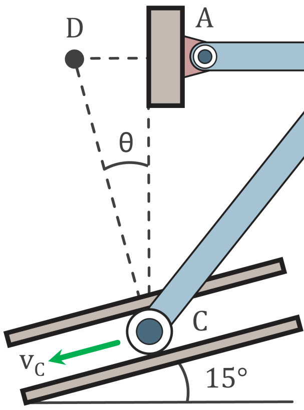
Wheel in a Slit



Point C has a wheel attached to it and is moving in the slit as shown in the figure. While trying to calculate the velocities in the system, you figured out it is easier to use the instantaneous point of zero velocity (an imaginary point that the rigid body appears to be rotating about). If D is the instantaneous point of zero velocity. Find the ratio $\frac{\omega_{CB}}{\omega_{AB}}$.

Using known expressions:

$$\boldsymbol{v_B} = \boldsymbol{v_A} + \boldsymbol{\omega_{AB}} \times \boldsymbol{r_{B/A}} \tag{1}$$

Using Equation 1 where $v_A = 0$, the velocity at B can be written as.

$$\boldsymbol{v_B} = \boldsymbol{\omega_{AB}} \times \boldsymbol{r_{B/A}} \quad \Rightarrow \quad \boldsymbol{v_B} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \begin{pmatrix} L_{AB} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -L_{AB} \cdot \omega_{AB} \\ 0 \end{pmatrix} \tag{2}$$

Furthermore, from the fact that point D is the instantaneous point of zero velocity it is proven that $\omega_{CB} = \omega_{CD} = \omega_{BD}$. We can also write the velocity at B with respect to point D.

$$\boldsymbol{v_B} = \boldsymbol{\omega_{BD}} \times \boldsymbol{r_{B/D}} \quad \Rightarrow \quad \boldsymbol{v_B} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{BD} \end{pmatrix} \times \begin{pmatrix} L_{BD} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -L_{BD} \cdot \omega_{BD} \\ 0 \end{pmatrix} \tag{3}$$

From Equations 2 and 3, together with the fact that $\omega_{CB} = \omega_{BD}$ we can find a ratio for $\frac{\omega_{CB}}{\omega_{AB}}$:

$$-L_{AB} \cdot \omega_{AB} = -L_{BD} \cdot \omega_{BD} = -L_{BD} \cdot \omega_{CB} \quad \Rightarrow \quad \frac{\omega_{CB}}{\omega_{AB}} = \frac{L_{AB}}{L_{BD}} \tag{4}$$