

# Energy & Heat Transfer



## Lecture 4

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# Study Materials



- Slides
- Book : (Y. A. Cengel & A. J. Ghajar. Heat and Mass Transfer: Fundamental & Application )
- Heat Quiz (<https://heatquiz.wsa.rwth-aachen.de/> )
- Learning activities

# Recap of lecture 3

Forced and natural/free convection:

$$\dot{Q} = hA\Delta T$$

$$\dot{q} = h\Delta T$$

Newton's cooling law

Nusselt Number: dimensionless variety of heat transfer coefficient  $h$

$$\text{Nu} = \frac{hL_c}{k} \quad \text{With } L_c \text{ a characteristic length for the considered geometry}$$

Determining the Nusselt Number: (empirical) correlations

- Forced convection: Nu as function of Re, Pr a.k.a.:  $\text{Nu} = f(\text{Re}, \text{Pr})$
- Natural convection: This lecture

⇒ Relation Nu, Re, Pr dependent on geometry and flow regime (laminar/turbulent)

⇒ Nu, Re, Pr dimensionless numbers: reduction number of variables

# Recap of lecture 3

## Convection Resistance :

$$\dot{Q} = hA\Delta T = \frac{\Delta T}{R_{conv}} \quad \text{met} \quad R_{conv} = \frac{1}{hA} \quad (\text{K/W})$$

## Dimensionless Numbers :

Nusselt Number: 
$$\text{Nu} = \frac{hL_c}{k}$$

Reynolds Number: 
$$\text{Re} = \frac{\rho U L_c}{\mu}$$

Prandtl Number: 
$$\text{Pr} = \frac{\mu c_p}{k}$$

+ Background on boundary layers

# Recap of lecture 3

If  $\dot{Q}$  must be found:

- Calculate at film temperature :  $T_f = \frac{T_s + T_\infty}{2}$
- Pull out ingredients like  $\mu, \rho, k, \text{Pr}$  from tables – like assignment bundle:  
air or given fluid) at  $T_f = \frac{T_s + T_\infty}{2}$
- Calculate Re and choose appropriate correlation based on geometry and Re
- Calculate Nu
- Derive  $h$  from it
- Fill out Newton's cooling law:  $\dot{Q} = hA\Delta T$

# Learning objectives lecture 4

## ○ Heat transfer through natural convection

### ● Natural Convection

○ Grashof Number

○ Rayleigh Number

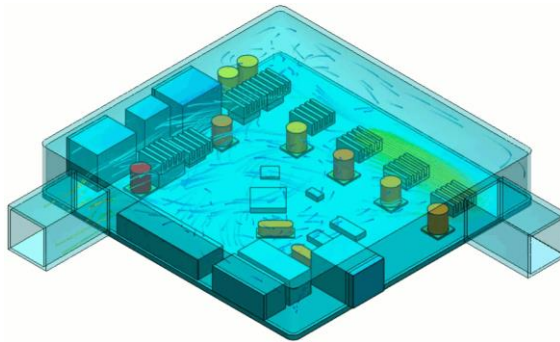
○ Nusselt Number

○ Using correlations for various configurations

○ Calculating natural convection with step-by-step plan

# TYPES OF CONVECTION (FROM LECTURE 3)

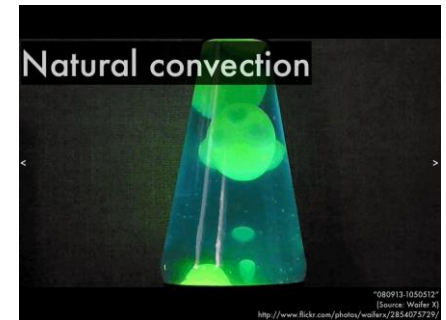
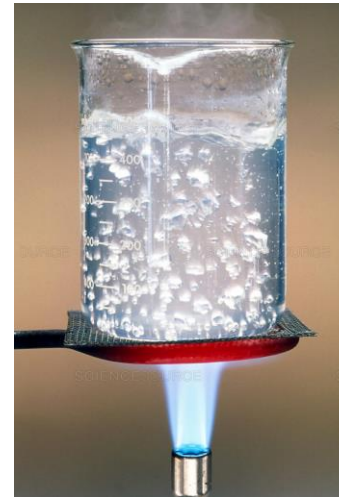
## Forced convection



Imposed flow (by pump, fan, ...)



