Energy & Heat Transfer

Lecture 4

By: Mohammad Mehrali

Study Materials

- **Slides**
- **Book : (Y. A. Cengel& A. J. Ghajar. Heat and Mass Transfer: Fundamental & Application)**
- **Heat Quiz**
- **Learning activities**

Recap of lecture 3

Forced and natural/free convection:

$\dot{Q} = hA\Delta T$ $\dot{q} = h\,\Delta T$ Newton's cooling law

Nusselt Number: dimensionless variety of heat transfer coefficient *h*

$$\mathrm{Nu} = \frac{hL_c}{k}$$ With L_c a characteristic length for the considered geometry

Determining the Nusselt Number: (empirical) correlations

- Forced convection: Nu as function of Re, Pr a.k.a.: Nu = *f*(Re, Pr)
- Natural convection: This lecture

⇒ Relation Nu, Re, Pr dependent on geometry and flow regime (laminar/ turbulent)

⇒ Nu, Re, Pr dimensionless numbers: reduction number of variables

Recap of lecture 3

Convection Resistance :

$$\dot{Q} = hA\Delta T = \frac{\Delta T}{R_{conv}} \quad \text{met} \quad R_{conv} = \frac{1}{hA} \quad (\text{K/W})$$

Dimensionless Numbers :

Nusselt Number: $\mathrm{Nu} = \frac{hL_c}{k}$

Reynolds Number: $\mathrm{Re} = \frac{\rho U L_c}{\mu}$

Prandtl Number: $\mathrm{Pr} = \frac{\mu c_p}{k}$

+ Background on boundary layers

Recap of lecture 3

If $\dot{Q}$ must be found:

- Calculate at film temperature : $T_f = \frac{T_s + T_\infty}{2}$
- Pull out ingredients like μ, ρ, k, Pr from tables – like assignment bundle: air or given fluid) at $T_f = \frac{T_s + T_\infty}{2}$

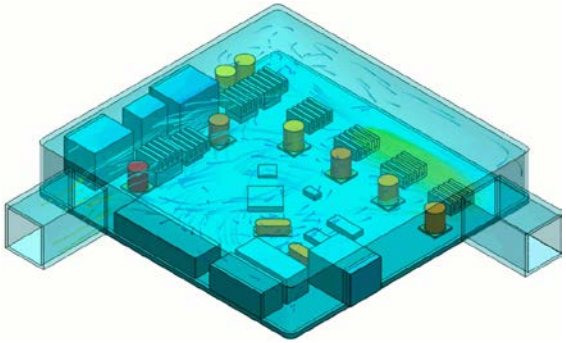
- Calculate Re and choose apprioprate correlation based on geometry and Re
- Calculate Nu
- Derive *h* from it
- Fill out Newton's cooling law: $\dot{Q} = hA\Delta T$

Learning objectives lecture 4

- **Heat transfer through natural convection**
  - **Natural Convection**
  - Grashof Number
  - Rayleigh Number
  - Nusselt Number
  - Using correlations for various configurations
  - Calculating natural convection with step-by-step plan

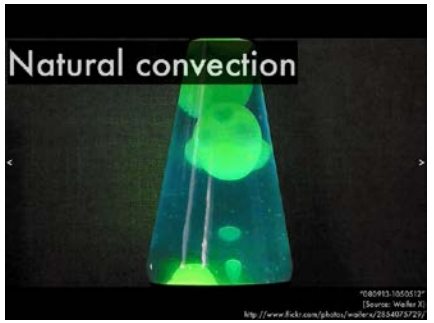
Types of convection (from Lecture 3)

Forced convection



Imposed flow (by pump, fan, …)

Natural/free convection



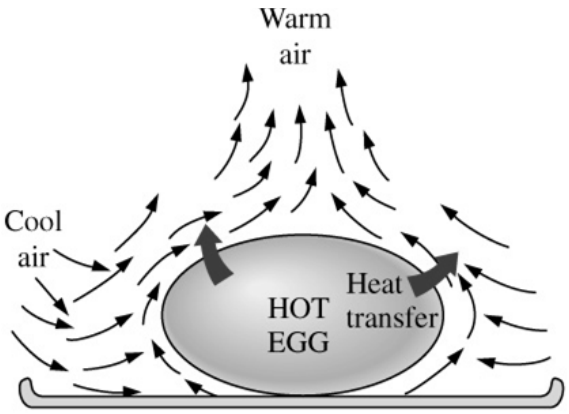
Temperature difference itself starts the flow

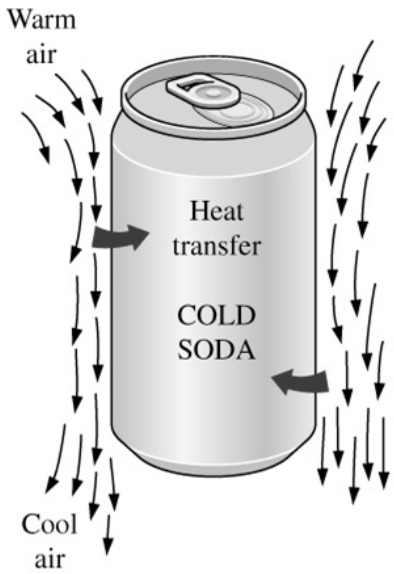
General: flow velocity and heat transfer rates are larger for forced convection

