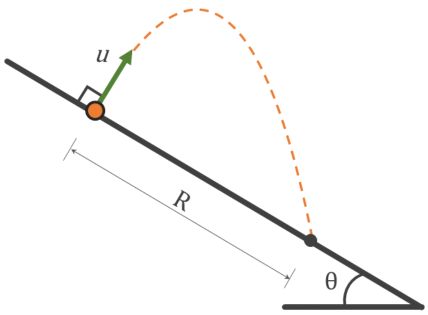
Projectile at a Right Angle



A projectile is fired with a velocity u at a right angle to the slope, which is inclined at an angle θ with the horizontal. Which of the following is the correct expression for the distance R to the point of impact.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a\int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (at + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (at + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given:

Initial velocity: $v_0 = u$
Displacement to point of impact: R

Solution:

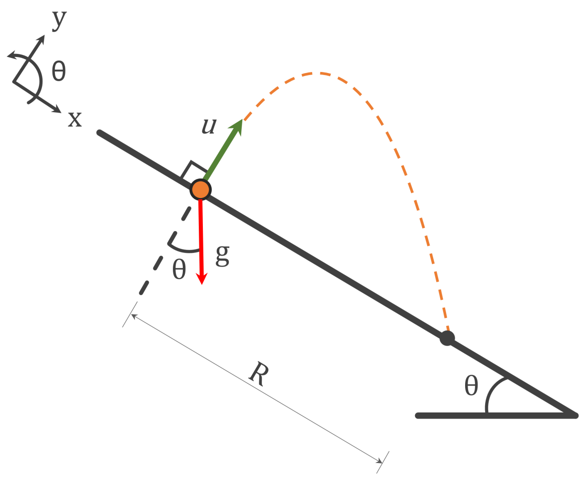


Figure 1: Projectile at Right Angle, with coordinate system

The coordinate system is defined such that y is defined positive upwards in the same direction of $\vec{u}$. Figure 1 shows the coordinate system, together with the gravitational acceleration $\vec{g}$. From Figure 1 the following two relations of x and y can be established.

$$\begin{cases} x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + x_0 \\ y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0 \end{cases} \tag{7}$$

Where $v_{x,0} = 0$ m/s, $v_{y,0} = u$ and $x_0 = y_0 = 0$ m. In addition, the x-and y-displacements after the projectile has landed on the slope are equal to R and 0 respectively. The acceleration in x-direction is $a_x = g\sin\theta$ and the acceleration in the y-direction is $a_y = -g\cos\theta$.

$$\begin{cases} x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + x_0 \\ y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0 \end{cases} \Rightarrow \begin{cases} x(t) = \frac{1}{2}gt^2\sin\theta = R \\ y(t) = -\frac{1}{2}gt^2\cos\theta + ut = 0 \end{cases} \tag{8}$$

From Equation (8), the time t can be rewritten as follows. The $t = 0$ solution is not considered.

$$t \neq 0 \quad \Rightarrow \quad u = \frac{1}{2}gt\cos\theta \quad \Rightarrow \quad t = \frac{2u}{g\cos\theta} \tag{9}$$

Inserting Equation (9) into Equation (8) for t gives the final expression for R.

$$R = \frac{1}{2}g\sin\theta \cdot \left(\frac{2u}{g\cos\theta}\right)^2 = \frac{2u^2}{g}\frac{\tan\theta}{\cos\theta} \tag{10}$$