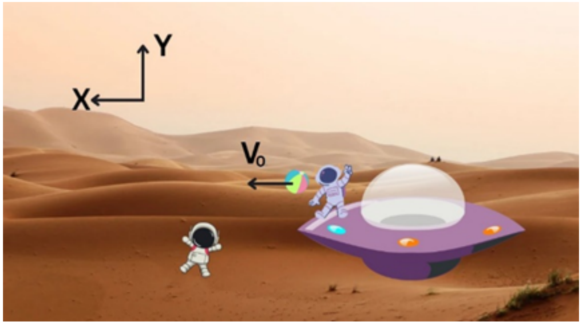
Mars Astronauts



Finally, the ball was in the air for 6 seconds and landed 5 meters away from the spaceship. Find the velocity v_0 of the ball. Take $g_{mars} = 4$ m/s^2.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given:

Gravitational acceleration on Mars: $g_{\text{mars}} = 4$ m/s^2
Airtime: $t_{\text{end}} = 6$ s
End position ball: $x(t_{\text{end}}) = 5$ m

Solution:

As can be seen in the image, the positive x-axis is taken to the left. The initial position of the ball is taken to be zero, hence $s_0 = 0$ m. The acceleration in the horizontal direction is also zero, thus Equation (6) becomes:

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \quad \Rightarrow \quad x(t) = v_0 t \tag{7}$$

Inserting $t = t_{\text{end}} = 6$ s and $x(t_{\text{end}}) = 5$ m gives:

$$x(t_{\text{end}}) = v_0 t_{\text{end}} \quad \Rightarrow \quad x(6) = v_0 \cdot 6 = 5 \quad \Rightarrow \quad v_0 = \frac{5}{6}\ \text{m/s} \tag{8}$$

Thus the initial velocity of the ball was $v_0 = \frac{5}{6}$ m/s.