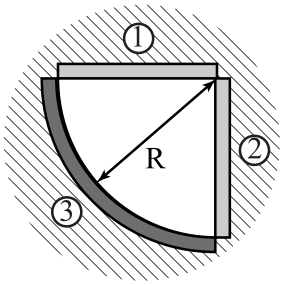
Section IV

Radiation solutions

Exercise IV.1 (Infinite pipe segment ⋆⋆)**:**

Consider an infinite long pipe segment as in the figure.



Tasks:

a) Specify the view factors Φ_{12}, Φ_{31} and Φ_{33} as a function of Φ_{13}.

b) Determine Φ_{13}.

Solution IV.1 (Infinite pipe segment ⋆⋆):

Task a)

When determining the view factors, recall the general rules that apply to view factors. First the summation rule:

$$\sum_{j=1}^{n} \Phi_{ij} = \Phi_{i1} + \Phi_{i2} + \Phi_{i3} + ... + \Phi_{in} = 1,$$

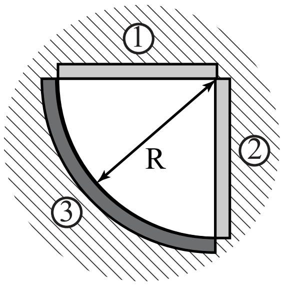
second, the reciprocity rule:

$$A_i \Phi_{ij} = A_j \Phi_{ji},$$

and third the symmetry rule:

$$\Phi_{ij} = \Phi_{ik},$$

which applies if two or more surfaces display symmetry about a third surface, they will have identical view factors from that surface.



From the figure, some information regarding the view factors can already be found. Surfaces 1 and 2 are flat plates, and therefore, cannot see themselves:

$$\Phi_{11} = 0 \tag{IV.1.1}$$

$$\Phi_{22} = 0 \tag{IV.1.2}$$

With this given, Φ_{12} can be expressed in terms of Φ_{13} by using the summation rule:

$$\cancelto{0}{\Phi_{11}} + \Phi_{12} + \Phi_{13} = 1 \tag{IV.1.3}$$
$$\Rightarrow \boldsymbol{\Phi_{12} = 1 - \Phi_{13}}$$

Furthermore, Φ_{31} can be expressed in terms of Φ_{13} by making use of the reciprocity rule, where L is the infinite length of the pipe segment:

$$\Phi_{31} A_3 = \Phi_{13} A_1 \tag{IV.1.4}$$

$$\Rightarrow \boldsymbol{\Phi_{31} = \Phi_{13} \cdot \frac{RL}{\frac{2\pi}{4} \cdot RL} = \frac{2}{\pi} \cdot \Phi_{13}} \tag{IV.1.5}$$

From the figure, it can be seen that surfaces 1 and 2 display symmetry about surface 3. Therefore Φ_{31} and Φ_{32} are equal to each other:

$$\Phi_{32} = \frac{2}{\pi} \cdot \Phi_{13} \tag{IV.1.6}$$

Lastly, with Φ_{31} and Φ_{32} known, the summation rule can be used to determine Φ_{33}

$$\Phi_{31} + \Phi_{32} + \Phi_{33} = 1 \tag{IV.1.7}$$

$$\Rightarrow \boldsymbol{\Phi_{33} = 1 - \frac{4}{\pi} \cdot \Phi_{13}}$$

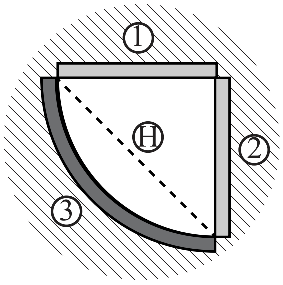
Conclusion

Which thus yields view factors Φ_{12}, Φ_{31}, and Φ_{33} as a function of Φ_{13}:

$$\Phi_{12} = 1 - \Phi_{13} \qquad \Phi_{31} = \frac{2}{\pi} \cdot \Phi_{13} \qquad \Phi_{33} = 1 - \frac{4}{\pi} \cdot \Phi_{13}$$

