Engineering Thermodynamics 2
Module 3, Energy and Sustainability (201700273)

Mechanical Engineering, B1
Faculty of Engineering Technology
University of Twente

Date: Sample exam (ET 2)
Time: 13.45 - 16.45 (+25% for students with the right on extra time)
Location: Therm
Teacher: G.G.M. Stoffels

Name Student:

Student number:

This exam consists of six (6) questions. Instructions for the exam and the answering of the assignments:

- It is allowed to use a thermodynamics textbook of choice at the exam. Also, a calculator may be used.

- Two formula sheets are handed out.

- Make diagrams, sketches and schemes when asked for in the blank space between the assignments, and fill out the tables given with the assignments. Use for the remaining questions the seperate exam sheets.

- The relative weight of each assignment is indicated. 300 points can be achieved in total.

- Use the correct symbols and do not forget the units (no units = wrong)!

- Give in your answers the formulas first in symbols, and fill out the numbers later.

- When you lack time or cannot find a value, then give the answers of successive questions in symbols.

- Make clear and neat tables, schemes and sketches which are not too small (unclear = wrong)!

- **At the end of the exam, hand in all paper with solutions and the assignments, folded in a standard exam sheet of double A4-format. Make sure that your name is on all papers.**

Good luck!

1 A Cooling Cycle (100)

To cool a room on -20°C a cooling cycle is used, which works according to the vapour compression cooling cycle. The refrigerant used in the cycle is 'Refrigerant 134a'.

Evaporation in the evaporator takes place at a pressure of 80 kPa (0.8 bar) and condensation in the condenser takes place at a pressure of 1200 kPa (12 bar). The processes in the evaporator and the condenser are ideal, the vapour compressor is not ideal. The superheated refrigerant has a temperature of 70°C at the start of the condenser.

The heat in the condenser is drained by a flow of cooling water. This cooling water is heated from 10°C to 15°C and the mass flow of the cooling water is 0.5 kg/s.

a (5): Give on the page next to this exercise the scheme of the cooling cycle described above, and number the characteristic points (call the point after the evaporator point 1). Indicate where heat / work is going in or out of the cycle.

b (5): Give in the table on the next page for every point the two known thermodynamic values that can be used to analyse the cycle, and **circle** them.

c (10): Make, on the page next to this exercise, a clear sketch of the Ts-diagram of this cooling cycle. Show the numbered points and give, where possible, values of pressures and temperatures.

d (15): Determine on every point in the cooling cycle the specific enthalpy, and collect all values found in the table. **(Remark: if you cannot find all h values then just continue and give in the questions below the desired formulas in symbols (h_1, h_2, etc.))**

e (10): Give the formulas with which the mass flow of the refrigerant in the cooling cycle can be determined, and calculate the mass flow of the refrigerant.

f (15): Give the formulas for the calculation of the power that the compressor needs and for the capacity of the cooling installation (the amount of heat that can be extracted from the cool room), and calculate them. Also give the formula for the coefficient of performance (COP) of the cooling cycle, and calculate it.

g (20): The surroundings that the cooling cycle is in has a temperature of 20°C. The cooling cycle keeps the room to be cooled at a constant temperature of -20°C by extracting heat from it (when the cooling cycle would be off, the room would heat up, while with the cooling cycle turned on there is an equilibrium between heating from outside and cooling by the cooling cycle).

Give the formula to calculate the entropy in- or decrease as a consequence of the heat transfer process of the hot surrounding air to the cold room, and calculate the in- or decrease of the entropy. Is entropy generated or destroyed during this process, and why? Is this in congruence with the second law of thermodyncamics, and explain why or why not. Does the entropy of the air in the cold room change, and why?

h (20): The owner of this installation is a real mechanical engineer, who has learned to use all energy as useful as possible and to save energy in this way. He wants to do something with the energy that is wasted in the cooling installation. He decides to adapt the installation a bit so that he can use the released heat to heat a swimming pool for his children.

How is an installation, with the primary goal of producing heat, called in general terms? What is the maximum temperature that the man can reach with the installation, and why? How much energy (in kW) does he save when he uses the installation? What is the coefficient of performance of the installation with the aim to produce heat?

a) Schematic overview of the installation

b) and d) Table with thermodynamic values

Remark: it is not required to fill in all values at all points. Some boxes can remain empty. If you do not need a value on a certain point, you do not need to fill in something in the table.

c) Ts-diagram

2   The Otto Engine (70)

Consider an internal combustion engine working according to the so-called 'air standard Otto cycle'. The compression ratio (so **not** the pressure ratio) is 9.5. The pressure and temperature at the inlet are respectively 95 kPa and 20°C. The temperature at the exit is 540°C. The specific heat may be taken independent of the temperature. Use a value at room temperature as can be found in several text books.

a (10): Give in the space below this exercise the Pv-diagram and the Ts-diagram of the cycle. Give in these diagrams the known pressures and temperatures. Also show where heat or work is added or taken from the cycle. Add the yet unknown pressures and temperatures later.

b (30): In the table below this exercise, fill in the known pressures and temperatures and calculate the remaining pressures and temperatures in the cycle (do not forget to add tem in the diagram).

c (10): Give the formula for the amount of specific heat added during the cycle, and calculate this added heat.

d (10): Give the formula for the nett specific work done by the cycle, and calculate the work.

e (10): Give the formula for the thermal efficiency of the cycle (expressed in temperatures) and calculate the thermal efficiency.

a) Pv and Ts-diagram of the Otto engine

b) Table with thermodynamic values

	P	T		
1				
2				
3				
4				

3 Derivation of Maxwell relation (20)

For a stretched rubber, the work term $-P\mathrm{d}v$ in the Gibbs equation for internal energy can be replaced by the term $+F\mathrm{d}l$, where F is the force exerted and l the change in length. The Gibbs equation then becomes: $\mathrm{d}u = T\mathrm{d}s + F\mathrm{d}l$.

