

# Energy & Heat Transfer

## Lecture 7

*By: Mohammad Mehrali*



# Recap of last lectures

## Heat Transfer Modes

### Conduction



- **Fourier Law**

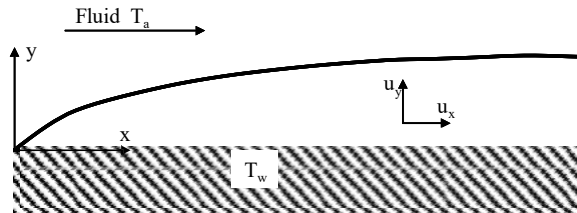
$$\dot{Q} = -kA \frac{\Delta T}{\Delta x} [W]$$

**Thermal  
Conductivity**  
[W/m.K]

Material properties

**Cross-  
Sectional Area**  
[m<sup>2</sup>]

### Convection



- **Newton's law of cooling**

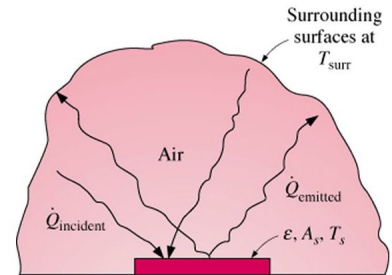
$$\dot{Q} = hA(T_w - T_a) [W]$$

Convective Heat  
Transfer Coefficient  
[W/m<sup>2</sup>K]

Flow dependent

- **Natural Convection**
- **Forced Convection**

### Radiation



- **Stefan-Boltzmann law**

$$\dot{Q} = \epsilon \sigma A (T_s^4 - T_\infty^4) [W]$$

Emissivity

Stefan-Boltzmann  
constant

$$\sigma = 5.670 \times 10^{-8} \frac{W}{m^2 K^4}$$

# STEP BY STEP PLAN

## Conduction:

1. Schematic (Steady state)
2. Negligible heat losses (insulation, adiabatic,...)
3. Geometry (flat plate, cylinder,...)
4. Arrangement of the layers
5. Resistance network
6. Calculation of resistances
7. Calculation of overall resistance
8. Calculation of heat flow
9. Calculation of temperatures for different layers

## Radiation:

1. Schematic (Steady state)
2. Negligible Radiations
3. Surrounding Temperature
4. Calculation of heat flow

## Convection:

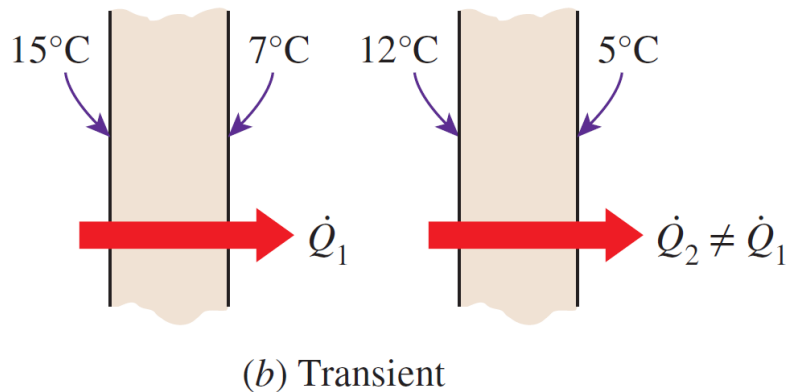
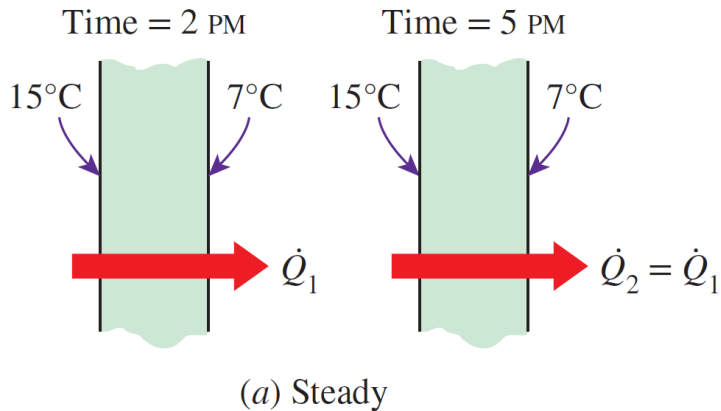
1. Schematic
2. Force of natural convection?
3. Geometry
4. Finding the film temperature
5. Finding ingredients from Table
6. Calculation of dimensionless numbers
7. Calculation of characteristic length
8. Finding correlations from tables to calculate Nu
9. Calculation of  $h$
10. Calculation of heat flow

# Learning objectives lecture 7

- **Time dependent heat transfer problems**
  - Unsteady/Transient heat transfer
  - Distinguish practical examples
  - “Derive” mathematical approximation
  - Determining validity of approximation



# UNSTEADY HEAT TRANSFER CONCEPT



- In **steady state heat transfer**, the **temperature** at any particular point in the system remains **constant** after equilibrium is attained.
- The **amount of heat entering** any section is then **equal** to the **amount of heat exiting** the section, because the driving force (temperature difference) is constant.
- In **unsteady state**, the **temperature** within an object itself keeps **changing with time**.
- The **heat entering a section** thus might **not be the same** as the **heat exiting the section**, as the temperature difference across the section keeps changing with time.

