Energy & Heat Transfer

Lecture 6

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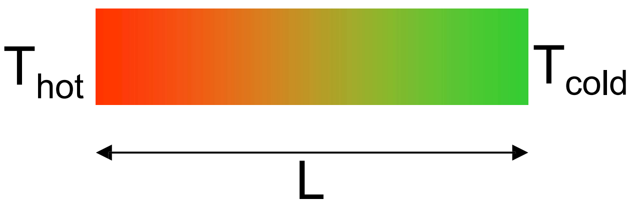
Recap of last lectures

Heat Transfer Modes

Conduction



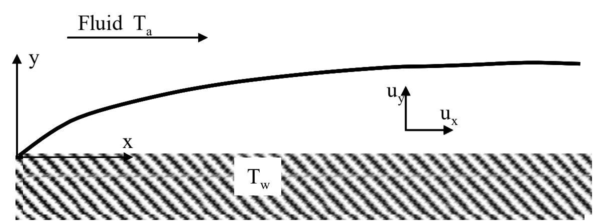
- **Fourier Law**

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x}\,[W]$$

Thermal Conductivity [W/m.K]
Material properties

Cross-Sectional Area $[m^2]$

Convection



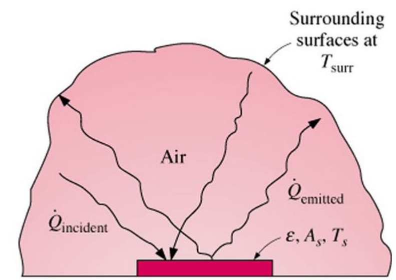
- **Newton's law of cooling**

$$\dot{Q} = hA(T_w - T_a)\,[W]$$

Convective Heat Transfer Coefficient
[W/m²K]
Flow dependent

- **Natural Convection**
- **Forced Convection**

Radiation



- **Stefan-Boltzmann law**

$$\dot{Q} = \varepsilon\sigma A(T_s^4 - T_\infty^4)[W]$$

Emissivity

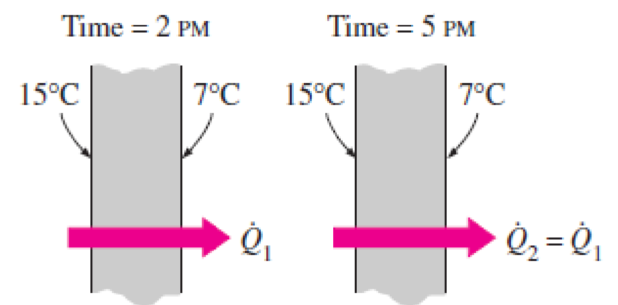
Stefan-Boltzmann constant

$$\sigma = 5.670 \times 10^{-8}\ \frac{W}{m^2K^4}$$

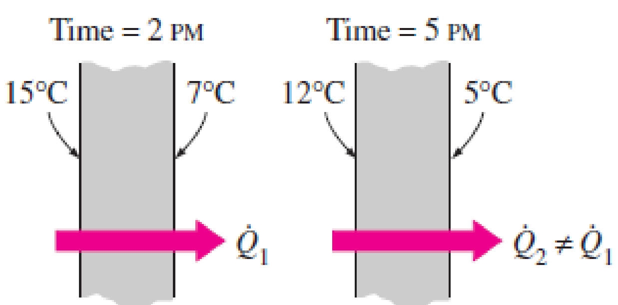
Learning objectives lecture 6

- **Time dependent heat transfer problems**
  - Distinguish practical examples
  - “Derive” mathematical approximation
  - Determining validity of approximation

UNSTEADY HEAT TRANSFER CONCEPT



Steady State



Unsteady State

- In **steady state heat transfer**, the temperature at any particular point in the system remains constant after equilibrium is attained.
- The amount of heat entering any section is then equal to the amount of heat exiting the section, because the driving force (temperature difference) is constant.

- In **unsteady state**, the temperature within an object itself keeps changing with time.
- The heat entering a section thus might not be the same as the heat exiting the section, as the temperature difference across the section keeps changing with time.

