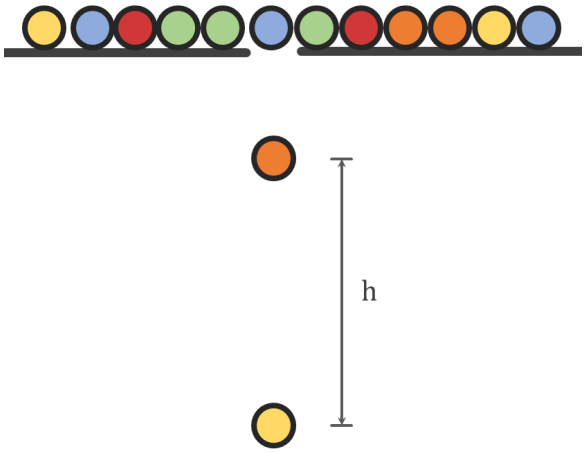
Falling Marbles



Marbles fall from rest through an opening at a steady rate of three per second. Determine the vertical separation h of two consecutive marbles when the lower one has dropped 7 meters. Neglect air resistance and take g = 10 m^2.

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_{0}^{t} \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_{0}^{t} (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

Given:
Distance: $h = 7m$

Gravitational acceleration: $g = 10m/s^2$

The marbles fall from rest, thus $v_0 = 0$. Seen from the top $s_0 = 0$, thus Equation 6 simplifies to:

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \Rightarrow s(t) = \frac{1}{2}a \cdot t^2 \tag{7}$$

Where $a = -g = -10m/s^2$ (since it points in the negative y-direction). The time it takes for the ball to drop 7m ($s = -7$) can be calculated using:

$$s(t) = \frac{1}{2}a \cdot t^2 \Rightarrow t = \sqrt{\frac{2 \cdot s}{a}} = \sqrt{\frac{2 \cdot -7}{-10}} \approx 1.18s \tag{8}$$

Three balls are dropped every second, thus every $\frac{1}{3}$s a ball is dropped. It took 1.18 seconds for the first ball to reach 7m, and after $\frac{1}{3}$s a second ball was dropped. The distance the second ball travelled is thus:

$$s(t) = \frac{1}{2}a \cdot t^2 \Rightarrow s(t_1 - t_2) = \frac{1}{2} \cdot -10 \cdot (1.18 - \frac{1}{3})^2 = -3.61m \tag{9}$$

Thus the distance between two consecutive marbles is $h = -3.61 - -7 = 3.39m$