



Formulary Heat Transfer: Convection

Version 1 from 2021

from 7th June 2021

1. Dimensionless numbers

Dimensionless numbers – Fluid dynamics

$$\text{Gr}_L = \frac{\beta g \rho^2 (T_W - T_\infty) L^3}{\eta^2} \quad (\text{Grashof number})$$

$$\text{Re}_L = \frac{\rho u L}{\eta} \quad (\text{Reynolds number})$$

Dimensionless numbers – Heat transfer

$$\text{Bi}_L = \frac{\alpha L}{\lambda} \quad (\text{Biot number})$$

$$\text{Fo} = \frac{at}{L^2} \quad \text{with} \quad a = \frac{\lambda}{\rho c_p} \quad (\text{Fourier number})$$

$$\text{Nu}_L = \frac{\alpha L}{\lambda} \quad (\text{Nusselt number})$$

$$\text{Pr} = \frac{\eta c_p}{\lambda} = \frac{\nu}{a} \quad (\text{Prandtl number})$$

$$\text{St}_L = \frac{\text{Nu}}{\text{Re}_L \text{Pr}} \quad (\text{Stanton number})$$

Dimensionless numbers – Mass transfer

$$\text{Le} = \frac{\lambda}{\rho D c_p} = \frac{a}{D} \quad (\text{Lewis number})$$

$$\text{Sc} = \frac{\eta}{\rho D} \quad (\text{Schmidt number})$$

$$\text{Sh}_L = \frac{g L}{\rho D} \quad (\text{Sherwood number})$$

4. Convection

$$\rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial y} + \rho w c_p \frac{\partial T}{\partial z} = \dots \quad (\text{Equation of energy conservation})$$

$$\dots = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) + \dot{\Phi}'''$$

Convective heat transfer

$$\frac{\dot{Q}}{A} = \dot{q}_w'' = \alpha(T_w - T_{fl}) \quad (\text{Convective heat flux})$$

$$W = \frac{1}{\alpha A} \quad (\text{Heat resistance})$$

$$\alpha = \frac{-\left(\lambda \frac{dT}{dy}\right)_{\text{Fluid, w}}}{T_w - T_{fl}} \quad (\text{Heat transfer coefficient})$$

$$\bar{\alpha} = \frac{1}{L} \int_0^L \alpha(x) dx \quad (\text{Average h.t. coefficient})$$

Boundary layer equations (Approximation with linear velocity profile)

$$\frac{\delta_u}{x} \approx \sqrt{\frac{12 \eta}{\rho u_\infty x}} = \sqrt{\frac{12}{\text{Re}_x}} \quad (\text{Thickness of the velocity boundary layer})$$

$$\frac{\delta_T}{\delta_u} \approx \left(\frac{\lambda}{\eta c_p} \right)^{1/3} = \frac{1}{\text{Pr}^{1/3}} \quad (\text{Thickness of the temperature boundary layer})$$

5. Heat transfer correlations

$$\Delta T_{\ln} = (T_w - T_{fl})_m = \frac{\Delta T_I - \Delta T_O}{\ln \frac{\Delta T_I}{\Delta T_O}} \quad (\text{Logarithmic temperature difference})$$

$$\dot{Q}_m = \bar{\alpha} A (T_w - T_{fl})_m \quad (\text{Average heat flow})$$

Forced convection flow along surfaces

$$Nu_x = f(\text{Re}_x, \text{Pr}, \dots) \quad (\text{Nusselt-correlation})$$

$$T_{\text{prop.}} = \frac{T_w + T_\infty}{2} \quad (\text{Temperature for determination of properties})$$

- **Flat plate – laminar flow, isothermal surface (1)**

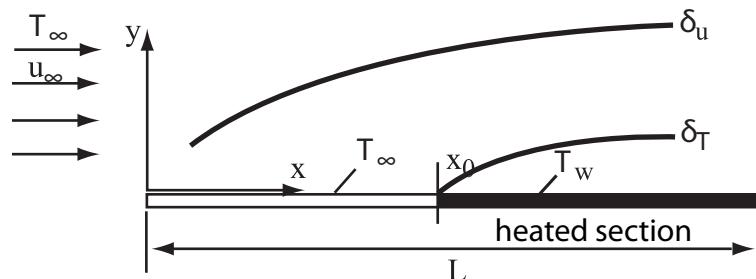
$$(0.6 < \text{Pr} < 10 \text{ and } \text{Re}_x < \text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5)$$

$$Nu_x = 0.332 \text{ Re}_x^{1/2} \text{Pr}^{1/3} \quad (\text{HTC.1})$$

$$\overline{Nu}_L = 0.664 \text{ Re}_L^{1/2} \text{Pr}^{1/3} \quad (\text{HTC.2})$$

