Fluid Mechanics 1

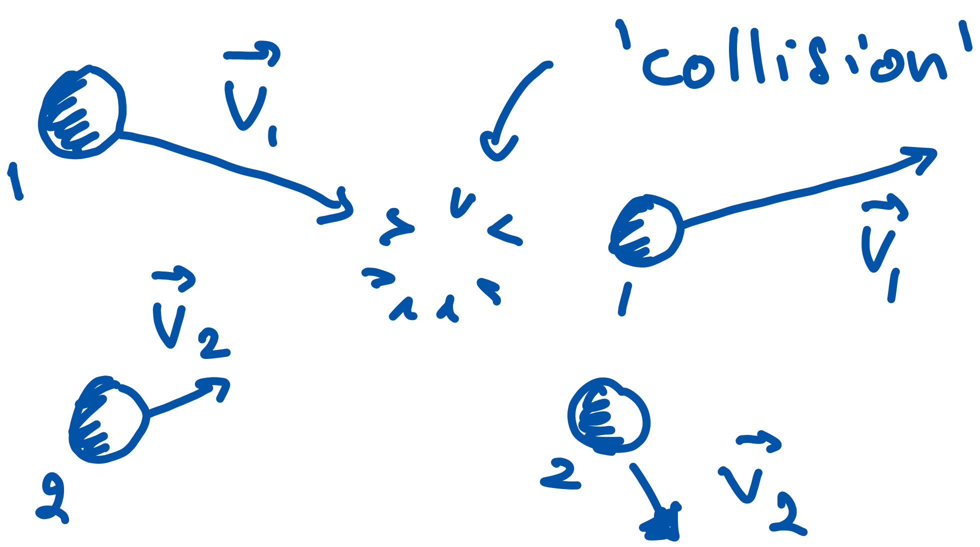
How to understand and describe the motion of fluids?

11 lectures (online, video) → Canvas.
6 Tutorials (online, live)
exam
resit

Reader : 11 chapters
+ exercises.

Fluid : liquid / gas

bunch of molecules.



How to compute the after-collision situation?

Conservation of:

<u>mass</u>: $m_1 + m_2 = \text{constant}$

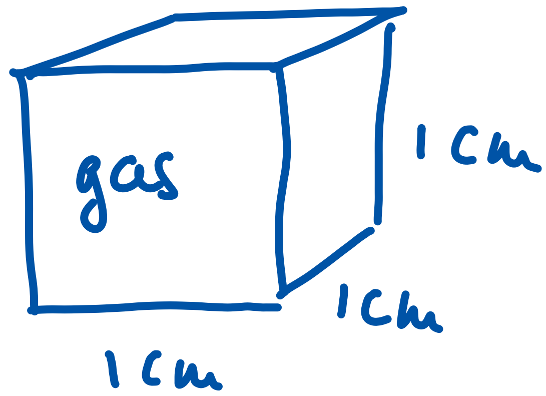
<u>momentum</u>: $m_1\vec{v}_1 + m_2\vec{v}_2 = \overrightarrow{\text{constant}}$

<u>energy</u>: $\left(\frac{1}{2}m_1v_1^2 + \epsilon_1\right) + \left(\frac{1}{2}m_2v_2^2 + \epsilon_2\right) = \text{constant}$

$v_1 \equiv |\vec{v}_1|$ ↳ internal energy.

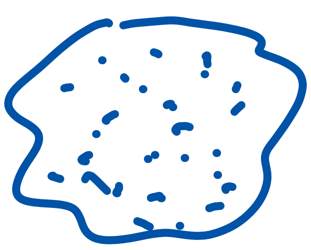
<u>Problem</u>:

$p = 1\,\text{bar}$

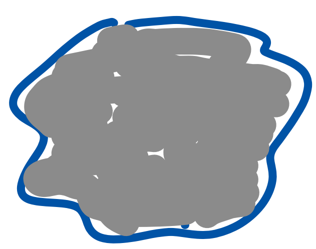


$\sim 10^{22}$ molecules!

<u>Solution</u>: continuum approach.



