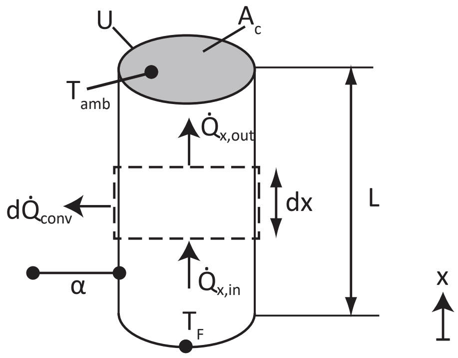
2.7 Pin-fin cooling on gas turbine blades ★★

All steps taken in tasks a), b) and c) work towards deriving the rate of heat loss from the fin. All heat conducted through the base will be dissipated to the surroundings. Therefore the dissipated heat equals the heat conducted through the base of the fin. From Fourier's law we know: $\dot{Q} = -\lambda A_{\mathrm{c}} \frac{dT}{dx}|_{x=0}$.
To get to this expression, we need to determine the temperature profile inside the fin to determine the heat flux that is carried away from the fin.
To do this, first the heat conduction equation inside the fin should be derived, which can be integrated later on to determine the temperature profile inside the fin. The heat conduction equation inside the fin can be derived based on the energy balance of infinitesimal element inside the fin.

a) *Derive the heat conduction equation for the given problem.*

1) Setting up the energy balance:



Energy balance for an infinitesimal element of the fin:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\mathrm{out}} \tag{2.79}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\mathrm{x,in}} - \dot{Q}_{\mathrm{x,out}} - d\dot{Q}_{\mathrm{conv}}} \tag{2.80}$$

2) Defining the fluxes:

Ingoing conductive heat flux:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{x,in}} = -\lambda \cdot A_{\mathrm{c}} \cdot \frac{dT}{dx}} \tag{2.81}$$

For an infinitesimal element, the outgoing conductive heat flux can be approximated by use of the Taylor series expansion:

$$\dot{Q}_{\text{x,out}} = \dot{Q}_{\text{x,in}} + \frac{\partial \dot{Q}_{\text{x,out}}}{\partial x} \cdot dx \tag{2.82}$$

$$\boxed{\rightarrow \dot{Q}_{\text{x,out}} = -\lambda \cdot A_{\text{C}} \cdot \frac{dT}{dx} + \frac{\partial}{\partial x}\left(-\lambda \cdot A_{\text{c}} \cdot \frac{\partial T}{\partial x}\right) \cdot dx} \tag{2.83}$$

Convective heat flux leaving the infinitesimal element:

$$\boxed{\rightarrow d\dot{Q}_{\text{conv}} = \alpha \cdot A_{\text{s}} \cdot (T(x) - T_{\text{amb}}) = \alpha \cdot U \cdot dx \cdot (T(x) - T_{\text{amb}})} \tag{2.84}$$

3) Inserting and rewriting:

Inserting the definitions of the heat fluxes yields:

$$0 = \dot{Q}_{\text{x,in}} - \dot{Q}_{\text{x,out}} - d\dot{Q}_{\text{conv}} \tag{2.85}$$

$$\lambda \cdot A_{\text{c}} \cdot 0 = \frac{d^2T}{dx^2} \cdot dx - \alpha \cdot U \cdot dx \cdot (T(x) - T_{\text{amb}}) \tag{2.86}$$

Rewriting yields:

$$\boxed{\rightarrow 0 = \frac{\partial^2 T}{\partial x^2} - \frac{\alpha \cdot U}{\lambda \cdot A_{\text{c}}} \cdot (T(x) - T_{\text{amb}})} \tag{2.87}$$

b) *Derive the function of the temperature profile inside the fin.*

4) Defining the boundary conditions:

The differential equation can be solved, by using the boundary conditions. Which can be described as:

The temperature at the fin base is:

$$\boxed{\rightarrow T(x = 0) = T_{\text{F}}} \tag{2.88}$$

The temperature at the fin head is:

$$\boxed{\rightarrow T(x = L) = T_{\text{amb}}} \tag{2.89}$$

5) Solving the equation:

As we are dealing with a 2$^{\text{nd}}$ order differential equation, it should be homogenized to solve this equation.