- **Flat plate – laminar boundary layer flow, isothermal surface (2)**

Heating or cooling starts at $x = x_0$



$$(0.6 < \text{Pr} < 10 \text{ and } \text{Re}_x < \text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5)$$

$$Nu_x = 0.332 \text{ Re}_x^{1/2} \text{Pr}^{1/3} \left[1 - \left(\frac{x_0}{x} \right)^{3/4} \right]^{-1/3} \quad (\text{HTC.3})$$

$$\begin{aligned}\overline{\text{Nu}}_L &= \frac{L}{L - x_0} \frac{1}{\lambda} \int_{x_0}^L \alpha(x) dx \\ &= 0.664 \text{Re}_L^{1/2} \text{Pr}^{1/3} \frac{\left[1 - \left(\frac{x_0}{L}\right)^{3/4}\right]^{2/3}}{\left[1 - \frac{x_0}{L}\right]} \quad (\text{HTC.4})\end{aligned}$$

- Flat plate – turbulent boundary layer flow, isothermal surface**

($\text{Re}_{L,\text{crit}} \approx 2 \cdot 10^5$ and $5 \cdot 10^5 < \text{Re} < 10^7$)

$$\text{Nu}_x = 0.0296 \text{Re}_x^{0.8} \text{Pr}^{0.43} \quad (\text{HTC.5})$$

$$\overline{\text{Nu}}_L \approx 0.036 \text{Pr}^{0.43} (\text{Re}_L^{0.8} - 9400) \quad (\text{HTC.6})$$

- Cylinders in a flow parallel to their longitudinal axis**

If the diameter of the body is much greater compared to the thickness of the boundary layer, cylinders in longitudinal flow can be regarded as flat plates.

- Cylinders in a flow perpendicular to their longitudinal axis**

$$\overline{\text{Nu}}_d = C \text{Re}_d^m \text{Pr}^{0.4} \quad (\text{HTC.7})$$

Re_d	C	m
0.4 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 4000	0.683	0.466
4000 – 40000	0.193	0.618
40000 – 400000	0.0266	0.805

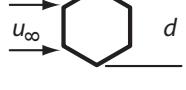
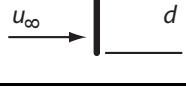
HTC.8 can be used as an alternative to HTC.7:

$$\overline{\text{Nu}}_d = \left[0.40 \text{Re}_d^{1/2} + 0.06 \text{Re}_d^{2/3}\right] \text{Pr}^{0.4} \left(\frac{\eta_\infty}{\eta_w}\right)^{1/4} \quad (\text{HTC.8})$$

here: $T_{\text{prop.}} = T_\infty$

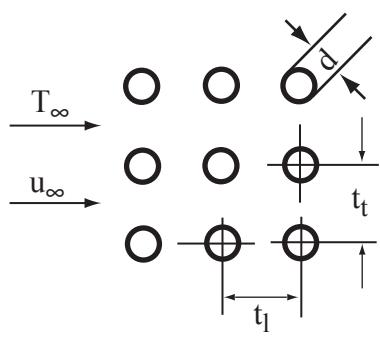
- Mean heat transfer for non circular cylinders

$$\overline{\text{Nu}}_d = C \text{ Re}_d^m \text{Pr}^{0.4} \quad (\text{HTC.9})$$

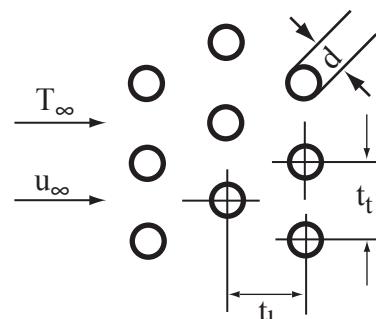
Geometry	Re_d	C	m
	$5 \cdot 10^3 - 1 \cdot 10^5$	0.246	0.588
	$5 \cdot 10^3 - 1 \cdot 10^5$	0.102	0.675
	$5 \cdot 10^3 - 1.95 \cdot 10^4$ $1.95 \cdot 10^4 - 1 \cdot 10^5$	0.160 0.0385	0.638 0.782
	$5 \cdot 10^3 - 1 \cdot 10^5$	0.153	0.638
	$4 \cdot 10^3 - 1.5 \cdot 10^4$	0.228	0.731

