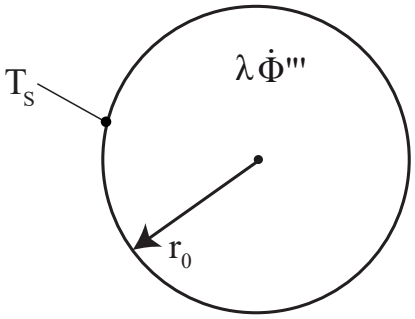
1.10 Resistance wire ★

A long homogeneous resistance wire is being used to heat the air in a room by the passage of electric current. Heat is generated in the wire uniformly at a constant rate $\dot{\Phi}'''$ as a result of resistance heating. Assume the problem to be one-dimensional steady-state heat transfer in radial direction.



Tasks:

a) Derive the heat conduction equation by setting up an energy balance.

b) Determine the temperature at r = 3.5 mm after steady operation conditions are reached.

Hints

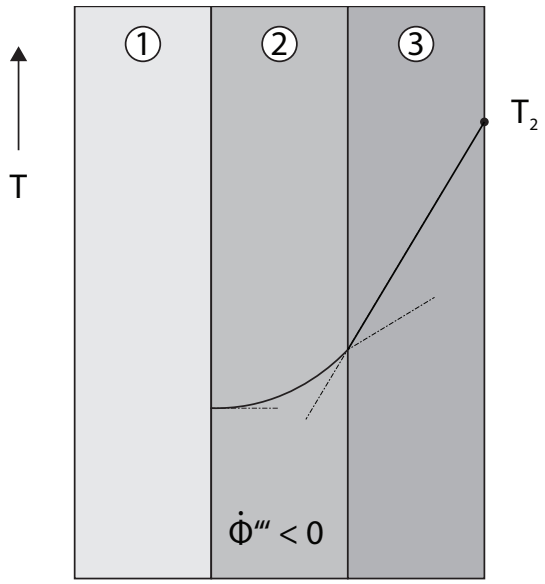
- Radiation can be neglected.

Given parameter

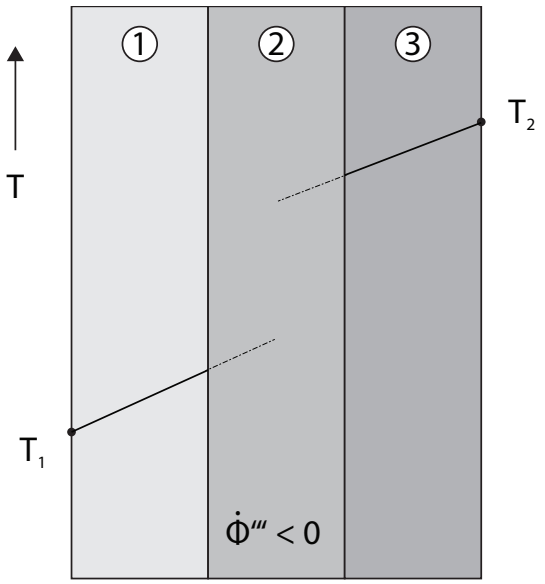
- Outer radius of the wire: $r_0 = 5$ mm
- Heat generation in the wire: $\dot{\Phi}''' = 5 \cdot 10^7$ W/m^3
- Temperature of the outer surface of the wire: $T_s = 180$ °C
- Thermal conductivity of the wire: $\lambda = 6$ W/mK

1.11 Draw temperature profiles ★★

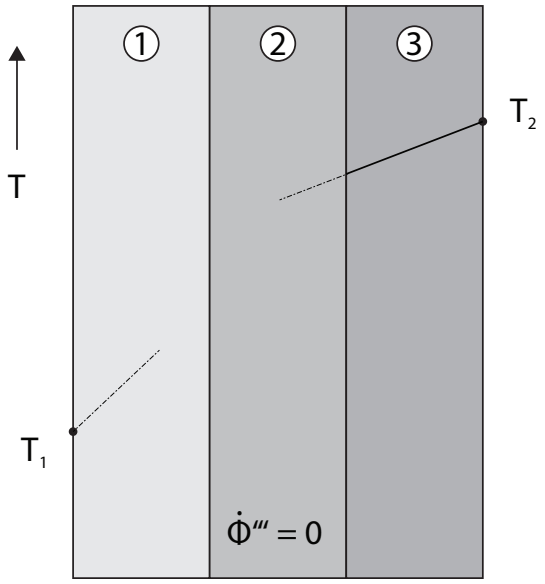
Complete the temperature profiles in the three-layered walls, shown below. Consider the information given in each drawing.



$\lambda_1 = \lambda_2 > \lambda_3$



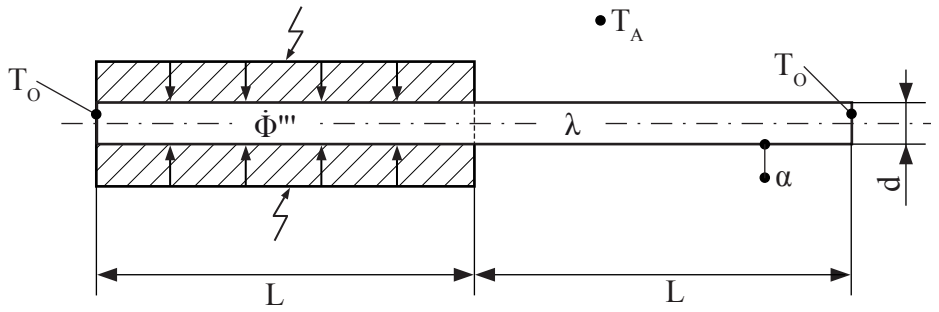
$\lambda_1 = \lambda_2 < \lambda_3$



$\lambda_1 = \lambda_2 < \lambda_3$

1.12 Copper rod ★★★

Both ends of a copper rod with a length L and a diameter d are kept at the same temperature T_O. The left half of the rod is insulated against all radial heat losses. An electric heating element generates Joule's heat of heat flux density $\dot{\Phi}'''$. The right half of the rod is subjected to a flow of the ambient air with an air temperature of T_A, yielding a heat transfer coefficient α. The thermal conductivity of the rod is given as λ.



Tasks:

a) Derive the equation for the temperature profile in the rod by setting up an energy balance.

b) Determine the value of $\dot{\Phi}'''$ so that the temperature in the center of the rod equals the temperature T_O at its ends.

c) Calculate $\dot{\Phi}'''$ using the following data: $L = 1\,\mathrm{m}$; $d = 5.2\,\mathrm{mm}$; $T_O = 120\,^\circ\mathrm{C}$; $T_A = 100\,^\circ\mathrm{C}$; $\alpha = 6\,\mathrm{W/m^2K}$; $\lambda = 372\,\mathrm{W/mK}$ for the conditions postulated in b).

d) Determine the extremes of the temperature distribution for the given values. Give their position and values, additionally, sketch the temperature profile.

Hint:

- Place the origin of the coordinate system in the middle of the rod.