The differential equation for a steady-state fin can be solved by introducing the parameter Θ and the fin parameter m:

$$\Theta = T(x) - T_{\text{amb}} \tag{2.90}$$

$$m^2 = \frac{\alpha \cdot U}{\lambda \cdot A_c} \tag{2.91}$$

Substitution into the differential equation yields:

$$0 = \frac{\partial^2 T}{\partial x^2} - \frac{\alpha \cdot U}{\lambda \cdot A_c} \cdot (T(x) - T_{amb}) \tag{2.92}$$

$$0 = \frac{\partial^2 \Theta}{\partial x^2} - m^2 \cdot \Theta \tag{2.93}$$

A standard solution for this differential equation is (see formula sheet):

$$\Theta(x) = A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x) \tag{2.94}$$

Furthermore, the boundary conditions can be rewritten to:

$$\Theta(x = 0) = T_F - T_{amb} = \Theta_F \tag{2.95}$$

$$\Theta(x = L) = T_{amb} - T_{amb} = 0 \tag{2.96}$$

The constants A and B can be determined by using the boundary conditions.

Constant A yields from:

$$\Theta(x = 0) = A = \Theta_F \tag{2.97}$$

And constant B yields from:

$$\Theta(x = L) = A \cdot \cosh(m \cdot L) + B \cdot \sinh(m \cdot L) = 0 \tag{2.98}$$

$$B = -A \cdot \frac{1}{\tanh(m \cdot L)} \tag{2.99}$$

Plugging in the constants yields the equation for the fin temperature:

$$\Theta(x) = A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x) \tag{2.100}$$

$$\Theta(x) = \Theta_F \cdot \cosh(m \cdot x) - \Theta_F \cdot \frac{1}{\tanh(m \cdot L)} \cdot \sinh(m \cdot x) \tag{2.101}$$

$$\boxed{\rightarrow \Theta(x) = \Theta_F \cdot \left(\cosh(m \cdot x) - \frac{\sinh(m \cdot x)}{\tanh(m \cdot L)}\right)} \tag{2.102}$$

Derive the expression for the heat flux as a function of the given variables.

6) Determining the rate of heat loss:

Dissipated heat through the fin:

$$\dot{Q} = -\lambda \cdot A_\mathrm{C} \cdot \frac{dT}{dx}|_{x=0} = -\lambda \cdot A_\mathrm{C} \cdot \frac{d\Theta}{dx}|_{x=0} \tag{2.103}$$

with

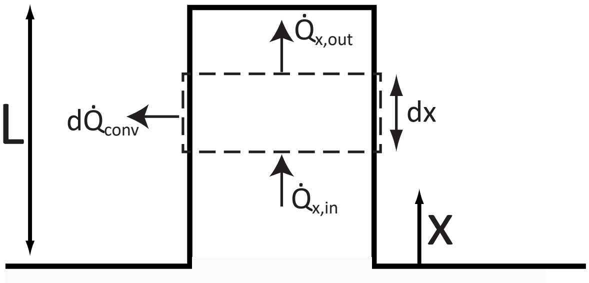
$$\frac{d\Theta}{dx}|_{x=0} = \Theta_\mathrm{F} \cdot \left(m \cdot \sinh(m \cdot 0) - \frac{m \cdot \cosh(m \cdot 0)}{\tanh(m \cdot L)} \right) = -\frac{\Theta_\mathrm{F} \cdot m}{\tanh(m \cdot L)} \tag{2.104}$$

So:

$$\boxed{\rightarrow \dot{Q} = \lambda \cdot A_\mathrm{C} \cdot \frac{\Theta_\mathrm{F} \cdot m}{\tanh(m \cdot L)}} \tag{2.105}$$