## Natural/free convection



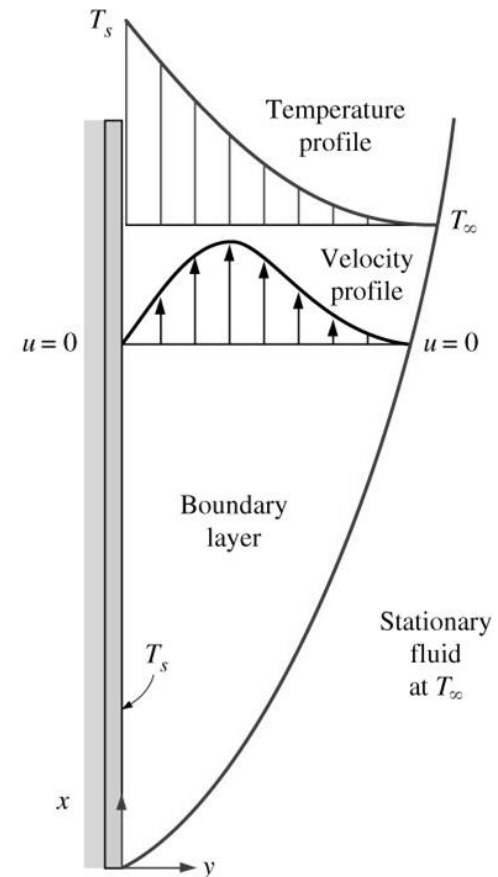
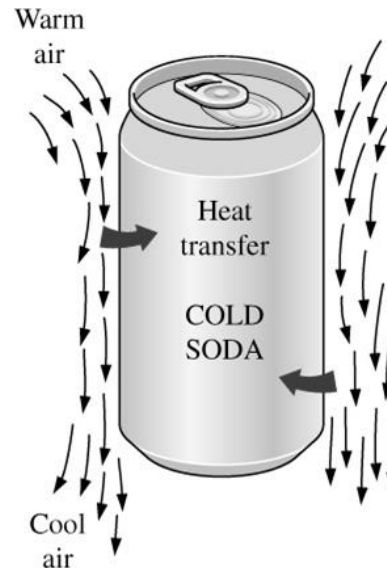
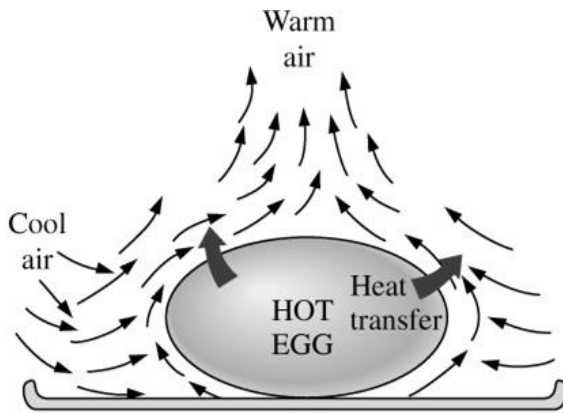
Temperature difference itself starts the flow

General: flow velocity and heat transfer rates are larger for forced convection

# Natural Convection

Also for natural convection velocity and thermal boundary layers!

- Velocity boundary layer is only area in which flow occurs
- **Hot** surface: **Upward flow**
- **Cold** surface: **Downward flow**





# Learning objectives lecture 4

## ○ Heat transfer through natural convection

- Natural Convection
- Grashof Number
- Rayleigh Number
- Nusselt Number
- Using correlations for various configurations
- Calculating natural convection with step-by-step plan

# Grashof Number

**Forced convection:** velocity  $U$  imposed

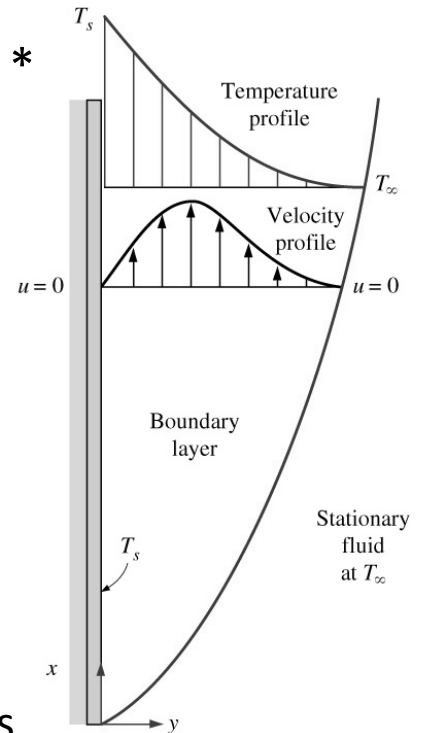
**Natural convection:** velocity follows from temp.difference  $T_s - T_\infty$  \*

