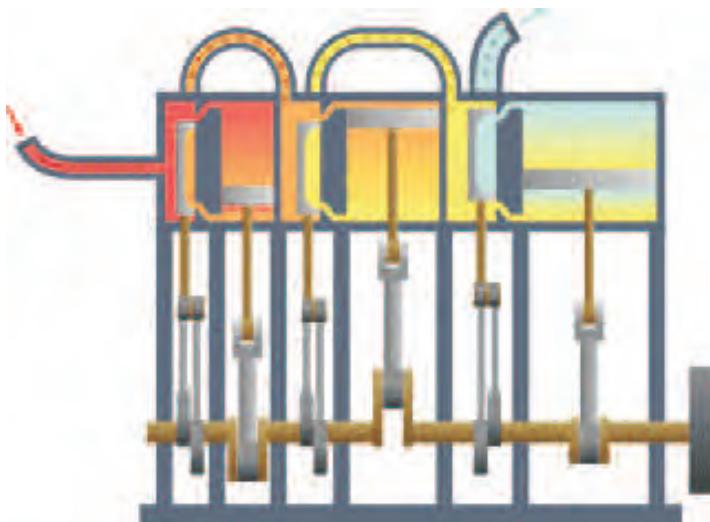


Engineering Thermodynamics 1 & 2

Mechanical Engineering
part of
ME module 2, Energy and Materials
ME module 3, Energy and Sustainability

Answers



University of Twente

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Preface

This document contains **answers** to the study problems for the course **Engineering Thermodynamics** for students **Mechanical Engineering at the University of Twente**. This course is part of the module 2, Energy and Materials (202000108) and module 3, Energy and Sustainability (202000114) of the Mechanical Engineering Bachelor programme. The book used in the course is: ***Thermodynamics, An Engineering Approach***, by Y.A. Çengel and M.A. Boles. Part of the problems in this document are based on problems of this book. Beside the book ***Thermodynamics: An Integrated Learning System***, by Philip S. Schmidt, Ofodike A. Ezekoye, John R. Howell en Derek K. Baker, is used as a source for the problems. The numbers of the tables refer to the tables in the book of Çengel and Boles. The problems are provided in a separate document.

Part of the problems will be treated during the tutorials. See the table on the next page for the division of the chapters and problems over the classes. The remaining problems can be used for extra practice. At the beginning of each chapter there are some conceptual questions about theoretical aspects. However, most of the questions are about application of the material. The answers to the problems, ordered in this document, are provided during the course. After the tutorial of a chapter is finished, the answers of that chapter are made available for the students.

The problems in the chapters 9, 10 and 11 are representative for the questions of the exam. However, the problems of the other chapters provide a basis to be able to understand the problems of the last chapters. Pay also special attention to the ones at the end of chapter 4. Chapter 8 about Exergy / Second Law Analysis is not covered in the course Engineering Thermodynamics. However, it is part of the projects *Analysis of an Energy System* and *Design of an Energy System* in modules 2 and 3, respectively.

Finally, I do have to mention that there can be mistakes in the answers, if you see mistakes please mention them to me. I hope the students can benefit from the answers in this document in the preparation of the exam.

Genie Stoffels
November 2021

Content of the course

The table below contains an overview of the division of the different chapters over the classes as well as an overview of the problems treated during the tutorials. Chapters refer to chapters in the book: **Thermodynamics, An Engineering Approach**, by Y.A. Çengel and M.A. Boles.

#	Subject	Chapter	Problems
1	Introduction, thermodynamic concepts and terminology	1	1.1, 1.2, 1.3, 1.6, 1.7, 1.8, 1.9, 1.15, 1.18, 1.10
2	Energy, work, heat transfer, enthalpy, entropy, efficiency	2, 4.1, 4.2	2.17, 2.18, 2.6, 2.11, 2.12, 2.13, 2.2, 2.7, 2.9, 2.14
3	State principle, phase transitions, liquids, saturated mixture, superheated steam, tables en diagrams	3.1 - 3.5	3.6, 3.9, 3.10, 3.11, 3.12, 3.13, 3.8
4	The first law of thermodynamics (conservation of energy) for open and closed systems	4.2, 5.1 - 5.4	4.3, 4.5, 4.9, 4.11, 4.12, 4.13, 4.17, 4.14
5	(Ir)reversible processes, entropy, the second law (entropy always increases) and second law applications	6.1, 6.6, 7.1 - 7.8, 7.12	5.2, 5.6, 5.9, 6.4, 6.5, 6.6, 6.7, 5.12, 5.4, 5.5, 5.8
6	Thermodynamic cycles for work, heat/cold, Carnot cycle, efficiencies and COP, perpetual mobiles	6.2 - 6.10, 9.2	7.5, 7.6, 7.7, 7.10, 7.11, 7.13, 7.12
7	Simple Rankine (vapor) cycle (steam turbine), compare to Carnot, design parameters	10.1 - 10.4	10.2, 10.4, 10.5, 10.6, 10.8
8	Advanced Rankine cycle (reheating, feed water heating), co-generation	10.4 - 10.5	10.11, 10.12, 10.15, 10.19, 10.16, 10.18, 10.9
	Exergy (second law analysis), quality of energy, Sankey and Grassmann diagrams	8, 9.12, 10.7	8.12, 8.2, 8.4, 8.7, 8.8, 8.11
9	Gases, ideal gas law, specific heat, internal energy and enthalpy of gases	3.6-3.8, 4.3-4.5, 7.9	3.17, 3.20, 3.22, 5.21 , 3.18
10	Gas cycles, simple Brayton cycle (gas turbine)	9.1, 9.3, 9.8	9.16, 9.17
11	Advanced Brayton cycle (inter cooling, reheating, regeneration), combined cycles, jet engines	9.9 - 9.11, 10.9	9.19, 9.18, 10.13, 10.14, 9.20, 9.15
13	Refrigeration and heat pump cycles, reversed Rankine cycle, coefficient of performance (COP)	11.1-11.4, 11.7	11.2, 11.3, 11.7, 11.5, 11.6
14	Reciprocating engines, Stirling, Otto, Diesel cycles	9.1 - 9.5	9.7, 9.12, 9.11, 9.6, 9.2, 9.10, 9.5, 9.9, 9.3
15	More about entropy	7.1 - 7.9, 7.13	5.10, 5.15, 5.28, 5.17, 5.16, 5.29, 5.21, 5.11

Subjects and problems covered in the lectures and tutorials (2020-2021).

Contents

Preface	iii
Course content	v
Contents	vi
1 Introduction, Thermodynamic Concepts and Terminology	1
1.1 Engineering Thermodynamics	1
1.2 Heat engines and refrigeration systems / heat pumps	1
1.3 Open, closed and isolated systems	2
1.4 State and thermodynamic variables or properties	3
1.5 Intensive and extensive properties	3
1.6 Processes	4
1.7 Dimensions	4
1.8 Mass and volume	5
1.9 Ideal gases	5
1.10 Galileo thermometer	6
1.11 Acceleration and force	6
1.12 Temperature	7
1.13 Pressure	7
1.14 Pressure on a diver	7
1.15 Pressure and height	7
1.16 Pressure in a tank	8
1.17 Pressure and manometer	8
1.18 Pressure in a cylinder	9
2 Energy, Work and Heat Transfer	11
2.1 Basics of Energy	11
2.2 Energy transfer mechanisms	11
2.3 Energy transfer by work and power	11
2.4 Work of a piston-cylinder device	12
2.5 Work done by compression	13
2.6 Work done by lifting a mass	13
2.7 Heat transfer	14
2.8 Work and heat transfer of a piston-cylinder device	14
2.9 Flow work and enthalpy 1	15
2.10 Flow work and enthalpy 2	16
2.11 Enthalpy of steam	16
2.12 Internal combustion engine	17
2.13 Hydroelectric power plant	17
2.14 Energy and power	17
2.15 Energy and Environment 1	18
2.16 Energy and Environment 2	18
2.17 Power needs of a car	19
2.18 Cooking with potential energy	19

3 Thermodynamic Properties of Pure Substances	21
3.1 Phases of matter	21
3.2 Tv -diagram	21
3.3 Incompressible liquid	22
3.4 Mixtures	22
3.5 Quality of a mixture	22
3.6 Use of tables: water	23
3.7 Use of tables, refrigerant 134a	25
3.8 Boiling of water	25
3.9 Vessel with 2 kg refrigerant 134a	25
3.10 Vessel with refrigerant 134a	25
3.11 Water mixture of liquid and vapor	26
3.12 Vaporization of water at constant pressure	27
3.13 Work done by heating 1	27
3.14 Work done by heating 2	28
3.15 Perfect gas	29
3.16 A balloon	29
3.17 An automobile tire	30
3.18 Two tanks with air	30
3.19 Enthalpy change of nitrogen	31
3.20 Heat transfer and work at constant enthalpy	31
3.21 Air in a turbine	32
3.22 Oxygen in a rigid tank	32
3.23 Two closed rigid system in thermal equilibrium	33
3.24 Two closed movable systems in equilibrium	33
4 The First Law of Thermodynamics, Conservation of Energy	35
4.1 First law of thermodynamics	35
4.2 Conservation of mass principle	35
4.3 Conservation of mass in a nozzle	36
4.4 Conservation of mass in a hair dryer	36
4.5 Work and power input for a compressor	36
4.6 Cooling a closed system	37
4.7 An insulated rigid tank	37
4.8 Nozzle	37
4.9 Steam Turbine	38
4.10 Refrigerant Compressor	40
4.11 Throttling valve	40
4.12 Feedwater heater	41
4.13 Steam/water heat exchanger	41
4.14 Steam turbine 2	42
4.15 Turbine with argon	43
4.16 Helium compressor	44
4.17 Mixing chamber	44
4.18 Refrigerant-134a heat exchanger	45
5 Entropy and the Second Law of Thermodynamics	47
5.1 Reversible and Irreversible Processes	47
5.2 Coffee	47
5.3 Insulated piston-cylinder device	48
5.4 Frictionless piston-cylinder device	48
5.5 Rigid adiabatic systems	49

5.6 Steady flow	50
5.7 Basics of entropy	50
5.8 Basics of the second law of thermodynamics	51
5.9 Isothermal expansion	51
5.10 Entropy change of an ideal gas in a rigid tank	51
5.11 Entropy change of compressed air	52
5.12 Radiator	53
5.13 Non-adiabatic compressor	53
5.14 Entropy change of a copper block	54
5.15 Entropy change of an iron block	55
5.16 Entropy change of an aluminium block	56
5.17 Entropy change of an ideal gas	56
5.18 Entropy change of ideal gases in an isothermal process	57
5.19 Entropy change of ideal gases in isentropic process	57
5.20 Entropy change of oxygen in a piston-cylinder device	58
5.21 Entropy change of nitrogen in a piston-cylinder device	58
5.22 Reversible condensation	59
5.23 Reversible isothermal expansion	60
5.24 Reversible processes for helium	60
5.25 Melting ice	61
5.26 Adiabatic steady flow device	63
5.27 Throttling valve	63
5.28 Stainless steel ball bearings	64
5.29 Egg	65
6 Second Law Applications	67
6.1 Neon in an insulated container	67
6.2 Air in the cylinder of an automobile engine	68
6.3 Isentropic efficiencies of steady flow devices	70
6.4 Adiabatic steam turbine 1	70
6.5 Adiabatic steam turbine 2	71
6.6 Refrigerant compressor	72
6.7 Turbine curve in Mollier diagram	73
6.8 Steam turbine coupled to an air compressor	74
6.9 Two stage compressor	75
6.10 Nozzle	77
6.11 Adiabatic device	78
6.12 Steady-state steady-flow adiabatic device	79
6.13 Venturi	80
7 Thermodynamic Cycles for work, Heat and cold, Carnot	81
7.1 Thermodynamic cycles	81
7.2 Thermal efficiency	81
7.3 Coefficient of performance	81
7.4 Carnot	82
7.5 Geothermal energy	83
7.6 A heat pump	83
7.7 Carnot engine	83
7.8 Air conditioner	84
7.9 Refrigerator	84
7.10 Heat engine	84
7.11 Nuclear power plant	85

7.12 Steam cycle	86
7.13 Perpetual - motion machines	88
7.14 Combined engine and heat pump	88
8 Exergy, Second Law Analysis of Systems: Answers	91
8.1 Exergy	91
8.2 A geothermal well	91
8.3 Steady-state steady-flow heater	91
8.4 A solar tower	92
8.5 Ammonia compressor	93
8.6 Rankine steam cycle	94
8.7 Solar pond	95
8.8 Turbine	96
8.9 Compressor	97
8.10 Nozzle	98
8.11 Heat Exchanger	100
8.12 Second Law Analysis of an actual Rankine cycle	102
9 Gas Power Cycles	107
9.1 Basics of the Stirling cycle	107
9.2 Stirling cycle 1	108
9.3 Stirling cycle 2	109
9.4 Basics of the ideal Otto cycle	111
9.5 Thermal efficiency Otto engine	112
9.6 Otto cycle 1	113
9.7 Otto cycle 2	114
9.8 Basics of the ideal Diesel cycle	115
9.9 Thermal efficiency Diesel cycle	117
9.10 Diesel cycle 1	117
9.11 Diesel cycle 2	119
9.12 Diesel engine with turbocharger	120
9.13 Closed Brayton Cycle	122
9.14 Carnot versus Stirling versus Brayton cycle	123
9.15 Jet propulsion cycle	127
9.16 Simple open Brayton cycle	128
9.17 The 7FA gas turbine	133
9.18 The 7FA gas turbine with regeneration	135
9.19 Brayton cycle with intercooling	136
9.20 Brayton cycle with reheating	139
10 Vapor and Combined Power Cycles	143
10.1 Rankine cycles	143
10.2 Power plant on a simple ideal Rankine cycle 1	145
10.3 Power plant on a simple ideal Rankine cycle 2	146
10.4 Power plant on an actual Rankine cycle	148
10.5 Coal fired steam power plant	150
10.6 Solar-pond power plant	151
10.7 Ideal power plant cooled by a lake 1	153
10.8 Actual power plant cooled by a lake	154
10.9 Reheat Rankine cycle	155
10.10 Ideal reheat Rankine cycle	158
10.11 Reheat-regenerative Rankine cycle with open feedwater heater	160
10.12 Reheat-regenerative Rankine cycle with closed feedwater heater	164

10.13 Combined gas-steam power cycle 1	168
10.14 Combined gas-steam power cycle 2	170
10.15 An ideal cogeneration power plant	173
10.16 An improved Rankine cycle	178
10.17 A Rankine cycle with and without regeneration	180
10.18 City Heating	181
10.19 Rankine cycle with vapor-liquid separator	182
10.20 Thomas Newcomens steam engines	185
11 Refrigeration and Heat Pumps	187
11.1 The vapor compression cooling cycle	187
11.2 An ideal refrigerator	188
11.3 A non-ideal refrigerator	189
11.4 Refrigerator on ammonia	190
11.5 Carnot engine driving an ideal refrigerator	192
11.6 Refrigerator on refrigerant	193
11.7 Heat pump	195
11.8 Reverse Brayton cycle	196
11.9 Aircraft cabin cooling	197
12 Matlab Stuff	201
12.1 Syntax gasprop	201
12.2 Answers for <i>Matlab</i> questions	203

1 Introduction, Thermodynamic Concepts and Terminology

1.1 Engineering Thermodynamics

1.1a Thermodynamics is often defined as the science of energy and deals with different forms of energy and their transformations, i.e. the analysis of transport and conversion of various forms of energy (work \leftrightarrow electricity \leftrightarrow heat \leftrightarrow cold). Note: energy is always conserved, it is never created or destroyed but only converted to a different form.

The application area is very broad ranging from chemical reactions and microscopic organism to common household applications, like heating and cooling as well as transportation vehicles and aircrafts or industrial installations like power plants for the generation of power and heat.

1.1b Engineering Thermodynamics is the application of thermodynamics to solve technological problems that engineers face in the development of new and innovative technologies.

In practice engineers use thermodynamics for the design of systems and installations used to transform energy into a different kind of energy. Think about steam power plants to produce electricity and heat, gas turbines for aircraft propulsion, internal combustion engines for cars and systems for cooling or heating, but also more complex systems with various functions such as the combined production of heat / cooling and electricity.

1.1c The two main objectives of the thermodynamic device are:

1. Conversion of heat (e.g. from combustion of fuel, the sun or geothermal) into work (electricity) using a heat engine.
2. Transportation of heat in a non-natural way using work (electricity) using a refrigeration system or a heat pump.

1.2 Heat engines and refrigeration systems / heat pumps

1.2a A heat engine is a device that converts energy in the form of heat (high temperature internal energy) from the combustion of a fuel, or a hot source like the sun, into mechanical work. The mechanical work is often in the form of an axis turning at high frequency, by using a generator this mechanical form of energy can be converted into electricity (which is a very valuable energy form that we need every day). Note: heat flows from a hot space to a cold space as it occurs spontaneously in nature. See also figure 1.2.1a.

Examples of heat engines are: power plants, aircraft propulsion systems, internal combustion engines for cars.

1.2b Refrigerators and heat pumps are devices that transport energy in the direction of **increasing** temperature. From experience it is known that heat is always transferred in the direction of decreasing temperature, i.e. from a high-temperature space to a low-temperature space. This heat transfer process occurs in nature in a spontaneous way without any human intervention. The reverse process, however, cannot occur without a special device, like a refrigerator or a heat pump.

Refrigerators and heat pumps both transfer the heat opposite to the natural direction, the difference, however, is the purpose of the devices. The purpose of a refrigerator is to cool a space while the purpose of the heat pump is to heat a space, e.g. a house. See also figure 1.2.1b and c.

1.2c In a heat engine heat is converted into work. In a refrigeration system or a heat pump work is used to transport heat from a cold to a hot space using work. The work is converted into heat.

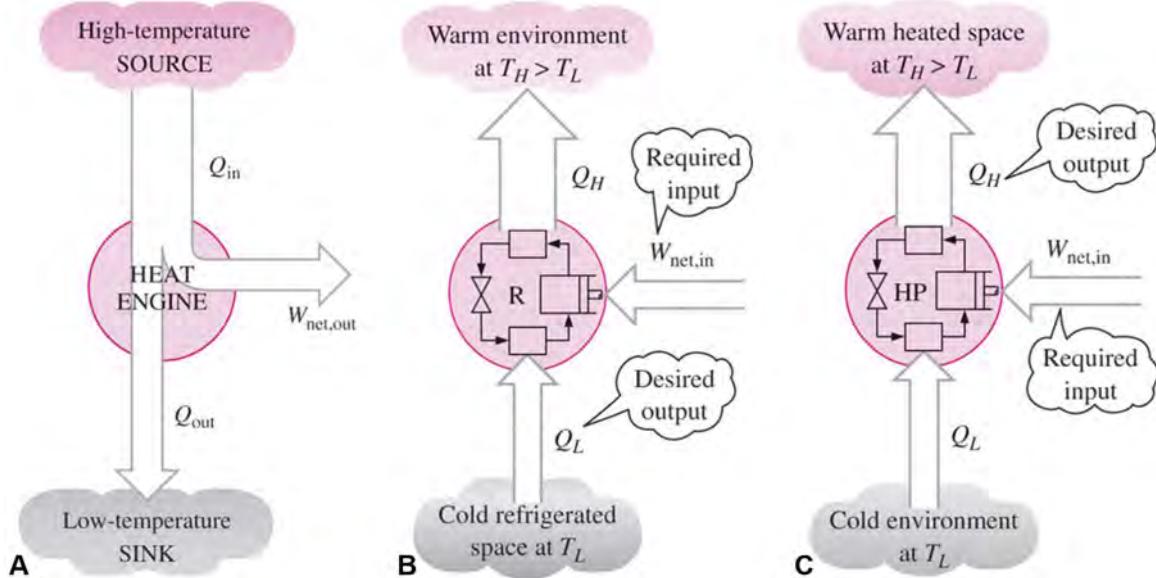


Figure 1.2.1: a: Heat engine, it produces work from heat while the heat is transferred from a hot to a cold space; b: Refrigeration system (R), it transfers heat from a cold to a hot space, the objective is to remove heat from the cooled space; c: Heat pump (HP), it transfers heat from a cold to a hot space, the objective is to supply heat to the warmer space.

1.2d The first law of thermodynamics states that energy is conserved:

$$E_{in} = E_{out} \rightarrow Q_{in} = W_{net} + Q_{out} \rightarrow W_{net} = Q_{in} - Q_{out} = 100 - 60 = 40 \text{ kJ.}$$

See also figure 1.2.1a, note that the subscripts, *in* and *out* refer to the heat engine and not to the high temperature source and the low temperature sink.

1.2e The first law of thermodynamics states that energy is conserved:

$$E_{in} = E_{out} \rightarrow Q_{in} + W_{net} = Q_{out} \rightarrow Q_{in} = Q_{out} - W_{net} = 45 - 10 = 35 \text{ kJ.}$$

See also figure 1.2.1b, note that the subscripts, *in* and *out* refer to the refrigeration system (R) and not to the cold and hot space.

1.2f Calculate the amount of heat, $Q_{out} = Q_H$ that is transferred from a cold space to a hot space by a heat pump which receives an amount of $Q_{in} = Q_L = 60 \text{ kJ}$ from the cold environment using a net work input of $W_{net} = 20 \text{ kJ}$.

The first law of thermodynamics states that energy is conserved:

$$E_{in} = E_{out} \rightarrow Q_{in} + W_{net} = Q_{out} \rightarrow Q_{out} = Q_{in} + W_{net} = 60 + 20 = 80 \text{ kJ.}$$

See also figure 1.2.1c, note that the subscripts, *in* and *out* refer to the heat pump (HP) and not to the cold and hot space.

1.3 Open, closed and isolated systems

1.3a A system is a part of the universe that has to be analysed. The system boundary separates the system from the rest of the universe (it is the boundary between the system and the surroundings/environment).

1.3b In a closed system energy can cross the boundary of a system, but the mass is fixed and no mass can go into or out the system. In an open system both mass and energy are allowed to cross the system boundary.

1.3c In an isolated system no energy or mass can cross the boundary.

1.3d This is an open system as mass (the cooling water) can leave the radiator (it crosses the boundary of the system). Note: the radiator of the car is only that part of the cooling system where the heat is rejected to the air. So the system boundary is assumed to be around the part of the cooling system that emits the heat to the air (called radiator) and the tubes that run through the engine block where the cooling water is heated (and that are not part of the radiator) cross the system boundary. If the tubes running through the engine block are assumed to be part of the radiator and the whole cooling system is assumed to be located within the system boundary the system would be closed.

1.3e The can is a closed system as only heat can cross the boundary of the system.

1.4 State and thermodynamic variables or properties

1.4a A property is any characteristic that defines a system, a thermodynamic property is a characteristic that describes the mass and energy of a thermodynamic system. Some examples of frequently used thermodynamic properties are: temperature, pressure, mass, volume, energy, entropy, enthalpy and the amount of mass. Some other more general properties are: viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, velocity, specific heat.

1.4b A thermodynamic state is a set of thermodynamic properties that describes completely the condition of the thermodynamic system. A thermodynamic state is mathematically described by a function consisting of several thermodynamic properties. At a given state all the properties have fixed values.

1.4c Two properties are independent if one property can vary while the other one is held constant. Dependent properties are properties that are defined in terms of other ones, one is a function of the other. Which are dependent and independent properties depends on the situation. For water for example the volume is independent of the pressure and the temperature. However for a gas the volume depends on the temperature and pressure.

1.4d The state postulate: Any two independent intensive thermodynamic properties are sufficient to describe the state of a system containing a single pure substance.

1.4e The change of a system to an other state is called a process.

1.4f A system of which the properties are not changing with time is said to be in a steady state. In thermodynamics this is called thermodynamic equilibrium. Thermodynamic equilibrium implies a state of balance of the system where there are no driving forces (like temperature differences) within the system.

1.5 Intensive and extensive properties

1.5a Intensive variables are independent of the size of the system. Examples are pressure, temperature, chemical potential, density.

1.5b Extensive properties are directly proportional to the mass or size of the system. They can be written as per kg [/kg]. Examples are mass, volume, energy, entropy, enthalpy.

1.5c Intensive properties.

1.5d Dividing an extensive property by mass gives a specific property. It is the property per unit mass. e.g. the specific volume (v) is the volume per unit mass, $v = V/m$ in m^3/kg or the specific energy is the energy per unit mass $e = E/m$ in J/kg .

1.5e The temperature and pressure are intensive properties, they do not change if the volume is multiplied. The mass and the energy are extensive properties, they scale with the system, if the volume is multiplied by a factor λ the mass and energy are also multiplied by a factor λ . The specific energy, which is the energy per unit mass, does not change as this is an intensive property. Compare if you double the size of a room the temperature and pressure remain the same but the amount of air and so the mass doubles and also the energy contained in the air doubles. The specific energy which is the energy per unit mass remains the same as the total energy doubles but also the mass.

1.6 Processes

1.6a An isothermal process is a process in which the temperature is constant. An isobaric process is a process in which the pressure is constant. An isochoric processes is a process in which the volume is constant.

1.6b An adiabatic process is a process during which there is no heat transfer to the surroundings.

1.6c A steady-flow process, is a process that remains constant in time, so time independent.

1.6d In a cyclic process one returns to the original starting point. After a cycle the properties of the state are not changed.

1.6e During a quasi-equilibrium (or quasi-static) process, the system remains practically in equilibrium at all times. Note that a quasi-equilibrium process is an idealized process and not a true representation of an actual process. But many actual processes closely approximate it and they can be modelled as quasi-equilibrium with negligible error. Engineers are interested in such processes as they are easy to analyse. Furthermore work-producing devices deliver the most work when they operate on quasi-equilibrium processes.

1.7 Dimensions

1.7a The seven fundamental dimensions are:

- 1: Mass, in general denoted by M in kilogram [kg].
- 2: Length, in general denoted by x in meters [m].
- 3: Temperature, in general denoted by T in Kelvin [K].
- 4: Time, in general denoted by t in seconds [s].
- 5: Amount of matter, in general denoted by n in mole [mol].
- 6: Electric current, in general denoted by I in ampere [A].
- 7: Amount of light, in general denoted by I in candela [cd].

1.7b Force = Mass \times acceleration $\Rightarrow F = M \times a$

$$[N] = [kg] \left[\frac{m}{s^2} \right] = \left[\frac{kgm}{s^2} \right] \quad (1.7.1)$$

Pressure = Force per area $\Rightarrow P = F/A$

$$[Pa] = \left[\frac{N}{m^2} \right] = \left[\frac{\frac{kgm}{s^2}}{m^2} \right] = \left[\frac{kg}{ms^2} \right] \quad (1.7.2)$$

Energy = Force × distance $\Rightarrow E = F \times x$

$$[J] = [N][m] = \left[\frac{k\text{gm}}{\text{s}^2} \right] [\text{m}] = \left[\frac{\text{kg}\text{m}^2}{\text{s}^2} \right] \quad (1.7.3)$$

Or energy = Mass × velocity of light squared $\Rightarrow E = M \times c^2$

$$[J] = [\text{kg}] \left[\left(\frac{\text{m}}{\text{s}} \right)^2 \right] = \left[\frac{\text{kg}\text{m}^2}{\text{s}^2} \right] \quad (1.7.4)$$

Power = energy per time $\Rightarrow \dot{E} = E/t$

$$[W] = \frac{[J]}{[\text{s}]} = \left[\frac{\text{kg}\text{m}^2}{\text{s}^2} \right] \times \frac{1}{\text{s}} = \left[\frac{\text{kg}\text{m}^2}{\text{s}^3} \right] \quad (1.7.5)$$

1.7c Kinetic energy = half × mass × velocity squared $\Rightarrow E_{kin} = \frac{1}{2}Mv^2$

$$[\text{kg}] \left[\left(\frac{\text{m}}{\text{s}} \right)^2 \right] = \left[\frac{\text{kg}\text{m}^2}{\text{s}^2} \right] = [J] \quad (1.7.6)$$

Potential energy = mass × acceleration of gravity × height $\Rightarrow E_{pot} = Mgh$

$$[\text{kg}] \left[\frac{\text{m}}{\text{s}^2} \right] [\text{m}] = \left[\frac{\text{kg}\text{m}^2}{\text{s}^2} \right] = [J] \quad (1.7.7)$$

1.8 Mass and volume

1.8a The mass of the substance is $M_{substance} = \rho V = \rho(lbh)$ [kg] and the mass of the combined system is $M_{comb} = M_{tank} + M_{substance} = M_{tank} + \rho V = M_{tank} + \rho(lbh)$ [kg].

The specific volume (v) is defined as the total volume (V) per mass (M). The specific volume of the substance is $v_{substance} = V_{substance}/M_{substance} = lbh/(\rho lbh) = 1/\rho$ [m^3/kg]. Note that the specific volume is the reciprocal of the density.

The specific volume of the combined system is the volume of the combined system divided by the mass of the combined system $v_{comb} = V_{comb}/M_{comb} = lbh/(M_{tank} + \rho(lbh))$ [m^3/kg].

1.8b $\rho_{water} = 1000 \text{ kg/m}^3 \implies M_{water} = 1000(1 \cdot 0.4 \cdot 0.5) = 200 \text{ kg}$ and $M_{comb} = 3 + 1000(1 \cdot 0.4 \cdot 0.5) = 3 + 200 = 203 \text{ kg}$.

The specific volume of the water is $v_{water} = 1/\rho = 1/1000 = 10^{-3} \text{ m}^3/\text{kg}$. The specific volume of the combined system is $v_{comb} = V_{comb}/M_{comb} = (1 \cdot 0.4 \cdot 0.5)/203 = 9.85 \cdot 10^{-4} \text{ m}^3/\text{kg}$.

1.8c $\rho_{air} = 1.225 \text{ kg/m}^3 \implies M_{air} = 1.225(1 \cdot 0.4 \cdot 0.5) = 0.245 \text{ kg}$ and $M_{comb} = 3 + 1.225(1 \cdot 0.4 \cdot 0.5) = 3 + 0.245 = 3.245 \text{ kg}$. In the case of air the weight of the air can be neglected compared to the weight of the tank. Whereas in the case of water the weight of the tank can be neglected compared to the water.

The specific volume of air is $v_{air} = 1/\rho_{air} = 0.816 \text{ m}^3/\text{kg}$ and the specific volume of the combined system is $v_{comb} = V_{comb}/M_{comb} = (1 \cdot 0.4 \cdot 0.5)/3.245 = 0.0616 \text{ m}^3/\text{kg}$.

1.9 Ideal gases

Use the ideal gas law to calculate the specific volume v , $v = RT/P$. In the formula T should be in Kelvin $42^\circ\text{C} = 315 \text{ K}$. The pressure should be in Pascal, $1.2 \text{ Bars} = 120 \text{ kPa} = 120 \text{ kN/m}^2$. This results in $v = (0.14304 \cdot 315)/120 = 0.375 \text{ m}^3/\text{kg}$. That the outcome is the specific volume in m^3/kg follows by looking to the dimensions, $(\text{kJ/kgK})(\text{K})(\text{m}^2/\text{kN}) = \text{m}^3/\text{kg}$. The total volume V is $V = v \cdot m = 0.375 \cdot 1.5 = 0.563 \text{ m}^3$. The density of the gas is the reciprocity of the specific volume $\rho = 1/v$, $\rho = 1/0.375 = 2.667 \text{ kg/m}^3$.

1.10 Galileo thermometer

1.10a If the outside temperature is 30 degree Celsius all bubbles are too heavy and they are all on the bottom. If the outside temperature is only 10 degree Celsius all bubbles are too light and all the bubbles are on the top.

1.10b The bubble that indicates the right temperature floats in the water somewhere in the middle between the bubbles that have a too low temperature tag on the bottom and the bubbles that have a too high temperature tag on the top.

1.10c The bubble with the 18 degree Celsius tag is the heaviest and the bubble with the 24 temperature tag is the lightest.

1.10d The 20 degree Celsius bubble is floating in the middle of the thermometer. Therefore, the upward force, F_u , that the bubble encounters is equal to the weight of water displaced by the bubble. The weight of the displaced water is the product of the density of water at 20 degree, ρ_{20} , the gravitational acceleration, g and the volume of the bubble, $V_b = 4/3\pi r^3$
 $\rightarrow F_u = \rho g V_b = 4/3\pi \rho_{20} g r^3$.

As the bubble floats the upward force is equal to the gravitational force, F_g , which is equal to the mass of the bubble, M_{b20} , times the gravitational acceleration

$$\rightarrow F_g = M_{b20} g.$$

Equate the two forces together results in: $F_u = 4/3\pi \rho_{20} g r^3 = F_g = M_{b20} g$
 $\rightarrow M_{b20} = 4/3\pi \rho_{20} r^3$. Filling in $r = \frac{1}{2d} = 0.0125$ m and $\rho_{20} = 998.2071$ kg/m³ gives
 $\rightarrow M_{b20} = 8.1666 \cdot 10^{-3}$ kg = 8.1666 grams.

1.10e Lowest temperature tag is the 18 degree bubble, its mass is: $M_{b18} = 4/3\pi \rho_{18} r^3 = 8.1698$ grams.
Highest temperature tag is the 24 degree bubble, its mass is: $M_{b24} = 4/3\pi \rho_{24} r^3 = 8.1591$ grams.
The mass difference between both bubbles is: $M_{b18} - M_{b24} = 0.0107$ grams = 10.7 milligrams.

The mass of the 19 degree bubble is: $M_{b19} = 4/3\pi \rho_{19} r^3 = 8.1682$ grams.

The mass of the 23 degree bubble is: $M_{b23} = 4/3\pi \rho_{23} r^3 = 8.1611$ grams.

Mass difference between 18 and 19 degree bubble: $M_{b18} - M_{b19} = 0.0016$ grams = 1.6 milligrams.

Mass difference between 23 and 24 degree bubble: $M_{b23} - M_{b24} = 0.0020$ grams = 2.0 milligrams.

As the relation between the temperature and the density of the water is not linear (you can easily see this in the graph, the line is not straight but a bit curved) the mass difference between two successive bubbles is not constant. The mass difference between the bubbles increases with temperature.

The relative mass difference of the 18 and 19 degree bubble is: $\frac{0.0016}{8.1682} \times 100\% = 0.0196\%$

The relative mass difference of the 23 and 24 degree bubble is: $\frac{0.0020}{8.1611} \times 100\% = 0.0245\%$

So, also the relative mass difference between the bubbles increases with temperature, and varies around 0.02% of the mass of the bubbles.

They could build these thermometers in Galileo's time, as the mass differences between the individual bubbles are in the order of milligrams. These differences can be determined using a simple balance scale which was already known in Galileo's time.

1.11 Acceleration and force

1.11a Force = Mass × acceleration, $F = M \times a$ with $a = 6g \Rightarrow F_{aircraft} = 6Mg$ [N].

1.11b $F_{moon} = 0.17Mg$ [N].

1.11c $g = 9.8$ m/s² $\Rightarrow F_{aircraft}^{man} = 5292$ N, $F_{aircraft}^{woman} = 3234$ N, $F_{moon}^{man} = 150$ N, $F_{aircraft}^{woman} = 91.6$ N.

1.12 Temperature

1.12a The absolute zero temperature is the lowest theoretically attainable possible temperature, this is the temperature at which all movements of molecules are frozen. Absolute temperatures use a reference state of zero at absolute zero. They can never be negative. The unit is Kelvin.

1.12b A relative temperature is a temperature measured relative to a non-absolute zero temperature. It can be negative.

1.12c The most common temperature scales in the SI system are the Celsius scale and the Kelvin scale. They are related by a factor of $273.15 \rightarrow T(K) = T(C) + 273.15$.

1.12d The boiling and freezing point of water.

1.13 Pressure

1.13a Pressure is defined as the normal force per unit acting on a surface, $P = \frac{\text{force} [\text{N}]}{\text{area} [\text{m}^2]} = \frac{F}{A}$. The unit of pressure is $\text{N/m}^2 = \text{Pa}$.

1.13b Vacuum pressure is zero pressure and atmospheric pressure is the pressure of the atmosphere outside. The absolute pressure is the pressure measured related to the vacuum pressure and the gauge pressure is the pressure measured related to the atmospheric pressure. They can be related $P_{\text{absolute}} = P_{\text{atmosphere}} + P_{\text{gauge}}$.

1.14 Pressure on a diver

1.14a The relative pressure on the diver is the pressure exerted by the water $P_{\text{rel}} = P_{\text{water}} = \rho gh$. The absolute pressure is the pressure of the water and the atmospheric pressure $P_{\text{abs}} = P_{\text{atm}} + P_{\text{water}} = P_{\text{atm}} + \rho gh$.

1.14b At $h = 15 \text{ m} \rightarrow P_{\text{rel}} = 1000 \cdot 9.81 \cdot 15 = 147150 \text{ Pa} = 147.2 \text{ kPa}$ and $P_{\text{abs}} = 248.2 \text{ kPa}$. At $h = 30 \text{ m} \rightarrow P_{\text{rel}} = 1000 \cdot 9.81 \cdot 30 = 294300 \text{ Pa} = 294.3 \text{ kPa}$ and $P_{\text{abs}} = 395.2 \text{ kPa}$.

1.14c The relative pressure is doubled if the diver goes two times deeper below the water level. The absolute pressure is not doubled.

1.14d The density of seawater is 1030 kg/m^3 .

At $h = 15 \text{ m} \rightarrow P_{\text{rel}} = 1030 \cdot 9.81 \cdot 15 = 151565 \text{ Pa} = 151.6 \text{ kPa}$ and $P_{\text{abs}} = 252.6 \text{ kPa}$.

At $h = 30 \text{ m} \rightarrow P_{\text{rel}} = 1030 \cdot 9.81 \cdot 30 = 303129 \text{ Pa} = 303.1 \text{ kPa}$ and $P_{\text{abs}} = 403.1 \text{ kPa}$.

1.14e $P_{\text{atm}} = P_{\text{read}} - \rho_{\text{seawater}}gh \rightarrow P_{\text{atm}} = 398800 - 1030 \cdot 9.81 \cdot 30 = 95671 \text{ Pa} = 95.7 \text{ kPa}$.

1.15 Pressure and height

1.15a $P_{\text{barometric}} = g\rho_{\text{air}}h$

1.15b $\Delta P = g\rho_{\text{air}}\Delta h \rightarrow \Delta h = \frac{\Delta P}{g\rho_{\text{air}}} = \frac{(930-780)\cdot 10^2}{9.81\cdot 1.2} = 1289 \text{ m}$.

1.15c Assume the tower is 40 m (13 floors and each floor about 3 meter) $\rightarrow \Delta P_{\text{barometric}} = g\rho_{\text{air}}\Delta h = 9.81 \cdot 1.2 \cdot 40 = 471 \text{ Pa} = 4.71 \text{ mBar}$.

1.16 Pressure in a tank

1.16a It is assumed that the pressure in the tank is uniform (i.e. its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the water-air interface. Starting with the pressure at the water-air interface and moving along the tube by adding or subtracting the ρgh terms until we reach the outside mercury-air interface and setting the result equal to P_{atm} since the tube is open to the atmosphere gives

$$P_{air-tank} + \rho_{water}gh_1 + \rho_{oil}gh_2 - \rho_{mercury}gh_3 = P_{atm}. \quad (1.16.1)$$

We are interested in the gauge pressure, which is the pressure in the tank relative to the atmospheric pressure and thus we write:

$$P_{gauge} = P_{air-tank} - P_{atm} = \rho_{mercury}gh_3 - \rho_{water}gh_1 - \rho_{oil}gh_2. \quad (1.16.2)$$

Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis considerably.

1.16b Filling in the numbers gives: $P_{gauge} = 13600 \cdot 9.81 \cdot 0.46 - 1000 \cdot 9.81 \cdot 0.2 - 850 \cdot 9.81 \cdot 0.3 = 56907.81 \text{ Pa} = 56.9 \text{ kPa}$.

1.16c If the oil in the tube is replaced by mercury and you assume that the values of the height stay the same the pressure inside the tank is lower $P_{gauge} = \rho_{mercury}g(h_3 - h_2) - \rho_{water}gh_1 = 13600 \cdot 9.81 \cdot (0.46 - 0.3) - 1000 \cdot 9.81 \cdot 0.2 = 19384.56 \text{ Pa} = 19.4 \text{ kPa}$.

If you assume that the pressure inside the tank does not change if you replace the oil by mercury the level of h_1 and h_3 changes (h_2 has no meaning anymore). How the levels change can not be calculated.

1.16d If both oil and mercury are taken out and the level h_1 stays the same the pressure inside the tank drops to $P_{gauge} = -\rho_{water}gh_1 = -1000 \cdot 9.81 \cdot 0.1 = -981 \text{ Pa} = -0.98 \text{ kPa}$. This pressure is negative, that means that the pressure inside the tank is larger than the atmospheric pressure. That is easily seen by the fact that the water in the tube is at a lower level than the water in the tank.

If is assumed that the pressure inside the tank does not change the water level h_1 becomes: $h_1 = -\frac{P_{gauge}}{\rho_{water}g} = -\frac{56907.81}{1000 \cdot 9.81} = -5.9 \text{ m}$. This is 5.9 meters above the water surface!

1.16e Oil has a lower density than water while mercury has a higher density than water. This implies that in a manometer tube the oil reaches a height higher than water while the mercury reaches a height lower than water. If the pressure is very high it is preferred to use mercury as the tube can be shorter than in the case of water. If the pressure is very low oil has the preference as the accuracy that can be reached is higher (the oil will be higher than water of mercury and thus is easier to read).

1.17 Pressure and manometer

1.17a The pressure in the duct is above the atmospheric pressure as the pressure in the duct is the fluid in the right leg of the manometer is higher. Pressure in the duct atmospheric pressure plus the pressure exerted by the fluid.

1.17b $P_{rel} = \rho gh$ and $P_{abs} = \rho gh + P_{atm}$.

1.17c $h = 15 \text{ mm} \rightarrow P_{rel} = 147.15 \text{ Pa}$ and $P_{abs} = 100147.15 \text{ Pa}$.

$h = 30 \text{ mm} \rightarrow P_{rel} = 294.30 \text{ Pa}$ and $P_{abs} = 100294.30 \text{ Pa}$.

The relative pressure doubles if the height doubles while the absolute pressure increases only a little bit, only about 0.15%. In the absolute pressure the atmospheric pressure dominates.

1.17d $h = 15 \text{ mm} \rightarrow P_{rel} = 2001.24 \text{ Pa}$ and $P_{abs} = 102001.24 \text{ Pa}$.

$h = 30 \text{ mm} \rightarrow P_{rel} = 4002.48 \text{ Pa}$ and $P_{abs} = 104002.48 \text{ Pa}$.

The relative pressure in the duct is much higher than in the case the fluid is water. This is because the density of mercury is about 14 times higher than the density of water. The relative pressure is about 14 times higher. The difference with the absolute pressure in c is less than with the relative pressure as the atmospheric pressure has a large contribution to the absolute pressure compared to the relative pressure.

1.17e If the absolute pressure in the duct is two times the atmospheric pressure the relative pressure in the duct is equal to the atmospheric pressure. $P_{rel} = P_{atm} = \rho gh \rightarrow h = \frac{P_{atm}}{\rho g}$.

For water $\rightarrow h = \frac{100000}{1000 \cdot 9.81} = 10.1 \text{ m}$.

For mercury $\rightarrow h = \frac{100000}{13600 \cdot 9.81} = 0.75 \text{ m}$.

For mercury you need a much shorter tube.

1.18 Pressure in a cylinder

$$\mathbf{1.18a} \quad P_{cylinder} = P_{atm} + P_{piston} + P_{spring} = P_{atm} + \frac{Mg}{A} + \frac{F}{A} = P_{atm} + \frac{Mg+F}{A}.$$

$$\mathbf{1.18b} \quad P_{cylinder} = 95000 + \frac{4 \cdot 9.81 + 60}{35 \cdot 10^{-4}} = 123354 \text{ Pa}. \text{ The pressure inside the cylinder is } 123.4 \text{ kPa}.$$

$$\mathbf{1.18c} \quad P_{cylinder} = P_{atm} + \frac{Mg+F}{A} = 2P_{atm} \rightarrow P_{atm} = \frac{Mg+F}{A} \rightarrow F = P_{atm}A - Mg.$$

$$F = 95000 \cdot 35 \cdot 10^{-4} - 4 \cdot 9.81 = 293.26 \text{ N}.$$

2 Energy, Work and Heat Transfer

2.1 Basics of Energy

2.1a Energy can exist in numerous forms such as thermal energy, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear. Their sum constitutes the total energy, E , of a system. Internal energy, U , kinetic energy, E_{kin} and potential energy E_{pot} , are the three forms of energy that are most important in thermodynamics. This gives for the total energy: $E = U + E_{kin} + E_{pot}$. The other forms of energy such as magnetic, electric, or surface tension effects can be significant in some specialized cases only but are usually ignored.

2.1b No, thermodynamics provides no information about the absolute value of the total energy. It deals only with the change of the total energy, which is what matters in engineering problems.

2.1c No, the total energy of a state only depends on its properties and not on the way followed to go there. Note that work and added heat are dependent of the path.

2.1d

- Sensible energy, which is energy that has to do with the kinetic energy of the molecules (this includes translation, rotation and vibration of the molecules).
- Latent energy, which is the internal energy associated with the phase of a system.
- Chemical energy, which is the energy associated with the atomic bonds in a molecule.
- Nuclear energy, which is the tremendous amount of energy associated with the strong bonds within the nucleus of the atom itself.

2.1e Kinetic energy $E_{kin} = \frac{1}{2}Mv^2$ and potential energy $E_{pot} = Mgh$.

2.2 Energy transfer mechanisms

2.2a In closed systems energy can be transferred by work and by heat. Their dimension is Joule, J.

2.2b Also in open systems energy can be transferred by work and heat (Yes).

2.2c Energy transfer by mass plays also a role in open systems (but not in closed). It is called flow work.

2.3 Energy transfer by work and power

2.3a $\delta W = \vec{F} \times d\vec{S}$, what does the δ mean? The δ symbol in the differential δW indicates that work is an inexact differential, or path function. That is, in general, there is no way to formally integrate the equation to achieve an analytical result for W itself, because the amount of work done in travelling between an initial location S_1 and a final location S_2 depends on the path followed between the points and the force applied in the direction \vec{S} at every point along the path.

$$\begin{aligned} \mathbf{2.3b} \quad W_{S_1-S_2} &= \int_{S_1}^{S_2} \vec{F} \cdot d\vec{S} \quad \text{with } \vec{F} = \vec{P} A \Rightarrow \\ W_{S_1-S_2} &= \int_{S_1}^{S_2} \vec{P} A \cdot d\vec{S} = \int_{V_1}^{V_2} P dV. \end{aligned}$$

2.3c The work is dependent on the integration path because it is an inexact differential (path function).

2.3d For work added to the system from the surroundings is done by compression, dv is negative and therefore W_{in} is negative. Work taken from the system to the surroundings is positive as dv is positive due to the expansion process.

2.3e $\dot{W} = dW/dt$.

2.4 Work of a piston-cylinder device

2.4a The sketch of the process is given in figure 2.4.1.

$$Pv^{1.4} = \text{Constant} \rightarrow P_1 V_1^{1.4} = P_2 V_2^{1.4} \Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{1.4}$$

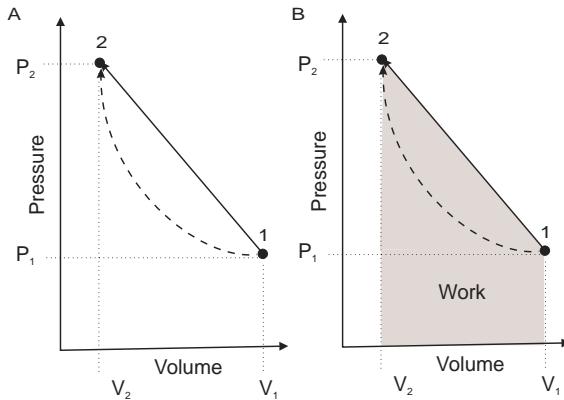


Figure 2.4.1: (A) Paths between the states 1 and 2 and (B) the work done in the case of the linear path, $\Delta P = c^* \Delta V$.

2.4b The work is the area below the line in the graph and can be calculated by integration of the pressure over the volume $\Rightarrow W = \int_{V_1}^{V_2} P dV$.

$$\text{With } P = cV^{-1.4} \text{ this gives } W = c \int_{V_1}^{V_2} (V^{-1.4}) dV = c \left[\frac{-1}{0.4} V^{-0.4} \right]_{V_1}^{V_2} = -\frac{c}{0.4} (V_2^{-0.4} - V_1^{-0.4}).$$

The constant c follows from $c = P_1 V_1^{1.4}$ with P_1 and V_1 filled in.

2.4c $P_2 = 100 \left(\frac{1}{0.1} \right)^{1.4} \text{ kPa} = 2512 \text{ kPa}$.

The constant c is $c = P_1 V_1^{1.4} \rightarrow c = 100 \cdot 1^{1.4} = 100 \text{ kPa(m}^3\text{)}^{1.4}$

Filling in all the numbers gives $W_{in} = -\frac{100 \cdot 10^3}{0.4} (0.1^{-0.4} - 1^{-0.4}) = -378 \cdot 10^3 \text{ J} = -378 \text{ kJ}$.

In this case the volume decrease ($V_2 < V_1$), the work is done by the surroundings on the system, therefore the work is denoted by W_{in} and the sign must be negative.

2.4d The differential relation between P and V is: $dP = c^* dV$. From this follows: $P = \int P = \int C^* dV = c^* V + c^{**}$, where c^{**} is a different constant following from the integration, $\Rightarrow P = c^* V + c^{**}$, the values of c^* and c^{**} follow from the known values of the volume and the pressure (two equations with two unknowns can be solved).

2.4e $W = \int_{V_1}^{V_2} P dV$.

$$\text{With } P = c^* V + c^{**} \text{ this gives } W = \int_{V_1}^{V_2} (c^* V + c^{**}) dV = \left[\frac{1}{2} c^* V^2 + c^{**} V \right]_{V_1}^{V_2} \\ W = \left(\frac{1}{2} c (V_2^2 - V_1^2) + c^* (V_2 - V_1) \right).$$

Now the constants have to be determined: $100 = c^{**} + c^*$ and $2512 = c^{**} + 0.1c^*$

Subtraction gives: $-2412 = 0.9c^* \rightarrow c^* = -2680 \text{ kPa}$ and then $c^{**} = 100 + 2680 = 2780 \text{ kPa}$.

By filling in the values for pressure, volume and the constants, the work can be calculated:

$$W = \left(\frac{1}{2} (-2680) (0.1^2 - 1^2) + 2780 (0.1 - 1) \right) = -1340(-0.99) + 2780(-0.9) = -1175 \text{ kJ}.$$

Alternatively the area under the graph can be calculated directly:

$$\Delta P = c^* \Delta V \Rightarrow W = -P_1(V_1 - V_2) + -\frac{1}{2}(P_2 - P_1)(V_1 - V_2) = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(100 \cdot 10^3 + 2512 \cdot 10^3)(1 - 0.1) = 1175 \cdot 10^3 \text{ J} = 1175 \text{ kJ}$$

At this point you have to decide whether this is positive or negative. It is negative as the work is added from the system to the surroundings by the compression of the volume, dv is negative.

2.4f The work done depends on the path. There are several other paths, every path that connects the begin and end state can be used. However some paths will be very unlikely.

2.4g The path with the minimum work is the path with the smallest area below the graph while the path with the maximum work is the one with the largest area.

Looking to the figure the you can see that the path with zero power is the path where you first decrease the pressure till 0, then decrease the volume till V_2 and finally increase the pressure till P_2 . However this path is practical not very handy as you have the lower the pressure till vacuum which will cost a lot of energy. A more realistic path is the one where the pressure is increased till P_2 before the volume is compressed till V_2 .

For the maximum energy you can find a path by first increasing the pressure a lot and then compress the volume. The amount of work will depend on the pressure rise you apply first. It is easily to imagine that at a high pressure you have to apply more force (is more work) to decrease a volume.

2.5 Work done by compression

2.5a The graph is shown in figure 2.5.1. The area under the graph by hand calculations is (divide the area into a square and a triangle) $A = 0.3m^3 \cdot 96kPa + 0.3m^3 \cdot 360kPa = 82,8m^3kPa = 82.8kJ$

2.5b The work is the area below the line in the graph and can be calculated by integration of the pressure over the volume change $\rightarrow W = \int_a^b P(V)dV = \int_{0.12}^{0.46} (aV + b)dV = [\frac{aV^2}{2} + bV] = 82.8kJ$
Both methods give the same answer.

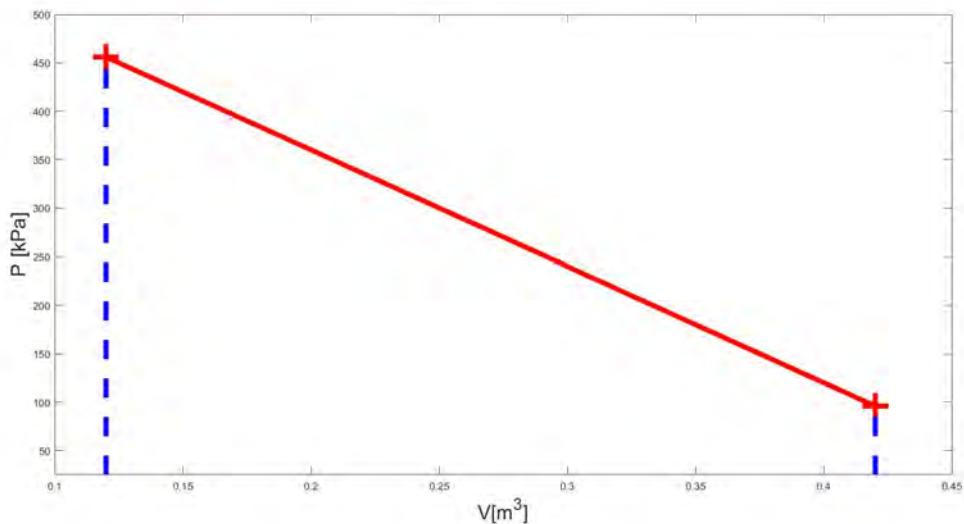


Figure 2.5.1: P-V diagramm with $P = aV + b$

2.6 Work done by lifting a mass

The situation is given in figure 2.6.1. The mass is given and equal with $m = 10 \text{ kg}$. The difference in height is $H = 10\text{m}$. It is assumed that the pulley, the dolly and the elevator are frictionless.

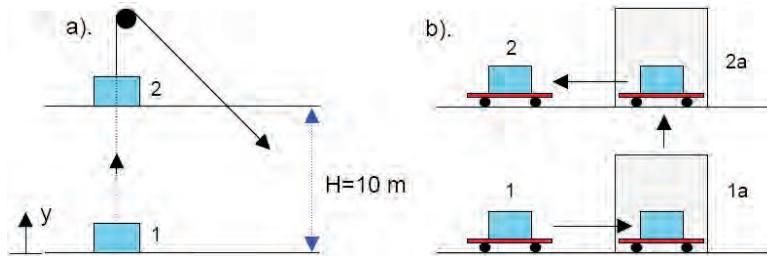


Figure 2.6.1: Paths between the states 1 and 2. In a) the pulley is used in b) the trolley and the elevator.

2.6a The work done can be calculated using $W_{1-2} = \int_1^2 F dS$ where F is the force given by $F = mg$ where $g = 9.8 \text{ m/s}^2$ is the gravitational acceleration and S is the distance over which the mass is moved. Therefore, it can be observed that the work done is equal with the difference in potential energy because our S will be equal with the change in height H . Therefore, the work is given by: $W_{1-2} = \int_1^2 F dS = \int_0^H mgdS = mgH = \Delta PE = 10 \cdot 9.8 \cdot 10 = 980 \text{ Nm} = 980 \text{ J}$.

2.6b If the mass is moved using a dolly and an elevator (both frictionless) the work is given by: $W_{1-2} = W_{1-1a} + W_{1a-2a} + W_{2a-2} = \int_1^{1a} F_{push} dS + \int_{1a}^{2a} F_{mass} dS + \int_{2a}^2 F_{push} dS = \int_0^L F_{push} dS + \int_0^H mgdS + \int_0^L F_{push} dS = \Delta PE + 2F \cdot L = 10 \cdot 9.8 \cdot 10 + 2 \cdot 2 \cdot 1 = 984 \text{ J}$. It can be observed that indeed, work is a path function and the results from the two cases differ. Both of the formulas for work include the difference in potential energy. The difference in potential energy is a state function as it can be calculated using only the initial and final states as gravitational force is conservative force (one can look up for more information).

2.7 Heat transfer

2.7a Heat transfer is defined as energy crossing the system boundary because of a temperature difference between the system and the surroundings.

2.7b No, in an isolated system heat can not cross the system boundary.

2.7c No, an adiabatic process is a process in which there is no heat transfer.

2.7d $\dot{Q} = \delta Q/dt$

2.8 Work and heat transfer of a piston-cylinder device

2.8a Adiabatic means there is no heat transfer. Quasi-static means that the process goes so slow that it can be assumed to be in equilibrium all the time. Therefore we can describe the state of the system at each point in time and we can model it. Remember a quasi-equilibrium process is an idealized process and not a true representation of an actual process, however many practical processes can be modeled as quasi-equilibrium.

Note that in all three cases $\Delta U = U_B - U_A$ is the same, i.e. the total energy of a state is independent of the path followed to reach it. However, the work, W and heat, Q are different for each case, i.e. they are path dependent. For each case $\Delta U = W_i + Q_i$ and $W_i = \int_{V_A}^{V_B} P dV$.

2.8b Path c: $\Delta U = W_c + Q_c \rightarrow Q_c = \Delta U - W_c$. For path c the process is adiabatic, i.e no heat transfer to the surroundings $\rightarrow Q_c = 0$ and therefore $W_c = \Delta U$.

$W_c = \int_{V_A}^{V_B} P dV$, with $P = cV^{-5/3}$ this gives $W = c \int_{V_1}^{V_2} (V^{-5/3}) dV = c \left[\frac{-3}{2} V^{-2/3} \right]_{V_1}^{V_2} = -\frac{3c}{2} (V_2^{-2/3} - V_1^{-2/3})$ with $c = P_A V_A^{5/3}$.

2.8c Path a: $\Delta U = W_a + Q_a \rightarrow Q_a = \Delta U - W_a = W_c - W_a$

and $W_a = \int_{V_A}^{V_B} P_A dV = P_A(V_B - V_A)$ as P_A is constant. Note that for the second part of the path v is constant and therefore dv is zero and no work is performed ($\delta w = Pdv$).

2.8d Path b: $\Delta U = W_b + Q_b \rightarrow Q_b = \Delta U - W_b = W_c - W_b$

W_b : The differential relation between P and V is: $dP = cdV$. From this follows: $P = \int dP = \int cdV = cV + c^*$, where c^* is a different constant following from the integration, $\Rightarrow P = cV + c^*$, the values of c^* and c follow from the known values of the volume and the pressure (two equations with two unknowns can be solved).

$$W_b = \int_{V_A}^{V_B} P dV, \text{ with } P = cV + c^* \text{ this gives } W = \int_{V_A}^{V_B} (cV + c^*) dV = \left[\frac{1}{2}cV^2 + c^*V \right]_{V_A}^{V_B}$$

$$W = \left(\frac{1}{2}c^*(V_B^2 - V_A^2) + c^{**}(V_B - V_A) \right).$$

Alternatively the area under the graph can be calculated directly:

$$W_b = P_B(V_B - V_A) + \frac{1}{2}(P_A - P_B)(V_B - V_A) = \frac{1}{2}(P_A + P_B)(V_B - V_A).$$

2.8e Above are 6 equations and 6 unknown variables, this can be solved.

$\Delta U = W_c$ follows from path c.

$$\text{The constant } c \text{ is given by: } c = P_A V_A^{5/3} \rightarrow c = (48 \cdot 10^5) \times (1 \cdot 10^{-3})^{5/3} = 48 \text{ (Nm)}^{5/3}.$$

$$\Delta U = W_c = -\frac{3c}{2} \left(V_2^{-2/3} - V_1^{-2/3} \right) = -\frac{3 \cdot 48}{2} \left((12 \cdot 10^{-3})^{-2/3} - (1 \cdot 10^{-3})^{-2/3} \right) = -72(19 - 100) = 7181 \text{ J} = 7.2 \text{ kJ.}$$

$$W_a = P_A(V_B - V_A) = (48 \cdot 10^5) \times (12 \cdot 10^{-3} - 1 \cdot 10^{-3}) = 52800 \text{ J} = 52.8 \text{ kJ.}$$

$$Q_a = \Delta U - W_a = 7.2 - 52.8 = -45.6 \text{ kJ.}$$

$$W_b = \left(\frac{1}{2}c(V_B^2 - V_A^2) + c^*(V_B - V_A) \right).$$

Now the constants have to be determined: $48 \cdot 10^5 = c^* + c \cdot 10^{-3}$ and $1 \cdot 10^5 = c^* + 12 \cdot 10^{-3}c$

$$\text{Subtraction gives: } 47 \cdot 10^5 = -11 \cdot 10^{-3}c \rightarrow c = -4273 \cdot 10^5 \text{ Pa and then } c^* = 48 \cdot 10^5 + 4273 \cdot 10^5 = 52.3 \cdot 10^5 \text{ Pa.}$$

By filling in the values for pressure, volume and the constants, the work can be calculated:

$$W_b = \left(\frac{1}{2}(-4273 \cdot 10^5) \left((12 \cdot 10^{-3})^2 - (1 \cdot 10^{-3})^2 \right) + 52.3 \cdot 10^5 (12 \cdot 10^{-3} - 1 \cdot 10^{-3}) \right) = 26978 \text{ J} = 27.0 \text{ kJ.}$$

Alternatively if the work is calculated by the area under the graph:

$$W_b = \frac{1}{2}(P_A + P_B)(V_B - V_A) = \frac{1}{2}(48 \cdot 10^5 + 1 \cdot 10^5)(12 \cdot 10^{-3} - 1 \cdot 10^{-3}) = 26950 \text{ J} = 27.0 \text{ kJ.}$$

$$Q_b = \Delta U - W_b = 7.2 - 27.0 = -19.8 \text{ kJ.}$$

To summarize: $\Delta u = 7.2 \text{ kJ}$, $W_a = 52.8 \text{ kJ}$, $Q_a = -45.6 \text{ kJ}$, $W_b = 27 \text{ kJ}$, $Q_b = -19.8 \text{ kJ}$, $W_c = 7.2 \text{ kJ}$ and $Q_c = 0 \text{ kJ}$.

Note that all work has a positive value, this means that work is done by the system on the surroundings which is correct as there was expansion ($V_b > V_a$). All heat has a negative sign which means that heat is added to the system, which is correct as the system by the expansion will cool down (except in the case it is adiabatic). The amount of work is the least for process c, this can also be seen in the graph. The amount of work is equal to the area under the line, which is the least for path c and the most for path a.

2.9 Flow work and enthalpy 1

2.9a Flow work is the work required to push mass across the system boundary into an open system.

2.9b $\delta W_{flow} = PAdx = PdV \rightarrow W_{flow} = PV$ and of course for the specific flow work $w_{flow} = Pv$.

2.9c The energy crossing the system boundary of an open system with mass transfer is the energy in the fluid and the flow work, $dE_{in-or-out} = (u + Pv + ke + pe)dm$ and the specific energy is $e_{in-or-out} = u + Pv + ke + pe$.

2.9d The combination of internal energy u and flow work Pv is called enthalpy, $h = u + Pv$. Its dimension is J/kg for the specific enthalpy h for the total enthalpy, $H = U + PV$, it is J.

Discussion: Specific enthalpy is a defined property, i.e., an algebraic combination of other properties (u, P , and v). In Chapter 4, we will find that it is sometimes convenient to lump properties together in this way, as they appear together frequently in certain types of problems.

2.10 Flow work and enthalpy 2

2.10a

$$dE_{mass-in} = \left(gh + \frac{1}{2}v^2 + u \right)_{in} dm. \quad (2.10.1)$$

2.10b

$$dW_{flow} = PAdx = PdV = Pvdm. \quad (2.10.2)$$

2.10c

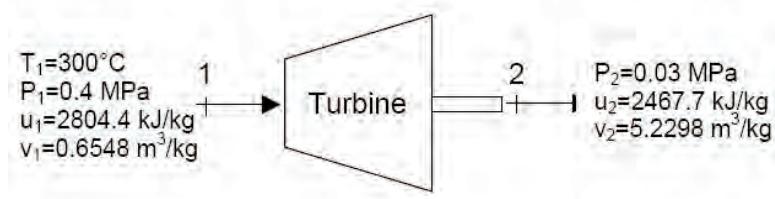
$$\begin{aligned} dE_{in} &= dE_{mass-in} + dE_{flow-in} = \left(gh + \frac{1}{2}v^2 + u \right)_{in} dm + Pvdm \\ &= dE_{mass-in} + dE_{flow-in} = \left(gh + \frac{1}{2}v^2 + u + Pv \right)_{in}. \end{aligned} \quad (2.10.3)$$

2.10d

$$\begin{aligned} dE_{out} &= dE_{mass-out} + dE_{flow-out} = \left(gh + \frac{1}{2}v^2 + u \right)_{out} dm + Pvdm \\ &= dE_{mass-out} + dE_{flow-out} = \left(gh + \frac{1}{2}v^2 + u + Pv \right)_{out}. \end{aligned} \quad (2.10.4)$$

2.10e Enthalpy in [kJ/kg]. So enthalpy is the internal energy plus the work necessary to push the flow through the system, $h = u + Pv$, and is a very useful property in analyzing flow systems as pumps, compressors or turbines. It is tabulated and can be looked up for different cases. Note that enthalpy, like internal energy, can be evaluated only indirectly. There is no "enthalpy meter" available.

2.11 Enthalpy of steam



2.11a The specific enthalpy is given by $h = u + Pv$. At the inlet it is $h_1 = u_1 + P_1v_1$ at the exit it is $h_2 = u_2 + P_2v_2$. The difference in specific enthalpy between the inlet and the exit is $\Delta h = h_2 - h_1 = u_2 + P_2v_2 - (u_1 + P_1v_1) = u_2 - u_1 + P_2v_2 - P_1v_1$.

2.11b $\Delta h = h_2 - h_1 = u_2 + P_2 v_2 - (u_1 + P_1 v_1) = u_2 - u_1 + P_2 v_2 - P_1 v_1 = 2467.7 - 2804.4 + 0.03 \cdot 10^3 \cdot 5.2298 - 0.4 \cdot 10^3 \cdot 0.6548 = -336.7 + 156.9 - 261.9 = -441.7 \text{ kJ/kg}$ (h decreases from inlet to outlet). Note the units: P is in Mpa and u in kJ.

Discussion: Specific enthalpy is a defined property, i.e., an algebraic combination of other properties (u , P , and v). In Chapter 4, we will find that it is sometimes convenient to lump properties together in this way, as they appear together frequently in certain types of problems.

2.12 Internal combustion engine

An internal combustion engine consumes fuel at a rate of $\dot{m} = 2.5$ gram/s. The heating value of the fuel is $h_{fuel} = 46 \text{ MJ/kg}$. The engine converts part of the energy from the fuel into a net work output and the rest of the energy is released as waste heat at the outlet. The power of the waste heat is, $\dot{Q}_{out} = 50 \text{ kW}$.

2.12a The power consumed by the engine is: $\dot{Q}_{in} = \dot{m} h_{fuel} = 2.5 \cdot 10^{-3} \cdot 46 \cdot 10^3 = 115 \text{ kW}$, that is consumed by the engine.

2.12b The net power output of the engine is calculated using the energy balance: $\dot{E}_{in} = \dot{E}_{out} \rightarrow \dot{Q}_{in} = \dot{W}_{net} + \dot{Q}_{out} \rightarrow \dot{W}_{net} = \dot{Q}_{in} - \dot{Q}_{out} = 115 - 50 = 65 \text{ kW}$.

2.12c The thermal efficiency of the engine is: $\eta_{thermal} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{65}{115} = 0.565 \rightarrow 56.5\%$.

2.13 Hydroelectric power plant

The total mechanical energy the water in a dam possesses is equivalent to the potential energy of water at the free surface of the dam (relative to free surface of discharge water), and it can be converted to work entirely. Therefore, the power potential of water is its potential energy, which is gz per unit mass (specific potential energy), and $\dot{m}gz$, for a given mass flow rate.

The specific potential energy is: $e_{pot} = g \cdot z = g \cdot h = 9.8 \cdot 90 = 882 \text{ J/kg} = 0.88 \text{ kJ/kg}$.

The mass flow rate is: $\dot{m} = \rho \cdot \dot{v} = 1000 \cdot 65 \text{ m}^3/\text{s} = 65000 \text{ kg/s}$.

The maximum power generation is therefore: $\dot{W}_{max} = \dot{m} e_{pot} (= \dot{m} \cdot g \cdot h) = 65000 \cdot 0.88 = 57200 \text{ kW} = 57.2 \text{ MW}$.

For the actual electric power generation the efficiency of the turbine-generator has to be taken into account: $\dot{W}_{electric} = \eta_{overall} \cdot \dot{W}_{max} = 0.48 \cdot 57.2 = 27.5 \text{ MW}$.

2.14 Energy and power

2.14a A conversion efficiency, η of 30% means that for every unit of nuclear heat per second 0.3 units of electric energy per second are produced, in a formula this looks like:

$$\eta = \frac{\text{desired output}}{\text{required input}} = \frac{\text{electric energy per second}}{\text{nuclear heat per second}} = \frac{\dot{Q}_{out}}{\dot{Q}_{nuclear-heat}} (= 0.3)$$

$$\rightarrow \dot{Q}_{nuclear-heat} [\text{MJ/s}] = \frac{\dot{Q}_{out}}{\eta} = \frac{\dot{Q}_{out}}{0.3} [\text{MW}].$$

2.14b The amount of coal can be calculated using the heating value of coal ($h = 28 \text{ MJ/kg}$). The heating value is the amount of energy released by burning one kg of fuel. The energy released by burning fuel is, $Q = m \cdot h$ or in rate form for the power, $\dot{Q} = \dot{m} \cdot h$. For the coal power plant also the efficiency of 50% (0.5) should be taken into account.

$$\rightarrow \dot{m}_{coal} [\text{kg/s}] = \frac{\dot{Q}_{out} [\text{MW}]}{0.5h [\text{MJ/kg}]} = \frac{\dot{Q}_{out}}{0.5h} [\text{kg/s}]$$

2.14c

- The nuclear power plant consumes an amount of nuclear heat:
 $\dot{Q}_{in-nuclear-heat} = \frac{1000}{0.3} = 3333 \text{ MJ/s} = 3333 \text{ MW}$. A year is $365 \times 24 \times 60 \times 60 = 31536 \cdot 10^3$ seconds $\rightarrow \dot{Q}_{in-nuclear-heat} = 3333 \times 31536 \cdot 10^3 = 10.51 \cdot 10^{10} \text{ MJ/year}$.
- The coal power plant consumes an amount of coal:
 $\dot{m}_{coal} = \frac{1000}{0.5 \cdot 28} = 71.4 \text{ kg/s} \rightarrow \dot{m}_{coal} = 71.4 \times 31536 \cdot 10^3 = 2.25 \cdot 10^9 \text{ kg/year}$.

With these numbers comparing both power plants is difficult as the input is of a different form units are different). The nuclear power plant requires a heat power input in MJ/year (coming from a nuclear reaction) while the coal power plant requires coal as input in kg/year. Therefore the dimensions of the input are different. For the nuclear power plant the input is $1.0512 \cdot 10^{11} \text{ MJ/year}$ while the input for the coal power plant is $2.25 \cdot 10^9 \text{ kg/year}$. Those two numbers cannot be compared easily.

To compare them the amount of coal can be converted into an amount of power using the heating value of coal: $\dot{Q}_{in-coal} = \dot{m}_{coal}h = 71.4 \times 28 = 1999.2 \text{ MJ/s} = 1999 \text{ kW}$. For a year this has to be multiplied by the seconds in a year: $\dot{Q}_{in-coal} = 1999.2 \times 31536 \cdot 10^3 = 6.30 \cdot 10^{10} \text{ MJ/year}$. Now they can be compared:

- Nuclear power plant: $\dot{Q}_{in-nuclear-heat} = 10.51 \cdot 10^{10} \text{ MJ/year}$.
- Coal power plant: $\dot{Q}_{in-coal} = 6.30 \cdot 10^{10} \text{ MJ/year}$.

It can be concluded that the nuclear power plant consumes more power to produce the same amount of output power. However this does not imply that the coal power plant is better. To say something about that also the impact on the environment or the nature or price of the fuel should be taken into account.

2.15 Energy and Environment 1

2.15a $Q_{CO_2} = XpQ$ [kg CO₂/year].

2.15b $Q_{CO_2} = 200000 \cdot 300 \cdot 0.59 = 35.4 \cdot 10^6 \text{ kg CO}_2/\text{year}$.

2.15c $Q_{CO_2} = 200000 \cdot 300 \cdot 1.1 = 66.6 \cdot 10^6 \text{ kg CO}_2/\text{year}$.

2.16 Energy and Environment 2

2.16a $Q_{NO_x-cars} = XaQ_{car}$ [kg NO_x/year].

$Q_{NO_x-electric} = Y_e Q_{electric}$ [kg NO_x/year].

$Q_{NO_x-gas} = Y_g Q_{gas}$ [kg NO_x/year].

$Q_{NO_x-total} = XaQ_{car} + Y_e Q_{electric} + Y_g Q_{gas}$ [kg NO_x/year].

2.16b $Q_{NO_x-cars} = 2 \cdot 15000 \cdot 0.57 = 1.71 \cdot 10^6 \text{ kg NO}_x/\text{year}$.

$Q_{NO_x-electric} = 3000 \cdot 7.1 = 21300 \text{ kg NO}_x/\text{year}$.

$Q_{NO_x-gas} = 2500 \cdot 4.3 = 10750 \text{ kg NO}_x/\text{year}$.

$Q_{NO_x-total} = 1742050 \text{ kg NO}_x/\text{year}$.

2.16c Driving the cars.

2.17 Power needs of a car

2.17a The work needed to accelerate a body is the change in the kinetic energy of the body:

$$W = \frac{1}{2}m(v^2 - v_i^2)$$

2.17b The average power is the work per unit time and is determined from:

$$\dot{W} = \frac{W}{\Delta t}$$

2.17c The additional power required is the work that needs to be done per unit time to raise the elevation of the car, which is equal to the change in potential energy of the car per unit time:

$$\dot{W} = mg \frac{\Delta h}{\Delta t} = mgv_{vertical} = mg \sin(\alpha)v$$

2.17d Fill in the values: $m = 1200 \text{ kg}$, $v_i = 20 \text{ km/h} = 5.56 \text{ m/s}$, $v = 80 \text{ km/h} = 22.22 \text{ m/s}$, $t = 20 \text{ seconds}$ and $\alpha = 30^\circ$.

$$W = \frac{1}{2}m(v^2 - v_i^2) = \frac{1}{2} \cdot 1200(22.22^2 - 5.56^2) = 277748 \text{ J} = 278 \text{ kJ}$$

$$\dot{W} = \frac{W}{\Delta t} = \frac{278}{20} = 13.9 \text{ kW}$$

Note that this is in addition to the power to overcome friction, rolling resistance and other imperfections.

$$\dot{W} = mg \frac{\Delta h}{\Delta t} = mgv_{vertical} = mg \sin(\alpha)v$$

$$\dot{W} = 1200 \cdot 9.8 \cdot \sin(30^\circ) \cdot 22.22 = 130654 \text{ J/s} = 131 \text{ kJ/s} = 131 \text{ kW}$$

Note that the car engine will have to produce 131 kW of additional power while climbing the hill if the car is to maintain its velocity. This is much more than the car needs to accelerate from 20 km/h to 80 km/h.

2.18 Cooking with potential energy

2.18a The potential energy of the turkey at the top of the building is: $PE = mgh$. This is converted into kinetic energy: $KE = \frac{1}{2}mv^2$. When the turkey hits the ground all kinetic energy is converted:

$$\Delta PE = \Delta KE \rightarrow mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh}$$

2.18b When the turkey hits the ground all kinetic energy is converted into internal energy, which is $U = mc\Delta T$.

$$\Delta KE = \Delta U \rightarrow \frac{1}{2}mv^2 = mc\Delta T \rightarrow \Delta T = \frac{v^2}{2c}$$

Alternatively the change in internal energy can be set equal to the change in potential energy directly.

$$\Delta PE = \Delta U \rightarrow mgh = mc\Delta T \rightarrow \Delta T = \frac{gh}{c}$$

So one time performing the experiment gives a temperature rise of the turkey of: $\Delta T = \frac{v^2}{2c} = \frac{gh}{c}$. To determine how often the experiment should be repeated one should assume the temperature rise needed to cook the turkey ΔT_{cook} . How often the experiment should be repeated follows from: $\# = \frac{\Delta T_{cook}}{\Delta T}$

2.18c Fill in the values: $h = 100 \text{ m}$, $c = 3000 \text{ J/kgK}$ and $\Delta T_{\text{cook}} = 210 - 10 = 200^\circ\text{C}$. The velocity when the turkey hits the ground is: $v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 100} = 44.3 \text{ m/s} = 159 \text{ km/h} !!!$ The temperature rise by performing the experiment once is: $\Delta T = \frac{v^2}{2c} = \frac{44.3^2}{2 \cdot 3000} = 0.33^\circ\text{C}$. The experiment should be repeated: $\# = \frac{\Delta T_{\text{cook}}}{\Delta T} = \frac{200}{0.33} = 606 \text{ x}$. At 0.33°C per fall it will require repeating the experiment more than 600 times to reach the rise in cooking temperature of 200°C .

2.18d From the answers we can conclude that the magnitude of the change in temperature is very small, only 0.33°C compared to a change in height of 100 m and a change in velocity of almost 160 km/h . We consider a change of height of 100 m as quite high and a velocity change of 160 km/h is really high, while a temperature change of 0.33° is almost neglectable. This means that you need a lot more energy to heat something substantially than to rise or move something.

2.18e If the building is two times higher the temperature rise per fall will be 2 times higher, i.e. 0.66°C , while the velocity will be increased by a factor of $\sqrt{2}$ to 225 km/h .

If we have a turkey with a different mass the answers do not change, the mass of the turkey is not important, the mass cancels in the equations.

2.18f No, it is not necessary to know the velocity in order to calculate how often you need to repeat the experiment to cook the turkey. You can set the change in potential energy equal to the change in internal energy directly.

3 Thermodynamic Properties of Pure Substances

3.1 Phases of matter

3.1a The three phases that matter can have are: the solid phase, the liquid phase and the gas phase.

3.1b The six phase transitions that can occur are:

- 1) From solid to liquid called melting.
- 2) From solid to gas called sublimation.
- 3) From liquid to solid called freezing.
- 4) From liquid to gas called vaporization.
- 5) From gas to solid called desublimation.
- 6) From gas to liquid called condensation.

3.1c If a substance starts to boil the pressure and temperature are fixed, they are at their saturation point. This is that at a given pressure, the temperature at which a pure substance changes phases is called the saturation temperature, T_{sat} . Likewise, at a given temperature, the pressure at which a substance changes phases is called the saturation pressure, P_{sat} . For example for water at a pressure of 101.325 kPa, $T_{sat} = 99.97^\circ\text{C}$. Conversely, at a temperature of 99.97°C , $P_{sat}=101.325$ kPa. For practically all substances tables that list the saturation pressure against the temperature (or the saturation temperature against the pressure) are available.

3.1d The area in which substances exist in two phases is called the co-existent area or simply the two phase area or the mixture phase.

3.1e A substance that exists in two phases is called a mixture.

3.1f The temperature of the liquid is equal to the temperature of the gas in the mixture, $T_l = T_v$, also the pressure of the liquid is equal to the pressure of the gas, $P_l = P_v$

3.1g The pressure and temperature of a mixture in an open system do not change if heat is added to the mixture. The heat added is used to vaporize the liquid in the mixture. However, once all of the liquid is vaporized the temperature of the vapor will rise (this is called the superheated vapor phase).

3.1h In general, the amount of heat absorbed or released during a phase-change process is called the latent heat. The energy (heat) necessary to vaporize the liquid is called the latent heat of vaporization. The amount of energy absorbed during melting is called the latent heat fusion.

3.1i The energy necessary to vaporize a liquid and the energy necessary to condense the vapor are equal. In other words the energy used for vaporization is equal to the energy that is released during condensation.

3.2 Tv -diagram

3.2a See figure 3.2.1.

3.2b Critical point → top of the dome curve where the line of saturated liquid (kooklijn) and the line of saturated vapor (damplijn) though each other.

3.2c Saturated liquid → on the left part of the dome curve (only the line!), this is the saturated liquid line (kooklijn).

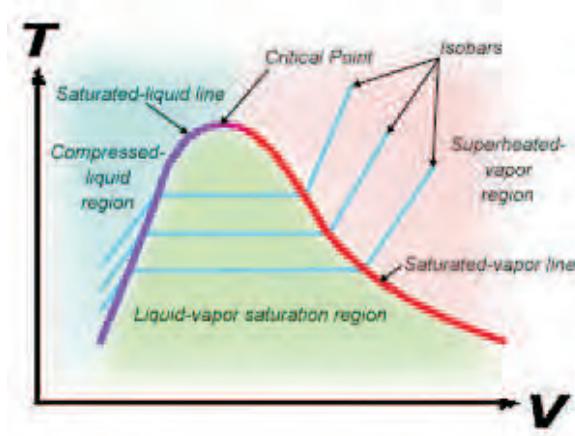


Figure 3.2.1: The $T - v$ diagram with the vapor dome and the different regions.

3.2d Saturated vapor → on the right part of the dome curve (only the line!), this is the saturated vapor line (damplijn).

3.2e Compressed liquid → in the area left of the saturated liquid line, thus left of the dome.

3.2f Saturated mixture → in the area below the dome.

3.2g Superheated vapor → in the area right of the saturated vapor line, thus right of the dome.

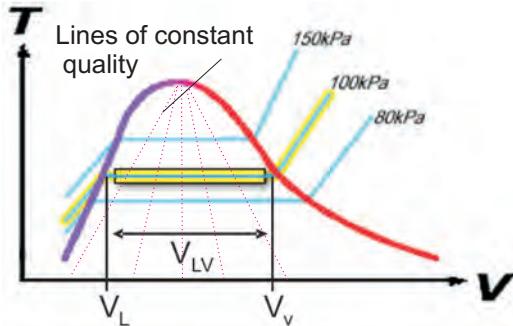
3.3 Incompressible liquid

3.3a An incompressible liquid is a substance of which the volume hardly changes when the pressure or temperature changes. That is it can not be compressed when pressure is applied to it, water is an example. The most important property is: $v = \text{constant} \rightarrow dv = 0$. This means no change in volume.

3.3b $\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = 0$ as $dv = 0$.
 $\kappa = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T = 0$ as $dv = 0$.

3.4 Mixtures

3.4a Tv - diagram.



3.4b $v_{lv} = v_v - v_l$.

3.5 Quality of a mixture

3.5a The total mass is the mass of the liquid plus the mass of the vapor, $m = m_l + m_v$.

3.5b The quality is defined as,

$$x = \text{Mass fraction of vapor} = \frac{\text{Mass of vapor}}{\text{Total mass of mixture}} = \frac{m_v}{m}$$

$$1 - x = \text{Mass fraction of liquid} = \frac{\text{Mass of liquid}}{\text{Total mass of mixture}} = \frac{m_l}{m}.$$

3.5c No, outside the mixture region x is not defined and has no meaning.

3.5d The quality can have values between zero and one, $0 \leq x \leq 1$.

3.5e For saturated liquid the quality is $x = 0$.

3.5f For saturated vapor the quality is $x = 1$.

3.5g Temperature and pressure do not change, they are coupled and constant during vaporization or condensation. Volume, internal energy, enthalpy and entropy do change. In a vaporization process the volume, internal energy, enthalpy and entropy increase. For a condensation process these properties decrease.

The quality of a mixtures also changes: during vaporization it increases, during condensation it decreases.

3.5h See figure at exercise 3.4 a (the dotted red lines, they go down from the critical point).

3.6 Use of tables: water

The values for the tables are found in the tables with the properties of water, see tables A-4, A-5, A-6, and A-7 from the thermodynamic textbook by Cengel and Boles. Note that tables A-4 and A-5 contain the same information, however, A-4 is sorted on temperature and A-5 on pressure. Also note, different books can have slightly different values. In this problem the required values come from tables A-4, A-5 and A-6 from the textbook by Cengel and Boles. Given values are underlined.

T ($^{\circ}\text{C}$)	P (kPa)	v (m^3/kg)	Phase description
50	12.352	4.16	mixture
120.21	200	0.88578	saturated vapor
250	400	0.59520	superheated vapor
110	600	0.001052	compressed liquid

T ($^{\circ}\text{C}$)	P (kPa)	h (kJ/kg)	x	Phase description
120.21	200	2045.82	0.7	mixture
140	361.53	1800	0.56	mixture
177.66	950	752.74	0.0	saturated liquid
80	500	335.49	-	compressed liquid
350	800	3161.4	-	superheated vapor

Explanation about the tables: The required values come from table A-4, A-5 and A-6 from the textbook by Cengel and Boles (8th edition). Note that tables A-4 and A-5 contain the same information: A-4 is sorted on temperature and A-5 on pressure.

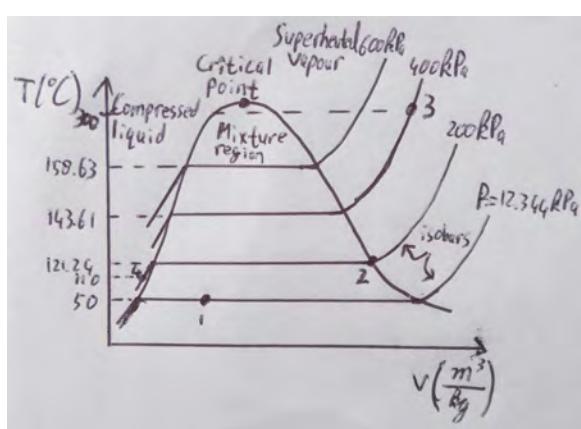
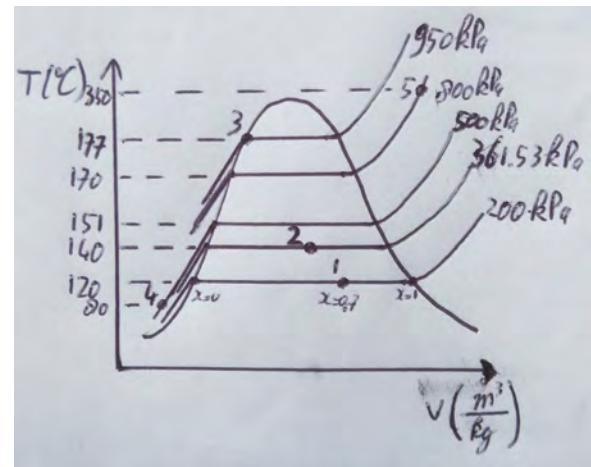
Table 1: see also figure 3.6.1.

First row: Given is the temperature of $T = 50^\circ\text{C}$. Look in table A-4. It states that the specific volume of the saturated liquid phase is $v_f = 0.001012 \text{ m}^3/\text{kg}$ and the specific volume of the saturated vapour phase is $v_g = 12.026 \text{ m}^3/\text{kg}$. The given specific volume of $v = 4.16 \text{ m}^3/\text{kg}$ is between these values, therefore we are in the mixture phase. The pressure is therefore the saturation pressure at 50°C : $P_{sat@50^\circ\text{C}} = 12.352 \text{ kPa}$.

Second row: The saturated vapour is given: at $P = 200 \text{ kPa}$. So, the point is on the vapour line. In table A-5 the temperature belonging to this saturation pressure can be found: $T_{sat@200\text{kPa}} = 120.21^\circ\text{C}$. The specific volume is the specific volume of the saturated vapour: $v_g = 0.88578 \text{ m}^3/\text{kg}$.

Third row: Temperature and pressure are given. The first thing that we need to find is at which temperature water boils for the given pressure. Table A-5 gives $T_{sat@400\text{kPa}} = 143.61^\circ\text{C}$. The given temperature is higher, so the water has already boiled and we now have a superheated vapour. The specific volume is found in table A-6 at $T = 250^\circ\text{C}$.

Fourth row: As in part 3, we look in table A-6: $T_{sat@600\text{kPa}} = 158.83$. This is higher than the given temperature of $T = 110^\circ\text{C}$. The water is therefore a compressed liquid. Since specific volume is independent of pressure, we take the specific volume at $T = 110^\circ\text{C}$ in table A-4.

Figure 3.6.1: T - v diagram for the first table (not to scale)Figure 3.6.2: T - v diagram for the second table (not to scale)**Table 2:** see also figure 3.6.2.

First row: Given is $x = 0.7$. x is the vapour fraction ($x = \frac{\text{number of parts vapour}}{\text{total parts}}$, not defined outside the mixture region), so we have a mixture of liquid and vapour. Therefore, $T = T_{sat@200\text{kPa}} = 120.21^\circ\text{C}$ (table A-5). h follows from $h = h_f + x(h_g - h_f)$ with $h_f = 504.71 \text{ kJ/kg}$ and $h_g = 2706.3 \text{ kJ/kg} \Rightarrow h = 2045.8 \text{ kJ/kg}$.

Second row: Given is $T = 140^\circ\text{C}$. From table A-4 it follows that $h_f = 589.16 \text{ kJ/kg}$ and $h_g = 2733.5 \text{ kJ/kg}$. The given h is between these values, so it is a mixture. Therefore, from table A-4, $p = P_{sat@140^\circ\text{C}} = 361.19 \text{ kPa}$. x is then determined as $x = \frac{h-h_f}{h_g-h_f} = 0.56$.

Third row: $x = 0.0$, so there is no vapour, only liquid. So, it is a saturated liquid (outside the mixture region x does not have a value). From table A-5: $T = T_{sat@950\text{kPa}} = 177.66^\circ\text{C}$ and $h = h_f = 752.74 \text{ kJ/kg}$.

Fourth row: At $P = 500 \text{ kPa}$, the saturation temperature is 151.83°C (table A-5). The given temperature of 80°C is lower, so we have a compressed liquid. Using $dh = vdP$, we find $h = h_f@80^\circ\text{C} + v(P - P_{sat@80^\circ\text{C}}) = 335.02 + 0.001029(500 - 47.416) = 335.49 \text{ kJ/kg}$. We are not in the mixture region, so x is not defined.

Fifth row: At $P = 800 \text{ kPa}$, $h_g = 2768.3 \text{ kJ/kg}$ (table A-5). The given value for h is higher, so it is a superheated vapour. In table 12 at $P = 800 \text{ kPa} = 0.8 \text{ Mpa}$ and $h = 3161.4 \text{ kJ/kg}$, we find that $T = 350^\circ\text{C}$. Since we are not in the mixture region, x is not defined.

3.7 Use of tables, refrigerant 134a

The values for this problem were obtained from the textbook by Cengel and Boles (8th edition), namely from tables A-11, A-12 and A-13 with the properties of refrigerant 134a. Note: different books can have slightly different values.

T ($^{\circ}$)	P (kPa)	v (m^3/kg)	Phase description
-8	500	0.0007570	compressed liquid
30	770.64	0.022	mixture
2.46	320	0.063681	saturated vapor
100	600	0.047900	superheated

T ($^{\circ}$)	P (kPa)	u (kJ/kg)	Phase description
20	572.07	95	mixture
-12	185.37	35.76	saturated liquid
86.2	400	300	superheated vapor
6	600	59.66	compressed liquid

3.8 Boiling of water

3.8a The temperature at which the water starts to boil is dependent on the pressure inside the cylinder. The pressure inside the cylinder is a combination of the outside pressure (P_{atm}) and the pressure of the piston ($P_{piston} = F/A = (M \cdot g)/A$, with A the surface and M the mass of the piston). $P_{cylinder} = P_{atm} + M \cdot g/A = (20 \cdot 9.8)/100 \cdot 10^{-4} + 100 \cdot 10^3 = 19600 + 100000 = 119600 \text{ Pa} = 119.6 \text{ kPa}$. The pressure inside the cylinder is 119.6 kPa. In table A-5 the saturation temperature of this pressure can be found. It is seen that at 100 kPa that water boils at $T_{sat@100kPa} = 99.61$ degree Celsius and at 125 kPa the water boils at $T_{sat@125kPa} = 105.97$ degree Celsius. By interpolation the temperature at a pressure of 119.6 kPa can be found. This temperature is $T_{sat@119.6kPa} = 104.6$ degree Celsius.

3.8b If you go up in the mountains the outside pressure (P_{atm}) will decrease. Therefore the pressure inside the cylinder will decrease. At a lower pressure the water starts to boil at a lower temperature.

3.8c If the pressure outside the cylinder decreases to 95 kPa the pressure inside the cylinder will also decrease with 5 kPa and becomes 114.6 kPa. The water will boil then at $T_{sat@114.6kPa} = 103.3$ degree Celsius.

3.9 Vessel with 2 kg refrigerant 134a

Given: Refrigerant 134a, $m=2$ kg, in a vessel at $P=900$ kPa and $T=80^{\circ}\text{C}$.

Both table A-12 and A-13 state that at a pressure of 900 kPa the saturation temperature of refrigerant 134a is 35.51°C . The temperature in the vessel is 80°C , therefore the refrigerant in the vessel is superheated vapor.

From table A-13: $P=900$ kPa and $T=80^{\circ}\text{C} \rightarrow v=0.02861 \text{ m}^3/\text{kg}$ and $u=288.87 \text{ kJ/kg}$.

The total volume of the vessel is $V = mv = 2 \cdot 0.02863 = 0.05726 \text{ m}^3$ and the total internal energy is $U = mu = 2 \cdot 289.88 = 579.76 \text{ kJ}$.

3.10 Vessel with refrigerant 134a

Given: Refrigerant 134a, $m=10$ kg, in a vessel, $V=0.5\text{m}^3$, at $T=-20^{\circ}\text{C}$.

3.10a The density of the refrigerent inside the vessel is $\rho = m/V = 10/0.5 = 20 \text{ kg/m}^3$. The specific volume is $v = 1/\rho = 1/20 = 0.05 \text{ m}^3/\text{kg}$. This is a mixture as 10 kg of the fluid takes less space than 0.5m^3 and 10 kg of gas takes more space than 0.5m^3 . This can be seen from table A-11 $\rightarrow v_l < v < v_v$ with $v_l = 0.0007361 \text{ m}^3/\text{kg}$ and $v_v = 0.14735 \text{ m}^3/\text{kg}$. Therefore the pressure is the saturation pressure $P = P_{sat@-20^\circ C} = 132.82 \text{ kPa}$ (table A-11).

3.10b The total internal energy U can be determined by $U = mu = m(u_l + x(u_v - u_l))$. The mixture fraction x follows from the volumes as $x = \frac{v - v_l}{v_v - v_l} = \frac{0.05 - 0.0007361}{0.14735 - 0.0007361} = 0.336$. The values for u_l and u_v follow from table A-11 $\rightarrow u_l = 25.37 \text{ kJ/kg}$ and $u_v = 218.86 \text{ kJ/kg}$. $U = mu = m(u_l + x(u_v - u_l)) = 10(25.37 + 0.336(218.86 - 25.37)) = 903.83 \text{ kJ}$.

3.10c The mixture fraction is defined as $x = m_v/m \rightarrow m_v = xm = 3.36 \text{ kg}$ and $m_l = m - m_v = 6.64 \text{ kg} \rightarrow V_l = v_l m_l = 0.0007361 \cdot 6.64 = 0.00489 \text{ m}^3$.

3.11 Water mixture of liquid and vapor

We have 0.1m^3 of liquid water and 0.9m^3 of water vapor in equilibrium at $P = 800 \text{ kPa}$. The substance is in the mixture phase! Heat is added at a constant pressure till the temperature is $350^\circ C$.

3.11a See figure 3.11.1.

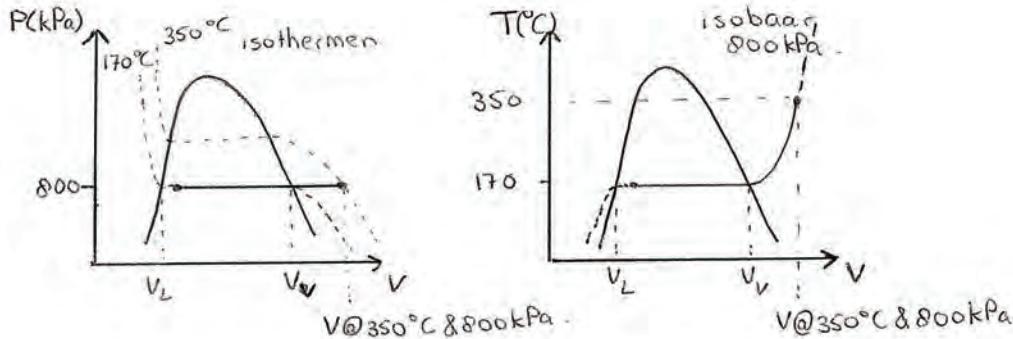


Figure 3.11.1: P-v and T-v diagrams

3.11b Initially the substance is in the mixture phase. In this phase the temperature is coupled to the pressure. It can be found in table A-5 $\rightarrow T = T_{sat@800kPa} = 170.41^\circ C$.

3.11c Table A-5 gives for the mixture phase at $800 \text{ kPa} \rightarrow v_l = 0.001115 \text{ m}^3/\text{kg}$ and $v_v = 0.24035 \text{ m}^3/\text{kg}$.

$$\text{Liquid water} \rightarrow V_{lw} = 0.1\text{m}^3 \rightarrow M_{lw} = \frac{V_{lw}}{v_l} = \frac{0.1}{0.001115} = 89.7 \text{ kg.}$$

$$\text{Water vapor} \rightarrow V_{wv} = 0.9\text{m}^3 \rightarrow M_{wv} = \frac{V_{wv}}{v_v} = \frac{0.9}{0.24035} = 3.7 \text{ kg.}$$

$$M_{total} = M_{lw} + M_{wv} = 89.7 + 3.7 = 93.4 \text{ kg.}$$

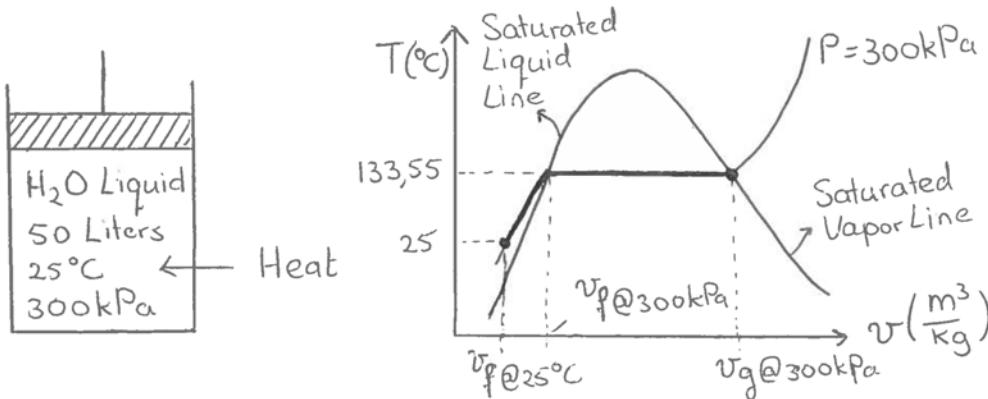
The mixture fraction $x = \frac{M_{wv}}{M_{total}} = \frac{3.7}{93.4} = 0.04 \rightarrow$ initially only 4% of the mixture is water vapor and 96% of the mixture is liquid water.

3.11d At $T = 350^\circ C$ and $P = 800 \text{ kPa}$ the substance is in the superheated vapor phase. The final (absolute) volume can be found using table A-6. At $T = 350^\circ C$ en $P = 800 \text{ kPa}$ the specific volume is $v = 0.35442 \text{ m}^3/\text{kg} \rightarrow V_{final} = M_{total} \cdot v = 93.4 \cdot 0.35442 = 33.1 \text{ m}^3$.

