Energy & Heat Transfer

Lecture 4

By: Mohammad Mehrali

Recap of lecture 3

Forced and natural/free convection:

$$\dot{Q} = hA\Delta T$$ $$\dot{q} = h\Delta T$$ Newton's cooling law

Nusselt Number: dimensionless variety of heat transfer coefficient *h*

$$\mathrm{Nu} = \frac{hL_c}{k}$$ With L_c a characteristic length for the considered geometry

Determining the Nusselt Number: (empirical) correlations

- Forced convection: Nu as function of Re, Pr : Nu = *f*(Re, Pr)
- Natural convection: This lecture

⇒ Relation Nu, Re, Pr dependent on geometry and flow regime (laminar/ turbulent)

⇒ Nu, Re, Pr dimensionless numbers: reduction number of variables

Recap of lecture 3

Convection Resistance :

$$\dot{Q} = hA\Delta T = \frac{\Delta T}{R_{conv}} \quad \text{met} \quad R_{conv} = \frac{1}{hA} \quad (\text{K/W})$$

Dimensionless Numbers :

Nusselt Number: $$\text{Nu} = \frac{hL_c}{k}$$

Reynolds Number: $$\text{Re} = \frac{\rho U L_c}{\mu}$$

Prandtl Number: $$\text{P}r = \frac{\mu c_p}{k}$$

+ Background on boundary layers

Recap of lecture 3

If $\dot{Q}$ must be found:

- Calculate at film temperature : $T_f = \frac{T_s + T_\infty}{2}$
- Pull out ingredients like *μ*, *ρ*, *k*, Pr from tables – like assignment bundle: air or given fluid) at $T_f = \frac{T_s + T_\infty}{2}$

- Calculate Re and choose apprioprate correlation based on geometry and Re
- Calculate Nu
- Derive *h* from it
- Fill out Newton's cooling

law: $\dot{Q} = hA\Delta T$

Learning objectives lecture 4

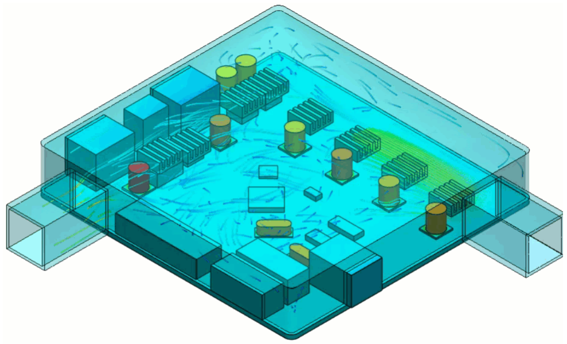
Heat transfer through natural convection

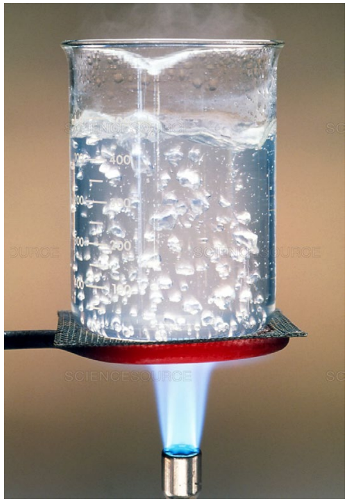
- **Natural Convection**
- Grashof Number
- Rayleigh Number
- Nusselt Number
- Using correlations for various configurations
- Calculating natural convection with step-by-step plan

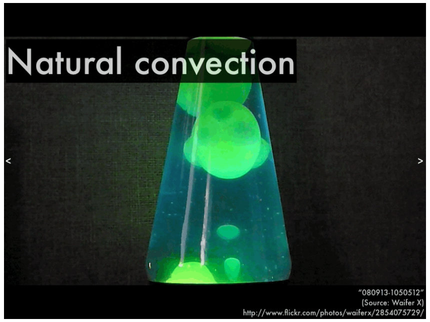
Types of convection (from Lecture 3)

