustrate the answers wherever necessary.

### Group - A

Answer any four questions.

4×15

**1.** (a) Show that the set  $\{a+b\sqrt{2};a,b\in Q\}$  where Q is the set of rational numbers, forms a group with respect to addition.



$$x_1x_2\overline{x}_3 + x_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2x_3 + \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2x_3$$
 8

- **2.** (a) Prove that every proper subgroup of a group of order 6 is commutative.
  - (b) Show that an integral domain that has a finite number of elements is a field.
- 3. (a) Solve the recurrance relation

$$a_r + 3a_{r-1} + 2a_{r-2} = f(r)$$

where 
$$f(r) = \begin{cases} 1 & where \ r = 2 \\ 0 & otherwise \end{cases}$$
 8

- (b) Find the discrete numeric function corresponding to the generating  $function \ A(z) = \frac{z^5}{5-6z-z^2}.$
- **4.** (a) Prove that a pendant edge in a connected graph G is contained in every spanning tree of G.
  - (b) In how many arrangements of COMPUTER be arranged whether vowels are adjacent.
- **5.** (a) Examine the mapping  $f: z \to z$  defined by f(x) = 4x + 7,  $x \in z$  is bijective or not.

- (b) Use mathematical induction to prove that  $16^n + 10n 1$  is divisible by  $25 \forall n \ge 1$ .
- **6.** (a) Prove that for a simple graph with n vertices and m components can have at most (n m) (n m + 1) / 2 edges.
  - (b) Define Hasse diagram of a poset (S, R) where S is a non-empty set with the relation R. https://chat.openai.com/share/4e26ed94-fd1a-44f2-bc75-5415e68cdd83

    Let X = {1, 2, 3, 4, 5, 6} and '/' (divided by) is a partial order relation on X. Draw the Hasse diagram on (X, /).
- **7.** (a) Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology and 30 do not study any of the three subjects
  - (i) Find the number of students studying all three subjects. 8
  - (ii) Find the number of students studying exactly one of the three subjects.
  - (b) Prove that a graph is a tree if and only if there is a unique path between every pair of vertices in G.
- **8.** (a) Prove that the roots of  $x^n 1 = 0$ ,  $n \in \mathbb{Z}$  form a subgroup of the multiplicative group of non-zero complex numbers. Is the subgroup cylic?
  - (b) Prove that an undirected graph possesses an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree.

7

### Group - B

Answer any one question.

 $1 \times 10$ 

- 9. (a) Define Euler graph with an example.
  - (b) For A =  $\{a, b, \{a, c\}, \phi\}$  determine  $\{\{a,c\}\} A$ .
  - (c) Construct truth table for the statement.

$$p \leftrightarrow (\overline{p} \vee \overline{q}).$$

3+3+4

- **10.** (a) Let A = {3, 7, 5} and B = {4, 5, 8}. Write down total number of distinct relations from A to B.
  - (b) Given that  $A \subseteq C$  and  $B \subseteq D$ , show that

 $A X B \subseteq C X D.$ 

(c) A tree has 2 vertices of degree 2, one vertex of degree 3 and 3 vertices of degree 4. How many vertices of degree I does it have? 3+4+3

(Internal Assessment: 30)