⇒ Alternative for Reynolds number, with temp.diff. instead  $U$ :

$$\text{Grashof number: } Gr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (-)$$

Greek letter “nu”

The **Grashof number (Gr)** is a **dimensionless** number in fluid dynamics and heat transfer which approximates the **ratio of** the **buoyancy** to **viscous force** acting on a fluid.



\*Choose  $\Delta T$  positive for convenience

# Grashof Number

$$\text{Grashof number: } Gr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (-)$$

- Gravitational acceleration :  $g = 9,81 \text{ m/s}^2$
- Volume expansion coefficient  $\beta \text{ (K}^{-1}\text{)}$ ; for most gases:  $\beta = \frac{2}{T_s + T_\infty}$   
(temperature in Kelvin;  $0^\circ\text{C} = 273,15 \text{ K}$ )
- Length  $L_c$  characteristic for geometry (length  $L$  for plate, diameter  $D$  for sphere/cylinder)
- Kinematic viscosity :  $\nu = \frac{\mu}{\rho} \text{ (m}^2\text{/s)}$  at avg. Temp:  $T_f = \frac{T_s + T_\infty}{2}$

# Learning objectives lecture 4

## ○ Heat transfer through natural convection

- Natural Convection
- Grashof Number
- **Rayleigh Number**
- Nusselt Number
- Using correlations for various configurations
- Calculating natural convection with step-by-step plan

# Rayleigh Number

Laminar / turbulent:



**Forced convection:**

Determined by Reynolds number : **Re**

**Natural convection:**

Determined by Grashof number **Gr**

Often combined with Prandtl number:

Rayleigh number **Ra = Gr · Pr**

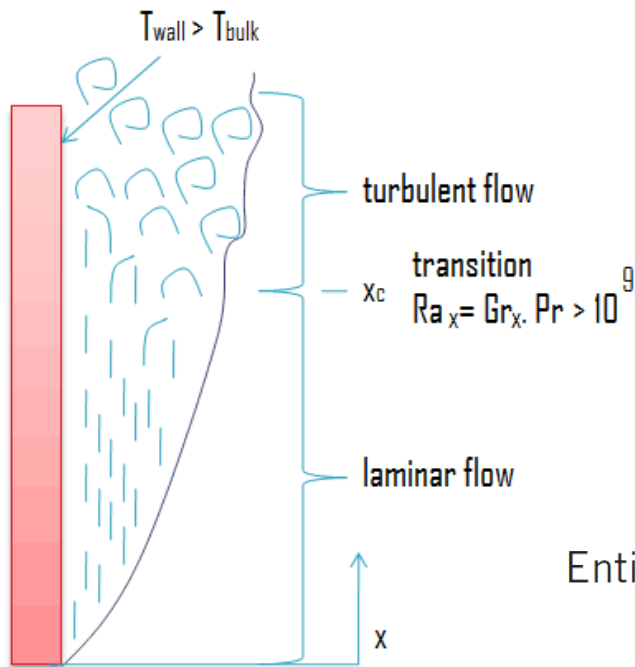
$$\text{Rayleigh Number : } Ra = Gr \cdot Pr = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} Pr$$

**Rayleigh Number** is a dimensionless number associated with buoyancy-driven flow, also known as free or natural convection

# Rayleigh Number

$$\text{Rayleigh Number : } Ra = Gr \cdot Pr = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} Pr$$

Vertical, flat plate (ex. radiator)



$$Nu = 0,59 Ra_L^{1/4} \text{ with } L_c = L \quad (10^4 < Ra_L < 10^9)$$

$$Nu = 0,1 Ra_L^{1/3} \text{ with } L_c = L \quad (10^{10} < Ra_L < 10^{13})$$

$10^9 < Ra_L < 10^{10}$  : find intermediate

$$\text{Entire range} \rightarrow Nu = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$$

(complex but more accurate)