Task b)

To determine Φ_{13} the following auxiliary plane H is introduced:



From symmetry it can be seen that:

$$\Phi_{H1} = \frac{1}{2}$$

Now the reciprocity rule can be used to find the numerical value for Φ_{1H}:

$$\Phi_{1H} A_1 = \Phi_{H1} A_H \tag{IV.1.8}$$

$$\Rightarrow \Phi_{1H} = \Phi_{H1} \frac{\sqrt{2} RL}{RL} = \frac{\sqrt{2}}{2}$$

From the figure it can be seen that surface 3 now only sees the auxiliary plane H, and therefore:

$$\Phi_{H3} = 1 \tag{IV.1.9}$$

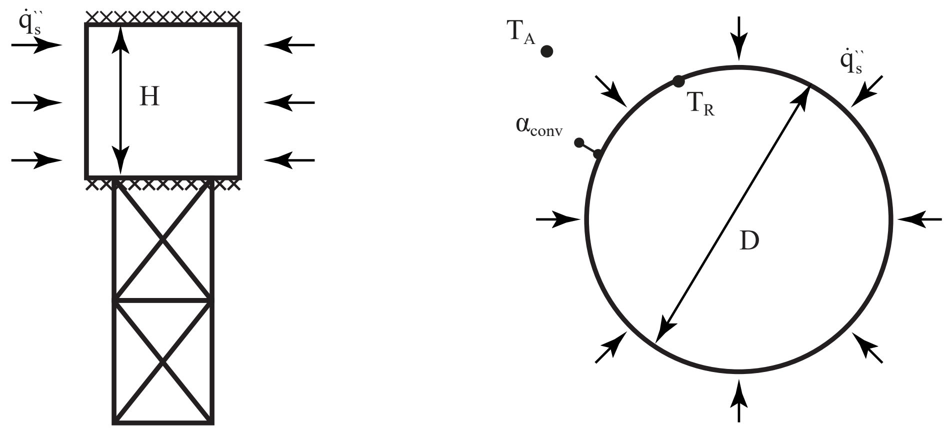
Conclusion

Thus, all radiation emitted from body 1 and transferred to surface H is directed towards body 3:

$$\Phi_{13} = \Phi_{1H} = \frac{\sqrt{2}}{2} \tag{IV.1.10}$$

Exercise IV.2 (Solar power tower ⋆)**:**

Solar radiation is uniformly and radially redirected toward a central cylindrical receiver in a solar tower plant by a surrounding mirror field (radiation density $\dot{q}''_{\mathrm{S}}$). Consequently, the surface of the receiver is heated to a temperature of T_{R}, and the thermal power output of the plant is $\dot{Q}_{\mathrm{th}}$.



(a) Side view (b) Top view

Given parameters:

- Receiver height: H
- Receiver outer diameter: D
- Receiver surface temperature: T_{R}
- Receiver emissivity of the surface: ϵ
- Heat transfer coefficient: α_{conv}
- Ambient temperature: T_{A}

Hints:

- Heat losses in the interior of the receiver as well as at its ends can be neglected.
- Radiation from the ambient can be neglected.
- The receiver can be considered as a grey body.

Tasks:

a) From a balance around the receiver, determine the mean radiation density $\dot{q}''_{\mathrm{S}}$ as a function of the thermal load $\dot{Q}_{\mathrm{th}}$.

Solution IV.2 (Solar power tower ⋆):

Task a)

To determine the mean radiation density $\dot{q}''_{\mathrm{S}}$ as a function of the thermal load $\dot{Q}_{\mathrm{th}}$, an energy balance around the receiver should be established.

