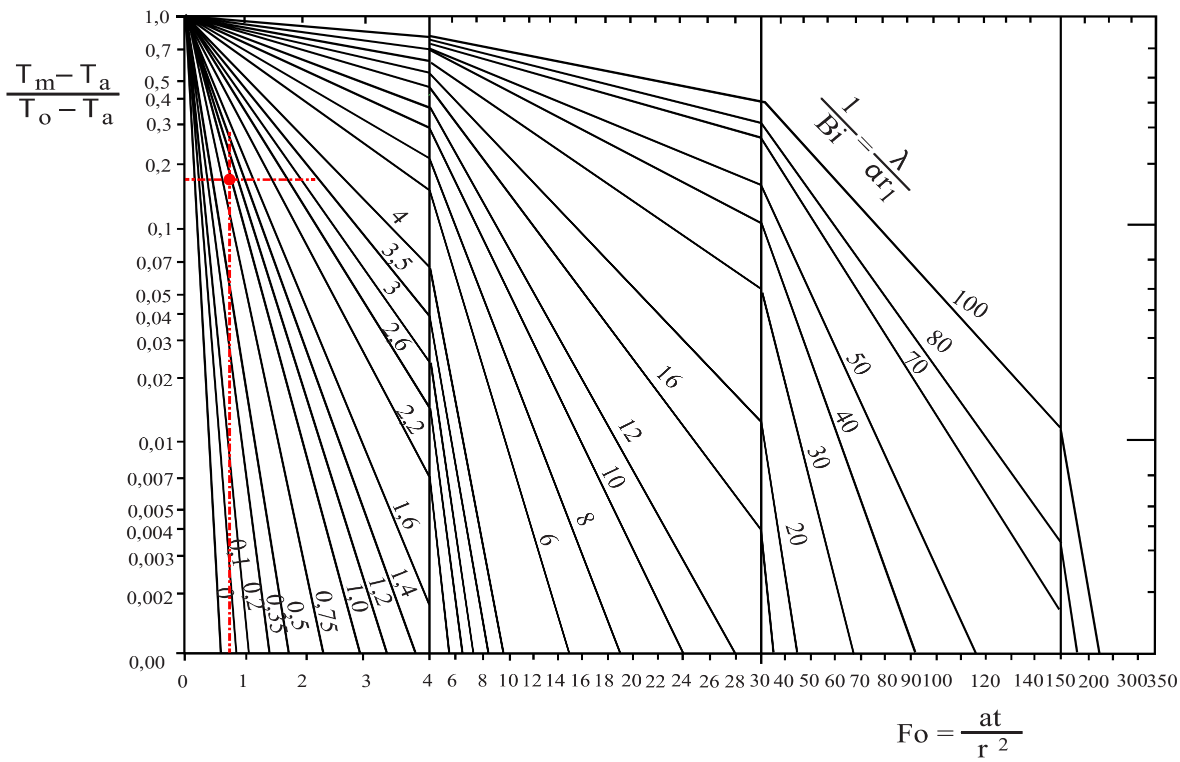
2.15 Heating and quenching of a sphere

a) *Determine T_m after 3 minutes:*

$$\frac{1}{\mathrm{Bi}} = \frac{\lambda}{\alpha \cdot r_1} = \frac{1.52\ [\mathrm{Wm^{-1}K^{-1}}]}{110\ [\mathrm{Wm^{-2}K^{-1}}] \cdot 0.015\ [\mathrm{m}]} = 0.9212 \tag{2.255}$$

$$\mathrm{Fo} = \frac{a \cdot t}{{r_1}^2} = \frac{9.5 \cdot 10^7\ [\mathrm{m^2s^{-1}}] \cdot 180\ [\mathrm{s}]}{0.015^2\ [\mathrm{m^2}]} = 0.76 \tag{2.256}$$

Using the Heisler diagram for the temperature in the centre of a sphere:



Results in:

$$\frac{T_\mathrm{m} - T_\mathrm{a}}{T_0 - T_\mathrm{a}} \approx 0.18 \tag{2.257}$$

$$T_\mathrm{m} = 0.18\,(T_0 - T_\mathrm{a}) + T_\mathrm{a} = 0.18\,(25\ [^\circ\mathrm{C}] - 200\ [^\circ\mathrm{C}]) + 200\ [^\circ\mathrm{C}] \tag{2.258}$$

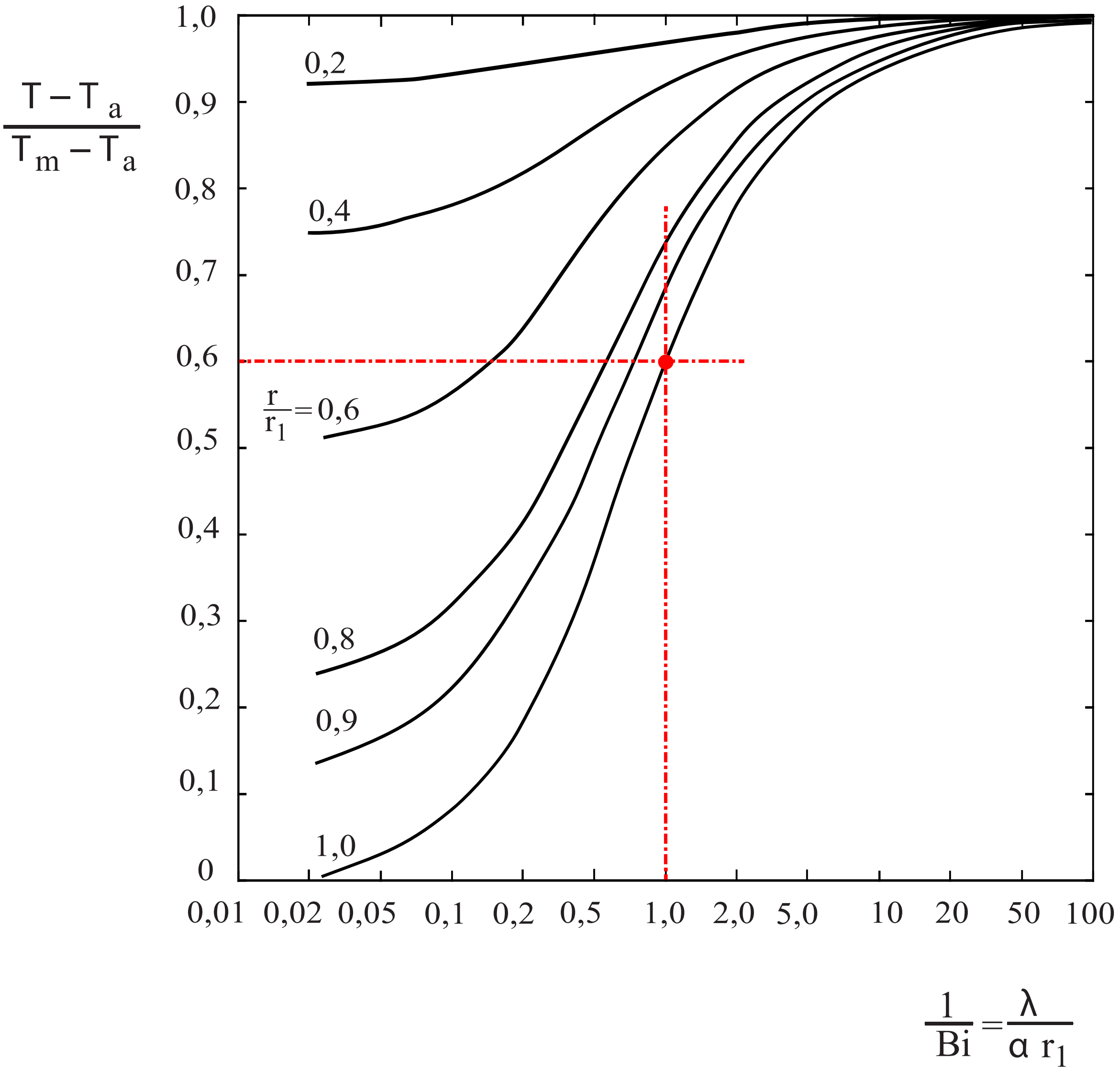
$$\boxed{\rightarrow T_\mathrm{m} = 168.5\ [^\circ\mathrm{C}]} \tag{2.259}$$

