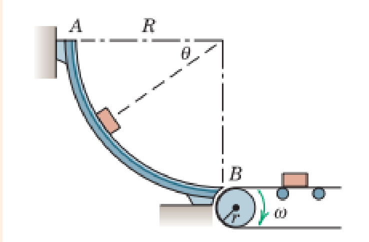
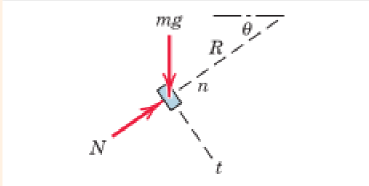
Sample Problem 3/7

Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B. Determine the expression for the normal contact force N between the guide and each object in terms of θ and specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.





Solution. The free-body diagram of the object is shown together with the coordinate directions n and t. The normal force N depends on the n-component of the acceleration which, in turn, depends on the velocity. The velocity will be cumulative according to the tangential acceleration a_t. Hence, we will find a_t first for any general position.

$$[\Sigma F_t = ma_t] \qquad mg\cos\theta = ma_t \qquad a_t = g\cos\theta$$

① Now we can find the velocity by integrating

$$[v\,dv = a_t\,ds] \quad \int_0^v v\,dv = \int_0^\theta g\cos\theta\,d(R\theta) \qquad v^2 = 2gR\sin\theta$$

We obtain the normal force by summing forces in the positive n-direction, which is the direction of the n-component of acceleration.

$$[\Sigma F_n = ma_n] \qquad N - mg\sin\theta = m\frac{v^2}{R} \qquad N = 3mg\sin\theta \qquad \textit{Ans.}$$

The conveyor pulley must turn at the rate $v = r\omega$ for $\theta = \pi/2$, so that

$$\omega = \sqrt{2gR}/r \qquad \textit{Ans.}$$

Helpful Hint

① It is essential here that we recognize the need to express the tangential acceleration as a function of position so that v may be found by integrating the kinematical relation $v\,dv = a_t\,ds$, in which all quantities are measured along the path.