Forced convection

Natural/free convection







Imposed flow (by pump, fan, …)

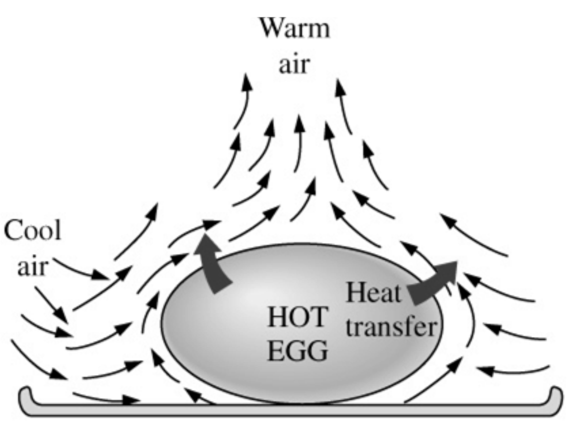
Temperature difference itself starts the flow

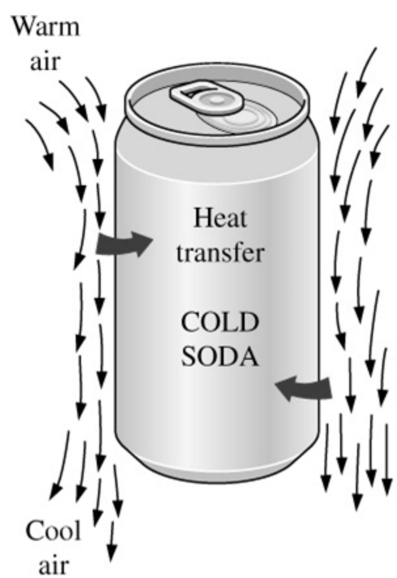
General: flow velocity and heat transfer rates are larger for forced convection

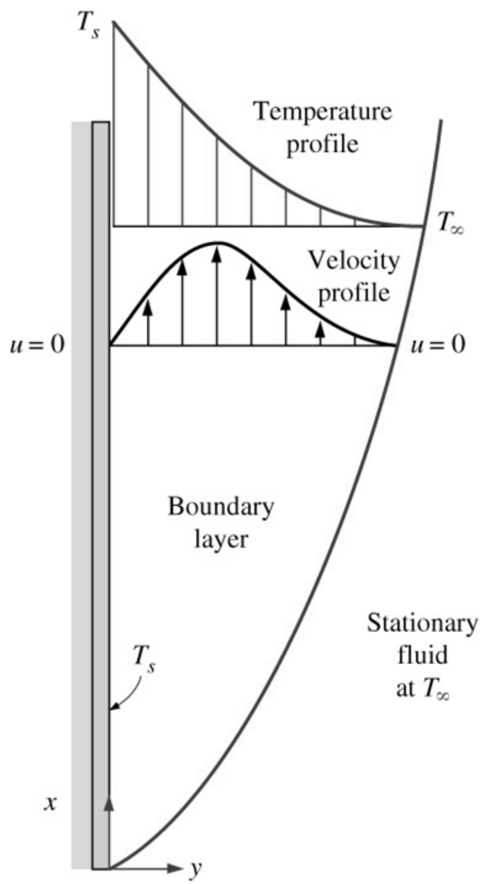
Natural Convection

Also for natural convection velocity and thermal boundary layers!

