Section III

Convection solutions

Exercise III.1 (Walking man ⋆):

A man has a body surface area of A and a skin temperature of T_{s}, with an average surface temperature of the clothed person of T_{c}. The convection heat transfer coefficient α for a clothed man walking in the air with temperature T_{A} is expressed as:

$$\alpha = C \cdot \sqrt{V},$$

for $0.5 < V < 5$ m/s, and where $C = 8.2\ \frac{\mathrm{J}}{\mathrm{m}^{2.5}\mathrm{s}^{0.5}\mathrm{K}}$, and V is the relative velocity of the man with respect to the air.

Given parameters:

- Surface area of the body: $A = 1.8\ \mathrm{m}^2$
- Thermal conductivity of the skin: $\lambda_{\mathrm{s}} = 0.25$ W/mK
- Thermal conductivity of clothes: $\lambda_{\mathrm{s}} = 0.03$ W/mK
- Thermal conductivity of the air: $\lambda_{\mathrm{a}} = 0.026$ W/mK
- Skin temperature of the man: $T_{\mathrm{s}} = 33$ °C
- Surface temperature of the clothed man: $T_{\mathrm{c}} = 30$ °C
- Air temperature: $T_{\mathrm{A}} = 15$ °C

Hints:

- Assume steady-state operating conditions.
- Assume the heat transfer coefficient to be constant over the entire surface.

Tasks:

a) Determine the rate of heat loss from the man by convection while walking in still air at a speed of 1 m/s.

b) Determine the rate of heat loss from the man walking in the air when walking in the same direction of the wind with a velocity of 1.5 m/s, while the wind is blowing at a velocity of 2 m/s.

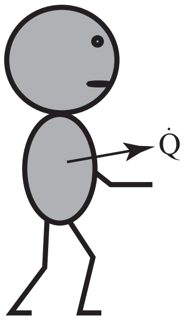
c) Determine the rate of heat loss and the relative velocity from the man while walking in still air with a Nusselt number of $\mathrm{Nu} = 510$, and a characteristic length of $L = 1$ m.

Solution III.1 (Walking man ⋆):

Task a)

1 **Setting up the balance:**

The man walks and therefore loses heat to the environment by convection.



The rate of heat loss by convection can be expressed as: The rate of heat loss due to convection can be described by:

$$\dot{Q} = \alpha A_s \left(T_c - T_A\right) \tag{III.1.1}$$

2 **Defining the elements within the balance:**

The relative velocity in the given problem is 1 m/s and thus the heat transfer coefficient yields to be:

$$\alpha = C \cdot \sqrt{V} \tag{III.1.2}$$

$$= 8.2 \left[\frac{\mathrm{J}}{\mathrm{m}^{2.5}\mathrm{s}^{0.5}\mathrm{K}}\right] \cdot \sqrt{1} \left[\frac{\mathrm{m}^{0.5}}{\mathrm{s}^{0.5}}\right] = 8.2 \left[\mathrm{W/m^2K}\right] \tag{III.1.3}$$

3 **Inserting and rearranging:**

Filling in all numerical values yields:

$$\dot{Q} = \alpha A_s (T_c - T_A) \tag{III.1.4}$$

$$= 8.2 \left[\mathrm{W/m^2K}\right] \cdot 1.8 \left[\mathrm{m^2}\right] \cdot (30 - 15) \left[{}^\circ\mathrm{C}\right] = 221 \left[\mathrm{W}\right]$$

Conclusion

The man loses 221 W of heat by convection while walking in still air at a speed of 1 m/s.

Task b)

1 **Setting up the balance:**

The rate of heat loss due to convection can be described by:

$$\dot{Q} = \alpha A_s \left(T_c - T_A\right) \tag{III.1.5}$$

2 **Defining the elements within the balance:**

The relative velocity of the man to the air in the given problem is 0.5 m/s and the heat transfer coefficient yields from:

$$\alpha = C \cdot \sqrt{V} \tag{III.1.6}$$

$$= 8.2 \left[\frac{\mathrm{J}}{\mathrm{m}^{2.5}\mathrm{s}^{0.5}\mathrm{K}}\right] \cdot \sqrt{0.5} \left[\frac{\mathrm{m}^{0.5}}{\mathrm{s}^{0.5}}\right] = 8.2 \; \left[\mathrm{W/m^2K}\right] \tag{III.1.7}$$

3 **Inserting and rearranging:**

Filling in all numerical values yields:

$$\dot{Q} = \alpha A_\mathrm{s}(T_\mathrm{c} - T_\mathrm{A}) \tag{III.1.8}$$
$$= 5.8 \; \left[\mathrm{W/m^2K}\right] \cdot 1.8 \; \left[\mathrm{m^2}\right] \cdot (30 - 15) \; [^\circ\mathrm{C}] = 157 \; [\mathrm{W}]$$

Conclusion

The man loses 157 W of heat by convection when walking in the same direction of the wind with a velocity of 1.5 m/s, while the wind is blowing at a velocity of 2 m/s.

