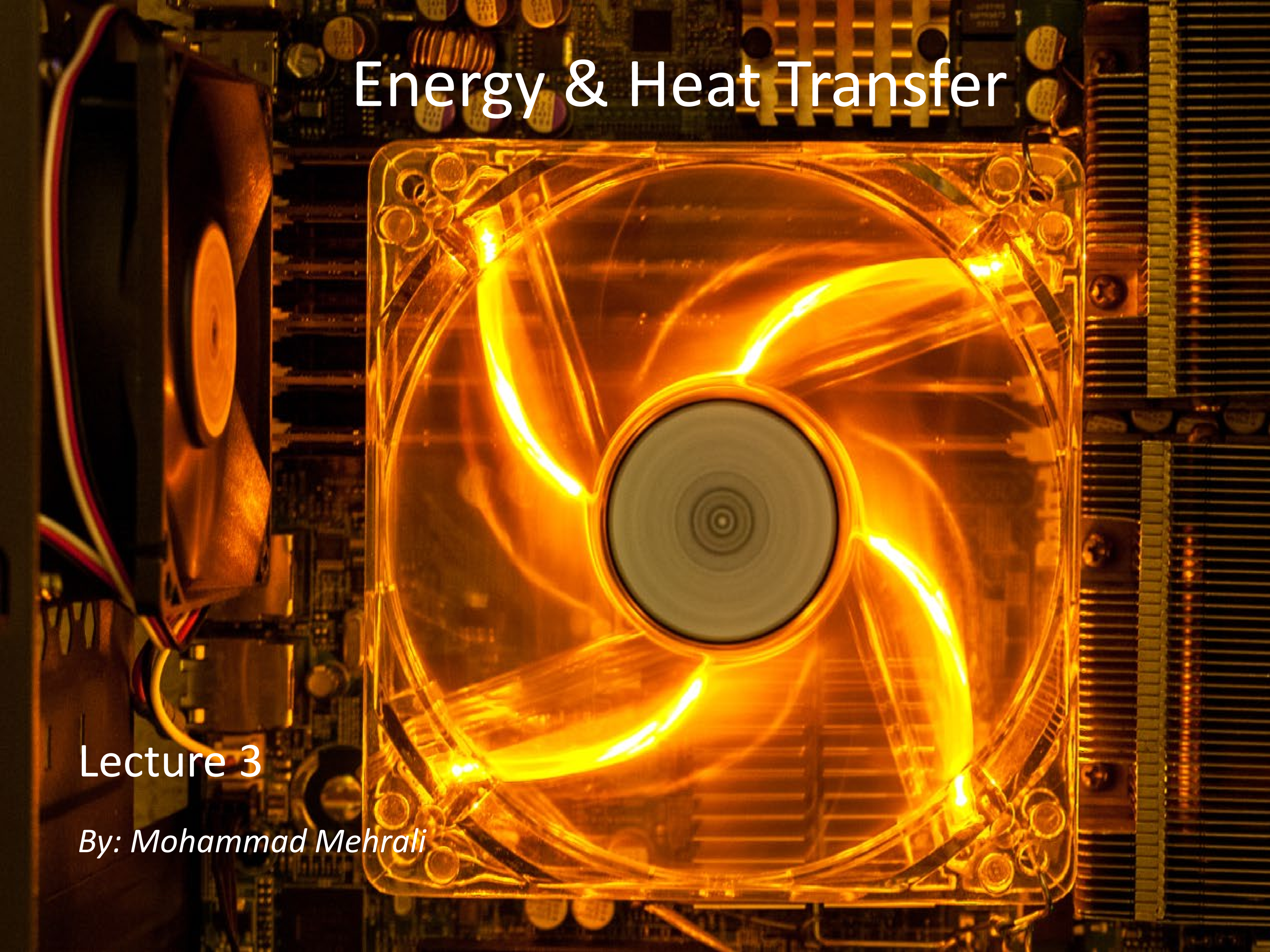
Energy & Heat Transfer



Lecture 3

By: Mohammad Mehrali

Recap of lecture 2

- **Efficiency** $\eta = \dfrac{\text{useful work}}{\text{inputted energy}} = \dfrac{\text{useful power}}{\text{inputted power}}$

  Energy is always conserved!

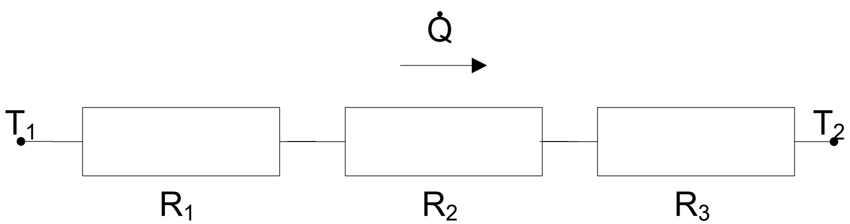
- **Temperature Differnce is the driving force for the transfer of heat**

- **Heat transfer rate**: $\dot{Q}$ (W) **;** **Heat flux**: $\dot{q} = \dot{Q}/A$ $(\mathrm{W/m^2})$

RECAP OF LECTURE 2

- **Conduction**
- **Fourier conduction equation for different geometries**
  - Plane surface: $\dot{Q} = -k\,A\dfrac{T_2 - T_1}{x_2 - x_1} = \dfrac{T_1 - T_2}{R}$ with $R = \dfrac{\Delta x}{kA}$ $\left(\dfrac{\mathrm{K}}{\mathrm{W}}\right)$
  - Cylindrical tube: $\dot{Q} = \dfrac{T_1 - T_2}{R}$ with $R = \dfrac{\ln(\frac{D_2}{D_1})}{2\pi L k}$
  - Spherical shell: $\dot{Q} = \dfrac{T_1 - T_2}{R}$ with $R = \dfrac{D_2 - D_1}{2\pi k D_1 D_2}$
- **Building resistance networks**