- Flow perpendicular to plain tube bundles

$$\overline{\text{Nu}}_d = 0.287 \text{ Re}_d^{0.6} \text{Pr}^{0.36} f_e \quad (\text{HTC.10})$$



in-line arrangement



staggered arrangement

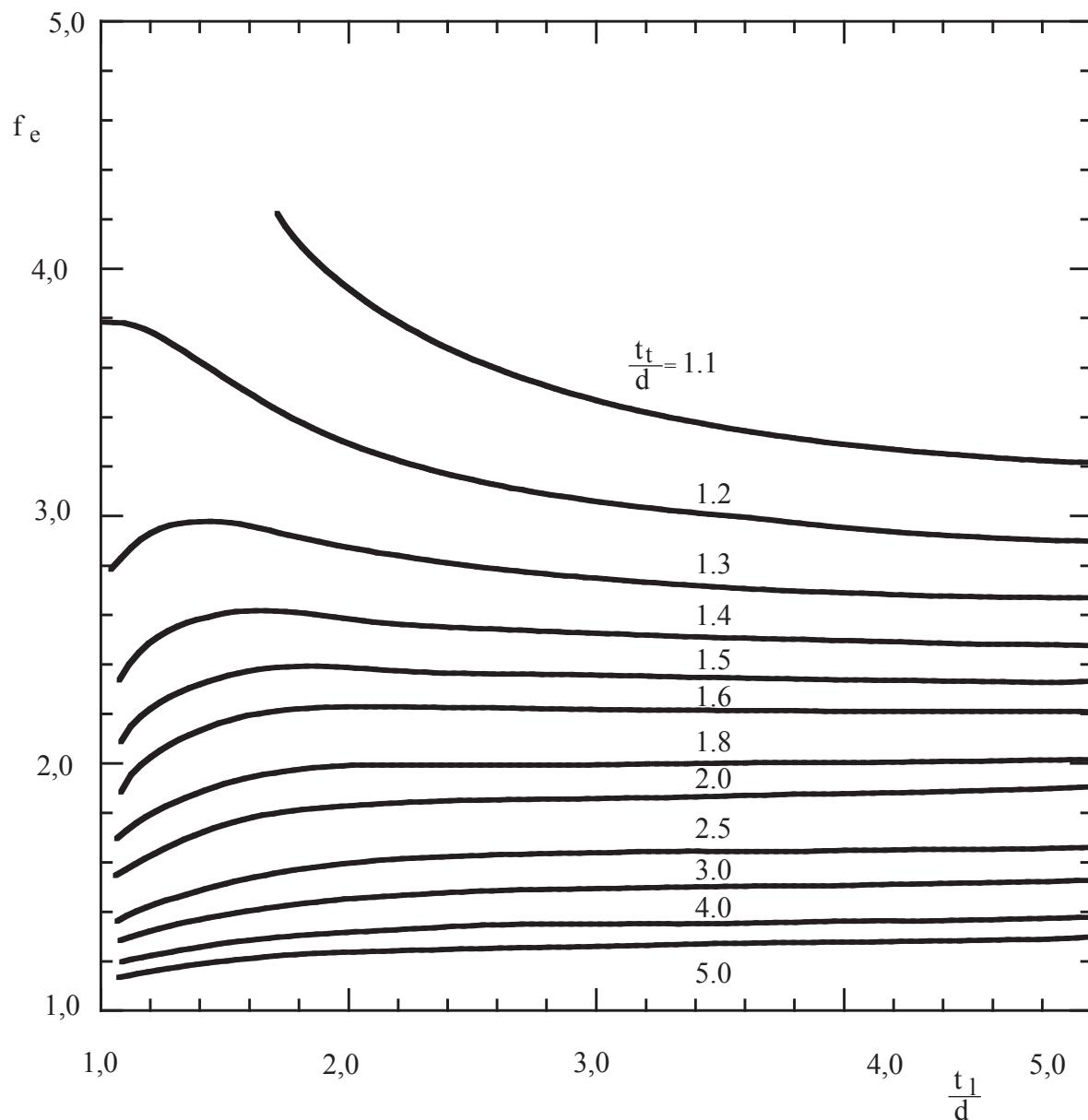


Figure 1: Arrangement factor, in-line arrangement

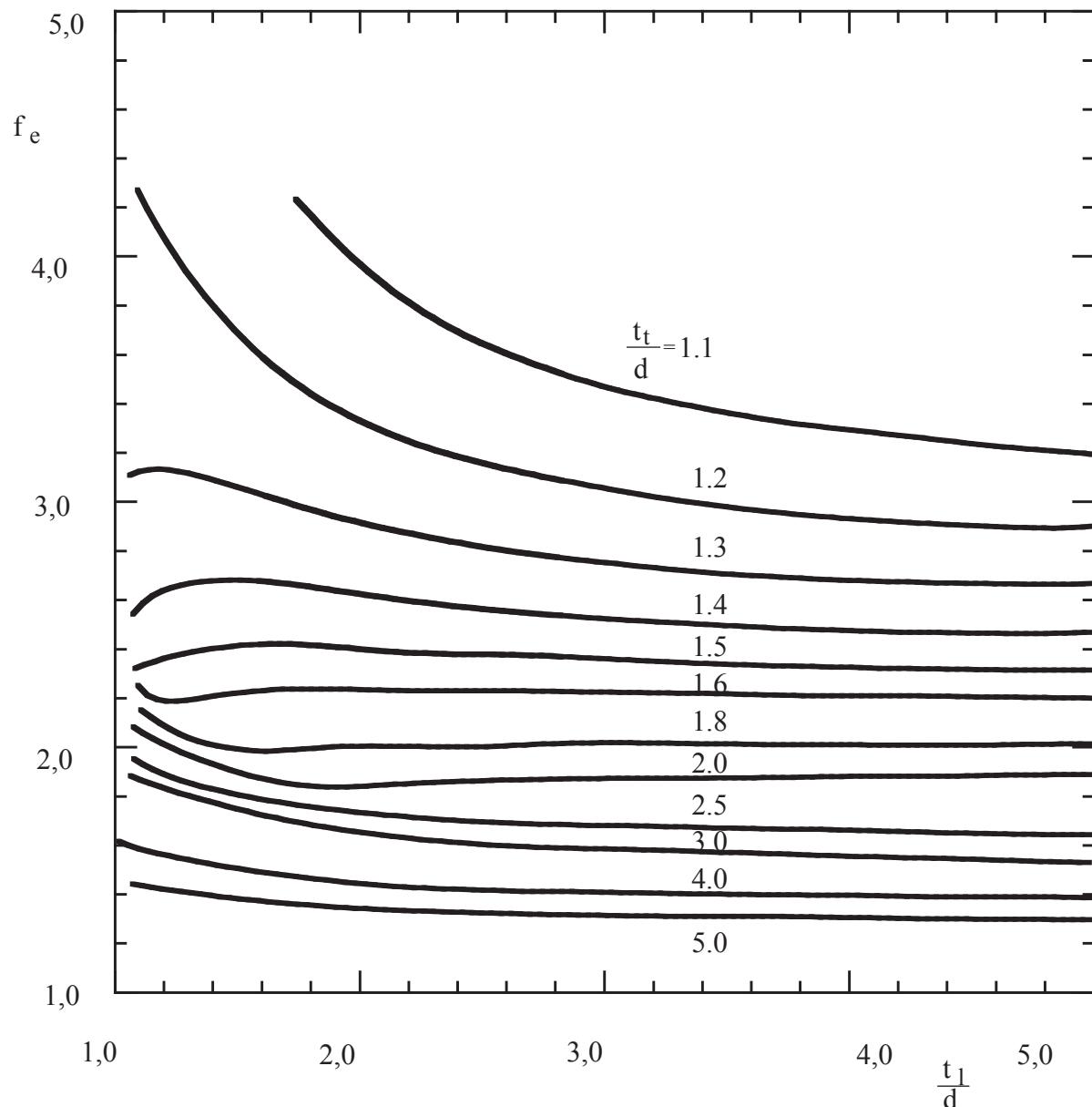


Figure 2: Arrangement factor, staggered arrangement

- Heat transfer from a surrounding fluid to spheres in the stream**

$$(0.7 < \text{Pr} < 380 \text{ and } 3.5 < \text{Re}_d < 8 \cdot 10^4)$$

$$\overline{\text{Nu}}_d = 2 + \left(0.4 \text{Re}_d^{1/2} + 0.06 \text{Re}_d^{2/3} \right) \text{Pr}^{0.4} \left(\frac{\eta_\infty}{\eta_w} \right)^{1/4} \quad (\text{HTC.11})$$

here: $T_{\text{prop.}} = T_\infty$

Forced convection in tubes, internal flow

$$\text{Nu}_x = f(\text{Re}_x, \text{Pr}, \dots) \quad (\text{Nusselt-correlation})$$

$$T_{\text{mat}} = \frac{T_{\text{fl,O}} + T_{\text{fl,I}}}{2} \quad (\text{Material property determination temperature})$$