When all liquid water has turned into water vapor the volume is $V_{total@sat} = M_{total} \cdot v_{v@800kPa} = 93.4 \cdot 0.24035 = 22.45 \text{ m}^3$.

3.12 Vaporization of water at constant pressure

Given is a piston-cylinder device that contains 50 liters of water at $T = 25^\circ\text{C}$ and 300 kPa. At constant pressure heat is added to the water until the entire liquid is vaporized.



3.12a Heat is added, the temperature of the water rises, but the water stays liquid until it reaches the temperature at which it starts to boil at a pressure of 300 kPa (point on the saturated liquid line). From there on the water starts to evaporate and the temperature stays constant until the entire liquid is vaporized (point on the saturated vapor line).

3.12b The end point is on the saturated vapor line, therefore the final temperature is the temperature at which the water boils at 300 kPa, $\rightarrow T = T_{sat@300kPa} = 133.52^\circ\text{C}$.

3.12c The mass of the water is the total volume, V in m^3 divided by the specific volume of the water at a temperature of $T = 25^\circ\text{C}$, $v_f@25C$.

$$V = 50 \text{ liters} = 50 \cdot 10^{-3} = 0.050 \text{ m}^3 \text{ and } v_f@25C = 0.001003 \text{ m}^3/\text{kg} \text{ (table 4).}$$

$$\text{Mass} \rightarrow m = \frac{V}{v_f@25C} = \frac{0.050 \text{ [m}^3\text{]}}{0.001003 \text{ [m}^3/\text{kg}\text{]}} = 49.85 \text{ kg.}$$

3.12d The total enthalpy change is:

$$\Delta H = H_{end} - H_{begin} = m \cdot h_{end} - m \cdot h_{begin} = m \cdot (h_{end} - h_{begin}) = m \cdot \Delta h.$$

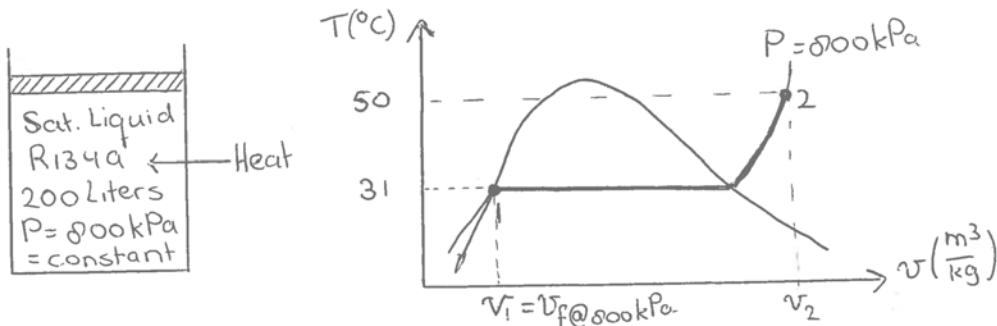
$$h_{begin} = h_f@25C = 104.83 \text{ kJ/kg} \text{ (table A-4)} \text{ and } h_{end} = h_g@300kPa = 2724.9 \text{ kJ/kg} \text{ (table A-5).}$$

$$\Delta H = m \cdot \Delta h = m \cdot (h_{end} - h_{begin}) = 49.85 \cdot (2724.9 - 104.83) = 49.85 \cdot 2620.07 = 130610 \text{ kJ} = 130.6 \text{ MJ.}$$

3.13 Work done by heating 1

Given is a piston-cylinder device that contains 200 liters of saturated liquid refrigerant 134a at 800 kPa. At a constant pressure of 800 kPa heat is added until the refrigerant reaches a temperature of $T = 50^\circ\text{C}$.

3.13a The process starts at the saturated liquid line at 800 kPa. By adding heat the liquid starts to vaporize and the volume increases. The temperature however stays constant till the entire liquid is vaporized (the added heat/energy is used to break the water bindings instead of heating the liquid). The temperature is the saturation temperature at 800 kPa which is: $T_{sat@800kPa} = 31.31^\circ\text{C}$ (table A-12). After all the liquid is vaporized the temperature starts to increase. Simultaneously, the volume increases while the pressure stays constant at 800 kPa. When the temperature $T_2 = 50^\circ\text{C}$ the process ends. At that point the refrigerant is superheated.



3.13b The specific work follows from the differential equation for the volume work $\delta w = Pdv$, when the pressure P is **constant** this is evaluated as:

$$\text{Specific work in kJ/kg} \rightarrow w = \int_{v_1}^{v_2} Pdv = P(v_2 - v_1).$$

For the total work in kJ this expression has to be multiplied by the mass: $W = m \cdot w$.

3.13c $v_1 = v_f @ 800kPa = 0.0008457 \text{ m}^3/\text{kg}$ (table A-12) and $v_2 = v @ 50C @ 800kPa = 0.028547 \text{ m}^3/\text{kg}$ (table A-13).

Specific work in $\text{kJ/kg} \rightarrow w = P(v_2 - v_1) = 800(0.028547 - 0.0008457) = 800 \cdot 0.0277 = 22.16 \text{ kJ/kg}$.

To determine the total work the mass of the liquid should be known. The mass of the liquid is the total volume, V in m^3 divided by the specific volume of the liquid at the saturation liquid line at 800 kPa.

$V = 200 \text{ liters} = 200 \cdot 10^{-3} = 0.200 \text{ m}^3$ and $v_f @ 800kPa = 0.0008458 \text{ m}^3/\text{kg}$ (table 12).

$$\text{Mass refrigerant} \rightarrow m = \frac{V}{v_f @ 800kPa} = \frac{0.200 [\text{m}^3]}{0.0008457 [\text{m}^3/\text{kg}]} = 236.49 \text{ kg}.$$

$$\text{Total work in kJ} \rightarrow W = m \cdot w = 236.49 \cdot 22.16 = 5240.6 \text{ kJ} = 5.24 \text{ MJ}.$$

The gas performs 5.24 MJ of total work on the environment (gas expands, volume gets larger).

3.14 Work done by heating 2

3.14a Given is a mass of 5 kg of saturated water at $P_1 = 200 \text{ kPa}$ that is heated at constant pressure to a temperature of $T_2 = 300^\circ\text{C}$.

The work done by the steam can be calculated as $W = \int PdV$, which is (since the pressure is kept constant) equal to $W = P(V_2 - V_1)$. From table A-5, we find that the specific volume is $v_1 = v_f @ 200kPa = 0.001061 \text{ m}^3/\text{kg}$. The volume is then $V_1 = mv_1 = 5 \cdot 0.001061 = 0.005305 \text{ m}^3$. The final specific volume can be determined from the tables for superheated water (table A-6) using $P_2 = P_1 = 200 \text{ kPa}$ and $T_2 = 300^\circ\text{C}$: $v_2 = 1.31623 \text{ m}^3/\text{kg}$. This results in a final volume of $V_2 = mv_2 = 5 \cdot 1.3162 = 6.581 \text{ m}^3$. Filling in the equation for the work then gives:

$$W = \int PdV = P(V_2 - V_1) = 200 \cdot (6.581 - 0.005305) = 1315.14 \text{ kJ}$$

What is happening in this situation? The water is saturated at the saturated liquid line at 200 kPa. Adding extra heat does not result in a temperature increase, but in the evaporation of the water. During evaporation the volume increases as molecules enter the gas phase, where they are further apart from each other. When all water has evaporated the temperature rises when more heat is added. The volume also increases. Hence, the volume increases constantly while the temperature is constant during evaporation.

3.15 Perfect gas

3.15a A perfect gas (or ideal gas) is an imaginary substance that obeys the relation $Pv = RT$. The ideal gas-relation closely approximates the $P - v - T$ behavior of real gases at low densities. At low pressures and high temperatures, the density of a gas decreases, and the gas behaves as an ideal gas under these conditions.

In the range of practical interest, many familiar gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, krypton, and even heavier gases such as carbon dioxide can be treated as ideal gases with negligible error. Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, however, should not be treated as ideal gases. Instead, the property tables should be used for these substances.

3.15b The ideal-gas equation of state, or simply the ideal-gas relation (the relation between $P - v - T$) can be written in different forms. With pressure P , temperature T , the gas constant R and the specific volume v its form is: $Pv = RT$.

If the volume is the total volume $V = mv$, with m the total mass, instead of the specific volume v this changes to: $PV = mRT$.

Instead of the gas constant R , which is different for every gas (see table A-2 in the book) also the universal gas constant, $R_u = 8.31 \text{ kJ/(kmol K)}$ (the same for every gas) can be used. In this case the ideal-gas relation changes to: $PV = NR_uT$, with N the number of molecules in mol.

The relation between R and R_u is: $R = \frac{R_u}{M}$ with M the molar mass (also called the molecular weight, see table A-1 in the book).

3.15c The ideal-gas relation with the specific volume is used, $Pv = RT$ which gives $v = \frac{RT}{P}$ or $P = \frac{RT}{v}$.

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = \frac{1}{v} \frac{R}{P} = \frac{R}{Pv} = \frac{1}{T}.$$

$$\kappa = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T = -\frac{1}{v} (-1) \frac{RT}{P^2} = \frac{RT}{v} \frac{1}{P^2} = \frac{1}{P}.$$

3.16 A balloon

3.16a Given: A balloon with diameter D and pressure P . What is the relation for the mole number? To determine the mole number the ideal gas law with the universal gas constant in it has to be used $PV = nR_uT \rightarrow n = \frac{PV}{TR_u}$. In this case the volume is the total volume, $V = 4/3\pi(1/2D)^3$ (and not the specific volume).

3.16b The total mass depends on the mole number and the molar mass, M , of the gas inside the balloon, $m = nM$.

3.16c The mole number is independent of the gas in the balloon, R_u is the universal gas constant which is the same for every gas, $R_u = 8.314 \text{ J/(molK)} \rightarrow n = \frac{P(4/3\pi(1/2D)^2)}{TR_u} = \frac{200 \cdot 10^3 \cdot 4/3 \cdot \pi(1/2 \cdot 6)^3}{8.314 \cdot 293} = 9.285 \cdot 10^3 \text{ mol} = 9.285 \text{ kmol}$.

For helium the molar mass is $M_{helium} = 4.003 \text{ kg/mol} \rightarrow m_{helium} = nM_{helium} = 9.285 \cdot 4.003 = 37.17 \text{ kg}$.

3.16d The mole number is the same for every gas, therefore also in the case of nitrogen it is 9.285 kmol. For nitrogen the molar mass is $M_{nitrogen} = 28.013 \text{ kg/kmol} \rightarrow m_{nitrogen} = nM_{nitrogen} = 9.285 \cdot 28.013 = 260.10 \text{ kg}$.

3.17 An automobile tire

3.17a Given: A tire, pressure P_1 , temperature T_1 and volume V_1 . It is assumed that the tire is rigid and therefore the volume is constant.

Find the relation for the pressure rise ΔP when the temperature rises to T_2 .

Ideal gas law $PV = mRT \rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow P_2 = P_1 \left(\frac{T_2}{T_1} \right)$ as the volume and the mass are constant.

$\Delta P = P_2 - P_1 = P_1 \left(\frac{T_2}{T_1} \right) - P_1 = P_1 \left(\frac{T_2}{T_1} - 1 \right)$. In which R is the specific gas constant for air, $R_{air} = 287 \text{ J/kgK}$ (table A-1). Also $PV = nR_uT$ can be used but the first one is preferred in this case as in b) the amount of air has to be calculated that has to be bled off to restore the pressure.

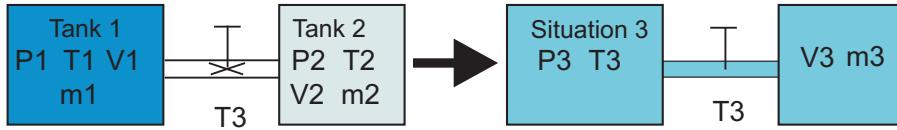
3.17b The mass of the air that must be bled off to restore the pressure to its original pressure P_1 at temperature T_2 must be determined. You should not determine the change in volume as the volume is not defined properly, it depends on the conditions as pressure and temperature.

The original mass is $m_1 = \frac{P_1 V_1}{R T_1}$, the new mass after bleeding of the air is $m_2 = \frac{P_1 V_1}{R T_2}$. The mass that must be bled off is $\Delta m = m_1 - m_2 = \frac{P_1 V_1}{R T_1} - \frac{P_1 V_1}{R T_2} = \frac{P_1 V_1}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$.

3.17c Given: $P_1 = 210 \text{ kPa}$ gauge pressure, $T_1 = 25^\circ\text{C} = 298 \text{ K}$, $V_1 = 0.025 \text{ m}^3$, $T_2 = 50^\circ\text{C} = 323 \text{ K}$ and $P_{atm} = 100 \text{ kPa}$. Calculate the pressure rise. Note $P_{1-absolute} = 210 + 100 = 310 \text{ kPa}$.
 $\Delta P = P_2 - P_1 = P_1 \left(\frac{T_2}{T_1} - 1 \right) = 310 \left(\frac{323}{298} - 1 \right) = 26 \text{ kPa}$.

3.17d The mass that must be bled off is $\Delta m = \frac{P_1 V_1}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) = \frac{310 \cdot 10^3 \cdot 0.025}{287} \left(\frac{1}{298} - \frac{1}{323} \right) = 7.01 \cdot 10^{-3} \text{ kg} = 7 \text{ gram}$.

3.18 Two tanks with air



3.18a Use the ideal gas law with the total volume and the mass in it because these numbers are given $PV = mRT \rightarrow V_2 = \frac{m_2 R T_2}{P_2}$.

3.18b Ideal gas law $PV = mRT \rightarrow m_1 = \frac{P_1 V_1}{R T_1}$.

The new pressure P_3 will be $P_3 = \frac{m_3 R T_3}{V_3}$. The mass and the volume are extensive value and can be added therefore the new mass $m_3 = m_1 + m_2$ and the new volume $V_3 = V_1 + V_2$.

$$P_3 = \frac{(m_1 + m_2) R T_3}{V_1 + V_2}$$

3.18c Given is $P_1 = 500 \text{ kPa}$, $T_1 = 25^\circ\text{C} = 298 \text{ K}$, $V_1 = 1 \text{ m}^3$, $P_2 = 200 \text{ kPa}$, $T_2 = 35^\circ\text{C} = 308 \text{ K}$, $m_2 = 5 \text{ kg}$ and $T_3 = 20^\circ\text{C} = 293 \text{ K}$. R is the specific gas constant for air which can be found in table 2 $\rightarrow R = 287 \text{ J/kgK}$.

$$V_2 = \frac{m_2 R T_2}{P_2} = \frac{5 \cdot 287 \cdot 308}{200 \cdot 10^3} = 2.21 \text{ m}^3$$

$$\text{3.18d } m_1 = \frac{P_1 V_1}{R T_1} = \frac{500 \cdot 10^3 \cdot 1}{287 \cdot 298} = 5.846 \text{ kg}$$

$$P_3 = \frac{(m_1 + m_2) R T_3}{V_1 + V_2} = \frac{(5.846 + 5) \cdot 287 \cdot 293}{1 + 2.21} = 284128 \text{ Pa} = 284.1 \text{ kPa}$$

3.19 Enthalpy change of nitrogen

This problem requires the change in enthalpy of nitrogen to be calculated (Δh) if it is heated from 600 K to 1000 K using three different values for c_p . The enthalpy change is given by $dh = c_p dT \rightarrow \Delta h = \int_{T_1}^{T_2} dh = \int_{T_1}^{T_2} c_p(T) dT$.

3.19a The empirical relation for c_p can be found in table A-2c and is $\bar{c}_p = a + bT + cT^2 + dT^3$ with T in Kelvin and \bar{c}_p in kJ/kmolK. For nitrogen the constants are $a = 28.9$, $b = -0.1571 \cdot 10^{-2}$, $c = 0.8081 \cdot 10^{-5}$ and $d = -2.873 \cdot 10^{-9}$.

$$\Delta h = \int_{T_1}^{T_2} \bar{c}_p(T) dT = \int_{T_1}^{T_2} (a + bT + cT^2 + dT^3) dT = [aT + \frac{1}{2}bT^2 + \frac{1}{3}cT^3 + \frac{1}{4}dT^4]_{T_1}^{T_2} = a(T_2 - T_1) + \frac{1}{2}b(T_2^2 - T_1^2) + \frac{1}{3}c(T_2^3 - T_1^3) + \frac{1}{4}d(T_2^4 - T_1^4).$$

Filling in $T_1 = 600$ K and $T_2 = 1000$ K and the values for a , b , c and d gives $\Delta h = 12544$ kJ/kmol. The molar weight of nitrogen is $M_{nitrogen} = 28.013$ kg/kmol $\rightarrow \Delta h = \frac{12544}{28.013} = 447.79$ kJ/kg.

3.19b The average value for c_p is $c_{p-avr} = \frac{1}{2}(c_p(T = 600\text{ K}) + c_p(T = 1000\text{ K})) = \frac{1}{2}(1.075 + 1.167) = 1.21$ kJ/kgK (the values of c_p follow from table A-2b).

$$\Delta h = \int_{T_1}^{T_2} c_p(T) dT = \int_{T_1}^{T_2} c_{p-avr} dT = [c_{p-avr} T]_{T_1}^{T_2} = c_{p-avr}(T_2 - T_1). \text{ Fill in the numbers and it follows that } \Delta h = 448.46 \text{ kJ/kg.}$$

3.19c The value of c_p at room temperature is $c_{p-room} = 1.039$ kJ/kgK (Table A-2a, 300 K).

$$\Delta h = \int_{T_1}^{T_2} c_p(T) dT = \int_{T_1}^{T_2} c_{p-room} dT = [c_{p-room} T]_{T_1}^{T_2} = c_{p-avr}(T_2 - T_1). \text{ Fill in the numbers and it follows that } \Delta h = 415.6 \text{ kJ/kg.}$$

3.19d The answer (c) significantly differs from the answers (a) and (b). Answer (a) is probably the most exact answer because the temperature dependence of c_p is taken into account. It seems that taking the average c_p value is not bad at all. Answer (b) is close to the answer (a). In part (c), taking the c_p at room temperature results in a large error in the answer; the calculated c_p is far from correct.

3.20 Heat transfer and work at constant enthalpy

3.20a For an ideal gas the enthalpy is only a function of the temperature $h = h(T) \neq h(P) \rightarrow dh = c_p dT$. If the enthalpy is constant $dh = 0$.

$$dh = c_p dT = 0 \rightarrow dT = 0 \rightarrow T_1 = T_2.$$

3.20b Ideal gas law $Pv = RT \rightarrow \frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \rightarrow P_1 v_1 = P_2 v_2$.

3.20c Also the internal energy of an ideal gas is only a function of temperature $u = u(T) \neq u(P) \rightarrow du = c_v dT$. From a it follows that dT is zero and therefore $du = c_v dT = 0$.

$$du = \delta q + \delta w = 0 \rightarrow \delta q = -\delta w \rightarrow q = -w \rightarrow Q = -W.$$

3.20d The specific work is, $\delta w = Pdv \rightarrow w = \int_{V_1}^{V_2} Pdv = \int_{V_1}^{V_2} \frac{RT}{v} dv = RT [\ln v]_{V_1}^{V_2} = RT \left(\ln \frac{V_2}{V_1} \right) = P_1 v_1 \left(\ln \frac{V_2}{V_1} \right)$. The total work is $W = mw = mP_1 v_1 \left(\ln \frac{V_2}{V_1} \right) = P_1 V_1 \left(\ln \frac{V_2}{V_1} \right)$.

3.20e $W = P_1 V_1 \left(\ln \frac{V_2}{V_1} \right) = 1 \cdot 10^5 \cdot 1 \cdot \ln 2 = 69315$ J = 69.3 kJ. The work is positive because it is an expansion process. Work is done by the system on the surroundings.

$$Q = -W \rightarrow Q = -69.315 \text{ J} = -69.3 \text{ kJ.}$$

3.21 Air in a turbine

The change in specific enthalpy (kJ/kg) of air through a turbine is computed in five different ways. To calculate the specific heat use $dh = c_p dT \rightarrow \Delta h = \int_{T_1}^{T_2} c_p dT$. For a constant c_p (method 1, 2 and 3) this results in $\Delta h = c_p(T_2 - T_1)$.

1. Assumed is a constant specific heat at the inlet temperature (table A-2b).

The inlet temperature is $1500^\circ\text{C} = 1773\text{ K}$. The specific heat at this temperature is 1.1466 kJ/kgK .

For constant c_p at the outlet temperature $\rightarrow \Delta h = c_p(T_2 - T_1) = 1.1337(773 - 1773) = -1133.7\text{ kJ/kg}$.

2. Assumed is a constant specific heat at the outlet temperature (table A-2b).

The outlet temperature is $500^\circ\text{C} = 773\text{ K}$. The specific heat at this temperature is 1.0411 kJ/kgK .

For constant c_p at the outlet temperature $\rightarrow \Delta h = c_p(T_2 - T_1) = 1.0411(773 - 1773) = -1041.1\text{ kJ/kg}$.

3. Assumed is a constant specific heat at the mean temperature (table A-2b).

The mean temperature is $1000^\circ\text{C} = 1273\text{ K}$. The specific heat at this temperature is 1.0935 kJ/kgK .

For constant c_p at the mean temperature $\rightarrow \Delta h = c_p(T_2 - T_1) = 1.0935(773 - 1773) = -1093.5\text{ kJ/kg}$.

4. A variable specific heat dependent on temperature is used by integrating the polynomial function in table A-2c. Table 2 gives $\overline{c_p} = a + bT + cT^2 + dT^3$ with T in Kelvin and $\overline{C_p}$ in kJ/kmolK . For air the constants are $a = 28.11$, $b = 0.1967 \cdot 10^{-2}$, $c = 0.4802 \cdot 10^{-5}$ and $d = -1.966 \cdot 10^{-9}$.

$$\Delta h = \int_{T_1}^{T_2} \overline{c_p}(T) dT = \int_{T_1}^{T_2} (a + bT + cT^2 + dT^3) dT = [aT + \frac{1}{2}bT^2 + \frac{1}{3}cT^3 + \frac{1}{4}dT^4]_{T_1}^{T_2} = a(T_2 - T_1) + \frac{1}{2}b(T_2^2 - T_1^2) + \frac{1}{3}c(T_2^3 - T_1^3) + \frac{1}{4}d(T_2^4 - T_1^4).$$

Filling in $T_1 = 1500^\circ\text{C} = 1773\text{ K}$ and $T_2 = 500^\circ\text{C} = 773\text{ K}$ and the values for a , b , c and d gives $\Delta h = -34114.6\text{ kJ/kmol}$. The molar weight of air is $M_{air} = 28.97\text{ kg/kmol} \rightarrow \Delta h = \frac{-34114.6}{28.97} = -1177.58\text{ kJ/kg}$.

5. The h values tabulated in table A-17 are used.

$1500^\circ\text{C} \rightarrow h_{1500} = 1971.3\text{ kJ/kg}$.

$500^\circ\text{C} \rightarrow h_{500} = 793.56\text{ kJ/kg}$.

$$\Delta h = h_{500} - h_{1500} = -1177.74\text{ kJ/kg}.$$

By comparing the results for methods 1, 2, 3 and 4 with those of method 5 it can be concluded that results of method 4 and 5 are equal. Method 4 takes into account the variable specific heat. Therefore it can be concluded that also table A-17 takes the variable specific heat into account. The first three methods use a constant specific heat and it can be seen that the result is less accurate.

3.22 Oxygen in a rigid tank

3.22a From the ideal gas law it follows that the pressure rises as the temperature rises while the volume is constant (which is the case as the tank is rigid). The tank is insulated therefore no heat transfer can take place through the walls of the tank. The heat has to come from the fan. The power to drive the fan is converted into heat and heats the oxygen.

3.22b Oxygen can be assumed to be an ideal gas under these circumstances. In the case of an ideal gas the internal energy change can be calculated using the specific heat of oxygen $\rightarrow du = c_v dT \rightarrow \Delta u = \int_{T_1}^{T_2} c_v dT = c_v(T_2 - T_1)$. The specific heat of oxygen at the mean temperature can be found in table 7.

The temperature T_2 follows from the ideal gas law $\rightarrow T_2 = \frac{P_2 T_1}{P_1}$.

The mass of the oxygen also follows from the ideal gas law $PV = mRT \rightarrow m = \frac{P_1 V}{R T_1}$.

Finally the change in internal energy in kJ is $\rightarrow \Delta U = m \Delta u = \frac{P_1 V}{R T_1} c_v (T_2 - T_1) = \frac{P_1 V}{R T_1} c_v (\frac{P_2 T_1}{P_1} - T_1) = \frac{P_1 V c_v}{R} (\frac{P_2}{P_1} - 1)$.

3.22c Given $V = 0.3 \text{ m}^3$, $P_1 = 100 \text{ kPa}$, $P_2 = 150 \text{ kPa}$ and $T_1 = 27^\circ\text{C} = 300 \text{ K}$.

$T_2 = \frac{P_2 T_1}{P_1} = 450 \text{ K} = 177^\circ\text{C}$. The mean temperature is $375 \text{ K} = 102^\circ\text{C}$ and in table 7 it is found that $c_v = 0.6742 \text{ kJ/kgK}$ at 100 degree Celsius. The specific gas constant for oxygen is 0.260 kJ/kgK (table 2) $\rightarrow \Delta U = \frac{P_1 V c_v}{R} \left(\frac{P_2}{P_1} - 1 \right) = 38.9 \text{ kJ}$.

3.23 Two closed rigid system in thermal equilibrium

3.23a In thermal equilibrium the temperature at both sides should be equal $T_{a2} = T_{b2} = T_2$.

The total internal energy of the whole system does not change as no heat or power is added to the system. Only the internal energy is distributed different over both parts of the system (this is because the system goes from an non-equilibrium to an equilibrium state)

$$\rightarrow U_1 = U_{a1} + U_{b1} = U_{a2} + U_{b2} = U_2$$

3.23b N_a and N_b do not change during the process in which the system goes to thermal equilibrium from state 1 to state 2 as the mass cannot pass the wall. But the temperature and the internal energy (which is a function of temperature) will change.

$$U_{a2} = \frac{3}{2} R_u N_a T_2 \quad \text{and} \quad U_{b2} = \frac{5}{2} R_u N_b T_2 \rightarrow U_2 = U_{a2} + U_{b2} = R_u T_2 \left(\frac{3}{2} N_a + \frac{5}{2} N_b \right). \quad (3.23.1)$$

3.23c $R_u = 8.314 \text{ J/molK}$

$$U_{a1} = \frac{3}{2} R_u N_a T_{a1} = \frac{3}{2} \cdot 8.314 \cdot 2 \cdot 250 = 6235.5 \text{ J}.$$

$$U_{b1} = \frac{5}{2} R_u N_b T_{b1} = \frac{5}{2} \cdot 8.314 \cdot 3 \cdot 350 = 21824.3 \text{ J}.$$

$$U_1 = U_{a1} + U_{b1} = 28059.8 \text{ J}.$$

$$U_2 = R_u T_2 \left(\frac{3}{2} N_a + \frac{5}{2} N_b \right) = U_1 \rightarrow T_2 = \frac{U_1}{R_u \left(\frac{3}{2} N_a + \frac{5}{2} N_b \right)} = 321.4 \text{ K}.$$

3.23d $U_{a2} = \frac{3}{2} R_u N_a T_2 = \frac{3}{2} \cdot 8.314 \cdot 2 \cdot 321.4 = 8017 \text{ J}$.

$$U_{b2} = \frac{5}{2} R_u N_b T_2 = \frac{5}{2} \cdot 8.314 \cdot 3 \cdot 321.4 = 20041 \text{ J}.$$

3.24 Two closed movable systems in equilibrium

3.24a In this case in equilibrium an additional condition has to be fulfilled. Besides the temperatures also the pressures at both sides of the wall should be equal therefore $\rightarrow P_{2a} = P_{2b} = P_2$ and $T_{a2} = T_{b2} = T_2$.

3.24b For the equilibrium temperature is does not matter if the wall can move or not, the heat transfer is not influenced by the movement of the wall. As in problem 3.23 the equilibrium temperature is 321.4 K. The internal energies of the subsystems are also the same as in problem 3.23 as the internal energy of an ideal gas is a function of the temperature only. As the equilibrium temperature is still $T_2 = 321.4 \text{ K}$ the internal energies are still $U_{a2} = 8017 \text{ J}$ and $U_{b2} = 20041 \text{ J}$.

3.24c In the case of state 2 the state equations become:

$$\frac{P_2}{T_2} = R_u \frac{N_a}{V_{a2}} \rightarrow V_{a2} = \frac{R_u N_a T_2}{P_2} \quad \text{and} \quad \frac{P_2}{T_2} = R_u \frac{N_b}{V_{b2}} \rightarrow V_{b2} = \frac{R_u N_b T_2}{P_2}. \quad (3.24.1)$$

3.24d It follows that $\frac{N_a}{V_{a2}} = \frac{N_b}{V_{b2}} \rightarrow \frac{V_{b2}}{V_{a2}} = \frac{N_b}{N_a} = \frac{3}{2}$.

$$V_t = V_{a2} + V_{b2} = 80 \text{ liter}.$$

Together this gives: $V_{a2} + \frac{3}{2} V_{a2} = 80 \rightarrow V_{a2} = \frac{80}{1+3/2} = 32 \text{ liter}$. $V_{b2} = 80 - 32 = 48 \text{ liter}$.

3.24e The equilibrium pressure is $P_2 = \frac{R_u T_2 N_a}{V_{a2}} = \frac{8.314 \cdot 321.4 \cdot 2}{32 \cdot 10^{-3}} = 167006 \text{ Pa} = 167 \text{ kPa}$.

4 The First Law of Thermodynamics, Conservation of Energy

4.1 First law of thermodynamics

Note that the derivative to the time of a variable (the flow of the variable) is often denoted with a dot above the symbol: Energy flow, $\frac{\partial E}{\partial t} = \dot{E}$; Mass flow, $\frac{\partial m}{\partial t} = \dot{m}$; Work flow (power), $\frac{\partial W}{\partial t} = \dot{W}$ and Heat flow $\frac{\partial Q}{\partial t} = \dot{Q}$.

4.1a The first law of thermodynamics, also known as the conservation of energy principle, states that energy can be neither created nor destroyed; it can only change form.

4.1b $\frac{\partial E}{\partial t} = \sum_{i=1}^n \frac{\partial m_i}{\partial t} (h_i + \frac{1}{2}v_i^2 + gz_i) + \frac{\partial W}{\partial t} + \frac{\partial Q}{\partial t}$ where the summation is over all in- and outlets of the system, note that $h_i = u_i + P_i v_i = u_i + P_i \frac{1}{\rho_i}$.

In words: the change of the energy of a system during time is the difference of the internal, kinetic and potential energy of at the in- and outlet plus the flow work $P_i v_i$ necessary to push the flow in and out the system plus the work and heat added to or delivered by the system.

4.1c $\frac{\partial E}{\partial t} = 0$ and therefore $\sum_{i=1}^n \frac{\partial m_i}{\partial t} (h_i + \frac{1}{2}v_i^2 + gz_i) = \frac{\partial W}{\partial t} + \frac{\partial Q}{\partial t}$.

In a system with only one in- and outlet the first law can be written as: $\dot{m}(h_1 + \frac{1}{2}v_1^2 + gz_1) - \dot{m}(h_2 + \frac{1}{2}v_2^2 + gz_2) = \dot{W} + \dot{Q}$.

4.1d Potential (gz) and kinetic energy ($\frac{1}{2}v^2$) can be neglected. Adiabatic behavior is assumed ($\frac{\partial Q}{\partial t} = 0$). If there is only one in- and one outlet then the first law simplifies to: $\frac{\partial W}{\partial t} = \frac{\partial m}{\partial t}(h_2 - h_1)$ or $\dot{W} = \dot{m}(h_2 - h_1)$.

4.1e Potential (gz) and kinetic energy ($\frac{1}{2}v^2$) can be neglected. No heat transfer ($\frac{\partial Q}{\partial t} = 0$) or work ($\frac{\partial W}{\partial t} = 0$). The first law simplifies to: $\left(\frac{\partial m_i}{\partial t} h_i\right)_{inlet} = \left(\frac{\partial m_i}{\partial t} h_i\right)_{outlet}$

4.1f Potential energy (gz) can be neglected. No heat transfer ($\frac{\partial Q}{\partial t} = 0$) or work ($\frac{\partial W}{\partial t} = 0$). If there is only one in- and one outlet then the first law simplifies to:
 $\frac{\partial m}{\partial t}(h_1 + \frac{1}{2}v_1^2) = \frac{\partial m}{\partial t}(h_2 + \frac{1}{2}v_2^2)$.

4.1g In a closed system there is no mass transfer $\frac{\partial m}{\partial t} = 0$. Therefore the first law simplifies to $\frac{\partial E}{\partial t} = \frac{\partial W}{\partial t} + \frac{\partial Q}{\partial t}$, by neglecting the potential and kinetic energy $E = U$ and therefore $\frac{\partial U}{\partial t} = \frac{\partial W}{\partial t} + \frac{\partial Q}{\partial t} \rightarrow dU = \delta Q + \delta W$ or in specific parameters $du = \delta q + \delta w$.

4.2 Conservation of mass principle

4.2a The conservation of mass principle states that the net mass to or from a system during a process is equal to the net change (increase or decrease) in the total mass of the system during that process.

4.2b In formula the conservation of mass is expressed as: $m_{in} - m_{out} = \Delta m_{system}$.

4.2c The mass flow rate is the amount of mass flowing through a cross section per unit time and is expressed as: $\dot{m} = \frac{\partial m}{\partial t} = \rho v A$ where ρ is the density, v is the velocity and A is the cross-sectional area normal to the flow direction. The mass flow rate can also be used in the conservation of mass principle, then its expression is written as: $\frac{\partial m_{in}}{\partial t} - \frac{\partial m_{out}}{\partial t} = \frac{\partial m_{system}}{\partial t}$ or $\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{system}$.

4.2d The volume flow rate is the volume of the fluid flowing through a cross section per unit time and is expressed as: $\dot{V} = \frac{\partial V}{\partial t} = vA = \frac{\dot{m}}{\rho}$, where v is the velocity, A is the cross-sectional area normal to the flow direction, ρ is the density and \dot{m} is the mass flow rate.

4.2e $\frac{\partial m_{system}}{\partial t} = \dot{m}_{system} = 0$ and therefore $\dot{m}_{in} = \dot{m}_{out}$.

4.3 Conservation of mass in a nozzle

The parameters that state this problem are: Mass flow, \dot{m} [kg/s]; density ρ [kg/m³]; velocity v [m/s] and area A [m²].

The main equations are: Conservation of mass, $\dot{m}_{in} = \dot{m}_{out}$ and $\dot{m} = \rho_{in}v_{in}A_{in}$.

4.3a $\dot{m} = \rho_{in}v_{in}A_{in} = 2.21 \times 30 \times 80 \cdot 10^{-4} = 0.53$ kg/s.

4.3b Conservation of mass: $A_{out} = \frac{\dot{m}}{\rho_{out}v_{out}} = \frac{0.53}{180 \times 0.762} = 38.6 \cdot 10^{-4}$ m² = 38.6 cm².

4.4 Conservation of mass in a hair dryer

The parameters that state this problem are: Mass flow, \dot{m} [kg/s]; density ρ [kg/m³]; velocity v [m/s] and area A [m²].

The main equations are: Conservation of mass, $\dot{m}_{in} - \dot{m}_{out}$ and $\dot{m} = \rho_{in}v_{in}A_{in}$.

4.4a There is conservation of mass, the mass flow rate going in (1) is equal to the mass flow rate going out (2), $\dot{m}_1 = \dot{m}_2$ with $\dot{m}_i = \rho_i v_i A_i$. The in- and outlet area, A are the same. This results in: $\rho_1 v_1 = \rho_2 v_2$. The percentage of increase in velocity follows from: $\frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = \frac{1.20}{1.05} = 1.14$. The increase in velocity is 14 percent.

4.5 Work and power input for a compressor

The parameters that state this problem are: Mass flow, \dot{m} [kg/s] and the enthalpy of the in- and outlet h_i [kJ/kg].

The main equation is conservation of energy in an open system (first law of thermodynamics): $\dot{m}(h_1 + \frac{1}{2}v_1^2 + gz_1) - \dot{m}(h_2 + \frac{1}{2}v_2^2 + gz_2) = \dot{W}_{net} + \dot{Q}_{net} = (\dot{W}_{out} - \dot{W}_{in}) + (\dot{Q}_{in} - \dot{Q}_{out}) = -\dot{W}_{in}$ as $\dot{W}_{out} = 0$ and $\dot{Q}_{in} = \dot{Q}_{out} = 0$. By neglecting kinetic and potential energy the first law is reduced to: $\dot{W}_{in} = \dot{m}(h_2 - h_1)$, where the subscript 1 denotes the situation at the inlet and 2 the situation at the outlet.

4.5a The specific work input required is $w_{in} = h_2 - h_1$. h_1 and h_2 have to be looked up in the tables. From $P_1 = 0.14$ MPa and the fact that the refrigerant is a saturated vapor at the inlet, follows from table 12 that: $h_1 = 236.04$ kJ/kg. From the fact that the refrigerant leaves as superheated vapor at $P_2 = 0.8$ MPa and 50°C follows from table 13 that $h_2 = 284.39$ kJ/kg. With these data the specific work input required for the compressor can be calculated: $w_{in} = h_2 - h_1 = 284.39 - 236.04 = 48.35$ kJ/kg.

4.5b The power required is $\dot{W}_{in} = \dot{m}(h_2 - h_1)$. Given is $\dot{m} = 0.04$ kg/s, h_1 and h_2 are known from part a). With these data the power required for the compressor can be calculated: $\dot{W}_{in} = \dot{m}(h_2 - h_1) = 1.93$ kW.

Note that: $\dot{W} = \dot{m}w$ so it can also be calculated by: $\dot{W} = \dot{m}w = 0.04 \cdot 48.35 = 1.93$ kW.

4.6 Cooling a closed system

The parameters that state this problem are: Pressure P [pa], Temperature T [K], mass m [kg], Internal energy u [kJ/kg], Heat q [kJ/kg] and Work w [kJ/kg].

The main equations are: Conservation of energy in a closed system (first law of thermodynamics), $du = \delta q + \delta w$, ideal gas relation $Pv = RT$ and the internal energy of an ideal gas is $du = c_v dT$.

4.6a The final pressure follows from the equation of state for the perfect gas, $Pv = RT \rightarrow \frac{P_i v_i}{RT_i} = \frac{P_f v_f}{RT_f}$.

As the tank is rigid $dv = 0$ ($v_i = v_f$) and therefore, $\frac{P_i}{T_i} = \frac{P_f}{T_f} \rightarrow P_f = P_i \left(\frac{T_f}{T_i} \right) = P_f = 250 \left(\frac{300}{500} \right) = 150 \text{ kPa}$.

4.6b The first law of thermodynamics for a closed system (energy balance) is $du = \delta q + \delta w$.

The last term is zero because $\delta w = Pdv = 0$ as $dv = 0$ (rigid tank), $\rightarrow \delta q = du$.

For a perfect gas $du = c_v dT$, with $c_v = 10.286 \text{ kJ/kgK}$ for hydrogen (tabel A-2, the average between 300 and 500 K). Therefore $q = c_v(T_f - T_i)$ and $Q = mc_v(T_f - T_i)$. The mass m follows from the equation of state for the perfect gas $Pv = RT$, (note $v = \frac{1}{\rho}$). $m = \rho V = \frac{V}{v_i} = \frac{P_i V}{R T_i}$ with V the volume en $R = 4.124 \text{ kJ/kgK}$ for hydrogen (tabel 2). This results in $Q = \left(\frac{P_i V}{R T_i} \right) c_v (T_f - T_i) = \left(\frac{250 \cdot 3}{4.124 \cdot 500} \right) 10.183 (300 - 500) = 740.8 \text{ kJ}$.

4.7 An insulated rigid tank

The parameters that state this problem are: Pressure P [pa], Temperature T [K], mass m [kg], Internal energy u [kJ/kg], Heat q [kJ/kg] and Work w [kJ/kg].

The main equations are: Conservation of energy in a closed system (first law of thermodynamics), $du = \delta q + \delta w$, ideal gas relation $Pv = RT$ and the internal energy of an ideal gas is $du = c_v dT$.

4.7a The first law of thermodynamics for a closed system (energy balance) is $du = \delta q + \delta w$. The tank is insulated $\rightarrow \delta q = 0$ and as the tank is rigid, $dv=0$, there is no work $\rightarrow \delta w = 0$. This results in $du = 0$. As $du = c_v dT$, also $dT = 0$. Therefore the final temperature is equal to the initial temperature and is $T_f = 50^\circ\text{C}$.

The final pressure follows from the equation of state for the perfect gas $Pv = RT$. With $dT = 0$ follows $P_i v_i = P_f v_f \rightarrow P_f = P_i \left(\frac{v_i}{v_f} \right)$. Using $v_f = 2v_i$ gives $P_f = P_i \left(\frac{v_i}{2v_i} \right) = 800 \frac{1}{2} = 400 \text{ kPa}$.

4.8 Nozzle

The variables involved in this problem are shown graphically in figure 4.8.1. The main equations in this problem are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)
- $P = \rho RT$ (ideal gas law)
- $\dot{m}_i = v_i \rho_i A_i$
- $\dot{E}_i = \dot{m} \left(h_i + \frac{1}{2} v_i^2 + g z_i \right) + \dot{Q}_i + \dot{W}_i$
- $dh = c_p dT$

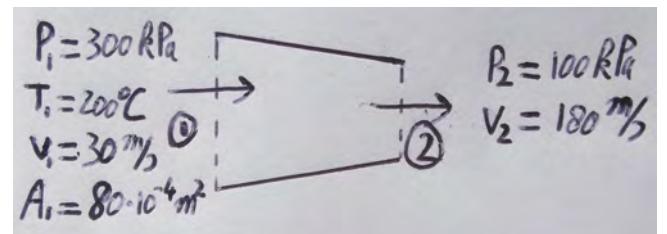


Figure 4.8.1: Situation of exercise 4.8

4.8a The mass flow rate is calculated as $\dot{m}_1 = v_1 \rho_1 A_1$, where we calculate the density from the ideal gas law: $\rho_1 = \frac{P_1}{RT_1}$. Substituting this in the mass flow equation gives: $\dot{m}_1 = v_1 \frac{P_1}{RT_1} A_1$. From table A-2, we find that $R = 0.2870 \text{ kJ/(kgK)}$. Substituting all values then gives:

$$\dot{m}_1 = v_1 \frac{P_1}{RT_1} A_1 = 30 \cdot \frac{300}{0.2870 \cdot (200 + 273)} \cdot 80 \cdot 10^{-4} = 0.530 \text{ kg/s}$$

4.8b The temperature change follows from $dh = c_p dT$, so we need to determine the enthalpy change. Use conservation of energy: $\dot{E}_{in} = \dot{E}_{out} \Rightarrow \dot{m}_1 (h_1 + \frac{1}{2}v_1^2 + gz_1) = \dot{m}_2 (h_2 + \frac{1}{2}v_2^2 + gz_2) + \dot{Q} + \dot{W}$. From conservation of mass, it follows that $\dot{m}_1 = \dot{m}_2$. Furthermore, the nozzle is placed horizontally ($z_1 = z_2$), is adiabatic ($\dot{Q} = 0$) and does not use or produce work ($\dot{W} = 0$). This results in $h_1 - h_2 = \frac{1}{2} (v_2^2 - v_1^2)$. So, $dh = c_p dT \Rightarrow h_1 - h_2 = c_p (T_1 - T_2) \Rightarrow T_2 = T_1 - \frac{h_1 - h_2}{c_p} = T_1 - \frac{v_2^2 - v_1^2}{2c_p} = 473 - \frac{180^2 - 30^2}{2 \cdot 1.029 \cdot 10^3} = 457.7 \text{ K} = 184.7^\circ\text{C}$, where $c_p = 1.029 \text{ kJ/(kgK)}$ was found in table A-2 at a value of $T = 500 \text{ K}$.

4.8c From conservation of mass we can calculate the exit area: $\dot{m}_1 = \dot{m}_2 \Rightarrow v_1 \rho_1 A_1 = v_2 \rho_2 A_2 \Rightarrow A_2 = \frac{v_1}{v_2} \frac{P_1 T_2}{P_2 T_1} A_1 = \frac{30}{180} \frac{300 \cdot 458}{473 \cdot 100} \cdot 80 \cdot 10^{-4} = 3.87 \cdot 10^{-3} \text{ m}^2 = 38.7 \text{ cm}^2$

4.9 Steam Turbine

The variables involved in this problem are shown graphically in figure 4.9.1. The main equations in this problem are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)
- $\dot{E}_i = \frac{1}{2} \dot{m} v_i^2$ (total kinetic energy)
- $\dot{E}_i = \dot{m} (h_i + \frac{1}{2}v_i^2 + gz_i) + \dot{Q}_i + \dot{W}_i$
- $\dot{m} = v_i \rho_i A_i$

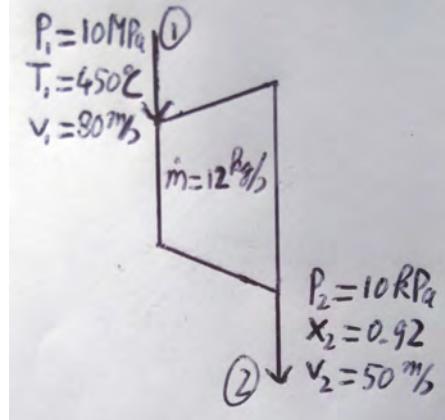


Figure 4.9.1: Situation of exercise 4.9

4.9a The change in kinetic energy is calculated as: $\Delta E_{kin} = E_{kin,1} - E_{kin,2} = \frac{1}{2} \dot{m} (v_1^2 - v_2^2) = \frac{1}{2} \cdot 12 \cdot (80^2 - 50^2) = 12 \cdot 1.950 = 23400 \text{ J/s} = 23.4 \text{ kW}$.

4.9b The power output follows from the conservation of energy:

$$\dot{m} \left(h_1 + \frac{1}{2}v_1^2 + gz_1 \right) = \dot{m} \left(h_2 + \frac{1}{2}v_2^2 + gz_2 \right) + \dot{Q} + \dot{W}.$$

The potential energy can be neglected ($z_1 = z_2$) and the turbine is adiabatic ($\dot{Q} = 0$). Therefore, the power output is equal to $\dot{W} = \dot{m} (h_1 - h_2 + \frac{1}{2} (v_1^2 - v_2^2))$. The values for the enthalpy can be determined from the tables: $h_1 = 3242.4 \text{ kJ/kg}$ (table 6) and $h_2 = h_f @ 10kPa + x(h_g @ 10kPa - h_f @ 10kPa) = 191.81 + 0.92(2583.9 - 191.81) = 2392.5 \text{ kJ/kg}$. This results in the power output:

$$\dot{W}_{out} = \dot{m} \left(h_1 - h_2 + \frac{1}{2} (v_1^2 - v_2^2) \right) = 12 \left(3242.4 \cdot 10^3 - 2392.5 \cdot 10^3 + \frac{1}{2} (80^2 - 50^2) \right) = 10.22 \text{ MW}$$

Note that the kinetic energy contributes only very little to the total power output. Therefore in most of the calculations the kinetic energy may be neglected.

4.9c The inlet area follows from $\dot{m} = u_1 \rho_1 A_1$, with $\rho_1 = \frac{1}{v_1}$ (v_1 the specific volume of the ingoing flow) (note that the velocity is now called u_1). From table 12, we find that $v_1 = 0.029782 \text{ m}^3/\text{kg}$. So, $A_1 = \frac{\dot{m}v_1}{u_1} = \frac{12 \cdot 0.029782}{80} = 4.47 \cdot 10^{-3} \text{ m}^2 = 44.7 \text{ cm}^2$.

4.10 Refrigerant Compressor

The variables involved in this problem are shown graphically in figure 4.10.1. The main equations in this problem are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)
- $\dot{E}_i = \dot{m} (h_i + \frac{1}{2}v_i^2 + gz_i) + \dot{Q}_i + \dot{W}_i$
- $\dot{Q}_i = \frac{\dot{m}_i}{\rho_i}$ (volume flow rate)

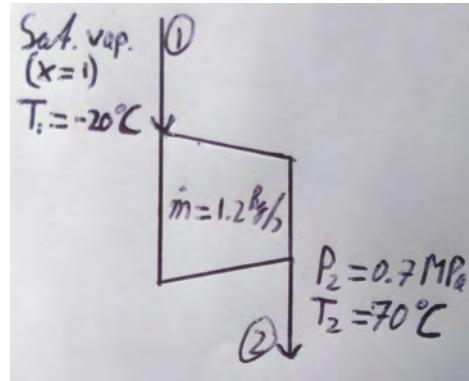


Figure 4.10.1: Situation of exercise 4.10

4.10a The power output follows from the conservation of energy: $\dot{E}_{in} = \dot{E}_{out} \Rightarrow \dot{m} (h_1 + \frac{1}{2}v_1^2 + gz_1) = \dot{m} (h_2 + \frac{1}{2}v_2^2 + gz_2) + \dot{Q} + \dot{W}$. Neglecting kinetic energy and potential energy and applying that the compressor is adiabatic reduces the power to: $\dot{W} = \dot{m}(h_2 - h_1)$. The values for the enthalpies can be determined from the tables: $h_1 = h_{g@-20^\circ C} = 238.41 \text{ kJ/kg}$ (table 14) and $h_2 = 308.33 \text{ kJ/kg}$ (table 16). So, the power input for the compressor is: $\dot{W} = \dot{m}(h_2 - h_1) = 1.2(308.33 - 238.41) = 83.9 \text{ kW}$.

4.10b The volume flow follows from $\dot{Q}_1 = \frac{\dot{m}_1}{\rho_1}$. The density is determined from the specific volume: $\rho_1 = \frac{1}{v_1}$. From table 14 it follows that $v_1 = v_{g@-20^\circ C} = 0.14729 \text{ m}^3/\text{kg}$, such that the volume flow at inlet becomes: $\dot{Q}_1 = \dot{m}_1 v_1 = 1.2 \cdot 0.14729 = 0.1767 \text{ m}^3/\text{s}$.

4.11 Throttling valve

The variables involved in this problem are shown graphically in figure 4.11.1. The main equation in this problem are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)
- $\dot{E}_i = \dot{m} (h_i + \frac{1}{2}v_i^2 + gz_i) + \dot{Q}_i + \dot{W}_i$

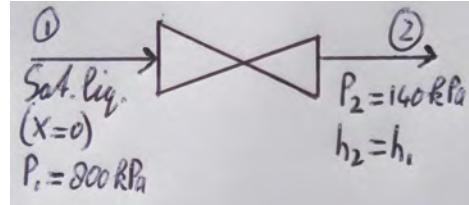


Figure 4.11.1: Situation of exercise 4.11

4.11a For a throttle valve the energy relation for the conservation of energy reduces to $h_1 = h_2$ since there are no work interactions, no heat transfer processes and changes in kinetic and potential energy can be neglected.

Table 12 $\rightarrow h_1 = h_{f@800kPa} = 95.47 \text{ kJ/kg}$. The temperature at inlet is found to be $T_1 = T_{sat@800kPa} = 31.31^\circ C$. At $P_2 = 140 \text{ kPa}$, the h -values for saturated liquid and saturated vapour are $h_f@140kPa = 27.08 \text{ kJ/kg}$ and $h_g@140kPa = 239.16 \text{ kJ/kg}$, respectively. Since $h_2 = h_1$ and $h_f < h_2 < h_g$, there is a mixture at the outlet, so $T_2 = T_{sat@140kPa} = -18.77^\circ C$. So, the temperature change is $\Delta T = T_1 - T_2 = 31.31 - (-18.77) = 50.08^\circ C$.

4.11b The final specific volume follows from the quality of the mixture: $x = \frac{h_2 - h_f}{h_g - h_f} = \frac{95.47 - 27.08}{239.16 - 27.08} = 0.322$. From the table, we find that $v_f@140kPa = 0.0007383 \text{ m}^3/\text{kg}$ and $v_g@140kPa = 0.14014 \text{ m}^3/\text{kg}$. So, $v_2 = v_f + x(v_g - v_f) = 0.0007383 + 0.322(0.14014 - 0.0007383) = 0.0456 \text{ m}^3/\text{kg}$.

4.12 Feedwater heater

The variables involved in this problem are shown graphically in figure 4.12.1. The main equations in this problem are:

- $\dot{E}_1 + \dot{E}_2 = \dot{E}_3$ (conservation of energy)
- $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ (conservation of mass)

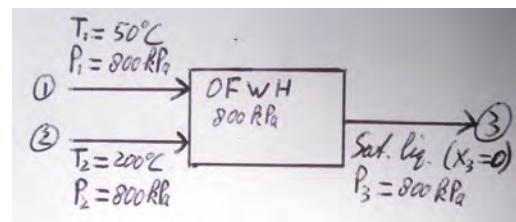


Figure 4.12.1: Situation of exercise 4.12

4.12a With saturated liquid water at the boiling line (saturated liquid line) is meant. The water contains exactly that amount of energy that, when a little bit of heat is added, a part of the water will evaporate. When a little bit of heat is extracted, it becomes a compressible liquid. The heater exit data are therefore only on the saturated liquid line.

4.12b When you heat above the saturated liquid temperature, you start to make steam again (water evaporates). This needs to be condensed before you can increase the pressure using a pump. So, you will loose energy when heating above the saturated liquid temperature.

4.12c The energy in each flow is given as $\dot{E}_i = \dot{m}_i h_i$. Conservation of energy gives $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$, and conservation of mass gives $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. Combining the two gives $\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2)h_3 \Rightarrow \frac{\dot{m}_1}{\dot{m}_2}h_1 + h_2 = \frac{\dot{m}_1 + \dot{m}_2}{\dot{m}_2}h_3 \Rightarrow \frac{\dot{m}_1}{\dot{m}_2}h_1 - \frac{\dot{m}_1}{\dot{m}_2}h_3 = h_3 - h_2 \Rightarrow \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_3 - h_2}{h_1 - h_3}$. The values for h follow from the tables: $h_1 = 209.35 \text{ kJ/kg}$ (table A-4), $h_2 = 2839.8 \text{ kJ/kg}$ (table A-6) and $h_3 = h_f@800kPa = 720.87 \text{ kJ/kg}$ (table A-5). This results in the ratio of mass flow rates:

$$\frac{\dot{m}_1}{\dot{m}_2} = \frac{720.87 - 2839.8}{209.35 - 720.87} = 4.14 \Rightarrow \dot{m}_1 = 4.14\dot{m}_2.$$

4.13 Steam/water heat exchanger

The variables involved in this problem are shown graphically in figure 4.13.1. The main equations are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)
- $\Delta h = c_p \Delta T$ (for a compressed liquid)

The water of the lake (cooling water) will warm up as a result of the heat released by the condensation of the steam. The energy lost by the steam during condensation will be gained by the water of the lake i.e. the heat transfer from the hot fluid is equal to the heat transfer to the cold fluid).

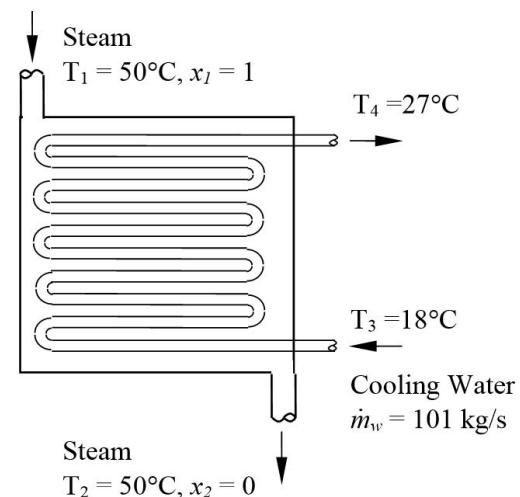


Figure 4.13.1: Situation of exercise 4.13

$h_1 = h_g@50^\circ C = 2592.1 \text{ kJ/kg}$ (table A-4) and $h_2 = h_f@50^\circ C = 209.33 \text{ kJ/kg}$ (table A-4). The specific heat for water of $25^\circ C$ is: 4.18 kJ/kgK (Table A-3).

4.13a Since the kinetic and potential energy changes may be neglected and there are no work and heat interactions between the condenser and the surroundings the energy balance for the *cooling water*

can be expressed as:

$$\dot{E}_{in,w} = \dot{E}_{out,w} \rightarrow \dot{Q}_{in,w} + \dot{m}_w h_3 = \dot{m}_w h_4 \rightarrow \dot{Q}_{in,w} = \dot{m}_w(h_4 - h_3) = \dot{m}_w c_p(T_4 - T_3).$$

Filling in the values yields for the rate of heat transfer to the cooling water:

$$\dot{Q}_{in,w} = \dot{m}_w c_p(T_4 - T_3) = 101 \cdot 4.18 \cdot (27 - 18) = 3799.6 \text{ kJ/s} = 3.8 \text{ MW}.$$

4.13b Noting that heat gain by the water $\dot{Q}_{in,w}$, is equal to the heat loss by the condensing steam $\dot{Q}_{out,s}$, the rate of condensation of the steam \dot{m}_s , in the condenser is determined from:

$$\begin{aligned} \dot{Q}_{out,s} &= \dot{Q}_{in,w} \rightarrow \dot{m}_s(h_1 - h_2) = \dot{m}_w c_p(T_4 - T_3) \\ \rightarrow \dot{m}_s &= \frac{\dot{m}_w c_p(T_4 - T_3)}{(h_1 - h_2)} = \frac{101 \cdot 4.18 \cdot (27 - 18)}{2592.1 - 209.33} = 1.59 \text{ kg/s}. \end{aligned}$$

4.14 Steam turbine 2

The variables involved in this problem are shown graphically in figure 4.14.1. The main equations are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)
- $\dot{E}_i = \dot{m}(h_i + \frac{1}{2}v_i^2 + gz_i) + \dot{Q}_i + \dot{W}_i$

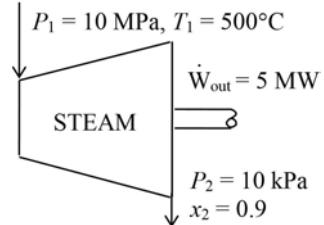


Figure 4.14.1: Situation of exercise 4.14

4.14a The power output of an adiabatic steam turbine where changes in kinetic and potential energies are neglected is given by the energy balance: $\dot{W}_{out} = \dot{m}(h_{in} - h_{out})$, where $\dot{W}_{out} = 5 \text{ MW} = 5000 \text{ kW}$. From this follows for the mass flow rate:

$$\dot{m} = \frac{\dot{W}_{out}}{(h_{in} - h_{out})} = \frac{\dot{W}_{out}}{(h_1 - h_2)}.$$

The fluid enters the adiabatic turbine at: $P_{in} = 10 \text{ MPa}$ and $T_{in} = 500^\circ\text{C}$. This is a superheated vapor, the value of the enthalpy can be found in table A6: $h_{in} = 3375.1 \text{ kJ/kg}$.

The fluid leaves at $P_{out} = 10 \text{ kPa}$ with a quality of $x = 0.9$.

The enthalpy can be found as: $h_{out} = h_f + x(h_g - h_f)$ where h_f and h_g can be found in table A5 at $P_{out} = 10 \text{ kPa} \rightarrow h_{out} = 191.81 + 0.9(2583.9 - 191.81) = 2344.7 \text{ kJ/kg}$

So for the mass flow rate follows:

$$\dot{m} = \frac{\dot{W}_{out}}{(h_1 - h_2)} = \frac{5000}{(3375.1 - 2344.7)} = 4.85 \text{ kg/s}.$$

4.14b If the turbine is not adiabatic but losses heat, the rate of heat transfer, \dot{Q}_{out} should be taken into account and the energy balance changes to: $\dot{W}_{out} + \dot{Q}_{out} = \dot{m}(h_{in} - h_{out})$.

For the mass flow rate follows:

$$\dot{m} = \frac{\dot{W}_{out} + \dot{Q}_{out}}{(h_{in} - h_{out})} = \frac{\dot{W}_{out} + \dot{Q}_{out}}{(h_1 - h_2)} = \frac{5000 + 500}{(3375.1 - 2344.7)} = 5.34 \text{ kg/s}.$$

Note: the rate of heat loss is 10% of the power output, this results in a 10% higher mass flow rate.

4.15 Turbine with argon

The variables involved in this problem are shown graphically in figure 4.15.1. The main equations are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{E}_i = \dot{m} (h_i + \frac{1}{2}v_i^2 + gz_i) + \dot{Q}_i + \dot{W}_i$
- $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$ (conservation of mass)
- $\dot{m} = \rho A v = \frac{1}{v} A v$ (mass flow rate, v = velocity)
- $Pv = RT$ (ideal gas law, v = specific volume)
- $dh = c_p dT$ (for an ideal gas)
- $\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT = c_p(T_2 - T_1)$
(under the assumption of constant c_p).

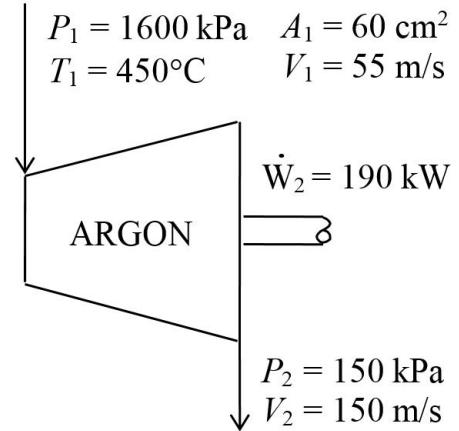


Figure 4.15.1: Situation of exercise 4.15

The constant pressure specific heat of argon is $c_p = 0.5203 \text{ kJ/kg}\cdot\text{K}$. The gas constant of argon is $R = 0.2081 \text{ kJ/kgk}$ (Table A-2a).

4.15a The mass flow rate follows from: $\dot{m} = \rho_1 A_1 v_1 = \frac{1}{v_1} A_1 v_1$, with $\rho_1 = \frac{1}{v_1}$ following from the ideal gas law: $P_1 v_1 = RT_1 \rightarrow v_1 = \frac{RT_1}{P_1} = \frac{0.2081(450+273)}{1600} = 0.09404 \text{ m}^3/\text{kg}$.

Thus the mass flow rate is: $\dot{m} = \frac{1}{v_1} A_1 v_1 = \frac{1}{0.09404} 0.006 \cdot 55 = 3.509 \text{ kg/s}$.

4.15b The exit temperature of argon follows from the energy balance for a steady flow system:

$$\dot{m} \left(h_1 + \frac{1}{2}v_1^2 + gz_1 \right) + \dot{Q}_1 + \dot{W}_1 = \dot{m} \left(h_2 + \frac{1}{2}v_2^2 + gz_2 \right) + \dot{Q}_2 + \dot{W}_2.$$

With $\dot{Q}_1 = \dot{Q}_2 = 0$ (no heat in- and output), $\dot{W}_1 = 0$ (no power input) and neglecting the potential energy change this reduces to:

$$\dot{m} \left(h_1 + \frac{1}{2}v_1^2 \right) = \dot{W}_2 + \dot{m} \left(h_2 + \frac{1}{2}v_2^2 \right) \rightarrow \dot{W}_2 = \dot{m} \left(h_1 - h_2 + \frac{v_1^2 - v_2^2}{2} \right) = \dot{m} \left(c_p(T_1 - T_2) + \frac{v_1^2 - v_2^2}{2} \right).$$

From this follows for the exit temperature of argon, T_2 :

$$T_2 = T_1 - \left(\frac{\left(\frac{\dot{W}_2}{\dot{m}} \right) - \left(\frac{v_1^2 - v_2^2}{2} \right)}{c_p} \right) = 450 - \left(\frac{\left(\frac{190 \cdot 10^3}{3.509} \right) - \left(\frac{55^2 - 150^2}{2} \right)}{0.5203 \cdot 10^3} \right) = 327^\circ\text{C}.$$

4.16 Helium compressor

The variables involved in this problem are shown graphically in figure 4.16.1. The main equations are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$ (conservation of mass)
- $\dot{E}_i = \dot{m}(h_i + \frac{1}{2}v_i^2 + gz_i) + \dot{Q}_i + \dot{W}_i$
- $dh = c_p dT$ (for an ideal gas)
- $\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT = c_p(T_2 - T_1)$
(under the assumption of constant c_p).

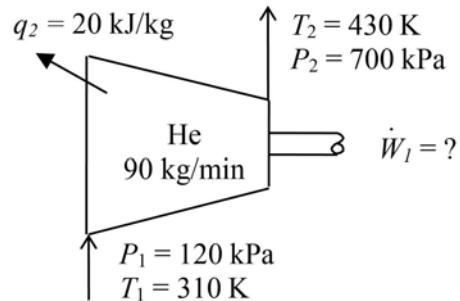


Figure 4.16.1: Situation of exercise 4.16

The constant pressure specific heat of helium is $c_p = 5.192 \text{ kJ/kg}\cdot\text{K}$ (Table A-2a). The mass flow rate is: $\dot{m} = 90 \text{ kg/min} = 1.5 \text{ kg/s}$.

4.16a The power input follows from the energy balance for a steady flow system:

$$\dot{m} \left(h_1 + \frac{1}{2}v_1^2 + gz_1 \right) + \dot{Q}_1 + \dot{W}_1 = \dot{m} \left(h_2 + \frac{1}{2}v_2^2 + gz_2 \right) + \dot{Q}_2 + \dot{W}_2.$$

With $\dot{Q}_1 = 0$ (no heat input), $\dot{W}_2 = 0$ (no power output) and neglecting kinetic and potential energy changes this reduces to $\dot{W}_1 + \dot{m}h_1 = \dot{Q}_2 + \dot{m}h_2$, which results in a power input of:

$$\dot{W}_1 = \dot{Q}_2 + \dot{m}(h_2 - h_1) = \dot{m}q_2 + \dot{m}c_p(T_2 - T_1) = 1.5 \cdot 20 + 1.5 \cdot 5.192(430 - 310) = 965 \text{ kW}.$$

4.17 Mixing chamber

The variables involved in this problem are shown graphically in figure 4.17.1. The main equations are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)

A hot (1) and a cold (2) stream enter a mixing chamber and leave as a stream (3) with intermediate temperature.

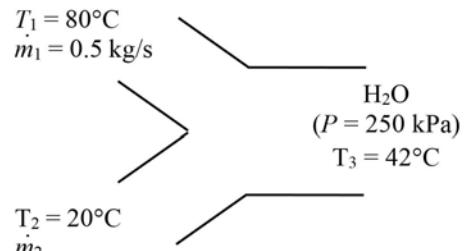


Figure 4.17.1: Situation of exercise 4.17

Conservation of energy and conservation of mass give:

1. Mass balance for mixing chamber: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$.
2. Energy balance for mixing chamber: $\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_3h_3$.

Combining them results in:

$$\dot{m}_1h_1 + \dot{m}_2h_2 = (\dot{m}_1 + \dot{m}_2)h_3 \rightarrow \dot{m}_2(h_2 - h_3) = \dot{m}_1(h_3 - h_1) \rightarrow \dot{m}_2 = \frac{\dot{m}_1(h_3 - h_1)}{(h_2 - h_3)}.$$

Noting that $T < T_{sat@250 \text{ kPa}} = 127.41^\circ\text{C}$, the water in all three streams exists as a compressed liquid, which can be approximated as a saturated liquid at the given temperature (Table A4). This is not at the right pressure but the pressure has almost no influence on the enthalpy, this approximation is known as the incompressible liquid approximation.

Thus, $\rightarrow h_{1@80^\circ\text{C}} = 335.02 \text{ kJ/kg}$, $h_{2@20^\circ\text{C}} = 83.951 \text{ kJ/kg}$ and $h_{3@42^\circ\text{C}} = 175.90 \text{ kJ/kg}$.

Substituting, the mass flow rate of cold water stream and the stream leaving the mixing chamber are determined to be $\rightarrow \dot{m}_2 = 0.865 \text{ kg/s}$ and $\dot{m}_3 = 1.365 \text{ kg/s}$.

4.18 Refrigerant-134a heat exchanger

The variables involved in this problem are shown graphically in figure 4.18.1. The main equations are:

- $\dot{E}_{in} = \dot{E}_{out}$ (conservation of energy)
- $\dot{m}_{in} = \dot{m}_{out}$ (conservation of mass)
- $\Delta h = c_p \Delta T$ (for a compressed liquid)

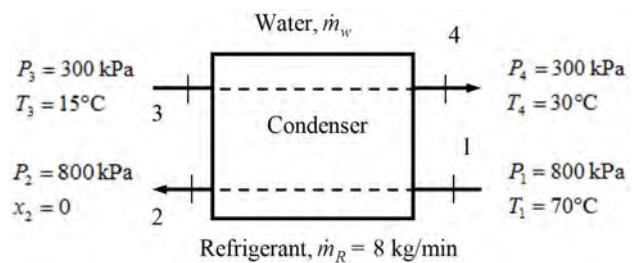


Figure 4.18.1: Situation of exercise 4.18

$h_1 = 306.88 \text{ kJ/kg}$ (table A-13) and $h_2 = h_f@800\text{kPa} = 95.47 \text{ kJ/kg}$ (table A-12).
The specific heat for water of 25°C is: 4.18 kJ/kgK (Table A-3)

4.18a Since the kinetic and potential energy changes may be neglected and there are no work and heat interactions between the condenser and the surroundings the energy balance for this steady-flow system can be expressed as:

$$\dot{E}_{in} = \dot{E}_{out} \rightarrow \dot{m}_R h_1 + \dot{m}_w h_3 = \dot{m}_R h_2 + \dot{m}_w h_4 \rightarrow \dot{m}_R (h_1 - h_2) = \dot{m}_w (h_4 - h_3) = \dot{m}_w c_p (T_4 - T_3).$$

$$\dot{m}_w = \frac{\dot{m}_R (h_1 - h_2)}{c_p (T_4 - T_3)} = \frac{0.13(306.88 - 95.47)}{4.18(30 - 15)} = 0.44 \text{ kg/s} = 26.2 \text{ kg/min.}$$

4.18b The rate of heat transfer to the cooling water equals the rate of heat rejected by the refrigerant:

$$\dot{Q}_{in,water} = \dot{Q}_{out,refrigerant} = \dot{m}_R (h_1 - h_2) = 0.13(306.88 - 95.47) = 27.48 \text{ kJ/s} = 27.48 \text{ kW.}$$

5 Entropy and the Second Law of Thermodynamics

5.1 Reversible and Irreversible Processes

5.1a A reversible process is a process that, having taken place, can be reversed, and in so doing, leaves no change in either the system or the surroundings. Or it is a process which can be reversed without leaving any trace on the surroundings.

5.1b A reversible process is possible only if the net heat and net work exchange between the system and surroundings is zero for the combined (original and reverse) process. A reversible process is an ideal process.

5.1c The process opposite to a reversible process is called an irreversible process.

5.1d For irreversible process, the surroundings usually do some work and therefore the system returns to its initial state. For a reversible process, this is not necessary as the system returns to its initial state by itself (that is why the process is reversible).

5.1e In the process should be no friction involved. Heat transfer occurs only due to infinitesimal temperature difference, no unrestrained expansion, no mixing of gases, no chemical reaction etc. Usually a reversible process is quasi-steady.

5.1f E.g. a moving piston, lifting blocks, a ball rolling down and up a hill with or without friction.

5.1g No actual process in nature is completely reversible, always there is some friction or temperature difference.

5.1h They are easy to analyze since a system passes through a series of equilibrium states during the reversible process, second, they serve as ideal models to which actual process can be compared. In the reversible processes are no losses, therefore can give the theoretical limits for the investigated phenomena.

5.1i No, as it is not possible to return to the previous original temperature of the drink without providing any refrigeration which requires some work input.

5.2 Coffee

Only the Second Law would be violated - this is why entropy is sometimes called time's arrow. Time advances in the direction of increasing entropy. The broken glass is an example of an irreversible process, where mechanical energy is converted into heat, or if heat flows from a hot to a cooler region. A reversible process is one in which there is no change in entropy, and the system and the surroundings can be returned to the initial state.

5.3 Insulated piston-cylinder device

5.3a The path for the expansion process is simply the reverse of the path for the compression process, see line a in figure 5.3.1B.

5.3b The expansion and compression processes follow different paths as a result of frictional losses to the wall of the cylinder. The graphs can be found in figure 5.3.1B. Note that in the case of friction the pressure and temperature after compression to V_2 are lower than in the case without friction. This is because part of the available work (which is the same as in the case without friction) has to be used to overcome friction (some of the work is lost in friction). The temperature is also lower in this case because of the ideal gas law, $Pv = RT \rightarrow$ lower pressure \rightarrow lower temperature at the same volume. Or you can argue that due to friction the walls are heated and less heat is available for the gas. Therefore also the pressure is lower at the same volume $\rightarrow Pv = RT$.

5.3c The sequence of paths would look in figure 5.3.1C. The pressure is decreasing every cycle with the final pressure being asymptotic to P_0 (pressure of the surroundings). Because of the ideal gas law also the temperature decreases, but it does not necessarily decrease to T_0 .

As frictional losses occur, the pressure in the gas for each successive sequence of processes gets lower until it eventually reaches the ambient pressure, P_0 . No additional expansion can now occur, since the gas is now in mechanical equilibrium with its surroundings.

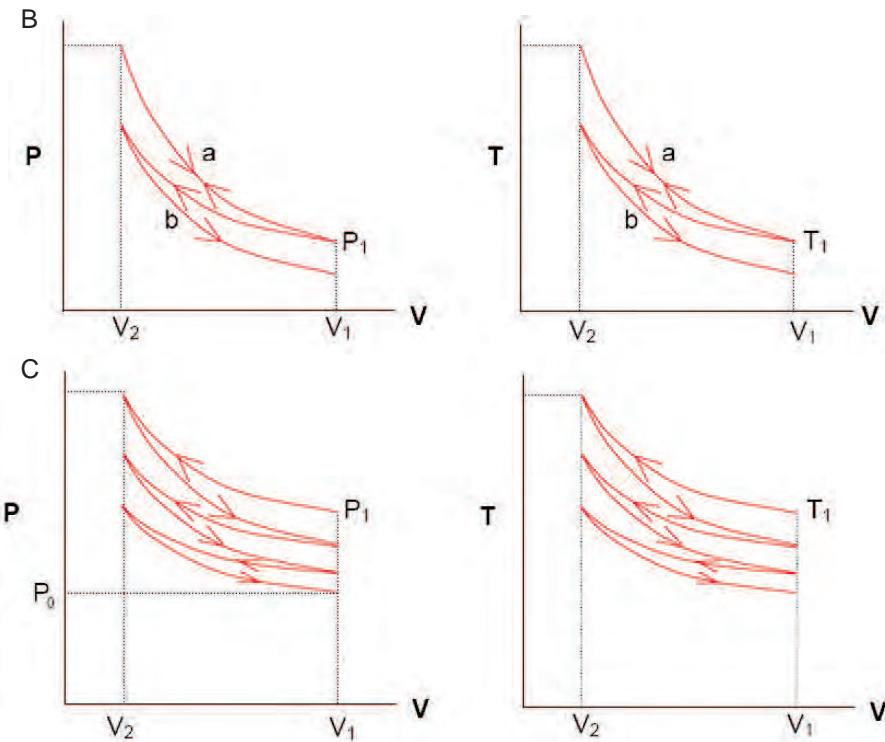


Figure 5.3.1: Graphs for exercise 5.3

5.4 Frictionless piston-cylinder device

5.4a The path for the expansion process is simply the reverse of the path for the compression process, see line a in figure 5.4.1B.

5.4b The expansion and compression processes follow different paths as a result of heat transfer to the surroundings. Every cycle the pressure will be a bit lower as a result of the heat transfer to the surroundings. The graphs can be found in figure 5.4.1B.

5.4c The sequence of paths would look like in figure 5.4.1C. Temperature and pressure decrease every cycle with the final temperature being asymptotic to T_0 (but the pressure not necessarily decreasing to P_0).

Heat is continuously being transferred from the system to the surroundings so long as the temperature of the surroundings, T_0 , is less than the temperature of the gas. As the gas temperature approaches the environment temperature, the potential for heat transfer approaches zero.

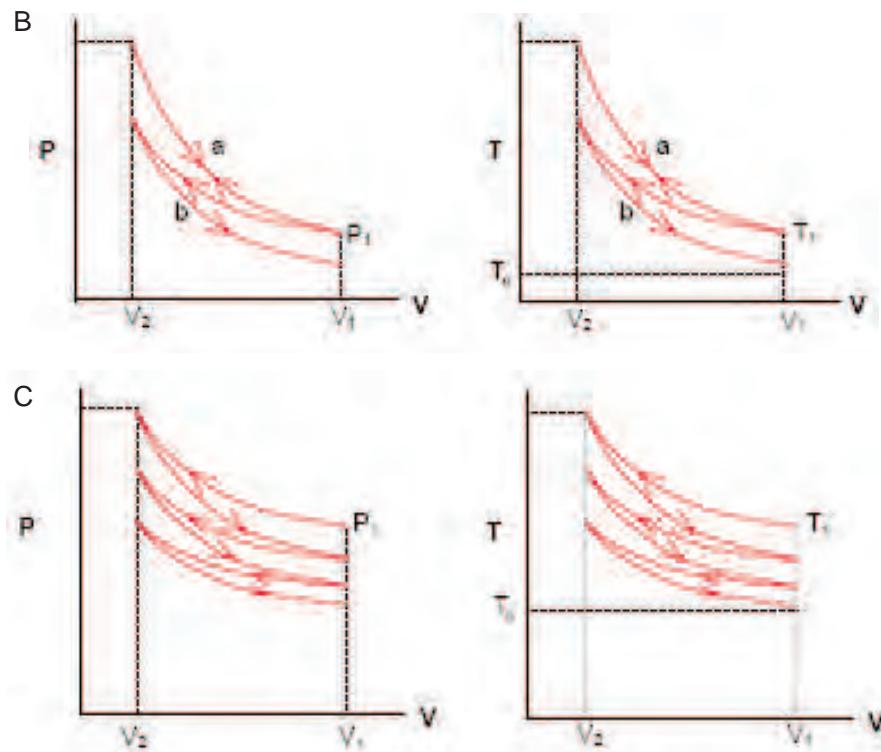


Figure 5.4.1: Graphs for exercise 5.4

5.5 Rigid adiabatic systems

5.5a There will be a heat flow from the smaller reservoir into the larger reservoir, temperature of the smaller reservoir will approach T_2 .

5.5b The flywheel will eventually come to rest, with the electrical energy generated becoming heat in the resistor that then diffuses into the reservoir. The heat generated is equal to the original kinetic energy of the flywheel.

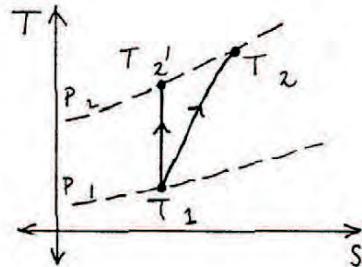
5.5c If the membrane is punctured, gas in the left part of the container performs a free expansion into the evacuated region, and will come to some final equilibrium state at a larger volume.

All three systems demonstrate different irreversible processes. Non of these processes can be reversed. The total energy in all cases remains constant, thus there is no violation of the first law (conservation of energy). The principle which determines the direction in which a process can take place is called the second law of thermodynamics. This law was invented to explain why processes have a direction and can not be reversed.

5.6 Steady flow

See the figure below. The reversible process is the process in which the temperature rises from T_1 to T_2' . The isentropic process over the same pressure difference but from T_1 to T_2' is by definition reversible, as we know, isentropic processes are adiabatic and reversible. The irreversible process is from T_1 to T_2 . The difference between temperature T_2 and T_2' can be due to the friction.

Furthermore, the irreversible process can be viewed as two processes: isentropic temperature rise from pressure p_1 to p_2 and temperature T_1 to T_2' , and a second process that further heats the gas at constant pressure to get the required state 2.



5.7 Basics of entropy

5.7a Irreversible process.

5.7b Reversible process.

5.7c Entropy, S in [kJ/K] or in the case of specific entropy s in [kJ/kgK].

5.7d Entropy is an extensive property which means that it is additive. Note that specific entropy is an intensive property.

5.7e In equilibrium the entropy has an extreme value, mostly the maximum value is used (instead of the minimum).

5.7f The entropy change is given by the net heat transfer divided by the temperature for the process that is reversible, $\rightarrow dS = \left(\frac{dQ}{T}\right)_{net,rev}$

5.7g If the process is not reversible the equality sign should be replaced by a larger than sign $>$.

5.7h Entropy change is caused by heat transfer, mass flow, and irreversibilities.

5.7i Heat transfer to a system increases the entropy, and heat transfer from a system decreases it.

5.7j The effect of irreversibilities is always to increase the entropy.

5.7k The entropy change between specified states is the same whether the process is reversible or irreversible. This is because entropy is a property, and like all other properties, it has fixed values at fixed states. Therefore, the entropy change between two states is the same no matter what path, reversible or irreversible, is followed during a process.

5.7l A process for which $dS = 0$, no change in entropy.

5.7m No, the entropy of the iron decreases but the entropy of the environment will increase as the environment heats up. The increase in entropy of the environment is larger than the decrease of the entropy of the iron as the temperature of the iron is higher than the temperature of the environment. Therefore the total entropy of the iron and the environment increase and thus the principle of entropy increase is not violated.

5.7n No, an isothermal process is not necessarily reversible, if there is friction involved the process is irreversible.

5.7o Yes, a reversible adiabatic process is isentropic.

5.8 Basics of the second law of thermodynamics

5.8a In the second law of thermodynamics it is stated that the entropy always increases.

5.8b The second law was necessary to explain why processes can not be reversed.

5.8c This is not a violation the second law of thermodynamics although the room is more ordered after the organizing. The entropy of the room has decreases but this did not happen spontaneously. Energy input of the person was needed to get a more ordered room. If the person was not there it would not have happened.

5.9 Isothermal expansion

Given: Refrigerant 134a, Isothermal process $\rightarrow T_1 = T_2$, pressure and temperature at state 1 and 2 are known.

\rightarrow At state 1: $P_1 = 300 \text{ kPa} < P_{sat@40}$ \rightarrow superheated \rightarrow Table A13:

$P = 0.28 \text{ MPa}$ and $T = 40^\circ\text{C} \rightarrow s = 1.0577 \text{ kJ/kgK}$.

$P = 0.32 \text{ MPa}$ and $T = 40^\circ\text{C} \rightarrow s = 1.0452 \text{ kJ/kgK}$.

Linear interpolation gives for $P_1 = 300 \text{ kPa}$ and $T_1 = 40^\circ\text{C}$: $s_1 = 1.0515 \text{ kJ/kgK}$.

\rightarrow At state 2: Table A13 gives:

$P = 0.14 \text{ MPa}$ and $T = 40^\circ\text{C} \rightarrow s = 1.1196 \text{ kJ/kgK}$.

$P = 0.18 \text{ MPa}$ and $T = 40^\circ\text{C} \rightarrow s = 1.0976 \text{ kJ/kgK}$.

Linear interpolation gives for $P_2 = 150 \text{ kPa}$ and $T_2 = 40^\circ\text{C}$: $s_2 = 1.1141 \text{ kJ/kgK}$.

The change in specific entropy is: $\Delta s = s_2 - s_1 = 1.1141 - 1.0515 = 0.626 \text{ kJ/kgK}$.

Even though there is no temperature change (isothermal process), there is an increase in entropy due to the drop in pressure.

5.10 Entropy change of an ideal gas in a rigid tank

The variables involved in this problem are shown graphically in figure 5.10.1. In this situation, the temperature of the gas does not change. The work added to the gas is lost as heat to the surroundings. Therefore, there is an equilibrium. The main equations in this problem are:

- $\Delta U = Q + W$ (conservation of energy closed system)
- $dU = c_v dT$ (for an ideal gas)
- $dU = TdS - PdV$ (Gibbs equation)
- $dS \geq \frac{Q}{T}$ (inequality of Clausius)
- $dS_{tot} = dS_{system} + dS_{surroundings}$

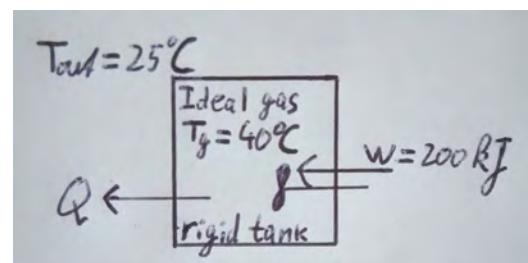


Figure 5.10.1: Situation of exercise 5.10

5.10a A rigid tank is considered, so V is constant, and therefore $dV = 0$. Also, the temperature of the gas is constant, such that $dT = 0$. From $dU = c_v dT$, it then follows that $dU = 0$. From the conservation of energy, we then find that $Q = -W = -200 \text{ kJ}$. So, 200 kJ of heat is flowing from the tank to the environment. For the entropy loss of the gas, we then find, with the inequality of Clausius:

$\Delta S_{gas} \geq \frac{Q}{T_{gas}} \Rightarrow \Delta S_{gas} \geq \frac{-200}{40+273} = -0.64 \text{ kJ/K}$. So, the minimum amount of entropy lost by the gas is $\Delta S_{gas} = -0.64 \text{ kJ/K}$.

5.10b For the entropy change of the environment, we use that an amount of heat of $Q = 200 \text{ kJ}$ is added at a temperature of T_{out} : $\Delta S_{out} \geq \frac{Q}{T_{out}} \Rightarrow \Delta S_{out} \geq \frac{200}{25+273} = 0.67 \text{ kJ/K}$.

The minimum total change of entropy in the universe is now calculated as the sum of the entropies of the gas and the surroundings: $\Delta S_{tot} = \Delta S_{gas} + \Delta S_{out} = -0.64 + 0.67 = 0.03 \text{ kJ/K}$. (Note that, due to the inequalities used, this is a "worst case" scenario. This is a minimum change and can also be higher, but definitely not lower.) Since the minimum total change of entropy is positive, this means that entropy is created and hence the second law is satisfied.

5.10c The total change of entropy of the ideal gas in the tank is calculated with the Gibbs equation: $dU = TdS - PdV \Rightarrow dS = \frac{1}{T}dU + \frac{P}{T}dV = 0$, since both dU and dV are 0. Since the gas in the tank is in equilibrium, we could also have expected this result.

Discussion of results Actually for the gas in the tank nothing happens during the long time. On the one hand heat comes into the system from the paddle wheel, but on the other hand this heat goes out through the walls of the tank. The temperature of the gas is constant and you can not measure any change in the gas if you put in a thermometer. This means that the gas is in equilibrium. In equilibrium the entropy of a system is at a maximum (the chaos (amount of disorder) is maximal) and there is no change in entropy, so $ds = 0$ as is calculated in part c.

However if the total process is considered there is actually something happening, namely heat is produced by the paddle wheel and flowing out the system from a high temperature (40 degree Celsius) to a low temperature (25 degree Celsius). This is a process that is irreversible. The heat can never be converted to work to turn the paddle wheel and also the heat can not flow in the reversed direction. During this process entropy is generated, as found in questions a and b.

5.11 Entropy change of compressed air

The variables involved in this problem are shown graphically in figure 5.11.1. In this situation, the temperature of the gas does not change. Any heat created is directly being transferred to the environment. The main equations in this problem are:

- conservation of energy open system
- $dh = c_p dT$
- $dS \geq \frac{Q}{T}$ (inequality of Clausius)

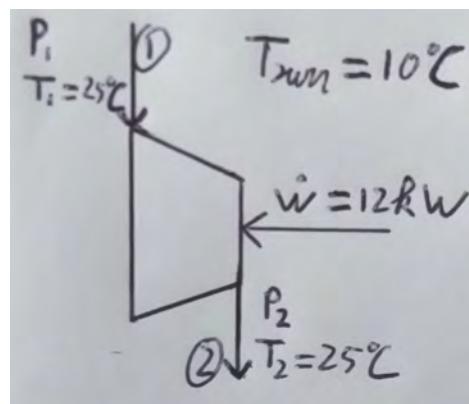


Figure 5.11.1: Situation of exercise 5.11

5.11a Conservation of energy in an open system: $\dot{m}(h_1 + \frac{1}{2}v_1^2 + gz_1) = \dot{m}(h_2 + \frac{1}{2}v_2^2 + gz_2) + \dot{Q} + \dot{W}$. Neglecting kinetic and potential energy reduces this equation to: $\dot{m}(h_2 - h_1) = -\dot{Q} - \dot{W}$. From $dh = c_p dT$, it follows that $dh = 0$, as the temperature is constant in this process. So, $\dot{Q} = -\dot{W}$, so 12 kW of heat is lost to the environment.

5.11b We determine the minimum rate of entropy change of the air with the inequality of Clausius: $\Delta \dot{S} = \frac{\dot{Q}}{T} \Rightarrow \Delta \dot{S}_{air} = \frac{-12}{273+25} = -0.0403 \text{ kW/K}$.

5.11c The minimum rate of entropy change of the environment is calculated as: $\Delta \dot{S}_{surr} = \frac{\dot{Q}_{surr}}{T_{surr}} = \frac{12}{273+10} = 0.042 \text{ kW/K}$. Since the sum of the minimum entropy changes is positive, we in total produce entropy and therefore satisfy the second law of thermodynamics.

5.12 Radiator

There is superheated steam in the radiator. It is closed ($dV = 0$) and the steam is cooling down to $T_2 = 80^\circ\text{C}$ due to heat transfer to the environment. Besides the volume, also the mass does not change (no mass transfer), such that the specific volume remains constant ($v_1 = v_2$). This means that we go down vertically in the T-v diagram (see figure 5.12.1) from P_1 to P_2 . This pressure is however unknown, so it needs to be found (using the specific volume) whether the steam is in the mixture phase at the second state, or still superheated or saturated vapour.

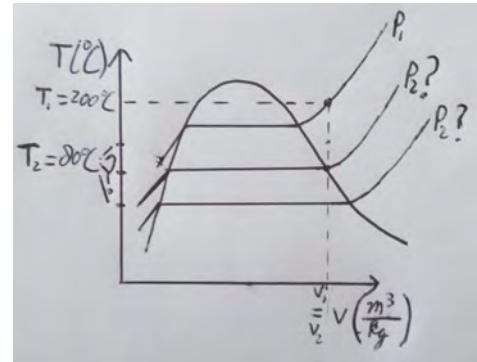


Figure 5.12.1: T-v diagram of exercise 5.12

5.12a From table A-6, we find that $v_1 = 1.08049 \text{ m}^3/\text{kg}$ (pressure and temperature known). From table A-4, we look up the values of v_f and v_g at a saturation temperature of $T_2 = 80^\circ\text{C}$: $v_f@T_{sat}=80^\circ\text{C} = 0.001029 \text{ m}^3/\text{kg}$ and $v_g@T_{sat}=80^\circ\text{C} = 3.4053 \text{ m}^3/\text{kg}$. So, $v_f < v_2 < v_g$ ($v_2 = v_1$), such that v_2 is in the mixture phase. The steam pressure is therefore equal to the saturation pressure at $T_2 = 80^\circ\text{C}$: $P_2 = 47.416 \text{ kPa}$. The vapour mass fraction follows from the lever rule: $x = \frac{v_2 - v_f}{v_g - v_f} = \frac{1.08049 - 0.001029}{3.4053 - 0.001029} = 0.317$.

5.12b We determine the initial entropy from table A-6: $s_1 = 7.5081 \text{ kJ/(kgK)}$. The final entropy s_2 is determined from table A-4 using the vapour mass fraction and the lever rule: $s_2 = s_f + x(s_g - s_f) = 1.0756 + 0.317(7.6111 - 1.0756) = 3.147 \text{ kJ/(kgK)}$. The entropy change in kJ/K is calculated using the volume and the specific volume to determine the mass: $\Delta s = m(s_2 - s_1) = \frac{V}{v_1}(s_2 - s_1) = \frac{20 \cdot 10^{-3}}{1.08049}(3.147 - 7.5081) = -0.081 \text{ kJ/K}$.

5.13 Non-adiabatic compressor

The variables involved in this problem are shown graphically in figure 5.13.1. Air is being compressed, and there is heat transfer to the environment. The main equations in this problem are:

- $\dot{E}_i = \dot{m}(h_i + \frac{1}{2}v_i^2 + gz_i) + \dot{Q}_i + \dot{W}_i \rightarrow \text{conservation of energy open system, potential and kinetic energy negligible}$
- $dh = c_p dT$
- $dS \geq \frac{\delta Q}{T}$ (inequality of Clausius)
- $Pv = RT$ (ideal gas law)
- $dh = Tds + vdP$ (Gibbs equation)

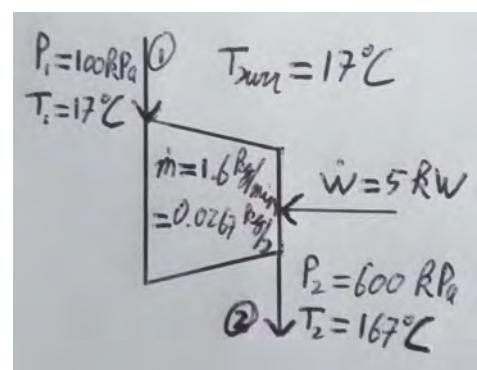


Figure 5.13.1: Situation of exercise 5.13

5.13a Conservation of energy with potential and kinetic energy neglected gives: $\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \rightarrow \dot{Q}_{out} = \dot{W}_{in} - \dot{m}(h_2 - h_1)$. We calculate the enthalpy change from $dh = c_p dT$, where $c_p = 1.005 \text{ kJ/(kgK)}$. Substituting this gives the rate of heat loss: $\dot{Q}_{out} = \dot{W}_{in} - \dot{m}c_p(T_2 - T_1) = 5 - 0.0267 \cdot 1.005 \cdot (167 - 17) = 5 - 4.025 = 0.975 \text{ kW}$. So, the heat loss is 0.975 kW, about 20% of the power input. (Note: $1.6 \text{ kg/min} = 0.0267 \text{ kg/s}$.)

5.13b Gibbs: $dh = Tds + vdP \rightarrow dS = \frac{1}{T}dh - \frac{v}{T}dP = \frac{c_p}{T}dT - \frac{R}{P}dP$, where we used the ideal gas law to rewrite $\frac{v}{T}$ and $dh = c_p dT$ to rewrite the first term. Integrating from state 1 to state 2 now gives the specific entropy change: $\Delta s = \int_{T_1}^{T_2} \frac{c_p}{T}dT - \int_{P_1}^{P_2} \frac{R}{P}dP = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) = 1.005 \ln\left(\frac{167+273}{17+273}\right) - 0.287 \ln\left(\frac{600}{100}\right) = -0.097 \text{ kJ/(kgK)}$. The rate of entropy change is calculated as: $\Delta \dot{S} = \dot{m} \Delta s = 0.0267 \cdot -0.097 = -0.0026 \text{ kW/K}$.

Alternatively, one can calculate the minimum rate of entropy change using the inequality of Clausius and the average temperature of the compressed air: $\Delta \dot{S} = \frac{\dot{Q}}{T} = \frac{-0.975}{0.5(T_1+T_2)} = \frac{-0.975}{0.5((17+273)+(167+273))} = \frac{-0.975}{365} = -0.0027 \text{ kW/K}$.

5.14 Entropy change of a copper block

5.14a The block will transfer heat energy to the water. This will decrease the temperature of the block, and the temperature of the water increases. This will happen until there is 1 new equilibrium temperature (since there is no heat transfer to the environment while the tank is insulated). From conservation of energy, we know that $\Delta U = Q + W$. While no work is done, this means that $\Delta U = Q$. Using $\Delta U = m \cdot C \cdot \Delta T$ and using that all heat of the block is transferred to the water ($\Delta U_c = -\Delta U_w$) gives:

$$\begin{aligned} \Delta U_c = -\Delta U_w &\Rightarrow m_c c_c (T_{equi} - T_c) = -m_w c_w (T_{equi} - T_w) \\ (m_c \cdot c_c + m_w \cdot c_w) T_{equi} &= m_w \cdot c_w \cdot T_w + m_c \cdot c_c \cdot T_c \\ T_{equi} &= \frac{m_w c_w T_w + m_c c_c T_c}{m_c c_c + m_w c_w} \end{aligned}$$

5.14b The entropy change follows from the Gibbs equation: $du = Tds - Pdv$. Both the water and the copper are incompressible, so $dv = 0$. Rewriting now gives the entropy change:

$$dS = \frac{1}{T} dU \Rightarrow S_2 - S_1 = \int_1^2 \frac{1}{T} dU = mC \int_1^2 \frac{1}{T} dT = mC [\ln T]_1^2 = mC \ln\left(\frac{T_2}{T_1}\right)$$

Note that the mass m and the specific heat C may only be taken out of the integral if they are independent of T . For C this is actually not the case, but for this relatively small change in temperature it is taken constant. For the copper block, state 1 is the initial temperature T_c and state 2 is the equilibrium temperature T_{equi} . For the water, state 1 is the initial temperature T_w and state 2 is also the equilibrium temperature. So, the total entropy change, which is the sum of the entropy change of the block and the entropy change of the water, is given by:

$$\Delta S = \Delta S_c + \Delta S_w = m_c c_c \ln\left(\frac{T_{equi}}{T_c}\right) + m_w c_w \ln\left(\frac{T_{equi}}{T_w}\right)$$

The copper is loosing heat, so $\frac{T_{equi}}{T_c} < 1$, so the ln is negative and there is an entropy loss. For the water, $\frac{T_{equi}}{T_w} > 1$, so the ln is positive and there is an increase in entropy.

5.14c The mass of the water is determined from the volume and the density: $m_w = \rho_w V_w = 1000 \cdot 120 \cdot 10^{-3} = 120$ kg. Substitution now gives the equilibrium temperature and the entropy changes:

$$T_{equi} = \frac{m_w c_w T_w + m_c c_c T_c}{m_c c_c + m_w c_w} = \frac{120 \cdot 4.18 \cdot 298 + 50 \cdot 0.386 \cdot 353}{50 \cdot 0.386 + 120 \cdot 4.18} = 300 \text{ K} = 27^\circ\text{C}$$

$$\Delta S_c = m_c c_c \ln \left(\frac{T_{equi}}{T_c} \right) = 50 \cdot 0.386 \cdot \ln \left(\frac{300}{353} \right) = -3.140 \text{ kJ/K}$$

$$\Delta S_w = m_w c_w \ln \left(\frac{T_{equi}}{T_w} \right) = 120 \cdot 4.18 \cdot \ln \left(\frac{300}{298} \right) = 3.355 \text{ kJ/K}$$

$$\Delta S_{total} = \Delta S_c + \Delta S_w = -3.140 + 3.355 = 0.215 \text{ kJ/K}$$

So, in total there is an increase in entropy, as it should be for spontaneous processes.

5.15 Entropy change of an iron block

5.15a The block will transfer heat energy to the water. This will decrease the temperature of the block, and the temperature of the water increases. This will happen until there is 1 new equilibrium temperature (since there is no heat transfer to the environment while the tank is insulated). Conservation of energy, gives $\Delta U = Q + W$. While no work is done, this means that $\Delta U = Q$. Using $\Delta U = m \cdot c \cdot \Delta T$ and using that all heat of the block is transferred to the water ($\Delta U_i = -\Delta U_w$) gives:

$$\begin{aligned} \Delta U_i = -\Delta U_w &\Rightarrow m_i c_i (T_{equi} - T_i) = -m_w c_w (T_{equi} - T_w) \\ (m_i \cdot c_i + m_w \cdot c_w) T_{equi} &= m_w \cdot c_w \cdot T_w + m_i \cdot c_i \cdot T_i \\ T_{equi} &= \frac{m_w c_w T_w + m_i c_i T_i}{m_i c_i + m_w c_w} \end{aligned}$$

The entropy change follows from the Gibbs equation: $du = Tds - Pdv$. Both the water and the iron are incompressible, so $dv = 0$. Rewriting now gives the entropy change:

$$dS = \frac{1}{T} dU \Rightarrow S_2 - S_1 = \int_1^2 \frac{1}{T} dU = mc \int_1^2 \frac{1}{T} dT = mc [\ln T]_1^2 = mc \ln \left(\frac{T_2}{T_1} \right)$$

Note that the mass m and the specific heat c may only be taken out of the integral if they are independent of T . For c this is actually not the case, but for this relatively small change in temperature it is assumed to be constant. For the iron block, state 1 is the initial temperature T_i and state 2 is the equilibrium temperature T_{equi} . For the water, state 1 is the initial temperature T_w and state 2 is also the equilibrium temperature. So, the total entropy change, which is the sum of the entropy change of the block and the entropy change of the water, is given by:

$$\Delta S = \Delta S_i + \Delta S_w = m_i c_i \ln \left(\frac{T_{equi}}{T_i} \right) + m_w c_w \ln \left(\frac{T_{equi}}{T_w} \right)$$

The iron is loosing heat, so $\frac{T_{equi}}{T_i} < 1$, so the ln is negative and there is an entropy loss. For the water, $\frac{T_{equi}}{T_w} > 1$, so the ln is positive and there is an increase in entropy.