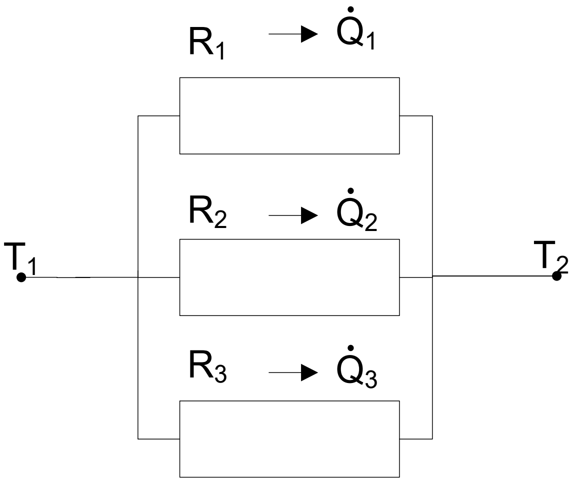
Series Resistors



$$R_{tot} = \sum_i R_i$$

(Add Resistors)

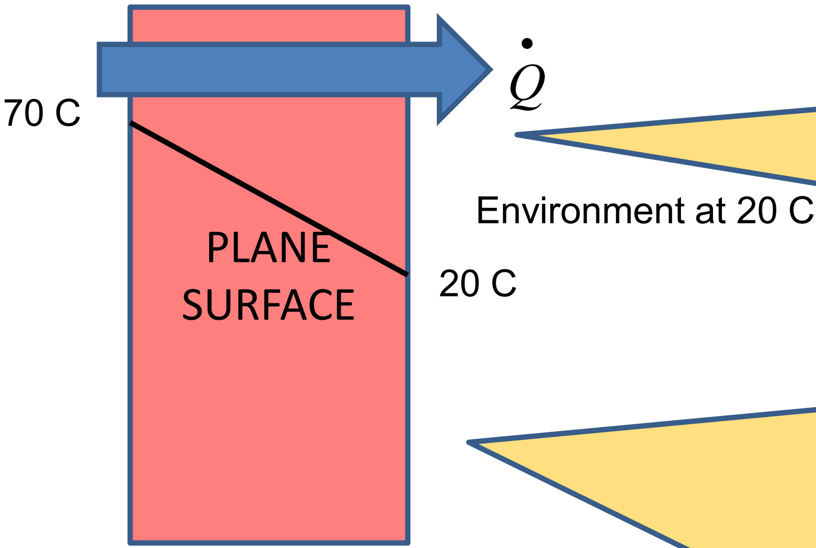
Parallel Resistors



$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

(Add Heat Flows)

WHY HEAT TRANSFER



Engineers are interested to know the rate at the which heat was transferred. In other words, Rate of Heat transfer $\dot{Q}$ is of importance in engineering applications.

$\dot{Q}$

Depends on mode of heat transfer and various factors.

In the case of conduction

$\dot{Q}$

Depends on

1> Temperature Difference

2> Thermal conductivity of the object

3> Surface area

4> Thickness of the object

$$\dot{Q} = -k\,A\frac{T_2 - T_1}{x_2 - x_1} = \frac{T_1 - T_2}{R} \quad \text{with} \quad R = \frac{\Delta x}{kA} \qquad \left(\frac{\mathrm{K}}{\mathrm{W}}\right)$$

Learning objectives lecture 3

- **Defining Convective Heat Transfer**
- Convective Heat Transfer Types
- Heat Transfer Rate in Convection
  - Newton's Law
  - Convection Resistance
  - Nusselt Number
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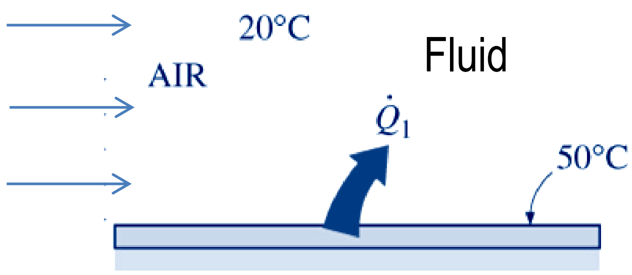
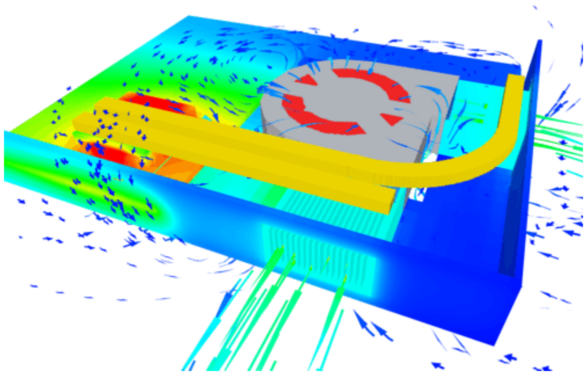
Convection:

Is the mode of energy transfer between a **solid surface** and the **adjacent liquid or gas (Fluid)** that is in motion, and it involves the combined effects of ***conduction and fluid motion.***





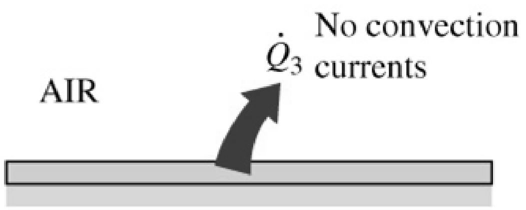
Convection and conduction are similar in that both mechanisms require the presence of a material medium.

