3.5 Exercises

Problem 3.1. *For incompressible flow, indicate for each of the following velocity fields whether they are steady/unsteady, and whether they satisfy mass conservation.*

(a) $u = x + y + z^2$, $v = x - y + z$, $w = 2xy + y^2 + 4$,
(b) $u = xyzt$, $v = -xyzt^2$, $w = \frac{1}{2}z^2(xt^2 - yt)$,
(c) $u = y^2 + 2xz$, $v = -2yz + x^2yz$, $w = \frac{1}{3}x^2z^2 + x^3y^4$.

Problem 3.2. *For a flow in the xy plane, the x component of velocity is given by $u = ax(y - b)$.*

(a) Find the y component of the velocity, v, for steady, incompressible flow.
(b) Explain why it is also valid for unsteady, incompressible flow.

Problem 3.3. *The x component of velocity in a steady, incompressible flow field in the xy plane is $u = A/x$. Find the simplest y component of velocity for this flow field.*

Problem 3.4. *For the following velocity fields, determine whether the continuity equation for incompressible flow is satisfied:*

(a) $\boldsymbol{u} = (ax, ay, -2az)^T$
(b) $\boldsymbol{u} = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}, 0\right)^T$

Problem 3.5. *For the two-dimensional velocity field $\boldsymbol{u} = (ax, by)^T$, and taking V with boundary S as a box defined by $0 \leq x \leq p$, $0 \leq y \leq q$, compute*

(a) $\int_V \frac{\partial u_j}{\partial x_j}\, dV$,
(b) $\int_S u_j n_j\, dS$.

Problem 3.6. *For the two-dimensional velocity field $\boldsymbol{u} = (ax, by)^T$, and taking V with boundary S as a disk defined by $0 \leq \sqrt{x^2 + y^2} \leq R$, compute*

(a) $\int_V \frac{\partial u_j}{\partial x_j}\, dV$,
(b) $\int_S u_j n_j\, dS$.

Problem 3.7. *For one-dimensional steady compresible flow ($v = w = 0$),*

(a) derive an expression for ρu if Φ, the mass flow rate per unit area is given.
(b) derive an expression for u in case the flow is incompressible.

Problem 3.8. *For one-dimensional compressible flow ($v = w = 0$) with constant velocity u,*

(a) show that $\rho(x,t) = \rho_o \sin(x - ut)$ satisfies the continuity equation.
(b) make a sketch of $\rho_o \sin(x - ut)$ at $t = 0$ and $t = 1/u$.
(c) show that $\rho(x,t) = f(x - ut)$ satisfies the continuity equation for any function f.

Problem 3.9. *It is known that the integral of the outward unit normal vector over an arbitrary but closed surface (3D) is the null-vector. This means that the integral of each component of the outward unit normal is zero. Proof this for the first component by taking a velocity field $\boldsymbol{u} = (1, 0, 0)^T$ and by using Gauss' divergence theorem.*

4.7 Exercises

Problem 4.1. *By using the integral formulation of momentum conservation, show that the law of Archimedes (287 BC - 212 BC) holds: in water which is not flowing the (upward) force on a blob of water by the surrounding water is equal to the (downward) gravity force on the blob.*

Problem 4.2. *An incompressible fluid flows steadily into a T-junction of diameter D at uniform velocity U, at the opposite outlet the fluid leaves at uniform velocity V. At the lateral exit the flow leaves at unknown uniform velocity. The pressure in the T-junction is uniform: p. Compute the force (in all directions) by the fluid on the pipe, neglect viscosity and gravity.*

Problem 4.3. *An incompressible fluid flows steadily into a pipe of diameter D at uniform velocity U and pressure p_1. At the end of the pipe is a contraction of diameter d, and the fluid leaves the contraction at uniform velocity V and pressure p_2. Compute the force (in all directions) by the fluid on the pipe, neglect viscosity and gravity.*

Problem 4.4. *Incompressible water is flowing steadily through a $180°$ elbow. At the inlet the pressure is p_1 and the cross section area is A_1, at the outlet the pressure is p_2 and the cross section area is A_2. The averaged velocity at the inlet is V_1. Find the horizontal component of the force by the fluid on the elbow, neglecting viscosity and gravity.*

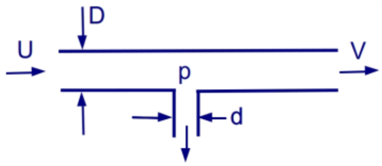
Problem 4.5. *An incompressible fluid flows steadily in the entrance region of a two-dimensional channel of height $2h$ and width w. At the entrance the presure is $p1$ and the uniform velocity is U_1. At the exit the pressure is p_2 and the velocity distribution is*

$$\frac{u}{u_{max}} = 1 - \left(\frac{y}{h}\right)^2. \tag{4.40}$$

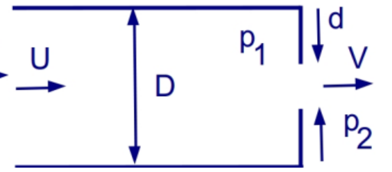
(a) Derive an expression for the maximum velocity at the downstream section.
(b) Derive an expression for the force on the walls in x-direction, neglecting gravity, and neglecting viscosity at entrance and exit.

Problem 4.6. *A small round object is tested in a wind tunnel with circular cross section with diameter D. The pressure is uniform across sections 1 and 2 and known: p_1 and p_2. At the entrance the uniform velocity is U. The velocity profile at section 2 is linear: it varies from zero at the tunnel centerline to a maximum at the tunnel wall. The viscosity effects on the wall of the wind tunnel can be neglected and the flow can be treated as incompressible.*

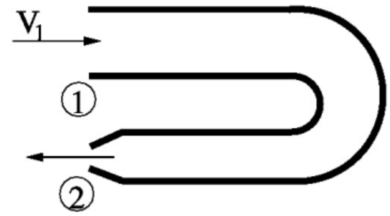
(a) Derive an expression for the mass flow rate in the wind tunnel,
(b) Derive an expression for the maximum velocity at section 2
(c) Derive an expression for the drag of the object and its supporting vane.



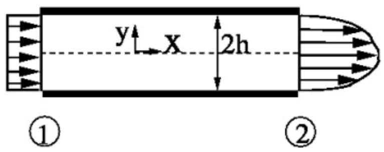
(a) Problem 4.2



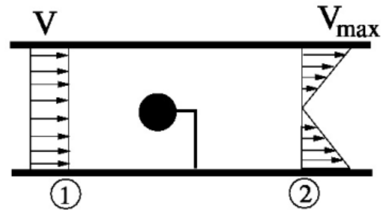
(b) Problem 4.3



(c) Problem 4.4



(a) Problem 4.5



(b) Problem 4.6