Task c)

1 **Setting up the balance:**

The rate of heat loss due to convection can be described by:

$$\dot{Q} = \alpha A_\mathrm{s} \left(T_\mathrm{c} - T_\mathrm{A}\right) \tag{III.1.9}$$

2 **Defining the elements within the balance:**

The heat transfer coefficient can be found by rewriting the definition of the Nusselt number:

$$\alpha = \frac{\mathrm{Nu}\lambda_\mathrm{f}}{L} \tag{III.1.10}$$
$$= \frac{510 \; [-] \cdot 0.026 \; \left[\mathrm{W/m^2K}\right]}{1 \; [\mathrm{m}]} = 13.3 \; \left[\mathrm{W/m^2K}\right]$$

Rewriting the given equation of the heat transfer coefficient yields the velocity of the man:

$$V = \left(\frac{\alpha}{8.2}\right)^2 \tag{III.1.11}$$

$$\left(\frac{13.3 \; \left[\frac{\mathrm{J}}{\mathrm{m^2sK}}\right]}{8.2 \; \left[\frac{\mathrm{J}}{\mathrm{m}^{2.5}\mathrm{s}^{0.5}\mathrm{K}}\right]}\right)^2 = 2.6 \; [\mathrm{m/s}] \tag{III.1.12}$$

3 **Inserting and rearranging:**

Filling in all numerical values yields:

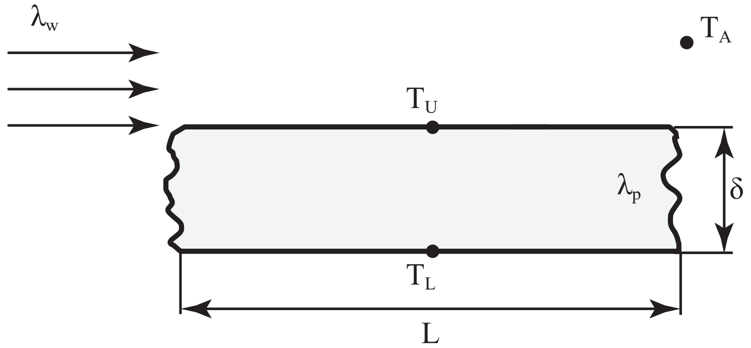
$$\dot{Q} = \alpha A_\mathrm{s}(T_\mathrm{c} - T_\mathrm{A}) \tag{III.1.13}$$
$$= 13.3 \; \left[\mathrm{W/m^2K}\right] \cdot 1.8 \; \left[\mathrm{m^2}\right] \cdot (30 - 15) \; [^\circ\mathrm{C}] = 358 \; [\mathrm{W}]$$

Conclusion

The man loses 358 W of heat by convection when walking at a velocity of 2.6 m/s with a Nusselt number of Nu = 510, and a characteristic length of $L = 1$ m.

Exercise III.2 (Thick solid plate ⋆):

The top surface of a thick solid plate is cooled by water flowing. The upper and lower surfaces of the solid plate are maintained at constant temperatures T_{U} and T_{L} respectively.



Given parameters:

- Thickness of the plate: δ
- Length of the plate: L
- Thermal conductivity of the plate: λ_{p}
- Thermal conductivity of the water: λ_{w}
- Upper surface temperature of the plate: T_{U}
- Lower surface temperature of the plate: T_{L}
- Ambient temperature: T_{A}

Hints:

- Assume steady-state operating conditions.
- Assume the heat transfer coefficient to be constant over the entire surface.
- $T_{\mathrm{L}} > T_{\mathrm{U}}$

Tasks:

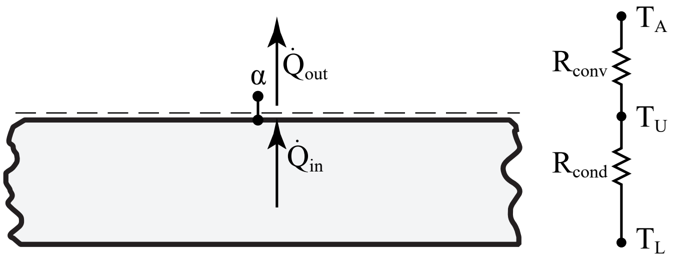
a) Determine the Nusselt number in terms of the given variables, using the length L of the plat as the characteristic length.

b) Determine the temperature gradient inside the water at the interface in terms of the given variables.

Solution III.2 (Thick solid plate ⋆⋆):

Task a)

1 Setting up the balance:



The energy balance at the interface reads:

$$0 = \dot{Q}_{\text{cond}} - \dot{Q}_{\text{conv}} \tag{III.2.1}$$

2 Defining the elements within the balance:

The rate of heat transfer by conduction can be described by use of Fourier's law:

$$\dot{Q}_{\text{cond}} = \frac{T_{\text{L}} - T_{\text{U}}}{R_{\text{cond}}} \tag{III.2.2}$$

and the rate of heat transfer by convection:

$$\dot{Q}_{\text{conv}} = \frac{T_{\text{L}} - T_{\text{U}}}{R_{\text{conv}}} \tag{III.2.3}$$

where the thermal resistances can be defined as:

$$R_{\text{cond}} = \frac{\delta}{A\lambda_{\text{p}}} \tag{III.2.4}$$

$$R_{\text{conv}} = \frac{1}{A\alpha} \tag{III.2.5}$$

3 Inserting and rearranging:

Inserting and rewriting yields:

$$\alpha = \frac{\lambda_{\text{p}}}{\delta} \cdot \frac{T_{\text{L}} - T_{\text{U}}}{T_{\text{U}} - T_{\text{A}}} \tag{III.2.6}$$

Using the definition of the Nusselt number it yields that:

$$\begin{aligned} \text{Nu} &= \frac{\alpha L}{\lambda_{\text{w}}} \\ &= \frac{\lambda_{\text{p}} L}{\lambda_{\text{w}}\delta} \cdot \frac{T_{\text{L}} - T_{\text{U}}}{T_{\text{U}} - T_{\text{A}}} \end{aligned} \tag{III.2.7}$$

Task b)

1 **Setting up the balance:**

Taking the same energy balance at the interface again:

$$0 = \dot{Q}_{\mathrm{cond}} - \dot{Q}_{\mathrm{conv}} \tag{III.2.8}$$

2 **Defining the elements within the balance:**

The rate of heat transfer by conduction in a plane wall can be described by use of Fourier's law:

$$\begin{aligned}\dot{Q}_{\mathrm{cond}} &= \left(-\lambda_{\mathrm{p}} A \frac{\partial T_{\mathrm{p}}}{\partial x}\right)_{\mathrm{int}} \\ &= -\lambda_{\mathrm{p}} A \frac{T_{\mathrm{L}} - T_{\mathrm{U}}}{\delta}\end{aligned} \tag{III.2.9}$$