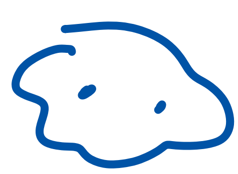
$\approx$



V V

$$\text{mass density} \equiv \frac{\sum_i m_i}{V} = \rho$$

If V is <u>too small</u>:



ρ will strongly depend on time

If V is <u>too large</u>:

you loose information,
real density varies strongly over the airplane.

$\Rightarrow$ V has to be chosen intermediate.

$\Rightarrow$ Choose V sufficiently large such that it contains sufficiently many molecules (10^4 ?) but sufficiently small w.r.t to the object of interest. (airplane).

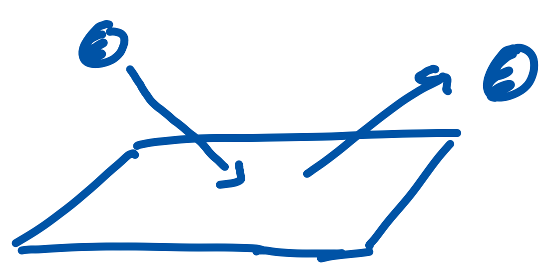
$\Rightarrow$ mm^3 ?

What is the velocity of the fluid?

$$\vec{u} \equiv \sum_i m_i v_i \Big/ \sum_i m_i$$ mass-averaged velocity.

↑ def.

$T \sim$ average energy of molecules.

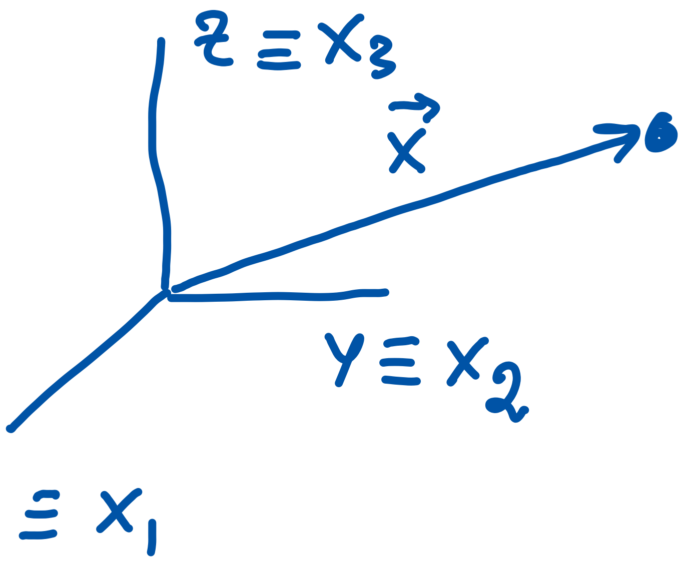
p :  $\Rightarrow$ force

very high frequency: 'continuous' force

Goal of Fluid Mechanics:

to describe $p(\vec{x},t)$, $\vec{u}(\vec{x},t)$, $\rho(\vec{x},t)$ and so on.

Reference system:
(position)