Flowing Fluid removes heat from the hot surface
Fluid: flowable medium (gas / liquid)

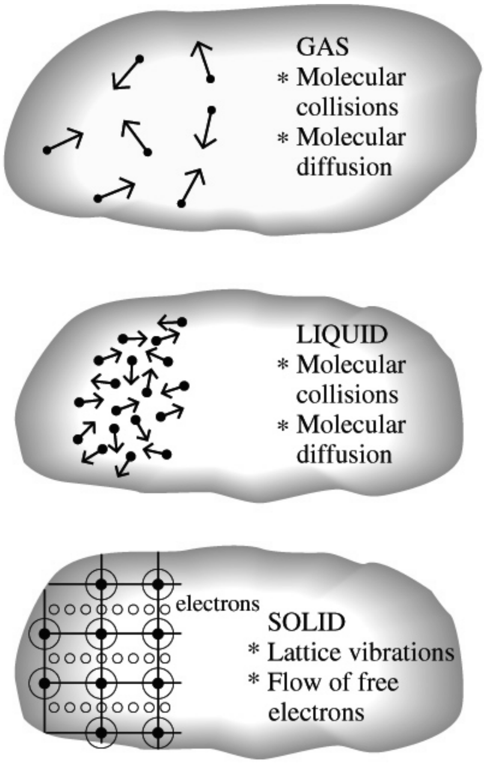
Convective heat transfer

Conduction: heat transfer between molecules

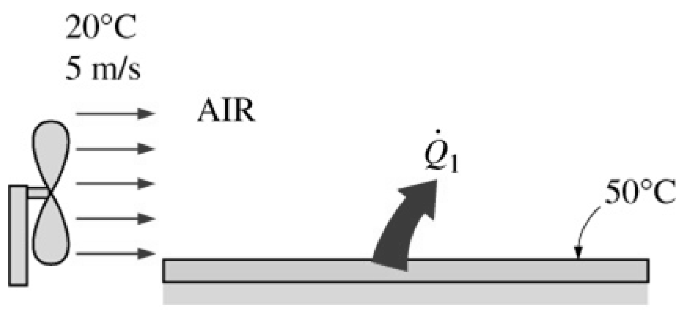
("bulk speed" equals zero)



- Stationary air conducts heat away from the surface.



Convection:



- Flowing air removes more heat
- Conduction stil exists, supply "fresh" molecules and discharge "heated" ones accelerates process

Fluid: flowable medium (gas / liquid)

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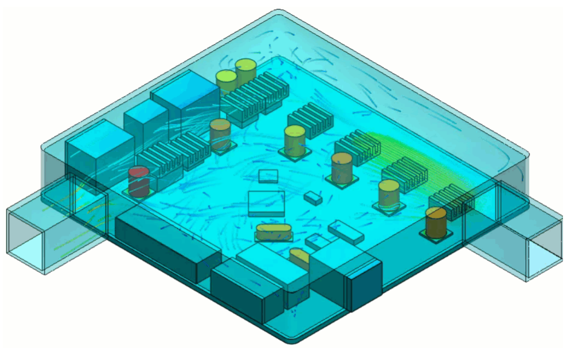
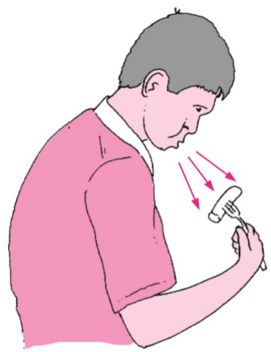
Convective heat transfer types

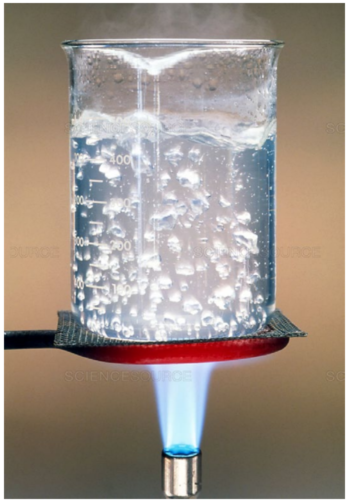
Forced convection

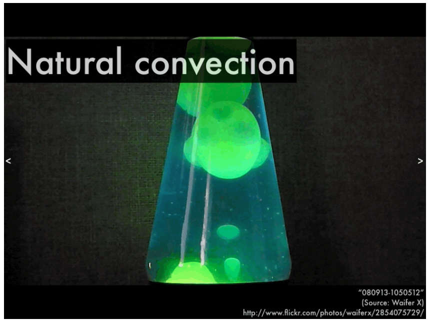




Imposed flow (by pump, fan, …)

Natural/free convection





Temperature difference itself starts the flow

General: flow velocity and heat transfer rates are larger for forced convection

LEARNING OBJECTIVES LECTURE 3

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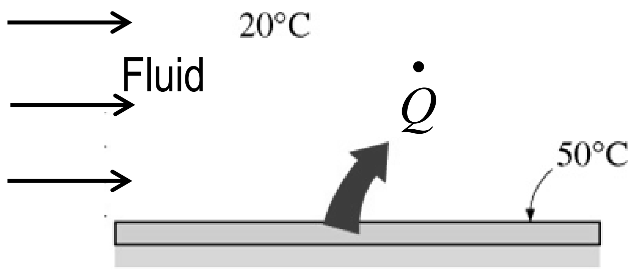
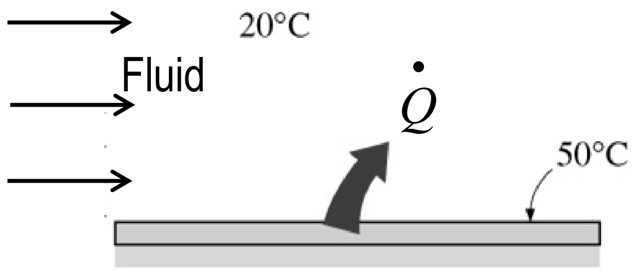
Heat Transfer Rate in Convection



Steady State Heat Transfer

HEAT TRANSFER RATE IN CONVECTION



In the case of Convection

$\dot{Q}$

Depends on :

1) Temperature Difference
2) convection heat transfer coefficient
3) Surface area of the object

Newton's Law:

$$\dot{Q} = h.A.\Delta T\,(\mathrm{W})$$

h is the **"convection heat transfer coefficient"** which basically takes care of various effects of fluid properties and flow properties