Exactly at the interface, the fluid is stagnant and thus the the rate of convection can be written as:

$$\dot{Q}_{\mathrm{conv}} = \left(-\lambda_{\mathrm{w}} A \frac{\partial T_{\mathrm{w}}}{\partial x}\right)_{\mathrm{int}} \tag{III.2.10}$$

Conclusion

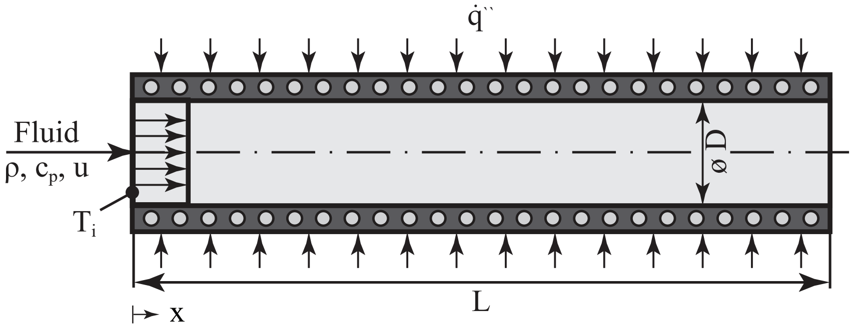
3 **Inserting and rearranging:**

Inserting and rewriting gives:

$$\left(\frac{\partial T_{\mathrm{w}}}{\partial x}\right)_{\mathrm{int}} = \frac{\lambda_{\mathrm{p}}}{\lambda_{\mathrm{w}}} \frac{T_{\mathrm{L}} - T_{\mathrm{U}}}{\delta}, \tag{III.2.11}$$

Exercise III.3 (Pipe flow ⋆⋆)**:**

A fluid flows through a long cylindrical tube. A constant heat flux density $\dot{q}''$ is imposed on the fluid.



Given parameters:

- Diameter of the pipe: D
- Length of the plate: L
- Heat flux density: $\dot{q}''$
- Density of the fluid: ρ
- Specific heat capacity of the fluid: c_p
- Average velocity of the fluid: u
- Fluid inlet temperature: T_i

Hints:

- Assume one-dimensional heat transfer in the axial direction.
- Assume steady-state operating conditions.
- Conduction in the fluid is negligible.

Tasks:

a) Determine the temperature profile of the fluid.

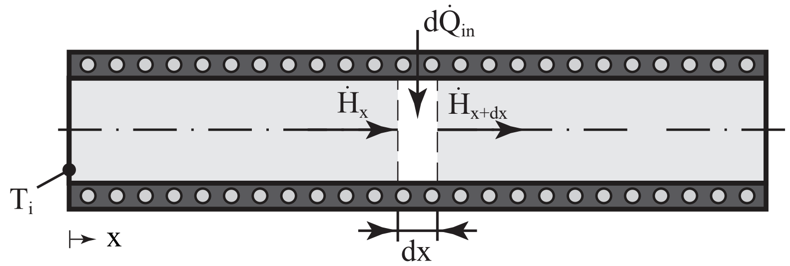
b) Determine the temperature of the fluid at 75% of the pipe length.

Solution III.3 (Pipe flow ★★):

Task a)

The temperature profile can be derived from the equation of energy conservation for a specific system. To derive this equation, the energy balance for an infinitesimal element within a specified domain should be derived and solved.

1 **Setting up the balance:**



The energy balance for an infinitesimal element within the pipe reads:

$$0 = \dot{H}_{\mathrm{x}} - \dot{H}_{\mathrm{x+dx}} + d\dot{Q}_{\mathrm{in}} \tag{III.3.1}$$

2 **Defining the elements within the balance:**

The rate of heat transport entering due to the motion of the fluid can be stated as:

$$\begin{aligned} \dot{H}_{\mathrm{x}} &= \dot{m} c_{\mathrm{p}} T(x) \\ &= \rho u \frac{\pi D^2}{4} c_{\mathrm{p}} T(x) \end{aligned} \tag{III.3.2}$$

and the rate of heat transfer transport leaving due to the motion of the fluid can be approximated by use of the Taylor series expansion:

$$\begin{aligned} \dot{H}_{\mathrm{x+dx}} &= \dot{H}_{\mathrm{x}} + \frac{\partial \dot{H}_{\mathrm{x}}}{\partial x} \cdot dx \\ &= \rho u \frac{\pi D^2}{4} c_{\mathrm{p}} T(x) + \frac{\partial}{\partial x} \left(\rho u \frac{\pi D^2}{4} c_{\mathrm{p}} T(x) \right) \cdot dx \end{aligned} \tag{III.3.3}$$

The incoming rate of heat transfer from the uniform heat flux can be stated as:

$$\begin{aligned} d\dot{Q}_{\mathrm{in}} &= \dot{q}'' \cdot dA_{\mathrm{s}} \\ &= \dot{q}'' \cdot \pi D \cdot dx \end{aligned} \tag{III.3.4}$$

3 **Inserting and rearranging:**

Inserting and rewriting yields:

$$\frac{\partial T}{\partial x} = \frac{4 \dot{q}''}{\rho u c_{\mathrm{p}} D} \tag{III.3.5}$$

4 **Defining the boundary and/or initial conditions:**

To solve the differential equation, one boundary condition is required since the temperature has been differentiated once with respect to position.

