Energy & Heat Transfer

Lecture 2

By: Mohammad Mehrali

Recap of lecture 1

- Work (W), energy (E), power (P)
- Work (*W)*, Energy (*E)* in $\mathrm{J} = \mathrm{N} \cdot \mathrm{m} = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} \cdot \mathrm{m}$
- Power : $P = \frac{\Delta E}{\Delta t}$ in Watt (W) = J / s
- Units kWh, kcal, hp, ….

Recap of lecture 1

- Potential energy : $E = F \cdot h$
- Thermal energy : $Q = m \cdot c \cdot \Delta T$
  - c is specific heat capacity in J / (kg · K)
- Electrical power: $P = U \cdot I$
  - Electrical energy: $E = P \cdot t = U \cdot I \cdot t$
  - Ohm's law: $U = I \cdot R$

$$\left.\begin{array}{l} P = U \cdot I \\ E = P \cdot t = U \cdot I \cdot t \\ U = I \cdot R \end{array}\right\} \quad \begin{array}{l} P = I^2 \cdot R \\ E = I^2 \cdot R \cdot t \end{array}$$

Learning objectives lecture 2

- **Energy in general**
  - Using an energy balance
  - Determine efficiency
  - Calculate losses for electric resistors

- **Conductive heat transfer**
  - *Explaining* conduction principles
  - *Calculate* conductive heat transfer
  - Using *thermal resistances* and insulation values

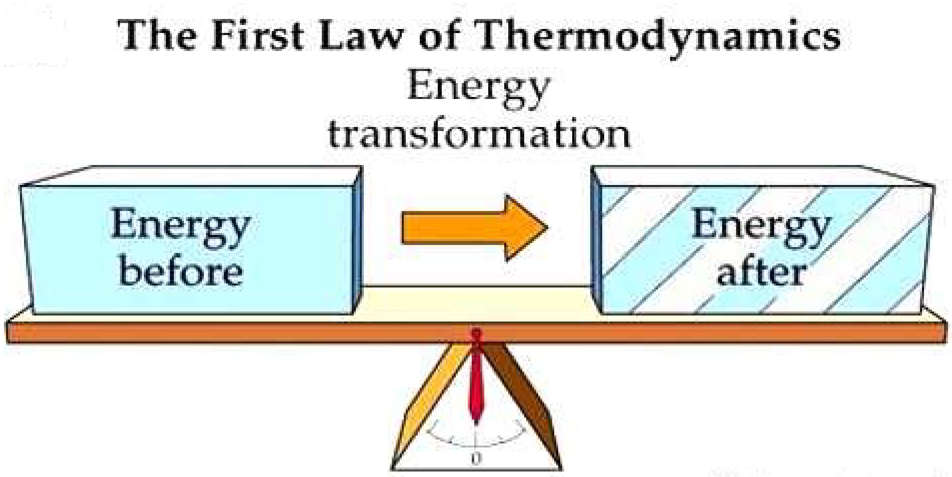
Learning objectives lecture 2

- **Energy in general**
  - **Using an energy balance**
  - Determine efficiency
  - Calculate losses for electric resistors
- **Conductive heat transfer**
  - *Explaining* conduction principles
  - *Calculate* conductive heat transfer
  - Using *thermal resistances* and insulation values

ENERGY BALANCE

- Energy is **always conserved**!

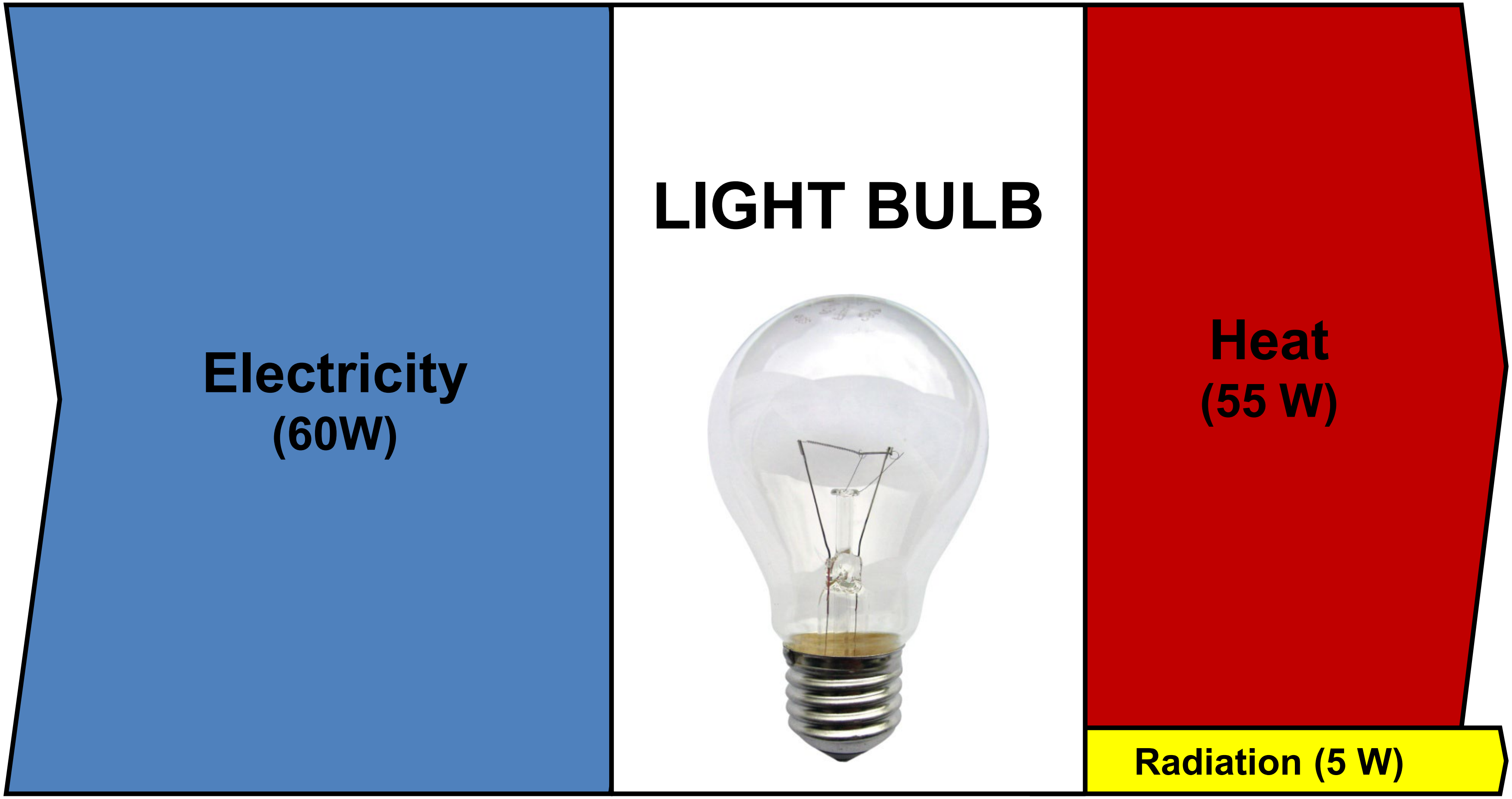
- First law of Thermodynamics:



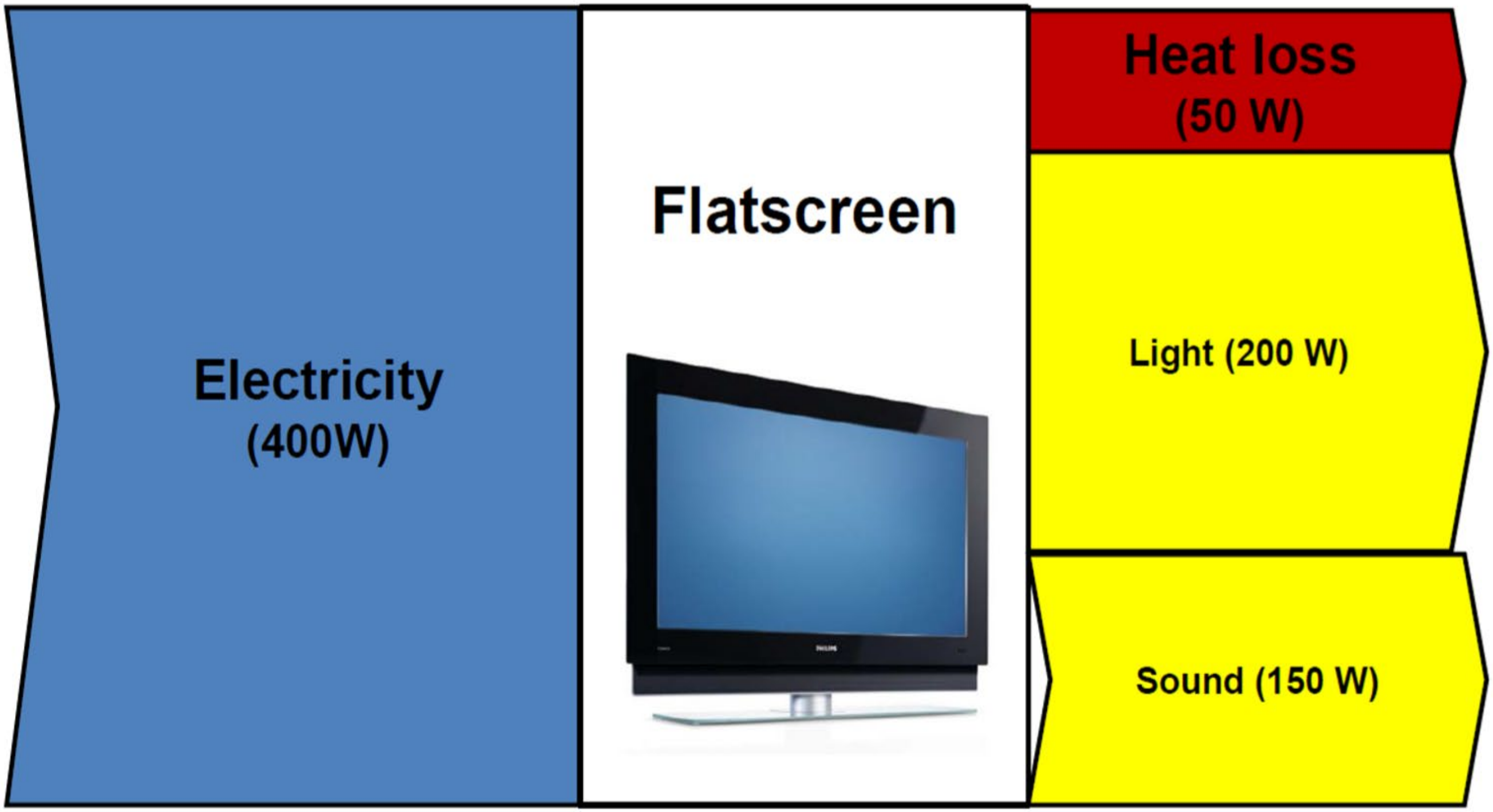
- https://www.youtube.com/watch?v=kP7q28wQ2P8&ab_channel=BBCEarthLab

Energy balance

- Sankey diagram light bulb



Energy balance



Energy balance

Learning objectives lecture 2

Energy in general

- Using an energy balance
- **Determine efficiency**
- Calculate losses for electric resistors

Conductive heat transfer

- *Explaining* conduction principles
- *Calculate* conductive heat transfer
- Using *thermal resistances* and insulation values

Efficiency

- Efficiency = fraction of “useful” work/power used
  - What is defined as useful?
  - What is the reference?

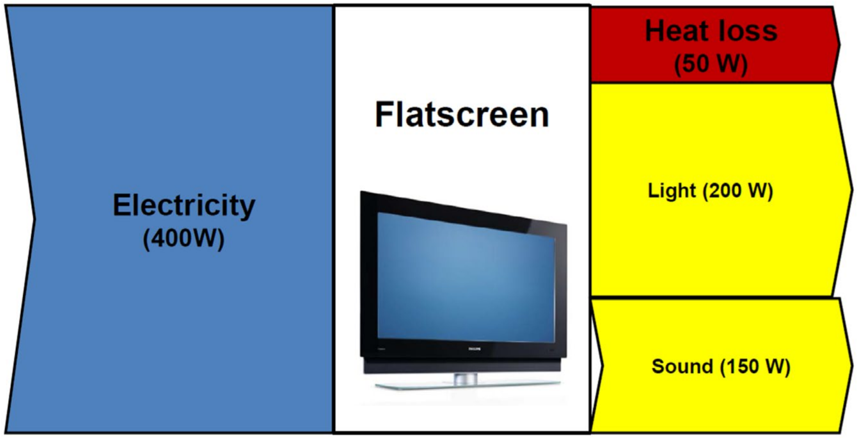
- Efficiency $$\eta = \frac{\textbf{useful work}}{\textbf{input energy}} = \frac{\textbf{useful power}}{\textbf{input power}}$$ (-)

- Use a Sankey diagram!

Efficiency

Efficiency definition not clear:

- What is useful? What is the input? Give the reasons for the choices made



Flatscreen: useful spent power?

So always explain how you have defined efficiency!

Efficiency – practical values

Product	Efficiency	Remark
Solar panel	12%	Part of sun’s radiation converted to electricity
Vacuum cleaner	20%	Part of electrical power converted to suction power
Petrol engine	35%	Part of chemical energy converted to kinetic energy
Electric kettle	84%	Part of electrical energy converted to useful heat in the water

Always state the used definition when talking about efficiency

Learning objectives lecture 2

Energy in general

- Using an energy balance
- Determine efficiency
- **Calculate losses for electric resistors**

Conductive heat transfer

- *Explaining* conduction principles
- *Calculate* conductive heat transfer
- Using *thermal resistances* and insulation values

Resistors

Electric resistors: barrier in electric circuit

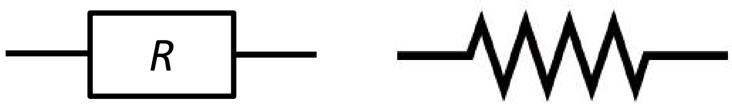
→ heat generation

Heat: purpose

Heat: "byproduct"

Resistors

Schematical symbols:



Resistance *R* determined by:

- Length of the wire (*L*)
- Through-flow area (*A*)
- Material (resistivity ρ)

