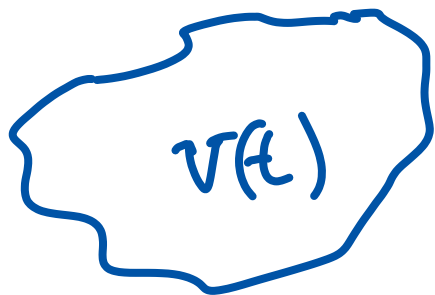


Lecture #10 Energy Conservation



$$\text{Energy}(t) \equiv \int_{V(t)} \rho E d\bar{V}$$

E? $E = e + \underbrace{\frac{1}{2} u_k u_k}_{\text{kinetic}}$

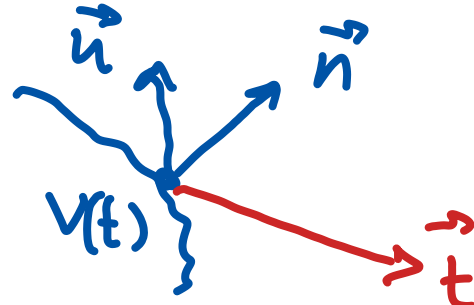
total (J/kg) internal (thermal)

1st law of thermodynamics:

$$\frac{d}{dt}(\text{Energy}) = \underbrace{\frac{d}{dt}(\text{Work})}_{\text{on } V} + \underbrace{\frac{d}{dt}(\text{Heat})}_{\text{to } V}$$

$$\frac{d}{dt}(\text{Work}) = \int_{S(t)} t_i u_i dA$$

$$+ \int_{V(t)} \rho g_i u_i d\bar{V}$$

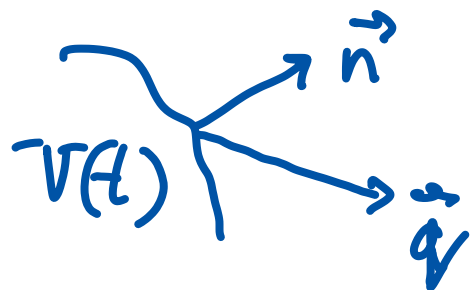


J/s ρ

$$\left[t_i u_i dS \right] = \frac{N}{\cancel{m^2}} \frac{m}{S} \cancel{u^2} = \frac{Nm}{S} = \frac{p}{S}$$

if $\vec{t} \perp \vec{u}$: Work done is zero

$$\frac{d}{dt}(\text{Heat}) = - \int_{S(t)} q_i n_i dS$$



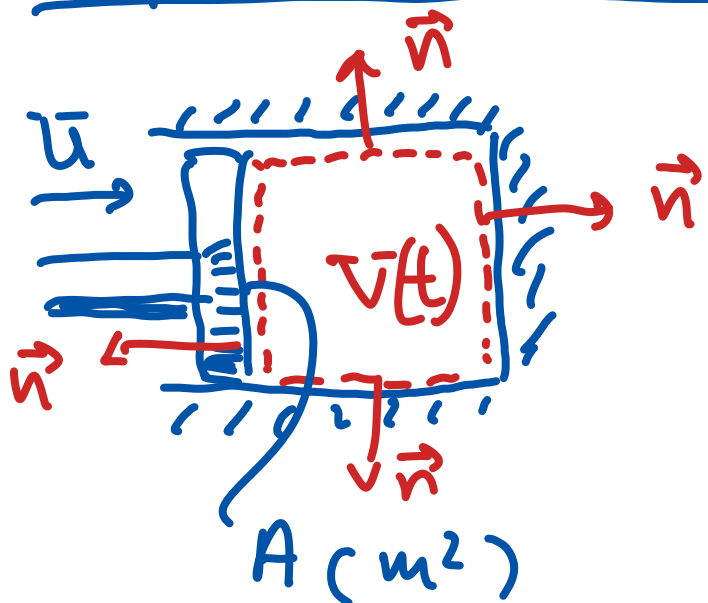
$$\frac{d}{dt} \text{Energy} = \frac{d}{dt} \int_{V(t)} \rho E dV$$

$$= \int_{V(t)} \frac{\partial(\rho E)}{\partial t} dV + \int_{S(t)} \rho E u_j n_j dS'$$

$$\Rightarrow \left\{ \begin{aligned} & \int_{V(t)} \frac{\partial(\rho E)}{\partial t} dV + \int_{S(t)} \rho E u_j n_j dS' \\ &= \int_{S(t)} \sigma_{ij} u_i n_j dS + \int_{V(t)} \rho g_i u_i dV \\ &+ \int_{S(t)} k \frac{\partial T}{\partial x_i} n_i dS \end{aligned} \right.$$

Energy conservation in integral form.