The inlet temperature is specified, which yields the required boundary condition:

$$T(x=0) = T_{\text{in}} \tag{III.3.6}$$

5 **Solving the equation:**

To solve the differential equation, it should be integrated once with respect to x:

$$T(x) = \frac{4\dot{q}''}{\rho u c_{\text{p}} D} \cdot x + C_1 \tag{III.3.7}$$

Using the defined boundary condition yields that:

$$\begin{aligned} T(x=0) &= \frac{4\dot{q}''}{\rho u c_{\text{p}} D} \cdot 0 + C_1 T_{\text{in}} \\ &\Rightarrow C_1 = T_{\text{in}} \end{aligned} \tag{III.3.8}$$

Conclusion

Which yields the temperature profile inside the fluid:

$$T(x) = \frac{4\dot{q}''}{\rho u c_{\text{p}} D} \cdot x + T_{\text{in}} \tag{III.3.9}$$

Task b)

To determine the temperature at 75% of the length of the pipe, the derived temperature profile can be used.

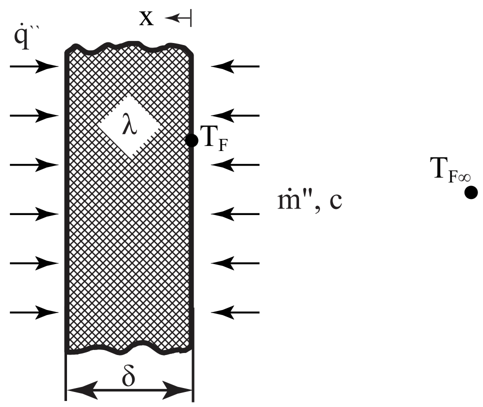
Conclusion

This yields:

$$T\left(x = \frac{3}{4}L\right) = \frac{3\dot{q}''L}{\rho u c_{\text{p}} D} + T_{\text{in}} \tag{III.3.10}$$

Exercise III.4 (Porous wall ⋆ ⋆ ⋆):

The surface of a porous wall, impermeable to radiation, absorbs a radiative heat flux. For cooling purposes, a coolant is circulated through the wall with an inlet temperature is T_F.



Given parameters:

- Imposed radiative heat flux: $\dot{q}'' = 150 \cdot 10^3$ W/m^2
- Wall thickness: $\delta = 50$ mm
- Wall Thermal conductivity: $\lambda = 8$ W/mK
- Coolant specific heat capacity: $c = 1000$ J/kgK
- Coolant inlet temperature: $T_F = -15$ °C
- Coolant area specific mass flux: $\dot{m}'' = 0.6$ kg/m$^2 \cdot$ s

Hints:

- Within the wall, conduction of the imposed radiative heat flux is negligible.
- The local fluid and wall temperatures can be assumed to be identical.

Tasks:

a) Determine the temperature profile $T(x)$ for the porous wall.

b) Determine the maximum temperature T_{max} reached within the wall.

c) Determine the heat flux $\dot{q}''_F$ per unit area, which is transmitted into the fluid at $x = 0$.

d) Which temperature $T_{F,\infty}$ does the fluid reach far away from the wall?

e) Sketch the temperature profiles for two different mass fluxes and mark each curve.

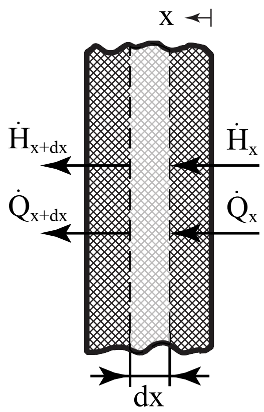
Solution III.4 (Porous wall ⋆ ⋆ ⋆):

Task a)

The temperature profile can be derived from the equation of energy conservation for a specific system. To derive this equation, the energy balance for an infinitesimal element within a specified domain should be derived and solved.

1 **Setting up the balance:**

Heat will be transferred by means of diffusion and fluid motion. Therefore the rate of diffusion and advection need to be incorporated within the steady-state balance.



The energy balance for the steady-state infinitesimal element can be written as:

$$0 = \dot{Q}_{\mathrm{x}} + \dot{H}_{\mathrm{x}} - \dot{Q}_{\mathrm{x+dx}} - \dot{H}_{\mathrm{x+dx}} \tag{III.4.1}$$

Note that it is assumed that energy transfer happens in the positive x-direction by convention. The direction is accounted for by the definition of the fluxes and the boundary conditions.

2 **Defining the elements within the balance:**

Diffusive heat transport can be written by Fourier's law:

$$\dot{Q}_{\mathrm{x}} = -\lambda A \frac{\partial T}{\partial x} \tag{III.4.2}$$

Rate of energy transport due to the fluid motion yields from:

$$\begin{aligned} \dot{H}_{\mathrm{x}} &= \dot{m}\, c\, T(x) \\ &= \dot{m}''\, A\, c\, T(x) \end{aligned} \tag{III.4.3}$$

Where the outgoing diffusive heat flow and enthalpy flow for an infinitesimal element can be approximated by use of Taylor series expansion:

$$\begin{aligned} \dot{Q}_{\mathrm{x+dx}} &= \dot{Q}_{\mathrm{x}} + \frac{\partial \dot{Q}_{\mathrm{x}}}{\partial x} \cdot dx \\ &= -\lambda A \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-\lambda A \frac{\partial T}{\partial x}\right) \cdot dx \end{aligned} \tag{III.4.4}$$

$$\dot{H}_{\mathrm{x+dx}} = \dot{H}_{\mathrm{x}} + \frac{\partial \dot{H}_{\mathrm{x}}}{\partial x} \cdot dx \tag{III.4.5}$$
$$= \dot{m}''\, A\, c\, T(x) + \frac{\partial}{\partial x}\left(\dot{m}''\, A\, c\, T(x)\right) \cdot dx$$

3 **Inserting and rearranging:**

Inserting all terms and rewriting the equation yields:

$$0 = \frac{\partial^2 T}{\partial x^2} - \frac{\dot{m}''\, c}{\lambda} \cdot \frac{\partial T}{\partial x} \tag{III.4.6}$$

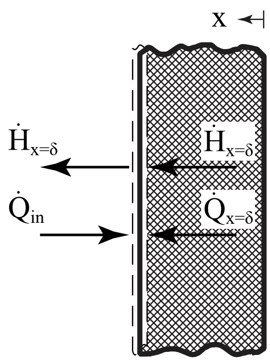
4 **Defining the boundary and/or initial conditions:**

As we are dealing with a 2nd order differential equation, we need two boundary conditions to be able to solve it.

