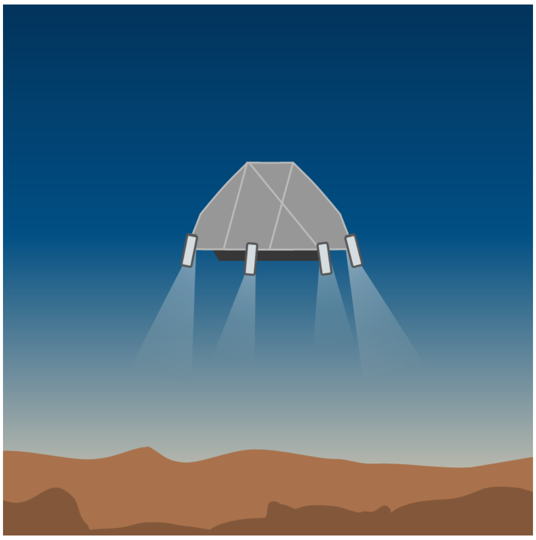
Mars Rover Landing



In the final stages of a Mars landing, a rover descends under retrothrust of its descent engine to a height of 5 m above the Martian surface where it has a downward speed of 2 m/s. Compute the impact speed v_{impact} of the rover with the surface if the engine is then immediately shut down.

Assume gravity on Mars is 0.38 times stronger than on Earth and take $g_{\text{earth}} = 10$ m/s^2.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a\int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \tag{6}$$

Given quantities:

Gravitational acceleration on Earth: $g_{\text{earth}} = 10 \text{ m/s}^2$
Gravitational acceleration on Mars: $g_{\text{mars}} = 0.38 g_{\text{earth}} = 0.38 \cdot 10 = 3.8 \text{ m/s}^2$
Initial downward velocity: $v_0 = -2 \text{ m/s}$
Distance above surface: $h = s_0 = 5 \text{ m}$

Solution:

The time it takes to reach the surface can be calculated using Equation (6), where the gravity on Mars is pointing downward. Take the origin of the coordinate system at the ground (hence $s(t_{\text{end}}) = 0$ m).

$$s(t_{\text{end}}) = \frac{1}{2} a t_{\text{end}}^2 + v_0 t_{\text{end}} + s_0 \quad \Rightarrow \quad \frac{1}{2} \cdot -3.8 \cdot t_{\text{end}}^2 + -2 \cdot t_{\text{end}} + 5 = 0 \tag{7}$$

From this, t_{end} can be calculated using the quadratic formula:

$$t_{\text{end}} = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot \frac{-3.8}{2} \cdot 5}}{-3.8} \approx -0.53 \mp 1.71 \Rightarrow t_{\text{end}} = -2.23 \ \vee \ t_{\text{end}} = 1.18 \tag{8}$$

We only consider positive values for t_{end}. Inserting $t_{\text{end}} = 1.18$ s in Equation (3) gives the velocity at the surface.

$$v(t_{\text{end}}) = a t_{\text{end}} + v_0 \quad \Rightarrow \quad v(1.18) = -3.8 \cdot 1.18 - 2 \approx -6.48 \text{ m/s} \tag{9}$$

Thus the impact speed at the surface is $v_{\text{impact}} = 6.48$ m/s and the velocity is pointing in the downward direction.