Unit: $\dfrac{\mathbf{W}}{\mathbf{m^2 \cdot K}}$

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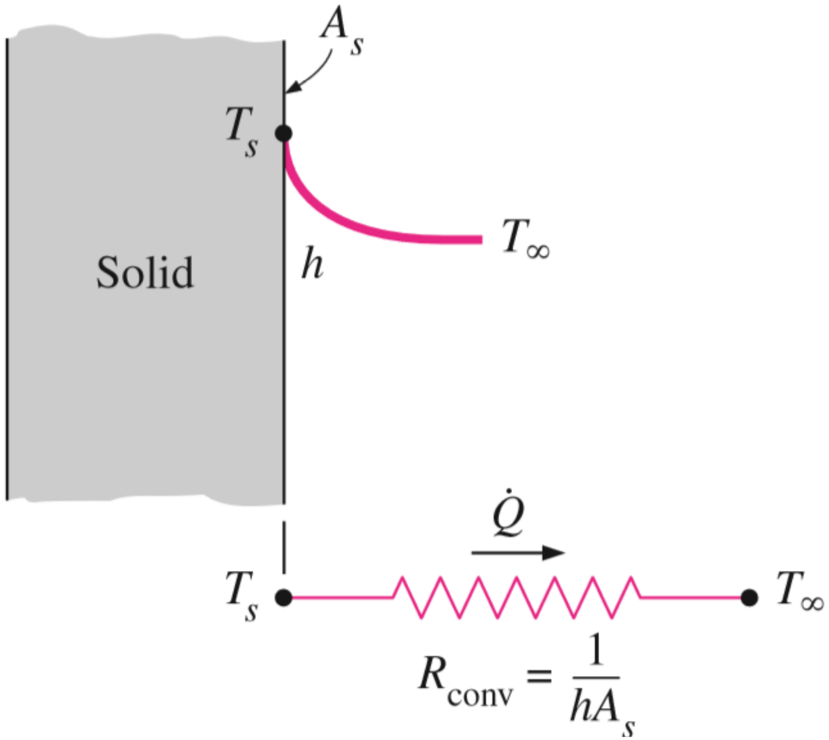
CONVECTION REISTANCE

$$\dot{Q} = hA\Delta T = \frac{\Delta T}{\frac{1}{hA}} \text{ with } \Delta T = T_S - T_\infty$$

$$\Rightarrow \dot{Q} = \frac{\Delta T}{R_{conv}}$$



Where **convection resistance**:

$$R_{conv} = \frac{1}{hA} \left(\frac{\mathrm{K}}{\mathrm{W}}\right)$$

Remember $$R_{Cond}, plane = \frac{\Delta x}{k\,A} \quad \left(\frac{\mathrm{K}}{\mathrm{W}}\right)$$

Example : The heat loss through windows

Given :

Area : 0.8m-high and 1.5m-wide

Thermal conductivity of Glass: **$k = 0.78$ W/m.˚C**

The room temperature: **20˚C**

The temperature of the outdoor: **-10˚C**

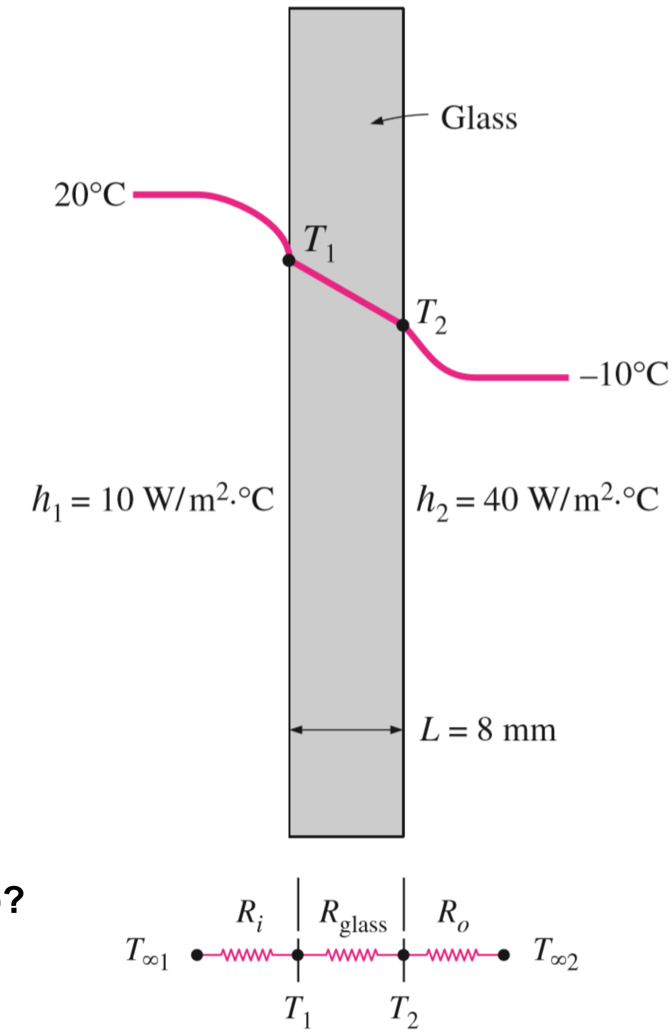
Convection heat transfer coefficient (Inside): **$h_1 = 10 W/m^2.˚C$**

Convection heat transfer coefficient (outside): **$h_2 = 40 W/m^2.˚C$**

Asked:

Determine the heat loss?

Determine the inner surface temperature of the window glass (T_1)?



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DIMENSIONLESS NUMBERS

HEAT TRANSFER DIMENSION LESS NUMBER

REYNOLDS NUMBER

STANTON NUMBER

NUSSET NUMBER

GRASHOFF NUMBER

BIOT NUMBER

FOURIER NUMBER

PECLET NUMBER

RAYLEIGHS NUMBER

GRAETZ NUMBER

LEWIS NUMBER

PRANDTL NUMBER

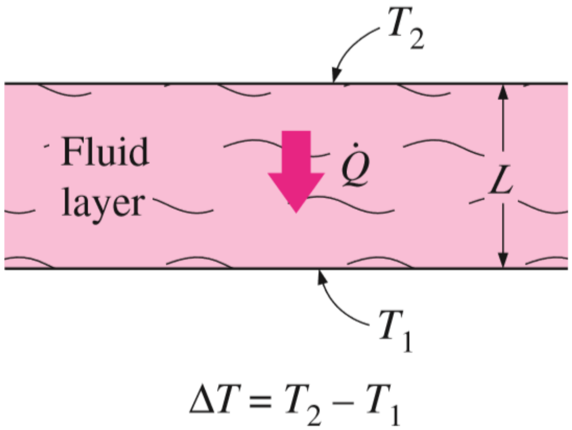
DIMENSIONLESS NUMBERS

- Dimensionless numbers allow for comparisons between very different systems.
- Dimensionless numbers tell you how the system will behave.
- Many useful relationships exist between dimensionless numbers that tell you how specific things influence the system.
- Dimensionless numbers allow you to solve a problem more easily.
- When you need to solve a problem numerically, dimensionless groups help you to scale your problem.

