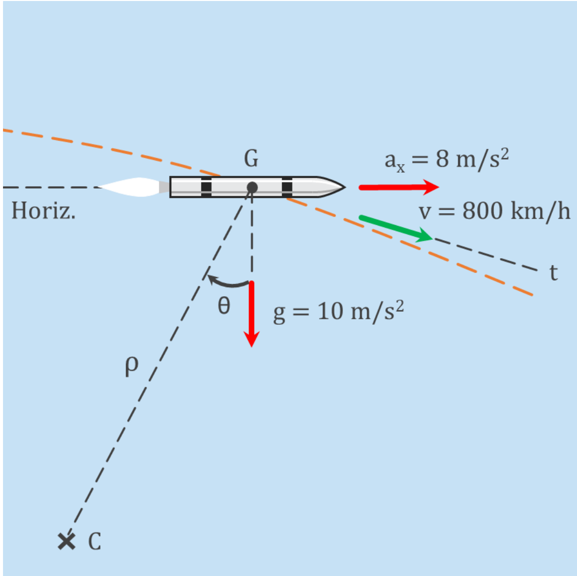
Rocket Accelerates



A rocket maintains at horizontal attitude of its axis during the powered phase of its flight (see the Figure). The acceleration due to horizontal thrust is 8 m/s^2, and the downward acceleration due to gravity is g = 10 m/s^2. At the instant represented, the velocity of the centre of mass G of the rocket along the $\theta = 15°$ direction of its trajectory is 800 km/h. Determine the radius of curvature ρ in meters of the flight trajectory. Round to the nearest hundred (e.g. 8700 m).

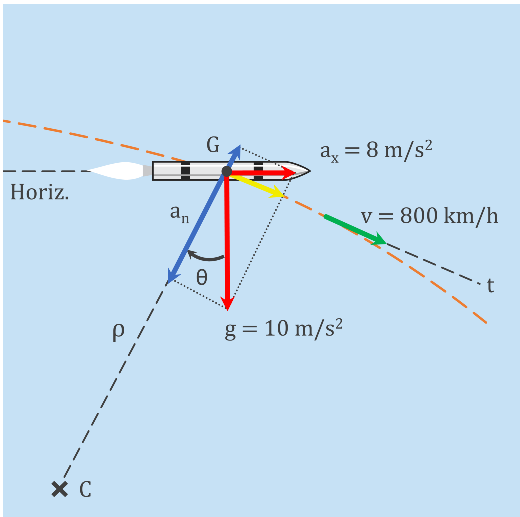


Figure 1: Rocket Accelerates

The normal acceleration a_n points towards the centre of curvature C. Figure 1 shows the acceleration vectors a_x and g deconstructed in the normal direction (blue) and the tangential direction (yellow). It can be easily seen that g and a_x deconstructed in the normal-direction are equal to $g \cos\theta$ and $a_x \sin\theta$, respectively. However, in this case $a_x \sin\theta$ points in the opposite way of a_n (points $\nearrow$ instead of $\swarrow$). This means that to determine the final value of a_n, the term $a_x \sin\theta$ should be subtracted. Resulting in the final answer:

$$a_n = g\cos\theta - a_x \sin\theta \tag{1}$$

Given quantities:

Angle: $\theta = 15°$
Gravitational acceleration: $g = 10\ \mathrm{m/s^2}$
Horizontal acceleration: $a_x = 8\ \mathrm{m/s^2}$
Velocity: $v = 800\ \mathrm{km/h} \approx 222.22\ \mathrm{m/s}$

Solution:

Inserting θ, g and a_x into Equation (1) results in:

$$a_n = g\cos\theta – a_x \sin\theta \quad \Rightarrow \quad a_n = 10 \cdot \cos(15°) – 8 \cdot \sin(15°) \approx 7.59\ \mathrm{m/s^2} \tag{2}$$

Rearranging results in the final answer:

$$\rho = \frac{v^2}{a_n} \quad \Rightarrow \quad \rho = \frac{222.22^2}{7.59} = 6507.40\ \mathrm{m} \approx 6500\ \mathrm{m} \tag{3}$$