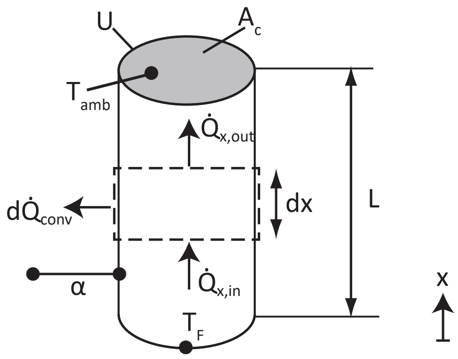
2.8 Heat flux material ★★

a) *Calculate the ratio between the heat flow of the aluminium and copper fin.*



In the case of a fin with an adiabatic head, the temperature profile and heat flux can be directly found on the formula sheet. If this not had been the case it could have been derived in the following way (step 1-6):

1) Setting up the energy balance:



Energy balance for an infinitesimal element of the fin:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{2.106}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\text{x,in}} - \dot{Q}_{\text{x,out}} - d\dot{Q}_{\text{conv}}} \tag{2.107}$$

2) Defining the fluxes:

Ingoing conductive heat flux:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{x,in}} = -\lambda \cdot A_{\mathrm{c}} \cdot \frac{dT}{dx}} \tag{2.108}$$

For an infinitesimal element, the outgoing conductive heat flux can be approximated by use of the Taylor series expansion:

$$\dot{Q}_{\mathrm{x,out}} = \dot{Q}_{\mathrm{x,in}} + \frac{\partial \dot{Q}_{\mathrm{x,out}}}{\partial x} \cdot dx \tag{2.109}$$

$$\boxed{\rightarrow \dot{Q}_{\mathrm{x,out}} = -\lambda \cdot A_{\mathrm{C}} \cdot \frac{dT}{dx} + \frac{\partial}{\partial x}\left(-\lambda \cdot A_{\mathrm{c}} \cdot \frac{\partial T}{\partial x}\right) \cdot dx} \tag{2.110}$$

Convective heat flux leaving the infinitesimal element:

$$\boxed{\rightarrow d\dot{Q}_{\mathrm{conv}} = \alpha \cdot A_{\mathrm{s}} \cdot (T(x) - T_{\mathrm{amb}}) = \alpha \cdot U \cdot dx \cdot (T(x) - T_{\mathrm{amb}})} \tag{2.111}$$

3) Inserting and rewriting:

Inserting the definitions of the heat fluxes yields:

$$0 = \dot{Q}_{\mathrm{x,in}} - \dot{Q}_{\mathrm{x,out}} - d\dot{Q}_{\mathrm{conv}} \tag{2.112}$$

$$\lambda \cdot A_{\mathrm{c}} \cdot 0 = \frac{d^2T}{dx^2} \cdot dx - \alpha \cdot U \cdot dx \cdot (T(x) - T_{\mathrm{amb}}) \tag{2.113}$$

Rewriting yields:

$$\boxed{\rightarrow 0 = \frac{\partial^2 T}{\partial x^2} - \frac{\alpha \cdot U}{\lambda \cdot A_{\mathrm{c}}} \cdot (T(x) - T_{\mathrm{amb}})} \tag{2.114}$$

4) Defining the boundary conditions:

The differential equation can be solved, by using the boundary conditions. Which can be described as:

The temperature at the fin base is:

$$\boxed{\rightarrow T(x=0) = T_{\mathrm{F}}} \tag{2.115}$$

The tip of the fin is adiabatic, and therefore the rate of heat transfer is zero at the tip ($\dot{Q} = \lambda A_{\mathrm{c}} \frac{\partial T}{\partial x}|_{x=L} = 0$):

$$\boxed{\rightarrow \frac{\partial T}{\partial x}\Big|_{x=L} = 0} \tag{2.116}$$

5) Solving the equation:

As we are dealing with a 2nd order differential equation, it should be homogenized to solve this equation.

