

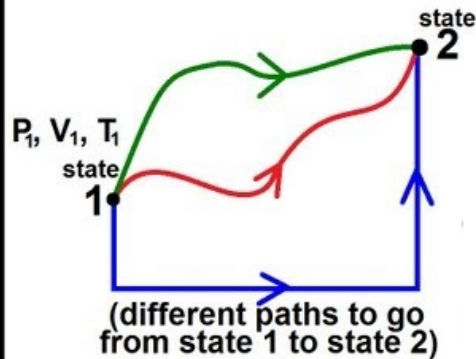
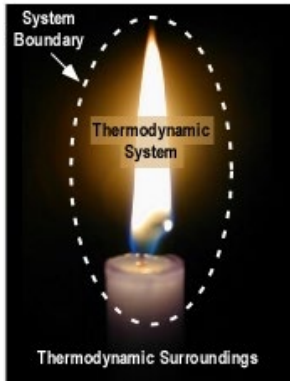
Class 9: Thermodynamic Properties of Pure Substances: Gases

A hot air balloon rises as the hot air inside the balloon is less dense and therefore lighter than the air outside the balloon.

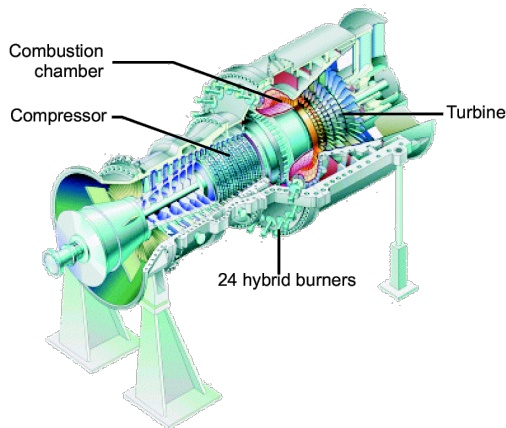


Roadmap Engineering Thermodynamics

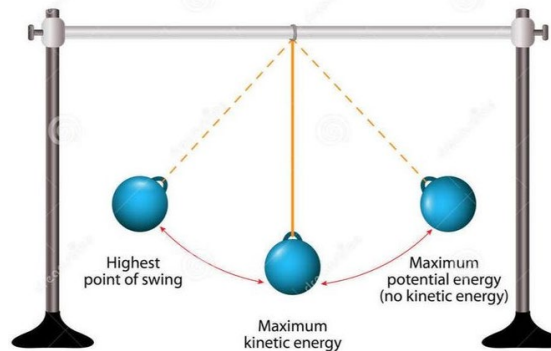
- Using thermodynamics for practical applications requires knowledge of:
Concepts and definitions (Class 1) → Various forms of energy (Class 2)



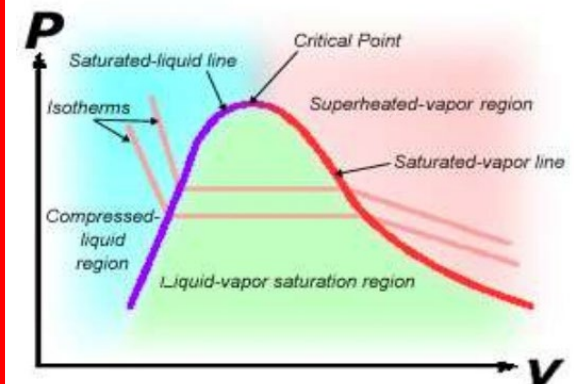
Power cycles
(Class 6 – 11)



Laws of Thermo
(Class 4 and 5)

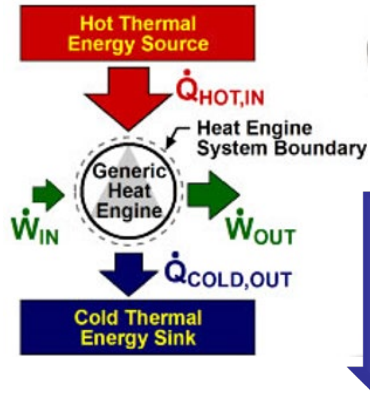


Properties of Substances
(Class 3, 9)

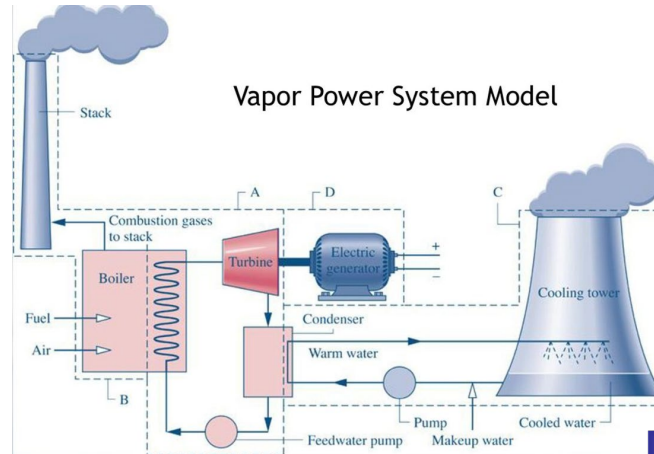


Roadmap Engineering Thermodynamics

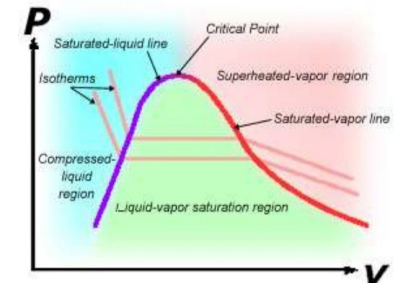
Thermodynamic cycles (Class 6)



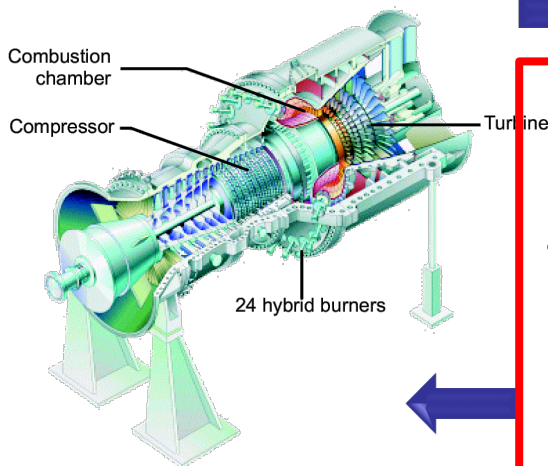
Vapor power cycles – Rankine cycle (Class 7, 8)



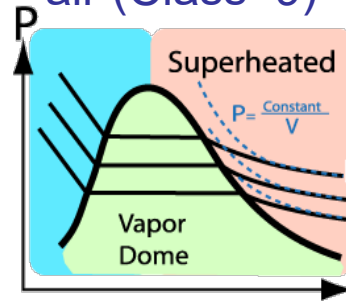
Properties of water (Class 3)



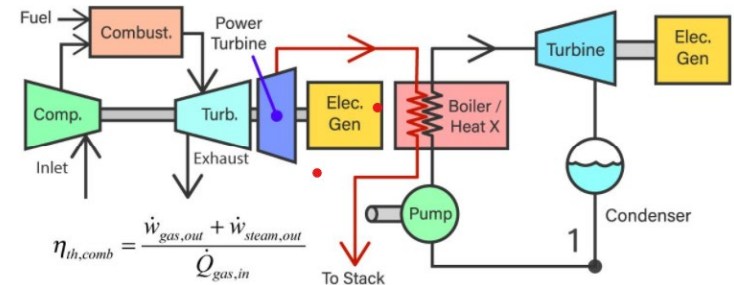
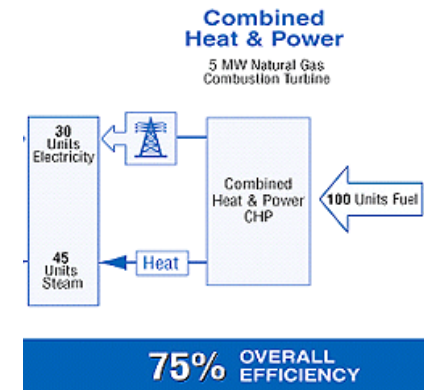
Gas power cycles – Brayton cycle (Class 10, 11)



Properties of air (Class 9)



Combined cycles
Combined heat & power (Class 8, 11)



Classes 9-10-11: Thermodynamic Power Cycles

- In class 1 to 6 we introduced all the (basic) tools we need to study thermodynamic systems that generate power or heat / cold
- Done, class 7 & 8: **vapor power cycles**, cycles using a **working fluid that undergoes a phase transition** (mostly water) through the cycle (Rankine cycle)
- To do, class 10 & 11: **gas power cycles**, cycles using **gas as working fluid** through the whole cycle (Brayton cycle)
- **First class 9: properties for gases** (Like we did properties for water in class 3)
- **Refrigeration and heat pump cycles**, cycles moving heat opposite to the natural direction using power are treated in module 3



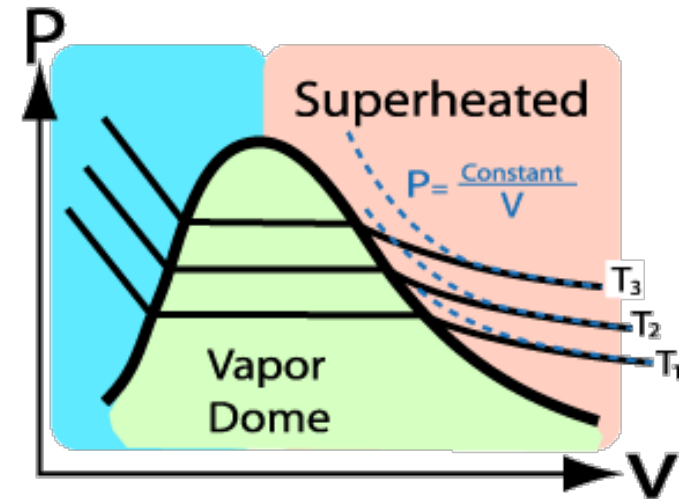
Jet engines are an example of gas power cycles



Power plants typically use vapor power cycles to generate electricity, e.g., the power plant in Geertruidenberg (NL)