Compare: flow through straw

Pouillet's law: $$R = \rho \frac{L}{A}$$

***Not* density:**
$\Omega \cdot m \neq kg/m^3$

Resistor as heat source

Resistor converts electic energy into thermal energy:

$$E = I^2 R \Delta t = Q \qquad [\mathrm{J}]$$

So electric power becomes thermal power:

Notation: $\dot{Q}$ ("Heat-flow" or heat transfer rate)

$$P = I^2 R = \dot{Q} \qquad [\mathrm{J/s}] = [\mathrm{W}]$$

Sidestep

Dot sign means: per unit time (time derivative)

$$\text{Power} = \frac{\text{Energy change}}{\text{Time}}$$

$$P = \frac{dE}{dt} \approx \frac{\Delta E}{\Delta t}$$

$$\dot{Q} = \frac{dQ}{dt} \approx \frac{\Delta Q}{\Delta t}$$

$$(\text{Velocity } v = \frac{dx}{dt} = \dot{x} \approx \frac{\text{displacement } \Delta x}{\Delta t})$$

$$(\text{Acceleration } a = \frac{dv}{dt} = \dot{v} = \ddot{x} \approx \frac{\text{velocity change } \Delta v}{\Delta t})$$

Resistivity

$$R = \rho \frac{L}{A}$$

Material	ρ ($\Omega \cdot m$) at 20 °C	
Copper	$1{,}67 \cdot 10^{-8}$	
Tungsten	$5{,}5 \cdot 10^{-8}$	Conductors
Aluminum	$2{,}65 \cdot 10^{-8}$	
Glass	$1 \cdot 10^{12}$	
PVC	$1 \cdot 10^{14}$	Insulators
Rubber	$1 \cdot 10^{15}$	

CONDUCTORS / INSULATORS

Also thermal!

Insulators?

Conductors?

Learning objectives lecture 2

- **Energy in general**
  - Using an energy balance
  - Determine efficiency
  - Calculate losses for electric resistors
- **Conductive heat transfer**
  - ***Explaining* conduction principles**
  - *Calculate* conductive heat transfer
  - Using *thermal resistances* and insulation values

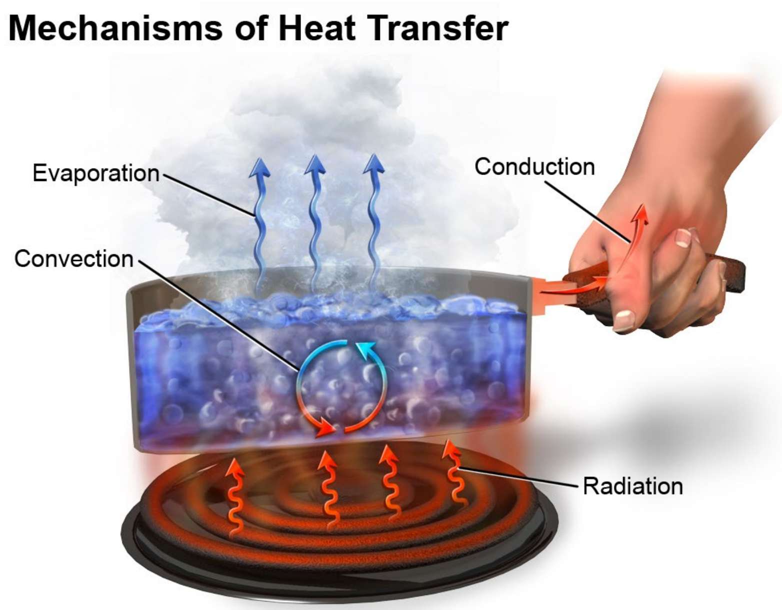
HEAT TRANSFER MECHANISMS

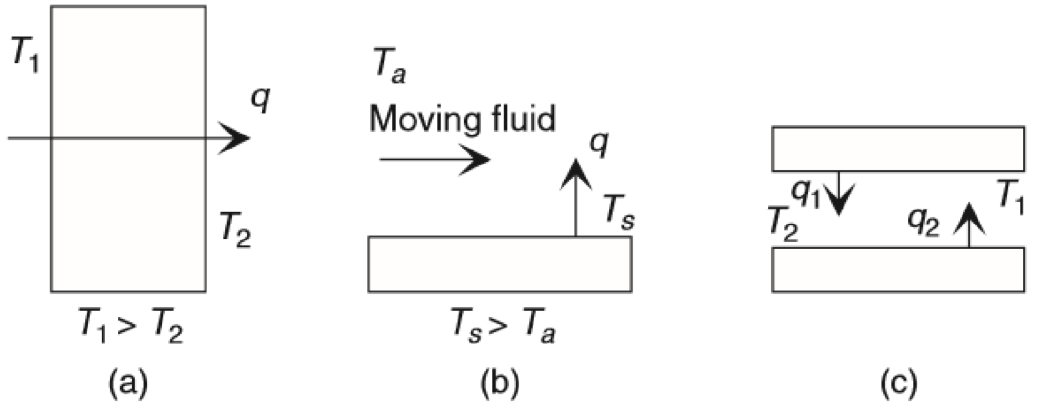
Representations of heat transfer modes:

(a) conduction through a solid, liquid or gas,

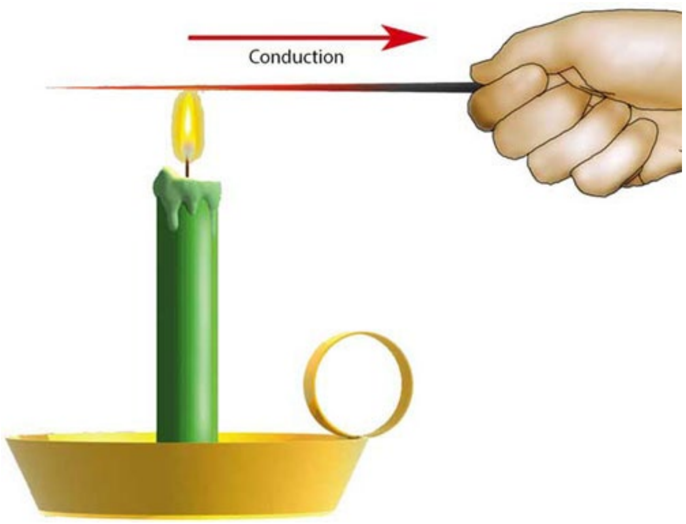
(b) convection from a surface to a moving fluid,