5.15b Substitution gives:

$$T_{equi} = \frac{m_w c_w T_w + m_i c_i T_i}{m_i c_i + m_w c_w} = \frac{100 \cdot 4.18 \cdot 295 + 12 \cdot 0.45 \cdot 623}{12 \cdot 0.45 + 100 \cdot 4.18} = 299.2 \text{ K} = 26.2^\circ\text{C}$$

$$\Delta S_i = m_i c_i \ln \left(\frac{T_{equi}}{T_i} \right) = 12 \cdot 0.45 \cdot \ln \left(\frac{299.2}{623} \right) = -3.96 \text{ kJ/K}$$

$$\Delta S_w = m_w c_w \ln \left(\frac{T_{equi}}{T_w} \right) = 100 \cdot 4.18 \cdot \ln \left(\frac{299.2}{295} \right) = 5.63 \text{ kJ/K}$$

$$\Delta S_{total} = \Delta S_i + \Delta S_w = -3.96 + 5.63 = 1.67 \text{ kJ/K}$$

So, in total there is an increase in entropy, as it should be for spontaneous processes.

5.16 Entropy change of an aluminium block

5.16a The iron block and aluminium block are brought into contact in an insulated environment (no heat transferred over system boundaries). So, all heat is kept in the system. From conservation of energy (and no work done) we can then determine the equilibrium temperature:

$$\begin{aligned}\Delta U &= Q + W \Rightarrow \Delta U = Q \\ \Delta U_i + \Delta U_a &= 0 \Rightarrow m_i c_i (T_{equi} - T_i) = -m_a c_a (T_{equi} - T_a) \\ (m_i \cdot c_i + m_a \cdot c_a) T_{equi} &= m_a \cdot c_a \cdot T_a + m_i \cdot c_i \cdot T_i \\ T_{equi} &= \frac{m_a c_a T_a + m_i c_i T_i}{m_i c_i + m_a c_a}\end{aligned}$$

5.16b The entropy change follows from the Gibbs relation, with $dV = 0$ because the volumes are constant.

$$dU = TdS - PdV \Rightarrow dS = \frac{1}{T}dU = \frac{mc}{T}dT \Rightarrow \Delta S = \int_1^2 \frac{mc}{T}dT = mc \int_1^2 \frac{1}{T}dT = mc \ln\left(\frac{T_2}{T_1}\right).$$

Where 1 is the initial situation and 2 is the equilibrium situation. So, the total entropy change is given as:

$$\Delta S_{total} = \Delta S_i + \Delta S_a = m_i c_i \ln\left(\frac{T_{equi}}{T_i}\right) + m_a c_a \ln\left(\frac{T_{equi}}{T_a}\right).$$

5.16c Substitution gives:

$$\begin{aligned}T_{equi} &= \frac{m_a c_a T_a + m_i c_i T_i}{m_i c_i + m_a c_a} = \frac{20 \cdot 903 \cdot 473 + 20 \cdot 447 \cdot 373}{20 \cdot 447 + 20 \cdot 903} = 440 \text{ K} = 167^\circ\text{C} \\ \Delta S_i &= m_i c_i \ln\left(\frac{T_{equi}}{T_i}\right) = 20 \cdot 447 \cdot \ln\left(\frac{440}{373}\right) = 1477 \text{ J/K} \\ \Delta S_a &= m_a c_a \ln\left(\frac{T_{equi}}{T_a}\right) = 20 \cdot 903 \cdot \ln\left(\frac{440}{473}\right) = -1306 \text{ J/K} \\ \Delta S_{total} &= \Delta S_i + \Delta S_a = 1477 - 1306 = 171 \text{ J/K}\end{aligned}$$

So again, an entropy loss in the block that is cooling and an entropy increase in the heating block. In total there is an increase in entropy, as it should be for spontaneous processes. Note that the equilibrium temperature is not the average of the initial temperatures, even though the mass of the blocks is equal. This is due to the difference in specific heat.

5.17 Entropy change of an ideal gas

5.17a $du = Tds - Pdv \rightarrow ds = \frac{du}{T} + \frac{Pdv}{T}$ and for an ideal gas $du = c_v dT$. Combining gives: $ds = \frac{c_v dT}{T} + \frac{Pdv}{T}$.

$$\begin{aligned}s_2 - s_1 &= \Delta s = \int_1^2 ds = \int_{T_1}^{T_2} \frac{c_v dT}{T} + \int_{v_1}^{v_2} \frac{Pdv}{T} = \int_{T_1}^{T_2} \frac{c_v dT}{T} + \int_{v_1}^{v_2} \frac{Rdv}{v} \\ s_2 - s_1 &= c_{v,avg} [\ln T]_{T_1}^{T_2} + R [\ln v]_{v_1}^{v_2} = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}. \quad (5.17.1)\end{aligned}$$

Note: In general c_v depends on temperature. Therefore, in general it should not be taken as a constant during integration. However, often the change in c_v is so small that it may be considered constant. This is called the constant-specific-heat assumption.

5.17b $dh = Tds + vdP \rightarrow ds = \frac{dh}{T} - \frac{vdP}{T}$ and $dh = c_p dT$. Combining gives: $ds = \frac{c_p dT}{T} - \frac{vdP}{T}$.

$$s_2 - s_1 = \Delta s = \int_1^2 ds = \int_{T_1}^{T_2} \frac{c_p dT}{T} - \int_{P_1}^{P_2} \frac{vdP}{T} = \int_{T_1}^{T_2} \frac{c_p dT}{T} - \int_{P_1}^{P_2} \frac{RdP}{P}$$

$$s_2 - s_1 = c_{P,\text{avg}} [\ln T]_{T_1}^{T_2} - R [\ln P]_{P_1}^{P_2} = c_{P,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (5.17.2)$$

5.17c To prove that the two relations for entropy change of ideal gases

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{and} \quad s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (5.17.3)$$

under the constant-specific-heat assumptions are equivalent the ideal-gas relation, $Pv = RT$, and the Nernst Lindemann relation for an ideal gas, $c_p - c_v = R$, is used. In the first relation eliminate c_v using, $c_v = c_p - R$ and eliminate v using $v_i = \frac{RT_i}{P_i}$. This gives:

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = (c_{p,\text{avg}} - R) \ln \frac{T_2}{T_1} + R \ln \frac{RT_2}{P_2} \frac{P_1}{RT_1} = \quad (5.17.4)$$

$$(c_{p,\text{avg}} - R) \ln \frac{T_2}{T_1} + R \ln \frac{T_2}{T_1} + \ln \frac{P_1}{P_2} = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}. \quad (5.17.4)$$

The result can also be obtained by starting with the second relation and eliminate c_p and P_i .

5.18 Entropy change of ideal gases in an isothermal process

5.18a In an isothermal process the temperature is constant, $dT = 0$, and thus $T_1 = T_2$.

5.18b Yes, the entropy of an ideal gas can change during an isothermal process.

5.18c An isothermal process, $\rightarrow dT = 0$, and thus $T_1 = T_2$ which gives $\ln \frac{T_2}{T_1} = \ln 0 = 1$.

$$s_1 - s_2 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = R \ln \frac{v_2}{v_1}.$$

$$s_1 - s_2 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = -R \ln \frac{P_2}{P_1}.$$

5.18d At constant pressure ($P_1 = P_2$) the relation for the change of entropy changes to:

$$s_1 - s_2 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} \text{ while at constant volume } (v_1 = v_2) \text{ it changes to } s_1 - s_2 = c_{v,\text{avg}} \ln \frac{T_2}{T_1}.$$

For an ideal gas, $c_p = c_v - R$, and as R is always positive this implies that c_p is always larger than c_v . Therefore the entropy change is larger for the process at constant pressure.

5.19 Entropy change of ideal gases in isentropic process

5.19a In an isentropic process the entropy is constant $ds = 0$, and thus $s_1 = s_2$.

5.19b If $ds = 0$ the relations found in exercise 5.17a and b can be written as: $\ln \frac{T_2}{T_1} = \frac{R}{c_{v,\text{avg}}} \ln \frac{v_2}{v_1}$ and as $\ln \frac{T_2}{T_1} = -\frac{R}{c_{p,\text{avg}}} \ln \frac{P_2}{P_1}$. Combining gives: $\frac{c_{p,\text{avg}}}{c_{v,\text{avg}}} \ln \frac{v_2}{v_1} = -\ln \frac{P_2}{P_1}$. This is equal to:

$\left(\ln \frac{v_2}{v_1} \right)^{\frac{c_{p,\text{avg}}}{c_{v,\text{avg}}}} = \ln \frac{P_1}{P_2}$ from which follows: $\left(\frac{v_2}{v_1} \right)^{\frac{c_{p,\text{avg}}}{c_{v,\text{avg}}}} = \frac{P_1}{P_2}$ which results in: $P_1 v_1^\gamma = P_2 v_2^\gamma$ with $\gamma = \frac{c_{p,\text{avg}}}{c_{v,\text{avg}}}$ and thus Pv^γ is constant.

5.20 Entropy change of oxygen in a piston-cylinder device

5.20a We can calculate the entropy change using the Gibbs equation:

$$Tds = du + Pdv \Rightarrow ds = \frac{du}{T} + \frac{P}{T}dv$$

From the ideal gas law ($Pv = RT$), we find that $\frac{P}{T} = \frac{R}{v}$. Furthermore, for an ideal gas it holds that $du = c_v dT$. Hence, the Gibbs equation can be rewritten to:

$$ds = \frac{c_v}{T}dT + \frac{R}{v}dv$$

Integrating from the initial to the final state, while assuming constant specific heats, gives the entropy change:

$$\Delta s = c_v \int_{T_1}^{T_2} \frac{1}{T}dT + R \int_{v_1}^{v_2} \frac{1}{v}dv = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

5.20b In order to calculate the entropy change for the given numbers, we need to find R and c_v for oxygen. From table 2 we find that $R = 0.260 \text{ kJ/(kgK)}$. We can find the value of c_v in table 7. We take the value at 150°C (approximately the average of the two temperatures) as the constant value of c_v : $c_v = 0.6882 \text{ kJ/(kgK)}$. Substituting the given numbers, we find that:

$$\begin{aligned} \Delta s &= c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) = 0.6882 \ln\left(\frac{287 + 273}{25 + 273}\right) + 0.260 \ln\left(\frac{0.1}{0.8}\right) \\ &= 0.4341 - 0.5407 = -0.1066 \text{ kJ/(kgK)} \end{aligned}$$

Discussion: The entropy of the system decreases. Maybe you didn't expect this as the temperature increases from 25 to 287°C . A higher temperature results in a higher entropy, however the pressure in the system also increases $\rightarrow P_1 v_1 / T_1 = P_2 v_2 / T_2 \rightarrow P_2 / P_1 = v_1 / v_2 \cdot T_2 / T_1 = 15 \rightarrow P_2 = 15 P_1$.

In contrast to the temperature a higher pressure results in a lower entropy. Here the effect of the decrease in entropy due to an increase in pressure seems to be larger than the increase of the entropy due to an increase of the temperature. This also follows from the formula and the numbers. The first term, accounting for the temperature change, results in a positive entropy change while the second term, accounting for the pressure change, results in a negative entropy change. The negative value is larger than the positive one. Therefore the total entropy change is negative.

The compression of the gas is a process in which work is added so the entropy of the system is allowed to decrease (it is NOT a spontaneous process). In order to meet the second law the entropy of the environment should rise by an amount more than or equal to the entropy drop.

5.21 Entropy change of nitrogen in a piston-cylinder device

5.21a The entropy change is given by: $\Delta s = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$, where $c_{v,\text{avg}}$ is the average specific heat (see exercise 5.17). The terms $\frac{v_2}{v_1}$ and $\frac{T_2}{T_1}$ need to be found to calculate the entropy change.

The first one is easy as the end volume is half the starting volume: $v_2 = \frac{1}{2}v_1$ or $2v_2 = v_1$.

Left is the $\frac{T_2}{T_1}$ term that needs to be evaluated. This can be written in terms of v .

Using the ideal gas relation ($PvRT$), $\frac{T_2}{T_1}$ can be written as: $\frac{T_2}{T_1} = \frac{P_2 v_2}{P_1 v_1} \frac{R}{R} = \frac{v_2}{v_1} \frac{P_2}{P_1}$.

The pressures from $\frac{P_2}{P_1}$ can be eliminated using the polytropic relation:

$Pv^{1.3} = \text{constant} \rightarrow P_1 v_1^{1.3} = P_2 v_2^{1.3}$. This gives: $\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{1.3}$ and therefore $\frac{T_2}{T_1} = \frac{v_2}{v_1} \left(\frac{v_1}{v_2}\right)^{1.3}$.

Now the found relations can be filled in, in the expression for the entropy change:

$$\begin{aligned}\Delta s &= c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = c_{v,avg} \ln \left[\frac{v_2}{v_1} \left(\frac{v_1}{v_2} \right)^{1.3} \right] + R \ln \frac{v_2}{v_1} = c_{v,avg} \ln \left[\frac{v_2}{2v_2} \left(\frac{2v_2}{v_2} \right)^{1.3} \right] + R \ln \frac{v_2}{2v_2} \\ &= c_{v,avg} \ln \left[\frac{1}{2} (2^{1.3}) \right] + R \ln \frac{1}{2} = c_{v,avg} (-\ln 2 + 1.3 \ln 2) - R \ln 2 = 0.3c_{v,avg} \ln 2 - R \ln 2 \\ &= \ln 2(0.3c_{v,avg} - R).\end{aligned}$$

5.21b The specific entropy change, Δs (in kJ/(kgK)) is independent of the pressure and temperature as the relation found above has no pressure or temperature in it anymore. It only depends on the type of gas.

For nitrogen: $R = 0.2968 \text{ kJ}/(\text{kgK})$ and $c_{v,avg} = 0.743 \text{ kJ}/(\text{kgK})$. This gives $\Delta s = -0.0512 \text{ kJ}/(\text{kgK})$ and for 1.2 kg gas the total entropy change is: $\Delta S = m \times \Delta s = -0.0615 \text{ kJ/K}$.

5.22 Reversible condensation

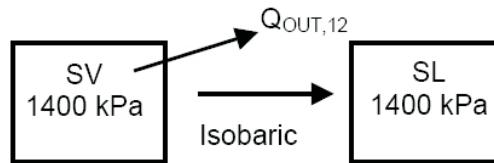


Figure 5.22.1: Situation for exercises 5.22

Given: Reversible isobaric process, $P_2 = P_1$, water $m = 3.5 \text{ kg}$.

State 1: Saturated vapor, $P_1 = 1400 \text{ kPa}$.

State 2: Saturated liquid, $P_2 = 1400 \text{ kPa}$.

Reversible heat transfer is given by $Q_{net,rev,1-2} = \int_1^2 \delta Q_{net,rev,1-2} = \int_1^2 T ds = m \int_1^2 T ds$.

Since the water is saturated throughout the process and P and T are also constant, $T = T_{sat@1400kPa} = 195.08^\circ\text{C} = 468.1 \text{ K}$ (table A-5).

$Q_{net,rev,1-2} = m \int_1^2 T ds = m T_{sat} (s_l - s_v) = -ms_{lv} T_{sat}$.

The value for $s_{lv} = 4.184 \text{ kJ/kgK}$ (table A-5)).

Filling in the numbers $Q_{net,rev,1-2} = -ms_{lv} T_{sat} = -3.5 \cdot 468.1 \cdot 4.184 = -6855 \text{ kJ}$ (out of system, entropy in the system decreases).

This result could also have been obtained by using: $T ds = dh - vdP = dh$ as $dP = 0 \rightarrow dh = T ds = \delta q_{net,rev,1-2}$. Therefore also $Q_{net,rev,1-2} = m \int_1^2 dh = (h_l - h_v) = -mh_{lv}$.

The value for $h_{lv} = 1959.1 \text{ kJ/kg}$ (table A-5) $\rightarrow Q_{net,rev,1-2} = -mh_{lv} = -3.5 \cdot 1959.1 = -6856.85 \text{ kJ}$ (out of system, entropy in the system decreases). This answer is equal to the answer via the other way.

Entropy decreases when a substance condenses from the saturated vapor to the saturated liquid state, even though the temperature and pressure remain constant. This makes sense, because $v_l < v_v$ (molecules occupy a smaller volume in the liquid state) and hence they are less randomly distributed spatially than in the vapor state.

5.23 Reversible isothermal expansion

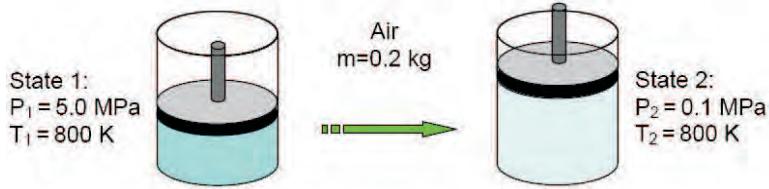


Figure 5.23.1: Situation for exercises 5.23

Given: Reversible isothermal process, $T_2 = T_1$, air, $m = 0.2 \text{ kg}$.

State 1: $P_1 = 5.0 \text{ MPa}$, $T_1 = 800 \text{ K}$.

State 2: $P_2 = 0.1 \text{ MPa}$, $T_2 = 800 \text{ K}$.

The air is assumed to be an ideal gas with constant specific heat and constant mass.

First law for a closed system $\Delta U = Q_{net,1-2} + W_{net,1-2}$.

Second law for reversible heat transfer is given by $Q_{net,rev,1-2} = \int_1^2 \delta Q_{net,rev,1-2} = \int_1^2 T dS = m \int_1^2 T ds$.

$dh = Tds + vdp \rightarrow ds = \frac{dh}{T} - \frac{vdp}{T}$. For an ideal gas $dh = c_p dT \rightarrow ds = c_p \frac{dT}{T} - R \frac{dp}{P}$. In the case of an isothermal process ($dT = 0$) the first term at the right hand side is zero. Integration from 1 → 2 yields $s_{gen} = s_2 - s_1 = -R \ln \frac{P_2}{P_1}$.

$Q_{net,rev,1-2} = m \int_1^2 T ds = mT(s_2 - s_1)$ if T is constant.

Combining the two equations results in $\rightarrow Q_{net,rev,1-2} = -mRT \ln \frac{P_2}{P_1}$.

For air $R = 0.287 \text{ kJ/kgK}$ (table 1).

Filling in the numbers results in $Q_{net,rev,1-2} = -0.2 \cdot 0.287 \cdot 800 \cdot \ln \frac{0.1}{5} = 179.6 \text{ kJ}$ (the + sign means that the heat transfer is into the control volume).

The first law states that $\Delta U = Q_{net,1-2} + W_{net,1-2} = \Delta U = U_2 - U_1 - m(u_2 - u_1) = 0$, since $du = c_v dT$ and $dT = 0$.

$W_{net,1-2} = -Q_{net,1-2} = -179.63 \text{ kJ}$. The - sign indicates that the nett work $W_{net,1-2} = W_{in,1-2} - W_{out,1-2}$ is negative. Therefore the work is out of the system, $W_{out,1-2} = 179.63 \text{ kJ}$.

The fact that the final result for $W_{out,1-2}$ has a positive sign confirms that the work was, indeed, out of the system as expected as the cylinder expanded. Had the sign turned out negative, it would have indicated that the actual work was into, not out of, the system.

5.24 Reversible processes for helium

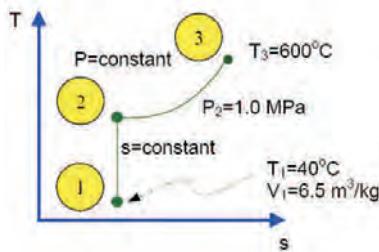


Figure 5.24.1: Situation for exercises 5.24

Given: Helium undergoing a reversible process in a closed (non-flow) device, $c_p = 5/2R$.

State 1: $T_1 = 313 \text{ K}$, $v_1 = 6.5 \text{ m}^3/\text{kg}$.

State 2: $P_2 = 1000 \text{ kPa}$.

State 3: $T_3 = 873 \text{ K}$, $P_3 = 1000 \text{ kPa}$.

Assumed is that helium behaves as an ideal gas, $Pv = RT$.

Which equations do we need?

The heat transfer and work for the processes can be found using the first law for a closed system $\Delta u = q_{net,1-2} + w_{net,1-2}$.

The difference in internal energy can be calculated using: $du = c_v dT$.

Work can be determined using: $\delta w = Pdv$.

For reversible processes the heat transfer is: $\delta q = Tds$.

For the first process 1-2 the heat transfer $q_{net,1-2} = 0$ as the process is isentropic, $ds = 0$ (reversible and adiabatic) $\rightarrow w_{net,1-2} = u_2 - u_1 = c_v(T_2 - T_1)$.

The internal energy change for the second process 2-3 is $u_3 - u_2 = c_v(T_3 - T_2)$.

For the second process 2-3 the work can be calculated by $w_{net,2-3} = \int_{v_2}^{v_3} Pdv = P_2(v_3 - v_2) = P_2 R \left(\frac{T_3}{P_3} - \frac{T_2}{P_2} \right) = R(T_3 - T_2)$ (P can be taken out of the integral because P is constant, $P_2 = P_3$).

If the internal energy and the work are known the heat transfer for the second process follows: $q_{net,2-3} = (u_3 - u_2) - w_{net,2-3} = c_v(T_3 - T_2) - w_{net,2-3} = c_v(T_3 - T_2) - R(T_3 - T_2) = (c_v - R)(T_3 - T_2)$.

At this moment all the equations are known. The next step is to determine the properties from the states that are not known yet. For this the ideal gas law, $Pv = RT$, can be used and for isentropic processes (note only the process 1-2!) also $Pv^k = \text{constant}$ with $k = c_p/c_v$ can be used.

Substitution of the ideal gas law in this last relation gives $\frac{P}{T}^{\frac{k-1}{k}} = \text{constant}$.

Furthermore we need the property data for c_v , k and R .

$R = 2.077 \text{ kJ/kgK}$ (table 1), the values of c_v and k follow from $c_p = 5/2R$, $c_v = c_p - R$ and $k = c_p/c_v$ $\rightarrow c_p = 5/2 \cdot 2.077 = 5.193 \text{ kJ/kgK} \rightarrow c_v = c_p - R = 3.116 \text{ kJ/kgK} \rightarrow k = c_p/c_v = 1.667$.

P_1 follows from the ideal gas law $P_1 = \frac{RT}{v_1} = \frac{2.077 \cdot 313}{6.5} = 100 \text{ kPa}$.

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 313 \left(\frac{1000}{100} \right)^{\frac{1.667-1}{1.667}} = 787 \text{ K.}$$

At this moment we know sufficient property data to calculate the heat transfer and work.

Process 1-2:

$$q_{net,1-2} = 0.$$

$$w_{net,1-2} = c_v(T_2 - T_1) = 3.116(787 - 313) = 1477 \text{ kJ/kg.}$$

$$w_{net,2-3} = R(T_3 - T_2) = 2.077(873 - 787) = 179 \text{ kJ/kg.}$$

$$q_{net,2-3} = (c_v - R)(T_3 - T_2) = (3.116 - 2.077)(873 - 787) = 89 \text{ kJ/kg.}$$

5.25 Melting ice

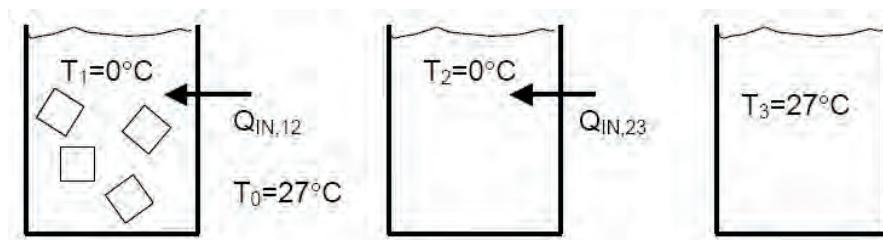


Figure 5.25.1: Situation of the different states for exercises 5.25

Given is: water, with $h_{ls} = 335 \text{ kJ/kg}$. At state 1 $\frac{m_l}{m} = 0.5$ and $\frac{m_s}{m} = 0.5$, where m is the total mass, m_l the mass of the liquid and m_s the mass of the ice (solid). The outside temperature is 27°C .

What happens here?

The bucket of ice reaching equilibrium with the surroundings is a process that takes place in two parts (1) melting of the ice at 0 degree Celsius (Process 1-2), and (2) heating of the water to equilibrium

with the environment at 27 degree Celsius (Process 2-3), see figure 5.25.1.

It is assumed that the water is at atmospheric pressure and water is an incompressible substance. At state 1 there is ice and water, at state 2 (the state between the two processes) the ice has just melted while at state 3 equilibrium with the surroundings is reached. As there is a mixture at state 1 and a saturated liquid at state 2 the temperature $T_1 = T_2 = 0^\circ\text{C}$.

In equilibrium the temperature is $T_3 = 27^\circ\text{C}$. (Note: $dh = Tds + vdp = Tds$ as $dp = 0$ and therefore $ds = dh/T$. This is used in the derivation in equation 5.25a).

5.25a For process 1-2 (melting of ice)

$$S_{gen,1-2} = S_2 - S_1 - \frac{Q_{in,1-2}}{T_0} \quad \text{with} \quad Q_{in,1-2} = m_s h_{ls} = \frac{m}{2} h_{ls}. \quad (5.25.0)$$

For melting ice, it can be approximated

$$S_2 - S_1 = m_s(s_2 - s_1) = m_s(s_l - s_s) = m_s s_{ls} = m_s \frac{h_{ls}}{T_{sat,s}} = \frac{m h_{ls}}{2 T_{sat,s}}. \quad (5.25.0)$$

Combining the two equations gives

$$S_{gen,1-2} = \frac{m h_{ls}}{2} \left(\frac{1}{T_{sat,s}} - \frac{1}{T_0} \right). \quad (5.25.0)$$

For process 2-3 (heating of water)

$$S_{gen,2-3} = S_3 - S_2 - \frac{Q_{in,2-3}}{T_0} \quad \text{with} \quad Q_{in,2-3} = m(u_3 - u_2) = mc_{p,w}(T_3 - T_2). \quad (5.25.0)$$

In the calculation of $Q_{in,2-3}$ the first law is used, $\Delta u = u_3 - u_2 = q_{in,2-3} + w_{2-3}$. $W_{2,3}$ is zero as no work is performed on the bucket. Approximating water to be an incompressible liquid (for incompressible liquid $s_a - s_b = c \ln \frac{T_b}{T_a}$)

$$S_3 - S_2 = m(s_3 - s_2) = mc_{p,w} \ln \frac{T_3}{T_2}. \quad (5.25.0)$$

Adding process 1-2 and 2-3,

$$S_{gen,1-3} = m \left[\frac{h_{ls}}{2} \left(\frac{1}{T_{sat,s}} - \frac{1}{T_0} \right) + c_{p,w} \ln \frac{T_3}{T_2} - \frac{c_{p,w}(T_3 - T_2)}{T_0} \right] = 3 \text{ kJ/K}. \quad (5.25.0)$$

$$m = S_{gen,1-3} \left[\frac{h_{ls}}{2} \left(\frac{1}{T_{sat,s}} - \frac{1}{T_0} \right) + c_{p,w} \ln \frac{T_3}{T_2} - \frac{c_{p,w}(T_3 - T_2)}{T_0} \right]^{-1}. \quad (5.25.0)$$

$c_{p,w} \approx 1.0 \text{ kJ/kgK}$ for temperatures between 0 and 27 degree Celsius (table 2),

$$m = 3 \left[\frac{335}{2} \left(\frac{1}{273} - \frac{1}{300} \right) + 1.0 \ln \frac{300}{273} - \frac{1.0(300 - 272)}{300} \right]^{-1} \rightarrow m = 50.33 \text{ kg}. \quad (5.25.0)$$

5.25b From the $S_{gen,1-3}$ equation above, it can easily be seen that if $T_0 = T_{sat,s} = 0^\circ\text{C}$, the melting term goes to zero. Furthermore, if $T_0 = T_{sat,s} = 0^\circ\text{C}$, there is no heating of the water after melting (i.e., $T_3 = T_2 = T_1 = 0^\circ\text{C}$), and therefore the two heating terms both are also zero. Hence, $S_{gen,1-3} = 0$ if $T_0 = T_{sat,s}$.

The result shown in part (b) shows that the only way S_{gen} can be 0 is if the system is always in equilibrium with the environment, i.e., if the heat transfer process is reversible. Of course, this is impossible to achieve in reality, since a temperature difference is required for heat transfer to actually occur, but it is a useful conceptual device as a limiting case.

5.26 Adiabatic steady flow device

Given: Air in an adiabatic ($Q = 0$) steady flow device.

State A: $P_A = 1 \text{ MPa}$, $T_A = 200^\circ\text{C}$.

State B: $P_B = 0.01 \text{ MPa}$, $T_B = 30^\circ\text{C}$.

Air is assumed to be an ideal gas. There is work done (no heat transfer), but its direction is unknown and the direction of the air flowing through the device has to be determined.

It is assumed that the flow direction is from $A \rightarrow B$ and the second law is applied to confirm or reject this assumption.

$dh = Tds + vdP \rightarrow ds = \frac{dh}{T} - \frac{vdP}{T}$. For an ideal gas $dh = c_p dT \rightarrow ds = c_p \frac{dT}{T} - R \frac{dP}{P}$. Integration from $A \rightarrow B$ yields $s_{gen} = s_B - s_A = c_p \ln \frac{T_B}{T_A} - R \ln \frac{P_B}{P_A}$.

For air $R = 0.287 \text{ kJ/kgK}$ (table 1) and $c_p = 1.01 \text{ kJ/kgK}$ (table 2).

Filling in the numbers results in $s_{gen} = 1.01 \ln \frac{303}{473} - 0.287 \ln \frac{0.01}{1} = 0.87 \text{ kJ/kgK}$.

Since the value of the entropy generation is positive, it means that the initial assumption of the flow direction is correct. The direction of the work must be out of the system, since $h_A > h_B$ and the flow is from A to B .

5.27 Throttling valve

Given: Incompressible flow through throttling valve, $P_1 > P_2$.

Assumed is that a throttling valve is an adiabatic device that doesn't produce work.

The first law for a steady flow steady state with neglectable kinetic and potential energy is: $\dot{Q}_{in} - \dot{W}_{out} = \dot{m}(h_2 - h_1)$. As there is no work and no heat transfer this reduces to $h_1 = h_2$ for a throttling valve.

5.27a $dh = Tds + vdP \rightarrow Tds = -vdP$. Integrating both sides gives $\int Tds = \int vdP \rightarrow \int Tds = v(P_1 - P_2)$ (v can be taken out of the integration as it is a constant for an incompressible liquid). $P_1 > P_2 \rightarrow (P_1 - P_2) > 0 \rightarrow Tds > 0 \rightarrow ds > 0$. Therefore the entropy increases in a throttling process.

5.27b $du = Tds - Pdv \rightarrow du = Tds \rightarrow ds = \frac{du}{T} \rightarrow ds = c \frac{dT}{T}$. (Note for an incompressible liquid $c_v = c_p = c$ and $du = cdT$).

$$s_{gen} = s_2 - s_1 = \int ds = \int c \frac{dT}{T} = c \ln \frac{T_2}{T_1}.$$

Next step is to eliminate T_2 . $h = u + Pv \rightarrow dh = du + d(Pv) = du + pdv + vdP = du + vdP = 0 \rightarrow du = -vdP$. As $du = cdT \rightarrow cdT = -vdP$. Integration gives $C(T_2 - T_1) = v(P_1 - P_2) \rightarrow T_2 = \frac{v}{C}(P_1 - P_2) + T_1$.

Combining the expression for s_{gen} and T_2 gives: $s_{gen} = c \ln \left[1 + \frac{v}{T_1 C} (P_1 - P_2) \right]$.

5.28 Stainless steel ball bearings

Assumptions:

The thermal properties of the bearing balls are constant.

The kinetic and potential energy changes of the balls are negligible.

The balls are at a uniform temperature at the end of the process.

Properties:

The density and specific heat of the ball bearings are given to be $\rho = 8085 \text{ kg/m}^3$ and $c_p = 0.480 \text{ kJ/kg}^\circ\text{C}$.

5.28a We take a single bearing ball as the system. The energy balance for this closed system can be expressed as:

$$E_{in} - E_{out} = \Delta E_{system}$$

(The net energy transfer by heat, work and mass equals the change in internal, kinetic, potential, etc. energies.)

$$-Q_{out} = \Delta U_{ball} = m(u_2 - u_1)$$

$$Q_{out} = mc(T_1 - T_2)$$

The total amount of heat transfer from a ball is:

$$m = \rho V = \rho \frac{\pi D^3}{6} = (8085 \text{ kg/m}^3) \frac{\pi (0.018m)^3}{6} = 0.02469 \text{ kg}$$

$$Q_{out} = mc(T_1 - T_2) = (0.02469 \text{ kg})(0.480 \text{ kJ/kg}^\circ\text{C})(900 - 850)^\circ\text{C} = 0.5925 \text{ kJ/ball}$$

Then the rate of heat transfer from the balls to the air becomes the production rate times the heat transfer per ball:

$$\dot{Q}_{total} = \dot{n}_{ball} * Q_{out(\text{per ball})} = (1100 \text{ balls/min}) * (0.5925 \text{ kJ/ball}) = 651.8 \text{ kJ/min} = 10.86 \text{ kW}$$

Therefore, heat is lost to the air at a rate of 10.86 kW.

5.28b We again take a single bearing ball as the system. The entropy generated during this process can be determined by applying an entropy balance on an extended system that includes the ball and its immediate surroundings so that the boundary temperature (T_b) of the extended system is at 20°C at all times:

$$S_{in} - S_{out} + S_{gen} = \Delta S_{system}$$

(The net entropy transfer by heat and mass plus the entropy generation equals the change in entropy.)

$$-\frac{Q_{out}}{T_b} + S_{gen} = \Delta S_{system} \rightarrow S_{gen} = \frac{Q_{out}}{T_b} + \Delta S_{system}$$

Where

$$\begin{aligned} \Delta S_{system} &= m(s_2 - s_1) = mc_{avg} \ln\left(\frac{T_2}{T_1}\right) \\ &= (0.02469 \text{ kg})(0.480 \text{ kJ/kg.K}) \ln\left(\frac{850 + 273}{900 + 273}\right) = -0.0005162 \text{ kJ/K} \end{aligned}$$

Substituting, we find the entropy generation per ball:

$$S_{gen} = \frac{Q_{out}}{T_b} + \Delta S_{system} = \frac{0.5925 \text{ kJ}}{293 \text{ K}} - 0.0005162 \text{ kJ/K} = 0.001506 \text{ kJ/K.ball}$$

Then the rate of entropy generation becomes the production rate times the entropy generation per ball.

$$\dot{S}_{gen} = \dot{n}_{ball} S_{gen} = (1100 \text{ balls/min}) * (0.001506 \text{ kJ/K.ball}) = 1.657 \text{ kJ/min.K} = 0.02761 \text{ kW/K}$$

5.29 Egg

Assumptions:

The thermal properties of the egg are constant.

There are no changes in kinetic and potential energies.

Energy absorption or release associated with any chemical and/or phase changes within the egg is negligible.

5.29a We take the egg as the system. This is a closed system since no mass enters or leaves the egg. The energy balance for this closed system can be expressed as:

$$E_{in} - E_{out} = \Delta E_{system}$$

(The net energy transfer by heat, work and mass equals the change in internal, kinetic, potential, etc. energies.)

$$Q_{in} = \Delta U_{egg} = m(u_2 - u_1) = mc(T_2 - T_1)$$

Then the mass of the egg and the amount of heat transfer become

$$m = \rho V = \rho \frac{\pi D^3}{6} = (1020 \text{ kg/m}^3) \frac{\pi (0.055\text{m})^3}{6} = 0.0889 \text{ kg}$$

$$Q_{in} = mc(T_2 - T_1) = (0.0889 \text{ kg})(3.32 \text{ kJ/kg°C})(70 - 8)^\circ\text{C} = 18.3 \text{ kJ}$$

5.29b We again take a single egg as the system. The entropy generated during this process can be determined by applying an entropy balance on an extended system that includes the ball and its immediate surroundings so that the boundary temperature (T_b) of the extended system is at 97°C at all times:

$$S_{in} - S_{out} + S_{gen} = \Delta S_{system}$$

(The net entropy transfer by heat and mass plus the entropy generation equals the change in entropy.)

$$\frac{Q_{in}}{T_b} + S_{gen} = \Delta S_{system} \rightarrow S_{gen} = -\frac{Q_{in}}{T_b} + \Delta S_{system}$$

Where

$$\Delta S_{system} = m(s_2 - s_1) = mc_{avg} \ln\left(\frac{T_2}{T_1}\right)$$

$$= (0.0889 \text{ kg})(3.32 \text{ kJ/kg.K}) \ln\left(\frac{70 + 273}{8 + 273}\right) = 0.0588 \text{ kJ/K}$$

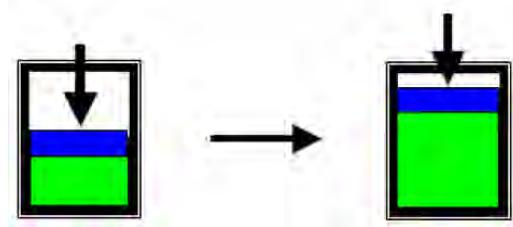
Substituting, we find the entropy generation:

$$S_{gen} = -\frac{Q_{in}}{T_b} + \Delta S_{system} = -\frac{18.3 \text{ kJ}}{370 \text{ K}} + 0.0588 \text{ kJ/K} = 0.00934 \text{ kJ/K}$$

6 Second Law Applications

6.1 Neon in an insulated container

A gas is initially in state 1 given by P_1 and T_1 . Work is done until state 2 is reached, which is given by P_2 and T_2 (see image on the right). The two states are represented by state A ($P_A = 1 \text{ atm}$, $T_A = 25^\circ\text{C}$) and state B ($P_B = 2 \text{ atm}$, $T_B = 139.7^\circ\text{C}$). It is asked which state, A or B, is the final state.



In order to find the initial state, we use the Gibbs equation:

$$Tds = dh - v dP$$

Since the container is insulated, we assume that the system is adiabatic. Furthermore, we assume neon to be an ideal gas. For an ideal gas, it holds that:

$$dh = c_P dT$$

and

$$v = \frac{RT}{P}$$

Substituting these relations in the Gibbs equation yields:

$$ds = \frac{c_P}{T} dh - \frac{R}{P} dP$$

In table 2, we find that $R = 0.412 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$. For a monatomic ideal gas, c_P is constant and equals $c_P = \frac{5}{2}R$. Hence, we can integrate the relation to obtain the change from initial to end state:

$$s_2 - s_1 = c_P \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

Since we have an adiabatic system, it must hold that $s_2 - s_1 \geq 0$. We will now try if this holds if state B as the initial state (and state A as the final state):

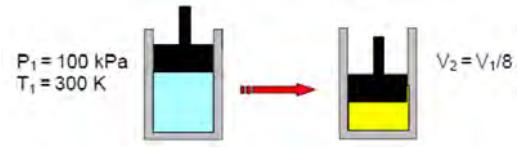
$$\begin{aligned} s_2 - s_1 &= c_P \ln \left(\frac{T_A}{T_B} \right) - R \ln \left(\frac{P_A}{P_B} \right) \\ &= \frac{5}{2} \cdot 0.412 \ln \left(\frac{25 + 273}{139.7 + 273} \right) - 0.412 \ln \left(\frac{1}{2} \right) \\ &= -0.05 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{aligned}$$

Since $s_2 - s_1 < 0$, our guess was incorrect and hence state A is the initial state. (So $P_1 = P_A = 1 \text{ atm}$ and $T_1 = T_A = 25^\circ\text{C}$).

Note that the entropy in a closed system can only be changed in two ways: irreversibility (which always causes entropy to increase) and heat transfer (into the system causes entropy to increase, while out of the system causes entropy to decrease). Therefore, if the system is adiabatic (= no heat transfer), the entropy change must be ≥ 0 .

6.2 Air in the cylinder of an automobile engine

Given is air in a cylinder at $T_1 = 300 \text{ K}$ and $P_1 = 100 \text{ kPa}$. The air is compressed adiabatically to one-eighth of the original volume. See also the image. Specific heats of air are assumed constant for this process.



6.2a An adiabatic and reversible process is also isentropic, thus the following relation is valid:

$$P_1 v_1^k = P_2 v_2^k \Rightarrow P_2 = P_1 \left(\frac{v_1}{v_2} \right)^k$$

From the ideal gas table for air (table 5s) we find that $k = 1.4$ at $T = 300 \text{ K}$. P_2 can now be calculated to be:

$$P_2 = P_1 \left(\frac{v_1}{v_2} \right)^k = 100 \cdot \left(\frac{8}{1} \right)^{1.4} = 1837.92 \text{ kPa}$$

We now calculate the final temperature T_2 using the given volumetric ratio and the ideal gas law ($Pv = RT$):

$$\frac{P_1 v_1}{T_1} = R = \frac{P_2 v_2}{T_2} \Rightarrow T_2 = T_1 \cdot \frac{P_2}{P_1} \cdot \frac{v_2}{v_1} = 300 \cdot \frac{1837.92}{100} \cdot \frac{1}{8} = 689.22 \text{ K}$$

The work of compression is calculated using the governing relation:

$$\begin{aligned} w_{in,1-2} &= - \int_{v_1}^{v_2} P dv = - \int_{v_1}^{v_2} P_1 \frac{v_1^k}{v^k} dv = -P_1 v_1^k \int_{v_1}^{v_2} \frac{1}{v^k} dv = -P_1 v_1^k \left[\frac{1}{1-k} \cdot \frac{1}{v^{k-1}} \right]_{v_1}^{v_2} \\ &= \frac{-P_1 v_1^k}{1-k} \left(\frac{1}{v_2^{k-1}} - \frac{1}{v_1^{k-1}} \right) = \frac{-P_1 v_1^k}{1-k} (v_2^{1-k} - v_1^{1-k}) \\ &= \frac{1}{k-1} (P_2 v_2 v_2^{1-k} - P_1 v_1 v_1^{1-k}) = \frac{P_2 v_2 - P_1 v_1}{k-1} \\ &= \frac{R(T_2 - T_1)}{k-1} = \frac{0.287 \cdot (689.22 - 300)}{1.4 - 1} = 279.27 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Note that, to get from the second line to the third line, we use the isentropic relation $P_1 v_1^k = P_2 v_2^k$, while we use the ideal gas law ($Pv = RT$) to get from the third to the fourth line. In addition, the denominator is switching from $1 - k$ to $k - 1$ by taking the $-$, that was in front of the term until then, into it.

6.2b If the compression occurs along an adiabatic path defined by $Pv^{1.5} = \text{constant}$, we can calculate the final pressure in the same way as in a, but with k replaced by 1.5:

$$P_1 v_1^n = P_2 v_2^n \Rightarrow P_2 = P_1 \left(\frac{v_1}{v_2} \right)^n = 100 \cdot \left(\frac{8}{1} \right)^{1.5} = 2262.7 \text{ kPa}$$

The final temperature can be calculated using the ideal gas law:

$$\frac{P_1 v_1}{T_1} = R = \frac{P_2 v_2}{T_2} \Rightarrow T_2 = T_1 \cdot \frac{P_2}{P_1} \cdot \frac{v_2}{v_1} = 300 \cdot \frac{2262.7}{100} \cdot \frac{1}{8} = 848.51 \text{ K}$$

The same derivation for work of compression can be performed, giving:

$$w_{in,1-2} = \frac{R(T_2 - T_1)}{n - 1} = \frac{0.287 \cdot (848.51 - 300)}{1.5 - 1} = 314.8 \frac{\text{kJ}}{\text{kg}}$$

It is seen that $w_{in,actual} > w_{in,isentropic}$, so the process is irreversible.

The compression efficiency is the ratio of the work of compression for an isentropic process to the work of the actual process. Based on the calculations, the compression efficiency is:

$$\eta_c = \frac{w_{in,isentropic}}{w_{in,actual}} = \frac{279.27}{314.8} = 0.887$$

Note: The isentropic efficiency of a device is used to relate the actual performance to the theoretical performance. As this problem illustrates, another way to characterize actual performance is to specify the polytropic exponent, n , that represents the actual process path, in comparison with the exponent k which characterizes an adiabatic and reversible path. The polytropic exponent is often used in place of the isentropic efficiency in reciprocating machines, such as car engines, while the isentropic efficiency is the preferred parameter for turbomachines such as jet engines.

6.3 Isentropic efficiencies of steady flow devices

6.3a In an ideal compressor or turbine, the entropy will not increase during compression or expansion. So $s_1 = s_2$ and in an $h-s$ or $T-s$ diagram, this is seen as a vertical line. See figures 6.3.1 and 6.3.2 for the $h-s$ diagrams of a compressor and a turbine, respectively.

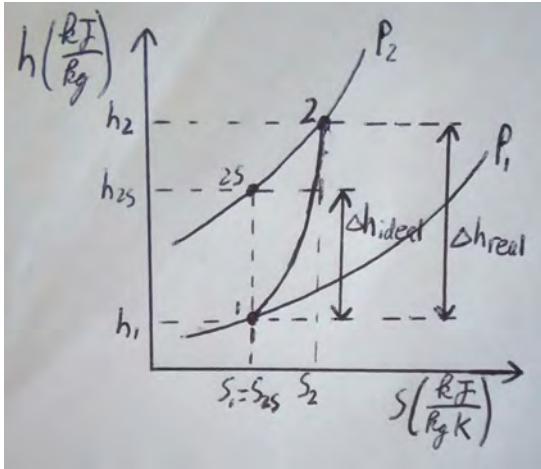


Figure 6.3.1: $h-s$ diagram of a compressor

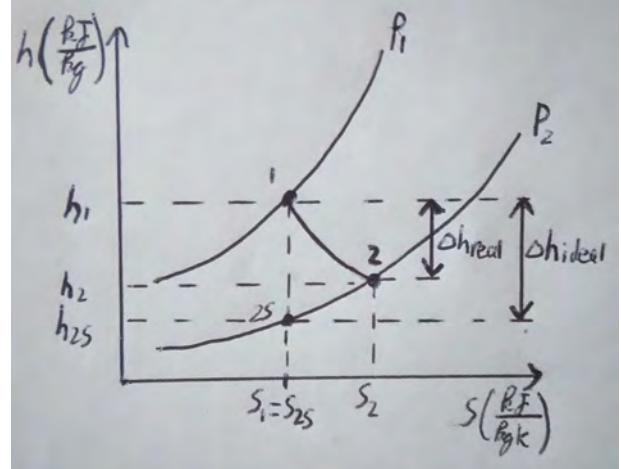


Figure 6.3.2: $h-s$ diagram of a turbine

$$\eta_{s,comp} = \frac{\Delta h_{ideal}}{\Delta h_{real}}$$

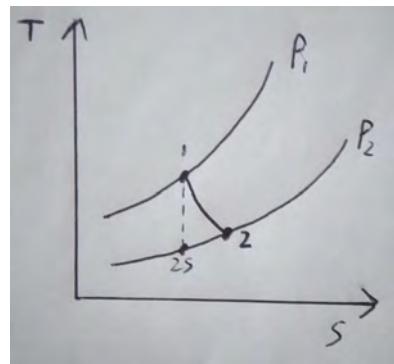
$$\eta_{s,comp} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\eta_{s,turb} = \frac{\Delta h_{real}}{\Delta h_{ideal}}$$

$$\eta_{s,turb} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Note that the isentropic efficiency can never be larger than 1, and note that the equations for the isentropic efficiencies are "reversed" because of the reversed function of the device.

6.3b Yes, because there is an increase in entropy, also for an adiabatic turbine. $\Delta s \geq \frac{Q}{T}$, so even if Q is 0, the entropy change will be larger than (or equal to) 0. In the case of $\Delta s = 0$, the process is ideal (and reversible). See figure 6.3.3 for a sketch of a $T-s$ diagram of an adiabatic turbine.



6.4 Adiabatic steam turbine 1

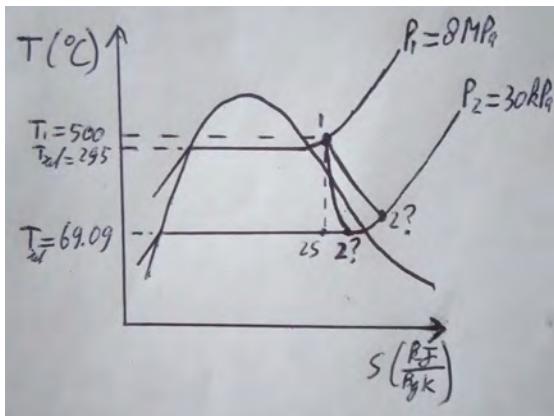
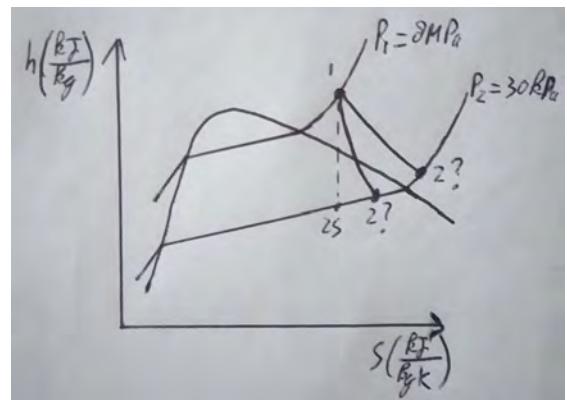
6.4a See figures 6.4.1 and 6.4.2.

6.4b At first, it needs to be determined in which region the outlet is. This is done using s . From table A-6, it is found that $s_1 = 6.7266 \text{ kJ/(kgK)}$. For an ideal process, $s_1 = s_{2s}$. At $P_2 = 30 \text{ kPa}$, $s_f = 0.9441 \text{ kJ/(kgK)}$ and $s_g = 7.7675 \text{ kJ/(kgK)}$ (from table A-5). Since $s_f < s_{2s} < s_g$, the ideal isentropic outlet point is in the mixture region. The vapour mass fraction is determined using the lever rule:

$$s_{2s} = s_f + x(s_g - s_f) \Rightarrow x = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.7266 - 0.9441}{7.7675 - 0.9441} = 0.85$$

Using the value of x and the values for h_f and h_g from the tables, we can also determine h_{2s} :

$$h_{2s} = h_f + x(h_g - h_f) = 289.27 + 0.85(2624.6 - 289.27) = 2273.935 \text{ kJ/kg}$$

Figure 6.4.1: T - s diagram of exercise 6.4Figure 6.4.2: h - s diagram of exercise 6.4

Now, we use the definition of the isentropic efficiency to determine h_2 :

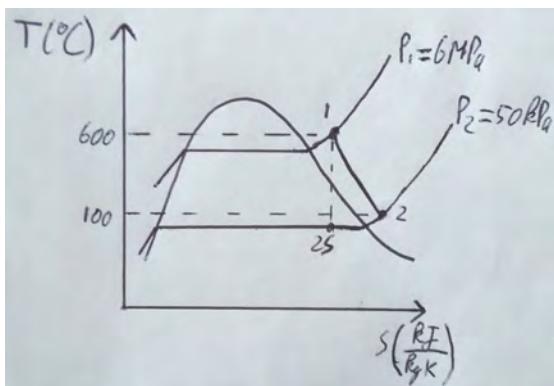
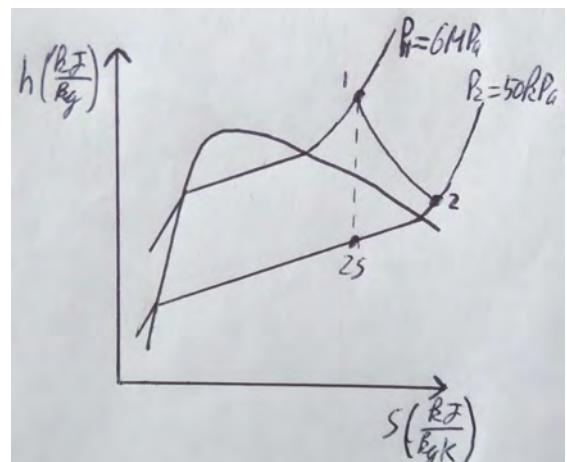
$$h_2 = h_1 - \eta_s(h_1 - h_{2s}) = 3399.5 - 0.9(3399.5 - 2273.94) = 2386.5 \text{ kJ/kg}$$

Since $h_f < h_2 < h_g$, we are still in the mixture region, so $T_{out} = T_{sat@30\text{kPa}} = 69.09^\circ\text{C}$.

6.4c The power output is given by $\dot{W} = \dot{m}(h_1 - h_2)$ (neglect potential and kinetic energy, and the turbine is adiabatic). So: $\dot{W} = 3(3399.5 - 2386.5) = 3039.0 \text{ kJ/s} = 3039.0 \text{ kW}$.

6.5 Adiabatic steam turbine 2

6.5a See figures 6.5.1 and 6.5.2.

Figure 6.5.1: T - s diagram of exercise 6.5Figure 6.5.2: h - s diagram of exercise 6.5

6.5b Use conservation of energy: $\dot{m}(h_1 + \frac{1}{2}v_1^2 + gz_1) = \dot{m}(h_2 + \frac{1}{2}v_2^2 + gz_2) + \dot{W} + \dot{Q}$. The turbine is adiabatic, so $\dot{Q} = 0$, and we neglect potential energy, such that the mass flow becomes:

$$\dot{m} = \frac{\dot{W}}{(h_1 - h_2) + \frac{1}{2}(v_1^2 - v_2^2)} = \frac{5 \cdot 10^6}{(3658.8 \cdot 10^3 - 2682.4 \cdot 10^3) + \frac{1}{2}(80^2 - 140^2)} = 5.16 \text{ kg/s}$$

Note that, since the velocity is used in m/s, that both h and \dot{W} must be filled in in J/kg and J/s and not in kJ/kg and kJ/s.

6.5c The isentropic efficiency is given as: $\eta_s = \frac{h_1 - h_2}{h_1 - h_{2s}}$. At a pressure of 50 kPa, $s_f = 1.0912 \text{ kJ/(kgK)}$ and $s_g = 7.5931 \text{ kJ/(kgK)}$. Since $s_f < s_{2s} < s_g$ and $s_{2s} = s_1$, we know that the ideal point is in the mixture phase, so $h_{2s} = h_f + x(h_g - h_f)$. From the tables at 50 kPa, we find that $h_f = 340.54 \text{ kJ/kg}$ and $h_g = 2645.2 \text{ kJ/kg}$. We determine x from the entropy:

$$x = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.1693 - 1.0912}{7.5931 - 1.0912} = 0.93$$

This leads to:

$$h_{2s} = h_f + x(h_g - h_f) = 340.54 + 0.93(2645.2 - 340.54) = 2483.87 \text{ kJ/kg}$$

Looking up the values for h_1 and h_2 in the tables then allows to calculate the efficiency:

$$\eta_s = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{3658.8 - 2682.4}{3658.8 - 2483.9} = 0.83$$

6.6 Refrigerant compressor

6.6a See figures 6.6.1 and 6.6.2.

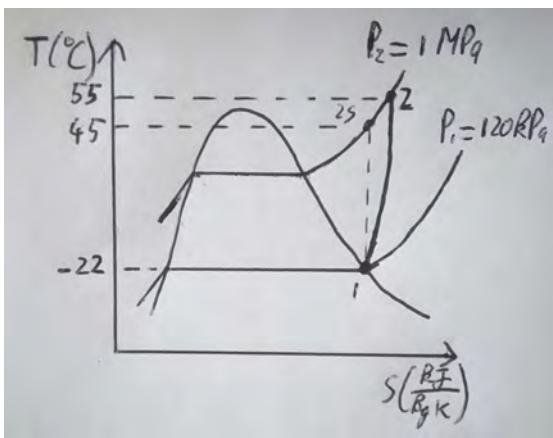


Figure 6.6.1: T - s diagram of exercise 6.6

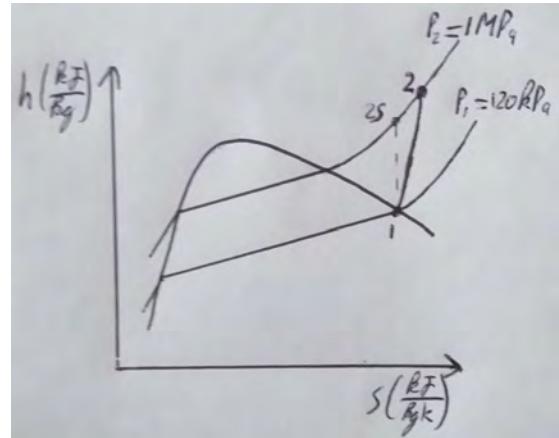


Figure 6.6.2: h - s diagram of exercise 6.6

6.6b To determine the temperature at the exit, we first use the isentropic efficiency to determine the enthalpy at the exit: $\eta_s = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = \frac{h_{2s} - h_1}{\eta_s} + h_1$. So, we need to determine h_{2s} . Ideal, so $s_{2s} = s_1 = s_g@120\text{kPa} = 0.9345 \text{ kJ/(kgK)}$. Then we go to table A-13 for superheated refrigerant 134A, and we search which enthalpy belongs to an entropy of $s_{2s} = 0.9345 \text{ kJ/(kgK)}$ at $P_2 = 1 \text{ MPa}$. Interpolating gives $h_{2s} = 277 \text{ kJ/kg}$ (and $T_{2s} \approx 45^\circ\text{C}$). So, $h_2 = \frac{h_{2s} - h_1}{\eta_s} + h_1 = \frac{277 - 237}{0.8} + 237 = 287 \text{ kJ/kg}$. In table A-13, we then search for the temperature belonging to this enthalpy and pressure: $T_2 = 55^\circ\text{C}$.

6.6c Use conservation of energy with $\dot{Q} = 0$ (adiabatic) and neglecting kinetic and potential energy: $\dot{W} = \dot{m}(h_2 - h_1)$. We calculate \dot{m} using the volume flow and the specific volume: $\dot{m} = \frac{\dot{v}}{v_g@120\text{kPa}}$. This results in:

$$\dot{W} = \dot{m}(h_2 - h_1) = \frac{\dot{v}}{v_g@120\text{kPa}} (h_2 - h_1) = \frac{0.3/60}{0.16212} (287 - 237) = 1.54 \text{ kW}$$

6.7 Turbine curve in Mollier diagram

The pressure and temperature at the inlet are given, such that we can look up the enthalpy at the inlet from the Mollier diagram: $h_1 = 3440 \text{ kJ/kg}$. The enthalpy at the exit follows from the isentropic efficiency:

$$\eta_{s,t} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{s,t}(h_1 - h_{2s})$$

At all outlet pressures, the ideal enthalpy h_{2s} is determined by drawing a vertical line ($s_{2s} = s_1$) from point 1 to the desired outlet pressure. The results are given in the table below. The Mollier diagram is shown in figure 6.7.1. It can clearly be seen that the turbine process is not a linear line, but has a curve in it.

$P_2 \text{ (bar)}$	$h_{2s} \text{ (kJ/kg)}$	$h_2 \text{ (kJ/kg)}$
30	3285	3316
10	2980	3072
5	2830	2952
2	2660	2816
1	2540	2720
0.1	2220	2464

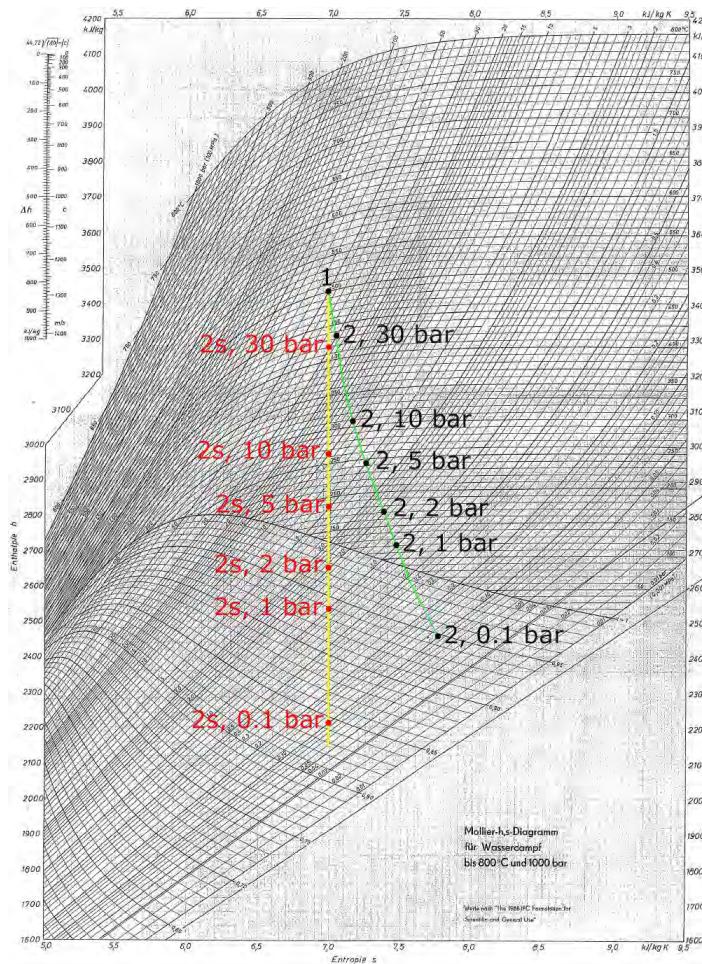
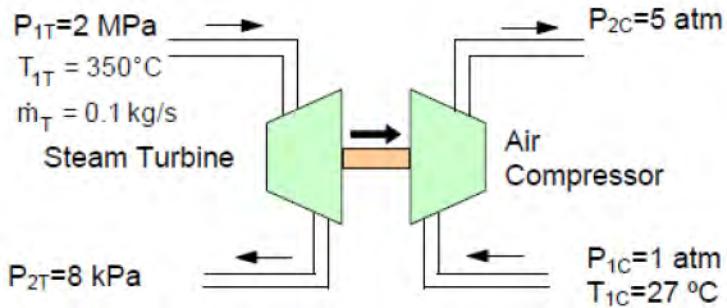


Figure 6.7.1: Mollier diagram diagram of exercise 6.7. It is clearly visible that the turbine process is described a curved line.

6.8 Steam turbine coupled to an air compressor

Given is a steam turbine with an efficiency of $\eta_{s,T} = 72\%$ that is driving an air compressor with an efficiency of $\eta_{s,C} = 80\%$. The pressures and temperatures are given in the figure on the right. We want to know the mass flow rate of air (in kg/h) that can be compressed.



We assume that both the turbine and the compressor are steady state steady flow processes, and we neglect any differences in kinetic and potential energy. Furthermore, we assume that both the turbine and the compressor are adiabatic and that air is an ideal gas with constant specific heats.

First of all, we make a table in which the known properties are underlined.

	P [kPa]	T [°C]	h [kJ/kg]	s [kJ/(kgK)]	other	\dot{m} [kg/s]
1T	<u>2000</u>	<u>350</u>	3136.6	6.9556		0.1
2Ts	<u>8</u>		2176.2	<u>$s_{2sT} = s_{1T}$</u>	$x = 0.8335$	0.1
2T	<u>8</u>		2445.1		<u>$\eta_{s,t} = 0.72$</u>	0.1
1C	<u>101.325</u>	<u>27</u>				
2Cs	<u>506.625</u>			<u>$s_{2sC} = s_{1C}$</u>		
2C	<u>506.625</u>				<u>$\eta_{s,c} = 0.80$</u>	

In order to calculate the mass flow rate of air that can be compressed, we first need to know the power that is delivered to the compressor. Hence, we first have to analyse the steam turbine. Using $P_{1T} = 2000 \text{ kPa}$ and $T_{1T} = 350^\circ\text{C}$, we find from the tables (or with the Mollier diagram for water) that $h_{1T} = 3136.6 \text{ kJ/kg}$ and $s_{1T} = 6.9556 \text{ kJ/(kgK)}$. With the entropy, we can calculate the vapour mass fraction for the isentropic case:

$$x_{2Ts} = \frac{s_{2Ts} - s_L}{s_V - s_L} = \frac{6.9556 - 0.5925}{8.2267 - 0.5925} = 0.8335$$

where s_L and s_V represent the entropy of saturated liquid and saturated vapour, respectively. The values of s_L and s_V are obtained from the tables for saturated water at $P_{2Ts} = 8 \text{ kPa}$. Using the vapour mass fraction, we can determine the output enthalpy of the isentropic case:

$$h_{2Ts} = h_L + x_{2Ts} (h_V - h_L) = 173.85 + 0.8335 \cdot (2576.1 - 173.85) = 2176.2 \text{ kJ/kg}$$

where h_L and h_V represent the enthalpy of saturated liquid and saturated vapour, respectively. From the isentropic efficiency, we can now calculate the actual enthalpy at the outlet of the turbine:

$$\eta_{s,T} = \frac{h_{1T} - h_{2T}}{h_{1T} - h_{2Ts}} \Rightarrow h_{2T} = h_{1T} - \eta_{s,t} \cdot (h_{1T} - h_{2Ts}) = 3136.6 - 0.72 \cdot (3136.6 - 2176.2) = 2445.1 \text{ kJ/kg}$$

(Note that the value for h_{2Ts} can also (quicker) be determined from the Mollier diagram by looking up point 1T and going down vertically (isentropic) to a pressure of 8 kPa. The values found may

differ slightly from the ones obtained from the table.) Now, we can calculate the power output of the turbine:

$$\dot{W}_{out,T} = \dot{m}_T (h_{1T} - h_{2T}) = 0.1 \cdot (3136.6 - 2445.1) = 69.15 \text{ kW}$$

The power input of the compressor should be equal to the power output of the turbine, so:

$$\dot{W}_{in,C} = \dot{W}_{out,T} = \frac{\dot{m}_C (h_{2Cs} - h_{1C})}{\eta_C}$$

For ideal gases, it holds that $dh = c_p dT$, so:

$$h_{2Cs} - h_{1C} = c_p (T_{2Cs} - T_{1C})$$

Hence, we can express the mass flow rate of air through the compressor as:

$$\dot{m}_C = \frac{\eta_C \dot{W}_{in,C}}{c_p (T_{2Cs} - T_{1C})}$$

For isentropic processes of an ideal gas, the following holds:

$$\frac{P^{\frac{k-1}{k}}}{T} = \text{constant} \Rightarrow T_{2Cs} = T_{1C} \left(\frac{P_{2C}}{P_{1C}} \right)^{\frac{k-1}{k}} = (273 + 27) \left(\frac{5}{1} \right)^{\frac{1.4-1}{1}} = 475.15 \text{ K}$$

Using a constant value of $c_p = 1.005 \text{ kJ/(kgK)}$, we can calculate the mass flow of air that can be compressed:

$$\dot{m}_C = \frac{\eta_C \dot{W}_{in,C}}{c_p (T_{2Cs} - T_{1C})} = \frac{0.80 \cdot 69.15}{1.005 \cdot (475.15 - 300)} = 0.314 \text{ kg/s} = 1131.4 \text{ kg/h}$$

Note: This is a good example of a "systems" problem, in which multiple components interact with each other and the performance of one component (in this case the compressor) is related to the performance of another (the turbine). The assumption of constant specific heat for the air compressor analysis gives reasonably accurate results in this case, since the temperature range over which the compressor operates is small. More accurate results could be obtained by using functions that do account for variable specific heat.

6.9 Two stage compressor

Given is a two-stage air compressor that is compressing air at a rate of 2 kg/s. The conditions of the process are shown in figure 6.9.1. The first stage is adiabatic, the second isothermal. The power to drive the compressor is 650 kW.

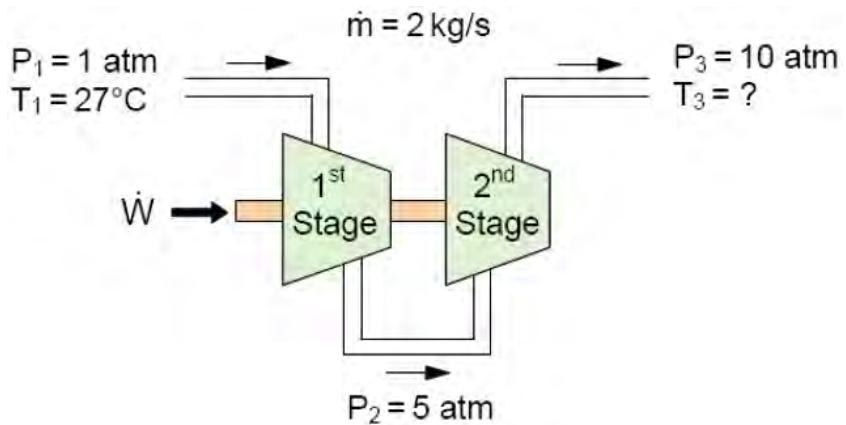


Figure 6.9.1: Two stage compressor of exercise 6.9

6.9a We want to know T_3 if both stages of the compressor were reversible. It was given that the first stage is adiabatic, and an adiabatic reversible process is isentropic. In the Mollier diagram (see figure 6.9.2), we look up point 1 ($P_1 = 1 \text{ atm} \approx 1 \text{ bar}$, $T_1 = 27^\circ\text{C}$). By going up vertically ($s_{2s} = s_1$) to the line $P_2 = 5 \text{ atm}$ we find that $T_{2s} = 200^\circ\text{C}$. Since the second stage is isothermal, this means that in the reversible case $T_3 = T_{2s} = 200^\circ\text{C}$.

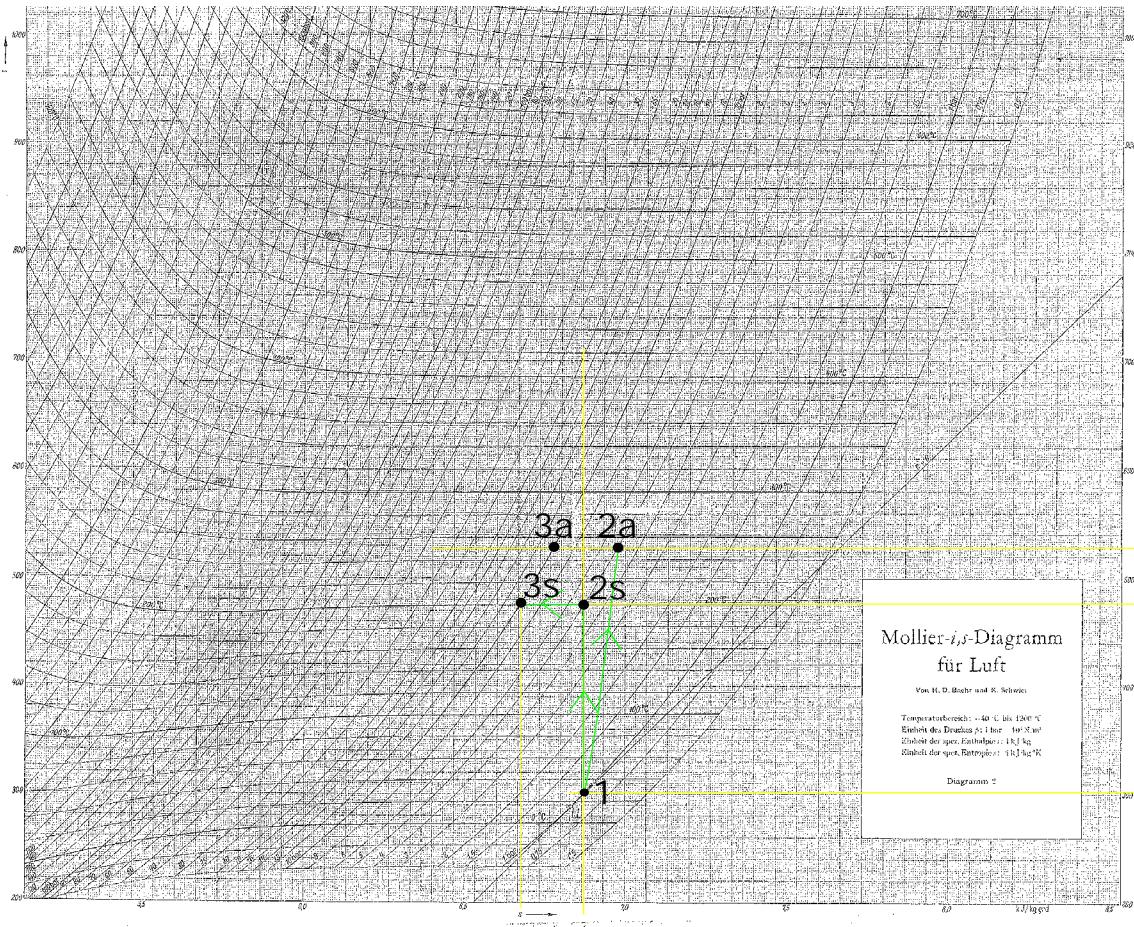


Figure 6.9.2: Mollier diagram of exercise 6.9

6.9b We now want to know the required power input for the reversible case. First of all, we construct the energy balance to determine how we can calculate the work for each stage. The energy balance for an open system is:

$$q_{in} + w_{in} + (h + ke + pe)_{in} = q_{out} + w_{out} + (h + ke + pe)_{out}$$

The first stage is adiabatic (so $q_{in} - q_{out} = 0$), and we neglect differences in kinetic and potential energy, such that the energy balance is reduced to:

$$w_{in,1-2} = h_2 - h_1$$

The second stage is not adiabatic, so we can only neglect differences in kinetic and potential energy:

$$w_{in,2-3} - q_{out,2-3} = h_3 - h_2$$

From the Mollier diagram, we read that $h_1 \approx 300$ kJ/kg and $h_{2s} \approx 477$ kJ/kg, such that

$$w_{in,1-2,rev} = h_{2s} - h_1 = 477 - 300 = 177 \text{ kJ/kg}$$

For the second stage, we use that

$$dh = c_p dT = 0$$

for an ideal gas in an isothermal process, and hence we find that $w_{in,2-3,rev} = q_{out,2-3,rev}$. The reversible heat transfer can be calculated as follows:

$$q_{out,2-3,rev} = -q_{in,2-3,rev} = - \int_{s_2}^{s_3} T ds = T_2 (s_2 - s_3)$$

where we used that $T_2 = T_3$ (isothermal). We can find the values of s_{2s} and s_{3s} in the Mollier diagram: $s_{2s} = 6.871 \text{ kJ/(kgK)}$ and $s_{3s} = 6.676 \text{ kJ/(kgK)}$. Therefore:

$$w_{in,2-3,rev} = q_{out,2-3,rev} = T_2(s_2 - s_3) = 473 \cdot (6.871 - 6.676) = 92.2 \text{ kJ/kg}$$

The total power input for the reversible compressor can now be calculated to equal:

$$\dot{W}_{in,rev} = \dot{m}(w_{in,1-2,rev} + w_{in,2-3,rev}) = 2 \cdot (177 + 92.2) = 538 \text{ kW}$$

6.9c The exit temperature of the first stage is given to be $T_2 = 250^\circ\text{C}$. In the Mollier diagram, we can now look up the final state of the first compression stage: $P_2 = 5 \text{ atm}$ and $T_2 = 250^\circ\text{C}$ gives $h_{2a} \approx 530 \text{ kJ/kg}$. The work done for the first stage is now calculated to be:

$$\dot{W}_{in,1-2,act} = \dot{m}(h_{2a} - h_1) = 2 \cdot (530 - 300) = 460 \text{ kW}$$

Thus, the actual power for the second stage is:

$$\dot{W}_{in,2-3,act} = \dot{W}_{in,1-3} - \dot{W}_{in,1-2,act} = 650 - 460 = 190 \text{ kW}$$

Note: Isothermal compression of a gas can only be carried out if there is heat removal simultaneously, as adiabatic compression will raise the gas temperature. Real compressors are, in fact, often cooled to reduce the required work of compression (i.e. the power input).

6.10 Nozzle

6.10a See figure 6.10.1 for a schematic overview. The isentropic efficiency of a nozzle is given as the real kinetic energy over the isentropic kinetic energy, so $\eta_s = \frac{v_2^2}{v_{2s}^2}$. We use conservation of energy: $\dot{m}(h_1 + \frac{1}{2}v_1^2 + gz_1) = \dot{m}(h_2 + \frac{1}{2}v_2^2 + gz_2) + \dot{W} + \dot{Q}$. The nozzle does not deliver or require work or heat, and we neglect potential energy, such that: $h_1 - h_2 = \frac{v_2^2 - v_1^2}{2}$. Using $dh = c_p dT$, this results in $\frac{v_2^2 - v_1^2}{2} = c_p(T_1 - T_2)$.

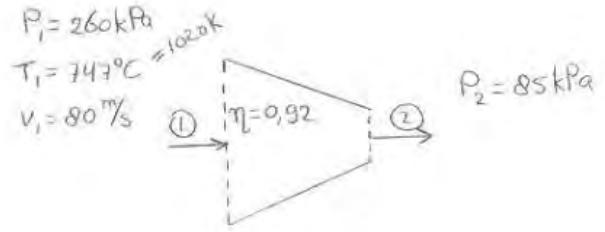


Figure 6.10.1: Schematic overview of exercise 6.10

For isentropic processes, it holds that $Pv^k = \text{constant}$, with $v = \frac{RT}{P}$ (note that v is in this case the specific volume and not the velocity). Rewriting this gives T_{2s} :

$$P \left(\frac{RT}{P} \right)^k = \text{constant} \Rightarrow P^{\frac{1-k}{k}} T = \text{constant} \Rightarrow P_1^{\frac{1-k}{k}} T_1 = P_{2s}^{\frac{1-k}{k}} T_{2s} \Rightarrow T_{2s} = \frac{P_1}{P_{2s}}^{\frac{1-k}{k}} T_1 = 766 \text{ K}$$

So, we can now also determine the isentropic outlet velocity:

$$\frac{v_{2s}^2 - v_1^2}{2} = c_p(T_1 - T_{2s}) \Rightarrow v_{2s} = \sqrt{2c_p(T_1 - T_{2s} + v_1^2)} = \sqrt{2 \cdot 1.121 \cdot 10^3 \cdot (1020 - 766) + 80^2} = 758.9 \text{ m/s}$$

Now, we can determine the real velocity from the definition of the isentropic efficiency:

$$\eta_s = \frac{v_2^2}{v_{2s}^2} \Rightarrow v_2 \sqrt{\eta_s} v_{2s} = \sqrt{0.92} \cdot 758.9 = 727.9 \text{ m/s}$$

6.10b The exit temperature is calculated as:

$$\frac{v_2^2 - v_1^2}{2} = c_p(T_1 - T_2) \Rightarrow T_2 = \frac{v_1^2 - v_2^2}{2c_p} + T_1 = \frac{80^2 - 727.9^2}{2 \cdot 1.121 \cdot 10^3} + 1020 = 786.5 \text{ K}$$

6.10c In the nozzle, we do not loose heat: $\dot{Q} = 0$.

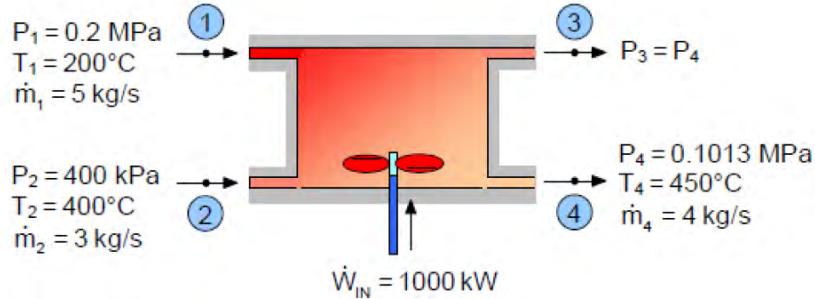
6.10d Gibbs equation: $dh = Tds + vdP \Rightarrow ds = \frac{1}{T}dh - \frac{v}{T}dP = \frac{c_p}{T}dT - \frac{R}{P}dP$. Integrating from state 1 to state 2 now gives the change in specific entropy, where we assume c_p to be constant with temperature:

$$\Delta s = \int_{T_1}^{T_2} \frac{c_p}{T} dT - \int_{P_1}^{P_2} \frac{R}{P} dP = c_p \ln\left(\frac{T_1}{T_2}\right) - R \ln\left(\frac{P_1}{P_2}\right) = 1.121 \ln\left(\frac{1020}{786.5}\right) - 0.2870 \ln\left(\frac{260}{85}\right) = 0.029 \frac{\text{kJ}}{\text{kgK}}$$

where the value for R is found in table A-2. To calculate the specific rate of entropy change \dot{s} , we need to multiply Δs with the mass flow \dot{m} . However, we do not know masses or areas, so we cannot calculate the mass flow.

6.11 Adiabatic device

Given is an adiabatic device using H_2O as medium. 2 streams are going in and 2 streams are going out of the device, as shown in the figure.



6.11a T_3 needs to be determined. In order to do so, we first want to know the mass flow. From conservation of mass, it follows that:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 + \dot{m}_4 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2 - \dot{m}_4 = 5 + 3 - 4 = 4 \text{ kg/s}$$

Next, we use the tables to determine the enthalpy at state 1, 2 and 4: $h_1 = 2870.0 \text{ kJ/kg}$, $h_2 = 3273.3 \text{ kJ/kg}$ and $h_4 = 3382.3 \text{ kJ/kg}$. The first law of thermodynamics for an open system (conservation of energy) gives:

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}_{in} (h + \text{ke} + \text{pe})_{in} = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}_{out} (h + \text{ke} + \text{pe})_{out}$$

Since the device is adiabatic, $\dot{Q}_{in} - \dot{Q}_{out} = 0$. Furthermore, differences in kinetic and potential energy can be neglected and the work output is 0. Hence, the equation for conservation of energy becomes:

$$\begin{aligned} W_{in} + \dot{m}_1 h_1 + \dot{m}_2 h_2 &= \dot{m}_3 h_3 + \dot{m}_4 h_4 \\ \Rightarrow h_3 &= \frac{\dot{W}_{in} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_4 h_4}{\dot{m}_3} \\ &= \frac{1000 + 5 \cdot 2870 + 3 \cdot 3273.3 - 4 \cdot 3382.3}{4} = 2910.18 \text{ kJ/kg} \end{aligned}$$

Interpolating in the tables for superheated water, we get that $T_3 = 217.9^\circ\text{C}$.

6.11b We want to determine whether the device violates the second law of thermodynamics. The second law of thermodynamics gives that (for adiabatic processes), the following must hold:

$$(\dot{m}s)_{out} - (\dot{m}s)_{in} \geq 0 \Rightarrow \dot{m}_3 s_3 + \dot{m}_4 s_4 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \geq 0$$

The values for the specific entropy can be obtained from the tables: $s_1 = 7.5059 \text{ kJ/(kgK)}$, $s_2 = 7.8982 \text{ kJ/(kgK)}$, $s_3 = 7.8985 \text{ kJ/(kgK)}$ and $s_4 = 8.6865 \text{ kJ/(kgK)}$. Hence, the total entropy change is calculated as:

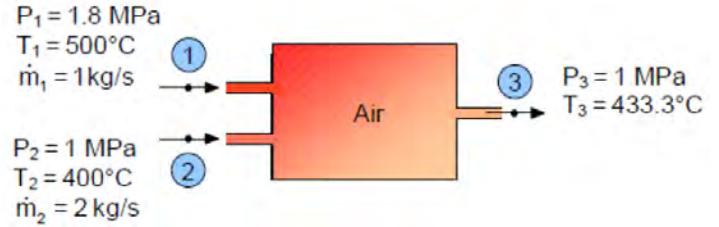
$$\Delta \dot{S} = \dot{m}_3 s_3 + \dot{m}_4 s_4 - \dot{m}_1 s_1 - \dot{m}_2 s_2 = 4 \cdot 7.8985 + 4 \cdot 8.6865 - 5 \cdot 7.5059 - 3 \cdot 7.8982 = 5.1159 \text{ kW/K}$$

Since this value is larger than 0, the second law is satisfied.

Note: This problem is a good illustration of how all three fundamental principles of thermodynamics, along with property relationships, are applied simultaneously: mass balance (to determine the unknown mass flow rate), energy balance (to determine the unknown temperature) and second law (entropy imbalance) to determine feasibility.

6.12 Steady-state steady-flow adiabatic device

Given is the steady-state steady-flow adiabatic device for air on the right. It is asked whether this device is possible. The specific heats are assumed to be constant.



The device is possible when both the first and the second law of thermodynamics are satisfied. We start by checking the first law (energy conservation). For an open system, it is given by:

$$\dot{Q}_{in} + \dot{W}_{in} + \sum (\dot{m}_{in} h_{in} + \dot{m}_{in} ke_{in} + \dot{m}_{in} pe_{in}) = \dot{Q}_{out} + \dot{W}_{out} + \sum (\dot{m}_{out} h_{out} + \dot{m}_{out} ke_{out} + \dot{m}_{out} pe_{out})$$

We neglect the differences in kinetic and potential energy. Furthermore, the device is adiabatic ($\dot{Q}_{in} - \dot{Q}_{out} = 0$) and no work is being done, so the conservation of energy for this device reads:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

From conservation of mass we know that

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

and hence the conservation of energy can be rewritten to:

$$\dot{m}_1 (h_1 - h_3) + \dot{m}_2 (h_2 - h_3) = 0$$

Assuming air to be an ideal gas, it holds that $dh = c_p dT$. Since the specific heats may be assumed constant, this gives:

$$\dot{m}_1 c_p (T_1 - T_3) + \dot{m}_2 c_p (T_2 - T_3) = 1 \cdot c_p \cdot (500 - 433.3) + 2 \cdot c_p \cdot (400 - 433.3) = 0$$

and hence the first law is satisfied.

Next, we have to verify the second law, which is (for an adiabatic device) given as:

$$\sum (\dot{m}_{out} s_{out}) - \sum (\dot{m}_{in} s_{in}) \geq 0 \Rightarrow \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \geq 0$$

This can be rewritten using conservation of mass ($\dot{m}_3 = \dot{m}_1 + \dot{m}_2$) to:

$$(\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 = \dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_3 - s_2) \geq 0$$

We use the Gibbs equation to calculate the entropy difference:

$$Tds = dh - vdP \Rightarrow ds = \frac{c_p}{T}dT - \frac{R}{P}dP$$

where we used that, for an ideal gas, $dh = c_p dT$ and $v = \frac{RT}{P}$. Integrating these relations over the desired ranges gives $s_3 - s_1$ and $s_3 - s_2$:

$$\begin{aligned}s_3 - s_1 &= c_p \ln\left(\frac{T_3}{T_1}\right) - R \ln\left(\frac{P_3}{P_1}\right) = c_p \ln\left(\frac{706.3}{773}\right) - 0.287 \cdot \ln\left(\frac{1}{1.8}\right) = -0.09c_p + 0.1687 \text{ kW/(K)} \\ s_3 - s_2 &= c_p \ln\left(\frac{T_3}{T_2}\right) - R \ln\left(\frac{P_3}{P_2}\right) = c_p \ln\left(\frac{706.3}{673}\right) - 0.287 \cdot \ln\left(\frac{1}{1}\right) = 0.0483c_p \text{ kW/K}\end{aligned}$$

Hence, the total entropy change is given by:

$$\dot{m}_1(s_3 - s_1) + \dot{m}_2(s_3 - s_2) = 1 \cdot (-0.09c_p + 0.1687) + 2 \cdot 0.0483c_p = 0.0066c_p + 0.1687 \text{ kW/K} \geq 0$$

The second law is thus satisfied. Since both the first and the second law are satisfied, the device is possible.

6.13 Venturi

6.13a See figure 6.13.1.

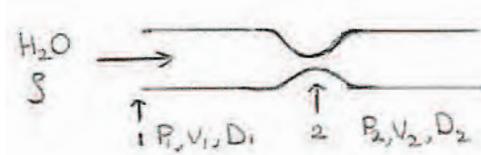


Figure 6.13.1: Situation of exercise 6.13

6.13b The main equations needed for this problem are the mass flow rate, the volume flow rate and the Bernoulli equation.

The mass flow rate is $\dot{m} = A\rho v$ and the volume flow rate is $\dot{v} = Av$, where $A = \frac{1}{4}\pi D^2$.

The Bernoulli equation is $\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 + gz_2$.

Conservation of mass states $\dot{m}_1 = \dot{m}_2 \rightarrow A_1 v_1 = A_2 v_2 \rightarrow v_2 = \frac{A_1}{A_2} v_1 = (\frac{D_1}{D_2})^2 v_1$.

In the Bernoulli equation the terms from the potential energy, gz , cancel as the tube is horizontal $\rightarrow 2(P_1 - P_2) = \rho(v_2^2 - v_1^2)$.

Combining the Bernoulli equation with the outcome of mass balance for the relation between v_1 and v_2 gives

$$2(P_1 - P_2) = \rho v_1^2 \left(\left(\frac{D_1}{D_2} \right)^4 - 1 \right) \Rightarrow v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho((\frac{D_1}{D_2})^4 - 1)}}.$$

Filling in this in the mass flow rate results in:

$$\dot{m} = \frac{1}{4}\pi D_1^2 \rho v_1 = \frac{1}{4}\pi D_1^2 \rho \sqrt{\frac{2(P_1 - P_2)}{\rho((\frac{D_1}{D_2})^4 - 1)}}.$$

And the volume flow rate is given by:

$$\dot{v} = \frac{1}{4}\pi D_1^2 v_1 = \frac{1}{4}\pi D_1^2 \sqrt{\frac{2(P_1 - P_2)}{\rho((\frac{D_1}{D_2})^4 - 1)}}.$$

7 Thermodynamic Cycles for work, Heat and cold, Carnot

7.1 Thermodynamic cycles

7.1a In a gas cycle the working medium is in the gas phase during the whole cycle. In a vapor cycle the working medium undergoes a phase change. In part of the cycle it is in the liquid phase, by adding heat to it, it evaporates to vapor, later in the cycle it turns back to the liquid phase by releasing heat (condensation in the condenser).

7.1b In a closed cycle the medium is recycled. In an open cycle the medium is released to the outside and a fresh fluid/gas is used for the next cycle.

7.1c In an internal combustion engine the combustion takes place in the engine itself, the working fluid is heated by combustion of the medium. In an external combustion engine the combustion takes place outside the engine in a separate device and only the heat is transferred to the engine.

7.1d In an ideal cycle all processes are isentropic.

7.2 Thermal efficiency

Thermal efficiency is a fraction of the heat input that is converted to net work output of a heat engine. Thermal efficiency measures the performance of a heat engine and it is denoted as η_{th} . For heat engines the thermal efficiency can be expressed as:

$$\eta_{th} = \frac{\text{net work output}}{\text{total heat input}} = \frac{W_{\text{nett}}}{Q_{in}} = \frac{(Q_{in} - Q_{out})}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}. \quad (7.2.0)$$

Depending on what data you have available you can use one of the different forms appearing in the relations above.

7.3 Coefficient of performance

7.3a The general definition for the coefficient of performance is

$$COP = \frac{\text{desired output}}{\text{required input}}. \quad (7.3.0)$$

7.3b For a refrigerator the desired output is Q_L , the required input is W_{in}

$$COP_R = \frac{\text{desired output}}{\text{required input}} = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1} = \frac{1}{\frac{T_H}{T_L} - 1}. \quad (7.3.0)$$

7.3c For a heat pump the desired output is Q_H , the required input is W_{in}

$$COP_{HP} = \frac{\text{desired output}}{\text{required input}} = \frac{Q_H}{W_{in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}} = \frac{1}{1 - \frac{T_L}{T_H}}. \quad (7.3.0)$$

7.3d

$$COP_{HP} = \frac{1}{1 - \frac{Q_L}{Q_H}} \rightarrow \frac{Q_H}{Q_L} = \frac{1}{COP_R} + 1. \quad (7.3.0)$$

$$COP_R = \frac{1}{\frac{Q_H}{Q_L} - 1} \rightarrow \frac{Q_L}{Q_H} = \frac{COP_R}{1 + COP_R}. \quad (7.3.0)$$

Insert the last equation into the equation for COP_{HP}

$$COP_{HP} = \frac{1}{\frac{COP_R}{1+COP_R}} = 1 + COP_R. \quad (7.3.0)$$

7.4 Carnot

7.4a The Carnot efficiency is the maximum thermodynamically possible heat engine efficiency.

7.4b The Carnot efficiency is defined as:

$$\eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}} \quad (7.4.0)$$

where T_{cold} and T_{hot} are the temperatures of the two heat reservoirs the engine is working between.

7.4c A Carnot cycle is an ideal heat engine cycle with the highest thermodynamically possible heat engine efficiency. It requires reversible heat transfer and no other irreversibilities.

7.4d See figure 7.4.1 for the $P - v$ and $T - s$ diagram of a Carnot cycle.

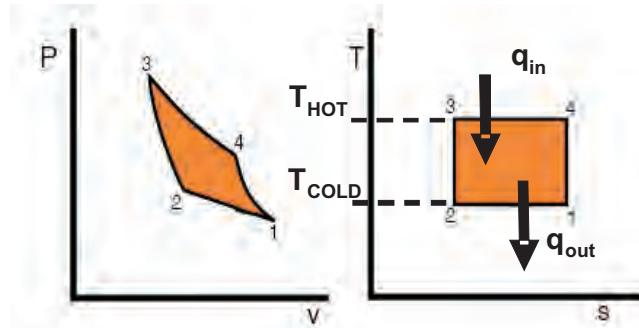


Figure 7.4.1: Change of the piston-cylinder device from state A to state B via three different paths.

7.4e Two processes are isothermal, $1 \rightarrow 2$ and $3 \rightarrow 4$ and two processes are adiabatic $2 \rightarrow 3$ and $4 \rightarrow 1$.

7.4f Heat is given on the diagram above, heat is added between 1 and 2 and rejected between 3 and 4. Work W_{1-2-3} is the work done by the gas during the expansion, work W_{3-4-1} is the work during compression. The net work $W_{net} = W_{1-2-3} - W_{3-4-1}$.

7.4g The Carnot cycle is reversible. It is a cyclic device and at the end of each cycle it returns to its initial state and the net work, thus cycle efficiency, is maximized by using processes that require the least amount of work and deliver the most, that is, by using reversible processes.

7.4h Yes, it is possible to use it also for refrigeration. The Carnot heat engine is a totally reversible cycle, therefore it is possible to reverse it, in which case it becomes the Carnot refrigeration cycle. The cycle remains exactly the same, except that the directions of any heat and work interactions are reversed.

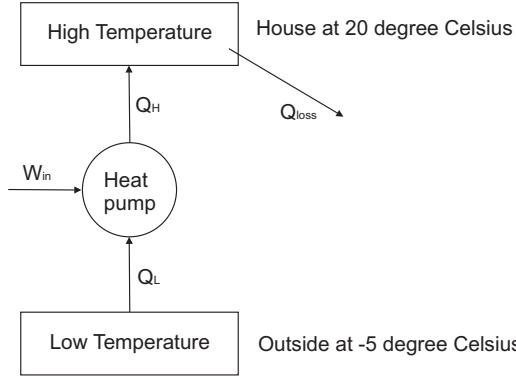
7.5 Geothermal energy

The maximum thermal efficiency can be calculated using the Carnot efficiency

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{293}{413} = 0.29. \quad (7.5.0)$$

The maximum thermodynamically possible efficiency is 29%. However in reality the efficiency will be lower. In winter when the environmental temperature (T_L) is lower the efficiency will be higher.

7.6 A heat pump



7.6a The coefficient of performance for a reversible heat pump is $COP_{hp,rev} = \frac{1}{1 - \frac{T_L}{T_H}}$.

7.6b The minimum power required to drive the heat pump is $\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{hp,rev}}$.

7.6c $T_{house} = T_H = 20^\circ\text{C} = 293 \text{ K}$.

$T_{outside} = T_L = -5^\circ\text{C} = 268 \text{ K}$.

$\dot{Q}_{loss} = \dot{Q}_H = 75000 \text{ kJ/h} = 20.8 \text{ kW}$.

$$COP_{hp,rev} = \frac{1}{1 - \frac{T_L}{T_H}} = \frac{1}{1 - \frac{268}{293}} = 11.72. \quad (7.6.0)$$

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{hp,rev}} = \frac{20.8}{11.72} = 1.77 \text{ kW}. \quad (7.6.0)$$

7.7 Carnot engine

7.7a $\eta_{th} = \frac{\text{desired output}}{\text{required input}} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = \frac{T_H - T_L}{T_H} = 1 - \frac{T_L}{T_H} = 1 - \frac{20 + 273.15}{200 + 273.15} = 0.38 \rightarrow 38\%$.

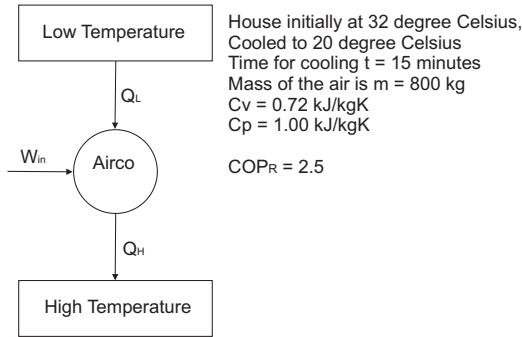
7.7b $\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}} = \frac{15}{0.38} = 39.47 \text{ kW}$ and $\dot{Q}_H = \dot{W}_{net} + \dot{Q}_L \rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W}_{net}$
 $\dot{Q}_L = 39.47 - 15 = 24.47 \text{ kW}$.

7.7c The inequality of Clausius for a single proces is: $\frac{\delta q}{T} \leq ds$, where the $=$ sign holds for the reversilbe proces, thus for the Carnot proces which is designed to be reversible. For a cycle one should take into account all processes in which heat transfer plays a role and after returning to the initial state the change in entropy, $ds = 0$ as s is a state function that returns to its original value in a cyclic proces. The inequality of Clausius can be written as: $\sum_{i=1}^n \left(\frac{q_i}{T_i} \right) = 0$ (if this was not a Carnot cycle

the = sign should be replaced by a \leq sign).

For this case the summation leads to: $\frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = \frac{39.47}{473} - \frac{24.47}{293} = 0.083 - 0.083 = 0 \text{ kJ/kgK}$. The 0 shows that indeed the second law (the inequality of Clausius) is satisfied

7.8 Air conditioner



7.8a The power drawn by the air conditioner is $\dot{W}_{in} = \frac{\dot{Q}_L}{COP_R}$ with $COP_R = \frac{Q_L}{Q_H - Q_L}$. Here Q_L is the heat that has to be removed from the house $Q_L = \Delta U = c_v m \Delta T$. The power of this heat is $\dot{Q}_L = \frac{Q_L}{t}$.

7.8b The energy of the cooled air is $Q_L = \Delta U = c_v m \Delta T = 0.72 \cdot 800 \cdot (32 - 20) = 6912 \text{ kJ}$.

The power of the cooled air is $\dot{Q}_L = \frac{Q_L}{t} = \frac{6912}{15 \cdot 60} = 7.68 \text{ kW}$.

The power drawn by the air conditioner is $\dot{W}_{in} = \frac{\dot{Q}_L}{COP_R} = \frac{7.68}{2.5} = 3.07 \text{ kW}$.

7.8c The amount of heat extracted from the house is $\dot{Q}_H = \dot{W}_{in} + \dot{Q}_L = 3.07 + 7.68 = 10.75 \text{ kW}$.

7.9 Refrigerator

7.9a The coefficient of performance is: $COP_R = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{1}{\frac{T_H}{T_L} - 1}$.

For cooling to T_{L1} the work required is: $W_1 = Q_L \left(\frac{T_H}{T_{L1}} - 1 \right)$.