b) *Determine t_1:*

$$\frac{T - T_a}{T_m - T_a} = \frac{44.4\ [^\circ C] - 30\ [^\circ C]}{54\ [^\circ C] - 30\ [^\circ C]} = 0.6 \tag{2.260}$$

$$\frac{r}{r_1} = 1 \tag{2.261}$$

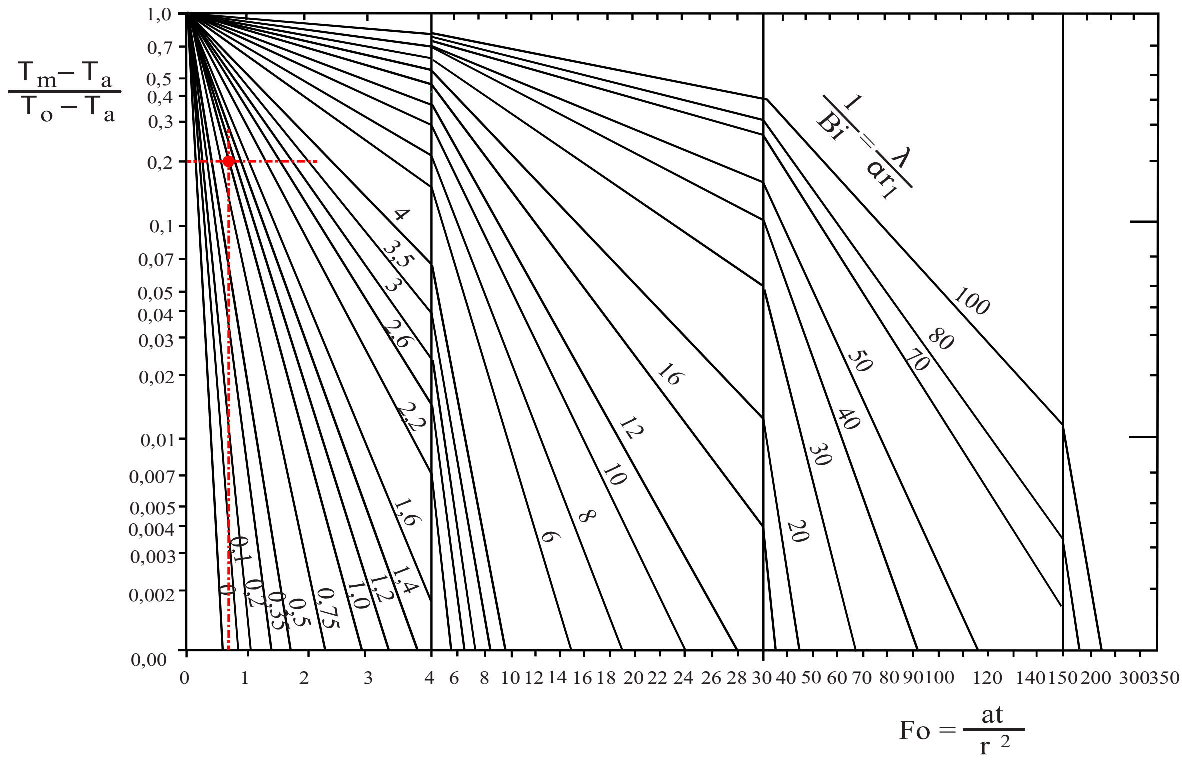
Using the Heisler diagram for the temperature distribution in a sphere to determine $\frac{1}{\mathrm{Bi}}$: (Note that this diagram is only valid when $\mathrm{Fo} > 0.2$)



$$\rightarrow \frac{1}{\mathrm{Bi}} \approx 1 \tag{2.262}$$

$$\frac{T_\mathrm{m} - T_\mathrm{a}}{T_0 - T_\mathrm{a}} = \frac{54\ [^\circ\mathrm{C}] - 30\ [^\circ\mathrm{C}]}{150\ [^\circ\mathrm{C}] - 30\ [^\circ\mathrm{C}]} = 0.2 \tag{2.263}$$