Derive the Maxwell relation that follows from this differential expression for the internal energy.

4 Enthalpy change of an ideal gas (40)

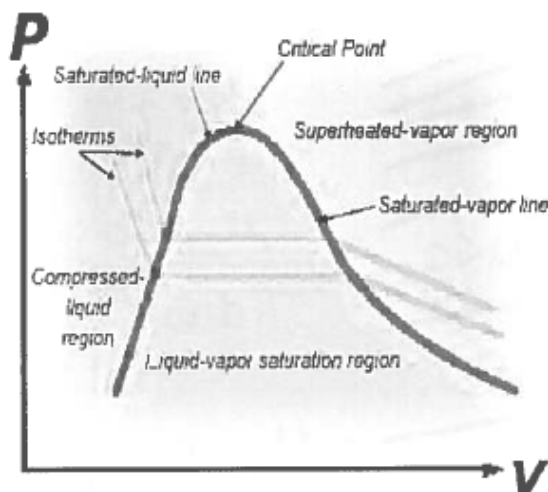Show that for the enthalpy of an ideal gas it holds that $\mathrm{d}h = c_p\mathrm{d}T$.

Hint: Start with the total differential of h as a function of P and T. Work out the partial differential using the systematic method, calculate them for an ideal gas and substitute them.

5 Pv-diagram van water (40)

Consider the Pv-diagram of water below. At first, water is considered compressible.
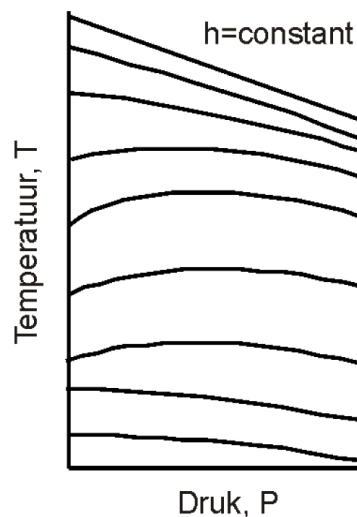
d) Pv-diagram ideal gas

a (10): Give the expression that desribes the slope of the isotherms in the Pv-diagram, and determine to which measurable combination of properties or thermodynamic coefficients this expression is equal. What does the minus sign indicate?

b (5): Why is the slope of the isotherms in the liquid region steeper than in the vapour region?

c (5): What happens with the isotherms when water is considered incompressible in the liquid region?

d (20): Now consider an ideal gas. Give the formula that describes the slope of the isotherms in the Pv-diagram (in terms of for example P, T, v and R). Give, in the space next to the Pv-diagram for water, a Pv-diagram for air with some isotherms in it.

Continue on the backside

6 Joule Thomson Coefficient (30)

a (5): Give the definition of the Joule Thomson coefficient, μ_{JT} and describe the meaning of the partial differential in words.

b (5): What is the value of the Joule Thomson coefficient of a substance when the pressure changes but the temperature stays the same.

c (10): The graph below shows the isenthalps in a temperature-pressure diagram. Draw the curve where the Joule Thomson coefficient equals 0 in the graph, and indicate in which regions the Joule Thomson coefficient is larger or higher than 0. Explain why you draw the curve in a certain way, and why the Joule Thomson coefficient is larger than 0 in the one region and smaller than 0 in the other region.

d (5): How does the temperature change at an increase in pressure of a gas when the Joule Thomson coefficent is respectively larger and smaller than 0, and why?

e (5): In which region is a refrigerant (that is used in for example a refrigerator), and why?

——————————————————- **End of the test** ——————————————————-

Engineering Thermodynamics 1
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1   A Cooling Cycle (100)

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b (5): Give in the table on the next page for every point the two known thermodynamic values that can be used to analyse the cycle, and **circle** them.

c (10): Make, on the page next to this exercise, a clear sketch of the Ts-diagram of this cooling cycle. Show the numbered points and give, where possible, values of pressures and temperatures.

d (15): Determine on every point in the cooling cycle the specific enthalpy, and collect all values found in the table. **(Remark: if you cannot find all h values then just continue and give in the questions below the desired formulas in symbols (h_1, h_2, etc.))**

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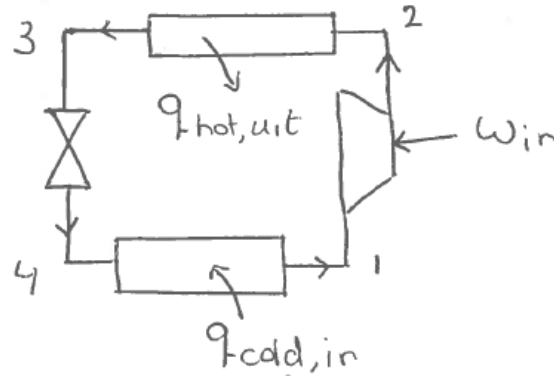
Give the formula to calculate the entropy in- or decrease as a consequence of the heat transfer process of the hot surrounding air to the cold room, and calculate the in- or decrease of the entropy. Is entropy generated or destroyed during this process, and why? Is this in congruence with the second law of thermodyncamics, and explain why or why not. Does the entropy of the air in the cold room change, and why?

h (20): The owner of this installation is a real mechanical engineer, who has learned to use all energy as useful as possible and to save energy in this way. He wants to do something with the energy that is wasted in the cooling installation. He decides to adapt the installation a bit so that he can use the released heat to heat a swimming pool for his children.