# Transient processes



$$t = 0 \text{ s}$$

$$T_{\infty} = 0^{\circ} \text{ C}$$

$$T_s = 25^{\circ} \text{ C}$$

$$\dot{Q} \approx 300 \text{ W}$$

$T_s$  drops -  $1^{\circ} \text{ C / min}$



$$t > 0 \text{ s}$$

$$T_{\infty} = 0^{\circ} \text{ C}$$

$$T_s = 12,5^{\circ} \text{ C}$$

$$\dot{Q} \approx 150 \text{ W}$$

$T_s$  drops -  $0.5^{\circ} \text{ C / min}$

- Assume: bottle has uniform temperature

- The lower the temperature difference is, the lower and slower the drop in temperature (difference) will be.

# Learning objectives lecture 7

- **Time dependent heat transfer problems**
  - Unsteady/Transient heat transfer
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  - “Derive” mathematical approximation
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# steady / transient

So far: steady state

- Constant in time
- Equilibrium (approximately)



Transient: time dependent

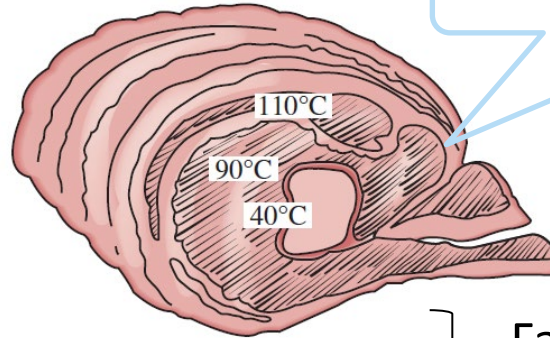
- Heating
- Cooling



# Learning objectives lecture 7

- Time dependent heat transfer problems
  - Unsteady/Transient heat transfer
  - Distinguish practical examples
  - “Derive” mathematical approximation
  - Determining validity of approximation

# UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS



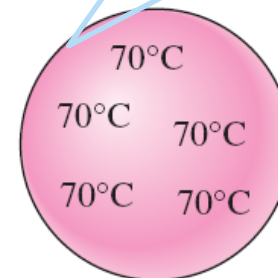
Temperature is a function of time and space

**Convection:** heat from outer layer  
**Conduction:** heat transferred from outer layer to core

Factors:  
 $h$ ,  $k$ ,  
geometry

- **Interior temperature** of some bodies remains essentially **uniform** at **all times** during a heat transfer process.
- The **temperature** of such bodies can be taken to be a **function of time only**,  $T(t)$ .
- Heat transfer analysis that utilizes this idealization is known as **lumped system analysis**.

Temperature is only function of Time



# UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

$$\left( \begin{array}{c} \text{Heat transfer into the body} \\ \text{during } dt \end{array} \right) = \left( \begin{array}{c} \text{The increase in} \\ \text{the of the body} \\ \text{energy during } dt \end{array} \right)$$

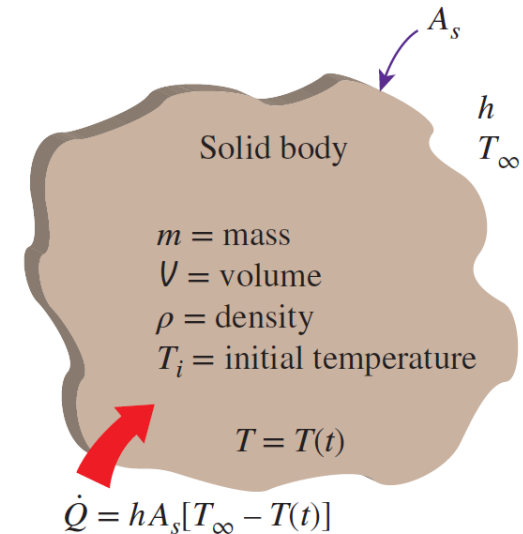
$$hA_s(T_\infty - T)dt = mc_p dT$$

$$m = \rho V \quad dT = d(T - T_\infty)$$

$$\frac{d(T - T_\infty)}{T - T_\infty} = \frac{hA_s}{\rho V c_p} dt$$

Integrating from  $t = 0$ , at which  $T = T_i$ , to any time  $t$ , at which  $T = T(t)$

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p} t$$



**The geometry and parameters involved in the lumped system analysis.**

# UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

$$\ln \frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = -\frac{hA_s}{\rho V c_p} t$$