For cooling to T_{L2} the work required is: $W_2 = Q_L \left(\frac{T_H}{T_{L2}} - 1 \right)$.

If it is assumed that W_1 is 100% then the increase $W_2 - W_1 = x\%$ $\rightarrow x = \frac{W_2 - W_1}{W_1} \cdot 100\%$.

7.9b $W_1 = Q_L \left(\frac{T_H}{T_{L1}} - 1 \right) = Q_L \left(\frac{273.15+20}{273.15-5} - 1 \right) = 0.093 Q_L$.

$W_2 = Q_L \left(\frac{T_H}{T_{L2}} - 1 \right) = Q_L \left(\frac{273.15+20}{273.15-25} - 1 \right) = 0.181 Q_L$.

This results in $x\%$ is 94.6%.

7.10 Heat engine

The question is if the power output of this heat engine with 27% efficiency is more than 1.8 kW when it can transfer at maximum 50 kJ/min to a low temperature reservoir. From this information follows $\eta_{he} = 27\%$ and $\dot{Q}_{out} = 50 \text{ kJ/min} = 0.833 \text{ kJ/s}$ and $\dot{W}_{net,required} \geq 1.8 \text{ kW}$.

The thermal efficiency for the heat engine is: $\eta_{he} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} \rightarrow \dot{W}_{net} = \eta_{he} \dot{Q}_{in}$.

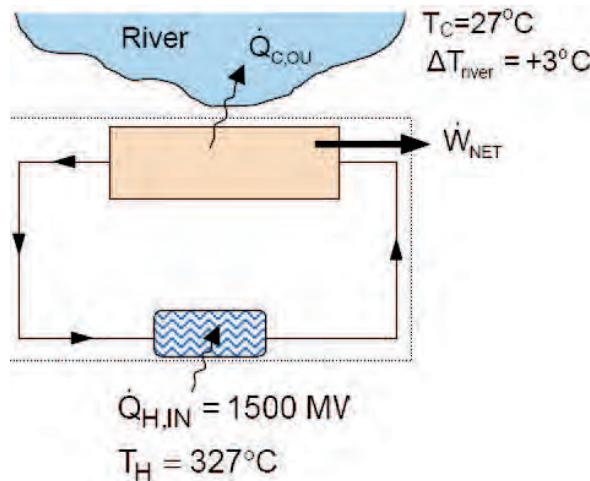
The first law (conservation of energy) gives: $\dot{W}_{net} = \dot{Q}_{net} = \dot{Q}_{in} - \dot{Q}_{out} \rightarrow \dot{Q}_{in} = \dot{W}_{net} + \dot{Q}_{out}$.

Combining these two equations gives: $\dot{W}_{net} = \eta_{he} \dot{Q}_{in} = \eta_{he} (\dot{W}_{net} + \dot{Q}_{out}) \rightarrow \dot{W}_{net} = \frac{\eta_{he} \dot{Q}_{out}}{1 - \eta_{he}}$.

Filling in the numbers gives: $\dot{W}_{net} = \frac{0.27 \cdot 0.833}{1 - 0.27} = 0.308 \text{ kW}$.

Since $\dot{W}_{net} < \dot{W}_{net,required}$ the engine will not provide sufficient power. This problem illustrates a case where environmental constraints set limits on the performance of a machine. "Thermal pollution" is a matter of environmental concern and power companies are required to submit environmental impact statements to regulatory agencies prior to building and operating power plants.

7.11 Nuclear power plant



7.11a The efficiency of a Carnot engine is: $\eta_{th,Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{27}{327} = 0.5 \rightarrow \eta_{th,Carnot} = 50\%$.

7.11b The net power output follows from the thermal efficiency which is $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$, in this case the thermal efficiency is the Carnot efficiency $\rightarrow \dot{W}_{net} = \eta_{th,Carnot} \dot{Q}_{in} = 0.5 \cdot 1500 = 750 \text{ MW}$.

7.11c Conservation of energy gives that the power rejected by the Carnot engine to the river as thermal waste heats the river. Therefore the rate of heat gained by the river equals the power rejected by the Carnot engine $\rightarrow \dot{Q}_{out,power\ plant} = \dot{Q}_{in,river}$, in which: $\dot{Q}_{in,river} = (\dot{m}\Delta h)_{river} = (\dot{m}c_p\Delta T)_{river}$

$$\rightarrow \dot{m}_{river} = \frac{\dot{Q}_{out,power\ plant}}{(c_p\Delta T)_{river}} = \frac{750 \cdot 10^3}{4.2 \cdot 3} = 59524 \text{ kg/s.}$$

The Carnot efficiency of the power plant analysed in this problem is 50%, and results in a rise in river temperature of 3°C . A real nuclear power plant would typically have a thermal efficiency of about 30% (roughly 60% of the Carnot efficiency), which would produce about 5°C rise in river temperature. While this might seem like a small temperature rise, it in fact can be quite significant with respect to the ecology of the river (imagine how different your living environment would feel if the temperature were increased by 5°C year-round!)

7.12 Steam cycle

7.12a See figure 7.12.1.

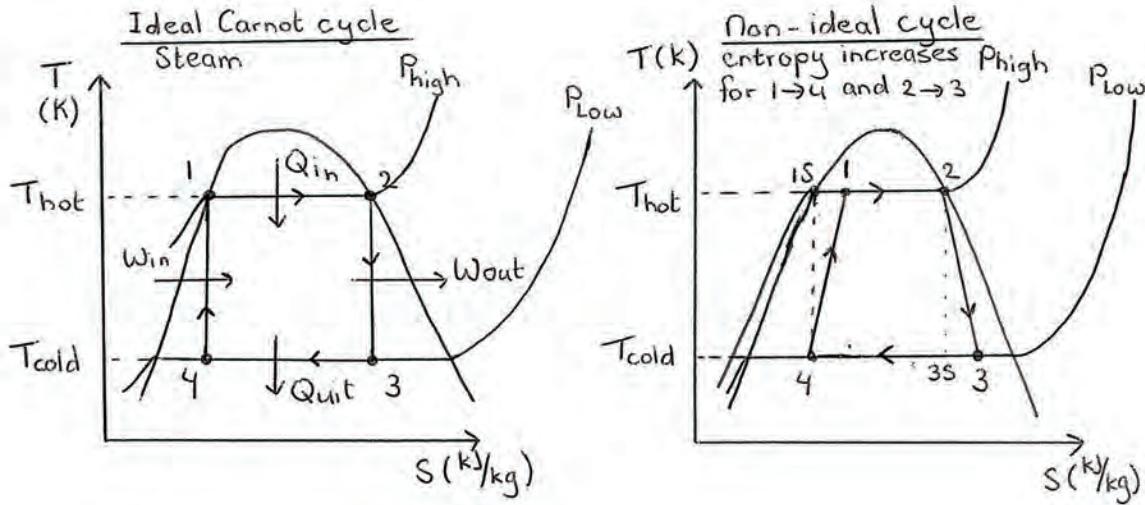


Figure 7.12.1: T - s diagrams for the ideal (left) and non-ideal Carnot cycle. Note: the unit of entropy (s) should be kJ/kgK instead of kJ/kg .

7.12b The heat is added during the vaporization process at 40 bar and rejected during the condensation process at 0.05 bar. Therefore the highest and lowest temperatures are the saturation temperatures at 40 bar and 0.05 bar, respectively $\rightarrow T_{hot} = T_{sat@40bar} = 250.39^\circ\text{C}$ and $T_{cold} = T_{sat@0.05bar} = 32.881^\circ\text{C}$.

7.12c In order to determine the heat and work transfers and the efficiency of the cycle, we need to determine the enthalpy values. Before doing so, we first collect all the known parameters in a table: see the underlined values in table 7.12.1.

Table 7.12.1: Flow characteristics of exercise 7.12

	P [kPa]	T [$^\circ\text{C}$]	h [kJ/kg]	x [-]	s [kJ/(kgK)]	other
1	<u>4000</u>	T_{hot}	1087	<u>0</u>	2.797	
2	<u>4000</u>	T_{hot}	2801	<u>1</u>	6.07	
3	<u>5</u>	T_{cold}	1851	0.707	$s_3 = s_2$	
4	<u>5</u>	T_{cold}	847.6	0.293	$s_4 = s_1$	

Using the tables for saturated water, we find that $h_1 = h_{f@P=4000\text{kPa}} = 1087 \text{ kJ/kg}$ and $s_1 = s_{f@P=4000\text{kPa}} = 2.797 \text{ kJ/(kgK)}$. Similarly, the values at point 2 can be determined: $h_2 = h_{g@P=4000\text{kPa}} = 2801 \text{ kJ/kg}$ and $s_2 = s_{g@4000\text{kPa}} = 6.07 \text{ kJ/(kgK)}$. For point 3, it is given that $s_3 = s_2$ and $P_3 = 5 \text{ kPa}$. It is seen that $s_f < s_3 < s_g$, and hence we need to compute the vapour mass fraction x :

$$x_3 = \frac{s_3 - s_{f@5\text{kPa}}}{s_{g@5\text{kPa}} - s_{f@5\text{kPa}}} = \frac{6.07 - 0.4761}{8.3930 - 0.4761} = 0.707$$

Now we can compute the enthalpy at point 3:

$$h_3 = h_{f@5\text{kPa}} + x_3(h_{g@5\text{kPa}} - h_{f@5\text{kPa}}) = 137.72 + 0.707 \cdot (2560.5 - 137.72) = 1851 \text{ kJ/kg}$$

In a similar way, we can determine the enthalpy at point 4, using the given pressure and $s_4 = s_1$:

$$x_4 = \frac{s_4 - s_f@5kPa}{s_g@5kPa - s_f@5kPa} = \frac{2.797 - 0.4761}{8.3930 - 0.4761} = 0.293$$

$$h_4 = h_f@5kPa + x_4(h_g@5kPa - h_f@5kPa) = 137.72 + 0.293 \cdot (2560.5 - 137.72) = 847.6 \text{ kJ/kg}$$

Now all the values for the enthalpy are known, we can compute the heat in (q_{in}), heat out q_{out} , work input of the compressor ($w_{c,in}$), work output of the turbine ($w_{t,out}$) and the thermal efficiency of the cycle (η_{th}):

$$\begin{aligned} q_{in} &= h_2 - h_1 = 2801 - 1087 = 1714 \text{ kJ/kg} \\ q_{out} &= h_3 - h_4 = 1851 - 847.6 = 1003.4 \text{ kJ/kg} \\ w_{c,in} &= h_1 - h_4 = 1087 - 847.6 = 239.4 \text{ kJ/kg} \\ w_{t,out} &= h_2 - h_3 = 2801 - 1851 = 950 \text{ kJ/kg} \\ \eta_{th} &= \frac{w_{nett}}{q_{in}} = \frac{w_{t,out} - w_{c,in}}{q_{in}} = \frac{950 - 239.4}{1714} = 0.415 = 41.5\% \end{aligned}$$

7.12d Now the turbine and the compressor may no longer be assumed ideal, the enthalpy values at point 1 and at point 3 change. The values used in c are the values for the ideal case and will be denoted with an extra s. The actual values can be computed from the given isentropic efficiency:

$$\begin{aligned} \eta_{s,c} &= \frac{h_{1s} - h_4}{h_{1a} - h_4} \Rightarrow h_{1a} = h_4 + \frac{h_{1s} - h_4}{\eta_{s,c}} = 847.6 + \frac{1087 - 847.6}{0.8} = 1146.9 \text{ kJ/kg} \\ \eta_{s,t} &= \frac{h_2 - h_{3a}}{h_2 - h_{3s}} \Rightarrow h_{3a} = h_2 - \eta_{s,t}(h_2 - h_{3s}) = 2801 - 0.8 \cdot (2801 - 1851) = 2041 \text{ kJ/kg} \end{aligned}$$

The heat and work transfers and the efficiency for the non-ideal cycle can now be calculated similarly as in c:

$$\begin{aligned} q_{in} &= h_2 - h_{1a} = 2801 - 1146.9 = 1654.1 \text{ kJ/kg} \\ q_{out} &= h_{3a} - h_4 = 2041 - 847.6 = 1193.4 \text{ kJ/kg} \\ w_{c,in} &= h_{1a} - h_4 = 1146.9 - 847.6 = 299.3 \text{ kJ/kg} \\ w_{t,out} &= h_2 - h_{3a} = 2801 - 2041 = 760.0 \text{ kJ/kg} \\ \eta_{th} &= \frac{w_{nett}}{q_{in}} = \frac{w_{t,out} - w_{c,in}}{q_{in}} = \frac{760.0 - 299.3}{1654.1} = 0.279 = 27.9\% \end{aligned}$$

Discussion of the results, in the ideal case the work output is larger and the work input is higher than in the non ideal case as expected. The heat input is lower in the non ideal case. This is because due to the increase in entropy during the compression part of the liquid is already vaporised. The heat output is larger in the non ideal case. This is because due to the increase in entropy during the expansion the saturated mixture has a higher quality, (x).The efficiency of the non ideal cycle is lower than the ideal cycle because the net work is lower (although the net heat input is also lower).

7.12e The inequality of Clausius for a cycle is: $\sum_{i=1}^n \left(\frac{q_i}{T_i} \right) \leq 0$ (see question 7.7c), for this situation with two reservoirs with heat transfer this leads to: $\frac{q_H}{T_H} - \frac{q_L}{T_L} \leq 0$.

For the ideal case: $\frac{q_H}{T_H} - \frac{q_L}{T_L} = \frac{1714}{523} - \frac{1003}{306} = 3.28 - 3.28 = 0$.

For the non-ideal case: $\frac{q_H}{T_H} - \frac{q_L}{T_L} = \frac{1654}{523} - \frac{1193}{306} = 3.16 - 3.90 = -0.74 \leq 0$.

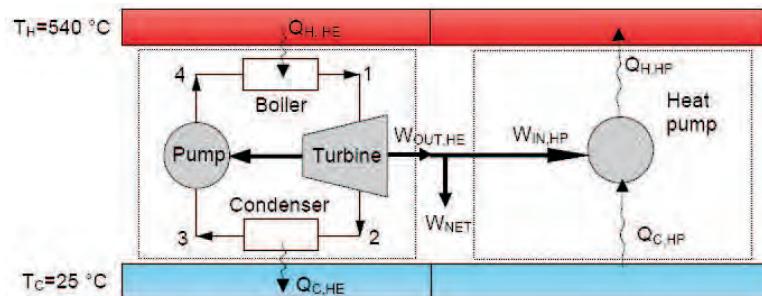
It follows that for the ideal case, which is the reversible case the result is 0, so no entropy is generated. This is expected on base of the second law for a reversible process. For the non-ideal case the change in entropy is negative, this is in agreement with the second law applied to a cycle, as $\sum_{i=1}^n \left(\frac{q_i}{T_i} \right) \leq 0$ for the non-reversible (non-ideal) case.

7.13 Perpetual - motion machines

7.13a The inventor has developed a perpetual - motion machine of the first kind, it violates the first law of thermodynamics which states that energy is conserved and can not be created or destroyed. A careful examination of this invention reveals that the system enclosed by the shaded area is continuously supplying energy to the outside at a rate of $\dot{Q}_{out} + \dot{W}_{out}$ without receiving any energy. That is this device is creating energy at a rate of $\dot{Q}_{out} + \dot{W}_{out}$, which is clearly a violation of the first law. Based on the first law a system that supplies energy at a rate of $\dot{Q}_{out} + \dot{W}_{out}$ should get heat in at a rate of \dot{Q}_{in} of $\dot{Q}_{out} + \dot{W}_{out}$ as conservation of energy requires: $\dot{W}_{net} = \dot{Q}_{net} = \dot{Q}_{in} - \dot{Q}_{out} \rightarrow \dot{Q}_{in} = \dot{W}_{net} + \dot{Q}_{out}$. But this system does not have any energy transfer to it over the system boundary.

7.13b The possibility of doubling the efficiency would certainly be very attractive to plant managers and, if not properly trained, they would probably give this idea a chance, since intuitively they see nothing wrong with it. However if you have some knowledge about the Kelvin - Planck statement of the second law you will immediately label this device as an impossible device. It works on a cycle and does a net amount of work while exchanging heat with a single reservoir (the boiler) only. It satisfies the first law of thermodynamics but violates the second law, and therefore will not work. This machine is a perpetual - motion machine of the second kind as it does not obey the second law in the form of the Kelvin Planck statement that states that two temperature reservoirs are needed in order to complete a cycle that provides work.

7.14 Combined engine and heat pump



Given is for the heat engine: $w_{out,he} = 0.5 \text{ kJ/kg}$, $q_{h,he} = 1 \text{ kJ/kg}$.

Given is for the heat pump: $w_{in,hp} = 0.25 \text{ kJ/kg}$, $q_{h,hp} = 1 \text{ kJ/kg}$, $COP_{hp} = 4$.

For the total system: $w_{net} = 0.25 \text{ kJ/kg}$, $T_C = 298 \text{ K}$, $T_H = 813 \text{ K}$.

For the heat engine the thermal efficiency is

$$\eta_{he} = \frac{w_{out,he}}{q_{h,he}} = \frac{0.5}{1} = 0.5 \rightarrow \eta_{he} = 50\%$$

while the corresponding thermal Carnot efficiency (maximum possible on basis of the second law) is

$$\eta_{he,Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{298}{813} = 0.633 \rightarrow \eta_{he,Carnot} = 63.3\%.$$

Thus, the heat engine itself does not violate the second law. The heat pump, however, has a specified coefficient of performance, COP_{hp} , of 4. For an ideal (Carnot) heat pump, COP_{hp} is given by

$$COP_{hp,Carnot} = \frac{1}{1 - \frac{T_C}{T_H}} = \frac{1}{\eta_{hp,Carnot}} = 1.58$$

which is lower than the specified COP of 4 therefore, the heat pump cannot provide the stated performance. The work required by the ideal (Carnot) heat pump is

$$COP_{hp} = \frac{q_{h,hp}}{w_{in,hp}} \rightarrow w_{in,hp} = \frac{q_{h,hp}}{COP_{hp}} = \frac{1}{1.58} = 0.633 \text{ kJ/kg.}$$

The invention as specified provides useful work output while drawing all of its energy from a single reservoir (at T_H) and no net heat is added or rejected at T_C . This violates the second law requirement that states that energy transfer to a low temperature reservoir is necessary.

Discussion: If the engine were actually a Carnot engine, it could provide a work output of 0.633 kJ/kg, and the ideal heat pump needs exactly that amount of work to provide the needed heat to the engine; however, no useful work is left over. This situation would result in net heat transfer of zero to the low temperature reservoir, which does not violate the second law (which states that the heat transfer is ≤ 0 where the zero applies for the ideal case).

8 Exergy, Second Law Analysis of Systems: Answers

8.1 Exergy

8.1a Exergy is *the energy that can be used to produce work or the energy available for work.*

8.1b Anergy is the non-useful energy, the part of the total energy that can not be used to produce work. For example heat that has a temperature that is (close to) the temperature of the environment.

8.1c $\psi_i = h_i - h_0 - T_0(s_i - s_0)$, note: $h_i - h_0$ is the total energy, $T_0(s_i - s_0)$ is the part of the energy that can not be used to produce work (the anergy). Subtracting both gives the useful energy, the exergy.

$$\mathbf{8.1d} \quad \psi_i = T_0(s_i - s_0)$$

8.1e T_0 is a reference temperature used to calculate the exergy, often the temperature of the environment.

8.1f No, T_0 is not fixed it is a reference temperature that can be chosen yourself, often the temperature of the environment is the most logic choice.

8.2 A geothermal well

8.2a As the reference state, we choose to use $P_0 = 0.1013 \text{ MPa}$ and $T_0 = 25^\circ\text{C} = 298 \text{ K}$ (which can be seen as surrounding conditions). Furthermore, we assume water at the reference state to be incompressible, such that $h \approx h_L(t)$ and $s \approx s_L(T)$. The flow exergy from the source is calculated by $\psi = (h_1 - h_0) - T_0(s_1 - s_0)$. Hence, we need to find the values of the enthalpies and entropies. They can be obtained from the property tables for water:

$$\begin{aligned} h_0 &= h_L(T = 25^\circ\text{C}) = 104.75 \text{ kJ/kg (table10s)} \\ s_0 &= s_L(T = 25^\circ\text{C}) = 0.3670 \text{ kJ/(kgK) (table10s)} \\ h_1 &= h(P = 1000 \text{ kPa}, T = 220^\circ\text{C}) = 2873.20 \text{ kJ/kg (table12s)} \\ s_1 &= s(P = 1000 \text{ kPa}, T = 220^\circ\text{C}) = 6.7853 \text{ kJ/(kgK) (table12s)} \end{aligned}$$

where interpolation is used for h_1 and s_1 to obtain the values for superheated steam at the correct temperature of 220°C . The exergy flow can now be computed:

$$\psi = (h_1 - h_0) - T_0(s_1 - s_0) = (2873.20 - 104.75) - 298(6.7853 - 0.3670) = 855.80 \text{ kJ/kg}$$

8.2b The reversible work is $(w_{out} - w_{in})_{rev} = \psi_{in} - \psi_{out}$. Since ψ_{out} is at the dead state, it equals zero. Therefore, the reversible work is:

$$(w_{out} - w_{in})_{rev} = 855.80 \text{ kJ/kg}$$

8.3 Steady-state steady-flow heater

We take the reference state as $P_0 = 0.1 \text{ MPa}$ and $T_0 = 20^\circ\text{C} = 293 \text{ K}$. The reversible work is calculated as:

$$(w_{out} - w_{in})_{rev} = q_{in,rev} \left(1 - \frac{T_0}{T_{in}}\right) - q_{out,rev} \left(1 - \frac{T_0}{T_{out}}\right) + \left(\psi + \frac{v^2}{2} + gz\right)_{in} - \left(\psi + \frac{v^2}{2} + gz\right)_{out}$$

Since the process is carried out adiabatically, $q_{in,rev} = q_{out,rev} = 0$. We also neglect potential and kinetic energy differences, such that the equation is reduced to:

$$(w_{out} - w_{in})_{rev} = \psi_{in} - \psi_{out}$$

The flow exergy is calculated by:

$$\psi = (h - h_0) - T_0(s - s_0)$$

Substituting this in the equation for the reversible work gives:

$$(w_{out} - w_{in})_{rev} = (h_1 - h_0) - T_0(s_1 - s_0) - [(h_2 - h_0) - T_0(s_2 - s_0)] = h_1 - h_2 - T_0(s_1 - s_2)$$

We can determine the values of h_1 , h_2 , s_1 and s_2 from the Mollier diagram of air:

$$\begin{aligned} h_1 &= 295 \text{ kJ/kg} \\ s_1 &= 6.38 \text{ kJ/(kgK)} \\ h_2 &= 480 \text{ kJ/kg} \\ s_2 &= 6.87 \text{ kJ/(kgK)} \end{aligned}$$

Hence, the reversible work output is:

$$(w_{out} - w_{in})_{rev} = h_1 - h_2 - T_0(s_1 - s_2) = 295 - 480 - 293(6.38 - 6.87) = -41.43 \text{ kJ/kg}$$

The negative sign indicates that reversible work input would be required to raise the temperature of the air from 20 to 200 °C. The heating process might hypothetically be carried out with a reversible heat pump using the dead state as a heat source. For the actual process, $q_{in} = h_2 - h_1 = 185 \text{ kJ/kg}$. The difference represents the irreversibility associated with heat transfer from a high temperature to a lower temperature gas.

8.4 A solar tower

8.4a The flow exergy of the tower-generated steam is calculated by: $\psi_1 = h_1 - h_0 - T_0(s_1 - s_0)$. We take the reference state as $T_0 = 25 \text{ }^\circ\text{C}$ and $P_0 = 101.3 \text{ kPa}$. Assuming water to be incompressible, the dead state properties are given by:

$$h_0 = h_L(T = 25 \text{ }^\circ\text{C}) = 104.75 \text{ kJ/kg}$$

$$s_0 = s_L(T = 25 \text{ }^\circ\text{C}) = 0.3670 \text{ kJ/kgK}$$

(following from the tables of saturated water.) From the tables for superheated steam (and interpolating), we find h_1 and s_1 :

$$h_1 = h(T = 515 \text{ }^\circ\text{C}, P = 1000 \text{ kPa}) = 3511.30 \text{ kJ/kg}$$

$$s_1 = s(T = 515 \text{ }^\circ\text{C}, P = 1000 \text{ kPa}) = 7.8032 \text{ kJ/kgK}$$

Hence, the flow exergy of the tower-generated steam is:

$$\psi_1 = h_1 - h_0 - T_0(s_1 - s_0) = 3511.30 - 104.75 - 298(7.8032 - 0.3670) = 1190.6 \text{ kJ/kg}$$

8.4b The flow exergy of steam at 300 °C and 1 MPa is calculated similar as the exergy of the tower-generated steam. First, we need to determine the enthalpy and the exergy:

$$h_2 = h(T = 300 \text{ }^\circ\text{C}, P = 1000 \text{ kPa}) = 3050.60 \text{ kJ/kg}$$

$$s_2 = s(T = 300 \text{ }^\circ\text{C}, P = 1000 \text{ kPa}) = 7.1219 \text{ kJ/kgK}$$

Now, the flow exergy of steam at 300 °C and 1 MPa is calculated as:

$$\psi_2 = h_2 - h_0 - T_0(s_2 - s_0) = 3050.60 - 104.75 - 298(7.1219 - 0.3670) = 932.9 \text{ kJ/kg}$$

8.4c The difference in exergy between both flows is:

$$\Delta\psi = \psi_1 - \psi_2 = 1190.6 - 932.9 = 257.7 \text{ kJ/kg}$$

This amount of exergy is lost. The exergy loss is the price we pay for smoothing out the variation of input energy rate.

8.5 Ammonia compressor

The reference state used in this exercise is given by $T_0 = 25^\circ\text{C} = 298 \text{ K}$ and $P_0 = 101.3 \text{ kPa}$.

8.5a The isentropic efficiency of the compressor is given. It is related to the work as:

$$\eta_{s,c} = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow w_a = \frac{h_{2s} - h_1}{\eta_{s,c}}$$

The properties can be obtained from table 19s: $h_1 = 1470.50 \text{ kJ/kg}$, $s_1 = 6.3369 \text{ kJ/(kgK)} = s_{2s}$, $h_{2s} = 1599.53 \text{ kJ/kg}$. Hence, the actual work of the compressor is:

$$w_a = \frac{h_{2s} - h_1}{\eta_{s,c}} = \frac{1599.53 - 1470.50}{0.7} = 184.33 \text{ kJ/kg}$$

8.5b The reversible work is calculated as:

$$(w_{out} - w_{in})_{rev} = q_{in,rev} \left(1 - \frac{T_0}{T_{in}}\right) - q_{out,rev} \left(1 - \frac{T_0}{T_{out}}\right) + \left(\psi + \frac{v^2}{2} + gz\right)_{in} - \left(\psi + \frac{v^2}{2} + gz\right)_{out}$$

The compressor is adiabatic (such that $q_{in,rev} = q_{out,rev} = 0$) and we neglect potential and kinetic energy differences. The formula for the reversible work is therefore reduced to:

$$(w_{out} - w_{in})_{rev} = \psi_1 - \psi_2$$

The flow exergy is calculated as $\psi = h - h_0 - T_0(s - s_0)$. This can be substituted in the (reduced) equation for the reversible work, giving:

$$(w_{out} - w_{in})_{rev} = h_1 - h_0 - T_0(s_1 - s_0) - [h_2 - h_0 - T_0(s_2 - s_0)] = h_1 - h_2 - T_0(s_1 - s_2)$$

The actual enthalpy at state 2 is:

$$h_2 = h_1 + w_a = 1470.50 + 184.33 = 1654.83 \text{ kJ/kg}$$

The entropy at state 2 can now be found by interpolating with the enthalpy at the correct pressure in table 19s: $s_2 = 6.501 \text{ kJ/(kgK)}$. Now, the reversible work is calculated as:

$$(w_{out} - w_{in})_{rev} = h_1 - h_2 - T_0(s_1 - s_2) = 1470.50 - 1654.83 - 298(6.337 - 6.501) = -135.46 \text{ kJ/kg}$$

The negative sign denotes that there is a net input of reversible work.

8.5c From table 19s, the enthalpy and the entropy at the reference state are determined to be: $h_0 = 1546.57 \text{ kJ/kg}$, $s_0 = 6.6024 \text{ kJ/(kgK)}$. The flow exergy of the ammonia entering the compressor is thus:

$$\psi_{in} = h_1 - h_0 - T_0(s_1 - s_0) = 1470.50 - 1546.57 - 298(6.3369 - 6.6024) = 3.049 \text{ kJ/kg}$$

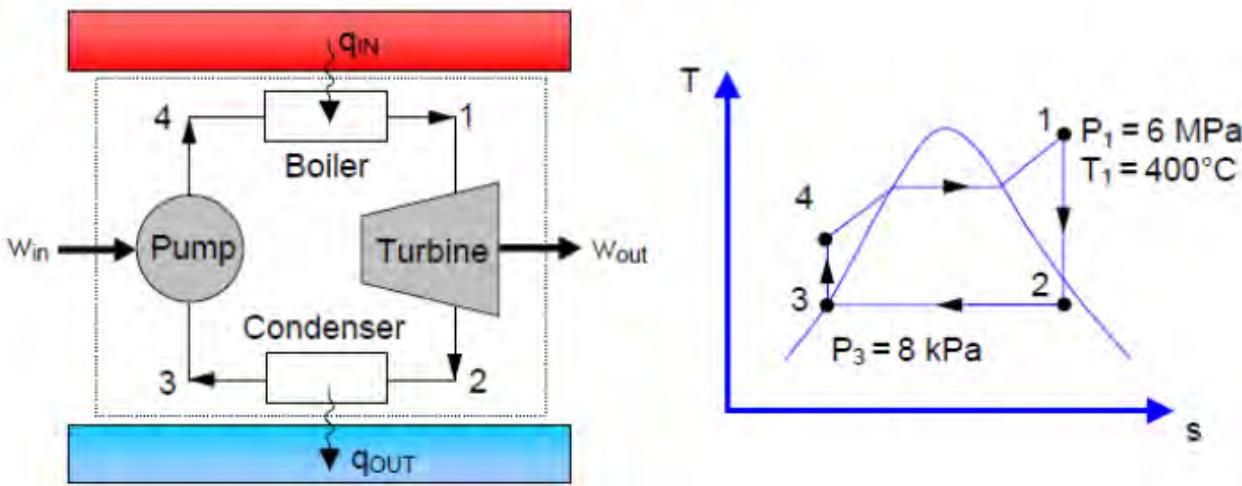
8.5d The flow exergy of the ammonia leaving the compressor is:

$$\psi_{out} = h_2 - h_0 - T_0(s_2 - s_0) = 1654.83 - 1546.57 - 298(6.501 - 6.6024) = 138.48 \text{ kJ/kg}$$

The reversible work computed from the exergies is slightly higher than the isentropic work ($h_{2s} - h_1$) because the outlet exergy is based on the actual exit temperature whereas the isentropic work is based on the isentropic exit temperature. Hence the exit fluid has more work-producing potential than it would at the isentropic exit temperature.

8.6 Rankine steam cycle

Given is the Rankine cycle as shown in the figure:



We want to find the second law efficiency, which is calculated as:

$$\eta_{secondlaw} = \frac{(w_{out} - w_{in})_a}{(w_{out} - w_{in})_{rev}}$$

Therefore, we first need to determine the actual nett work of the cycle, which is done by performing an energy analysis of the Rankine cycle. The nett work of the cycle can be expressed as:

$$(w_{out} - w_{in})_a = w_t - w_p = (h_1 - h_2) - v_3(P_4 - P_3)$$

With the pressure and the temperature at point 1 known, we can find in the tables that: $h_1 = 3177.0 \text{ kJ/kg}$ and $s_1 = 6.5404 \text{ kJ/(kgK)}$. Since the turbine is isentropic, $s_2 = s_1$ and hence we have the required two known parameters at point 2 as well (entropy and pressure). From the saturated water tables, we find that $h_2 = 2045.40 \text{ kJ/kg}$. At point 3, we can find the properties of saturated liquid at $P_3 = 8 \text{ kPa}$, giving $h_3 = 173.85 \text{ kJ/kg}$ and $v_3 = 0.001008 \text{ m}^3/\text{kg}$. Hence, the actual nett work output of the Rankine cycle is:

$$(w_{out} - w_{in})_a = (h_1 - h_2) - v_3(P_4 - P_3) = (3177.0 - 2045.4) - 0.001008(6000 - 8) = 1125.6 \text{ kJ/kg}$$

Now we will determine the reversible work output of the cycle. The reversible work is calculated as:

$$(w_{out} - w_{in})_{rev} = q_{in,rev} \left(1 - \frac{T_0}{T_{in}} \right) - q_{out,rev} \left(1 - \frac{T_0}{T_{out}} \right) + \left(\psi + \frac{v^2}{2} + gz \right)_{in} - \left(\psi + \frac{v^2}{2} + gz \right)_{out}$$

The flow exergy terms are zero because there is no mass crossing the control volume boundary (where the whole system is the control volume). Now, neglecting potential and kinetic energy differences yields:

$$(w_{out} - w_{in})_{rev} = q_{in,rev} \left(1 - \frac{T_0}{T_{in}} \right) - q_{out,rev} \left(1 - \frac{T_0}{T_{out}} \right)$$

T_{in} is the saturation temperature at the operating pressure in the boiler, while T_{out} is the saturation temperature in the condenser at P_3 : $T_{sat}(P_1) = 275.62^\circ\text{C} = 548.62\text{ K}$ and $T_{sat}(P_3) = 41.518^\circ\text{C} = 314.518\text{ K}$. Hence, the reversible work output is:

$$(w_{out} - w_{in})_{rev} = (h_1 - h_4) \left(1 - \frac{T_0}{T_{sat}(P_1)} \right) - (h_2 - h_3) \left(1 - \frac{T_0}{T_{sat}(P_3)} \right)$$

At state 4, we know that $s_4 = s_3$ (since the pump is isentropic), such that $s_4 = s_3 = s_L(P = 8\text{ kPa}) = 0.5925\text{ kJ/(kgK)}$. With $P_4 = 6000\text{ kPa}$, this results in $h_4 = 179.66\text{ kJ/kg}$ (interpolated from table 13s). So, the reversible work is:

$$\begin{aligned} (w_{out} - w_{in})_{rev} &= (h_1 - h_4) \left(1 - \frac{T_0}{T_{sat}(P_1)} \right) - (h_2 - h_3) \left(1 - \frac{T_0}{T_{sat}(P_3)} \right) \\ &= (3177.0 - 179.66) \left(1 - \frac{298}{548.62} \right) - (2045.40 - 173.85) \left(1 - \frac{298}{314.518} \right) \\ &= 1270.6\text{ kJ/kg} \end{aligned}$$

Finally, we can calculate the second law efficiency:

$$\eta_{secondlaw} = \frac{(w_{out} - w_{in})_a}{(w_{out} - w_{in})_{rev}} = \frac{1125.6}{1270.6} = 0.886$$

The conventional first law efficiency for this system,

$$\eta_{th} = \frac{(w_{out} - w_{in})_a}{q_{in}} = \frac{(w_{out} - w_{in})_a}{h_1 - h_4} = 0.376$$

is substantially lower than the second law efficiency. The two efficiencies, while somewhat related, are not the same. One measures actual work derived from a given amount of heat input, while the other simply compares the actual work with what could have been derived from a perfect cycle. The first law efficiency is the standard measure of a power system, since work output and heat output have very realistic economic meaning. The second law efficiency is less commonly used and is primarily of interest in theoretical evaluation of sources of loss in the system.

8.7 Solar pond

8.7a The flow exergy is calculated as $\psi = h - h_0 - T_0(s - s_0)$. For the reference state, we use $P_0 = 0.1013\text{ MPa}$ and $T_0 = 25^\circ\text{C} = 298\text{ K}$. The enthalpy and the entropy of the reference state can be found in table 10s: $h_0 = h_L(T = 25^\circ\text{C}) = 104.75\text{ kJ/kg}$ and $s_0 = s_L(T = 25^\circ\text{C}) = 0.3670\text{ kJ/(kgK)}$. The pressure at the bottom of the pond (state 1) is:

$$P_1 = P_{atm} + \frac{gz_1}{v_1} = 101.3 \cdot 10^3 + \frac{9.81 \cdot 3}{8.83 \cdot 10^{-4}} = 134.63\text{ kPa}$$

(pay attention with the units, as the term $\frac{gz_1}{v_1}$ gives a pressure in Pa when SI units are used.) The enthalpy and the entropy at state 1 are obtained from the tables as saturated liquid at P_1 : $h_1 = h_L(P = 134.63\text{ kPa}) = 453.16\text{ kJ/kg}$ and $s_1 = s_L(P = 134.63\text{ kPa}) = 1.3971\text{ kJ/(kgK)}$. Now, the flow exergy is:

$$\psi_1 = h_1 - h_0 - T_0(s_1 - s_0) = 453.16 - 104.75 - 298(1.3971 - 0.3670) = 41.44\text{ kJ/kg}$$

8.7b The reversible work is given by:

$$(w_{out} - w_{in})_{rev} = q_{in,rev} \left(1 - \frac{T_0}{T_{in}} \right) - q_{out,rev} \left(1 - \frac{T_0}{T_{out}} \right) + \left(\psi + \frac{v^2}{2} + gz \right)_{in} - \left(\psi + \frac{v^2}{2} + gz \right)_{out}$$

The system is adiabatic, we neglect kinetic energy differences and the potential energy of the input is zero. Hence, the equation is reduced to:

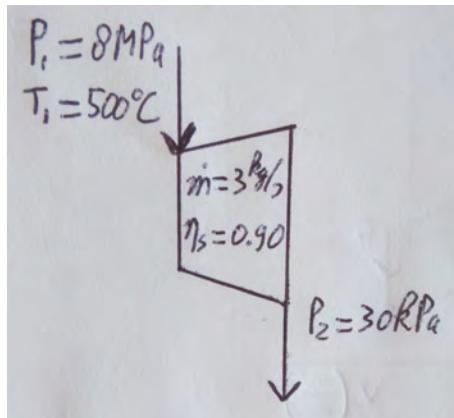
$$(w_{out} - w_{in})_{rev} = \psi_{in} - (\psi + gz)_{out}$$

Since ψ_{out} is at the dead state, it equals 0. Thus, the reversible work is:

$$(w_{out} - w_{in})_{rev} = \psi_{in} - (gz)_{out} = 41.44 \cdot 10^3 - 9.81 \cdot 3 = 41.41 \text{ kJ/kg}$$

The additional z term (potential energy) represents the reversible work required to lift the water from the bottom to the surface of the pond.

8.8 Turbine



	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))
1	8000	500	3398.3	6.724
2s	30		2274.9	6.724
2	30	69.10	2387.2	7.059
0	100	20	83.96	0.2966

0 is the reference point for exergy.

8.8a First, we need to determine the enthalpy and entropy values. At the inlet, we have superheated vapour. From table A6, we find that $h_1 = 3398.3 \text{ kJ/kg}$ and $s_1 = 6.724 \text{ kJ/kg}$. For the outlet, we first need to determine the ideal outlet conditions: $s_{2s} = s_1$. At $P_2 = 30 \text{ kPa}$, we find that $s_f = 0.9439$ and $s_g = 7.7686 \text{ kJ/(kgK)}$, so we are in the mixture region. $x = \frac{s_{2s}-s_f}{s_g-s_f} = \frac{6.724-0.9439}{7.7686-0.9439} = 0.85$. So, $h_{2s} = h_f + x(h_g - h_f) = 289.23 + 0.85(2625.3 - 283.23) = 2274.89 \text{ kJ/kg}$. From the isentropic efficiency, we now find the actual enthalpy: $h_2 = h_1 - \eta_s(h_1 - h_{2s}) = 2387.23 \text{ kJ/kg}$. From the tables, we now find that $s_2 = 7.059 \text{ kJ/kg}$. The Mollier diagram is given in figure 8.8.1.

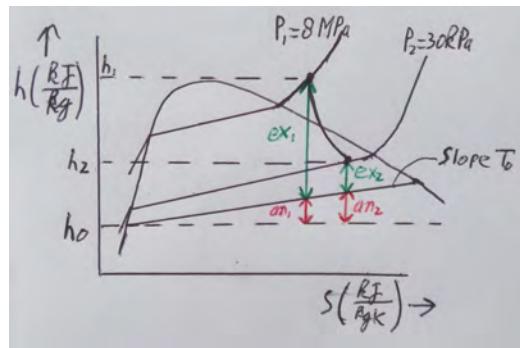


Figure 8.8.1: Mollier diagram of exercise 8.8. Note that the reference line for exergy has a slope T_0 (see also the exergy lines in the Mollier diagram)

8.8b The exergy at the inlet is:

$$\Psi_1 = \dot{m}((h_1 - h_0) - T_0(s_1 - s_0)) = 3((3398.3 - 83.96) - 293(6.724 - 0.2966)) = 4293.3 \text{ kJ/s}$$

The exergy at the outlet is:

$$\Psi_2 = \dot{m}((h_2 - h_0) - T_0(s_2 - s_0)) = 3((2387.2 - 83.96) - 342.1(2387.2 - 83.96)) = 965.6 \text{ kJ/s}$$

8.8c The Sankey diagram is a diagram with energy flows. Energy is conserved, so the total arrow remains the same thickness. See figure 8.8.2.

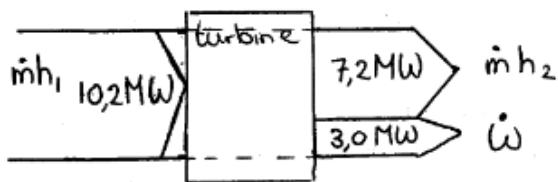


Figure 8.8.2: Sankey diagram of exercise 8.8

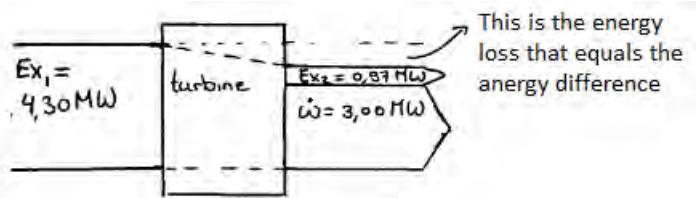
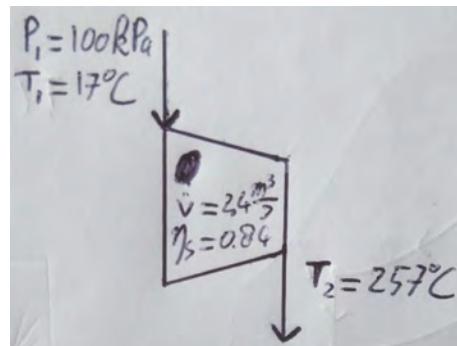


Figure 8.8.3: Grassmann diagram of exercise 8.8

8.8d See figure 8.8.3. The Grassmann diagram is the diagram with exergy flows. Exergy is not conserved, so the total thickness of the arrows becomes smaller.

8.9 Compressor



	P (bar)	T ($^\circ\text{C}$)	h (kJ/kg)	s (kJ/(kgK))
0=1	1	17	290	6.83
2s	7		496	6.83
2	7	257	535	6.90

0 is the reference point for exergy.

8.9a See figure 8.9.1. Since the inlet conditions are chosen as the reference state, the exergy and anergy at the inlet are 0.

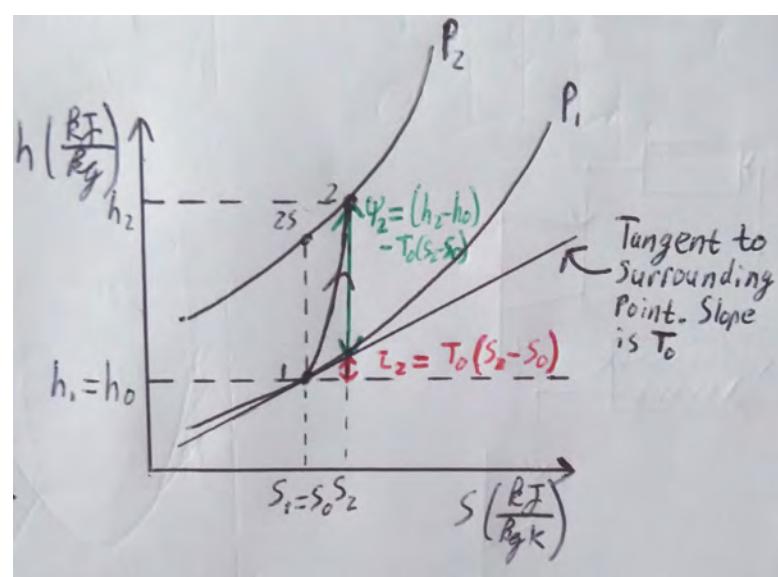


Figure 8.9.1: Mollier diagram (h-s diagram) for exercise 8.9

8.9b To determine the exergy at the outlet, the conditions at the outlet need to be determined. First of all, we read h_1 and s_1 from the Mollier diagram for air: $h_1 = 290 \text{ kJ/kg}$ and $s_1 = 6.83 \text{ kJ/(kgK)}$. For the second point, we only know the temperature. Since the temperature of an ideal compressor is not equal to the non-ideal compressor, $T_{2s} \neq T_2$ (unlike the pressure which is usually given), the output is harder to determine. However, since the isotherms are horizontal in the region of interest, it is possible to determine the actual output enthalpy: $h_2 = 535 \text{ kJ/kg}$. Using the isentropic efficiency, we can now determine the enthalpy for the ideal compressor: $h_{2s} = h_1 + \eta_s(h_{2a} - h_1) = 496 \text{ kJ/kg}$. Using this value and $s_{2s} = s_1$, it is now possible to determine the output pressure: $P_{2s} = P_2 = 7 \text{ bar}$. This gives for the actual entropy output the value $s_2 = 6.90 \text{ kJ/(kgK)}$. Now, we can calculate the exergy at the outlet (at the inlet it is 0, as it is chosen as the reference state):

$$\psi_2 = (h_2 - h_0) - T_0(s_2 - s_0) = 224.7 \frac{\text{kJ}}{\text{kgK}}$$

$$\Psi_2 = \dot{m}\psi_2 = \rho\dot{v}\psi_2 = 1.22 \cdot 2.4 \cdot 224.7 = 658 \text{ kW}$$

The anergy at the outlet is equal to:

$$\text{Anergy}_2 = T_0(s_2 - s_0) = 20.3 \frac{\text{kJ}}{\text{kg}}$$

$$\text{ANERGY}_2 = \dot{m}\text{Anergy}_2 = \rho\dot{v}\text{Anergy}_2 = 60 \text{ kW}$$

8.9c The incoming energy is $\dot{E}_{in} = \dot{m}h_1 = 849.7 \text{ kW}$ and the outgoing energy is $\dot{E}_{out} = \dot{m}h_2 = 1567.6 \text{ kW}$.

The work is $\dot{W} = \dot{m}(h_2 - h_1) = 717.9 \text{ kW}$.

Energy balance: $\dot{E}_{in} + \dot{W} = \dot{E}_{out}$, which is indeed the case. See the diagram in figure 8.9.2.

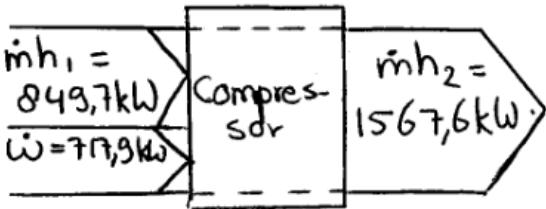


Figure 8.9.2: Sankey diagram exercise 8.9

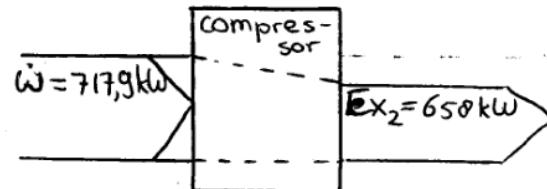
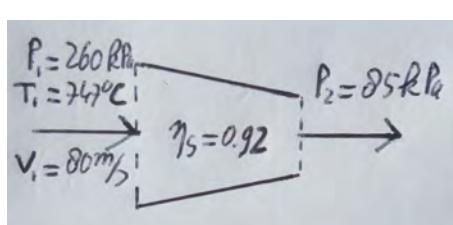


Figure 8.9.3: Grassmann diagram exercise 8.9

8.9d See figure 8.9.3. The deficit on the right side is the loss in exergy, or the production in anergy.

8.10 Nozzle



	$P \text{ (kPa)}$	$T \text{ (}^\circ\text{C)$	$h \left(\frac{\text{kJ}}{\text{kg}} \right)$	$s \left(\frac{\text{kJ}}{\text{kgK}} \right)$	$v \left(\frac{\text{m}}{\text{s}} \right)$
1	260	747	1070	7.90	80
2s	85	495	790	7.90	
2	85	515	812	7.95	722
0	100	17	290	6.83	0

0 is the reference point for exergy.

The enthalpy and entropy at point 1 is determined by looking in the Mollier diagram for air at $P_1 = 260 \text{ kPa}$ and $T_1 = 747^\circ\text{C}$. This gives $h_1 = 1070 \text{ kJ/kg}$ and $s_1 = 7.90 \text{ kJ/kg}$. Since $s_{2s} = s_1$ and $P_2 = P_{2s} = 85 \text{ kPa}$, it is now possible to determine that $h_{2s} = 790 \text{ kJ/kg}$. The isentropic efficiency of

a nozzle is given as $\eta_s = \frac{h_1 - h_2}{h_1 - h_{2s}}$, such that h_2 can be determined: $h_2 = h_1 - \eta_s(h_1 - h_{2s}) = 812 \text{ kJ/kg}$. Using this value and $P_2 = 85 \text{ kPa}$, it is found that $s_2 = 7.95 \text{ kJ/(kgK)}$. The outlet velocity follows from the energy balance over the nozzle:

$$\dot{m} \left[\left(h_1 + \frac{1}{2}v_1^2 + gz_1 \right) - \left(h_2 + \frac{1}{2}v_2^2 + gz_2 \right) \right] = \dot{Q} + \dot{W} \quad (8.10.0)$$

Neglecting potential differences ($z_1 = z_2$), applying that the nozzle does not deliver work ($\dot{W} = 0$) and assuming that the nozzle is adiabatic ($\dot{Q} = 0$) then results in $h_1 - h_2 = \frac{1}{2}(v_2^2 - v_1^2)$, from which it follows that $v_2 = \sqrt{2(h_1 - h_2) + v_1^2} = 722 \text{ m/s}$.

The reference state is assumed to have a pressure of $P_0 = 100 \text{ kPa}$ and a temperature $T_0 = 17^\circ\text{C}$. The other values can then be found in the Mollier diagram.

8.10a See figure 8.10.1.

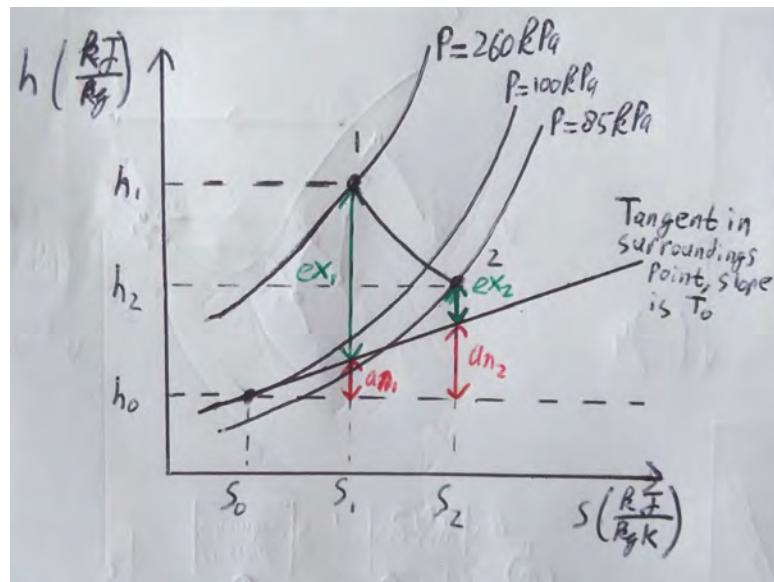


Figure 8.10.1: Mollier diagram (h - s diagram) for exercise 8.10

8.10b The kinetic energy should be taken into account in the exergy. It is fully mechanical energy and therefore 100% exergy. It is independent of the temperature and pressure.

At the inlet:

$$ex_1 = (h_1 - h_0) - T_0(s_1 - s_0) + \frac{1}{2}v_1^2 = 472900 \text{ J/kg} = 472.9 \text{ kJ/kg}$$

$$an_1 = T_0(s_1 - s_0) = 310.3 \text{ kJ/kg}$$

At the outlet:

$$ex_1 = (h_2 - h_0) - T_0(s_2 - s_0) + \frac{1}{2}v_2^2 = 457842 \text{ J/kg} = 457.8 \text{ kJ/kg}$$

$$an_1 = T_0(s_s - s_0) = 324.8 \text{ kJ/kg}$$

8.10c The Sankey diagram is the diagram with energy flows. For the nozzle, we need to take into account the velocities, besides the enthalpies (see energy balance in equation 8.10). Therefore, the incoming energy is equal to $\dot{E}_1 = h_1 + \frac{1}{2}v_1^2 = 1070 \cdot 10^3 + 3200 = 1073200 \text{ W} = 1.073 \text{ MW}$ and the outgoing energy is $\dot{E}_2 = h_2 + \frac{1}{2}v_2^2 = 812 \cdot 10^3 + 260642 = 1.073 \text{ MW}$. It can be seen that the kinetic energy at the inlet is very small. See the Sankey diagram in figure 8.10.2.

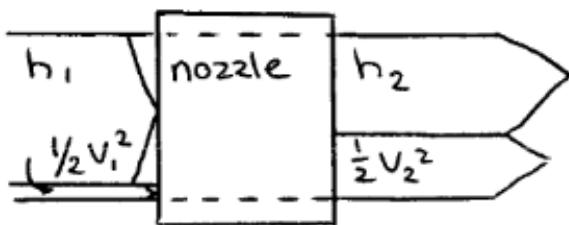


Figure 8.10.2: Sankey diagram exercise 8.10

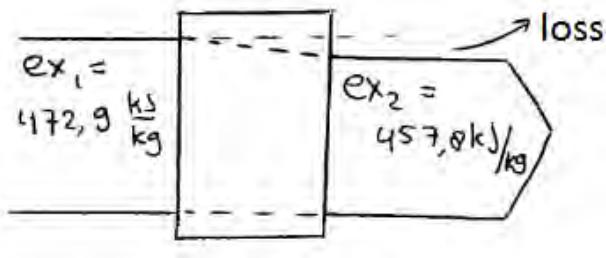
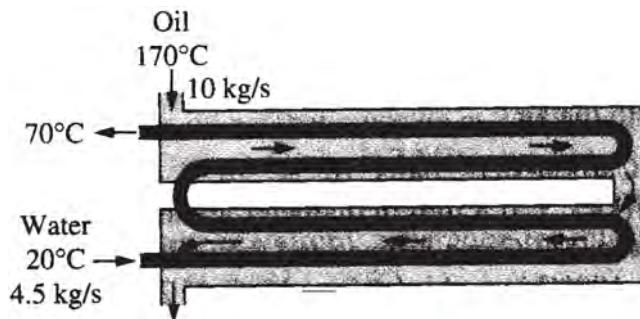


Figure 8.10.3: Grassmann diagram exercise 8.10

8.10d The Grassmann diagram is the diagram with exergy flows. See figure 8.10.3.

8.11 Heat Exchanger

Schematic overview of the heat exchanger:



8.11a The heat (energy) is transferred from the oil to the water. There is conservation of energy, so $\Delta E_{oil} = \Delta E_{water}$. Since kinetic and potential energy are negligible, it holds that $\Delta H_{oil} = \Delta H_{water}$. Using $\Delta H = \dot{m}C_p\Delta T$, this results in:

$$\begin{aligned} C_{p,o}(T_{in,o} - T_{out,o}) &= \dot{m}_w C_{p,w}(T_{in,w} - T_{out,w}) \\ T_{out,o} &= \frac{-\dot{m}_w}{\dot{m}_o} \frac{C_{p,w}}{C_{p,o}} (T_{out,w} - T_{in,w}) + T_{in,o} \\ T_{out,o} &= \frac{-4.5 \cdot 4.18}{10 \cdot 2.30} (70 - 20) + 170 = 129 \text{ }^\circ\text{C} \end{aligned}$$

8.11b Use $dh = C_p dT$, which is (from energy conservation) equal to dq . For reversible heat transfer, $dq = Tds$, such that, when all relations are combined, we find that $ds = \frac{C_p}{T}dT$, or $d\dot{S} = \frac{\dot{m}C_p}{T}dT$. Work this out for both the water and the oil, and add them up to find the total entropy change:

$$\begin{aligned} \dot{S}_w &= \int_{T_{in,w}}^{T_{out,w}} \frac{\dot{m}_w C_{p,w}}{T} dT = \dot{m}_w C_{p,w} \ln \left(\frac{T_{out,w}}{T_{in,w}} \right) = 4.5 \cdot 4.18 \ln \left(\frac{70 + 273}{20 + 273} \right) = 2.96 \frac{\text{kJ}}{\text{s}} \\ \dot{S}_o &= \int_{T_{in,o}}^{T_{out,o}} \frac{\dot{m}_o C_{p,o}}{T} dT = \dot{m}_o C_{p,o} \ln \left(\frac{T_{out,o}}{T_{in,o}} \right) = 10 \cdot 2.30 \ln \left(\frac{129 + 273}{170 + 273} \right) = -2.23 \frac{\text{kJ}}{\text{s}} \\ \Delta\dot{S}_{total} &= \Delta\dot{S}_w + \Delta\dot{S}_o = 2.96 - 2.23 = 0.73 \frac{\text{kJ}}{\text{s}} \end{aligned}$$

So, as expected, the total entropy increases.

8.11c The exergy is given as $ex = (h_1 - h_0) - T_0(s_1 - s_0) = C_p(T_1 - T_0) - T_0C_p \ln \frac{T_1}{T_0}$. See for the last part also b, where you worked out that $s_2 - s_1 = C_p \ln \frac{T_2}{T_1}$. For this assignment, we choose the temperature of the surroundings as $T_0 = 20^\circ\text{C}$.

$$ex_{out,w} = \dot{m}_w C_{p,w} \left(T_{out,w} - T_0 - T_0 \ln \frac{T_{out,w}}{T_0} \right) = 4.5 \cdot 4.18 \left(343 - 293 - 293 \ln \frac{343}{293} \right) = 72.1 \text{ kW}$$

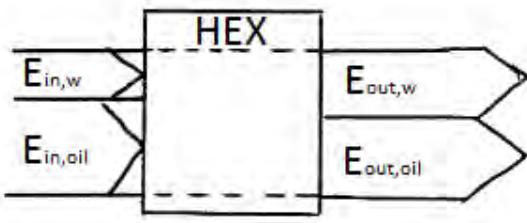
$$ex_{in,w} = \dot{m}_w C_{p,w} \left(T_{in,w} - T_0 - T_0 \ln \frac{T_{in,w}}{T_0} \right) = 0 \text{ kW}$$

$$ex_{out,o} = \dot{m}_o C_{p,o} \left(T_{out,o} - T_0 - T_0 \ln \frac{T_{out,o}}{T_0} \right) = 10 \cdot 2.30 \left(402 - 293 - 293 \ln \frac{402}{293} \right) = 375.6 \text{ kW}$$

$$ex_{in,o} = \dot{m}_o C_{p,o} \left(T_{in,o} - T_0 - T_0 \ln \frac{T_{in,o}}{T_0} \right) = 10 \cdot 2.30 \left(443 - 293 - 293 \ln \frac{443}{293} \right) = 664.1 \text{ kW}$$

Note that the exergy of the incoming water equals 0 because $T_{in,w} = T_0$.

8.11d The Sankey diagram is the diagram with energy flows. Energy is conserved, so $\dot{E}_{in} = \dot{E}_{out}$. See figure 8.11.1.



$$\dot{E}_{in,w} = \dot{m}_w C_{p,w} T_{in,w} = 5.51 \text{ MW}$$

$$\dot{E}_{out,w} = \dot{m}_w C_{p,w} T_{out,w} = 6.45 \text{ MW}$$

$$\dot{E}_{in,o} = \dot{m}_o C_{p,o} T_{in,o} = 10.19 \text{ MW}$$

$$\dot{E}_{out,o} = \dot{m}_o C_{p,o} T_{out,o} = 9.2 \text{ MW}$$

Figure 8.11.1: Sankey diagram of exercise 8.11

Check: $\dot{E}_{in,tot} = 15.7 \text{ MW}$, $\dot{E}_{out,tot} = 15.7 \text{ MW}$, so energy is indeed conserved.

8.11e The Grassmann diagram is the diagram with exergy flows. Exergy is not conserved, so the total width of the arrows is smaller at the outlet than at the inlet. See figure 8.11.2.

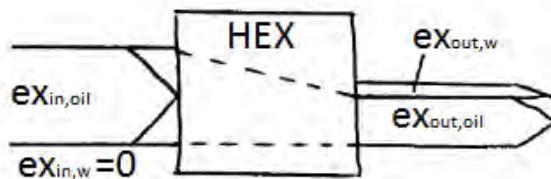


Figure 8.11.2: Grassmann diagram of exercise 8.11

8.12 Second Law Analysis of an actual Rankine cycle

First the results of the first law analysis are summarized:

Table 8.12.1: Summary of the results of exercise 10.4.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))
1	10	45.81	191.83	0.6494
2	10000		203.7	0.6549
3	10000	500	3373.7	6.5976
4	10	45.81	2298.3	7.2535

$$\dot{m} = 197.5 \text{ kg/s}$$

$$\dot{W}_{out,t} = \dot{m}(h_3 - h_4) = 197.5(3373.7 - 2298.3) = 212.4 \text{ MW}$$

$$\dot{W}_{in,p} = \dot{m}(h_2 - h_1) = 197.5(203.7 - 191.83) = 2.3 \text{ MW}$$

$$\dot{W}_{nett} = \dot{W}_{out,t} - \dot{W}_{in,p} = 212.4 - 2.3 = 210.1 \text{ MW}$$

$$\dot{Q}_{in,boiler} = \dot{m}(h_3 - h_2) = 197.5(3373.7 - 203.7) = 626.1 \text{ MW}$$

$$\dot{Q}_{out,condenser} = \dot{m}(h_4 - h_1) = 197.5(2298.3 - 191.83) = 416.0 \text{ MW}$$

$$\dot{Q}_{nett} = \dot{Q}_{in,boiler} - \dot{Q}_{out,condenser} = 626.1 - 416.0 = 210.1 \text{ MW}$$

$$\eta_{th} = \frac{\dot{W}_{out,t} - \dot{W}_{in,p}}{\dot{Q}_{in,boiler}} = \frac{210.0}{626.1} = 33.5\%$$

Note that $\dot{W}_{nett} = \dot{Q}_{nett}$ and hence the first law of thermodynamics is satisfied.

8.12a As is commonly done, we choose to use the surrounding conditions as reference point: $T_0 = 20^\circ\text{C} = 293 \text{ K}$ and $P_0 = 1 \text{ bar} = 100 \text{ kPa}$, resulting in $h_0 = 84.01 \text{ kJ/kg}$ and $s_0 = 0.2968 \text{ kJ/(kgK)}$.

8.12b The energy flow in a water or vapour flow can be calculated by multiplying the mass flow with the specific enthalpy: $\dot{E} = \dot{m}h$. This results in the following energy flows:

$$\dot{E}_1 = \dot{m}h_1 = 197.5 \cdot 191.83 = 37.9 \text{ MW}$$

$$\dot{E}_2 = \dot{m}h_2 = 197.5 \cdot 203.7 = 40.2 \text{ MW}$$

$$\dot{E}_3 = \dot{m}h_3 = 197.5 \cdot 3373.7 = 666.3 \text{ MW}$$

$$\dot{E}_4 = \dot{m}h_4 = 197.5 \cdot 2298.3 = 453.9 \text{ MW}$$

8.12c The exergy can be calculated from the enthalpy and the entropy as: $\psi(h - h_0) - T_0(s - s_0)$. The exergy flows are obtained by multiplying this with the mass flow:

$$\dot{\psi}_1 = \dot{m}[(h_1 - h_0) - T_0(s_1 - s_0)] = 197.5[(191.83 - 84.01) - 293(0.6494 - 0.2968)] = 0.9 \text{ MW}$$

$$\dot{\psi}_2 = \dot{m}[(h_2 - h_0) - T_0(s_2 - s_0)] = 197.5[(203.7 - 84.01) - 293(0.6549 - 0.2968)] = 2.9 \text{ MW}$$

$$\dot{\psi}_3 = \dot{m}[(h_3 - h_0) - T_0(s_3 - s_0)] = 197.5[(3373.7 - 84.01) - 293(6.5976 - 0.2968)] = 285.1 \text{ MW}$$

$$\dot{\psi}_4 = \dot{m}[(h_4 - h_0) - T_0(s_4 - s_0)] = 197.5[(2298.3 - 84.01) - 293(7.2535 - 0.2968)] = 34.8 \text{ MW}$$

Furthermore, we need to know the exergy going into or out of the boiler, the turbine, the condenser and the pump. The exergy in a flow with constant temperature can be calculated with $\dot{\psi} = \dot{Q} \left(1 - \frac{T_0}{T}\right)$. This is used to calculate the exergy that is given to the cooling water in the condenser:

$$\dot{\psi}_{condenser} = \dot{Q}_{out,condenser} \left(1 - \frac{T_0}{T}\right) = 416.0 \left(1 - \frac{293}{45.81 + 273}\right) = 33.7 \text{ MW}$$

For the boiler, we assume that we need to add fuel with 6% more energy than used in the boiler, as we will loss some exergy during combustion:

$$\dot{\psi}_{boiler} = 1.06 \dot{Q}_{boiler} = 1.06 \cdot 626.1 = 663.7 \text{ MW}$$

The amount of destroyed exergy in a process is calculated by $\dot{\psi}_{destroyed} = \dot{m} (T_0 \cdot s_{gen})$. This is used to calculate the destroyed exergy in the turbine and the pump:

$$\dot{\psi}_{des,t} = \dot{m} (T_0 (s_4 - s_3)) = 197.5 (293 (7.2535 - 6.5976)) = 38 \text{ MW}$$

$$\dot{\psi}_{des,p} = \dot{m} (T_0 (s_2 - s_1)) = 197.5 (293 (0.6549 - 0.6494)) = 0.3 \text{ MW}$$

The work coming out of the turbine, or going into the pump, is already the exergy flow, since work is 100% exergy.

8.12d The Carnot efficiency is:

$$\eta_{carnot} = 1 - \frac{T_{max}}{T_{min}} = 1 - \frac{20 + 273}{500 + 273} = 0.621$$

The second law efficiency is:

$$\eta_{secondlaw} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{0.335}{0.621} = 0.539$$

8.12e A second law efficiency 53.9% means that the cycle has a performance of almost 54% of its theoretically best possible performance, with is the Carnot efficiency. The thermal efficiency is 33.5%. The second law efficiency is higher as in that case the cycle is compared to the maximum theoretical efficiency (Carnot efficiency) which is always lower than 100% and depends on the highest and lowest temperature in the cycle. The thermal efficiency does not take into account that a cycle can never have an efficiency of 100% (due to restrictions of the second law). In this case, the second law efficiency is shows that the cycle still can be improved by almost a factor of 2.

8.12f The Sankey diagram is shown in figure 8.12.1. Note: Use different colors for different categories of flows. Flows are conserved, before and after a device the total amount of energy should be the same. Give the amount in absolute values and in percentages. The heat input is often taken as 100%. This way you see immediately the thermal efficiency of the cycle, in this case 33.5% (which also follows from the calculations).

8.12g The Grassmann diagram is shown in figure 8.12.2. Note: Use the same structure, lay-out and colors as in the Sankey diagram. On the contrary to the Sankey diagram the exergy flows are NOT conserved. Before and after a device the values are not the same. Give the amount of exergy in absolute values and in percentages. The heat input is often taken as 100%. Absolute values of work are equal to those in the Sankey diagram.

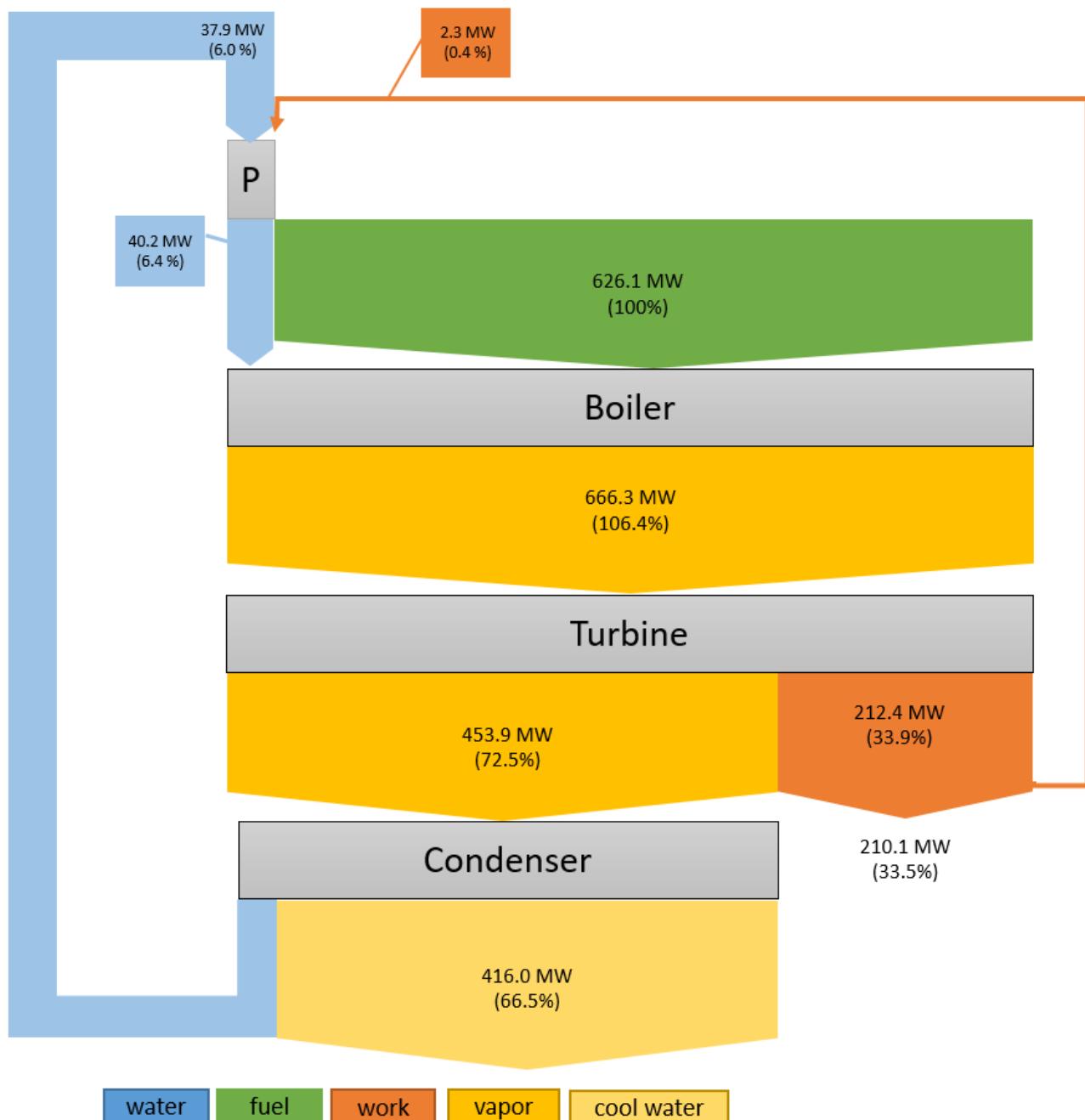


Figure 8.12.1: Sankey diagram of exercise 8.12.

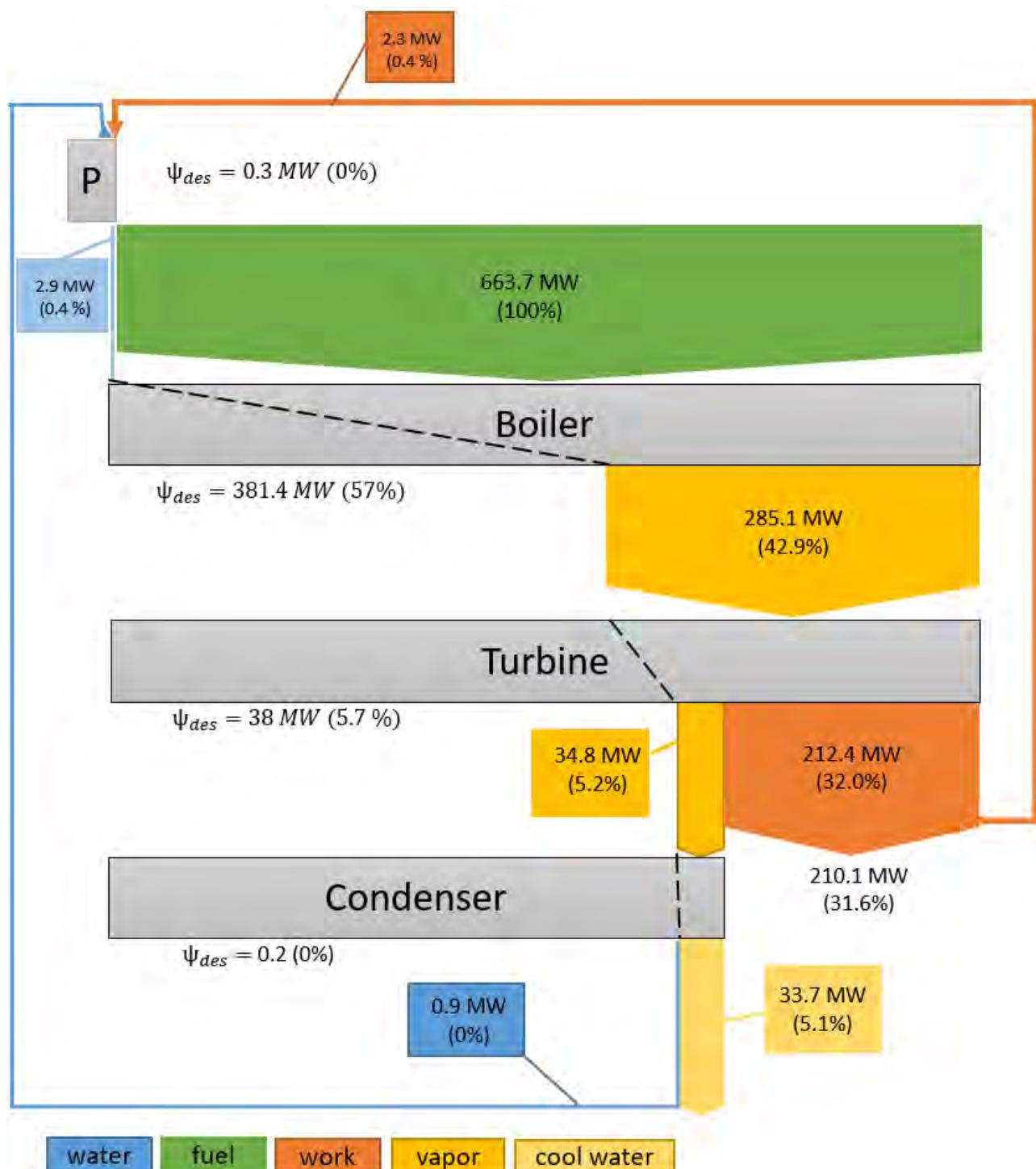


Figure 8.12.2: Grassmann diagram of exercise 8.12.

8.12h Both diagrams can be put upon each other. Doing so it is clear where losses in the system occur that could have been used useful. The work output of the turbine and the work input of the pump have the same absolute value in both diagrams, however the percentages are different.

The Sankey diagram shows a large amount of energy lost in the condenser (416 MW, 66.5%). However, in the Grassmann diagram can be seen that that the exergy of this energy flow is much lower (33.7 MW, 5.1%). So the energy loss in the condenser does not contain a lot of useful energy (exergy) anymore.

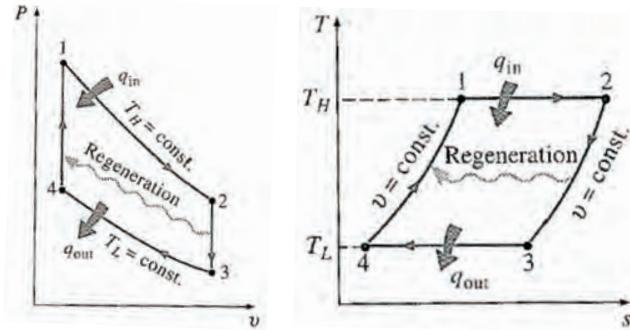
In the condenser only 0.1% is lost. The exergy lost in the turbine is 6.1%, this is because the turbine is not ideal and generates entropy. The exergy gain in the pump is 0.4%. There is an increase in exergy after the pump as work is supplied to the pump. The changes in exergy in the condenser, turbine and boiler are relative small. The largest exergy loss is in the boiler. The exergy lost in the boiler is 57.3% ($100.4 - 42.9 = 57.3\%$). This is a large amount. So improvements in the system should be made here rather than in the turbine or the condenser.

The Sankey and Grassmann diagram shows that the energy and exergy concerned with the pump process is small compared to the energy and exergy flows in the whole cycle. The pump work is almost negligible compared to the turbine output.

9 Gas Power Cycles

9.1 Basics of the Stirling cycle

9.1a The $P - v$ and $T - s$ diagram of the Stirling cycle, indicated are the constant volume and constant temperature processes as well as the heat transfer in and out the engine and the regenerated heat.



9.1b The Stirling cycle consists of four processes:

- 1 → 2 : isothermal expansion.
- 2 → 3 : constant volume heating.
- 3 → 4 : isothermal compression.
- 4 → 1 : constant volume cooling.

9.1c See figure at a.

9.1d The Stirling cycle is a closed system. There are two pistons in a cylinder device. In analyzing the heat is transferred over the system boundary into or out of the piston cylinder device by heat transfer. There is no mass transfer into or out of the piston cylinder device.

9.1e Heat addition and rejection occur isothermal, the second law for an isothermal process with heat transfer gives:

$$\delta q = T ds \rightarrow q_{2-1} = T_1 \Delta s_{2-1} = T_1(s_2 - s_1)$$

The entropy change follows form the Gibbs equations for entropy: $T ds = dh - v dP$. For an ideal gas, as is assumed in the Stirling cycle, it changes to $T ds = c_p dT - (RT)/P dP$. For the heat addition at constant $T_H = T_1 = T_2$ the entropy change Δs_{2-1} is given by

$$\Delta s_{2-1} = \int_1^2 ds = \int_{T_1}^{T_2} \frac{c_p}{T} dT - \int_{P_1}^{P_2} \frac{R}{P} dP = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) = -R \ln\left(\frac{P_2}{P_1}\right).$$

Similar follows for the entropy change Δs_{3-4} for heat rejection at constant $T_C = T_3 = T_4$,

$$\Delta s_{3-4} = \int_4^3 ds = \int_{T_4}^{T_3} \frac{c_p}{T} dT - \int_{P_4}^{P_3} \frac{R}{P} dP = c_p \ln\left(\frac{T_4}{T_3}\right) - R \ln\left(\frac{P_4}{P_3}\right) = -R \ln\left(\frac{P_4}{P_3}\right).$$

The heat added q_{in} and rejected q_{out} is

$$q_{in} = q_{2-1} = T_1 \Delta s_{2-1} = -T_1 R \ln\left(\frac{P_2}{P_1}\right) \quad \text{and} \quad q_{out} = q_{3-4} = T_3 \Delta s_{3-4} = -R T_3 \ln\left(\frac{P_4}{P_3}\right).$$

Work is delivered between 3 and 4 where the volume increases

$$w_{out} = w_{3-4} = \int_{v_3}^{v_4} P dv = RT \ln\left(\frac{v_4}{v_3}\right) = RT \ln\left(\frac{P_3}{P_4}\right).$$

The work input is between 1 and 2 where the volume decreases

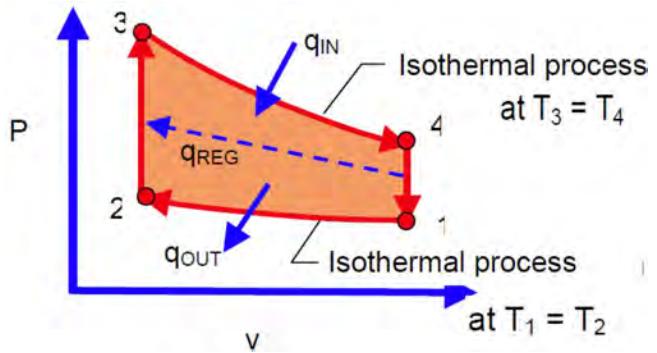
$$w_{in} = w_{1-2} = - \int_{v_1}^{v_2} P dv = -RT \ln\left(\frac{v_2}{v_1}\right) = -RT \ln\left(\frac{P_1}{P_2}\right).$$

As heat is added to the working fluid isothermally the efficiency of the Stirling cycle is equal to the efficiency of a Carnot cycle

$$\eta_{Stirling} = \eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{T_1}{T_3}.$$

9.2 Stirling cycle 1

9.2a Between state 1 and 2, the air is isothermally compressed. Heat is rejected at low temperature to maintain the temperature at T_1 . From state 2 to 3, heat is added isochorically. This heat is regenerated from the process between state 4 and 1. Between state 3 and 4, the air expands isothermally. Heat is added to keep the temperature constant. Between state 4 and 1, heat is rejected isochorically towards the regenerator.



9.2b First of all, we make a table with the two known (underlined) variables at each point.

	P (MPa)	T (K)	s (kJ/(kgK))	v (m^3/kg)	other
1	<u>3.5</u>	<u>423</u>			
2		<u>$T_2 = T_1$</u>			<u>$q_{out,41} = q_{in,23}$</u>
3		705		<u>$v_3 = v_2$</u>	<u>$\eta_s = 40\%$</u>
4		<u>$T_4 = T_3$</u>		<u>$v_4 = v_1$</u>	

To find the highest temperature, we set the thermal efficiency η_{th} equal to the Carnot Thermal Efficiency: $\eta = 1 - \frac{T_L}{T_H}$. This can be done because, for the ideal Stirling cycle, the thermal efficiency is the same as the Carnot efficiency since all the external heat input and heat rejection is assumed to occur at constant temperature. Hence:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

In order to determine q_{in} and q_{out} , we can use the inequality of Clausius: $ds \leq \frac{\delta q_{net}}{T_{res}}$. For reversible processes, it holds that $ds = \frac{\delta q_{net}}{T_{res}}$, such that $q_{out} = T_1(s_1 - s_2)$ and $q_{in} = T_3(s_4 - s_3)$ (note that in both processes the temperature is constant). Therefore, we can write:

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_1(s_1 - s_2)}{T_3(s_4 - s_3)}$$

This (the entropy differences) can be rewritten using the Gibbs equation $Tds = dh - vdP$, combined with the ideal gas relation $dh = c_p dT$, such that:

$$Tds = dh - vdP = Tds = c_p dT - vdP \Rightarrow ds = \frac{c_p}{T} dT - \frac{v}{T} dP$$

Using the ideal gas law, $\frac{v}{T}$ can be expressed in terms of P : $\frac{v}{T} = \frac{R}{P}$. Integrating from state 2 to state 1, this results in:

$$s_1 - s_2 = c_p \ln \left(\frac{T_1}{T_2} \right) - R \ln \left(\frac{P_1}{P_2} \right) = -R \ln \left(\frac{P_1}{P_2} \right)$$

Since the process is isothermal ($T_1 = T_2$). Similarly, $s_4 - s_3$ is determined to equal:

$$s_4 - s_3 = c_p \ln \left(\frac{T_4}{T_3} \right) - R \ln \left(\frac{P_4}{P_3} \right) = -R \ln \left(\frac{P_4}{P_3} \right)$$

Therefore, the thermal efficiency equals:

$$\eta_{th} = 1 - \frac{T_1(s_1 - s_2)}{T_3(s_4 - s_3)} = 1 - \frac{T_1(-R \ln \left(\frac{P_1}{P_2} \right))}{T_3(-R \ln \left(\frac{P_4}{P_3} \right))}$$

From the ideal gas law, it follows that:

$$\frac{P_1}{P_2} = \frac{RT_1/v_1}{RT_2/v_2} = \frac{v_2}{v_1}$$

since $T_1 = T_2$. Similarly, we can rewrite $\frac{P_4}{P_3}$, but now we also use that $v_3 = v_2$ and $v_4 = v_1$:

$$\frac{P_4}{P_3} = \frac{RT_4/v_4}{RT_3/v_3} = \frac{v_3}{v_4} = \frac{v_2}{v_1} = \frac{P_1}{P_2}$$

Therefore, both terms with \ln are the same, and the thermal efficiency reduces to:

$$\eta_{th} = 1 - \frac{T_1}{T_3} \Rightarrow T_3 = \frac{T_1}{1 - \eta_{th}} \Rightarrow T_3 = \frac{423}{0.6} = 705 \text{ K}$$

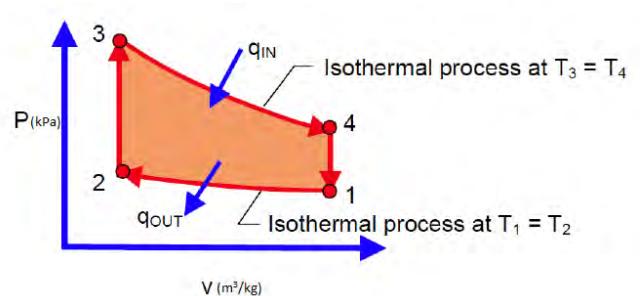
Remark: The external heat input and heat rejection can only occur at constant temperature by using an ideal regenerator to transfer heat internally to raise the temperature from T_2 to T_3 . In an actual Stirling cycle with non-ideal components, the efficiency will be substantially lower than the Carnot efficiency, but still relatively high compared with other piston-cylinder cycles, such as Otto and Diesel.

9.2c $q_{out,41}$ is used as the regenerated energy. Therefore $q_{reg} = q_{out,41} = c_v(T_4 - T_1)$. c_v can be found in Table A-2 at 150°C = 423 K, $c_v = 0.7295 \text{ kJ/kgK}$.

$$q_{reg} = c_v(T_3 - T_1) = 0.7295(705 - 423) = 205.7 \text{ kJ/kg.}$$

9.3 Stirling cycle 2

9.3a Between state 1 and 2, the air is isothermally compressed. Heat is rejected at low temperature to maintain the temperature at T_1 . From state 2 to 3, heat is added isochorically. This heat is regenerated from the process between state 4 and 1. Between state 3 and 4, the air expands isothermally. Heat is added to keep the temperature constant. Between state 4 and 1, heat is rejected isochorically towards the regenerator.



9.3b First of all, we make a table with the two known (underlined) variables at each point.

	P (MPa)	T (K)	s (kJ/(kgK))	v (m ³ /kg)	other
1	<u>0.8</u>	<u>373</u>			
2	<u>1.6</u>	<u>$T_2 = T_1$</u>			
3	<u>3.2</u>	746		<u>$v_3 = v_2$</u>	
4	1.6	<u>$T_4 = T_3$</u>		<u>$v_1 = v_4$</u>	

The net work is calculated as follows:

$$w_{net} = w_{out} - w_{in}$$

Work must be performed on the system in process 1→2 ($= w_{in}$), while work is delivered by the system in process 3→4 ($= w_{out}$). In the other processes, no work is done or delivered, since the volume remains constant. Work in an isothermal process for an ideal gas is given as:

$$w_{out,ab} = \int_a^b P dv$$

Furthermore, the ideal gas law, $Pv = RT$ can be used to express P in terms of v . Hence, w_{in} is expressed as follows:

$$w_{in} = - \int_{v_1}^{v_2} P dv = - \int_{v_1}^{v_2} \frac{RT}{v} dv = -RT_1 \ln\left(\frac{v_2}{v_1}\right)$$

Note that the temperature can only be taken out of the integral since it is constant (as the process is isothermal). The volume ratio can be determined from the ideal gas law $\frac{Pv}{T} = \text{constant}$. Since $T_1 = T_2$ (isothermal), this results in:

$$\frac{Pv}{T} = \text{constant} \Rightarrow P_1 v_1 = P_2 v_2 \Rightarrow \frac{v_2}{v_1} = \frac{P_1}{P_2}$$

Hence, w_{in} can be calculated:

$$w_{in} = -RT \ln\left(\frac{v_2}{v_1}\right) = -RT \ln\left(\frac{P_1}{P_2}\right) = -287 \cdot 373 \cdot \ln\left(\frac{0.8 \cdot 10^6}{1.6 \cdot 10^6}\right) = 74.2 \cdot 10^3 \text{ J/kg} = 74.2 \text{ kJ/kg}$$

Similarly, w_{out} is expressed as:

$$w_{out} = \int_{v_3}^{v_4} P dv = RT_3 \ln\left(\frac{v_4}{v_3}\right) = RT_3 \ln\left(\frac{P_3}{P_4}\right)$$

In order to determine the temperature at T_3 , we again use the ideal gas law, combined with a constant specific volume between 2 and 3:

$$\frac{Pv}{T} = \text{constant} \Rightarrow \frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow T_3 = \frac{T_2 P_3}{P_2} = \frac{T_1 P_3}{P_2} = \frac{373 \cdot 3.2 \cdot 10^6}{1.6 \cdot 10^6} = 746 \text{ K}$$

Furthermore, the pressure at state 4 needs to be determined (use $v_4 = v_1$):

$$\frac{Pv}{T} = \text{constant} \Rightarrow \frac{P_4}{T_4} = \frac{P_1}{T_1} \Rightarrow P_4 = \frac{P_1 T_4}{T_1} = \frac{P_1 T_3}{T_1} = \frac{0.8 \cdot 10^6 \cdot 746}{373} = 1.6 \cdot 10^6 \text{ Pa} = 1.6 \text{ MPa}$$

Substituting these numbers in the expression for w_{out} gives:

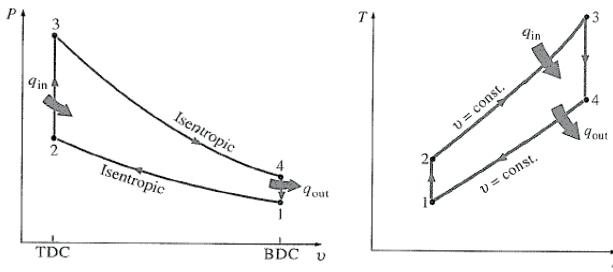
$$w_{out} = 287 \cdot 746 \cdot \ln\left(\frac{3.2}{1.6}\right) = 148.4 \cdot 10^3 \text{ J/kg} = 148.4 \text{ kJ/kg}$$

And hence the net work is calculated to be:

$$w_{net} = w_{out} - w_{in} = 148.4 - 74.2 = 74.2 \text{ kJ/kg}$$

9.4 Basics of the ideal Otto cycle

9.4a The $P-v$ and $T-s$ diagram of the Otto cycle, indicated are the constant volume and the constant entropy processes as well as the heat transfer in and out the engine. TDC stands Top Dead Center the highest piston position and BDC stands Bottom Dead Center the lowest piston position



9.4b The Otto cycle consists of four processes:

1 → 2 : isentropic compression.

2 → 3 : constant volume reversible heat transfer to the air in the engine from the surroundings.

3 → 4 : isentropic expansion.

4 → 1 : constant volume reversible heat transfer from the air in the engine to the surroundings.

9.4c See figure at a.

9.4d The Otto cycle is a closed system. There is a piston in a cylinder device. In analyzing the heat is transferred over the system boundary into or out of the piston cylinder device by heat transfer. There is no mass transfer into or out of the piston cylinder device.

9.4e The compression ratio is the ratio of the maximum volume (at state 1 and 4) over the minimum volume (at state 2 and 3) and is defined as $r_v = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{V_4}{V_3}$.

9.4f The cycle efficiency is given by:

$$\eta_{otto} = \frac{\text{what we get}}{\text{what we pay for}} = \frac{w_{net}}{q_{in}} = \frac{q_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

According to the second law $q = \Delta u - w$. In the steps where heat is added the volume is constant, therefore $\delta w = Pdv = 0$ and $q = \delta u = \int c_v dT$. In general the specific heat is dependent on temperature and should be evaluated by analyzing the integral. By taking the mean heat capacity of air between the designated temperatures the efficiency is

$$\eta_{otto} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{u_4 - u_1}{u_3 - u_2} = 1 - \frac{c_{v,mean,T1-T4}(T_4 - T_1)}{c_{v,mean,T3-T2}(T_3 - T_2)}$$

For an ideal air-standard cycle where the specific heats are assumed to be independent from the temperature the specific heats cancel. If in addition the isentropic relations for the expansion (3-4) and compression (1-2) process

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} = \frac{1}{r_v^{k-1}} = \left(\frac{V_2}{V_1}\right)^{k-1} = \frac{T_1}{T_2}$$

are taken into account the efficiency is given by

$$\eta_{otto} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_4 - T_1}{(T_4 - T_1)r_v^{k-1}} = 1 - \frac{1}{r_v^{k-1}}.$$

The heat added is $q_{in} = c_v(T_3 - T_2)$ and the heat rejected is $q_{out} = c_v(T_4 - T_1)$.

The net work is $w_{net} = q_{net} = q_{in} - q_{out} = c_v(T_3 - T_2) - c_v(T_4 - T_1) = c_v(T_3 + T_1 - T_4 - T_2)$.

9.5 Thermal efficiency Otto engine

9.5a Gas power cycles with air as the working fluid and the combustion process replaced by an equivalent amount of heat transfer from the surroundings are called *air-standard* cycles. The processes making up the cycle may be reversible or irreversible, with the irreversibilities usually accounted for by use of the component efficiencies. If the specific heats are assumed to be independent of temperature, the cycle is then called an *ideal air-standard* cycle. If in addition the specific heat values are evaluated at room temperature, the cycle is referred to as a *cold ideal air-standard cycle*.

9.5b The minimum cylinder volume is the clearance volume of $V_c = 21.3 \text{ cm}^3$.

The maximum volume is the clearance volume added to the swept volume V_s .

The swept volume is $V_s = 0.25 \cdot \pi \cdot d^2 \cdot h = 0.25 \cdot \pi 50^2 \cdot 75 = 147187 \text{ mm}^3 = 147.2 \text{ cm}^3$.

This results in a maximum volume of $\rightarrow V_{max} = V_c + V_s = 21.3 + 147.2 = 168.5 \text{ cm}^3$.

9.5c The compression ratio is $r_v = \frac{V_{max}}{V_{min}} = \frac{168.5}{21.3} = 7.91$.

9.5d The efficiency of the cold ideal air-standard cycle for an Otto engine is given by

$$\begin{aligned} \eta &= \frac{\text{what we get}}{\text{what we pay for}} = \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \\ &= 1 - \frac{C_v(T_4 - T_1)}{C_v(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_4 - T_1}{(T_4 - T_1)r_v^{k-1}} = 1 - \frac{1}{r_v^{k-1}}. \end{aligned}$$

Here is used that for isentropic processes (1-2 and 3-4) $Tv^{k-1} = \text{constant}$ and therefore

$$T_3 = T_4 \left(\frac{V_4}{V_3}\right)^{k-1} = T_4 r_v^{k-1} \quad \text{and} \quad T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1} = T_1 r_v^{k-1}$$

9.5e The thermal efficiency is

$$\eta = 1 - \frac{1}{r_v^{k-1}} = 1 - \frac{1}{7.91^{1.4-1}} = 0.563.$$

9.6 Otto cycle 1

9.6a Between state 1 and 2, air is compressed isentropically. Between 2 and 3, heat is added at constant volume. Between 3 and 4 the air is expanded isentropically, after which (4 to 1) heat is given to the surroundings.

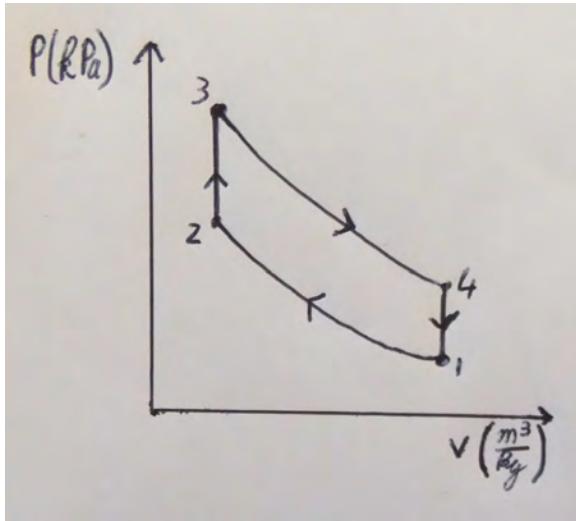


Figure 9.6.1: P-v diagram of the Otto cycle

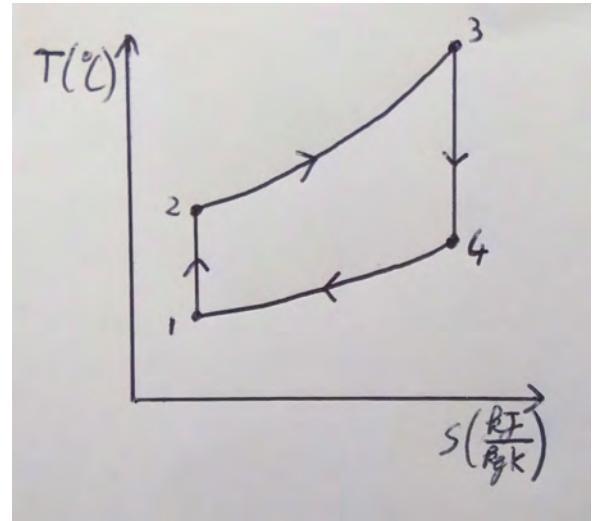


Figure 9.6.2: T-s diagram of the Otto cycle

9.6b To determine the thermal efficiency of this Otto cycle, you have to make a few assumptions.

- The working fluid is air, which always behaves as an ideal gas.
- All the processes that make up the cycle are internally reversible.
- The combustion process is replaced by a heat-addition process from an external source.
- The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

The thermal efficiency for an Otto cycle, with these assumptions, is: $\eta_{Otto,ias} = \frac{w_{net}}{q_{in}}$.
 $q_{in} = c_v(T_3 - T_2)$. c_v can be found in Table A-2 at 25°C, $c_v = 0.718 \text{ kJ/kgK}$.

$$q_{in} = c_v(T_3 - T_2) = 0.718(3273 - 873) = 1722.96 \text{ kJ/kg}$$

$$\eta_{Otto,ias} = \frac{w_{net}}{q_{in}} = \frac{900}{1722.96} = 0.522$$

9.6c The compression ratio can be calculated using the efficiency of the engine. In this Otto cycle: $w_{net} = q_{in} - q_{out}$. Both heat transfer processes occur at constant volume so $q_{in} = c_v(T_3 - T_2)$ and $q_{out} = c_v(T_4 - T_1)$. With this knowledge, the efficiency can be rewritten as (see assignment 9.5):

$$\eta_{Otto,ias} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{r^{k-1}}$$

The value for k can be found in table A-2 at 25°C ($k = 1.4$). This leads to the following result:

$$1 - \frac{1}{r^{k-1}} = \eta_{Otto,ias} \rightarrow r = \left(\frac{1}{1 - \eta_{Otto,ias}} \right)^{\frac{1}{k-1}} = \left(\frac{1}{1 - 0.522} \right)^{\frac{1}{1.4-1}} = 6.33$$

This problem did not explicitly state that the cycle was to be assumed as cold air standard, so it was left to the engineer to decide at what temperature to evaluate properties. A more accurate answer might have been achieved by assuming a mean value of temperature somewhere between ambient and the maximum of 3000 degrees Celsius. In any case, the assumption of constant specific heat during the heating process from 2-3 is only an approximation, since this process occurs over a large temperature differential (> 2000 degrees Celsius!). Nonetheless, this simplification in the analysis is useful to provide a rough idea of the compression ratio needed to achieve desired performance.

