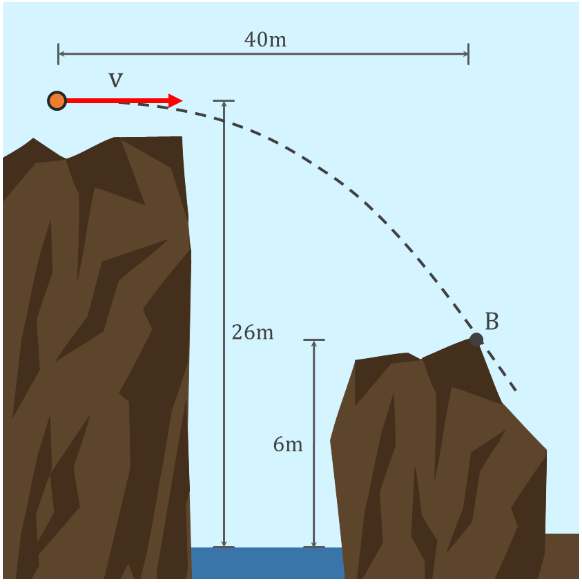
Rock Thrown from Cliff



With what minimum horizontal velocity v can a boy throw a rock at A and have it just clear the obstruction at B? Neglect air resistance and take g = 10 m/s^2.

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (a \cdot t + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t (a \cdot t + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2} a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

For the displacement in x-and y-direction, this results in:

$$x(t) = \frac{1}{2} a_x \cdot t^2 + v_{x,0} \cdot t + s_{x,0} \tag{7}$$

$$y(t) = \frac{1}{2}a_y \cdot t^2 + v_{y,0} \cdot t + s_{y,0} \tag{8}$$

Given:
Initial height of the ball (with respect to the cliff): $H_0 = s_{y,0} = s_{x,0} = 0m$
Gravitational constant: $g = 10m/s^2$

Point B is located at $x = 40m$ and $y = -20m$. Together with the the fact that there is no horizontal acceleration ($a_x = 0m/s^2$) and vertical initial velocity ($v_{y,0} = 0m/s$), we can solve for $v_{x,0}$:

$$x(t) = \frac{1}{2}a_x \cdot t^2 + v_{x,0} \cdot t + s_{x,0} \quad \Rightarrow \quad 40 = v_{x,0} \cdot t \tag{9}$$

$$y(t) = \frac{1}{2}a_y \cdot t^2 + v_{y,0} \cdot t + s_{y,0} \quad \Rightarrow \quad -20 = -\frac{1}{2}g \cdot t^2 \tag{10}$$

For the last equation the time can be calculated before the ball reaches point B:

$$t = \sqrt{\frac{2 \cdot v_{y,0}}{g}} \quad \Rightarrow \quad t = \sqrt{\frac{2 \cdot 20}{10}} = 2s \tag{11}$$

Inserting t = 2s into Equation 9 results in:

$$40 = v_{x,0} \cdot 2 \quad \Rightarrow \quad v_{x,0} = 20m/s \tag{12}$$