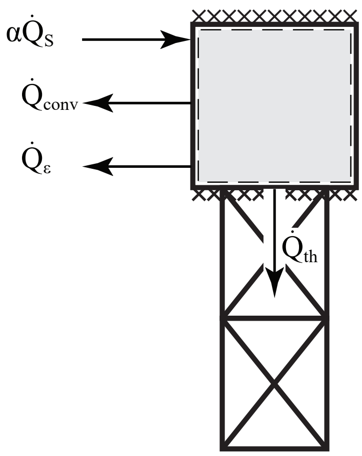
Since it is not known what the values are, it is the most straightforward to set an inner energy balance, which its general definition reads:

$$\frac{\partial U}{\partial t} = \alpha \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\epsilon} - \sum \dot{Q}_{\mathrm{out}},$$

where $\sum \dot{Q}_{\mathrm{out}}$ describes the losses that are not due to radiative heat losses, e.g. convection and thermal power generation.

1 **Setting up the balance:**

The sun is radiating on the solar power tower ($\dot{Q}_{\mathrm{S}}$) with a heat flux density $\dot{q}''_{\mathrm{S}}$, but at the same time the receiver is losing heat due to emitting radiation as a grey body ($\dot{Q}_{\epsilon}$) and convection ($\dot{Q}_{\mathrm{conv}}$). The remaining thermal load ($\dot{Q}_{\mathrm{th}}$) is being transferred inside the solar power tower where it is converted into electrical power.



Where the inner energy balance reads:

$$0 = \alpha \dot{Q}_{\mathrm{S}} - \dot{Q}_{\mathrm{conv}} - \dot{Q}_{\epsilon} - \dot{Q}_{\mathrm{th}} \tag{IV.2.1}$$

2 **Defining the elements within the balance:**

From Kirchoff's law it is stated that for a grey body:

$$\alpha = \epsilon \tag{IV.2.2}$$

The solar radiation absorbed by the receiver can be expressed as:

$$\begin{aligned} \alpha \dot{Q}_{\mathrm{S}} &= \epsilon \dot{q}''_{\mathrm{S}} A_{\mathrm{S}} \\ &= \epsilon \dot{q}''_{\mathrm{S}} \pi D H \end{aligned} \tag{IV.2.3}$$

The rate of heat loss due to convection yields from Newton's law of cooling:

$$\begin{aligned}\dot{Q}_{\text{conv}} &= \alpha_{\text{conv}} A_{\text{s}} \left(T_{\text{R}} - T_{\text{A}}\right) \\ &= \alpha_{\text{conv}} \pi D H \left(T_{\text{R}} - T_{\text{A}}\right)\end{aligned} \tag{IV.2.4}$$

The emitted radiation by the solar receiver yields from the product of the emissivity and the Stefan-Boltzmann law:

$$\begin{aligned}\dot{Q}_{\epsilon} &= \epsilon \sigma A_{\text{s}} T_{\text{R}}^4 \\ &= \epsilon \sigma \pi D H T_{\text{R}}^4\end{aligned} \tag{IV.2.5}$$

Conclusion

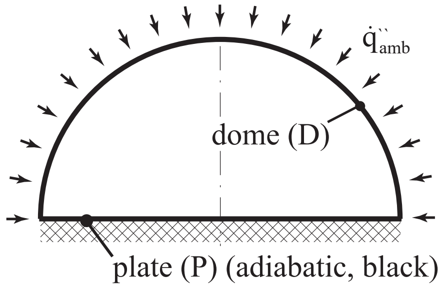
3 **Inserting and rearranging:**

Inserting the expressions of the fluxes into the inner energy balance and rewriting yields:

$$\dot{q}_{\text{S}}'' = \frac{\alpha_{\text{conv}}}{\epsilon} \left(T_{\text{R}} - T_{\text{A}}\right) + \sigma T_{\text{R}}^4 + \frac{\dot{Q}_{\text{th}}}{\epsilon \pi D H} \tag{IV.2.6}$$

Exercise IV.3 (Hemispherical dome ⋆⋆):

A thin circular plate (P) is covered by a hemispherical, transparent, grey dome (D). A radiative heat flux from the ambient $\dot{q}''_{\mathrm{amb}}$ is uniformly falling on the dome.