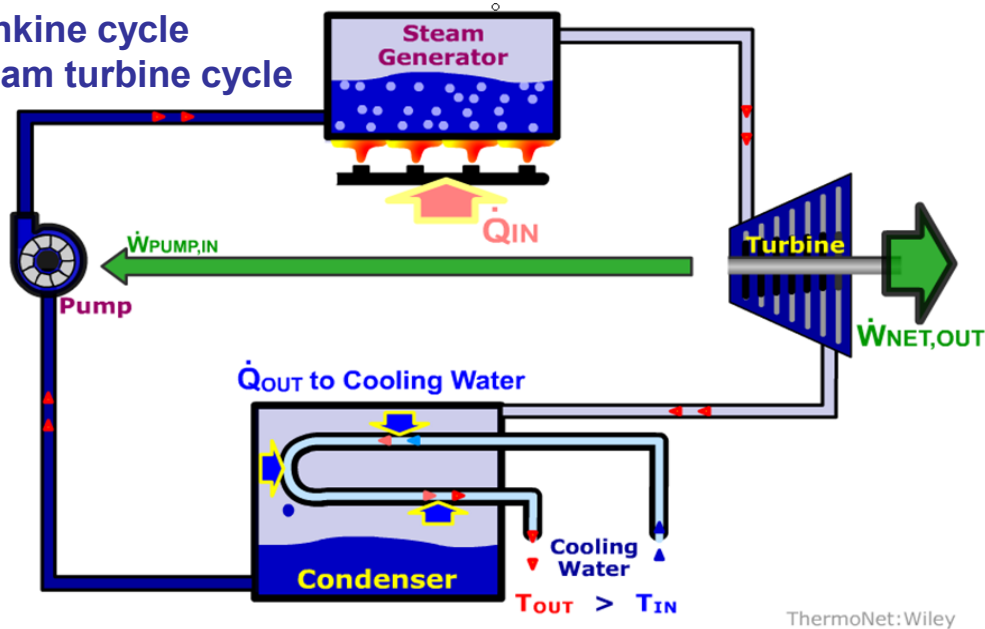
Content Class 9

- **Properties of gases**
- How can we get energy values for air or other gases, like we learned in class 3 for water
 - Diagrams and tables
 - Ideal gas law, $Pv = RT$
- Internal energy for ideal gas
- Enthalpy for ideal gas
- Entropy changes for ideal gas
- Specific heat capacities for gases
 - C_p : for constant pressure
 - C_v for constant volume
- **Learning goal:** declare and explain the behavior of gases at different temperatures and pressures

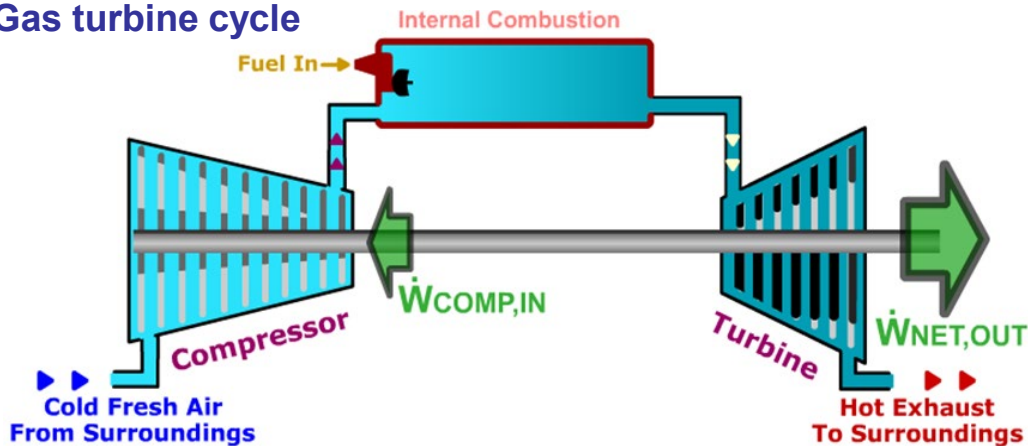


Steam turbine cycle versus gas turbine cycle

Rankine cycle
Steam turbine cycle



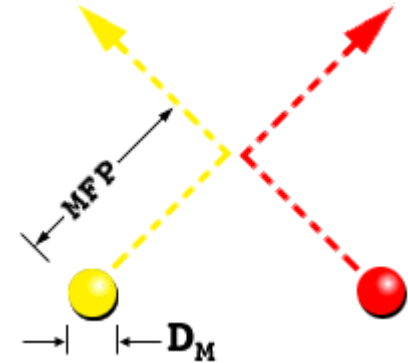
Open Brayton cycle
Gas turbine cycle



- The **Rankine cycle** (also steam turbine or vapor power cycle) uses water as working fluid
- Water undergoes a phase transition during the cycle
- We know how they work and how to analyse them
- The **Brayton cycle** (also gas turbine or gas power cycle) is a similar power cycle using air as working fluid
- The air is always in the gas phase, no phase transition
- Before we can analyze them, we need to know how we can find data for gas

Gases

- How is a **gas** different from a solid or a liquid ?
- In gases molecules are relatively far apart compared to liquids and solids
- The molecules do not feel one another's presence except during collisions
- This has a few implications
 - Gases have a low density
 - Gases are highly compressible
- They behave different than liquids and solid and therefore the way to find their properties and how to handle the formulas is different



Molecules are far apart if the mean distance they travel before hitting another one is much larger than their diameter



Molecules do not feel their presence as their molecular fields are localized close around their centers

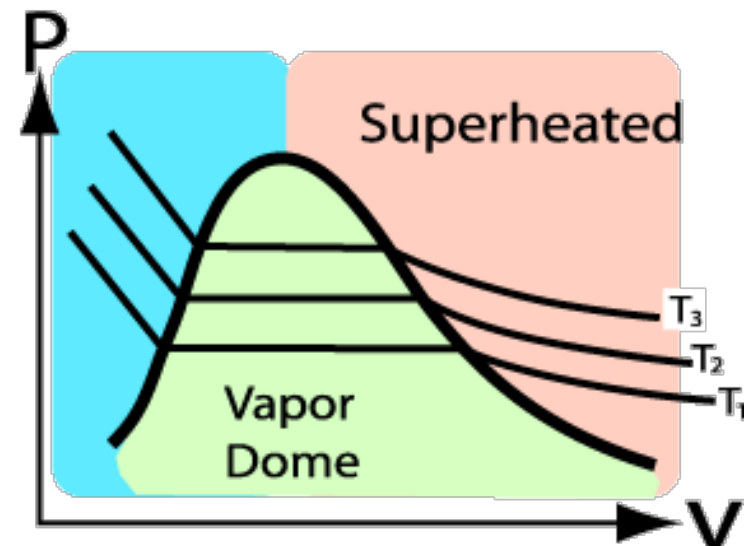
Tables, Diagrams and Equations of State

- Two properties known?
 - All other properties can be found using a table, diagram or formula
(This is stated in the State principle)
- For water properties, like v , T and P can be looked up in tables or diagrams

Table of Steam Properties

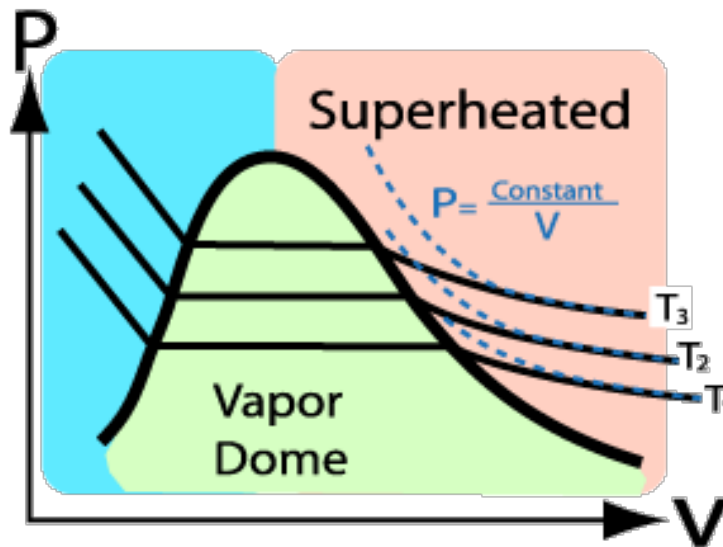
P (kPa)	T ($^{\circ}\text{C}$)	v (m^3/kg)
100	100	1.696
100	120	1.793
100	160	1.984

ThermoNet: John Wiley Publishers



Tables, Diagrams and Equations of State

- A relation describing the curves would be handy in e.g., computer programs (Matlab)
- Curves in the diagrams are not continuous and this makes a mathematical description difficult for the whole region
- However, in the superheated region the isotherms can be represented by a mathematical relation: $P = \frac{\text{Constant}}{V}$



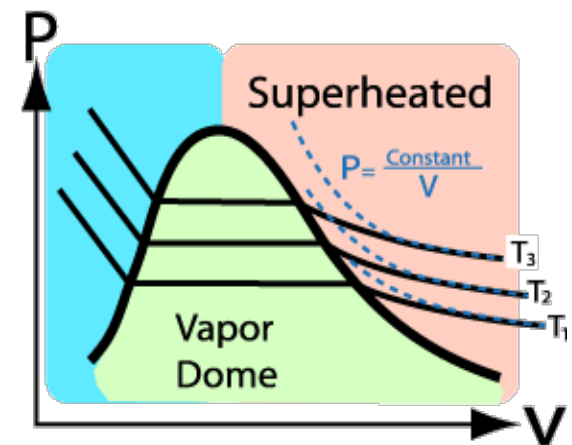
- Note: Xsteam.m can be used in Matlab to program a vapor power cycle. XSteam is based on data from a table, it returns the value you want based on the data implemented. Data in between are calculated based upon interpolation.

Ideal gas equation

- Unlike liquids for gases the relation between P , T and v of an ideal gas can be specified in an **equation of state for an ideal gas**
- An **equation of state** is any equation that relates the pressure, temperature, and specific volume of a substance
- The simplest and best-known equation of state for substances in the gas phase is **the ideal-gas equation of state**
- This equation predicts the P - v - T behavior of a gas quite accurately within some properly selected region
- In the superheated region the isotherms in a PV -diagram can be represented by the mathematical relation, $P = \frac{\text{Constant}}{v}$, which leads to the **Ideal gas equation of state:**

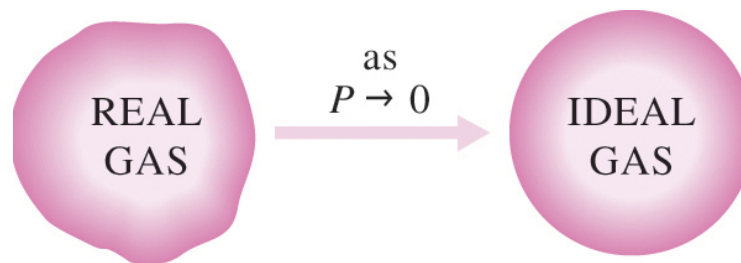
$$Pv = RT$$

T in Kelvin and P absolute pressure in Pascal (or kPa)



Ideal gas equation

- The ideal gas equation is only valid under specific conditions
- Validity
 - Real gases only behave as an ideal gas at low densities (i.e., low pressure, high temperature)
 - At very low pressures, all gases approach ideal-gas behavior (regardless of their temperature)