Transient processes

$t = 0$ s

$T_\infty = 0\,^\circ C$

$T_s = 25\,^\circ C$

$\dot{Q} \approx 300\,\mathrm{W}$

T_s drops $-10\,^\circ C$ / min

$t > 0$ s

$T_\infty = 0\,^\circ C$

$T_s = 12{,}5\,^\circ C$

$\dot{Q} \approx 150\,\mathrm{W}$

T_s drops $-5\,^\circ C$ / min

- Assume: bottle has uniform temperature

- The lower the temperature difference is, the lower and slower the drop in temperature (difference) will be.

steady / transient

So far: steady state
- Constant in time
- Equilibrium (approximately)

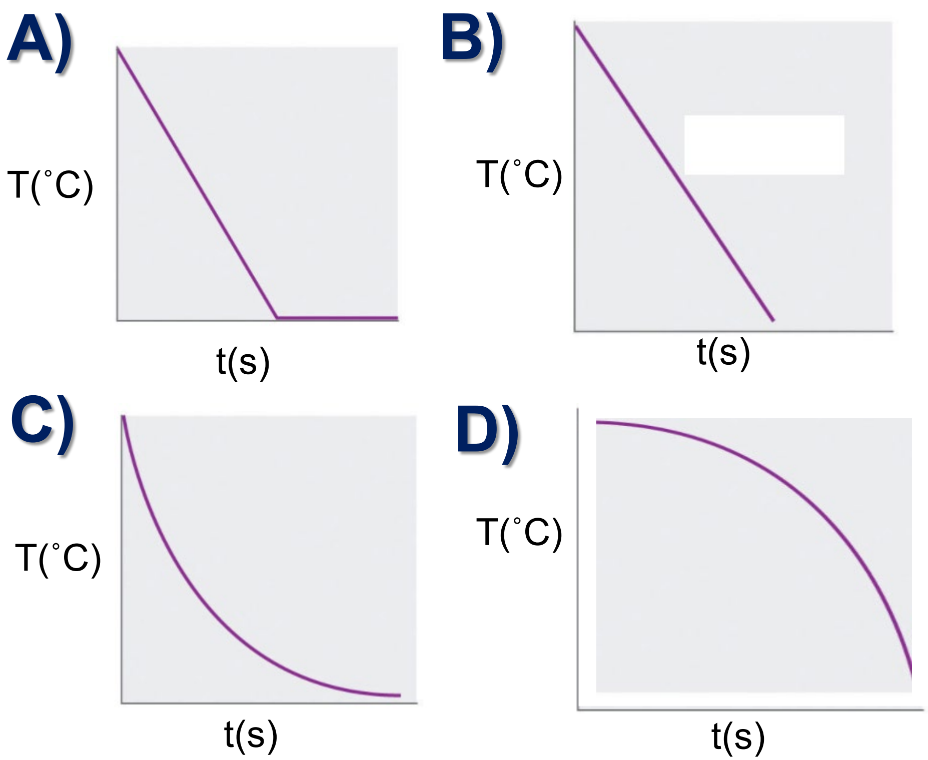



Transient: time dependent
- Heating
- Cooling

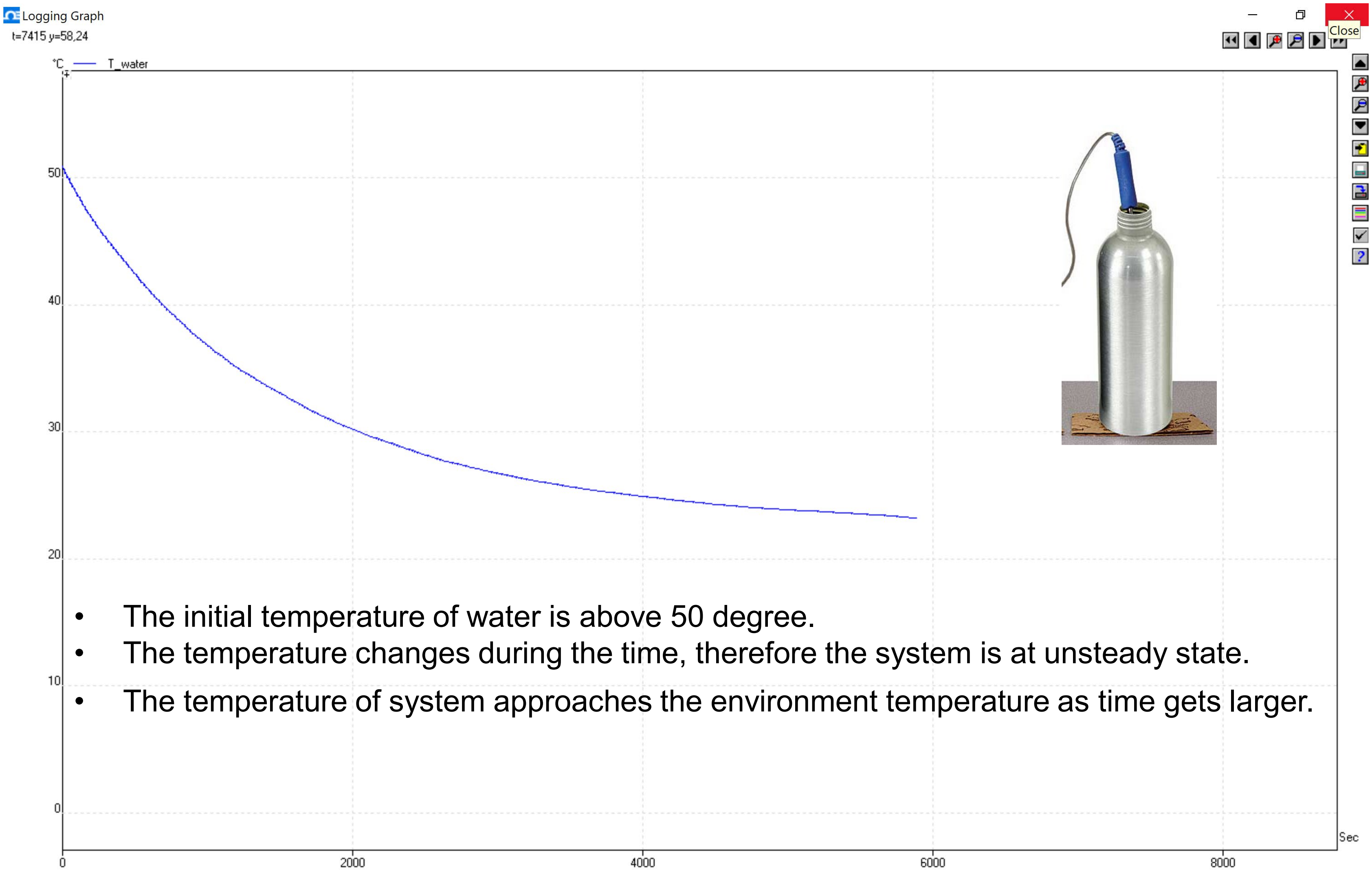



Transient processes

- The initial temperature of water is above 50 degree.
- The temperature changes during the time, therefore the system is at unsteady state.



Temperature transient analysis of a hot water

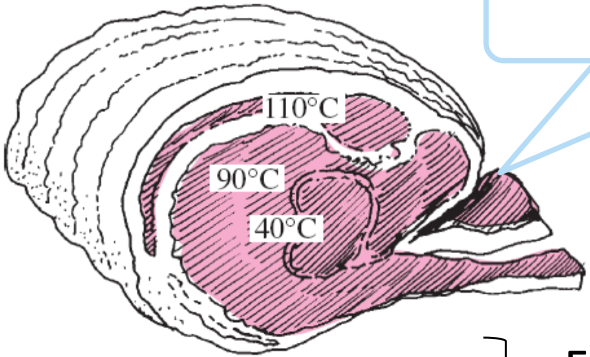


- The initial temperature of water is above 50 degree.
- The temperature changes during the time, therefore the system is at unsteady state.
- The temperature of system approaches the environment temperature as time gets larger.

Learning objectives lecture 6

