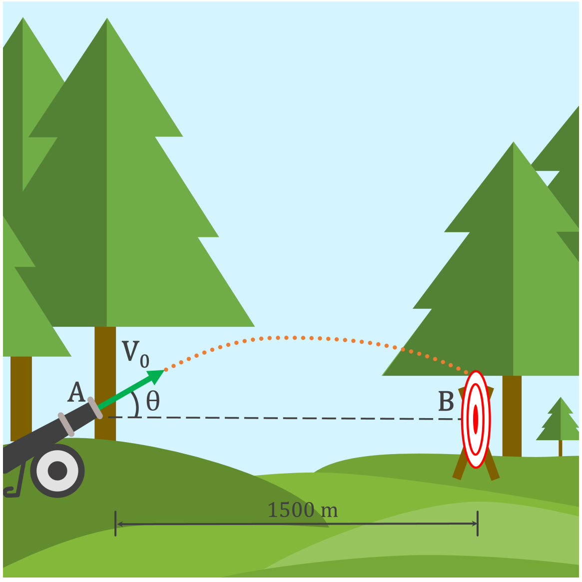
Bullet on Target



A cannon fires a bullet from A toward a target B. Find an expression for the time t_{end} it takes for the bullet to reach the target, in terms of d, v_0 and θ.

The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is $d = 1500$ m. Neglect air resistances and assume that the bullet is directed along the vertical centreline of the target. Take $g = 10$ m/s^2.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Distance A-B: $s_{\mathrm{AB}} = 1500$ m
Gravitational acceleration: $g = 10$ m/s^2
Initial velocity: $v_0 = 900$ m/s
Target diameter: $D = 2$ m

Solution:

Filling in Equation (6) gives an relation for the x-position with respect to time. Where $a = x_0 = 0$, since there is no acceleration in the x-direction and the coordinate system is chosen at the end of the cannon barrel.

$$x(t) = v_{0,x} t_{\mathrm{end}} = s_{\mathrm{AB}} \quad \Rightarrow \quad v_0 t_{\mathrm{end}} \cos\theta = s_{\mathrm{AB}} \tag{7}$$

Rewriting gives a relation for the time t_{end} with respect to θ and v_0.

$$v_0 t_{\mathrm{end}} \cos\theta = s_{\mathrm{AB}} \quad \Rightarrow \quad t_{\mathrm{end}} = \frac{s_{\mathrm{AB}}}{v_0 \cos\theta} \tag{8}$$