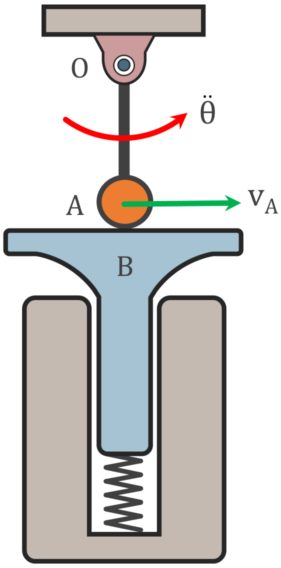
Acceleration of Crankshaft



Determine the acceleration of the blue shaft B **if the crank** OA **has a angular acceleration of** $\ddot{\theta} = 12$ rad/s^2 **and ball** A **has a velocity of** $v_A = 12$ m/s **at this position. The spring maintains contact between the roller and the surface of the plunger.** $L_{OA} = 0.5$ m**.**

Using known expressions:

$$\boldsymbol{v_A} = \boldsymbol{\omega} \times \boldsymbol{r_{A/O}} \tag{1}$$

$$\boldsymbol{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \tag{2}$$

$$\boldsymbol{a}_t = \boldsymbol{\alpha} \times \boldsymbol{r} \tag{3}$$

Given:
Angle: $\theta = 90°$
Angular acceleration: $\ddot{\theta} = 12$ rad/s^2
Velocity: $v_A = 12$ m/s
Distance from O to A: $L_{OA} = 0.5$ m.

The blue shaft B can only move up and down, thus it has a zero acceleration in the horizontal direction. At this time instant, the angle of the crankshaft is exactly 90°, which means, point A has a normal and tangential acceleration in the y-and x-direction respectively. Since the roller remains in contact with the shaft, its normal acceleration must be equal to the acceleration of the shaft. This is visualized in the kinematic diagram of Figure 1. As can be seen in Equation 2, the angular velocity of the crankshaft is needed. Using Equation 1 the angular velocity of the crankshaft can be calculated as follows:

$$\boldsymbol{v_A} = \boldsymbol{\omega} \times \boldsymbol{r_{A/O}} \quad \Rightarrow \quad \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\omega \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

Thus $\omega = 6$ rad/s.
Using Equation 2 the acceleration of shaft B becomes:

$$\boldsymbol{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \quad \Rightarrow \tag{5}$$

$$\begin{pmatrix} 0 \\ a_B \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left(\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -0.5 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0.5\omega \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5\omega^2 \\ 0 \end{pmatrix}$$

Thus $a_B = 0.5 \cdot \omega^2 = 0.5 \cdot 6^2 = 18$ m/s^2.

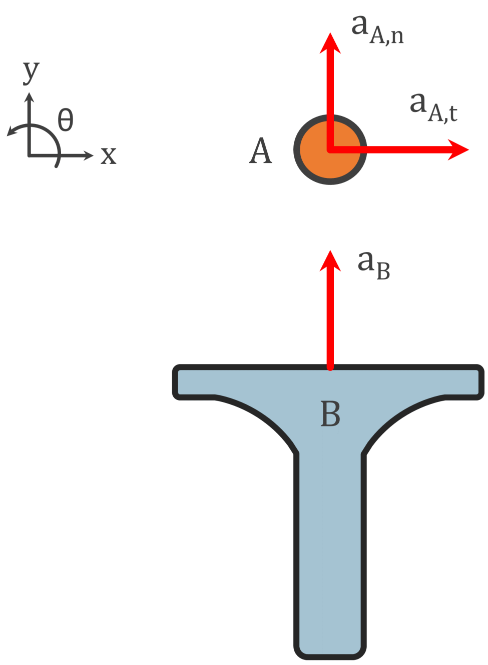


Figure 1: Kinematic diagram of the ball and the plunger.