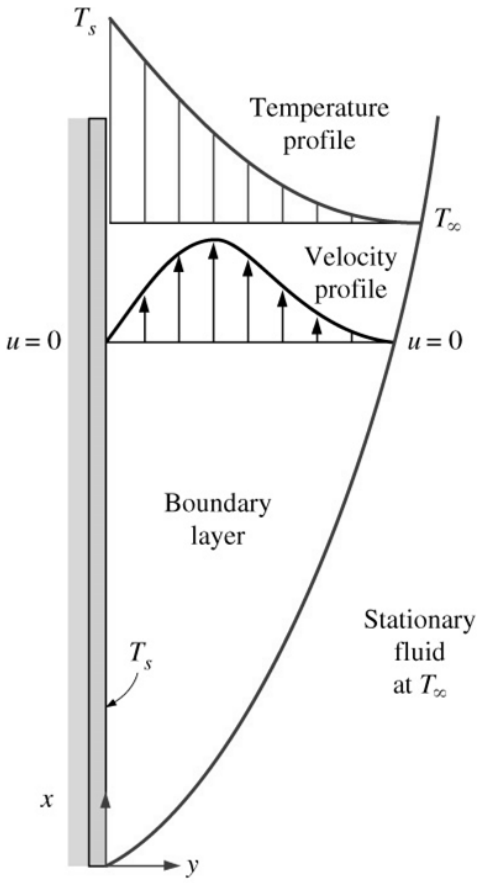
Natural Convection

Also for natural convection velocity and thermal boundary layers!

- Velocity boundary layer is only area in which flow occurs
- **Hot** surface: **Upward flow**
- **Cold** surface: **Downward flow**







Natural Convection

https://www.youtube.com/watch?v=0mUU69ParFM&ab_channel=ScienceUpwiththeSingingZoologist

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Grashof Number

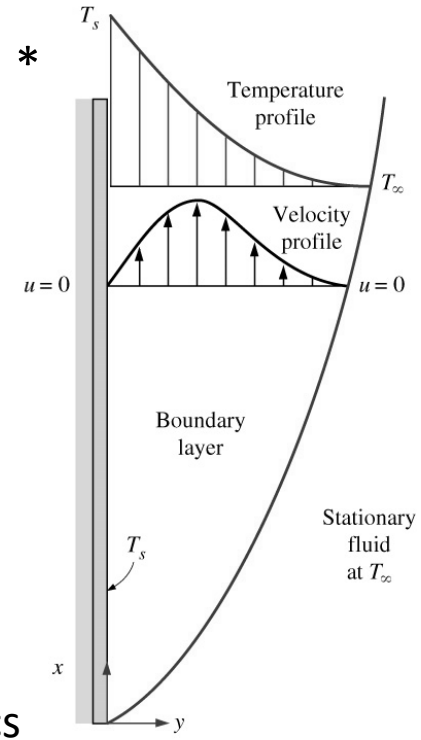
Forced convection: velocity *U* imposed

Natural convection: velocity follows from temp.difference T_s - T_∞ *

$\Rightarrow$ Alternative for Reynolds number, with temp.diff. instead *U*:

Grashof number: $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}$ (–)

Greek letter "nu"



The **Grashof number (Gr)** is a **dimensionless** number in fluid dynamics and heat transfer which approximates the **ratio of** the **buoyancy** to **viscous force** acting on a fluid.

*Choose ΔT positive for convenience

Grashof Number

Grashof number: $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}$ $(-)$

- Gravitational acceleration : **g = 9,81 m/s²**
- Volume expansion coefficient **β (K⁻¹);** **for most gases:** $\beta = \dfrac{2}{T_s + T_\infty}$

  (temperature in Kelvin; 0 ˚C = 273,15 K)

- Length **L_c** characteristic for geometry **(length *L* for plate, diameter *D* for sphere/cylinder)**
- Kinematic viscosity : $\nu = \dfrac{\mu}{\rho}(\mathrm{m}^2/\mathrm{s})$ at avg. Temp: $T_f = \dfrac{T_s + T_\infty}{2}$