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt}$$

Where :

$$b = \frac{hA_s}{\rho V c_p} \quad (1/s)$$

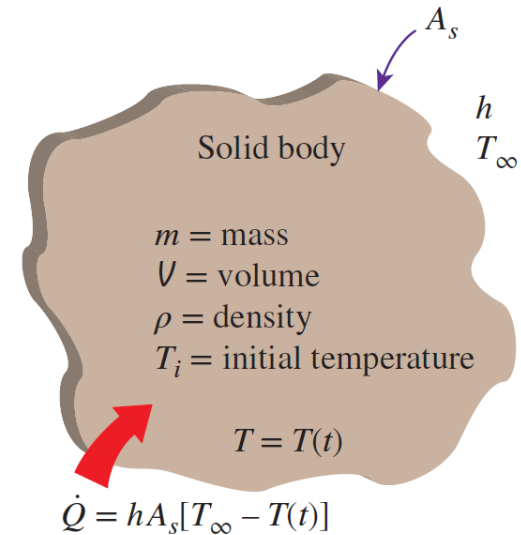
$h$  : heat transfer coefficient around object ( $W \cdot m^{-2} \cdot K^{-1}$ )

$A_s$ : external surface area ( $m^2$ )

$\rho$  : density of object ( $kg \cdot m^{-3}$ )

$c_p$  : specific heat of object ( $J \cdot kg^{-1} \cdot K^{-1}$ )

$V$  : volume of object, ( $m^3$ )



Only for lumped system analysis

# UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

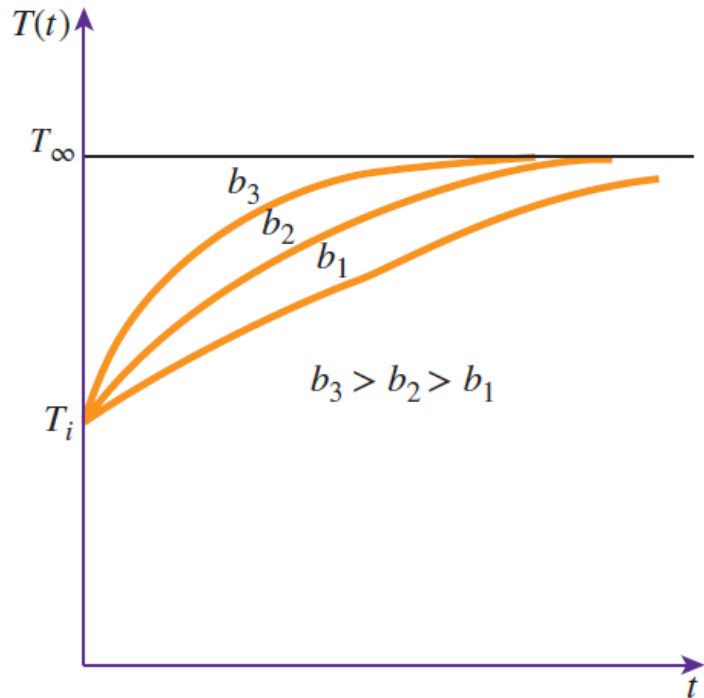
$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} \quad \text{Where} \quad b = \frac{hA_s}{\rho V c_p} \quad (1/s)$$

**What is this equation representing ?**

- This equation enables us to **determine** the **temperature  $T(t)$**  of a body **at time  $t$** , or alternatively, the **time  $t$  required** for the temperature to **reach** a specified **value  $T(t)$** .
- The **temperature of a body approaches** the ambient temperature  **$T_{\infty}$  exponentially**.
- What is the **effect of  $b$**  on duration ( $t$ ) of the temperature of a body to reach the ambient temperature  **$T_{\infty}$**  ?

# UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} \quad b = \frac{hA_s}{\rho V c_p} \quad (1/s)$$



- The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of  $b$  indicates that the body approaches the environment temperature in a short time.
- The temperature of a lumped system approaches the environment temperature as time gets larger.



# Activity(1)

## Predicting the Time of Death

A person is found dead at 5PM in a room whose temperature is  $20^{\circ}\text{C}$ . The temperature of the body is measured to be  $25^{\circ}\text{C}$  when found, and the heat transfer coefficient is estimated to be  $h = 8 \text{ W/m}^2 \cdot ^{\circ}\text{C}$ . Modelling the body as a 30-cm-diameter, 1.70-m-long cylinder, estimate the time of death of that person.