- Velocity boundary layer is only area in which flow occurs
- **Hot** surface: **Upward flow**
- **Cold** surface: **Downward flow**







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Grashof Number

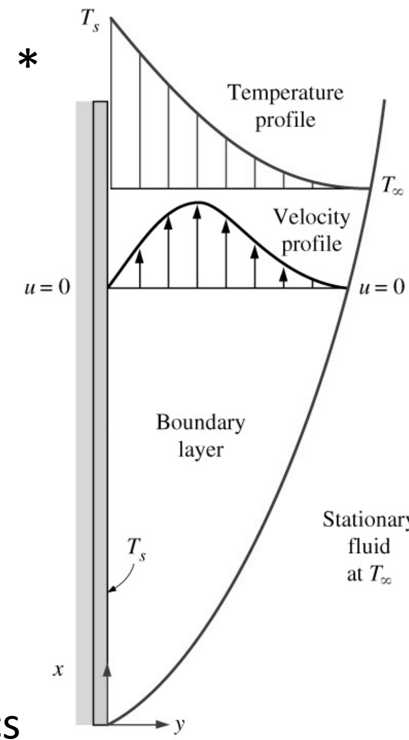
Forced convection: velocity U imposed

Natural convection: velocity follows from temp.difference T_s - T_∞ *

$\Rightarrow$ Alternative for Reynolds number, with temp.diff. instead U:

Grashof number: $$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (-)$$

Greek letter "nu"



The **Grashof number (Gr)** is a **dimensionless** number in fluid dynamics and heat transfer which approximates the **ratio of** the **buoyancy** to **viscous force** acting on a fluid.

*Choose ΔT positive for convenience

Grashof Number

Grashof number: $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}$ $(-)$

- Gravitational acceleration : **$g = 9{,}81\ m/s^2$**
- Volume expansion coefficient ***β*** **(K^{-1});** **for most gases:** $\beta = \dfrac{2}{T_s + T_\infty}$

  (temperature in Kelvin; 0 ˚C = 273,15 K)

- Length **L_c** characteristic for geometry **(length *L* for plate, diameter *D* for sphere/cylinder)**
- Kinematic viscosity : $\nu = \dfrac{\mu}{\rho}\,(\mathrm{m^2/s})$ at avg. Temp: $T_f = \dfrac{T_s + T_\infty}{2}$

Learning objectives lecture 4

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Rayleigh Number

Laminar / turbulent:

$\leftrightarrow$

Forced convection:
Determined by Reynolds number : **Re**

Natural convection:
Determined by Grashof number **Gr**

Often combined with Prandtl number:
Rayleigh number **Ra = Gr · Pr**

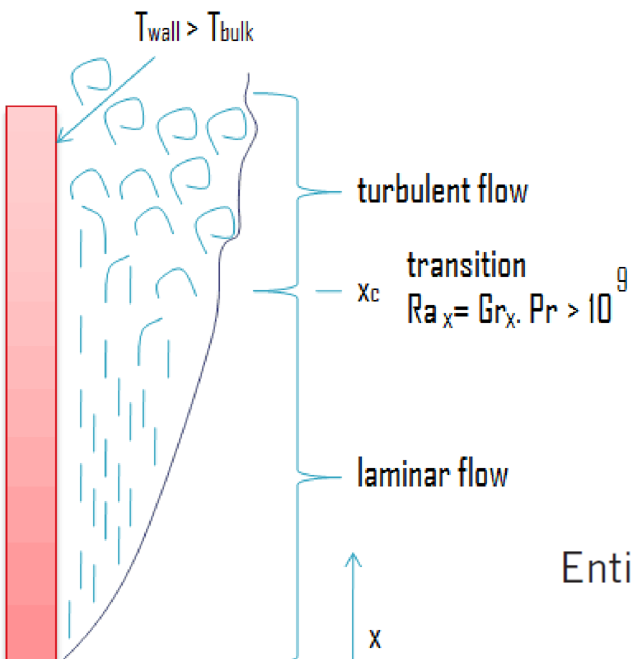
Rayleigh Number : $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \dfrac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr}$

Rayleigh Number is a dimensionless number associated with buoyancy-driven flow, also known as free or natural convection

Rayleigh Number

$$\text{Rayleigh Number}: \mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\ 3}}{\nu^2}\mathrm{Pr}$$

Vertical, flat plate (ex. radiator)



$\mathrm{Nu} = 0{,}59\ \mathrm{Ra}_L^{\ 1/4}$ with $L_c = L$ $(10^4 < \mathrm{Ra}_L < 10^9)$