# Learning objectives lecture 4

## ● Heat transfer through natural convection

- Natural Convection
- Grashof Number
- Rayleigh Number
- **Nusselt Number**
- Using correlations for various configurations
- Calculating natural convection with step-by-step plan

# NUSSELT NUMBER

## Forced Convection

$$\frac{hL}{k} = a \left( \frac{\rho U L}{\mu} \right)^b \frac{\mu c_p}{k}$$

$\downarrow$                        $\downarrow$                        $\downarrow$

$$\text{Nu} = a \cdot \text{Re}^b \cdot \text{Pr}^c$$

## Natural Convection

Constant  
coefficient

Constant  
exponent

$$\text{Nu} = C \text{Ra}_L^n$$

Nusselt number      Rayleigh number

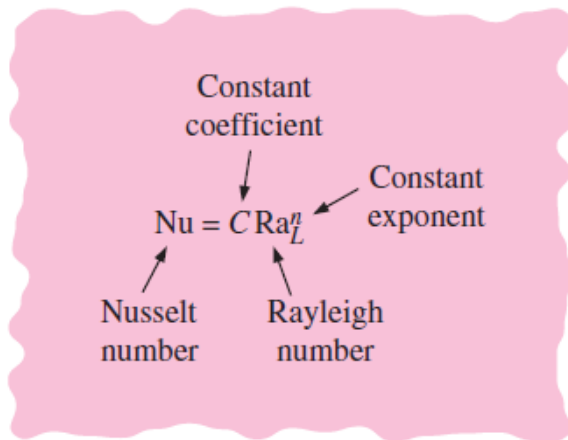
$$\text{Ra} = \text{Gr} \cdot \text{Pr} = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} \text{Pr}$$



# NUSSELT NUMBER

$$\text{Rayleigh Number : } Ra = Gr \cdot Pr = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} Pr$$

**Rayleigh number**, which is the product of the Grashof and Prandtl numbers



**The values of the constants C and n depend on the**

- ✓ geometry of the surface
- ✓ the flow regime

**The value of n is usually**

- ✓ 0,25 for laminar flow
- ✓ 0,33 for turbulent flow.

**The value of C**

- ✓ is normally less than 1.

All fluid properties are to be evaluated at the film temperature  $T_f = (T_s + T_{inv})/2$ .

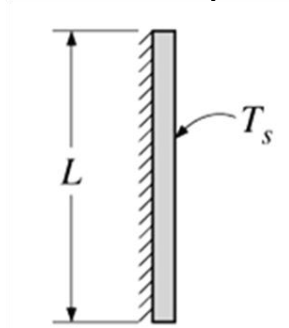
# Learning objectives lecture 4

## ● Heat transfer through natural convection

- Natural Convection
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- Rayleigh Number
- Nusselt Number
- **Using correlations for various configurations**
- Calculating natural convection with step-by-step plan

# Free Convection Correlations

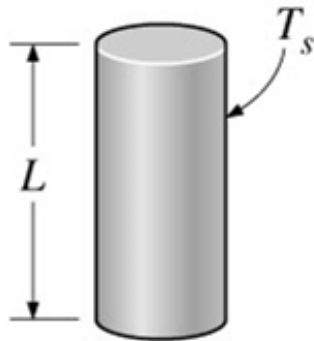
Vertical, flat plate



$$\text{Nu} = 0,59 \text{ Ra}_L^{1/4} \text{ with } L_c = L \quad (10^4 < \text{Ra}_L < 10^9)$$
$$\text{Nu} = 0,1 \text{ Ra}_L^{1/3} \text{ with } L_c = L \quad (10^{10} < \text{Ra}_L < 10^{13})$$

Entire range  $\Rightarrow$  
$$\text{Nu} = \left\{ 0.825 + \frac{0.387 \text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}} \right\}^2$$

(complex but more accurate)

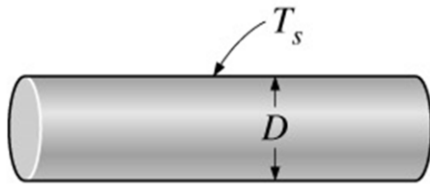


Treat a vertical plate with height  $L$  and surface  $\pi DL$ , if diameter  $D$  sufficiently large with respect to  $L$  ( $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$ )

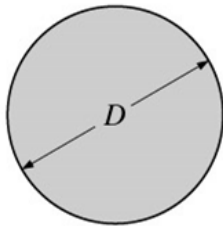
Vertical cylinder (e.g. can)