(c) radiation between two surfaces





Heat transfer mechanisms



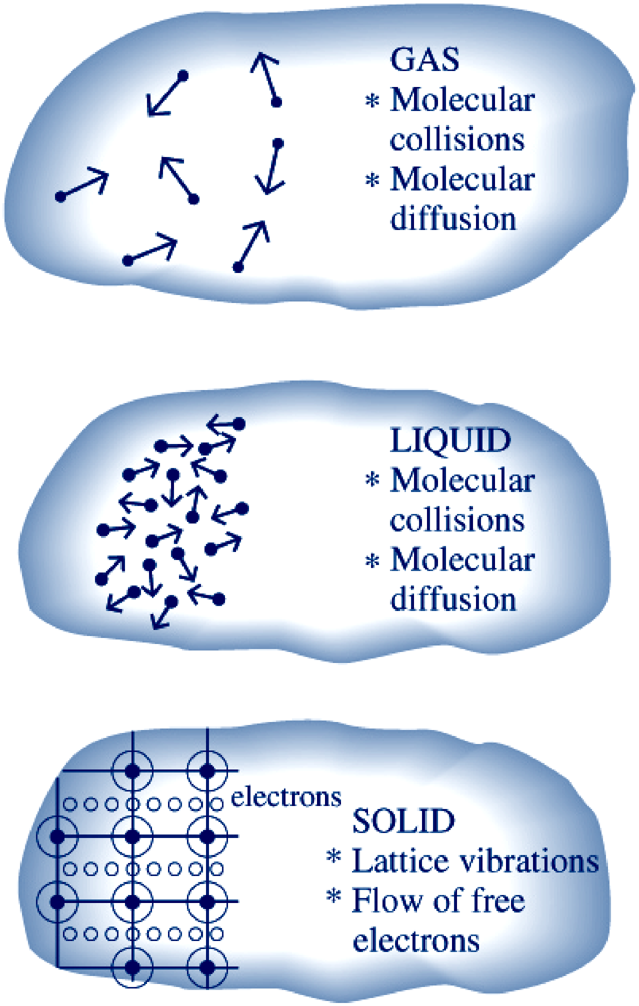
- Conduction
- Convection
- Radiation

Conduction

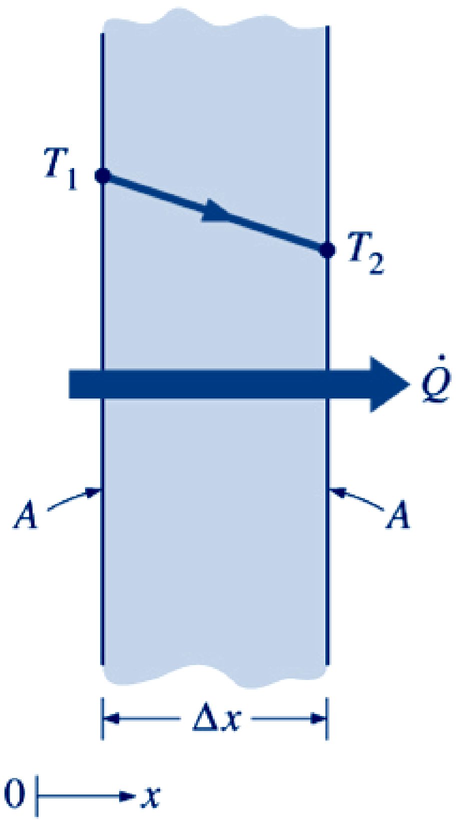
Conduction:

Transfer of thermal energy between (non flowing) molecules

- Always from more energetic (warm) to less energetic (cold) particles!



CONDUCTION(FOURIER'S LAW)



x_1 x_2

Cross section of part of a wall (almost flat)

Heat transfer rate : $\dot{Q}$

- Steady state condition-1D
- Proportional to area *A*
- Proportional to temp. Difference : $T_2 - T_1$
- Inversely proportional to thickness: $\Delta x = x_2 - x_1$
- Dependent on material → thermal conductivity: *k*

$$\dot{Q} = -k\,A\,\frac{T_2 - T_1}{x_2 - x_1} = -k\,A\,\frac{\Delta T}{\Delta x} \quad \text{(W)}$$

Negative sign since heat flow is positive at a negative temperature gradient.

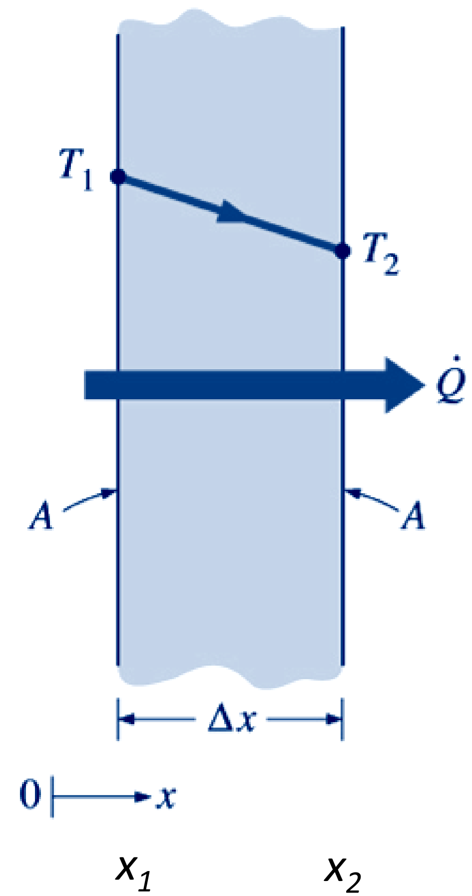
CONDUCTION(FOURIER'S LAW)

$$\dot{Q} = -kA\frac{T_2 - T_1}{x_2 - x_1} = -kA\frac{\Delta T}{\Delta x} \text{(W)}$$

- Fourier's law simplified for **plane surface**

- Per m^2: **Heat flux**

$$\dot{q} = \frac{\dot{Q}}{A} = -k\frac{T_2 - T_1}{x_2 - x_1} = -k\frac{\Delta T}{\Delta x} \text{(W/m}^2\text{)}$$



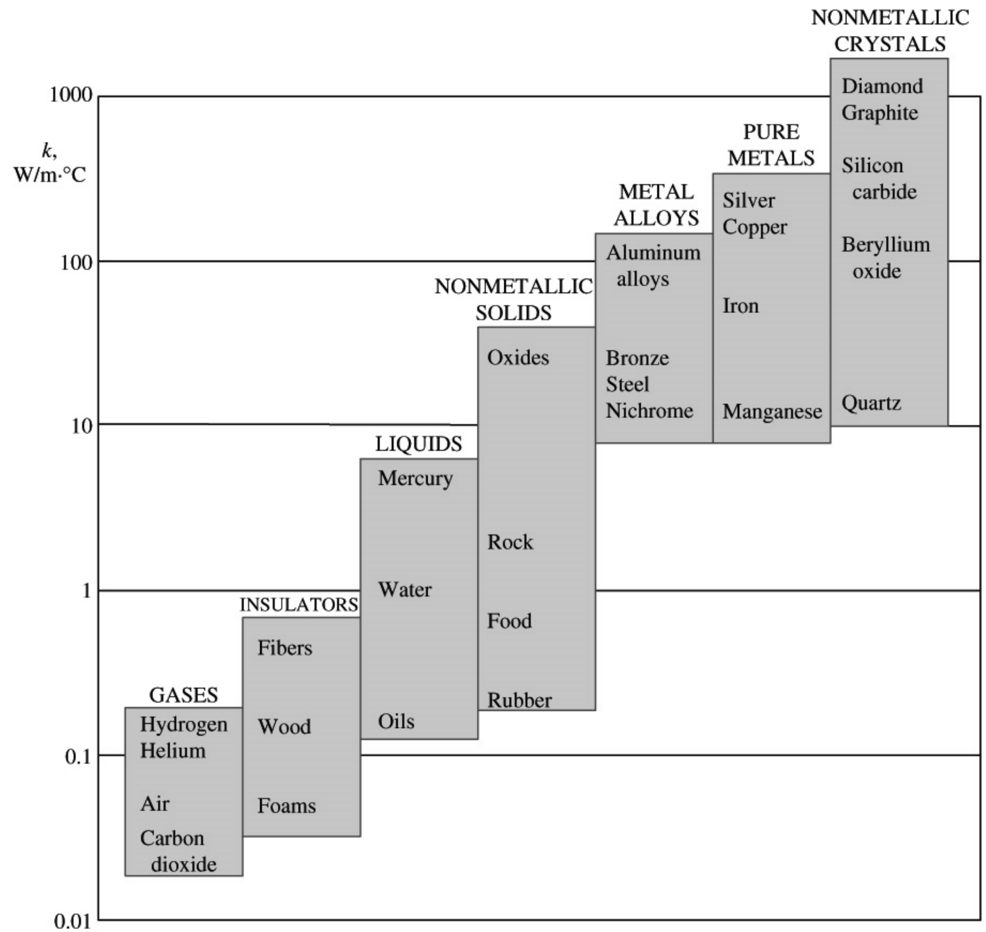
Thermal conductivity (*k*)