$\mathrm{Nu} = 0{,}1\ \mathrm{Ra}_L^{\ 1/3}$ with $L_c = L$ $(10^9 < \mathrm{Ra}_L < 10^{13})$

$10^9 < \mathrm{Ra}_L < 10^{10}$: find intermediate

Entire range $\Rightarrow$ $$\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

(complex but more accurate)

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Nusselt Number

Forced Convection

$$\frac{hL}{k} = a\left(\frac{\rho U L}{\mu}\right)^b \frac{\mu c_p}{k}^c$$

$$\mathrm{Nu} = a \cdot \quad \mathrm{Re}^b \cdot \quad \mathrm{Pr}^c$$

Natural Convection

Constant coefficient

Constant exponent

$$\mathrm{Nu} = C\,\mathrm{Ra}_L^n$$

Nusselt number

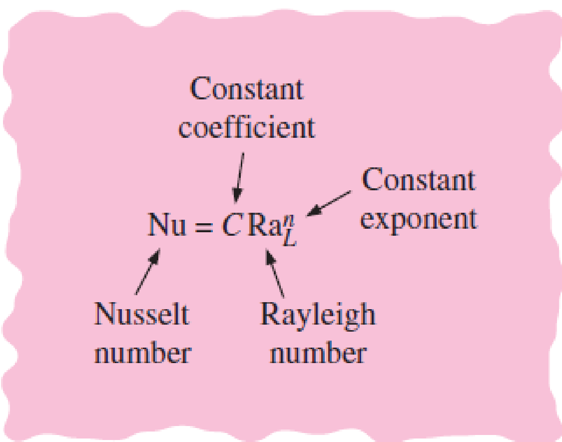
Rayleigh number

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$$

NUSSELT NUMBER

$$\text{Rayleigh Number}: \mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$$

Rayleigh number, which is the product of the Grashof and Prandtl numbers



The values of the constants C and n depend on the

- ✓ geometry of the surface
- ✓ the flow regime

The value of n is usually

- ✓ 0,25 for laminar flow
- ✓ 0,33 for turbulent flow.

The value of C

- ✓ is normally less than 1.

All fluid properties are to be evaluated at the film temperature Tf =(T_s+T_{inv})/2.

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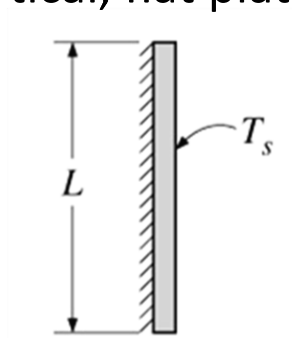
Free Convection Correlations

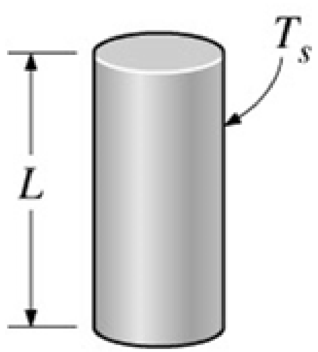
Vertical, flat plate



Nu = 0,59 $Ra_L^{1/4}$ with $L_c = L$ ($10^4 < Ra_L < 10^9$)
Nu = 0,1 $Ra_L^{1/3}$ with $L_c = L$ ($10^{10} < Ra_L < 10^{13}$)

Entire range ⟹ $$Nu = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$$

(complex but more accurate)

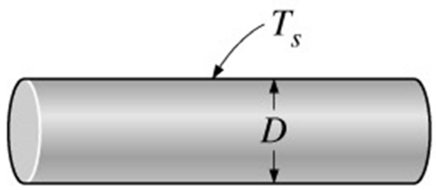


Vertical cylinder (e.g. can)

