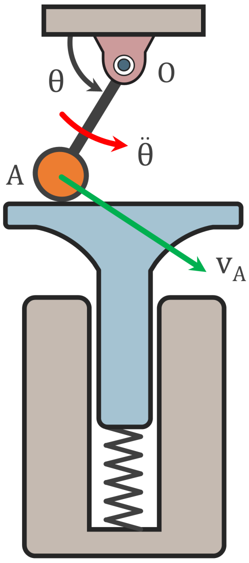
Acceleration of Crankshaft



Determine the acceleration of the blue shaft B if the crank OA has a angular acceleration of $\ddot{\theta} = 12\ \mathrm{rad/s^2}$ and ball A has a velocity of $v_A = 12\ \mathrm{m/s}$ at this position. The spring maintains contact between the roller and the surface of the plunger. $L_{OA} = 0.5\ \mathrm{m}$ and $\theta = 60°$.

Using known expressions:

$$\boldsymbol{v_A} = \boldsymbol{\omega} \times \boldsymbol{r_{A/O}} \tag{1}$$

$$\boldsymbol{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \tag{2}$$

$$\boldsymbol{a}_t = \boldsymbol{\alpha} \times \boldsymbol{r} \tag{3}$$

Given:
Angle: $\theta = 60°$
Angular acceleration: $\alpha = \ddot{\theta} = 12\ \mathrm{rad/s^2}$
Velocity: $v_A = 12\ \mathrm{m/s}$
Distance from O to A: $L_{OA} = 0.5\ \mathrm{m}$.

The blue shaft B can only move up and down, thus it has a zero acceleration in the x-direction. A kinematic diagram of the situation is drawn in Figure 1. The total acceleration in the y-direction of the ball must be equal to the acceleration in the y-direction of the shaft, since they remain in contact. Both the normal and tangential acceleration can be decomposed in an acceleration in the x-and y-direction.

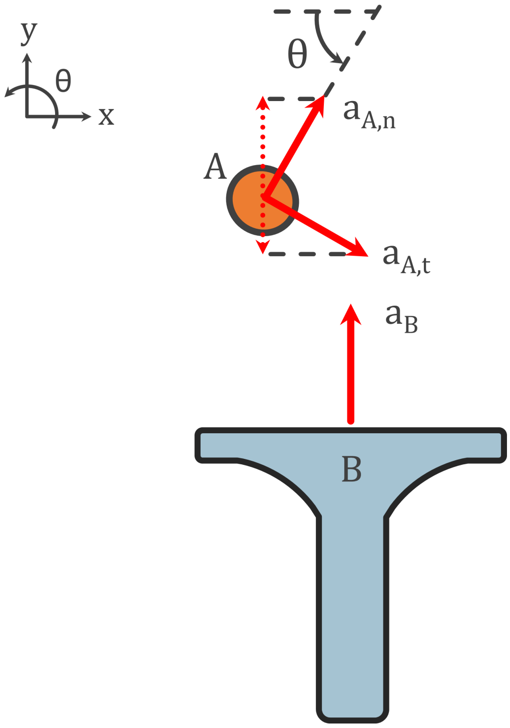


Figure 1: Kinematic diagram of the ball and the plunger.

As can be seen in Equation 2, the angular velocity of the crankshaft is needed. Using Equation 1 the angular velocity of the crankshaft can be calculated as follows:

$$\boldsymbol{v_A} = \boldsymbol{\omega} \times \boldsymbol{r_{A/O}} \quad \Rightarrow \quad \begin{pmatrix} 12\sin\theta \\ -12\cos\theta \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -0.5\cos\theta \\ -0.5\sin\theta \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\omega\sin\theta \\ -0.5\omega\cos\theta \\ 0 \end{pmatrix} \tag{4}$$

Thus $\omega = 6$ rad/s.

Using Equation 2 the normal acceleration of ball A becomes:

$$\boldsymbol{a}_{A,n} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r_{A/O}}) \quad \Rightarrow \tag{5}$$

$$\boldsymbol{a}_{A,n} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left(\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -0.5\cos\theta \\ -0.5\sin\theta \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0.5\omega\sin\theta \\ -0.5\omega\cos\theta \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\omega^2\cos\theta \\ 0.5\omega^2\sin\theta \\ 0 \end{pmatrix}$$

Thus $a_{A,n,y} = 0.5\omega^2 \sin\theta$.

Using Equation 6 the tangential acceleration of ball A becomes:

$$\boldsymbol{a}_t = \ddot{\boldsymbol{\theta}} \times \boldsymbol{r} \quad \Rightarrow \quad \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} -0.5\cos\theta \\ -0.5\sin\theta \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\alpha\sin\theta \\ -0.5\alpha\cos\theta \\ 0 \end{pmatrix} \tag{6}$$

Thus $a_{A,t,y} = -0.5\alpha\cos\theta$.

The total acceleration in y-direction of ball A is equal to the total acceleration of the blue shaft B, and this is equal to $a_B = a_{A,n,y} + a_{A,t,y} = 0.5\omega^2\sin\theta - 0.5\alpha\cos\theta = 9\sqrt{3} - 3 \approx 12.6\ \mathrm{m/s^2}$.