NUSSELT NUMBER

$$\dot{q}_{\text{conv}} = h\Delta T$$

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L}$$



Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

- **The larger the Nusselt number, the more effective the convection.**
- **A Nusselt number of Nu=1 for a fluid layer represents heat transfer across the layer by pure conduction.**

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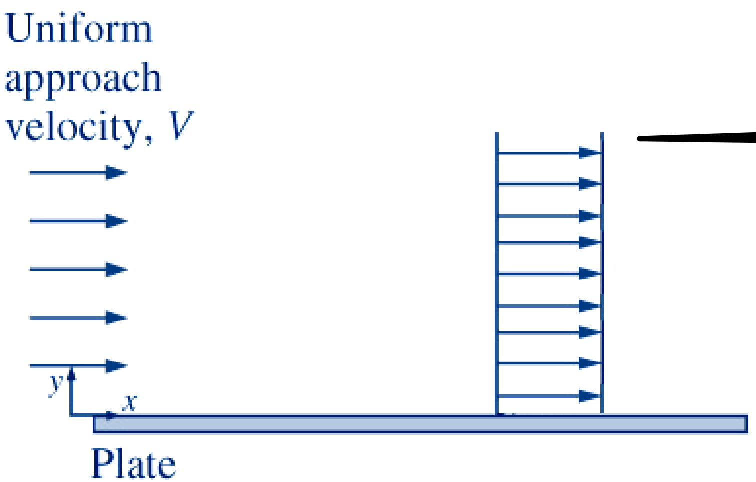
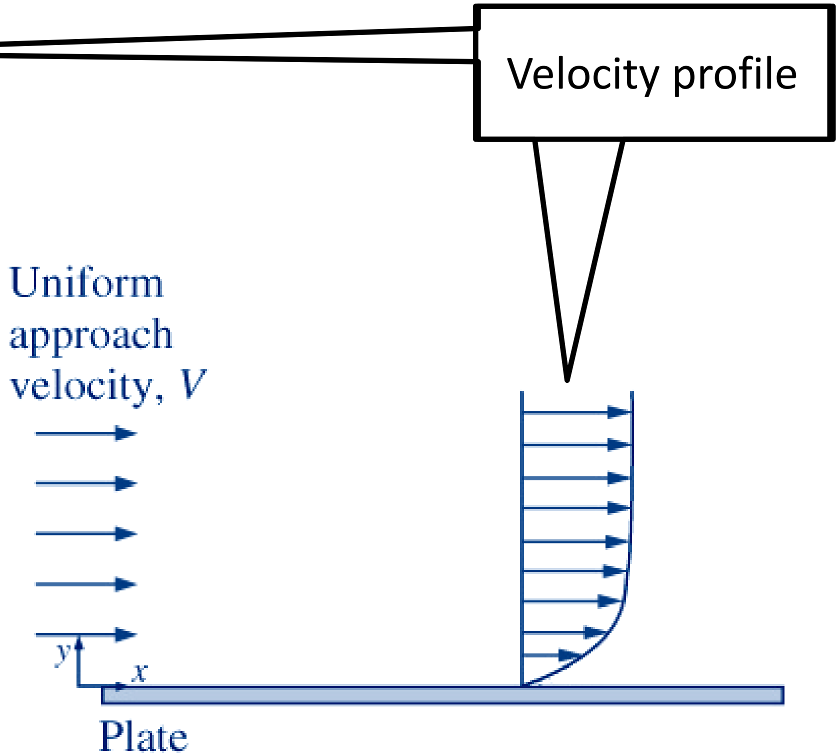
Flow Parameters

First consider forced convection over a flat plate (2D)

External Flow



Idealized (non physical)

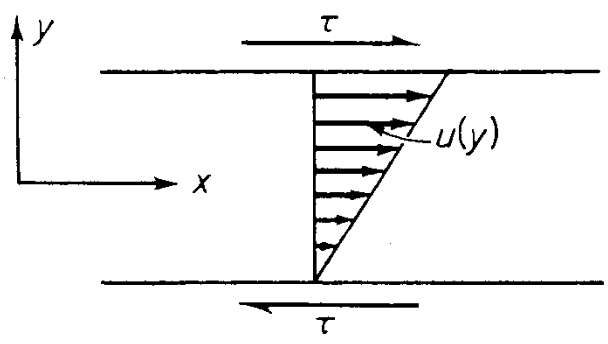


Reality

Why the velocity profile is not uniform ?

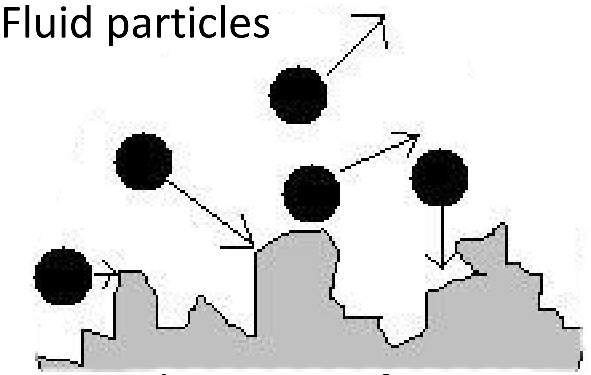
- **No slip condition** (velocity zero at surface)
- **Viscosity**

Viscosity μ: “stickiness”, resistance to deformation (shear)



