2.18 Heating and quenching of a sphere

- Problem type

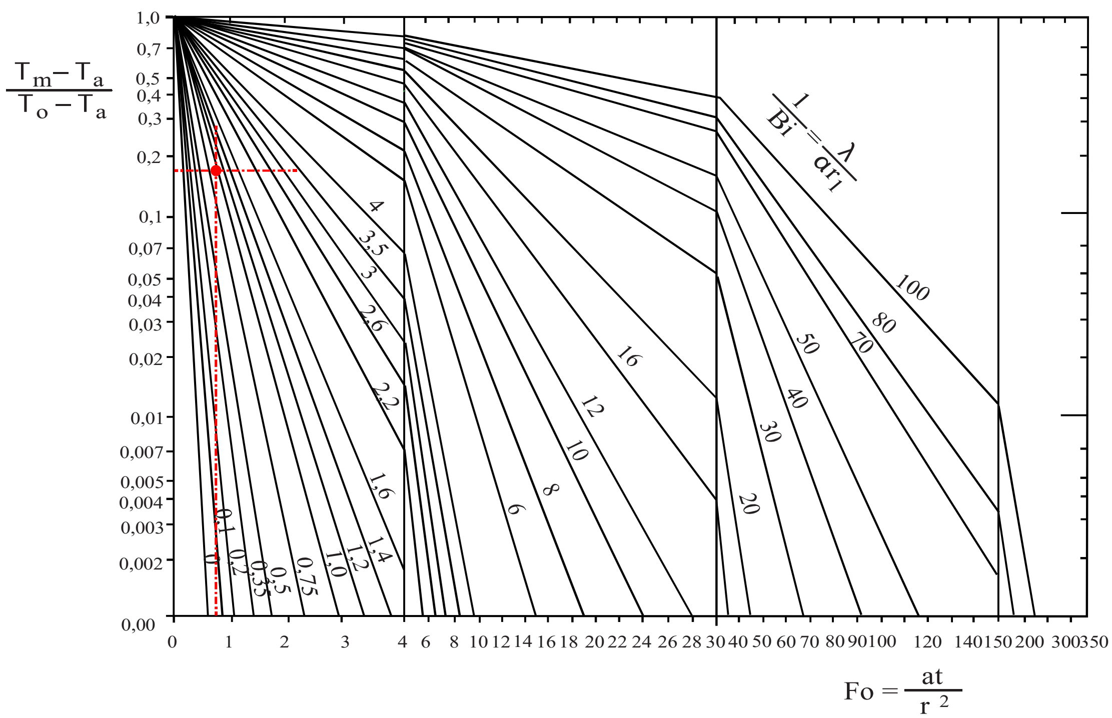
  Transient heat transfer that is non-homogeneous and does penetrate.

a) Determining T_m after 3 minutes:

$$\frac{1}{\mathrm{Bi}} = \frac{\lambda}{\alpha \cdot r_1} = \frac{1.52\ [\mathrm{Wm^{-1}K^{-1}}]}{110\ [\mathrm{Wm^{-2}K^{-1}}] \cdot 0.015\ [\mathrm{m}]} = 0.9212 \tag{2.177}$$

$$\mathrm{Fo} = \frac{a \cdot t}{{r_1}^2} = \frac{9.5 \cdot 10^{7}\ [\mathrm{m^2s^{-1}}] \cdot 180\ [\mathrm{s}]}{0.015^2\ [\mathrm{m^2}]} = 0.76 \tag{2.178}$$

Using the Heisler diagram for the temperature in the centre of a sphere:



Results in:

$$\frac{T_\mathrm{m} - T_\mathrm{a}}{T_0 - T_\mathrm{a}} \approx 0.18 \tag{2.179}$$