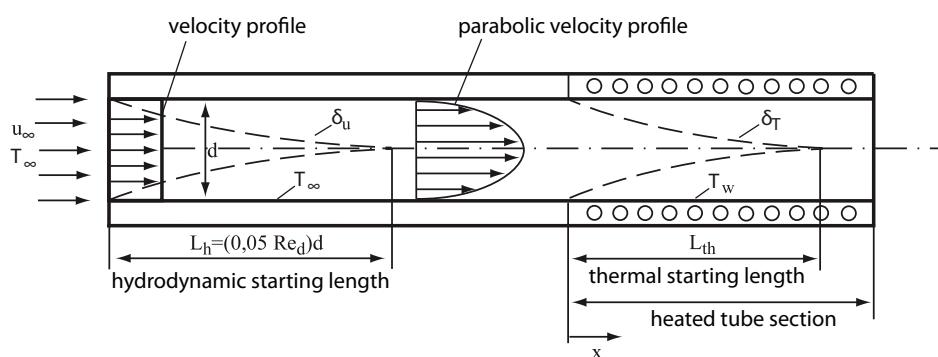
$$d_h = 4 \frac{A_c}{U} \quad (\text{Hydraulic mean diameter})$$

here: A_c cross-section area

U wetted perimeter

- Laminar flow in tubes – isothermal surface (1)**

Fully developed flow at the start of the heat transferring section of a tube ($\text{Re}_{d,\text{crit}} \approx 2300$)



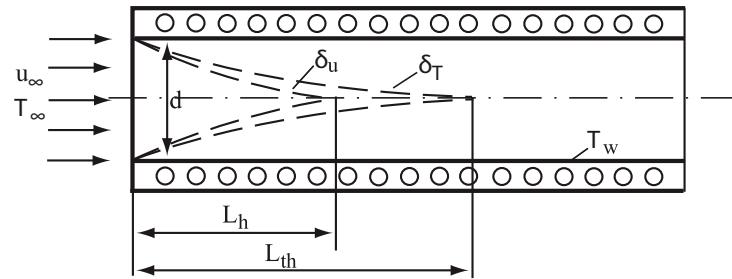
$$\overline{\text{Nu}}_d = \left(3.66 + \frac{0.19 \left(\text{Re}_d \text{Pr} \frac{d}{L} \right)^{0.8}}{1 + 0.117 \left(\text{Re}_d \text{Pr} \frac{d}{L} \right)^{0.467}} \right) \left(\frac{\eta}{\eta_w} \right)^{0.14} \quad (\text{HTC.12})$$

$$\frac{L_{th}}{d} \approx 0.05 \text{Re}_d \text{Pr} \quad (\text{Thermal starting length})$$

After L_{th} , the Nusselt number has a final value of $\overline{\text{Nu}}_\infty = 3.66 \left(\frac{\eta}{\eta_w} \right)^{0.14}$.

- **Laminar flow in tubes – isothermal surface (2)**

Simultaneous hydrodynamic and thermal start ($\text{Re}_{d,\text{crit}} \approx 2300$)



$$\overline{\text{Nu}}_d = \left(3.66 + \frac{0.0677 (\text{Re}_d \text{Pr} \frac{d}{L})^{1.33}}{1 + 0.1 \text{Pr} (\text{Re}_d \frac{d}{L})^{0.83}} \right) \left(\frac{\eta}{\eta_w} \right)^{0.14} \quad (\text{HTC.13})$$

$$\frac{L_{\text{th}}}{d} \approx 0.05 \text{ Re}_d \text{Pr} \quad (\text{Thermal starting length})$$

After L_{th} , the Nusselt number has a final value of $\overline{\text{Nu}}_\infty = 3.66 (\eta/\eta_w)^{0.14}$.

- **Laminar flow in tubes – impressed heat flow**

If instead of the wall temperature, the heat flow at the wall remains constant, then the heat transfer coefficients have values increased by 20%.