9.7 Otto cycle 2

9.7a Between state 1 and 2, air is compressed isentropically. Between 2 and 3, heat is added at constant volume. Between 3 and 4 the air is expanded isentropically, after which (4 to 1) heat is given to the surroundings.

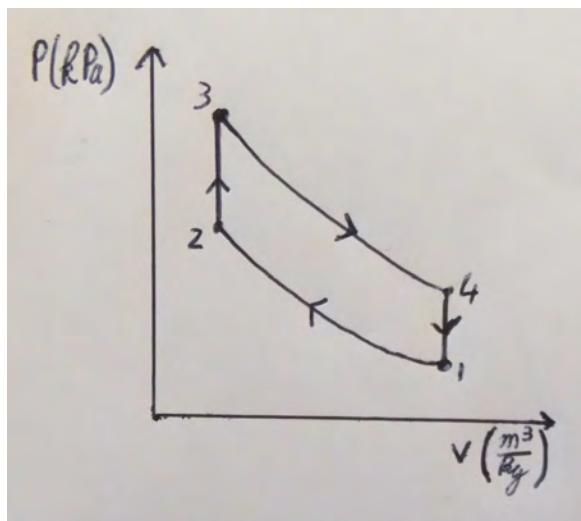


Figure 9.7.1: P-v diagram of the Otto cycle

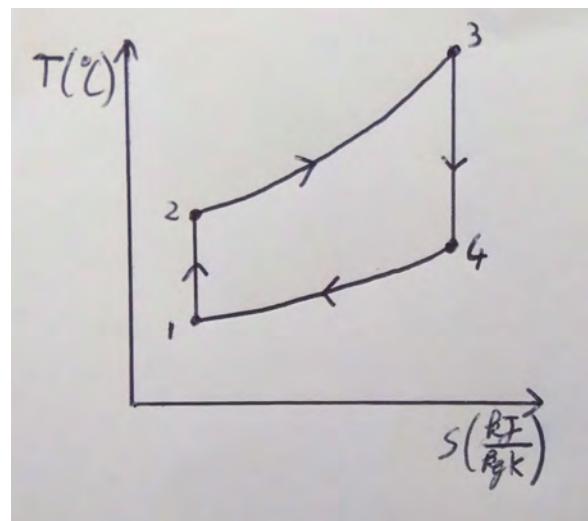


Figure 9.7.2: T-s diagram of the Otto cycle

9.7b First of all, we make a table with the two known (underlined) variables at each point.

	P (kPa)	T (K)	s (kJ/(kgK))	v (m^3/kg)	other
1	<u>95</u>	<u>300</u>		0.9063	
2	1746	689.2	<u>$s_2 = s_1$</u>	<u>$\frac{v_1}{v_2} = 8$</u>	
3	4393	1733.6		<u>$v_3 = v_2$</u>	<u>$q_{in} = 750 \text{ kJ/kg}$</u>
4		754.6	<u>$s_4 = s_3$</u>	<u>$v_1 = v_4$</u>	

It is asked to determine the pressure at the end of the heat addition process, so T_3 and P_3 are asked. Before we can do that, we need to determine P_2 and T_2 . We use that, for an ideal gas during isentropic processes, $Pv^k = \text{constant}$ (with $k = \frac{c_p}{c_v}$, for air, $k = 1.4$), so:

$$P_1 v_1^k = P_2 v_2^k \rightarrow P_2 = P_1 \left(\frac{v_1}{v_2} \right)^k = 95 \cdot 8^{1.4} = 1746 \text{ kPa}$$

Furthermore we have the ideal gas law: $Pv = RT$, so $\frac{Pv}{T} = \text{constant}$. So, $P = \text{const} \cdot \frac{T}{v}$, such that

$Pv^k = \text{constant}$ can be rewritten to $Tv^{k-1} = \text{const}$. Using this we can calculate T_2 :

$$T_1 v_1^{k-1} = T_2 v_2^{k-1} \rightarrow T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{k-1} = 300 \cdot 8^{0.4} = 689.2 \text{ K}$$

For a closed system, we have $\Delta u = (q_{in} - q_{out}) + (w_{in} - w_{out})$. For process 2-3, this means that $\Delta u = q_{in}$, and for ideal gases we also know that $\Delta u = c_v dT$, such that

$$q_{in} = c_v(T_3 - T_2) \rightarrow T_3 = \frac{q_{in}}{c_v} + T_2 = \frac{750}{0.718} + 689 = 1733.6 \text{ K}$$

With the ideal gas law we then calculate the pressure in point 3: $\frac{P_2 v_2}{T_2} = \frac{P_3 v_3}{T_3} \rightarrow P_3 = P_2 \frac{T_3}{T_2} = 1746 \cdot \frac{1733.6}{689} = 4393 \text{ kPa}$, where we use the fact that $v_2 = v_3$.

9.7c The net work output is calculated as: $w_{net} = q_{in} - q_{out}$. $q_{out} = -q_{4-1} = c_v(T_4 - T_1)$, so we need to determine T_4 :

$$T_4 v_4^{k-1} = T_3 v_3^{k-1} \rightarrow T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{k-1} = 1733.6 \left(\frac{1}{8} \right)^{0.4} = 754.6 \text{ K}$$

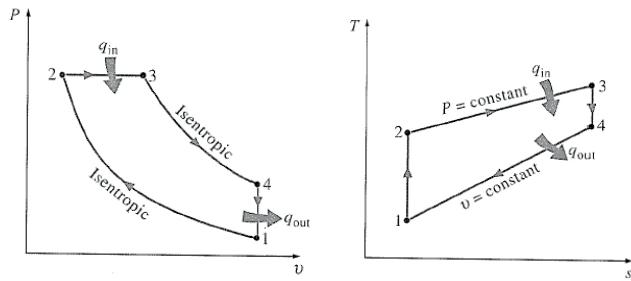
So, we get $w_{net} = q_{in} - c_v(T_4 - T_1) = 750 - 0.718(754.6 - 300) = 423.6 \text{ kJ/kg}$.

9.7d $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)} = \frac{423.6}{750} = 0.564$

9.7e The mean effective pressure is calculated as the ratio of the work done per stroke to the volume of a stroke, so $MEP = \frac{w_{net}}{v_1 - v_2}$. With the ideal gas law, we can determine v_1 and v_2 : $v_1 = \frac{RT_1}{P_1} = \frac{0.287 \cdot 300}{95} = 0.9063 \text{ m}^3/\text{kg}$ and $v_2 = \frac{RT_2}{P_2} = \frac{0.287 \cdot 689.2}{1746} = 0.1133 \text{ m}^3/\text{kg}$, so $MEP = \frac{w_{net}}{v_1 - v_2} = \frac{423.6}{0.9063 - 0.1133} = 534 \text{ kPa}$.

9.8 Basics of the ideal Diesel cycle

9.8a The $P-v$ and $T-s$ diagram of the Diesel cycle, indicated are the constant volume, the constant pressure and the constant entropy processes as well as the heat transfer in and out the engine.



9.8b The Diesel cycle consists of four processes:

$1 \rightarrow 2$: isentropic compression.

$2 \rightarrow 3$: constant pressure reversible heat transfer to the air in the engine from the surroundings.

$3 \rightarrow 4$: isentropic expansion.

$4 \rightarrow 1$: constant volume reversible heat transfer from the air in the engine to the surroundings.

9.8c See figure at a.

9.8d The Diesel cycle is a closed system. There is a piston in a cylinder device. In analyzing the heat is transferred over the system boundary into or out of the piston cylinder device by heat transfer. There is no mass transfer into or out of the piston cylinder device.

9.8e The compression ratio is the ratio of the maximum volume (at state 1 and 4) over the minimum volume (at state 2) and is defined as $r_v = \frac{V_{max}}{V_{min}} = \frac{V_2}{V_1}$.

9.8f The cut off ratio is the ratio between the volume at state 3 (where fuel injection ends) to the maximum volume (at state 2) and is defined as $r_c = \frac{V_3}{V_2}$. Note the difference with the compression

9.8g The cycle efficiency is given by:

$$\eta_{diesel} = \frac{\text{what we get}}{\text{what we pay for}} = \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

According to the second law $Q = \Delta U - W$. In the step where heat is added the pressure is constant, and the heat addition can be described by the difference in the enthalpy values, $Q_{in} = h_3 - h_2 = \int C_p dT$. In the step where the heat is rejected the volume is constant, therefore $\delta W = PdV = 0$ and $Q = u_4 - u_1 = \int C_v dT$. In general the specific heat is dependent on temperature and should be evaluated by analyzing the integral. By taking the mean heat capacity of air between the designated temperatures the efficiency is

$$\eta_{diesel} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{u_4 - u_1}{h_3 - h_2} = 1 - \frac{C_{v,mean,T1-T4}(T_4 - T_1)}{C_{p,mean,T3-T2}(T_3 - T_2)}$$

For an ideal air-standard cycle where the specific heats are assumed to be independent from the temperature and the ration between C_p and C_v is $k = \frac{C_p}{C_v}$. If in addition the isentropic relation for compression (1-2) process

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{k-1} = \frac{1}{r_v^{k-1}}$$

is taken into account the efficiency is given by

$$\eta_{diesel} = 1 - \left(\frac{C_v}{C_p} \right) \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \left(\frac{1}{k} \right) \left(\frac{T_1}{T_2} \right) \frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)} = 1 - \left(\frac{1}{k} \right) \left(\frac{v_2}{v_1} \right)^{k-1} \frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)}.$$

Observe that $r_v = V_1/V_2$ and that for the constant pressure process (2-3)

$$\frac{T_3}{T_2} = \frac{P_3 V_3}{P_2 V_2} = \frac{V_3}{V_2} = r_c$$

and

$$\frac{T_4}{T_1} = \frac{P_4 V_4}{P_1 V_1} = \frac{P_4}{P_1} = \frac{P_4}{P_3} \frac{P_3}{P_1} = \frac{P_4}{P_3} \frac{P_2}{P_1} = \left(\frac{V_3}{V_4} \right)^k \left(\frac{V_1}{V_2} \right)^k = \left(\frac{V_3}{V_2} \right)^k = r_c^k$$

Than the efficiency becomes

$$\eta_{diesel} = 1 - \frac{1}{k} \frac{1}{r_v^{k-1}} \frac{[r_c^k - 1]}{[r_c - 1]}.$$

The heat added is $q_{in} = c_p(T_3 - T_2)$ and the heat rejected is $q_{out} = c_v(T_4 - T_1)$.

The net work is $w_{net} = q_{net} = q_{in} - q_{out} = c_p(T_3 - T_2) - c_v(T_4 - T_1)$.

9.8h The differences between an Otto and a Diesel engine are:

Heat addition in a Diesel engine occurs at constant pressure while it in a Otto engine occurs at constant volume. This gives a characteristic difference in the P_V diagram.

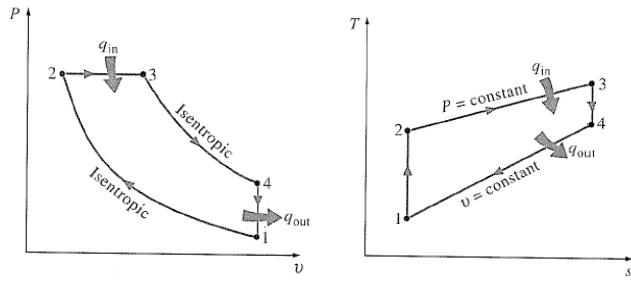
In a Diesel engine an additional parameter, the cut off ratio, plays a role.

Mostly pressures are higher in a Diesel engine and they have a higher efficiency.

On average Diesel engines are more expensive and heavier than Otto engine.

9.9 Thermal efficiency Diesel cycle

9.9a The $P - v$ and $T - s$ diagram of the Diesel cycle are



9.9b Given is $T_1 = 15^\circ\text{C} = 288\text{ K}$ and $T_3 = 1100^\circ\text{C} = 1373\text{ K}$ and $r_v = \frac{v_1}{v_2} = 12$. The temperature after compression can be found via the relation between temperature and volume for an isentropic process: $T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{k-1} = r_v^{k-1} = 288 \cdot 12^{0.4} = 288 \cdot 2.7 = 778\text{ K} = 505^\circ\text{C}$.

The temperature after compression can also be found via the relation for temperature and volume for an isentropic process $T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{k-1}$. However, in this case the ratio v_3/v_4 is more difficult to find

$$\frac{v_3}{v_4} = \frac{v_3}{v_1} = \frac{v_3 v_2}{v_1 v_2} = \frac{v_2 v_3}{v_1 v_2} = \frac{1}{r_v} \frac{P_2}{T_2} \frac{T_3}{P_3} = \frac{1}{r_v} \frac{T_3}{T_2} = \frac{1}{12} \frac{1373}{778} = \frac{1}{6.8} \rightarrow T_4 = 1373 \cdot \left(\frac{1}{6.8} \right)^{0.4} = 638\text{ K} = 365^\circ\text{C}.$$

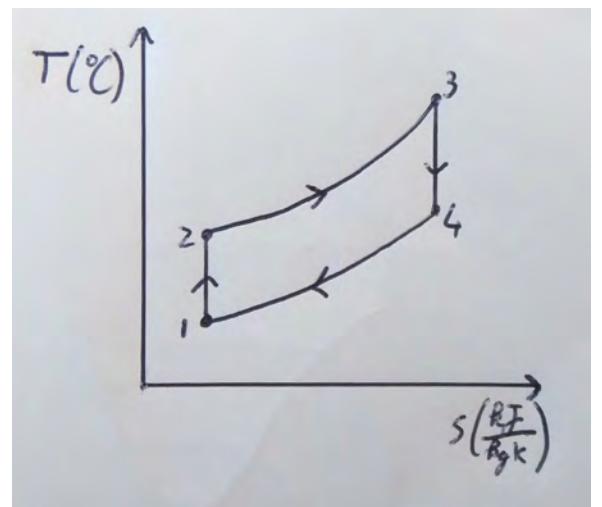
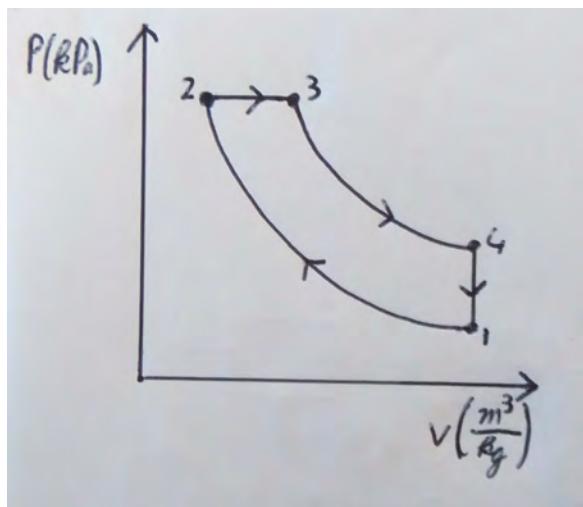
9.9c The air-standard thermal efficiency is

$$\eta_{diesel} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{0.717}{1.004} \frac{(638 - 288)}{(1373 - 778)} = 0.58.$$

9.9d The second law efficiency is $\eta_\psi = \frac{\eta_{diesel}}{\eta_{Carnot}}$ where the Carnot efficiency is $\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{288}{1373} = 0.83 \rightarrow \eta_\psi = \frac{0.58}{0.83} = 0.70$.

9.9e The net work is $w_{net} = q_{net} = q_{in} - q_{out} = c_p(T_3 - T_2) - c_v(T_4 - T_1) = 1.004(1373 - 778) - 0.717(638 - 288) = 346.4\text{ kJ/kg}$.

9.10 Diesel cycle 1



9.10a Between state 1 and 2, the air is compressed isentropically. No heat is added or rejected. From state 2 to 3, isobaric heat addition occurs, and work is being delivered. Between state 3 and 4, the air expands isentropically, and work is being delivered. Between state 4 and 1, heat is rejected isochorically.

9.10b First of all, we make a table with the two known (underlined) variables at each point.

	P (MPa)	T (K)	s (kJ/(kgK))	v (m ³ /kg)	other
1	<u>0.1</u>	<u>298</u>			
2		903.4	<u>s₂ = s₁</u>	<u>v₁/₂ = r_v = 16</u>	
3	<u>P₃ = P₂</u>	1855.8			<u>q_{in} = 1000 kJ/kg</u>
4		816.5	<u>s₄ = s₃</u>	<u>v₁ = v₄</u>	

Now we want to find the temperature after compression (T_2), heat addition (T_3) and expansion (T_4). For isentropic processes, it holds that $Tv^{k-1} = \text{constant}$, such that:

$$T_1 v_1^{k-1} = T_2 v_2^{k-1} \Rightarrow T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{k-1} = T_1 r_v^{k-1} = 298 \cdot 16^{0.4} = 903.4 \text{ K}$$

To determine the temperature after heat addition (T_3), we use the first law for closed systems:

$$\Delta u = \Delta q + \Delta w \Rightarrow q_{in} = u_3 - u_2 + (P_3 v_3 - P_2 v_2) = h_3 - h_2 = c_p(T_3 - T_2)$$

where the relation for ideal gases $dh = c_p dT$ and the definition of enthalpy $h = u + Pv$ are used. Rewriting now gives T_3 :

$$q_{in} = c_p(T_3 - T_2) \Rightarrow T_3 = \frac{q_{in}}{c_p} + T_2 = \frac{1000}{1.005} + 903.4 = 1898.4 \text{ K}$$

To determine T_4 , we again use the relation for isentropic processes:

$$T_3 v_3^{k-1} = T_4 v_4^{k-1} \Rightarrow T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{k-1}$$

The volume v_4 is equal to v_1 , but v_3 is still unknown. It can be expressed in terms of v_2 by using the ideal gas law:

$$\frac{Pv}{T} = \text{constant} \Rightarrow \frac{v_2}{T_2} = \frac{v_3}{T_3} \Rightarrow v_3 = \frac{v_2 T_3}{T_2}$$

Hence:

$$T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{k-1} = T_3 \left(\frac{v_2}{v_1} \cdot \frac{T_3}{T_2} \right)^{k-1} = T_3 \left(\frac{1}{r_v} \cdot \frac{T_3}{T_2} \right)^{k-1} = 1898.4 \cdot \left(\frac{1}{16} \cdot \frac{1898.4}{903.4} \right)^{0.4} = 842.8 \text{ K}$$

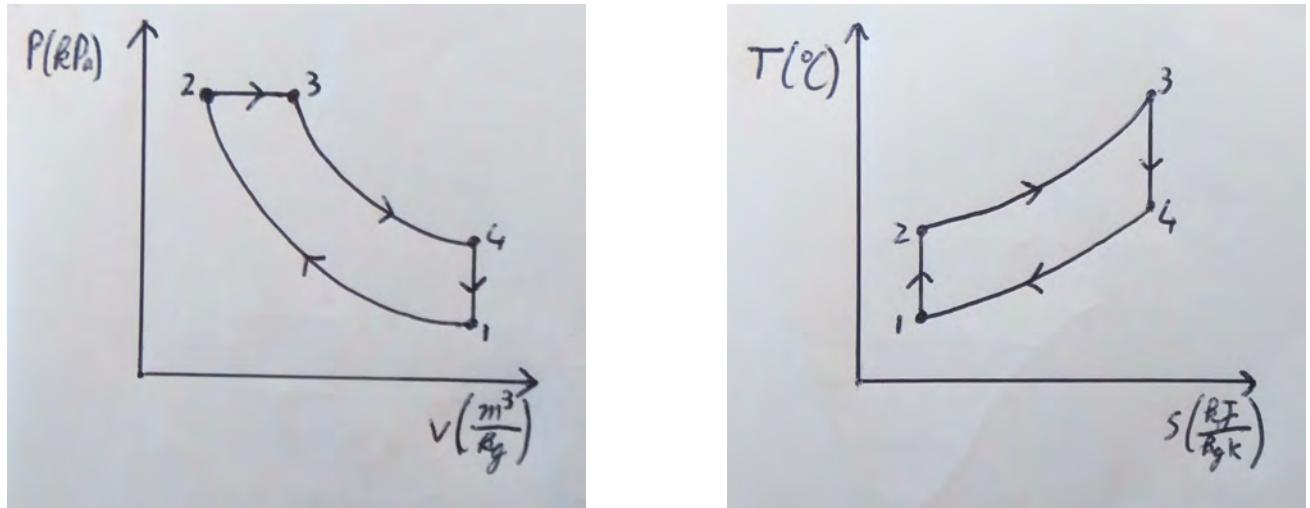
9.10c The efficiency is given by:

$$\begin{aligned} \eta_{Diesel} &= \frac{w_{net}}{q_{in}} = \frac{w_{23} + w_{34} - w_{12}}{q_{in}} = \frac{(P_3 v_3 - P_2 v_2) + (u_3 - u_4) - (u_2 - u_1)}{(u_3 - u_2) + (P_3 v_3 - P_2 v_2)} \\ &= \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)} \\ &= 1 - \frac{c_v(T_4 - T_1)}{q_{in}} = 1 - \frac{0.7580 \cdot (842.8 - 298)}{1000} = 0.413 = 41.3\% \end{aligned}$$

It is seen that the efficiency of the Diesel cycle is actually lower than that of an Otto cycle for the same compression ratio. However, the compression ratio of 16 given in this problem would not be practical in an Otto engine due to the problem of preignition of the fuel in the cylinder during compression. Since, in a Diesel engine, the fuel is not injected after compression, higher compression ratios are feasible, yielding higher efficiencies.

9.11 Diesel cycle 2

9.11a Between the starting state 1 and state 2, air is compressed isentropic. Between 2 and 3 we have isobaric combustion, after which (3-4) the air expands isentropic. Lastly, between 4 and 1, we have isochoric exhaust of heat.



9.11b First a tabel is made to collect all data in a systematic way.

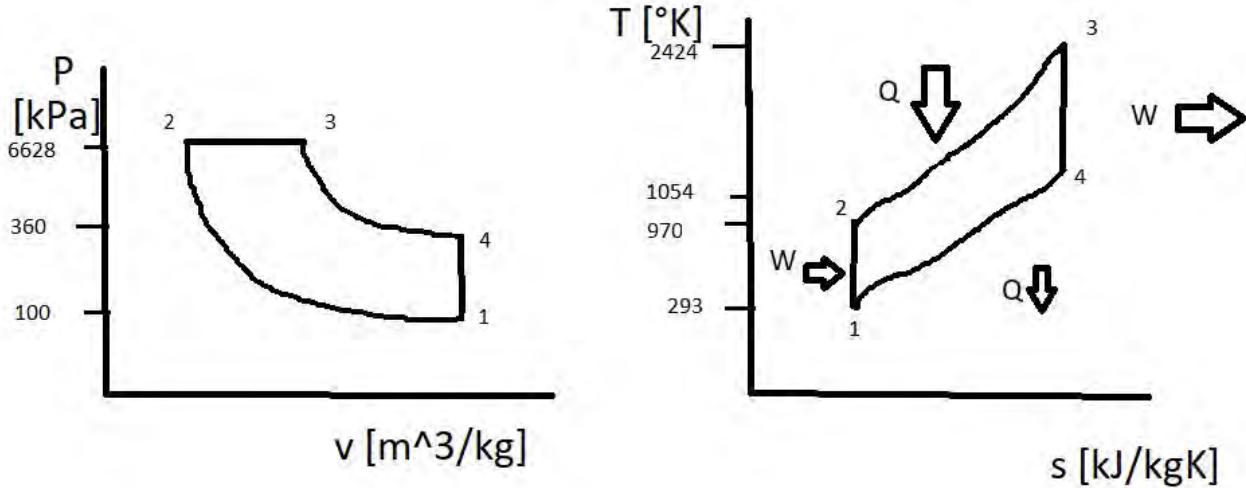
	P (kPa)	T (K)	s (kJ/(kgK))	v (m ³ /kg)	other
1	95	293		0.8852	
2		971	<u>s₂ = s₁</u>	<u>v₁/v₂ = 20</u>	
3	<u>P₃ = P₂</u>	2200			
4		921	<u>s₄ = s₃</u>	<u>v₁ = v₄</u>	

$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$, so we need to have all temperatures. $T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1} = 293 \cdot 20^{0.4} = 971$ K. $T_4 = T_3 \left(\frac{v_3}{v_4}\right)^{k-1}$, but we don't know v_3 yet. Use the ideal gas law: $\frac{P_2 v_2}{T_2} = \frac{P_3 v_3}{T_3} \rightarrow v_3 = v_2 \frac{T_3}{T_2}$. Substituting in the expression for T_4 gives $T_4 = T_3 \left(\frac{T_3}{T_2} \frac{v_2}{v_4}\right)^{k-1} = 2200 \left(\frac{2200}{971} \cdot \frac{1}{20}\right)^{0.4} = 921$ K. So, $\eta_{th} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = \frac{1.005(2200 - 971) - 0.718(921 - 293)}{1.005(2200 - 971)} = 0.635$.

9.11c $MEP = \frac{w_{net}}{v_{max} - v_{min}} = \frac{q_{in} - q_{out}}{v_1 - v_2} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{v_1 \left(1 - \frac{v_2}{v_1}\right)}$. Use the ideal gas law to calculate v_1 : $P_1 v_1 = RT_1 \rightarrow v_1 = \frac{RT_1}{P_1} = \frac{0.287 \cdot 293}{95} = 0.8852$ m³/kg. Furthermore, $\frac{v_2}{v_1}$ is given as one over the compression ratio, so $MEP = \frac{1.005(2200 - 971) - 0.718(921 - 293)}{0.8852 \left(1 - \frac{1}{20}\right)} = \frac{784}{0.84} = 932$ kPa.

9.12 Diesel engine with turbocharger

9.12a Pv -diagram and Ts -diagram of the cycle



9.12b At first, the minimum volume must be determined:

$$v_{min} = v_{max} - v_{swept} = 500 \text{ cm}^3 - 475 \text{ cm}^3 = 25 \text{ cm}^3$$

The compression ratio follows from:

$$r_{comp} = \frac{v_{max}}{v_{min}} = \frac{500 \text{ cm}^3}{25 \text{ cm}^3} = 20$$

The specific volume is calculated using the ideal gas law:

$$v = \frac{RT}{P} = \frac{0.287 \text{ kJ/kgK} \cdot 293 \text{ K}}{100 \text{ kPa}} = 0.841 \text{ m}^3/\text{kg}$$

9.12c Table with properties, the known properties are underlined.

	P [kPa]	T [K]	v [m^3/kg]	others
1	<u>100</u>	<u>293</u>	0.841	
2	6628	970	$v_2 = v_1 / r_{comp} = 0.042$	<u>$s_2 = s_1$</u>
3	<u>$P_3 = P_2$</u>	2424	$v_3 = r_{cutoff} \cdot v_2 = 0.105$	
4	360	1054	$v_4 = v_1 = 0.841$	<u>$s_4 = s_3$</u>

The missing pressures, temperatures and volumes are calculated as follows. For the processes $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$, isentropic relationships are used.

$$v_2 = \frac{v_1}{r_{comp}} = 0.04204 \text{ m}^3/\text{kg}$$

$$P_2 = P_1 \cdot \left(\frac{v_1}{v_2}\right)^k = 6628 \text{ kPa}$$

$$T_2 = \frac{P_2 v_2}{R} = 970 \text{ K}$$

$$v_3 = r_{cutoff} \cdot v_2 = 0.105 \text{ m}^3/\text{kg}$$

$$T_3 = \frac{P_3 v_3}{R} = 2424 \text{ K}$$

$$P_4 = P_3 \cdot \left(\frac{v_3}{v_4}\right)^k = 360 \text{ kPa}$$

$$T_4 = \frac{P_4 v_4}{R} = 1054 \text{ K}$$

9.12d The specific net work output can be calculated with the following equation:

$$w_{net} = c_p \cdot (T_3 - T_2) - c_v \cdot (T_4 - T_1) = 906.1 \text{ kJ/kg}$$

9.12e At first, the specific heat input must be determined:

$$q_{in} = c_p \cdot (T_3 - T_2) = 1454 \text{ kJ/kg}$$

Using this, the thermal efficiency follows from:

$$\eta = \frac{w_{net}}{q_{in}} = 0.623$$

9.12f The density of the gas in the cylinder is equal to $\frac{1}{v}$. At point 1, the total volume in the cylinder is 500 cm^3 . Hence, the mass follows from:

$$m = V_1 \cdot \frac{1}{v_1} = 5.95 \cdot 10^{-4} \text{ kg}$$

9.12g To find the power output of the engine, one must take all cylinders into account. Further, the revolution speed must be converted to revolutions per second. The engine only produces work once per two complete revolutions. So the power output follows from:

$$\dot{W}_{net} = w_{net} \cdot \frac{1}{2} \cdot \frac{RPM}{60} \cdot 4m = 77.34 \text{ kW}$$

9.12h The compression ratio is unaffected.

$$v_1 = \frac{RT_1}{P_1} = 0.46 \text{ m}^3/\text{kg}$$

	P [kPa]	T [K]	v [m^3/kg]	others
1	<u>180</u>	<u>293</u>	0.46	
2	11932	956	$v_2 = v_1 / r_{comp} = 0.023$	<u>$s_2 = s_1$</u>
3	<u>$P_3 = P_2$</u>	2390	$v_3 = v_{cutoff} \cdot v_2 = 0.0575$	
4	649	1040	<u>$v_4 = v_1 = 0.46$</u>	<u>$s_4 = s_3$</u>

The missing values are found in the same way as in question b to g.

$$w_{net} = 896.2 \text{ kJ/kg}$$

$$q_{in} = 1434 \text{ kJ/kg}$$

$$\eta = 0.63$$

$$m = \frac{1}{v_1} \cdot V_1 = \frac{1}{920} \text{ kg}$$

$$\dot{W}_{net} = w_{net} \cdot \frac{1}{2} \cdot \frac{RPM}{60} \cdot 4m = 139.63 \text{ kW}$$

9.12i Thermal efficiency and specific net work output are barely affected by the turbocharger. However, the power output is highly affected. This is due to the very much higher mass flow rate.

9.12j The main reason for that is friction. A turbocharger makes it possible to build smaller engines, with a less amount of cylinders, but the same power output. So the total surface area subjected to friction is reduced, and therefore, the losses are reduced, too. Another reason is the weight reduction due to the smaller engine.

9.13 Closed Brayton Cycle

9.13a A closed Brayton cycle consist of a compressor, combustor, turbine and cooler. These are linked as shown in figure 9.13.1.

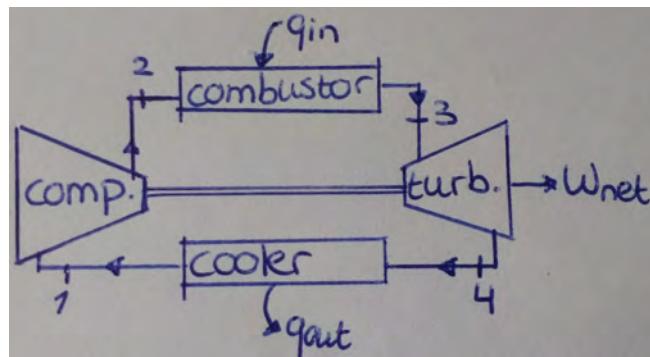


Figure 9.13.1: Schematic overview of a closed Brayton cycle

9.13b The Brayton cycle consists of the following processes:

- 1-2 Isentropic compression
- 2-3 Constant-pressure heat addition
- 3-4 Isentropic expansion
- 4-1 Constant-pressure heat rejection

Displayed in h-s and P-v diagrams, this looks as following:

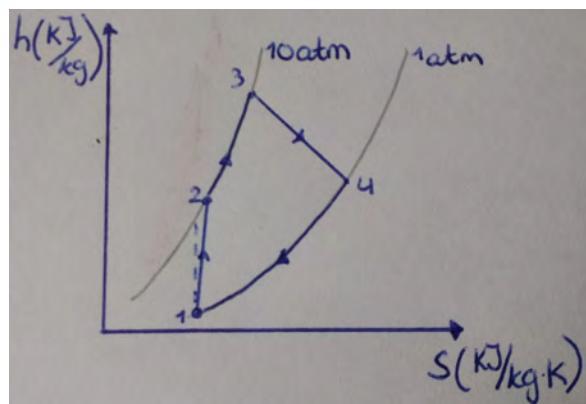


Figure 9.13.2: hs-diagram Brayton cycle

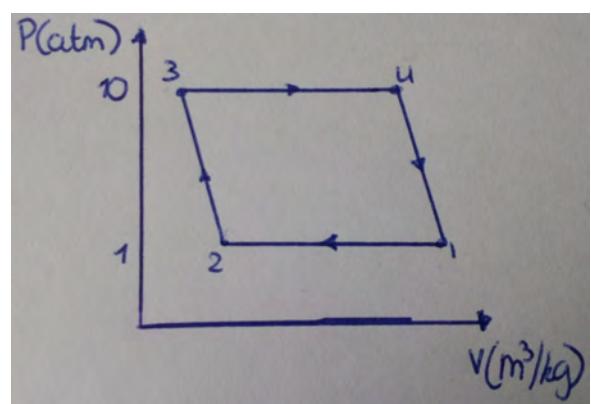


Figure 9.13.3: Pv-diagram Brayton cycle

9.13c We make a table with the two known (underlined) variables at each point.

	P (atm)	T (K)	s (kJ/(kgK))	v (m^3/kg)	other
1	<u>1</u>	<u>299</u>			
2	<u>$P_2 = P_1$</u>				<u>$w_{net} = 0$</u>
3		<u>1174</u>			<u>$r_p = 10$</u>
4	<u>$P_4 = P_3$</u>				<u>$\eta_{turb} = 0.6$</u>

9.13d Zero work output means that the net work is zero. Therefore,

$$w_{net} = w_{out,turb} - w_{in,comp} = 0 \Rightarrow w_{out,turb} = w_{in,comp}$$

Because the turbine and compressor are not ideal, we have to deal with two efficiencies.

$$\eta_{turb} = \frac{w_{out,turb}}{w_{out,turb,s}}$$

$$\eta_{comp} = \frac{w_{in,comp,s}}{w_{in,comp}}$$

With the use of the net work equation, the efficiencies can be simplified to:

$$\eta_{turb} = \frac{w_{out,turb}}{w_{out,turb,s}} \Rightarrow w_{out,turb} = \eta_{turb} * w_{out,turb,s} = w_{in,comp}$$

$$\eta_{comp} = \frac{w_{in,comp,s}}{w_{in,comp}} = \frac{w_{in,comp,s}}{\eta_{turb} * w_{out,turb,s}}$$

To calculate $w_{in,comp,s}$ and $w_{out,turb,s}$, we have to use the First law for a control volume: $(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$. In the compressor, only w_{in} have to be considered. The rest is zero. $h_{exit} - h_{inlet}$ can be rewritten with the use of: $dh = c_p dT$. This leads in total to:

$$\eta_{comp} = \frac{c_p(T_{2,s} - T_1)}{\eta_{turb} * c_p(T_3 - T_{4,s})} = \frac{T_{2,s} - T_1}{\eta_{turb}(T_3 - T_{4,s})}$$

$T_{2,s}$ and $T_{4,s}$ can be calculated with the formula for temperature for isentropic processes of an ideal gas

with constant specific heats: $T_{b,s} = T_a \left(\frac{P_b}{P_a} \right)^{\frac{k-1}{k}}$. k can be found in Table 5s at a temperature of 25 degrees Celsius (=1.4001).

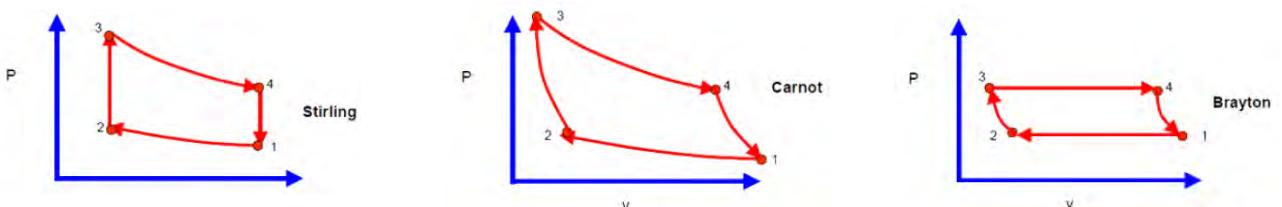
$$T_{2,s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 298 \left(\frac{10atm}{1atm} \right)^{\frac{1.4001-1}{1.4001}} = 575.4 K$$

$$T_{4,s} = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = 1173 \left(\frac{1atm}{10atm} \right)^{\frac{1.4001-1}{1.4001}} = 607.5 K$$

$$\eta_{comp} = \frac{T_{2,s} - T_1}{\eta_{turb}(T_3 - T_{4,s})} = \frac{575.4 - 298}{0.6(1173 - 607.5)} = 81.8\%$$

9.14 Carnot versus Stirling versus Brayton cycle

9.14a See the figures below. Note that process 2-3 (heat addition) and process 4-1 (heat rejection) in the Stirling engine are isochoric, while proces 1-2 (heat rejection from exhaust gases) and proces 3-4 (heat addition in combustion chamber) in the Brayton engine are isobaric. The Carnot engine does not have isobaric or isochoric processes, and hence there are no straight lines in a $P-v$ -diagram.



9.14b The temperatures T_2 and T_4 can be found using the ideal gas law:

$$Pv = RT \Rightarrow T = \frac{Pv}{R}$$

This gives:

$$T_2 = \frac{P_2 v_2}{R} = \frac{10 \cdot 10^5 \cdot 0.05}{287} = 174.2 \text{ K}$$

and

$$T_4 = \frac{P_4 v_4}{R} = \frac{1 \cdot 10^5 \cdot 0.40}{287} = 139.4 \text{ K}$$

Remark: be consistent with the units! Fill in the pressure in Pa and specific gas constant in J/(kgK) (as was done here), or use kPa and kJ/(kgK).

9.14c First of all, we will make a table where we fill in all known parameters of the Carnot cycle. Recall the Carnot cycle: process 1-2 denotes isothermal compression, process 2-3 is isentropic compression, process 3-4 is isothermal expansion and process 4-1 is isentropic expansion.

	P (kPa)	T (K)	s (kJ/(kgK))	v (m^3/kg)	other
1		$T_1 = T_2$	$s_1 = s_4$		
2	1000	174.2		$v_2 = 0.05$	
3		$T_3 = T_4$	$s_2 = s_3$		
4	100	139.4		$v_4 = 0.40$	

The thermal efficiency of this cycle is calculated as:

$$\eta_{th,c} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

The step from rewriting the work to heat is obtained by making the energy balance for each process, and neglecting the kinetic and potential energy in all cases:

1-2 $w_{in} + h_{in} = q_{out} + h_{out} \Rightarrow w_{in} + (h_{in} - h_{out}) = q_{out}$. Use of the ideal gas relation $dh = c_p dT$ combined with the fact that this process is isothermal ($dT = 0$) yields $w_{in} = q_{out}$.

2-3 Adiabatic ($dq = 0$), so the energy balance becomes: $w_{in,2-3} = (h_{out} - h_{in})$. Use of the ideal gas relation $dh = c_p dT$ gives $w_{in,2-3} = c_p(T_3 - T_2) = c_p(T_H - T_L)$.

3-4 $h_{in} + q_{in} = w_{out} + h_{out} \Rightarrow w_{out} = q_{in} + (h_{in} - h_{out})$. Use of the ideal gas relation $dh = c_p dT$ combined with the fact that this process is isothermal ($dT = 0$) yields $w_{out} = q_{in}$.

4-1 Adiabatic ($dq = 0$), so the energy balance becomes: $w_{out,4-1} = -(h_{out} - h_{in})$. Use of the ideal gas relation $dh = c_p dT$ gives $w_{out,4-1} = -c_p(T_4 - T_1) = -c_p(T_H - T_L)$.

Hence, the work input in process 2-3 is exactly delivered by the work output in process 4-1, and the thermal efficiency could indeed be written in terms of heat input and output as done above.

From the inequality of Clausius for reversible processes, we find that $q_{out} = T_L(s_1 - s_2)$ and $q_{in} = T_H(s_4 - s_3)$. Since the processes 2-3 and 4-1 are isentropic, this means that the thermal efficiency is (as expected) rewritten to the well-known Carnot efficiency:

$$\eta_{th,c} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_L(s_1 - s_2)}{T_H(s_4 - s_3)} = 1 - \frac{T_L(s_4 - s_3)}{T_H(s_4 - s_3)} = 1 - \frac{T_L}{T_H} = 1 - \frac{139.4}{174.2} = 0.1998$$

9.14d First of all, we will make a table where we fill in all known parameters of the Stirling cycle. Recall the Stirling cycle: process 1-2 denotes isothermal compression, process 2-3 is isochoric heat addition, process 3-4 is isothermal expansion and process 4-1 is isochoric heat rejection.

	P (kPa)	T (K)	s (kJ/(kgK))	v (m ³ /kg)	other
1		$T_1 = T_2$		$v_1 = v_4$	
2	<u>1000</u>	174.2		<u>$v_2 = 0.05$</u>	
3		$T_3 = T_4$		<u>$v_3 = v_2$</u>	
4	<u>100</u>	139.4		<u>$v_4 = 0.40$</u>	

The efficiency of this cycle is calculated as:

$$\eta_{th,s} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

In order to determine q_{in} and q_{out} , we can use the inequality of Clausius: $ds \leq \frac{\delta q_{net}}{T_{res}}$. For reversible processes, it holds that $ds = \frac{\delta q_{net}}{T_{res}}$, such that $q_{out} = T_1(s_1 - s_2)$ and $q_{in} = T_3(s_4 - s_3)$ (note that in both processes the temperature is constant). Therefore, we can write:

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_1(s_1 - s_2)}{T_3(s_4 - s_3)}$$

This (the entropy differences) can be rewritten using the Gibbs equation $Tds = dh - vdP$, combined with the ideal gas relation $dh = c_p dT$, such that:

$$Tds = dh - vdP = Tds = c_p dT - vdP \Rightarrow ds = \frac{c_p}{T} dT - \frac{v}{T} dP$$

Using the ideal gas law, $\frac{v}{T}$ can be expressed in terms of P : $\frac{v}{T} = \frac{R}{P}$. Integrating from state 2 to state 1, this results in:

$$s_1 - s_2 = c_p \ln\left(\frac{T_1}{T_2}\right) - R \ln\left(\frac{P_1}{P_2}\right) = -R \ln\left(\frac{P_1}{P_2}\right)$$

Since the process is isothermal ($T_1 = T_2$). Similarly, $s_4 - s_3$ is determined to equal:

$$s_4 - s_3 = c_p \ln\left(\frac{T_4}{T_3}\right) - R \ln\left(\frac{P_4}{P_3}\right) = -R \ln\left(\frac{P_4}{P_3}\right)$$

Therefore, the thermal efficiency equals:

$$\eta_{th} = 1 - \frac{T_1(s_1 - s_2)}{T_3(s_4 - s_3)} = 1 - \frac{T_1 \left(-R \ln\left(\frac{P_1}{P_2}\right) \right)}{T_3 \left(-R \ln\left(\frac{P_4}{P_3}\right) \right)}$$

From the ideal gas law, it follows that:

$$\frac{P_1}{P_2} = \frac{RT_1/v_1}{RT_2/v_2} = \frac{v_2}{v_1}$$

since $T_1 = T_2$. Similarly, we can rewrite $\frac{P_4}{P_3}$, but now we also use that $v_3 = v_2$ and $v_4 = v_1$:

$$\frac{P_4}{P_3} = \frac{RT_4/v_4}{RT_3/v_3} = \frac{v_3}{v_4} = \frac{v_2}{v_1} = \frac{P_1}{P_2}$$

Therefore, both terms with ln are the same, and the thermal efficiency reduces to:

$$\eta_{th} = 1 - \frac{T_1}{T_3} = 1 - \frac{T_L}{T_H} = \eta_{th,c} = 0.1998$$

Since the efficiency of the ideal Stirling cycle is equal to the Carnot efficiency, the ratio of the Carnot efficiency over the Stirling cycle efficiency is 1.

9.14e First of all, we will make a table where we fill in all known parameters of the Brayton cycle. Recall the closed ideal Brayton cycle: process 1-2 denotes isentropic compression, process 2-3 is isobaric heat addition, process 3-4 is isentropic expansion and process 4-1 is isobaric heat rejection.

	P (kPa)	T (K)	s (kJ/(kgK))	v (m ³ /kg)	other
1	$P_1 = P_4$	90.2	$s_1 = s_2$		
2	1000	174.2		$v_2 = 0.05$	
3	$P_3 = P_2$	269.1	$s_3 = s_4$		
4	100	139.4		$v_4 = 0.40$	

The thermal efficiency is calculated as:

$$\eta_{th,b} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} = \frac{c_p((T_3 - T_4) - (T_2 - T_1))}{c_p(T_3 - T_2)} = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_2}$$

where the enthalpy changes can be rewritten to the temperatures using the ideal gas relation $dh = c_p dT$. In order to determine the temperature at state 1, we use the following isentropic ideal gas relation:

$$\frac{P^{\frac{k-1}{k}}}{T} = \text{constant} \Rightarrow \frac{P_1^{\frac{k-1}{k}}}{T_1} = \frac{P_2^{\frac{k-1}{k}}}{T_2} \Rightarrow T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{\frac{k-1}{k}} = 174.2 \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} = 90.2 \text{ K}$$

Similarly, T_3 can be determined:

$$\frac{P^{\frac{k-1}{k}}}{T} = \text{constant} \Rightarrow \frac{P_3^{\frac{k-1}{k}}}{T_3} = \frac{P_4^{\frac{k-1}{k}}}{T_4} \Rightarrow T_3 = T_4 \left(\frac{P_3}{P_4} \right)^{\frac{k-1}{k}} = 139.4 \left(\frac{10}{1} \right)^{\frac{1.4-1}{1.4}} = 269.1 \text{ K}$$

Hence, the efficiency is calculated to be:

$$\eta_{th,b} = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_2} = \frac{269.1 - 139.4 - 174.2 + 90.2}{269.1 - 174.2} = 0.4816$$

Therefore, the ratio is calculated to be:

$$\frac{\eta_{th,b}}{\eta_{th,c}} = \frac{0.4816}{0.1998} = 2.41$$

Is this possible? Yes, this answer is, in fact, possible. For the Carnot and Stirling cycles, since all external heat transfers into and out of the cycle are isothermal, $T_1 = T_2$ and $T_3 = T_4$, so the maximum and minimum temperatures occur at states 4 and 2, respectively. However, for the Brayton cycle, heating and cooling occur at constant pressure, not at constant temperature. The maximum temperature T_3 (based on isentropic compression) is higher than T_4 and the minimum temperature, T_1 , is lower than T_2 . Hence the Brayton cycle for the given states has a higher efficiency.

9.15 Jet propulsion cycle

9.15a First of all, we summarize all given values in a table. The relation for the enthalpy at 4 follows from the fact that the turbine solely drives the compressor, but has a heat loss as well.

	P (MPa)	T (K)	s (kJ/(kgK))	h (kJ/kg)	other
1	0.1	270			
2	0.57	523			
3	0.57	1098			
4			$s_4 = s_3$	$h_4 - h_3 - q_{out} = h_2 - h_1$	$q_{out} = 20 \text{ kJ/kg}$
5		553	$s_5 = s_4$		$q_{out} = 25 \text{ kJ/kg}$

Now, the isentropic efficiency of the compressor will be determined. This can be done in 2 ways: using the Mollier diagram or using formulas.

First of all, the Mollier diagram method is worked out. The enthalpy h_1 can be determined at $P_1 = 1 \text{ bar}$ and $T_1 = -3^\circ\text{C}$, resulting in $h_1 \approx 260 \text{ kJ/kg}$. Going up vertically ($s_{2s} = s_1$) to $P_2 = 5.7 \text{ bar}$ gives $h_{2s} \approx 440 \text{ kJ/kg}$. Looking up the point $P_2 = 5.7 \text{ bar}$ and $T_2 = 250^\circ\text{C}$ gives $h_2 \approx 530 \text{ kJ/kg}$. The isentropic efficiency of the compressor is now given as:

$$\eta_{s,comp} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{440 - 260}{530 - 260} = 0.667 = 66.7\%$$

Now, the isentropic efficiency will be determined using formulas. From the energy balance over the compressor, where the velocities and the potential energy are neglected, and the process is assumed adiabatic, it follows that $w_{in} = h_2 - h_1$ (as is used in the Mollier diagram analysis as well). For ideal gases, it holds that $dh = c_p dT$, such that $w_{in} = c_p(T_2 - T_1)$. For the ideal situation, this means that $w_{in,s} = c_p(T_{2s} - T_1)$. Furthermore, for isentropic processes, it holds that $\frac{1}{T} P^{\frac{k-1}{k}} = \text{constant}$. Hence, the ideal output temperature can be determined:

$$\frac{P^{\frac{k-1}{k}}}{T} = \text{constant} \Rightarrow \frac{P_1^{\frac{k-1}{k}}}{T_1} = \frac{P_{2s}^{\frac{k-1}{k}}}{T_{2s}} \Rightarrow T_{2s} = T_1 \frac{P_{2s}}{P_1}^{\frac{k-1}{k}}$$

Now, the isentropic efficiency can be determined as:

$$\eta_{s,comp} = \frac{w_{in,s}}{w_{in}} = \frac{c_p \left(T_1 \cdot \frac{P_{2s}}{P_1}^{\frac{k-1}{k}} - T_1 \right)}{c_p(T_2 - T_1)} = \frac{270 \cdot \frac{5.7}{1}^{\frac{1.4-1}{1.4}} - 270}{523 - 270} = 0.688 = 68.8\%$$

It is noted that there exists a difference between the answers of the two methods, due to reading errors in using the Mollier diagram.

9.15b Using the determined enthalpies, the work input in the compressor is determined to be:

$$w_{in} = h_2 - h_1 = 530 - 260 = 270 \text{ kJ/kg}$$

Using the formulas, the work input in the compressor is determined to be:

$$w_{in} = c_p(T_2 - T_1) = 1.004 \cdot (523 - 270) = 254 \text{ kJ/kg}$$

Again there is a difference between the answers, due to reading errors.

9.15c In order to determine the exit velocity of the nozzle, the energy balance over the nozzle is considered:

$$q_{in} + w_{in} + (h + ke + pe)_{in} = q_{out} + w_{out} + (h + ke + pe)_{out}$$

No work is delivered to or by the nozzle, so $w_{in} = w_{out} = 0$. Furthermore, potential energies can be neglected, and there is no heat input ($q_{in} = 0$). The specific kinetic energy is expressed as $\frac{1}{2}v^2$, and the inlet velocity (v_4) is neglected. Hence, the energy balance reduces to:

$$h_4 = q_{out,2} + h_5 + \frac{1}{2}v_5^2 \Rightarrow (h_4 - h_5) - q_{out,2} = \frac{1}{2}v_5^2$$

Using the ideal gas relation $dh = c_p dT$, the enthalpies are rewritten to temperatures. In order to calculate the outlet velocity, first T_4 needs to be determined. This follows from the (simplified) energy balance over the turbine, combined with the fact that all work is used to power the compressor:

$$q_{out,1} + w_{out} = c_p(T_3 - T_4) \Rightarrow T_4 = T_3 - \frac{q_{out,1} + w_{in}}{c_p} = 1098 - \frac{20 \cdot 10^3 + 254 \cdot 10^3}{1.004 \cdot 10^3} = 825 \text{ K}$$

Now, the outlet velocity is calculated. Note that the velocity is in m/s, corresponding to the square root of J/kg (and not kJ/kg!).

$$v_5 = \sqrt{2(c_p(T_4 - T_5) - q_{out,2})} = \sqrt{2(1.004 \cdot 10^3 \cdot (825 - 553) - 25 \cdot 10^3)} = 705 \text{ m/s}$$

9.16 Simple open Brayton cycle

9.16a See figure 9.16.1.

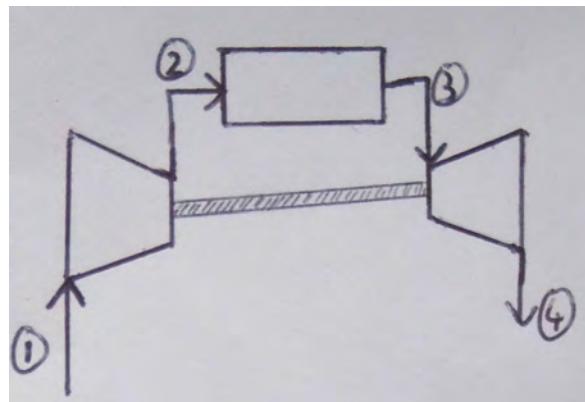


Figure 9.16.1: Schematic overview

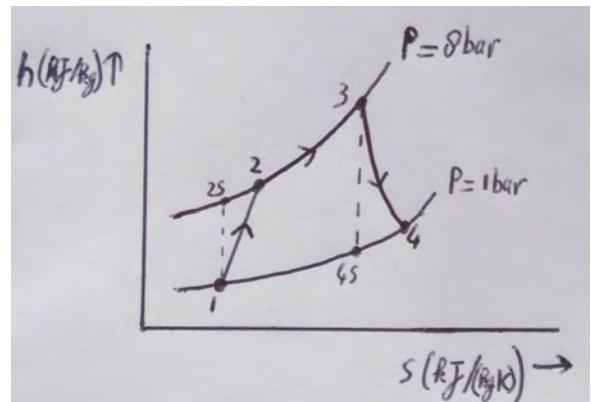


Figure 9.16.2: T-s diagram

9.16b See figure 9.16.2.

9.16c See the table below. The known variables are underlined. The pressures are known since an open cycle is used and hence the inlet and outlet pressures are the atmospheric pressure.

	$P(\text{bar})$	$T(\text{°C})$	$h(\text{kJ/kg})$	$s(\text{kJ/(kgK)})$	other
1	<u>1</u>	<u>37</u>	310		
2s	<u>8</u>	280	560	<u>s_1</u>	
2	<u>8</u>	360	643		<u>$\eta_c = 75\%$</u>
3	<u>8</u>	<u>887</u>	1230		
4s	<u>1</u>	400	690	<u>s_3</u>	
4	<u>1</u>	492	787		<u>$\eta_t = 82\%$</u>

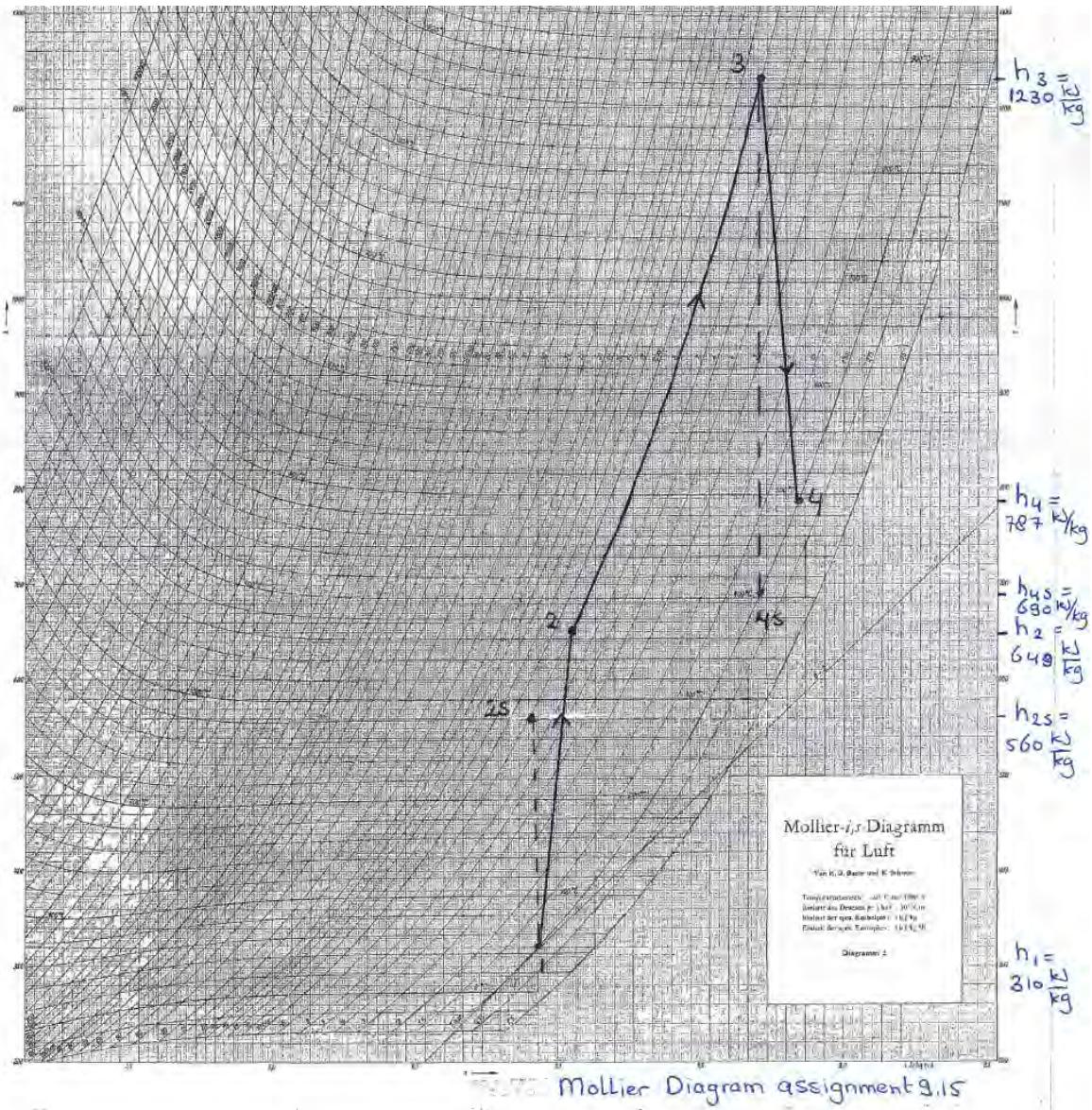


Figure 9.16.3: Mollier diagram of the simple open Brayton cycle of assignment 9.16.

9.16d In order to determine the outlet temperature, first h_3 is determined. Reading from the Mollier diagram in figure 9.16.3 gives $h_3 = 1230 \text{ kJ/kg}$. Going down vertically ($\Delta s = 0$) to $P = 1 \text{ bar}$ the ideal enthalpy is determined: $h_{4s} = 690 \text{ kJ/kg}$. Now

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 1230 - 0.82(1230 - 690) = 787 \text{ kJ/kg}$$

Substituting the determined values gives $h_4 = 787 \text{ kJ/kg}$. The temperature at the exit can now be determined using the Mollier diagram at $P = 1 \text{ bar}$ and $h = 787 \text{ kJ/kg}$: $T_4 = 492^\circ\text{C} = 765 \text{ K}$.

9.16e The net work output is calculated as $w_{net} = w_{turb} - w_{comp} = (h_3 - h_4) - (h_2 - h_1)$. h_1 can be determined directly from the Mollier diagram at $P = 1 \text{ bar}$ and $T = 37^\circ\text{C}$: $h_1 = 310 \text{ kJ/kg}$. Going up vertically we find $h_{2s} = 560 \text{ kJ/kg}$. Using the definition of the isentropic efficiency we can calculate h_2 :

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 310 + \frac{560 - 310}{0.75} = 643.3 \text{ kJ/kg}$$

Now all values are known the net work can be calculated: $w_{net} = (1230 - 787) - (643 - 310) = 110 \text{ kJ/kg}$.

9.16f The thermal efficiency is given by: $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} = \frac{110}{1230 - 643} = 0.187$. So the thermal efficiency is 18.7%.

9.16g The second law efficiency is given by $\eta_{2nd\,law} = \frac{\eta_{th}}{\eta_{carnot}}$, in which $\eta_{carnot} = 1 - \frac{T_{low}}{T_{high}} = 1 - \frac{T_1}{T_3} = 1 - \frac{310}{1160} = 0.73$. This gives that $\eta_{2nd\,law} = \frac{0.187}{0.73} = 0.26$.

Alternative method using the table instead of the Mollier diagram

Above the values for the enthalpy, h_1, h_2, h_3 and h_4 are found using the Mollier diagram for air. They can also be found using the table with the ideal gas properties of air (table A-17). This yields a bit more accurate values but takes more time and gives less insight in the process. Below the values for the enthalpies are determined using the table in order to compare both methods and their results.

- Point 1: $T_1 = 310 \text{ K}$ & $P_1 = 1 \text{ Bar} \rightarrow$ Table A-17: $h_1 = 310.24 \text{ kJ/kg}$ & $P_{r1} = 1.5546$.
- Point 2s: $P_2 = 8 \text{ Bar}$ & $s_2 = s_1 \rightarrow P_{r2} = \frac{P_2}{P_1} P_{r1} = 8 \cdot 1.5546 = 12.4368 \rightarrow$ Table A-17: $h_{2s} = 562.54 \text{ kJ/kg}$ & $T_{2s} = 557.21 \text{ K}$.
- Point 2: For h_2 the formula for the isentropic efficiency for a compressor is used just like is done in case the h values are read from the Mollier diagram.

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 310.24 + \frac{562.54 - 310.24}{0.75} = 646.67 \text{ kJ/kg}$$

Table A-17 gives $T_2 = 637.59 \text{ K}$.

- Point 3: $T_3 = 1160 \text{ K}$ & $P_3 = 8 \text{ Bar} \rightarrow$ Table A-17: $h_3 = 1230.92 \text{ kJ/kg}$ & $P_{r3} = 207.2$.
- Point 4s: $P_4 = 1 \text{ Bar}$ & $s_4 = s_3 \rightarrow P_{r4} = \frac{P_4}{P_3} P_{r3} = \frac{1}{8} \cdot 207.2 = 25.9 \rightarrow$ Table A-17: $h_{4s} = 692.19 \text{ kJ/kg}$ & $T_{4s} = 680.35 \text{ K}$.
- Point 4: For h_4 the formula for the isentropic efficiency for a turbine is used just like is done in case the h values are read from the Mollier diagram.

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 1230.92 - 0.82(1230.92 - 692.19) = 78.169 \text{ kJ/kg}$$

Table A-17 gives $T_4 = 770.05 \text{ K}$.

Table 9.16.1 summarizes the results found using the Mollier diagram and the table (underlined are the two known variables). It can be concluded that both methods yields similar values for h and T . Using these values to determine the net work output and the thermal efficiency results in:

$$w_{net} = w_{turb} - w_{comp} = (h_3 - h_4) - (h_2 - h_1) = (1231 - 789) - (647 - 310) = 105 \text{ kJ/kg.}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} = \frac{105}{1231 - 647} = 0.1798.$$

So the net work output is 105 kJ/kg and the thermal efficiency is 18.0%. Using the Mollier diagram this was 110 kJ/kg and 18.7% respectively.

Table 9.16.1: Comparison of the three different methods.

	$P(\text{bar})$ diagram	$T(\text{°C})$ diagram	$h(\text{kJ/kg})$ diagram	other	$T(\text{K})$ table	$P_r(-)$ table	$h(\text{kJ/kg})$ table	$T(\text{K})$ formula	$h(\text{kJ/kg})$ formula
1	<u>1</u>	<u>37</u>	310		<u>310</u>	1.5546	310	<u>310</u>	310
2s	<u>8</u>	280	560	<u>$s_{2s} = s_1$</u>	557	12.4368	563	555	559
2	<u>8</u>	360	643	<u>$\eta_c = 75\%$</u>	638		647	637	642
3	<u>8</u>	<u>887</u>	1230		<u>1160</u>	207.2	1231	<u>1160</u>	1223
4s	<u>1</u>	400	690	<u>$s_{4s} = s_3$</u>	680	25.9	692	684	695
4	<u>1</u>	492	787	<u>$\eta_t = 82\%$</u>	770		789	770	790

Alternative method using formulas instead of the tables or the Mollier diagram

Besides determining the enthalpies using either the tables or the Mollier diagram, it is also possible to determine them using formulas. This takes more time, but can be a good solution when other gases are used (e.g. helium), for which there are often no tables or diagrams available. The formula's can also well be used when you use Matlab. Below, the values for the enthalpies are determined using the ideal gas formula's, and they are compared to the previous methods in table 9.16.1. The values of c_p and k are taken from table A-2b at the average temperature.

- Point 1: We cannot determine the actual enthalpy at point 1. Since we are only interested in enthalpy differences, it does not matter too much. For comparison with the other methods, we will now choose it to equal $h_1 = 310 \text{ kJ/kg}$. This value is also approximately obtained by using the relation for ideal gases $dh = c_p dT$ and integrating this from $T = 273 \text{ K}$ to T_1 . To get a precise answer as possible, c_p should be taken as a function of the temperature. For many gases, this function is given in the tables (e.g. table A-2c).

- Point 2s: For isentropic processes with an ideal gas, it holds that $\frac{P^{k-1}}{T} = \text{constant}$, such that

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1} \right)^{\frac{k-1}{k}} \Rightarrow T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = (273 + 37) \left(\frac{8}{1} \right)^{\frac{1.4-1}{1}} = 554.9 \text{ K}$$

Using that for ideal gases $dh = c_p dT$, we can determine h_{2s} :

$$dh = c_p dT \Rightarrow h_{2s} = c_p(T_{2s} - T_1) + h_1 = 1.016(555 - 310) = 559 \text{ kJ/kg}$$

c_p and k are taken from table A-2b at the average temperature, which is around 433 K.

- Point 2: We use the definition of the isentropic efficiency to determine h_2 from h_{2s} :

$$\eta_{s,c} = \frac{h_2 - h_1}{h_{2s} - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_s} = 642 \text{ kJ/kg}$$

T_2 also follows from $dh = c_p dT$:

$$dh = c_p dT \Rightarrow T_2 = T_1 + \frac{(h_2 - h_1)}{c_p} = 310 + \frac{(642 - 310)}{1.016} = 637 \text{ K}$$

- Point 3: We again use the ideal gas relation $dh = c_p dT$ to determine h_3 :

$$dh = c_p dT \Rightarrow h_3 = c_p(T_3 - T_2) + h_2 = 1.11(887 - 360) + 642 = 1223 \text{ kJ/kg}$$

In this case, c_p was taken as the average value of c_p at $T_2 = 360^\circ\text{C}$ and $T_3 = 887^\circ\text{C}$ from table A-2b. When using Matlab, or more accurate answers are desired, the function $c_p(T)$ (table A-2c) can be used, which then should be integrated from T_2 to T_3 .

- Point 4s: We use, once more, that for isentropic processes with an ideal gas $\frac{P^{\frac{k-1}{k}}}{T} = \text{constant}$:

$$T_{4s} = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = 1160 \left(\frac{1}{8} \right)^{0.254} = 684 \text{ K}$$

The value of k was taken at 1.34 (table A-2b), because at higher temperatures it does become significantly lower than the commonly used 1.4. To be more precise, one can use the function $c_p(T)$ (Table A-2c) and with $c_v(T) = c_p(T) - R$, calculate the ratio of the two (remember that $k = c_p/c_v$) and determine what T_{4s} should be. This is doable using Matlab, but takes far too much time to do by hand, and therefore it has been chosen to use a rough estimation of k here. From this temperature, h_{4s} can be determined:

$$dh = c_p dT \Rightarrow h_{4s} = h_3 + c_p(T_{4s} - T_3) = 1223 + 1.11(684 - 1160) = 695 \text{ kJ/kg}$$

- Point 4: We use the definition of the isentropic efficiency to determine h_4 from h_{4s} :

$$\eta_{s,t} = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_{s,t}(h_3 - h_{4s}) = 1223 - 0.82(1223 - 695) = 790 \text{ kJ/kg}$$

T_4 also follows from $dh = c_p dT$:

$$dh = c_p dT \Rightarrow T_4 = T_3 - \frac{(h_3 - h_4)}{c_p} = 1160 - \frac{(1223 - 790)}{1.11} = 770 \text{ K}$$

Using the h -values found using the formulas results in:

$$w_{net} = w_{turb} - w_{comp} = (h_3 - h_4) - (h_2 - h_1) = (1223 - 790) - (642 - 310) = 111 \text{ kJ/kg.}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} = \frac{111}{1223 - 642} = 0.188.$$

So the net work output is 111 kJ/kg and the thermal efficiency is 18.8%. Using the Mollier diagram this was 110 kJ/kg and 18.7% respectively and using the tables they were 105 kJ/kg and 18.0% respectively.

It is observed that the values correspond relatively well with the values obtained by using the tables or the Mollier diagram. Hence, this method seems to offer a good method for calculating the enthalpy values of other **ideal** gases, where no tables or diagrams are available. Also, it is a method that is good to use in Matlab, as all formulas can be programmed, and the effect of changes can be quickly determined afterwards. However, in order to get precise results, it is essential that the values of c_p and k are taken realistic, and not constant, as the variation is too large within the used temperature ranges.

9.17 The 7FA gas turbine

9.17a See figure 9.17.1.

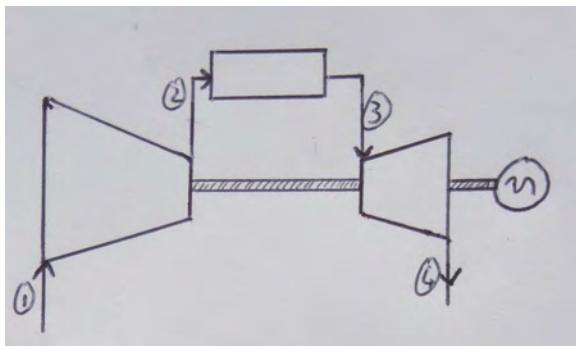


Figure 9.17.1: Overview of exercise 9.17

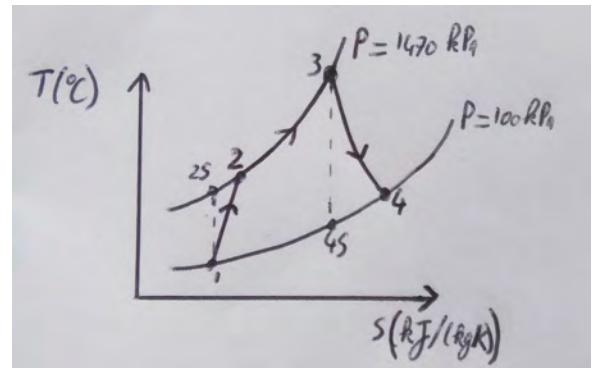


Figure 9.17.2: *Ts*-diagram

9.17b The known variables are the underlined values in the table below. The thermal efficiency and the netto power output will be used to determine the second parameter.

	$P(\text{kPa})$	$T(\text{°C})$	$h(\text{kJ/kg})$	$s(\text{kJ}/(\text{kgK}))$	other
1	<u>100</u>	<u>20</u>	295	6.83	
2s	<u>1470</u>	345	625	<u>s_1</u>	
2	<u>1470</u>	405	682	6.90	<u>η_{th}, \dot{W}_{net} and \dot{m}</u>
3	<u>1470</u>	<u>1288</u>	1720	7.90	
4s	<u>100</u>	540	830	<u>s_3</u>	
4	<u>100</u>	655	960	8.04	<u>\dot{W}_{net} and \dot{m}</u>

9.17c See figure 9.17.2. Note, this is an open cycle, therefore the line 1-4 should not be drawn.

9.17d

$$\eta_{s,comp} = \frac{\Delta h_{isentropic}}{\Delta h_{real}} = \frac{h_{2s} - h_1}{h_2 - h_1} \quad \text{and} \quad \eta_{s,turb} = \frac{\Delta h_{real}}{\Delta h_{isentropic}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

9.17e In order to calculate the isentropic efficiencies all h-values are needed. h_1 and h_3 can be found directly in the Mollier diagram given in figure 9.17.3: $h_1 = 295 \text{ kJ/kg}$ and $h_3 = 1720 \text{ kJ/kg}$. By going down vertically ($\Delta s = 0$) in the Mollier diagram from point 1, one finds $h_{2s} = 625 \text{ kJ/kg}$, and going down vertically from point 3 gives $h_{4s} = 830 \text{ kJ/kg}$. From the thermal efficiency and the netto power h_2 can be determined: $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$ with $\dot{Q}_{in} = \dot{m}(h_3 - h_2)$. Rewriting gives:

$$\dot{Q}_{in} = \frac{\dot{W}_{net}}{\eta_{th}} = \dot{m}(h_3 - h_2) \rightarrow h_2 = h_3 - \frac{\dot{W}_{net}}{\dot{m}\eta_{th}} = 1720 - \frac{159 \cdot 10^3}{\frac{1536 \cdot 10^3}{3600} \cdot 0.359} = 682 \text{ kJ/kg}$$

h_4 is now calculated from the netto power:

$$\dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{comp} = \dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1) \rightarrow h_4 = h_3 - h_2 + h_1 - \frac{\dot{W}_{net}}{\dot{m}} = 1333 - \frac{159 \cdot 10^3}{\frac{1536 \cdot 10^3}{3600}} = 960 \text{ kJ/kg}$$

Substituting these values in the equations from e we find the isentropic efficiencies:

$$\eta_{s,comp} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{625 - 295}{682 - 295} = 0.85 \quad \text{and} \quad \eta_{s,turb} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{1720 - 960}{1720 - 830} = 0.85$$

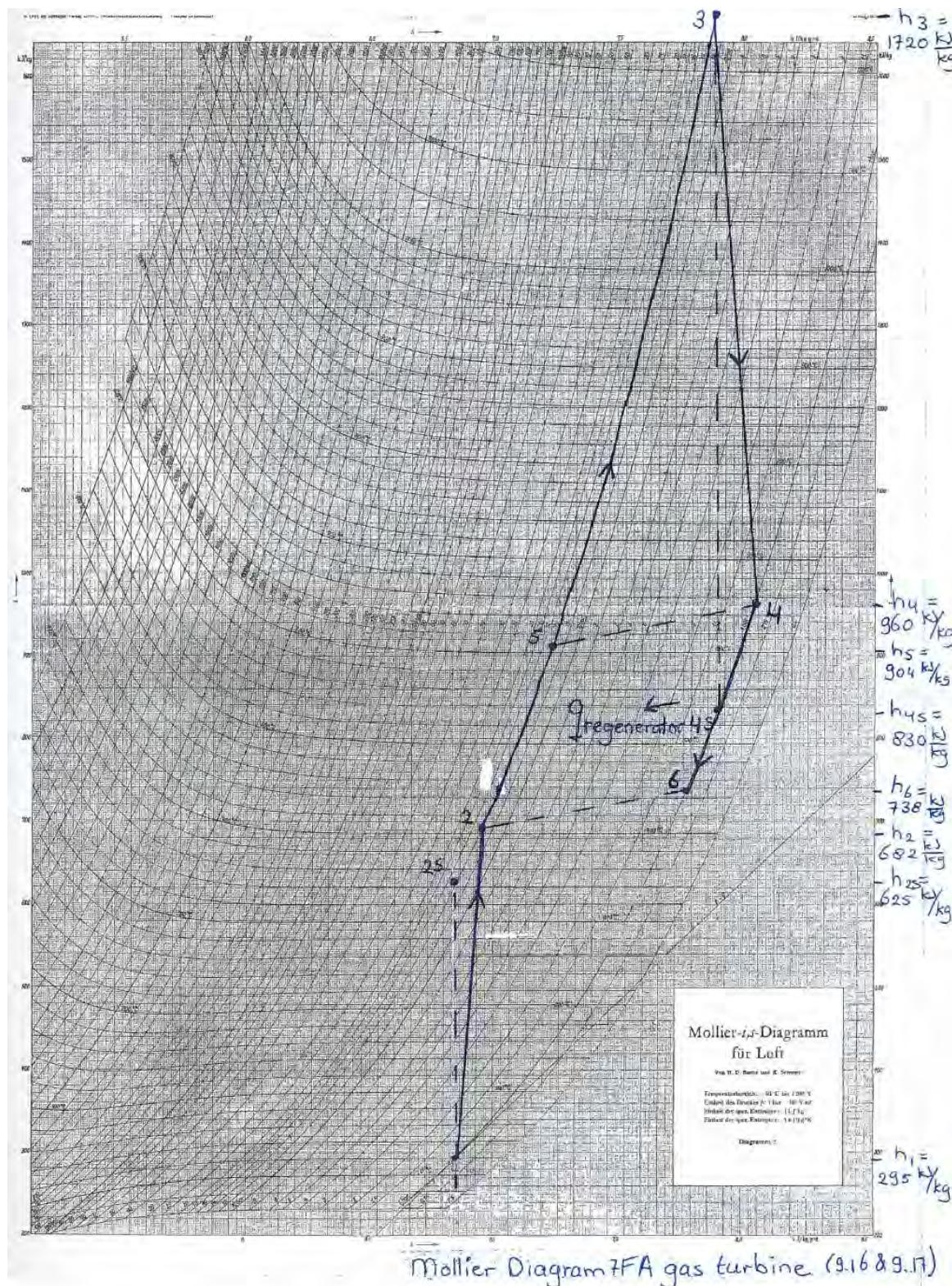


Figure 9.17.3: Mollier diagram of the 7FA gas turbine of assignment 9.17 and 9.18. For assignment 9.17 only the points 1, 2, 3 and 4 are of importance. The points 5 and 6 are added for assignment 9.18 where a regenerator is added to the cycle. The regenerated heat for assignment 9.18 is denoted by $q_{\text{regenerator}}$.

9.18 The 7FA gas turbine with regeneration

9.18a See figure 9.18.1. Note that this is the same setup as in exercise 9.16 and many values found in that exercise are valid in this cycle as well. Points 1, 2, 3 and 4 do not change by the regeneration, but points 5 (after the regenerator) and point 6 (at the other side after the regenerator) are added. Also the Mollier diagram of assignment 9.17 can be used again, points 1, 2, 3 and 4 are the same (points 5 and 6 are added later after they have been calculated).

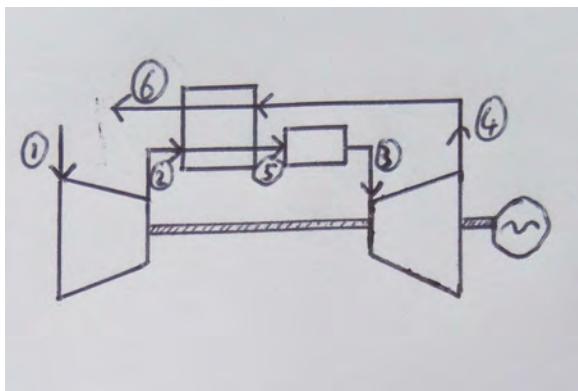


Figure 9.18.1: Schematic overview

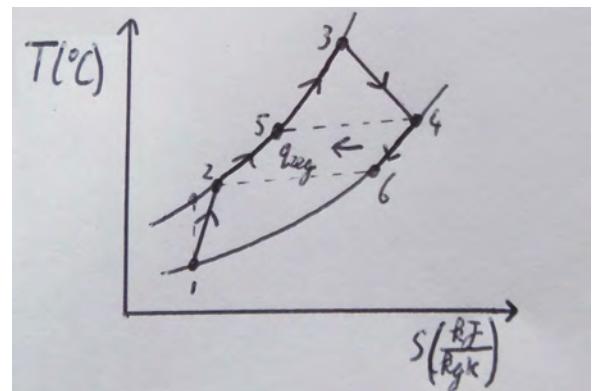


Figure 9.18.2: T-s diagram

9.18b See figure 9.18.2. Compared to the situation without regenerator, there is now a transfer of heat that is still in the flow at the exit of the turbine (point 4) to the compressed cold air (point 2). In the T-s diagram point 5 and 6 are added, and the line 4-6 is extra compared to exercise 9.16. Points 1, 2, 3 and 4 remain unchanged. The transfer of heat is also denoted in the Mollier diagram in figure 9.17.3.

9.18c The net power output is calculated as $\dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{comp} = \dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1)$. Since these h-values do not change, the net power output is equal to the one in exercise 9.16 (195 MW, given in the exercise). So, the regenerator does not influence the power output.

9.18d The heat output is the heat that is still in the exhaust gases, with regenerator given by $h_6 - h_1$. Since energy is conserved (and the mass flows are constant throughout the cycle), $h_5 - h_2 = h_4 - h_6$. Furthermore, the effectiveness of the regenerator is given by $\epsilon = \frac{q_{reg}}{q_{reg,max}} = \frac{h_5 - h_2}{h_4 - h_2}$. Rewriting gives $h_5 = h_2 + \epsilon(h_4 - h_2) = 682 + 0.8(960 - 682) = 904.4$ kJ/kg and hence $h_6 = h_4 - h_5 + h_2 = 960 - 904.4 + 682 = 737.6$ kJ/kg.

So, the heat output is with regenerator $q_{out} = h_6 - h_1 = 737.6 - 295 = 442.6$ kJ/kg, compared to $q_{out} = h_4 - h_1 = 665$ kJ/kg without regenerator. As expected, the heat output is lower ($665 - 442.6 = 665$ kJ/kg lower), which was of course the goal of the regenerator.

The heat input, the heat added in the combustion chamber, with regenerator is given by $q_{in} = h_3 - h_5 = 815.6$ kJ/kg. Without regenerator this is $q_{in} = h_3 - h_2 = 1038$ kJ/kg. Also this value has decreased ($1038 - 815.6 = 222.4$ kJ/kg), which was the purpose of using the heat from the exhaust gases in the cycle.

The regenerated heat is the amount of heat that the heat input is decreased or the amount of heat the the heat output is decreased (these are equal), which is $q_{reg} = h_4 - h_6 = h_5 - h_2 = 222.4$ kJ/kg.

The points 5 and 6 are, after they have been calculated, also added to the Mollier diagram given in figure 9.17.3, also the regenerated heat is denoted in the Mollier diagram. Note that if the effectiveness of the regenerator would have been 100% the dashed lines 4-5 and 6-2 would have been horizontal where point 5 would go up along the isobar and point 6 would go down along the isobar resulting in a higher amount (the maximum) of regenerated heat.

9.18e

$$\eta_{th,reg} = \frac{w_{net}}{q_{in}} = \frac{h_3 - h_4 - (h_2 - h_1)}{h_3 - h_5} = 0.46$$

In the situation without regenerator, the thermal efficiency was $\eta_{th} = 0.359$, so the efficiency with regenerator is higher (as expected).

$$\textbf{9.18f } \eta_{2nd\ law} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{\eta_{th}}{1 - \frac{T_{low}}{T_{high}}} = \frac{0.46}{1 - \frac{293}{1561}} = \frac{0.46}{0.81} = 0.57.$$

9.19 Brayton cycle with intercooling

9.19a See figure 9.19.1.

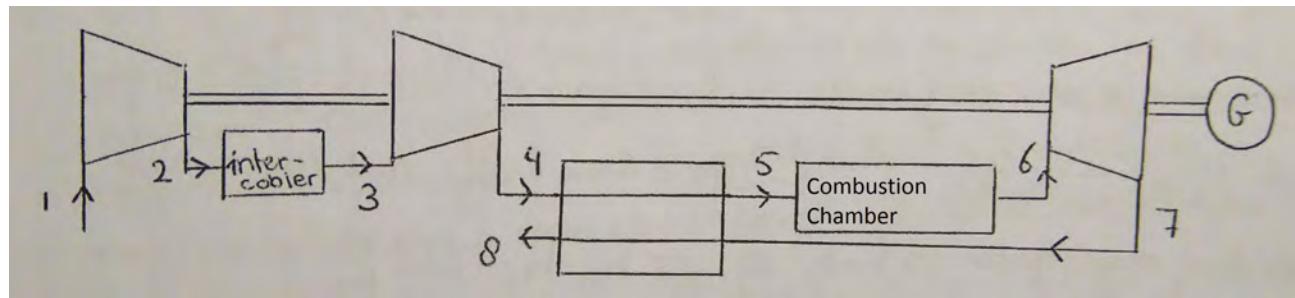


Figure 9.19.1: Schematic overview of the cycle

9.19b See the table below. The two known values at the characteristic points are underlined. Temperatures are read from the Mollier diagram. At point 5, ϵ stands for the effectiveness of the regenerator.

	P (bar)	T ($^{\circ}\text{C}$)	h (kJ/kg)	s (kJ/kgK)	other
1	<u>1</u>	<u>15</u>	290		
2s	<u>4</u>	155	430	<u>$s_{2s} = s_1$</u>	
2	<u>4</u>	195	465		$\eta_{sc} = 0.8$
3	<u>4</u>	<u>40</u>	310		
4s	<u>12</u>	155	430	<u>$s_{4s} = s_3$</u>	
4	<u>12</u>	195	460		$\eta_{sc} = 0.8$
5	<u>12</u>	440	728		$\underline{\epsilon} = 1$
6	<u>12</u>	<u>860</u>	1200		
7s	<u>1</u>	330	610	<u>$s_{7s} = s_6$</u>	
7	<u>1</u>	440	728		$\eta_{st} = 0.8$
8	<u>1</u>	195	460		$h_5 - h_4 = h_7 - h_8$

9.19c See figure 9.19.2.

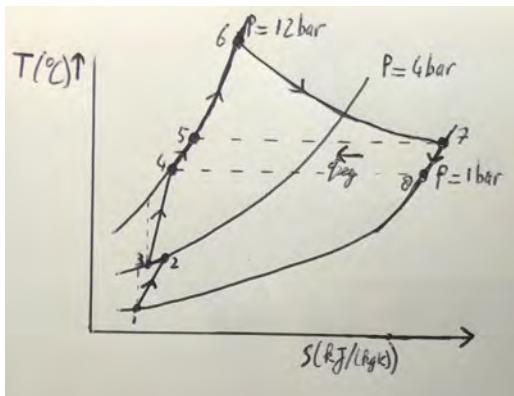


Figure 9.19.2: Ts-diagram of the cycle. The figure is not to scale. Point 6 should have been drawn much higher. Important is that point 5 and 7 are on the same temperature and point 4 and 8 as well (effectiveness of the regenerator is 1).

The h -values in point 1, 2s, 4s, 6 and 7s can be read directly from the Mollier diagram, see figure 9.19.3. Using the isentropic efficiency of the compressors and the turbine, h_2 , h_4 and h_7 can be determined:

$$\eta_{sc} = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{sc}} = 290 + \frac{430 - 290}{0.8} = 465 \text{ kJ/kg}$$

$$\eta_{sc} = \frac{h_{4s} - h_3}{h_4 - h_3} \rightarrow h_4 = h_3 + \frac{h_{4s} - h_3}{\eta_{sc}} = 310 + \frac{430 - 310}{0.8} = 460 \text{ kJ/kg}$$

$$\eta_{st} = \frac{h_6 - h_7}{h_6 - h_{7s}} \rightarrow h_7 = h_6 - \eta_{st}(h_6 - h_{7s}) = 1200 - 0.8(1200 - 610) = 728 \text{ kJ/kg}$$

The value h_5 is determined using the effectiveness of the regenerator:

$$\epsilon = \frac{h_5 - h_4}{h_7 - h_4} \rightarrow h_5 = h_4 + \epsilon(h_7 - h_4) = 460 + 1(728 - 460) = 728 \text{ kJ/kg}$$

Now conservation of energy is used to determine h_8 :

$$h_7 - h_8 = h_5 - h_4 \rightarrow h_8 = h_7 - (h_5 - h_4) = 728 - (728 - 460) = 460 \text{ kJ/kg}$$

9.19d The work delivered by the turbine is given as $w_{turb} = h_6 - h_7 = 1200 - 728 = 472 \text{ kJ/kg}$. The temperature at the exit of the turbine can be determined using the Mollier diagram: $T_7 = 440^\circ\text{C}$.

9.19e The thermal efficiency is:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{\eta_m w_{turb} - \frac{w_{comp1} + w_{comp2}}{\eta_m}}{q_{in}} = \frac{\eta_m(h_6 - h_7) - \frac{(h_2 - h_1) + (h_4 - h_3)}{\eta_m}}{h_6 - h_5} = \frac{0.98 \cdot 472 - \frac{325}{0.98}}{472} = 0.277.$$

9.19f The power supplied to the generator is:

$$\begin{aligned} \dot{W}_{gen} &= \dot{W}_{net} = \dot{m} w_{net} = \dot{m} \left(\eta_m w_{turb} - \frac{w_{comp1} + w_{comp2}}{\eta_m} \right) = \dot{m} \left(\eta_m(h_6 - h_7) - \frac{(h_2 - h_1) + (h_4 - h_3)}{\eta_m} \right) \\ &= 7 \left(0.98(1200 - 728) - \frac{(465 - 290) + (460 - 310)}{0.98} \right) = 7 \left(0.98 \cdot 472 - \frac{325}{0.98} \right) = 916.49 \text{ kW} = 0.92 \text{ MW}. \end{aligned}$$

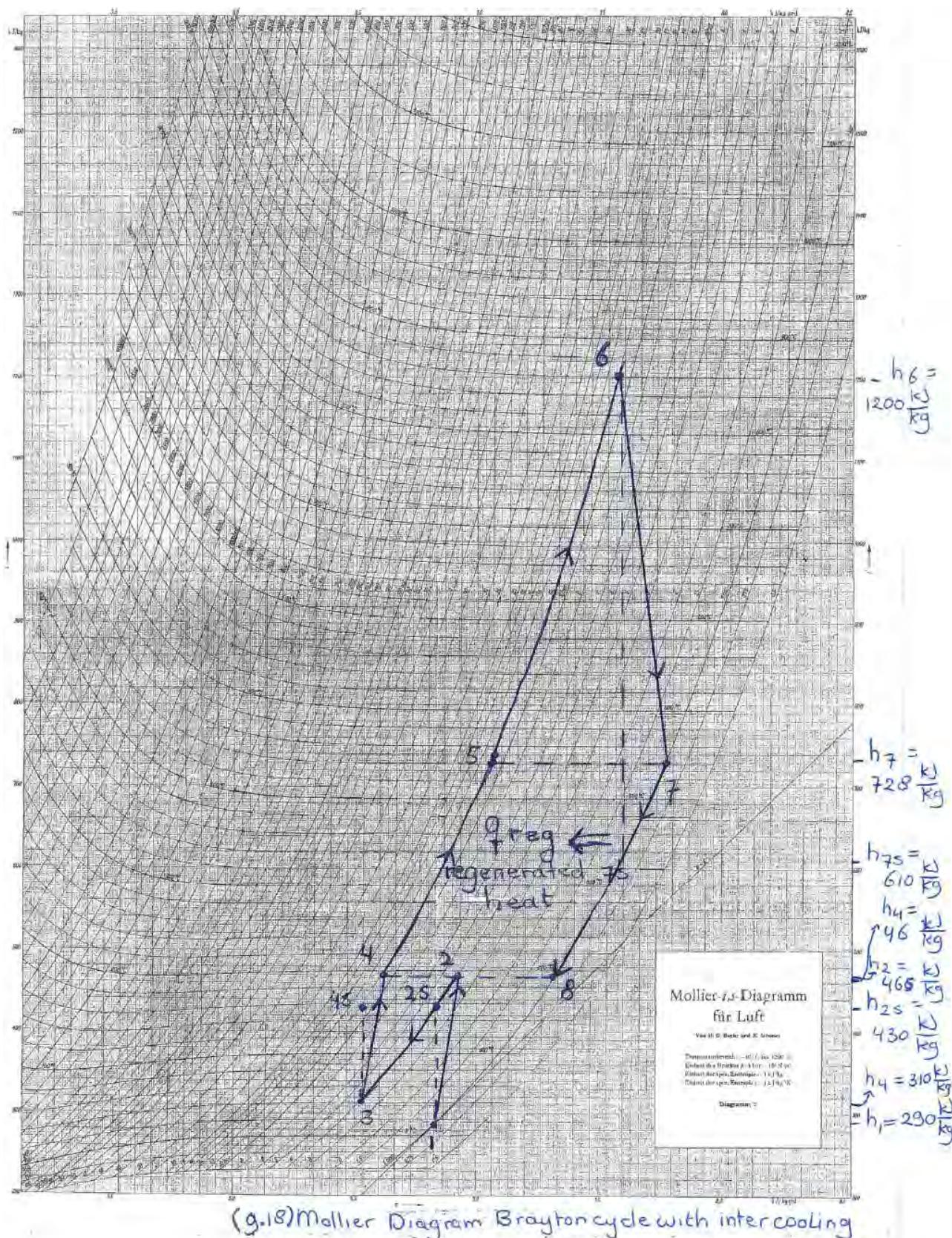


Figure 9.19.3: Mollier diagram of the Brayton cycle with intercooling of assignment 9.19.

9.20 Brayton cycle with reheating

9.20a See figure 9.20.1.

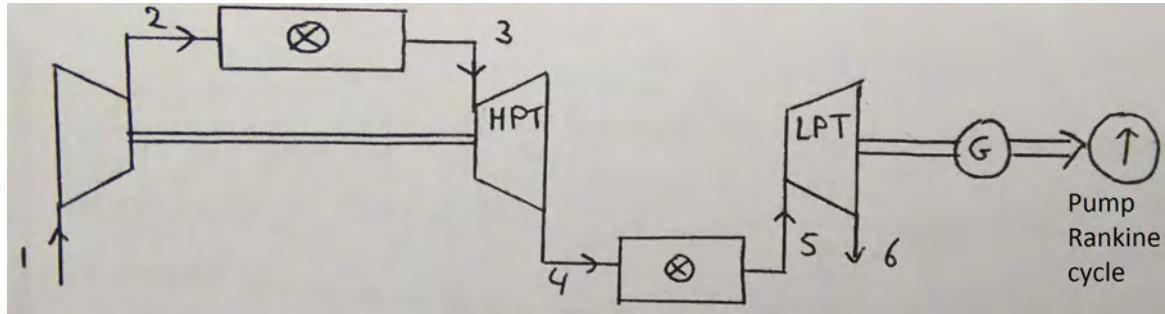


Figure 9.20.1: Overview of the Brayton cycle with reheating in assignment 9.20

9.20b The two known given values at the characteristic points are the underlined values in the table below. Temperatures are read from the Mollier diagram.

	P (bar)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/kgK)	other
1	<u>1</u>	<u>15</u>	290	6.83	
2s	<u>20</u>	410	690	<u>$s_{2s} = s_1$</u>	
2	<u>20</u>	500	790	6.97	<u>$\eta_{sc} = 0.8$</u>
3	<u>20</u>	<u>1060</u>	1440	7.62	
4s	3.5	585	884	<u>$s_{4s} = s_3$</u>	<u>$\eta_{st} = 0.9$</u>
4	<u>$P_4 = P_{4s} = 3.5$</u>	630	940	7.66	<u>$\dot{W}_{out,HPT} = \dot{W}_{in,comp}$</u>
5	<u>$P_5 = P_4 = 3.5$</u>	<u>980</u>	1340	8.05	
6s	<u>1</u>	650	960	<u>$s_{6s} = s_5$</u>	
6	<u>1</u>	685	998	8.10	<u>$\eta_{st} = 0.9$</u>

9.20c See figure 9.20.2.

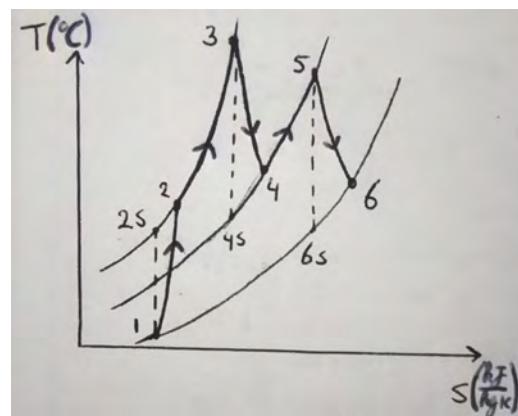


Figure 9.20.2: T-s diagram of exercise 9.20.

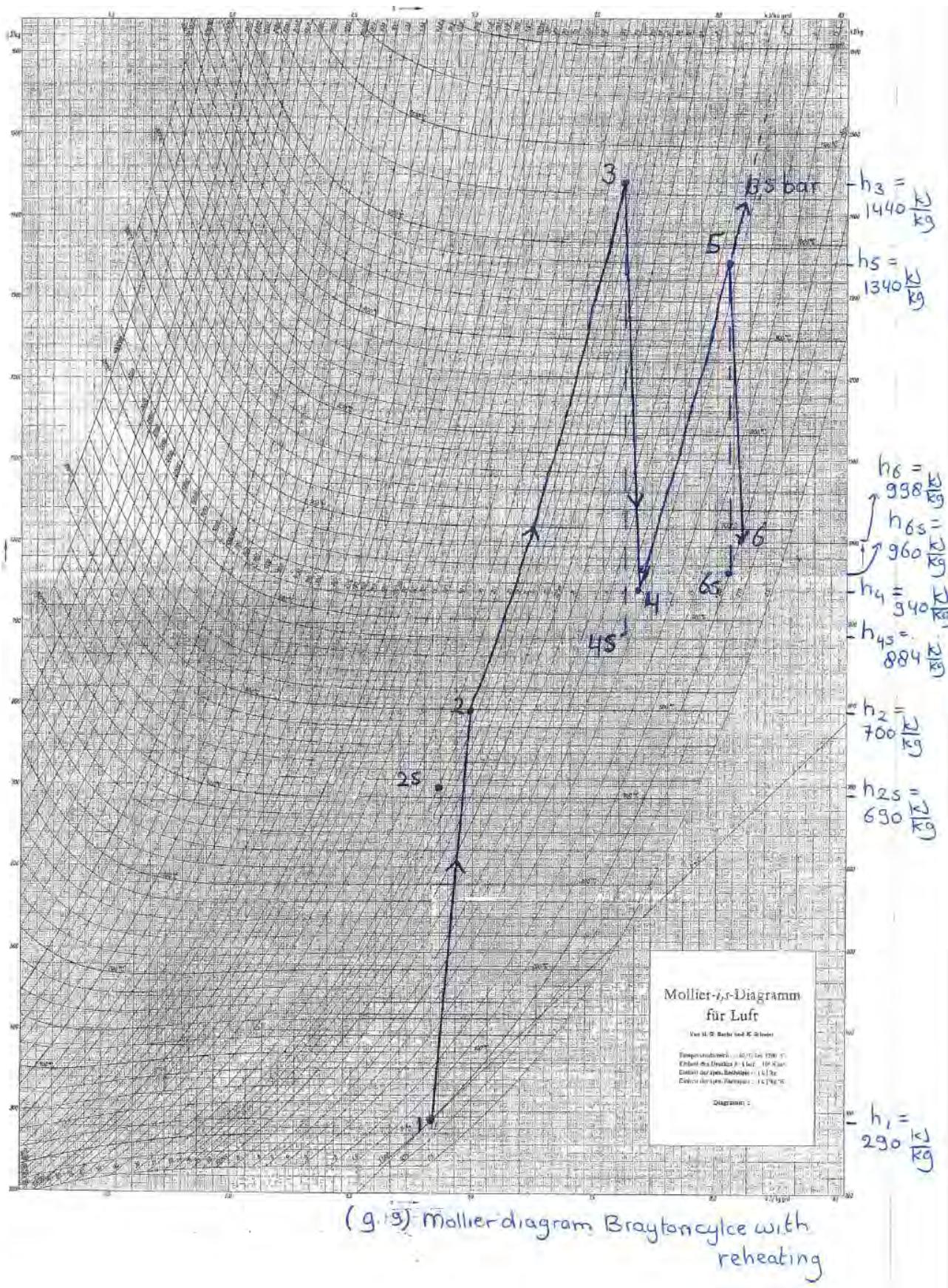


Figure 9.20.3: Mollier diagram of the Brayton cycle with reheating of assignment 9.20.

9.20d To find the required values the cycles had to be analyzed and all h -values have to be determined:

- h_1 can be read out in the Mollier diagram, see figure 9.20.3.
- By going up vertically ($\Delta s = 0$) to $P_{2s} = 20$ bar, one finds the value of h_{2s} . Using the isentropic efficiency of the compressor, h_2 is determined: $\eta_{sc} = \frac{h_{2s}-h_1}{h_2-h_1} \rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_{sc}} = 790$ kJ/kg.
- h_3 can be found in the Mollier diagram directly.
- h_4 follows from the fact that the high pressure turbine is used only to power the compressor: $\dot{W}_{out,HPT} = \dot{W}_{in,comp} \rightarrow \dot{m}(h_3 - h_4) = \dot{m}(h_2 - h_1) \rightarrow h_4 = h_3 - h_2 + h_1 = 940$ kJ/kg. This value is used to calculate h_{4s} via the isentropic efficiency of the turbine: $\eta_{st} = \frac{h_3-h_4}{h_{3s}-h_{4s}} \rightarrow h_{4s} = h_3 - \frac{h_3-h_4}{\eta_{st}} = 884$ kJ/kg. Using the Mollier diagram, the pressure can now be determined (vertically below point 3 and the enthalpy is known): $P_{4s} = P_4 = P_5 = 3.5$ bar.
- h_5 can now be read from the Mollier diagram: $h_5 = 1340$ kJ/kg. Now h_{6s} is determined using the Mollier diagram as well (vertically below point 5 at a pressure of 1 bar): $h_{6s} = 960$ kJ/kg.
- h_6 is determined with the isentropic efficiency of the turbine: $\eta_{st} = \frac{h_5-h_6}{h_5-h_{6s}} \rightarrow h_6 = h_5 - \eta_{st}(h_5 - h_{6s}) = 998$ kJ/kg.