Unit: $$\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}}$$

$\Rightarrow$ The amount of power conducted through 1 m of material at a 1 K temperature difference

High *k*: Conductor

Low *k*: Insulator



Learning objectives lecture 2

Energy in general

- Using an energy balance
- Determine efficiency
- Calculate losses for electric resistors

Conductive heat transfer

- *Explaining* conduction principles
- ***Calculate* conductive heat transfer**
- Using *thermal resistances* and insulation values

ASSIGNMENT: ELECTRIC KETTLE

*What is the **efficiency** of an electric kettle when the water is almost boiling (T=100 °C)?*

Data/assumptions/simplifications:

- Wall temperature outside 50°C
- Diameter 10 cm
- Height 20 cm
- Plastic wall:
  - Thickness 2,7 mm
  - k = 0,20 W / (m · K)
- Neglect losses through top and bottom
- It can be treated as plane wall
- Electric power P_{el} = 930 W

$$\dot{Q} = -k\,A\frac{\Delta T}{\Delta x}$$

Conduction flat wall- summarized

- Fourier's law for plane surface:
  - Temperature difference $\Delta T = T_2 - T_1$ (K)
  - Thickness $\Delta x = x_2 - x_1$ (m)
  - Surface area A (m^2)
  - Thermal conductivity k (W / (m · K))

Heat transfer rate: $\dot{Q} = -k\,A\,\dfrac{\Delta T}{\Delta x}$ (W)

Heat flux: $\dot{q} = -k\,\dfrac{\Delta T}{\Delta x}$ (W / m^2)

Next: model heat flow as thermal resistance network

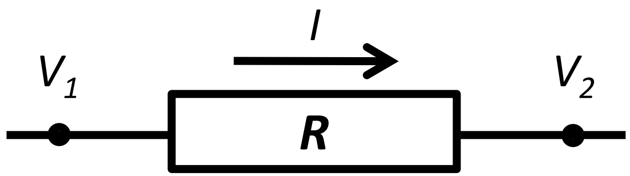
(Make it easier for calculations)

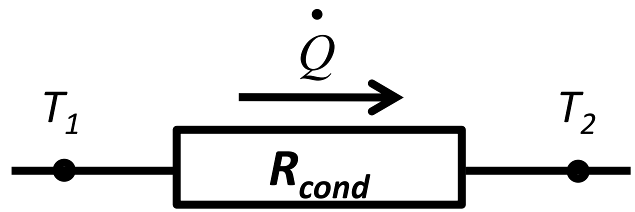
Learning objectives lecture 2

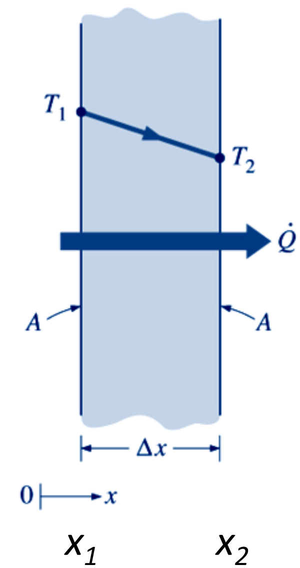
- **Energy in general**
  - Using an energy balance
  - Determine efficiency
  - Calculate losses for electric resistors
- **Conductive heat transfer**
  - *Explaining* conduction principles
  - *Calculate* conductive heat transfer
  - **Using *thermal resistances* and insulation values**

Analogy electricity - heat

Electric resistor ↔ Flat wall as thermal resistor







Voltage difference	$V_1 - V_2$	↔	Temperature difference	$T_1 - T_2$
Current flow	I	↔	Heat flow	$\dot{Q}$
Electrical resistance	R	↔	Thermal resistance	R_{cond}

unit: Ω **unit: K/W**

Analogy electricity - heat

Heat conduction through plane wall

Fourier's law:

$$\dot{Q} = -k\,A\,\frac{T_2 - T_1}{\Delta x}$$

$$= +k\,A\,\frac{T_1 - T_2}{\Delta x}$$

$$= \frac{T_1 - T_2}{\dfrac{\Delta x}{k\,A}}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{cond}} \text{ with } R_{cond} = \frac{\Delta x}{kA} \quad \text{(K/W)}$$

Heat transfer in "Ohmic way"!

Electrical resistance

Ohm's law:

$$I = \frac{V_1 - V_2}{R}$$

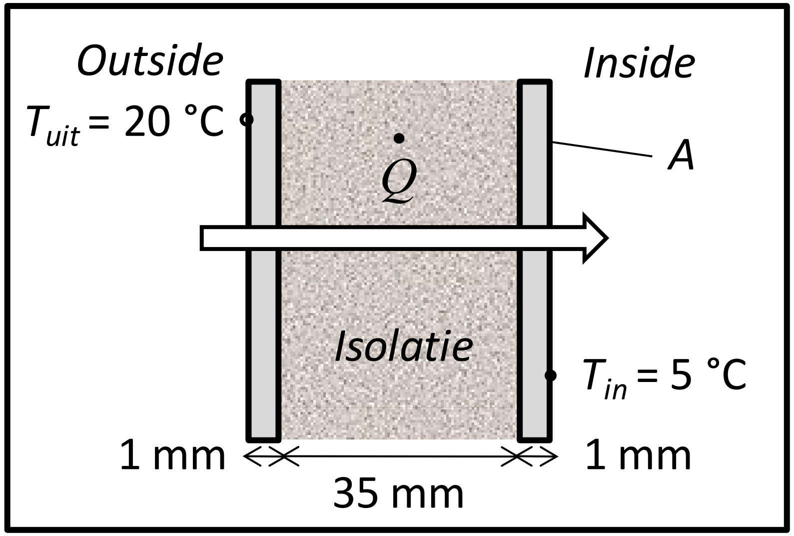
$V_1 - V_2$	$\leftrightarrow$	$T_1 - T_2$
I	$\leftrightarrow$	$\dot{Q}$
R	$\leftrightarrow$	R_{cond}

