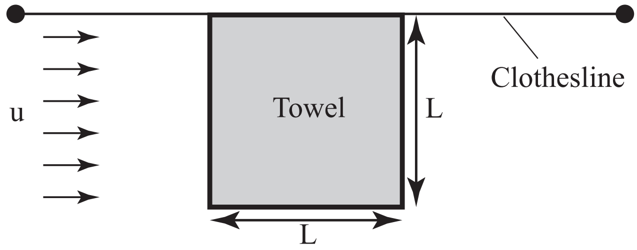
Exercise V.2: (Soaked towel ⋆⋆)

A square towel (edge length L) soaked with water is hung up on a clothesline. The wind is blowing against the towel from the side with a velocity of u. The air and the towel have the same temperature T that remains constant over time.



Given parameters:

- Temperature of towel and air: $T = 20\ °C$
- Velocity of the wind: $u = 1\ m/s$
- Edge length of the towel: $L = 0.5\ m$

Hints:

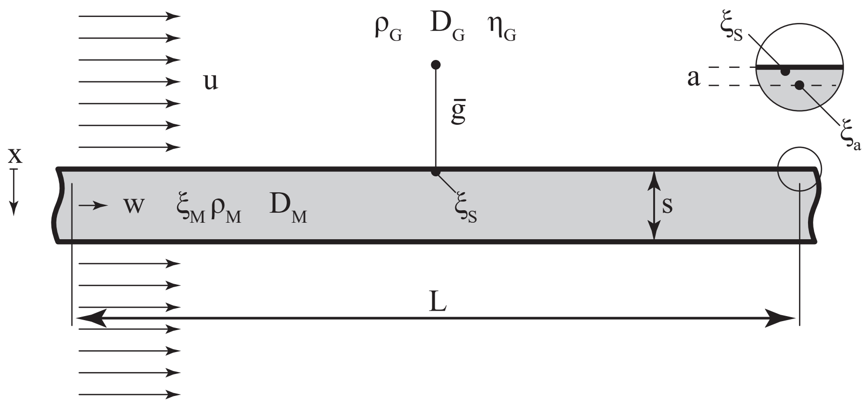
- The influence of radiation on the problem is to be neglected.
- The Lewis relation is valid.

Tasks:

a) Calculate the average mass transfer coefficient $\overline{g}$.

Exercise V.3: (Metal strap ⋆⋆⋆)

A metal strap (M) with a sheet-thickness s is pulled at a velocity w through an oven of length L. Thereby a CO_2-containing gas atmosphere (G) flows over the strap with a velocity u. On entering the furnace, the metal strap has a homogeneous fraction of carbon ξ_M. Under quasi-steady conditions, a constant fraction of carbon ξ_S exists just below the surface of the metal strip, while carbon constantly diffuses into its interior.



Given parameters:

- Diffusion coefficient of carbon in metal: $D_M = 5 \cdot 10^{-10}$ m²/s
- Diffusion coefficient of carbon in gas: $D_G = 1 \cdot 10^{-4}$ m²/s
- Combined density of metal: $\rho_M = 8 \cdot 10^3$ kg/m³
- Combined density of the gases: $\rho_G = 0.8$ kg/m³
- Combined viscosity of the gases: $\eta_G = 5.6 \cdot 10^{-5}$ Pa · s
- Flow velocity: $u = 10$ m/s
- Sheet velocity: $w = 2 \cdot 10^{-2}$ m/s
- Initial carbon fraction: $\xi_M = 0.2 \cdot 10^{-4}$
- Surface carbon fraction: $\xi_S = 1 \cdot 10^{-4}$
- Oven length: $L = 10$ m
- Strap thickness: $s = 50 \cdot 10^{-3}$ m
- Layer depth: $a = 0.2 \cdot 10^{-3}$ m

Hints:

- Use a co-moving coordinate system.
- Neglect diffusion in the direction of the sheet.
- The strap thickness is significantly greater than twice the depth of penetration of the carbon.

Tasks:

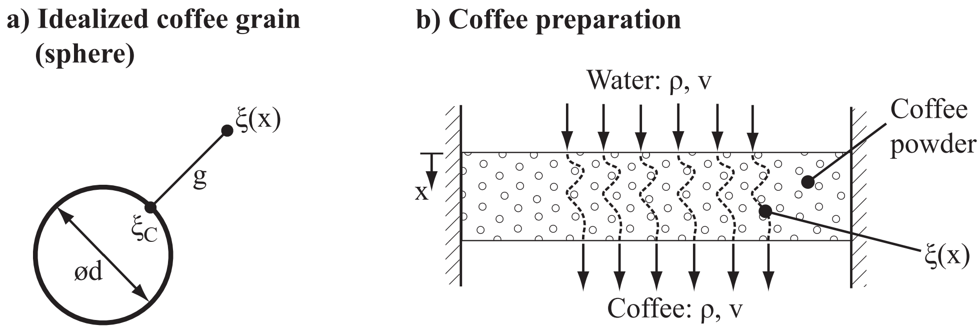
a) Calculate the mean mass transfer coefficient $\bar{g}$ on the strip's surface. Show that the assumption of a constant carbon content of ξ_S just below the surface is valid.

b) Provide the differential equation for the carbon transport in the strap as well as the appropriate initial and boundary conditions. For this, provide a suitable equation for the carbon content in the strap $\xi(x,t)$.

c) Calculate the carbon content of ξ_a which is established at the end of the furnace at a depth a from the strap's surface. Show that the assumption of a low penetration depth is justified.

Exercise V.4: (Coffee brewing ⋆⋆⋆)

Coffee powder consists of many coffee grains, forming a highly concentrated coffee solution during brewing. The flavors come out of the coffee grains (ξ_{C}) through diffusion into the passing water flow ($\xi(x)$). This diffusion process is well described by the mass-transfer coefficient g.



Given parameters:

- Diffusion coefficient of the flavors in water: D
- Water flow velocity: v
- Total density of flavored water: ρ
- Concentration of highly concentrated coffee solution: ξ_{C}
- Mass transfer coefficient: g
- Diameter of a coffee grain: d
- Packing density in the coffee powder: p

Hints:

- The problem is stationary and the Stefan flow is negligible.
- All coffee grains have the same diameter and are homogeneously distributed in the powder.
- If you cannot solve part a), you may use $\dot{m}_{\mathrm{C}}''' = \frac{g}{d} \cdot (\xi_{\mathrm{C}} - \xi(x))$.

Tasks:

a) Determine the mass flow out of the grains per unit volume $\dot{m}_{\mathrm{C}}'''$, which is transferred from the coffee grains to the passing water flow.

b) Derive the differential equation for the mass concentration of flavors $\xi(x)$ in the passing water flow. Consider the packing density p of the coffee grains within the ground coffee powder in the source term:$\dot{m}''' = p \cdot \dot{m}_{\mathrm{C}}'''$.

c) Write down the required differential equation and the boundary conditions, when diffusion is negligible in the flow direction, and determine the concentration profile of $\xi(x)$ for this case.

d) Draw the concentration profiles $\xi(x)$ for two different grain diameters $d_1 < d_2$ over the length of the ground coffee powder in the given chart and mark the curves clearly.