Task 2 Shaping Machine (Whitworth Mechanism)

Solution

The shaping machine in Task 1 is reconsidered. The masses of the sliders are negligible. The shaft rotates with constant rpm n_1.

a) The frame torque should be analytically determined. The centre of gravity S_3 has the distance h to the x-axis.

The frame torque is the resultant inertial torque around the z-axis. The total frame torque is the sum of the mass moments of the components around the z-axis, i.e.:

$$\vec{M}_z = \sum_i \vec{M}_{z,i} = \vec{M}_{z1} + \vec{M}_{z2} + \vec{M}_{z3} \tag{1.1}$$

The moment around the z-axis not only results from the angular acceleration of the components but also from the inertial forces and the resulting displacement torque:

$$\vec{M}_z = \sum_{i=1}^{n} \vec{M}_{\alpha i} + \sum_{i=1}^{n} \vec{M}_{ai} \tag{1.2}$$

From the principle of angular momentum, the moment due to the rotational inertia can be derived:

$$\vec{M}_{\alpha i} = -\frac{d}{dt}(J_{Si} \cdot \vec{\omega}_i) = -J_{Si} \cdot \vec{\alpha}_i \tag{1.3}$$

The resultant moment due to the inertial forces (displacement torque) is:

$$\vec{M}_{ai} = \vec{l}_i \times \vec{F}_i \tag{1.4}$$

1. Crank (Component 1):

$$M_{z1,\alpha} = -J_{S1} \cdot \ddot{\varphi} = 0 \tag{1.5}$$

$$M_{z1,a} = -F_{1,x} \cdot \big(l_0 - l_{S1} \cdot cos(\varphi)\big) + F_{1,y} \cdot l_{S1} \cdot sin(\varphi) \tag{1.6}$$

From task 1:

$$F_{1,x} = m_1 \cdot \omega^2 \cdot l_{S1} \cdot sin(\varphi) \tag{1.7}$$

$$F_{1,y} = -m_1 \cdot \omega^2 \cdot l_{S1} \cdot cos(\varphi) \tag{1.8}$$

Thus:

$$M_{z1,a} = -m_1 \cdot \omega^2 \cdot l_{S1} \cdot sin(\varphi) \cdot \big(l_0 - l_{S1} \cdot cos(\varphi)\big) - m_1 \cdot \omega^2 \cdot l_{S1} \cdot cos(\varphi) \cdot sin(\varphi) = -m_1 \cdot \omega^2 \cdot l_0 \cdot l_{S1} \cdot sin(\varphi) \tag{1.9}$$

$$\Rightarrow M_{z1} = -m_1 \cdot \omega^2 \cdot l_0 \cdot l_{S1} \cdot sin(\varphi) \tag{1.10}$$

Alternative solution (A_0 als reference point for the inertial forces):

$$M_{z1,\alpha} = -\left(J_{S1} + m_1 \cdot {l_{S1}}^2\right) \cdot \ddot{\varphi} = 0 \tag{1.11}$$

$$M_{z1,a} = F_{1,x} \cdot h = -m_1 \cdot \omega^2 \cdot l_{S1} \cdot sin(\varphi) \cdot h \tag{1.12}$$

($F_{1,y}$ has no lever arm related to B_0)

2. Rocker (Component 2):

Since $\ddot{\psi}$ is defined into the opposite direction of the related coordinate system and positive direction of rotation, respectively, it needs to be negated:

$$M_{z2,\alpha} = -J_{S2} \cdot \left(-\ddot{\psi}\right) \tag{1.13}$$

$$M_{z2,a} = -F_{2,x} \cdot l_{S2} \cdot cos(\psi) + F_{2,y} \cdot l_{S2} \cdot sin(\psi) \tag{1.14}$$

From task 1:

$$F_{2,x} = m_2 \cdot l_{S2} \cdot \dot{\varphi}^2 \cdot \left(\psi'^2 \cdot sin(\psi) - \psi'' \cdot cos(\psi)\right) \tag{1.15}$$

$$F_{2,y} = m_2 \cdot l_{S2} \cdot \dot{\varphi}^2 \cdot \left(\psi'^2 \cdot cos(\psi) + \psi'' \cdot sin(\psi)\right) \tag{1.16}$$

$$\Rightarrow M_{z2,a} = m_2 \cdot {l_{S2}}^2 \cdot \dot{\varphi}^2 \cdot \psi'' \tag{1.17}$$

$$\begin{aligned}\Rightarrow M_{z2} = J_{S2} \cdot \ddot{\psi} + m_2 \cdot {l_{S2}}^2 \cdot \dot{\varphi}^2 \cdot \psi'' &= \left(J_{S2} + m_2 \cdot {l_{S2}}^2\right) \cdot \dot{\varphi}^2 \cdot \psi'' \\ &= \left(J_{S2} + m_2 \cdot {l_{S2}}^2\right) \cdot \ddot{\psi} \\ &= \left(J_{S2} + m_2 \cdot {l_{S2}}^2\right) \cdot \frac{\omega^2 \cdot \lambda \cdot sin(\varphi) \cdot (\lambda^2 - 1)}{(1 - 2 \cdot \lambda \cdot cos(\varphi) + \lambda^2)^2}\end{aligned} \tag{1.18}$$

3. Slider (Component 3):

$$M_{z3,\alpha} = 0 \text{ (No rotation!)} \tag{1.19}$$

$$M_{z3,a} = -F_{3,x} \cdot h = -(-m_3 \cdot \ddot{x}_{S3}) \cdot h = m_3 \cdot \ddot{x}_{S3} \cdot h \tag{1.20}$$

From task 1:

$$\ddot{x}_{S3} = \frac{h \cdot \lambda \cdot \omega^2 \cdot sin(\varphi) \cdot (2 \cdot \lambda^2 - \lambda \cdot cos(\varphi) - 1)}{\left(1 - \lambda \cdot cos(\varphi)\right)^3} \tag{1.21}$$

$$\Rightarrow M_{z3} = \frac{m_3 \cdot h^2 \cdot \lambda \cdot \omega^2 \cdot sin(\varphi) \cdot (2 \cdot \lambda^2 - \lambda \cdot cos(\varphi) - 1)}{\left(1 - \lambda \cdot cos(\varphi)\right)^3} \tag{1.22}$$

b) For the external dead position and for the position $\varphi = 38°$ the frame torque should be numerically determined ($J_{B0} = 1{,}6\ \mathrm{kgm}^2$).

Moment of inertia of the rocker around B_0:

$$J_{B_0} = J_{S2} + m_2 \cdot {l_{S2}}^2 \tag{1.23}$$

From that it follows:

$$M_z(\varphi = 68°) = -1181{,}4\ Nm$$

$$M_z(\varphi = 38°) = -1181{,}4\ Nm$$