How is an installation, with the primary goal of producing heat, called in general terms? What is the maximum temperature that the man can reach with the installation, and why? How much energy (in kW) does he save when he uses the installation? What is the coefficient of performance of the installation with the aim to produce heat?

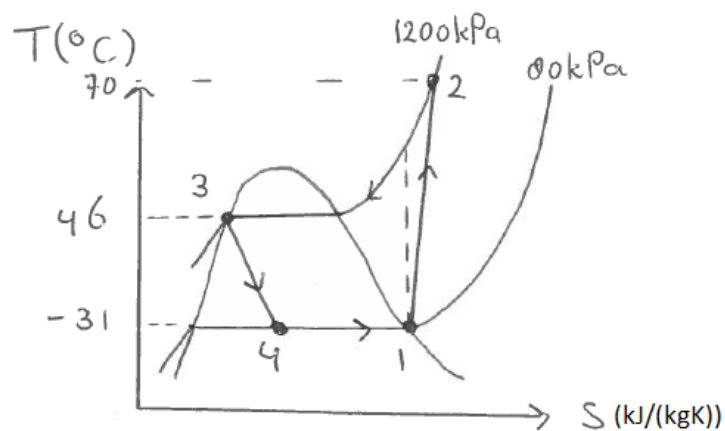# A Cooling Cycle (100): Answers

a) Schematic overview of the installation

b) and d) Table with thermodynamic values

	P [kPa]	T[°C]	x [-]	s [kJ/(kgK)]	h^* [kJ/kg]	h^{**} [kJ/kg]
1	80	-31.13	1 (sat. vap.)	0.957	231.46	228.39
2	1200	70			300.61	298.96
3	1200	46.29	0 (sat. liq.)		117.77	115.76
4	80	-31.13	0.48		$h_4 = h_3$	$h_4 = h_3$

* Values taken from book by Cengel & Boles
** Values taken from book by Schmidt

c) Ts-diagram

3
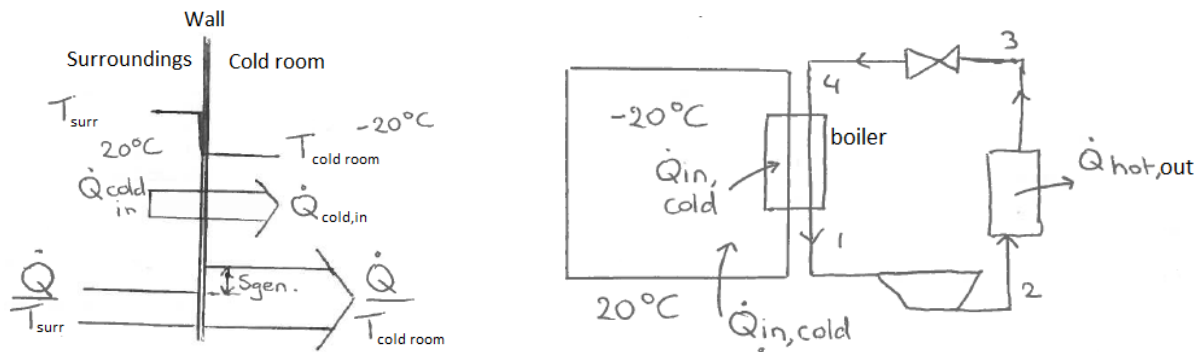
e) (cw stands for cooling water.)

$$\dot{Q}_{hot,out} = \dot{Q}_{cw,in}$$
$$\dot{m}_{ref}(h_2 - h_3) = \dot{m}_{cw}c_P\Delta T$$
$$\dot{m}_{ref} = \frac{\dot{m}_{cw} \cdot c_P \cdot \Delta T}{h_2 - h_3}$$
$$\dot{m}_{ref} = \frac{0.5 \cdot 4.2 \cdot 5}{300.6 - 117.8} = \frac{10.5}{182.8}$$
$$\dot{m}_{ref} = 0.057 \ \frac{\text{kg}}{\text{s}}$$

f) $\dot{W}_{in,comp} = \dot{m}_{ref}(h_2 - h_1) = 0.057(300.6 - 231.46) = 3.94 \text{ kW}$
$\dot{Q}_{in,cold} = \dot{m}_{ref}(h_1 - h_4) = 0.057(231.46 - 117.77) = 6.48 \text{ kW}$
$COP_{ref} = \frac{\dot{Q}_{in,cold}}{\dot{W}_{in,comp}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{6.48}{3.94} = 1.64$

g) The figures below shows the heat transfer process (left) and the situation of the air in the cold room.