Ideal gas equation

- Ideal gas equation of state: $Pv = RT$
- There are various versions of the ideal gas equation, but they all describe the same phenomena

$$PV = nR_u T \quad \leftarrow \quad n = m/M$$

$$PV = (m/M)R_u T \quad \leftarrow \quad R_u = RM$$

$$PV = mRT \quad \leftarrow \quad v = V/m$$

$$Pv = RT \quad \leftarrow \quad v = 1/\rho$$

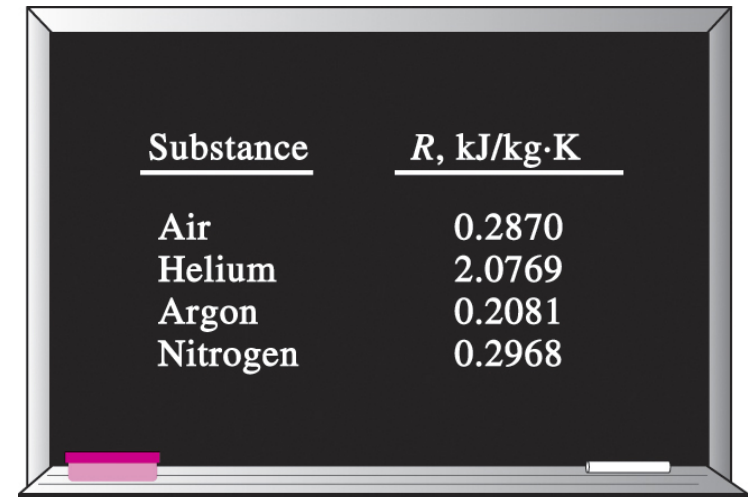
$$P = \rho RT$$

- R_u = universal gas constant (J/molK)
- R = specific gas constant (kJ/kgK)
- n = number of particles (mol)
- m = total mass (kg)
- M = molar mass (kg/mol)
- V = total volume (m³)
- v = specific volume (m³/kg)
- ρ = density (kg/m³)

- Last two versions mostly used in mechanical engineering
- The first one is mostly known from high school

Universal and Specific Gas Constant

- The ideal gas equation
 - In chemistry mostly $PV = nR_u T$ is used
 - In thermodynamics mostly $Pv = RT$ (fluid dynamics $P/\rho = RT$)
- R_u is the **universal gas constant**, same for every gas
- R is the **specific gas constant**, unique for every gas
- Both are related by: $R_u = RM$
where M is the molar mass
- Values for R can be found in tables: 1 and 2a



<u>Substance</u>	<u>R, kJ/kg·K</u>
Air	0.2870
Helium	2.0769
Argon	0.2081
Nitrogen	0.2968

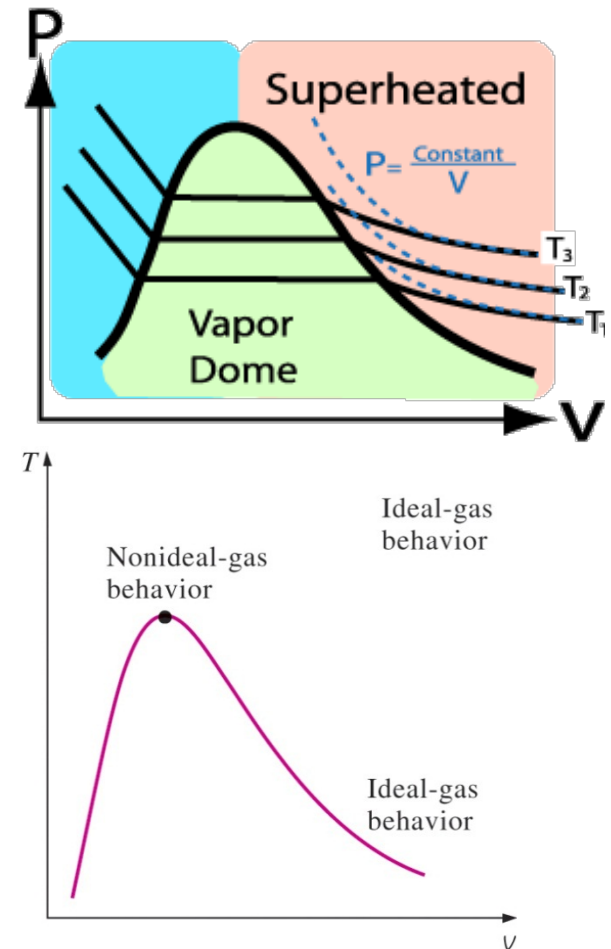
Different substances have different specific gas constants.

Universal and Specific Gas Constant

- The universal gas constant (R_u) and the specific gas constant (R), are related by $R_u = RM$, where M is the molar mass
- **Example**
- **Universal Gas Constant (R_u):** All gases have same value
 - $R_u = 8.314 \text{ kJ}/(\text{kmol.K})$
- **Specific Gas Constant (R):** Each gas has a unique value, e.g., air:
 - $R_{\text{air}} = 0.287 \text{ kJ}/(\text{kg.K})$ (Table 1 or 2)
 - $M_{\text{air}} = 28.97 \text{ kmol/kg}$
 - $R_{\text{air}} M_{\text{air}} = 0.287 \times 28.97 = 8.314 \text{ kJ}/(\text{kmol.K})$

Ideal Gas and Compressibility Factor

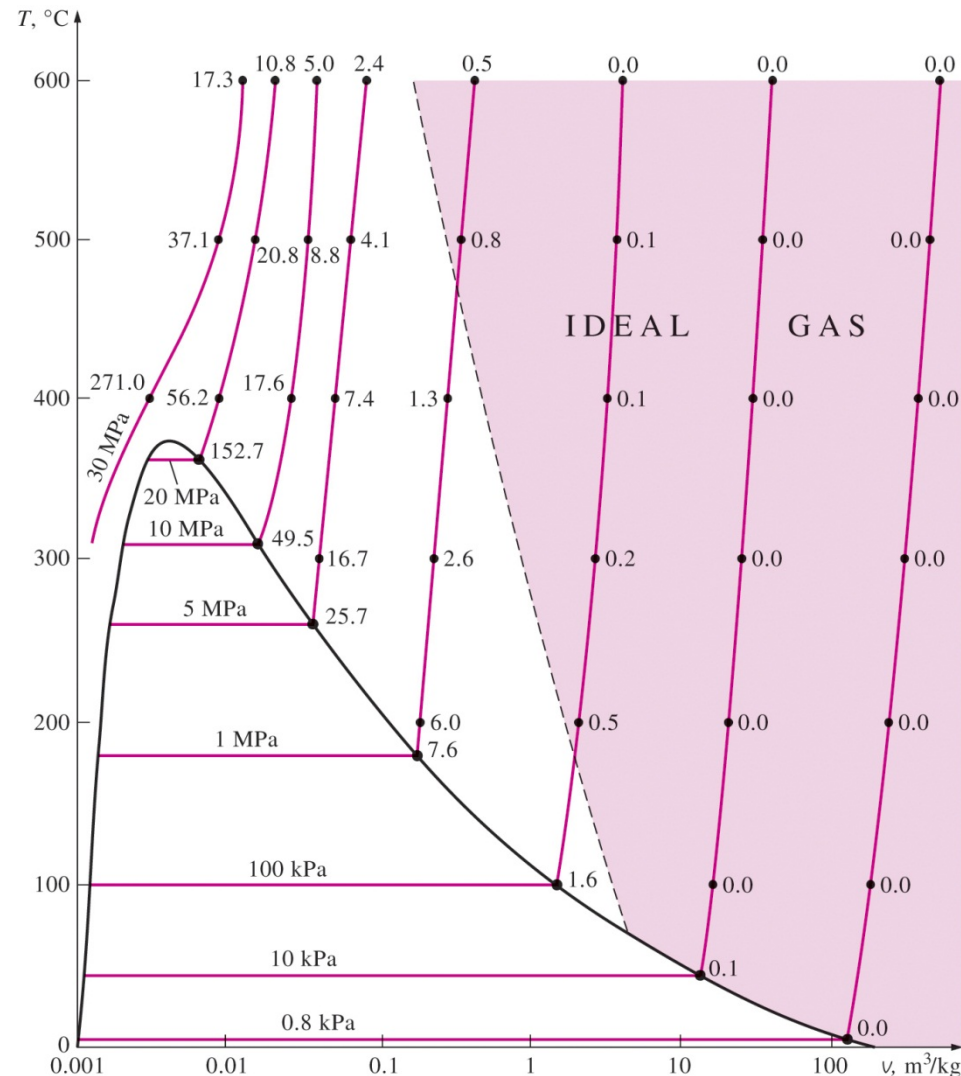
- When is it reasonable to assume that a substance behaves as an ideal gas?
- An ideal gas is one that follows the ideal gas law, $Pv = RT$
- All real gases deviate significantly from the ideal gas law near their saturation region (see graph), but they behave more and more like an ideal gas farther into the superheated region
- The generalized compressibility factor, Z , is used to quantify how closely a substance behaves to an ideal gas
- Z is dimensionless and defined as: $Z = \frac{Pv}{RT}$
- The closer Z approaches 1 the better the gas approximates an ideal gas



Gases deviate from the ideal-gas behavior the most in the neighborhood of the critical point.

Is Water Vapor an Ideal Gas?

- At pressures below 10 kPa, water vapor can be treated as an ideal gas, regardless of its temperature, with negligible error (less than 0.1 percent)
- At higher pressures, however, the ideal gas assumption yields unacceptable errors, particularly in the vicinity of the critical point and the saturated vapor line
- In air-conditioning applications, the water vapor in the air can be treated as an ideal gas
- In steam power plant applications, however, the pressures involved are usually very high; therefore, ideal-gas relations cannot not be used



Percentage of error ($[(v_{\text{table}} - v_{\text{ideal}})/v_{\text{table}}] \times 100$) involved in assuming steam to be an ideal gas, and the region where steam can be treated as an ideal gas with less than 1 percent error.

Other Equations of State

- Besides the ideal gas equation of state there are other equations of state for specific situations to relate P, T and v for gases
- The **Van der Waals' equation** (1873), is an attempt to remove restrictions of the ideal gas law, if $a = 0$ and $b = 0$ this reduces to the ideal gas equation

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

- The **Redlich-Kwong Equation** is accurate at temperatures above T_{CR}

$$P = \frac{RT}{v-b_{RK}} - \frac{a_{RK}}{v(v+b_{RK})T^{1/2}}$$

- Benedict-Webb-Rubin Equation of State** (1940), describes the PvT-behavior of hydro-carbons, constants are given in table 3-4

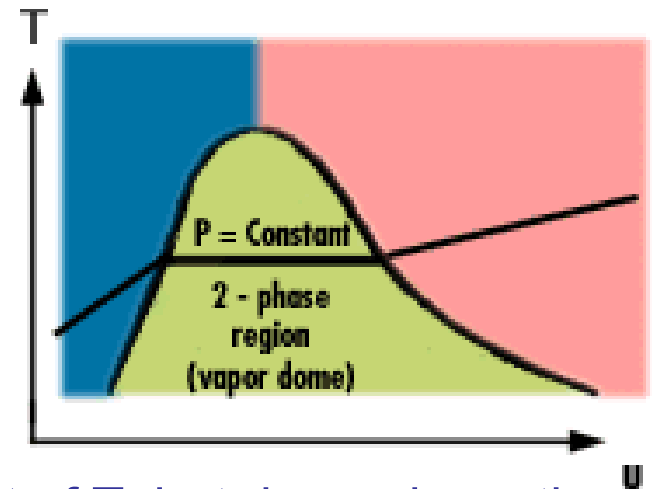
$$P = \frac{RT}{v} + (B_0RT - A_0 - \frac{C_0}{T^2}) \left(\frac{1}{v^2} \right) + \frac{(bRT-a)}{v^3} + \frac{\alpha a}{v^6} + c \frac{1+\gamma/v^2}{v^3 T^2} \exp \left(\frac{-\gamma}{v^2} \right)$$