The differential equation for a steady-state fin can be solved by introducing the parameter Θ and the fin parameter m:

$$\Theta = T(x) - T_{\mathrm{amb}} \tag{2.117}$$

$$m^2 = \frac{\alpha \cdot U}{\lambda \cdot A_{\mathrm{c}}} \tag{2.118}$$

Substitution into the differential equation yields:

$$0 = \frac{\partial^2 T}{\partial x^2} - \frac{\alpha \cdot U}{\lambda \cdot A_{\mathrm{c}}} \cdot (T(x) - T_{\mathrm{amb}}) \tag{2.119}$$

$$0 = \frac{\partial^2 \Theta}{\partial x^2} - m^2 \cdot \Theta \tag{2.120}$$

A standard solution for this differential equation is (see formula sheet):

$$\Theta(x) = A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x) \tag{2.121}$$

With its derivative with respect to x:

$$\frac{d\Theta(x)}{dx} = A \cdot m \cdot \sinh(m \cdot x) + B \cdot m \cdot \cosh(m \cdot x) = 0 \tag{2.122}$$

Furthermore, the boundary conditions can be rewritten to:

$$\Theta(x = 0) = T_{\mathrm{F}} - T_{\mathrm{amb}} = \Theta_{\mathrm{F}} \tag{2.123}$$

$$\frac{\partial \Theta}{\partial x}\Big|_{x=L} = 0 \tag{2.124}$$

The constants A and B can be determined by using the boundary conditions.

Constant A yields from:

$$\Theta(x = 0) = A = \Theta_{\mathrm{F}} \tag{2.125}$$

And constant B yields from:

$$\frac{d\Theta(x)}{dx}\Big|_{x=L} = A \cdot m \cdot \sinh(m \cdot L) + B \cdot m \cdot \cosh(m \cdot L) = 0 \tag{2.126}$$

$$B = -A \cdot \tanh(m \cdot L) \tag{2.127}$$

Plugging in the constants yields the equation for the fin temperature:

$$\Theta(x) = A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x) \tag{2.128}$$

$$\Theta(x) = \Theta_{\mathrm{F}} \cdot \cosh(m \cdot x) - \Theta_{\mathrm{F}} \cdot \tanh(m \cdot L) \cdot \sinh(m \cdot x) \tag{2.129}$$

$$\boxed{\rightarrow \Theta(x) = \Theta_{\mathrm{F}} \left(\cosh(m \cdot x) - \tanh(m \cdot L) \cdot \sinh(m \cdot x)\right)} \tag{2.130}$$

Or its alternative notation:

$$\boxed{\rightarrow \Theta(x) = \Theta_{\mathrm{F}} \cdot \frac{\cosh(m \cdot (L - x))}{\cosh(m \cdot L)}} \tag{2.131}$$

6) Determining the rate of heat loss:

Dissipated heat through the fin:

$$\dot{Q} = -\lambda \cdot A_{\mathrm{C}} \cdot \frac{dT}{dx}\Big|_{x=0} = -\lambda \cdot A_{\mathrm{C}} \cdot \frac{d\Theta}{dx}\Big|_{x=0} \tag{2.132}$$

with

$$\frac{d\Theta}{dx}\Big|_{x=0} = \Theta_{\mathrm{F}} \cdot \left(m \cdot \sinh(m \cdot 0) - m \cdot \tanh(m \cdot L) \cdot \cosh(m \cdot 0)\right) \tag{2.133}$$

$$\frac{d\Theta}{dx}\Big|_{x=0} = -m \cdot \Theta_{\mathrm{F}} \cdot \tanh(m \cdot L) \tag{2.134}$$

So:

$$\boxed{\rightarrow \dot{Q} = \lambda \cdot A_{\mathrm{C}} \cdot m \cdot \Theta_{\mathrm{F}} \cdot \tanh(m \cdot L)} \tag{2.135}$$

<u>Aluminium fin:</u>

Heat flux:

$$\dot{Q}_{\mathrm{A}} = \lambda_{\mathrm{A}} \cdot A_{\mathrm{c}} \cdot m_{\mathrm{A}} \cdot \Theta_{\mathrm{F}} \cdot \tanh(m_{\mathrm{A}} \cdot L_{\mathrm{A}}) \tag{2.136}$$

Fin parameter (plane fin):

$$m_{\mathrm{A}} = \sqrt{\frac{2 \cdot \alpha}{\lambda_{\mathrm{A}} \cdot \delta}} \tag{2.137}$$