- **Time dependent heat transfer problems**
  - Distinguish practical examples
  - “Derive” mathematical approximation
  - Determining validity of approximation

UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS



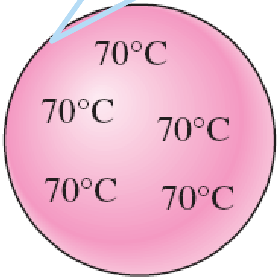
Temperature is a function of time and space

Convection: heat from outer layer
Conduction: heat transferred from outer layer to core

Factors:
h, *k*,
geometry

- **Interior temperature** of some bodies remains essentially uniform at all times during a heat transfer process.
- The **temperature** of such bodies can be taken to be a function of time only, $T(t)$.
- Heat transfer analysis that utilizes this idealization is known as **lumped system analysis**.

Temperature is only function of Time



$$\begin{pmatrix}\text{Heat transfer into the body}\\ \text{during } dt\end{pmatrix} = \begin{pmatrix}\text{The increase in the}\\ \text{energy of the body}\\ \text{during } dt\end{pmatrix}$$

$$hA_s(T_\infty - T)\,dt = mc_p\,dT$$

$$m = \rho V \quad dT = d(T - T_\infty)$$

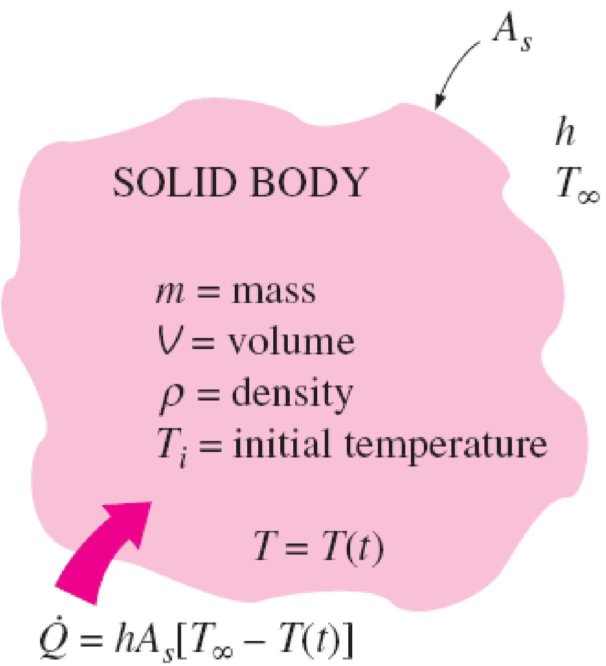
$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p}\,dt$$