With the h -values and the Mollier diagram the required values can be calculated:

- The pressure ratios of the turbines are now $\frac{P_3}{P_4} = 5.7$ for the HPT and $\frac{P_5}{P_6} = 3.5$ for the LPT.
- The temperature at the exit of the turbines can be determined from the Mollier-diagram: $T_4 = 630^\circ\text{C}$ for the HPT and $T_6 = 685^\circ\text{C}$ for the LPT.
- The power delivered by the turbines is $\dot{W}_{out,HPT} = \dot{m}(h_3 - h_4) = 1500$ kW = 1.5 MW for the HPT and $\dot{W}_{out,LPT} = \dot{m}(h_5 - h_6) = 1026$ kW = 1.0 MW for the LPT.

9.20e The netto power delivered to the pump is $\dot{W}_{net,pump} = \eta_{gear} \cdot \dot{m}(h_5 - h_6) = 1005$ kW = 1.0 MW.

9.20f The thermal efficiency is $\eta_{th} = \frac{\dot{W}_{net,pump}}{\dot{Q}_{in}} = \frac{\dot{m}(h_5 - h_6)\eta_{gear}}{\dot{m}(h_3 - h_2) + \dot{m}(h_5 - h_4)} = 0.31$.

10 Vapor and Combined Power Cycles

10.1 Rankine cycles

10.1a The four processes that make up an ideal Rankine cycle are isentropic compression (in an ideal pump), isobaric heat addition (in a boiler), isentropic expansion (in an ideal turbine) and isobaric heat rejection (in a condenser).

10.1b In actual vapor power cycles, the compression and expansion of the fluid is not isentropic. Instead, entropy is created during these processes, such that the processes are irreversible and hence not ideal.

10.1c See figures 10.1.1 and 10.1.2 below.

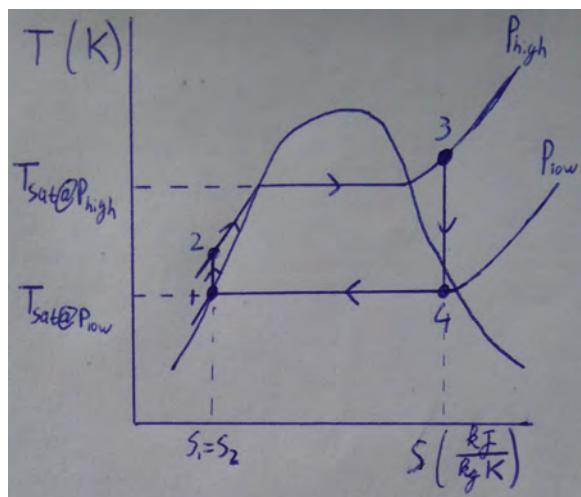


Figure 10.1.1: T - s -diagram of an ideal Rankine cycle

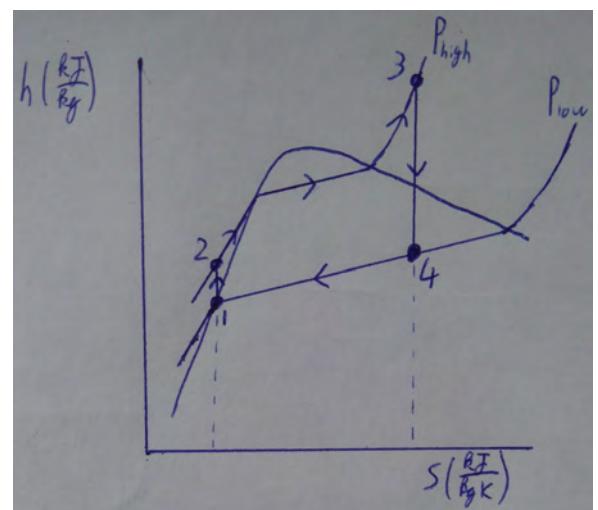


Figure 10.1.2: h - s -diagram of an ideal Rankine cycle

10.1d See figures 10.1.3 and 10.1.4 below.

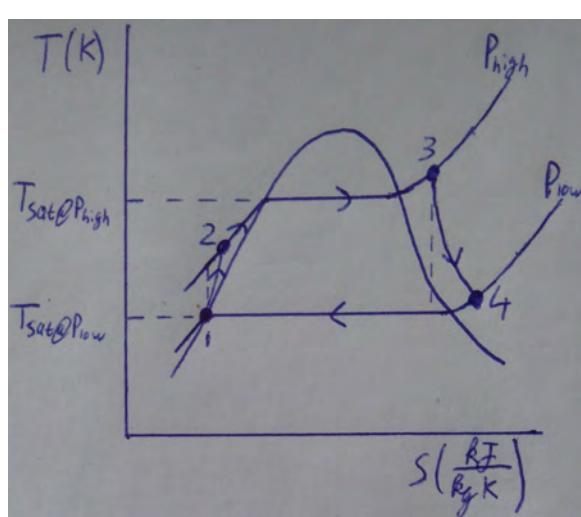


Figure 10.1.3: T - s -diagram of an actual Rankine cycle

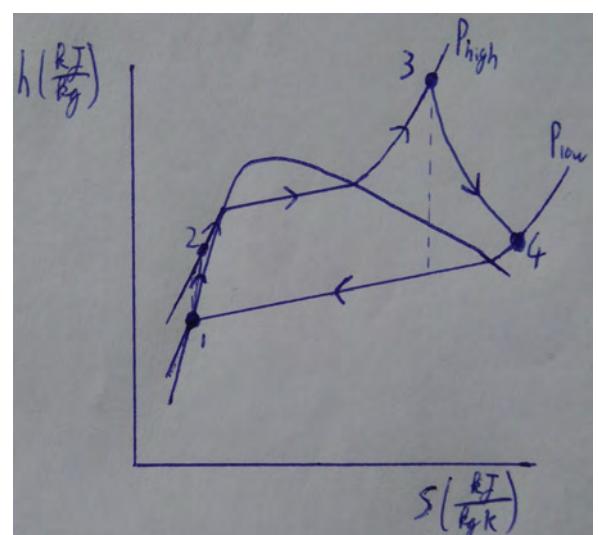


Figure 10.1.4: h - s -diagram of an actual Rankine cycle

10.1e The temperature of 10 °C is the lowest temperature that can be reached with this condenser. In the tables, we find that this is the saturation temperature belonging to 1.2281 kPa.

10.1f In the summer, the lowest reachable temperature is 20 °C, which is the saturation temperature belonging to 2.3388 kPa. In the winter, the operating pressure is 0.8726 kPa (saturation temperature of 5 °C).

10.1g Lowering the condenser pressure will have the following effects:

- Pump work input is increased
- Turbine work output is increased
- No effect on supplied heat
- Heat rejected is increased
- Cycle efficiency is increased
- Moisture content at the turbine exit is increased (i.e. quality of the saturated mixture is decreased)

10.1h Increasing the boiler pressure will have the following effects:

- Pump work input is increased
- Turbine work output remains the same (approximately)
- Supplied heat remains the same (approximately), although it takes place at a higher temperature
- Rejected heat is decreased
- Cycle efficiency is increased
- Moisture content at the turbine exit is increased (i.e. quality of the saturated mixture is decreased)

10.1i Increasing the steam temperature before the turbine will have the following effects:

- Pump work input remains the same
- Turbine work output is increased
- Supplied heat is increased
- Rejected heat is increased
- Cycle efficiency is increased
- Moisture content at the turbine exit is decreased (i.e. quality of the saturated mixture is increased)

10.2 Power plant on a simple ideal Rankine cycle 1

10.2a See figure 10.2.1.

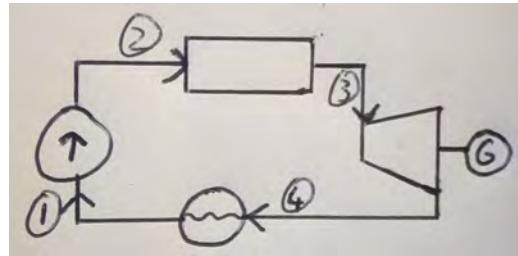


Figure 10.2.1: Overview of exercise 10.2

10.2b See figures 10.2.2 and 10.2.3.

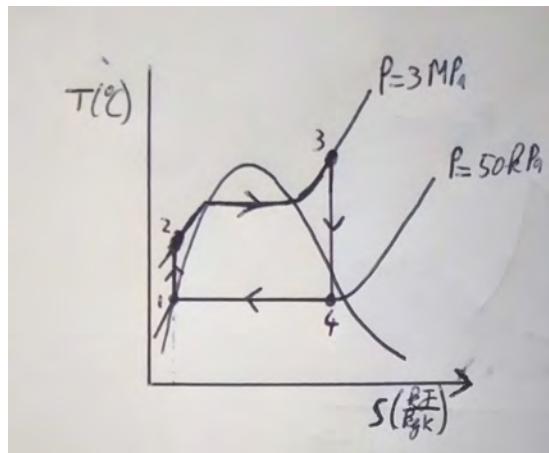


Figure 10.2.2: T-s diagram of exercise 10.2

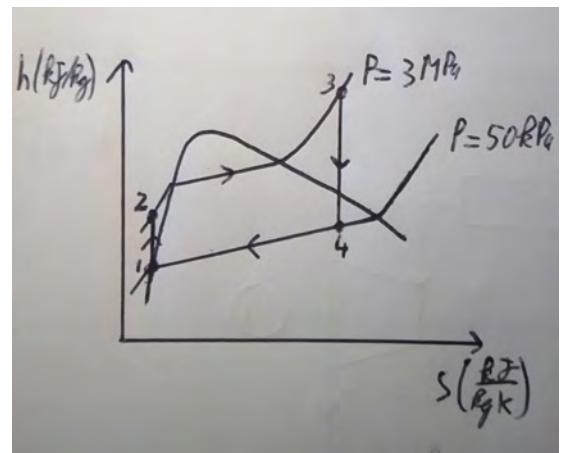


Figure 10.2.3: h-s diagram of exercise 10.2

10.2c The known variables are the underlined values in the table below.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	phase
1	<u>50</u>	81.32	340.54	1.0912	<u>sat. liq.</u>
2	<u>3000</u>		343.58	<u>$s_2 = s_1$</u>	compr. liq.
3	<u>3000</u>	<u>400</u>	3231.7	6.9235	superheated
4	<u>50</u>	81.32	2407.8	<u>$s_4 = s_3$</u>	mix, $x = 0.9$

10.2d From the first law, it follows that the power delivered by the (adiabatic) turbine is calculated as $\dot{W}_{turb} = \dot{m}(h_3 - h_4)$, the power needed for the (adiabatic) pump is $\dot{W}_{pump} = \dot{m}(h_2 - h_1)$ and the heat input for the combustion chamber is $\dot{Q}_{in} = \dot{m}(h_3 - h_2)$. So, all h -values need to be determined.

In point 1, we have a saturated liquid at $P_1 = 50$ kPa. In table A-5 (at the correct pressure), we find $h_1 = 340.54$ kJ/kg, $T_1 = 81.32$ °C and $s_1 = 1.0912$ kJ/(kgK). For point 2, we use the relation $dh = Tds + vdP$. For an isentropic pump, this reduces to $dh = vdP$, which gives $h_2 = h_1 + v(P_2 - P_1)$ for constant v . $v = v_1 = 0.001029$ m³/kg (from table A-5 as well), so $h_2 = 340.54 + 0.001029(3000 - 50) =$

343.576 kJ/kg. h_3 and s_3 can be found in table A-6, for superheated vapour: $h_3 = 3231.7$ kJ/kg and $s_3 = 6.9235$ kJ/(kgK). For point 4, we now need to find the phase. The entropy s_4 is known to equal the entropy in point 3, so $s_3 = s_4 = 6.9235$ kJ/(kgK). From table A-5, we can find that $s_{f,50 \text{ kPa}} = 1.0912$ kJ/(kgK) and $s_{g,50 \text{ kPa}} = 7.5931$ kJ/(kgK). Since $s_f < s_4 < s_g$, we have a mixture, so we need to determine the vapour fraction: $x = \frac{s_4 - s_f}{s_g - s_f} = \frac{6.9235 - 1.0912}{7.5931 - 1.0912} = 0.897$. We can now determine h_4 using h_f , h_g and x . From the table we find $h_f = 340.54$ kJ/kg and $h_g = 2645.2$ kJ/kg. Therefore, $h_4 = h_f + x(h_g - h_f) = 340.54 + 0.9(2645.2 - 340.54) = 2407.82$ kJ/kg.

Now we have all h -values, we can calculate what was asked:

$$\dot{W}_{turb} = \dot{m}(h_3 - h_4) = 60(3231.7 - 2407.8) = 49434 \text{ kW} = 49.4 \text{ MW}$$

$$\dot{W}_{pump} = \dot{m}(h_2 - h_1) = 60(343.58 - 340.54) = 182 \text{ kW} = 0.18 \text{ MW}$$

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = 60(3231.7 - 343.50) = 173292 \text{ kW} = 173.3 \text{ MW}$$

10.2e The net power output of the plant is $\dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{pump} = \dot{m}(h_3 - h_4 - (h_2 - h_1)) = 49.22$ MW.

10.2f $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{m}(h_3 - h_4 - (h_2 - h_1))}{\dot{m}(h_3 - h_2)} = 0.28$.

10.3 Power plant on a simple ideal Rankine cycle 2

10.3a See figure 10.3.1.

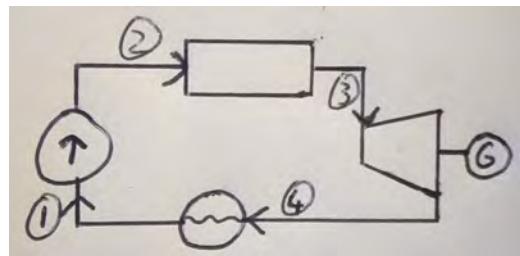


Figure 10.3.1: Schematic overview of the cycle

10.3b See figures 10.3.2 and 10.3.3.

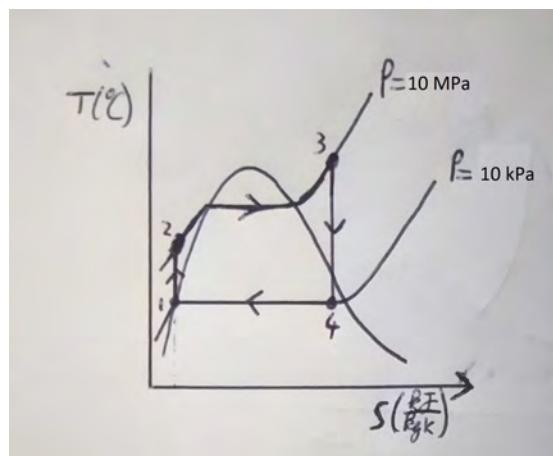


Figure 10.3.2: T-s diagram of exercise 10.3

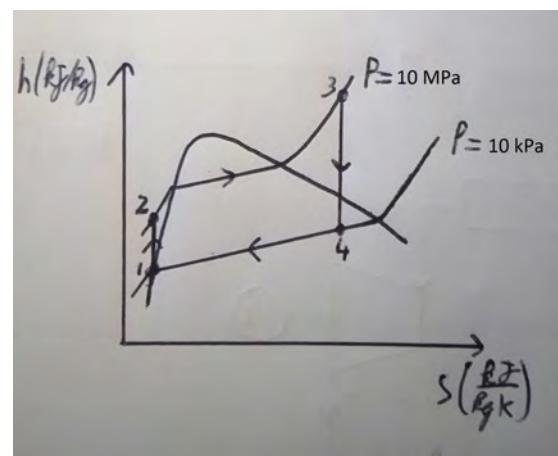


Figure 10.3.3: h-s diagram of exercise 10.3

10.3c See table 10.3.1.

Table 10.3.1: Table of exercise 10.3. The two known variables are underlined.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	phase	v (m³/kg)
1	<u>10</u>	45.81	191.83		<u>sat. liq.</u>	0.001010
2	<u>10000</u>		201.92	<u>$s_2 = s_1$</u>	compr. liq.	
3	<u>10000</u>	<u>500</u>	3373.7	6.6576	superheated	
4	<u>10</u>	45.81	2108.54	<u>$s_4 = s_3$</u>	mix	

10.3d The vapor mass fraction at the turbine exit can be calculated as $x = \frac{s_4 - s_f}{s_g - s_f}$. Since an isentropic turbine is used (ideal cycle), $s_4 = s_3$. The value for s_3 can be found in table A-6 at $P_3 = 10$ MPa and $T_3 = 500$ °C: $s_3 = 6.6576$ kJ/kgK. In table A-5, we now find that, for point 4 ($P_4 = 10$ kPa), $s_f = 0.6493$ and $s_g = 8.1502$ kJ/(kgK). So, $x = \frac{6.6576 - 0.6493}{8.1502 - 0.6493} = 0.80$.

10.3e To determine the work and the heat input, we need to determine all h -values. h_1 can be found in table A-5 for a saturated liquid at $P_1 = 10$ kPa: $h_1 = 191.83$ kJ/kg. h_2 is calculated using $dh = Tds + vdP$, which reduces to $h_2 - h_1 = v(P_2 - P_1)$ since the pump is isentropic. Since the specific volume is constant at $v_1 = 0.001010$ m³/kg (table A-5), we find that $h_2 = h_1 + v(P_2 - P_1) = 201.92$ kJ/kg. h_3 is determined from table A-6 at $P_3 = 10$ MPa and $T_3 = 500$ °C: $h_3 = 3373.7$ kJ/kg. h_4 is calculated using table A-5 and the vapor mass fraction from d: $h_4 = h_f + x(h_g - h_f) = 191.83 + 0.8(2584.7 - 191.83) = 2108.54$ kJ/kg.

Using this, we can calculate the work delivered by the turbine, the work required by the pump and the heat input to the boiler:

$$w_{turb} = h_3 - h_4 = 3373.7 - 2108.54 = 1265.2 \text{ kJ/kg}$$

$$w_{pump} = h_2 - h_1 = 201.92 - 191.83 = 10.09 \text{ kJ/kg}$$

$$q_{in} = h_3 - h_2 = 3373.7 - 201.92 = 3171.8 \text{ kJ/kg}$$

10.3f We can determine the mass flow using the given net power output ($\dot{W}_{net} = 210$ MW): $\dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{pump} = \dot{m}((h_3 - h_4) - (h_2 - h_1)) \rightarrow \dot{m} = \frac{\dot{W}_{net}}{(h_3 - h_4) - (h_2 - h_1)} = 167.3 \text{ kg/s}$.

$$\mathbf{10.3g} \quad \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{m}((h_3 - h_4) - (h_2 - h_1))}{\dot{m}(h_3 - h_2)} = \frac{h_3 - h_4 - (h_2 - h_1)}{h_3 - h_2} = 0.396.$$

Above the cycle is analysed using the enthalpy and entropy values from the tables and by calculating the mixture fraction for point 4 in the saturated mixture region. Alternatively the enthalpy value in the mixture region can be read from the Mollier diagram. This method is less accurate but much faster and gives a better overview of the process. Specially for cycles with more devices that have reheating and/or feed water heating and where more points are located in the mixture region.

See figure 10.4.3 for the Mollier diagram. The vertical line between point 3 and 4s represents the ideal process. For point 3 is found from the diagram: $h_3 = 3375$ kJ/kg, and for point 4: $h_4 = 2100$ kJ/kg and $x_4 = 0.80$. These values are close to the calculated values.

10.4 Power plant on an actual Rankine cycle

10.4a See figure 10.3.1.

10.4b See figures 10.4.1 and 10.4.2.

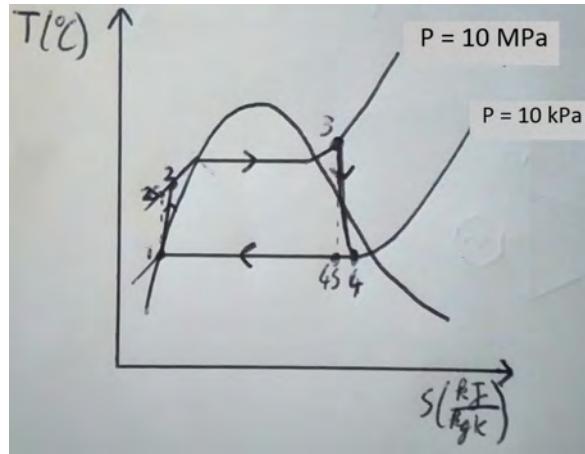


Figure 10.4.1: T-s diagram of exercise 10.4

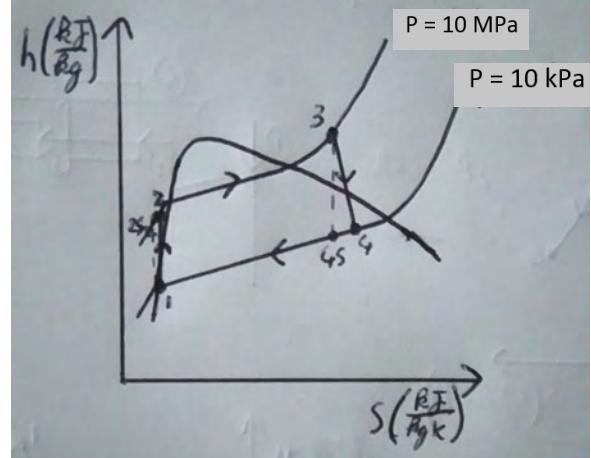


Figure 10.4.2: h-s diagram of exercise 10.4

10.4c See table 10.4.1.

Table 10.4.1: Table of exercise 10.4. The two known variables at each point are underlined.

	P (kPa)	T(°C)	h (kJ/kg)	s (kJ/(kgK))	phase	v (m ³ /kg)	other
1	<u>10</u>	45.81	191.83		sat. liq.	0.001010	
2s	<u>10000</u>		201.92	<u>$s_2 = s_1$</u>	compr. liq.		
2	<u>10000</u>		203.7		comp. liq.		<u>$\eta_{sp} = 0.85$</u>
3	<u>10000</u>	<u>500</u>	3373.7	6.6576	superheated		
4s	<u>10</u>	45.81	2108.54	<u>$s_4 = s_3$</u>	mix		
4	<u>10</u>	45.81	2298.3		mix		<u>$\eta_{st} = 0.85$</u>

10.4d Point 2 and 4 from 10.3 are now point 2s and 4s. The h -values of point 2 and 4 can be calculated with the isentropic efficiency: $\eta_{sp} = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{sp}} = 203.7 \text{ kJ/kg}$ and $\eta_{st} = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta_{st}(h_3 - h_{4s}) = 2298.3 \text{ kJ/kg}$. The new quality is then $x = \frac{h_4 - h_f}{h_g - h_f} = 0.88$.

$$w_{turb} = h_3 - h_4 = 3373.7 - 2298.3 = 1075.4 \text{ kJ/kg}$$

$$w_{pump} = h_2 - h_1 = 203.7 - 191.83 = 11.87 \text{ kJ/kg}$$

$$q_{in} = h_3 - h_2 = 3373.7 - 203.7 = 3170.0 \text{ kJ/kg}$$

10.4f We can determine the mass flow using the given net power output ($\dot{W}_{net} = 210 \text{ MW}$): $\dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{pump} = \dot{m}((h_3 - h_4) - (h_2 - h_1)) \rightarrow \dot{m} = \frac{\dot{W}_{net}}{(h_3 - h_4) - (h_2 - h_1)} = 197.5 \text{ kg/s}$.

$$10.4g \quad \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{m}((h_3-h_4)-(h_2-h_1))}{\dot{m}(h_3-h_2)} = \frac{h_3-h_4-(h_2-h_1)}{h_3-h_2} = 0.335.$$

Mollier Diagram, assignment 10.3 & 10.4. Power plant on a simple ideal and a real Rankine cycle

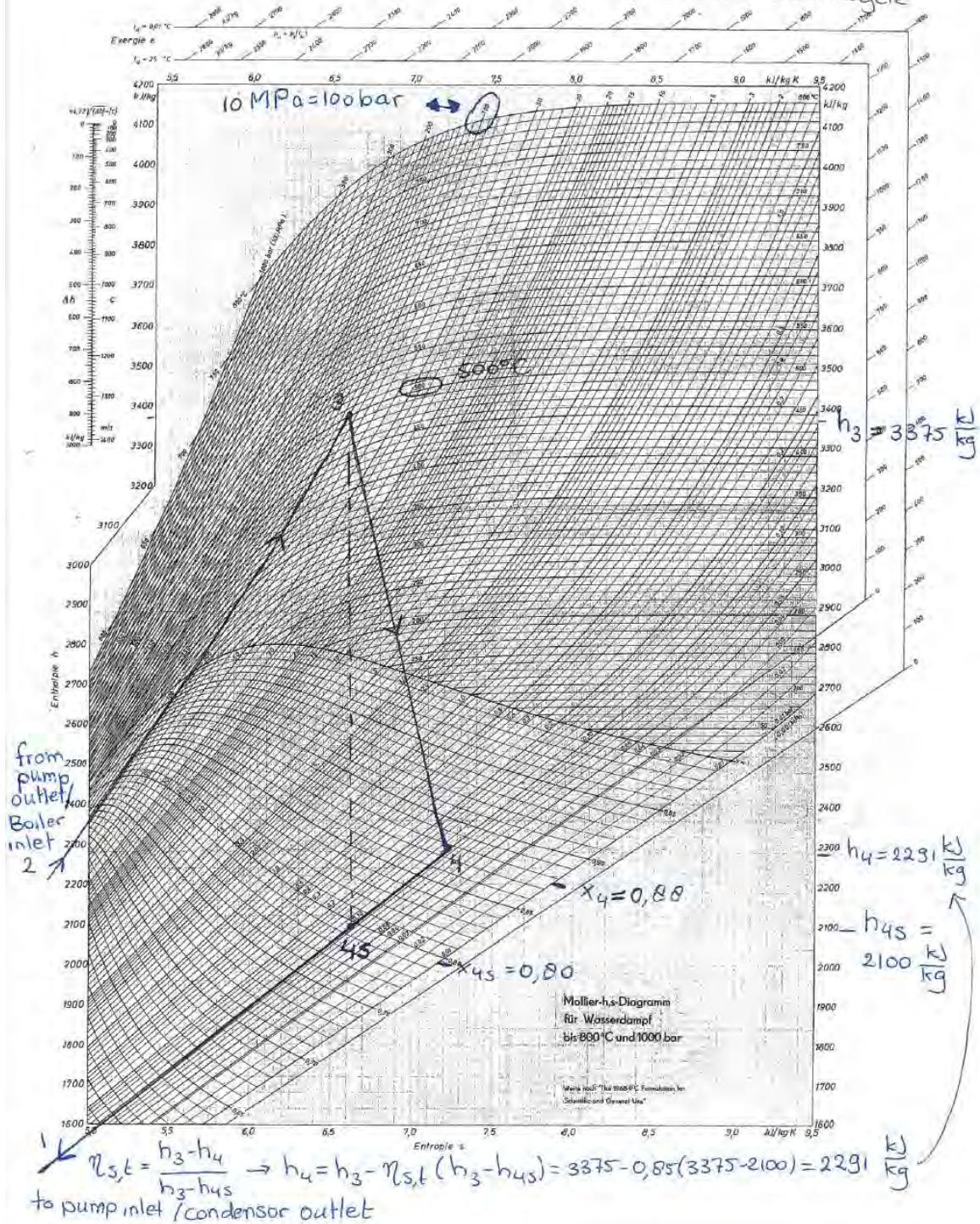


Figure 10.4.3: Mollier diagram of the Rankine cycle of assignment 10.3 and 10.4. The dashed line between point 3 and 4s represents the isentropic process of the ideal cycle.

Above the cycle is analysed using the enthalpy and entropy values from the tables and by calculating the mixture fraction for points in the saturated mixture region. Alternatively the enthalpy values in the mixture region can be read from the Mollier diagram, see figure 10.4.3. This method is less accurate but much faster and gives a better overview of the process. Specially for cycles with more devices that

have reheating and/or feed water heating and where more points are located in the mixture region.

The dashed vertical line between point 3 and 4s represents the ideal process. For point 3 is found from the diagram: $h_3 = 3375 \text{ kJ/kg}$, for point 4s: $h_{4s} = 2100 \text{ kJ/kg}$ and $x_{4s} = 0.80$ and for point 4: $h_4 = 2291 \text{ kJ/kg}$ and $x_{4s} = 0.88$. These values are close to the calculated values and fine to use in the analysis.

10.5 Coal fired steam power plant

10.5a See figure 10.5.1.

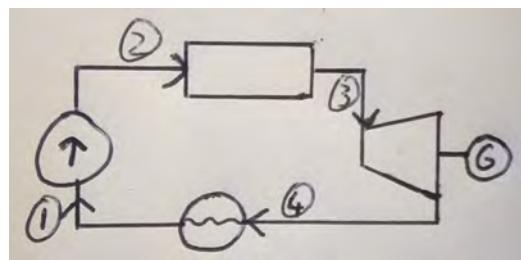


Figure 10.5.1: Overview of exercise 10.5

10.5b See figures 10.5.2 and 10.5.3.

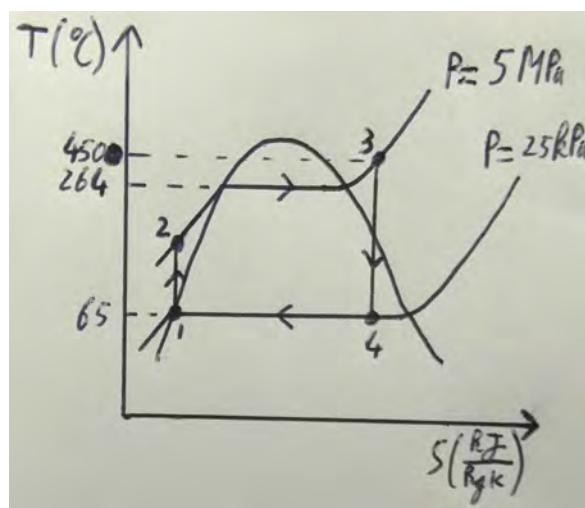


Figure 10.5.2: T-s diagram of exercise 10.5

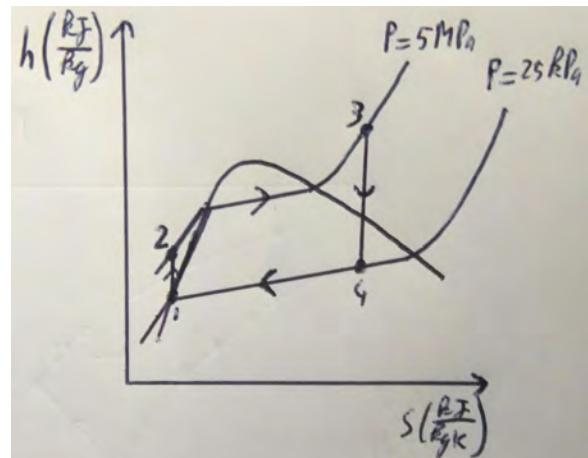


Figure 10.5.3: h-s diagram of exercise 10.5

10.5c See table 10.5.1. The known variables are underlined.

Table 10.5.1: Table of exercise 10.5. The two known variables at each point are underlined.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	phase	v (m^3/kg)
1	<u>25</u>	64.97	272		sat. liquid	0.001020
2	<u>5000</u>		277	$s_2 = s_1$	comp. liq	
3	<u>5000</u>	<u>450</u>	3320		superh. vap.	
4	<u>25</u>	64.97	2275	$s_4 = s_3$	mixture	

10.5d In order to determine the work delivered by the turbine, the work required by the pump and the heat input into the boiler, we need to determine the values of h . Reading in table A-5 (saturated liquid, $P_1 = 25 \text{ kPa}$) gives $h_1 = 272 \text{ kJ/kg}$. For the isentropic pump, $dh = v dP$ holds, so $h_2 = h_1 + v(P_2 - P_1) = 272 + 0.001020(5000 - 25) = 277 \text{ kJ/kg}$. h_3 can be determined using either the Mollier diagram or the table for superheated vapor: $h_3 = 3320 \text{ kJ/kg}$. Using the Mollier diagram, going down vertically ($\Delta s = 0$) to $P = 25 \text{ kPa}$ gives $h_4 = 2275 \text{ kJ/kg}$. Alternatively, one can look in the table for saturated water at $P_4 = 25 \text{ kPa}$ and $s_4 = s_3$.

Now we can answer the question:

$$w_{out,turb} = h_3 - h_4 = 3320 - 2275 = 1045 \text{ kJ/kg}$$

$$w_{in,pump} = h_2 - h_1 = 277 - 272 = 5 \text{ kJ/kg}$$

$$q_{in,boiler} = h_3 - h_2 = 3320 - 277 = 3043 \text{ kJ/kg}$$

10.5e

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in,boiler}} = \frac{\dot{m}((h_3 - h_4) - (h_2 - h_1))}{\dot{m}(h_3 - h_2)} = \frac{(3320 - 2275) - (277 - 272)}{3320 - 277} = 0.342$$

10.5f

$$\eta_{total} = \frac{\dot{W}_{out}}{\dot{Q}_{in,coal}} = \frac{\dot{m}\eta_{gen}w_{net}}{\dot{m}\frac{q_{in}}{\eta_{boiler}}} = \frac{\eta_{gen}((h_3 - h_4) - (h_2 - h_1))}{\frac{h_3 - h_2}{\eta_{boiler}}} = 0.75 \cdot 0.96 \cdot \frac{1045 - 5}{3043} = 0.246$$

$$10.5g \quad \dot{m}_{steam} = \frac{\dot{W}_{out}}{\eta_{gen}w_{net}} = \frac{\dot{W}_{out}}{\eta_{gen}((h_3 - h_4) - (h_2 - h_1))} = \frac{300000}{0.96 \cdot (1045 - 5)} = 300.5 \text{ kg/s.}$$

10.5h Let c_c denote the heating value of the coal (so $c_c = 30 \text{ MJ/kg}$), then $\dot{Q}_{in,coal} = \dot{m}_{coal}c_c$. We can find \dot{m}_{coal} using $\dot{m}_{coal} = \frac{\dot{m}_{steam} q_{in,boiler}}{c_c} = \frac{300.5 \cdot 3320 - 277}{30000 \cdot 0.75} = 40.6 \text{ kg/s} = 146.3 \text{ ton/h.}$

10.6 Solar-pond power plant

10.6a See figure 10.6.1.

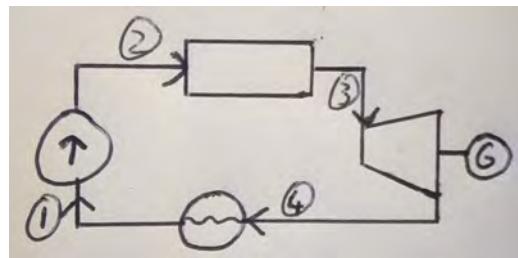


Figure 10.6.1: Overview of exercise 10.6

10.6b See figures 10.6.2 and 10.6.3.

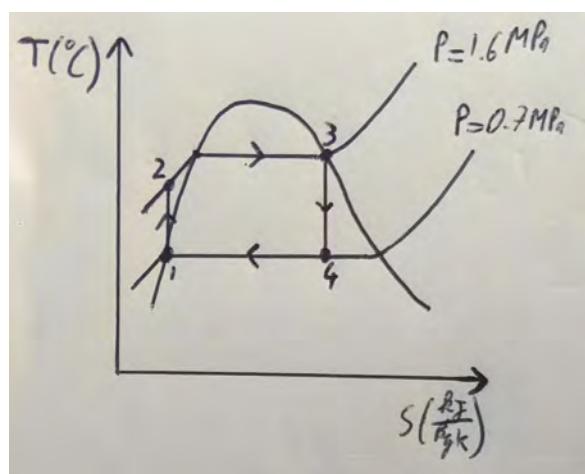


Figure 10.6.2: T-s diagram of exercise 10.6

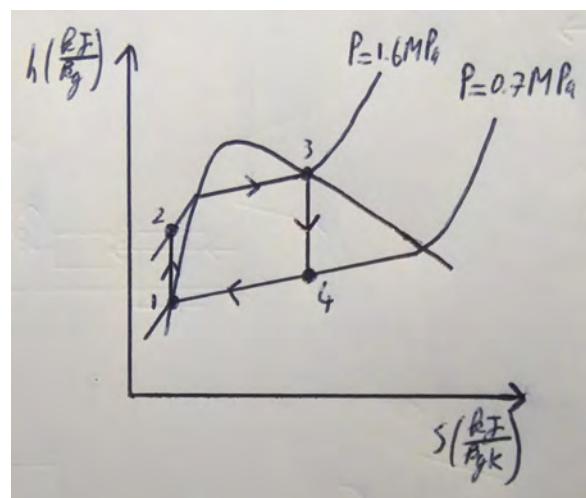


Figure 10.6.3: h-s diagram of exercise 10.6

10.6c See table 10.6.1.

Table 10.6.1: Table of exercise 10.6. The 2 known variables at each point are underlined.

	P (MPa)	h (kJ/kg)	s (kJ/(kgK))	Phase
1	<u>0.7</u>	86.78		sat. liq.
2	<u>1.6</u>	87.53	<u>$s_2 = s_1$</u>	comp. liq.
3	<u>1.6</u>	275.33	0.8982	sat. vapor
4	<u>0.7</u>	258.91	<u>$s_4 = s_3$</u>	mixture

10.6d First the h -values need to be determined. $h_1 = h_{f@0.7 \text{ MPa}} = 86.78 \text{ kJ/kg}$, which can be read directly from table A-12 (and using the fact that at the exit of the condenser we have saturated liquid). $h_3 = h_{g@1.6 \text{ MPa}} = 275.33 \text{ kJ/kg}$, also following directly from table A-12 (and the given fact that the refrigerant enters the turbine as saturated vapor). For the isentropic pump, $dh = v dP$, such that $h_2 = h_1 + v(P_2 - P_1) = 86.78 + 0.0008328(1600 - 700) = 87.53 \text{ kJ/kg}$ (where $v = v_{f@0.7 \text{ MPa}}$). From table A-12, we can also find that $s_3 = 0.8982 \text{ kJ/(kgK)}$. Since the turbine is isentropic and hence $s_4 = s_3$, we can now calculate x : $x = \frac{s_4 - s_{f@0.7 \text{ MPa}}}{s_{g@0.7 \text{ MPa}} - s_{f@0.7 \text{ MPa}}} = \frac{0.8982 - 0.3242}{0.9080 - 0.3242} = 0.98$, so now h_4 can be determined: $h_4 = h_{f@0.7 \text{ MPa}} + x(h_{g@0.7 \text{ MPa}} - h_{f@0.7 \text{ MPa}}) = 258.91 \text{ kJ/kg}$. With the given mass flow of $\dot{m} = 6 \text{ kg/s}$ follows:

$$\dot{W}_{turb} = \dot{m}(h_3 - h_4) = 98.52 \text{ kW}$$

$$\dot{W}_{pump} = \dot{m}(h_2 - h_1) = 4.5 \text{ kW}$$

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = 1126.8 \text{ kW}$$

$$\mathbf{10.6e} \quad \dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{pump} = 98.52 - 4.5 = 94.02 \text{ kW.}$$

10.6f $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{94.02}{1126.8} = 0.083 \rightarrow 8.3\%$. Note: this is a very bad installation. This is due to the fact that the refrigerant enters the turbine as saturated vapor (instead of superheated) and can therefore not expand much.

10.7 Ideal power plant cooled by a lake 1

10.7a See figure 10.7.1.

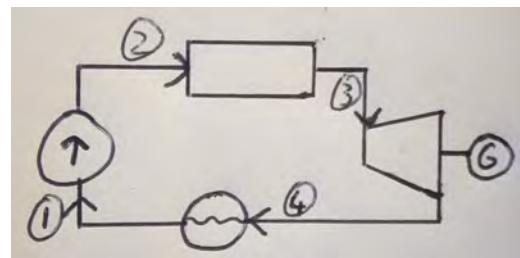


Figure 10.7.1: Overview of exercise 10.7

10.7b See figures 10.7.2 and 10.7.3.

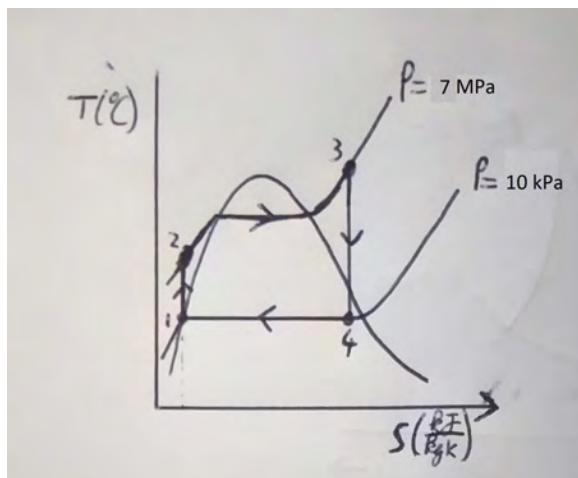


Figure 10.7.2: T-s diagram of exercise 10.7

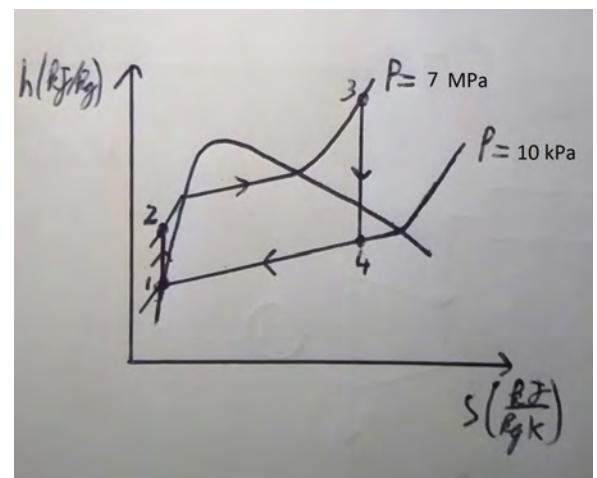


Figure 10.7.3: h-s diagram of exercise 10.7

10.7c See table 10.7.1.

Table 10.7.1: Characteristics of exercise 10.7. The two known variables at each point are underlined.

	P (kPa)	T(°C)	h (kJ/kg)	s (kJ/(kgK))	Phase
1	<u>10</u>	45.81	191.81		sat. liq.
2	<u>7000</u>		198.87	<u>$s_2 = s_1$</u>	compr. liq.
3	<u>7000</u>	<u>500</u>	3410.3	6.7975	superh. vap.
4	<u>10</u>	45.81	2153.2	<u>$s_4 = s_3$</u>	mixture

10.7d We first need to determine the values of h . The flow out of the condenser is saturated liquid, so $h_1 = h_{f@10\text{ kPa}} = 191.81 \text{ kJ/kg}$ (from table A-5). $dh = Tds + vdP = vdP$ (isentropic), so $h_2 = h_1 + v(P_2 - P_1) = 191.81 + 0.001010(7000 - 10) = 198.87 \text{ kJ/kg}$. Table A-6 gives us the value of h_3 : $h_3 = 3410.3 \text{ kJ/kg}$. Using $s_4 = s_3 = 6.7975$, we can determine the vapor mass fraction in point 4: $x = \frac{s_4 - s_f}{s_g - s_f} = \frac{6.7975 - 0.6493}{8.1502 - 0.6493} = 0.82$, such that $h_4 = h_f + x(h_g - h_f) = 2153.2 \text{ kJ/kg}$.

$$w_{turb} = h_3 - h_4 = 1257.1 \text{ kJ/kg}$$

$$w_{pump} = h_2 - h_1 = 7.06 \text{ kJ/kg}$$

$$q_{in} = h_3 - h_2 = 3211.43 \text{ kJ/kg}$$

$$\text{10.7e } \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{turb} - \dot{W}_{pump}}{\dot{Q}_{in}} = \frac{\dot{m}((h_3 - h_4) - (h_2 - h_1))}{\dot{m}(h_3 - h_2)} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = 0.389.$$

$$\text{10.7f } \dot{W}_{net} = \dot{m}_{steam}((h_3 - h_4) - (h_2 - h_1)) \rightarrow \dot{m}_{steam} = \frac{\dot{W}_{net}}{(h_3 - h_4) - (h_2 - h_1)} = \frac{45000}{1257.1 - 7.06} = 35.998 \text{ kg/s.}$$

10.7g Energy must be conserved, so $\Delta\dot{U}_{water,lake} = \dot{Q}_{out}$ and furthermore $du = cdT \rightarrow \Delta\dot{U} = c\Delta T \dot{m}_{lake}$ and $\dot{Q}_{out} = \dot{m}_{steam}(h_4 - h_1)$. $c = 4.18 \text{ kJ/(kgK)}$, which is the specific heat of water. Combining gives $c\Delta T \dot{m}_{lake} = \dot{m}_{steam}(h_4 - h_1) \rightarrow \Delta T = \frac{\dot{m}_{steam}}{\dot{m}_{lake}} \frac{(h_4 - h_1)}{c} = 8.45 \text{ }^{\circ}\text{C}$.

10.8 Actual power plant cooled by a lake

10.8a See exercise 10.7.

10.8b See figure 10.8.1 and 10.8.2.

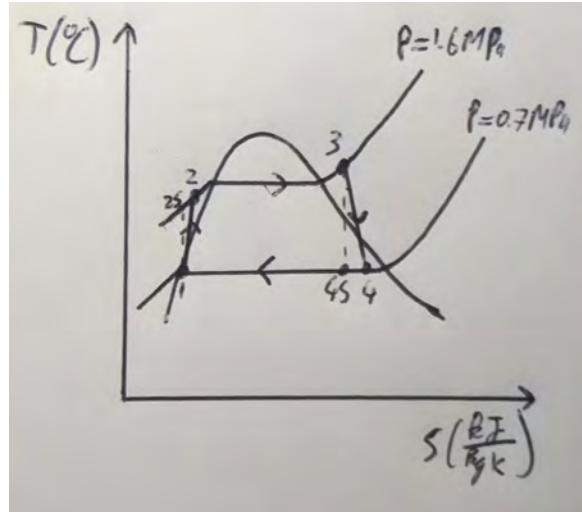


Figure 10.8.1: T-s diagram of exercise 10.8

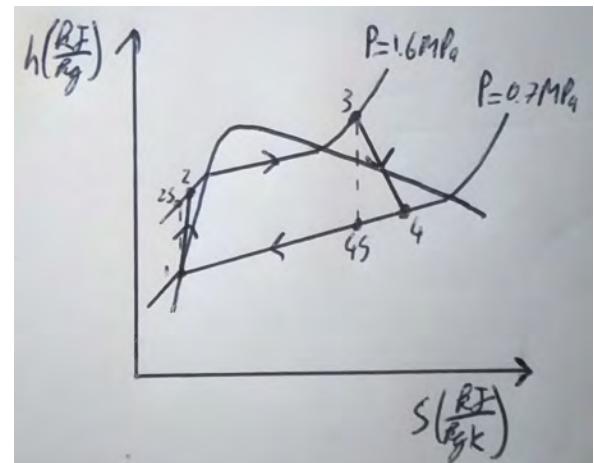


Figure 10.8.2: h-s diagram of exercise 10.8

10.8c See table 10.8.1.

10.8d The old values (from 10.7) for point 2 and 4 are now the values for point 2s and 4s. We can calculate the new h -values using the isentropic efficiency: $\eta_{sp} = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{sp}} = 199.92 \text{ kJ/kg}$ and $\eta_{st} = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta_{st}(h_3 - h_{4s}) = 2316.6 \text{ kJ/kg}$.

$$w_{turb} = h_3 - h_4 = 1093.7 \text{ kJ/kg}$$

Table 10.8.1: Table of exercise 10.8. The two known variables at each point are underlined.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	Phase	other
1	<u>10</u>	45.81	191.81		sat. liq.	
2s	<u>7000</u>		198.87	<u>$s_2 = s_1$</u>	compr. liq.	
2	<u>7000</u>		199.92		compr. liq.	<u>$\eta_{sp} = 0.87$</u>
3	<u>7000</u>	<u>500</u>	3410.3	6.7975	superh. vap.	
4s	<u>10</u>	45.81	2153.2	<u>$s_{4s} = s_3$</u>	mixture	
4	<u>10</u>		2316.6		mixture	<u>$\eta_{st} = 0.87$</u>

$$w_{pump} = h_2 - h_1 = 8.11 \text{ kJ/kg}$$

$$q_{in} = h_3 - h_2 = 3210.4 \text{ kJ/kg}$$

10.8e $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{turb} - \dot{W}_{pump}}{\dot{Q}_{in}} = \frac{\dot{m}((h_3 - h_4) - (h_2 - h_1))}{\dot{m}(h_3 - h_2)} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} = 0.338.$

10.8f $\dot{W}_{net} = \dot{m}_{steam}(h_3 - h_4) - (h_2 - h_1) \rightarrow \dot{m}_{steam} = \frac{\dot{W}_{net}}{(h_3 - h_4) - (h_2 - h_1)} = \frac{45000}{1093.7 - 8.11} = 41.45 \text{ kg/s.}$

10.8g Energy must be conserved, so $\Delta\dot{U}_{water,lake} = \dot{Q}_{out}$ and furthermore $du = cdT \rightarrow \Delta\dot{U} = c\Delta T \dot{m}_{lake}$ and $\dot{Q}_{out} = \dot{m}_{steam}(h_4 - h_1)$. $c = 4.18 \text{ kJ/(kgK)}$, which is the specific heat of water. Combining gives $c\Delta T \dot{m}_{lake} = \dot{m}_{steam}(h_4 - h_1) \rightarrow \Delta T = \frac{\dot{m}_{steam}(h_4 - h_1)}{\dot{m}_{lake}c} = 10.53 \text{ }^{\circ}\text{C.}$

10.9 Reheat Rankine cycle

10.9a See figure 10.9.1.

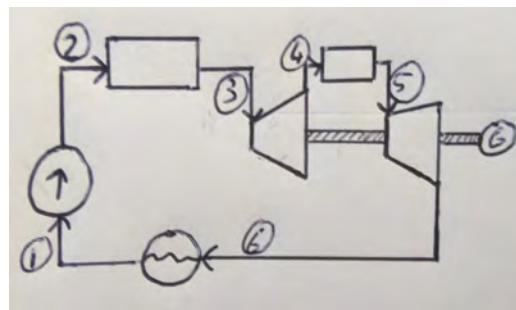


Figure 10.9.1: Overview of exercise 10.9

10.9b See figures 10.9.2 and 10.9.3. Note that point 6 can also be in the mixture region, but that this is not known on beforehand.

10.9c See table 10.9.1.

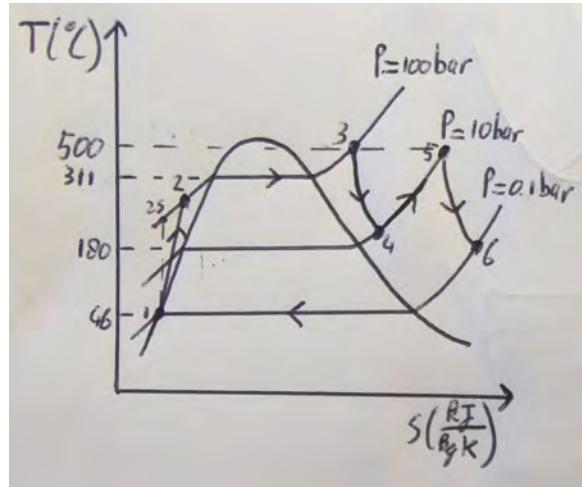


Figure 10.9.2: T-s diagram of exercise 10.9

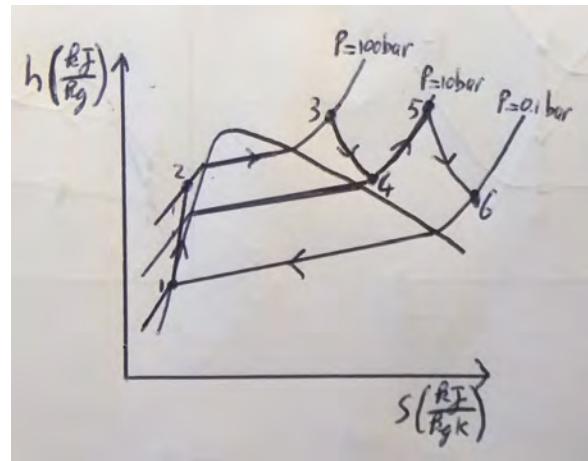


Figure 10.9.3: h-s diagram of exercise 10.9

Table 10.9.1: Table of exercise 10.9. The two known variables at each point are underlined.

	P (bar)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	Phase	other
1	<u>0.1</u>	46	191.83		sat. liq.	
2s	<u>100</u>		201.92	<u>$s_2 = s_1$</u>	compr. liq.	
2	<u>100</u>		202.45		compr. liq.	$\eta_{sp} = 0.95$
3	<u>100</u>	<u>500</u>	3370		superh. vap.	
4s	<u>10</u>	180	2780	<u>$s_{4s} = s_3$</u>	superh. vap.	
4	<u>10</u>	230	2898		superh. vap.	$\eta_{st} = 0.8$
5	<u>10</u>	<u>500</u>	3500		superh. vap.	
6s	<u>0.1</u>	46	2460	<u>$s_{6s} = s_5$</u>	mix, $x = 0.95$	
6	<u>0.1</u>	90	2668		superh. vap.	$\eta_{st} = 0.8$

10.9d In order to determine the vapor mass fraction or temperature at the exit of the LPT, we need to determine h_5 , h_{6s} and h_6 . h_5 can be found in the Mollier diagram in figure 10.9.4: $h_5 = 3500 \text{ kJ/kg}$. By going down vertically ($\Delta s = 0$) to $P_{6s} = 0.1 \text{ bar}$ we find $h_{6s} = 2460 \text{ kJ/kg}$. Using the isentropic efficiency, we find $h_6 = h_5 - \eta_{st}(h_5 - h_{6s}) = 2668 \text{ kJ/kg}$. In the diagram we can see that this is in the superheated region and the steam has a temperature of $T_6 = 90^\circ\text{C}$.

10.9e In order to determine the thermal efficiency of the cycle, we need to determine the other h -values as well. h_1 can be read in table A-5: $h_1 = 191.83 \text{ kJ/kg}$. For the isentropic pump, we find $h_{2s} = h_1 + v(P_2 - P_1) = 191.83 + 0.001010(10000 - 10) = 201.92 \text{ kJ/kg}$. The isentropic efficiency of the pump then gives $h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{sp}} = 202.45 \text{ kJ/kg}$. h_3 can be found in the Mollier diagram in figure 10.9.4: $h_3 = 3370 \text{ kJ/kg}$. Going down vertically gives $h_{4s} = 2780 \text{ kJ/kg}$, which results in $h_4 = h_3 - \eta_{st}(h_3 - h_{4s}) = 2898 \text{ kJ/kg}$.

Now we can calculate the thermal efficiency:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)} = \frac{1294}{3770} = 0.34$$

10.9f $\dot{W}_{net} = \dot{m}w_{net} \rightarrow \dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{150000}{1294} = 115.9 \text{ kg/s.}$

10.9g $\eta_{2nd\ law} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{\eta_{th}}{1 - \frac{T_1}{T_3}} = \frac{0.34}{1 - \frac{46+273}{500+273}} = 0.59$

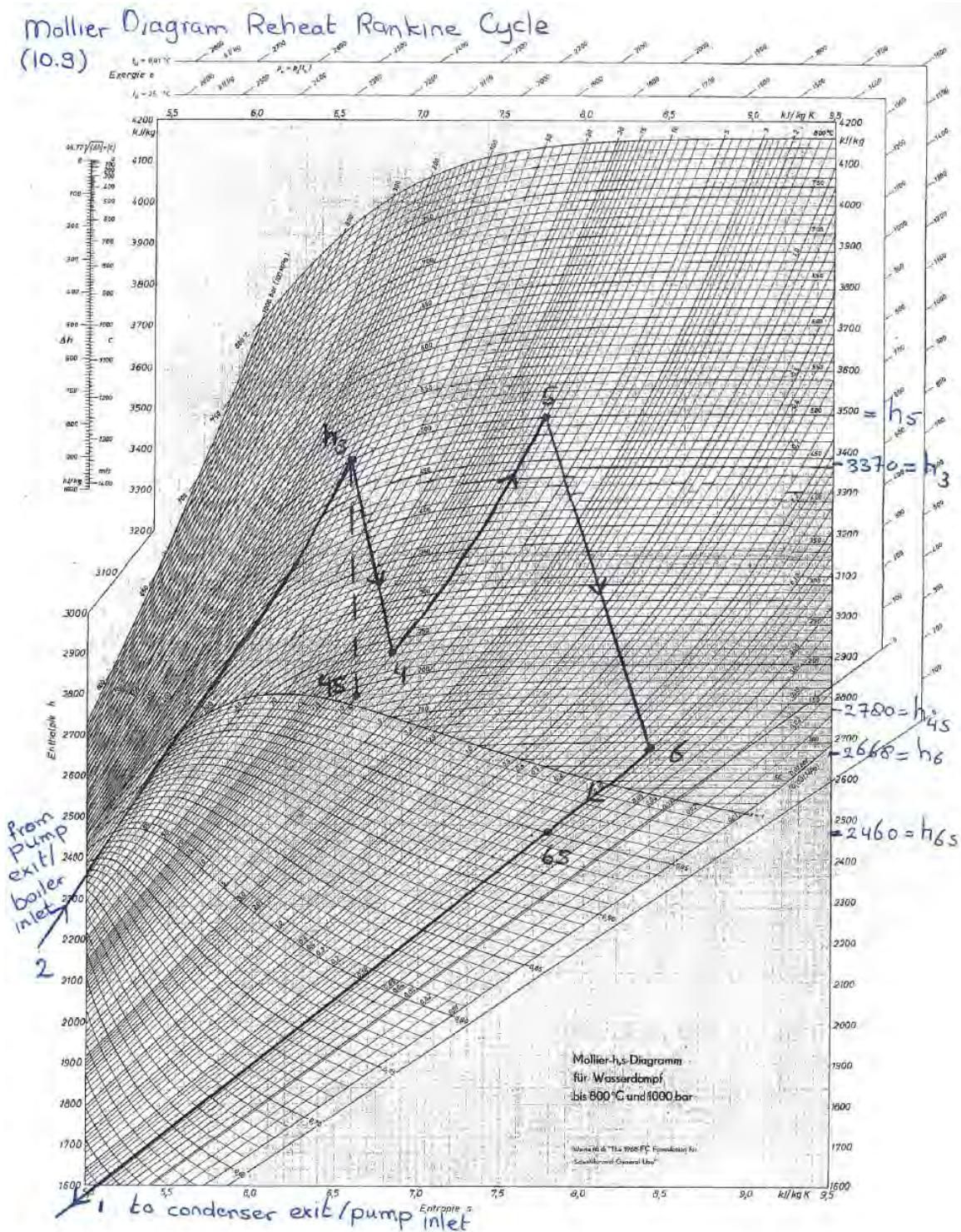


Figure 10.9.4: Mollier diagram of the reheat Rankine cycle of assignment 10.9.

10.10 Ideal reheat Rankine cycle

10.10a See figure 10.10.1.

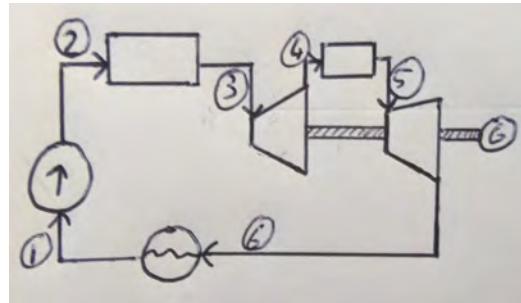


Figure 10.10.1: Schematic overview of exercise 10.10

10.10b See figures 10.10.2 and 10.10.3.

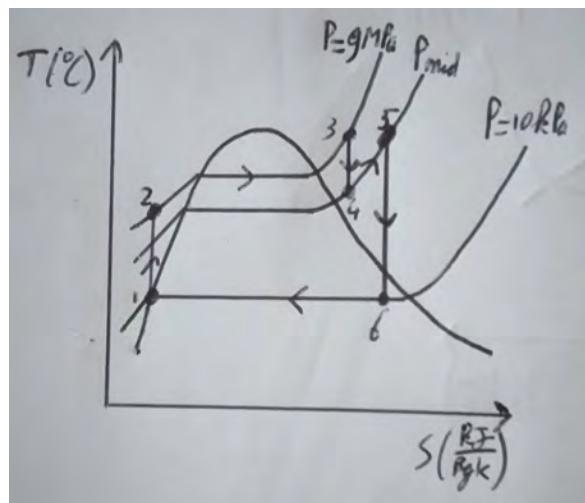


Figure 10.10.2: T-s diagram of exercise 10.10

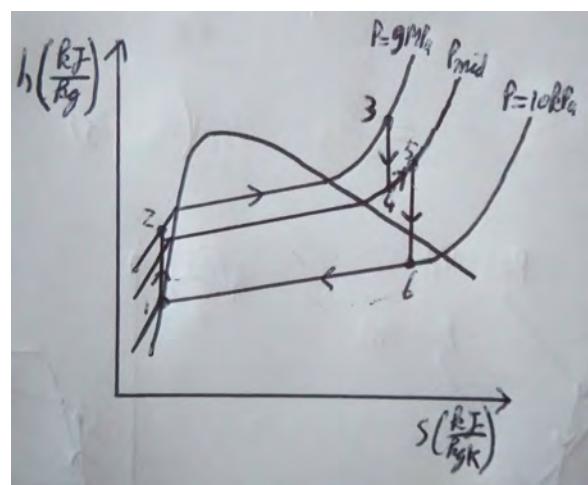


Figure 10.10.3: h-s diagram of exercise 10.10

10.10c See table 10.10.1.

Table 10.10.1: Table of exercise 10.10. The two known variables at each point are underlined.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	Phase	other
1	<u>10</u>	45.81	191.8		sat. liq.	$v_f = 0.001010 \text{ m}^3/\text{kg}$
2	<u>9000</u>		200.9	<u>$s_2 = s_1$</u>	compr. liq.	
3	<u>9000</u>	<u>500</u>	3380		superh. vap.	
4	<u>$P_4 = P_5$</u>	287	2990	<u>$s_{4s} = s_3$</u>	superh. vap.	
5	2200	<u>500</u>	3460	<u>$s_5 = s_6$</u>	superh. vap.	
6	<u>10</u>	45.81	2350		mix, $x = 0.9$	

10.10d We determine the pressure at which reheating takes place with the Mollier diagram, see figure 10.10.4. Looking up point 6 ($P = 10 \text{ kPa}$, $x = 0.9$) and going up vertically ($\Delta s = 0$) to $T_5 = 500^\circ\text{C}$ gives us point 5. We can now read out the enthalpy and the pressure in this point: $h_5 = 3460 \text{ kJ/kg}$ and $P_5 = 2200 \text{ kPa}$ (= 22 bar). So the pressure at which reheating takes place is 22 bar.

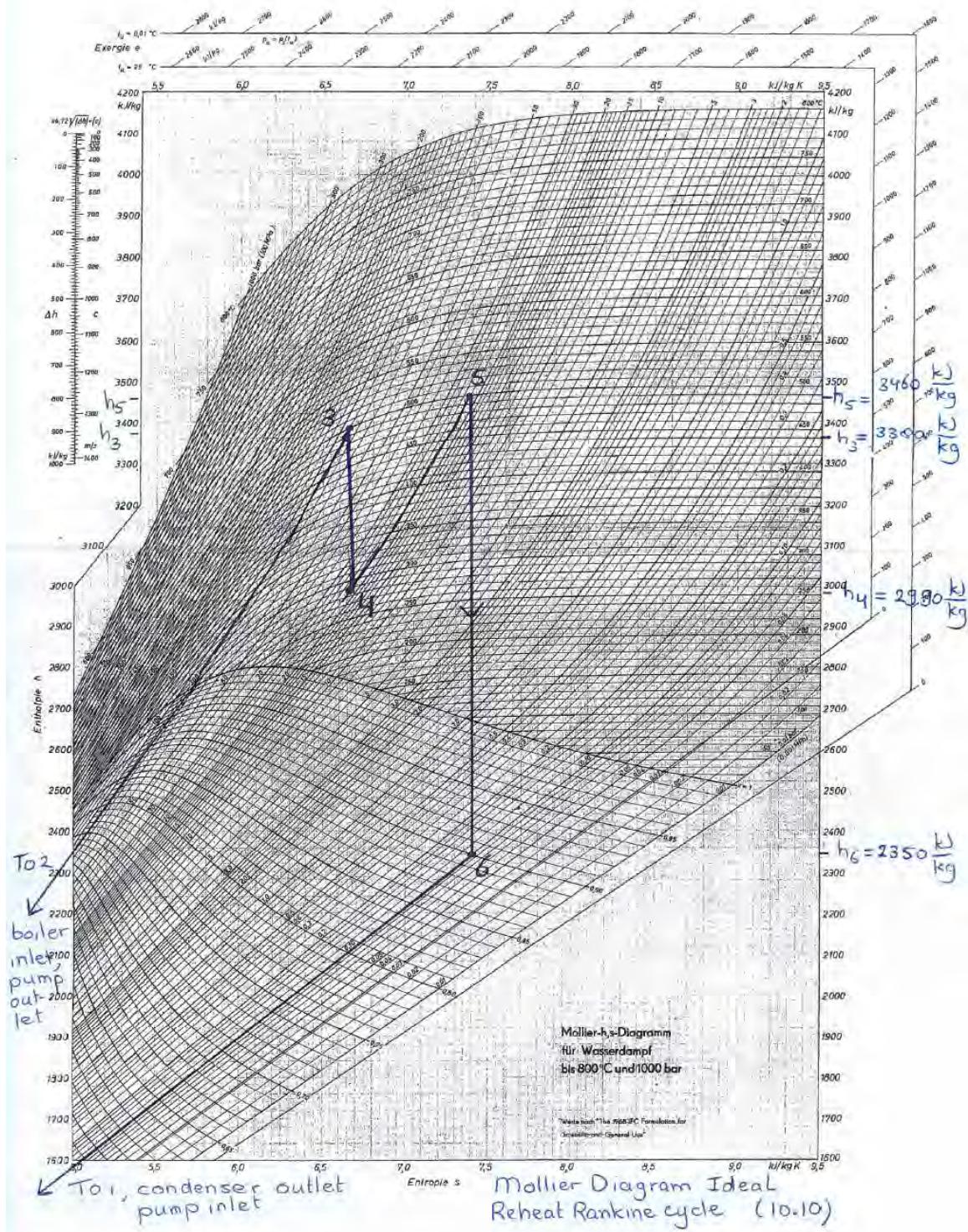


Figure 10.10.4: Mollier diagram of the ideal reheat Rankine cycle of assignment 10.10.

10.10e The heat input in the boiler is calculated as $\dot{Q}_{in} = \dot{m}(h_3 - h_2 + h_5 - h_4)$, so we need to determine the h -values in point 2, 3, 4 and 5. For an isentropic pump, $h_2 = h_1 + v(P_2 - P_1)$. Looking up h_1 in table 10 gives $h_1 = 191.8 \text{ kJ/kg}$ and $v = v_f@10 \text{ kPa} = 0.001010 \text{ m}^3/\text{kg}$. So, $h_2 = 191.8 + 0.001010(9000 - 10) = 200.88 \text{ kJ/kg}$. Using the Mollier diagram we find $h_3 = 3380 \text{ kJ/kg}$. Going down vertically to $P_4 = 22 \text{ bar}$ (determined in b) we find $h_4 = 2990 \text{ kJ/kg}$. In d, we already found that $h_5 = 3460 \text{ kJ/kg}$. So, $\dot{Q}_{in} = \dot{m}(h_3 - h_2 + h_5 - h_4) = 25(3380 - 201 + 3460 - 2990) = 91225 \text{ kW} = 91.2 \text{ MW}$.

10.10f To determine the thermal efficiency we need to read out h_6 in the Mollier diagram: $h_6 = 2350 \text{ kJ/kg}$. Now $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{HPT} + \dot{W}_{LPT} - \dot{W}_{pump}}{\dot{Q}_{in}} = \frac{\dot{m}(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{\dot{m}(h_3 - h_2) + (h_5 - h_3)} = \frac{35.23 \text{ MW}}{91.2 \text{ MW}} = 0.386$.

10.10g The second law efficiency is given as $\eta_{2nd\,law} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{\eta_{th}}{1 - \frac{T_L}{T_H}} = \frac{\eta_{th}}{1 - \frac{46+273}{500+273}} = \frac{0.386}{0.59} = 0.65$.

10.11 Reheat-regenerative Rankine cycle with open feedwater heater

10.11a See figure 10.11.1.

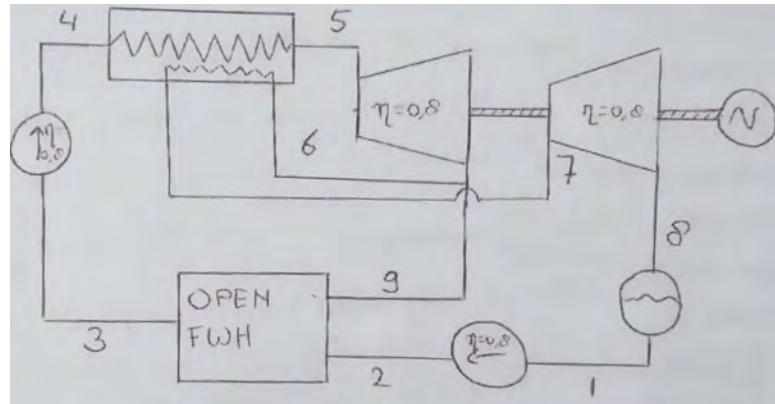


Figure 10.11.1: Overview of exercise 10.11

10.11b See figures 10.11.2 and 10.11.3.

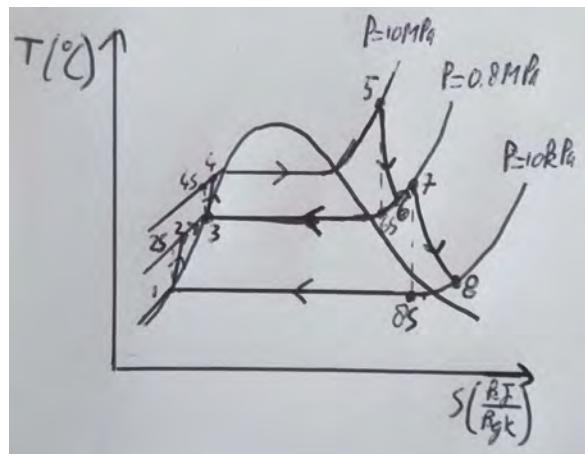


Figure 10.11.2: T-s diagram of exercise 10.11

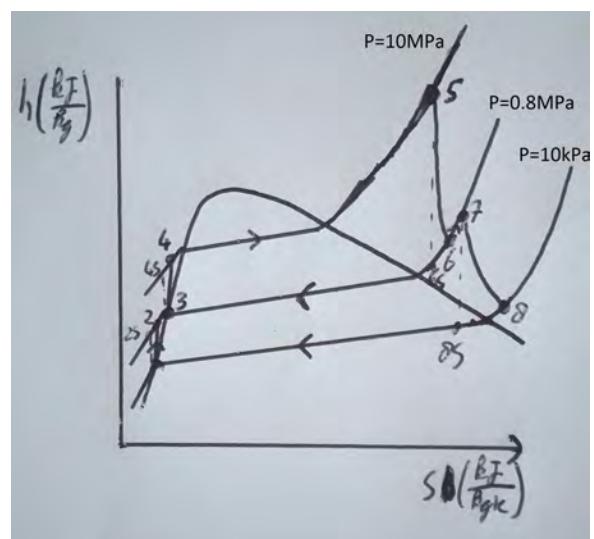


Figure 10.11.3: h-s diagram of exercise 10.11

10.11c See table 10.11.1.

Table 10.11.1: Table of exercise 10.11. The two known variables at each point are underlined.

	P (kPa)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/(kgK))	phase	\dot{m}	other
1	<u>10</u>	45.81	191.81		<u>sat. liq.</u>	\dot{m}_1	
2s	<u>800</u>		192.61	<u>$s_{2s} = s_1$</u>	comp. liq.	\dot{m}_1	
2	<u>800</u>		192.81		comp. liq.	\dot{m}_1	<u>$\eta_s = 0.8$</u>
3	<u>800</u>	170.41	720.87	2.0457	<u>sat. liq.</u>	\dot{m}_3	
4s	<u>10000</u>		731.13	<u>$s_{4s} = s_3$</u>	comp. liq.	\dot{m}_3	
4	<u>10000</u>		733.69	2.0515	comp. liq.	\dot{m}_3	<u>$\eta_s = 0.8$</u>
5	<u>10000</u>	<u>550</u>	3500	6.65	superh.	\dot{m}_3	
6s	<u>800</u>		2800	<u>$s_{6s} = s_5$</u>	superh.	\dot{m}_1	
6	<u>800</u>	240	2940	7.0	superh.	\dot{m}_1	<u>$\eta_s = 0.8$</u>
7	<u>800</u>	<u>500</u>	3480	7.85	superh.	\dot{m}_1	
8s	<u>10</u>	45.81	2500	<u>$s_{8s} = s_7$</u>	mix	\dot{m}_1	
8	<u>10</u>	105	2696		superh.	\dot{m}_1	<u>$\eta_s = 0.8$</u>
9	<u>800</u>		<u>$h_9 = h_6 = 2940$</u>		superh.	\dot{m}_9	

10.11d In this assignment there are three different unknown mass flows \dot{m}_1, \dot{m}_3 and \dot{m}_9 . In table 10.11.1 is denoted which mass flows are the same (column \dot{m}). In addition there are 9 different h values. The h values can be found using the two known variables per h value given in table 10.11.1. In order to find the three mass flows three equations are needed (3 unkowns \rightarrow 3 equations).

The net power output of the cycle $\dot{W}_{net} = 80$ MW, is given and leads to the first equation that is needed to determine the mass flow rates:

$$\dot{W}_{net} = \dot{W}_{HPT} + \dot{W}_{LPT} - \dot{W}_{pumps} = \dot{m}_3(h_5 - h_6) + \dot{m}_1(h_7 - h_8) - \dot{m}_1(h_2 - h_1) - \dot{m}_3(h_4 - h_3).$$

The other two equations are the mass balance and the energy balance for the feed water heater:

$$\dot{m}_3 = \dot{m}_2 + \dot{m}_9, \dot{m}_2 = \dot{m}_1 \rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_9$$

$$\dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_9 h_9, \dot{m}_2 = \dot{m}_1 \rightarrow \dot{m}_3 h_3 = \dot{m}_1 h_2 + \dot{m}_9 h_9$$

In order to determine all mass flow rates, first all h -values have to be determined (see also the summarizing table at c). Once these are known the mass flow rates can be determined.

- Point 1: Table A-5 gives $h_1 = h_{f@10kPa} = 191.81$ kJ/kg.
- Point 2: For an isentropic pump (point 2s) $dh = vdP$ holds, so $h_{2s} = h_1 + v_1(P_2 - P_1) = 191.81 + 0.001010(800 - 10) = 192.61$ kJ/kg. We use $\eta_{sp} = \frac{h_{2s} - h_1}{h_2 - h_1}$ to determine h_2 :

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{sp}} = 191.81 + \frac{192.61 - 191.81}{0.8} = 192.81$$
 kJ/kg.

- Point 3: At the outlet of the OFWH, the phase of the working fluid is a saturated liquid (when this is different, extra data would be given). The pressure in an OFWH remains constant, so $h_3 = h_{f@800\text{ kPa}} = 720.87 \text{ kJ/kg}$.
- Point 4: The same method as for point 2 is used: $h_{4s} = h_3 + v_3(P_4 - P_3) = 720.87 + 0.001115(10000 - 800) = 731.13 \text{ kJ/kg}$, which gives $h_4 = h_3 + \frac{h_{4s}-h_3}{\eta_{sp}} = 720.87 + \frac{731.13-720.87}{0.8} = 733.69 \text{ kJ/kg}$.
- Point 5: The Mollier diagram gives for $P_5 = 10 \text{ MPa} = 100 \text{ bar}$ and $T_5 = 550^\circ\text{C}$: $h_5 = 3500 \text{ kJ/kg}$.
- Point 6: In the Mollier diagram go down vertically from point 5, which gives $h_{6s} = 2800 \text{ kJ/kg}$. This would be the enthalpy for an ideal turbine, but for a not-ideal turbine we have the isentropic efficiency: $\eta_{st} = \frac{\Delta h_{rea}}{\Delta h_s} = \frac{h_5-h_6}{h_5-h_{6s}} \rightarrow h_6 = h_5 - \eta_{st}(h_5 - h_{6s}) = 3500 - 0.8(3500 - 2800) = 2940 \text{ kJ/kg}$.
- Point 7: Read out in the Mollier diagram again with $P_7 = 0.8 \text{ MPa} = 8 \text{ bar}$ and $T_7 = 500^\circ\text{C}$: $h_7 = 3480 \text{ kJ/kg}$.
- Point 8: The same method as for point 6 is used, first the enthalpy is determined for an ideal turbine using point 7 and the Mollier diagram: $h_{8s} = 2500 \text{ kJ/kg}$. With the isentropic efficiency can be calculated $h_8 = h_7 - \eta_{st}(h_7 - h_{8s}) = 3480 - 0.8(3480 - 2500) = 2696 \text{ kJ/kg}$.
- Point 9: Point 9 is identical to point 6 (except for the mass flow) and hence $h_9 = h_6 = 2940 \text{ kJ/kg}$.

Now all h -values are known, these can be used to determine the mass flow rates using the equations given above (\dot{W}_{net} and the mass and energy balance for the feed water heater). In the expression for \dot{W}_{net} mass flow 1 and 3 are still unknown. It is necessary to express \dot{m}_1 in \dot{m}_3 (or the other way around) using the mass and energy balance. First \dot{m}_9 is eliminated

$$\dot{m}_3 = \dot{m}_2 + \dot{m}_9, \dot{m}_2 = \dot{m}_1 \rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_9 \rightarrow \dot{m}_9 = \dot{m}_3 - \dot{m}_1$$

$$\dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_9 h_9 \rightarrow \dot{m}_3 h_3 = \dot{m}_1 h_2 + \dot{m}_9 h_9$$

Rewriting the mass balance to $\dot{m}_9 = \dot{m}_3 - \dot{m}_1$ and substituting in the energy balance gives an expression for \dot{m}_1 in terms of \dot{m}_3 :

$$\dot{m}_3 h_3 = \dot{m}_1 h_2 + (\dot{m}_3 - \dot{m}_1) h_9 \rightarrow \dot{m}_3(h_3 - h_9) = \dot{m}_1(h_2 - h_9) \rightarrow \dot{m}_1 = \dot{m}_3 \left(\frac{h_9 - h_3}{h_9 - h_2} \right)$$

Now \dot{m}_1 is expressed in \dot{m}_3 and known h values this expression for \dot{m}_1 can be substituted in the equation for \dot{W}_{net} to give an equation with only \dot{m}_3 and known h values:

$$\begin{aligned} \dot{W}_{net} &= \dot{m}_3(h_5 - h_6) + \dot{m}_3 \left(\frac{h_9 - h_3}{h_9 - h_2} \right) [(h_7 - h_8) - (h_2 - h_1)] - \dot{m}_3(h_4 - h_3) \\ \dot{m}_3 &= \frac{\dot{W}_{net}}{(h_5 - h_6 - h_4 + h_3) + \left(\frac{h_9 - h_3}{h_9 - h_2} \right) [(h_7 - h_8) - (h_2 - h_1)]} \\ &= \frac{80 \cdot 10^3}{547 + 0.81 \cdot 782} = 67.77 \text{ kg/s} \end{aligned}$$

Substituting back in the relations that followed from the mass and energy balances, it is found that $\dot{m}_1 = 54.89 \text{ kg/s}$ and $\dot{m}_9 = 12.88 \text{ kg/s}$.

$$\mathbf{10.11e} \quad \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{m}_3(h_5 - h_4) + \dot{m}_1(h_7 - h_6)} = \frac{80000}{67.77(3500 - 734) + 54.89(3480 - 2940)} = \frac{80000}{217092.4} = 0.37$$

$$\mathbf{10.11f} \quad \eta_{2nd law} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{\eta_{th}}{1 - \frac{T_L}{T_H}} = \frac{0.37}{1 - \frac{273+46}{273+550}} = \frac{0.37}{0.61} = 0.607.$$

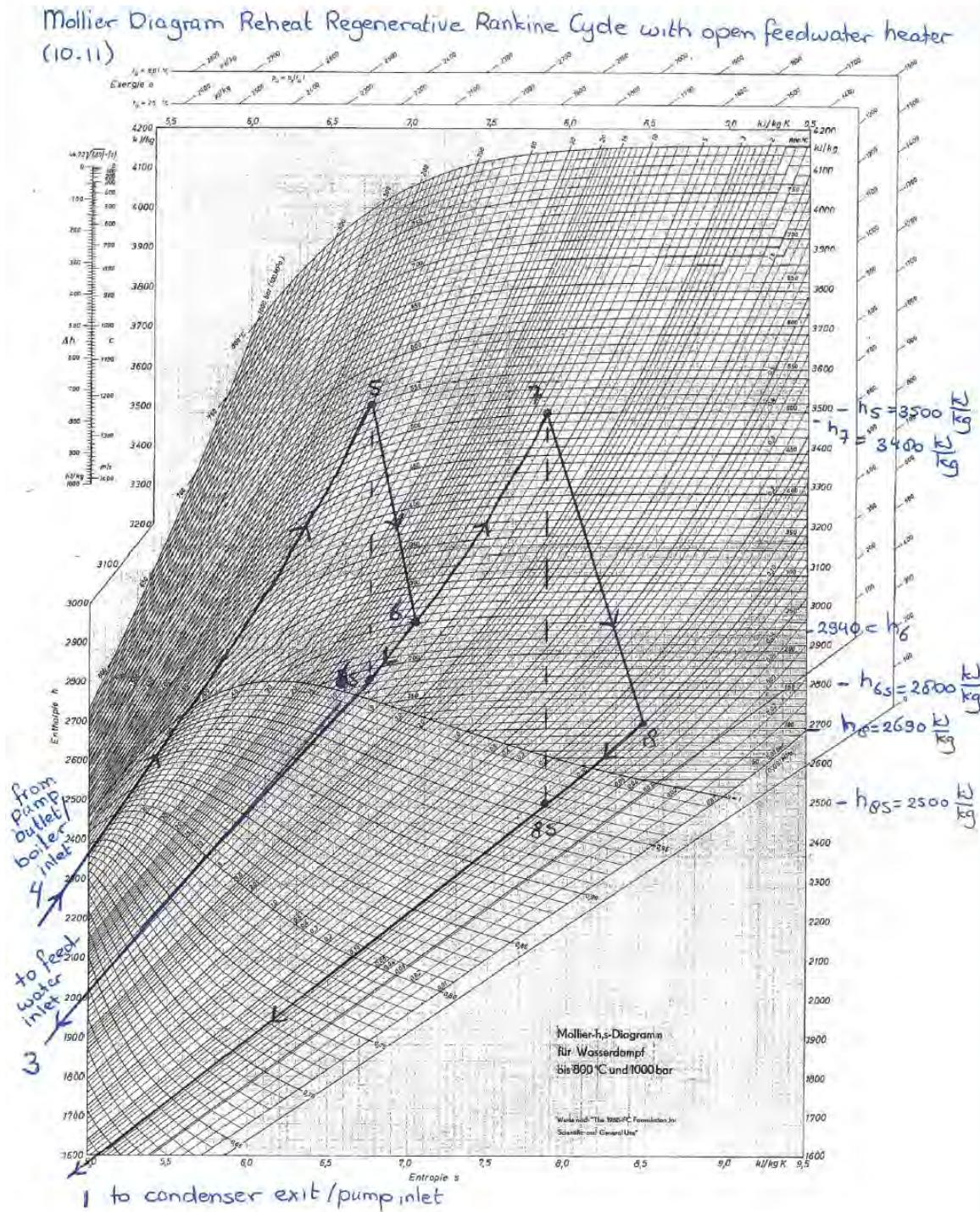


Figure 10.11.4: Mollier diagram of the reheat-regenerative Rankine cycle with open feedwater heater of assignment 10.11.

10.12 Reheat-regenerative Rankine cycle with closed feedwater heater

10.12a See figure 10.12.1.

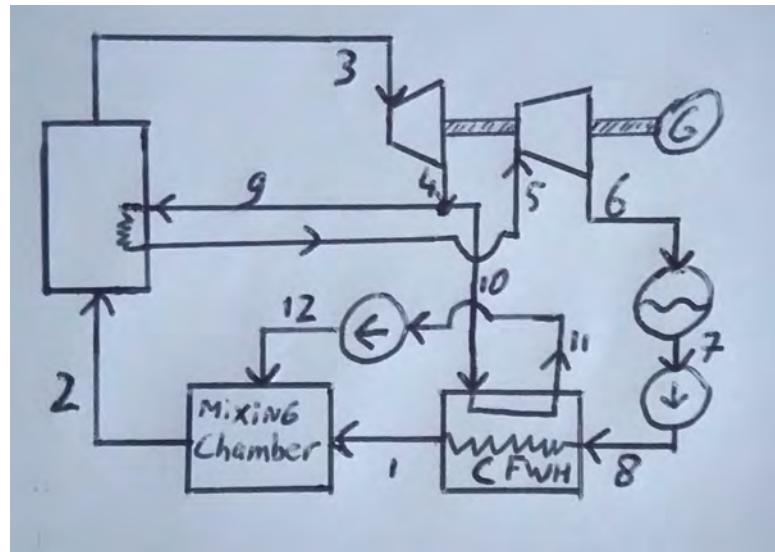


Figure 10.12.1: Overview of exercise 10.12

10.12b See figures 10.12.2 and 10.12.3.

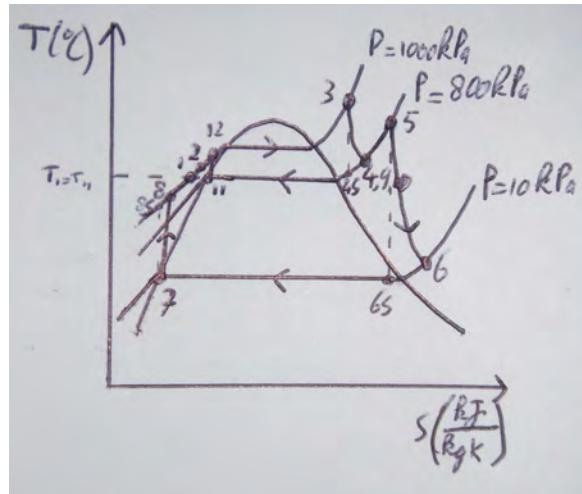


Figure 10.12.2: T-s diagram of exercise 10.12

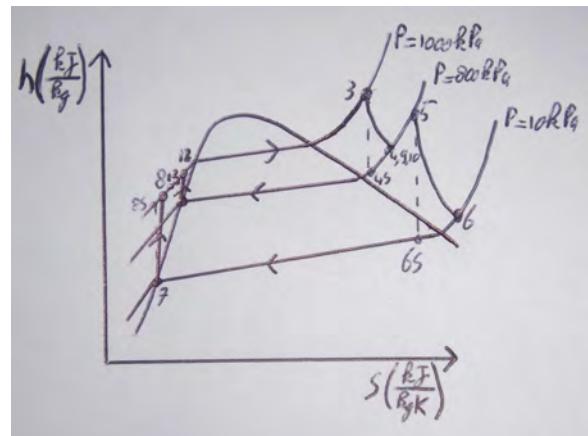


Figure 10.12.3: h-s diagram of exercise 10.12

10.12c See table 10.12.1.

10.12d In comparison to assignment 10.11, the points 3, 4, 5, 6, 7, 9 and 10 are identical concerning P , T , h and s (so NOT concerning the mass flows). So h -values need to be found for 5 points 1, 2, 8, 11 and 12. In addition 3 different mass flows have to be determined (\dot{m}_1 , \dot{m}_2 and \dot{m}_{12}). Compared to assignment 10.11 there are two energy balances that have to be taken into account, the energy balance over the mixing chamber and the energy balance over the closed feed water heater. Like in assignment 10.11 also the mass balance and the equation for the net power output (\dot{W}_{net}) have to be taken into account.

Table 10.12.1: Table of exercise 10.12. The two known variables at each point are underlined.

	P (kPa)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/(kgK))	phase	\dot{m}	other
1	<u>10000</u>	<u>$T_1 = T_{11}$</u>	721.11		comp. liq.	\dot{m}_1	Energy bal. mix. cham.
2	<u>10000</u>		723.55		comp. liq.	\dot{m}_2	
3	<u>10000</u>	<u>550</u>	3500	6.65	superh.	\dot{m}_2	
4s	<u>800</u>		2800	<u>$s_{4s} = s_3$</u>	superh.	\dot{m}_2	
4	<u>800</u>	240	2940	7.0	superh.	\dot{m}_2	<u>$\eta_s = 0.8$</u>
5	<u>800</u>	<u>500</u>	3480	7.85	superh.	\dot{m}_1	
6s	<u>10</u>	45.81	2500	<u>$s_{6s} = s_5$</u>	mix	\dot{m}_1	
6	<u>10</u>	105	2696		superh.	\dot{m}_1	<u>$\eta_s = 0.8$</u>
7	<u>10</u>	45.81	191.83		<u>sat. liq.</u>	\dot{m}_1	$v_f = 0.001010 \text{ m}^3/\text{kg}$
8s	<u>10000</u>		201.92	<u>$s_{8s} = s_7$</u>	comp. liq.	\dot{m}_1	
8	<u>10000</u>		204.44		comp. liq.	\dot{m}_1	<u>$\eta_s = 0.8$</u>
9	<u>800</u>	240	2940	7.0	superh.	\dot{m}_1	<u>$\eta_s = 0.8$</u>
10	<u>800</u>	240	2940	7.0	superh.	\dot{m}_{12}	<u>$\eta_s = 0.8$</u>
11	<u>800</u>	170.43	721.11		<u>sat. liq.</u>	\dot{m}_{12}	$v_f = 0.001115 \text{ m}^3/\text{kg}$
12s	<u>10000</u>		731.37	<u>$s_{12s} = s_{11}$</u>	comp. liq.	\dot{m}_{12}	
12	<u>10000</u>		733.93		comp. liq.	\dot{m}_{12}	<u>$\eta_s = 0.8$</u>

This gives four equations:

1. Energy balance over the mixing chamber: $\dot{m}_2 h_2 = \dot{m}_{12} h_{12} + \dot{m}_1 h_1$.
2. Energy balance over the closed feed water heater: $\dot{m}_1 h_8 + \dot{m}_{12} h_{10} = \dot{m}_1 h_1 + \dot{m}_{12} h_{11}$
3. Mass balance: $\dot{m}_2 = \dot{m}_{12} + \dot{m}_1$
4. Net power output: $\dot{W}_{net} = \dot{m}_2(h_3 - h_4) + \dot{m}_1(h_5 - h_6) - \dot{m}_1(h_8 - h_7) - \dot{m}_{12}(h_{12} - h_{11})$

Four equations means that there are also 4 unknown variables that have to be determined using these 4 equations. These variables are the three mass flows, \dot{m}_1 , \dot{m}_2 and \dot{m}_{12} and the enthalpy of point 2, h_2 . The second known variable for point 2 is the energy balance over the mixing chamber (if an other thing would be assumed for point 2 e.g. saturated liquid then the system would be over determined and the equations cannot be solved).

To determine the mass flow rates first the missing h values have to be determined, once they are known the four equations above are used to determine h_2 and the mass flows.

- Point 11: From the table: $h_{11} = h_f@800 \text{ kPa} = 721.11 \text{ kJ/kg}$. The temperature belonging to this point is $T_{11} = 170.43 \text{ }^{\circ}\text{C}$.

- Point 1: The temperature in point 1 is the same as the temperature in point 11 (given in the exercise). It appears that for this pressure we cannot use the tables for compressed liquid, so we use the Incompressible Liquid Approximation and look up the value of h_1 at the correct temperature (not pressure) for saturated liquid, which is then equal to h_{11} : $h_1 = 721.11 \text{ kJ/kg}$.
- Point 12: Used is $dh = vdP \rightarrow h_{12s} = h_{11} + v(P_{12} - P_{11}) = 721.11 + 0.001115(10000 - 800) = 731.37 \text{ kJ/kg}$. Use of the isentropic efficiency gives $h_{12} = h_{11} + \frac{h_{12s} - h_{11}}{\eta_{sp}} = 733.93 \text{ kJ/kg}$.
- Point 8: The same method is used as for point 12: $h_{8s} = h_7 + v(P_8 - P_7) = 191.83 + 0.001010(10000 - 10) = 201.92 \text{ kJ/kg}$. The isentropic efficiency gives $h_8 = h_7 + \frac{h_{8s} - h_7}{\eta_s} = 204.44 \text{ kJ/kg}$.
- Point 2: h_2 follows from the energy balance over the mixing chamber: $\dot{m}_2 h_2 = \dot{m}_{12} h_{12} + \dot{m}_1 h_1 \rightarrow h_2 = \frac{\dot{m}_{12} h_{12} + \dot{m}_1 h_1}{\dot{m}_2}$. However, to calculate h_2 the mass flows are needed. To find them the other equations have to be used also.

Now first the mass flows are determined:

- Rewriting the mass balance $\dot{m}_2 = \dot{m}_{12} + \dot{m}_1$ gives an expression for $\dot{m}_{12} \rightarrow \dot{m}_{12} = \dot{m}_2 - \dot{m}_1$.
- Substitution of the expression for \dot{m}_{12} in the energy balance over the closed feed water heater, $\dot{m}_1 h_8 + \dot{m}_{12} h_{10} = \dot{m}_1 h_1 + \dot{m}_{12} h_{11}$, gives an expression for $\dot{m}_1 \rightarrow \dot{m}_1 h_8 + (\dot{m}_2 - \dot{m}_1) h_{10} = \dot{m}_1 h_1 + (\dot{m}_2 - \dot{m}_1) h_{11} \rightarrow \dot{m}_1 = \dot{m}_2 \left(\frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right)$.
- Combining this expression with the mass balance leads to an expression for $\dot{m}_{12} \rightarrow \dot{m}_{12} = \dot{m}_2 \left(1 - \frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right)$.
- Substitution of the expressions for \dot{m}_1 and \dot{m}_{12} in the equation for the net power output leads to an expression for \dot{m}_2 :

$$\begin{aligned} \dot{W}_{net} &= \dot{m}_2 \left[(h_3 - h_4 + \left(\frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right) (h_5 - h_6)) \right. \\ &\quad \left. - \left[\left(\frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right) (h_8 - h_7) - \left(1 - \frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right) (h_{12} - h_{11}) \right] \right] \rightarrow \\ \dot{m}_2 &= \frac{\dot{W}_{net}}{\left[(h_3 - h_4 + \left(\frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right) (h_5 - h_6) - \left(\frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right) (h_8 - h_7) - \left(1 - \frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right) (h_{12} - h_{11})) \right]} \end{aligned}$$

- Filling in the h values gives the mass flow rate through the boiler: $\dot{m}_2 = 59.65 \text{ kg/s}$.
- Hence $\dot{m}_1 = \dot{m}_2 \left(\frac{h_{11} - h_{10}}{h_8 - h_{10} - h_1 + h_{11}} \right) = 48.32 \text{ kg/s}$ and $\dot{m}_{12} = \dot{m}_2 - \dot{m}_1 = 11.33 \text{ kg/s}$

Finally also h_2 can be found $h_2 = \frac{\dot{m}_{12} h_{12} + \dot{m}_1 h_1}{\dot{m}_2} = 723.55 \text{ kJ/kg}$.

$$\mathbf{10.12e} \quad \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{m}_2(h_3 - h_2) + \dot{m}_1(h_5 - h_9)} = \frac{80000}{59.24(3500 - 721.16) + 47.85(3480 - 2940)} = 0.420.$$

$$\mathbf{10.12f} \quad \eta_{2^{nd} law} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{\eta_{th}}{1 - \frac{T_L}{T_H}} = \frac{0.42}{1 - \frac{273+46}{273+550}} = \frac{0.42}{0.61} = 0.689.$$

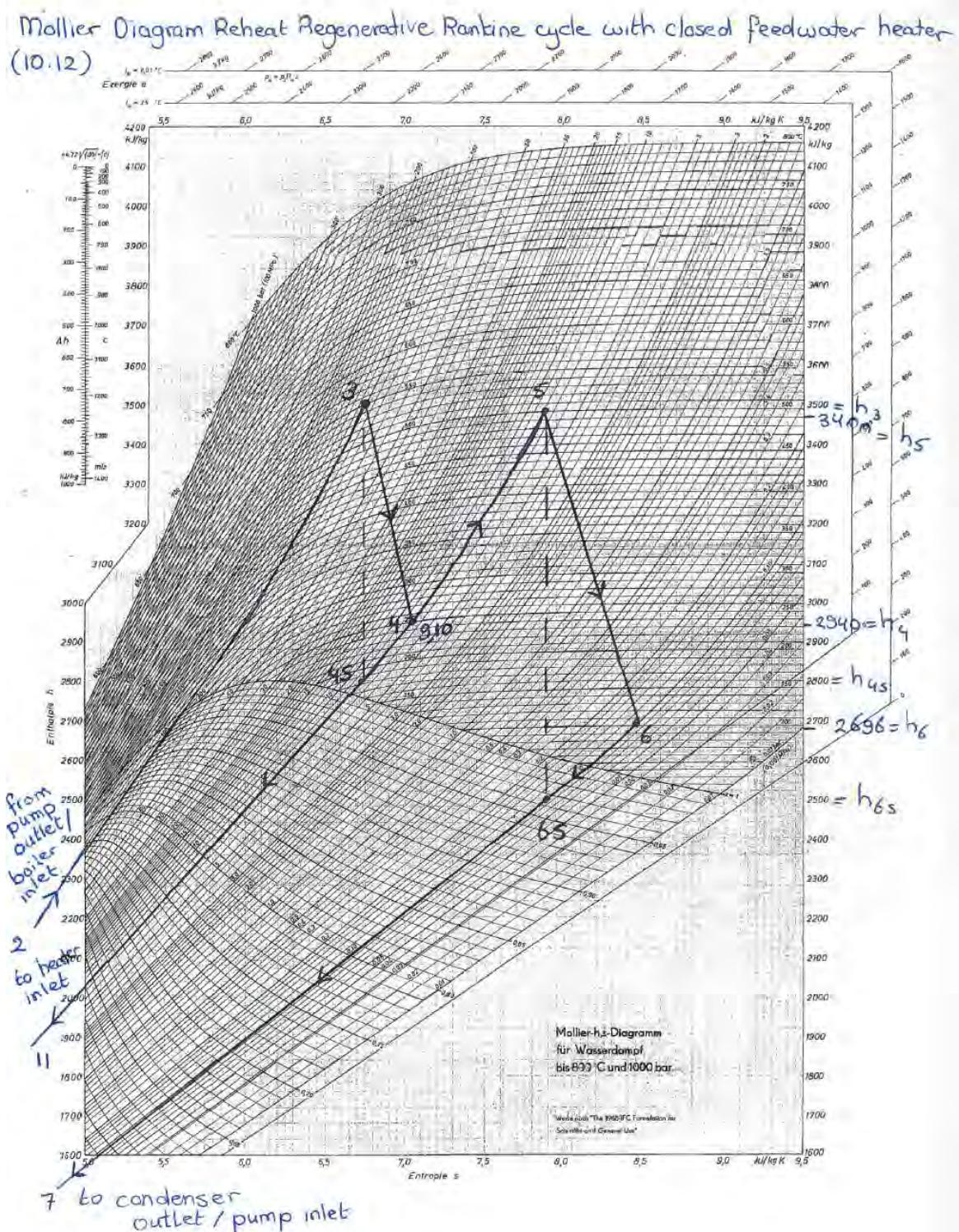


Figure 10.12.4: Mollier diagram of the reheat-regenerative Rankine cycle with closed feedwater heater of assignment 10.12.