- The **Virial Equation of State**, where B, C, .. are functions of temperature)

$$P = \frac{RT}{v} \left(1 + \frac{B}{v} + \frac{C}{v^2} + \dots \right)$$

Internal Energy

- The **internal energy** is the energy associated with intermolecular motion, electronic and translational kinetic energy
- Internal energy (like other energies) is measured relative to a reference state (in contrast to e.g., temperature measured relative to the lowest possible temperature and pressure measured relative to vacuum)
- Internal energy is a property and therefore it can be described by two other independent properties e.g., T and P
 - For a real (not ideal) gas, $u = u(T,P)$
 - For an **ideal gas**, $u = u(T) \neq u(P)$ (prove in module 3)
- Outside the two-phase region there is a relation between u and T
- Inside the two-phase region u is independent of T , but depends on the quality of the mixture, x

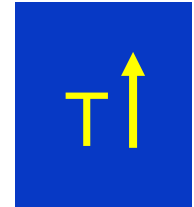


Enthalpy

- **Enthalpy** is like internal energy
- Enthalpy is also measured relative to a reference state also
- Enthalpy is also a property and therefore it can be described by two other independent properties, e.g., T and P
- **Enthalpy**
 - Recall $h = u + Pv$
 - For a real (not ideal) gas, $h = h(T, P)$
 - For an ideal gas, $Pv = RT \rightarrow h = u(T) + RT$
 - Therefore, for an **ideal gas**, $h = h(T) \neq h(P)$ (prove in module 3)
- T, u and h are **dependent** properties for ideal gases
- Changes of u & h with respect to T are very important as thermodynamics deals with energy changes and differences
- For an isothermal process u and h are constant for ideal gases

Specific heat

- **Specific heat, c [kJ/kgK]:** the amount of energy required to raise the temperature of a unit mass of a substance by one degree in a specified way
- Approximately specific heat is defined as: $c = \frac{de}{dT}$
- The specific heat depends on:
 - the substance (water, oil, air, helium, steel, wood,...)
 - the temperature
 - the process (constant pressure or constant volume)
- Values for the specific heat of solid, liquids and various gases can be found in tables 2 and 3

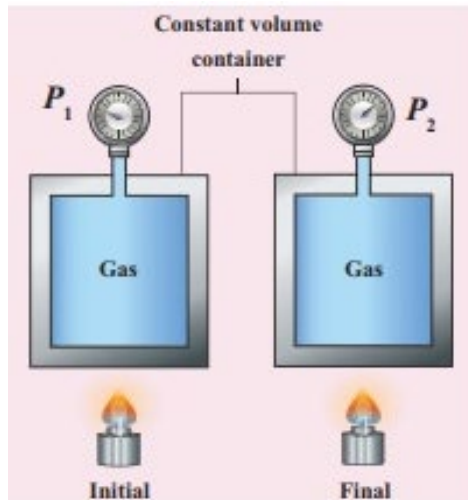


Specific heat – different process

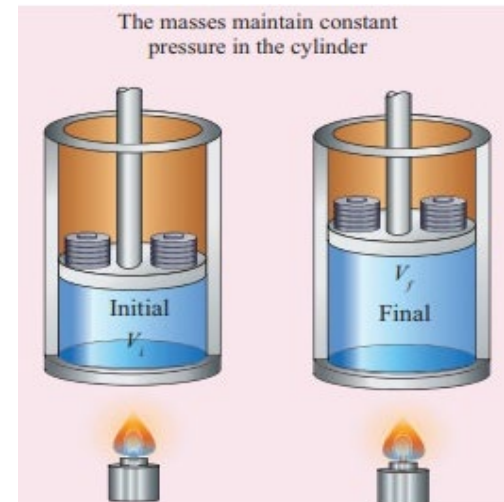
- Depending on the **process** different forms of heat capacity are defined:
 - Specific heat at constant volume, c_v :** The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant
 - Specific heat at constant pressure, c_p :** The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant
- Specific heat capacity (c_v) for **constant volume** process: $c_v = \left(\frac{\partial u}{\partial T}\right)_v$
- Specific heat capacity (c_p) for **constant pressure** process: $c_p = \left(\frac{\partial h}{\partial T}\right)_p$
- In general $c_p > c_v$ as extra energy is needed to expand the volume in order to keep the pressure constant ($\delta w = Pdv$)

Specific heat – different process

- The **specific heat** depends on the **substance** and the **process** involved, imagine a volume with gas, which process costs more energy?
 - Heating at constant volume (isochoric) → P as well as T will increase
 - Heating at constant pressure (isobaric) → v as well as T will increase



Adding heat in an isochoric process results in an increase in temperature, also the pressure increases but no work is performed.



Adding heat in an isobaric process results in an increase in temperature as well as in an expansion of the volume. Volume work is performed on the environment, $\delta w = P dv$.

- To achieve the same rise in temperature for the isobaric process more energy is needed as part of it is converted into work done on the environment, therefore $c_p > c_v$

Specific Heats for Ideal Gases

- In general, internal energy and enthalpy are functions of T and P, however, for an ideal gas, these properties are functions of temperature only (prove in module 3), therefore specific heats are functions of temperature only
- **Enthalpy** for an **ideal gas**, $h = h(T) \neq h(P)$ and

$$c_P = \left(\frac{\partial h}{\partial T} \right)_P = \frac{dh}{dT} \rightarrow dh = c_P dT \rightarrow \Delta h = \int_{T_1}^{T_2} c_P dT$$

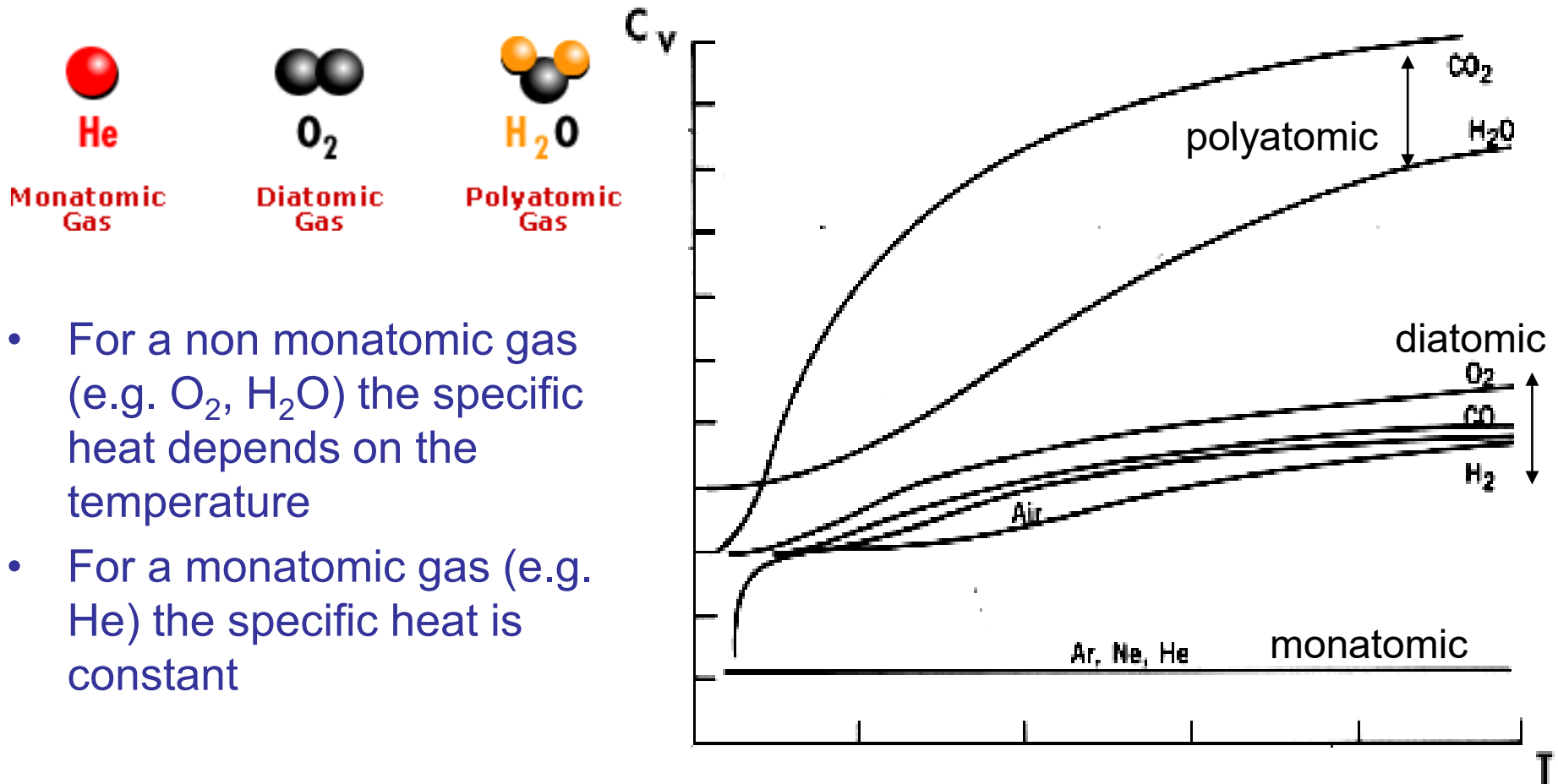
- Similarly, **internal energy** for **ideal gas**, $u = u(T) \neq u(P)$ and

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v = \frac{du}{dT} \rightarrow du = c_v dT \rightarrow \Delta u = \int_{T_1}^{T_2} c_v dT$$

- In general, the specific heat depends on temperature ($c_i = c_i(T)$) which makes the integral touch to evaluate

Specific Heats for Ideal Gases

- Specific heat at constant volume versus temperature for three types of gases



Not for the exam

Specific Heats for Ideal Gases

- For ideal gases always: $\Delta u = \int_{T_1}^{T_2} c_v dT$ & $\Delta h = \int_{T_1}^{T_2} c_p dT$ as the specific heat does not depend on temperature
- For ideal gases, the specific heat at constant pressure and constant volume are related (use $h = u + Pv$)
 - $dh = du + d(Pv) = du + d(RT) = du + RdT$ (as $Pv = RT$)
 - $c_p dT = c_v dT + RdT$ (as $dh = c_p dT$ and $du = c_v dT$)
 - $(c_p - c_v)dT = RdT$
 - and therefore: $R = c_p - c_v$ ($c_p > c_v$)

Specific Heats for Solids and Liquids

- Unlike gases, **solids and liquids** are **incompressible**, and therefore there is no difference in the specific heat for constant volume or pressure
- The volume does not change $\rightarrow v = \text{constant} \rightarrow d\mathbf{v} = 0$
- No difference between specific heat for constant pressure and constant volume
$$\left(\frac{\partial h}{\partial T}\right)_P = \left(\frac{\partial(u + Pv)}{\partial T}\right)_P = \left(\frac{\partial u}{\partial T}\right)_P + \left(v\frac{\partial P}{\partial T}\right)_P + \left(P\frac{\partial v}{\partial T}\right)_P \approx \frac{\partial u}{\partial T}$$
 - $c_P = c_V$ (often denoted as c) and therefore: $\Delta u = \Delta h = \int_{T_1}^{T_2} c dT$
 - Values of c_P and c_V can be found in tables 2 and 3
- Example:** water enters a cooling tower at 10 degree C and leaves it at 15 degree C. How much energy is transferred to the water per kg?

