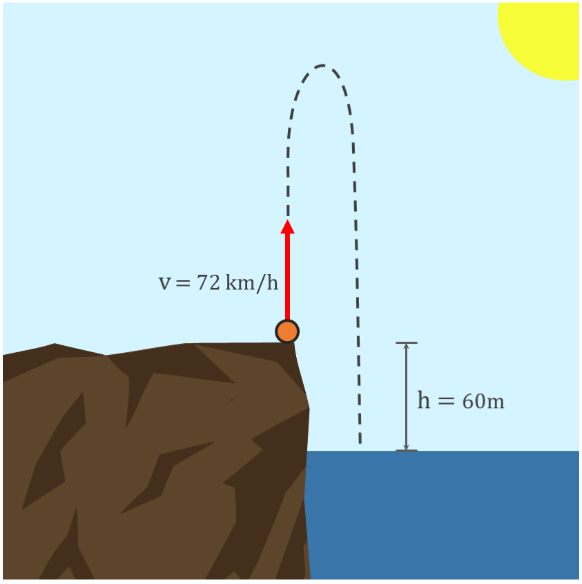
Ball Thrown off a Cliff



A ball is thrown vertically up with an initial vertical velocity of 72 km/h at the edge of a 60 meter high cliff. What is the total time t_{end} in seconds that the ball is in the air?

Neglect air resistance and take $g = 10$ m/s^2.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a\int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (at + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (at + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Initial vertical velocity: $v_{y,0} = 72$ km/h $= 20$ m/s
Initial height of the ball (with respect to the cliff): $y_0 = s_{y,0} = 0$ m
Gravitational constant: $g = 10$ m/s^2

Solution:

For the vertical displacement in y-direction, Equation (6) results in:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \tag{7}$$

At the instant the ball reaches the sea, its vertical displacement is $y = -60$ m. The acceleration on the ball is the gravitational acceleration: $a_y = -g$. Combining this into Equation (7) yields an equation for the total time of the ball to reach the sea:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \quad \Rightarrow \quad y(t) = -\frac{1}{2}gt^2 + v_{y,0}t \tag{8}$$

Inserting $y(t_{\text{end}}) = -60$ m, $g = 10$ m/s^2 and $v_{y,0} = 20$ m/s results in:

$$-60 = -\frac{1}{2} \cdot 10 \cdot t_{\text{end}}^2 + 20 \cdot t_{\text{end}} \quad \Rightarrow \quad t_{\text{end}}^2 - 4 \cdot t_{\text{end}} - 12 = 0 \tag{9}$$

$$(t_{\text{end}} + 2)(t_{\text{end}} - 6) = 0 \quad \Rightarrow \quad t_{\text{end}} = -2 \text{ s} \quad \vee \quad t_{\text{end}} = 6 \text{ s} \tag{10}$$

Since t_{end} cannot be negative, the final answer is $t_{\text{end}} = 6$ s.