10.13 Combined gas-steam power cycle 1

10.13a See figure 10.13.1.

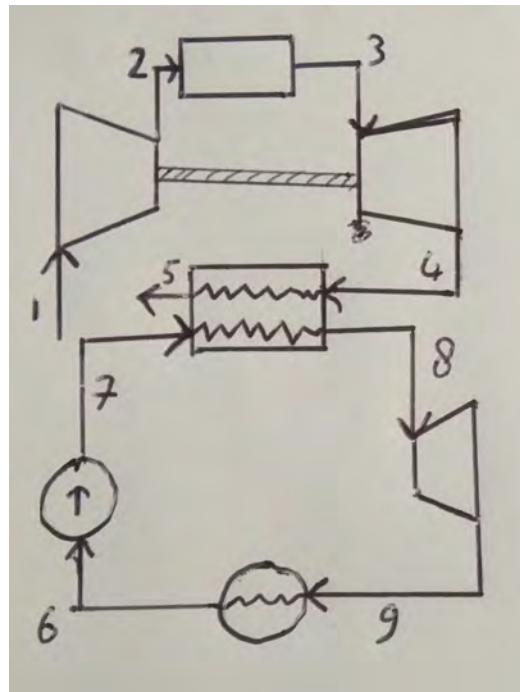


Figure 10.13.1: Schematic overview of question 10.13

10.13b See table 10.13.1. The two known variables at each point are underlined.

Table 10.13.1: Table with parameters of question 10.13

	P (kPa)	T (K)	h (kJ/kg)	s (kJ/(kgK))	phase	other
1	<u>100</u>	<u>300</u>	305		air	
2s	<u>800</u>		560	<u>$s_{2s} = s_1$</u>	air	
2	<u>800</u>		624		air	<u>$\eta_{sc} = 0.8$</u>
3	<u>800</u>	<u>1300</u>	1400		air	
4s	<u>100</u>		795	<u>$s_{4s} = s_3$</u>	air	
4	<u>100</u>		886		air	<u>$\eta_{st} = 0.85$</u>
5	<u>100</u>	<u>450</u>	450		air	
6	<u>5</u>		137.82		<u>sat. liq.</u>	$v_f = 0.001005 \text{ m}^3/\text{kg}$
7	<u>7000</u>		144.85	<u>$s_7 = s_6$</u>	comp. liq.	
8	<u>7000</u>	<u>773</u>	3410.3	6.7975	superh.	
9	<u>5</u>		2071.9	<u>$s_9 = s_8$</u>	mix	

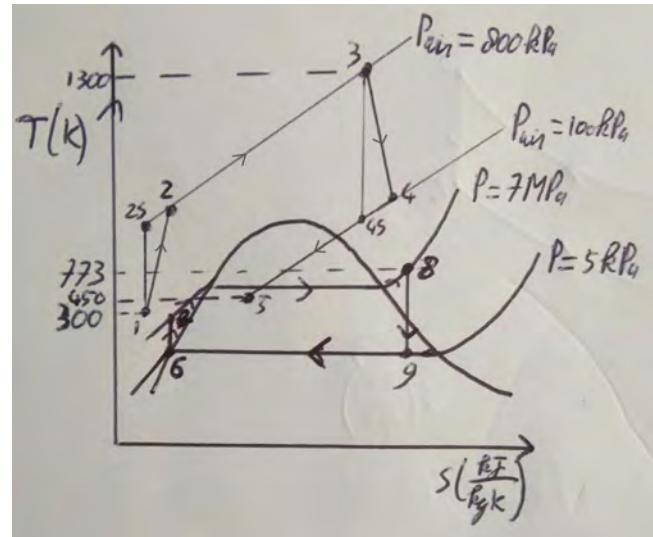


Figure 10.13.2: T-s diagram of question 10.13

10.13c See figure 10.13.2.

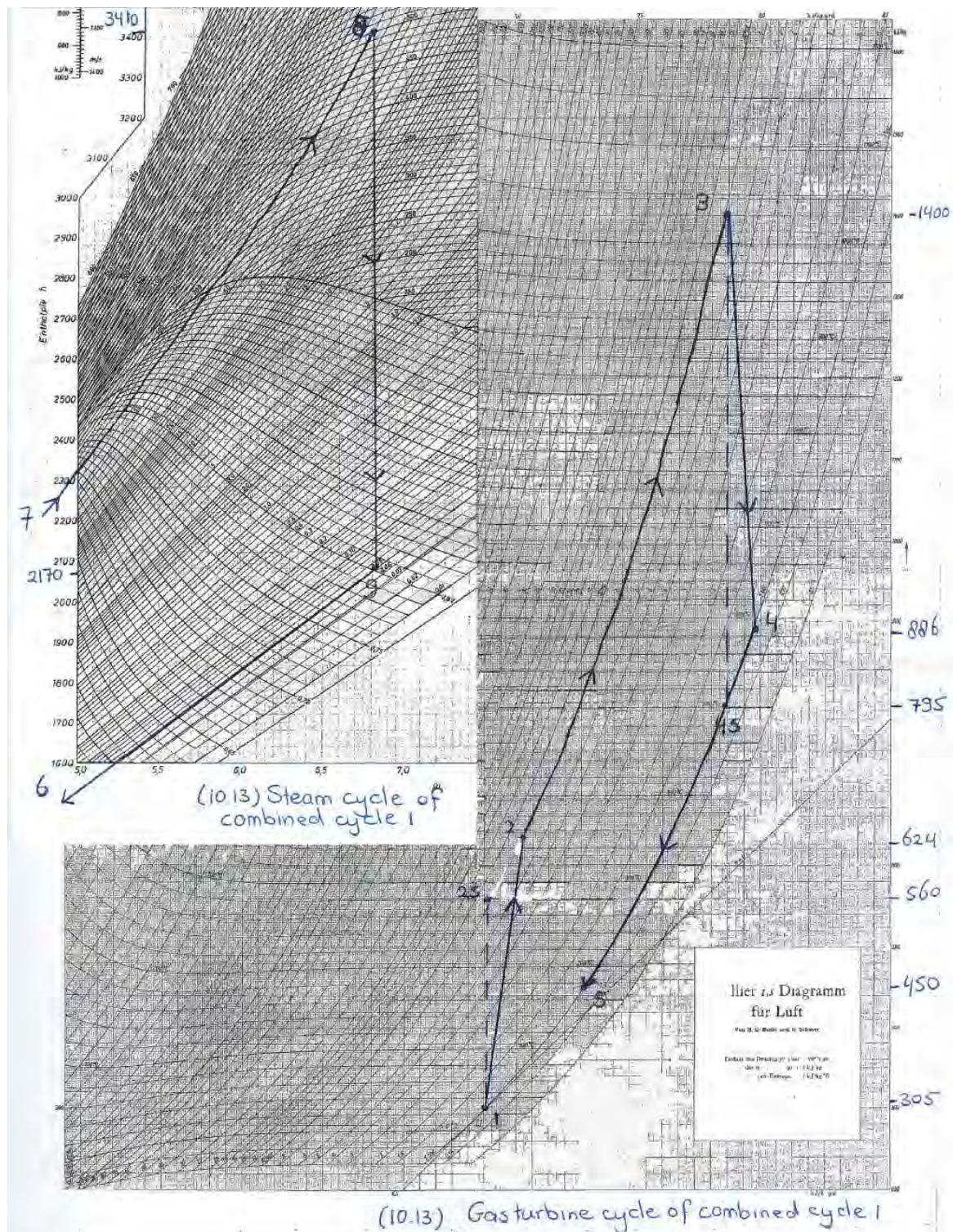


Figure 10.13.3: Mollier diagram of the gas turbine cycle of the combined cycle of assignment 10.13. The Mollier diagram of the steam turbine cycle is given at the upper left.

10.13d To determine the ratio of the mass flows of the air, \dot{m}_g , and of the water/steam, \dot{m}_s , we need to determine the energy balance over the heat exchanger:

$$\dot{Q}_{in} = \dot{Q}_{out} \Rightarrow \dot{m}_g h_4 + \dot{m}_s h_7 = \dot{m}_g h_5 + \dot{m}_s h_8 \Rightarrow \dot{m}_g (h_4 - h_5) = \dot{m}_s (h_8 - h_7) \Rightarrow \frac{\dot{m}_s}{\dot{m}_g} = \frac{h_4 - h_5}{h_8 - h_7}$$

So, the h-values need to be determined. For point 1, we use the Mollier diagram, see figure 10.13.3: $h_1 = 305 \text{ kJ/kg}$. Going vertically ($\Delta s = 0$) up we find $h_{2s} = 560 \text{ kJ/kg}$, such that $h_2 = \frac{h_{2s}-h_1}{\eta_{sc}} + h_1 = 624 \text{ kJ/kg}$. Point 3 can be read from the Mollier diagram: $h_3 = 1400 \text{ kJ/kg}$, and going down vertically we find $h_{4s} = 795 \text{ kJ/kg}$. Using the isentropic efficiency of the turbine we then get $h_4 = h_3 - \eta_{st}(h_3 - h_{4s}) = 886 \text{ kJ/kg}$. Point 5 is again read from the Mollier diagram: $h_5 = 450 \text{ kJ/kg}$.

Point 6 is determined from the tables for water: $h_6 = h_f@5 \text{ kPa} = 137.82 \text{ kJ/kg}$. For a pump, we then use that $dh = Tds + vdp$, such that, for an isentropic pump ($ds = 0$) $h_7 = h_6 + v(P_7 - P_6) = 144.85 \text{ kJ/kg}$. For point 8 we look in the tables for superheated vapor: $h_8 = 3410.3 \text{ kJ/kg}$ and $s_8 = 6.7975 \text{ kJ/(kgK)}$. For point 9, we now look in the tables at 5 kPa, and we see that $s_f < s_9 < s_g$, and hence we have a mixture. Determining x gives $x = \frac{s_9-s_f}{s_g-s_f} = \frac{6.7975-0.4764}{8.3951-0.4764} = 0.798$, such that $h_9 = h_f + x(h_g - h_f) = 137.82 + 0.798(2561.5 - 137.82) = 2071.9 \text{ kJ/kg}$. Points 8 and 9 can also be found using the Mollier diagram of water given in figure 10.13.3.

Substituting the values in the expression for the mass ratio gives: $\frac{\dot{m}_s}{\dot{m}_g} = \frac{h_4-h_5}{h_8-h_7} = \frac{886-450}{3410.3-144.85} = 0.13$. So, with 1 kg exhaust gases we can heat up only 0.13 kg steam.

10.13e The thermal efficiency of the combined cycle is given as:

$$\begin{aligned}\eta_{th} &= \frac{\dot{W}_{out,gas} + \dot{W}_{out,steam} - \dot{W}_{in,gas} - \dot{W}_{in,steam}}{\dot{Q}_{in}} \\ &= \frac{\dot{m}_g(h_3 - h_4) + \dot{m}_s(h_8 - h_9) - \dot{m}_g(h_2 - h_1) - \dot{m}_s(h_7 - h_6)}{\dot{m}_g(h_3 - h_2)} \\ &= \frac{(h_3 - h_4) - (h_2 - h_1) + \frac{\dot{m}_s}{\dot{m}_g}(h_8 - h_9 - (h_7 - h_6))}{h_3 - h_2} = 0.47\end{aligned}$$

10.13f $\eta_{2^{nd}law} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{\eta_{th}}{1 - \frac{T_{low}}{T_{high}}} = \frac{0.47}{1 - \frac{300}{1300}} = \frac{0.47}{0.77} = 0.61$

10.14 Combined gas-steam power cycle 2

10.14a See figure 10.14.1

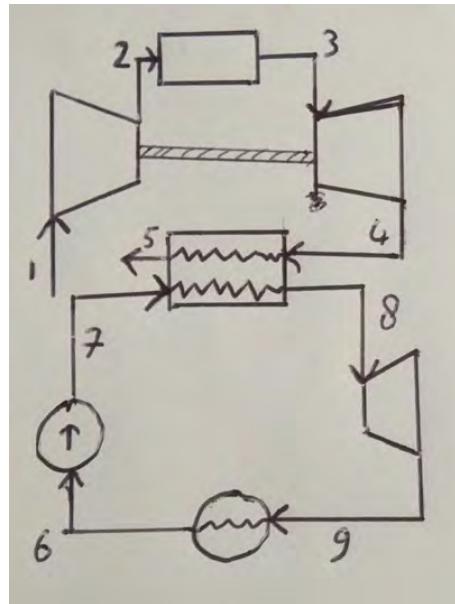


Figure 10.14.1: Schematic overview of exercise 10.14

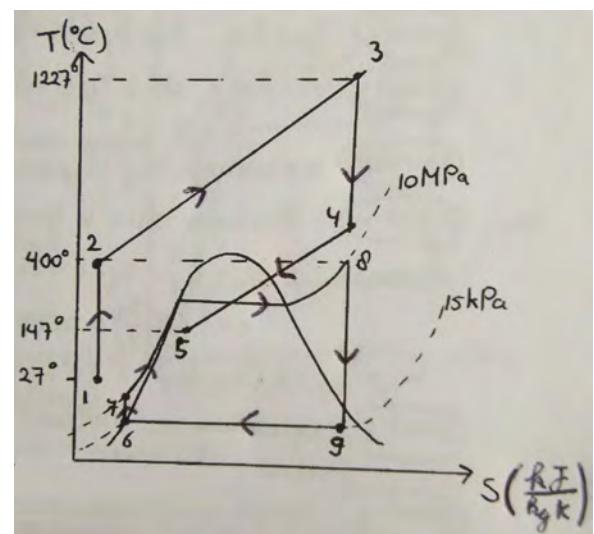


Figure 10.14.2: T-s diagram of exercise 10.14

Table 10.14.1: Table with parameters of exercise 10.14

	P (kPa)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/(kgK))	phase	other
1	<u>100</u>	<u>27</u>	305		air	
2	<u>1600</u>	390	670	$s_2 = s_1$	air	
3	<u>1600</u>	<u>1227</u>	1620		air	
4	<u>100</u>	480	770	$s_4 = s_3$	air	
5	<u>100</u>	<u>147</u>	430		air	
6	<u>15</u>	53.97	225.94		<u>sat. liq.</u>	$v_f = 0.001014 \text{ m}^3/\text{kg}$
7	<u>10000</u>		236.06	$s_7 = s_6$	comp. liq.	
8	<u>10000</u>	<u>400</u>	3096.5	6.2120	superh.	
9	<u>15</u>		2005.8	$s_9 = s_8$	mix, $x = 0.75$	

10.14b See table 10.14.1. The two known variables at each point are underlined.

10.14c See figure 10.14.2.

10.14d The mass flow of the steam follows from the energy balance over the heat exchanger:

$$\dot{Q}_{in} = \dot{Q}_{out} \Rightarrow \dot{m}_g h_4 + \dot{m}_s h_7 = \dot{m}_g h_5 + \dot{m}_s h_8 \Rightarrow \dot{m}_g (h_4 - h_5) = \dot{m}_s (h_8 - h_7) \Rightarrow \dot{m}_s = \dot{m}_g \frac{h_4 - h_5}{h_8 - h_7}$$

So, the enthalpies need to be determined. For point 1, we read in the Mollier diagram for air, given in figure 10.14.3, at $P_1 = 1$ bar and $T_1 = 27$ $^{\circ}$ C, giving $h_1 = 305$ kJ/kg. Going vertically ($\Delta s = 0$) up to $P_2 = 16$ bar gives $h_2 = 670$ kJ/kg. Point 3 is found by going to $T_3 = 1227$ $^{\circ}$ C at $P_3 = 16$ bar: $h_3 = 1620$ kJ/kg. Going down vertically ($\Delta s = 0$) to $P_4 = 1$ bar then gives $h_4 = 770$ kJ/kg, and going to $T_5 = 147$ $^{\circ}$ C at the same pressure gives $h_5 = 430$ kJ/kg.

For point 6, the tables for water are used: $h_6 = h_{f@15 \text{ kPa}} = 225.94$ kJ/kg, and $v_6 = 0.001014 \text{ m}^3/\text{kg}$. For a pump, we use that $dh = Tds + vdp$. Since the pump is isentropic, $ds = 0$ and hence $h_7 = h_6 + v(P_7 - P_6) = 225.94 + 0.001014(10000 - 15) = 236.06$ kJ/kg. Point 8 is found in the table for superheated vapor: $P_8 = 10$ MPa, $T_8 = 400$ $^{\circ}$ C, giving $h_8 = 3096.5$ kJ/kg and $s_8 = 6.2120$ kJ/(kgK). For point 9, we make use of the fact that the turbine is isentropic, and hence $s_9 = s_8$. At $P_9 = 15$ kPa it is found that, for this entropy, we must have a mixture, so we calculate $x = \frac{s_9 - s_f}{s_g - s_f} = \frac{6.2120 - 0.7549}{8.0085 - 0.7549} = 0.75$, such that $h_9 = h_f + x(h_g - h_f) = 225.94 + 0.75(2599.1 - 225.94) = 2005.81$ kJ/kg. Points 8 and 9 can also be found using the Mollier diagram of water given in figure 10.14.3.

Filling in the values in the equation for the mass flow of the steam gives: $\dot{m}_s = \dot{m}_g \frac{h_4 - h_5}{h_8 - h_7} = 14 \frac{770 - 430}{3096.5 - 236.06} = 1.66 \text{ kg/s}$.

$$\mathbf{10.14e} \quad \dot{W}_{net} = \dot{m}_g [(h_3 - h_4) - (h_2 - h_1)] + \dot{m}_s [(h_8 - h_9) - (h_7 - h_6)] = 8583.76 \text{ kW} = 8.85 \text{ MW}$$

$$\mathbf{10.14f} \quad \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{m}_g [(h_3 - h_4) - (h_2 - h_1)] + \dot{m}_s [(h_8 - h_9) - (h_7 - h_6)]}{\dot{m}_g (h_3 - h_2)} = \frac{8583.76}{14(1620 - 670)} = 0.65$$

$$\mathbf{10.14g} \quad \eta_{2nd\ law} = \frac{\eta_{th}}{\eta_{carnot}} = \frac{\eta_{th}}{1 - \frac{T_L}{T_H}} = \frac{0.65}{1 - \frac{300}{1500}} = \frac{0.65}{0.8} = 0.81$$

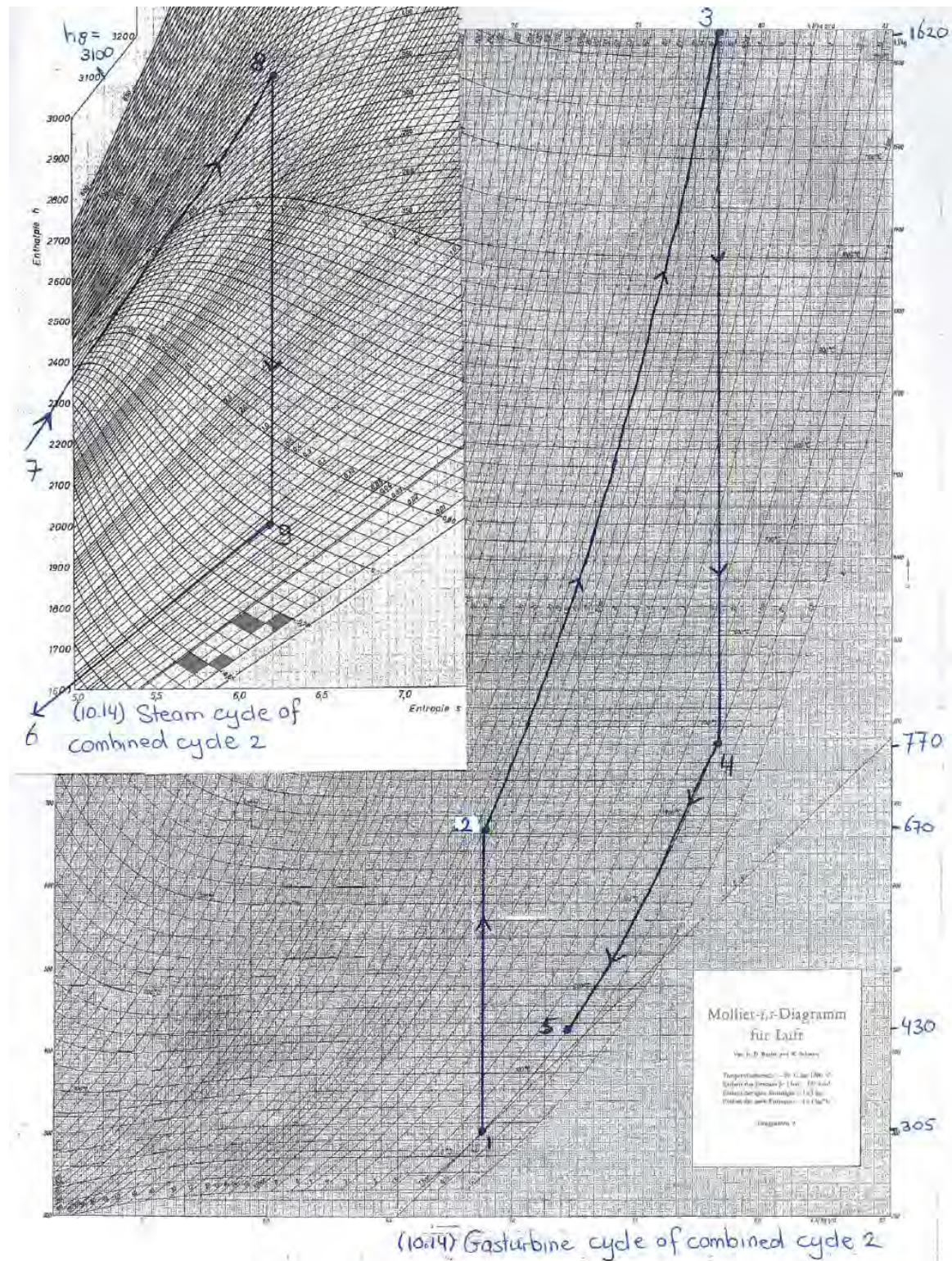


Figure 10.14.3: Mollier diagram of the gas turbine cycle of the combined cycle of assignment 10.14. The Mollier diagram of the steam turbine cycle is given at the upper left.

10.15 An ideal cogeneration power plant

10.15a The schematic overview of the cogeneration power plant is shown in figure 10.15.1 and the $T-s$ -diagram of the this power plant is shown in figure 10.15.2.

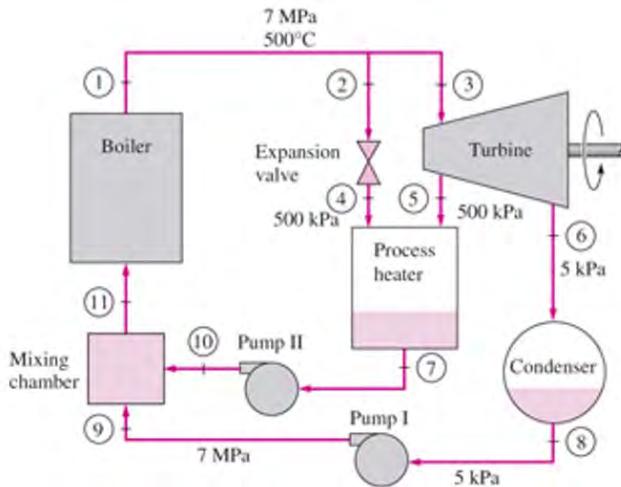


Figure 10.15.1: Schematic overview of the cogeneration powerplant of assignment 10.15

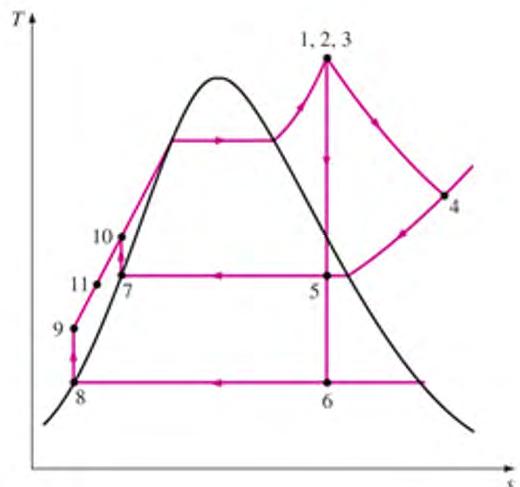


Figure 10.15.2: T-s diagram of the cogeneration powerplant of assignment 10.15

10.15b The power plant operates on an ideal cycle and thus the pumps and the turbine are isentropic. There are no pressure drops in the boiler, the process heater and the condenser. Steam leaves the process heater and the condenser as saturated liquid. The two known variables are collected in table 10.15.1 (underlined variables). Using them the h -values in each point can be determined.

The mass flows through the turbine and the process heater change depending on the way the plant is used. There are three modes:

1. Maximum rate of process heat: all mass flows through the process chamber via point 2 and no mass flows through the turbine (point 3), $\dot{m}_3 = 0 \text{ kg/s}$ and $\dot{m}_2 = \dot{m}_1$.
2. Maximum power output: all mass flows through the turbine via point 3 and 6 and no mass flows through the process chamber (point 2 and 5), $\dot{m}_2 = 0 \text{ kg/s}$ and $\dot{m}_3 = \dot{m}_1$.
3. A combination of process heat and power output: the mass flow is splitted after the boiler, $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$.

However, the changing mass flows have no influence on the h value of most of the points as they in general do not depend on the mass flow. The only exception is the point after the mixing chamber, point 11. This point depends on the ratio of the mass flows through the turbine and the process heater and can be determined using the energy balance over the mixing chamber ($\dot{m}_9 h_9 + \dot{m}_{10} h_{10} = \dot{m}_{11} h_{11}$). Note that in case that the maximum rate of process heat is achieved $\dot{m}_9 = 0 \text{ kg/s}$ thus $\dot{m}_{10} = \dot{m}_{11}$ and $h_{11} = h_{10}$. In the case of maximum power $\dot{m}_{10} = 0 \text{ kg/s}$ thus $\dot{m}_9 = \dot{m}_{11}$ and $h_{11} = h_9$.

The h values are determined using the Mollier diagram for water, see figure 10.15.3 and the tables in the book. They are added to table 10.15.1.

- Point 1, 2 and 3: $h_1 = 3410 \text{ kJ/kg}$, can be read from the Mollier diagram directly at $P = 7 \text{ Bar}$ and $T = 500^\circ\text{C}$. h_2 and h_3 have the same value as for them only the mass flow changes.
- Point 4: h_4 also has the same value as 1, 2 and 3 as the throttling process does not change the value of the enthalpy.
- Point 5: The turbine is isentropic, $s_5 = s_3$. From point 3, vertically down to the $P_5 = 5 \text{ Bar}$ isobar, $h_5 = 2740 \text{ kJ/kg}$.

- Point 6: The turbine is isentropic, $s_6 = s_3$. From point 3, vertically down to the $P_5 = 0.05$ Bar isobar, $h_6 = 2075$ kJ/kg.
- Point 7: Saturated liquid at $P_7 = 500$ kPa, from table A-5 follows: $h_7 = 640.09$ kJ/kg.
- Point 8: Saturated liquid at $P_8 = 5$ kPa, from table A-5 follows: $h_8 = 137.75$ kJ/kg.
- Point 9: For a pump is used that $dh = Tds + vdp$. Since the pump is isentropic, $ds = 0$ and

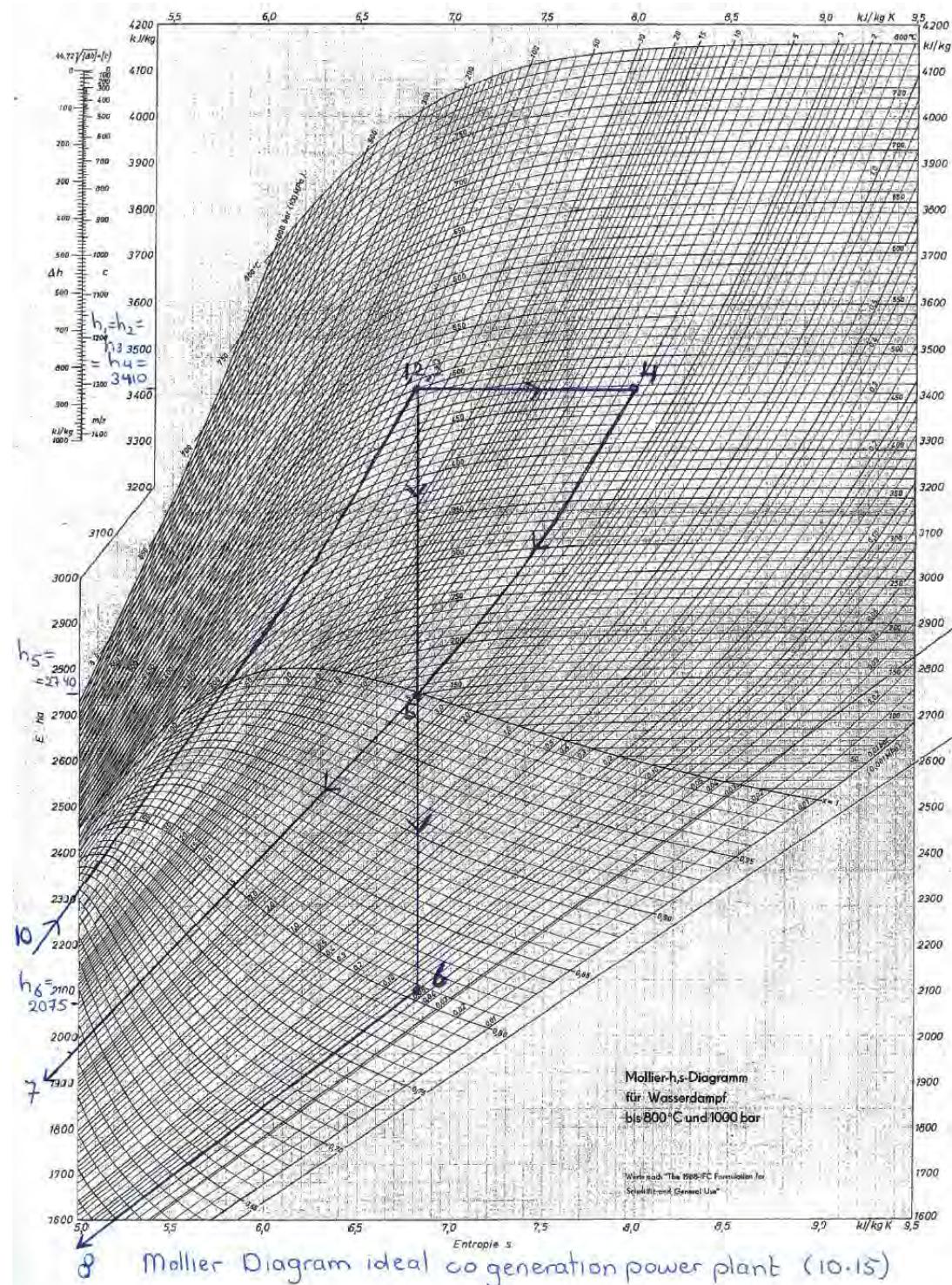


Figure 10.15.3: Mollier diagram of the co-generation power plant of assignment 10.15. Note: the line between point 2 and 4 represents the throttling process, $\Delta h = 0$ and is horizontal to the x -axis.

Table 10.15.1: Table with parameters of exercise 10.15. The two known variables are underlined. Temperature and x -values, not known before, are read from the Mollier diagram. At point 11: 1, 2 and 3 refer to the mode of use: maximum rate of process heat (1), maximum power output (2) and the combination of process heat and power output (3). * The mass flows in the last column are given for mode 3, the combination of process heat and power.

	P (kPa)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/kgK)	phase	other	\dot{m} (kg/s)*
1	<u>7000</u>	<u>500</u>	3410		sup. vapor		15
2	<u>7000</u>	<u>500</u>	3410		sup. vapor		1.5
3	<u>7000</u>	<u>500</u>	3410		sup. vapor		13.5
4	<u>500</u>	465	<u>$h_4 = h_3$</u>		sup. vapor		\dot{m}_2
5	<u>500</u>	150	2740	<u>$s_5 = s_3$</u>	$x = 0.998$		10.5
6	<u>5</u>	35	2075	<u>$s_6 = s_3$</u>	$x = 0.805$		3
7	<u>500</u>	150	640.09		<u>sat. liq., $x = 0$</u>		12
8	<u>5</u>	35	137.75		<u>sat. liq., $x = 0$</u>		\dot{m}_6
9	<u>7000</u>		144.78	<u>$s_9 = s_8$</u>	comp. liq.	$v_f = 0.001005 \text{ m}^3/\text{kg}$	\dot{m}_6
10	<u>7000</u>		647.19	<u>$s_{10} = s_7$</u>	comp. liq.	$v_f = 0.001093 \text{ m}^3/\text{kg}$	\dot{m}_7
11 ¹	<u>7000</u>		<u>$h_{11} = h_{10}$</u>		comp. liq.		\dot{m}_1
11 ²	<u>7000</u>		<u>$h_{11} = h_9$</u>		comp. liq.		\dot{m}_1
11 ³	<u>7000</u>		547		comp. liq.	<u>Energy bal. mix. cham.</u>	\dot{m}_1

$$\text{hence } h_9 = h_8 + v(P_9 - P_8) = 137.75 + 0.001005(7000 - 5) = 144.78 \text{ kJ/kg.}$$

- Point 10: For a pump is used that $dh = Tds + vdP$. Since the pump is isentropic, $ds = 0$ and hence $h_{10} = h_7 + v(P_{10} - P_7) = 640.09 + 0.001093(7000 - 500) = 647.19 \text{ kJ/kg.}$
- Point 11: h_{11} is found using the energy balance over the process heater and therefore depends on the mode of use that determines the ratio between the mass flows through the process heater ($\dot{m}_7 = \dot{m}_{10}$) and the turbine ($\dot{m}_6 = \dot{m}_9$).

$$\dot{m}_9 h_9 + \dot{m}_{10} h_{10} = \dot{m}_{11} h_{11} \rightarrow h_{11} = \frac{\dot{m}_9 h_9 + \dot{m}_{10} h_{10}}{\dot{m}_{11}} \text{ where } \dot{m}_{11} = \dot{m}_9 + \dot{m}_{10}.$$

For the different modes of use this yields:

1. Maximum rate of process heat: $\dot{m}_9 = 0 \text{ kg/s}$ thus $\dot{m}_{10} = \dot{m}_{11}$ and $h_{11} = h_{10}$.
2. Maximum power: $\dot{m}_{10} = 0 \text{ kg/s}$ thus $\dot{m}_9 = \dot{m}_{11}$ and $h_{11} = h_9$.
3. A combination of power en process heat: \dot{m}_9 and \dot{m}_{10} depend on the required process heat and power output of the cogeneration power plant. When the ratio between the mass flows is known h_{11} can be calculated.

Now all h -values are known the questions can be answered.

10.15c The maximum rate of process heat (mode 1) is achieved when all steam leaving the boiler is throttled and sent to the process heater and none is sent to the turbine (that is $\dot{m}_3 = \dot{m}_5 = \dot{m}_6 = 0 \text{ kg/s}$ and $\dot{m}_2 = \dot{m}_4 = \dot{m}_7 = \dot{m}_{10} = \dot{m}_{11} = \dot{m}_1 = 15 \text{ kg/s}$). Thus

$$\dot{Q}_{process,max} = \dot{m}_1(h_4 - h_7) = 15(3410 - 640) = 41.55 \text{ MW.}$$

10.15d The utilization factor at the mode of maximum process heat (mode 1) is almost 100% since no heat is rejected in the condenser (all other heat losses are assumed to be negligible). All heat supplied in the boiler is used as process heat ($\dot{Q}_{in} = \dot{Q}_{process}$). There is no power output since there is no mass flow through the turbine. The power input to the pump is $\dot{W}_{pump} = \dot{m}_1(h_{10} - h_7) = 0.11 \text{ MW}$.

$$\epsilon_u = \frac{\dot{W}_{net} + \dot{Q}_{process}}{\dot{Q}_{in}} = \frac{-0.11 + 41.55}{41.55} = 0.997 \text{ or } 99.7\%.$$

That is, almost 100% of the energy put in the boiler is utilized for a useful purpose.

10.15e When no process heat is supplied (mode 2), all steam leaving the boiler passes through the turbine and expands to the condenser pressure, $P_8 = 5 \text{ kPa}$ (that is $\dot{m}_2 = \dot{m}_5 = \dot{m}_4 = \dot{m}_7 = \dot{m}_{10} = 0 \text{ kg/s}$ and $\dot{m}_3 = \dot{m}_1 = \dot{m}_6 = \dot{m}_8 = \dot{m}_9 = \dot{m}_{11} = 15 \text{ kg/s}$). Maximum power is produced in this mode, which is determined to be:

$$\dot{W}_{turbine} = \dot{m}_3(h_3 - h_6) = 15(3410 - 2075) = 20.03 \text{ MW}$$

$$\dot{W}_{pump} = \dot{m}_3(h_9 - h_8) = 15(145 - 138) = 0.11 \text{ MW}$$

$$\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{pump} = 19.92 \text{ MW.}$$

10.15f To calculate the utilization factor and the thermal efficiency \dot{Q}_{in} is required, note that for this mode $h_{11} = h_9$:

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_{11}) = 15(3410 - 145) = 48.98 \text{ MW.}$$

This results in the utilization factor to be:

$$\epsilon_u = \frac{\dot{W}_{net} + \dot{Q}_{process}}{\dot{Q}_{in}} = \frac{19.92 + 0}{48.98} = 0.407 \text{ or } 40.7\%.$$

That is, 40.7% of the input energy is utilized for a useful purpose. The thermal efficiency is:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{19.92}{48.98} = 0.407 \text{ or } 40.7\%.$$

The utilization factor is equivalent to the thermal efficiency in this case since no process heat is produced. The cogeneration power plant is used as a 'normal' power plant, producing power only. The utilization factor is much lower than the one found in the case the maximum process heat is produced. That is because the heat supplied cannot all be converted to work (due to the restrictions put by the second law of thermodynamics). Some heat has to be rejected to the environment at a low temperature (usually in the condenser), that heat cannot be utilized useful.

10.15g In this case the power plant is used as a cogeneration power plant (mode 3). The mass flow is splitted after the boiler and also in the turbine some steam is extracted. This results in 6 different mass flows in the power plant. They have to be determined before the process heat and the power output of the plant in this mode can be calculated (see also table: 10.15.1).

1. The main mass flow, the flow through the boiler: $\dot{m}_1 = \dot{m}_{11} = 15 \text{ kg/s}$.
2. 10% of the flow is extracted before the turbine $\rightarrow \dot{m}_2 = \dot{m}_4 = 0.1\dot{m}_1 = 0.1 \cdot 15 = 1.5 \text{ kg/s}$.

3. The mass balance at the junction of point 1, 2 and 3 gives: $\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 15 - 1.5 = 13.5 \text{ kg/s}$.
4. 70% of the flow is extracted in the turbine at $P_5 = 500 \text{ kPa} \rightarrow \dot{m}_5 = 0.7\dot{m}_1 = 0.1 \cdot 15 = 10.5 \text{ kg/s}$.
5. From the mass balance over the process heater follows: $\dot{m}_7 = \dot{m}_4 + \dot{m}_5 = 1.5 + 10.5 = 12 \text{ kg/s}$. Also $\dot{m}_{10} = \dot{m}_7 = 12 \text{ kg/s}$.
6. From the mass balance over the turbine follows: $\dot{m}_6 = \dot{m}_3 - \dot{m}_5 = 13.5 - 10.5 = 3 \text{ kg/s}$. Also $\dot{m}_8 = \dot{m}_9 = \dot{m}_6 = 3 \text{ kg/s}$.

The process heat follows from the energy balance over the process heater:

$$\dot{m}_4 h_4 + \dot{m}_5 h_5 = \dot{Q}_{process,heat} + \dot{m}_7 h_7 \rightarrow \dot{Q}_{process,heat} = \dot{m}_4 h_4 + \dot{m}_5 h_5 - \dot{m}_7 h_7$$

$$\dot{Q}_{process,heat} = 1.5 \cdot 3410 + 10.5 \cdot 2740 - 12 \cdot 640 = 26.2 \text{ MW}.$$

The net power output produced in this mode is:

$$\dot{W}_{turbine} = \dot{m}_3(h_3 - h_5) + \dot{m}_6(h_5 - h_6) = 13.5(3410 - 2740) + 3(2740 - 2075) = 11.04 \text{ MW}$$

$$\dot{W}_{pumps} = \dot{m}_8(h_9 - h_8) + \dot{m}_7(h_{10} - h_7) = 3(145 - 138) + 12(647 - 640) = 0.11 \text{ MW}$$

$$\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{pumps} = 11.04 - 0.11 = 10.93 \text{ MW}.$$

10.15h For the utilization factor and the thermal efficiency $\dot{Q}_{in} = h_1 - h_{11}$ is required, so h_{11} is needed. For this mode of operation it is not known yet and depends on the ratios of the mass flows. It is determined from the energy balance over the mixing chamber:

$$\dot{m}_9 h_9 + \dot{m}_{10} h_{10} = \dot{m}_{11} h_{11} \rightarrow h_{11} = \frac{\dot{m}_9 h_9 + \dot{m}_{10} h_{10}}{\dot{m}_{11}} = \frac{3 \cdot 145 + 12 \cdot 648}{15} = 547.4 \text{ kJ/kg}.$$

Now, the heat input can be calculated:

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_{11}) = 15(3410 - 547) = 42.99 \text{ MW}.$$

This results in the utilization factor to be:

$$\epsilon_u = \frac{\dot{W}_{net} + \dot{Q}_{process}}{\dot{Q}_{in}} = \frac{10.93 + 26.2}{42.99} = 0.864 \text{ or } 86.4\%.$$

That is, 86.4% of the energy is utilized for a useful purpose. The energy that is not utilized is rejected in the condenser: $\dot{Q}_{condenser} = \dot{m}_6(h_6 - h_8) = 3(2075 - 138) = 5.81 \text{ MW}$, which is 13.5% of the energy supplied to the boiler. The difference of 0.1% is probably due to rounding errors. The thermal efficiency is:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{10.93}{42.99} = 0.254 \text{ or } 24.4\%.$$

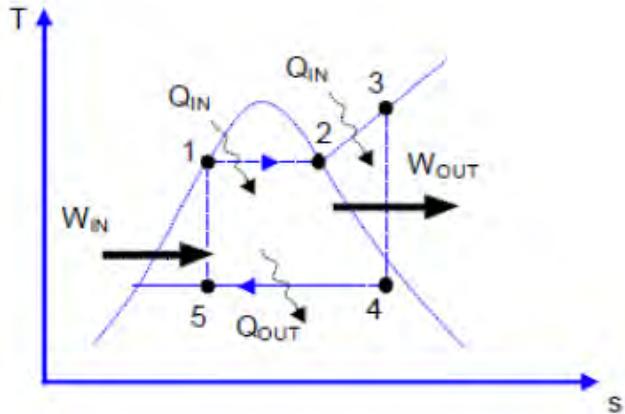
The utilization factor is higher than the thermal efficiency. This is not surprising as it is the purpose of this mode to produce process heat also to utilize more of the input energy for a useful purpose. That it is so much higher is because a large part of the mass flow is sent to the process heater.

If instead of 70% only 30% of the flow is extracted in the turbine the values change and come closer together: $\epsilon_u = 0.62$ or 62% and $\eta_{th} = 0.35$ or 35% ($\dot{m}_5 = 4.5 \text{ kg/s}$, $\dot{m}_6 = 9 \text{ kg/s}$, $\dot{m}_7 = 6 \text{ kg/s}$, $h_{11} = 346 \text{ kJ/kg}$, $\dot{Q}_{process,heat} = 16.60 \text{ MW}$, $\dot{W}_{turbine} = 15.03 \text{ MW}$, $\dot{W}_{pumps} = 0.11 \text{ MW}$, $\dot{W}_{net} = 14.92 \text{ MW}$ and $\dot{Q}_{in} = 45.96 \text{ MW}$).

Compared to question d the utilization factor is lower because also work is produced and some heat is rejected to the environment in the condenser. Compared to question f is it higher as in f no process heat was produced at all and the system was used in the mode of producing maximum power. The thermal efficiency is lower than in case that the maximum power output is produced as also process heat is produced which lowers the mass flow through the turbine and thus the power output. However in total more of the input energy is utilized useful.

10.16 An improved Rankine cycle

10.16a First of all, we draw a Ts -diagram, as shown on the right. It is seen that point 1 is a saturated liquid at the high pressure, and that a mixture comes out of the condenser. Both the turbine and the compressor are isentropic, and hence the processes are characterised by vertical lines in the diagram.



Now, we make a table with all characteristic points, see table 10.16.1. In order to determine the efficiency of the cycle, we need to determine the h -values. $h_1 = h_L@6000\text{kPa} = 1213.3 \text{ kJ/kg}$. Furthermore, $s_1 = s_L@6000\text{kPa} = 3.0266 \text{ kJ/(kgK)}$. $h_2 = h_v@6000\text{kPa} = 2783.9 \text{ kJ/kg}$. At point 3, we will need both the enthalpy and the entropy (for point 4). In the tables for superheated water at $P = 6000 \text{ kPa}$ and $T = 400^\circ\text{C}$ it is found that $h_3 = 3177.0 \text{ kJ/kg}$ and $s_3 = 6.5404 \text{ kJ/(kgK)}$. The quality at point 4 can now be determined from the entropy:

$$x_4 = \frac{s_4 - s_{L@8\text{kPa}}}{s_{v@8\text{kPa}} - s_{L@8\text{kPa}}} = \frac{6.5404 - 0.5925}{8.2267 - 0.5925} = 0.779$$

Now, h_4 can be determined:

$$h_4 = h_{L@8\text{kPa}} + x_4(h_{v@8\text{kPa}} - h_{L@8\text{kPa}}) = 173.85 + 0.779(2576.1 - 173.85) = 2045.2 \text{ kJ/kg}$$

Similarly, h_5 is determined by first determining the quality using the entropy, and using that to calculate h_5 :

$$x_5 = \frac{s_5 - s_{L@8\text{kPa}}}{s_{v@8\text{kPa}} - s_{L@8\text{kPa}}} = \frac{3.0266 - 0.5925}{8.2267 - 0.5925} = 0.319$$

$$h_5 = h_{L@8\text{kPa}} + x_5(h_{v@8\text{kPa}} - h_{L@8\text{kPa}}) = 173.85 + 0.319(2576.1 - 173.85) = 940.2 \text{ kJ/kg}$$

The cycle efficiency is now calculated as:

$$\eta_{Ra} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{(h_3 - h_4) - (h_1 - h_5)}{h_3 - h_1} = \frac{(3177.0 - 2045.2) - (1213.3 - 940.2)}{3177.0 - 1213.3} = 0.437$$

Table 10.16.1: Parameters of exercise 10.16a. The 2 known variables per point are underlined.

	P (kPa)	T ($^\circ\text{C}$)	h (kJ/kg)	s (kJ/(kgK))	x (-)	other
1	<u>6000</u>	275.59	1213.3	3.0266	<u>0</u>	
2	<u>6000</u>	275.59	2783.9	5.8886	<u>1</u>	
3	<u>6000</u>	<u>400</u>	3177.0	6.5404		
4	<u>8</u>	41.52	2045.2	<u>$s_4 = s_3$</u>	0.779	
5	<u>8</u>	41.52	940.2	<u>$s_5 = s_1$</u>	0.319	

10.16b For the usual Rankine cycle under the same conditions, the points 1 and 5 change. h_5 is now on the saturated liquid line and equal to $h_{L@8\text{kPa}} = 173.85 \text{ kJ/kg}$, and $s_5 = s_{L@8\text{kPa}} = 0.5925 \text{ kJ/(kgK)}$. To determine h_1 we use the formula for an isentropic pump: $dh = vdP \rightarrow h_1 - h_5 = v(P_1 - P_5)$. This gives $h_1 = h_5 + v(P_1 - P_5) = 173.85 + 0.001(6000 - 8) = 179.84 \text{ kJ/kg}$. The table with the parameters for the usual Rankine cycle is seen in table 10.16.2. Now, the cycle efficiency is calculated as:

$$\eta_{Rb} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{(h_3 - h_4) - (h_1 - h_5)}{h_3 - h_1} = \frac{(3177.0 - 2045.2) - (179.84 - 173.85)}{3177.0 - 179.84} = 0.376$$

Table 10.16.2: Parameters of exercise 10.16b. The 2 known variables per point are underlined.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	x (-)	other
1	<u>6000</u>	275.59	179.84	<u>$s_1 = s_5$</u>		
2	<u>6000</u>	275.59	2783.9	5.8886	<u>1</u>	
3	<u>6000</u>	<u>400</u>	3177.0	6.5404		
4	<u>8</u>	41.52	2045.2	<u>$s_4 = s_3$</u>	0.779	
5	<u>8</u>	41.52	173.85	0.5925	<u>0</u>	

10.16c The cycle operates between $T_{low} = 41.52^\circ\text{C}$ (saturation temperature at 8 kPa) and $T_{high} = 400^\circ\text{C}$, such that:

$$\eta_{carnot} = 1 - \frac{T_{low}}{T_{high}} = 1 - \frac{41.52 + 273}{400 + 273} = 0.533$$

The ratio of the proposed Rankine cycle to the appropriate Carnot efficiency is:

$$\frac{\eta_{Ra}}{\eta_{carnot}} = \frac{0.437}{0.533} = 0.820$$

The ratio of the usual Rankine cycle to the appropriate Carnot efficiency is:

$$\frac{\eta_{Rb}}{\eta_{carnot}} = \frac{0.376}{0.533} = 0.707$$

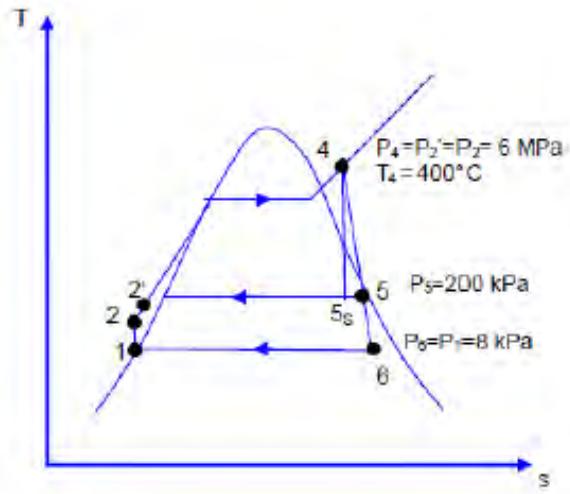
10.16d Without superheating, the quality at state 3 will be 1 (i.e. point 3 and 2 will be the same), such that $s_3 = 5.8886 \text{ kJ/(kgK)}$. Hence, the new h_4 can be calculated (similarly as was done in a) to be $h_4 = 1840.37 \text{ kJ/kg}$. The efficiency of that cycle will then be:

$$\eta_{Rd} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{(h_3 - h_4) - (h_1 - h_5)}{h_3 - h_1} = \frac{(2783.9 - 1840.37) - (1213.3 - 940.2)}{2783.9 - 1213.3} = 0.427$$

Discussion of the results: If the compression process for the steam cycle was entirely within the mixture region, leaving the compressor as saturated liquid, and the steam left the boiler as saturated vapor, such that the boiler heat input occurred at constant temperature, the $T-s$ -diagram would be a rectangle, as in the case of a Carnot cycle, and the Rankine efficiency would approach the Carnot efficiency. As a practical matter, turbines and compressors which operate entirely within the wet region are extremely difficult to build and have low isentropic efficiencies. For this reason, it is more practical and efficient to design Rankine cycles to pressurize the fluid as a liquid (with a pump) and expand it as a gas (with a turbine).

10.17 A Rankine cycle with and without regeneration

First of all, we draw a $T-s$ diagram, as shown on the right. Now, we make a table with all characteristic points in it, as shown in table 10.17.1. In order to determine the efficiency of the cycle without regeneration, we need to determine the enthalpy values of point 1, 2, 4 and 6, as the other points are not involved in the cycle without regeneration. h_1 can be found directly in the tables: $h_1 = h_{L@8\text{kPa}} = 173.85 \text{ kJ/kg}$. The work input in the pump is calculated using $w_{in,pump} = \Delta h = v\Delta P$, where the specific volume is assumed constant at $v_1 = v_{L@8\text{kPa}} = 0.001008 \text{ m}^3/\text{kg}$. Hence, $w_{in,pump} = v_1(P_2 - P_1) = 0.001008 \cdot (6000 - 8) = 6.04 \text{ kJ/kg}$, such that $h_2 = h_1 + w_{in,pump} = 173.85 + 6.04 = 179.89 \text{ kJ/kg}$.



The properties at state 4 are determined from the tables for superheated water: $h_4 = 3177.0 \text{ kJ/kg}$ and $s_4 = 6.5404 \text{ kJ/(kgK)}$. The quality at state 6s can be determined using the entropy:

$$x_{6s} = \frac{s_4 - s_{L@8\text{kPa}}}{s_{v@8\text{kPa}} - s_{L@8\text{kPa}}} = \frac{6.5404 - 0.5925}{8.2267 - 0.5925} = 0.779$$

Hence,

$$h_{6s} = h_{L@8\text{kPa}} + x_{6s}(h_{v@8\text{kPa}} - h_{L@8\text{kPa}}) = 173.85 + 0.779(2576.1 - 173.85) = 2045.2 \text{ kJ/kg}$$

Using the isentropic efficiency, h_6 is determined:

$$h_6 = h_4 - \eta_s(h_4 - h_{6s}) = 3177.0 - 0.88(3177.0 - 2045.2) = 2181.0 \text{ kJ/kg}$$

It is now possible to determine the efficiency of the cycle without regeneration:

$$\eta_{th,noregen} = \frac{w_{net}}{q_{in}} = \frac{(h_4 - h_6) - (h_2 - h_1)}{h_4 - h_2} = \frac{(3177.0 - 2181.0) - (179.89 - 173.85)}{3177.0 - 179.89} = 0.330$$

To determine the efficiency of the cycle with regeneration, we also need to determine h'_2 and h_5 . The values of all other points remain unchanged. First, h_{5s} is determined similar as h_{6s} was determined:

$$x_{5s} = \frac{s_4 - s_{L@200\text{kPa}}}{s_{v@200\text{kPa}} - s_{L@200\text{kPa}}} = \frac{6.5404 - 1.5304}{7.1272 - 1.5304} = 0.895$$

$$h_{5s} = h_{L@200\text{kPa}} + x_{5s}(h_{v@200\text{kPa}} - h_{L@200\text{kPa}}) = 504.8 + 0.895(2706.5 - 504.8) = 2475.3 \text{ kJ/kg}$$

The isentropic efficiency of the turbine is used to calculate h_5 :

$$h_5 = h_4 - \eta_s(h_4 - h_{5s}) = 3177.0 - 0.88(3177.0 - 2475.3) = 2559.5 \text{ kJ/kg}$$

In order to determine h'_2 , we make use of the energy balance over the closed feedwater heater, where we assume that the condensing steam leaves as a saturated liquid at 200 kPa (the extraction pressure). The energy balance therefore becomes:

$$0.1\dot{m}_4(h_5 - h_{L@200\text{kPa}}) = \dot{m}_4(h'_2 - h_2)$$

which gives:

$$h'_2 = \frac{0.1\dot{m}_4(h_5 - h_{L@200\text{kPa}})}{\dot{m}_4} + h_2 = 0.1(2559.5 - 504.8) + 179.89 = 385.4 \text{ kJ/kg}$$

Table 10.17.1: Table with parameters of exercise 10.17. The 2 known variables per point are underlined.

	P (kPa)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/(kgK))	x (-)	\dot{m}	other
1	<u>8</u>		173.85		<u>0</u>	\dot{m}_4	$v_1 = 0.001008 \text{ m}^3/\text{kg}$
2	<u>6000</u>		179.89			\dot{m}_4	$w_{in,pump} = \Delta h = v\Delta P$
2'	<u>6000</u>		385.4			\dot{m}_4	energy balance CFWH
4	<u>6000</u>	<u>400</u>	3177.0	6.5404		\dot{m}_4	
5s	<u>200</u>		2457.3	<u>$s_{5s} = s_4$</u>	0.895	$0.1\dot{m}_4$	
5	<u>200</u>		2559.5			$0.1\dot{m}_4$	<u>$\eta_s = 0.88$</u>
6s	<u>8</u>		2045.2	<u>$s_{6s} = s_4$</u>	0.779	$0.9\dot{m}_4$	
6	<u>8</u>		2181.0			$0.9\dot{m}_4$	<u>$\eta_s = 0.88$</u>

Now, the efficiency of the cycle with regeneration can be calculated:

$$\begin{aligned}\eta_{th,regen} &= \frac{w_{net}}{q_{in}} = \frac{0.9(h_4 - h_6) + 0.1(h_4 - h_5) - (h_2 - h_1)}{h_4 - h'_2} \\ &= \frac{0.9(3177.0 - 2181.0) + 0.1(3177.0 - 2559.5) - (179.89 - 173.85)}{3177.0 - 385.4} = 0.341\end{aligned}$$

Hence, the efficiency is improved by 1.1%, which is a very significant improvement representing major savings in fuel costs for operation of the power plant. In large high-pressure steam plants, as many as 8-10 feedwater heaters may be used in series to effect even greater efficiency gains. The gains, however, are not directly proportional to the number of heaters used. Ultimately, there is a point of diminishing returns at which the added capital and maintenance costs of more heaters outweighs the incremental fuel savings.

10.18 City Heating

The amount of heat added to the power plant is:

$$\dot{Q}_{in} = \frac{\dot{W}_{net}}{\eta_{th}} = \frac{40}{0.25} = 160 \text{ MW}$$

Now, from the schematic diagram, we can derive the following:

$$\dot{Q}_{river} + (\text{Thermal demand}) = \dot{Q}_{in} - \dot{W}_{elect}$$

Thus, the amount of heat rejected to the river is given by:

$$\dot{Q}_{river} = \dot{Q}_{in} - \dot{W}_{elect} - (\text{Thermal demand}) = 160 - 40 - 50 = 70 \text{ MW.}$$

10.19 Rankine cycle with vapor-liquid separator

10.19a See figure 10.19.1.

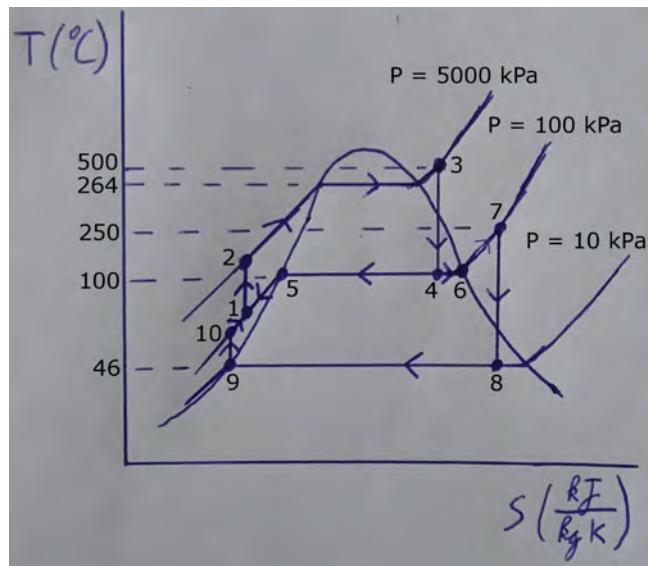


Figure 10.19.1: Ts-diagram of assignment 10.19. The lines for the processes in the pumps and the turbines are ideal (isenropic efficiency of 100%).

10.19b See table 10.19.1 and the Mollier diagram in figure 10.19.2.

Table 10.19.1: Table with parameters of exercise 10.19. The 2 known variables per point are underlined.

	P (kPa)	T (°C)	x	h (kJ/kg)	other	\dot{m}
1	<u>100</u>			204.38	$\dot{m}_5 h_5 + \dot{m}_{10} h_{10} = \dot{m}_1 h_1$	\dot{m}_1
2	<u>5000</u>			209.34	$s_2 = s_1$	\dot{m}_1
3	<u>5000</u>	<u>500</u>		3430		\dot{m}_1
4	<u>100</u>		0.9446	2550	$s_4 = s_3$	\dot{m}_1
5	<u>100</u>		0	417.51		$\dot{m}_5 = (1 - x_4) \dot{m}_1$
6	<u>100</u>		1	2675.1		$\dot{m}_{10} = x_4 \dot{m}_1$
7	<u>100</u>	<u>250</u>		2975		$\dot{m}_{10} = x_4 \dot{m}_1$
8	<u>10</u>			2550	$s_8 = s_7$	$\dot{m}_{10} = x_4 \dot{m}_1$
9	<u>10</u>		0	191.83		$\dot{m}_{10} = x_4 \dot{m}_1$
10	<u>100</u>			191.92	$s_{10} = s_9$	$\dot{m}_{10} = x_4 \dot{m}_1$

Below for every point is explained how it is derived.

- Point 3: In order to determine the enthalpies, we start at point 3. With the given pressure and temperature, the value h_3 can be found in the Mollier diagram: $h_3 = 3430$ kJ/kg.
- Point 4, 5 and 6: Going down vertically ($s_4 = s_3$) to $P_4 = 100$ kPa gives $h_4 = 2550$ kJ/kg. This is in the mixture region, and the saturated liquid (point 5) and saturated vapor (point 6) are separated. From the tables, we find that $h_5 = 417.51$ kJ/kg and $h_6 = 2675.1$ kJ/kg (this last value can also be found in the Mollier diagram).

- Point 7: h_7 is found in the Mollier diagram at $P_7 = 100 \text{ kPa}$ and $T_7 = 250^\circ\text{C}$: $h_7 = 2975 \text{ kJ/kg}$.
- Point 8: Going down vertically ($s_8 = s_7$) to $P_8 = 10 \text{ kPa}$ gives $h_8 = 2550 \text{ kJ/kg}$.
- Point 9: In the tables for saturated water, we find that $h_9 = h_{L@10kPa} = 191.83 \text{ kJ/kg}$.
- Point 10: In order to determine h_{10} , we use the relation $dh = Tds + vdP$. For an isentropic pump, this reduces to $dh = vdP$, which gives $h_{10} = h_9 + v(P_{10} - P_9)$ for constant v . We use $v = v_9 = v_{L@10kPa} = 0.001010 \text{ m}^3/\text{kg}$, such that $h_{10} = 191.92 \text{ kJ/kg}$.
- Point 1: For point 1, we have to use the energy balance over the mixing chamber:

$$\dot{m}_5 h_5 + \dot{m}_{10} h_{10} = \dot{m}_1 h_1 \rightarrow h_1 = \frac{\dot{m}_5 h_5 + \dot{m}_{10} h_{10}}{\dot{m}_1}$$

The mass flows can be expressed in each other using the quality (the vapor mass fraction) at point 4: $\dot{m}_{10} = x_4 \dot{m}_1$ and $\dot{m}_5 = (1 - x_4) \dot{m}_1$. Substituting these relations into the equation for h_1 yields an expression independent of the mass flows:

$$h_1 = \frac{\dot{m}_5 h_5 + \dot{m}_{10} h_{10}}{\dot{m}_1} = \frac{(1 - x_4) \dot{m}_1 h_5 + x_4 \dot{m}_1 h_{10}}{\dot{m}_1} = (1 - x_4) h_5 + x_4 h_{10}$$

The quality at point 4 can be determined as:

$$x_4 = \frac{h_4 - h_L}{h_V - h_L} = \frac{2550 - 417.51}{2675.1 - 417.51} = 0.9446$$

such that $h_1 = 204.38 \text{ kJ/kg}$.

- Point 2: For point 2, we again use the relation $dh = vdP$, which gives $h_2 = h_1 + v(P_2 - P_1)$ for constant v . The specific volume is, corresponding to the incompressible liquid approximation, determined at the correct temperature: the enthalpy is matched to the saturated liquid enthalpy in the temperature saturated water tables. This results, after interpolating, in $v_1 = 0.0010115 \text{ m}^3/\text{kg}$, such that $h_2 = 209.34 \text{ kJ/kg}$.

10.19c The thermal efficiency of the cycle is calculated as:

$$\begin{aligned} \eta_{th} &= \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{m}_1(h_3 - h_4) + \dot{m}_{10}(h_7 - h_8) - \dot{m}_{10}(h_{10} - h_9) - \dot{m}_1(h_2 - h_1)}{\dot{m}_1(h_3 - h_2) + \dot{m}_{10}(h_7 - h_6)} \\ &= \frac{\dot{m}_1(h_3 - h_4) + x_4 \dot{m}_1(h_7 - h_8) - x_4 \dot{m}_1(h_{10} - h_9) - \dot{m}_1(h_2 - h_1)}{\dot{m}_1(h_3 - h_2) + x_4 \dot{m}_1(h_7 - h_6)} \\ &= \frac{(h_3 - h_4) + x_4(h_7 - h_8) - x_4(h_{10} - h_9) - (h_2 - h_1)}{(h_3 - h_2) + x_4(h_7 - h_6)} \\ &= \frac{(3430 - 2550) + 0.9446(2975 - 2550) - 0.9446(191.92 - 191.83) - (209.34 - 204.38)}{(3430 - 209.34) + 0.9466(2975 - 2675.1)} \\ &= 0.3645 = 36.45\% \end{aligned}$$

Note: it is not necessary to know the absolute value of the mass flows to calculate the thermal efficiency. The ratio of the mass flows that follows from x_4 is sufficient.

10.19d The net power output is given by $\dot{W}_{net} = \dot{m}_1(h_3 - h_4) + x_4 \dot{m}_1(h_7 - h_8) - x_4 \dot{m}_1(h_{10} - h_9) - \dot{m}_1(h_2 - h_1)$. Hence, the required mass flow \dot{m}_1 is determined as:

$$\begin{aligned} \dot{m}_1 &= \frac{\dot{W}_{net}}{(h_3 - h_4) + x_4(h_7 - h_8) - x_4(h_{10} - h_9) - (h_2 - h_1)} \\ &= \frac{80000}{(3430 - 2550) + 0.9446(2975 - 2550) - 0.9446(191.92 - 191.83) - (209.34 - 204.38)} = 62.63 \text{ kg/s} \end{aligned}$$

From the earlier determined relations between \dot{m}_1 , \dot{m}_{10} , \dot{m}_5 and x_4 the other mass flows are determined: $\dot{m}_{10} = x_4 \dot{m}_1 = 0.9466 \cdot 62.63 = 59.29 \text{ kg/s}$ and $\dot{m}_5 = (1 - x_4) \dot{m}_1 = (1 - 0.9466) \cdot 62.63 = 3.34 \text{ kg/s}$.

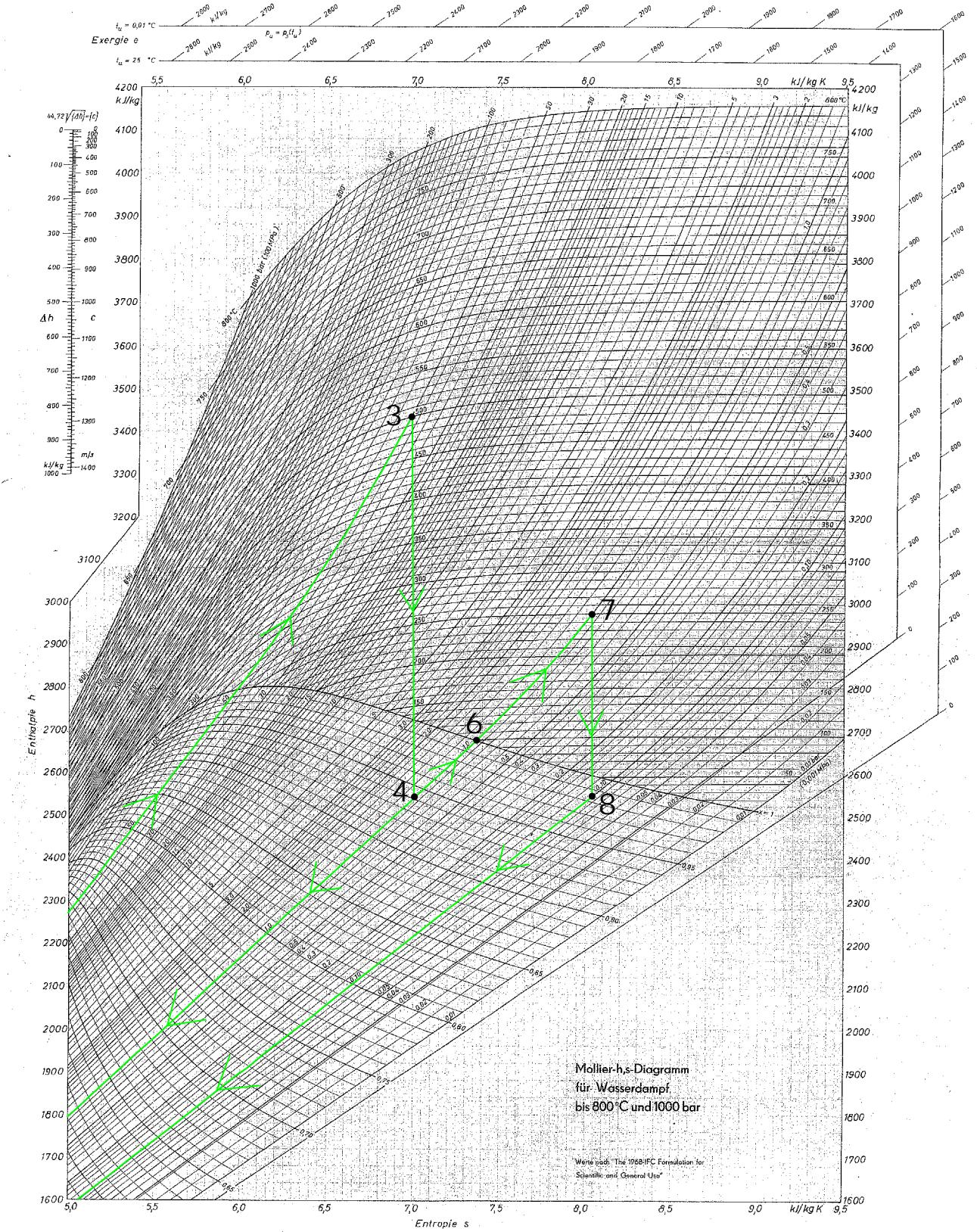
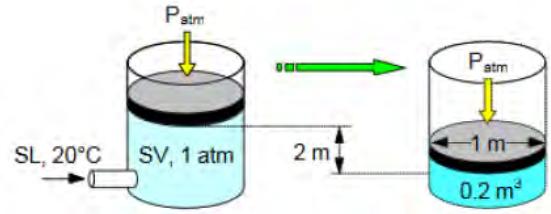


Figure 10.19.2: Mollier diagram of the Rankine cycle with steam separator of assignment 10.19.

10.20 Thomas Newcomens steam engines

10.20a The situation is shown in the figure on the right. First, we want to determine the initial volume of saturated steam in the cycle:

$$V_1 = 0.2 + L_p A_p = 0.2 + L_p \frac{\pi}{4} D_P^2 = 0.2 + \frac{p_i}{4} 1^2 = 1.77 \text{ m}^3$$



The initial mass of the steam is therefore:

$$m_1 = \frac{V_1}{v_1} = \frac{V_1}{v_{v@1bar}} = \frac{1.77}{1.6737} = 1.06 \text{ kg}$$

Also, the specific energy of the saturated steam can be found to be $u_1 = u_{v@1bar} = 2506.1 \text{ kJ/kg}$. At the end of the stroke, the volume will be 0.2 m^3 , and the system will be saturated at 1 bar. The mass of the system will be $m_2 = m_1 + m_{in}$, where m_{in} is the mass sprayed into the cylinder. The specific enthalpy of the sprayed cold water is $h_{in} = h_{L,20^\circ C} = 83.835 \text{ kJ/kg}$. The quality at the end of the stroke is given by:

$$x_2 = \frac{v_2 - v_{L@P=1bar}}{v_{v@1bar} - v_{L@1bar}} = \frac{\frac{0.2}{m_2} - 0.001043}{1.6727} = \frac{0.11957}{m_2} - 0.00062$$

The first law for the control volume can be written as:

$$W_{in} + m_{in}h_{in} = U_2 - U_1 = m_2u_2 - m_1u_1$$

where the work done by the atmosphere on the control volume is given by:

$$W_{in} = - \int_{V_1}^{V_2} P dV = P_{atm}(V_2 - V_1)$$

and the internal energy at state 2 is given by:

$$\begin{aligned} U_2 &= m_2(u_L + x_2(u_v - u_L)) = (m_1 + m_{in}) \left(418.96 + \left(\frac{0.11957}{m_1 + m_{in}} - 0.00062 \right) \cdot 2087.1 \right) \\ &= 417.659(m_1 + m_{in}) + 249.549 \end{aligned}$$

Substituting the expression for the internal energy into the First Law equation gives:

$$\begin{aligned} m_{in}h_{in} &= P_{atm}(V_2 - V_1) + (417.659(m_1 + m_{in}) + 249.549) - U_1 \\ \Rightarrow m_{in} &= \frac{m_1u_1 - P_{atm}(V_2 - V_1) - 249.549 - 417.659m_1}{417.659 - h_{in}} \\ &= \frac{(1.06 \cdot 2506.1) - 101.325(0.2 - 1.77) - 249.549 - (417.659 \cdot 1.06)}{417.659 - 83.835} = 6.36 \text{ kg} \end{aligned}$$

10.20b The piston is connected to a linkage such that the work output of the engine per stroke is given by:

$$w_{out,engine} = W_{in,atm} = P(V_1 - V_2) = 101.325 \cdot (1.77 - 0.2) = 159.1 \text{ kJ}$$

10.20c When operating at 1 cycle per minute, the power output in kW is:

$$\dot{W}_{out,engine} = 159.1 \cdot \frac{1}{60} = 2.65 \text{ kW}$$

11 Refrigeration and Heat Pumps

11.1 The vapor compression cooling cycle

11.1a The cycle consists of compression (increase of pressure in a vapor compressor), heat rejection in a condenser at high pressure, throttling (decrease of pressure in a throttle valve) and heat absorption in an evaporator at low pressure.

11.1b

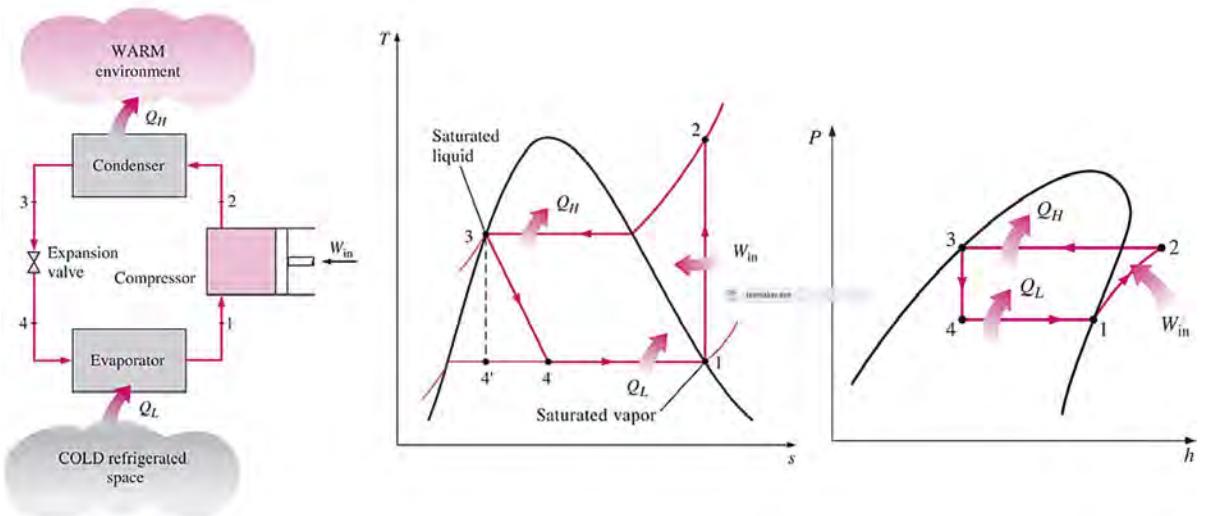


Figure 11.1.1: Schematic overview of the vapor compression refrigeration cycle and the Ts and Ph -diagram and ideal cycle.

11.1c The goal of a refrigerator is to cool a space at low temperature using work. Hence, a high input of heat (at low temperature coming from the cold space) in the evaporator is desired, and a low net work input. The coefficient of performance for a refrigerator is defined as:

$$COP_R = \frac{q_{in,cold}}{w_{net,in}} = \frac{h_1 - h_4}{h_2 - h_1}$$

The goal of a heat pump is to warm a space at high temperature using work. Hence, a high heat rejection at high temperature in the condenser is desired, and a low net work input. The coefficient of performance for a heat pump is defined as:

$$COP_{HP} = \frac{q_{out,hot}}{w_{net,in}} = \frac{h_2 - h_3}{h_2 - h_1}$$

11.1d From conservation of energy, it follows that the following relation for $\dot{Q}_{in,cold}$, $\dot{Q}_{out,hot}$ and $\dot{W}_{in,comp}$ must hold:

$$\dot{Q}_{in,cold} + \dot{W}_{in,comp} = \dot{Q}_{out,hot}$$

11.1e In an ideal vapor compression cycle, the compression is done isentropically. In an actual vapor compression cycle, this is not the case. In addition, an actual vapor cycle may lose pressure between the different components, or the refrigerant may not enter the compressor as saturated vapor, but as a superheated vapor.

11.2 An ideal refrigerator

11.2a See figure 11.2.1.

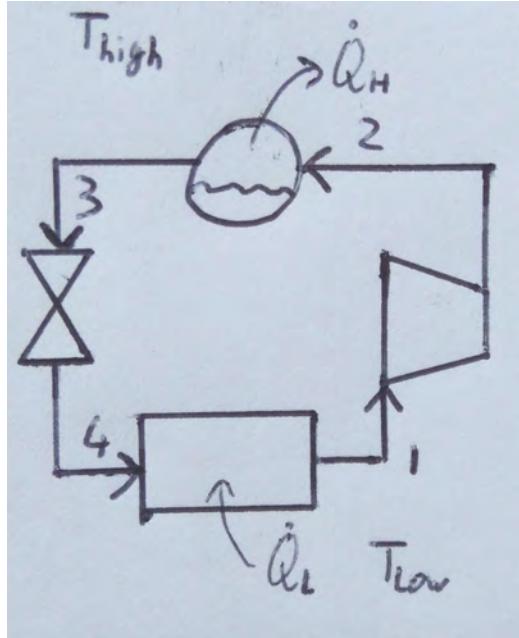


Figure 11.2.1: Schematic overview of question 11.2

11.2b See figure 11.2.2.

11.2c The two known variables at every characteristic position are given in table 11.2.1.

Table 11.2.1: Table with parameters of question 11.2

	P (MPa)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/(kgK))	phase
1	0.12	-22.36	233.86	0.9354	Sat. vap.
2	0.7		270	$s_2 = s_1$	superh. vap.
3	0.7	26.72	86.19		sat. liq.
4	0.12	-22.36	$h_4 = h_3$		mix

11.2d In order to determine the rate of heat removal, we need to determine the enthalpies h_1 and h_4 . From table A12, we can directly read h_1 : $h_1 = h_g@P=0.12\text{MPa} = 233.86 \text{ kJ/kg}$. Also h_3 can be read directly from the tables: $h_3 = h_f@P=0.7\text{MPa} = 86.19 \text{ kJ/kg}$. Since we have a throttling valve between 3 and 4, we know that $h_4 = h_3 = 86.19 \text{ kJ/kg}$. So, the rate of heat removal is: $\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.05(233.86 - 86.19) = 7.4 \text{ kW}$.

11.2e The power input of the compressor is given by $\dot{W}_{in} = \dot{m}(h_2 - h_1)$. So, we need to determine h_2 . Since the compressor is ideal, we know that $s_2 = s_1 = 0.9354 \text{ kJ/(kgK)}$ (can be found in the tables). In table A13, for superheated refrigerant, we then look at $P_2 = 0.7 \text{ MPa}$ at $s_2 = 0.9354 \text{ kJ/(kgK)}$. Interpolating then gives that $h_2 = 270 \text{ kJ/kg}$. So, $\dot{W}_{in} = \dot{m}(h_2 - h_1) = 0.05(270 - 233.86) = 1.8 \text{ kW}$.

11.2f The rate of heat rejection is given as $\dot{Q}_H = \dot{m}(h_2 - h_3) = 0.05(270 - 86.19) = 9.2 \text{ kW}$. Note that also $\dot{Q}_H = \dot{Q}_L + \dot{W}_{in}$ (conservation of energy).

11.2g The coefficient of performance is $\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.4}{1.8} = 4.1$.

11.3 A non-ideal refrigerator

11.3a See figure 11.3.1.

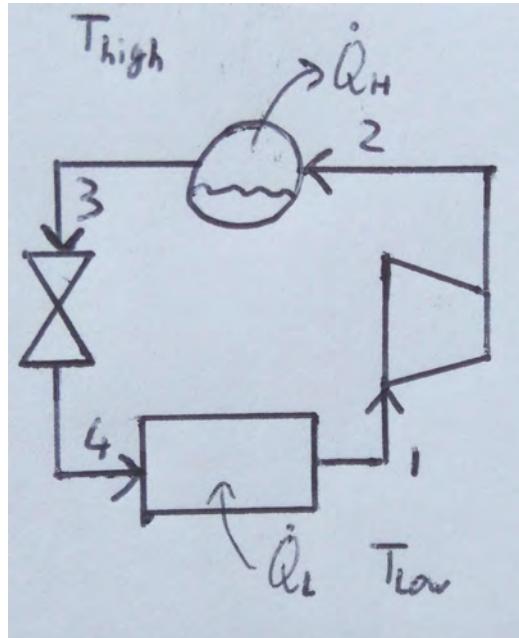


Figure 11.3.1: Schematic overview of question 11.3

11.3b See figure 11.3.2.

11.3c The two known parameters at each characteristic point are shown in table 11.3.1.

Table 11.3.1: Table with parameters of question 11.3

	P (kPa)	T ($^{\circ}\text{C}$)	h (kJ/kg)	s (kJ/(kgK))	phase
1	140	-10	243.40		Superh. vap.
2s	700		277	$s_{2s} = s_1$	Superh. vap.
2	700	50	286.08		Superh. vap.
3	650	24	82.90		Compr. liq
4	150		$h_4 = h_3$		mix

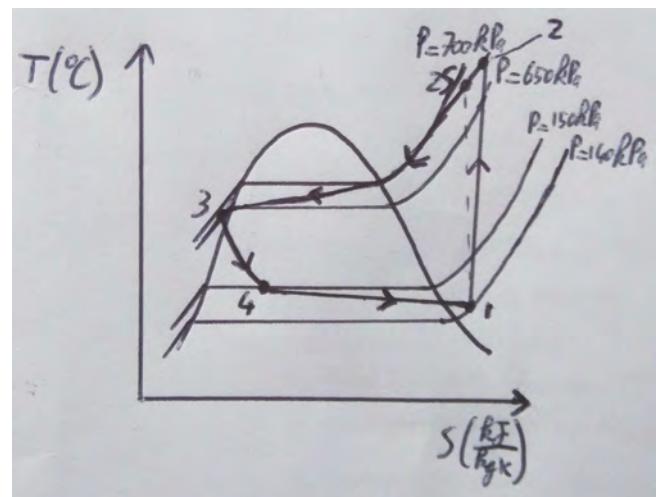


Figure 11.3.2: T-s diagram of question 11.3

11.3d The heat removal is calculated as $\dot{Q}_L = \dot{m}(h_1 - h_4)$, so we need to determine the enthalpies h_1 and h_4 . h_1 can be read directly from table A13 (superheated refrigerant) by looking at the correct pressure and temperature: $h_1 = 243.40 \text{ kJ/kg}$. Because we have a throttling valve, $h_4 = h_3$, which can be read from the table as well. Because $T_{sat@650kPa} = 24.15^\circ\text{C} \approx 24^\circ\text{C}$, we look up h_3 as saturated liquid at $P = 650 \text{ kPa}$: $h_3 = 82.90 \text{ kJ/kg}$, so $h_4 = 82.90 \text{ kJ/kg}$. Hence, $\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.12(243.40 - 82.90) = 19.26 \text{ kW}$.

11.3e The power input is calculated as $\dot{W}_{in} = \dot{m}(h_2 - h_1)$, so we need to determine h_2 . We have the pressure and the temperature, so from table A13 (superheated refrigerant) we find that $h_2 = 286.08 \text{ kJ/kg}$. Therefore, $\dot{W}_{in} = \dot{m}(h_2 - h_1) = 0.12(286.08 - 243.40) = 5.12 \text{ kW}$.

11.3f The isentropic efficiency of the compressor is given by $\eta_s = \frac{h_{2s} - h_1}{h_2 - h_1}$. In table A13, we find that $s_1 = 0.9606 \text{ kJ/(kgK)}$, such that we can find h_{2s} : $h_{2s} = h_{s1@700\text{kPa}} = 277 \text{ kJ/kg}$. Substituting these values gives $\eta_s = \frac{277 - 243}{286 - 243} = 0.79$.

11.3g The coefficient of performance is $\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{19.26}{5.12} = 3.76$.

11.4 Refrigerator on ammonia

11.4a See figure 11.4.1.

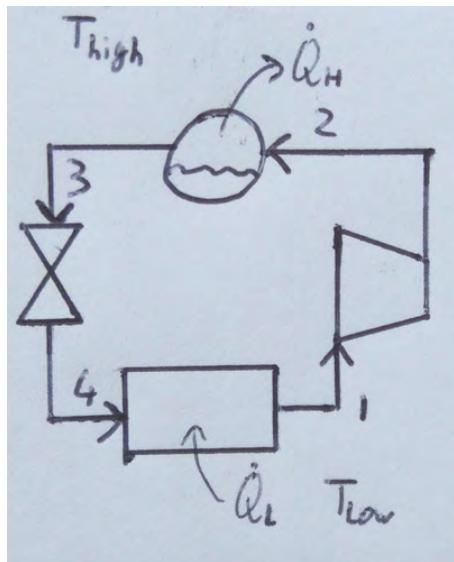


Figure 11.4.1: Schematic overview of question 11.4

11.4b See figure 11.4.2

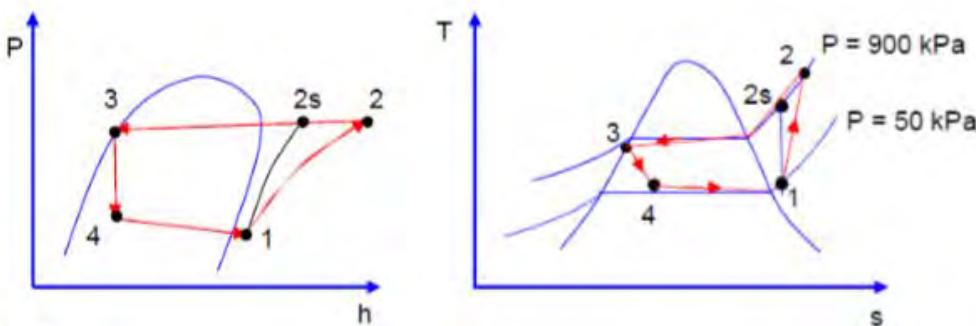


Figure 11.4.2: P-h and T-s diagram of question 11.4

11.4c The two known parameters at each characteristic point are shown in table 11.4.1.

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	phase	other
1	50	-40				
2	900				superh. vap.	$\eta_{compressor} = 70\%$
3	750					$x_3 = 0$
4	60			$h_4 = h_3$		

Table 11.4.1: Table with parameters of question 11.4

The rate of heat rejection is given as $\dot{Q}_H = \dot{m}(h_2 - h_3)$, so we need to determine the enthalpies h_2 , h_3 and the mass flow.

State 1 is superheated, so h_1 can be read directly from Table 17s for ammonia by looking at the correct pressure and temperature. $h_1 = 1411.9 \text{ kJ/kg}$ and $s_1 = 6.4332 \text{ kJ/kgK}$. $h_{2s} = h_{@900 \text{ kPa} @ s = 6.4332} = 1860 \text{ kJ/kg}$ by interpolating.

State 3 is saturated at a pressure of 750 kPa. The enthalpy can be found in Table 17s as well. So $h_3 = h_4 = 274.6 \text{ kJ/kg}$. h_2 can be calculated with the use of the compressor efficiency.

$$\eta_{comp} = \frac{h_{2,s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2,s} - h_1}{\eta_{comp}} = 1411.9 + \frac{1860 - 1411.9}{0.70} = 2052.0 \text{ kJ/kg.}$$

To calculate the mass flow, we have to use the First law for a control volume:

$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$. This leads to: $w_{in,comp} = h_2 - h_1 = 2052.0 - 1411.9 = 640.1 \text{ kJ/kg}$

$$\dot{m} = \frac{\dot{W}_{in,comp}}{w_{in,comp}} = \frac{2(hp) * 0.7457(\text{kW}/hp)}{640.1(\text{kJ/kg})} = 2.33 * 10^{-3} \text{ kg/hp.}$$

The rejected heat transfer is:

$$\dot{Q}_{out,cond} = \dot{m}(h_2 - h_3) = 2.33 * 10^{-3}(2052.0 - 274.6) = 4.14 \text{ kW}$$

11.4d The coefficient of performance is given by

$$COP = \frac{\text{Desired output}}{\text{Required input}} = \frac{\text{Cooling effect}}{\text{Work input}} = \frac{q_{in,evap}}{w_{in,comp}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1411.9 - 274.6}{2052.0 - 1411.9} = 1.78$$

11.4e The refrigeration capacity is as following.

$$\dot{Q}_{in,evap} = \dot{m} * q_{in,evap} = 2.33 * 10^{-3}(1411.9 - 274.6) = 2.65 \text{ kW}$$

11.5 Carnot engine driving an ideal refrigerator

11.5a A schematic overview of the setup can be found in figure 11.5.1.

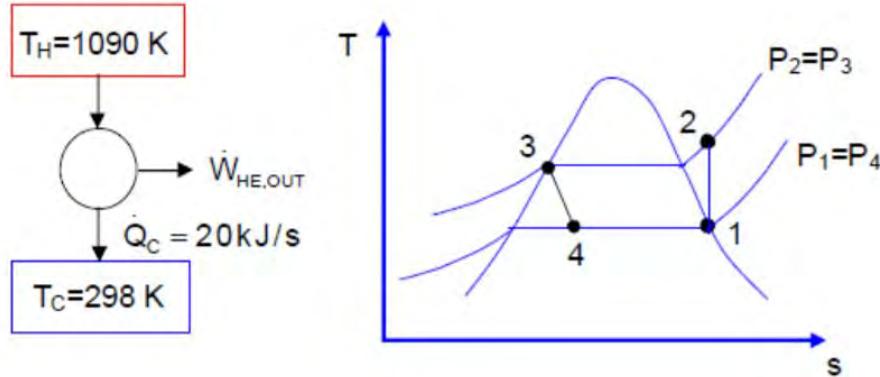


Figure 11.5.1: Schematic overview and T-s diagram of question 11.5

11.5b A Carnot engine consists of the following processes:

State 1 \Rightarrow 2: Reversible isothermal expansion

State 2 \Rightarrow 3: Reversible adiabatic expansion

State 3 \Rightarrow 4: Reversible isothermal compression

State 4 \Rightarrow 1: Reversible adiabatic compression

Figure 11.5.1 shows how this looks in a T-s diagram.

11.5c The two known parameters at each characteristic point are shown in table 11.5.1.

	P (MPa)	T ($^{\circ}$ C)	h (kJ/kg)	s (kJ/(kgK))	phase	other
1	0.17		242			sat. vapor
2	1.1		272	$s_2 = s_1$		
3	$P_3 = P_2$		113			sat. liquid
4	$P_4 = P_1$		$h_4 = h_3$			

Table 11.5.1: Table with parameters of question 11.5

The coefficient of performance (COP) can be calculated using:

$$COP = \frac{q_{in,evap}}{w_{in,comp}} = \frac{h_1 - h_4}{h_2 - h_1}$$

h_1 and h_3 can be read directly from Table A-12. They are 242 and 113 kJ/kg. s_1 can be read from Table A-12 as well. $s_1 = 0.914 \text{ kJ/kgK}$ $s_2 = s_1$ so h_2 can be found in the Table A-13 using the pressure and entropy. $h_2 = 272$.

$$COP = \frac{q_{in,evap}}{w_{in,comp}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{242 - 113}{272 - 242} = 4.3$$

11.5d The refrigerator's capacity is calculated using $\dot{Q}_{in,evap} = \dot{W}_{in,comp} * COP$. We have to determine the power output of the Carnot engine.

$$\eta_{he,carn} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_c}{T_h} = 1 - \frac{298}{1090} = 0.727$$

$$\eta_{he,carn} = 1 - \frac{Q_L}{Q_H} \Rightarrow \dot{Q}_H = \frac{\dot{Q}_C}{1 - \eta_{he}}$$

$$\dot{W}_{he,out} = \dot{Q}_H - \dot{Q}_C = \dot{Q}_C \left(\frac{\eta_{he,carn}}{1 - \eta_{he,carn}} \right) = 20 \left(\frac{0.727}{1 - 0.727} \right) = 53.26 \text{ kW}$$

$$\dot{Q}_{in,evap} = \dot{W}_{in,comp} * COP = 53.26 * 4.3 = 229 \text{ kW}$$

This problem illustrates how cooling capacity of a refrigeration system can be related back to the fundamental energy required to obtain that cooling. By assuming a best-case scenario for the initial energy converter (the Carnot engine), the maximum amount of primary fuel consumption, \dot{Q}_H , could be estimated.

11.6 Refrigerator on refrigerant

11.6a See figure 11.6.1.

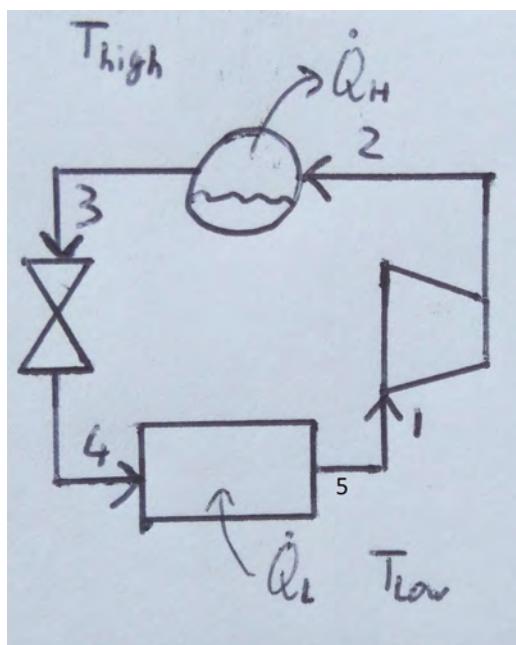


Figure 11.6.1: Schematic overview of question 11.6

11.6b See figure 11.6.2.

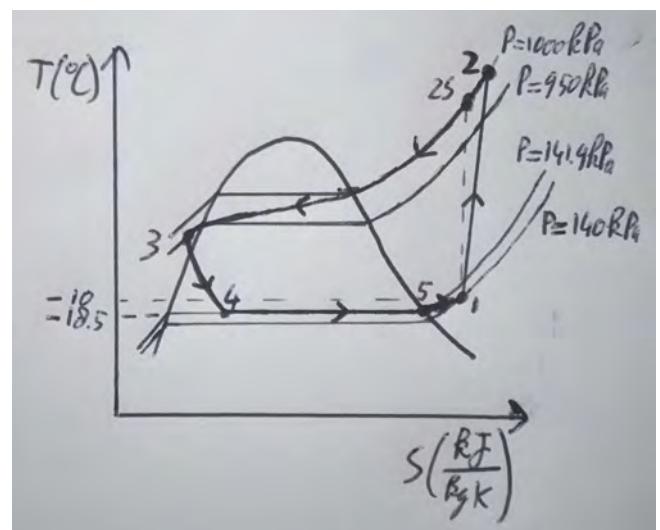


Figure 11.6.2: T-s diagram of question 11.6

11.6c The two known parameters at each characteristic point are shown in table 11.6.1.

Table 11.6.1: Table with parameters of question 11.6

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	phase	other
1	<u>140</u>	<u>-10</u>	243.40	0.9606	Superh. vap.	
2s	<u>1000</u>		286	<u>$s_{2s} = s_1$</u>	Superh. vap.	
2	<u>1000</u>		298		Superh. vap.	<u>$\eta_s = 0.78$</u>
3	<u>950</u>	<u>30</u>	91.49		Compr. liq	
4	<u>$P_4 = P_5$</u>	-18.5	<u>$h_4 = h_3$</u>		mix	
5	141.9	<u>-18.5</u>	236.2		<u>sat. vap.</u>	

11.6d The rate of heat removal from the refrigerated space is calculated as $\dot{Q}_L = \dot{m}(h_5 - h_4)$, so the enthalpies need to be determined, as well as the mass flow.

From table A13, we can find that $h_1 = 243.40$ kJ/kg, and $s_1 = 0.9606$ kJ/(kgK). Using $s_{2s} = s_1$ and $P_2 = 1000$ kPa we also find $h_{2s} = 286$ kJ/kg in this table. From the isentropic efficiency, we then calculate h_2 : $\eta_s = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_s} = 243 + \frac{286-243}{0.78} = 298$ kJ/kg. h_3 can be found in table A11 at $T = 30$ °C: $h_3 = 91.49$ kJ/kg. $h_4 = h_3$, because for a throttling valve $\Delta h = 0$, and lastly we find $h_5 = h_g@-18.5^\circ\text{C} = 236.2$ kJ/kg.

We can calculate the mass flow using the given volume flow and using the density. At point 1, $v_1 = 0.1455 \text{ m}^3/\text{kg}$ (from table A13), so $\rho_1 = \frac{1}{v_1} = 6.87 \text{ kg/m}^3$. The volume flow is $\dot{V}_1 = 0.3 \text{ m}^3/\text{min} = 0.005 \text{ kg/s}$. So, the mass flow is $\dot{m}_1 = \dot{V}\rho_1 = 0.005 \cdot 6.87 = 0.034 \text{ kg/s}$.

So, the rate of heat removal is $\dot{Q}_L = \dot{m}(h_5 - h_4) = 0.034(236.2 - 91) = 4.93 \text{ kW}$.

11.6e The power input is $\dot{W}_{comp} = \dot{m}(h_2 - h_1) = 0.034(298 - 243) = 1.87 \text{ kW}$.

11.6f The pressure at point 5, which is equal to the one at point 4, can be found in the tables for saturated vapor at $T_5 = -18.5$ °C: $P_5 = 0.1419$ MPa. So, $\Delta P = P_5 - P_1 = 141.9 - 140 = 1.9$ kPa.

11.6g The rate of heat gain is calculated as $\Delta\dot{Q} = \dot{m}(h_1 - h_5) = 0.034(243.4 - 236.2) = 0.24 \text{ kW}$.

11.7 Heat pump

11.7a See figure 11.7.1.