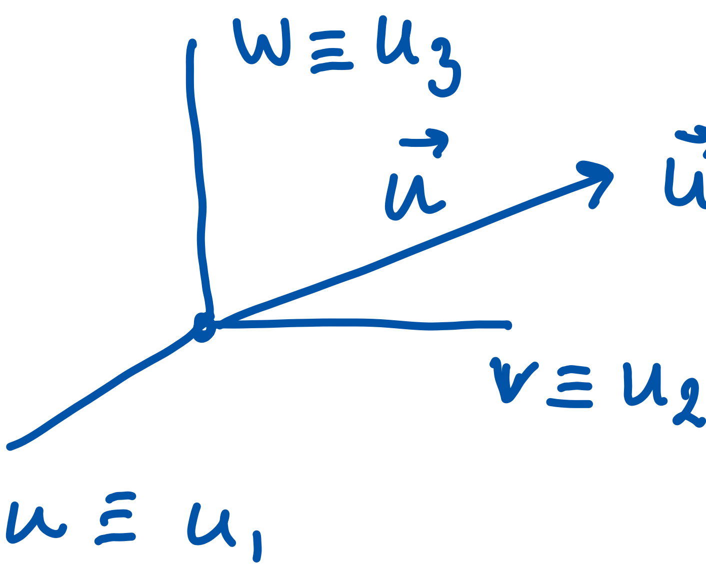


alphabetical notation

$$\vec{x} \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

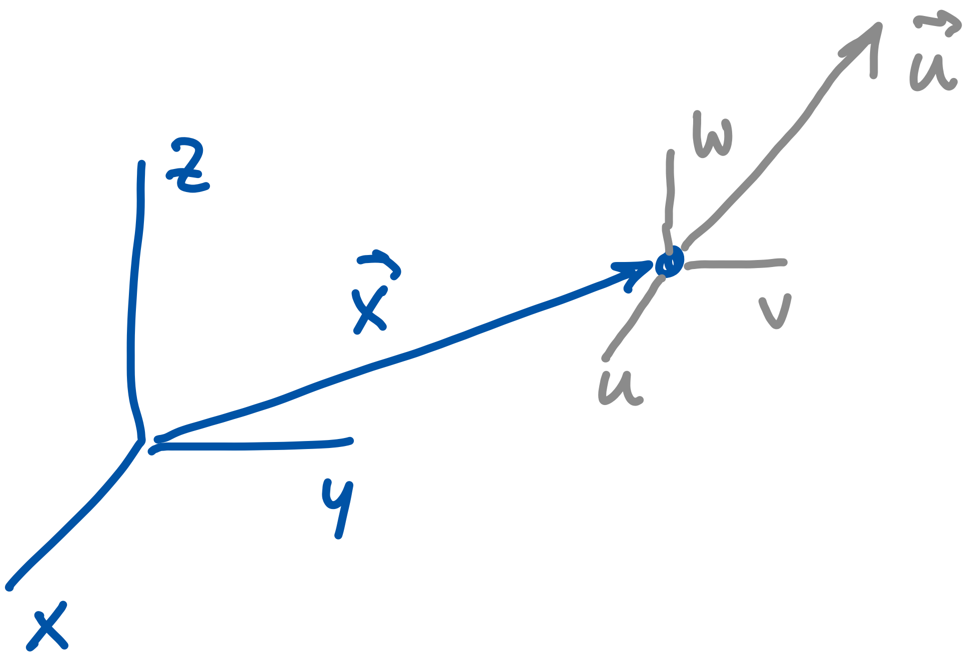
index notation.

Similar: (velocity)



$$\vec{u} \equiv \begin{pmatrix} u \\ v \\ w \end{pmatrix} \equiv \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

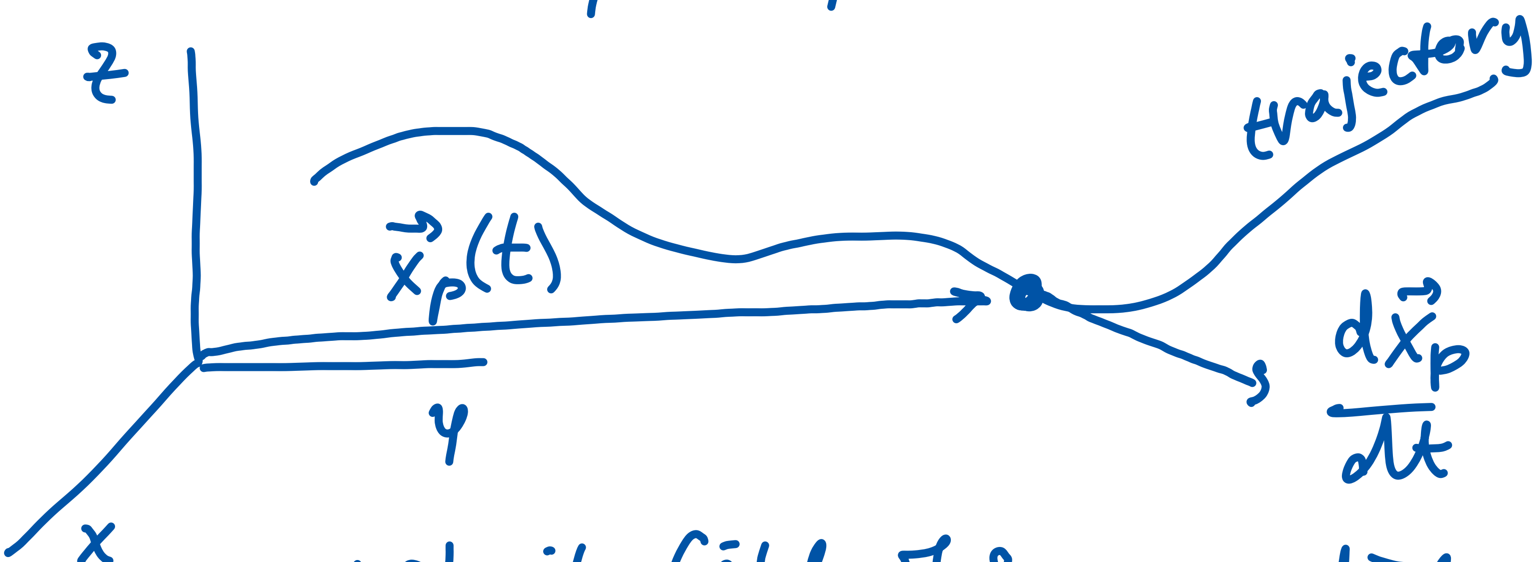
Usage:



'Visualization' of Flows:

Particle Trajectories.

Assume particle that exactly follows the flow
↳ same mass density as fluid
- sufficiently small.



velocity field of surrounding fluid: $\vec{u}(\underbrace{\vec{x},t})$

independent parameters: we can choose their value

$$\frac{d\vec{x}_p}{dt} = \begin{pmatrix} \frac{dx_p}{dt} \\ \frac{dy_p}{dt} \\ \frac{dz_p}{dt} \end{pmatrix} \qquad \vec{x}_p(t) \equiv \begin{pmatrix} x_p(t) \\ y_p(t) \\ z_p(t) \end{pmatrix}$$

What is the connection between the particle and the fluid velocity?

particle moves exactly with the fluid

$\Rightarrow$ particle velocity = fluid velocity

at all times.

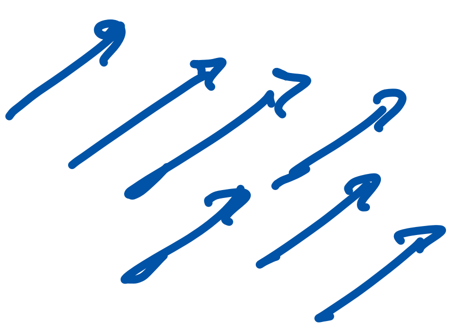
at the particle position!

$$\boxed{\frac{d\vec{x}_p}{dt}(t) = \vec{u}(\vec{x}_p(t), t) \qquad \forall\, t}$$

$\uparrow$!

(for all)

Example #1: parallel uniform flow:



$$\vec{u}(\vec{x},t) = \begin{pmatrix} U \\ V \\ W \end{pmatrix} = \overrightarrow{\text{constant}}$$

$$\frac{d\vec{x}_p}{dt} = \vec{u}(\vec{x}_p(t), t)$$

3 equations.

$$\frac{dx_p}{dt} = u(\vec{x}_p(t), t) = U \quad \Rightarrow \frac{dx_p}{dt} = U = \text{const}$$

$$\Rightarrow \quad x_p(t) = Ut + x_p^0$$

integration constant

similarly: $y_p(t) = Vt + y_p^0 \qquad z_p(t) = Wt + z_p^0$

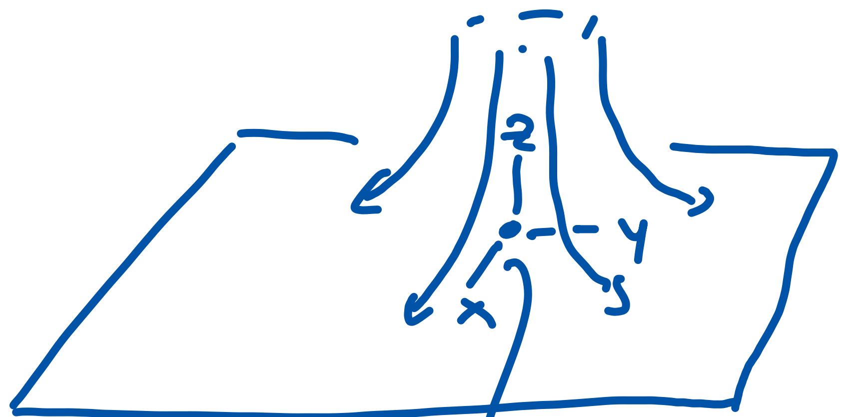
$$\Rightarrow \quad \vec{x}_p(t) = \begin{pmatrix} U \\ V \\ W \end{pmatrix} t + \vec{x}_p^{\,0}$$

$\vec{x}_p^{\,0}$ — constant vector.