Temperature profile:

$$\Theta_{\mathrm{A}}(x) = \Theta_{\mathrm{F}} \cdot \frac{\cosh[m_{\mathrm{A}} \cdot (L_{\mathrm{A}} - x)]}{\cosh[m_{\mathrm{A}} \cdot L_{\mathrm{A}}]} \tag{2.138}$$

7) Determining the ratio between the heat flow of the aluminium and copper fin:

Head temperature:

$$\Rightarrow \Theta_{\mathrm{A}}(L) = \frac{\Theta_{\mathrm{F}}}{\cosh\left[m_{\mathrm{A}} \cdot L_{\mathrm{A}}\right]} \tag{2.139}$$

Copper fin:

Heat flux:

$$\dot{Q}_{\mathrm{C}} = \lambda_{\mathrm{C}} \cdot A_{\mathrm{c}} \cdot m_{\mathrm{C}} \cdot \Theta_{\mathrm{F}} \cdot \tanh\left(m_{\mathrm{C}} \cdot L_{\mathrm{C}}\right) \tag{2.140}$$

Fin parameter (plane fin):

$$m_{\mathrm{c}} = \sqrt{\frac{2 \cdot \alpha}{\lambda_{\mathrm{c}} \cdot \delta}} \tag{2.141}$$

Temperature profile:

$$\Theta_{\mathrm{C}}(x) = \Theta_{\mathrm{F}} \cdot \frac{\cosh\left[m_{\mathrm{C}} \cdot (L_{\mathrm{C}} - x)\right]}{\cosh\left[m_{\mathrm{C}} \cdot L_{\mathrm{C}}\right]} \tag{2.142}$$

Head temperature:

$$\Rightarrow \Theta_{\mathrm{C}}(L) = \frac{\Theta_{\mathrm{F}}}{\cosh\left[m_{\mathrm{C}} \cdot L_{\mathrm{C}}\right]} \tag{2.143}$$

Length ratio:

Head temperatures remains unchanged:

$$\Theta_{\mathrm{A}}(L) = \Theta_{\mathrm{C}}(L) \tag{2.144}$$

$$\Rightarrow m_{\mathrm{A}} \cdot L_{\mathrm{A}} = m_{\mathrm{C}} \cdot L_{\mathrm{C}} \tag{2.145}$$

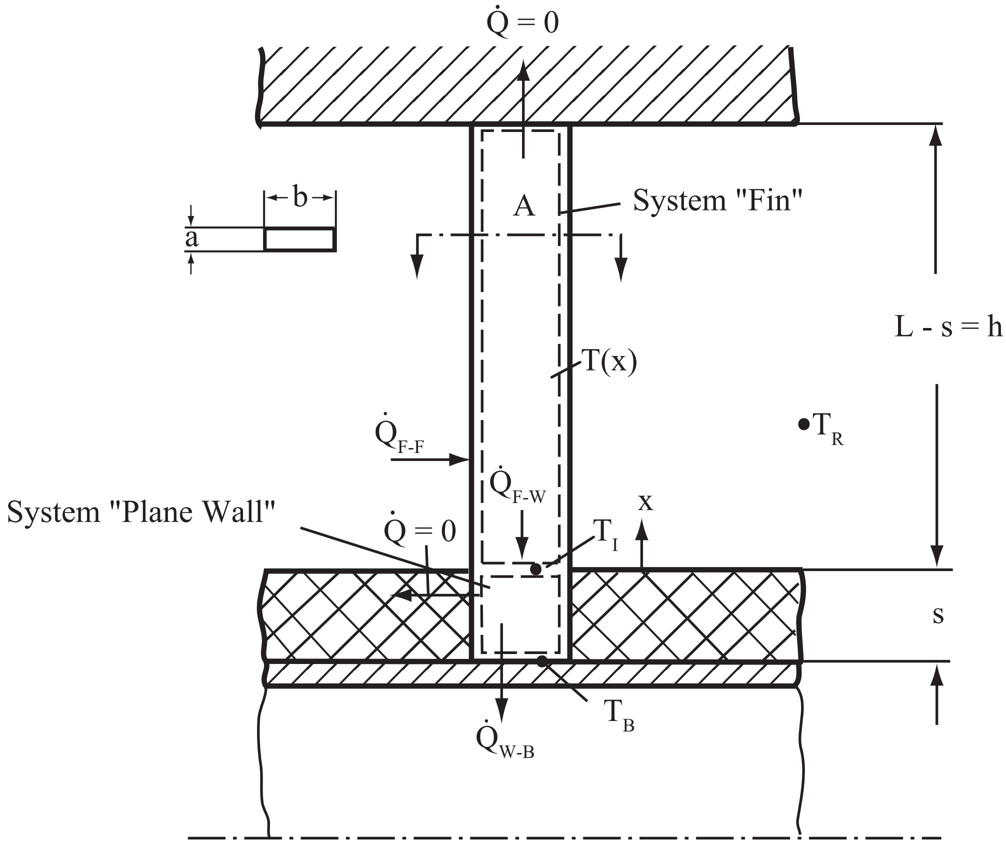
Heat flux ratio:

$$\boxed{\rightarrow \frac{\dot{Q}_{\mathrm{A}}}{\dot{Q}_{\mathrm{C}}} = \frac{\lambda_{\mathrm{A}} \cdot A_{\mathrm{c}} \cdot \sqrt{\frac{2 \cdot \alpha}{\lambda_{\mathrm{A}} \cdot \delta}} \cdot \Theta_{\mathrm{F}} \cdot \tanh\left(m_{\mathrm{A}} \cdot L_{\mathrm{A}}\right)}{\lambda_{\mathrm{C}} \cdot A_{\mathrm{c}} \cdot \sqrt{\frac{2 \cdot \alpha}{\lambda_{\mathrm{c}} \cdot \delta}} \cdot \Theta_{\mathrm{F}} \cdot \tanh\left(m_{\mathrm{C}} \cdot L_{\mathrm{C}}\right)} = \frac{\lambda_A^{1/2}}{\lambda_C^{1/2}}} \tag{2.146}$$

2.9 Pipe fastening ★ ★ ★

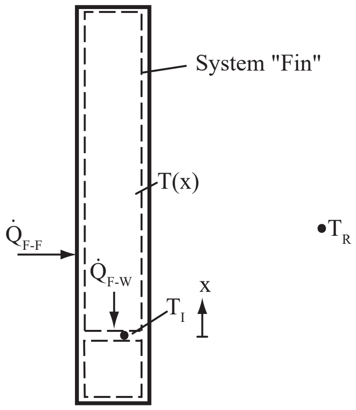
a) *Calculate the heat $\dot{Q}$ from one steel band absorbed by the brine.*

Note that we are dealing with two systems that are connected in series. Where the first system **Fin** conducts heat towards system **Plane Wall**, but the **Fin** system gains heat at the same time as a consequence of convection. Once at the intersection, only conduction will take place through the **Plane Wall** system.



1) Setting up energy balances:

System **Fin**:

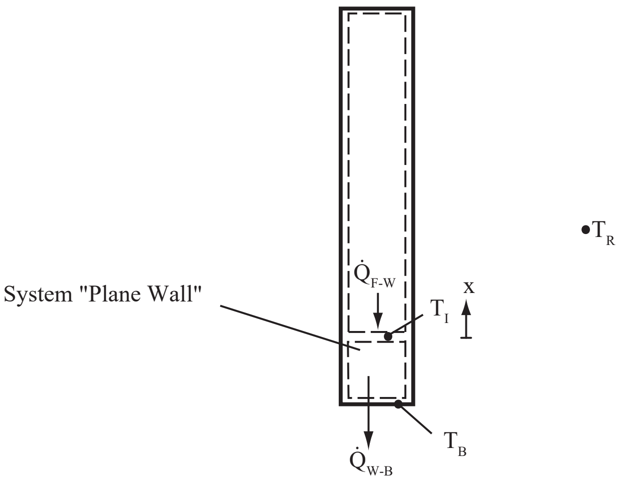


Energy balance of the system:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{2.147}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{F-F} - \dot{Q}_{F-W}} \tag{2.148}$$