## Assumptions

- 1) The body can be modelled as a 30-cm-diameter, 1.70-m-long cylinder.
- 2) The thermal properties of the body and the heat transfer coefficient are constant.
- 3) The radiation effects are negligible.
- 4) The person was healthy(!) when he or she died with a body temperature of  $37^{\circ}\text{C}$ .
- 5) The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of  $(37+25)/2 = 31^{\circ}\text{C}$ ;  $k = 0.617 \text{ W/m} \cdot ^{\circ}\text{C}$ ,  $\rho = 996 \text{ kg/m}^3$ , and  $C_p = 4178 \text{ J/kg} \cdot ^{\circ}\text{C}$

# Activity(1)

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} \quad b = \frac{hA_s}{\rho V c_p}$$

$$A_s = 2\pi r^2 + 2\pi rL, \quad V = \pi r^2 L$$

$$\begin{aligned} b &= \frac{hA_s}{\rho V C_p} = \frac{8 \left( \frac{W}{m^2 \cdot ^\circ C} \right) \times (2\pi \times (0.15m)^2 + 2\pi \times (0.15m) \times 1.7m)}{996 \left( \frac{kg}{m^3} \right) \times (\pi \times (0.15m)^2 \times 1.7m) \times 4178 \left( \frac{J}{kg \cdot ^\circ C} \right)} \\ &= 2,79 \times 10^{-5} (s^{-1}) \end{aligned}$$

# Activity(1)

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} \longrightarrow \frac{25 - 20}{37 - 20} = e^{-(2,79 \times 10^{-5} \text{ s}^{-1})t}$$

$$\longrightarrow t = 43,860 \text{ s} = 12.2 \text{ h}$$

Therefore, as a rough estimate, the person died about 12 h before the body was found, and thus the time of death is

**5AM.**

# Learning objectives lecture 7

- Time dependent heat transfer problems
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  - Distinguish practical examples
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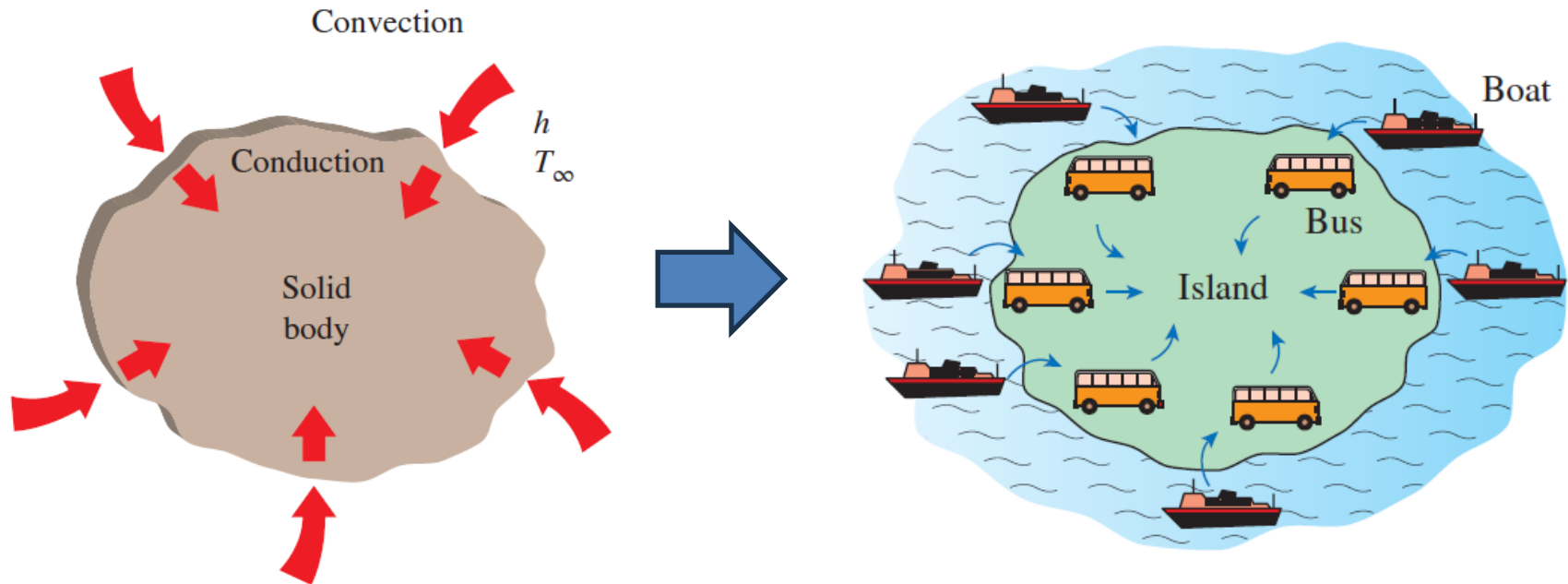
# UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} \quad b = \frac{hA_s}{\rho V c_p} \quad (1/s)$$

