Passenger Plane Take-Off

The pilot of a passenger plane brings the engines to full take-off power before releasing the brakes as the aircraft is standing still on the runway. The jet thrust remains constant, and the aircraft has a near-constant acceleration of $0.4g$. If the take-off speed is $v_1 = 216$ km/h, what is the distance s_1 in meters the plane needs to take off?

Neglect air resistance and take $g = 10$ m/s^2.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Gravitational acceleration: $g = 10\ \text{m/s}^2$
Acceleration: $a = 0.4g = 0.4 \cdot 10 = 4\ \text{m/s}^2$
Initial velocity: $v_0 = 0\ \text{m/s}$
Final velocity: $v_1 = 216\ \text{km/h} \approx 60\ \text{m/s}$

Solution:

In the case of this exercise, s_0 and v_0 are equal to zero, resulting in:

$$v(t) = at \tag{7}$$

$$s(t) = \frac{1}{2}at^2 \tag{8}$$

Filling in $v_1 = 60\ \text{m/s}$ and $a = 4\ \text{m/s}^2$ in Equation (7) results in:

$$v(t_1) = at_1 = v_1 \quad \Rightarrow \quad 60 = 4 \cdot t_1 \quad \Rightarrow \quad t_1 = 15\ \text{s} \tag{9}$$

Inserting $t_1 = 15$ s in Equation (8) results in the final answer s_1:

$$s_1 = s(15) = \frac{1}{2} \cdot 4 \cdot 15^2 = 450\ \text{m} \tag{10}$$