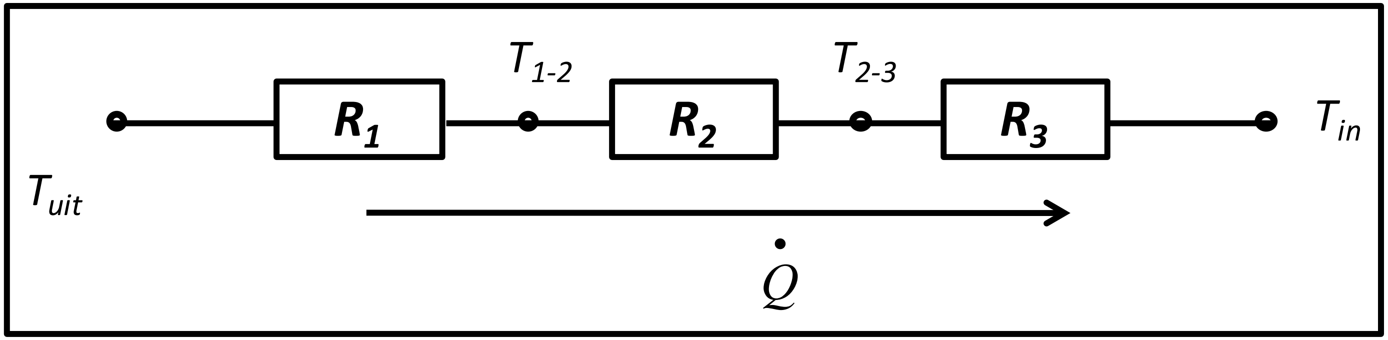
$$\Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{cond}}$$

THERMAL RESISTANCE NETWORKS

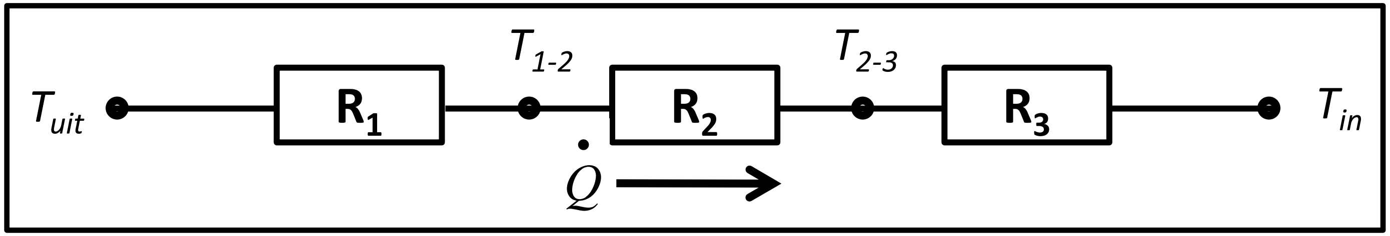
Fridge wall: heat flow $\dot{Q}$ through three resistors

- Outer wall (R_1)
- Insulation (R_2)
- Inner wall (R_3)

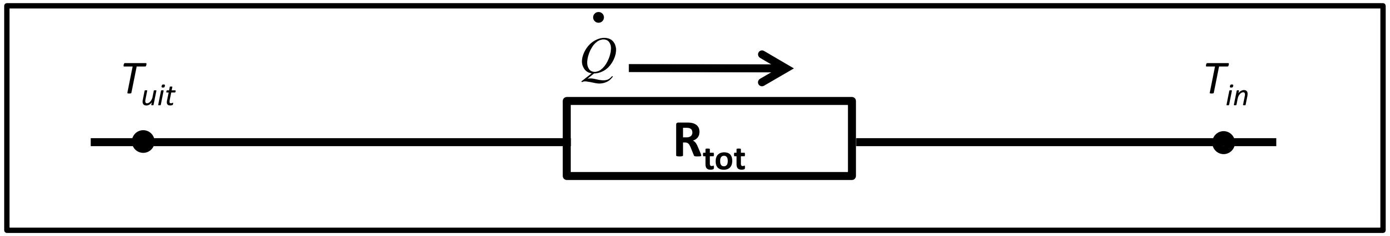




THERMAL RESISTANCE NETWORKS



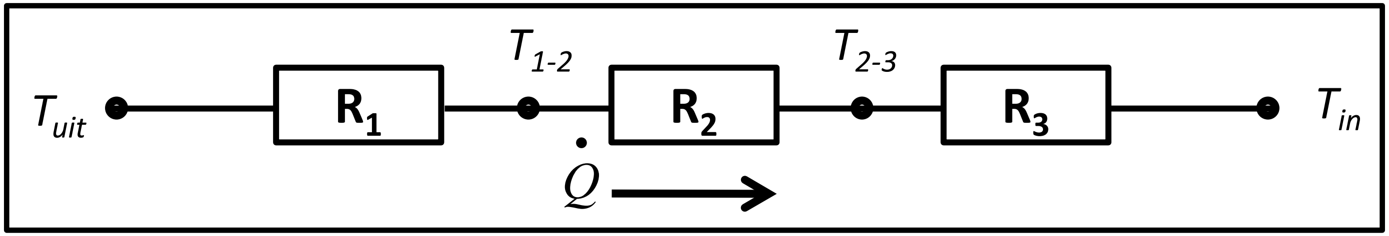
Total resistance $R_{tot} = R_1 + R_2 + R_3$ (series)



$\Rightarrow$ heat flow $$\dot{Q} = \frac{T_{uit} - T_{in}}{R_{tot}}$$

Resistors in series: total resistance equals the sum of the individual resistances

Thermal resistance networks



$T_{1\text{-}2}$, $T_{2\text{-}3}$? Use: $\dot{Q}$ = constant throughout the resistors

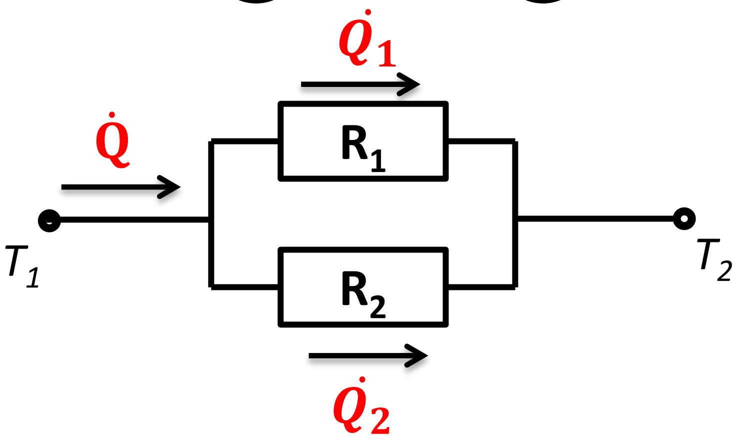
$$\dot{Q} = \frac{T_{uit} - T_{in}}{R_{tot}}$$