$$\mathrm{d}s \geq \frac{\delta q}{T} \Rightarrow \Delta s = \frac{q}{T} \Rightarrow \Delta\dot{S} = \frac{\dot{Q}}{T}$$

$$\dot{S}_{gen} = \Delta\dot{S}_{coldroom} + \Delta\dot{S}_{surr}$$
$$\dot{S}_{gen} = \frac{\dot{Q}_{cold,in}}{T_{coldroom}} - \frac{\dot{Q}_{cold,in}}{T_{surr}}$$
$$\dot{S}_{gen} = \frac{6.48}{273 - 20} - \frac{6.48}{273 + 20}$$
$$\dot{S}_{gen} = 0.026 - 0.022$$
$$\dot{S}_{gen} = 0.004 \ \frac{\text{kW}}{\text{K}}$$

or specific: $s_{gen} = \frac{113.7}{273-20} - \frac{113.7}{273+20} = 0.45 - 0.39 = 0.06 \ \frac{\text{kJ}}{\text{kgK}}$

$\dot{S}_{gen}$ or s_{gen} is postive and entropy is generated. This is in agreement with the second law of thermodynamics that states that for a spontanouos process entropy is created. The transfer of heat from the hot envoronment to the cold space is a spontanouos process.

The air in the cold room is in equilibrium. $\dot{Q}_{in,cold}$ goes from the surroundings to the room that is to be cooled, and this heat is taken from this room by the cooling cycle. Hence, the

temperature of the room does not change, i.e. $T = constant = -20$ °C. From the Gibbs relation, it now follows that:

$$\mathrm{d}u = T\mathrm{d}s - P\mathrm{d}v$$

$$c_v\cancel{\mathrm{d}T}^{\;0} = T\mathrm{d}s - P\cancel{\mathrm{d}v}^{\;0}$$

$$\Rightarrow \mathrm{d}s = 0$$

So the entropy of the cold air does not change.

h) This is called a heat pump.
   The maximum temperature that can be reached is $T_{max} = 70$°C.
   The energy that is saved is equal to $\dot{Q}_{out,hot} = \dot{m}_{ref}(h_2 - h_3) = 0.057(300.61 - 117.77) = 10.42$ kW.
   The coefficient of performance of the heat pump is given by:

$$COP_{hp} = \frac{\dot{Q}_{out,hot}}{\dot{W}_{in,comp}} = \frac{10.42}{3.94} = 2.64$$

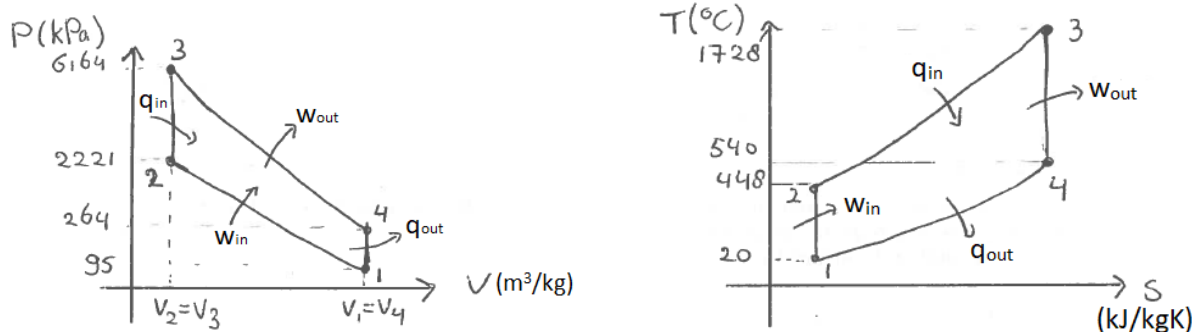Check: $COP_{hp} - COP_{ref} = 2.64 - 1.64 = 1$, which is correct.

2   The Otto Engine (70)

Consider an internal combustion engine working according to the so-called 'air standard Otto cycle'. The compression ratio (so **not** the pressure ratio) is 9.5. The pressure and temperature at the inlet are respectively 95 kPa and 20°C. The temperature at the exit is 540°C. The specific heat may be taken independent of the temperature. Use a value at room temperature as can be found in several text books.

a (10): Give in the space below this exercise the Pv-diagram and the Ts-diagram of the cycle. Give in these diagrams the known pressures and temperatures. Also show where heat or work is added or taken from the cycle. Add the yet unknown pressures and temperatures later.

b (30): In the table below this exercise, fill in the known pressures and temperatures and calculate the remaining pressures and temperatures in the cycle (do not forget to add tem in the diagram).

c (10): Give the formula for the amount of specific heat added during the cycle, and calculate this added heat.

d (1x): Give the formula for the nett specific work done by the cycle, and calculate the work.

e (10): Give the formula for the thermal efficiency of the cycle (expressed in temperatures) and calculate the thermal efficiency.

The Otto Engine (70): Answers

a) Pv and Ts-diagram of the Otto engine

b) Table with thermodynamic values

	P (kPa)	T (°C)	v	other
1	95	20	v_1	
2	2221	448	$\frac{v_1}{v_2} = 9.5$	$s_1 = s_2$
3	6164	1728	v_2	$s_3 = s_4$
4	264	540	v_1	

b) Process 1→2 is isentropic expansion, so

$$P_2 v_2^k = P_1 v_1^k \Rightarrow P_2 = P_1 \left(\frac{v_1}{v_2}\right)^k = 95 \cdot (9.5)^{1.4} = 95 \cdot 23.38 = 2221 \text{ kPa}$$

Furthermore, from the ideal gas law, the temperature can be calculated:

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \Rightarrow T_2 = T_1 \frac{P_2 v_2}{P_1 v_1} = 293 \frac{2221 \cdot 1}{95 \cdot 9.5} = 721 \text{ K}$$

