

## Learning activities before - Lecture 4

To make the session more efficient we would like to ask you to do some preparation beforehand. This will help you to be prepared for the lecture in advance by having the overall view of the content.

The learning objective of the session are:

- Heat transfer through natural convection
- Natural Convection
- Grashof Number
- Rayleigh Number
- Nusselt Number
- Using correlations for various configurations
- Calculating natural convection with step-by-step plan

Please prepare yourself in the following way:

Question to make you think:

How long for a can of soda to get cold in fridge?

In **natural convection**, the fluid motion occurs by natural means such as buoyancy. Since the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encountered in natural convection is also low.

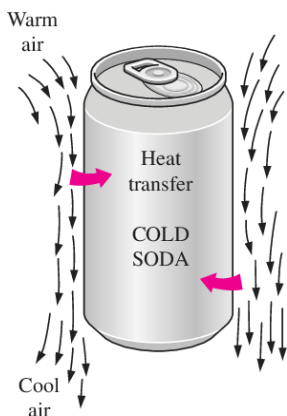
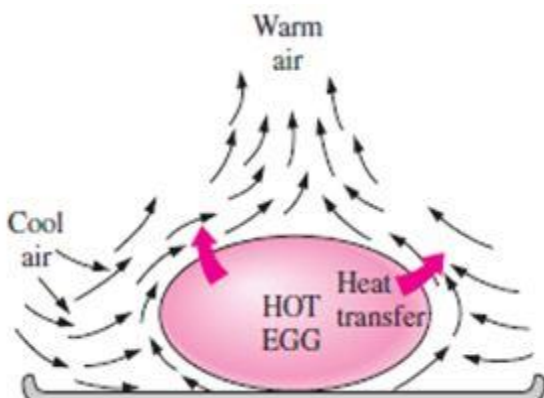


FIGURE 9-2

### Mechanisms of Natural Convection :

Consider a hot object exposed to cold air. The temperature of the outside of the object will drop (as a result of heat transfer with cold air), and the temperature of adjacent air to the object will rise. Consequently, the object is surrounded with a thin layer of warmer air and heat will be transferred from this layer to the outer layers of air.

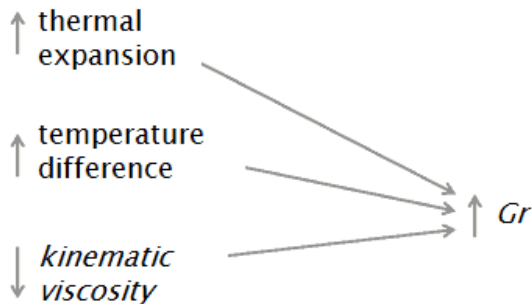


The temperature of the air adjacent to the hot object is higher, thus its density is lower. As a result, the heated air rises. This movement is called the natural convection current. Note that in the absence of this movement, heat transfer would be by conduction only and its rate would be much lower.

## Grashof Number:

Grashof number is a dimensionless group. It represents the ratio of the buoyancy force to the viscous force acting on the fluid:

$$Gr = \frac{\text{buoyant forces}}{\text{viscous forces}} = \frac{g\beta(T_{wall} - T_{\infty})L^3}{\nu^2}$$



The role played by Reynolds number in forced convection is played by the Grashof number in natural convection. The critical Grashof number is observed to be about  $10^9$  for vertical plates. Thus, the flow regime on a vertical plate becomes turbulent at Grashof number greater than  $10^9$ .

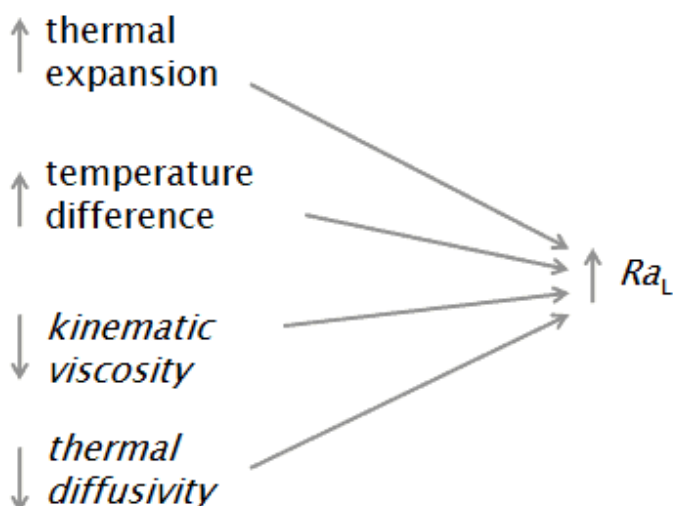
### Natural Convection over Surfaces

Natural convection on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid.