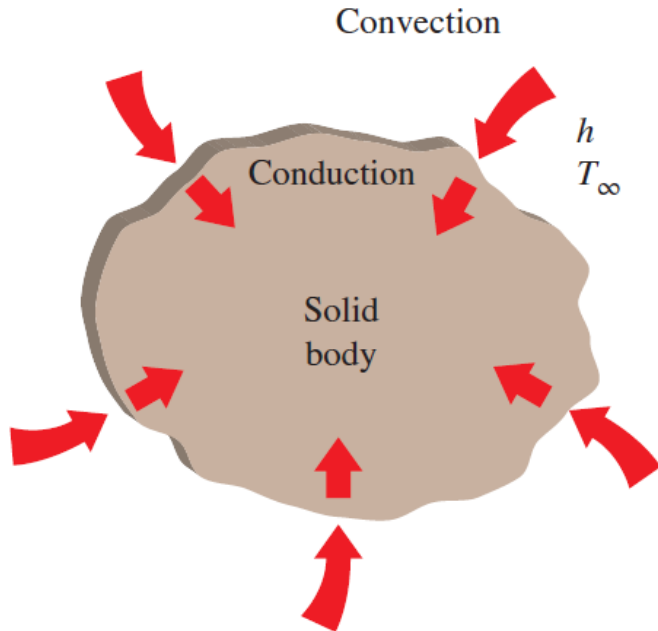
But when is this equation applicable ?

What is the criterion ?

# LUMPED SYSTEM



# BIOT NUMBER (Bi)



$$Bi = \frac{\text{heat convection}}{\text{heat conduction}}$$

$$Bi = \frac{h}{k/L_c} \frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

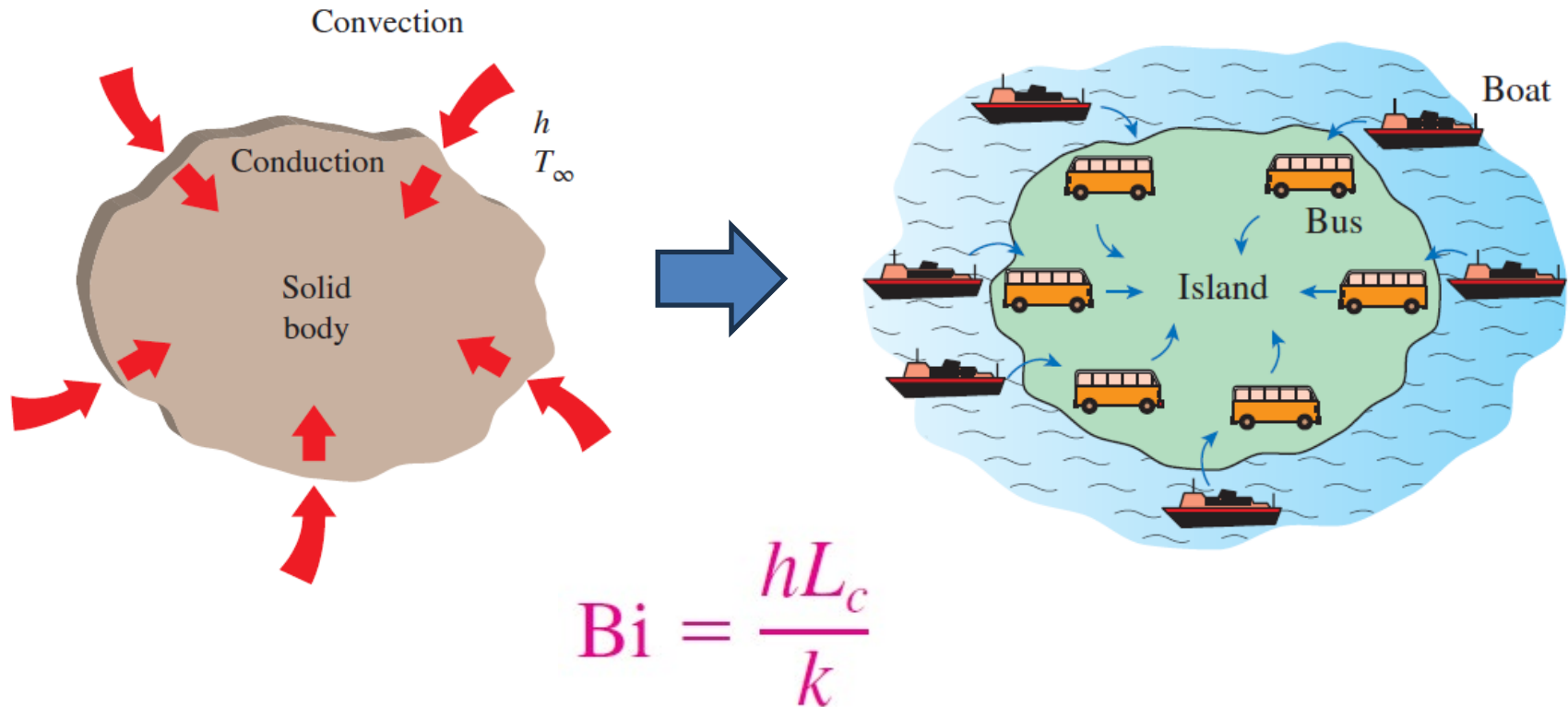
or

$$Bi = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$$Bi = \frac{hL_c}{k} \quad \text{When characteristic length is : } L_c = \frac{V}{A_s}$$

