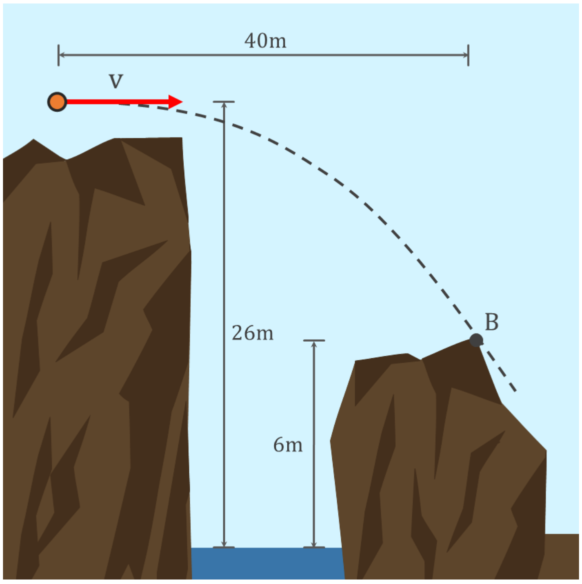
Rock Thrown from Cliff



Determine the minimum horizontal initial speed $v_{\min}$ necessary to throw a rock at point A towards point B and have it clear the obstruction at point B.

Neglect air resistance and take $g = 10\ \mathrm{m/s^2}$.

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (at + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (at + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Initial height of the ball (with respect to the cliff): $y_0 = s_{y,0} = s_{x,0} = 0$ m
Gravitational constant: $g = 10$ m/s^2

Solution:

For the displacement in x-and y-direction, Equation (6) results in:

$$\begin{cases} x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + s_{x,0} \\ y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \end{cases} \tag{7}$$

Point B is located at $x = 40$ m and $y = -20$ m. Together with the the fact that there is no horizontal acceleration ($a_x = 0$ m/s^2) and vertical initial velocity ($v_{y,0} = 0$ m/s), we can solve for $v_{x,0}$:

$$\begin{cases} x(t) = 40 = \frac{1}{2}a_x t^2 + v_{x,0}t + s_{x,0} \\ y(t) = -20 = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \end{cases} \quad \Rightarrow \quad \begin{cases} 40 = v_{x,0}t \\ -20 = -\frac{1}{2}gt^2 \end{cases} \tag{8}$$

For the last equation the time t can be calculated before the ball reaches point B:

$$t = \sqrt{\frac{2v_{y,0}}{g}} = \sqrt{\frac{2 \cdot 20}{10}} = 2 \text{ s} \tag{9}$$

Inserting $t = 2$ s into Equation (8) results in:

$$40 = v_{x,0} \cdot 2 \quad \Rightarrow \quad v_{x,0} = 20 \text{ m/s} \tag{10}$$