- **Turbulent flow in tubes – isothermal surface**

Simultaneous hydrodynamic and thermal start

($\text{Re}_{d,\text{crit}} \approx 2300$, $\text{Re}_d > 2300$, $0.6 < \text{Pr} < 500$ and $L/d > 1$)

$$\overline{\text{Nu}}_d = 0.0235 (\text{Re}_d^{0.8} - 230) (1.8 \text{Pr}^{0.3} - 0.8) \left(1 + \left(\frac{d}{L} \right)^{2/3} \right) \left(\frac{\eta}{\eta_w} \right)^{0.14} \quad (\text{HTC.14})$$

Simplified Nusselt law for the *fully developed* turbulent pipe flow
($\text{Re}_{d,\text{crit}} \approx 2300$, $3000 < \text{Re}_d < 10^5$ and $L/d > 40$)

$$\overline{\text{Nu}}_d = 0.027 \text{Re}_d^{0.8} \text{Pr}^{1/3} \left(\frac{\eta}{\eta_w} \right)^{0.14} \quad (\text{HTC.15})$$

- **Turbulent flow in tubes – impressed heat flow**

The heat transfer coefficients at impressed heat flows are comparable to the coefficients obtained at constant wall temperatures.

Natural convection

$$\text{Nu}_x = f(\text{Gr}_x, \text{Pr}, \dots) \quad (\text{Nusselt-correlation})$$

$$T_{\text{prop.}} = \frac{T_w + T_\infty}{2} \quad (\text{Temperature for determination of properties})$$

$$\beta = \frac{1}{T_\infty} \quad (\text{Isobaric expansion coefficient})$$

- **Vertical plate – laminar boundary layer flow, isothermal surface**

$$\text{Nu}_x = 0.508 \left(\frac{\text{Pr}}{0.952 + \text{Pr}} \right)^{1/4} (\text{Gr}_x \text{Pr})^{1/4} \quad (\text{HTC.16})$$

$$\overline{\text{Nu}}_L = C (\text{Gr}_L \text{Pr})^{1/4} \quad (\text{HTC.17})$$

for $\text{Gr}_L \text{Pr} < \text{Gr}_{L,crit} \text{Pr} = 4 \cdot 10^9$

Pr	0.003	0.01	0.03	0.72	1	2	10	100	1000	∞
C	0.182	0.242	0.305	0.516	0.535	0.568	0.620	0.653	0.665	0.670

- **Vertical plate – laminar boundary layer flow, impressed heat flow**

$$\text{Nu}_x = 0.60 (\text{Gr}_x^* \text{Pr})^{1/5} \quad (\text{HTC.18})$$

for $10^5 < \text{Gr}_x^* < 10^{11}$

with $\text{Gr}_x^* = \text{Gr}_x \text{Nu}_x = \frac{\rho^2 g \beta \dot{q}_w'' x^4}{\lambda \eta^2}$

- **Vertical plate – turbulent boundary layer flow, isothermal surface**

$$\overline{\text{Nu}}_L = 0.13 (\text{Gr}_L \text{Pr})^{1/3} \quad (\text{HTC.19})$$

for $10^9 < (\text{Gr}_L \text{Pr}) < 10^{12}$

- **Vertical cylinder – laminar and turbulent boundary layer flow**

For diameter-length-ratios of $d/L > 35 \text{Gr}_L^{-1/4}$, the relationships valid for the vertical plate can be applied.

- **Horizontal cylinder – isothermal surface**

laminar: $\overline{\text{Nu}}_d = 0.53 (\text{Gr}_d \text{Pr})^{1/4}$ (HTC.20)

for $10^4 < \text{Gr}_d \text{Pr} < 10^9$

turbulent: $\overline{\text{Nu}}_d = 0.13 (\text{Gr}_d \text{Pr})^{1/3}$ (HTC.21)

for $10^9 < \text{Gr}_d \text{Pr} < 10^{12}$

- **Horizontal plate – isothermal surface**

Free upper side with $T_w > T_\infty$ or free lower side with $T_w < T_\infty$

laminar: $\overline{\text{Nu}}_L = 0.54 (\text{Gr}_L \text{Pr})^{1/4}$ (HTC.22a)

for $10^4 < \text{Gr}_L \text{Pr} < 10^7$

turbulent: $\overline{\text{Nu}}_L = 0.15 (\text{Gr}_L \text{Pr})^{1/3}$ (HTC.23a)

for $10^7 < \text{Gr}_L \text{Pr} < 10^9$

Free upper side with $T_w < T_\infty$ or free lower side with $T_w > T_\infty$

laminar: $\overline{\text{Nu}}_L = 0.27 (\text{Gr}_L \text{Pr})^{1/4}$ (HTC.24a)

for $10^5 < \text{Gr}_L \text{Pr} < 10^{10}$

- **Horizontal plate – impressed heat flow**

Free upper side with $T_w > T_\infty$ or free lower side with $T_w < T_\infty$

laminar: $\overline{\text{Nu}}_L = 0.16 (\text{Gr}_L \text{Pr})^{1/4}$ (HTC.22b)