Almost uniform temperature for  $Bi \leq 0,1$   
“lumped system”

# LUMPED SYSTEM



- Relatively high  $h \rightarrow \dots \rightarrow Bi$  high, non uniform
- Relatively high  $L_c = V/A \rightarrow \dots \rightarrow Bi$  high, non uniform
- Relatively high  $k \rightarrow \dots \rightarrow Bi$  low, uniform



# BIOT NUMBER (Bi)

$$L_c = \text{Characteristic length} = \frac{\text{Volume of the solid (V)}}{\text{Surface area of the solid (A}_s\text{)}}$$

The values of characteristic length ( $L_c$ ), for simple geometric shapes, are given below:

$$\text{Flat plate : } L_c = \frac{V}{A_s} = \frac{LBH}{2BH} = L/2 = \text{semi-thickness}$$

where  $L$ ,  $B$  and  $H$  are thickness, width and height of the plate.

$$\text{Cylinder (long) : } L_c = \frac{\pi R^2 L}{2\pi RL} = \frac{R}{2} \quad \text{where, } R = \text{radius of the cylinder.}$$

$$\text{Sphere: } L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} \quad \text{where, } R = \text{radius of the sphere.}$$

$$\text{Cube: } L_c = \frac{L^3}{6L^2} = \frac{L}{6} \quad \text{where, } L = \text{Side of the cube.}$$

# LUMPED SYSTEM

- i. Lumped system analysis is exact when  $Bi = 0$  and approximate when  $Bi > 0$ . Of course, the smaller the  $Bi$  number, the more accurate the lumped system analysis.
- ii. The first step in the application of lumped system analysis is the calculation of the Biot number, and the assessment of the applicability of this approach.
- iii. One may still wish to use lumped system analysis even when the criterion  $Bi < 0.1$  is not satisfied, if high accuracy is not a major concern.
- iv. Note that the Biot number is the ratio of the convection at the surface to conduction within the body, and this number should be as small as possible for lumped system analysis to be applicable.
- v. Small bodies with high thermal conductivity are good candidates for lumped system analysis, especially when they are in a medium that is a poor conductor of heat (such as air or another gas) and motionless. Thus, the hot small copper ball placed in quiescent air, is most likely to satisfy the criterion for lumped system analysis.

# Nusselt vs. Biot

Nusselt number

$$Nu = \frac{h L_c}{k}$$



≠



Biot number

$$Bi = \frac{h L_c}{k}$$

Dimensionless measure for convection so increase of heat transfer due to flow

*k of fluid!*

Dimensionless measure for degree of temperature distribution within body

*k of surrounded object!*

Definitions seem similar but are substantially different!

# Lumped or not?



$$h = 15 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$$

$$k = 0,6 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$$

$$V = 0,45 \text{ L} = 4,5 \cdot 10^{-4} \text{ m}^3$$

$$A \approx 7,5 \cdot 10^{-2} \text{ m}^2$$

$$\left. \begin{array}{l} V = 0,45 \text{ L} = 4,5 \cdot 10^{-4} \text{ m}^3 \\ A \approx 7,5 \cdot 10^{-2} \text{ m}^2 \end{array} \right\} L_c \approx 0,006 \text{ m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \times 0,006}{0,6} = 0,15 > 0,1$$

**So no lumped system!**



$$\longleftrightarrow$$
$$D \approx 0,06 \text{ m}$$

$$h = 5 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$$
$$k = 0,6 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$$

$$L_c = \frac{V}{A} = \frac{\frac{1}{6} \pi D^3}{\pi D^2} = \frac{D}{6} = 0,01 \text{ m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{5 \cdot 0,01}{0,6} = 0,083 < 0,1$$

**So "lumped system"**

# Activity(2)

## Predicting the Time of Death

A person is found dead at 5PM in a room whose temperature is  $20^{\circ}\text{C}$ . The temperature of the body is measured to be  $25^{\circ}\text{C}$  when found, and the heat transfer coefficient is estimated to be  $h = 8 \text{ W/m}^2 \cdot ^{\circ}\text{C}$ . Modelling the body as a 30-cm-diameter, 1.70-m-long cylinder.