Using the Heisler diagram for the temperature in the centre of a sphere to determine Fo:

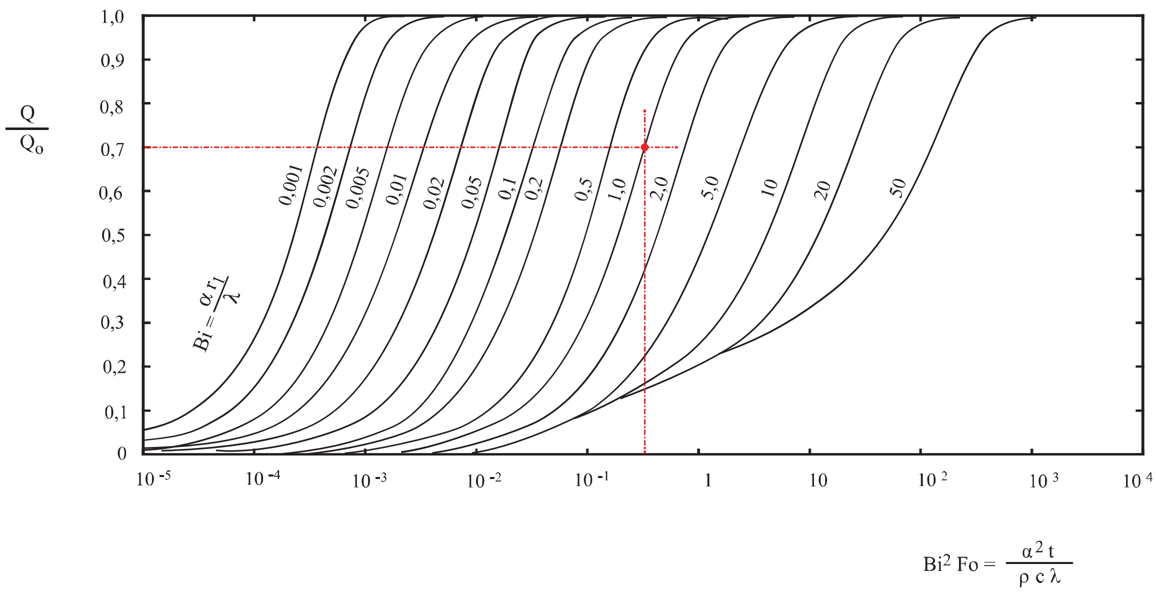


$$\boxed{\rightarrow \mathrm{Fo} \approx 0.7 \ \rightarrow t_1 \approx 165.79\ [\mathrm{s}]} \tag{2.264}$$

c) *Determine the dissipated heat Q at time instant t_1.*

$$\mathrm{Bi}^2 \cdot \mathrm{Fo} = 0.7 \tag{2.265}$$

Using the Heisler diagram for the heat loss of a sphere:



$$\frac{Q}{Q_o} \approx 0.92 \tag{2.266}$$

$$Q \approx 0.92 \cdot \left(\rho \cdot \frac{4}{3} \cdot \pi \cdot {r_1}^2 \cdot c_{\mathrm{p}} \cdot (T_0 - T_a) \right) = \tag{2.267}$$

$$Q = 0.92 \cdot \left(1.45 \cdot 10^3 \ [\mathrm{kg/m^3}] \cdot \frac{4}{3} \cdot \pi \cdot 0.015^2 \ [\mathrm{m^2}] \cdot 880 \ [\mathrm{J/kgK}] \cdot (150 - 30) \ [\mathrm{K}] \right) \tag{2.268}$$

$$\boxed{\rightarrow Q = 132.76 \ [\mathrm{kJ}]} \tag{2.269}$$