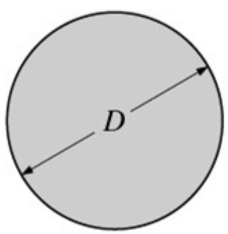
Treat a vertical plate with height L and surface πDL, if diameter D sufficiently large with respect to L ($D \geq \frac{35L}{Gr_L^{1/4}}$)

FREE CONVECTION CORRELATIONS

Horizontal cylinder (e.g. pipe)



$$\mathrm{Nu} = \left\{ 0.6 + \frac{0.387 \mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}} \right\}^2$$



$$\mathrm{Nu} = 2 + \frac{0.589 \mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$$

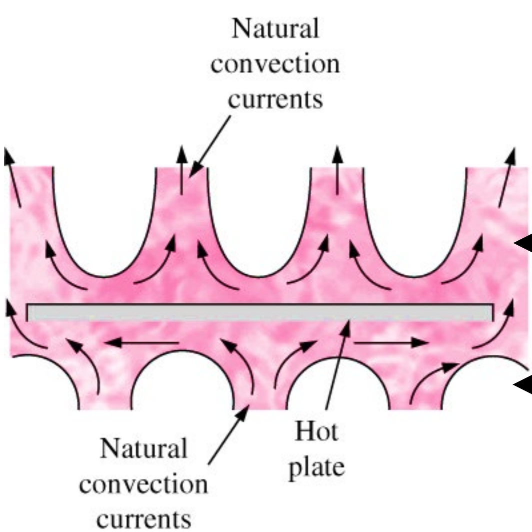
Sphere (e.g. light bulb)

Notation:
Ra_L is Ra with $L_c = L$;
Ra_D is Ra with $L_c = D$

Free Convection Correlations

Horizontal plate: upward / downward flow perpendicular to surface → plumes instead of adjacent boundary layer

Hot surface



Fluid away from surface

Effective heat transfer

$Nu = 0{,}54\ Ra^{1/4}$ $(10^4 < Ra < 10^7)$

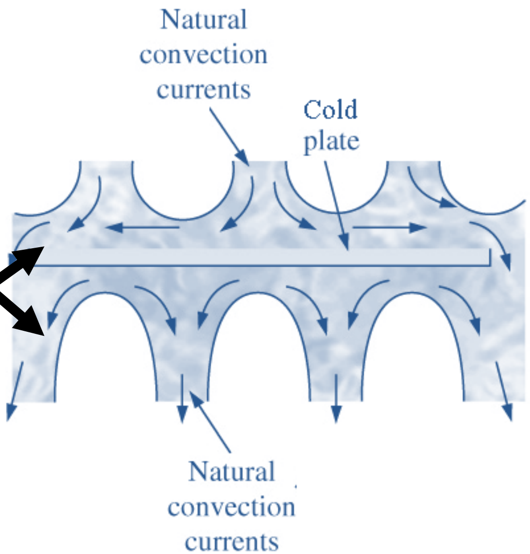
$Nu = 0{,}15\ Ra^{1/3}$ $(10^7 < Ra < 10^{11})$

Fluid towards surface

Reduced heat transfer

$Nu = 0{,}27\ Ra^{1/4}$ $(10^5 < Ra < 10^{11})$

Cold surface



N.B.: Characteristic length

So actually: $Ra_{A/p}$

$$L_c = \frac{\text{Area}}{\text{Perimeter}} = \frac{A}{p}$$

Free Convection Correlations

Plate at angle *θ* with vertical axis

(*θ* < 60º)

Hot surface:

- Top side: advanced methods
- Bottom side: same as vertical plate, except for using *g* cos *θ* instead of *g*:

$$\mathrm{Ra}_L = \mathrm{Gr}_L \cdot \mathrm{Pr} = \frac{g \cos\theta\, \beta (T_s - T_\infty) L^3}{\nu^2} \cdot \mathrm{Pr}$$

Cold surface: everything upside down

- Bottom side: advanced methods
- Top side: same as vertical plate, except for using *g* cos *θ* instead of *g*