Integrating with

$T = T_i$* at *$t = 0$

$T = T(t)$* at *$t = t$

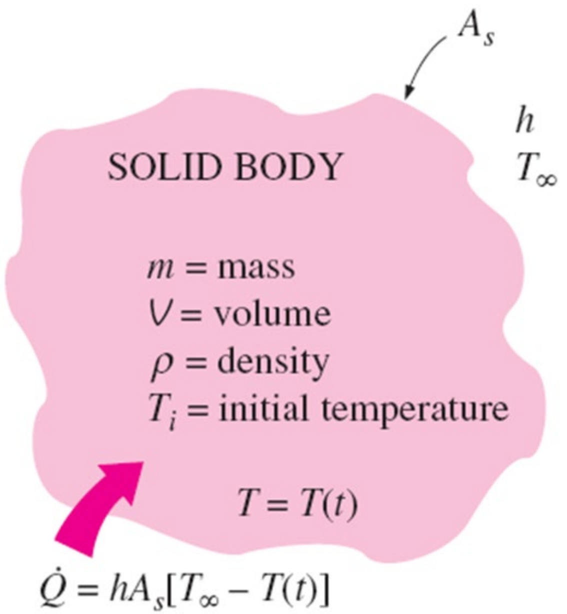
$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}\,t$$



The geometry and parameters involved in the lumped system analysis.

Unsteady Heat Transfer Lumped System Analysis

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$ Where $b = \frac{hA_s}{\rho V c_p}$



- *h :* heat transfer coefficient around object ($W \cdot m^{-2} \cdot K^{-1}$)
- A_s: external surface area (m^2)
- *ρ :* density of object ($kg \cdot m^{-3}$)
- c_p : specific heat of object ($J \cdot kg^{-1} \cdot K^{-1}$)
- *V* : volume of object, (m^3)

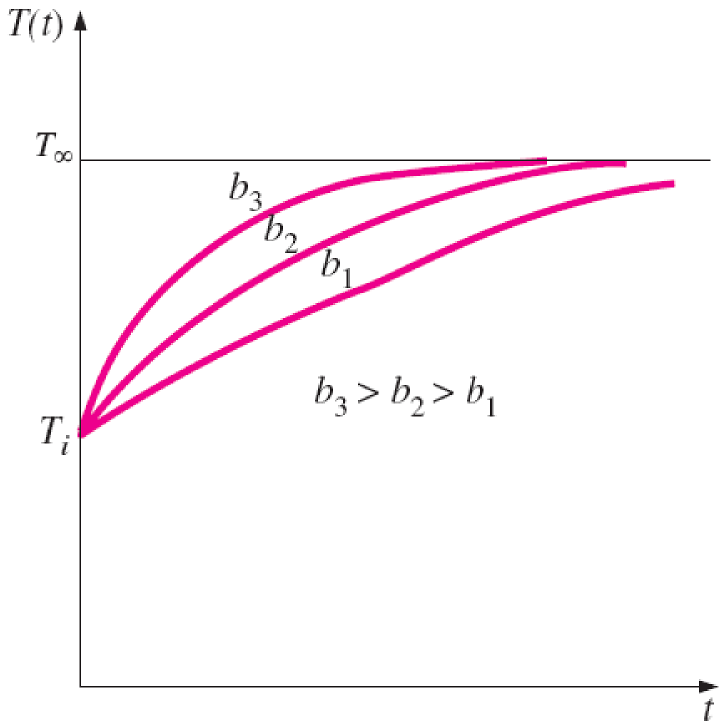
Only for lumped system analysis

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$ Where $b = \frac{hA_s}{\rho V c_p}$

What is this equation representing ?

- This equation enables us to determine the temperature $T(t)$ of a body at time t, or alternatively, the time t required for the temperature to reach a specified value $T(t)$.
- The temperature of a body approaches the ambient temperature T_∞ exponentially.
- What is the effect of b on duration (t) of the temperature of a body to reach the ambient temperature T_∞ ?

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V c_p}$$



- The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of *b* indicates that the body approaches the environment temperature in a short time.

