Energy & Heat Transfer

Lecture 7

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Heat Transfer Modes

Conduction

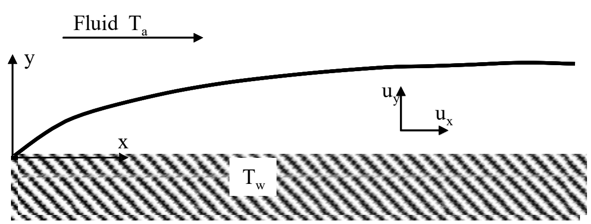
- **Fourier Law**

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x}\,[W]$$

Thermal Conductivity [W/m.K]
Material properties

Cross-Sectional Area $[m^2]$

Convection



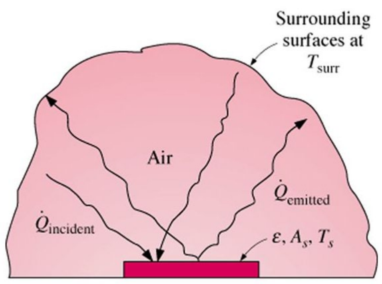
- **Newton's law of cooling**

$$\dot{Q} = hA(T_w - T_a)\;[W]$$

Convective Heat Transfer Coefficient [W/m^{2}K]
Flow dependent

- **Natural Convection**
- **Forced Convection**

Radiation



- **Stefan-Boltzmann law**

$$\dot{Q} = \varepsilon\sigma A(T_s^4 - T_\infty^4)[W]$$

Emissivity

Stefan-Boltzmann constant

$$\sigma = 5.670 \times 10^{-8}\,\frac{W}{m^2K^4}$$

STEP BY STEP PLAN

Conduction:

1. Schematic (Steady state)

2. Negligible heat losses (insulation, adiabatic,..)

3. Geometry (flat plate, cylinder,…)

4. Arrangement of the layers

5. Resistance network

6. Calculation of resistances

7. Calculation of overall resistance

8. Calculation of heat flow

9. Calculation of temperatures for different layers

Radiation:

1. Schematic (Steady state)

2. Negligible Radiations

3. Surrounding Temperature

4. Calculation of heat flow

Convection:

1. Schematic

2. Force of natural convection?

3. Geometry

4. Finding the film temperature

5. Finding ingredients from Table

6. Calculation of dimensionless numbers

7. Calculation of characteristic length

8. Finding correlations from tables to calculate Nu

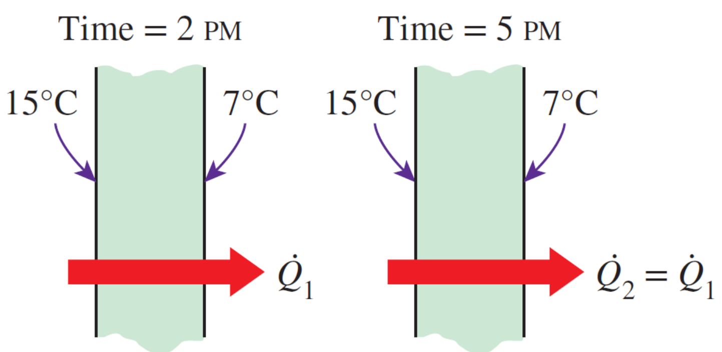
9. Calculation of h

10. Calculation of heat flow

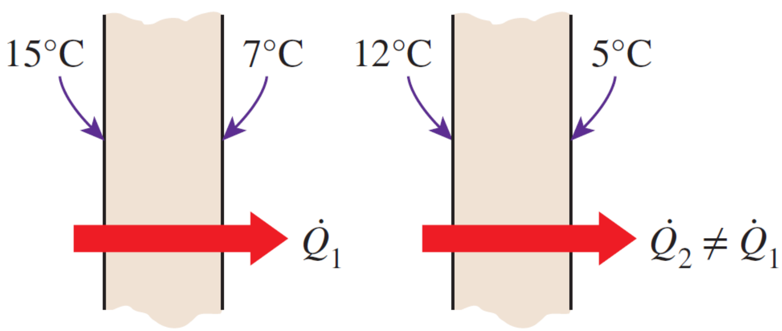
Learning objectives lecture 7

- **Time dependent heat transfer problems**
  - Unsteady/Transient heat trasnfer
  - Distinguish practical examples
  - “Derive” mathematical approximation
  - Determining validity of approximation

