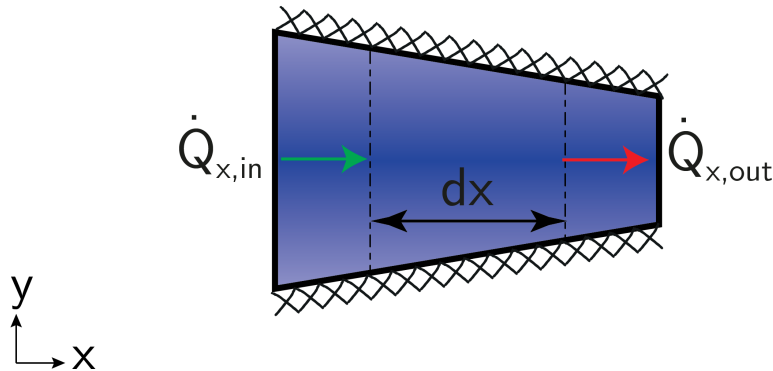


EB - Cond. - IE 4

Setup the energy balance to calculate the one-dimensional steady-state temperature profile inside the cone.

1 Setting up the balance:

To derive the one-dimensional steady-state temperature profile, an energy balance around an infinitesimal element is needed. Heat is conducted in and out of the element.



Hence, the steady-state balance reads:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} = 0,$$

the sum of the in- and outgoing fluxes should equal zero, because of steady-state conditions.

2 Defining the elements within the balance:

The ingoing flux described by use of Fourier's law:

$$\dot{Q}_{x,in} = -\lambda A(x) \frac{\partial T}{\partial x},$$

and the outgoing flux is approximated by the use of the Taylor series expansion.

$$\begin{aligned} \dot{Q}_{x,out} &= \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx \\ &= -\lambda A \frac{\partial T}{\partial x} - \lambda A(x) \frac{\partial T}{\partial x} + \left(-\lambda A(x) \frac{\partial T}{\partial x} \right) \cdot dx. \end{aligned}$$

3 Inserting and rearranging:

$$-\lambda A \frac{\partial T}{\partial x} + \lambda A \frac{\partial T}{\partial x} - \frac{\partial}{\partial x} \left(-\lambda A \frac{\partial T}{\partial x} \right) dx = 0,$$

λ is constant, hence cancel out:

$$A(x) \frac{\partial^2 T}{\partial x^2} = 0.$$



4 Defining the boundary and/or initial conditions:

$$T(x = 0) = T_1,$$

and

$$T(x = L) = T_2.$$

The boundary conditions above describe that the temperature of the body equals T_1 on the left side and T_2 on the right side, as can be seen in the sketched situation.