### Natural Convection Correlations

The complexities of the fluid flow make it very difficult to obtain simple analytical relations for natural convection. Thus, most of the relationships in natural convection are based on experimental correlations. The Rayleigh number is defined as the product of the Grashof and Prandtl numbers:

$$Ra_L = Gr_L \cdot Pr = \frac{g\beta(T_{wall} - T_{\infty})L^3}{\nu \cdot \alpha}$$



The Nusselt number in natural convection is in the following form:

$$Nu = hL_k = C Ra^n$$

where the constants  $C$  and  $n$  depend on the geometry of the surface and the flow. Table 14-1 in Cengel book lists these constants for a variety of geometries.

These relationships are for isothermal surfaces, but could be used approximately for the case of non-isothermal surfaces by assuming surface temperature to be constant at some average value.

### Free Convection- Horizontal Plates

Horizontal plate: upward / downward flow perpendicular to surface → plumes instead of adjacent boundary layer

Please watch below video to get more details about natural convection around horizontal plates :

[https://www.youtube.com/watch?v=MMzHP3vPXnA&feature=emb\\_imp\\_woyt](https://www.youtube.com/watch?v=MMzHP3vPXnA&feature=emb_imp_woyt)

**We are going to discuss more about natural convection heat transfer in this session.**

## Natural Convection

In natural convection, the fluid motion occurs by natural means such as buoyancy. Since the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encountered in natural convection is also low.

### Mechanisms of Natural Convection

Consider a hot object exposed to cold air. The temperature of the outside of the object will drop (as a result of heat transfer with cold air), and the temperature of adjacent air to the object will rise. Consequently, the object is surrounded with a thin layer of warmer air and heat will be transferred from this layer to the outer layers of air.

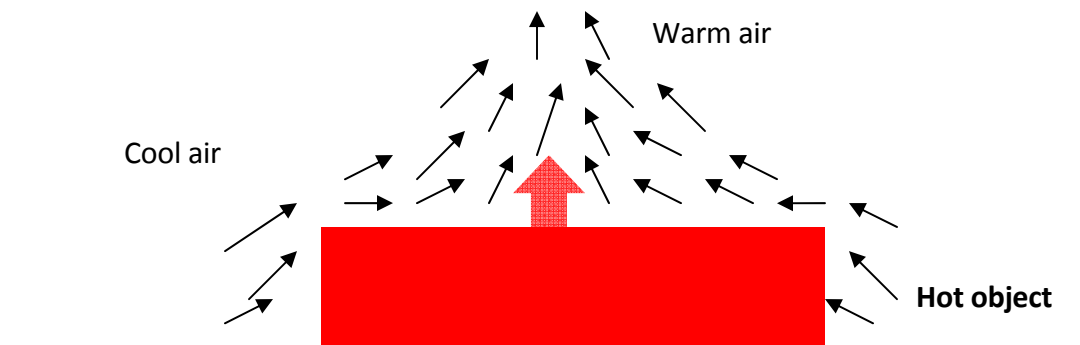


Fig. 1: Natural convection heat transfer from a hot body.

The temperature of the air adjacent to the hot object is higher, thus its density is lower. As a result, the heated air rises. This movement is called the *natural convection current*. Note that in the absence of this movement, heat transfer would be by conduction only and its rate would be much lower.

In a gravitational field, there is a net force that pushes a light fluid placed in a heavier fluid upwards. This force is called the *buoyancy force*.