Unsteady Heat Transfer concept



(*a*) Steady



(*b*) Transient

- In **steady state heat transfer**, the temperature at any particular point in the system remains constant after equilibrium is attained.
- The amount of heat entering any section is then equal to the amount of heat exiting the section, because the driving force (temperature difference) is constant.

- In **unsteady state**, the temperature within an object itself keeps changing with time.
- The heat entering a section thus might not be the same as the heat exiting the section, as the temperature difference across the section keeps changing with time.

Transient processes

$t = 0$ s

$T_\infty = 0\ ^\circ\mathrm{C}$

$T_s = 25\ ^\circ\mathrm{C}$

$\dot{Q} \approx 300\,\mathrm{W}$

T_s drops - 1 °C / min

$t > 0$ s

$T_\infty = 0\ ^\circ\mathrm{C}$

$T_s = 12{,}5\ ^\circ\mathrm{C}$

$\dot{Q} \approx 150\,\mathrm{W}$

T_s drops – 0.5 °C / min

- Assume: bottle has uniform temperature

- The lower the temperature difference is, the lower and slower the drop in temperature (difference) will be.

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steady / transient

So far: steady state
- Constant in time
- Equilibrium (approximately)




Transient: time dependent
- Heating
- Cooling




Learning objectives lecture 7

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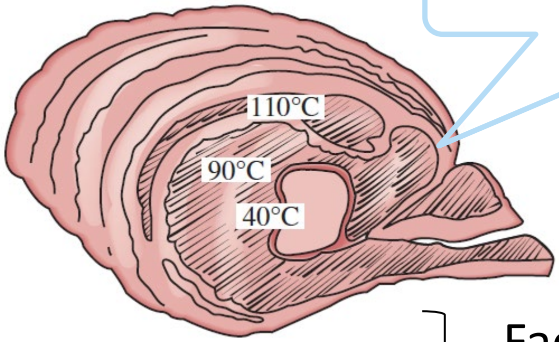
UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS



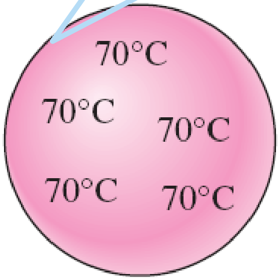
Temperature is a function of time and space

Convection: heat from outer layer
Conduction: heat transferred from outer layer to core

Factors:
h, *k*,
geometry

- **Interior temperature** of some bodies remains essentially uniform at all times during a heat transfer process.
- The **temperature** of such bodies can be taken to be a function of time only, $T(t)$.
- Heat transfer analysis that utilizes this idealization is known as **lumped system analysis**.

Temperature is only function of Time



UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

$$\begin{pmatrix}\text{Heat transfer into the body}\\ \text{during } dt\end{pmatrix} = \begin{pmatrix}\text{The increase in}\\ \text{the of the body}\\ \text{energy during } dt\end{pmatrix}$$

