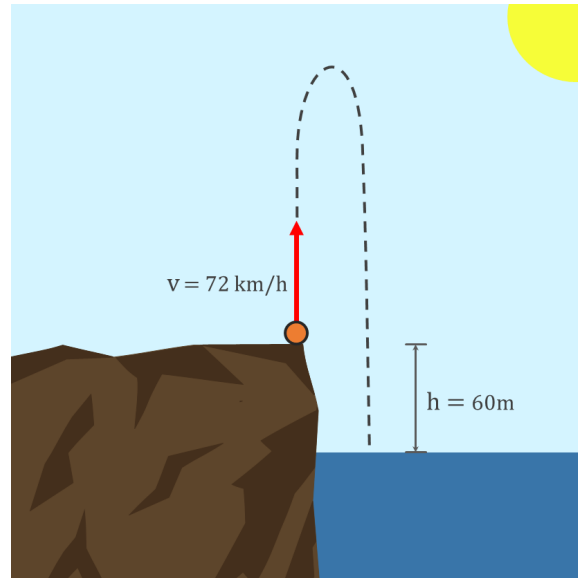


Ball Thrown from Cliff



A ball is thrown vertically up with an initial vertical velocity of 72 km/h at the edge of a 60 meter high cliff. What is the total time t_{end} in seconds that the ball is in the air?

Neglect air resistance and take $g = 10 \text{ m/s}^2$.

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (at + v_0) dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (at + v_0) dt \quad (5)$$

$$s(t) = \frac{1}{2} at^2 + v_0 t + s_0 \quad (6)$$

Given quantities:

Initial vertical velocity: $v_{y,0} = 72 \text{ km/h} = 20 \text{ m/s}$

Initial height of the ball (with respect to the cliff): $y_0 = s_{y,0} = 0 \text{ m}$

Gravitational constant: $g = 10 \text{ m/s}^2$

Solution:

For the vertical displacement in y -direction, Equation (6) results in:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \quad (7)$$

At the instant the ball reaches the sea, its vertical displacement is $y = -60 \text{ m}$. The acceleration on the ball is the gravitational acceleration: $a_y = -g$. Combining this into Equation (7) yields an equation for the total time of the ball to reach the sea:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \Rightarrow y(t) = -\frac{1}{2}gt^2 + v_{y,0}t \quad (8)$$

Inserting $y(t_{\text{end}}) = -60 \text{ m}$, $g = 10 \text{ m/s}^2$ and $v_{y,0} = 20 \text{ m/s}$ results in:

$$-60 = -\frac{1}{2} \cdot 10 \cdot t_{\text{end}}^2 + 20 \cdot t_{\text{end}} \Rightarrow t_{\text{end}}^2 - 4 \cdot t_{\text{end}} - 12 = 0 \quad (9)$$

$$(t_{\text{end}} + 2)(t_{\text{end}} - 6) = 0 \Rightarrow t_{\text{end}} = -2 \text{ s} \quad \vee \quad t_{\text{end}} = 6 \text{ s} \quad (10)$$

Since t_{end} cannot be negative, the final answer is $t_{\text{end}} = 6 \text{ s}$.