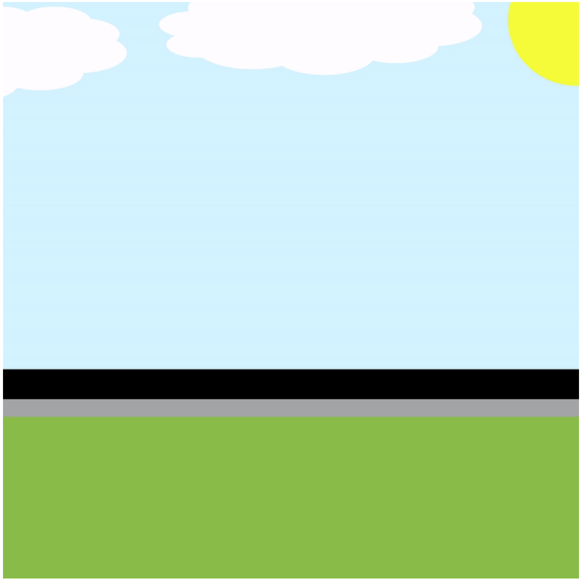
Passenger Plane Take-Off



The pilot of a passenger plane brings the engines to full take-off power before releasing the brakes as the aircraft is standing on the runway. The jet thrust remains constant, and the aircraft has a near-constant acceleration of 0.4g. If the take-off speed is 216km/h, what is the distance s in meters the plane needs to take off? Neglect air resistance and take $g = 10$ m/s^2.

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

Given:
Acceleration: $a = 0.4 \cdot g = 4m/s^2$
Initial velocity: $v_0 = 0m/s$
End velocity: $v_1 = 200km/h = 60m/s$

In the case of this exercise, s_0 and v_0 are equal to zero, resulting in:

$$v(t) = a \cdot t \tag{7}$$

$$s(t) = \frac{1}{2}a \cdot t^2 \tag{8}$$

Filling in $v_1 = 60m/s$ and $a = 4m/s^2$ in Equation 7 results in:

$$60 = 4 \cdot t \quad \Rightarrow \quad t = 15s \tag{9}$$

Inserting t= 15s in Equation 8 results in the final answer:

$$s(15) = \frac{1}{2} \cdot 4 \cdot 15^2 = 450m \tag{10}$$