Also process 3→4 is isentropic, such that

$$T_3 v_3^{k-1} = T_4 v_4^{k-1} \Rightarrow T_3 = T_4 \left(\frac{v_4}{v_3}\right)^{k-1} = 813(9.5)^{0.4} = 2001 \text{ K}$$

The pressure at point 3 is now calculated using the ideal gas law:

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \Rightarrow P_3 = P_2 \frac{v_2 T_3}{v_3 T_2} = P_2 \frac{T_3}{T_2} = 2221 \frac{2001}{721} = 6164 \text{ kPa}$$

The pressure at point 4 is also calculated using the ideal gas law:

$$\frac{P_4 v_4}{T_4} = \frac{P_1 v_1}{T_1} \Rightarrow P_4 = P_1 \frac{v_1 T_4}{v_4 T_1} = P_1 \frac{T_4}{T_1} = 95 \frac{813}{293} = 263.6 \text{ kPa}$$

c) $q_{in} = u_3 - u_2 = c_v(T_3 - T_2) = 0.718(1728 - 448) = 919 \text{ } \frac{\text{kJ}}{\text{kg}}$

d)

$$\begin{aligned}
w_{nett} &= w_{out} - w_{in} \\
&= (u_3 - u_4) - (u_2 - u_1) \\
&= c_v \left[(T_3 - T_4) - (T_2 - T_1)\right] \\
&= 0.718 \left[(1728 - 540) - (448 - 20)\right] \\
&= 0.718 \cdot 760 = 545.7 \text{ } \frac{\text{kJ}}{\text{kg}}
\end{aligned}$$

e)

$$\eta_{th} = \frac{w_{nett}}{q_{in}} = \frac{546}{919} = 0.59$$

or:

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{540 - 20}{1728 - 448} = 1 - \frac{520}{1280} = 1 - 0.41 = 0.59$$

Hence, $\eta_{th} = 59\%$.

3   Derivation of Maxwell Relation (20)

For a stretched rubber, the work term $-P\mathrm{d}v$ in the Gibbs equation for internal energy can be replaced by the term $+F\mathrm{d}l$, where F is the force exerted and l the change in length. The Gibbs equation then becomes: $\mathrm{d}u = T\mathrm{d}s + F\mathrm{d}l$.

Derive the Maxwell relation that follows from this differential expression for the internal energy.

Derivation of Maxwell Relation (20): Answers

$$\mathrm{d}u = T\mathrm{d}s + F\mathrm{d}l \qquad\qquad \Rightarrow \left(\frac{\partial u}{\partial s}\right)_l = T, \left(\frac{\partial u}{\partial l}\right)_s = F$$

$$\left.\begin{array}{l}\dfrac{\partial^2 u}{\partial l \partial s} = \left(\dfrac{\partial}{\partial l}\left(\dfrac{\partial u}{\partial s}\right)_l\right)_s = \left(\dfrac{\partial T}{\partial l}\right)_s \\[2.5ex] \dfrac{\partial^2 u}{\partial s \partial l} = \left(\dfrac{\partial}{\partial s}\left(\dfrac{\partial u}{\partial l}\right)_s\right)_l = \left(\dfrac{\partial F}{\partial s}\right)_l\end{array}\right\} \dfrac{\partial^2 u}{\partial l \partial s} = \dfrac{\partial^2 u}{\partial s \partial l} \Rightarrow \left(\dfrac{\partial T}{\partial l}\right)_s = \left(\dfrac{\partial F}{\partial s}\right)_l$$

4   Enthalpy Change of an Ideal Gas (40)

Show that for the enthalpy of an ideal gas it holds that $\mathrm{d}h = c_p \mathrm{d}T$.

    Hint: Start with the total differential of h as a function of P and T. Work out the partial differential using the systematic method, calculate them for an ideal gas and substitute them.

Enthalpy Change of an Ideal Gas (40): Answers

$$\mathrm{d}h = \left(\frac{\partial h}{\partial P}\right)_T \mathrm{d}P + \left(\frac{\partial h}{\partial T}\right)_P \mathrm{d}T$$

$$\left(\frac{\partial h}{\partial P}\right)_T = T\left(\frac{\partial s}{\partial P}\right)_T + v\left(\frac{\partial P}{\partial P}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_P + v \cdot 1 = -Tv\beta + v$$

$$\left(\frac{\partial h}{\partial T}\right)_P = T\left(\frac{\partial s}{\partial T}\right)_P + v\left(\frac{\partial P}{\partial T}\right)_P = T\frac{C_P}{T} + 0 = C_P$$

For an ideal gas it holds that $Pv = RT$, such that $v = \frac{RT}{P}$ and hence:

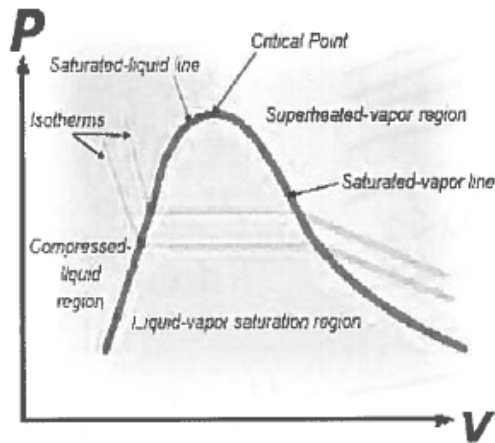$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{v}\frac{R}{P} = \frac{1}{T}$$