- The temperature of a lumped system approaches the environment temperature as time gets larger.

Transient processes

$t = 0$ s

$T_\infty = 0\ ^\circ C$

$T_s = 25\ ^\circ C$

$\dot{Q} \approx 300$ W

T_s drops - 10 °C / min

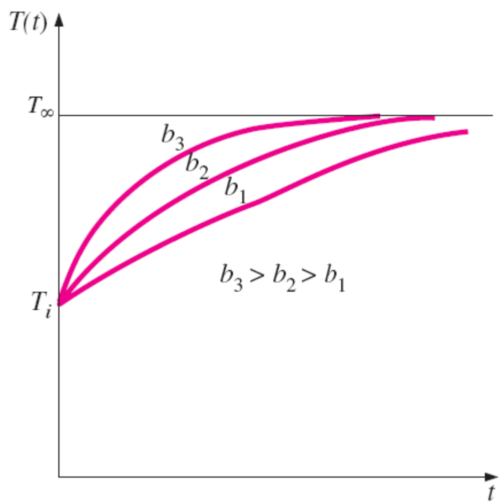
$t > 0$ s

$T_\infty = 0\ ^\circ C$

$T_s = 12{,}5\ ^\circ C$

$\dot{Q} \approx 150$ W

T_s drops - 5 °C / min



Activity(1)

Predicting the Time of Death

A person is found dead at 5PM in a room whose temperature is 20˚C. The temperature of the body is measured to be 25 ˚C when found, and the heat transfer coefficient is estimated to be h =8 W/m^2 · ˚C. Modelling the body as a 30-cm-diameter, 1.70-m-long cylinder, estimate the time of death of that person.

Assumptions

1) The body can be modelled as a 30-cm-diameter, 1.70-m-long cylinder.
2) The thermal properties of the body and the heat transfer coefficient are constant.
3) The radiation effects are negligible.
4) The person was healthy(!) when he or she died with a body temperature of 37 ˚C.
5) The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of (37+25)/2 =31 ˚C; k = 0.617 W/m · ˚C, ρ= 996 kg/m^3, and C_p =4178 J/kg · ˚C

Activity(1)

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V c_p}$$

$$A_s = 2\pi r^2 + 2\pi r L \quad , \ V = \pi r^2 L$$

$$b = \frac{hA_s}{\rho V C_P} = \frac{8\left(\frac{W}{m^2.°C}\right)\times(2\pi\times(0.15m)^2+2\pi\times(0.15m)\times1.7m))}{996(\frac{kg}{m^3})\times(\pi\times(0.15m)^2\times1.7m)\times4178(\frac{J}{kg.°C})}$$

$$= 2{,}79 \times 10^{-5}(s^{-1})$$

Activity(1)

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-bt} \longrightarrow \frac{25-20}{37-20}=e^{-(2{,}79\times10^{-5}\,s^{-1})t}$$

$\longrightarrow$ **t** = 43,860 s =**12.2 h**

Therefore, as a rough estimate, the person died about 12 h before the body was found, and thus the time of death is

5AM.

Learning objectives lecture 6

- **Time dependent heat transfer problems**
  - Distinguish practical examples
  - “Derive” mathematical approximation
  - Determining validity of approximation

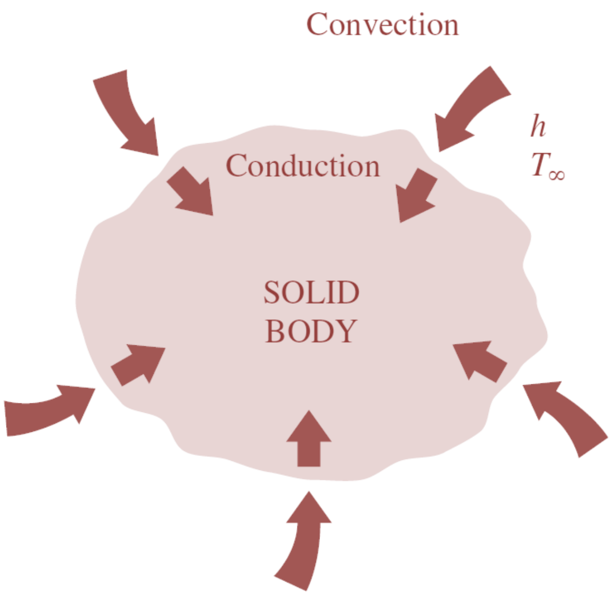
Unsteady Heat Transfer Lumped System Analysis

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$ Where $$b = \frac{hA_s}{\rho V c_p}$$

But when is this equation applicable ?

What is the criterion ?