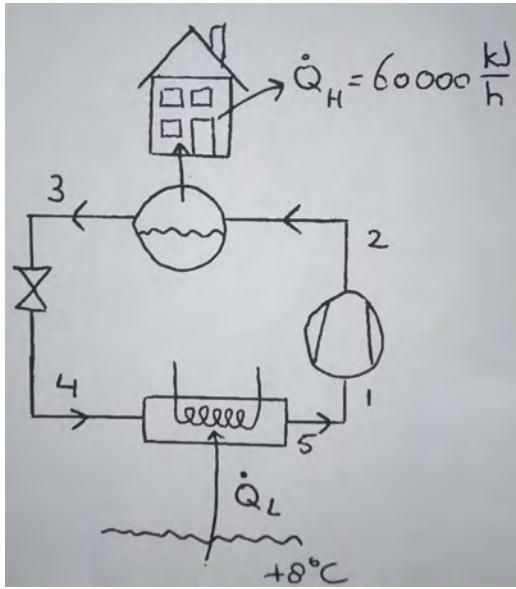


Figure 11.7.1: Schematic overview of question 11.7

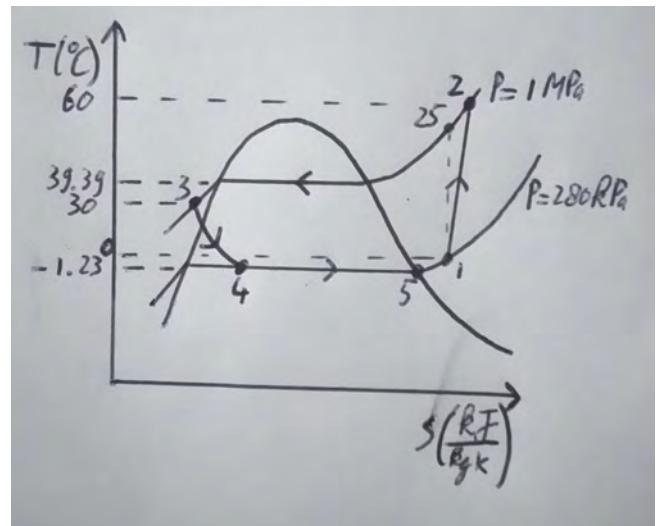


Figure 11.7.2: T-s diagram of question 11.7

11.7b See figure 11.7.2.

11.7c The two known parameters at each characteristic point are shown in table 11.7.1.

Table 11.7.1: Table with parameters of question 11.7

	P (kPa)	T (°C)	h (kJ/kg)	s (kJ/(kgK))	phase
1	280	0	247.64		Superh.
2	1000	60	291.36		Superh.
3	1000	30	91.49		Compr. liq
4	$P_4 = P_5$	-1.23	$h_4 = h_3$		mix

11.7d Before we can determine the power input, we need to determine the enthalpies. h_1 can be found in table A13 (it is superheated, because the temperature is above the saturation temperature at this pressure): $h_1 = 246.64 \text{ kJ/kg}$. Also h_2 can be found in table A13: $h_2 = 291.36 \text{ kJ/kg}$. At the exit of the condenser, we know that the temperature is $T_3 = 30^\circ\text{C}$ at a pressure of $P_3 = 1 \text{ MPa}$. In the tables, we can find that $T_{sat,1\text{MPa}} = 39.39^\circ\text{C}$, so we have a compressed liquid. Looking at a temperature of $T_3 = 30^\circ\text{C}$ in table A11 (assume h independent of pressure, since liquids are nearly incompressible), we find $h_3 = 91.49 \text{ kJ/kg}$. Between point 3 and 4 we have a throttling valve, so $h_4 = h_3 = 91.49 \text{ kJ/kg}$. At the exit of the evaporator we have a saturated vapor at $P_5 = 280 \text{ kPa}$, so $h_5 = 246.52 \text{ kJ/kg}$ (can be found in table A12).

The power input is now calculated as $\dot{W}_{in} = \dot{m}(h_2 - h_1)$, so we still need to determine \dot{m} . We use the heat loss of the house for this: $\dot{Q}_H = \dot{m}(h_2 - h_3) \Rightarrow \dot{m} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{60000}{3600 \cdot (291 - 91)} = 0.083 \text{ kg/s}$. So, $\dot{W}_{in} = \dot{m}(h_2 - h_1) = 0.083(291 - 246.64) = 3.58 \text{ kW}$.

11.7e The rate of heat absorption from the water is $\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.083(247.64 - 91.49) = 12.96 \text{ kW}$.

11.7f In that case, the heat that is now absorbed from the water would then come from electrical heating, so the increase in electric power would be 12.96 kW.

11.8 Reverse Brayton cycle

11.8a See figure 11.8.1.

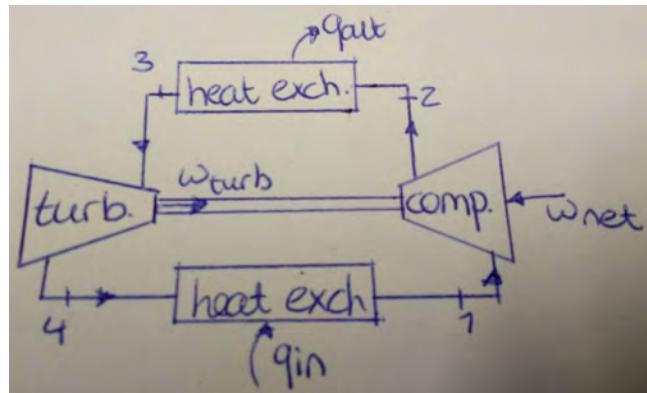


Figure 11.8.1: Schematic overview of question 11.8

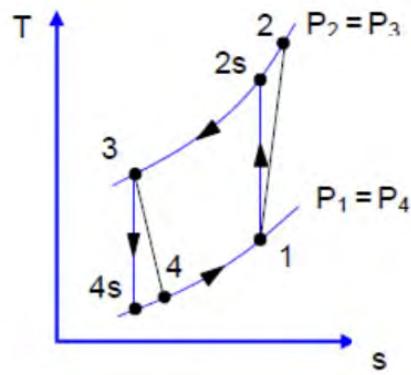


Figure 11.8.2: *Ts*-diagram of Reverse Brayton cycle

11.8b The Reverse Brayton Cycle consist of the following processes:

- 1 - 2 Isentropic compression in a compressor
- 2 - 3 Constant-pressure heat rejection
- 3 - 4 Isentropic expansion in a turbine
- 4 - 1 Constant-pressure heat addition

How this is displayed in a *Ts*-diagram, can be found in figure 11.8.2.

11.8c The two known variables at every characteristic position are given in table 11.8.1.

	P (atm)	T ($^{\circ}\text{C}$)	h (kJ/kg)	s (kJ/(kgK))	phase	other
1	1	0	275			
2s	$P_{2s} = P_2$		430	$s_{2s} = s_1$		
2	5		503			$\eta_{compr} = 68\%$
3	$P_3 = P_2$	80	350			
4	$P_4 = P_1$		256			$\eta_{turb} = 72\%$
4s	$P_{4s} = P_4$		220	$s_{4s} = s_3$		

Table 11.8.1: Table with parameters of question 11.8

The coefficient of performance can be calculated using: $COP = \frac{q_{in}}{w_{net}} = \frac{q_{in}}{w_{comp} - w_{turb}}$
 $q_{in} = h_1 - h_4$, $w_{comp} = h_2 - h_1$ and $w_{turb} = h_3 - h_4$ can be filled in at the formula for the COP.

$$COP = \frac{q_{in}}{w_{comp} - w_{turb}} = \frac{h_1 - h_4}{h_2 - h_1 - (h_3 - h_4)}$$

The only thing that is left to do, is to calculate the needed enthalpy values.

h_1 and h_3 can be found in the Mollier diagram for air. The other values can be calculated using the given efficiencies.

$$\eta_{comp} = \frac{h_{2s} - h_1}{h_2 - h_1} \text{ and } \eta_{turb} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

To find h_{2s} and h_{4s} we use that the entropy from 1 to 2s and 3 to 4s does not change. The enthalpy values can be found in the Mollier diagram. To find h_{2s} , use s_1 and the new pressure P_2 . To find h_{4s} , use s_3 and the new pressure P_4 . The results are $h_{2s} = 430 \text{ kJ/kg}$ and $h_{4s} = 220 \text{ kJ/kg}$.

$$\eta_{comp} = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{comp}} = 275 + \frac{430 - 275}{0.68} = 503 \text{ kJ/kg}$$

$$\eta_{turb} = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta(h_3 - h_{4s}) = 350 - 0.72(350 - 220) = 256 \text{ kJ/kg}$$

$$COP = \frac{h_1 - h_4}{h_2 - h_1 - (h_3 - h_4)} = \frac{275 - 220}{503 - 275 - (350 - 256)} = 0.14$$

Discussion

Reverse Brayton refrigeration systems typically have a low coefficient of performance compared with conventional vapor compression systems. This is due to the fact that the temperature of the working fluid changes rapidly in the heat exchangers, while a condensing or evaporating fluid releases and absorbs heat isothermally. In this particular problem, the coefficient of performance is also reduced due to the low efficiencies of the compressor and turbine.

11.9 Aircraft cabin cooling

Asked is to determine the heat transfer per kg of cabin air provided by the heat exchanger to meet the given requirements. Thus, $q_{out,23}$ in [kJ/kg]. The schematic overview of the system and the Ts -diagram of the system are given in figure 11.9.1.

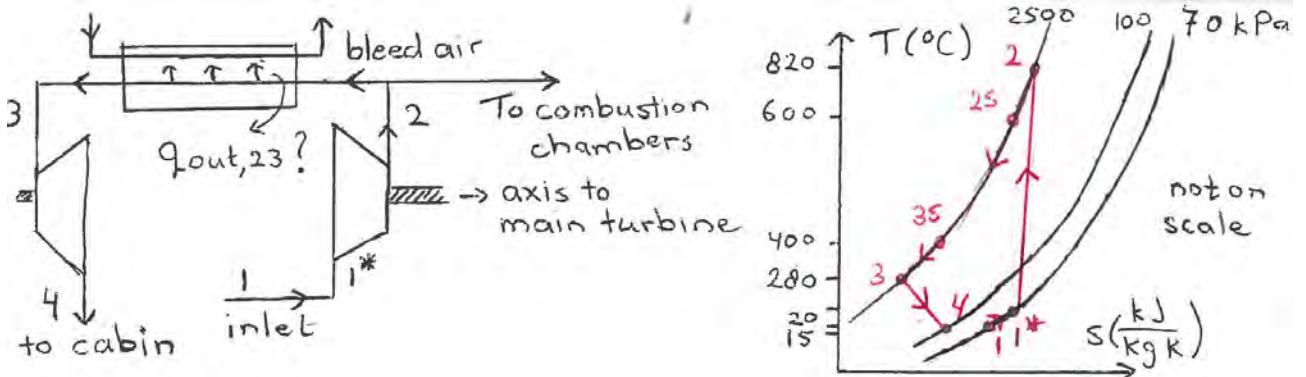


Figure 11.9.1: Schematic overview of the reversed Brayton cycle used to cool the aircraft cabin (left) and the Ts -diagram of the system (right).

In table 11.9.1, the two given variables at every point are collected. For the calculations an extra point 1^* needs to be added just before the compressor as the velocity of the air at the inlet has a high value of $v_1 = 250 \text{ m/s}$. Since the inlet velocity is not negligible define point 1^* to include the kinetic energy as part of the enthalpy of the air entering the compressor. The velocity at point 1^* is zero.

	P (kPa)	T ($^{\circ}$ C)	T (K)	h (kJ/kg)	other	T (K)**
1	<u>70</u>	<u>20</u>	293	290	<u>v=250 m/s</u>	293
1*	<u>70</u>	45	318	321.25	<u>v = 0 m/s</u>	324.55
2s	<u>2500</u>	600	873	905	<u>$s_{2s} = s_1$</u>	901.56
2	<u>2500</u>	820	1093	1155.18	<u>$\eta_c = 70\%$</u>	1148.98
3	<u>2500</u>	280	553	561.5	<u>$\eta_t = 70\%$</u>	496.55
3s	<u>2500</u>	400	673	680	<u>$s_{3s} = s_4$</u>	
4	<u>100</u>	<u>15</u>	288	285	<u>v=0 m/s</u>	288

Table 11.9.1: Table with parameters of question 11.9, known values are underlined. ** Temperatures resulting from using the formulas, see the alternative solution at the end of this problem.

The higher enthalpy at point 1* leads also to a higher temperature at point 1*, this is known as the 'stagnation temperature'.

The system will be solved using the Mollier diagram given in figure 11.9.2. The cycle is draw in the diagram, note the reversed flow direction compared to the Brayton power cycle.

- Point 1: Using the Mollier diagram for air, h_1 can be found 20°C on the 70 kPa isobar $\rightarrow h_1 = 290 \text{ kJ/kg}$.
- Point 1*: Conservation of energy gives (note between point 1 and 1* there is no work interaction or heat transfer and a negligible change in potential energy)

$$h_{1*} + \frac{1}{2}v_{1*}^2 = h_1 + \frac{1}{2}v_1^2 \rightarrow h_{1*} = h_1 + \frac{1}{2}(v_1^2 - v_{1*}^2) = 290 + \frac{1}{2}((250)^2 - (0)^2) \cdot 10^{-3} = 321.25 \text{ kJ/kg.}$$

The stagnation temperature T_{1*} can be read from the Mollier diagram $\rightarrow T_{1*} = 45^{\circ}\text{C}$.

- Point 2: h_2 can be determined using the isentropic efficiency of the compressor:

$$\eta_c = \frac{h_{2s} - h_{1*}}{h_2 - h_{1*}} \rightarrow h_2 = h_{1*} + \frac{h_{2s} - h_{1*}}{\eta_c} = 321.26 + \frac{905 - 321.25}{0.7} = 1155.18 \text{ kJ/kg.}$$

h_{2s} can be read from the Mollier diagram. The entropy of point 2s equals the entropy of point 1* and therefore the enthalpy of point 2s is vertically right above point 1* on the 2500 kPa isobar $\rightarrow h_{2s} = 905 \text{ kJ/kg}$.

- Point 4: To determine point 4 first point 4 is needed. h_4 can be determined directly from the Mollier diagram at 15°C on the 2500 kPa isobar $\rightarrow h_4 = 285 \text{ kJ/kg}$. Note that the velocity here is 0 so there is no need to take the kinetic energy into account.
- Point 3: Using the Mollier diagram for air h_{3s} can be found vertically below point 4 the isobar of 100 kPa as $s_{3s} = s_4 \rightarrow h_{3s} = 680 \text{ kJ/kg}$. h_3 can now be calculated using the turbine efficiency.

$$\eta_t = \frac{h_3 - h_4}{h_{3s} - h_4} \rightarrow h_3 = h_4 + \eta_t(h_{3s} - h_4) = 285 + 0.7 \cdot (680 - 285) = 589.5 \text{ kJ/kg.}$$

Finally the heat transfer per kg of cabin air provided by the heat exchanger can be calculated $q_{out,23} = h_2 - h_3 = 1155.18 - 589.5 = 565.68 \text{ kJ/kg}$.

Discussion

Cabin and avionics cooling in jet aircraft represents one of the most common applications of the reverse Brayton cycle. This is convenient, since a large amount of compressed air is already available in the engine, and so a small amount can simply be bled off the compressor to divert to the refrigeration cycle. Also, ambient air is readily available to cool the compressor bleed air before expanding it through the turbine. Since at high altitudes the ambient air temperature is quite low (around -50°C at 10 km), only a small volume of ambient air is needed in the heat exchanger.

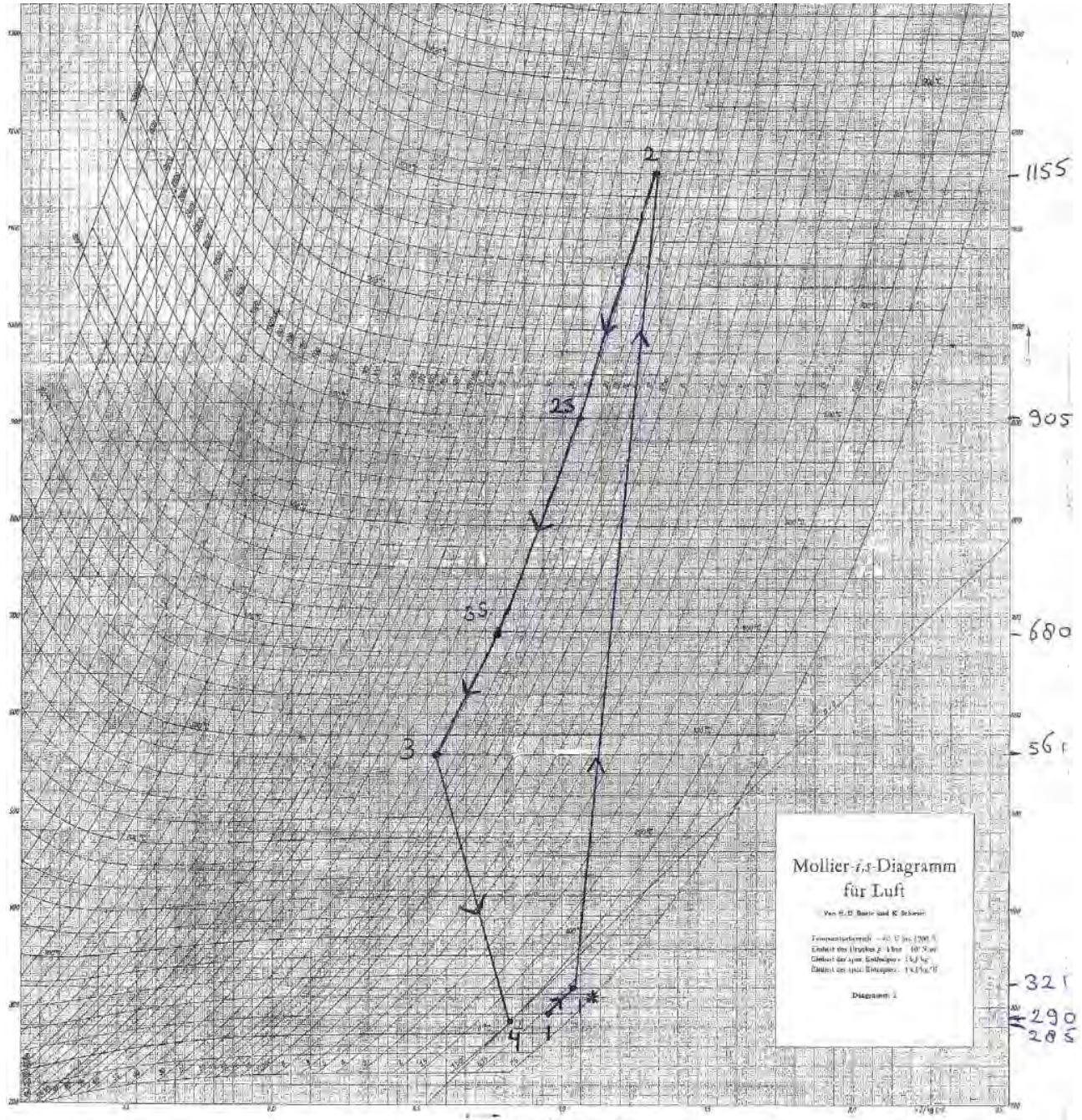


Figure 11.9.2: Mollier diagram the reverse Brayton cycle to cool the aircraft cabin. Note the reversed direction of the flow compared to the Brayton power cycle.

Alternatively this problem can be solved by determining the temperatures using the formulas of reading h values from the Mollier diagram. This is briefly worked out below. The temperatures found using the formulas are also given in table 11.9.1 Constant specific heats at 20°C are assumed, $c_p = 1.005 \text{ kJ/kgK}$ and $k = 1.4$.

- Point 1 and 4: Given are $T_1 = 20^\circ\text{C} = 293 \text{ K}$ and $T_4 = 15^\circ\text{C} = 288 \text{ K}$.
- Point 1*: The energy balance over point 1 and 1* can be used to find the stagnation temperature T_{1*} (note between point 1 and 1* there is no work or heat transfer and a negligible change in potential energy)

$$h_{1*} + \frac{1}{2}v_{1*}^2 = h_1 + \frac{1}{2}v_1^2 \rightarrow h_{1*} - h_1 = c_p(T_{1*} - T_1) = \frac{1}{2}(v^2 - v_{1*}^2) \rightarrow T_{1*} = T_1 + \frac{v^2 - v_{1*}^2}{2 \cdot c_p}$$

$$T_{1*} = 293 + \frac{1}{2}((250)^2 - (0)^2) \cdot 10^{-3} = 324.25 \text{ kJ/kg.}$$

- Point 2: T_2 can be determined using the isentropic efficiency of the compressor:

$$\eta_c = \frac{h_{2s} - h_{1*}}{h_2 - h_{1*}} = \frac{c_p(T_{2s} - T_{1*})}{c_p(T_2 - T_{1*})} \rightarrow T_2 = T_{1*} + \frac{T_{2s} - T_{1*}}{\eta_c}.$$

The temperature at point 2s can be calculated using the relation for an isentropic process:

$$T_{2s} = T_{1*} \left(\frac{P_2}{P_1} \right)^{\left(\frac{k-1}{k}\right)} = 324.25 \left(\frac{2500}{70} \right)^{\left(\frac{1.4-1}{1.4}\right)} = 901.56 \text{ K}$$

Combining gives for T_2 :

$$T_2 = T_{1*} + \frac{T_{2s} - T_{1*}}{\eta_c} = 324.25 + \frac{901.56 - 324.25}{0.7} = 1148.98 \text{ K.}$$

- Point 3: T_3 can be determined using the isentropic efficiency of the turbine:

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})} \rightarrow T_3 = \frac{T_4 - \eta_t T_{4s}}{1 - \eta_t}.$$

The temperature at point 4s can be expressed in T_3 using the relation for an isentropic process:

$$T_{4s} = T_3 \left(\frac{P_4}{P_3} \right)^{\left(\frac{k-1}{k}\right)} = T_3 \left(\frac{100}{2500} \right)^{\left(\frac{1.4-1}{1.4}\right)} = 0.4T_3$$

Combining both relations gives for T_3 :

$$T_3 = \frac{T_4 - \eta_t T_{4s}}{1 - \eta_t} = \frac{T_4 - 0.4\eta_t T_3}{1 - \eta_t} \rightarrow T_3 = \frac{T_4}{1 - 0.6\eta_t} = \frac{288}{1 - 0.6 \cdot 0.7} = 496.55 \text{ K.}$$

Finally the heat transfer per kg of cabin air provided by the heat exchanger can be calculated $q_{out,23} = h_2 - h_3 = c_p(T_2 - T_3) = 1.005(1148.98 - 496.55) = 655.69 \text{ kJ/kg.}$

Discussion

This value is different from the one found using the Mollier diagram (593.68 kJ/kg). This is due to the fact that in the last method constant specific heats are assumed at 20°C = 293 K. The specific heat is dependent on the temperature and changes considerably in the temperature range of this system. At a temperature of 1000 K $c_p = 1.124 \text{ kJ/kgK}$ and $k = 1.336$ (remember $k = c_p/c_v$). Taking into account the temperature dependence of the specific heats would give a value closer to the value found using the Mollier diagram. This is something that can be done using e.g. a Matlab script.

Using an average value of $k = 1.37$ and $c_p = 1.065 \text{ kJ/kgK}$ gives $q_{out,23} = 584 \text{ kJ/kg}$ which is close to the Mollier diagram value (note that this value also has an error margin as the reading of the diagram is not very accurate).

12 Matlab Stuff

12.1 Syntax gasprop

Description

gasprop is a small program to determine the c_p values and the difference in enthalpy as a function of temperature of various common gases. The purpose of this function is to make it very easy to use very precise values of the c_p value and enthalpy difference in MatLab. It is also made for educational purposes. The program can be used in optimization problems.

The range of temperatures are between 273K to 1500K for most gases. Some are in the range of 273K to 1800K and a few in the range of 273 to 1000K/1300K. When the range is exceeded there will be a warning indicating whether the temperature was too low or too high. In that case respectively the lowest and highest temperature in the range will be used to calculate the c_p value. Note that these results will be inaccurate and may not be valid. Keep this in your mind when using the program with very low and very high temperatures. When calculating the enthalpy difference at temperatures higher than the maximum temperature, it has no point because the enthalpy difference is a function of the c_p value. Because the c_p value, in this program, remains constant above a certain temperature, it has no purpose to determine the enthalpy change.

The program is made by a student of the Mechanical Engineering BSc Programme of the University of Twente. Everybody is free to use the program. If there are any questions or corrections, please contact e.s.j.beitler@student.utwente.nl.

Input

The syntax is very simple, there are two input arguments needed for the c_p value and three arguments for the enthalpy change. The first input argument is the desired gas. The second input argument is the desired temperature in Kelvin. For calculating the enthalpy change, the third input argument is the second, a higher, temperature.

For example, the command `cp = gasprop('N2',300)` gives the c_p value of nitrogen at 300K. The command `[dh,mean_cp] = gasprop('air',1300,1500)` gives the enthalpy difference and the mean c_p value. When you only define `dh = gasprop('air',1300,1500)`, only the difference in enthalpy is returned.

All of the supported gases with the proper input syntax can be found below in table 12.1.1. The input is not case sensitive.

Reference

B.G. Kyle, *Chemical and Process Thermodynamics* (Englewood Cliffs, NJ: Prentice-Hall, 1984).

Table 12.1.1: Overview of the gases with the input.

Substance	Formula	Input Syntax	Temperature range (K)
Nitrogen	N_2	N2	273-1800
Oxygen	O_2	O2	273-1800
Air	-	Air	273-1800
Hydrogen	H_2	H2	273-1800
Carbon monoxide	CO	CO	273-1800
Carbon dioxide	CO_2	CO2	273-1800
Water vapor	H_2O	H2O	273-1800
Nitric oxide	NO	NO	273-1500
Nitrous oxide	N_2O	N2O	273-1500
Nitrogen dioxide	NO_2	NO2	273-1500
Ammonia	NH_3	NH3	273-1500
Sulfur	S_2	S2	273-1800
Sulfur dioxide	SO_2	SO2	273-1800
Sulfur trioxide	SO_3	SO3	273-1300
Acetylene	C_2H_2	C2H2	273-1500
Benzene	C_6H_6	C6H6	273-1500
Methanol	CH_4O	CH4O	273-1000
Ethanol	C_2H_6O	C2H6O	273-1500
Hydrogen chloride	HCl	HCl	273-1500
Methane	CH_4	CH4	273-1500
Ethane	C_2H_6	C2H6	273-1500
Propane	C_3H_8	C3H8	273-1500
n-Butane	C_4H_{10}	C4H10_n	273-1500
i-Butane	C_4H_{10}	C4H10_i	273-1500
n-Pentane	C_5H_{12}	C5H12	273-1500
n-Hexane	C_6H_{14}	C6H14	273-1500
Ethylene	C_2H_4	C2H4	273-1500
Propylene	C_3H_6	C3H6	273-1500

12.2 Answers for *Matlab* questions

Answer *Matlab* question 9.2

```

1 %% Stirling cycle 1, 9.2
2 %% Constants
3 T3 = 705;
4 T1 = 423;
5 n = T3-T1;           %Determine number of steps
6 R = 0.287;            %The specific gas constant for air
7 qreg = 0;              %Initialization
8 T = T1;
9 for k = 1:n          %Recall the integral of cv dT
10    cv = gasprop('air',T)-R;    %Calculating the Cv value
11    qreg = qreg+cv;           %dT = 1
12    T = T + 1;
13 end
14 qreg % = 2.150519123473088e+02 kJ

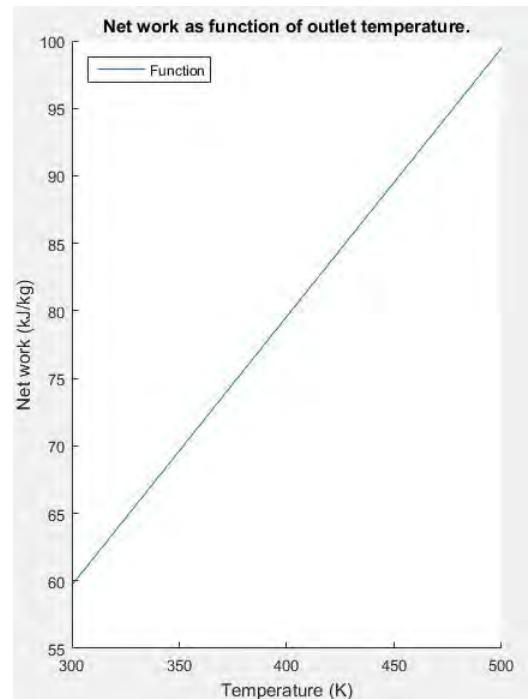
```

Answer *Matlab* question 9.3

```

1 %% Stirling cycle 2, 9.3
2 %% Constants
3 R = 0.287;
4 T1 = linspace(300,500,50); % Range of
5 P1 = 0.8;                  % temperature
6 P2 = 1.6;
7 P3 = 3.2;
8 %% Calculations
9 w_in = - R.*T1.*log(P1./P2); %kJ
10 T3 = T1.*P3./P2; %K
11 P4 = P1.*T3./T1; %MPa
12 w_out = R.*T3.*log(P3./P2);
13 w_net = w_out-w_in;
14 %% Plotting the figure
15 figure(1), clf(1), hold on
16 plot(T1,w_net)
17 title('Net work as function of outlet temperature.')
18 xlabel('Temperature (K)')
19 ylabel('Net work (kJ/kg)')
20 legend('Function','Location','NorthWest')
21 hold off

```

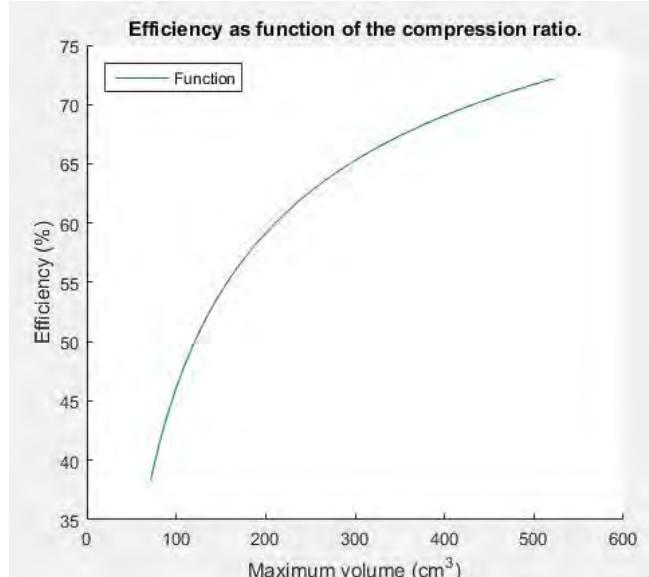


Answer Matlab question 9.5

```

1 %% Thermal efficiency Otto engine , 9.5
2 %% Constants
3 Vc = 21.3; %cm^3
4 R = 0.2870;
5 k = 1.4;
6 %% Calculations
7 Vs = linspace(50,500,50); %cm^3
8 Vmax = Vc + Vs;
9 rv = Vmax/Vc;
10 eta = (1 - 1./(rv.^^(k-1))).*100;
11 %% Plotting the figure
12 figure(1), clf(1), hold on
13 plot(Vmax, eta)
14 title('Efficiency as function of the compression ratio.')
15 xlabel('Maximum volume (cm^3)')
16 ylabel('Efficiency (%)')
17 legend('Function', 'Location', 'NorthWest')
18 hold off

```

**Answer Matlab question 9.7**

```

1 %% Otto , cycle 2, 9.7
2 %% Constants
3 T1 = 300; %K
4 T2 = 689.2;
5 T3 = 1733.6;
6 T4 = 754.6;
7 q_in = 750; %kJ/kg
8 R = 0.2870;
9 q_out = 0; %initialization
10 n = T4 - T1;
11 T = T1;
12 for k = 1:n
13     cv = gasprop('air',T)-R; %Calculating the Cv value
14     q_out = q_out+cv; %dT = 1
15     T = T + 1;
16 end
17 eta = (q_in - q_out)/q_in %0.5422

```

Answer Matlab question 9.16, Simple Brayton cycle

```

1  %% Simple open Brayton Cycle , see exercise 9.16 in the assignment bundle
2  %Optimization for pressure ratio
3  %This Matlab programm is an example of a simple Brayton cycle modelled in
4  %Matlab. It uses the program gasprop made by a student of the Mechanical
5  %Engineering BSc Programme. See further details about that programm in
6  %its manual. The temperature dependency of c_p and k is taken into
7  %account. However, note that the relations used for the isentropic
8  %processes in the compressor and the turbine ( $T/P^{((k-1)/k)}$ ) are derived
9  %under the assumption that  $c_p$  is independent of the temperature.
10 %This gives some small errors in the results. It is estimated that these
11 %errors are less than 5 K or 5 kJ/kg.
12 %The problem solved is exercise 9.16 in the assignment bundle of the
13 %thermodynamics course. Please look at the answer of that exercise for
14 %more information about the formula's used and the comparison with the
15 %results found using the Mollier diagram or the table (A-17).
16 clear, clc
17 %% Constants (P in Bar, T in Kelvin)
18 P1 = 1; %pressure inlet compressor (Bar)
19 P2 = linspace(1.01,20,1000); %pressure ratio , 8 Bar (before
20 P3 = P2; %optimization)
21 P4 = P1;
22 T1 = 37 + 273; %temperature inlet compressor (K)
23 T3 = 887 + 273; %temperature inlet turbine (K)
24 R = 0.2870; %Specific gas constant kJ/kgK
25 eta2 = 0.75; %Isentropic efficiency compressor
26 eta4 = 0.82; %Isentropic efficiency turbine
27 %%
28 for i = 1:length(P2)
29 %% Determine h1
30 h1 = T1; %reference value for h_1: 310 kJ/kg
31 %for temperatures in the range of 200 to 350 K the value of h is equal
32 %to the temperature value (see table A-17)
33 %% Determine T2s and h2s
34 %Note: k_2 should be taken at the average temperature between T1 and T2s,
35 %however T2s is not known before hand -> iterative process starting
36 %with k at room temperature (k = 1.4)
37 %It is important to have k as accurate as possible , k has a large
38 %influence
39 k_start = 1.4; %Start value of k = k at room temperature
40 T2s_start = T1*(P2(i)/P1)^((k_start-1)/k_start); %T2s start iter. 561.55 K
41 Tav_1_2s = [0 (T1 + T2s_start)/2]; %Average T between comp. inlet/outlet
42 x=2;
43 %%iteration loop
44 while abs(Tav_1_2s(x)-Tav_1_2s(x-1)) > 0.01
45 kav_1_2s = gasprop('air',Tav_1_2s(x))/(gasprop('air',Tav_1_2s(x))-R);
46 T2s = T1*(P2(i)/P1)^((kav_1_2s-1)/kav_1_2s);
47 x=x+1;
48 Tav_1_2s(x) = (T1 + T2s)/2;
49 end
50 Tav_1_2s=Tav_1_2s(x); %432.40 K

```

```

51 % After this loop the correct average temperature is found. The accepted
52 %error is 0.01 K (look for the condition in the while loop). The
53 %corresponding value of kav_1_2s and T2s must be calculated once more:
54 kav_1_2s = gasprop('air',Tav_1_2s)/(gasprop('air',Tav_1_2s)-R);%1.3887
55 T2s = T1*(P2(i)/P1)^((kav_1_2s-1)/kav_1_2s); %554.80 K
56 %Note: in the derivation of this formula Cp is taken independent of the
57 %temperature. This gives a small error in the results (for this case T2s
58 %should be 557 K).
59 %With the T2s found after the iteration process h2s can be calculated
60 h2s = h1 + gasprop('air',T1,T2s); %561.10 kJ/kg
61 %% Determine h2 and T2
62 h2 = h1 + (h2s-h1)/eta2; %644.81 kJ/kg
63 %To calculate T2 c_p should be calculated at the average temperature
64 %between T1 and T2 -> iterative process starting at the average
65 %temperature between T1 and T2s
66 cp_2 = [0 gasprop('air',Tav_1_2s)];
67 x=2;
68 %iteration loop
69 while abs(cp_2(x)-cp_2(x-1))>0.0001
70 T2 = T1 + (h2-h1)/cp_2(x);
71 Tav_1_2 = (T1 + T2)/2;
72 x=x+1;
73 cp_2(x) = gasprop('air',Tav_1_2);
74 end
75 cp_2=cp_2(x); %1.0324 kJ/kgK
76 % After this loop the correct average c_p is found. The accepted error
77 %is 0.0001 kJ/kgK (look for the condition in the while loop).
78 %The corresponding value of T_2 and Tav_1_2 must be calculated once more:
79 T2 = T1 + (h2-h1)/cp_2; %634.31 K
80 Tav_1_2 = (T1 + T2)/2; %472.15 K
81 %% Determine h3
82 h3 = h2 + gasprop('air',T2,T3); %1231.2 kJ/kg
83 %% Determine T4s and h4s
84 %To determine T4s and h4 the same method is used as for T2s and h2s
85 T4s_start= T3*(P4/P3(i))^((k_start-1)/k_start); %T4s start iter. 640.37 K
86 Tav_3_4s = [0 (T3 + T4s_start)/2]; %Average T between turb. inlet/outlet
87 x=2;
88 %iteration loop
89 while abs(Tav_3_4s(x)-Tav_3_4s(x-1)) > 0.01
90 kav_3_4s = gasprop('air',Tav_3_4s(x))/(gasprop('air',Tav_3_4s(x))-R);
91 T4s = T3*(P4/P3(i))^((kav_3_4s-1)/kav_3_4s);
92 x=x+1;
93 Tav_3_4s(x) = (T3 + T4s)/2;
94 end
95 Tav_3_4s=Tav_3_4s(x); %920.50 K
96 % After this loop the correct average temperature is found. The accepted
97 %error is 0.01 K (look for the condition in the while loop). The
98 %corresponding value of kav_3_4s and T4s must be calculated once more:
99 kav_3_4s = gasprop('air',Tav_3_4s)/(gasprop('air',Tav_3_4s)-R); %1.3443
100 T4s = T3*(P4/P3(i))^((kav_3_4s-1)/kav_3_4s); %681.01 K
101 %Note: in the derivation of this formula Cp is taken independent of the
102 %temperature. This gives a small error in the results (for this case T4s

```

```

103 %should be 680 K).
104 %with the T4s found after the iteration process h4s can be calculated
105 h4s = h3 - gasprop('air',T4s,T3);                                %694.66 kJ/kg
106 %% Determine h4 and T4
107 h4 = h3 - eta4*(h3-h4s);                                         %791.24 kJ/kg
108 %To calculate T4 c_p should be calculated at the average temperature
109 %between T3 and T4 -> iterative process starting at the average
110 %temperature between T4 and T4s
111 cp_4 = [0 gasprop('air',Tav_3_4s)];;
112 x=2;
113 %%iteration loop
114 while abs(cp_4(x)-cp_4(x-1))>0.0001
115 T4 = T3 + (h4-h3)/cp_4(x);
116 Tav_3_4 = (T3 + T4)/2;
117 x=x+1;
118 cp_4(x) = gasprop('air',Tav_3_4);
119 end
120 cp_4=cp_4(x);                                                 %1.1295 kJ/kgK
121 %After this loop the correct average c_p is found. The accepted error
122 %is 0.0001 kJ/kgK (look for the condition in the while loop).
123 %The corresponding value of T_4 and Tav_3_4 must be calculated once more:
124 T4 = T3 + (h4-h3)/cp_4;                                         %770.47 K
125 Tav_3_4 = (T3 + T4)/2;                                         %965.24 K
126 %% All enthalpy values are determined and can be used to determine the
127 %work input, work output, heat input and thermal efficiency.
128 %% Calculate specific work output (turbine output)
129 w_out = (h3-h4);                                              %439.96 kJ/kg
130 %% Calculate specific work input (compressor input)
131 w_in = (h2-h1);                                              %334.81 kJ/kg
132 %% Calculate back work
133 back_work_ratio(i) = w_in / w_out;
134 %% Calculate net specific work output (turbine - compressor)
135 w_net(i) = (h3-h4)-(h2-h1);                                    %105.15 kJ/kg
136 %% Calculate net specific heat input
137 q_in = (h3-h2);                                               %586.39 kJ/kg
138 %% Calculate efficiency
139 e_th(i) = ((h3-h4)-(h2-h1))/(h3-h2)*100;                      %17.93%
140 %%
141 end
142 %
143 %Make graph Thermal Effiency
144 figure(1),clf(1),hold on
145 plot(P2,e_th)
146 title('Efficiency as function of the pressure ratio')
147 xlabel('Pressure ratio (-)')
148 ylabel('Thermal Effiency (%)')
149 grid on, grid minor, hold off
150 %With the function max you can find the maximum value of the efficiency
151 %and the indice (for finding at which value this occurs).
152 [e_th,index_P] = max(e_th); %finding from every column the highest e_th
153 optimum_P_e_th = P2(index_P) % 6.81 Bar & its index
154 e_th_max = e_th           % 18.13%

```

```

155 %
156 %Make graph Net specific work output
157 figure(2), clf(2), hold on
158 plot(P2, w_net)
159 title('Net specific work output as function of the pressure ratio')
160 xlabel('Pressure ratio (-)')
161 ylabel('Net specific work output (kJ/kg)')
162 grid on, grid minor, hold off
163 %{With the function max you can find the maximum value of net specific
164 %work output and the indice (for finding at which value this occurs).
165 [w_net, index_P] = max(w_net); %finding from every column the highest
166 optimum_P_w_net = P2(index_P) % 4.70 Bar w_net & its index
167 w_net_max = w_net % 191.11 kJ/kg
168 %

```

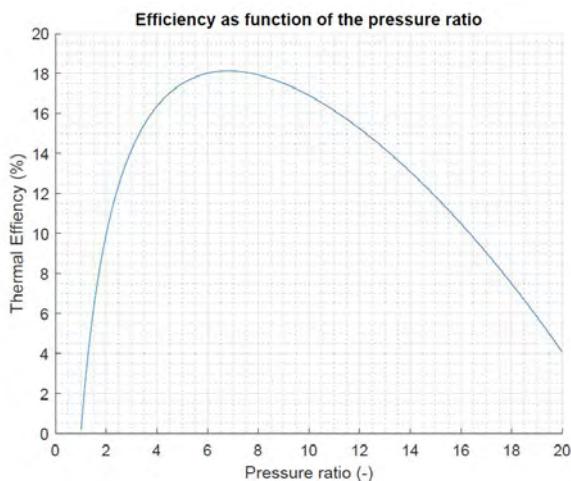


Figure 12.2.1: Graph showing the thermal efficiency as function of the pressure ratio (isentropic efficiency turbine 82% and pump 75%). Maximum efficiency, 18.1% at a pressure ratio of 6.8 Bar.

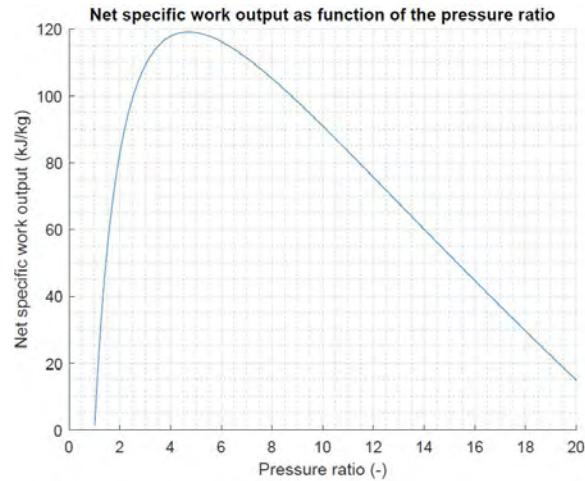


Figure 12.2.2: Graph showing the net specific work output as function of the pressure ratio (isentropic efficiency turbine 82% and pump 75%). Maximum net specific work output, 191.1 kJ/kg at a pressure ratio of 4.7 Bar

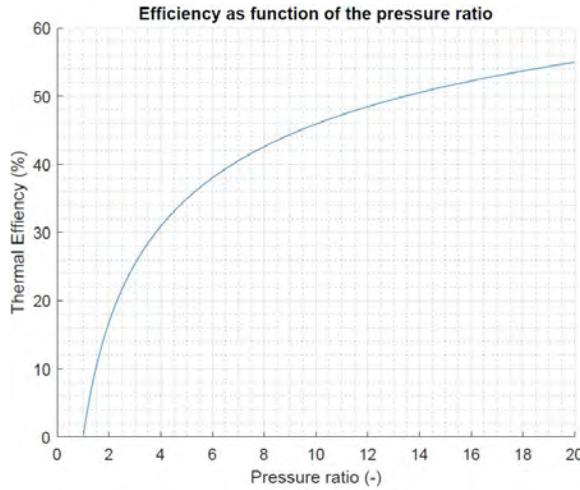


Figure 12.2.3: Graph showing the thermal efficiency as function of the pressure ratio (isentropic efficiencies turbine and pump are 100%). No maximum efficiency, higher pressure ratio higher efficiency.

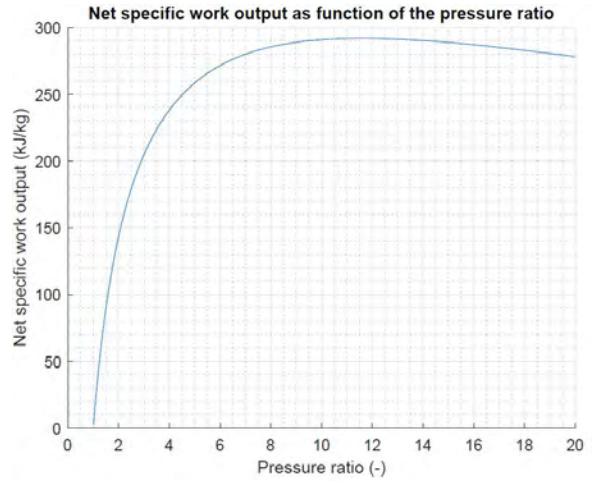


Figure 12.2.4: Graph showing the net specific work output as function of the pressure ratio (isentropic efficiencies turbine and pump are 100%). Maximum net specific work output, 291.9 kJ/kg at a pressure of 11.6 Bar.

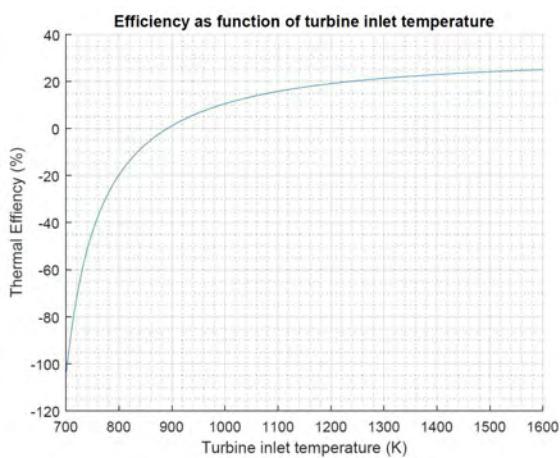


Figure 12.2.5: Graph showing the thermal efficiency as function of the turbine inlet temperature (isentropic efficiency turbine 82% and pump 75%). There is no maximum efficiency, the higher the inlet temperature the higher the efficiency. Note: the maximum inlet temperature is restricted by the materials.

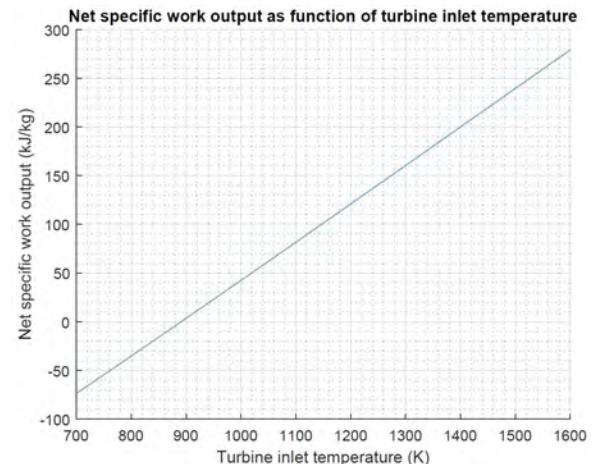


Figure 12.2.6: Graph showing the net specific work output as function of the turbine inlet temperature (isentropic efficiency turbine 82% and pump 75%). No maximum specific net work output, the higher the inlet temperature the higher the work output. Note: the maximum inlet temperature is restricted by the materials.

Answer Matlab question 9.16 with regenerator (with efficiency of 80%)

```

1  %% Simple open Brayton Cycle , exercise 9.16 in the assignment bundle with
2  %a regenerator add (efficiency of 80%)
3  %
4  %This Matlab programm is an example of a simple Brayton cycle modelled in
5  %Matlab. It uses the program gasprop made by a student of the Mechanical
6  %Engineering BSc Programme. See further details about that programm in
7  %its manual. The temperature dependency of c_p and k is taken into
8  %account. However, note that the relations used for the isentropic
9  %processes in the compressor and the turbine ( $T/P^{((k-1)/k)}$ ) are derived
10 %under the assumption that  $c_p$  is independent of the temperature.
11 %This gives some small errors in the results. It is estimated that these
12 %errors are less than 5 K or 5 kJ/kg.
13 %The problem solved is exercise 9.16 in the assignment bundle of the
14 %thermodynamics course. Please look at the answer of that exercise for
15 %more information about the formula's used and the comparison with the
16 %results found using the Mollier diagram or the table (A-17).
17 clear , clc
18 %% Constants (P in Bar, T in Kelvin)
19 P1 = 1; %pressure inlet compressor (Bar)
20 P2 = linspace(1.01,20,100); %pressure ratio (8 Bar before
21 P3 = P2; %optimization)
22 P4 = P1;
23 T1 = 37 + 273; %temperature inlet compressor (K)
24 T3 = 887 + 273; %temperature inlet turbine (K)
25 R = 0.2870; %Specific gas constant kJ/kgK
26 eta2 = 0.75; %Isentropic efficiency compressor
27 eta4 = 0.82; %Isentropic efficiency turbine
28 eta_reg = 0.8; %effectiveness regenerator
29 %%
30 for i = 1:length(P2)
31 %% Determine h1 %reference value for h_1: 310 kJ/kg
32 h1 = T1;
33 %for temperatures in the range of 200 to 350 K the value of h is equal
34 %to the temperature value (see table A-17)
35 %% Determine T2s and h2s
36 %Note: k_2 should be taken at the average temperature between T1 and T2s,
37 %however T2s is not known before hand -> iterative process starting
38 %with k at room temperature (k = 1.4)
39 %It is important to have k as accurate as possible, k has a large
40 %influence
41 k_start = 1.4; %Start value of k = k at room temperature
42 T2s_start= T1*(P2(i)/P1)^((k_start-1)/k_start);%T2s start iter. 561.55 K
43 Tav_1_2s = [0 (T1 + T2s_start)/2]; %Average T between comp. inlet/outlet
44 x=2;
45 %%iteration loop
46 while abs(Tav_1_2s(x)-Tav_1_2s(x-1)) > 0.01
47 kav_1_2s = gasprop('air',Tav_1_2s(x))/(gasprop('air',Tav_1_2s(x))-R);
48 T2s = T1*(P2(i)/P1)^((kav_1_2s-1)/kav_1_2s);
49 x=x+1;
50 Tav_1_2s(x) = (T1 + T2s)/2;

```

```

51 end
52 Tav_1_2s=Tav_1_2s(x); %432.40 K
53 % After this loop the correct average temperature is found. The accepted
54 %error is 0.01 K (look for the condition in the while loop). The
55 %corresponding value of kav_1_2s and T2s must be calculated once more:
56 kav_1_2s = gasprop('air',Tav_1_2s)/(gasprop('air',Tav_1_2s)-R);%1.3887
57 T2s = T1*(P2(i)/P1)^((kav_1_2s-1)/kav_1_2s); %554.80 K
58 %Note: in the derivation of this formula Cp is taken independent of the
59 %temperature. This gives a small error in the results (for this case T2s
60 %should be 557 K).
61 %With the T2s found after the iteration process h2s can be calculated
62 h2s = h1 + gasprop('air',T1,T2s); %561.10 kJ/kg
63 %% Determine h2 and T2
64 h2 = h1 + (h2s-h1)/eta2; %644.81 kJ/kg
65 %To calculate T2 c_p should be calculated at the average temperature
66 %between T1 and T2 -> iterative process starting at the average
67 %temperature between T1 and T2s
68 cp_2 = [0 gasprop('air',Tav_1_2s)];
69 x=2;
70 %iteration loop
71 while abs(cp_2(x)-cp_2(x-1))>0.0001
72 T2 = T1 + (h2-h1)/cp_2(x);
73 Tav_1_2 = (T1 + T2)/2;
74 x=x+1;
75 cp_2(x) = gasprop('air',Tav_1_2);
76 end
77 cp_2=cp_2(x); %1.0324 kJ/kgK
78 % After this loop the correct average c_p is found. The accepted error is
79 %0.0001 kJ/kgK (look for the condition in the while loop).
80 %The corresponding value of T_2 and Tav_1_2 must be calculated once more:
81 T2 = T1 + (h2-h1)/cp_2; %634.31 K
82 Tav_1_2 = (T1 + T2)/2; %472.15 K
83 %% Determine h3
84 h3 = h2 + gasprop('air',T2,T3); %1231.2 kJ/kg
85 %% Determine T4s and h4s
86 %To determine T4s and h4 the same method is used as for T2s and h2s
87 T4s_start= T3*(P4/P3(i))^((k_start-1)/k_start); %T4s start iter. 640.37 K
88 Tav_3_4s = [0 (T3 + T4s_start)/2]; %Average T between turb. inlet/outlet
89 x=2;
90 %iteration loop
91 while abs(Tav_3_4s(x)-Tav_3_4s(x-1)) > 0.01
92 kav_3_4s = gasprop('air',Tav_3_4s(x))/(gasprop('air',Tav_3_4s(x))-R);
93 T4s = T3*(P4/P3(i))^((kav_3_4s-1)/kav_3_4s);
94 x=x+1;
95 Tav_3_4s(x) = (T3 + T4s)/2;
96 end
97 Tav_3_4s=Tav_3_4s(x); %920.50 K
98 % After this loop the correct average temperature is found. The accepted
99 %error is 0.01 K (look for the condition in the while loop). The
100 %corresponding value of kav_3_4s and T4s must be calculated once more:
101 kav_3_4s = gasprop('air',Tav_3_4s)/(gasprop('air',Tav_3_4s)-R); %1.3443
102 T4s = T3*(P4/P3(i))^((kav_3_4s-1)/kav_3_4s); %681.01 K

```

```

103 %Note: in the derivation of this formula Cp is taken independent of the
104 %temperature. This gives a small error in the results (for this case T4s
105 %should be 680 K).
106 %with the T4s found after the iteration process h4s can be calculated
107 h4s = h3 - gasprop('air',T4s,T3); %694.66 kJ/kg
108 %% Determine h4 and T4
109 h4 = h3 - eta4*(h3-h4s); %791.24 kJ/kg
110 %To calculate T4 c_p should be calculated at the average temperature
111 %between T3 and T4 -> iterative process starting at the average
112 %temperature between T4 and T4s
113 cp_4 = [0 gasprop('air',Tav_3_4s)];;
114 x=2;
115 %%iteration loop
116 while abs(cp_4(x)-cp_4(x-1))>0.0001
117 T4 = T3 + (h4-h3)/cp_4(x);
118 Tav_3_4 = (T3 + T4)/2;
119 x=x+1;
120 cp_4(x) = gasprop('air',Tav_3_4);
121 end
122 cp_4=cp_4(x); %1.1295 kJ/kgK
123 % After this loop the correct average c_p is found. The accepted error
124 %is 0.0001 kJ/kgK (look for the condition in the while loop).
125 %The corresponding value of T4 and Tav_3_4 must be calculated once more:
126 T4 = T3 + (h4-h3)/cp_4; %770.47 K
127 Tav_3_4 = (T3 + T4)/2; %965.24 K
128 %% Determine h5
129 h5 = h2 + eta_reg*(h4-h2); %761.95 kJ/kg
130 %% Determine h6
131 h6 = h4-h5+h2; %674.09 kJ/kg
132 %% All enthalpy values are determined and can be used to determine
133 %the work input, work output, heat input and thermal efficiency.
134 %% Calculate specific work output (turbine output)
135 w_out = (h3-h4); %439.96 kJ/kg
136 %% Calculate specific work input (compressor input)
137 w_in = (h2-h1); %334.81 kJ/kg
138 %% Calculate net specific work output (turbine - compressor)
139 w_net(i) = (h3-h4)-(h2-h1); %105.15 kJ/kg
140 %% Calculate specific regenerated heat
141 q_reg(i) = (h5-h2); %117.14 kJ/kg
142 %% Calculate net specific heat input
143 q_in = (h3-h5); %469.24 kJ/kg
144 %% Calculate efficiency
145 e_th(i) = 100*((h3-h4)-(h2-h1))/(h3-h5); %22.41%
146 %%
147 end
148 %Make graph Thermal efficiency
149 figure(1), clf(1), hold on
150 plot(P2,e_th)
151 title('Efficiency as function of the pressure ratio')
152 xlabel('Pressure ratio (-)')
153 ylabel('Thermal Efficiency (%)')
154 grid on, grid minor, hold off

```

```
155 %With the function max you can find the maximum value of the efficiency
156 %and the indices (for finding at which value this occurs).
157 [ e_th , index_P ] = max(e_th); %finding from every column the highest eta
158 optimum_P_e_th = P2(index_P) % 3.18 Bar & its index
159 e_th_max = e_th % 30.07%
160 %Make graph Net specific work output
161 figure(2), clf(2), hold on
162 plot(P2, w_net)
163 title('Net specific work output as function of the pressure ratio')
164 xlabel('Pressure ratio (-)')
165 ylabel('Net specific work output (kJ/kg)')
166 grid on, grid minor, hold off
167 %With the function max you can find the maximum value of the efficiency
168 %and the indices (for finding at which value this occurs).
169 [ w_net , index_P ] = max(w_net); %finding from every column the highest
170 optimum_P_w_net = P2(index_P) % 4.70 Bar w_net & its index
171 w_net_max = w_net % 191.11 kJ/kg%
172 %Make graph Net specific regenerated heat
173 figure(3), clf(3), hold on
174 plot(P2, q_reg)
175 title('Net specific regenerated heat as function of the pressure ratio')
176 xlabel('Pressure ratio (-)')
177 ylabel('Regenerated heat (kJ/kg)')
178 grid on, grid minor, hold off
179 %
```

Answer Matlab question 9.19, Brayton cycle with intercooling and regeneration

```

1 %% Brayton with intercooling and regeneration , 9.19
2 clear , clc
3 %% Constants
4 P1 = 1; %Bar
5 P2 = 4; %Bar
6 P3 = P2;
7 P4 = 12; %Bar
8 P5 = P4;
9 P6 = P4;
10 P7 = P1;
11 P8 = P1;
12 T1 = 15+273; %K
13 T3 = 40+273; %K
14 T6 = 860+273; %K
15 eta2 = 0.8; %Isentropic efficiency compressor 1
16 eta4 = eta2; %Isentropic efficiency compressor 2
17 eta7 = eta2; %Isentropic efficiency turbine
18 eta_mech = 0.98; %Efficiency between compressor – turbine – generator
19 epsilon = 1; %Effectiveness regenerator
20 m = 7; %mass flow air in kg/s
21 R = 0.2870; %specifiec gas constant in kJ/kgK
22 %% Determine h1
23 h1 = gasprop('air',273,T1); %288 kJ/kg
24 h1 = T1;
25 %for temperatures in the range of 200 to 350 K the value of h is equal to
26 %the temperature value (see table A-17)
27 %% Determine T2s and h2s
28 %Note: k_2 should be taken at the average temperature between T1 and T2s,
29 %however T2s is not know before hand -> iterative process starting
30 %with k at room temperature (k = 1.4). It is important to have k as
31 %accurate as possible , k has a large influence
32 k_start = 1.4;
33 T2s_start= T1*(P2/P1)^((k_start-1)/k_start); %T2s to start iter. 428 K
34 Tav_1_2s = [0 (T1 + T2s_start)/2]; %Average T between comp. inlet/outlet
35 x=2;
36 %iteration loop
37 while abs(Tav_1_2s(x)-Tav_1_2s(x-1)) > 0.01
38 kav_1_2s = gasprop('air',Tav_1_2s(x))/(gasprop('air',Tav_1_2s(x))-R);
39 T2s = T1*(P2/P1)^((kav_1_2s-1)/kav_1_2s);
40 x=x+1;
41 Tav_1_2s(x) = (T1 + T2s)/2;
42 end
43 Tav_1_2s=Tav_1_2s(x); %357 K
44 % After this loop the correct average temperature is found. The accepted
45 %error is 0.01 K (look for the condition in the while loop). The
46 %corresponding value of kav_1_2s and T2s must be calculated once more:
47 kav_1_2s = gasprop('air',Tav_1_2s)/(gasprop('air',Tav_1_2s)-R); %1.3954
48 T2s = T1*(P2/P1)^((kav_1_2s-1)/kav_1_2s); %427 K
49 %Note: in the derivation of this formula Cp is taken independent of the
50 %temperature. This gives a small error in the results (for this case T2s

```

```

51 %should be 557 K).
52 %With the T2s found after the iteration process h2s can be calculated
53 h2s = h1 + gasprop('air',T1,T2s); %428 kJ/kg
54 %% Determine h2 and T2
55 h2 = h1 + (h2s-h1)/eta2; %463 kJ/kg
56 %To calculate T2 c_p should be calculated at the average temperature
57 %between T1 and T2 -> iterative process starting at the average
58 %temperature between T1 and T2s
59 cp_2 = [0 gasprop('air',Tav_1_2s)];;
60 x=2;
61 %%iteration loop
62 while abs(cp_2(x)-cp_2(x-1))>0.0001
63 T2 = T1 + (h2-h1)/cp_2(x);
64 Tav_1_2 = (T1 + T2)/2;
65 x=x+1;
66 cp_2(x) = gasprop('air',Tav_1_2);
67 end
68 cp_2=cp_2(x); %1.0156 kJ/kgK
69 %After this loop the correct average c_p is found. The accepted error
70 %is 0.0001 kJ/kgK (look for the condition in the while loop).
71 %The corresponding value of T_2 and Tav_1_2 must be calculated once more:
72 T2 = T1 + (h2-h1)/cp_2; %461 K
73 Tav_1_2 = (T1 + T2)/2; %374 K
74 %% Determine h3
75 h3 = h2-gasprop('air',T3,T2); %313 kJ/kg
76 %% Determine T4s and h4s
77 %To determine T4s and h4 the same method is used as for T2s and h2s
78 T4s_start= T3*(P4/P3)^((k_start-1)/k_start); %T4s to start iter. 428 K
79 Tav_3_4s = [0 (T3 + T4s_start)/2]; %Average T between compr. inlet/outlet
80 x=2;
81 %%iteration loop
82 while abs(Tav_3_4s(x)-Tav_3_4s(x-1)) > 0.01
83 kav_3_4s = gasprop('air',Tav_3_4s(x))/(gasprop('air',Tav_3_4s(x))-R);
84 T4s = T3*(P4/P3)^((kav_3_4s-1)/kav_3_4s);
85 x=x+1;
86 Tav_3_4s(x) = (T3 + T4s)/2;
87 end
88 Tav_3_4s=Tav_3_4s(x); %370 K
89 %After this loop the correct average temperature is found. The accepted
90 %error is 0.01 K (look for the condition in the while loop). The
91 %corresponding value of kav_3_4s and T4s must be calculated once more:
92 kav_3_4s = gasprop('air',Tav_3_4s)/(gasprop('air',Tav_3_4s)-R);%1.3943
93 T4s = T3*(P4/P3)^((kav_3_4s-1)/kav_3_4s); %427 K
94 %Note: in the derivation of this formula Cp is taken independent of the
95 %temperature. This gives a small error in the results (for this case T4s
96 %should be 680 K).
97 %with the T4s found after the iteration process h4s can be calculated
98 h4s = h3 + gasprop('air',T3,T4s); %429 kJ/kg
99 %% Determine h4 and T4
100 h4 = h3 + (h4s-h3)/eta4; %458 kJ/kg
101 %To calculate T4 c_p should be calculated at the average temperature
102 %between T3 and T4 -> iterative process starting at the average

```

```

103 %temperature between T4 and T4s
104 cp_4 = [0 gasprop('air',Tav_3_4s)]; %1.0172 kJ/kgK
105 x=2;
106 %iteration loop
107 while abs(cp_4(x)-cp_4(x-1))>0.0001
108 T4 = T3 + (h4-h3)/cp_4(x);
109 Tav_3_4 = (T3 + T4)/2;
110 x=x+1;
111 cp_4(x) = gasprop('air',Tav_3_4);
112 end
113 cp_4=cp_4(x); % After this loop the correct average c_p is found. The accepted error
114 %is 0.0001 kJ/kgK (look for the condition in the while loop).
115 %The corresponding value of T_4 and Tav_3_4 must be calculated once more:
116 T4 = T3 + (h4-h3)/cp_4; %455 K
117 Tav_3_4 = (T3 + T4)/2; %384 K
118 %% Determine h6
119 %h6 = gasprop('air',0,T6);
120 h6 = h4 + gasprop('air',T4,T6);
121 %% Determine T7s and h7s
122 %To determine T7s and h7 the same method is used as for T2s and h2s
123 T7s_start= T6*(P7/P6)^((k_start-1)/k_start); %T7s to start ite. 557 K
124 Tav_6_7s = [0 (T6 + T7s_start)/2]; %Average T between turb. inlet/outlet
125 x=2;
126 %iteration loop
127 while abs(Tav_6_7s(x)-Tav_6_7s(x-1)) > 0.01
128 kav_6_7s = gasprop('air',Tav_6_7s(x))/(gasprop('air',Tav_6_7s(x))-R);
129 T7s = T6*(P7/P6)^((kav_6_7s-1)/kav_6_7s);
130 x=x+1;
131 Tav_6_7s(x) = (T6 + T7s)/2;
132 end
133 Tav_6_7s=Tav_6_7s(x); %864 K
134 % After this loop the correct average temperature is found. The accepted
135 %error is 0.01 K (look for the condition in the while loop). The
136 %corresponding value of kav_6_7s and T7s must be calculated once more:
137 kav_6_7s = gasprop('air',Tav_6_7s)/(gasprop('air',Tav_6_7s)-R); %1.3491
138 T7s = T6*(P7/P6)^((kav_6_7s-1)/kav_6_7s); %596 K
139 %Note: in the derivation of this formula Cp is taken independent of the
140 %temperature. This gives a small error in the results (for this case T4s
141 %should be 680 K).
142 %with the T4s found after the iteration process h4s can be calculated
143 h7s = h6 - gasprop('air',T7s,T6); %604 kJ/kg
144 %% Determine h7 and T7
145 h7 = h6 - eta7*(h6-h7s); %723 kJ/kg
146 %To calculate T7 c_p should be calculated at the average temperature
147 %between T6 and T7 -> iterative process starting at the average
148 %temperature between T4 and T4s
149 cp_7 = [0 gasprop('air',Tav_6_7s)];
150 x=2;
151 %iteration loop
152 while abs(cp_7(x)-cp_7(x-1))>0.0001
153 T7 = T6 + (h7-h6)/cp_7(x);

```

```

155 Tav_6_7 = (T6 + T7) / 2;
156 x=x+1;
157 cp_7(x) = gasprop('air',Tav_6_7);
158 end
159 cp_7=cp_7(x); %1.1205 kJ/kgK
160 % After this loop the correct average c_p is found. The accepted error
161 %is 0.0001 kJ/kgK (look for the condition in the while loop).
162 %The corresponding value of T_7 and Tav_6_7 must be calculated once more:
163 T7 = T6 + (h7-h6)/cp_7; %708 K
164 Tav_6_7 = (T6 + T7) / 2; %920 K
165 %% Determine h5
166 h5 = h4 + epsilon*(h7-h4); %723 K
167 %% Determine h8
168 h8 = h7 - (h5-h4); %458 K
169 %% Calculate specific work output
170 w_out = eta_mech*(h6-h7); %467 kJ/kg
171 %% Calculate specific work input
172 w_in = ((h2-h1)+(h4-h3))/eta_mech; %327 kJ/kg
173 %% Calculate specific net work output
174 w_net = w_out - w_in; %141 kJ/kg
175 %% Calculate specific regenerated heat
176 q_reg = (h5-h4); %266 kJ/kg
177 %% Calculate specific heat input
178 q_in = (h6-h5); %477 kJ/kg
179 %% Determine the thermal efficiency
180 eta_thermal = (w_out-w_in)/q_in; %29.4%
181 %% Determine the power supplied by the system
182 W_net_power= m*(w_out-w_in); %983 kW
183 %% Determine the power input to the system
184 Q_in_power= m*(q_in); %3337 kW
185 %%

```

Answer Matlab question 10.3, Simple ideal Rankine cycle

```

1 %% Power plant on a simple ideal Rankine cycle 2, 10.3 using XSteam
2 clear, clc
3 %% Constants
4 P1 = 0.1; %bar
5 P2 = 100;
6 P3 = P2;
7 P4 = P1;
8 T3 = 500; %degrees Celsius
9 W_net = 210e3;
10 %% Determine h1
11 h1 = XSteam('hL_p',P1); %191.812 kJ/kg
12 %% Determine h2
13 s2 = XSteam('s_ph',P1,h1);
14 h2 = XSteam('h_ps',P2,s2); %201.925 kJ/kg
15 %% Determine h3
16 h3 = XSteam('h_pT',P3,T3); %3375.1 kJ/kg
17 %% Determine h4
18 s4 = XSteam('s_ph',P3,h3);
19 h4 = XSteam('h_ps',P4,s4); %2089.6 kJ/kg
20 %% Determine vapour mass fraction at 4
21 x4 = XSteam('x_ph',P4,h4); %0.7934
22 %% Work & heat input
23 w_tur = h3-h4; %1285.4 kJ/kg
24 w_pump = h2-h1; %10.1125 kJ/kg
25 w_net = w_tur-w_pump; %1275.3 kJ/kg
26 q_in = h3-h2; %3173.1 kJ/kg
27 %% Mass flow
28 m = W_net/w_net %164.67 kg/s
29 %% Efficiency
30 Q_in = m*q_in %522.5 kW
31 eta_th = W_net/Q_in %40.1%
32
33 %% Drawing a Ts diagram
34 T = linspace(0,375,1000);
35 s_1 = linspace(0.1,7,1000);
36 s_2 = linspace(0.1,9,1000);
37 for k = 1:length(T)
38 Tliq(k) = XSteam('sL_T',T(k));
39 Tvpap(k) = XSteam('sV_T',T(k));
40 isobar1(k) = XSteam('T_ps',P1,s_2(k));
41 isobar2(k) = XSteam('T_ps',P2,s_1(k));
42 end
43
44 s1 = XSteam('s_ph',P1,h1);
45 s3 = XSteam('s_ph',P3,h3);
46 T1 = XSteam('T_ph',P1,h1);
47 T2 = XSteam('T_ph',P2,h2);
48 T3 = XSteam('T_ph',P3,h3);
49 T4 = XSteam('T_ph',P4,h4);
50

```

```

51 figure(1), clf(1), hold on
52 title('Ts-diagram')
53 ylabel('T (degree Celsius)')
54 xlabel('s (kJ/kgK)')
55 plot(Tliq,T,'k'), plot(Tvap,T,'k')
56 plot([s1 s2 s3 s4 s1],[T1 T2 T3 T4 T1], 'ok')
57 plot(s_2,isobar1), plot(s_1,isobar2)
58 plot([s3 s4],[T3 T4]), plot([s1 s2],[T1 T2])
59 text(s1+0.1,T1-20,'1'), text(s2-0.1,T2+40,'2'), text(s3+0.15,T3,'3'),
60 text(s4+0.1,T4-20,'4'),
61 text(s3, T3+150,'P = 100 Bar'), text(s3+1.8, T1+200,'P = 0.1 Bar')
62 hold off

```

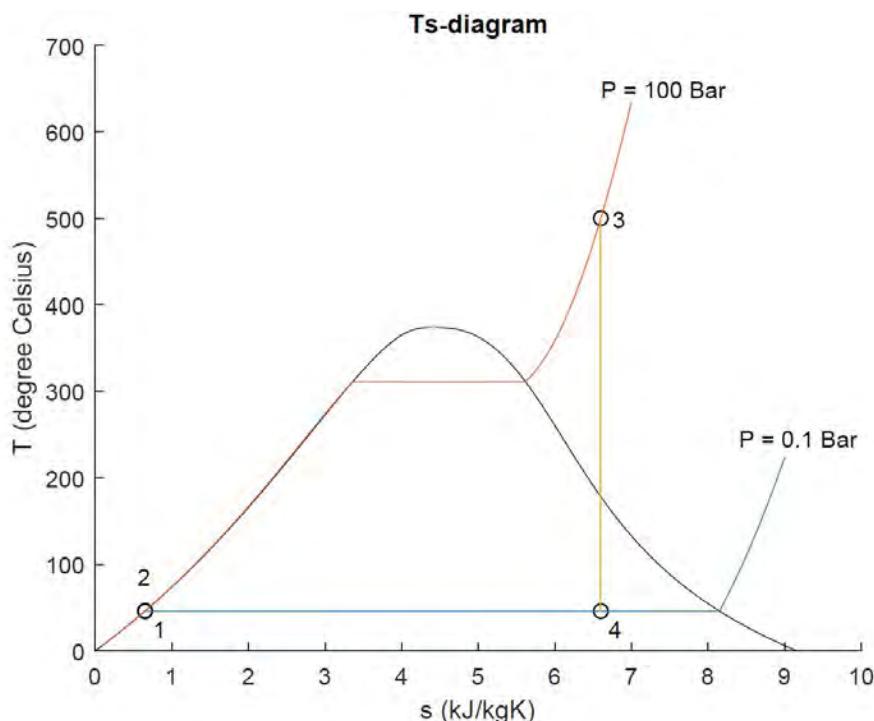


Figure 12.2.7: Ts-diagram of a simple ideal Rankine cycle.

Answer Matlab question 10.4, actual Rankine cycle

```

1 %% Power plant on an actual Rankine cycle , 10.4 using XSteam
2 %Optimization for turbine inlet temperature
3 clear , clf
4 %% Constants
5 P1 = 0.1; %bar
6 P2 = 100; %bar
7 P3 = P2;
8 P4 = P1;
9 T3 = linspace(XSteam( 'Tsat_p' ,P3) ,700 ,1000); %degrees C
10 eta2 = 0.85; %isentropic efficieny
11 eta4 = 0.85;
12 W_net = 210000; %kW
13 %% loop to determine optimal turbine inlet temperature
14 for i = 1:length(T3);
15 %% Determine h1
16 h1 = XSteam( 'hL_p' ,P1); %191.812 kJ/kg
17 %% Determine h2
18 s2s = XSteam( 's_ph' ,P1,h1);
19 h2s = XSteam( 'h_ps' ,P2,s2s); %201.925 kJ/kg
20 h2 = h1 + (h2s-h1)/eta2; %2013.710 kJ/kg
21 %% Determine h3
22 h3 = XSteam( 'h_pT' ,P3,T3(i)); %3375.1 kJ/kg
23 %% Determine h4
24 s4s = XSteam( 's_ph' ,P3,h3);
25 h4s = XSteam( 'h_ps' ,P4,s4s); %2089.6 kJ/kg
26 h4 = h3-eta4*(h3-h4s); %2282.5 kJ/kg
27 %% Determine vapour mass fraction at 4
28 x4 = XSteam( 'x_ph' ,P4,h4); %0.8740
29 %% Work & heat input
30 w_tur = h3-h4; %1092.6 kJ/kg
31 w_pump = h2-h1; %11.897 kJ/kg
32 w_net(i) = w_tur-w_pump; %1080.7 kJ/kg
33 q_in = h3-h2; %3171.3 kJ/kg
34 %% Efficiency
35 eta_th(i) = w_net(i)/q_in; %34.1%
36 %% Mass flow
37 m = W_net/(w_tur-w_pump); %194.31 kg/s
38 %% Power input
39 Q_in = m*q_in; %616.25 kW
40 %%
41 end
42 %Make graph Thermal efficiency
43 figure(1) , clf(1) , hold on
44 plot(T3,eta_th)
45 title('Efficiency as function of the temperature at the turbine inlet')
46 xlabel('Temperature at the turbine inlet (degree Celsius)')
47 ylabel('Thermal Effiency (%)')
48 grid on, grid minor, hold off
49 %With the function max you can find the maximum value of the efficiency
50 %and the indices (for finding at which value this occurs).

```

```

51 [ eta_th , index_T ] = max( eta_th ); %finding from every column the highest
52 optimum_T_eta_th = T3(index_T) %eta & its index
53 eta_th_max = eta_th
54 %Make graph Net specific work output
55 figure(2), clf(2), hold on
56 plot(T3,w_net)
57 title('Net specific work output as function of the temperature at the
      turbine inlet')
58 xlabel('Temperature at the turbine inlet (degree Celsius)')
59 ylabel('Net specific work output (kJ/kg)')
60 grid on, grid minor, hold off
61 %With the function max you can find the maximum value of the efficiency
62 %and the indices (for finding at which value this occurs).
63 [ w_net , index_T ] = max(w_net); %finding from every column the highest
64 optimum_T_w_net = T3(index_T) %w_net & its index
65 w_net_max = w_net

```

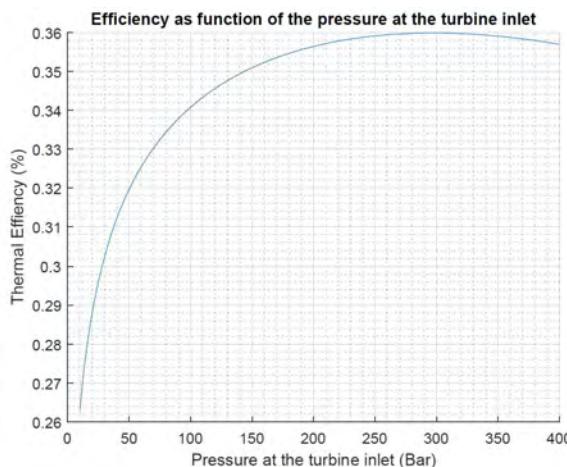


Figure 12.2.8: Graph showing the thermal efficiency as function of the pressure at the inlet of the turbine (isentropic efficiencies turbine and pump are 85%). Maximum efficiency, 35.99% at a turbine inlet pressure of $P_2 = 298$ Bar.

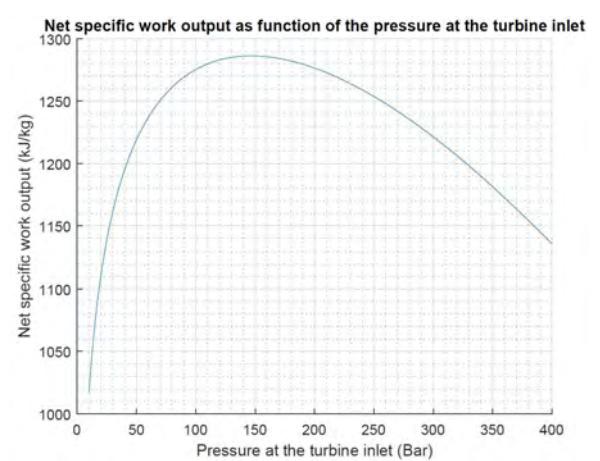


Figure 12.2.9: Graph showing the net specific work output as function of the pressure at the inlet of the turbine (isentropic efficiencies turbine and pump are 85%). Maximum net specific work output, 1088.55 kJ/kg at a turbine inlet pressure of $P_2 = 142$ Bar.

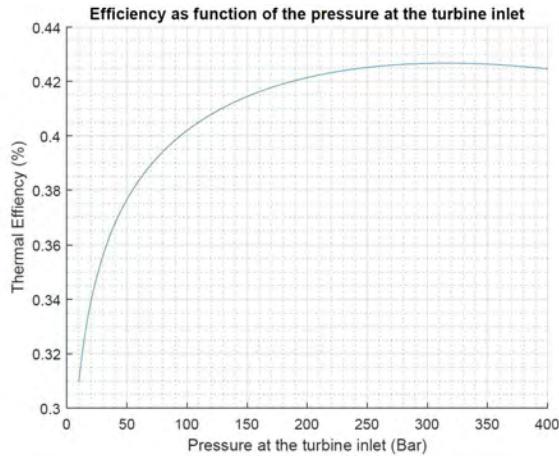


Figure 12.2.10: Graph showing the thermal efficiency as function of the pressure at the inlet of the turbine (isentropic efficiencies turbine and pump are 100%). Maximum efficiency, 42.67% at a turbine inlet pressure of $P_2 = 316$ Bar.

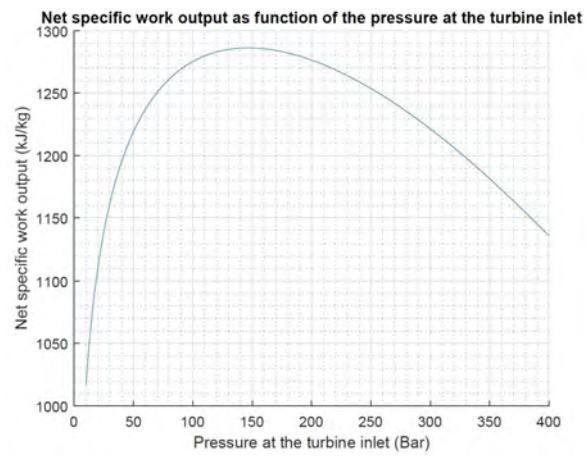


Figure 12.2.11: Graph showing the net specific work output as function of the pressure at the inlet of the turbine (isentropic efficiencies turbine and pump are 100%). Maximum net specific work output, 1286.25 kJ/kg at a turbine inlet pressure of $P_2 = 147$ Bar.

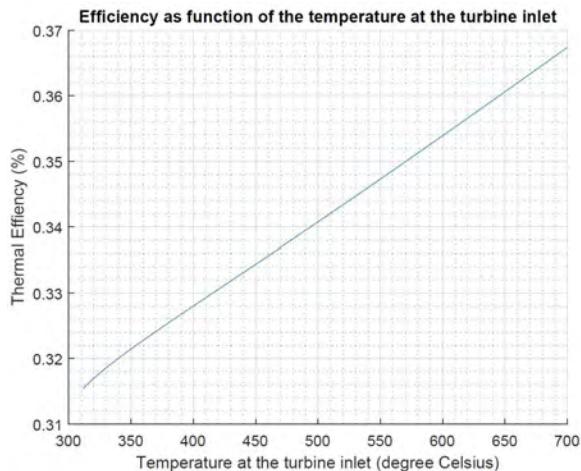


Figure 12.2.12: Graph showing the thermal efficiency as function of the temperature at the inlet of the turbine (isentropic efficiencies turbine and pump are 85%). There is no maximum efficiency, the higher the inlet temperature the higher the efficiency. Note: the maximum inlet temperature is restricted by the materials.

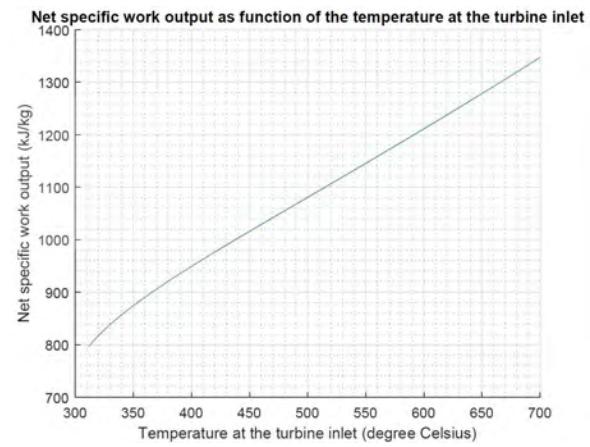


Figure 12.2.13: Graph showing the net specific work output as function of the temperature at the inlet of the turbine (isentropic efficiencies turbine and pump are 85%). There is no maximum efficiency, the higher the inlet temperature the higher the net specific work output. Note: the maximum inlet temperature is restricted by the materials.

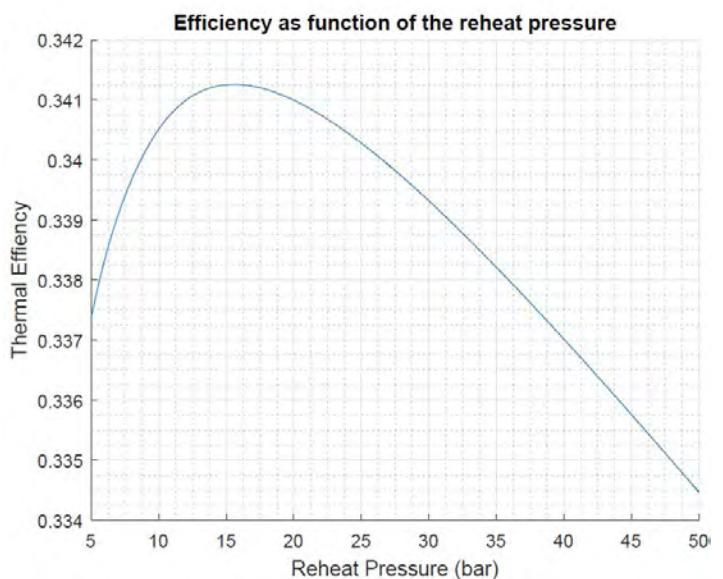
Answer Matlab question 10.9, reheat Rankine cycle

```

1  %% Optimization Reheat Rankine cycle , 10.9
2  %% Constants
3  P1 = 0.1;          %bar
4  P2 = 100;
5  P3 = P2;
6  P4 = linspace(5,50,100);
7  P5 = P4;
8  P6 = P1;
9  T3 = 500;          %degrees Celsius
10 T5 = T3;
11 eta2 = 0.95;       %isentropic efficiency
12 eta4 = 0.8;
13 eta6 = eta4;

14
15 for k = 1:length(P4)
16 %% Determine h1
17 h1 = XSteam('hL_p',P1);
18 %% Determine h2
19 s2s = XSteam('s_ph',P1,h1);
20 h2s = XSteam('h_ps',P2,s2s);
21 h2 = h1 + (h2s-h1)/eta2;
22 %% Determine h3
23 h3 = XSteam('h_pT',P3,T3);
24 %% Determine h4
25 s4s = XSteam('s_ph',P3,h3);
26 h4s = XSteam('h_ps',P4(k),s4s);
27 h4 = h3-eta4*(h3-h4s);
28 %% Determine h5
29 h5 = XSteam('h_pT',P5(k),T5);
30 %% Determine h6
31 s6s = XSteam('s_ph',P5(k),h5);
32 h6s = XSteam('h_ps',P6,s6s);
33 h6 = h5-eta4*(h5-h6s);
34 %% Determine efficiency
35 w_tur = (h3-h4) + (h5-h6);
36 w_pum = h2-h1;
37 w_net = w_tur-w_pum;
38 q_in = (h3-h2)+(h5-h4);
39 eta_th(k) = w_net/q_in;
40 end
41 figure(1),clf(1),hold on
42 plot(P4,eta_th)
43 title('Efficiency as function of the reheat pressure')
44 xlabel('Reheat Pressure (bar)')
45 ylabel('Thermal Efficiency')
46 grid on, grid minor, hold off
47 % answer = 15.57 bar

```



Answer Matlab question 10.11, reheat regenerative Rankine cycle

```

1 %% Optimization Reheat-regenerative Rankine cycle with open feedwater
2 heater , 10.11
3 clear , clc
4 %% Constants
5 P1 = 0.1; %bar
6 P2 = linspace(2,50,50);
7 P3 = P2;
8 P4 = 100;
9 P5 = P4;
10 P6 = P2;
11 P7 = P2;
12 P8 = P1;
13 P9 = P2;
14 T5 = 550; %degrees Celsius
15 T7 = 500;
16 eta2 = 0.8; %isentropic efficiency
17 eta4 = 0.8;
18 eta6 = 0.8;
19 eta8 = eta6;
20 m_tot = 100; %kg/s
21 m1 = linspace(20,90,50);
22 m2 = m_tot-m1;
23 for k = 1:length(m1);
24     for n = 1:length(P2)
25         %% Determine h1
26         h1 = XSteam('hL_p',P1);
27         %% Determine h2
28         s2s = XSteam('s_ph',P1,h1);
29         h2s = XSteam('h_ps',P2(n),s2s);
30         h2 = h1 + (h2s-h1)/eta2;
31         %% Determine h5
32         h5 = XSteam('h_pT',P5,T5);
33         %% Determine h6 = h9
34         s6s = XSteam('s_ph',P5,h5);
35         h6s = XSteam('h_ps',P6(n),s6s);
36         h6 = h5-eta6*(h5-h6s);
37         h9 = h6;
38         %% Determine h3
39         h3 = (m1(k)*h2+m2(k)*h9)/m_tot;
40         %% Determine h4
41         s4s = XSteam('s_ph',P3(n),h3);
42         h4s = XSteam('h_ps',P4,s4s);
43         h4 = h3 + (h4s-h3)/eta4;
44         %% Determine h7
45         h7 = XSteam('h_pT',P7(n),T7);
46         %% Determine h8
47         s8s = XSteam('s_ph',P7(n),h7);
48         h8s = XSteam('h_ps',P8,s8s);
49         h8 = h7-eta8*(h7-h8s);
50         %% Determine efficiency

```

```

50 W_tur = m_tot*(h5-h6) + m1(k)*(h7-h8);
51 W_pum = m1(k)*(h2-h1) + m_tot*(h4-h3);
52 W_net = W_tur-W_pum;
53 Q_in = m_tot*(h5-h4) + m1(k)*(h7-h6);
54 eta_th(n,k) = W_net/Q_in;
55 end
56 end
57 [P,M] = meshgrid(m1,P2);
58 figure(1), clf(1), hold on
59 surf(M,P,eta_th*100)
60 title('Efficiency')
61 ylabel('Mass flow through 2nd turbine (kg/s)')
62 xlabel('Pressure at reheating (bar)')
63 zlabel('Thermal Efficiency')
64 view(-70,20)
65 grid on, grid minor, axis equal, hold off
66 %With the function max you can find the maximum value of the efficiency
       and the indices (for finding at which value this occurs). The maximum
       efficiency is 37.78% at a pressure of 8.86 bar and with a m1 of 67.14 kg
       /s %
67 [etas,indices] = max(eta_th); %finding from every column the highest eta
       & its index
68 [eta,index_m] = max(etas); %finding the highest eta & the index of the
       mass
69 index_P = indices(index_m); %finding the index of the pressure
70 optimum_P = P2(index_P)      % Answer: P = 8.86 Bar
71 optimum_m = m1(index_m)     % Answer: m = 67.14 kg/s
72 eta_max = eta*100          % Answer: Eta = 37.8%

```

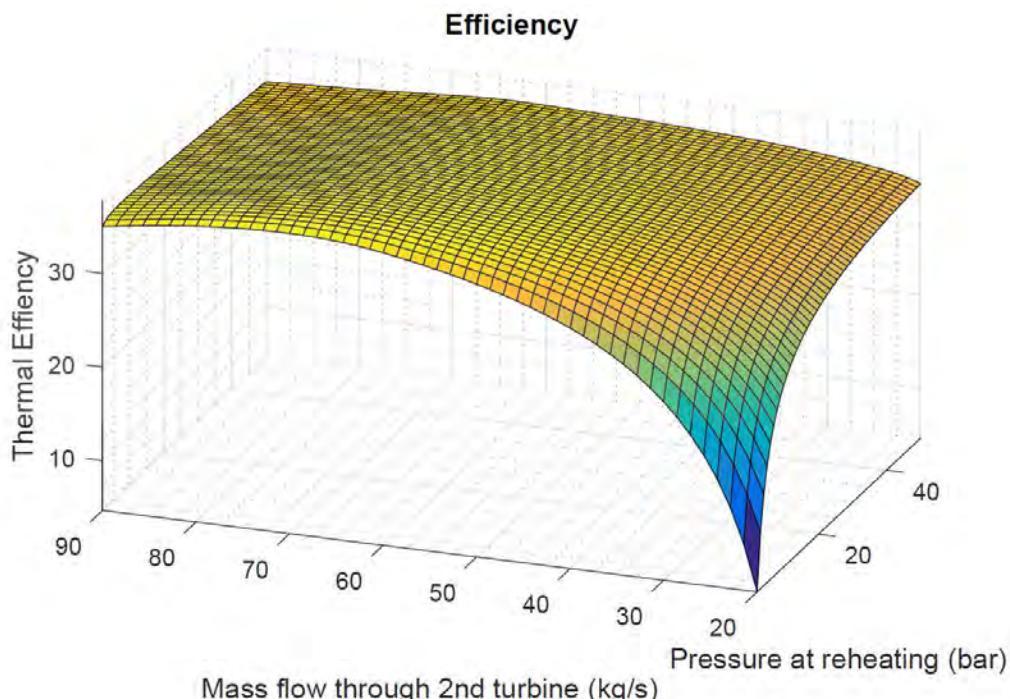


Figure 12.2.14: Graph to determine the optimal reheat pressure and mass flow of Matlab question 10.11.

Answer Matlab question 10.13, Combined cycle 1

```

1 %% 10.13 Combined gas-steam power cycle 1
2 clear ,clc
3 %% Constants
4 P1 = 1;
5 P2 = 8; %Bar
6 P3 = P2;
7 P4 = P1;
8 P5 = P1;
9 P6 = 0.05;
10 P7 = 70;
11 P8 = P7;
12 P9 = P6;
13 T1 = 300; %K
14 T3 = 1300; %K
15 T5 = 177+273; %K
16 T8 = 500; %Celcius
17 eta_sc = 0.8;
18 eta_st = 0.85;
19 R = 0.2870; %kJ/kgK
20 %% Determine h1
21 h1 = T1; %300 kJ/kg
22 %for temperatures in the range of 200 to 350 K the value of h is equal
23 %to the temperature value (see table A-17)
24 %% Determine T2s and h2s
25 %Note: k_2 should be taken at the average temperature between T1 and T2s,
26 %however T2s is not known before hand -> iterative process starting
27 %with k at room temperature (k = 1.4)
28 %It is important to have k as accurate as possible, k has a large
29 %influence
30 k_start = 1.4; %Start value of k = k at room temperature
31 T2s_start = T1*(P2/P1)^((k_start-1)/k_start);%T2s start iter. 543.43 K
32 Tav_1_2s = [0 (T1 + T2s_start)/2]; %Average T between comp. inlet/outlet
33 x=2;
34 %iteration loop
35 while abs(Tav_1_2s(x)-Tav_1_2s(x-1)) > 0.01
36 kav_1_2s = gasprop('air',Tav_1_2s(x))/(gasprop('air',Tav_1_2s(x))-R);
37 T2s = T1*(P2/P1)^((kav_1_2s-1)/kav_1_2s);
38 x=x+1;
39 Tav_1_2s(x) = (T1 + T2s)/2;
40 end
41 Tav_1_2s=Tav_1_2s(x); %418.81 K
42 % After this loop the correct average temperature is found. The accepted
43 %error is 0.01 K (look for the condition in the while loop). The
44 %corresponding value of kav_1_2s and T2s must be calculated once more:
45 kav_1_2s = gasprop('air',Tav_1_2s)/(gasprop('air',Tav_1_2s)-R);%1.3899
46 T2s = T1*(P2/P1)^((kav_1_2s-1)/kav_1_2s); %537.62 K
47 %Note: in the derivation of this formula Cp is taken independent of the
48 %temperature. This gives a small error in the results (for this case T2s
49 %should be 557 K).
50 %With the T2s found after the iteration process h2s can be calculated

```

```

51 h2s = h1 + gasprop('air',T1,T2s); %543.18 kJ/kg
52 %% Determine h2 and T2
53 h2 = h1 + (h2s-h1)/eta_sc; %603.98 kJ/kg
54 %To calculate T2 c_p should be calculated at the average temperature
55 %between T1 and T2 -> iterative process starting at the average
56 %temperature between T1 and T2s
57 cp_2 = [0 gasprop('air',Tav_1_2s)]; 
58 x=2;
59 %%iteration loop
60 while abs(cp_2(x)-cp_2(x-1))>0.0001
61 T2 = T1 + (h2-h1)/cp_2(x);
62 Tav_1_2 = (T1 + T2)/2;
63 x=x+1;
64 cp_2(x) = gasprop('air',Tav_1_2);
65 end
66 cp_2=cp_2(x); %1.0281 kJ/kgK
67 % After this loop the correct average c_p is found. The accepted error
68 %is 0.0001 kJ/kgK (look for the condition in the while loop).
69 %The corresponding value of T_2 and Tav_1_2 must be calculated once more:
70 T2 = T1 + (h2-h1)/cp_2; %595.68 K
71 Tav_1_2 = (T1 + T2)/2; %447.84 K
72 %% Determine h3
73 h3 = h2 + gasprop('air',T2,T3); %1396.3 kJ/kg
74 %% Determine T4s and h4s
75 %To determine T4s and h4 the same method is used as for T2s and h2s
76 T4s_start= T3*(P4/P3)^((k_start-1)/k_start); %T4s start iter. 717.66 K
77 Tav_3_4s = [0 (T3 + T4s_start)/2]; %Average T between turb. inlet/outlet
78 x=2;
79 %%iteration loop
80 while abs(Tav_3_4s(x)-Tav_3_4s(x-1)) > 0.01
81 kav_3_4s = gasprop('air',Tav_3_4s(x))/(gasprop('air',Tav_3_4s(x))-R);
82 T4s = T3*(P4/P3)^((kav_3_4s-1)/kav_3_4s);
83 x=x+1;
84 Tav_3_4s(x) = (T3 + T4s)/2;
85 end
86 Tav_3_4s=Tav_3_4s(x); %1035.7 K
87 % After this loop the correct average temperature is found. The accepted
88 %error is 0.01 K (look for the condition in the while loop). The
89 %corresponding value of kav_3_4s and T4s must be calculated once more:
90 kav_3_4s = gasprop('air',Tav_3_4s)/(gasprop('air',Tav_3_4s)-R); %1.3443
91 T4s = T3*(P4/P3)^((kav_3_4s-1)/kav_3_4s); %771.31 K
92 %Note: in the derivation of this formula Cp is taken independent of the
93 %temperature. This gives a small error in the results.
94 %With the T4s found after the iteration process h4s can be calculated
95 h4s = h3 - gasprop('air',T4s,T3); %792.39 kJ/kg
96 %% Determine h4 and T4
97 h4 = h3 - eta_st*(h3-h4s); %882.97 kJ/kg
98 %To calculate T4 c_p should be calculated at the average temperature
99 %between T3 and T4 -> iterative process starting at the average
100 %temperature between T4 and T4s
101 cp_4 = [0 gasprop('air',Tav_3_4s)];
102 x=2;

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103 %iteration loop
104 while abs( cp_4(x)-cp_4(x-1))>0.0001
105 T4 = T3 + (h4-h3)/cp_4(x);
106 Tav_3_4 = (T3 + T4)/2;
107 x=x+1;
108 cp_4(x) = gasprop('air',Tav_3_4);
109 end
110 cp_4=cp_4(x);                                %1.1512 kJ/kgK
111 %After this loop the correct average c_p is found. The accepted error
112 %is 0.0001 kJ/kgK (look for the condition in the while loop).
113 %The corresponding value of T_4 and Tav_3_4 must be calculated once more:
114 T4 = T3 + (h4-h3)/cp_4;                      %854.11 K
115 Tav_3_4 = (T3 + T4)/2;                        %1077.1 K
116 %% Determine h5
117 h5 = h1 + gasprop('air',T1,T5);             %452.38 kJ/kgK
118 %% Determine h6
119 h6 = XSteam('hL_p',P6);                     %137.77 kJ/kgK
120 %% Determine h7
121 s6 = XSteam('s_ph',P6,h6);                  %144.86 kJ/kgK
122 h7 = XSteam('h_ps',P7,s6);                  %3411.3 kJ/kgK
123 %% Determine h8
124 h8 = XSteam('h_pT',P8,T8);                  %2072.9 kJ/kgK
125 %% Determine h9
126 s8 = XSteam('s_ph',P8,h8);                  %0.1318
127 h9 = XSteam('h_ps',P9,s8);                  %48.57%
128 %% Determine mass ratios
129 ratio = (h4-h5)/(h8-h7);
130 %% Determine thermal efficiency
131 eta_th = (h3-h4-h2+h1+ratio*(h8-h9-h7+h6))/(h3-h2)*100;    %63.51%
132 %% Determine second law effiency
133 eta_2nd = eta_th/(1-((XSteam('T_ph',P9,h9)+273)/T3));      %63.51%

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