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Rayleigh Number

Laminar / turbulent: ↔

Forced convection:
Determined by Reynolds number : **Re**

Natural convection:
Determined by Grashof number **Gr**

Often combined with Prandtl number:
Rayleigh number **Ra = Gr · Pr**

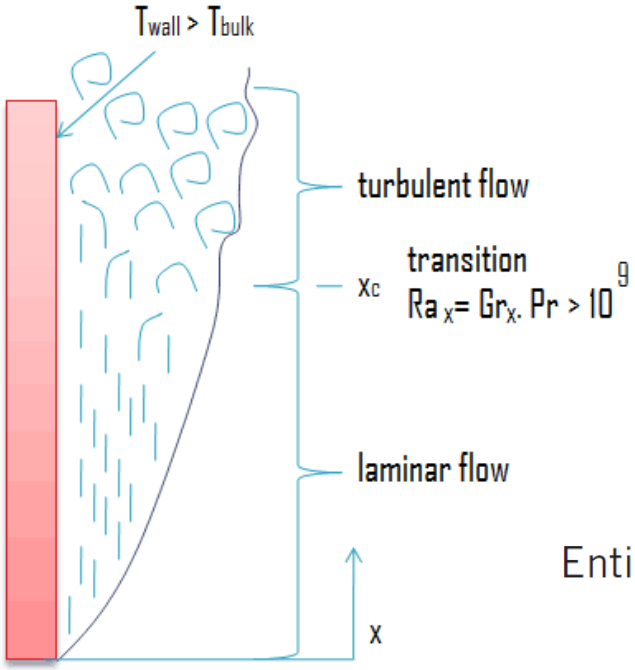
Rayleigh Number : $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$

Rayleigh Number is a dimensionless number associated with buoyancy-driven flow, also known as free or natural convection

Rayleigh Number

Rayleigh Number : $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$

Vertical, flat plate (ex. radiator)



$Nu = 0{,}59\ Ra_L^{1/4}$ with $L_c = L$ $(10^4 < Ra_L < 10^9)$

$Nu = 0{,}1\ Ra_L^{1/3}$ with $L_c = L$ $(10^9 < Ra_L < 10^{13})$

$10^9 < Ra_L < 10^{10}$: find intermediate

Entire range ⇒ $Nu = \left\{0.825 + \dfrac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$

(complex but more accurate)

https://www.youtube.com/watch?v=efLbY1dEmyo&feature=emb_logo&ab_channel=SpeedComfort

Learning objectives lecture 4

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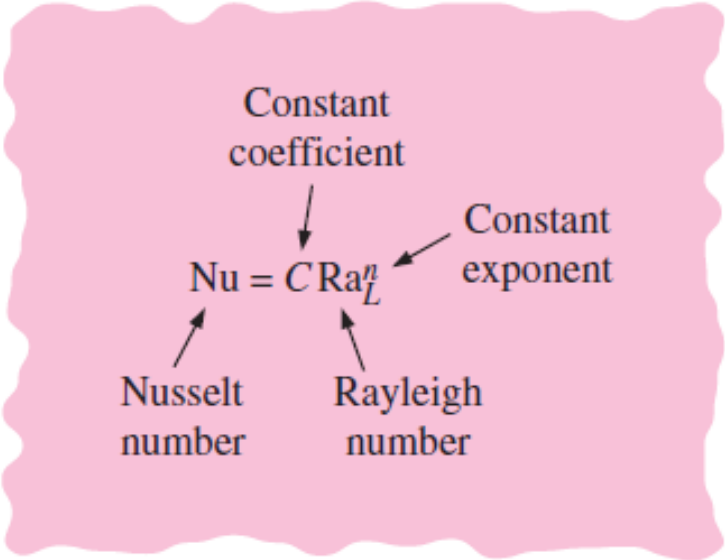
Nusselt Number

Forced Convection

$$\frac{hL}{k} = a\left(\frac{\rho U L}{\mu}\right)^{b}\left(\frac{\mu c_p}{k}\right)^{c}$$

$$\mathrm{Nu} = a \cdot \mathrm{Re}^{b} \cdot \mathrm{Pr}^{c}$$

Natural Convection

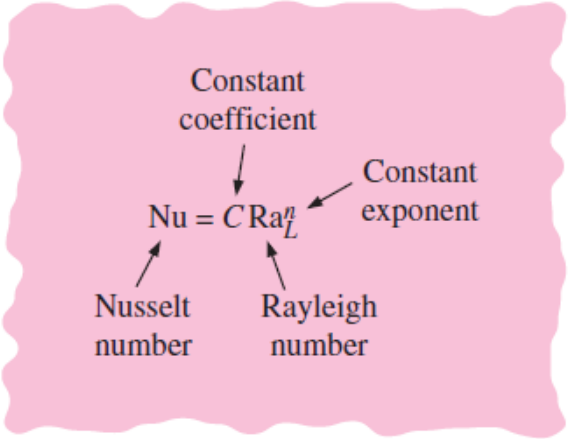


$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$$

Nusselt Number

Rayleigh Number : $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$

Rayleigh number, which is the product of the Grashof and Prandtl numbers



The values of the constants C and n depend on the

- ✓ geometry of the surface
- ✓ the flow regime

The value of n is usually

- ✓ 0,25 for laminar flow
- ✓ 0,33 for turbulent flow.

The value of C

- ✓ is normally less than 1.

All fluid properties are to be evaluated at the film temperature $T_f = (T_s + T_{inv})/2$.