Given parameters:

- Temperature of the dome: T_{D}
- Surfaces of the plate and dome: $A_{\mathrm{P}},\ A_{\mathrm{D}}$
- Radiative heat flux: $\dot{q}''_{\mathrm{amb}}$
- View factor: Φ_{DP}
- Absorptivity of the plate: $\alpha_{\mathrm{P}} = 1$
- Reflectivity of the dome: $\rho_{\mathrm{D}} = 0$
- Transmissivity of the dome: τ_{D}
- Emissivity of the dome: ϵ_{D}

Hints:

- Conduction and convection are to be neglected.
- All surfaces are radiating diffusely.

Tasks:

a) Derive an expression for the temperature of the plate T_{P}.

Solution IV.3 (Hemispherical dome ★★):

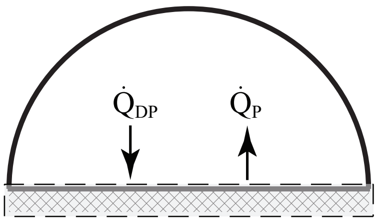
Task a)

The temperature of the plate T_P can be determined by setting up an energy balance around the plate. Both an inner and outer energy balance offer solution routes that result in an identical expression.

Outer energy balance:

1 Setting up the balance:

Partially the dome is radiating its surface brightness on the plate. Besides, the plate emits its surface brightness as well.



Therefore the steady-state outer balance reads:

$$0 = \dot{Q}_\mathrm{DP} - \dot{Q}_\mathrm{P} \tag{IV.3.1}$$

2 Defining the elements within the balance:

The plate acts as a black body radiator and therefore its surface brightness can be expressed as:

$$\dot{Q}_\mathrm{P} = \sigma A_\mathrm{P} T_\mathrm{P}^4 \tag{IV.3.2}$$

The radiative transport from the dome to the plate yields from the respective view factor and the surface brightness of the dome:

$$\dot{Q}_\mathrm{DP} = \Phi_\mathrm{DP} \dot{Q}_\mathrm{D} \tag{IV.3.3}$$

The dome emits radiation as a grey body, but it also transmits some of the ambient radiation. Therefore the surface brightness of its inner surface is expressed as:

$$\dot{Q}_\mathrm{D} = \dot{Q}_{\mathrm{D},\epsilon} + \cancelto{\rho_\mathrm{D} = 0}{\dot{Q}_{\mathrm{D},\rho}} + \dot{Q}_{\mathrm{D},\tau}, \tag{IV.3.4}$$

where the emission term is defined as:

$$\dot{Q}_{\mathrm{D},\epsilon} = \epsilon_\mathrm{D} \sigma A_\mathrm{D} T_\mathrm{D}^4, \tag{IV.3.5}$$

and the transmission term:

$$\dot{Q}_{\mathrm{D},\tau} = \tau_\mathrm{D} \dot{q}''_\mathrm{amb} A_\mathrm{D} \tag{IV.3.6}$$

Conclusion

3 Inserting and rearranging:

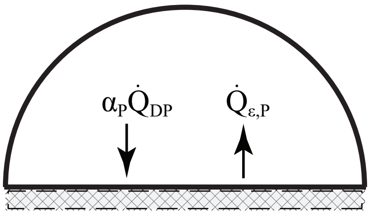
Inserting the definitions of the surface brightness of all bodies into the energy balance and rewriting gives:

$$T_\mathrm{P} = \sqrt[4]{\Phi_\mathrm{DP} \epsilon_\mathrm{D} \frac{A_\mathrm{D}}{A_\mathrm{P}} T_\mathrm{D}^4 + \Phi_\mathrm{DP} \tau_\mathrm{D} \dot{q}''_\mathrm{amb} \frac{A_\mathrm{D}}{\sigma A_\mathrm{P}}} \tag{IV.3.7}$$

Inner energy balance:

1 **Setting up the balance:**

Partially the dome is radiating its surface brightness on the plate. Besides, the plate emits its surface brightness as well.