## Lumped or not ?

### Assumptions

- 1) The body can be modelled as a 30-cm-diameter, 1.70-m-long cylinder.
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- 5) The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of  $(37+25)/2 = 31^{\circ}\text{C}$ ;  $k = 0.617 \text{ W/m} \cdot ^{\circ}\text{C}$ ,  $\rho = 996 \text{ kg/m}^3$ , and  $C_p = 4178 \text{ J/kg} \cdot ^{\circ}\text{C}$

# Activity(2)

$$A_s = 2\pi r^2 + 2\pi rL, \quad V = \pi r^2 L$$

$$L_c = \frac{V}{A_s} = \frac{\pi \times (0.15m)^2 \times 1.7m}{2\pi \times (0.15m)^2 + 2\pi \times (0.15m) \times 1.7m}$$
$$= 0,0689(m)$$

Then the Biot number becomes :

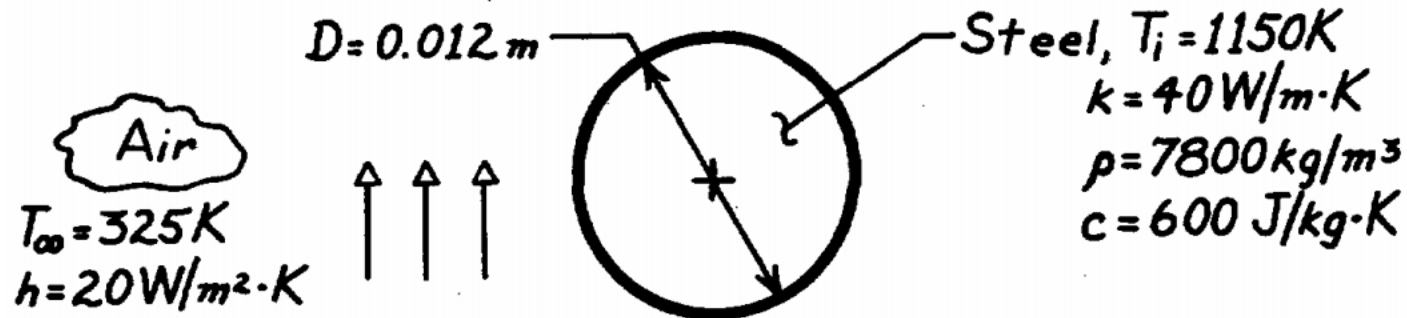
$$Bi = \frac{hL_c}{k} = \frac{8 \left( \frac{W}{m^2 \cdot ^\circ C} \right) \times 0,0689m}{0,617 \left( \frac{W}{m \cdot ^\circ C} \right)} = 0.89 > 0,1$$

Therefore, lumped system analysis is *not* applicable. However, we can still use it to get a “rough” estimate of the time of death.

# Activity(3)

**KNOWN:** Diameter and initial temperature of steel balls cooling in air.

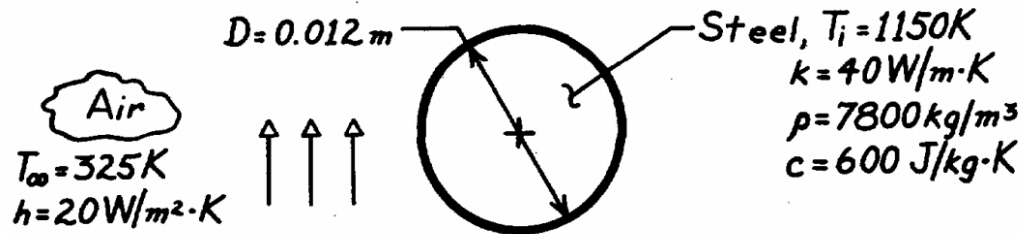
**FIND:** Time required to cool to 400K.



## Assumptions

- (1) Negligible radiation effects.
- (2) Constant properties.

# Activity(3)



## ANALYSIS:

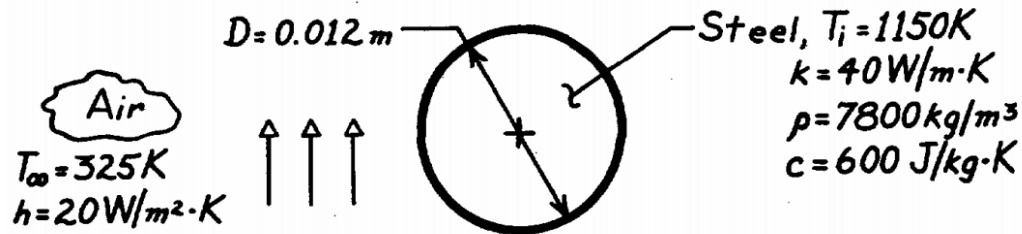
Calculating Biot number :  $Bi = \frac{hL_c}{k}$  Characteristic Length for sphere :  $L_c = r_o/3$

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{20\text{ W/m}^2\cdot\text{K} (0.002\text{ m})}{40\text{ W/m}\cdot\text{K}} = 0.001.$$

Hence, the temperature of the steel remains approximately uniform during the cooling process, and the lumped capacitance method may be used.