To compute $\vec{x}_p^{\,0}$ we need initial condition.

$$\vec{x}_p(0) = \vec{x}_p^{\,0}.$$

Example #2:



Stagnation point: fluid velocity is zero

$$\vec{u}(\vec{x},t) = \begin{pmatrix} ax \\ by \\ cz \end{pmatrix}$$

a, b, c satisfy certain condition.

$$\frac{d\vec{x}_p}{dt} = \vec{u}(\vec{x}_p(t), t) = \begin{pmatrix} a\,x_p(t) \\ b\,y_p(t) \\ c\,z_p(t) \end{pmatrix}$$

↑ !

x_p equation: $\quad \dfrac{dx_p}{dt}(t) = a\,x_p(t)$ — linear, 1st order ODE (ordinary differential equation)

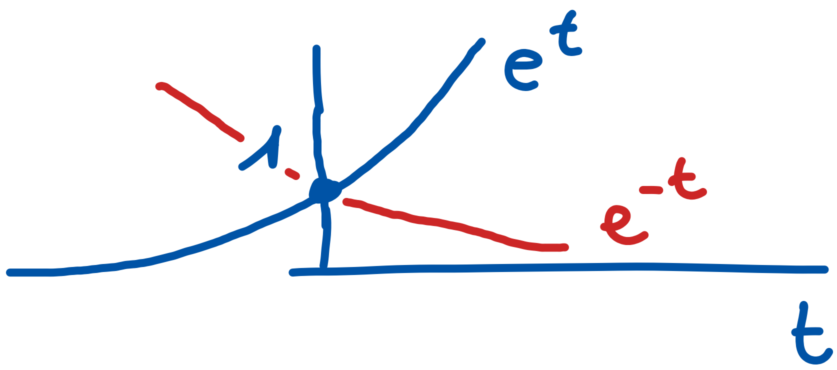
$$\Rightarrow \quad x_p(t) = x_p^0\, e^{at}$$

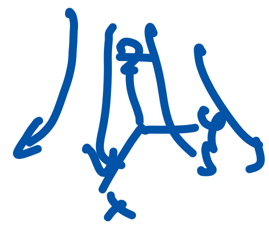
x_p^0 — integration constant.

$\Rightarrow \quad x_p(0) = x_p^0$: initial x-position.

Similar expressions for $y_p(t)$, $z_p(t)$.

Final answer: $\vec{x}_p(t) = \vec{x}_p^{\,0} \begin{pmatrix} e^{at} \\ e^{bt} \\ e^{ct} \end{pmatrix}$





$a, b > 0$
$c < 0$

Hints: $x_p \frac{dx_p}{dt} = \frac{d}{dt}\left(\frac{1}{2}x_p^2\right)$ $x_p(t)$

check: chain rule:

$$\frac{d}{dt}\left(\frac{1}{2}x_p^2\right) = \frac{d}{dx_p}\left(\frac{1}{2}x_p^2\right)\frac{dx_p}{dt}$$

$$2 \cdot \frac{1}{2} x_p \quad \frac{dx_p}{dt}$$

$$= x_p \frac{dx_p}{dt}$$

$$\frac{1}{x_p}\frac{dx_p}{dt} = \frac{d}{dt}\left(\ln x_p\right)$$

check: chain rule:

$$\frac{d}{dt}\ln x_p = \frac{d}{dx_p}\ln x_p \;\frac{dx_p}{dt}$$

$$= \frac{1}{x_p}\frac{dx_p}{dt}$$

Finally:

$$\cos(x_p)\frac{dx_p}{dt} = \frac{d}{dt}\left(\sin x_p(t)\right)$$

chain rule check:

$$\frac{d}{dt}\sin(x_p) = \frac{d}{dx_p}\sin(x_p)\cdot\frac{dx_p}{dt}$$

$$= \cos(x_p)\frac{dx_p}{dt}$$