At $x = 0$ the temperature is already specified, which gives us the first boundary condition:

$$T(x = 0) = T_{\mathrm{F}} \tag{III.4.7}$$

The other boundary condition can be obtained by setting up an energy balance at $x = \delta$



At this location, we have the following energy balance:

$$\dot{Q}_{\mathrm{x}=\delta} + \dot{Q}_{\mathrm{in}} + \cancelto{0}{\dot{H}_{x=\delta} - \dot{H}_{x=\delta}} = 0, \tag{III.4.8}$$

where:

$$\dot{Q}_{\mathrm{in}} = \dot{q}''\, A$$

and:

$$\dot{Q}_{\mathrm{x}=\delta} = -\lambda A \left.\frac{\partial T}{\partial x}\right|_{x=\delta}$$

Inserting and rewriting yields the second boundary condition::

$$\left.\frac{\partial T}{\partial x}\right|_{x=\delta} = \frac{\dot{q}''}{\lambda} \tag{III.4.9}$$

5 **Solving the equation:**

As we are dealing with a linear differential equation, one can try to solve this equation by "guessing" what the solution temperature profile will look like:

$$T(x) = \exp(sx) \tag{III.4.10}$$

Differentiating once with respect to x:

$$\frac{\partial T}{\partial x} = s\exp(sx) \tag{III.4.11}$$

Differentiating twice with respect to x:

$$\frac{\partial^2 T}{\partial x^2} = s^2\exp(sx) \tag{III.4.12}$$

Substitution of the 1st and 2nd derivative into the energy balance gives us:

$$0 = \frac{\partial^2 T}{\partial x^2} - \frac{\dot{m}''\,c}{\lambda}\cdot\frac{\partial T}{\partial x} \tag{III.4.13}$$

$$s^2\exp(sx) - \frac{\dot{m}''\,c}{\lambda}\cdot s\exp(sx) = 0 \tag{III.4.14}$$

Rewriting:

$$\exp(sx)\left(s^2 - \frac{\dot{m}''\,c}{\lambda}\right) = 0 \tag{III.4.15}$$

The standard solution would be $\exp(sx) = 0$, but we need a problem-specific solution and therefore it is not possible that$\exp(sx) = 0$, and thus it should be that:

$$s^2 - \frac{\dot{m}''\,c}{\lambda}\cdot s = 0 \tag{III.4.16}$$

This gives us:

$$s_1 = 0 \qquad \text{and} \qquad s_2 = \frac{\dot{m}''\,c}{\lambda} \tag{III.4.17}$$

Therefore the general solution will be:

$$T(x) = c_1\exp(s_1 x) + c_2\exp(s_2 x) \tag{III.4.18}$$

Which can be rewritten to:

$$T(x) = c_1 + c_2\exp\left(\frac{\dot{m}''\,c}{\lambda}x\right) \tag{III.4.19}$$

Now having found a general solution, the integration constants c_1 and c_2 should be determined by use of the boundary conditions.

From $T(x=0) = T_\mathrm{F}$ it is found that:

$$\begin{gathered} T(x=0) = c_1 + c_2\exp(0) = T_\mathrm{F} \\ \Rightarrow c_1 = T_\mathrm{F} - c_2 \end{gathered} \tag{III.4.20}$$

From $\left.\frac{\partial T}{\partial x}\right|_{x=\delta} = \frac{\dot{q}''}{\lambda}$ it yields that:

$$\begin{gathered} \left.\frac{\partial T}{\partial x}\right|_{x=\delta} = c_2\frac{\dot{m}''\,c}{\lambda}\cdot\exp\left(\frac{\dot{m}''\,c}{\lambda}\cdot\delta\right) = \frac{\dot{q}''}{\lambda} \\ \Rightarrow c_2 = \frac{\dot{q}''}{\dot{m}''\,c}\exp\left(-\frac{\dot{m}''\,c}{\lambda}\cdot\delta\right) \end{gathered} \tag{III.4.21}$$

Conclusion

Substitution of c_1 and c_2 into the expression of $T(x)$ gives:

$$T(x) = T_F - \frac{\dot{q}''}{\dot{m}''\,c} \cdot \exp\left(-\frac{\dot{m}''\,c}{\lambda} \cdot \delta\right) \cdot \left[1 - \exp\left(\frac{\dot{m}''\,c}{\lambda} \cdot x\right)\right] \tag{III.4.22}$$

Task b)