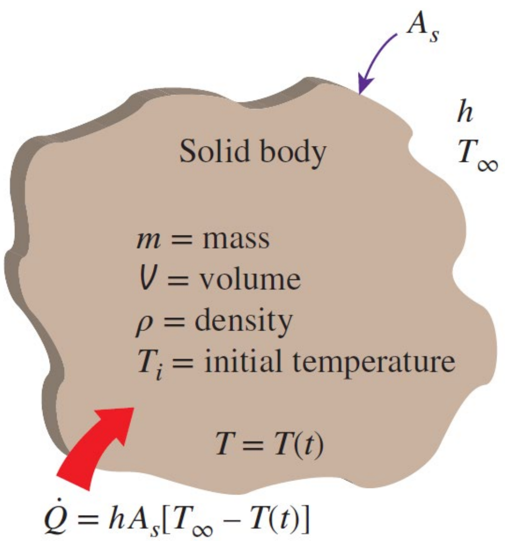
$$hA_s(T_\infty - T)\,dt = mc_p\,dT$$

$$m = \rho V \qquad dT = d(T - T_\infty)$$

$$\frac{d(T - T_\infty)}{T - T_\infty} = \frac{hA_s}{\rho V c_p}\,dt$$

Integrating from $t = 0$, at which $T = T_i$, to any time t, at which $T = T(t)$

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}t$$



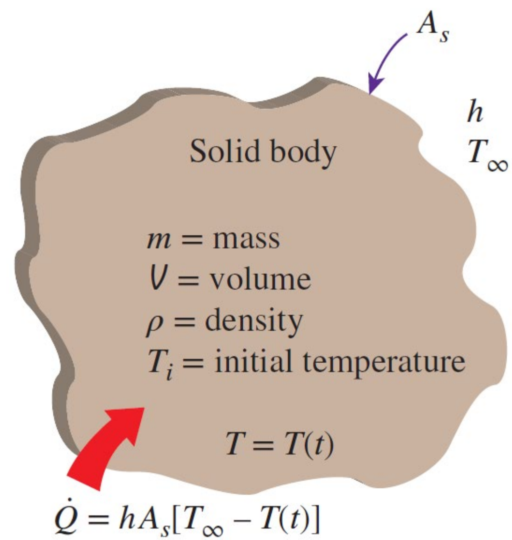
The geometry and parameters involved in the lumped system analysis.

UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p} t$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

Where : $$b = \frac{hA_s}{\rho V c_p} \qquad (1/\text{s})$$



h : heat transfer coefficient around object (W · m^{-2} · K^{-1})

A_s: external surface area (m^2)

ρ : density of object (kg · m^{-3})

c_p : specific heat of object (J · kg^{-1} · K^{-1})

V : volume of object, (m^3)

Only for lumped system analysis

Unsteady Heat Transfer Lumped System Analysis

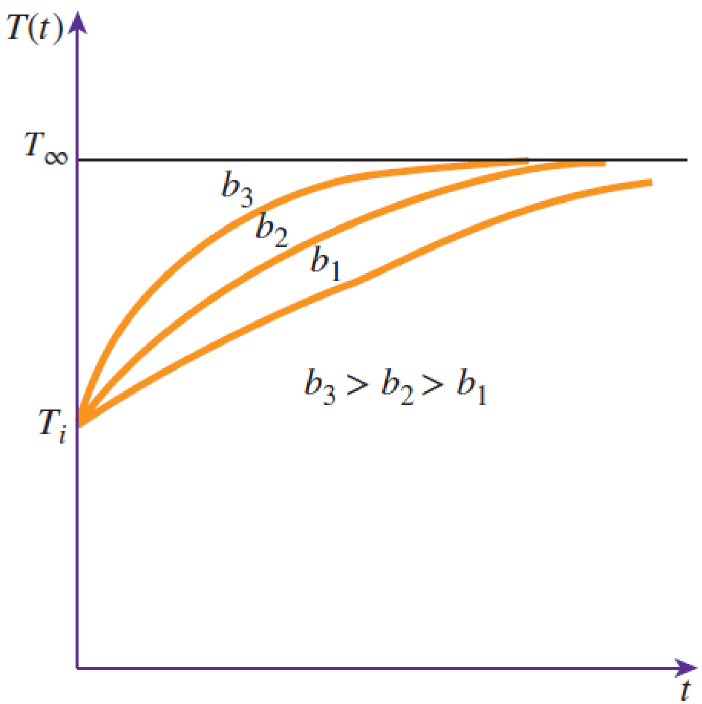
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \text{Where} \quad b = \frac{hA_s}{\rho V c_p} \qquad (1/\text{s})$$

What is this equation representing ?

