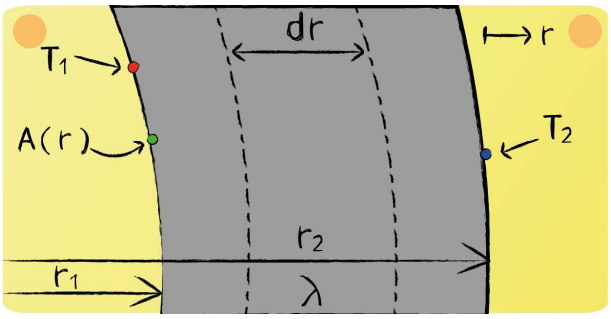
Lecture 5 - Question 5



Develop an energy balance to calculate the temperature profile inside the pipe wall and give the boundary conditions. Assume one-dimensional steady-state conditions. The expansion of the pipe in axial directions is L.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

In order to obtain the steady-state energy balance, the sum of the heat fluxes entering and leaving the system should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda 2\pi r L \frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} = \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r} dr = -\lambda 2\pi r L \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}(-\lambda 2\pi r L \frac{\partial T}{\partial r}) dr$$

The ingoing heat flux can be described by use of Fourier's law. The outgoing flux can be approximated by use of the Taylor series expansion.

Boundary conditions:

$$T(r = r_1) = T_1$$

$$T(r = r_2) = T_2$$

The boundary conditions describe that the inner and outer surface temperature equal T_1 and T_2 respectively.