Mass concentrations of air and water on the Lake Constance

Determination of mass concentrations of air and water on Lake Constance.

1. Task 5 in Stefan flow

In this task, the mass concentrations of air and water over the surface of Lake Constance must be calculated under the conditions given in Fig. (1) over Lake Constance.

2. Answer

2.1. *Given parameters*

- 1: Pressure $P = 1$ bar,
- 2: Temperature $T = 20\,°\mathrm{C}$,
- 3: Air molecular weight $M_a = 29\ \mathrm{g\,mol^{-1}}$,
- 4: Water molecular weight $M_{H_2O} = 18\ \mathrm{g\,mol^{-1}}$,
- 5: Saturation pressure of water under atmosphoric conditions $p_{H_2O}(x = 0) = 0.0234$ bar,
- 6: The entire lake surface $A = 536 \cdot 10^6\ \mathrm{m}^2$.

2.2. *Determination of the mass concentrations*

$$\xi_{H_2O} = \frac{m_{H_2O}}{m_{ges}} = \frac{m_{H_2O}}{m_{H_2O} + m_{air}} = \frac{1}{1 + \frac{m_{air}}{m_{H_2O}}}, \tag{2.1}$$

$$m_{H_2O} = \frac{p_{H_2O} V}{R_{H_2O} T}, \;\; m_{air} = \frac{p_{air} V}{R_{air} T}, \tag{2.2}$$

$$\xi_{H_2O} = \frac{1}{1 + \frac{\frac{p_{air} V}{R_{air} T}}{\frac{p_{H_2O} V}{R_{H_2O} T}}} = \frac{1}{1 + \frac{p_{air} R_{H_2O}}{p_{H_2O} R_{air}}}, \tag{2.3}$$

$$R_{H_2O} = \frac{R_m}{M_{H_2O}}, \;\; R_{air} = \frac{R_m}{M_{air}}, \tag{2.4}$$

$$\xi_{H_2O} = \frac{1}{1 + \frac{p_{air} M_{air}}{p_{H_2O} M_{H_2O}}}, \;\; p_{air} = 1 - p_{H_2O}. \tag{2.5}$$

$$\xi_{H_2O}(x = 0) = \frac{1}{1 + \frac{\left(1 - p_{H_2O}(x=0)\right) M_{air}}{p_{H_2O}(x=0) M_{H_2O}}} = \frac{1}{1 + \frac{(1-0.0234)\cdot 29}{0.0234 \cdot 18}} = 1.47 \cdot 10^{-2}, \tag{2.6}$$

$$p_{H_2O}(x \to \infty) = \phi \cdot p_{H_2O}(x = 0) = 0.6 \cdot 0.0234 = 0.01404\text{bar}, \tag{2.7}$$

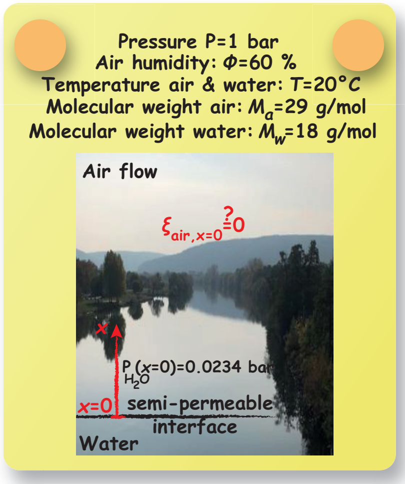

FIGURE 1. Problem definition.

$$\xi_{H_2O}(x \to \infty) = \frac{1}{1 + \frac{(1-0.01404)\cdot 29}{0.01404 \cdot 18}} = 8.761 \cdot 10^{-3}. \tag{2.8}$$

$$\text{Stefan Factor} = \frac{1}{1 - \xi_{H_2O}(x=0)} = 1.015. \tag{2.9}$$

$$\dot{m}'' = g \cdot \frac{1}{1 - \xi_{H_2O}(x=0)} \left(\xi_{H_2O}(x=0) - \xi_{H_2O}(x \to \infty)\right) = 1.41 \cdot 10^{-8}\,\mathrm{kg\,m^{-2}\,s^{-1}}. \tag{2.10}$$

$$\dot{m} = \dot{m}'' \cdot A = 1.41 \cdot 10^{-8} \cdot 536 \cdot 10^{6} = 7.548\,\mathrm{kg\,s^{-1}}. \tag{2.11}$$

$$\xi_{H_2O}(x=0) \neq 1 \text{ also Nein } \xi_{air}(x=0) \neq 0. \tag{2.12}$$