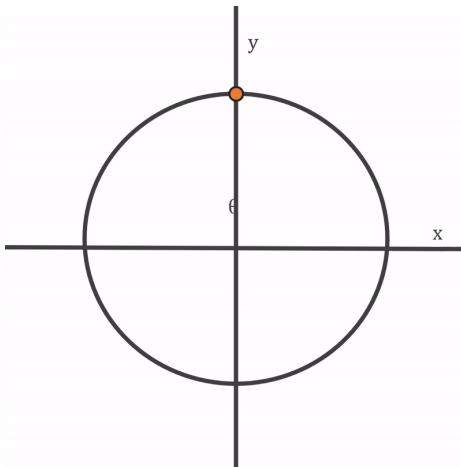


Particle in Circular Motion



Which of the following equations gives the correct relation for the acceleration vector \mathbf{a} in Cartesian coordinates for this particle following a circular motion with radius R ?

Hint: Pay close attention to the definition of θ .

Solution:

The standard formulas of polar coordinates cannot be directly applied, because θ is defined with respect to the vertical. The actual relations will be obtained ourselves.

The transformation to x - and y -coordinates is the following:

$$\begin{cases} x = R \sin \theta \\ y = R \cos \theta \end{cases} \quad (1)$$

Take the first and second derivatives of both components to get relations for the velocity and acceleration. The chain rule is applied.

$$\begin{cases} \dot{x} = R\dot{\theta} \cos \theta \\ \dot{y} = -R\dot{\theta} \sin \theta \end{cases} \quad (2)$$

$$\begin{cases} \ddot{x} = R\ddot{\theta} \cos \theta - R\dot{\theta}^2 \sin \theta \\ \ddot{y} = -R\ddot{\theta} \sin \theta - R\dot{\theta}^2 \cos \theta \end{cases} \quad (3)$$

Combining both results in an acceleration vector $\mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$:

$$\mathbf{a} = (R\ddot{\theta} \cos \theta - R\dot{\theta}^2 \sin \theta)\mathbf{i} + (-R\ddot{\theta} \sin \theta - R\dot{\theta}^2 \cos \theta)\mathbf{j} \quad (4)$$