Application Example



Slow: $\frac{1}{2} u_k u_k \ll e$

$\Rightarrow E \approx e$

Uniform: T, e, ρ, p
~ uniform.

" $\mu = 0$ " neglect shear stress

" $k = 0$ " " heat conduction

" $g = 0$ " " gravity.

Assume: ideal gas: $p = \rho R T$

R : specific gas constant $\frac{J}{kg, K}$
↳ depends on the gas.

also: $e = c_v T$

↳ specific heat at constant volume
($J/kg, K$)

$c_v \equiv \left(\frac{\partial e}{\partial T} \right)_v$

$c_p \equiv \left(\frac{\partial h}{\partial T} \right)_p$

$c_p - c_v = R$ $c_p / c_v \equiv \gamma$

c_v, c_p, R constant.

Apply Energy conservation:

$$\begin{aligned}
 \int_{V(t)} \underbrace{\frac{\partial}{\partial t}(\rho E)}_{\text{uniform.}} dV &\approx \frac{\partial}{\partial t}(\rho E) \cdot \int_V dV \\
 &= \frac{\partial}{\partial t}(\rho e) V \\
 &= \frac{\partial}{\partial t}(\rho C_v T) V
 \end{aligned}$$

$$\begin{aligned}
 \int_{S(t)} \rho E u_i n_i dS &\approx \rho E \int_{S(t)} u_i n_i dS \approx \rho e \frac{dV}{dt} \\
 &= \rho C_v T \frac{dV}{dt}
 \end{aligned}$$

$$\begin{aligned}
 \int_{S(t)} \sigma_{ij} u_i n_j dS &\approx \int_{S(t)} -p \delta_{ij} u_i n_j dS \\
 &= - \int_{S(t)} p u_j n_j dS \approx -p \frac{dV}{dt}
 \end{aligned}$$

$$\int_{V(t)} \rho g_i u_i dV \approx 0 \quad "g=0" \quad \text{uniform.}$$

$$\int_{S(t)} k \frac{\partial T}{\partial x_i} n_i dS \approx 0 \quad "k=0" \quad , \quad \frac{\partial T}{\partial x_i} = 0$$

$$V \frac{\partial}{\partial t}(\rho C_V T) + \rho C_V T \frac{d\bar{V}}{dt} = \frac{d}{dt}(\rho C_V T \bar{V})$$

mass conservation: $\rho \bar{V} = \text{constant}$

$$\Rightarrow \frac{d}{dt}(\rho C_V T \bar{V}) = \rho \bar{V} C_V \frac{dT}{dt}$$

Energy conv: $\rho \bar{V} C_V \frac{dT}{dt} = -p \frac{d\bar{V}}{dt} = -p R T \frac{d\bar{V}}{dt}$

$$\frac{1}{p}: \quad C_V \cdot V \frac{dT}{dt} = -R \cdot T \frac{d\bar{V}}{dt}$$

$$\frac{1}{V T}: \quad C_V \frac{1}{T} \frac{dT}{dt} = -R \frac{1}{V} \frac{d\bar{V}}{dt}$$