for $\text{Gr}_L \text{Pr} < 2 \cdot 10^8$

turbulent: $\overline{\text{Nu}}_L = 0.13 (\text{Gr}_L \text{Pr})^{1/3}$ (HTC.23b)

for $5 \cdot 10^8 < \text{Gr}_L \text{Pr} < 10^{11}$

Free upper side with $T_W < T_\infty$ or free lower side with $T_W > T_\infty$

$$\text{laminar: } \overline{\text{Nu}}_L = 0.58 (\text{Gr}_L \text{Pr})^{1/5} \quad (\text{HTC.24b})$$

$$\text{for } 10^6 < \text{Gr}_L \text{Pr} < 10^{11}$$

- **Fluid layers between isothermal, vertical walls**

Height/distance ratio $3.1 < H/s < 42.2$

$$\text{heat conduction only: } \overline{\text{Nu}}_s = 1 \quad \text{for } \text{Gr}_s < 2 \cdot 10^3$$

$$\text{laminar: } \overline{\text{Nu}}_s = 0.20 (H/s)^{-1/9} (\text{Gr}_s \text{Pr})^{1/4} \quad (\text{HTC.25})$$

$$\text{for } 2 \cdot 10^3 < \text{Gr}_s < 2 \cdot 10^4$$

$$\text{turbulent: } \overline{\text{Nu}}_s = 0.071 (H/s)^{-1/9} (\text{Gr}_s \text{Pr})^{1/3} \quad (\text{HTC.26})$$

$$\text{for } 2 \cdot 10^5 < \text{Gr}_s < 10^7$$

- **Fluid layers between isothermal, horizontal walls**

$$\text{heat conduction only: } \overline{\text{Nu}}_s = 1 \quad \text{for } \text{Gr}_s < 2 \cdot 10^3$$

$$\text{laminar: } \overline{\text{Nu}}_s = 0.21 (\text{Gr}_s \text{Pr})^{1/4} \quad (\text{HTC.27})$$

$$\text{for } 2 \cdot 10^3 < \text{Gr}_s < 3.2 \cdot 10^5$$

$$\text{turbulent: } \overline{\text{Nu}}_s = 0.075 (\text{Gr}_s \text{Pr})^{1/3} \quad (\text{HTC.28})$$

$$\text{for } 3.2 \cdot 10^5 < \text{Gr}_s < 10^7$$

If heated from above, the relationships for heat conduction only are valid.

Appendix A – Properties of various materials

Tabelle 1: Metals at 20°C

	ρ 10 ³ kg/m ³	c kJ/kg K	λ W/m K	a 10 ⁻⁶ m ² /s
Aluminum	2,70	0,888	237	98,80
Lead	11,34	0,129	35	23,90
Chromium	6,92	0,440	91	29,90
Iron	7,86	0,452	81	22,80
Gold	19,26	0,129	316	127,20
Copper	8,93	0,382	399	117,00
Magnesium	1,74	1,020	156	87,90
Manganese	7,42	0,473	21	6,00
Molybdenum	10,20	0,251	138	53,90
Sodium	9,71	1,220	133	11,20
Nickel	8,85	0,448	91	23,00
Platinum	21,37	0,133	71	25,00
Silver	10,50	0,235	427	173,00
Titanium	4,50	0,522	22	9,40
Wolfram	19,00	0,134	173	67,90
Zinc	7,10	0,387	121	44,00
Tin, white	7,29	0,225	67	40,80
Bronze	8,80	0,377	62	18,70
Cast iron	7,80	0,540	42...50	10...12
Carbon steel (<0,4% C)	7,85	0,465	42...50	12...15
Cr-Ni-steel (X12CrNi 18,8)	7,80	0,500	15	3,80