Energy balance: $e_{in} = e_{out} \rightarrow (h + ke + pe + q + w)_{in} = (h + ke + pe + q + w)_{out}$
neglecting ke, pe energie and realizing that w_{out-in} and q_{out} are zero this reduces to:

$$q_{in} = h_{out} - h_{in} = \Delta h = \int_{T_{in}}^{T_{out}} c dT = c(T_{out} - T_{in}) = 4.2(15 - 10) = 21 \text{ kJ/kg}$$

BREAK



<https://www.cafepress.com/+thermodynamics+mugs>

Entropy Change for Ideal Gases

- An expression for the entropy change of an ideal gas can be derived from the Gibbs equations
- First, we use the Gibbs equation with the internal energy
- For **internal energy**, the Gibbs equation is: $du = Tds - Pdv$
- This can be rewritten to: $ds = \frac{du}{T} + \frac{P}{T}dv$ (*)
- For an ideal gas it holds: $du = c_v dT$ and $Pv = RT$
- Substituting these relations in * results in: $ds = \frac{c_v dT}{T} + \frac{R}{v}dv$
- Integrating: $\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{c_v dT}{T} + \int_{v_1}^{v_2} \frac{R}{v} dv$
- The left-hand side and the second part at the right-hand side can be evaluated to: $s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v dT}{T} + R \ln \frac{v_2}{v_1}$ (this is allowed as R is constant)
- As c_v , in general, is a function of T ($c_v(T)$) the integral that is left cannot be evaluated very easy

Entropy Change for Ideal Gases

- A similar derivation can be done for the Gibbs equations with the enthalpy
- For **enthalpy**, the Gibbs equation is: $dh = Tds + vdp$
- This can be rewritten to: $ds = \frac{dh}{T} - \frac{v}{T} dp$ (*)
- For an ideal gas it holds: $dh = c_p dT$ and $Pv = RT$
- Substituting these relations in * results in: $ds = \frac{c_p dT}{T} - \frac{R}{P} dP$
- Integrating: $\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{c_p dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dP$
- The left-hand side and the second part at the right-hand side can be evaluated to: $s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{P_2}{P_1}$ (this is allowed as R is constant)
- As c_p in general is a function of T ($c_p(T)$) the integral that is left can not be evaluated very easy
- Is this one different from the one resulting from $du = Tds - Pdv$?

Entropy Change for Ideal Gases

- Gibbs for u, $du = Tds - Pd v$ gives: $s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v dT}{T} + R \ln \frac{v_2}{v_1}$
- Gibbs for h, $dh = Tds + v dP$ gives: $s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{P_2}{P_1}$
- Using $c_p - c_v = R$ it can be shown that both equations for $\Delta s = s_2 - s_1$ are equal:

$$\begin{aligned} s_2 - s_1 &= \int_{T_1}^{T_2} \frac{c_v dT}{T} + R \ln \frac{v_2}{v_1} = \int_{T_1}^{T_2} \frac{[c_p - R] dT}{T} + R \ln \frac{v_2}{v_1} \\ &= \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \int_{T_1}^{T_2} \frac{dT}{T} + R \ln \frac{v_2}{v_1} = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \\ &= \int_{T_1}^{T_2} \frac{c_p dT}{T} - R (\ln \frac{T_2}{T_1} - \ln \frac{v_2}{v_1}) = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{T_2 v_1}{T_1 v_2} = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{P_2}{P_1} = s_2 - s_1 \end{aligned}$$

- Both relations give the same results
- Which one to use depends on the situation, process and given properties

Entropy Change for Ideal Gases

- Gibbs for u, $du = Tds - Pd v$ gives: $s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v dT}{T} + R \ln \frac{v_2}{v_1}$
- Gibbs for h, $dh = Tds + v dP$ gives: $s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{P_2}{P_1}$
- If c_v or c_p can be assumed **independent** of the temperature, e.g., in a restricted / small temperature range, the integrals are easy to evaluate to give two expressions for the entropy difference

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad (\text{from } du)$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (\text{from } dh)$$

- Both can be used and will give similar results
- Which one is the best to use depends on the given properties of the process e.g., if P is constant, $P_1 = P_2$, then $\rightarrow \ln \frac{P_2}{P_1} = 0$, this can be a reason to choose the second one

Isentropic Process for Ideal Gases

- For an isentropic process, $\Delta s = 0$ and c_p and c_v **independent** of T the two equations for $s_2 - s_1$ reduce to $0 = \ln \frac{T_2}{T_1} + \frac{R}{c_v} \ln \frac{v_2}{v_1}$ and $0 = \ln \frac{T_2}{T_1} - \frac{R}{c_p} \ln \frac{P_2}{P_1}$

Rewriting the first one $0 = \ln \frac{T_2}{T_1} + \frac{R}{c_v} \ln \frac{v_2}{v_1} \longrightarrow \ln \left(\frac{T_2}{T_1} \right)_s = -\frac{R}{c_v} \ln \left(\frac{v_2}{v_1} \right)_s$

and using:

- Define $k = c_p/c_v$
- $R = c_p - c_v$
- $R/c_v = (c_p - c_v)/c_v = k - 1$
- Values for k table 2a, 2b
- Air at 300 K, $k = 1.4$

$$\ln \left(\frac{T_2}{T_1} \right)_s = \ln \left(\left(\frac{v_1}{v_2} \right)_s^{\frac{R}{c_v}} \right)$$
$$\exp \left(\ln \left(\frac{T_2}{T_1} \right)_s \right) = \exp \left(\ln \left(\left(\frac{v_1}{v_2} \right)_s^{\frac{R}{c_v}} \right) \right)$$
$$\left(\frac{T_2}{T_1} \right)_s = \left(\frac{v_1}{v_2} \right)_s^{\frac{R}{c_v}}$$

Results in: $\left(\frac{T_2}{T_1} \right)_s = \left(\frac{v_1}{v_2} \right)_s^{k-1} \Rightarrow T v^{k-1} = \text{constant}$

Isentropic Process for Ideal Gases

- From $0 = \ln \frac{T_2}{T_1} + \frac{R}{c_V} \ln \frac{v_2}{v_1}$ we derived $\left(\frac{T_2}{T_1}\right)_s = \left(\frac{v_1}{v_2}\right)_s^{k-1} \Rightarrow T v^{k-1} = \text{constant}$

- Similarly, the second one $0 = \ln \frac{T_2}{T_1} - \frac{R}{c_p} \ln \frac{P_2}{P_1}$ can be rewritten in

$$\left(\frac{T_2}{T_1}\right)_s = \left(\frac{P_2}{P_1}\right)_s^{\frac{k-1}{k}} \Rightarrow \frac{P^{\frac{k-1}{k}}}{T} = \text{constant}$$



- The right-hand side of the first equation equals the right-hand side of the second one and combining both equations results in

$$\left(\frac{P_2}{P_1}\right)_s = \left(\frac{v_1}{v_2}\right)_s^k \Rightarrow P v^k = \text{constant}$$

$$\begin{aligned} \frac{R}{c_p} \ln \left(\frac{P_2}{P_1}\right) &= -\frac{R}{c_V} \ln \left(\frac{v_2}{v_1}\right)_s \\ \left(\frac{P_2}{P_1}\right)_s^{\frac{R}{c_p}} &= -\left(\frac{v_2}{v_1}\right)_s^{\frac{R}{c_V}} \\ \left(\frac{P_2}{P_1}\right)_s &= -\left(\frac{v_2}{v_1}\right)_s^{\frac{c_p R}{R c_V}} \\ \left(\frac{P_2}{P_1}\right)_s &= \left(\frac{v_1}{v_2}\right)_s^{\frac{c_p}{c_V}} \end{aligned}$$

Isentropic Process for Ideal Gases

- For isentropic processes ($\Delta s = 0$) involving an ideal gas and assuming c_p and c_v independent of T the following three relations were derived
- All of them can be used, which one is the most convenient depends on the other known properties of the process

$$\left(\frac{T_2}{T_1}\right)_s = \left(\frac{v_1}{v_2}\right)_s^{k-1} \Rightarrow T v^{k-1} = \text{constant}$$

$$\left(\frac{T_2}{T_1}\right)_s = \left(\frac{P_2}{P_1}\right)_s^{\frac{k-1}{k}} \Rightarrow \frac{P^{\frac{k-1}{k}}}{T} = \text{constant}$$

$$\left(\frac{P_2}{P_1}\right)_s = \left(\frac{v_1}{v_2}\right)_s^k \Rightarrow P v^k = \text{constant}$$

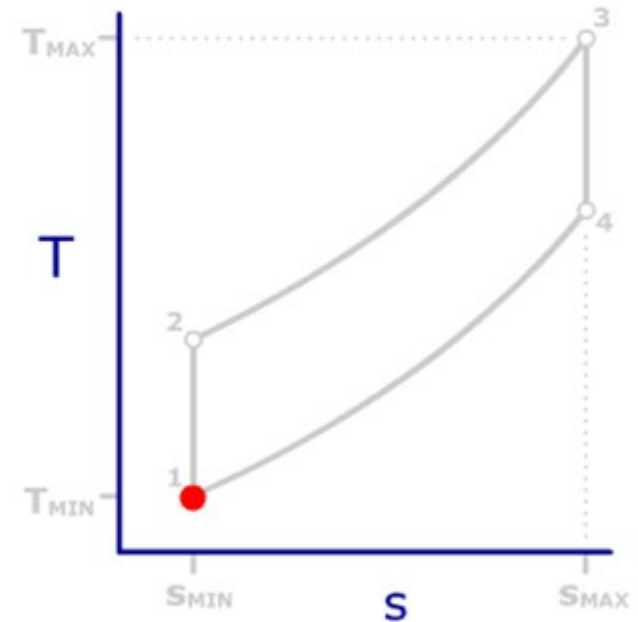
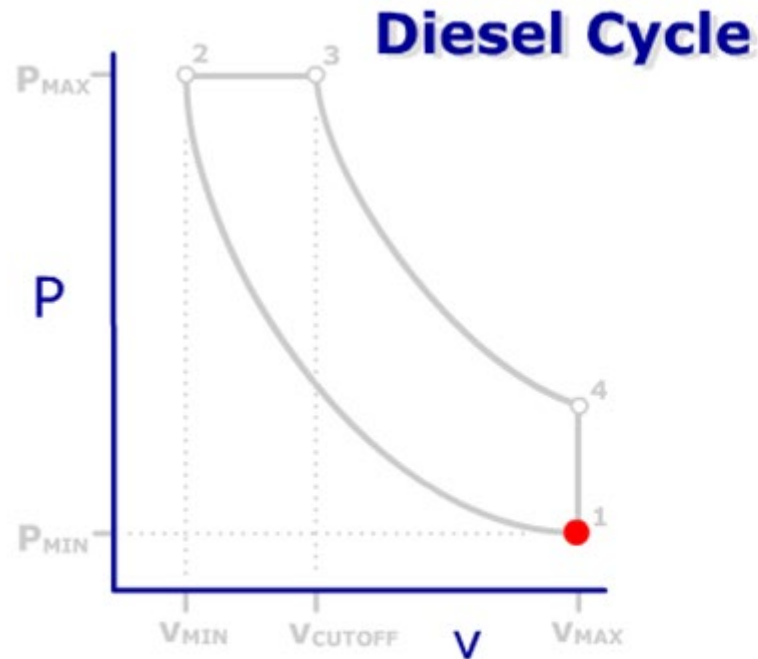
- $k = c_p/c_v$
- $R = c_p - c_v$
- $R/c_v = (c_p - c_v)/c_v = k - 1$

Example: isentropic turbine,
 $T_{\text{in}} = 800$ degree C, $P_{\text{in}} = 8$ bar,
 $P_{\text{out}} = 1$ bar. What is T_{out} ?