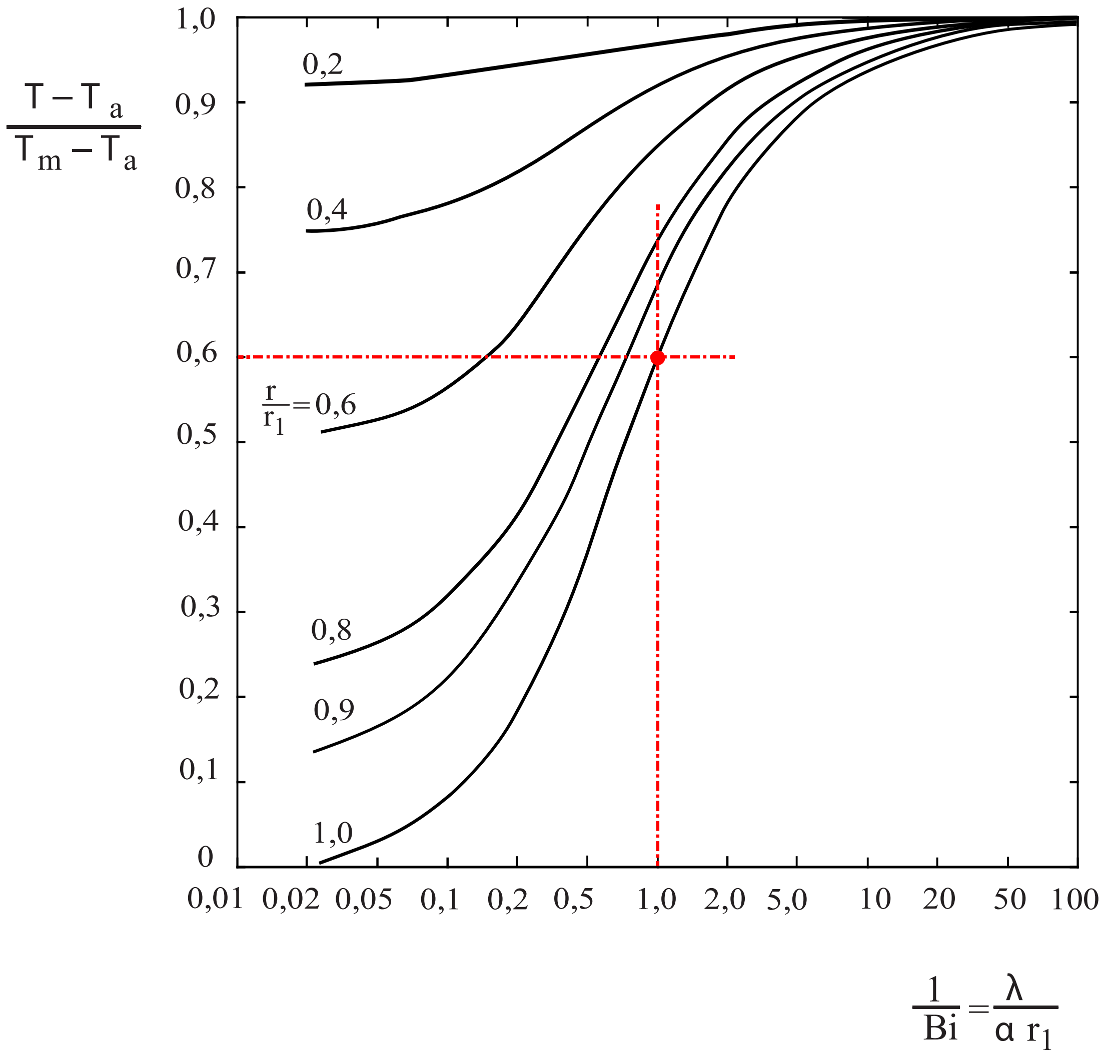
$$\rightarrow T_{\mathrm{m}} \approx 0.18\,(T_0 - T_{\mathrm{a}}) + T_{\mathrm{a}} = 0.18\,(25\ [^{\circ}\mathrm{C}] - 200\ [^{\circ}\mathrm{C}]) + 200\ [^{\circ}\mathrm{C}] = 168.5\ [^{\circ}\mathrm{C}] \tag{2.180}$$

b) Determining t_1:

$$\frac{T - T_{\mathrm{a}}}{T_{\mathrm{m}} - T_{\mathrm{a}}} = \frac{44.4\ [^{\circ}\mathrm{C}] - 30\ [^{\circ}\mathrm{C}]}{54\ [^{\circ}\mathrm{C}] - 30\ [^{\circ}\mathrm{C}]} = 0.6 \tag{2.181}$$