- This equation enables us to determine the temperature $T(t)$ of a body at time t, or alternatively, the time t required for the temperature to reach a specified value $T(t)$.
- The temperature of a body approaches the ambient temperature T_∞ exponentially.
- What is the effect of b on duration (t) of the temperature of a body to reach the ambient temperature T_∞ ?

UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V c_p} \quad (1/s)$$



- The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of *b* indicates that the body approaches the environment temperature in a short time.

- The temperature of a lumped system approaches the environment temperature as time gets larger.

Learning objectives lecture 7

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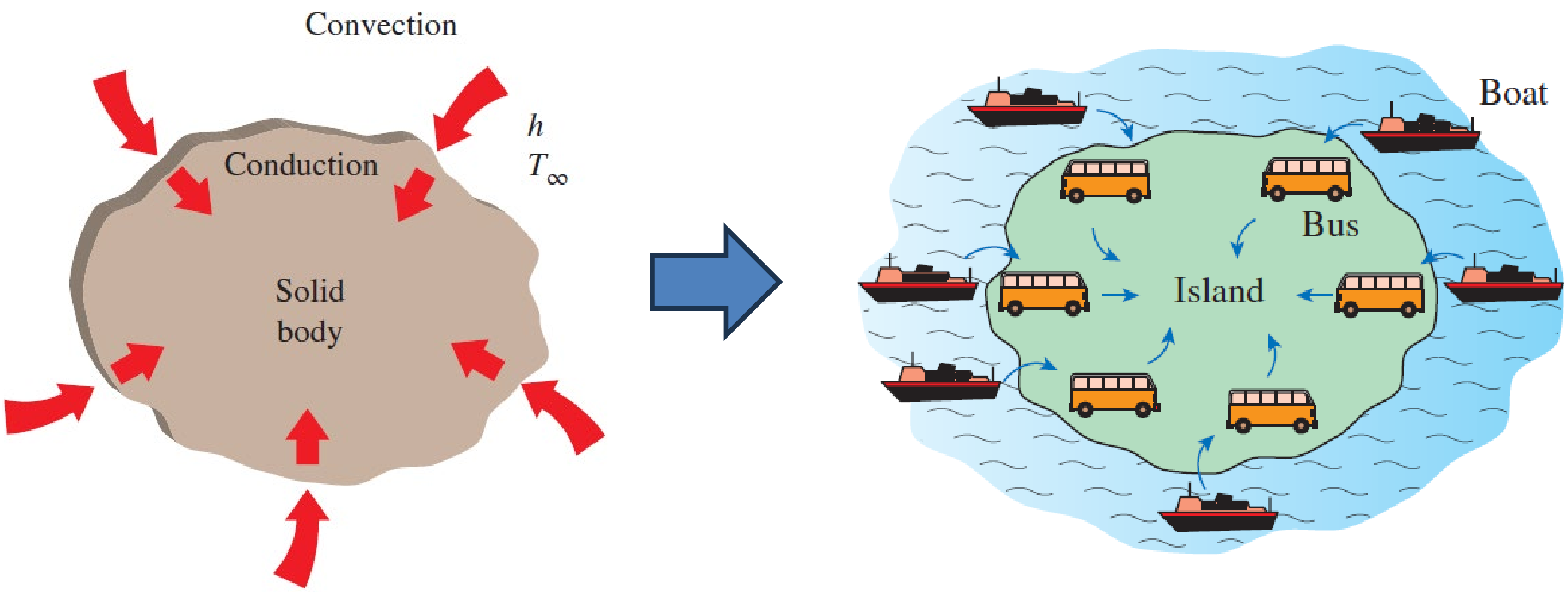
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V c_p} \qquad (1/\mathrm{s})$$