$$\Rightarrow C_V \frac{d}{dt} \ln T = -R \frac{d}{dt} \ln V$$

$$\Rightarrow \frac{d}{dt} \ln T + \frac{R}{C_V} \frac{d}{dt} \ln V = 0$$

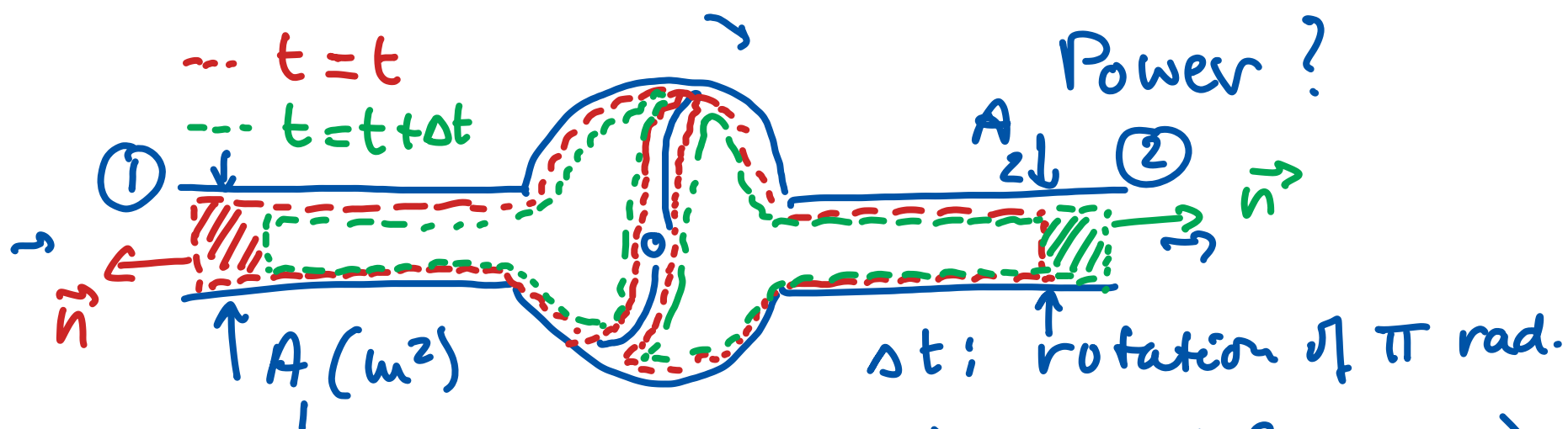
$$\frac{R}{C_V} = \gamma - 1 \quad \Rightarrow \frac{d}{dt} (\ln T + \ln V^{\gamma-1}) = 0$$

$$\Rightarrow \frac{d}{dt} \ln (T V^{\gamma-1}) = 0$$

$$\Rightarrow \frac{d}{dt} () = 0 \quad \Rightarrow \boxed{T V^{\gamma-1} = \text{constant}}$$

entropy is constant \Rightarrow reversible.

Application Example



$$\frac{d}{dt} \int_{V(t)} \rho E d\bar{V} \approx \frac{\left(\int_V \rho E d\bar{V} \right)_{t+\Delta t} - \left(\int_V \rho E d\bar{V} \right)_t}{\Delta t}$$

exact when $\Delta t \rightarrow 0$.

$$= \frac{1}{\Delta t} \left\{ (\rho E A u)_2 \Delta t - (\rho E A u)_1 \Delta t \right\}$$

$$= \dot{m} (E_2 - E_1), \quad \dot{m} \equiv \rho A u \left(\frac{\text{kg}}{\text{s}} \right)$$

$\mu = 0$ at ①, ②

$$\int_{S(t)} \sigma_{ij} u_i n_j dS \approx \int_{S_1} -p u_i n_i dS + \int_{S_1} -p u_i n_i dS$$

$$+ \underbrace{\int_{S_{\text{blade}}} \sigma_{ij} u_i n_j dS}_{=0} + \underbrace{\int_{S_{\text{wall}}} \sigma_{ij} u_i n_j dS}_{=0}$$

$\dot{W} \equiv$ Work by the blade on \bar{V}

Similar $\dot{Q} \equiv \int_{S(t)} -k \frac{\partial T}{\partial x_i} n_i dS$ to \bar{V}

neglect gravity.

note that $\int_{S_1} -p u_i n_i dS \stackrel{\text{uniform}}{=} (-p \cdot -U \cdot A)_1$

$$= \left(\frac{p}{\rho} \rho U A \right)_1 = \left(\frac{p}{\rho} \right)_1 \cdot \dot{m}$$

at S_2 : $-\left(\frac{p}{\rho} \right)_2 \cdot \dot{m}$ $\Delta(\cdot) \equiv (\cdot)_2 - (\cdot)_1$

$$\Rightarrow \dot{m} \Delta E + \dot{m} \Delta \left(\frac{p}{\rho} \right) = \dot{W} + \dot{Q}$$

$$\Rightarrow \boxed{\dot{m} (H_2 - H_1) = \dot{W} + \dot{Q}} \quad \text{Compressor equation.}$$

$$H \equiv \bar{E} + p/\rho \quad \text{Total enthalpy}$$