Fins - Flux 2
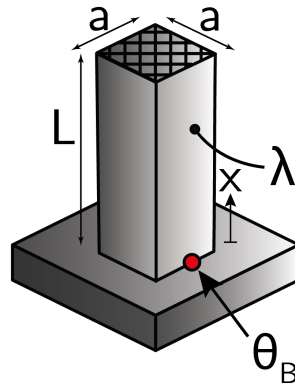
Calculate the rate of heat transfer for a fin with an adiabatic head.

Given the fin temperature profile:

$$\Theta(x) = \Theta_\text{B} \frac{\cosh\left(m\left(L - x\right)\right)}{\cosh\left(mL\right)}$$

Where $\Theta(x) = T(x) - T_\infty$ and $m = \sqrt{\frac{4\alpha}{\lambda a}}$.

The rate of heat transfer through the fin can be expressed as:

$$\dot{Q} = -\lambda A_\text{c} \frac{\partial T}{\partial x}\big|_{x=0} = -\lambda a^2 \frac{\partial \Theta}{\partial x}\big|_{x=0}$$

Differentiation of $\Theta(x)$ with respect to x yields:

$$\frac{\partial \Theta}{\partial x} = -m\Theta_\text{B} \frac{\sinh\left(m\left(L - x\right)\right)}{\tanh\left(mL\right)}$$

There for at $x = 0$ gives:

$$\frac{\partial \Theta}{\partial x}\big|_{x=0} = -\Theta_\text{B} \frac{\sinh\left(mL\right)}{\cosh\left(mL\right)} = -\Theta_\text{B} \cosh\left(mL\right)$$

Substitution into the expression for the rate of heat transfer yields:

$$\dot{Q} = \lambda a^2 m \Theta_\text{B} \tanh\left(mL\right)$$