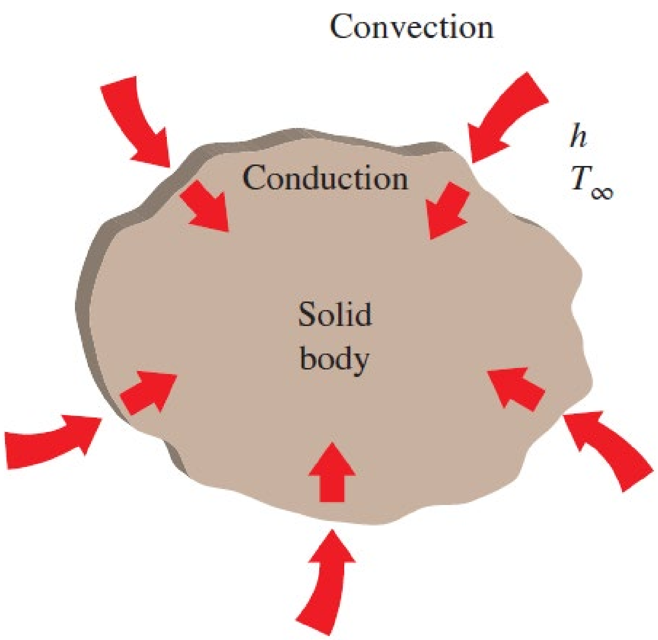
But when is this equation applicable ?

What is the criterion ?

LUMPED SYSTEM



Biot number (Bi)



$$\mathrm{Bi} = \frac{\text{heat convection}}{\text{heat conduction}}$$

$$\mathrm{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

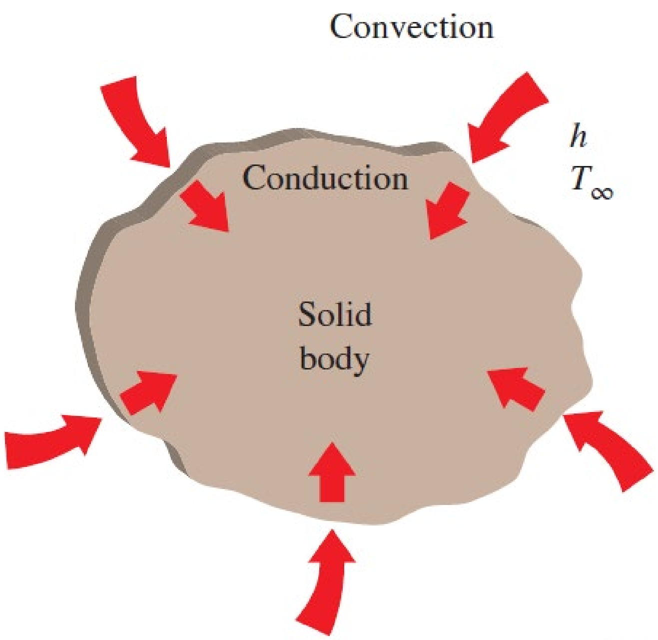
or

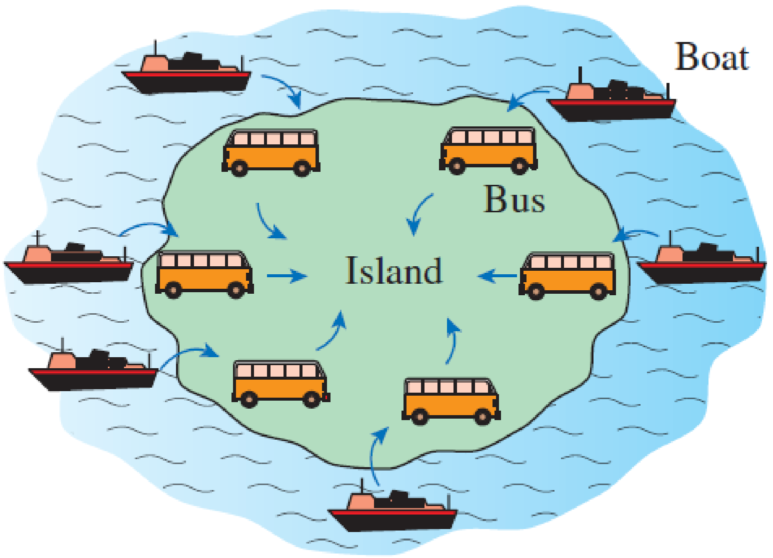
$$\mathrm{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$$\mathrm{Bi} = \frac{hL_c}{k}$$ **When characteristic length is :** $$L_c = \frac{V}{A_s}$$

