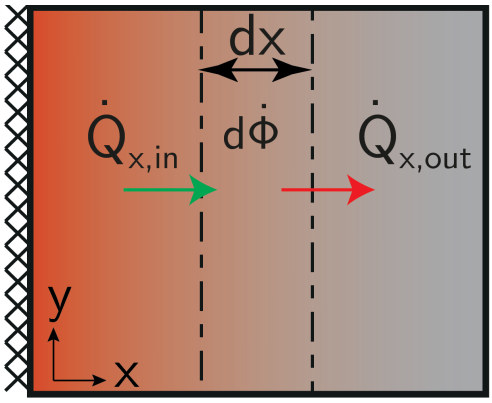
EB - Cond. - IE 15

Set up the energy balance for a one-dimensional steady-state heat transfer through the wall, which is adiabatic on the left-hand side with the cross-sectional area A and give the corresponding boundary conditions. There is a source $\dot{\Phi}'''$ in the wall.



Energy Balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} + d\dot{\Phi} = 0$$

Heat Fluxes:

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

$$d\dot{\Phi} = d\dot{\Phi} = \dot{\Phi}''' A dx$$

The in and outgoing flux should equal each other and are characterized by conductive heat transfer. The outgoing flux can be approximated by use of the Taylor series expansion.

Boundary conditions:

$$\frac{\partial T(x=0)}{\partial x} = 0$$

$$T(x = L) = T_1$$

The first boundary condition describes that the temperature gradient on the left should be zero. This due to the fact that heat transfer to the environment at the left side is zero, because of the insulation. The second describes that the temperature of the wall equals T_1 on the right side.