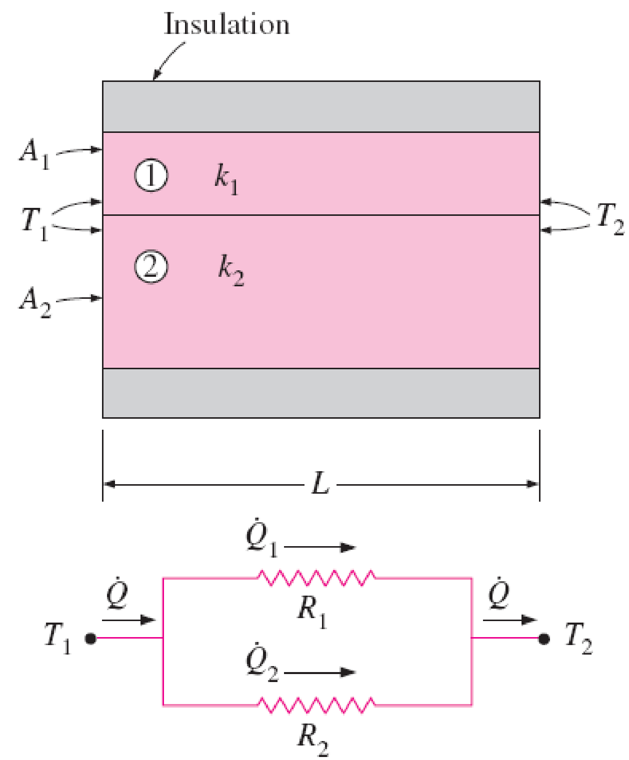
$$= \frac{T_{uit} - T_{1-2}}{R_1} = \frac{T_{1-2} - T_{2-3}}{R_2} = \frac{T_{2-3} - T_{in}}{R_3} = \frac{T_{uit} - T_{2-3}}{R_1 + R_2} = \frac{T_{1-2} - T_{in}}{R_2 + R_3}$$

⇒ Solve all unknown temperatures by comparing total and partial resistors in series, similar to voltage in an electrical circuit

⇒ **Next: parallel thermal resistors**

Thermal resistance networks





$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

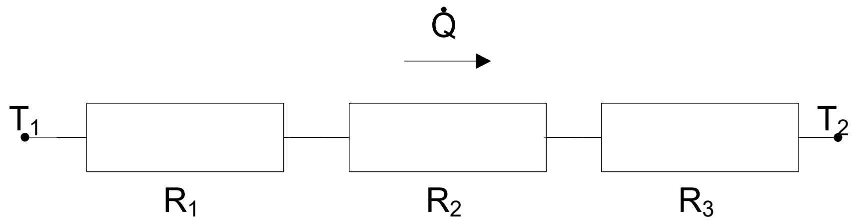
$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$

Thermal resistance networks

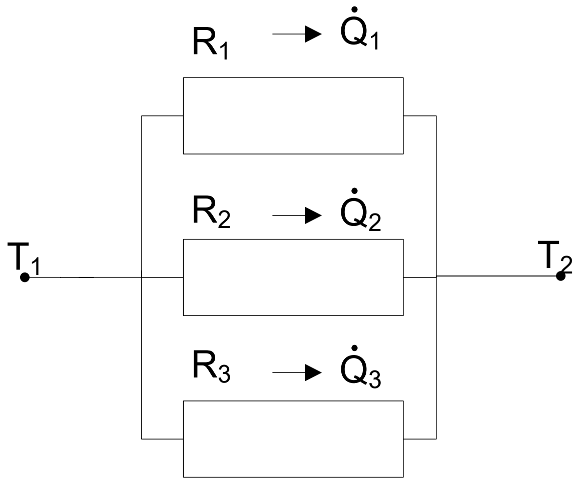
Series Resistors



$$R_{tot} = \sum_i R_i$$

(Add Resistors)

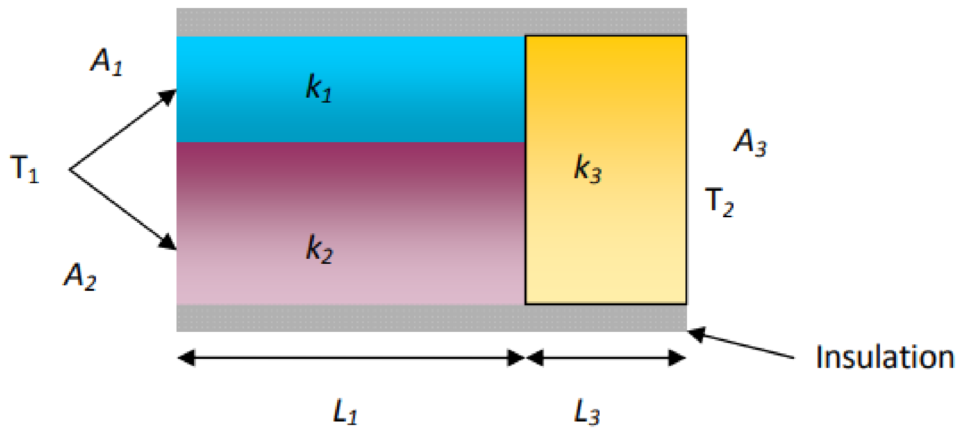
Parallel Resistors

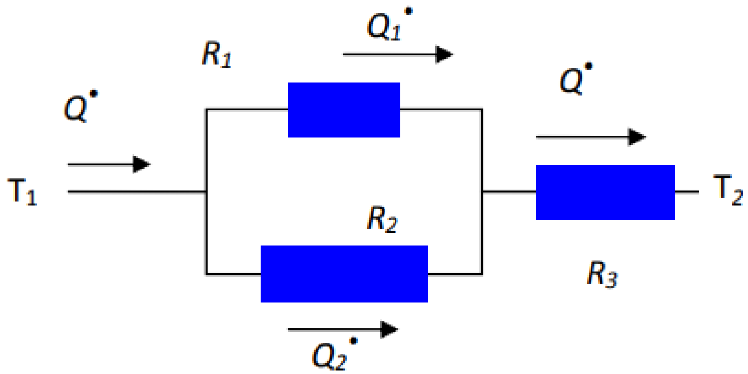


$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

(Add Heat Flows)

THERMAL RESISTANCE NETWORKS





$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

Resistance and insulation

Thermal resistance plane surface:

$$R = \frac{\Delta x}{kA} \qquad (\frac{\mathrm{K}}{\mathrm{W}})$$

Alternative concept: insulation value (building materials):

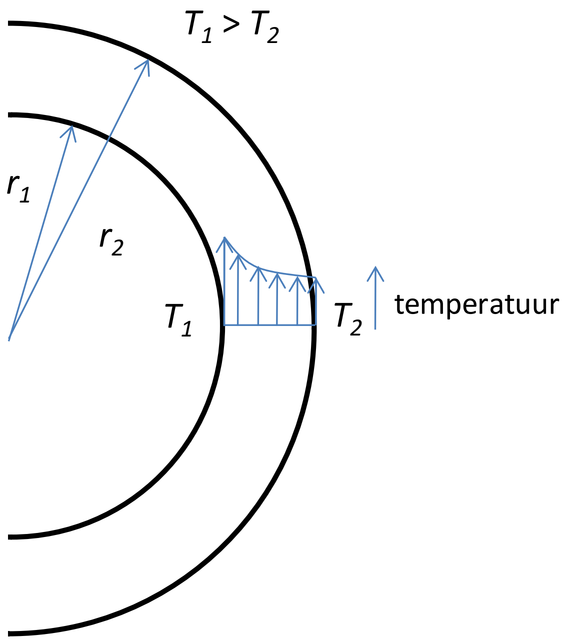
$$R - value = \frac{\Delta x}{k} = R \cdot A \qquad (\frac{\mathrm{m}^2 \cdot \mathrm{K}}{\mathrm{W}})$$

$\Rightarrow$ Insulation value is the heat resistance of 1 m^2 of material

Material	***R*-value (m^2 · K / W)**
Bricks (wall)	0,26
Rockwool 5 cm thick	1,45
Glasswool 11 cm thick	2,50

NB: Often the symbol *R* is used for both concepts. Look at the units!