P and T are given so the second one is the best one to use. For air $k = 1.4$ (table 2):

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)_s^{\frac{k-1}{k}} = 1073 \left(\frac{1}{8}\right)_s^{\frac{1.4-1}{1.4}} = 592 \text{ K} = 319 \text{ C}$$

Isentropic Processes in Diesel Cycle

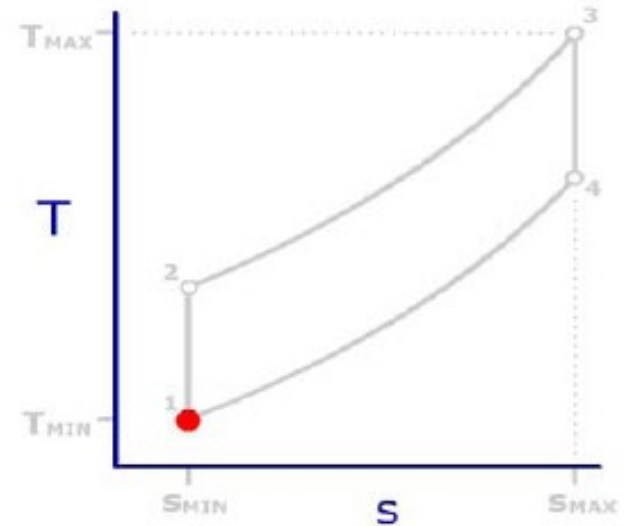


ThermoNet: Wiley

- In a Diesel cycle it is assumed that compression (1-2) and expansion (3-4) are isentropic processes ($\Delta s = 0$). For these processes the relations derived in the previous slides can be used.

Example: Isentropic Process in Diesel Cycle

- Assume a Diesel cycle working on air with the inlet conditions:
 $P_1 = 0.1 \text{ Mpa}$; $T_1 = 27 \text{ }^\circ\text{C}$; $r_v = v_1/v_2 = 20$; $k = 1.4$
- Calculate the pressure and temperature after compression
- Solution:** It is assumed that the compression process is isentropic, $ds = 0$
- Isentropic process $Pv^k = \text{constant}$: $P_2 = P_1 \left(\frac{v_1}{v_2} \right)_s^k = 0.1(20)^{1.4} = 6.6 \text{ MPa}$
- Also, $Tv^{(k-1)} = \text{constant}$: $T_2 = T_1 \left(\frac{v_1}{v_2} \right)_s^{k-1} = 300(20)^{0.4} = 994.3 \text{ K}$
- Also, the ideal gas law is valid for this process but only the ideal gas law is not sufficient to calculate all properties



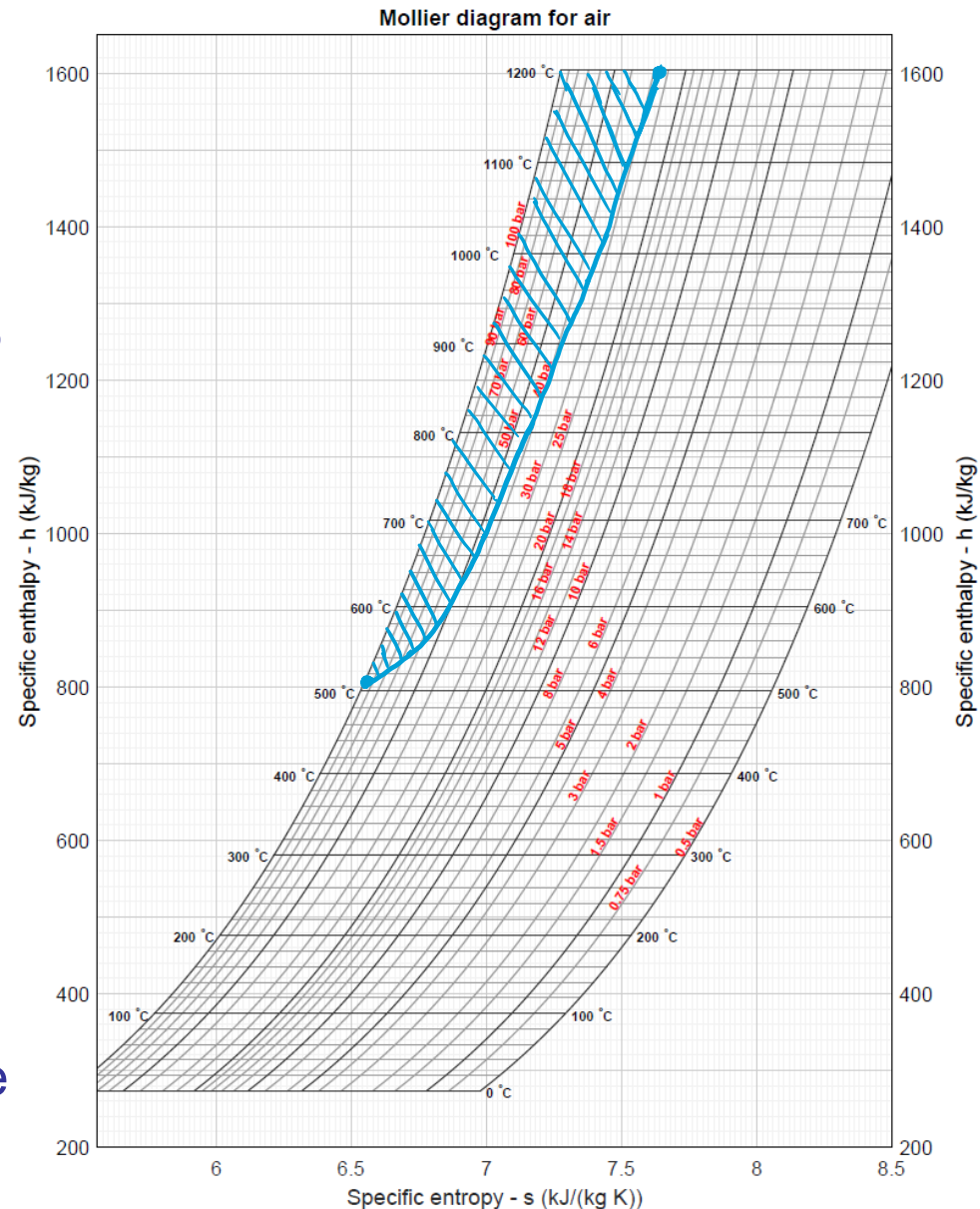
Ideal Gas Tables

- To calculate energy differences for gasses requires knowledge of the values of the specific heats, as a function of temperature
- Correlations exist that provide information on the variation of specific heats with temperature, but evaluating the integrals is tough if c_p and c_v cannot be assumed to be constant (larger temperature range)
- Tabulated data are often used to find changes in internal energy, u and enthalpy, h between states
- Tables relate T , u and h (e.g., table A-17 for air)
- Unlike steam tables, in gas tables the pressure is not tabulated
- Use $Pv = RT$ to relate P , v and T
- Note: energies are measured or given based on a reference state, absolute energies (like absolute pressure and temperature) do not exist
- Different sources can have different reference states
- Lucky, mostly only energy differences are important in thermodynamics



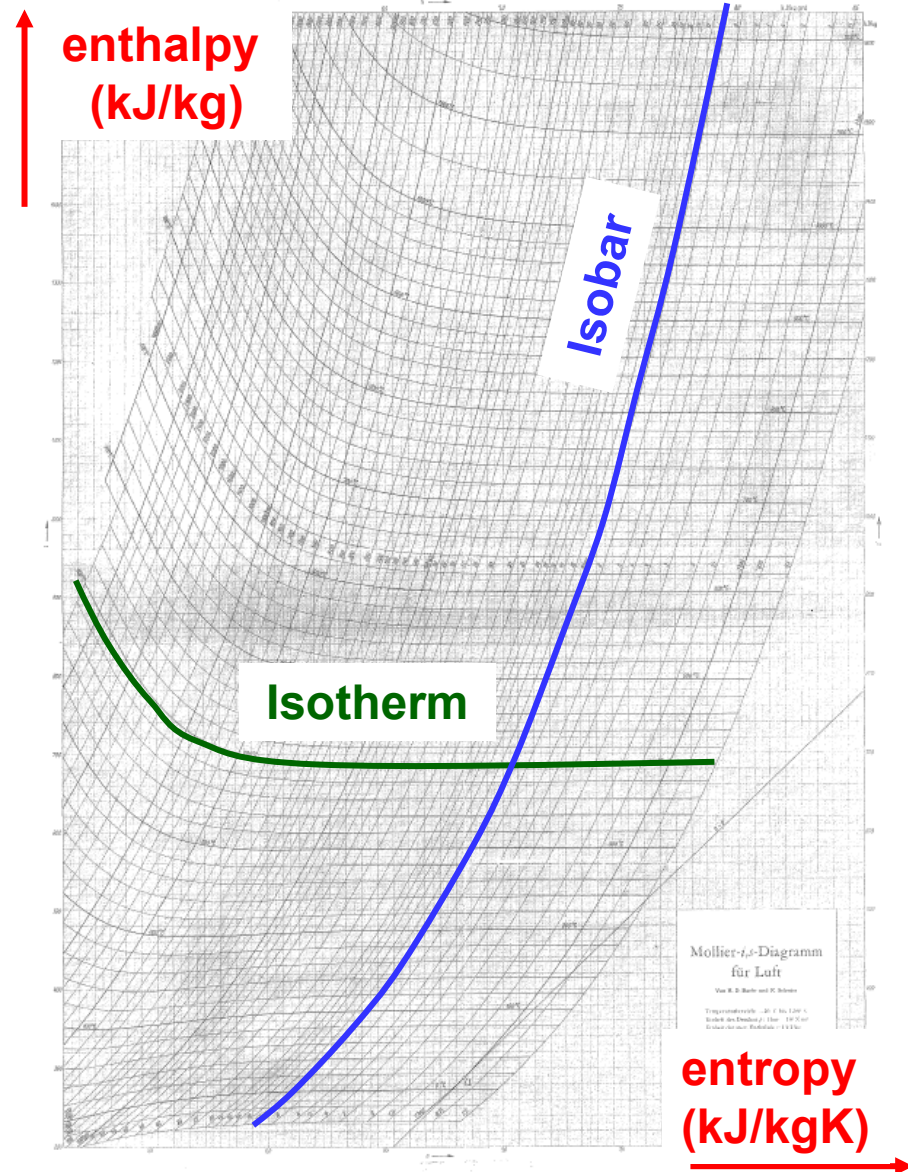
Mollier Diagram Air for Brayton Cycle

- Enthalpy values for air can also be found using the Mollier diagram
- Diagram is less exact, but gives a good overview of the process
- This diagram is made using Matlab
- In the program some assumptions are made, the most important one is that air is assumed to be an ideal gas, however the dependence of the specific heats on temperature is taken into account
- So, the diagram is valid for air assumed to be an ideal gas
- If the air cannot be assumed ideal (high P and high T indicated in bleu) the values slightly deviate
- We will use this diagram to analyse the gas power cycles (next class)