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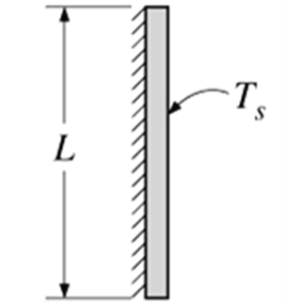
Free Convection Correlations

Vertical, flat plate

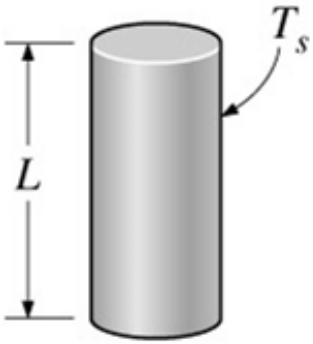


$Nu = 0{,}59\ Ra_L^{1/4}$ with $L_c = L$ $(10^4 < Ra_L < 10^9)$

$Nu = 0{,}1\ Ra_L^{1/3}$ with $L_c = L$ $(10^{10} < Ra_L < 10^{13})$

Entire range ➡ $$Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$$

(complex but more accurate)

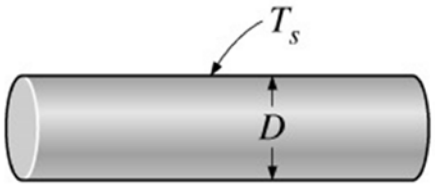


Vertical cylinder (e.g. can)

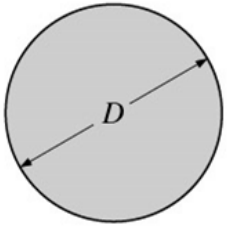
Treat a vertical plate with height L and surface πDL, if diameter D sufficiently large with respect to L $\left(D \geq \frac{35L}{Gr_L^{1/4}}\right)$

Free Convection Correlations

Horizontal cylinder (e.g. pipe)



$$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$



$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$$

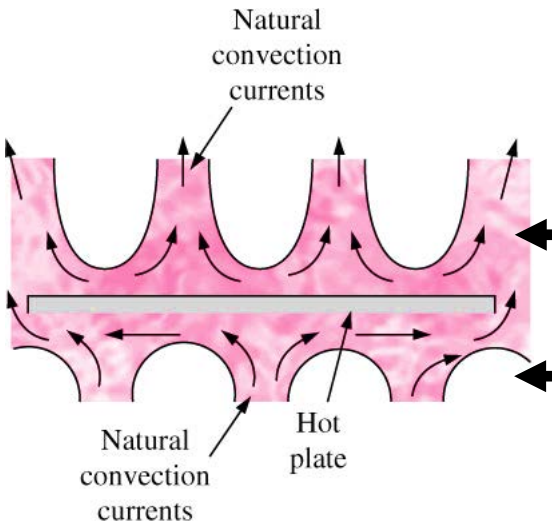
Sphere (e.g. light bulb)

Notation:
Ra_L is Ra with $L_c = L$;
Ra_D is Ra with $L_c = D$

Free Convection Correlations

Horizontal plate: upward / downward flow perpendicular to surface→ plumes instead of adjacent boundary layer

Hot surface



Fluid away from surface
Effective heat transfer

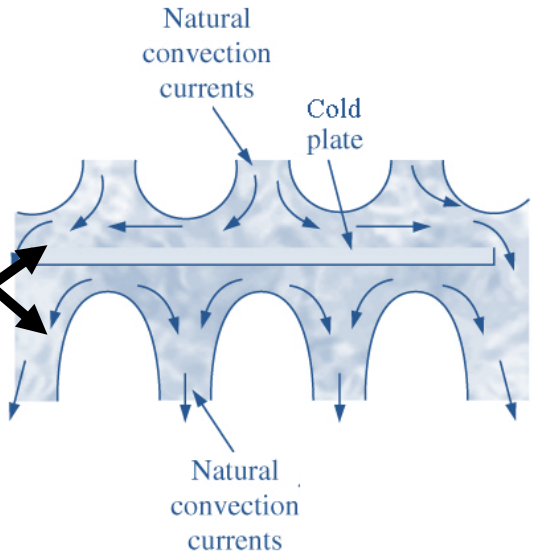
$Nu = 0{,}54\ Ra^{1/4}$ $(10^4 < Ra < 10^7)$

$Nu = 0{,}15\ Ra^{1/3}$ $(10^7 < Ra < 10^{11})$

Fluid towards surface
Reduced heat transfer

$Nu = 0{,}27\ Ra^{1/4}$ $(10^5 < Ra < 10^{11})$

Cold surface



N.B.: Characteristic length
So actually: $Ra_{A/p}$

$$L_c = \frac{\text{Area}}{\text{Perimeter}} = \frac{A}{p}$$

Free Convection Correlations

Plate at angle θ with vertical axis

($\theta < 60°$)

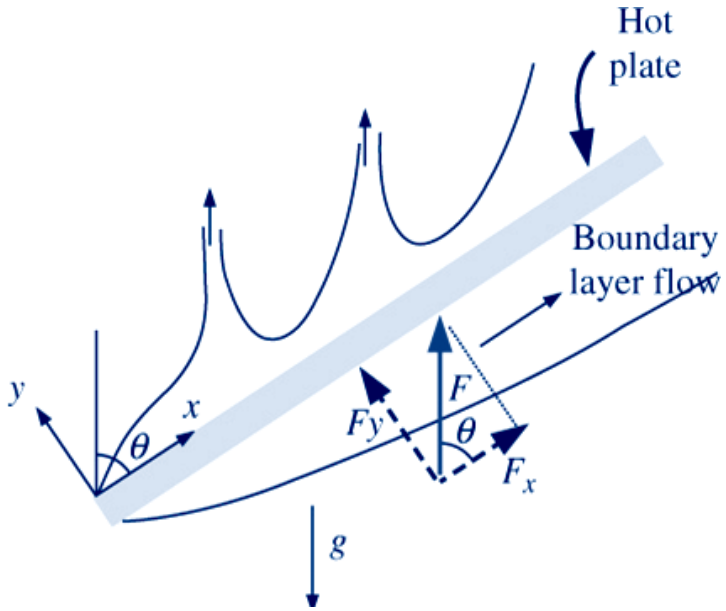


Plate at angle θ (ex. radiator in attic room)

