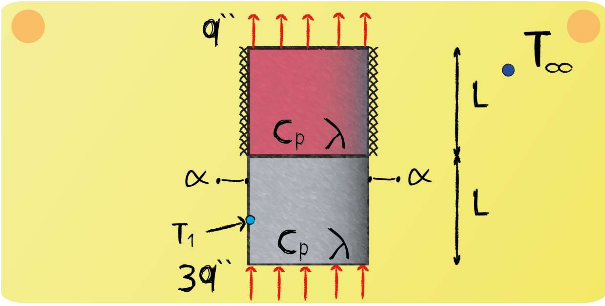
Lecture 14 - Question 9



Two cubes are placed on top of each other. The grey cube is experiencing steady-state heat transfer, which has a constant heat flux entering, but is loosing heat at the side surfaces at a constant rate as well. The pink cube is loosing heat at a constant rate at the top surface, but gaining heat by conduction from the grey cube. The side surfaces of this cube are fully adiabatic. Derive the differential equation that expresses the change in temperature of the pink cube over the course of time. Assume the temperature to be homogeneous and neglect radiation.

Energy balance:

$$\frac{\partial U}{\partial t} = \sum \dot{Q}_{in} - \sum \dot{Q}_{out}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{\partial U}{\partial t} = \rho \cdot c_p \cdot L^3 \cdot \frac{dT_w}{dt}$$

The internal energy of the control volume can be described as: $U = m \cdot c_p \cdot T$.

Heat fluxes:

$$\sum \dot{Q}_{in} = 3q''L^2 - 4\alpha L^2 (T_1 - T_\infty)$$

$$\sum \dot{Q}_{out} = q''L^2$$