2.16 Transient body

- Problem type

  Transient heat transfer.

a) Compiling a differential equation for the temporal variation of the body temperature T_{sb}:

Energy balance:

$$\frac{dU}{dt} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{conv}} \tag{2.168}$$

Temporal variation of the inner energy of the solid body:

$$\frac{dU}{dt} = m \cdot c_{\text{p}} \cdot \frac{dT_{\text{sb}}}{dt} \tag{2.169}$$

Heat flux entering the solid body:

$$\dot{Q}_{\text{in}} = \dot{q}'' \cdot A_{\text{s}} \tag{2.170}$$

Convection at the surface of the solid body:

$$\dot{Q}_{\text{conv}} = \alpha \cdot A_{\text{s}} \cdot (T_{\text{sb}} - T_{\text{a}}) \tag{2.171}$$

Insert and rearrange:

$$\frac{dT_{\text{sb}}}{dt} = \frac{1}{m \cdot c_{\text{p}}} \left(\dot{q}'' \cdot A_{\text{s}} - \alpha \cdot A_{\text{s}} \cdot (T_{\text{sb}} - T_{\text{a}})\right) \tag{2.172}$$

2.17 Cooling of a copper rod

- Problem type

  Transient heat transfer.

a) Determining the time it takes for the copper rod to cool to an average temperature of 25°C:

Checking whether the lumped capacity model can be used:

$$\mathrm{Bi} \;=\; \frac{\alpha \cdot V}{\lambda \cdot A} = \frac{\alpha \cdot d}{4\lambda} \;=\; 0.0025 \;<<\; 1 \tag{2.173}$$

Lumped capacity model:

$$\Theta^* = 1 - e^{-\frac{\alpha A}{\rho c_\mathrm{p} V} t} \tag{2.174}$$

Dimensionless temperature:

$$\Theta^* \;=\; \frac{T(t) - T_0}{T_\mathrm{A} - T_0} = \frac{25\ [^\circ\mathrm{C}] - 100\ [^\circ\mathrm{C}]}{20\ [^\circ\mathrm{C}] - 100\ [^\circ\mathrm{C}]} \;=\; 0.9375 \tag{2.175}$$

Rearrange the lumped capacity formula and insert:

$$t = -\frac{d\rho c_\mathrm{p}}{4\alpha} \ln\left(1 - \Theta^*\right) = \\ -\frac{0.02\ [\mathrm{m}] \cdot 8930\ [\mathrm{kg\ m^{-3}}] \cdot 382\ [\mathrm{J\ kg^{-1}\ K^{-1}}]}{4 \cdot 200\ [\mathrm{Wm^{-2}K^{-1}}]} \ln\left(1 - 0.9375\right) = 236.45\ [\mathrm{s}] \tag{2.176}$$