Hot surface:

- Top side: advanced methods
- Bottom side: same as vertical plate, except for using g cos θ instead of g:

$$\mathrm{Ra}_L = \mathrm{Gr}_L \cdot \mathrm{Pr} = \frac{g\cos\theta\,\beta(T_s - T_\infty)L^3}{\nu^2} \cdot \mathrm{Pr}$$

Cold surface: everything upside down

- Bottom side: advanced methods
- Top side: same as vertical plate, except for using g cos θ instead of g

Vertical plate:

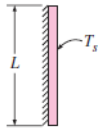
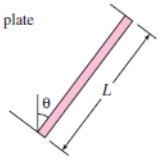
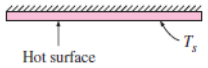
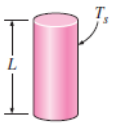
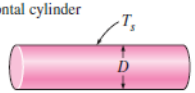
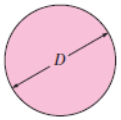
Nu = 0,59 $Ra_L^{1/4}$ $(10^4 < RaL < 10^9)$

Nu = 0,1 $Ra_L^{1/3}$ $(10^{10} < RaL < 10^{13})$

Free Convection Correlations

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20) $\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ ($\text{Pr} \geq 0.7$)	$\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)

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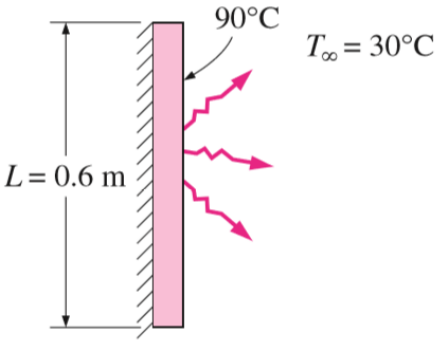
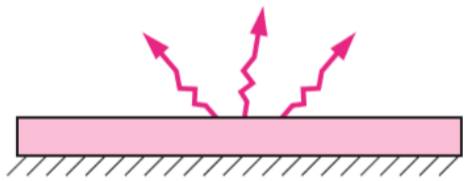
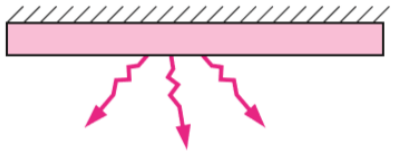
Example 1

Cooling of a Plate in Different Orientations



(*a*) Vertical


(*b*) Hot surface facing up


(*c*) Hot surface facing down

- A: 0.6-m x 0.6-m thin square plate
- room temperature: 30˚C.
- One side of the plate is maintained at a temperature of 90˚C, while the other side is insulated
- Determine the rate of heat transfer from the plate by natural convection if the plate is
  - (a) vertical,
  - (b) horizontal with hot surface facing up, and
  - (c) horizontal with hot surface facing down.

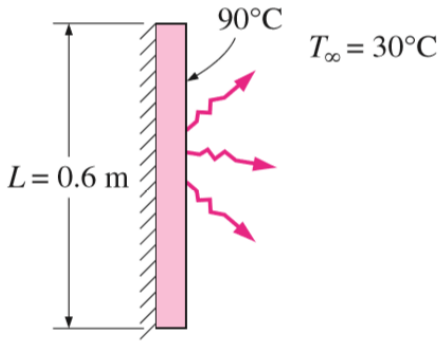
Properties of air (in the back of assigment bundle):

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m^2/s^2	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946

Properties of air (in the back of assigment bundle):

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m^2/s^2	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946

Example 1



(*a*) Vertical

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate	L	10^4–10^9 10^9–10^{13} Entire range	$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$ (9-19) $\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3}$ (9-20) $\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\mathrm{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate) (*b*) Lower surface of a hot plate (or upper surface of a cold plate)	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\mathrm{Nu} = 0.54\mathrm{Ra}_L^{1/4}$ (9-22) $\mathrm{Nu} = 0.15\mathrm{Ra}_L^{1/3}$ (9-23) $\mathrm{Nu} = 0.27\mathrm{Ra}_L^{1/4}$ (9-24)
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\mathrm{Gr}_L^{1/4}}$
Horizontal cylinder	D	$\mathrm{Ra}_D \leq 10^{12}$	$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere	D	$\mathrm{Ra}_D \leq 10^{11}$ ($\mathrm{Pr} \geq 0.7$)	$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1+(0.469/\mathrm{Pr})^{9/16}]^{4/9}}$ (9-26)