# Activity(3)



## ANALYSIS:

Calculating time :  $\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad b = \frac{hA_s}{\rho V c_p}$

$$t = \frac{\rho V c_p}{h A_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{\rho \left( \pi D^3 / 6 \right) c_p}{h \pi D^2} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

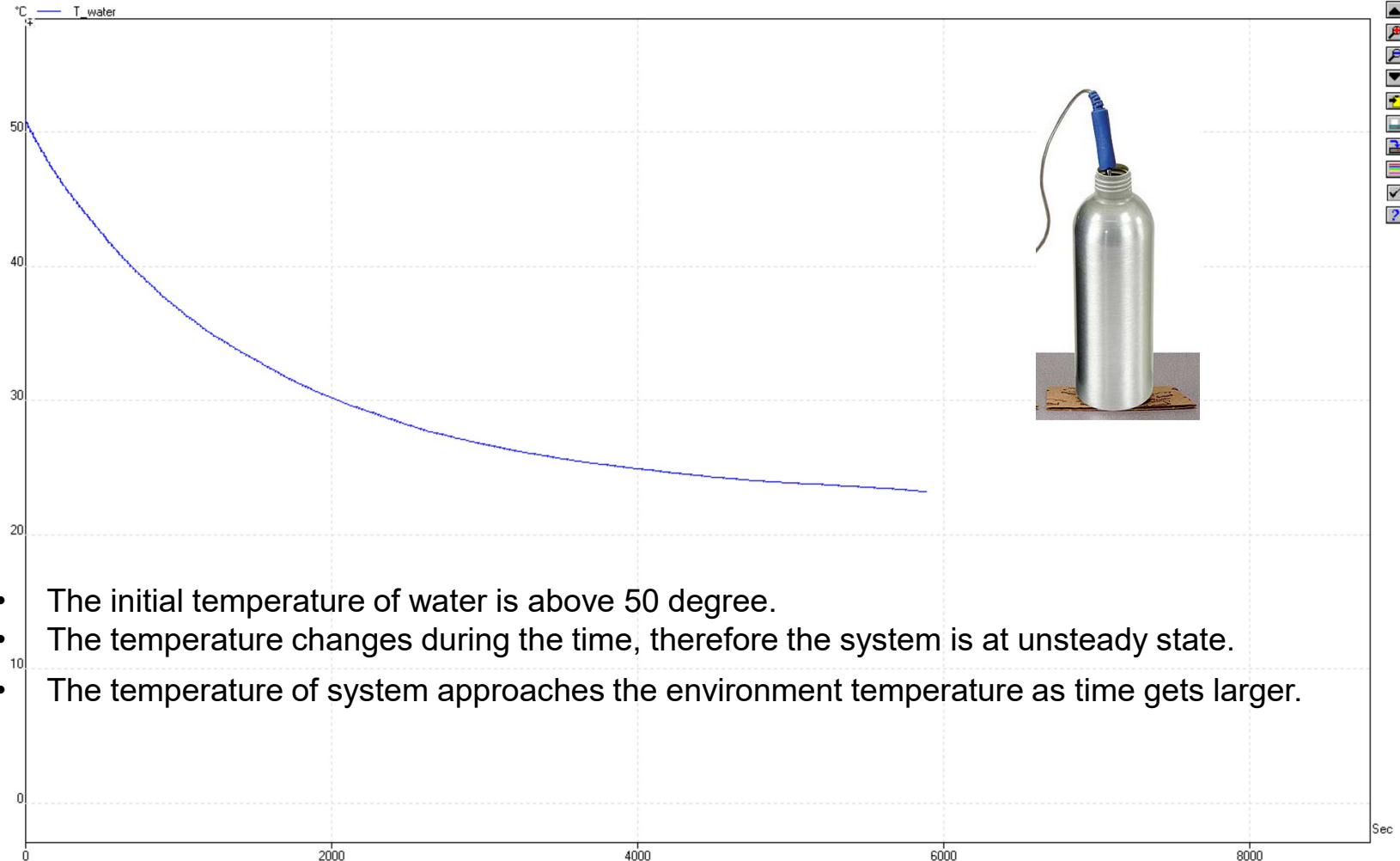
$$t = \frac{7800 \text{ kg/m}^3 (0.012 \text{ m}) 600 \text{ J/kg} \cdot \text{K}}{6 \times 20 \text{ W/m}^2 \cdot \text{K}} \ln \frac{1150 - 325}{400 - 325}$$

$$t = 1122 \text{ s} = 0.312 \text{ h}$$



# Temperature transient analysis of a hot water

Logging Graph  
t=7415 y=58.24



- The initial temperature of water is above 50 degree.
- The temperature changes during the time, therefore the system is at unsteady state.
- The temperature of system approaches the environment temperature as time gets larger.

# Summary transient processes

- Biot number  $Bi = \frac{h L_c}{k} \quad (-)$
- For  $Bi \leq 0,1$ : “lumped system” with  $L_c = \frac{V}{A} \quad (\text{m})$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V c_p}$$

- For  $Bi > 0,1$  : different approaches
- Approximations: check validity

# Reference



Y. A. Cengel & A. J. Ghajar. Heat and Mass Transfer: Fundamental & Application:

Chapter 4: Transient Heat Conduction

4–1 : LUMPED SYSTEM ANALYSIS

Criteria for Lumped System Analysis

Some Remarks on Heat Transfer in Lumped Systems