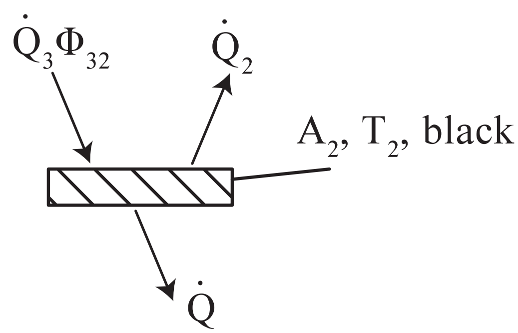
Chapter 1

Solutions radiative heat tansfer

1.1 Cupola

a) *Compute the amount of heat transferred through radiation between the surfaces A_1 and A_2.*

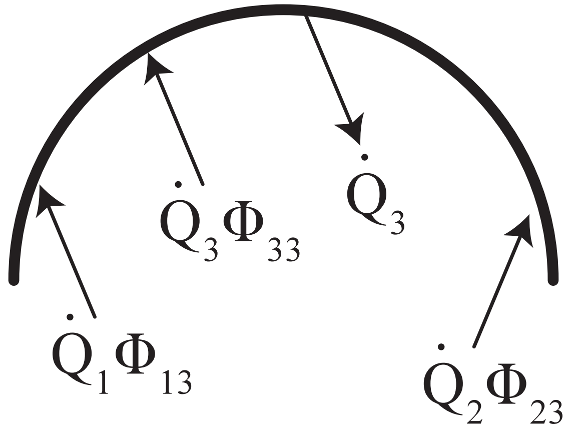


Energy balance around body 2:

$$\dot{Q} = \dot{Q}_3 \cdot \Phi_{32} - \dot{Q}_2 \tag{1.1}$$

Surface brightness body 2:

$$\dot{Q}_2 = \sigma \cdot T_2^4 \cdot \frac{\pi \cdot R^2}{2} = 5.67 \cdot 10^{-8}\ [\mathrm{W/m^2K^4}] \cdot 293.15^4\ [\mathrm{K^4}] \cdot \frac{\pi \cdot 3^2}{2}\ [\mathrm{m^2}] = 5920\ [\mathrm{W}] \tag{1.2}$$



Energy balance around body 3:

$$\dot{Q}_3 = \dot{Q}_1 \cdot \Phi_{13} + \dot{Q}_2 \cdot \Phi_{23} + \dot{Q}_3 \cdot \Phi_{33} \tag{1.3}$$

$$\Rightarrow \dot{Q}_3 = \frac{\dot{Q}_1 \cdot \Phi_{13} + \dot{Q}_2 \cdot \Phi_{23}}{1 - \Phi_{33}} \tag{1.4}$$

Surface brightness body 1 (note that we are dealing with an grey body and thus $\epsilon_1 = \alpha_1 = 0.6$ and that $\rho_1 = 1 - \alpha_1 - \cancelto{0}{\tau_1} = 0.4$):

$$\dot{Q}_1 = \epsilon_1 \cdot \sigma \cdot T_1^4 \cdot A_1 + \rho_1 \cdot \dot{Q}_3 \cdot \Phi_{31} \tag{1.5}$$

Sum rule:

$$\cancelto{0}{\Phi_{11}} + \cancelto{0}{\Phi_{12}} + \Phi_{13} = 1 \tag{1.6}$$

$$\cancelto{0}{\Phi_{21}} + \cancelto{0}{\Phi_{22}} + \Phi_{23} = 1 \tag{1.7}$$

$$\Phi_{31} + \Phi_{32} + \Phi_{33} = 1 \tag{1.8}$$

We know that $\Phi_{31} = \Phi_{32}$ due to symmetry.

Reciprocity rule:

$$\Phi_{31} A_3 = \Phi 13 A_1 \tag{1.9}$$

Rearranging and inserting:

$$\Rightarrow \Phi_{31} = \frac{A_1}{A_3} = \frac{\frac{R^2\pi}{2}}{\frac{4\pi R^2}{2}} = \frac{1}{4} \tag{1.10}$$

$$\Rightarrow \Phi_{32} = \Phi_{31} = \frac{1}{4} \tag{1.11}$$

$$\Rightarrow \Phi_{33} = 1 - \Phi_{31} - \Phi_{32} = \frac{1}{2} \tag{1.12}$$

Inserting Equation 1.5 into 1.3 results in:

$$\dot{Q}_3 = \frac{\left(\epsilon_1 \sigma T_1^4 A_1 + \rho_1 \dot{Q}_3 \Phi_{31}\right) \cancelto{1}{\Phi_{13}} + \dot{Q}_2 \cancelto{1}{\Phi_{23}}}{1 - \Phi_{33}} \tag{1.13}$$

$$\dot{Q}_3 = \frac{\epsilon_1 \sigma T_1^4 A_1 + \dot{Q}_2}{1 - \Phi_{33} - \rho_1 \cdot \Phi_{31}} \tag{1.14}$$