System **Plane Wall**:



Energy balance of the system:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{2.149}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\mathrm{F-W}} - \dot{Q}_{\mathrm{W-B}}} \tag{2.150}$$

2) Defining the fluxes:

Heat flux through the **Fin** under the assumption that the fin's head is adiabatic, which implies that all heat conducted through the fin is transferred towards the **Plane Wall** system.

$$\boxed{\rightarrow \dot{Q}_{\mathrm{F-F}} = \lambda A_{\mathrm{c}} m \Theta_{\mathrm{I}} \tanh\left(m\left(L-s\right)\right)} \tag{2.151}$$

with

$$m = \sqrt{\frac{\alpha \cdot U}{\lambda \cdot A}} = \sqrt{\frac{\alpha \cdot 2\left(a+b\right)}{\lambda \cdot a \cdot b}} = \sqrt{\frac{6\ [\mathrm{W/m^2K}] \cdot 2\left(0.025\ [\mathrm{m}] + 0.006\ [\mathrm{m}]\right)}{58\ [\mathrm{W/mK}] \cdot 0.025\ [\mathrm{m}] \cdot 0.006\ [\mathrm{m}]}} = 6.54\ [1/\mathrm{m}] \tag{2.152}$$

$$\Theta_{\mathrm{I}} = T_{\mathrm{R}} - T_{\mathrm{I}} \tag{2.153}$$

Note that $\Theta_{\mathrm{I}} \neq T_{\mathrm{I}} - T_{\mathrm{R}}$, but $\Theta_{\mathrm{I}} = T_{\mathrm{R}} - T_{\mathrm{I}}$ due to the fact that the fin system is receiving heat instead of loosing.

Furthermore, we can describe the flux through the **Plane Wall** by use of Fourier's law:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{W-B}} = -\lambda A_{\mathrm{c}} \frac{dT}{dx} = \frac{\lambda}{s} \cdot A_{\mathrm{c}} \cdot \left(T_{\mathrm{I}} - T_{\mathrm{B}}\right)} \tag{2.154}$$

3) Inserting and rearranging:

Inserting the found definitions into the energy balance of the **Fin** yields:

$$0 = \dot{Q}_{\mathrm{F-F}} - \dot{Q}_{\mathrm{F-W}} \tag{2.155}$$

$$\boxed{\rightarrow \dot{Q}_{\mathrm{F-W}} = \dot{Q}_{\mathrm{F-F}} = \lambda A_{\mathrm{c}} m \Theta_{\mathrm{I}} \tanh\left(m\left(L-s\right)\right)} \tag{2.156}$$

Inserting the found definitions into the energy balance of the **Plane Wall** and rearranging yields:

$$0 = \dot{Q}_{\mathrm{F-W}} - \dot{Q}_{\mathrm{W-B}} \tag{2.157}$$

$$\lambda A_{\mathrm{c}} m \left(T_{\mathrm{R}} - T_{\mathrm{I}}\right) \tanh\left(m\left(L-s\right)\right) = \frac{\lambda}{s} \cdot A_{\mathrm{c}} \cdot \left(T_{\mathrm{I}} - T_{\mathrm{B}}\right) \tag{2.158}$$

$$T_{\mathrm{I}} = \frac{T_{\mathrm{R}} s m \tanh\left(m\left(L-s\right)\right) + T_{\mathrm{B}}}{s m \tanh\left(m\left(L-s\right)\right) + 1} = -15.013\ ^{\circ}\mathrm{C} \tag{2.159}$$

$$T_{\mathrm{I}} = \frac{20\ [^{\circ}\mathrm{C}] \cdot 0.04\ [\mathrm{m}] \cdot 6.54\ [1/\mathrm{m}] \cdot \tanh\left(6.54\ [1/\mathrm{m}] \cdot \left(0.29 - 0.04\right)\ [\mathrm{m}]\right) - 23.5\ [^{\circ}\mathrm{C}]}{0.04\ [\mathrm{m}] \cdot 6.54\ [1/\mathrm{m}] \cdot \tanh\left(6.54\ [1/\mathrm{m}] \cdot \left(0.29 - 0.04\right)\ [\mathrm{m}]\right) + 1} \tag{2.160}$$

$$\boxed{\rightarrow T_{\mathrm{I}} = -15.013\ [^{\circ}\mathrm{C}]} \tag{2.161}$$

As we now do know the temperature T_I at the interface between the fin system and the plane wall system, we can plug it into one of the fluxes in order to determine the rate of heat transfer towards the brine.