Almost uniform temperature for Bi ≤ 0,1
"lumped system"

LUMPED SYSTEM





$$\mathrm{Bi} = \frac{hL_c}{k}$$

- Relatively high h → … → Bi high, non uniform
- Relatively high $L_c = V/A$ → … → Bi high, non uniform
- Relatively high k → … → Bi low, uniform

Biot number (Bi)

$$L_c = \text{Characteristic length} = \frac{\text{Volume of the solid } (V)}{\text{Surface area of the solid } (A_s)}$$

The values of characteristic length (L_c), for simple geometric shapes, are given below:

Flat plate : $L_c = \dfrac{V}{A_s} = \dfrac{LBH}{2BH} = L/2 = \text{semi-thickness}$

where L, B and H are thickness, width and height of the plate.

Cylinder (long) : $L_c = \dfrac{\pi R^2 L}{2\pi R L} = \dfrac{R}{2}$ where, R = radius of the cylinder.

Sphere: $L_c = \dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \dfrac{R}{3}$ where, R = radius of the sphere.

Cube: $L_c = \dfrac{L^3}{6L^2} = \dfrac{L}{6}$ where, L = Side of the cube.

LUMPED SYSTEM

i. Lumped system analysis is exact when Bi= 0 and approximate when Bi > 0. Of course, the smaller the Bi number, the more accurate the lumped system analysis.

ii. The first step in the application of lumped system analysis is the calculation of the Biot number, and the assessment of the applicability of this approach.

iii. One may still wish to use lumped system analysis even when the criterion Bi<0.1 is not satisfied, if high accuracy is not a major concern.

iv. Note that the Biot number is the ratio of the convection at the surface to conduction within the body, and this number should be as small as possible for lumped system analysis to be applicable.

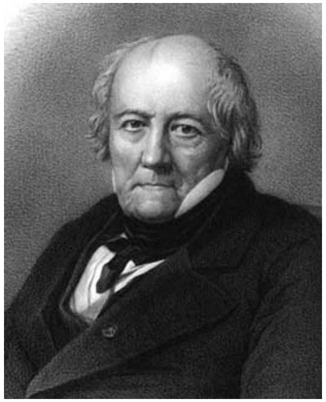
v. Small bodies with high thermal conductivity are good candidates for lumped system analysis, especially when they are in a medium that is a poor conductor of heat (such as air or another gas) and motionless. Thus, the hot small copper ball placed in quiescent air, is most likely to satisfy the criterion for lumped system analysis.

Nusselt vs. Biot

Nusselt number

$$\mathrm{Nu} = \frac{h\,L_c}{k}$$

$\neq$



Biot number

$$\mathrm{Bi} = \frac{h\,L_c}{k}$$

Dimensionless measure for convection so increase of heat transfer due to flow
<u>*k* of fluid!</u>

Dimensionless measure for degree of temperature distribution within body
<u>*k* of surrounded object!</u>

Definitions seem similar but are substantially different!

Summary transient processes

- Biot number $\mathrm{Bi} = \dfrac{h\,L_c}{k} \quad (-)$
- For Bi ≤ 0,1: "lumped system" with $L_c = \dfrac{V}{A} \quad (\mathrm{m})$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad\qquad b = \frac{hA_s}{\rho V c_p}$$

- For Bi > 0,1 : different approaches
- Approximations: check validity

Reference

Y. A. Cengel & A. J. Ghajar. Heat and Mass Transfer: Fundamental & Application:

Chapter 4: Transient Heat Conduction

4–1 : LUMPED SYSTEM ANALYSIS

Criteria for Lumped System Analysis

Some Remarks on Heat Transfer in Lumped Systems