# FREE CONVECTION CORRELATIONS

Horizontal cylinder (e.g. pipe)



$$Nu = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$$



Sphere (e.g. light bulb)

$$Nu = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$$

**Notation:**

$Ra_L$  is  $Ra$  with

$L_c = L$  ;

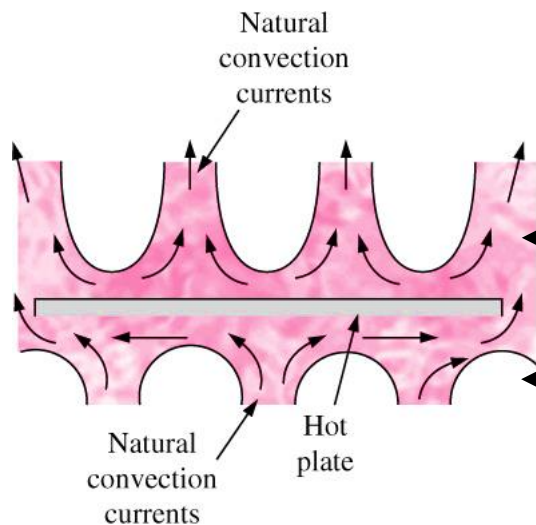
$Ra_D$  is  $Ra$  with

$L_c = D$

# Free Convection Correlations

Horizontal plate: upward / downward flow perpendicular to surface → plumes instead of adjacent boundary layer

**Hot surface**



**Fluid away from surface**

Effective heat transfer

$$Nu = 0,54 Ra^{1/4} \quad (10^4 < Ra < 10^7)$$

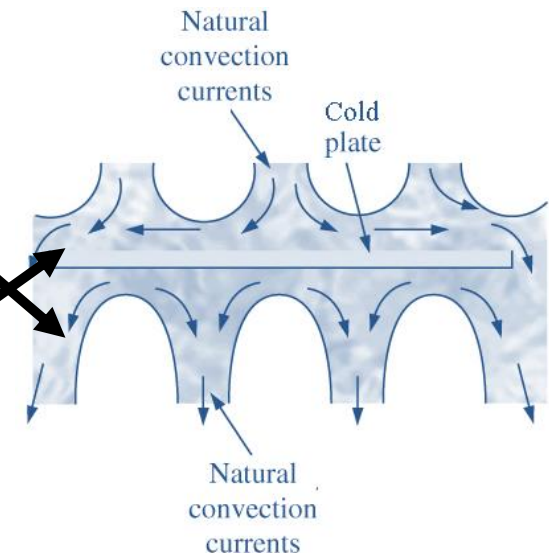
$$Nu = 0,15 Ra^{1/3} \quad (10^7 < Ra < 10^{11})$$

**Fluid towards surface**

Reduced heat transfer

$$Nu = 0,27 Ra^{1/4} \quad (10^5 < Ra < 10^{11})$$

**Cold surface**



N.B.: Characteristic length  
So actually:  $Ra_{A/p}$

$$L_c = \frac{\text{Area}}{\text{Perimeter}} = \frac{A}{p}$$

# Free Convection Correlations

Plate at angle  $\theta$  with vertical axis  
( $\theta < 60^\circ$ )

Hot surface:

- Top side: advanced methods
- Bottom side: same as vertical plate, except for using  $g \cos \theta$  instead of  $g$ :

$$Ra_L = Gr_L \cdot Pr = \frac{g \cos \theta \beta (T_s - T_\infty) L^3}{\nu^2} \cdot Pr$$

Cold surface: everything upside down

- Bottom side: advanced methods
- Top side: same as vertical plate, except for using  $g \cos \theta$  instead of  $g$