Tabelle 2: Non-metal solids at 20°C

	ρ 10 ³ kg/m ³	c kJ/kg K	λ W/m K	a 10 ⁻⁶ m ² /s
Acryl glass	1,18	1,44	0,184	0,108
Asphalt	2,12	0,92	0,7	0,36
Concrete	2,1	0,88	1	0,54
Ice (water 0°C)	0,917	2,04	2,25	1,203
Soil coarse gravel	2,04	1,84	0,52	0,14
Sand, dry	1,65	0,8	0,27	0,2
Sand, wet	1,75	1	0,58	0,33
Clay	1,45	0,88	1,28	1
Glass.				
window	2,48	0,7	0,87	0,5
mirror	2,7	0,8	0,76	0,35
quarz	2,21	0,73	1,4	0,87
Glass wool	1,2	0,66	0,046	0,58
Gypsum	1	1,09	0,51	0,47
Granite	2,75	0,89	2,9	1,18
Cork	0,19	1,88	0,041	0,115
Marble	2,6	0,8	2,8	1,35
Mortar	1,9	0,8	0,93	0,61
Paper	0,7	1,2	0,12	0,14
Polyethylene	0,92	2,3	0,35	0,17
Polytetrafluorethylene	2,2	1,04	0,23	0,1
PVC	1,38	0,96	0,15	0,11
Porcelain (95°C)	2,4	1,08	1,03	0,4
Hard coal	1,35	1,26	0,26	0,15
Fir wood (radial)	0,415	2,72	0,14	0,12
Plaster	1,69	0,8	0,79	0,58
Bricks	1,6...1,8	0,84	0,38...0,52	0,28...0,34

Tabelle 3: Liquids at 1 bar

	<i>T</i>	ρ	<i>c</i>	λ	ν	<i>a</i>	Pr
	°C	10^3 kg/m^3	kJ/kg K	W/m K	$10^{-6} \text{ m}^2/\text{s}$	$10^{-6} \text{ m}^2/\text{s}$	1
Nitrogen	-190	0,861	1,988	0,161	0,321	0,0939	3,42
Water	0	0,9998	4,218	0,561	1,793	0,133	13,48
	20	0,9982	4,181	0,598	1,004	0,1434	7,001
	40	0,9922	4,177	0,631	0,658	0,1521	4,3280
	60	0,9832	4,184	0,654	0,475	0,1591	2,983
	80	0,9718	4,197	0,67	0,365	0,1643	2,221
	99,63	0,9586	4,216	0,679	0,295	0,168	1,757
Aqueous non-organic solution							
21% NaCl	-10	1,187	3,312	0,528	4,02	0,136	29,5
Benzene	20	0,879	1,738	0,154	0,74	0,101	7,33
Methanol	20	0,792	2,495	0,22	0,737	0,111	6,57
Fuel oil	20	0,819	2	0,116	1,82	0,0709	25,7
	100	0,766	2,38	0,104	0,711	0,0572	12,4
Mercury	20	13,55	0,139	9,3	0,115	4,9	0,023

Tabelle 4: Gases at 1 bar

	<i>T</i>	ρ	<i>c</i>	λ	ν	<i>a</i>	Pr
	°C	kg/m^3	kJ/kg K	10^{-3} W/m K	$10^{-6} \text{ m}^2/\text{s}$	$10^{-6} \text{ m}^2/\text{s}$	1
Air	-200	5,106	1,186	6,886	0,979	1,137	0,8606
	-100	2,019	1,011	16,2	5,829	7,851	0,7423
	0	1,275	1,006	24,18	13,52	18,83	0,7179
	20	1,188	1,007	25,69	15,35	21,47	0,7148
	40	1,112	1,007	27,16	17,26	24,24	0,7122
	80	0,9859	1,01	30,01	21,35	30,14	0,7083
	100	0,9329	1,012	31,39	23,51	33,26	0,707
	200	0,7356	1,026	37,95	35,47	50,3	0,7051
	400	0,517	1,069	49,96	64,51	90,38	0,7137
	600	0,3986	1,116	61,14	99,63	137,5	0,7247
	800	0,3243	1,155	71,54	140,2	191	0,7342
	1000	0,2734	1,185	80,77	185,9	249,2	0,7458
Steam	100	0,5896	2,042	25,08	20,81	20,83	0,999
	200	0,4604	1,975	33,28	35,14	36,6	0,96
	400	0,3223	2,07	54,76	75,86	82,07	0,9243
	600	0,2483	2,203	79,89	131,4	146,1	0,8993
	800	0,2019	2,343	107,3	199,9	226,8	0,8816
	1000	0,1702	2,478	163,3	280	323,2	0,8665
Hydrogen	0	0,0886	14,24	176	95	139	0,68
	50	0,0748	14,36	202	126	188	0,67
	100	0,0649	14,44	229	159	244	0,65
Carbon dioxide	0	1,95	0,829	14,3	7,1	8,86	0,8
	50	1,648	0,875	17,8	9,8	12,3	0,8
	100	1,428	0,925	21,3	12,4	16,1	0,8
Helium	27	0,1625	5,193	155,7	122,6	184,5	0,655