Substitution of Equation 1.14 into 1.1:

$$\begin{aligned} \dot{Q} &= \frac{\epsilon_1 \sigma T_1^4 A_1 + \dot{Q}_2}{1 - \Phi_{33} - \rho_1 \cdot \Phi_{31}} \cdot \Phi_{32} - \dot{Q}_2 \\ &= \frac{0.6 \cdot 5.67 \cdot 10^{-8}\ [\mathrm{W/m^2K^4}] 423.15^4\ [\mathrm{K^4}] \cdot \frac{1}{2}\pi \cdot 3^2\ [\mathrm{m^2}] + 5920\ [\mathrm{W}]}{1 - \frac{1}{2} - 0.4 \cdot \frac{1}{4}} \cdot \frac{1}{4} - 5920\ [\mathrm{W}] = 7417\ [\mathrm{W}] \end{aligned} \tag{1.15}$$

b) *Which temperature T_3 is obtained for surface A_3.*

Due to the fact that body 3 is gray and adiabatic it acts like a black body. Therefore:

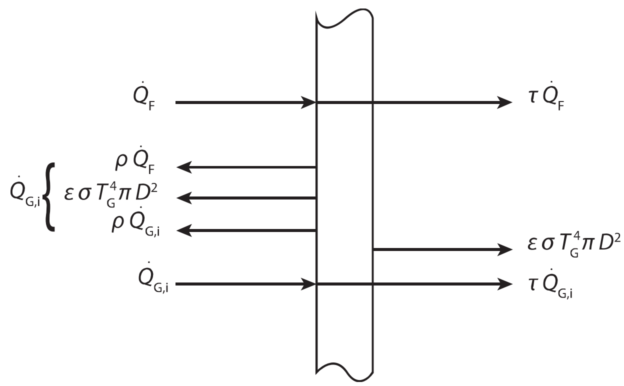
$$\dot{Q}_3 = \frac{\epsilon_1 \sigma T_1^4 A_1 + \dot{Q}_2}{1 - \Phi_{33} - \rho_1 \cdot \Phi_{31}} \cdot \Phi_{32} = A_3 \sigma T_3^4 \tag{1.16}$$

$$\begin{aligned} \Rightarrow T_3 &= \sqrt[4]{\frac{1}{A_3 \sigma} \frac{\epsilon_1 \sigma T_1^4 A_1 + \dot{Q}_2}{1 - \Phi_{33} - \rho_1 \cdot \Phi_{31}} \cdot \Phi_{32}} \\ &= \sqrt[4]{\frac{1}{\frac{4\pi 3^2}{2}\ [\mathrm{m^2}] 5.67 \cdot 10^{-8}\ [\mathrm{W/m^2K^4}]} \frac{0.6 \cdot 5.67 \cdot 10^{-8}\ [\mathrm{W/m^2K^4}]\ 423.15\ [\mathrm{K^4}] \frac{\pi 3^2}{2}\ [\mathrm{m^2}] + 5920\ [\mathrm{W}]}{1 - \frac{1}{2} - 0.4 \cdot \frac{1}{4}} \cdot \frac{1}{4}} \\ &= 359\ [\mathrm{K}] \end{aligned} \tag{1.17}$$

1.2 Light bulb

a) *Provide the energy balance for determining the glass temperature T_{G}, while neglecting radiation from the environment.*

Since the surface of the filament in comparison to the glass body is small ($\Phi_{\mathrm{GG,inner}} = 1$), all reflected and emitted radiation on the inside of the glass ball (surface brightness $\dot{Q}_{\mathrm{G,i}}$) interacts only with the glass body:



The outer energy balance for the glass is:

$$0 = \dot{Q}_{\mathrm{D}} + \dot{Q}_{\mathrm{G,i}} - \dot{Q}_{\mathrm{G,i}} - \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} - \tau\left(\dot{Q}_{\mathrm{D}} + \dot{Q}_{\mathrm{G,i}}\right) \quad \Rightarrow \quad \dot{Q}_{\mathrm{D}} = \frac{\epsilon\sigma T_{\mathrm{G}}^4 \pi D^2 + \tau\dot{Q}_{\mathrm{G,i}}}{1-\tau} \tag{1.18}$$

