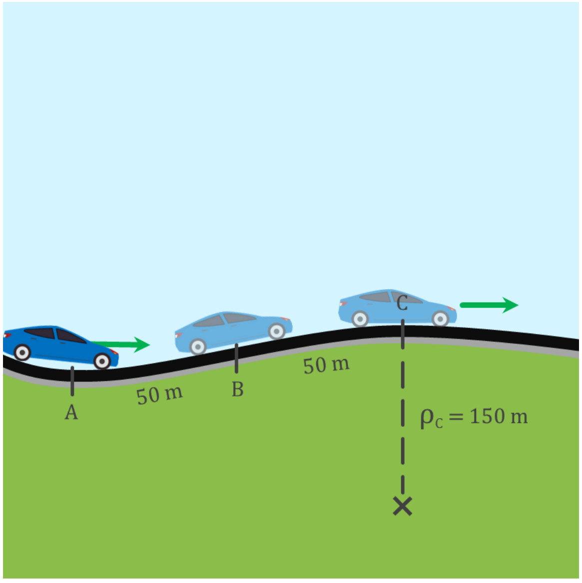
Dip and Hump in the Road



The driver of the car applies the brakes to produce a uniform deceleration, to go over the dip and hump in the road. The speed at the bottom A of the dip is 90 km/h and 54 km/h at the top C of the hump, which is 100m along the road from A. If the passengers experience a total acceleration of 3 m/s^2 at A and if the radius of curvature of the hump at C is 150 m, determine the radius of curvature ρ in meters at A. Round to the nearest integer (no decimals).

Using known expressions (for arbitrary acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}v}{a} \tag{1}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}s}{v} \tag{2}$$

$$\mathrm{d}t = \frac{\mathrm{d}v}{a} = \frac{\mathrm{d}s}{v} \quad \Rightarrow \quad v\mathrm{d}v = a\mathrm{d}s \tag{3}$$

$$\int_{v_0}^{v_1} v \,\mathrm{d}v = \int_{s_0}^{s_1} a \,\mathrm{d}s \tag{4}$$

$$a_n = \frac{{v_t}^2}{\rho} \quad \Rightarrow \quad \rho = \frac{{v_t}^2}{a_n} \tag{5}$$

Given quantities:

Velocity at A: $v_A = 90$ km/h $= 25$ m/s
Velocity at C: $v_{\mathrm{C}} = 54$ km/h $= 15$ m/s
Acceleration at A: $a_{\mathrm{A}} = 3$ m/s^2
Distance: $s_{\mathrm{AC}} = 100$ m

Solution:

Using Equation (6) for a constant acceleration a results in:

$$\int_{v_{\mathrm{A}}}^{v_{\mathrm{C}}} v \, \mathrm{d}v = a \int_{s_{\mathrm{A}}}^{s_{\mathrm{C}}} \mathrm{d}s \tag{6}$$

$$\frac{1}{2} v^2 \bigg|_{v=v_{\mathrm{A}}}^{v_{\mathrm{C}}} = a s|_{s=s_{\mathrm{A}}}^{s_{\mathrm{C}}} \tag{7}$$

$$\frac{1}{2}(v_{\mathrm{C}}^2 - v_{\mathrm{A}}^2) = a(s_{\mathrm{C}} - s_{\mathrm{A}}) = a s_{\mathrm{AC}} \tag{8}$$

Rewriting gives:

$$a s_{\mathrm{AC}} = \frac{1}{2}({v_{\mathrm{C}}}^2 - {v_{\mathrm{A}}}^2) \quad \Rightarrow \quad a = \frac{{v_{\mathrm{C}}}^2 - {v_{\mathrm{A}}}^2}{2 s_{\mathrm{AC}}} \tag{9}$$

Furthermore, Equation (5) gives:

$$a_{n,\mathrm{A}} = \frac{{v_{\mathrm{A}}}^2}{\rho_{\mathrm{A}}} \quad \Rightarrow \quad \rho_{\mathrm{A}} = \frac{{v_{\mathrm{A}}}^2}{a_{n,\mathrm{A}}} \tag{10}$$

The acceleration calculated using Equation (9) is the average acceleration, in this case deceleration, over the path the car travels. Since the deceleration is uniform, the acceleration equals the tangential acceleration a_t and is constant over the path.

$$a = a_t = a_{t,\mathrm{A}} = \frac{{v_{\mathrm{C}}}^2 - {v_{\mathrm{A}}}^2}{2 s_{\mathrm{AC}}} = \frac{15^2 - 25^2}{2 \cdot 100} = -2 \text{ m/s}^2 \tag{11}$$

To calculate the radius of curvature ρ_{A} at A, we need to find the normal acceleration $a_{n,\mathrm{A}}$. Because $\vec{a}_{t,\mathrm{A}}$ and $\vec{a}_{n,\mathrm{A}}$ are perpendicular, the Pythagorean theorem can be used:

$${a_{\mathrm{A}}}^2 = {a_{t,\mathrm{A}}}^2 + {a_{n,\mathrm{A}}}^2 \quad \Rightarrow \quad a_{n,\mathrm{A}} = \sqrt{{a_{\mathrm{A}}}^2 - {a_{t,\mathrm{A}}}^2} = \sqrt{3^2 - (-2)^2} \approx 2.236 \text{ m/s}^2 \tag{12}$$

Inserting this in Equation (10) gives:

$$\rho_{\mathrm{A}} = \frac{{v_{\mathrm{A}}}^2}{a_{n,\mathrm{A}}} = \frac{25^2}{2.236} = 279.5 \text{ m} \approx 280 \text{ m} \tag{13}$$