	μ (Pa·s)
Oil	0.10 - 0.86
Water	0.0010
Air	0.000018
Peanut butter	150 – 250

Fluid particles



Close-up surface

On small scale all surfaces are rough
→ fluid doesn’t flow there

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Convective Heat Transfer Coefficient

h is complexly related to fluid properties and fluid flow parameters. Experiments, formulations and research have lead to grouping these parameters as follows as. **This is particularly for the case of forced convection:**

$$\frac{hL}{k} = a\left(\frac{\rho U L}{\mu}\right)^b \left(\frac{\mu c_p}{k}\right)^c$$

$$\mathrm{Nu} = a \cdot \mathrm{Re}^b \cdot \mathrm{Pr}^c$$

With *a, b, c* constants dependent on geometry and flow type

Proof follows from laws of conservation of mass, momentum and energy

Nusselt Number : $\mathrm{Nu} = \frac{hL}{k}$

Reynolds number: $\mathrm{Re} = \frac{\rho U L}{\mu}$

Prandtl number: $Pr = \frac{\mu c_p}{k}$

Parameters:

Flow velocity : U (m/s)

Thermal conductivity : k (W/m.k)

Density : ρ (kg/m^3)

Distance from leading edge/length: x, L (m)

Dynamic viscosity : μ (N · s/m^2)

Specific heat capacity : Cp (J/kg · K)

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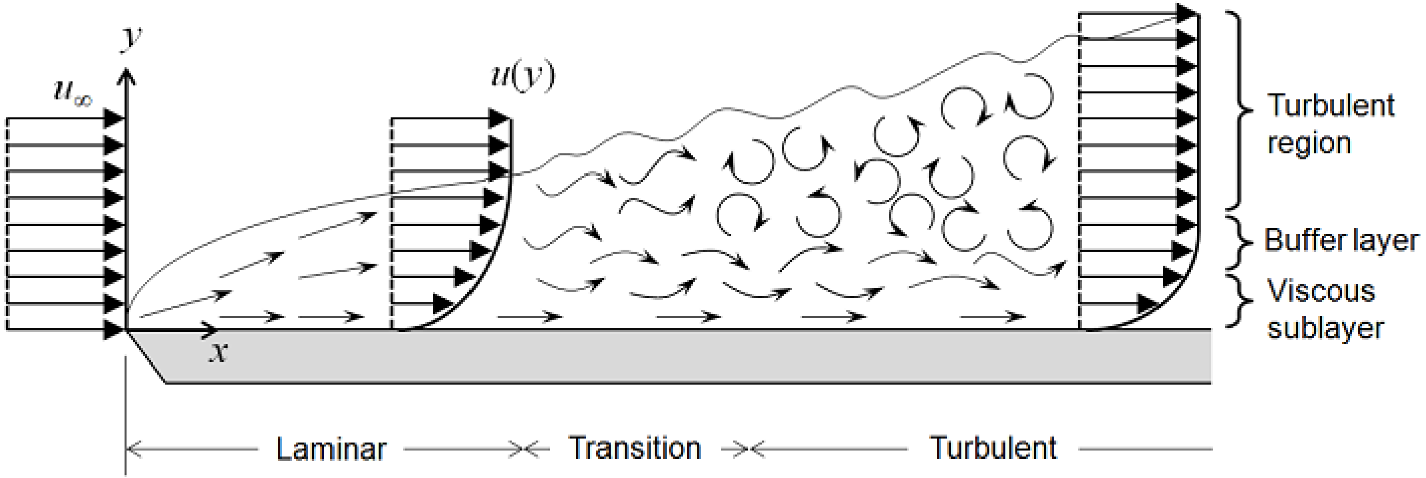
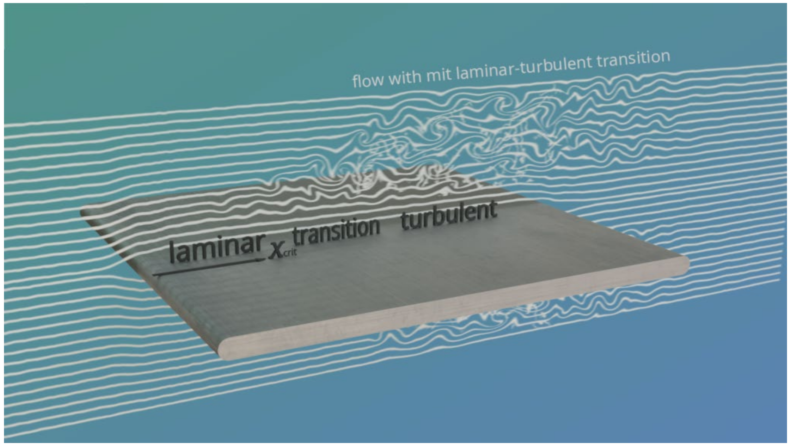
Laminar and Turbulent Flow

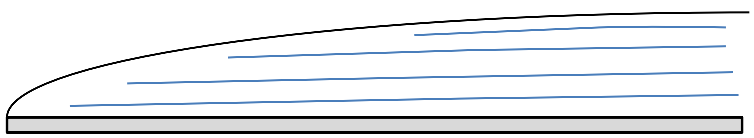




Influence of Reynolds number

Low Re: Laminar flow

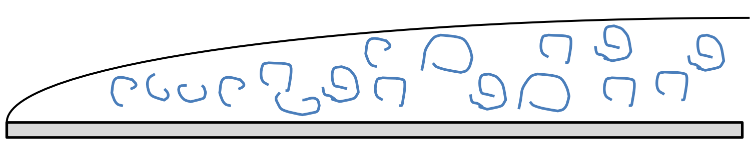
- Viscosity dominates momentum → neatly 'layered' flow



$$Re = \frac{\rho U L}{\mu}$$

High Re: turbulent flow

- Momentum dominates viscosity → flow starts to swirl (**chaos!**)



