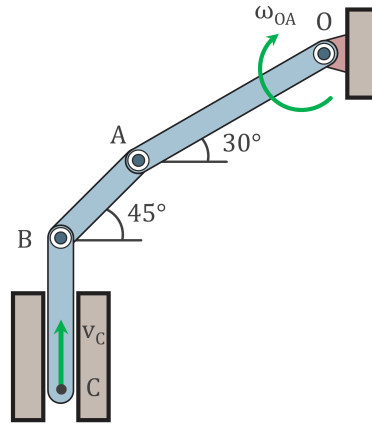# Two Bar System

A three-bar system is shown in the figure. Bar OA rotates with an angle velocity $\omega_{OA} = -4$ rad/s. The length of OA is equal to $L_{OA} = L_{BC} = 2$ m. The length of AB is 1 m. Find the angular velocity of bar AB.

Using known expressions (for rigid bodies):

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O} = \mathbf{v}_O + \boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O} \tag{1}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} \tag{2}$$

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C} = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} \tag{3}$$

Given quantities:

Velocity of point C: v_C
Angular velocity of bar OA: $\omega_{OA} = -4$ rad/s
Length OA: $L_{OA} = 2$ m
Length AB: $L_{AB} = 1$ m
Length BC: $L_{BC} = 2$ m
Angle OA with horizontal: $\theta = 30°$
Angle BA with horizontal: $\gamma = 45°$

Solution:

Looking at the image, it becomes apparent that point O is fixed, so $\mathbf{v}_O = 0$ m/s. Using Equation (1) gives:

$$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{OA} \end{pmatrix} \times \begin{pmatrix} -\sqrt{3} \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -\sqrt{3} \\ 0 \end{pmatrix} \omega_{OA} \qquad (4)$$

Inserting this in Equation (2) results in:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{pmatrix} 1 \\ -\sqrt{3} \\ 0 \end{pmatrix} \omega_{OA} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -\frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \\ 0 \end{pmatrix} = \qquad (5)$$

$$\begin{pmatrix} 1 \\ -\sqrt{3} \\ 0 \end{pmatrix} \omega_{OA} + \begin{pmatrix} \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \\ 0 \end{pmatrix} \omega_{AB} = \begin{pmatrix} \frac{1}{2}\sqrt{2} \cdot \omega_{AB} + \omega_{OA} \\ -\frac{1}{2}\sqrt{2} \cdot \omega_{AB} - \sqrt{3} \cdot \omega_{OA} \\ 0 \end{pmatrix}$$

Now we calculate the velocity of B seen from point C using Equation 3, this results in the following.

$$\mathbf{v}_B = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \begin{pmatrix} 0 \\ v_C \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \qquad (6)$$

Since bar BC slides inside a slot, it does not rotate, hence $\omega_{BC} = 0$ rad/s. This results in an upward velocity of B.

$$\mathbf{v}_B = \begin{pmatrix} 0 \\ v_C \\ 0 \end{pmatrix} \qquad (7)$$

From Equation (5) and (7), ω_{AB} can be solved.

$$\begin{pmatrix} \frac{1}{2}\sqrt{2} \cdot \omega_{AB} + \omega_{OA} \\ -\frac{1}{2}\sqrt{2} \cdot \omega_{AB} - \sqrt{3} \cdot \omega_{OA} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ v_C \\ 0 \end{pmatrix} \quad \Rightarrow \quad \omega_{AB} = -\omega_{OA} \cdot \frac{2}{\sqrt{2}} \qquad (8)$$

Inserting $\omega_{OA} = -4$ rad/s gives.

$$\omega_{AB} = -\omega_{OA} \cdot \frac{2}{\sqrt{2}} \quad \Rightarrow \quad \omega_{AB} = 4 \cdot \frac{2}{\sqrt{2}} = 4 \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2} \text{ rad/s}$$