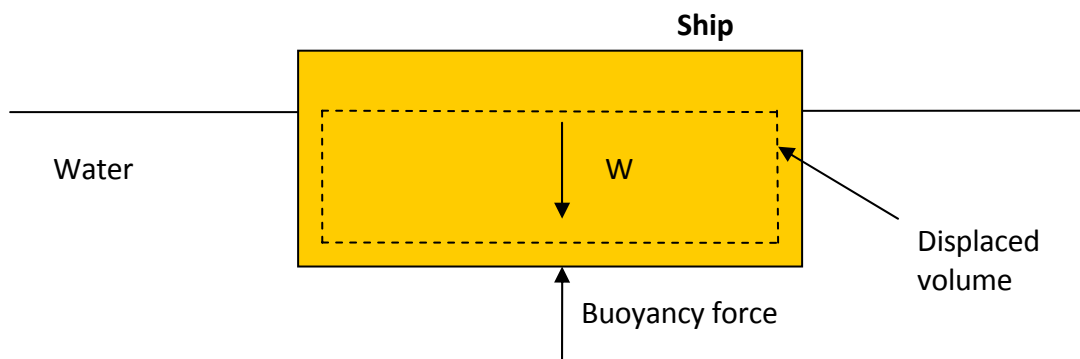


Fig. 2: Buoyancy force keeps the ship float in water.

The magnitude of the buoyancy force is the weight of the fluid displaced by the body.

$$F_{\text{buoyancy}} = \rho_{\text{fluid}} g V_{\text{body}}$$

where  $V_{\text{body}}$  is the volume of the portion of the body immersed in the fluid. The net force is:

$$F_{\text{net}} = W - F_{\text{buoyancy}}$$

$$F_{\text{net}} = (\rho_{\text{body}} - \rho_{\text{fluid}}) g V_{\text{body}}$$

Note that the net force is proportional to the *difference in the densities* of the fluid and the body. This is known as *Archimedes' principle*.

We all encounter the feeling of “weight loss” in water which is caused by the buoyancy force. Other examples are hot balloon rising, and the chimney effect.

Note that the buoyancy force needs the gravity field, thus in space (where no gravity exists) the buoyancy effects does not exist.

Density is a function of temperature, the variation of density of a fluid with temperature at constant pressure can be expressed in terms of the volume expansion coefficient  $\beta$ , defined as:

$$\beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p \quad \left( \frac{1}{K} \right)$$

$$\beta \approx -\frac{1}{\rho} \frac{\Delta \rho}{\Delta T} \rightarrow \Delta \rho \approx -\rho \beta \Delta T \quad (\text{at constant } P)$$

It can be shown that for an ideal gas

$$\beta_{\text{ideal gas}} = \frac{1}{T}$$

Since the buoyancy force is proportional to the density difference, the larger the temperature difference between the fluid and the body, the larger the buoyancy force will be.

Whenever two bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to that of the motion. Under steady conditions, the air flow rate driven by buoyancy is established by balancing the buoyancy force with the frictional force.

### Grashof Number

Grashof number is a dimensionless group. It represents the ratio of the buoyancy force to the viscous force acting on the fluid:

$$Gr = \frac{\text{buoyancy forces}}{\text{viscous forces}} = \frac{g \Delta \rho V}{\rho \nu^2} = \frac{g \beta \Delta T V}{\rho \nu}$$

It is also expressed as

$$Gr = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}$$

where

$g$  = gravitational acceleration,  $m/s^2$

$\beta$  = coefficient of volume expansion,  $1/K$

$\delta$  = characteristic length of the geometry,  $m$

$\nu$  = kinematics viscosity of the fluid,  $m^2/s$

The role played by Reynolds number in forced convection is played by the Grashof number in natural convection. The critical Grashof number is observed to be about  $10^9$  for vertical plates. Thus, the flow regime on a vertical plate becomes turbulent at Grashof number greater than  $10^9$ . The heat transfer rate in natural convection is expressed by Newton's law of cooling as:  $Q'_{conv} = h A (T_s - T_\infty)$

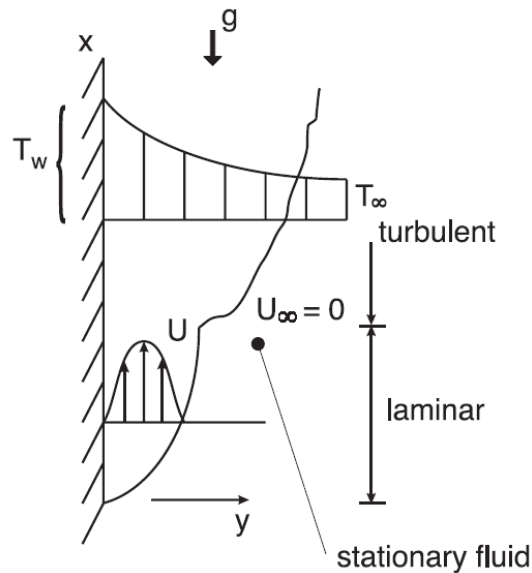


Fig. 3: Velocity and temperature profile for natural convection flow over a hot vertical plate.  $Gr_{critical} = 10^9$

### Natural Convection over Surfaces

Natural convection on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid.

