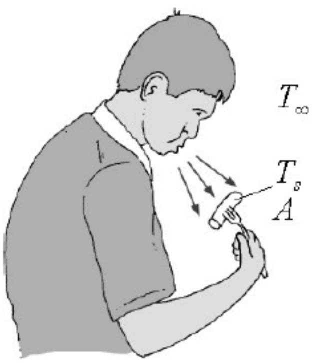
Solutions lecture 7

7.1 Blowing man with a carrot

Analysis

We need to determine the temperature of the carrot after one minute of blowing. Furthermore, we need to determine how long the man should blow to lower the temperature of the carrot to 80 °C. The initial temperature is 100 °C, with an ambient temperature of 30 °C. The heat transfer coefficient, thermal conductivity, density, and specific heat of the carrot are given.



Approach

Assumptions

We assume that the carrot is a perfect cylinder with a length of 7 cm and a diameter of 2 cm.

Route to solution

With the assumption that the carrot has a uniform temperature, we need to determine if the carrot can be considered a lumped system. To do this, first, the characteristic length needs to be determined. This can be done with

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4} \cdot D^2 \cdot l}{\pi \cdot D \cdot L + 2 \cdot \left(\frac{\pi}{4} \cdot D^2\right)}$$

This value can then be substituted in the formula for the Biot number

$$\mathrm{Bi} = \frac{hL_c}{k}$$

If Bi$<$ 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-b \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

For objects with a uniform temperature distribution, the constant b is defined as

$$b = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta$$

Substituting the known values and $t = 60$, the temperature is found.

For the second question, we again use the last-mentioned equation and check for which t on the right-hand side equals 80.

Elaboration

We start with determining the characteristic length:

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4} \cdot D^2 \cdot l}{\pi \cdot D \cdot L + 2 \cdot \left(\frac{\pi}{4} \cdot D^2\right)} = 0.004\,375\,\mathrm{m}$$

Substituting the characteristic length in the formula for the Biot number

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{15 \cdot 0.004375}{0.8} = 0.082[-]$$

Because Bi< 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-b \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

The constant b is now:

$$b = \frac{hA}{\rho c_p V} = \frac{15 \cdot \left(\pi \cdot 0.02 \cdot 0.07 + 2\left(\frac{\pi}{4} \cdot 0.02^2\right)\right)}{1100 \cdot 3.60 \cdot 10^3 \cdot \frac{\pi}{4} \cdot 0.02^{\cdot} 0.07} = 0.866 \times 10^{-3}\,\mathrm{s}^{-1}$$

Rewriting and substituting the values:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta = (100 - 20) \cdot e^{-0.866 \cdot 10^{-3} \cdot 60} + 20 = 95.96\,^\circ\mathrm{C}$$

For the second part, as the temperature has decreased from 100 °C to 80 °C:

$$\begin{aligned}
80 &= (100 - 20) \cdot e^{-0.866 \cdot 10^{-3} \cdot t} + 20 \\
60 &= 80 e^{-0.866 \cdot 10^{-3} \cdot t} \\
0.75 &= e^{-0.866 \cdot 10^{-3} \cdot t} \\
\ln(0.75) &= -0.866 \cdot 10^{-3} \cdot t \\
-02877 &= -0.866 \cdot 10^{-3} \cdot t \\
t &= 332\,\mathrm{s}
\end{aligned}$$

Evaluation

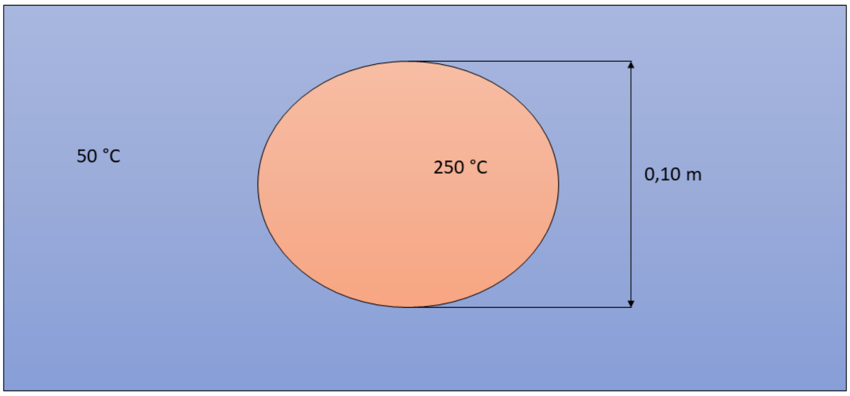
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

7.2 Cooling a copper sphere

Analysis

We need to determine the temperature of the copper block after it is immersed in a cold fluid, 5 minutes after immersion. The density, specific heat, and thermal conductivity of the copper sphere are given, as well as the diameter and the initial temperature. The heat transfer coefficient and the temperature of the fluid are also provided.