Substituting everything in the total differential of h yields:

$$\mathrm{d}h = v(1 - T\beta)\mathrm{d}P + C_P\mathrm{d}T = v\left(1 - \frac{T}{T}\right)\mathrm{d}P + C_P\mathrm{d}T$$
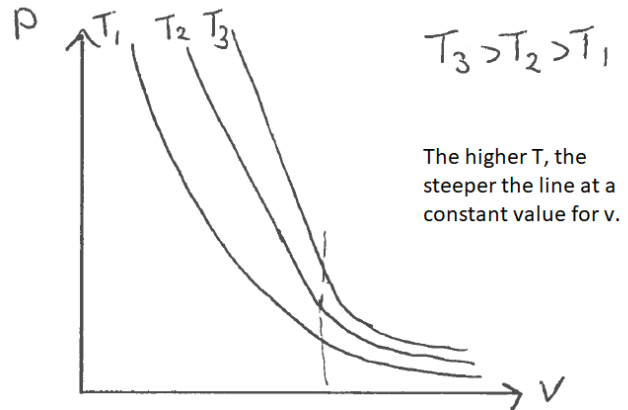
So, $\mathrm{d}h = C_p\mathrm{d}T$.

5 Pv-Diagram of Water (40)

Consider the Pv-diagram of water below. At first, water is considered compressible.

d) Pv-diagram ideal gas

a (10): Give the expression that desribes the slope of the isotherms in the Pv-diagram, and determine to which measurable combination of properties or thermodynamic coefficients this expression is equal. What does the minus sign indicate?

b (5): Why is the slope of the isotherms in the liquid region steeper than in the vapour region?

c (5): What happens with the isotherms when water is considered incompressible in the liquid region?

d (20): Now consider an ideal gas. Give the formula that describes the slope of the isotherms in the Pv-diagram (in terms of for example P, T, v and R). Give, in the space next to the Pv-diagram for water, a Pv-diagram for air with some isotherms in it.

Pv-Diagram of Water (40): Answers

a:

$$\left(\frac{\partial P}{\partial v}\right)_T = \frac{1}{\left(\frac{\partial v}{\partial P}\right)_T} = \frac{1}{-v\kappa} = \frac{-1}{v\kappa}$$

It is negative (a minus sign), so the slope is downwards (i.e. the value of P decreases with increasing v and constant T).

b: Because the volume of a vapour is much larger than the volume of a liquid ($v_{\text{vapour}} \gg v_{\text{liquid}}$)

c: If it is incompressible, v is constant. Hence, $\partial v = 0$ such that $\left(\frac{\partial P}{\partial v}\right)_T \to -\infty$

d: Ideal gas:

$$Pv = RT \Rightarrow P = \frac{RT}{v} \Rightarrow \left(\frac{\partial P}{\partial v}\right)_T = -\frac{RT}{v^2}$$

9

$v \to 0$ then slope $\to -\infty$
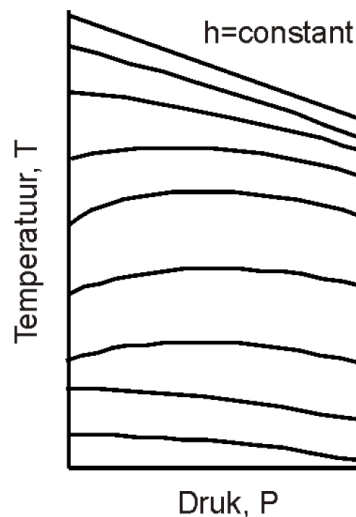$v \to \infty$ then slope $\to -0$
$-sign \Rightarrow$ draw decreasing line
At a constant value for v, it holds that the slope of the line is steeper at a higher temperature.

6   Joule Thomson Coefficient (30)

a (5): Give the definition of the Joule Thomson coefficient, μ_{JT} and describe the meaning of the partial differential in words.

b (5): What is the value of the Joule Thomson coefficient of a substance when the pressure changes but the temperature stays the same.

c (10): The graph below shows the isenthalps in a temperature-pressure diagram. Draw the curve where the Joule Thomson coefficient equals 0 in the graph, and indicate in which regions the Joule Thomson coefficient is larger or higher than 0. Explain why you draw the curve in a certain way, and why the Joule Thomson coefficient is larger than 0 in the one region and smaller than 0 in the other region.

d (5): How does the temperature change at an increase in pressure of a gas when the Joule Thomson coefficent is respectively larger and smaller than 0, and why?

e (5): In which region is a refrigerant (that is used in for example a refrigerator), and why?
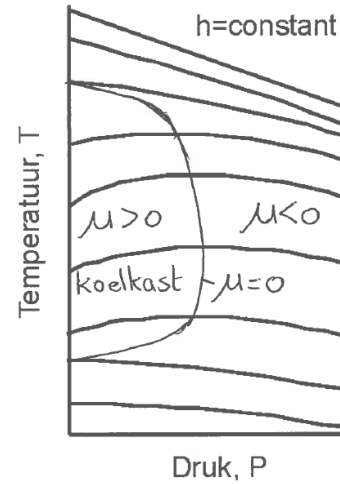
Joule Thomson Coefficient (30): Answers

a: $\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_h$ The Joule Thomson coefficient describes the change of the temperature for a change in pressure at a constant enthalpy.

b: $dT = 0 \Rightarrow \mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_h = 0$

c: The curve goes through the isenthalps at the point where their slope is 0 (i.e. the maxima of the isenthalps). At the left of the curve the slope is positive: $\mu_{JT} > 0$, and at the right of the curve the slope is negative: $\mu_{JT} < 0$

d: An increase in pressure, so $dP > 0$. For the Joule Thomson coefficient larger than 0, $dP > 0$ results in $dT > 0$ (because the slope is positive this has to be the case) and hence the temperature increases. For $\mu_{JT} < 0$ and $dP > 0$ one obtains $dT <$) (because the slope is negative) and hence the temperature decreases.

e: Refrigerant, it should cool (so $dT < 0$ when $dP < 0$, so $\mu_{JT} > 0$ and hence it is inside the circle (at the left of the curve).