Biot number (Bi)



$$\text{Bi} = \frac{\text{heat convection}}{\text{heat conduction}}$$

$$\text{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

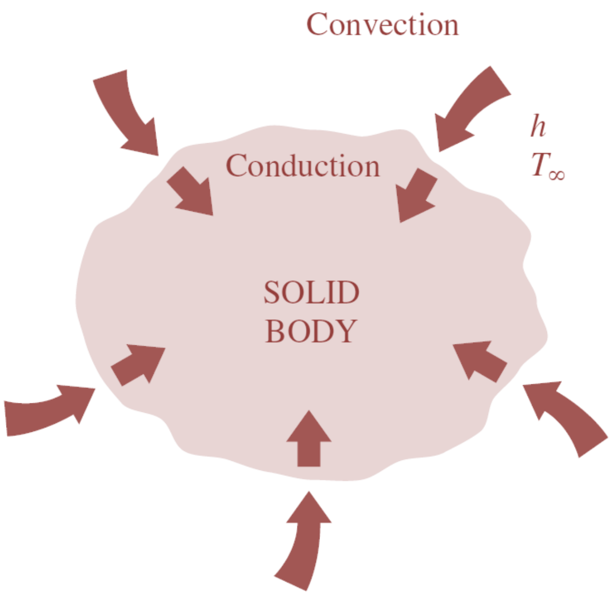
or

$$\text{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$\text{Bi} = \dfrac{hL_c}{k}$ **When characteristic length is :** $L_c = \dfrac{V}{A_s}$

Almost uniform temperature for Bi ≤ 0,1
“lumped system”

Biot number (Bi)



$$\text{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

or

$$\text{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$$\text{Bi} = \frac{hL_c}{k} \quad , \quad L_c = \frac{V}{A_s}$$

- A small Biot number represents small resistance to heat conduction, and thus small temperature gradients within the body.
- Lumped system analysis assumes a uniform temperature distribution throughout the body, which will be the case only when the thermal resistance of the body to heat conduction (the conduction resistance) is zero.

BIOT NUMBER (BI)

$$L_c = \text{Characteristic length} = \frac{\text{Volume of the solid } (V)}{\text{Surface area of the solid } (A_s)}$$

The values of characteristic length (L_c), for simple geometric shapes, are given below:

$$\textit{Flat plate}: \; L_c = \frac{V}{A_s} = \frac{LBH}{2BH} = L/2 = \text{semi-thickness}$$

where L, B and H are thickness, width and height of the plate.

$$\textit{Cylinder (long)}: \; L_c = \frac{\pi R^2 L}{2\pi R L} = \frac{R}{2} \qquad \text{where,} \quad R = \text{radius of the cylinder.}$$

$$\textit{Sphere}: \; L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} \qquad \text{where,} \quad R = \text{radius of the sphere.}$$

$$\textit{Cube}: \; L_c = \frac{L^3}{6L^2} = \frac{L}{6} \qquad \text{where,} \quad L = \text{Side of the cube.}$$

LUMPED SYSTEM

i. Lumped system analysis is exact when Bi= 0 and approximate when Bi > 0. Of course, the smaller the Bi number, the more accurate the lumped system analysis.

ii. The first step in the application of lumped system analysis is the calculation of the Biot number, and the assessment of the applicability of this approach.

iii. One may still wish to use lumped system analysis even when the criterion Bi<0.1 is not satisfied, if high accuracy is not a major concern.

iv. Note that the Biot number is the ratio of the convection at the surface to conduction within the body, and this number should be as small as possible for lumped system analysis to be applicable.

v. Small bodies with high thermal conductivity are good candidates for lumped system analysis, especially when they are in a medium that is a poor conductor of heat (such as air or another gas) and motionless. Thus, the hot small copper ball placed in quiescent air, is most likely to satisfy the criterion for lumped system analysis.

Nusselt vs. Biot

Nusselt number

$$\mathrm{Nu} = \frac{h\,L_c}{k}$$

$\neq$

Biot number

$$\mathrm{Bi} = \frac{h\,L_c}{k}$$

Dimensionless measure for convection so increase of heat transfer due to flow

k of fluid!