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°C$ and 1 atm are (Table A-15)

$$k = 0.02808\ \mathrm{W/m \cdot °C} \qquad \mathrm{Pr} = 0.7202$$

$$\nu = 1.896 \times 10^{-5}\ \mathrm{m^2/s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333\ \mathrm{K}}$$

Analysis (*a*) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}$$

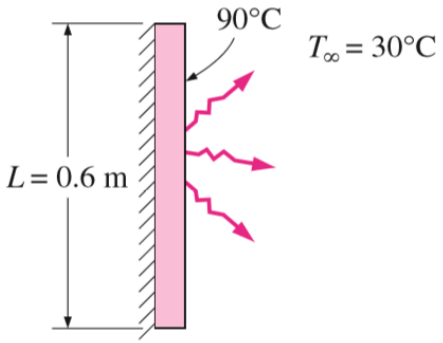
$$= \frac{(9.81\ \mathrm{m/s^2})[1/(333\ \mathrm{K})](90 - 30\ \mathrm{K})(0.6\ \mathrm{m})^3}{(1.896 \times 10^{-5}\ \mathrm{m^2/s})^2}(0.722) = 7.656 \times 10^8$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\mathrm{Nu} = \left\{0.825 + \frac{0.387\ \mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4$$

Example 1



(*a*) Vertical

Note that the simpler relation Eq. 9-19 would give $\text{Nu} = 0.59\,\text{Ra}_L^{1/4} = 98.14$, which is 13 percent lower. Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.02808\ \text{W/m}\cdot{}^\circ\text{C}}{0.6\ \text{m}}(113.4) = 5.306\ \text{W/m}^2\cdot{}^\circ\text{C}$$

$$A_s = L^2 = (0.6\ \text{m})^2 = 0.36\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306\ \text{W/m}^2\cdot{}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{115\ W}$$

Vertical plate:

Nu = 0,59 $Ra_L^{1/4}$ $(10^4 < Ra_L < 10^9)$

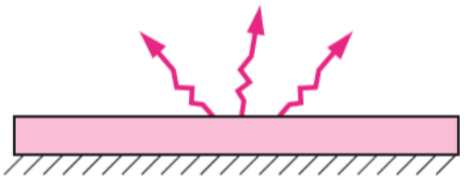
Nu = 0,1 $Ra_L^{1/3}$ $(10^9 < Ra_L < 10^{13})$

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20) $\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1+(0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) Hot surface (b) Lower surface of a hot plate (or upper surface of a cold plate) Hot surface	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere	D	$\text{Ra}_D \leq 10^{11}$ ($\text{Pr} \geq 0.7$)	$\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1+(0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)

Example 1



(*b*) Hot surface facing up

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate	L	10^4–10^9 10^9–10^{13} Entire range	$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$ (9-19) $\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3}$ (9-20) $\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\mathrm{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)	A_s/p	10^4–10^7 10^7–10^{11}	$\mathrm{Nu} = 0.54\mathrm{Ra}_L^{1/4}$ (9-22) $\mathrm{Nu} = 0.15\mathrm{Ra}_L^{1/3}$ (9-23)
(*b*) Lower surface of a hot plate (or upper surface of a cold plate)	A_s/p	10^5–10^{11}	$\mathrm{Nu} = 0.27\mathrm{Ra}_L^{1/4}$ (9-24)
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\mathrm{Gr}_L^{1/4}}$
Horizontal cylinder	D	$\mathrm{Ra}_D \leq 10^{12}$	$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere	D	$\mathrm{Ra}_D \leq 10^{11}$ ($\mathrm{Pr} \geq 0.7$)	$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1+(0.469/\mathrm{Pr})^{9/16}]^{4/9}}$ (9-26)

(*b*) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\ \text{m}}{4} = 0.15\ \text{m}$$

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr}$$

$$= \frac{(9.81\ \text{m/s}^2)[1/(333\ \text{K})](90 - 30\ \text{K})(0.15\ \text{m})^3}{(1.896 \times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$\mathrm{Nu} = 0.54\ \mathrm{Ra}_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

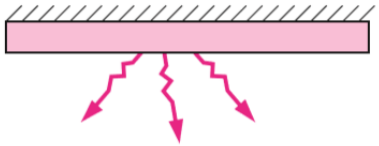
$$h = \frac{k}{L_c}\mathrm{Nu} = \frac{0.0280\ \text{W/m} \cdot {}^\circ\text{C}}{0.15\ \text{m}}(31.76) = 5.946\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

$$A_s = L^2 = (0.6\ \text{m})^2 = 0.36\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{128\ W}$$

Example 1



(*c*) Hot surface facing down

(*c*) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (*b*). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$\text{Nu} = 0.27\,\text{Ra}_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}^{1/3}$ (9-20) $\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) (b) Lower surface of a hot plate (or upper surface of a cold plate)	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}^{1/4}$ (9-24)
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere	D	$\text{Ra}_D \leq 10^{11}$ ($\text{Pr} \geq 0.7$)	$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.02808\ \text{W/m} \cdot {}^\circ\text{C}}{0.15\ \text{m}}(15.86) = 2.973\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{64.2\ W}$$

Step-by-step plan natural convection

If $\dot{Q}$ must be found:

- Determine ingredients necessary for dimensionless no. (Pr, *k* and *v* at average temperature $\frac{T_s + T_\infty}{2}$)
- Determine Ra: $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$
- Choose appropriate correlation based on geometry and Ra
- Determine Nu
- Resolve *h* from it
- Fill out Newton's cooling law: $\dot{Q} = hA\Delta T$

Example 2

How large is the heat flow?