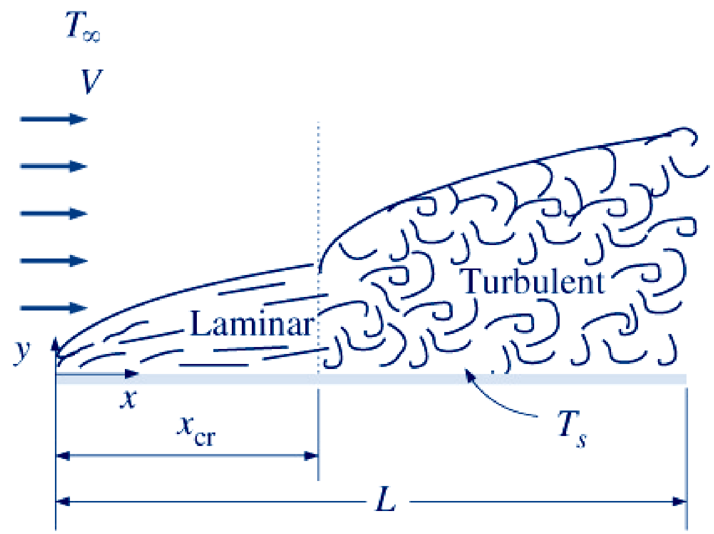
Turbulence: fluid particles have individual irregular deviations from the mean "bulk speed" because of high momentum

Influence of Reynolds number

LAMINAR VS. TURBULENT

Turbulent boundary layer has **higher *h***:

- Heat spreads better through chaotic mixing of particles



Laminar or turbulent?

- Close to leading edge always laminar
- transition laminar → turbulent
- Here only extremes: either totally laminar or totally turbulent

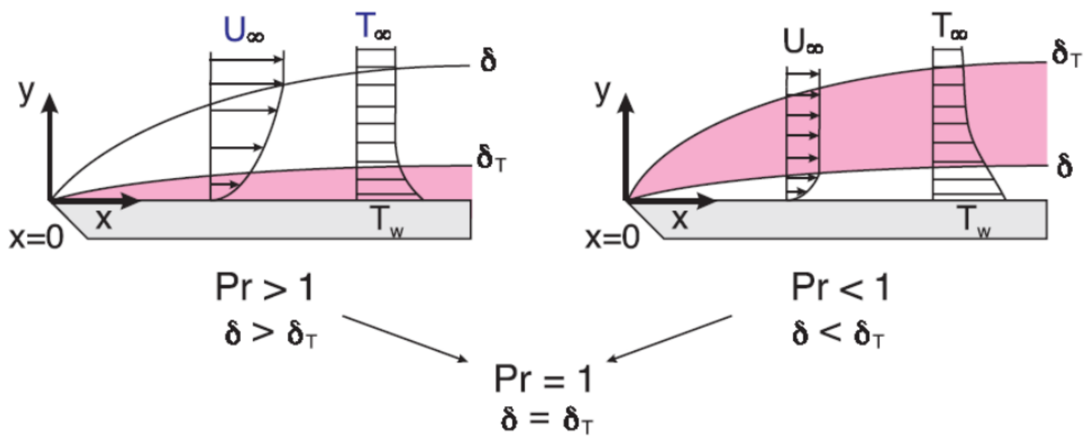
This also holds for surfaces other than a flate plate!

Thermal Boundary Layer

Similar to velocity boundary layer, a **thermal boundary layer** develops when a fluid at specific temperature flows over a surface which is at different temperature.

Prandtl number:

$$Pr = \frac{\mu c_p}{k}$$



The thickness of the thermal boundary layer δ_t is defined as the distance at which:

$$\frac{T - T_s}{T_\infty - T_s} = 0.99$$

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Characteristic length

Numbers sometimes based on *L*, sometimes *D*, …

General notation:

$$\mathrm{Nu} = \frac{h\,L_c}{k} \qquad \mathrm{Re} = \frac{\rho U\,L_c}{\mu}$$

Per geometry L_c is defined

- Flow over flat surface: length *L*
- Flow around sphere/cylinder: diameter *D*
- Other cases: Lecture 4

Subscripts:

Re_D, Re_L useful
Nu_D, Nu_L useful
Re_{Lc}, Nu_{Lc} not useful

- Numbers sometimes based on *L*, sometimes *D* (official notation: Re_L, Re_D)
- Per geometry distinction between Reynolds Numbers (flow regime)

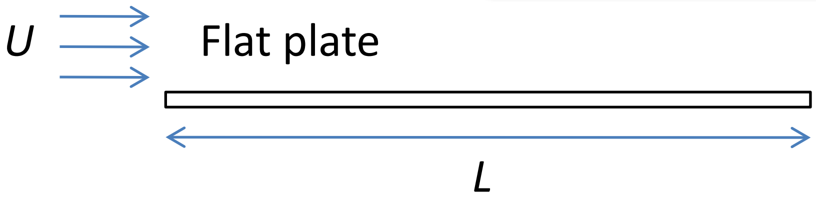
External flow $\mathrm{Nu} = a \cdot \mathrm{Re}^b \mathrm{Pr}^c$

where a, b, and c are constants. The properties of the fluid are usually evaluated at the **film temperature** defined as:

$$T_f = \frac{T_s + T_\infty}{2}$$

CORRELATIONS FOR *h* – FORCED CONVECTION

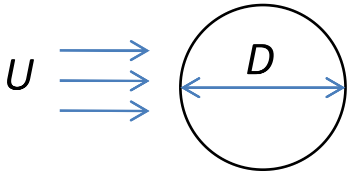
External flow $\mathrm{Nu} = a \cdot \mathrm{Re}^b \mathrm{Pr}^c$



Flat plate

$a = 0{,}664$; $b = 0{,}5$; $c = 1/3$ ($\mathrm{Re} < 5 \cdot 10^5$)
$a = 0{,}037$; $b = 0{,}8$; $c = 1/3$ ($\mathrm{Re} > 5 \cdot 10^5$)

$$\mathrm{Re} = \frac{\rho U L}{\mu}$$

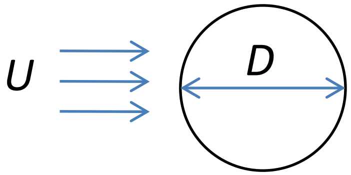


Cylinder

$$\mathrm{Re} = \frac{\rho U D}{\mu}$$

$a = 0{,}193$; $b = 0{,}618$; $c = 1/3$ (4000 < Re < 40.000)
$a = 0{,}027$; $b = 0{,}805$; $c = 1/3$ (40.000 < Re < 400.000)