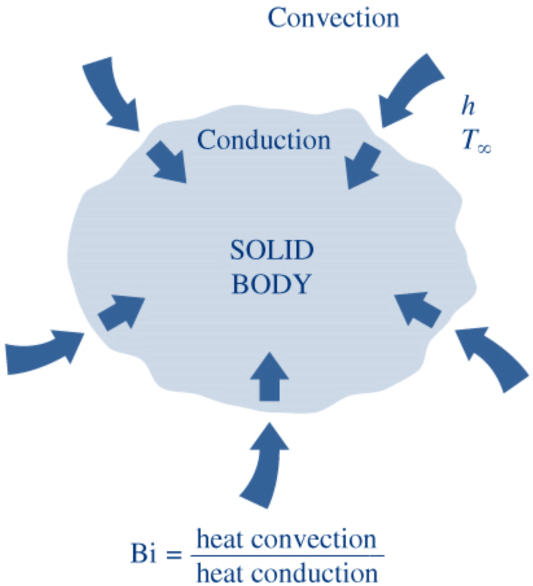
Dimensionless measure for degree of temperature distribution within body

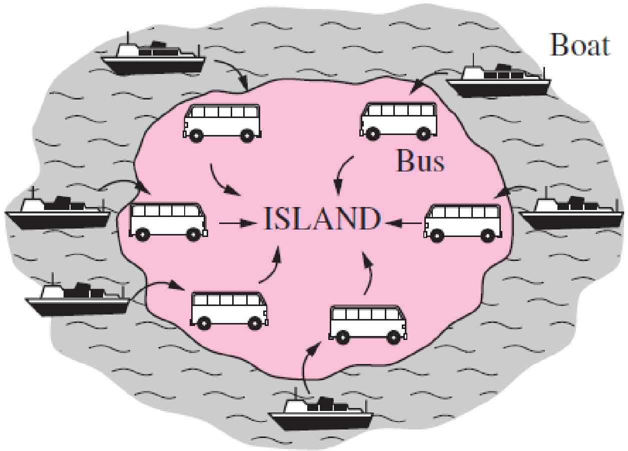
k of surrounded object!

Definitions seem similar but are substantially different!

Lumped system

$$Bi = \frac{h\,L_c}{k}$$





- Relatively high h → … → Bi high, non uniform
- Relatively high $L_c = V/A$ → … → Bi high, non uniform
- Relatively high k → … → Bi low, uniform

Lumped or not?

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{15 \times 0,006}{0,6} = 0,15 > 0,1$$

h = 15 W·m^{-2}·K^{-1}

So no lumped system!

k = 0,6 W·m^{-1}·K^{-1}

V = 0,45 L = 4,5 · 10^{-4} m^3

A ≈ 7,5 · 10^{-2} m^2

$\Big\}$ $L_c \approx 0{,}006$ m

$D \approx 0{,}06$ m

h = 5 W·m^{-2}·K^{-1}

k = 0,6 W·m^{-1}·K^{-1}

$$\mathrm{Bi} = \frac{h\,L_c}{k} = \frac{5 \times 0{,}01}{0{,}6} = 0{,}083 < 0{,}1$$

$$L_c = \frac{V}{A} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{D}{6} = 0{,}01 \text{ m}$$

So "lumped system"

Activity(2)

Predicting the Time of Death

A person is found dead at 5PM in a room whose temperature is 20˚C. The temperature of the body is measured to be 25 ˚C when found, and the heat transfer coefficient is estimated to be h =8 W/m^2 · ˚C. Modelling the body as a 30-cm-diameter, 1.70-m-long cylinder.

Lumped or not ?

Assumptions

1) The body can be modelled as a 30-cm-diameter, 1.70-m-long cylinder.
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5) The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of (37+25)/2 =31 ˚C; k = 0.617 W/m · ˚C, ρ= 996 kg/m^3, and C_p =4178 J/kg · ˚C

Activity(2)

$$A_s = 2\pi r^2 + 2\pi rL \quad , \text{V}= \pi r^2 L$$

$$L_C = \frac{V}{A_s} = \frac{\pi \times (0.15m)^2 \times 1.7m}{2\pi \times (0.15m)^2 + 2\pi \times (0.15m) \times 1.7m)}$$

$$= 0{,}0689(m)$$

Then the Biot number becomes :

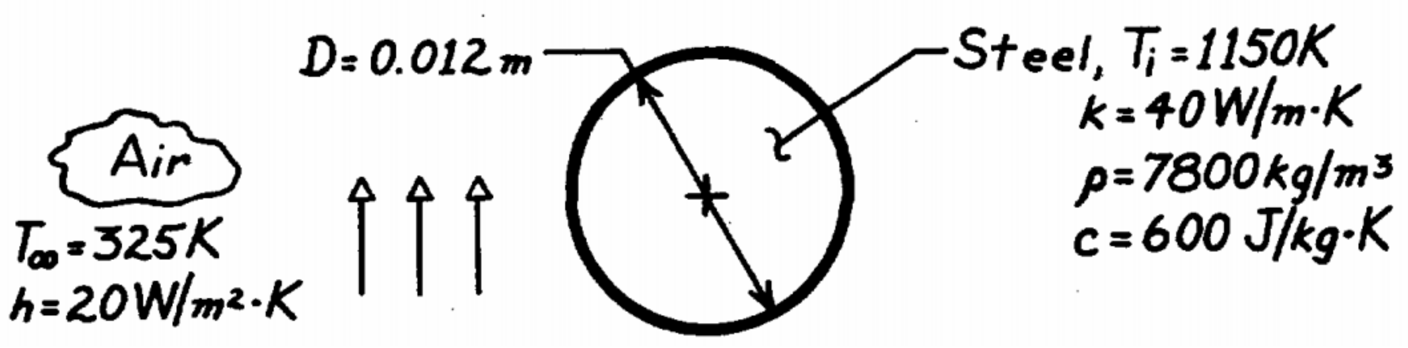
$$\text{Bi} = \frac{hL_C}{k} = \frac{8\left(\frac{W}{m^2.°\text{C}}\right)\times 0{,}0689m}{0{,}617\left(\frac{W}{m.°\text{C}}\right)} = 0.89 \; > 0{,}1$$