Surrounding temperature

T_∞ = 20 ˚C

T_s = 220 ˚C

L_1 = 0,20 m

L_2 = 0,35 m

Natural convection

Example 2

L_1 = 0,20 m

L_2 = 0,35 m

T_s = 220 °C

T_∞ = 20 °C

Horizontal, flat, warm plate:

Nu = 0,54 $Ra^{1/4}$ $(10^4 < Ra < 10^7)$

Nu = 0,15 $Ra^{1/3}$ $(10^7 < Ra < 10^{11})$

Characteristic length $L_c = A/p$

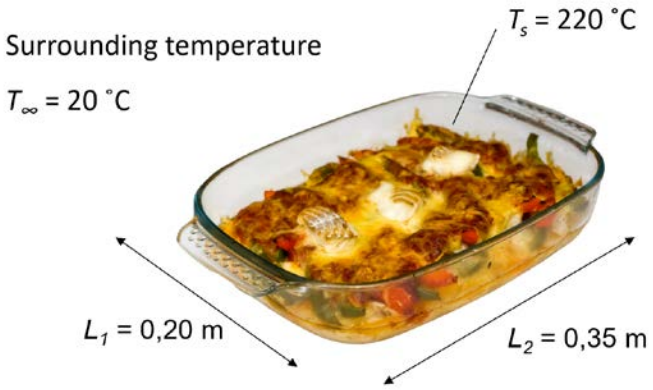
$$Ra = Gr \cdot Pr = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2} Pr$$

Properties of air (in the back of assignment module):

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m^2/s^2	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946

Example 2

(Intermediate) answers



Natural convection

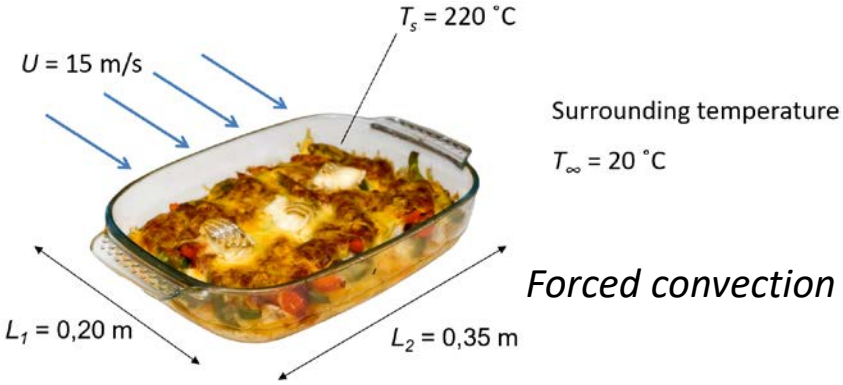
- Average temperature: 120 ˚C
- Ra = $1{,}45 \cdot 10^6$
- Nu = 18,7
- h = 9,48 W/(m^2 ·K)
- Heat flow 133 W (around 1/3 forced convection: 462 W)

Blowing from long side

- h = 33 W/(m^2 ·K)
- Heat flow 462 W

Blowing from short side:

- h = 24,9 W/(m^2 ·K)
- Heat flow 349 W



Forced convection

Example 3

Imagine: 25 W light bulb with diameter D = 0,08 m releases 22,5 W of heat. Determine T_s for T_∞ = 20 ˚C

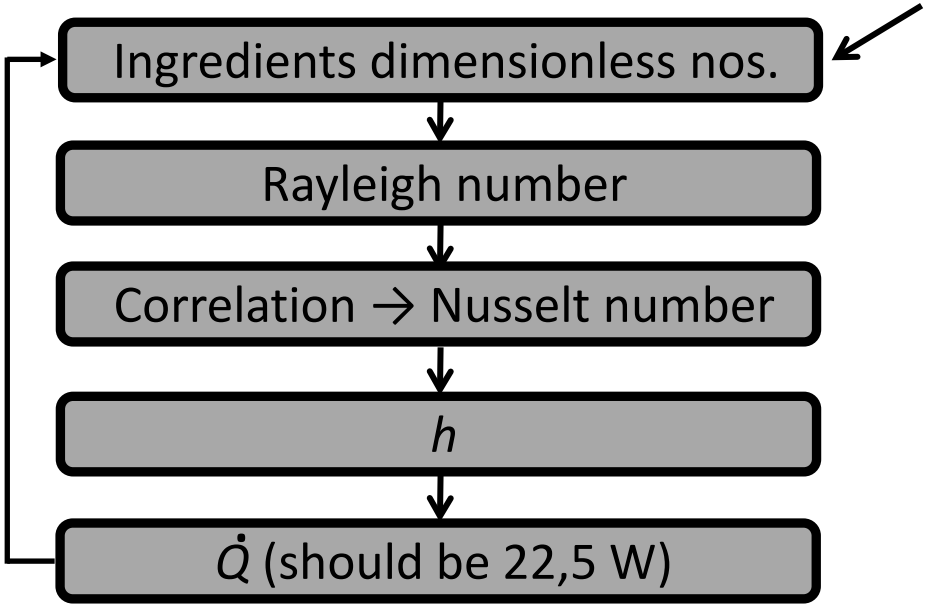
Problem:

To calculate T_s, h is necessary ($\dot{Q} = hA(T_s - T_\infty)$) but h depends on T_s (because Nu, Ra, Pr are dependent on temperature) and that is the one we are looking for…

$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$$

Example 3

Problem: T_s unknown → starting values for step-by-step plan unknown (Pr, k, ν, …)



At what avg. temp? $\frac{T_s + T_\infty}{2}$?

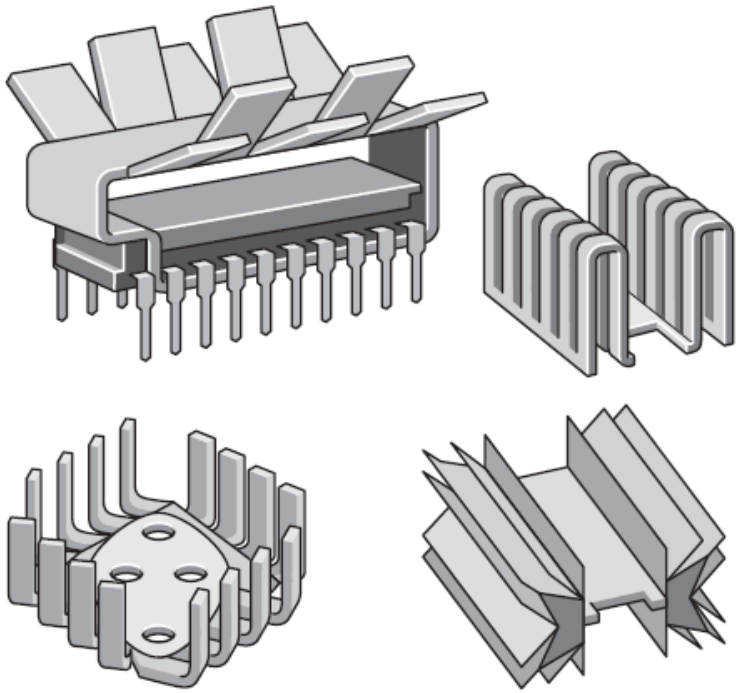
Unknown

<u>Approach: iterative solving</u>

- Choose an estimated T_s to start with
- Follow steps and check whether T_s yields the right heat flow
- Resulting heat flow ≠ 22,5 W?Adjust estimated T_s and repeat all steps till you find a T_s that (approximately) yields $\dot{Q}$ = 22,5 W

Fins

What is Purpose of the Fins on Engine Surfaces ? Why design such a thing ?



Fins are extended surfaces and they increase the surface area leading to increase in heat transfer rate. Useful in cooling applications

Summary natural convection

General (same as for forced convection)

$$\dot{Q} = hA\Delta T \quad (\mathrm{W})$$

Newton's cooling law

"Supporting" equations for *h*:

Nusselt number Nu as function of Rayleigh number Ra = Gr · Pr

Grashof number $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}$

- Forced convection: Re ↔ natural convection: Gr
- Ra and Nu just like Re based upon characteristic length of geometry (indicate using subscript)

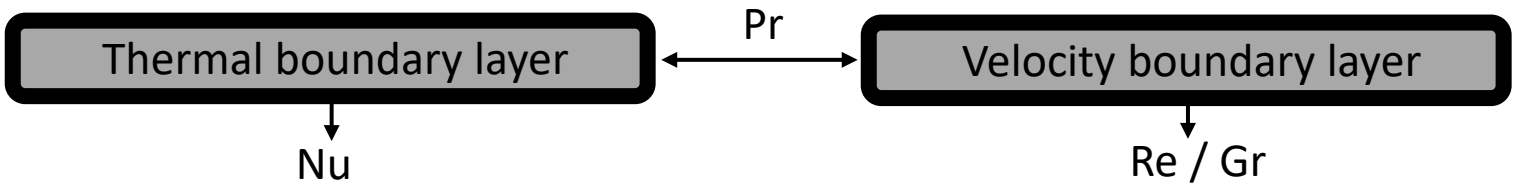
General conclusion convection

- Calculate heat flow using Newton's cooling law
- Determine heat transfer coefficient *h* in this using correlations between Nu (→ *h*) and other dimensionless nos.

⇒ step-by-step plans (+ iterative solving)

Other learning objectives convection: *flow phenomena*

- Being able to sum up what parameter are influencing *h*
- Knowing differences between laminar and turbulent ,
- Explaining how velocity and thermal boundary layers form
- Predicting/reasoning the temperature development



Reference

Chapter 9 of Heat Trasnfer, Cenegel.

Feedback Session

- Next Lectorial : Forced Convection
- Questions? Feel free to ask!