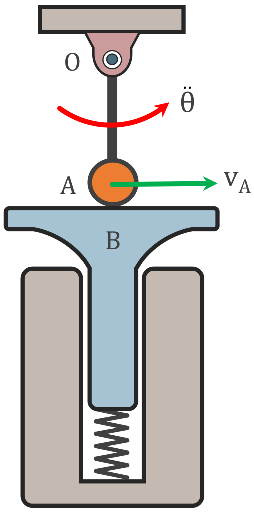
Acceleration of a Crankshaft



Determine the acceleration of the blue shaft B if the crank OA has a angular acceleration of $\ddot{\theta} = 12$ rad/s^2 and ball A has a velocity of $v_A = 3$ m/s at this position. The spring maintains contact between the roller and the surface of the plunger.

Take $L_{OA} = 0.5$ m.

Using known expressions (for rigid bodies and a (relative) fixed point O*)*:

$$\mathbf{v}_{\mathrm{A}} = \boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} \tag{1}$$

$$\mathbf{a}_{\mathrm{A}} = \mathbf{a}_{\mathrm{A},n} + \mathbf{a}_{\mathrm{A},t} \tag{2}$$

$$\mathbf{a}_{\mathrm{A},n} = \boldsymbol{\omega}_{\mathrm{OA}} \times (\boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}}) \tag{3}$$

$$\mathbf{a}_{\mathrm{A},t} = \boldsymbol{\alpha}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} \tag{4}$$

Given quantities:

Angle: $\theta = 90°$
Absolute angular acceleration: $\alpha_{\mathrm{OA}} = \ddot{\theta} = 12$ rad/s^2
Angular acceleration: $\boldsymbol{\alpha}_{\mathrm{OA}} = \alpha_{\mathrm{OA}}\mathbf{k}$
Velocity of A: $v_{\mathrm{A}} = 3$ m/s
Distance from O to A: $L_{\mathrm{OA}} = 0.5$ m.

Solution:

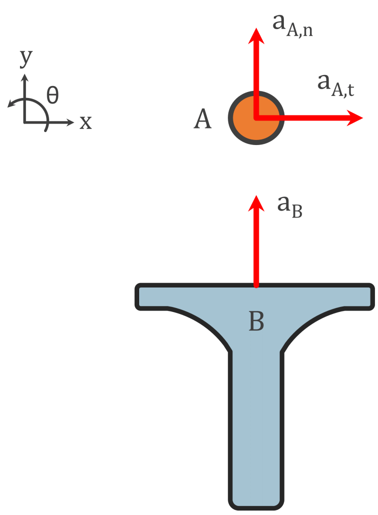


Figure 1: Kinematic diagram of the ball and the plunger.

The blue shaft B can only move up and down, thus it has zero acceleration in the x-direction ($\mathbf{a}_{\mathrm{B}} = a_{\mathrm{B}}\mathbf{j}$). At this time instant, the crankshaft angle is exactly 90°, which means point A has a normal and tangential acceleration in the y-and x-direction respectively. Since the roller remains in contact with the shaft, its normal acceleration must be equal to the acceleration of the shaft. This is visualized in the kinematic diagram of Figure 1. As seen in Equation (3), the angular velocity of the crankshaft is needed. Using Equation (1) the angular velocity of the crankshaft is calculated:

$$\mathbf{v}_{\mathrm{A}} = \boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} \quad \Rightarrow \quad \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \begin{pmatrix} 0 \\ -0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\omega_{\mathrm{OA}} \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

Thus $\omega_{\mathrm{OA}} = 6$ rad/s.

Using Equation (3) the acceleration of shaft B becomes:

$$\mathbf{a}_{\mathrm{A},n} = \boldsymbol{\omega}_{\mathrm{OA}} \times (\boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}}) = \mathbf{a}_{\mathrm{B}} \quad \Rightarrow \tag{6}$$

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \left(\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \begin{pmatrix} 0 \\ -0.5 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \begin{pmatrix} 0.5\omega_{\mathrm{OA}} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5\omega_{\mathrm{OA}}^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ a_{\mathrm{B}} \\ 0 \end{pmatrix}$$

Thus $a_{\mathrm{B}} = 0.5\omega_{\mathrm{OA}}^2 = 0.5 \cdot 6^2 = 18$ m/s^2.