Example Examination
Fluid Mechanics I
Module 7, 201500321, 2015

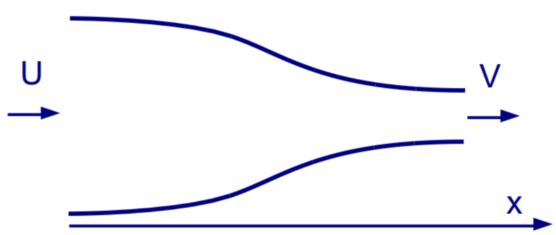
Problem 1 [2 points.]
Given is the flow field

$$u(x,y,z,t) = \frac{1+t}{e^x}, \quad v(x,y,z,t) = \frac{sin(t)}{y^2}, \quad w(x,y,z,t) = zt, \tag{1}$$

where u, v and w are the velocity components in x, y and z direction respectively.

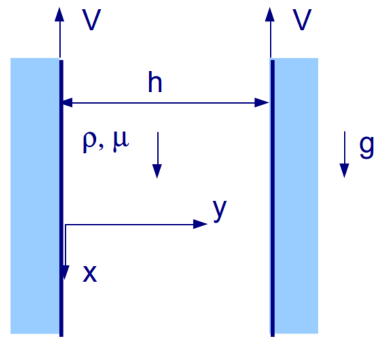
- *Compute the trajectory $x_p(t)$, $y_p(t)$, $z_p(t)$ of a dust particle that is located in the point $x = 0$, $y = 1$, $z = 1$ at $t = 0$.*

Problem 2 [2 points.]
An incompressible fluid with density ρ enters a construction through an opening of area size A at uniform velocity U and uniform pressure p_{in}, and leaves the construction through an opening of area size $\frac{1}{2}A$ at uniform velocity $V = 2U$ and uniform pressure p_{out}. Viscosity and gravity can be neglected and the flow is steady.



- *Compute the force in x-direction by the fluid on the pipe.*

Problem 3 [2 POINTS.]
Incompressible viscous water with density ρ falls steadily between two vertical parallel plates, both plates move upward with speed V. The flow is laminar and fully developed. The gap width between the plates is h, gravity acceleration is g, the viscosity is μ and the pressure is constant: p_o.



(a) *Derive an expression for the velocity profile $u(y)$, starting from the reduced Navier-Stokes equations, with $y = 0$ on the left plate.*
(b) *Derive an expression for the shear stress by the flow on the left plate.*

Problem 4 [1 POINT.]
Consider the following temperature distribution:

$$T(x,t) = [a\cos(\lambda x) - b\sin(\lambda x)]\exp(-\beta t).$$

Determine the parameter β such that $T(x,t)$ satisfies the following diffusion equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}.$$

Problem 5 [2 POINTS.]
In the steady, inviscid and compressible flow around an object of an ideal (perfect) gas with given γ and gas constant R, and given temperature T_∞ and Mach number M_∞ at infinity, the velocity is measured at some point not far from the object: u.

(a) *Compute the temperature T at that point.*
(b) *Show that the total enthalpy is constant along streamlines.*

Appendix A

Formulas available during the Exam

A.1 Introduction

A.1.1 Particle trajecories

$$\frac{dx_p}{dt} = u(x_p(t), y_p(t), z_p(t), t), \quad \frac{dy_p}{dt} = v(x_p(t), y_p(t), z_p(t), t), \quad \frac{dz_p}{dt} = w(x_p(t), y_p(t), z_p(t), t). \tag{A.1}$$

A.1.2 Streamlines

$$\frac{dx_p}{ds} = u(x_p(s), y_p(s), z_p(s), t), \quad \frac{dy_p}{ds} = v(x_p(s), y_p(s), z_p(s), t), \quad \frac{dz_p}{ds} = w(x_p(s), y_p(s), z_p(s), t). \tag{A.2}$$

A.2 Mass Conservation

A.2.1 Integral form

$$\int_{V(t)} \frac{\partial \rho}{\partial t}\, dV + \int_{A(t)} \rho\, (u_j n_j)\, dA = 0. \tag{A.3}$$

A.2.2 Differential form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0. \tag{A.4}$$

A.3 Momentum Conservation

A.3.1 Integral form

$$\int_{V(t)} \frac{\partial}{\partial t}(\rho u_i)\, dV + \int_{A(t)} \rho u_i \,(u_j n_j)\, dA = \int_{A(t)} \sigma_{ij} n_j \, dA + \int_{V(t)} \rho g_i \, dV, \quad i = 1, 2, 3. \tag{A.5}$$

A.3.2 Stress tensor

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \tag{A.6}$$

$$\tau_{ij} = \mu(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) - \frac{2}{3}\mu\delta_{ij}\frac{\partial u_k}{\partial x_k}, \quad \delta_{ij} = \begin{cases} 0, & i \neq j, \\ 1, & i = j. \end{cases} \tag{A.7}$$

A.3.3 Cauchy equation

Tension vector $\boldsymbol{t}$ by medium A on medium B, $\boldsymbol{n}$ pointing to A:

$$t_i = \sigma_{ij} n_j, \quad i = 1, 2, 3. \tag{A.8}$$

A.3.4 Differential form (Navier-Stokes)

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}\left(\rho u_i u_j - \sigma_{ij}\right) = \rho g_i. \tag{A.9}$$

A.3.5 Reduced Navier-Stokes

$$\frac{\partial p}{\partial x} - \mu\frac{\partial^2 u}{\partial y^2} = \rho g_1, \quad \frac{\partial p}{\partial y} = \rho g_2. \tag{A.10}$$

A.3.6 Material derivative

Time derivative while traveling with the flow:

$$\frac{Df}{Dt} \equiv \frac{\partial f}{\partial t} + u_j \frac{\partial f}{\partial x_j}, \quad \text{for any function } f(x, y, z, t) \tag{A.11}$$