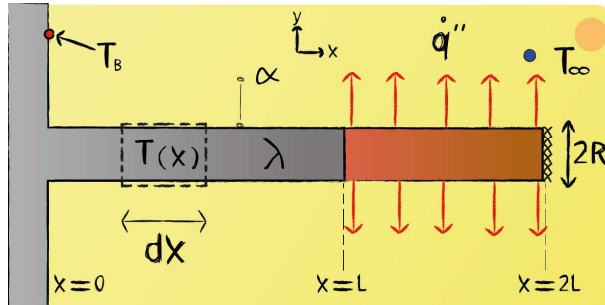


Lecture 11 - Question 8



Derive a homogeneous differential equation to describe the axial temperature distribution for the left fin. Assume one-dimensional, steady-state heat transfer in the x-direction with no sources/sinks.

Homogenization:

$$\Theta = T(x) - T_{\infty}$$

$$m^2 = \frac{2 \cdot \alpha}{\lambda \cdot R}$$

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} - \dot{Q}_{conv}(x) = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:



$$\dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \frac{\partial T}{\partial x} \rightarrow \dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \frac{\partial \Theta}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda \cdot \pi R^2 \cdot \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx \rightarrow \dot{Q}_{x,out} = \dot{Q}_{x,in} - \lambda \cdot \pi R^2 \cdot \frac{\partial^2 \Theta}{\partial x^2} dx$$

$$\dot{Q}_{conv} = \alpha \cdot 2\pi R dx \cdot (T(x) - T_{\infty}) \rightarrow \dot{Q}_{conv} = \alpha \cdot 2\pi R dx \cdot \Theta$$

Boundary Conditions:

$$\Theta(x=0) = T_B - T_{\infty}$$

$$\frac{\partial \Theta}{\partial x}(x=L) = -\frac{\dot{q}'' 2L}{\lambda R}$$

The first boundary condition results from the fact that $T(x=0) = T_B$ and the second one from the fact that $\dot{Q}_{cond}(x=L) = \dot{q}'' 2\pi RL$.

$$\frac{\partial^2 \Theta}{\partial x^2} - m^2 \Theta = 0$$