As an alternate solution, one can also use the inner energy balance for the glass:

$$0 = \alpha\left(\dot{Q}_{\mathrm{D}} + \dot{Q}_{\mathrm{G,i}}\right) - 2\epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} \tag{1.19}$$

With the surface brightness $\dot{Q}_{\mathrm{G,i}}$:

$$\dot{Q}_{\mathrm{G,i}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\left(\dot{Q}_{\mathrm{D}} + \dot{Q}_{\mathrm{G,i}}\right) \quad \Rightarrow \quad \dot{Q}_{\mathrm{G,i}} = \frac{\epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{D}}}{1-\rho} \tag{1.20}$$

1.3 Doner Kebap

a) *Determine the minimal distance x between the doner kebap and the electric heater so that the critical temprature T_{C} is not exceeded at the surface.*

The internal balance arround the doner kebap gives:

$$\dot{Q}_{\mathrm{D},\varepsilon} = \dot{Q}_{\mathrm{H}}\Phi_{\mathrm{HD}}\alpha_{\mathrm{D}} = \dot{Q}_{\mathrm{H}}\Phi_{\mathrm{HD}}\varepsilon_{\mathrm{D}} \tag{1.21}$$

The doner kebap radiates as a grey body and diffuses:

$$\dot{Q}_{\mathrm{D},\varepsilon} = \varepsilon_{\mathrm{D}}\sigma A_{\mathrm{D}}T_{\mathrm{D}}^4 \tag{1.22}$$

Applying the reciprocity rule yields:

$$\Phi_{\mathrm{HD}} = \frac{A_{\mathrm{D}}}{A_{\mathrm{H}}}\Phi_{\mathrm{DH}} = \frac{A_{\mathrm{D}}}{A_{\mathrm{H}}}a\exp(-bx) \tag{1.23}$$

Inserting this into the energy balance equation gives:

$$\varepsilon_{\mathrm{D}}\sigma A_{\mathrm{D}}T_{\mathrm{D}}^4 = \dot{Q}_{\mathrm{H}}\frac{A_{\mathrm{D}}}{A_{\mathrm{H}}}a\exp(-bx) \tag{1.24}$$

Rearranging the equation to give x:

$$x = \frac{1}{b}\ln\left(\frac{a\dot{Q}_{\mathrm{H}}}{\sigma A_{\mathrm{H}}T_{\mathrm{C}}^4}\right) \tag{1.25}$$

Alternative solution with external energy balance and surface brightness arround the Döner pack:

$$\dot{Q}_{\mathrm{D},\varepsilon} = \dot{Q}_{\mathrm{H}}\Phi_{\mathrm{HS}}\dot{Q}_{\mathrm{D},\varepsilon} = \varepsilon_{\mathrm{S}}\sigma A_{\mathrm{S}}T_{\mathrm{S}}^4 + (1-\varepsilon_{\mathrm{S}})\left(\dot{Q}_{\mathrm{H}}\Phi_{\mathrm{HS}}\right) \tag{1.26}$$

1.4 Solar Cell

a) *Determine the efficiency η of the thermal collector, and use the given numerical values to calculate the results.*

The efficiency of the solar collector describes the ratio of heat flow $\dot{Q}_{\text{Water}}$, which is added to the water for heating, and the radiant heat flux $\dot{Q}_{\text{rad}}$ on the solar collector.:

$$\eta = \frac{\dot{Q}_{\text{Water}}}{\dot{Q}_{\text{rad}}} \tag{1.27}$$

Since the solar collector is inclined by the angle α relative to the direction of the solar radiation, to calculate the incident radiative heat flux $\dot{Q}_{\text{rad}}$ from the radiance $\dot{q}''$ that is projected on A_{proj} of the collector can be calculated as:

$$\dot{Q}_{\text{rad}} = \dot{q}'' \cdot A_{\text{proj}} = \dot{q}'' \cdot A \cdot \cos\alpha = 2.569\,\text{kW} \tag{1.28}$$

The heat absorbed by the water can be determined using a global energy balance around the water:

$$H'_{\text{Water}} - H''_{\text{Water}} + \dot{Q}_{\text{Water}} = 0 \Leftrightarrow \dot{Q}_{\text{Water}} = \dot{m} \cdot c_p \cdot (T'' - T') = 2.424\,\text{kW} \tag{1.29}$$

Knowing these two heat fluxes, the efficiency can now be calculated:

$$\eta = \frac{\dot{Q}_{\text{Water}}}{\dot{Q}_{\text{rad}}} = 0.943 = 94.3\,\% \tag{1.30}$$