Determining the position of the maximum temperature can be achieved by either taking the derivative of the temperature profile, identifying its critical points (where the derivative equals zero), or by logical reasoning. Considering that the fluid should experience heating as it enters the porous wall and the point at which it exits the wall corresponds to the maximum temperature, denoted as $x = \delta$. Consequently, it follows that:

$$T_{\max} = T_F + \frac{\dot{q}''_s}{\dot{m}''\,c} \cdot \left[1 - \exp\left(-\frac{\dot{m}''\,c}{\lambda} \cdot \delta\right)\right] \tag{III.4.23}$$

$$= -15\ [^\circ\mathrm{C}] + \frac{150 \cdot 10^3\ [\frac{\mathrm{W}}{\mathrm{m}^2}]}{0.6\ [\frac{\mathrm{kg}}{\mathrm{m}^2\cdot\mathrm{s}}] \cdot 1000\ [\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}]} \cdot \left[1 - \exp\left(\frac{0.6\ [\frac{\mathrm{kg}}{\mathrm{m}^2\cdot\mathrm{s}}] \cdot 1000\ [\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}]}{8\ [\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}}]} \cdot 0.05\ [\mathrm{m}]\right)\right] = 229\ [^\circ\mathrm{C}]$$

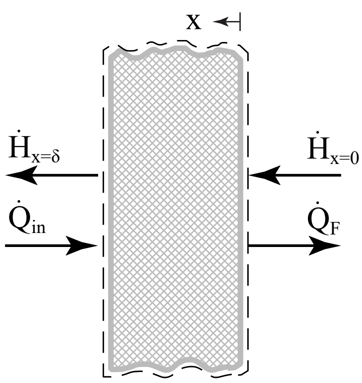
Conclusion

The maximum temperature reached within the wall is thus 229 °C.

Task c)

The heat flux $\dot{q}''_F$ at $x = 0$ is determined by setting up a global energy balance spanning from $x = 0$ to $x = \delta$.

1 **Setting up the balance:**



For the local energy balance exactly at $x = 0$, one finds the following energy balance:

$$0 = -\dot{H}_{x=0} - \dot{H}_{x=0} + \dot{Q}_{\mathrm{in}} - \dot{Q}_F \tag{III.4.24}$$

2 **Defining the elements within the balance:**

The rate of heat transferred into the fluid at $x = 0$ can be expressed in terms of the cross-sectional area and the heat flux $\dot{q}''_F$ per unit area:

$$\dot{Q}_F = \dot{q}''_F A \tag{III.4.25}$$

The rate of heat transferred into the fluid at $x = \delta$ can be expressed in terms of the cross-sectional area and the heat flux $\dot{q}''$ per unit area:

$$\dot{Q}_{\text{in}} = \dot{q}'' A \tag{III.4.26}$$

The energy transferred due to the motion of the fluid at $x = 0$ and $x = \delta$ is written as:

$$\begin{aligned} \dot{H}_{\text{x=0}} &= \dot{m}''\, A\, c\, T\,(x = 0) \\ &= \dot{m}''\, A\, c\, T_{\text{F}} \end{aligned} \tag{III.4.27}$$

and:

$$\dot{H}_{\text{x}=\delta} = \dot{m}''\, A\, c\, T\,(x = \delta) \tag{III.4.28}$$

3 **Inserting and rearranging:**

$$\begin{aligned} \dot{q}''_{\text{F}} &= \dot{q}'' - \dot{m}''\, c\, \left(T\,(x = \delta) - T_{\text{F}}\right) \\ &= 150 \cdot 10^3 \left[\frac{\text{W}}{\text{m}^2}\right] - 0.6 \left[\frac{\text{kg}}{\text{m}^2 \cdot \text{s}}\right] 1000 \left[\frac{\text{J}}{\text{kg} \cdot \text{K}}\right] (229 - 15)\ [\text{K}] = 3.5 \left[\frac{\text{kW}}{\text{m}^2}\right] \end{aligned} \tag{III.4.29}$$

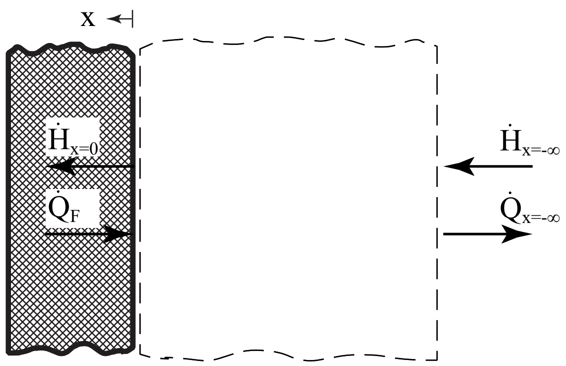
Conclusion

The heat flux per unit area transmitted into the fluid at $x = 0$ is thus $3.5\ \frac{\text{kW}}{\text{m}^2}$.

Task d)

$T_{\text{F},\infty}$ can be determined by setting up an energy balance. A global energy balance will be taken for $-\infty \leq x \leq 0$.

1 **Setting up the balance:**



The energy balance for the given domain is written as:

$$0 = \dot{Q}_{\text{F}} - \dot{Q}_{\text{x}=-\infty} - \dot{H}_{x=0} + \dot{H}_{x=-\infty} \tag{III.4.30}$$

2 **Defining the elements within the balance:**

The rate of heat transfer towards the fluid can be described as:

$$\dot{Q}_{\text{F}} = \dot{q}''_{\text{F}} A \tag{III.4.31}$$

The energy transferred due to the motion of the fluid at $x = 0$ and $x = -\infty$ is written as:

$$\begin{aligned} \dot{H}_{\text{x=0}} &= \dot{m}''\, A\, c\, T\,(x = 0) \\ &= \dot{m}''\, A\, c\, T_{\text{F}} \end{aligned} \tag{III.4.32}$$

and:

$$\dot{H}_{\text{x}=-\infty} = \dot{m}''\, A\, c\, T_{\text{F},\infty} \tag{III.4.33}$$

Lastly, from the convective temperature profile, it is known that sufficiently far away, the temperature gradient $\left.\frac{\partial T_{\text{F}}}{\partial x}\right|_{x=-\infty} = 0$, and therefore:

$$\dot{Q}_{x=-\infty} = -\lambda_{\text{F}} A \left.\frac{\partial T_{\text{F}}}{\partial x}\right|_{x=-\infty} = 0 \tag{III.4.34}$$