$$\frac{r}{r_1} = 1 \tag{2.182}$$

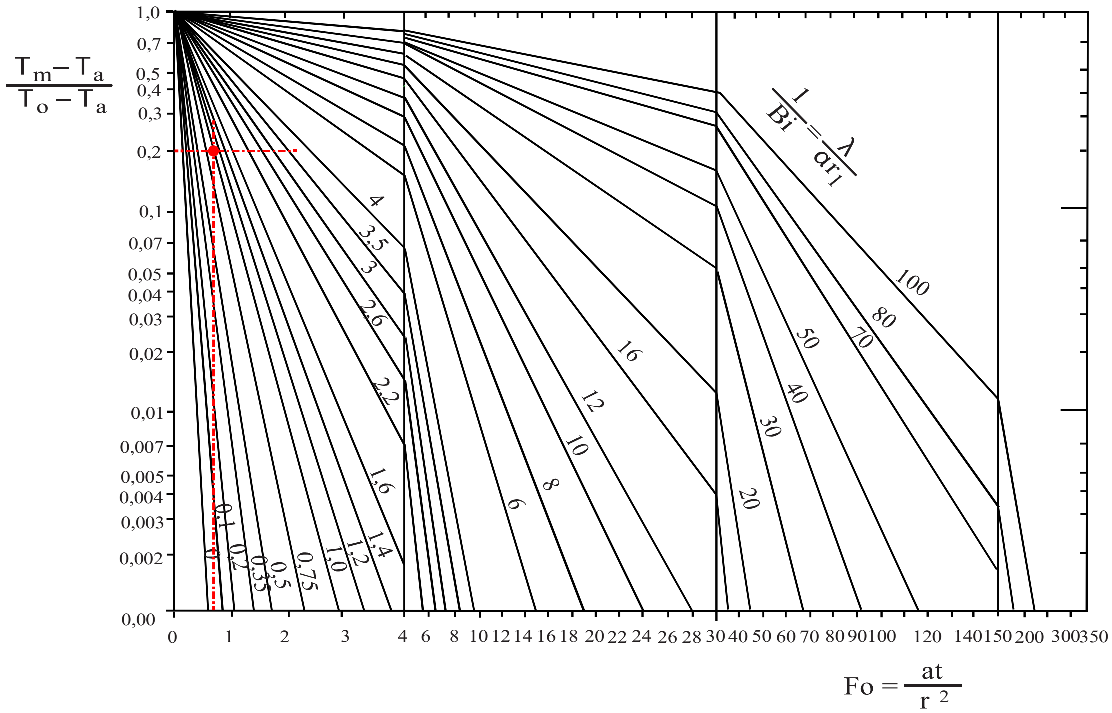
Using the Heisler diagram for the temperature distribution in a sphere to determine $\frac{1}{\mathrm{Bi}}$: (Note that this diagram is only valid when $\mathrm{Fo} > 0.2$)



$$\rightarrow \frac{1}{\mathrm{Bi}} \approx 1 \tag{2.183}$$

$$\frac{T_\mathrm{m} - T_\mathrm{a}}{T_0 - T_\mathrm{a}} = \frac{54\ [^\circ\mathrm{C}] - 30\ [^\circ\mathrm{C}]}{150\ [^\circ\mathrm{C}] - 30\ [^\circ\mathrm{C}]} = 0.2 \tag{2.184}$$

Using the Heisler diagram for the temperature in the centre of a sphere to determine Fo:

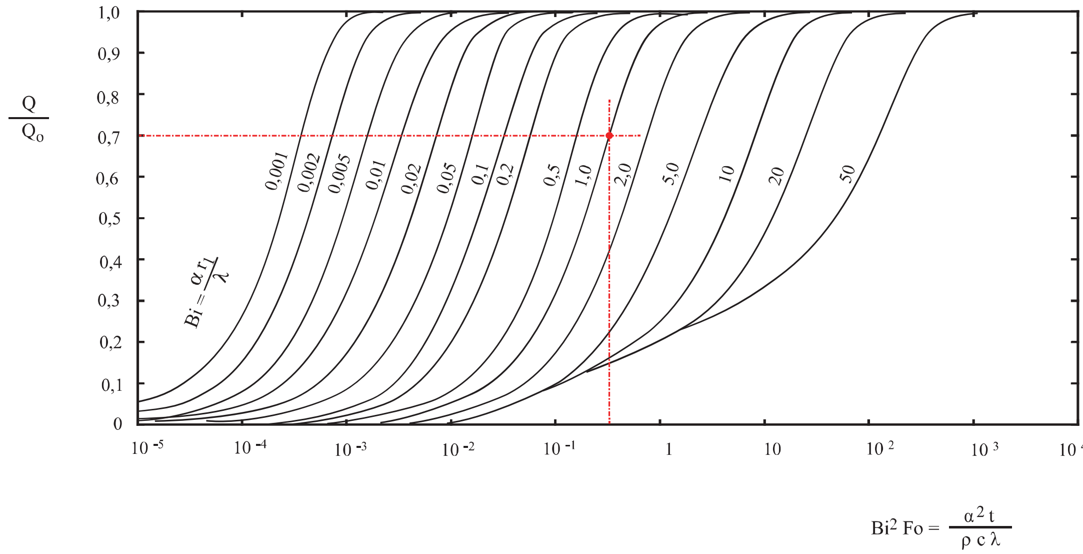


$$\text{Fo} \approx 0.7 \;\; \rightarrow t_1 \approx 165.79 \text{ [s]} \tag{2.185}$$

c) Determining the dissipated heat Q at time instant t_1.

$$\text{Bi}^2 \cdot \text{Fo} \;=\; 0.7 \tag{2.186}$$

Using the Heisler diagram for the heat loss of a sphere:



$$\frac{Q}{Q_o} \approx 0.92 \tag{2.187}$$

$$\rightarrow Q \approx 0.92 \cdot \left(\rho \cdot \frac{4}{3} \cdot \pi \cdot {r_1}^2 \cdot c_{\mathrm{p}} \cdot (T_0 - T_a) \right) = 132.76 \text{ [kJ]} \tag{2.188}$$

2.19 Pizza stone

- Problem type

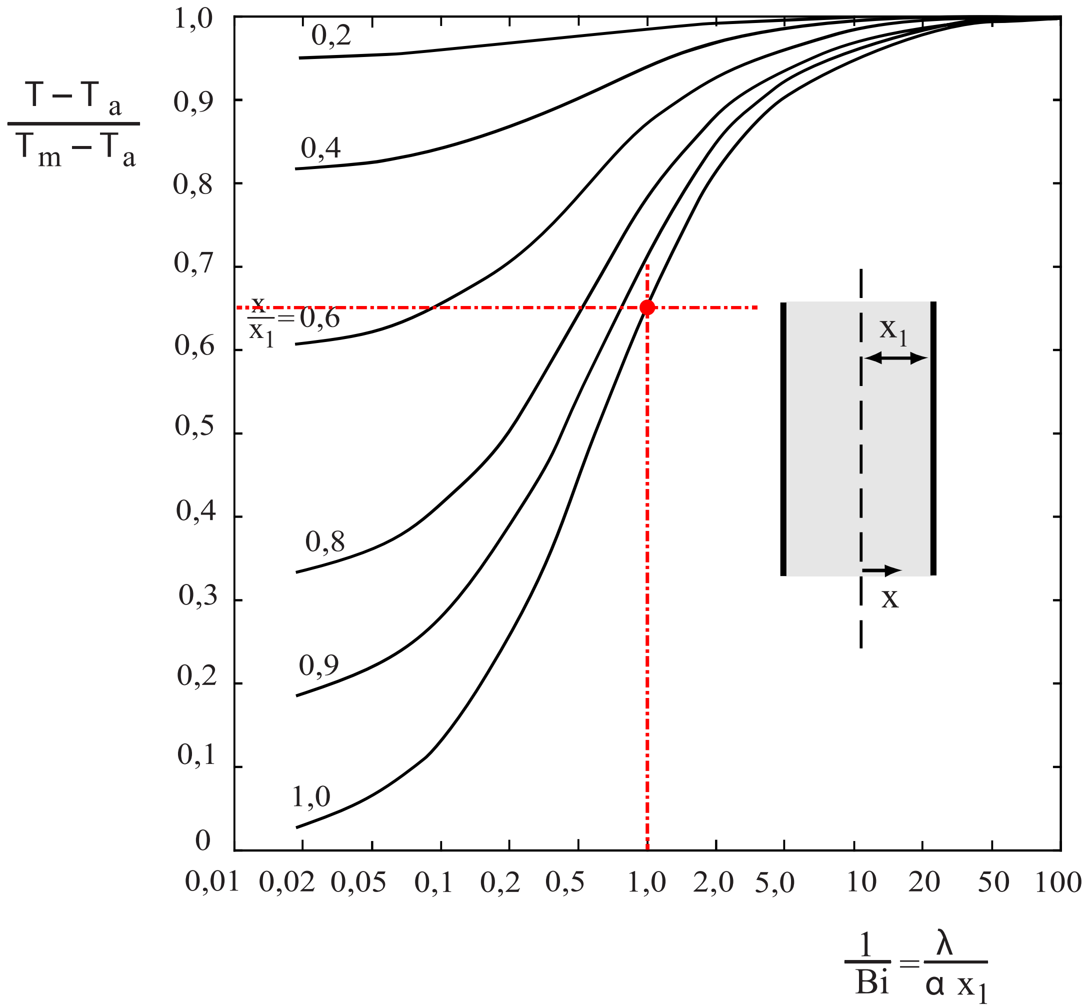
  Transient heat transfer that is non-homogeneous and does penetrate.

