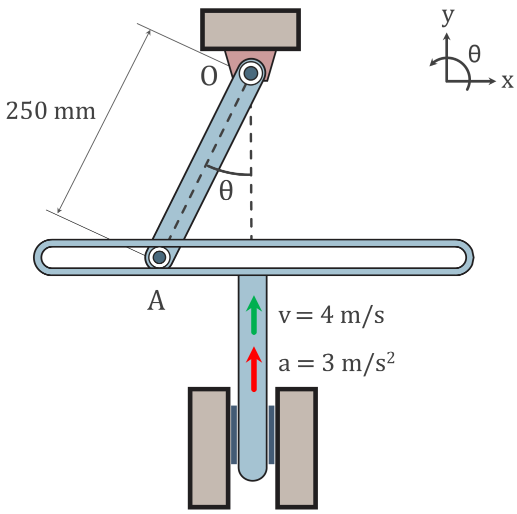
Link Motion



A link rotates clockwise shown in the picture. Give the correct expression for the acceleration of the roller inside slotted guide, a_A.

Using known expressions:

$$\boldsymbol{a_{A/O}} = \boldsymbol{a_O} + \boldsymbol{a_t} + \boldsymbol{a_n} = \boldsymbol{a_O} + \boldsymbol{\alpha} \times \boldsymbol{r_{A/O}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r_{A/O}}) \tag{1}$$

Given:
Vertical velocity of the slotted guide: $v = 4$ m/s
Vertical acceleration of the slotted guide: $a = 3$ m/s^2
Distance between O and A: $L_{OA} = 0.25$ m
Angle: θ
Angular velocity: $\omega = \dot{\theta}$
Angular acceleration: $\alpha = \ddot{\theta}$

First a kinematic diagram is made in Figure 1 which shows all accelerations acting on roller A. From this it can be seen that a_A can be calculated using a_n and a_t. Using Equation 1 $a_{A/O}$ becomes:

$$\boldsymbol{a_{A/O}} = \boldsymbol{a_O} + \boldsymbol{\alpha} \times \boldsymbol{r_{A/O}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r_{A/O}}) = \tag{2}$$

$$\begin{pmatrix} 0 \\ 0 \\ -\alpha \end{pmatrix} \times \begin{pmatrix} -0.25\sin\theta \\ -0.25\cos\theta \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left(\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -0.25\sin\theta \\ -0.25\cos\theta \\ 0 \end{pmatrix} \right) =$$

$$\begin{pmatrix} -0.25\alpha\cos\theta \\ 0.25\alpha\sin\theta \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -0.25\omega\cos\theta \\ 0.25\omega\sin\theta \\ 0 \end{pmatrix} =$$

$$\begin{pmatrix} -0.25\alpha\cos\theta \\ 0.25\alpha\sin\theta \\ 0 \end{pmatrix} + \begin{pmatrix} 0.25\omega^2\sin\theta \\ 0.25\omega^2\cos\theta \\ 0 \end{pmatrix} = \begin{pmatrix} -0.25\alpha\cos\theta + 0.25\omega^2\sin\theta \\ 0.25\alpha\sin\theta + 0.25\omega^2\cos\theta \\ 0 \end{pmatrix}$$

Where $\omega = \dot{\theta}$ and $\alpha = \ddot{\theta}$.
From Figure 1 follows:

$$\boldsymbol{a_{A/O}} = \boldsymbol{a_A} + \boldsymbol{a} \quad \Rightarrow \quad \begin{pmatrix} -0.25\alpha\cos\theta + 0.25\omega^2\sin\theta \\ 0.25\alpha\sin\theta + 0.25\omega^2\cos\theta \\ 0 \end{pmatrix} = \begin{pmatrix} -a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \tag{3}$$

This results in that $a_A = -0.25\alpha\cos\theta + 0.25\omega^2\sin\theta$. Since we have drawn a_A in Figure 1 in the negative x-direction. The acceleration of the roller in the positive x-direction becomes: $a_A = -0.25 \cdot \cos\theta \cdot \ddot{\theta} + 0.25 \cdot \sin\theta \cdot \dot{\theta}^2$.

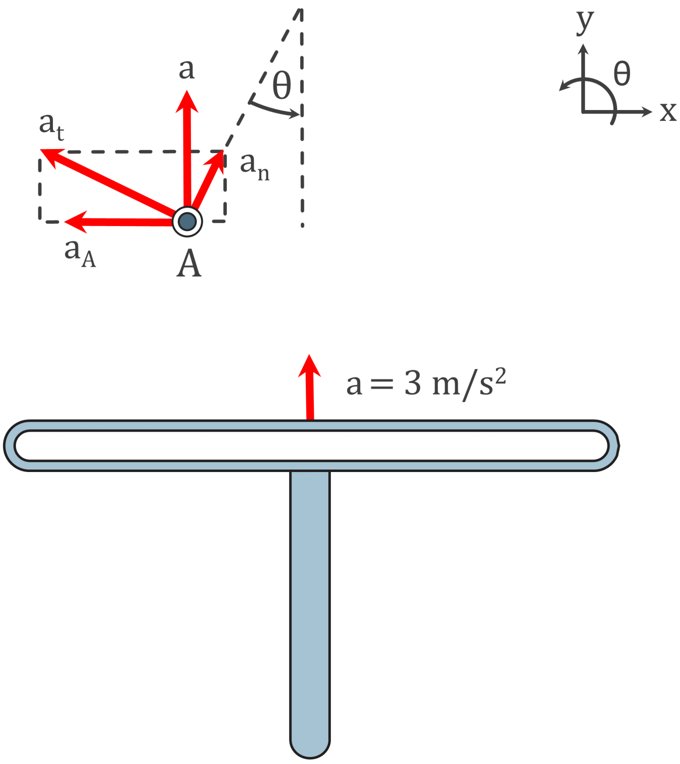


Figure 1: Kinematic diagram of the roller and the slotted guide.