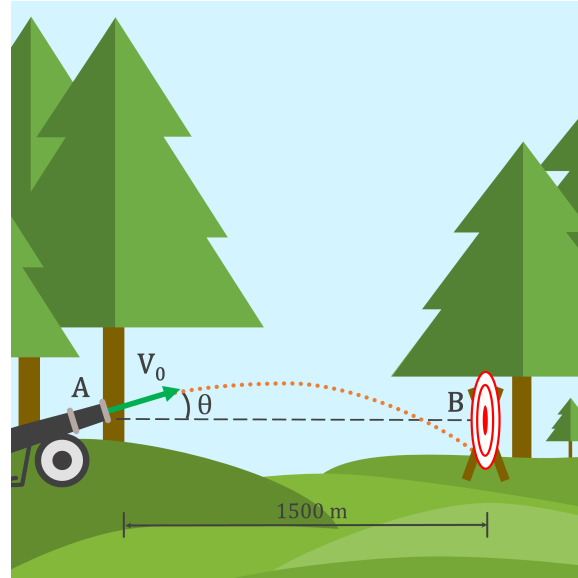




Bullet on Target Bottom



A cannon fires a bullet from A toward a target B. Find an expression for the vertical displacement $y(t)$ as a function of time, v_0 and θ , for which the round just hits the bottom of the target. The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is 1500 m. Neglect air resistances and assume that the bullet is directed along the vertical centreline of the target. Take $g = 10 \text{ m/s}^2$.
 $y(t) = \dots = \dots$

Using known expressions:

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^v dv = a \int_0^t dt \quad (2)$$

$$v(t) = a \cdot t + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (v_0 + at) dt \quad (4)$$

$$\int_{s_0}^s ds = \int_0^t (v_0 + at) dt \quad (5)$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \quad (6)$$

Given:

Distance A-B: $s = 1500m$

Gravitational acceleration: $g = 10m/s^2$

Initial velocity: $v_0 = 900m/s$

Diameter target: $D = 2m$

Filling in Equation 6 gives an relation for the y-position with respect to time. Where $a = -g$ and $y_0 = 0m$, since the cannon barrel is at the same altitude as the target centre.

$$y(t) = -\frac{1}{2} \cdot g \cdot t^2 + v_{0,y} \cdot t + y_0 \Rightarrow y(t) = -\frac{1}{2} \cdot g \cdot t^2 + \sin \theta \cdot v_0 \cdot t \quad (7)$$

The bottom of the target is at $1m$ from the centreline thus $y(t_{end}) = -1m$. Furthermore, inserting $g = 10m/s^2$ result in.

$$y(t) = -\frac{1}{2} \cdot 10 \cdot t^2 + \sin \theta \cdot v_0 \cdot t = 1 \Rightarrow -5 \cdot t^2 + \sin \theta \cdot v_0 \cdot t = -1 \quad (8)$$