Therefore, lumped system analysis is *not* applicable. However, we can still use it to get a “rough” estimate of the time of death.

Activity(3)

KNOWN: Diameter and initial temperature of steel balls cooling in air.

FIND: Time required to cool to 400K.

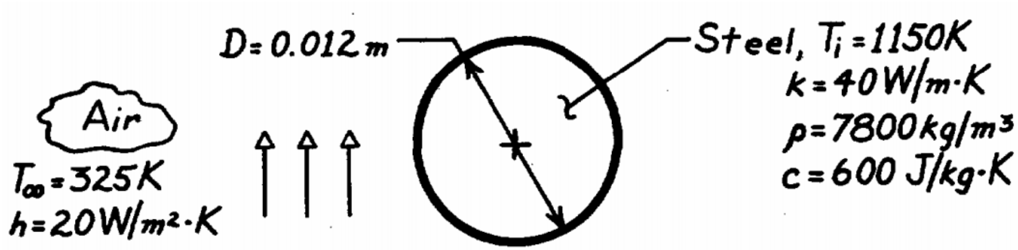


Assumptions

(1) Negligible radiation effects.

(2) Constant properties.

Activity(3)



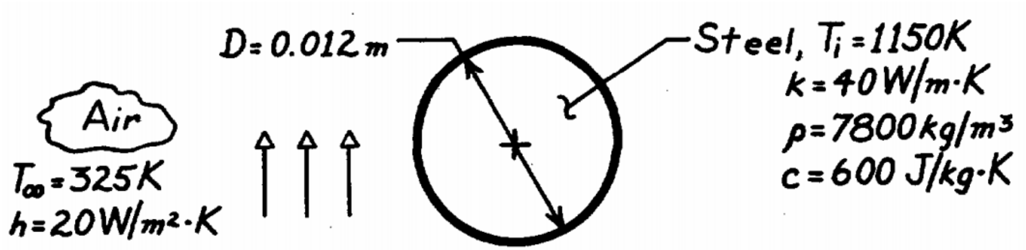
ANALYSIS:

Calculating Biot number : $Bi = \frac{hL_c}{k}$ **Characteristic Length for sphere :** $L_c = r_o/3$

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{20\ W/m^2 \cdot K\ (0.002m)}{40\ W/m \cdot K} = 0.001.$$

Hence, the temperature of the steel remains approximately uniform during the cooling process, and the lumped capacitance method may be used.

Activity(3)



ANALYSIS:

Calculating time : $$\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-bt} \qquad b=\frac{hA_s}{\rho V c_p}$$

$$t=\frac{\rho V c_p}{hA_s}\ln\frac{T_i-T_\infty}{T-T_\infty}=\frac{\rho\left(\pi D^3/6\right)c_p}{h\pi D^2}\ln\frac{T_i-T_\infty}{T-T_\infty}$$

$$t=\frac{7800\text{kg/m}^3\,(0.012\text{m})\,600\text{J/kg}\cdot\text{K}}{6\times 20\ \text{W/m}^2\cdot\text{K}}\ln\frac{1150-325}{400-325}$$

$$t=1122\ \text{s}\ =0.312\text{h}$$

Summary transient processes

- Biot number $\mathrm{Bi} = \dfrac{h\,L_c}{k}$ $(-)$
- For Bi ≤ 0,1: “lumped system” with $L_c = \dfrac{V}{A}$ (m)

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V c_p}$$

- For Bi > 0,1 : different approaches
- Approximations: check validity

Reference

Y. A. Cengel & A. J. Ghajar. Heat and Mass Transfer: Fundamental & Application:

Chapter 4: Transient Heat Conduction
4–1 : LUMPED SYSTEM ANALYSIS
    Criteria for Lumped System Analysis
    Some Remarks on Heat Transfer in Lumped Systems