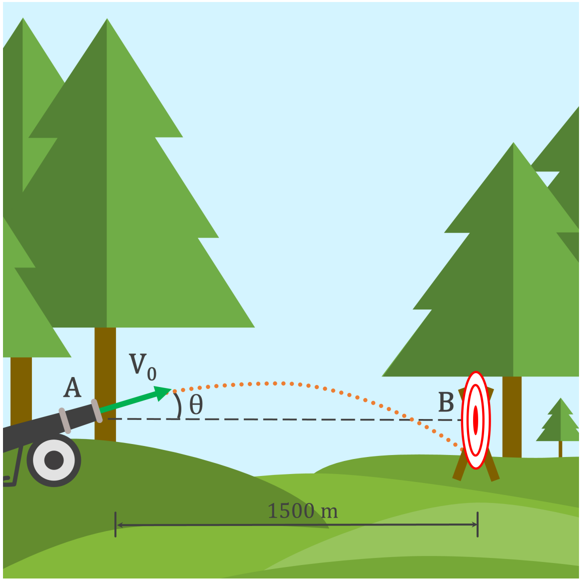
Bullet on Target Bottom



A cannon fires a bullet from A toward a target B. Find the equation which includes the vertical displacement $y(t)$ as a function of time, v_0 and θ, for which the round just hits the bottom of the target. The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is 1500 m. Neglect air resistances and assume that the bullet is directed along the vertical centreline of the target. Take $g = 10$ m/s^2.

$\boldsymbol{y(t) = \cdots = \ldots}$

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Distance A-B: $s_{\mathrm{AB}} = 1500$ m
Gravitational acceleration: $g = 10$ m/s^2
Initial velocity: $v_0 = 900$ m/s
Target diameter: $D = 2$ m

Solution:

Filling in Equation (6) gives an relation for the y-position with respect to time. Where $a = -g$ and $y_0 = 0$ m, since the cannon barrel is at the same altitude as the target centre.

$$y(t) = -\frac{1}{2}gt^2 + v_{0,y}t + y_0 = -\frac{1}{2}gt^2 + v_0 t \sin\theta \tag{7}$$

The bottom of the target is at 1 m from the centreline thus $y(t_{\mathrm{end}}) = -1$ m. Furthermore, inserting $g = 10$ m/s^2 result in.

$$y(t) = -\frac{1}{2} \cdot 10 \cdot t^2 + v_0 t \sin\theta = 1 \quad \Rightarrow \quad -5 \cdot t^2 + v_0 t \sin\theta = -1 \tag{8}$$