─────────────────────── - **End of the test** ─────────────────────────-

Formulas

Differential expressions for thermodynamic energies

Energy	Symbol, variabels	Differential
Internal energy	$u(s,v)$	$du = Tds - Pdv$
Enthalpy	$h(s,P) = u + Pv$	$dh = Tds + vdP$
Helmholtz energy	$a(T,v) = u - Ts$	$da = -sdT - Pdv$
Gibbs energy	$g(T,P) = u - Ts + Pv$	$dg = -sdT + vdP$

Maxwell relations

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial P}{\partial s}\right)_v$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial v}{\partial s}\right)_P$$

$$\left(\frac{\partial P}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

$$\left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$$

Thermodynamic coefficients

Isothermal compressibility coefficient: $\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T$

Thermal expansion coefficient: $\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P$

Joule-Thomson coefficient: $\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_h$

Specific heat for a constant volume process: $c_v = \left(\frac{\partial u}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v$
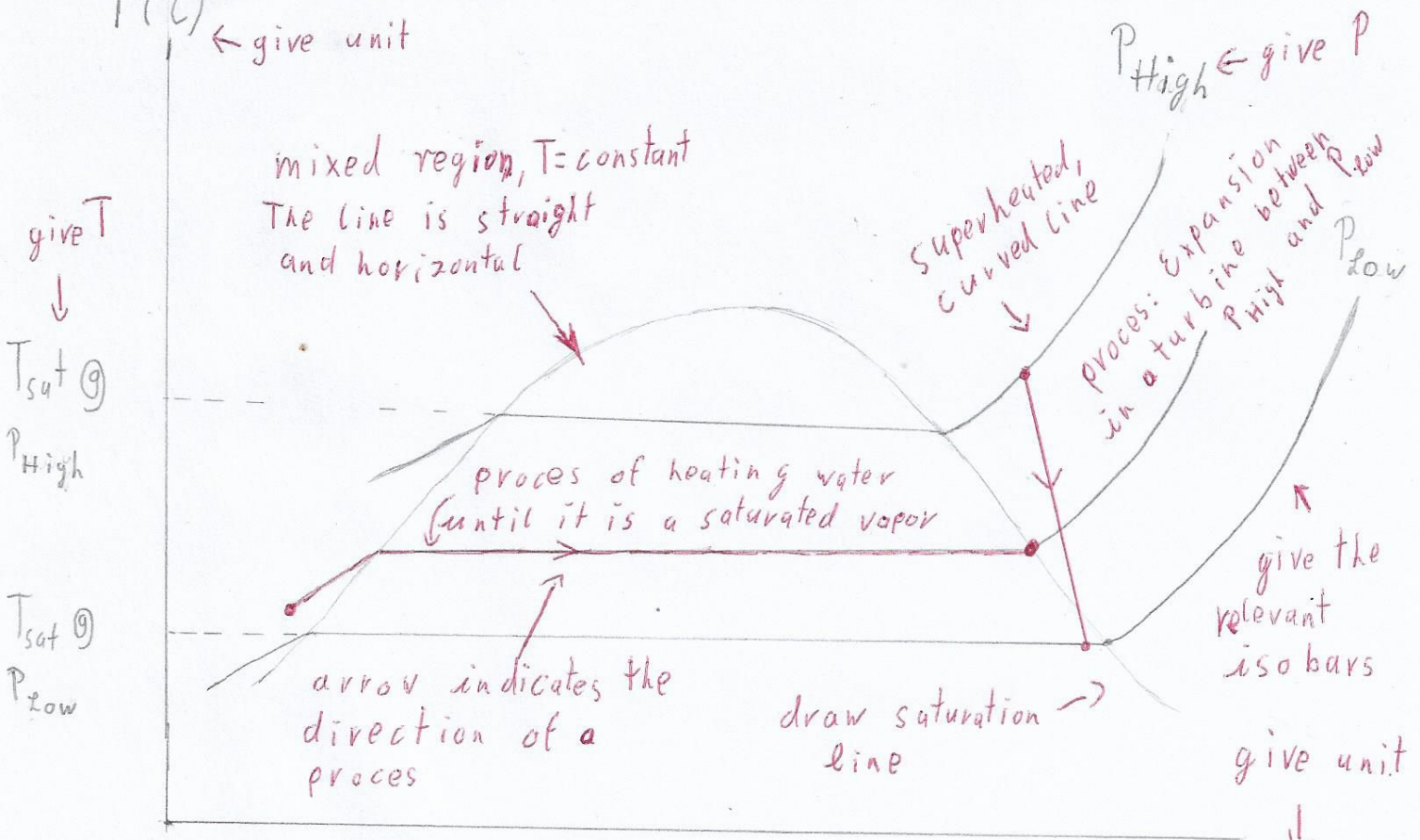
Specific heat for a constant pressure process: $c_p = \left(\frac{\partial h}{\partial T}\right)_P = T\left(\frac{\partial s}{\partial T}\right)_P$