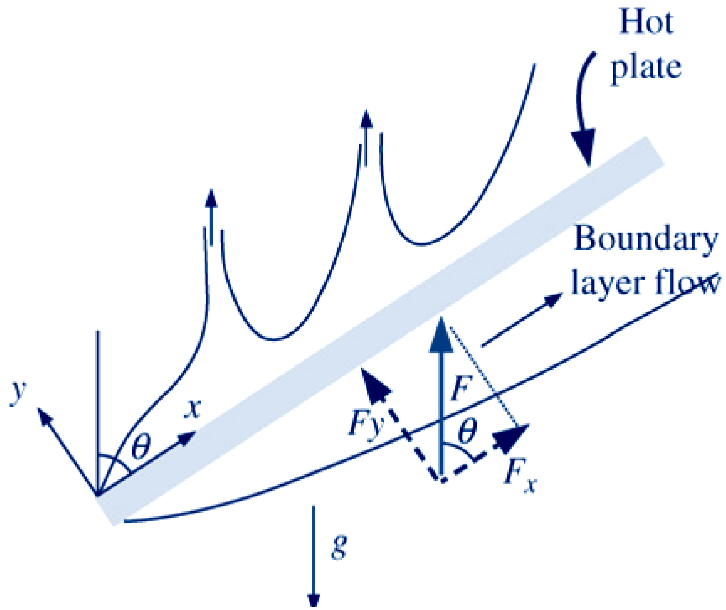


Plate at angle *θ* (ex. radiator in attic room)

Vertical plate:

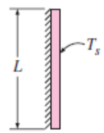
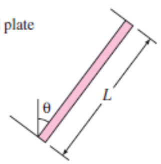
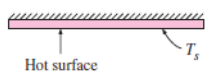
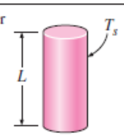
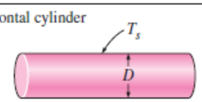
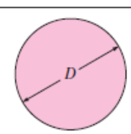
$Nu = 0{,}59\ Ra_L^{1/4}$ $(10^4 < Ra_L < 10^9)$

$Nu = 0{,}1\ Ra_L^{1/3}$ $(10^{10} < Ra_L < 10^{13})$

Free Convection Correlations

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20) $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \dfrac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \dfrac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)

Learning objectives lecture 4

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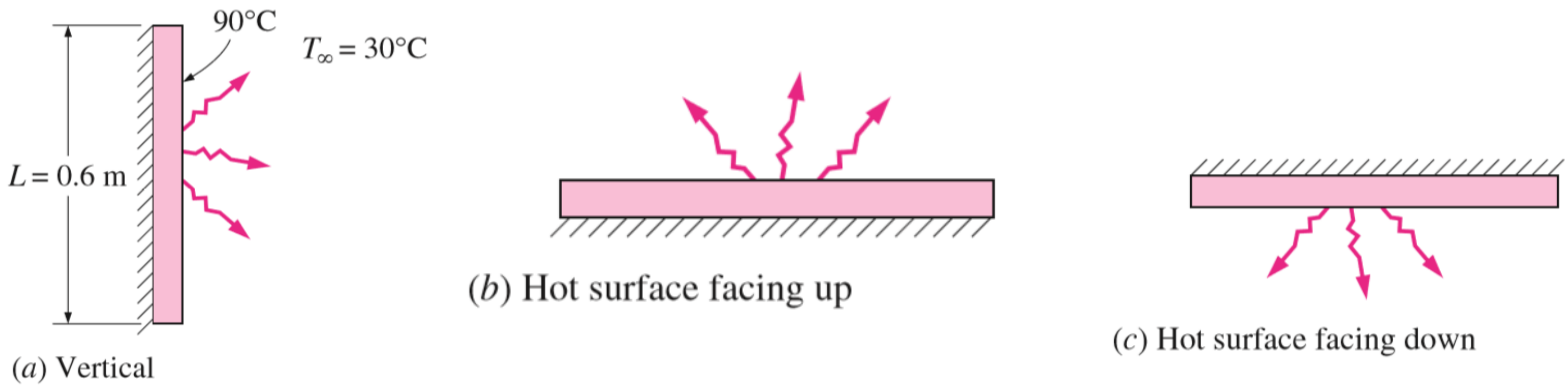
- Rayleigh Number
- Nusselt Number
- Using correlations for various configurations
- **Calculating natural convection with step-by-step plan**