Therefore the steady-state inner balance reads:

$$0 = \alpha_{\mathrm{P}} \dot{Q}_{\mathrm{DP}} - \dot{Q}_{\epsilon,\mathrm{P}} \tag{IV.3.8}$$

2 **Defining the elements within the balance:**

The radiation emitted by the plate can be described as the radiation emitted by a black body:

$$\dot{Q}_{\epsilon,\mathrm{P}} = \sigma A_{\mathrm{P}} T_{\mathrm{P}}^4$$

First, it is given that the absorptivity of the plate equals 1:

$$\alpha_{\mathrm{P}} = 1 \tag{IV.3.9}$$

The radiative transport from the dome to the plate yields from the respective view factor and the surface brightness of the dome:

$$\dot{Q}_{\mathrm{DP}} = \Phi_{\mathrm{DP}} \dot{Q}_{\mathrm{D}} \tag{IV.3.10}$$

The dome emits radiation as a grey body, but it also transmits some of the ambient radiation. Therefore the surface brightness of its inner surface is expressed as:

$$\dot{Q}_{\mathrm{D}} = \dot{Q}_{\mathrm{D},\epsilon} + \cancelto{\rho_{\mathrm{D}} = 0}{\dot{Q}_{\mathrm{D},\rho}} + \dot{Q}_{\mathrm{D},\tau}, \tag{IV.3.11}$$

where the emission term is defined as:

$$\dot{Q}_{\mathrm{D},\epsilon} = \epsilon_{\mathrm{D}} \sigma A_{\mathrm{D}} T_{\mathrm{D}}^4, \tag{IV.3.12}$$

and the transmission term:

$$\dot{Q}_{\mathrm{D},\tau} = \tau_{\mathrm{D}} \dot{q}''_{\mathrm{amb}} A_{\mathrm{D}} \tag{IV.3.13}$$

Conclusion

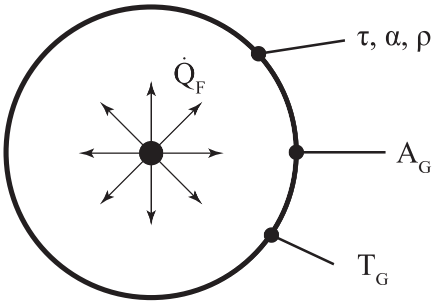
3 **Inserting and rearranging:**

Resulting in the same equation as found for the outer energy balance:

$$T_{\mathrm{P}} = \sqrt[4]{\Phi_{\mathrm{DP}} \epsilon_{\mathrm{D}} \frac{A_{\mathrm{D}}}{A_{\mathrm{P}}} T_{\mathrm{D}}^4 + \Phi_{\mathrm{DP}} \tau_{\mathrm{D}} \dot{q}''_{\mathrm{amb}} \frac{A_{\mathrm{D}}}{\sigma A_{\mathrm{P}}}} \tag{IV.3.14}$$

Exercise IV.4 (Light bulb ⋆⋆)**:**

The filament of a light bulb emits diffuse radiation $\dot{Q}_{\mathrm{F}}$. The glass of the bulb is thin, spherical, and acts as a gray body. The surface of the filament is small in comparison to the glass body and the problem is steady in time.



Given parameters:

- Power consumption of the filament: $\dot{Q}_{\mathrm{F}}$
- Glass properties: τ, α, ρ
- Surface of the glass sphere: A_{G}

Hints:

- The surface of the filament in comparison to the glass body is small.

Tasks:

a) Provide the energy balance in terms of given variables for determining the glass temperature T_{G}, while neglecting radiation from the environment.

Solution IV.4 (Light bulb ★★):

Task a)

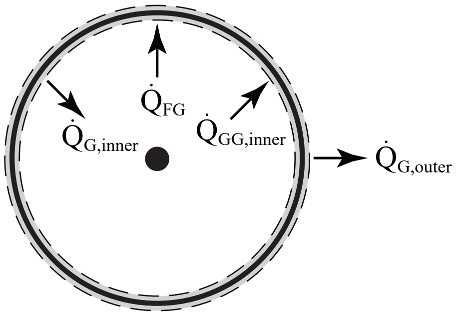
A possible way of deriving the glass temperature T_G is by setting up the outer or inner energy balance for the glass bulb.