Approach

Assumptions

Route to solution

We start with determining the characteristic length of the copper sphere. This can be done with the following equation:

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6}$$

This value can be substituted in the equation for the Biot number:

$$\text{Bi} = \frac{hL_c}{k}$$

When Bi < 0.1, the system is considered lumped, and hence the lump capacitance method may be applied for the solution:

$$\theta(t) = e^{-b\cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

For objects with a uniform temperature distribution, the constant b is defined as

$$b = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta$$

Substituting the known values and $t = 300$, the temperature is found.

Elaboration

We start with determining the characteristic length:

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6} = 0.0167\,\mathrm{m}$$

Substituting the characteristic length in the formula for the Biot number

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{200 \cdot 0.01667}{386} = 8.64 \cdot 10^{-3}[-]$$

Because Bi< 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-b \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

The constant b is now:

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200}{8954 \cdot 0.01667 \cdot 383} = 0.0035\,\mathrm{s}^{-1}$$

Rewriting and substituting the values:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta = (250 - 50) \cdot e^{-0.0035 \cdot 300} + 50 = 120\,^\circ\mathrm{C}$$

Evaluation

Check your answer:

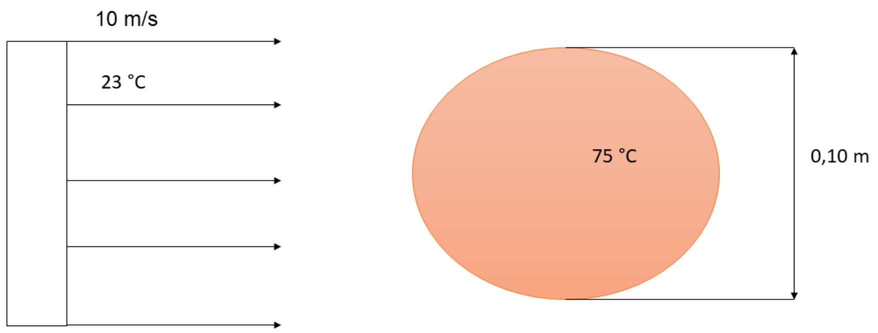
- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

7.3 Cooling a copper sphere under forced convection conditions

Analysis

A copper sphere is wrapped in a plastic film and placed in an oven at 75 °C. After removal from the oven, the sphere is exposed to an air stream at 23 °C and 10 m/s. Estimate the time taken to cool the sphere to 35 °C. The following information is given:

- Copper:
  - $\rho = 8933\ \mathrm{kg\,m^{-3}}$, $k = 400\ \mathrm{W\,m^{-1}\,K^{-1}}$, $c_p = 380\ \mathrm{J\,kg^{-1}\,K^{-1}}$
- For air at 23 °C
  - $\mu = 18.16 \cdot 10^{-6}\ \mathrm{N\,s\,m^{-2}}$, $\nu = 15.36 \cdot 10^{-6}\ \mathrm{m^2\,s^{-1}}$
  - $k = 0.0258\ \mathrm{W\,m^{-1}\,K}$, Pr=0.709
  - $\mu_s = 19.78 \cdot 10^{-6}\ \mathrm{N\,s\,m^{-2}}$ at 35 °C



The diameter in the figure above is wrong!

Approach

Assumptions

Assume the sphere is a lumped system.

Route to solution

A relation for the Nusselt number is provided. The only unknown in this is the Reynolds number, so we first need to determine this:

$$\mathrm{Re} = \frac{\rho U D}{\mu} = \frac{UD}{\nu}$$

Substitute all the variables in the correlation for the Nusselt number. Now, the Nusselt number is defined as

$$\mathrm{Nu} = \frac{hD}{k} \implies h = \frac{k}{D} \cdot \mathrm{Nu}$$

With h known, and the system being lumped, the following equations are valid:

$$\theta(t) = e^{-b \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

where b is defined as

$$b = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$\frac{T(t) - T_\infty}{T(0) - T_\infty} = e^{-b \cdot t}$$

Substituting the known values and rewriting will give an expression for t.

Elaboration

We start with determining the Reynolds number:

$$\mathrm{Re} = \frac{UD}{\nu} = \frac{10 \cdot 0.01}{15.36 \cdot 10^{-6}} = 6510[-]$$

Substitution of all variables in the correlation for the Nusselt number:

$$\begin{aligned}\mathrm{Nu} &= 2 + \left[0.4(6510)^{0.5} + 0.06(6510)^{2/3}\right](0.709)^{0.4}\left(\frac{18.16 \cdot 10^{-6}}{19.75 \cdot 10^{-6}}\right)^{0.25} \\ &= 2 + [32.27 + 20.92] \cdot 0.87 \cdot 0.979 = 47.3[-]\end{aligned}$$

Rewriting the definition of the Nusselt number and substituting:

$$h = \frac{k}{D} \cdot \mathrm{Nu} = \frac{0.0258}{0.01} \cdot 47.3 = 122\,\mathrm{W\,m^{-2}\,K^{-1}}$$