3 **Inserting and rearranging:**

$$\begin{aligned} T_{\text{F},\infty} &= -\frac{\dot{q}''_{\text{F}}}{\dot{m}''\, c} + T_{\text{F}} \\ &= -\frac{3.5 \cdot 10^3\ [\frac{\text{W}}{\text{m}^2}]}{0.6\ [\frac{\text{kg}}{\text{m}^2\cdot\text{s}}] \cdot 1000\ [\frac{\text{J}}{\text{kg}\cdot\text{K}}]} - 15\ [^\circ\text{C}] = -21\ [^\circ\text{C}] \end{aligned} \tag{III.4.35}$$

Conclusion

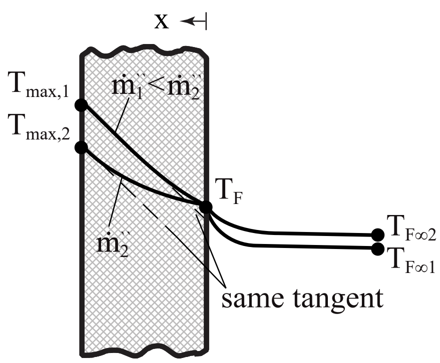
The temperature from the fluid far away from the wall is thus −15 °C.

Task e)

From the derived equation for the temperature profile inside the wall, it can be found that the maximum temperature is higher for a smaller mass flux. But at $x = 0$, the temperature is fixed to T_{F}. Therefore, inside the wall, it is steeper for a smaller mass flux.

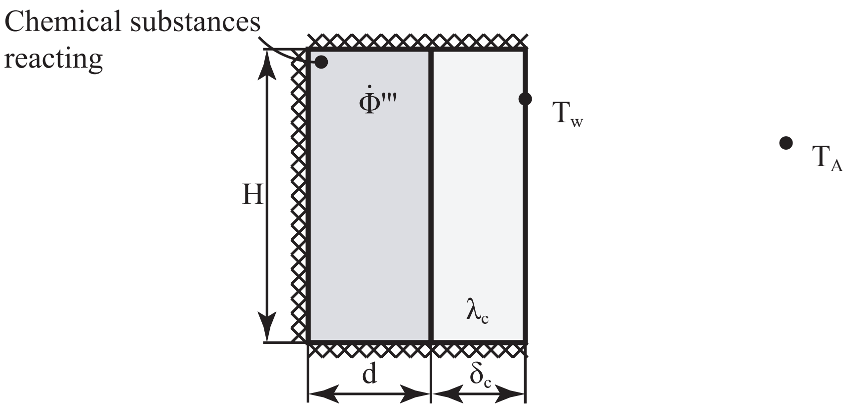
Furthermore, from the expression found for $T_{\text{F},\infty}$, it can be seen that this becomes higher in the case that the mass flux is smaller. Still, in both cases, it will have a zero-slope gradient for $x \to -\infty$.

Conclusion



Exercise III.5 (Substance container ⋆):

Imagine you are involved in the design of a chemical substance container. These containers house substances that generate heat during chemical reactions. The top and back are adiabatically insulated. During this reaction heat is dissipated to the surrounding air.



Given parameters:

- Height of the container: $H = 80$ cm
- Depth of the container: $d = 50$ cm
- Wall thickness of the container: $\delta_c = 10$ cm
- Thermal conductivity of the wall: $\lambda_c = 0.3$ W/mK
- Thermal conductivity of the air: $\lambda = 0.025$ W/mK
- Prandtl number of the air: $\mathrm{Pr} = 0.72$
- Kinematic viscosity of the air: $\nu = 1.5 \cdot 10^{-5}$ m^2/s
- Outside temperature of the wall: $T_w = 30$ °C
- Temperature of the ambient air: $T_A = 20$ °C

Hints:

- Assume one-dimensional heat transfer.
- Assume steady-state operating conditions.

Tasks:

a) Determine the interface temperature between the chemical substances and their container.

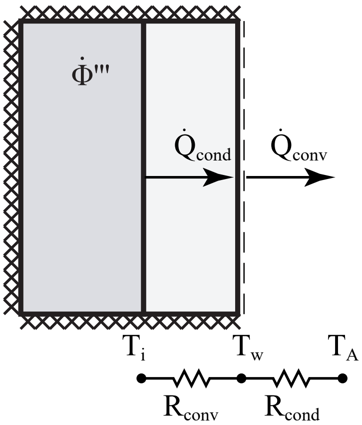
b) Determine the heat generated by the substances per unit volume $\dot{\Phi}'''$.

Solution III.5 (Substance container ⋆):

Task a)

The interface temperature between the chemical substances and the container wall can be determined by setting up an energy balance at the interface between the container and the ambient.

1 **Setting up the balance:**



The energy balance reads:

$$0 = \dot{Q}_{\mathrm{cond}} - \dot{Q}_{\mathrm{conv}} \tag{III.5.1}$$

2 **Defining the elements within the balance:**

The conductive flux can be written by use of the thermal resistance theorem:

$$\dot{Q}_{\mathrm{cond}} = \frac{T_{\mathrm{i}} - T_{\mathrm{w}}}{R_{\mathrm{cond}}} \tag{III.5.2}$$

The same goes for the convective flux:

$$\dot{Q}_{\mathrm{conv}} = \frac{T_{\mathrm{w}} - T_{\mathrm{A}}}{R_{\mathrm{conv}}} \tag{III.5.3}$$

Where the thermal resistances are defined as:

$$R_{\mathrm{cond}} = \frac{\delta_{\mathrm{c}}}{\lambda_{\mathrm{c}} A} \tag{III.5.4}$$

and:

$$R_{\mathrm{conv}} = \frac{1}{\overline{\alpha} A} \tag{III.5.5}$$

Nevertheless, the average heat transfer coefficient is not given. Given the absence of forced flow within the container, natural convection is the prevalent mode. Estimating the heat transfer coefficient involves employing a Nusselt correlation applicable to natural flow along a vertical plate. To initiate this process, it is imperative first to ascertain the value of the Grashof number and thus the flow regime.

