2.12 Solar collector ★

a) *Determine the rate of heat loss for $\theta = 0^o$*

1) Setup the definition of rate of heat loss:

The rate of heat loss by convection can be described by:

$$\boxed{\rightarrow \dot{Q} = \overline{\alpha} A_\mathrm{s} \left(T_\mathrm{A} - T_\mathrm{G}\right)} \tag{2.204}$$

2) Defining all required parameters:

The average heat transfer coefficient for natural internal convection can be expressed as:

$$\overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} \tag{2.205}$$

To the find the expression for the Nusselt number, we first should now what the value for the Grashof number. Which can be expressed as:

$$\mathrm{Gr}_\mathrm{s} = \frac{g\beta \left(T_\mathrm{a} - T_\mathrm{g}\right) s^3}{\nu^2} \tag{2.206}$$

The expansion coefficient β for an ideal gas can be approximated by the following relation: (where we take T_∞ to be the average temperature inside the collector.)

$$\boxed{\rightarrow \beta = \frac{1}{T_\infty} = \frac{1}{\left(\frac{40+80}{2} + 273\right)\ [\mathrm{K}]} = 0.0030\ [\mathrm{K}^{-1}]} \tag{2.207}$$

Filling in all numerical values into the expression of the Grashof number yields:

$$\mathrm{Gr}_\mathrm{s} = \frac{9.81\ [\mathrm{m/s^2}] \cdot 0.0030\ [\mathrm{K}^{-1}] \left(80 - 40\right)\ [\mathrm{K}] \cdot 0.02^3\ [\mathrm{m}^3]}{\left(1.9305 \cdot 10^{-5}\right)^2\ [\mathrm{m^4/s^2}]} \tag{2.208}$$

$$\boxed{\rightarrow \mathrm{Gr}_\mathrm{s} = 2.2584 \cdot 10^4} \tag{2.209}$$

With this we find that the correlation $\overline{\mathrm{Nu}}_\mathrm{s} = 0.075 \left(\mathrm{Gr}_\mathrm{s}\mathrm{Pr}\right)^{1/3}$ for the Nusselt number, as $\mathrm{Gr}_\mathrm{s} > 2 \cdot 10^3$:

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_\mathrm{s} = 0.075 \left(\mathrm{Gr}_\mathrm{s}\mathrm{Pr}\right)^{1/3} = 0.075 \left(2.2584 \cdot 10^4 \cdot 0.7103\right)^{1/3} = 1.9644} \tag{2.210}$$

Average heat transfer coefficient:

$$\boxed{\rightarrow \overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} = \frac{1.9644 \cdot 0.0286\ [\mathrm{W/mK}]}{0.02\ [\mathrm{m}]} = 2.81\ [\mathrm{W/m^2K}]} \tag{2.211}$$

3) Inserting and rearranging:

Rate of heat transfer:

$$\dot{Q} = \overline{\alpha} \cdot A_\mathrm{s} \cdot (T_\mathrm{a} - T_\mathrm{g}) = 2.81\ [\mathrm{W/m^2K}] \cdot\ 0.8\ [\mathrm{m}] \cdot\ 3\ [\mathrm{m}] \cdot (80 - 40)\ \ [\mathrm{K}] \tag{2.212}$$

$$\boxed{\rightarrow \dot{Q} = 269.7\ [\mathrm{W}]} \tag{2.213}$$

b) *Determine the rate of heat loss for* $\theta = 90^o$

1) Setup the definition of rate of heat loss:

The rate of heat loss by convection can be described by:

$$\boxed{\rightarrow \dot{Q} = \overline{\alpha} A_\mathrm{s} \left(T_\mathrm{A} - T_\mathrm{G}\right)} \tag{2.214}$$

2) Defining all required parameters:

The average heat transfer coefficient for natural internal convection can be expressed as:

$$\overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} \tag{2.215}$$

To the find the expression for the Nusselt number, we first should now what the value for the Grashof number. Which can be expressed as:

$$\mathrm{Gr_s} = \frac{g\beta \left(T_\mathrm{a} - T_\mathrm{g}\right) s^3}{\nu^2} \tag{2.216}$$

Filling in all numerical values, as in question a), into the expression of the Grashof number yields:

$$\boxed{\rightarrow \mathrm{Gr_s} = 2.2584 \cdot 10^4} \tag{2.217}$$

We find that the correlation $\overline{\mathrm{Nu}}_\mathrm{s} = 0.20\,(L/s)^{-1/9}\,(\mathrm{Gr_s Pr})^{1/4}$ for the Nusselt number does meet the criteria $3.1 < L/s < 42.2$, but is does not meet the requirement $2 \cdot 10^3 < \mathrm{Gr_s} < 2 \cdot 10^4$. As the Grashof number just falls outside this range, we decide still to use it, but we have to keep in mind that our found rate of heat transfer will have some error in it!

$$\overline{\mathrm{Nu}}_\mathrm{s} = 0.20\,(L/s)^{-1/9}\,(\mathrm{Gr_s Pr})^{1/4} = 0.20\,(0.8/0.02)^{-1/9}\left(2.2584 \cdot 10^4 \cdot 0.7103\right)^{1/4} \tag{2.218}$$

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_\mathrm{s} = 1.5369} \tag{2.219}$$

Average heat transfer coefficient:

$$\boxed{\rightarrow \overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} = \frac{1.5369 \cdot 0.0286\ [\mathrm{W/mK}]}{0.02\ [\mathrm{m}]} = 2.20\ [\mathrm{W/m^2K}]} \tag{2.220}$$

3) Inserting and rearranging:

Rate of heat transfer:

$$\dot{Q} = \overline{\alpha} \cdot A_{\mathrm{s}} \cdot (T_{\mathrm{a}} - T_{\mathrm{g}}) = 2.20\ [\mathrm{W/m^2K}] \cdot\ 0.8\ [\mathrm{m}] \cdot\ 3\ [\mathrm{m}] \cdot (80 - 40)\ \ [\mathrm{K}] \tag{2.221}$$

$$\boxed{\rightarrow \dot{Q} = 211.0\ [\mathrm{W}]} \tag{2.222}$$