a) Determining the time to heat up the pizza stone

$$\frac{T - T_\mathrm{a}}{T_\mathrm{m} - T_\mathrm{a}} = \frac{205\ [^\circ\mathrm{C}] - 250\ [^\circ\mathrm{C}]}{181\ [^\circ\mathrm{C}] - 250\ [^\circ\mathrm{C}]} = 0.65 \tag{2.189}$$

$$\frac{x}{x_1} = 1 \tag{2.190}$$

Using the Heisler diagram for the temperature distribution in a plate to determine $\frac{1}{\mathrm{Bi}}$: (Note that this diagram is only valid when $\mathrm{Fo} > 0.2$)



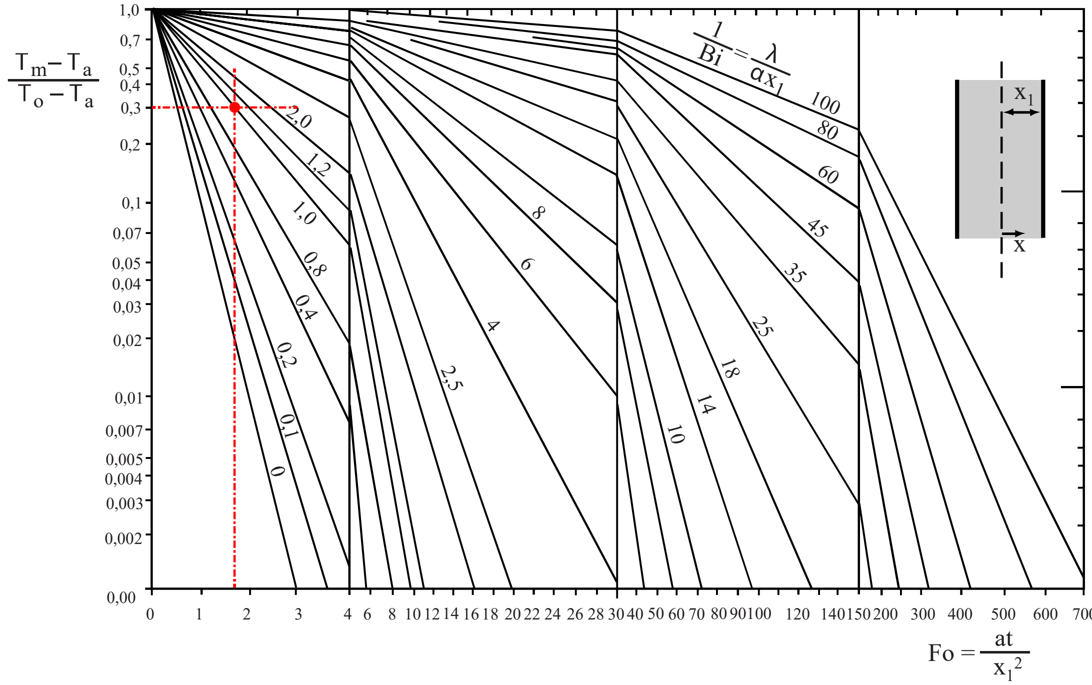
Results in:

$$\rightarrow \frac{1}{\mathrm{Bi}} \approx 1 \tag{2.191}$$

Determining the dimensionless temperature:

$$\frac{T_\mathrm{m} - T_\mathrm{a}}{T_0 - T_\mathrm{a}} = \frac{181\ [^\circ\mathrm{C}] - 250\ [^\circ\mathrm{C}]}{20\ [^\circ\mathrm{C}] - 250\ [^\circ\mathrm{C}]} = 0.3 \tag{2.192}$$

Using the Heisler diagram for the mid-plane temperature of a plate with thickness $2x_1$ to determine Fo:



$$Fo \approx 1.8 \tag{2.193}$$

$$t \approx 32 \text{ [min]} \tag{2.194}$$