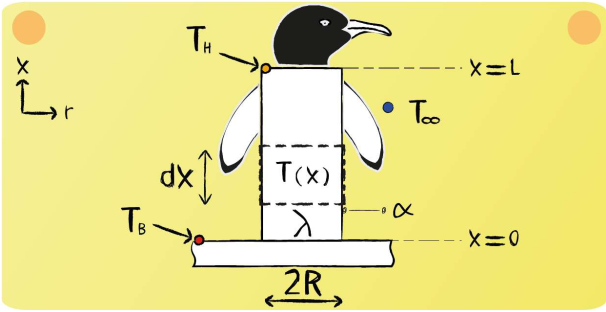
Lecture 11 - Question 7



Derive a homogeneous differential equation to describe the axial temperature distribution for the body of a penguin. Assume one-dimensional, steady-state heat transfer in the x-direction with no sources/sinks.

Homogenization:

$$\Theta \;=\; T(x) \;-\; T_\infty$$

$$m^2 = \;\tfrac{2\cdot\alpha}{\lambda\cdot R}$$

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} - \dot{Q}_{conv}(x) = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda\cdot\pi R^2\cdot\tfrac{\partial T}{\partial x} \;\rightarrow \dot{Q}_{x,in} = -\lambda\cdot\pi R^2\cdot\tfrac{\partial\Theta}{\partial x}$$

$$\dot{Q}_{x,out} \;=\; -\lambda\cdot\pi R^2\cdot\tfrac{\partial T}{\partial x} + \tfrac{\partial\dot{Q}_{x,in}}{\partial x}\cdot dx \;\rightarrow \dot{Q}_{x,out} \;=\; \dot{Q}_{x,in} - \lambda\cdot\pi R^2\cdot\tfrac{\partial^2\Theta}{\partial x^2}dx$$

$$\dot{Q}_{conv} \;=\; \alpha\cdot 2\pi R dx\cdot(T(x) - T_\infty) \rightarrow \dot{Q}_{conv} \;=\alpha\cdot 2\pi R dx\cdot\Theta$$

Boundary Conditions:

$$\Theta(x=0) \;= T_B - \;T_\infty$$

$$\tfrac{\partial\Theta}{\partial x}\,(x=L) = -\tfrac{\dot{q}''2L}{\lambda R}$$

The first boundary condition results from the fact that $T(x=0) = T_\mathrm{B}$ and the second one from the fact that $Q_\mathrm{cond}(x=L) = \dot{q}''2\pi RL$.