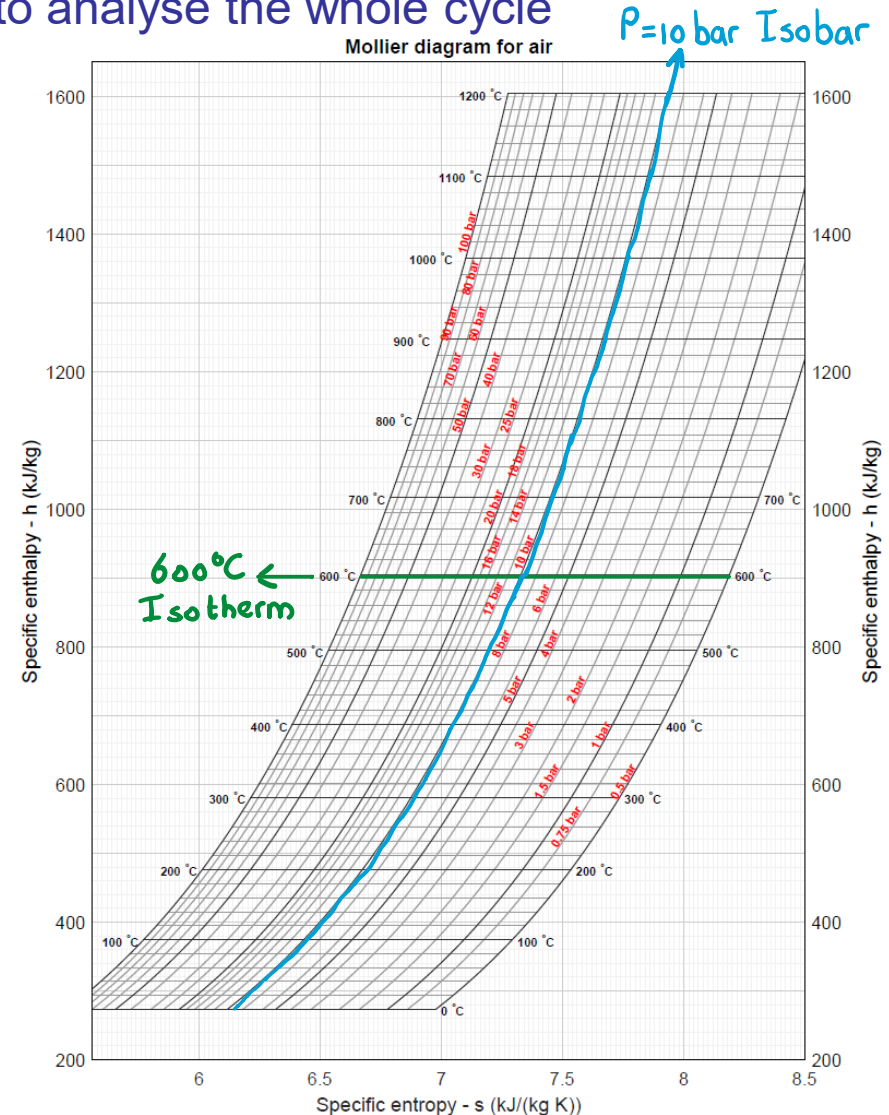
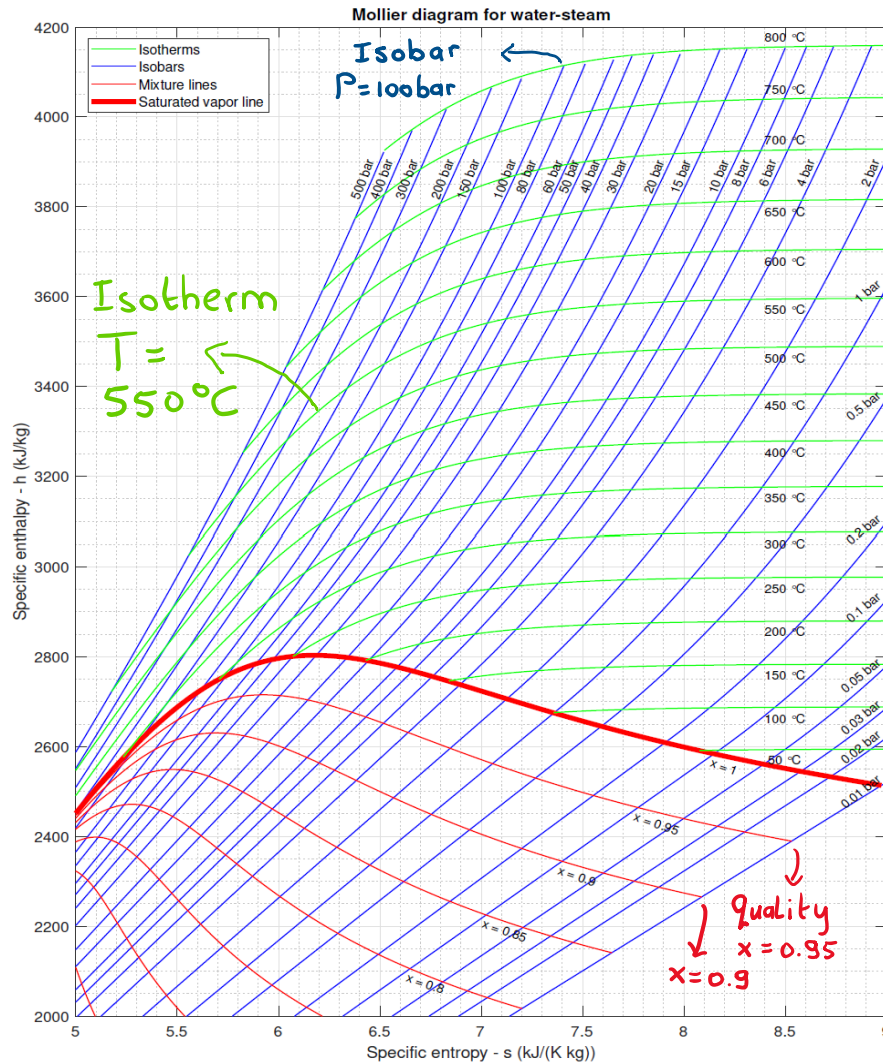
Mollier diagram air

- Scanned version of a Mollier diagram for real air
 - Enthalpy (kJ/kg) on the y-axis, horizontal line $\rightarrow dh = 0$
 - Entropy (kJ/kgK) on the x-axis, vertical line $\rightarrow ds = 0$
 - Isobars (in **Bar** !!)
 - Isotherms (in degree Celsius)
- The main difference with the one in which the air is assumed to be an ideal gas (previous page) is that the isotherms bend upwards in the high T and high P region where the air cannot be approximated as an ideal gas anymore
- This diagram is mostly used in the answers for the assignments