Outer energy balance:

1 **Setting up the balance:**

Inside the glass bulb, both the filament and the inner surface of the glass bulb emit their surface brightness onto the glass. Some of this radiation is reflected by the glass bulb. Finally, the inner surface emits radiation as a grey body radiator.

On the exterior, a portion of the radiated surface brightness from the inner surface of the glass bulb and the filament is transmitted. Additionally, the outer surface of the glass bulb emits radiation as a grey body radiator.



With this given, the outer energy balance for the glass bulb is:

$$0 = \dot{Q}_{FG} + \dot{Q}_{GG,inner} - \dot{Q}_{G,inner} - \dot{Q}_{G,outer} \tag{IV.4.1}$$

2 **Defining the elements within the balance:**

The radiative transport from the filament to the glass bulb yields from the respective view factor and the surface brightness of the filament:

$$\dot{Q}_{FG} = \Phi_{FG}\dot{Q}_F \tag{IV.4.2}$$

Since the surface of the filament in comparison to the glass body is small, it can be assumed that:

$$\Phi_{FG} = 1, \tag{IV.4.3}$$

and

$$\Phi_{GG,inner} = 1. \tag{IV.4.4}$$

The radiation emitted by the inner surface of the glass bulb itself is written as:

$$\dot{Q}_{GG,inner} = \Phi_{GG,inner}\dot{Q}_{G,inner} \tag{IV.4.5}$$

The surface brightness of the inside of the glass bulb is written as:

$$\dot{Q}_{G,inner} = \dot{Q}_{\epsilon,G,inner} + \dot{Q}_{\rho,G,inner} + \dot{Q}_{\tau,G,inner} \tag{IV.4.6}$$

The emission term can be written as:

$$\dot{Q}_{\epsilon,\mathrm{G,inner}} = \epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}} \tag{IV.4.7}$$

The reflection term yields from the reflected surface brightness of the filament and the inner surface of the glass bulb:

$$\dot{Q}_{\rho,\mathrm{G,inner}} = \rho\left(\dot{Q}_{\mathrm{FG}} + \dot{Q}_{\mathrm{G,inner}}\right) \tag{IV.4.8}$$

Lastly, since no radiation from the outside is transmitted, the inner surface its transmission term is equal to zero:

$$\dot{Q}_{\tau,\mathrm{G,inner}} = 0 \tag{IV.4.9}$$

Plugging in equations (IV.4.7) - (IV.4.9) into the definition of the surface brightness given in equation (IV.4.6):

$$\dot{Q}_{\mathrm{G,inner}} = \frac{\epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho} \tag{IV.4.10}$$

Furthermore, the surface brightness of the outer surface of the glass bulb is described as:

$$\dot{Q}_{\mathrm{G,outer}} = \dot{Q}_{\epsilon,\mathrm{G,outer}} + \dot{Q}_{\rho,\mathrm{G,outer}} + \dot{Q}_{\tau,\mathrm{G,outer}}, \tag{IV.4.11}$$

where the emission term is written as:

$$\dot{Q}_{\epsilon,\mathrm{G,outer}} = \epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}}, \tag{IV.4.12}$$

since no radiation from the ambient is received from the ambient, the outer surface does not reflect any radiation:

$$\dot{Q}_{\rho,\mathrm{G,outer}} = 0, \tag{IV.4.13}$$

and the transmitted radiation arises from fractions of the surface brightness of the filament and the inner surface of the glass that passes through the outer surface:

$$\dot{Q}_{\tau,\mathrm{G,outer}} = \tau\left(\dot{Q}_{\mathrm{FG}} + \dot{Q}_{\mathrm{GG,inner}}\right) \tag{IV.4.14}$$

