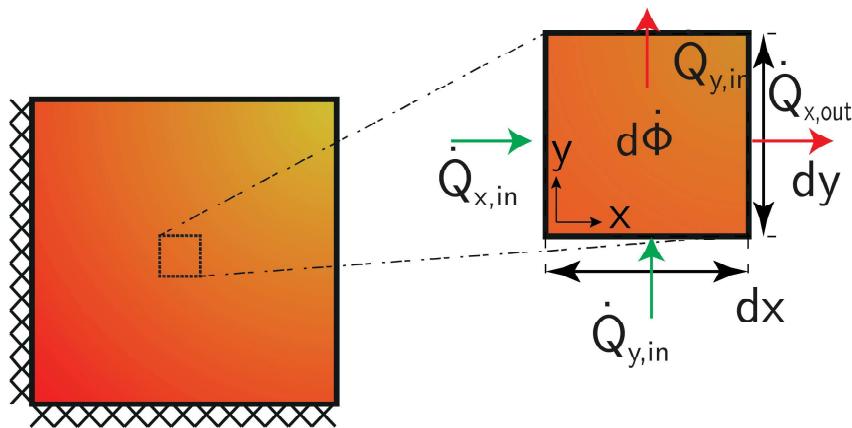


EB - Cond. - IE 1

Provide the governing two-dimensional energy balance in Cartesian coordinates to describe the heat conduction problem in the body with heat source $\dot{\Phi}'''$. Assume steady-state conditions.

1 Setting up the balance:

To derive the governing equation, an energy balance around a two-dimensional infinitesimal element is needed. Heat is conducted in and out of the element, while internal heat is generated.



Hence, the steady-state energy balance reads:

$$0 = \dot{Q}_{x,in} - \dot{Q}_{x,out} + \dot{Q}_{y,in} - \dot{Q}_{y,out} + d\dot{\Phi}.$$

2 Defining the elements within the balance:

The internal heat generation is written as:

$$d\dot{\Phi} = \dot{\Phi}''' \cdot dx \cdot dy \cdot dz.$$

The incoming rates of heat are described by Fourier's law of heat conduction:

$$\dot{Q}_{x,in} = -\lambda dy dz \frac{\partial T}{\partial x},$$

and:

$$\dot{Q}_{y,in} = -\lambda dx dz \frac{\partial T}{\partial y}.$$

The outgoing fluxes are approximated by the use of the Taylor series expansion:

$$\begin{aligned}\dot{Q}_{x,out} &= \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx \\ &= -\lambda dy dz \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(-\lambda dy dz \frac{\partial T}{\partial x} \right) \cdot dx,\end{aligned}$$

and:

$$\begin{aligned}\dot{Q}_{y,\text{out}} &= \dot{Q}_{y,\text{in}} + \frac{\partial \dot{Q}_{y,\text{in}}}{\partial y} \cdot dy \\ &= -\lambda dx dz \frac{\partial T}{\partial y} + \frac{\partial}{\partial y} \left(-\lambda dx dz \frac{\partial T}{\partial y} \right) \cdot dy.\end{aligned}$$

3 Inserting and rearranging:

$$0 = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \dot{\Phi}'''.$$