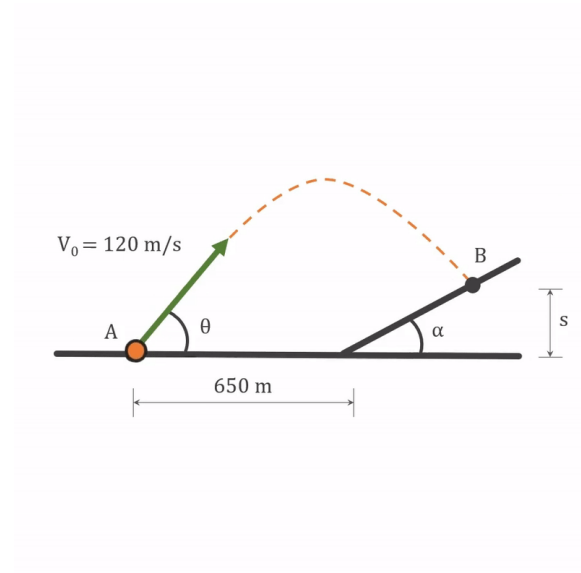


Trajectory of Ball



A ball is launched from point A with the initial conditions shown. Find an expression for the vertical displacement $y(t)$.

Neglect all air resistances.

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (at + v_0) dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (at + v_0) dt \quad (5)$$

$$s(t) = \frac{1}{2} at^2 + v_0 t + s_0 \quad (6)$$

Solution:

For the vertical displacement in y -direction, equation (6) results in:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \quad (7)$$

Since $s_{y,0} = 0$ m and $a_y = -g$, the resulting equation reduces to:

$$y(t) = -\frac{1}{2}gt^2 + v_{y,0}t \quad (8)$$

Where $v_{y,0} = v_0 \sin \theta$. Substituting for $v_{y,0}$ gives the final expression:

$$y(t) = -\frac{1}{2}gt^2 + v_0 t \sin \theta \quad (9)$$

Or written alternatively as:

$$y(t) = \frac{1}{2}a_y t^2 + v_0 t \sin \theta \quad (10)$$