The velocity and temperature distribution for natural convection over a hot vertical plate are shown in Fig. 3.

Note that the velocity at the edge of the boundary layer becomes zero. It is expected since the fluid beyond the boundary layer is stationary.

The shape of the velocity and temperature profiles, in the cold plate case, remains the same but their direction is reversed.

### Natural Convection Correlations

The complexities of the fluid flow make it very difficult to obtain simple analytical relations for natural convection. Thus, most of the relationships in natural convection are based on experimental correlations.

The Rayleigh number is defined as the product of the Grashof and Prandtl numbers:

$$Ra = Gr Pr = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} Pr$$

The Nusselt number in natural convection is in the following form:

$$Nu = \frac{h\delta}{k} = C Ra^n$$

where the constants  $C$  and  $n$  depend on the geometry of the surface and the flow. Table 14-1 in Cengel book lists these constants for a variety of geometries.

These relationships are for isothermal surfaces, but could be used approximately for the case of non-isothermal surfaces by assuming surface temperature to be constant at some average value.

### Isothermal Vertical Plate

For a vertical plate, the characteristic length is  $L$ .

$$Nu = \begin{cases} 0.59 Ra^{1/4} & 10^4 < Ra < 10^9 \\ 0.1 Ra^{1/3} & 10^9 < Ra < 10^{13} \end{cases}$$

Note that for ideal gases,  $\beta = 1/T_f$

### Isothermal Horizontal Plate

The characteristics length is  $A/p$  where the surface area is  $A$ , and perimeter is  $p$ .

a) Upper surface of a hot plate

$$Nu = \begin{cases} 0.54 Ra^{1/4} & 10^4 < Ra < 10^7 \\ 0.15 Ra^{1/3} & 10^7 < Ra < 10^{11} \end{cases}$$

b) Lower surface of a hot plate

$$Nu = 0.27 Ra^{1/4} \quad 10^5 < Ra < 10^{11}$$

### Example 1: isothermal vertical plate

A large vertical plate 4 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer if the plate is 10 m wide.

*Solution:*

We first determine the film temperature as

$$T_f = (T_s + T_\infty) / 2 = 35^\circ\text{C} = 308 \text{ K}$$

The properties are:

$$\beta = 1 / 308 = 3.25 \times 10^{-3}, k = 0.02685 \text{ (W/mK)}, \nu = 16.5 \times 10^{-6}, \text{Pr} = 0.7$$

The Rayleigh number can be found:

$$Ra = Gr \text{Pr} = \frac{9.8 \text{ m/s}^2 (3.25 \times 10^{-3} \text{ K}^{-1}) (60 - 10^\circ \text{C}) (4 \text{ m})^3}{(16.5 \times 10^{-6})^2} 0.7 = 3.743 \times 10^{11}$$

The Nusselt number can be found from:

$$Nu = 0.1 Ra^{1/3} = 0.1 (3.743 \times 10^{11})^{1/3} = 720.7$$

The heat-transfer coefficient is

$$h = \frac{Nu k}{L} = \frac{720.7 \times 0.02685}{4} = 4.84 \text{ (W/m}^2\text{K)}$$

The heat transfer is

$$Q^\circ = h A (T_s - T_\infty) = 7.84 \text{ W/m}^2\text{C}^2 (4 \times 10 \text{ m}^2) (60 - 10^\circ \text{C}) = 9.675 \text{ kW}$$

## Learning activities after - Lecture 4

You have joined the fourth Lecture of our E&H course. Below you will find some activities that will help you to bring your learning into practice and/or to learn more about the topics that were addressed.

### Natural convection Questions

Try to think and find out the answers for below questions :

1. What is Natural convection?
2. What is Grashof number ?
3. What is Rayleigh number?
4. How to calculate Nusselt number for natural convection cases?
5. Can you define the natural convection current for cold and hot horizontal plate ?
6. Can you define which correlations to use for different natural convection cases?

please watch below video which will show you how to use your knowledge of heat transfer till now to calculate the free convection for vertical isothermal plate:

<https://www.youtube.com/watch?v=0wioVJt4V2I>