Now, calculating constant b:

$$b = \frac{hA_s}{\rho V c_p} = \frac{122 \cdot 4\pi R^2}{\rho \cdot \frac{4}{3}\pi R^3 \cdot c_p} = \frac{122 \cdot 3}{8933 \cdot 0.005 \cdot 380} = 0.021\,56\,\mathrm{s^{-1}}$$

Substituting b and the temperatures into the rewritten equation for $\theta(t)$

$$\begin{aligned}\frac{T(t) - T_\infty}{T(0) - T_\infty} &= e^{-b \cdot t} \\ \frac{35 - 23}{75 - 23} &= e^{-0.02156 \cdot t} \\ 0.2308 &= e^{-0.02156 \cdot t} \\ \ln(0.2308) &= -0.02156 \cdot t \\ 1.466 &= 0.02156t \\ t &\approx 68\mathrm{s}\end{aligned}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

7.4 Marbles - Hand in

Analysis

One of the production steps of Marbles is letting the marbles cool down to room temperature. The marbles have been cooled down rapidly from a high temperature of 650 °C to 100 °C. In a further cooling chamber, the marbles are cooled down to 30 °C. Calculate how long the marbles need to be in this further cooling chamber to reach the 30 °C. In the cooling chamber, there is an air stream of 13 m/s at 20 °C. First, calculate the Biot number for the marble. The marble has a diameter of 13 mm.

Approach

Assumptions

We assume that the marble is a perfect sphere with a diameter of 13mm.

Route to solution

First, we need to find the material properties of the marbles. As we are making an approximation anyway, sources found on any search engine are probably good enough. For this example, we will use the following properties.
$\rho = 2200$ kg m^{-3}, $k = 6$ W m^{-1} K^{-1}, $c_p = 792$ J kg^{-1} K^{-1}
With the assumption that the marble has a uniform temperature, we need to determine if the marble can be considered a lumped system. To do this, the biot number is checked.

$$\text{Bi} = \frac{hL_c}{k}$$

If Bi< 0.1, the marble may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-b \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

For objects with a uniform temperature distribution, the constant b is defined as

$$b = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to the variable time:

$$t = \frac{ln\left(\dfrac{T(t) - T_\infty}{T(0) - T_\infty}\right)}{-c}$$

Substituting the known values, the cooling time is found.

Elaboration

We start with determining the Nusselt number used for the heat transfer coefficient h:

$$Nu = 2 + \left[0.4 \cdot Re^{1/2} + 0.06 \cdot Re^{2/3}\right] Pr^{0.4} \cdot \left(\frac{\mu_\infty}{\mu_f}\right)^{0.25}$$

For this we calculate Re:

$$Re = \frac{U \cdot D}{\nu_f}$$

Using the properties of the air we use:
$\nu_f = 1.946 \times 10^{-5}$, $k_{air} = 0.0284$, $Pr = 0.7189$, $\mu_\infty = 1.825 \times 10^{-5}$, $\mu_f = 2.03 \times 10^{-5}$

Then the Reynolds number equals:

$$Re = \frac{13 \cdot 0.013}{1.946 \cdot 10^{-5}} = 8687$$

Then the Nusselt number is:

$$Nu = 2 + [0.4 \cdot 8687^{1/2} + 0.06 \cdot 8687^{2/3}]0.7189^{0.4} \cdot (\frac{1.825}{2.03})^{0.25} = 55.45$$

With the Nusselt number, the heat transfer coefficient h is calculated as:

$$h = \frac{k_{air}}{L_c} \cdot Nu = \frac{0.0284}{0.013} \cdot 55.45 = 121.3$$

Note that in this calculation, the characteristic length L_c is the diameter of the marble. For calculating the characteristic length for the Biot number, the calculation is:

$$L_c = \frac{V}{A} = \frac{\frac{4}{3}\pi \cdot r^3}{4\pi \cdot r^2} = \frac{r}{3} = 0.0022\,\mathrm{m}$$

Substituting the characteristic length in the formula for the Biot number

$$\mathrm{Bi} = \frac{h_c}{k} = \frac{121.3 \cdot 0.0022}{6} = 0.0438[-]$$

Because Bi< 0.1, the marble may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-b \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

The constant b is now:

$$b = \frac{h \cdot A}{\rho \cdot c_p \cdot V} = \frac{h \cdot 3}{\rho \cdot c_p \cdot r} = \frac{121.3 \cdot 3}{2200 \cdot 792 \cdot 0.013} = 0.0321\,\mathrm{s}^{-1}$$

Rewriting and substituting the values:

$$t = \frac{ln\left(\frac{T(t) - T_\infty}{T(0) - T_\infty}\right)}{-c} = \frac{ln\left(\frac{30-20}{100-20}\right)}{-0.0321} = 64.7s$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?