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$



Sphere

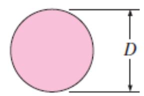
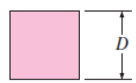
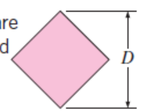
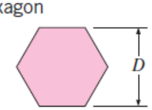
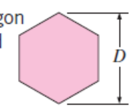
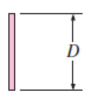
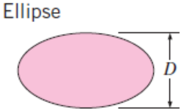
$$\mathrm{Re} = \frac{\rho U D}{\mu}$$

$\mathrm{Nu} \approx 2 + [0{,}4\mathrm{Re}^{1/2} + 0{,}06\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0{,}4}$
(optimal for Re < 80.000)

CORRELATIONS FOR *h* – FORCED CONVECTION

TABLE 7-1

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

Conclusion forced convection

General (also natural convection):

$$\dot{Q} = h A \Delta T \quad \text{(W)}$$ Newton's cooling law $$\dot{q} = h \Delta T \quad (\text{W/m}^2)$$

"Supporting" equations for *h (Forced Convection)*:

$$\text{Nu} = a \cdot \text{Re}^b \, \text{Pr}^c$$

a, b, c dependent on geometry and flow regime (laminar / turbulent)

Nusselt Number $$\text{Nu}_L = \frac{hL}{k}; \quad \text{Nu}_D = \frac{hD}{k} \quad (-)$$

Reynolds Number $$\text{Re}_L = \frac{\rho U L}{\mu}; \quad \text{Re}_D = \frac{\rho U D}{\mu} \quad (-)$$

Prandtl Number $$\text{P}r = \frac{\mu c_p}{k}$$

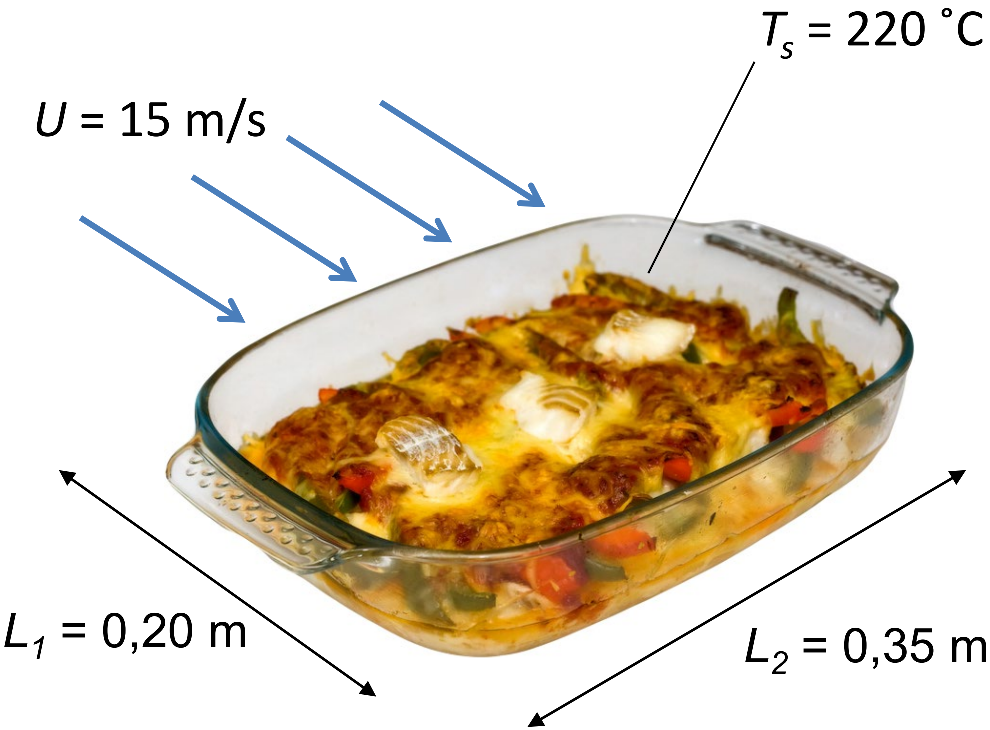
Dimensionless numbers make similar shaped situations comparable; "universal" parameters

LEARNING OBJECTIVES LECTURE 3

- Defining Convective Heat Transfer
- Convective Heat Transfer Types
- **Heat Transfer Rate in Convection**
  - Newton’s Law
  - Convection Resistance
  - Nusselt Number
- Forced Convection
  - Flow Parameters
  - Convective Heat Transfer Coefficient
  - Laminar and Turbulent Flow
  - Using additional correlations for various configurations
- **Step-by-step plan for convection calculations**

Example

Calculate the heat transfer rate?



Surrounding temperature

T_∞ = 20 ˚C

SUMMARY

- **Heat Transfer Equation**

$$\dot{Q} = h A \Delta T$$

Newton's cooling law

- **Convection resistance**

$$\dot{Q} = h A \Delta T = \frac{\Delta T}{R_{conv}} \quad with \;\; R_{conv} = \frac{1}{hA} \quad \text{(K/W)}$$

- **Heat Transfer corelations**

Forced convection: Nu as function of Re, Pr : Nu = *f*(Re, Pr)
Natural convection: next lecture

Dimensionless Numbers

Nusselt Number $$\mathrm{Nu}_L = \frac{hL}{k};\ \ \mathrm{Nu}_D = \frac{hD}{k}\ \ (-)$$

Reynolds Number $$\mathrm{Re}_L = \frac{\rho U L}{\mu};\ \ \mathrm{Re}_D = \frac{\rho U D}{\mu}\ (-)$$

Prandtl Number $$\mathrm{P}r = \frac{\mu c_p}{k}$$

Exercise:

Show Nu, Pr and Re are dimensionless

QUESTION TIME

Question Time