Conclusion

3 **Inserting and rearranging:**

Plugging the definitions of all terms into the outer energy balance yields:

$$0 = \dot{Q}_{\mathrm{F}} - \epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}} - \tau\left(\dot{Q}_{\mathrm{F}} + \frac{\epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho}\right), \tag{IV.4.15}$$

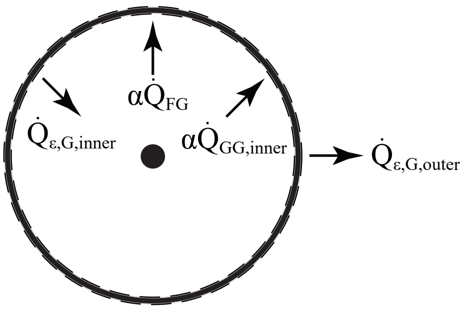
where T_{G} is the only unknown parameter and thus can be determined from this energy balance.

Inner energy balance:

1 Setting up the balance:

Inside the glass bulb, both the filament and the inner surface of the glass bulb emit radiation onto the glass that is partially absorbed.

On the exterior, also radiation is emitted by the outer surface of the glass.



The inner energy balance reads:

$$0 = \alpha\dot{Q}_{\mathrm{FG}} + \alpha\dot{Q}_{\mathrm{G,inner}} - \dot{Q}_{\epsilon,\mathrm{G,inner}} - \dot{Q}_{\epsilon,\mathrm{G,outer}} \tag{IV.4.16}$$

2 Defining the elements within the balance:

The radiative transport from the filament to the glass bulb yields from the respective view factor and the surface brightness of the filament:

$$\dot{Q}_{\mathrm{FG}} = \Phi_{\mathrm{FG}}\dot{Q}_{\mathrm{F}} \tag{IV.4.17}$$

Since the surface of the filament in comparison to the glass body is small, it can be assumed that:

$$\Phi_{\mathrm{FG}} = 1, \tag{IV.4.18}$$

and

$$\Phi_{\mathrm{GG,inner}} = 1. \tag{IV.4.19}$$

The emitted radiation by the in- and outside of the glass bulb can be written as:

$$\dot{Q}_{\epsilon,\mathrm{G,inner}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} \tag{IV.4.20}$$

$$\dot{Q}_{\epsilon,\mathrm{G,outer}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} \tag{IV.4.21}$$

Furthermore, from Kirchoff's law, it yields that:

$$\epsilon = \alpha \tag{IV.4.22}$$

The surface brightness of the inside of the glass bulb is written as:

$$\dot{Q}_{\mathrm{G,inner}} = \dot{Q}_{\epsilon,\mathrm{G,inner}} + \dot{Q}_{\rho,\mathrm{G,inner}} + \dot{Q}_{\tau,\mathrm{G,inner}} \tag{IV.4.23}$$

The emission term can be written as:

$$\dot{Q}_{\epsilon,\mathrm{G,inner}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} \tag{IV.4.24}$$

The reflection term yields from the reflected surface brightness of the filament and the inner surface of the glass bulb:

$$\dot{Q}_{\rho,\mathrm{G,inner}} = \rho\left(\dot{Q}_{\mathrm{FG}} + \dot{Q}_{\mathrm{G,inner}}\right) \tag{IV.4.25}$$

Lastly, since no radiation from the outside is transmitted, the inner surface its transmission term is equal to zero:

$$\dot{Q}_{\tau,\mathrm{G,inner}} = 0 \tag{IV.4.26}$$

Plugging in equations (IV.4.24) - (IV.4.26) into the definition of the surface brightness given in equation (IV.4.23):

$$\dot{Q}_{\mathrm{G,inner}} = \frac{\epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho} \tag{IV.4.27}$$

Conclusion

3 **Inserting and rearranging:**

Inserting and rewriting yields:

$$\epsilon\dot{Q}_{\mathrm{F}} + \frac{\epsilon^2\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \epsilon\rho\dot{Q}_{\mathrm{F}}}{1-\rho} - 2\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} = 0, \tag{IV.4.28}$$

where T_{G} is the only unknown parameter and thus can be determined from this energy balance.