Example 1

Cooling of a Plate in Different Orientations



- A: 0.6-m x 0.6-m thin square plate
- room temperature: 30˚C.
- One side of the plate is maintained at a temperature of 90˚C, while the other side is insulated
- Determine the rate of heat transfer from the plate by natural convection if the plate is
- (a) vertical,
- (b) horizontal with hot surface facing up, and
- (c) horizontal with hot surface facing down.

Step-by-step plan natural convection

If $\dot{Q}$ must be found:

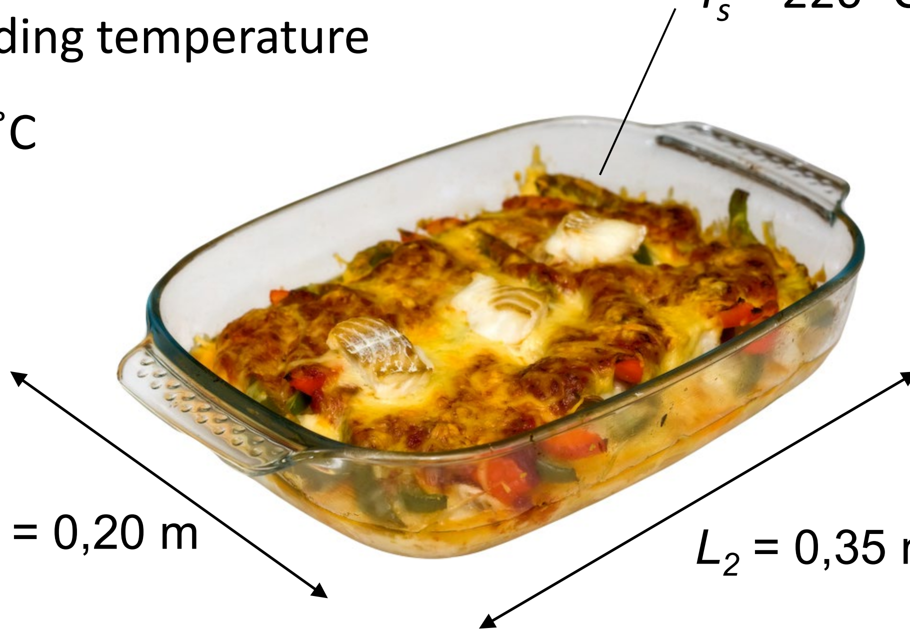
- Determine ingredients necessary for dimensionless no. (Pr, *k* and *v* at average temperature $\frac{T_s + T_\infty}{2}$)
- Determine Ra: $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$
- Choose appropriate correlation based on geometry and Ra
- Determine Nu
- Resolve *h* from it
- Fill out Newton's cooling law: $\dot{Q} = hA\Delta T$

Example 2

How large is the heat flow?

