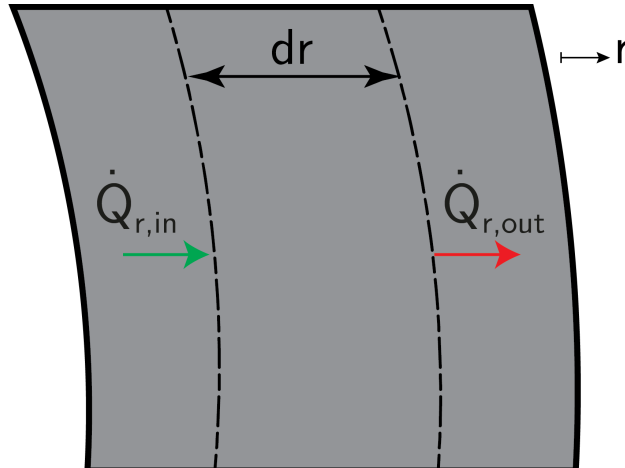


## EB - Cond. - IE 9

Hot water flows through a long pipe of length  $L$ . The water temperature and external surface temperature of the pipe are constant and equal to  $T_\infty$  and  $T_1$  respectively. Set up the energy balance for radial heat conduction in the pipe wall and give the appropriate boundary conditions.



**Energy balance:**

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

Since the type of heat transfer is steady-state, the sum of the in- and outgoing heat fluxes of the control volume should equal zero.

**Heat fluxes:**

$$\dot{Q}_{r,in} = -\lambda A(r) \frac{\partial T}{\partial r} = -\lambda 2\pi r L \frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} = \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r} dr = -\lambda 2\pi r L \frac{\partial T}{\partial r} + \frac{\partial}{\partial r} (-\lambda 2\pi r L \frac{\partial T}{\partial r}) dr$$

The ingoing flux can be described by use of Fourier's law and the outgoing flux can be approximated by use of the Taylor series expansion.

**Boundary conditions:**

$$\frac{\partial T(r=r_1)}{\partial r} = -\frac{\alpha}{\lambda} (T_\infty - T(r=r_1))$$

$$T(r=r_2) = T_1$$

The first boundary condition results from the fact that  $\dot{Q}_{r=r_1} = -\lambda A(r) \frac{\partial T(r=r_1)}{\partial r} = \alpha A(r) (T_\infty - T(r=r_1))$ , the second one describes that the temperature at the surface equals  $T_1$ .