Nernst-Lindeman Relation: $c_v = c_P - \frac{Tv\beta^2}{\kappa}$

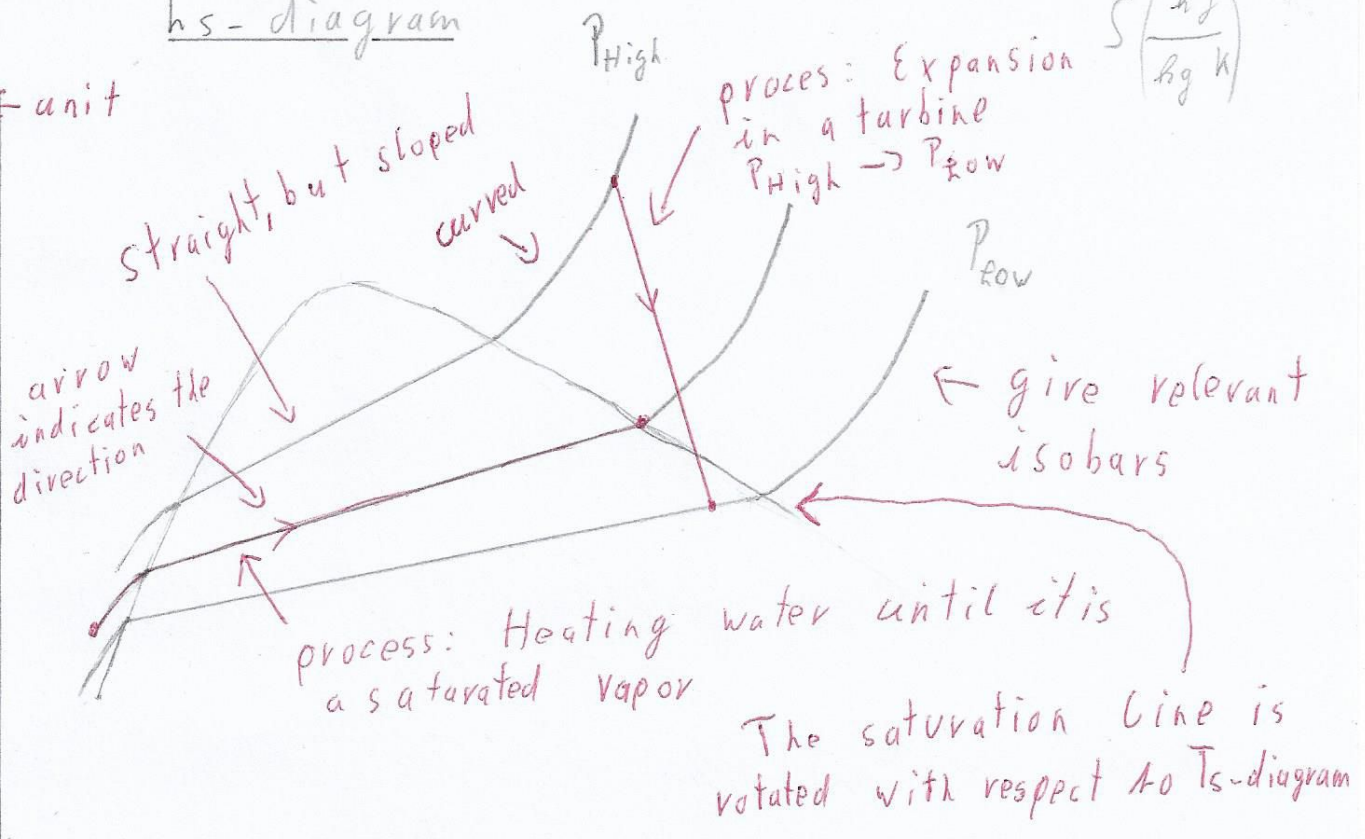**Clapeyron relation:** $\left(\frac{dP}{dT}\right)_{sat} = \left(\frac{h_{12}}{Tv_{12}}\right)$

Clausius-Clapeyron relation: $\left(\frac{dP}{P}\right)_{sat} = \frac{h_{lv}}{R}\left(\frac{dT}{T^2}\right)_{sat}$ of $\left(\frac{d(lnP)}{dT}\right)_{sat} = \frac{h_{lv}}{RT_{sat}^2}$

Ts - diagram

$T(°C)$ ← give unit

give T → T_{sat} @ P_{High}

T_{sat} @ P_{Low}

mixed region, T=constant
The line is straight
and horizontal

proces of heating water
(until it is a saturated vapor)

arrow indicates the
direction of a
proces

draw saturation →
line

P_{High} ← give P

Superheated,
curved line

proces: Expansion between P_{Low}
in a turbine
P_{High} and P_{Low}

give the
relevant
isobars

give unit
↓
$S\left(\dfrac{kJ}{kg\,k}\right)$

hs - diagram

$h\left(\dfrac{kJ}{kg}\right)$ ← unit

P_{High}

straight, but sloped

curved

proces: Expansion
in a turbine
$P_{High} → P_{Low}$

P_{Low}

arrow
indicates the
direction

← give relevant
isobars

process: Heating water until it is
a saturated vapor

The saturation line is
rotated with respect to Ts-diagram

$S\left(\dfrac{kJ}{kg\,k}\right)$

Notice the difference between the
Ts and hs diagram. Indicate known
temperatures and pressures.