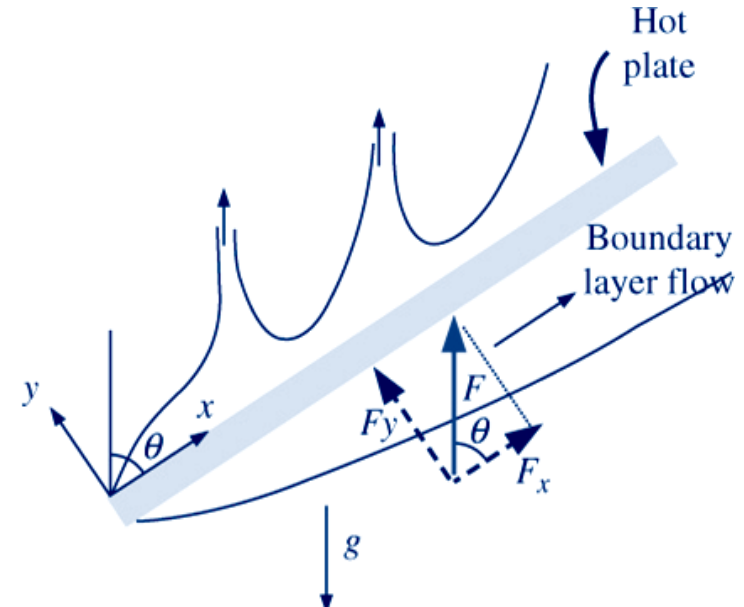


Plate at angle  $\theta$  (ex. radiator in attic room)

Vertical plate:

$$Nu = 0,59 Ra_L^{1/4} \quad (10^4 < RaL < 10^9)$$

$$Nu = 0,1 Ra_L^{1/3} \quad (10^{10} < RaL < 10^{13})$$



# Learning objectives lecture 4

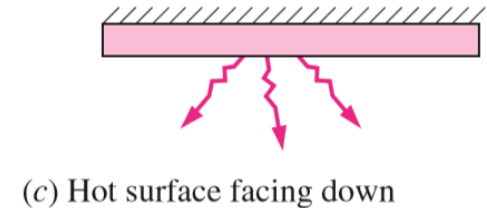
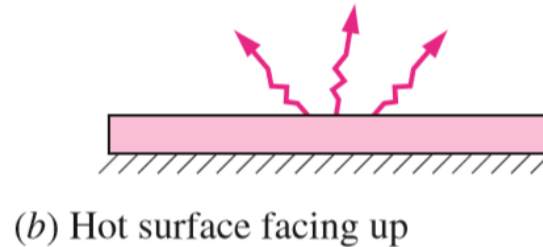
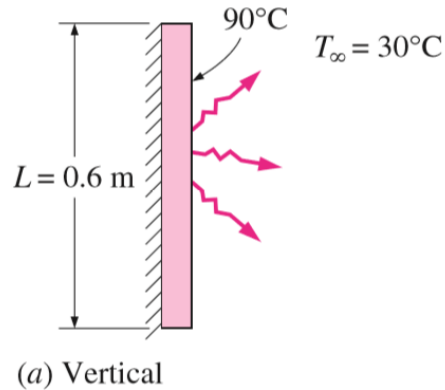
## ● Heat transfer through natural convection

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# Example 1

## Cooling of a Plate in Different Orientations

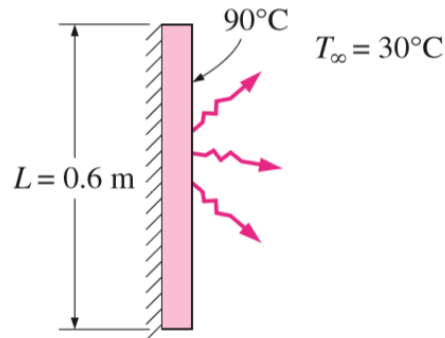


- A: 0.6-m x 0.6-m thin square plate
- room temperature:  $30^\circ\text{C}$ .
- One side of the plate is maintained at a temperature of  $90^\circ\text{C}$ , while the other side is insulated
- Determine the rate of heat transfer from the plate by natural convection if the plate is
  - (a) vertical,
  - (b) horizontal with hot surface facing up, and
  - (c) horizontal with hot surface facing down.

## Properties of air (in the back of assignment bundle):



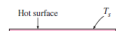
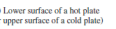