The property to evaluate the fluid properties at yields from the average temperature of the boundary layer:

$$T_{\text{prop}} = \frac{T_{\text{w}} - T_{\text{A}}}{2} \tag{III.5.6}$$
$$= \frac{(30-20)\ [^{\circ}\text{C}]}{2} = 25\ [^{\circ}\text{C}]$$

Assuming air to act as an ideal gas, the volume expansion coefficient yields from:

$$\beta = \frac{1}{T_{\text{prop}}} \tag{III.5.7}$$
$$= \frac{1}{(25+273)\ [\text{K}]} = 3.4 \cdot 10^{-3}\ \left[\text{K}^{-1}\right]$$

Besides, the height of the container is the characteristic length to assess all relevant parameters:

$$L = H \tag{III.5.8}$$
$$0.8\ [\text{m}]$$

The Grashof number yields from:

$$\text{Gr}_{\text{L}} = \frac{\beta g \left(T_{\text{w}} - T_{\text{A}}\right) L^3}{\nu^2} \tag{III.5.9}$$
$$= \frac{3.4 \cdot 10^{-3}\ \left[\text{K}^{-1}\right] \cdot 9.81\ \left[\text{m/s}^2\right] (30-20)\ [\text{K}] \cdot 0.8^3\ \left[\text{m}^3\right]}{\left(1.5 \cdot 10^{-5}\right)^2\ \left[\text{m}^4/\text{s}^2\right]} = 7.5 \cdot 10^8\ [-]$$

From this, it thus appears that HTC.17 can be used to determine the average Nusselt number. Where for $\text{Pr} = 0.72$ it yields that $C = 0.516$.

$$\overline{\text{Nu}_{\text{L}}} = C\left(\text{Gr}_{\text{L}}\ \text{Pr}\right)^{1/4} \tag{III.5.10}$$
$$= 0.516\left(\left(7.5 \cdot 10^8\right) \cdot 0.72\right)^{1/4} = 78.6\ [-]$$

Rewriting the definition of the Nusselt number yields the average heat transfer coefficient:

$$\overline{\alpha} = \frac{\overline{\text{Nu}_{\text{L}}}\lambda}{L} \tag{III.5.11}$$
$$= \frac{78.6\ [-] \cdot 0.025\ [\text{W/mK}]}{0.8\ [\text{m}]} = 2.5\ \left[\text{W/m}^2\text{K}\right]$$

3 **Inserting and rearranging:**

Inserting all definitions into the energy balance and rewriting yields:

$$T_{\text{i}} = T_{\text{w}} + \frac{\alpha \delta_{\text{c}}}{\lambda_{\text{c}}}\left(T_{\text{w}} - T_{\text{A}}\right) \tag{III.5.12}$$
$$30\ [^{\circ}\text{C}] + \frac{2.5\ \left[\text{W/m}^2\text{K}\right] \cdot 0.1\ [\text{m}]}{0.3\ [\text{W/mK}]}(30-20)\ [^{\circ}\text{C}] = 38.2\ [^{\circ}\text{C}]$$

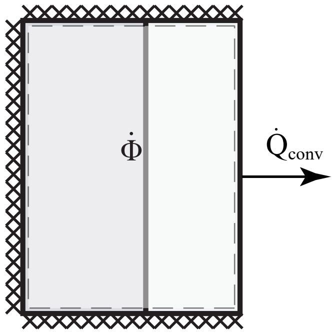
Conclusion

The interface temperature between the chemical substances and their container is thus 38.2 °C.

Task b)

To determine the heat generated by the substances per unit volume, a global energy balance around the container can be set.

1 **Setting up the balance:**



The steady-state energy balance reads:

$$0 = \dot{\Phi} - \dot{Q}_{\mathrm{conv}}, \tag{III.5.13}$$

which states that all heat generated by the substances is dissipated through convection.

2 **Defining the elements within the balance:**

The heat generated can be written as:

$$\begin{aligned} \dot{\Phi} &= \dot{\Phi}''' \, V \\ &\dot{\Phi}''' \, H \, d \, w, \end{aligned} \tag{III.5.14}$$

where w represents the width of the container.

The heat dissipated by convection can be stated as:

$$\begin{aligned} \dot{Q}_{\mathrm{conv}} &= \overline{\alpha} A \left(T_{\mathrm{w}} - T_{\mathrm{A}}\right) \\ &= \overline{\alpha} \, H \, w \, \left(T_{\mathrm{w}} - T_{\mathrm{A}}\right) \end{aligned} \tag{III.5.15}$$

3 **Inserting and rearranging:**

Inserting and rewriting yields:

$$\begin{aligned} \dot{\Phi}''' &= \frac{\overline{\alpha}}{d} \cdot \left(T_{\mathrm{w}} - T_{\mathrm{A}}\right) \\ &= \frac{2.5 \left[\mathrm{W/m^2K}\right]}{0.5 \left[\mathrm{m}\right]} \cdot \left(30 - 20\right) \left[{}^{\circ}\mathrm{C}\right] = 49.1 \left[\mathrm{W/m^3}\right] \end{aligned} \tag{III.5.16}$$

Conclusion

The rate of heat generation by the substances per unit volume is thus 49.1 $\mathrm{W/m^3}$.