Which results in:

$$\dot{Q}_\mathrm{W-B} = \frac{\lambda}{s} \cdot A_\mathrm{c} \cdot (T_\mathrm{I} - T_\mathrm{B}) = 1.85\ \mathrm{W} = \frac{56\ [\mathrm{W/mK}]}{0.04\ [\mathrm{m}]} \cdot (0.025 \cdot 0.006)\ [\mathrm{m}^2] \cdot (-15.013 + 23.5)\ [\mathrm{K}] \tag{2.162}$$

$$\boxed{\rightarrow \dot{Q} = 1.85\ \mathrm{W}} \tag{2.163}$$

b) *Up to which hight h_0 does frost form on the steel ban (h_0 is the distance from the surface of the brine pipe), if the steam content of the air in the surrounding room is above the saturation vapour pressure for the maximum steel band temperature?*

Frost can be formed at a temperature below 0 °C, therefore the following condition can be used:

$$T(x = h_0) = T_0 = 0\,°\mathrm{C} \tag{2.164}$$

To determine of h_0, the temperature profile within the **Fin** system should be determined. When having determined this profile, the position can be determined where the temperature of the fin equals 0 °C.

The temperature profile inside the **Fin** system could be derived by use of deriving the heat conduction equation. But for the scenario of a fin with an adiabatic head, it can also be found on the formula sheet.

Which is for our scenario:

$$\Theta(x) = \Theta_\mathrm{I} \frac{\cosh(m((L-s)-x))}{\cosh(m \cdot (L-s))} \tag{2.165}$$

For simplicity we use $h = L - s$

$$\Theta(x) = \Theta_\mathrm{I} \frac{\cosh(m(h-x))}{\cosh(m \cdot h)} \tag{2.166}$$

Where

$$\Theta_\mathrm{I} = T_\mathrm{R} - T_\mathrm{I} = (20 - -15.013) = 35.013\ [°\mathrm{C}] \tag{2.167}$$

Using the condition $T(x = h_0) = T_0$ yields:

$$\Theta(x = h_0) = \Theta_0 = T_\mathrm{R} - T_0 = 20\ [°\mathrm{C}] \tag{2.168}$$

This condition can be used to determine h_0:

$$\Theta(x = h_0) = \Theta_\mathrm{I} \frac{\cosh(m(h-h_0))}{\cosh(m \cdot h)} = \Theta_0 \tag{2.169}$$

Rewriting yields:

$$\cosh(m(h-h_0)) = \frac{\Theta_0}{\Theta_\mathrm{I}} \cdot \cosh(m \cdot h) \tag{2.170}$$

$$m(h-h_0) = \cosh^{-1}\left(\frac{\Theta_0}{\Theta_\mathrm{I}} \cdot \cosh(m \cdot h)\right) \tag{2.171}$$

$$h_0 = h - \frac{1}{m} \cdot \cosh^{-1}\left(\frac{\Theta_0}{\Theta_\mathrm{I}} \cdot \cosh(m \cdot h)\right) \tag{2.172}$$

Inserting numerical values yields:

$$h_0 = 0.25\ [\mathrm{m}] - \frac{1}{6.54\ [1/\mathrm{m}]} \cdot \cosh^{-1}\left(\frac{20\ [^\circ\mathrm{C}]}{35.013\ [^\circ\mathrm{C}]} \cdot \cosh(6.54\ [1/\mathrm{m}] \cdot 0.25\ [\mathrm{m}])\right) \tag{2.173}$$

$$\boxed{\rightarrow h_0 = 0.1\ [\mathrm{m}]} \tag{2.174}$$