Conduction in non-planar surface



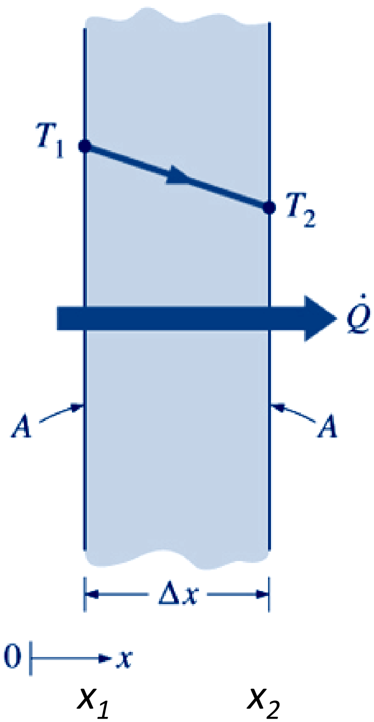
Small radius: small flow-through surface→ high resistance→ large temperature *gradient*

(In areas with a high resistance the temperature decreases more in the direction of the heat flow)

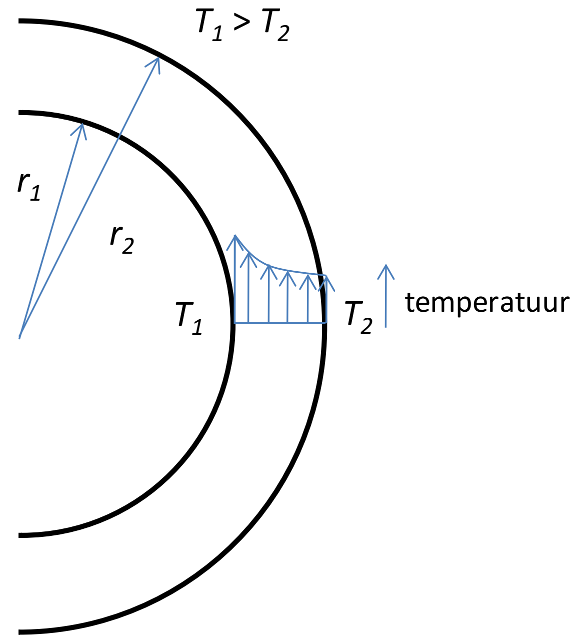
$$\dot{Q} = \frac{T_1 - T_2}{R}$$

CONDUCTION IN NON-PLANAR SURFACE

Plane wall



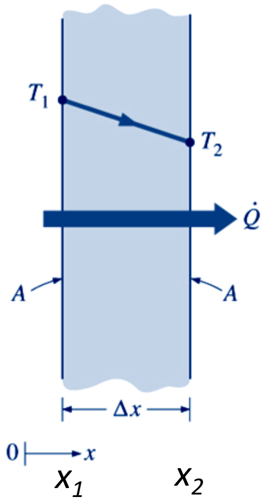
Curved wall



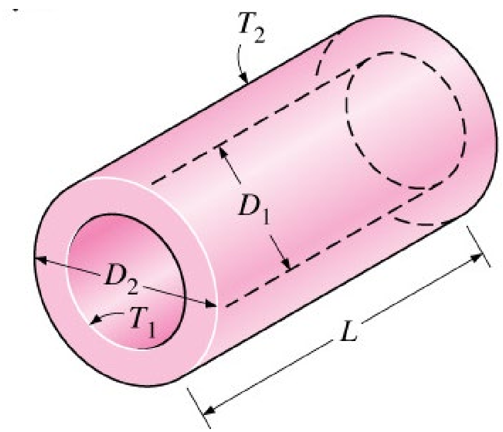
- Surface area constant over *x*
- Temperature profile linearly decreasing (Fourier's law)
- Temperature gradient (= 'slope') constant

- Surface area increases with *r*
- Temperature profile decreasing concave up
- Temperature gradient smaller with increasing radius *r*

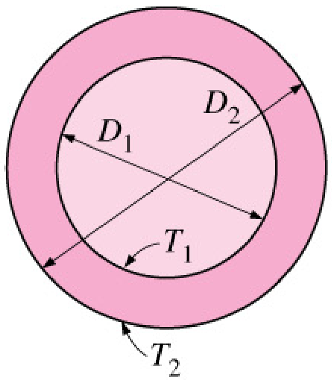
Various conduction resistances



Plane wall



Cilindrical pipe



Spherical shell

$$R = \frac{\Delta x}{kA}$$

$$R = \frac{\ln(\frac{D_2}{D_1})}{2\pi L k}$$

$$R = \frac{D_2 - D_1}{2\pi k D_1 D_2}$$

$$\boxed{\dot{Q} = \frac{T_1 - T_2}{R}}$$

Learning objectives lecture 2

- **Energy in general**
  - Using an energy balance
  - Determine efficiency
  - Calculate losses electric resistors

- **Conductive heat transfer**
  - *Explaining* conduction principles
  - *Calculate* conductive heat transfer
  - Using *thermal resistances* and insulation values

Summary lecture 2 (1/2)

- Efficiency $$\eta = \frac{\text{useful work}}{\text{inputted energy}} = \frac{\text{useful power}}{\text{inputted power}}$$

  Justify the applied definition!

- Heat transfer rate $\dot{Q}$ (W); Heat flux: $\dot{q} = \dot{Q}/A$ (W/m^2)

  $\uparrow$

  "Q-flux"

- Heat dissipation electric resistor

  $$\dot{Q} = I^2 R \quad (\text{W}) \qquad \text{with} \qquad R = \rho \frac{L}{A} \qquad (\Omega)$$

SUMMARY LECTURE 2 (2/2)

- Conduction: heat transfer between molecules
- Different appearances of Fourier's law
  - Plane surface: $\dot{Q} = -k\,A\,\dfrac{T_2 - T_1}{x_2 - x_1} = \dfrac{T_1 - T_2}{R}$ with $R = \dfrac{\Delta x}{kA}$ $\quad (\dfrac{\mathrm{K}}{\mathrm{W}})$
  - Cylindrical tube: $\dot{Q} = \dfrac{T_1 - T_2}{R}$ with $R = \dfrac{\ln(\dfrac{D_2}{D_1})}{2\pi\,L\,k}$
  - Spherical shell: $\dot{Q} = \dfrac{T_1 - T_2}{R}$ with $R = \dfrac{D_2 - D_1}{2\pi\,kD_1\,D_2}$
- Thermal resistors: for ease of calculation
- Insulation value: *R*-value = $R \cdot A \quad (\dfrac{\mathrm{m}^2 \cdot \mathrm{K}}{\mathrm{W}})$