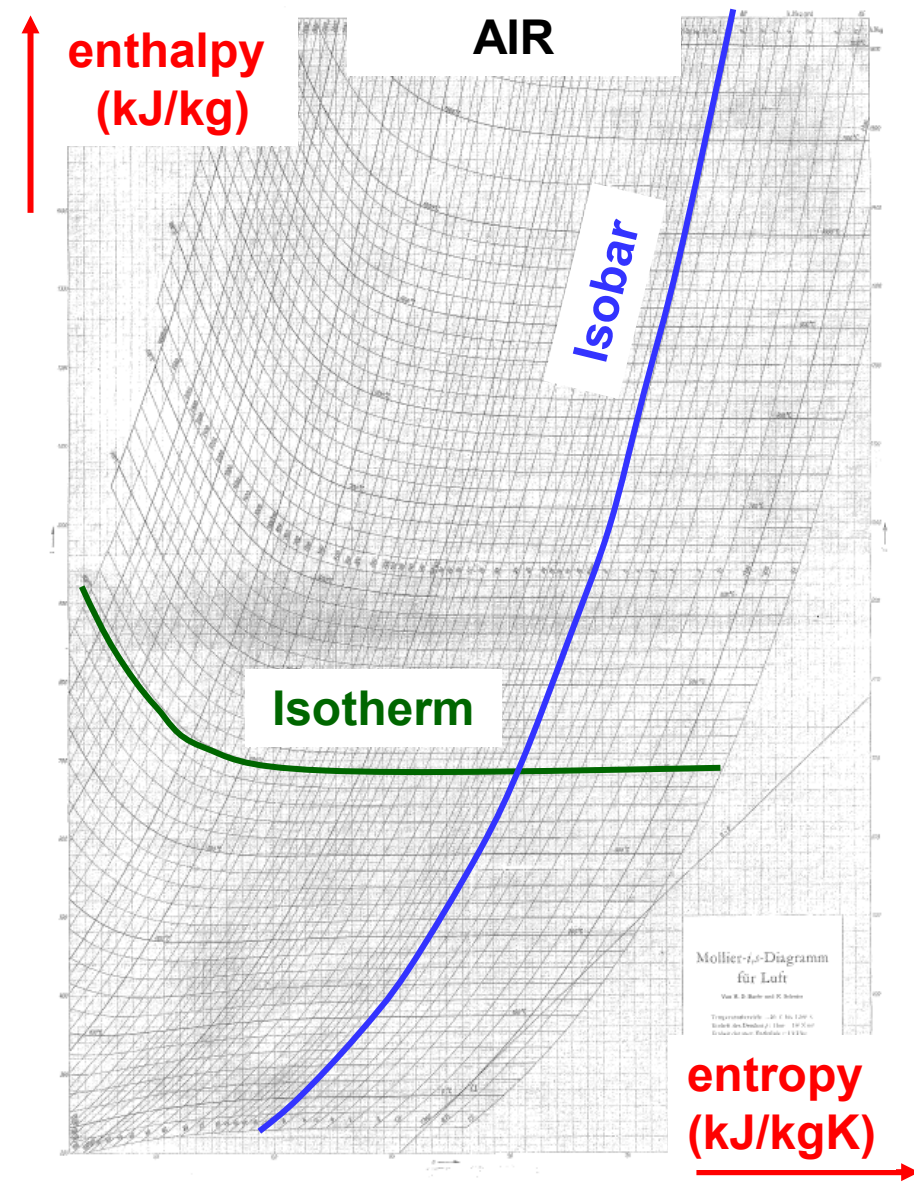
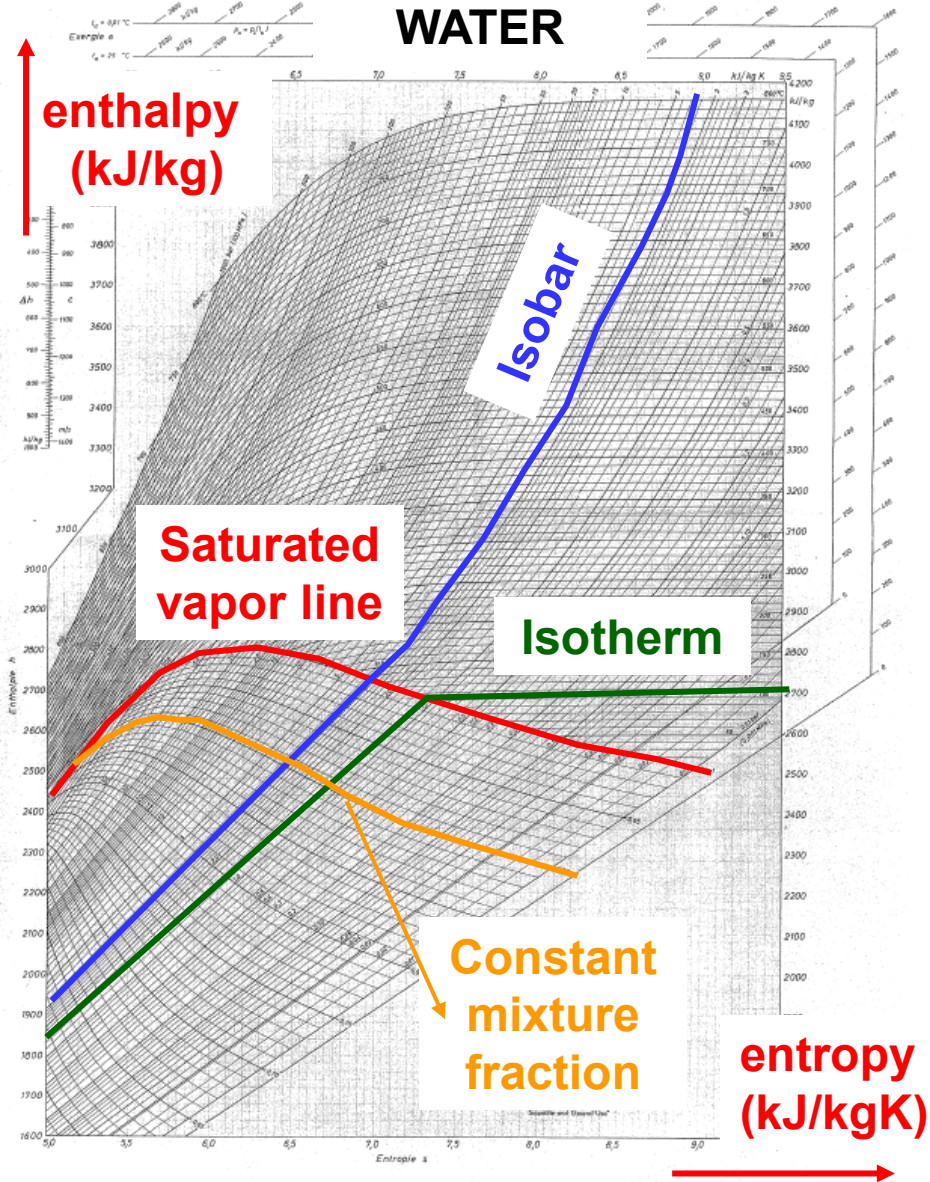


Compare Mollier Diagram Water and Air

- Air: no mixture region and in contrast to the water diagram also data at low T & P are present and the diagram can be used to analyse the whole cycle



Compare Mollier Diagram Water and Air



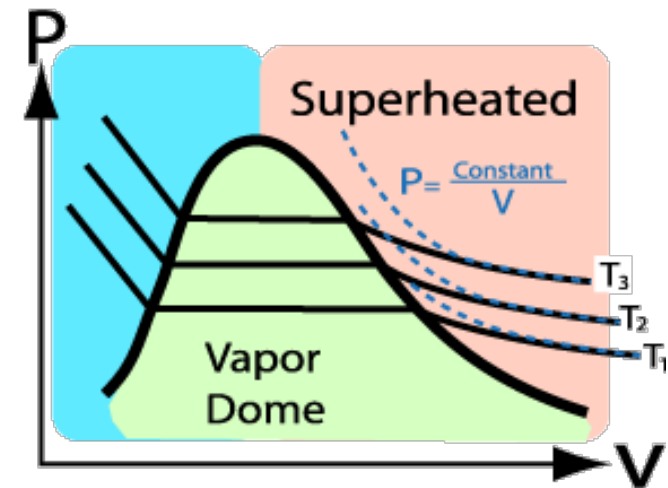
Some Other Thermodynamic Properties

- Besides the specific heat some other thermodynamic properties are needed to describe processes (see module 3)
- Isothermal compressibility (κ):** $\kappa = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T$
 - Fractional change in volume for an isothermal change in pressure
- Coefficient of Thermal Expansion (β):** $\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P$
 - Fractional change in volume for an isobaric change in temperature
- Together they give for example the change in volume if the pressure and / or temperature are changed: $\frac{dV}{V} = \beta dT - \kappa dP$
- Joule Thompson Coefficient (μ):** $\mu_{JT} = \left(\frac{\partial T}{\partial P} \right)_h$
 - Change of temperature for a constant enthalpy (isenthalpic) change of the pressure (throttling)

Not for the exam

Recapitulate class 9

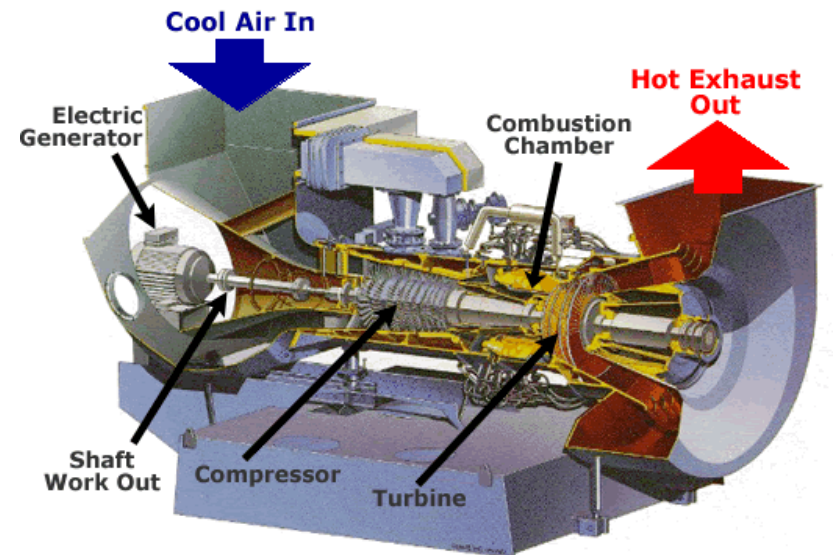
- Ideal gas equation of state: $Pv = RT$
- Tables for ideal gas
- Mollier diagram for air
- Specific heat
 - Constant pressure specific heat, c_p
 - Constant temperature specific heat, c_v
- Relation between them: $c_p - c_v = R$
- Internal energy and enthalpy of ideal gases depends on temperature only, u and h for ideal gas: $du = c_v dT$ and $dh = c_p dT$
- Entropy ideal gas: $s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$ & $s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$
- Isentropic processes for ideal gas:



$$Pv^k = \text{constant} \ \& \ Tv^{k-1} = \text{constant} \ \& \ \frac{P^{\frac{k-1}{k}}}{T} = \text{constant}$$

Next Class 10: Gas Power Cycles - Simple

- **Gas power cycles**, cycles using gas as working fluid through the whole cycle to produce power
 - Air-standard cycle
 - Open and closed Brayton cycle
 - Ideal and real Brayton cycle
 - Power in- and output
 - Thermal efficiency
 - Comparison to the Rankine cycle
 - Design parameters
 - Mollier diagram for air



A basic gas turbine engine

How do jet engines work?

<https://www.youtube.com/watch?v=eA699AKxT7s>

Keep in mind: Important Formulas

- Specific volume $v = V/m$ [m^3/kg] and density $\rho = 1/v = m/V$ [kg/m^3]
- Volume work $\delta w = P dv$
- Enthalpy $h = u + Pv$, (u internal energy, P pressure, v volume)
- Thermal efficiency $\eta_{\text{thermal}} = \frac{\text{Net electrical power output}}{\text{Rate of fuel energy input}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}$
- Mixture fraction $x = \frac{v - v_l}{v_v - v_l} \rightarrow v = v_l + x(v_v - v_l)$

- Ideal gas law $Pv = RT$, $c_p - c_v = R$
- For an ideal gas $du = c_v dT$ and $dh = c_p dT$

- Conservation of mass $m_{\text{in}} = m_{\text{out}}$, mass flow rate $\dot{m} = \rho v A$
- Conservation of energy, first law of thermodynamics
 - Closed system $du = \delta w - \delta q \rightarrow \Delta u = w - q$
 - Open system $q_{\text{in}} + w_{\text{in}} + (h + ke + pe)_{\text{in}} = q_{\text{out}} + w_{\text{out}} + (h + ke + pe)_{\text{out}}$
- S increases, second law $ds_{\text{total}} = ds_{\text{system}} + ds_{\text{surroundings}} = \delta s_{\text{gen}} \geq 0$
- Inequality of Clausius $ds \geq \frac{\delta q_{\text{net}}}{T_{\text{res}}}$ (= for reversible process)
- Reversible heat transfer $\delta q_{\text{net,rev}} = T ds$, irreversible $\delta q_{\text{net,irrev}} < T ds$
- Gibbs equations $T ds = du + P dv$ and $T ds = dh - v dP$
- Isentropic efficiencies $\eta_{\text{INPUT,S}} = \frac{w_{\text{IN,S}}}{w_{\text{IN,A}}}$, $\eta_{\text{OUTPUT,S}} = \frac{w_{\text{OUT,A}}}{w_{\text{OUT,S}}}$

- Isentropic processes ideal gas $Pv^k = \text{constant}$, $Tv^{k-1} = \text{constant}$, $P^{(k-1)/k}/T = \text{constant}$

- Thermal efficiency power cycles $\eta_{\text{he}} = \frac{w_{\text{out}} - w_{\text{in}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}}$
- Carnot efficiency $\eta_{\text{carnot}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$