Surrounding temperature

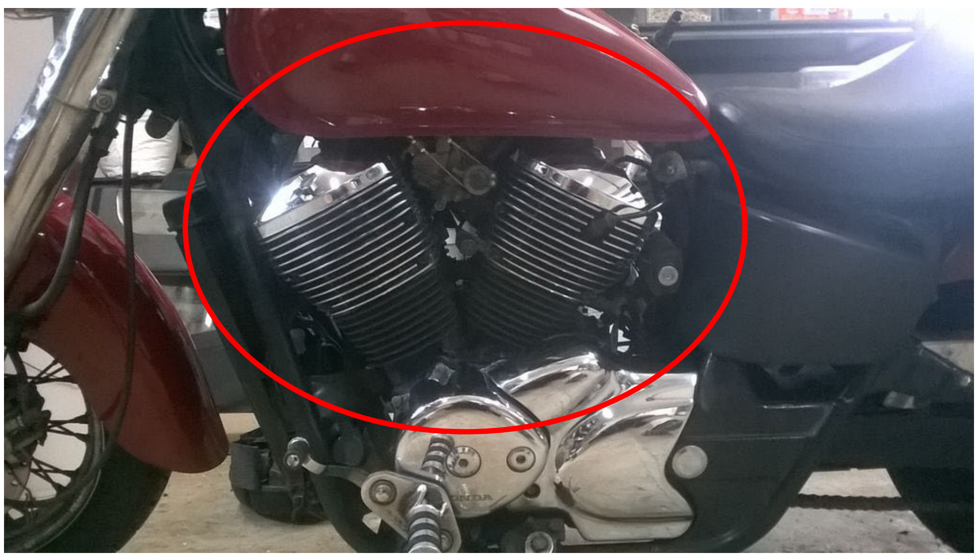
T_∞ = 20 ˚C

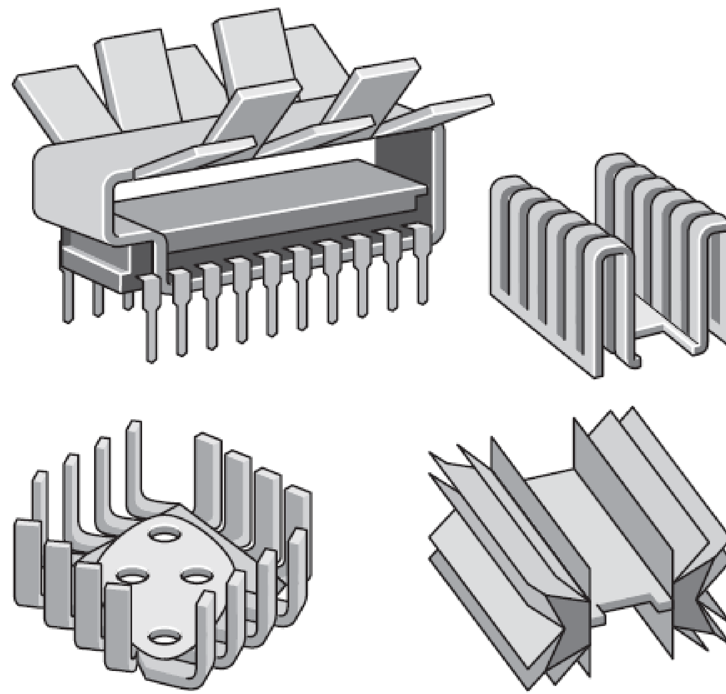


Natural convection

Fins

What is Purpose of the Fins on Engine Surfaces ? Why design such a thing ?





Fins are extended surfaces and they increase the surface area leading to increase in heat transfer rate. Useful in cooling applications

Summary natural convection

General (same as for forced convection)

$$\dot{Q} = hA\Delta T \quad \mathrm{(W)}$$

Newton's cooling law

"Supporting" equations for *h*:

Nusselt number Nu as function of Rayleigh number Ra = Gr · Pr

Grashof number $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}$

- Forced convection: Re ↔ natural convection: Gr
- Ra and Nu just like Re based upon characteristic length of geometry (indicate using subscript)

General conclusion convection

- Calculate heat flow using Newton's cooling law
- Determine heat transfer coefficient *h* in this using correlations between Nu (→ *h*) and other dimensionless nos.

⇒ step-by-step plans (+ iterative solving)

Other learning objectives convection: *flow phenomena*

- Being able to sum up what parameter are influencing *h*
- Knowing differences between laminar and turbulent ,
- Explaining how velocity and thermal boundary layers form
- Predicting/reasoning the temperature development