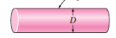

Temp. $T, ^\circ\text{C}$	Density $\rho, \text{kg/m}^3$	Specific Heat $c_p, \text{J/kg} \cdot \text{K}$	Thermal Conductivity $k, \text{W/m} \cdot \text{K}$	Thermal Diffusivity $\alpha, \text{m}^2/\text{s}^2$	Dynamic Viscosity $\mu, \text{kg/m} \cdot \text{s}$	Kinematic Viscosity $\nu, \text{m}^2/\text{s}$	Prandtl Number Pr
20	1.204	1007	0.02514	$2.074 \times 10^{-5}$	$1.825 \times 10^{-5}$	$1.516 \times 10^{-5}$	0.7309
25	1.184	1007	0.02551	$2.141 \times 10^{-5}$	$1.849 \times 10^{-5}$	$1.562 \times 10^{-5}$	0.7296
30	1.164	1007	0.02588	$2.208 \times 10^{-5}$	$1.872 \times 10^{-5}$	$1.608 \times 10^{-5}$	0.7282
35	1.145	1007	0.02625	$2.277 \times 10^{-5}$	$1.895 \times 10^{-5}$	$1.655 \times 10^{-5}$	0.7268
40	1.127	1007	0.02662	$2.346 \times 10^{-5}$	$1.918 \times 10^{-5}$	$1.702 \times 10^{-5}$	0.7255
45	1.109	1007	0.02699	$2.416 \times 10^{-5}$	$1.941 \times 10^{-5}$	$1.750 \times 10^{-5}$	0.7241
50	1.092	1007	0.02735	$2.487 \times 10^{-5}$	$1.963 \times 10^{-5}$	$1.798 \times 10^{-5}$	0.7228
60	1.059	1007	0.02808	$2.632 \times 10^{-5}$	$2.008 \times 10^{-5}$	$1.896 \times 10^{-5}$	0.7202
70	1.028	1007	0.02881	$2.780 \times 10^{-5}$	$2.052 \times 10^{-5}$	$1.995 \times 10^{-5}$	0.7177
80	0.9994	1008	0.02953	$2.931 \times 10^{-5}$	$2.096 \times 10^{-5}$	$2.097 \times 10^{-5}$	0.7154
90	0.9718	1008	0.03024	$3.086 \times 10^{-5}$	$2.139 \times 10^{-5}$	$2.201 \times 10^{-5}$	0.7132
100	0.9458	1009	0.03095	$3.243 \times 10^{-5}$	$2.181 \times 10^{-5}$	$2.306 \times 10^{-5}$	0.7111
120	0.8977	1011	0.03235	$3.565 \times 10^{-5}$	$2.264 \times 10^{-5}$	$2.522 \times 10^{-5}$	0.7073
140	0.8542	1013	0.03374	$3.898 \times 10^{-5}$	$2.345 \times 10^{-5}$	$2.745 \times 10^{-5}$	0.7041
160	0.8148	1016	0.03511	$4.241 \times 10^{-5}$	$2.420 \times 10^{-5}$	$2.975 \times 10^{-5}$	0.7014
180	0.7788	1019	0.03646	$4.593 \times 10^{-5}$	$2.504 \times 10^{-5}$	$3.212 \times 10^{-5}$	0.6992
200	0.7459	1023	0.03779	$4.954 \times 10^{-5}$	$2.577 \times 10^{-5}$	$3.455 \times 10^{-5}$	0.6974
250	0.6746	1033	0.04104	$5.890 \times 10^{-5}$	$2.760 \times 10^{-5}$	$4.091 \times 10^{-5}$	0.6946

# Example 1



(a) Vertical

TABLE 9-1  
Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length $L_c$	Range of Ra	Nu	
Vertical plate 	$L$	$10^4$ – $10^9$ $10^5$ – $10^{12}$ Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ $Nu = \left\{ 0.825 + \frac{0.387Ra_L^{1/4}}{[1 + (0.492/Pr)^{1/4}]^{1/4}} \right\}^2$ (complex but more accurate)	(9-19) (9-20) (9-21)
Inclined plate 	$L$		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace $g$ by $g \cos \theta$ for $Ra < 10^9$	
Horizontal plate (Surface area $A$ and perimeter $p$ ) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	$A_s/p$	$10^4$ – $10^7$ $10^5$ – $10^{11}$	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$	(9-22) (9-23)
(b) Lower surface of a hot plate (or upper surface of a cold plate) 	$A_s/p$	$10^5$ – $10^{11}$	$Nu = 0.27Ra_L^{1/4}$	(9-24)
Vertical cylinder 	$L$		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$	
Horizontal cylinder 	$D$	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/4}}{[1 + (0.559/Pr)^{1/4}]^{1/4}} \right\}^2$	(9-25)
Sphere 	$D$	$Ra_D \leq 10^{11}$ ( $Pr \geq 0.7$ )	$Nu = 2 + \frac{0.599Ra_D^{1/4}}{[1 + (0.469/Pr)^{1/4}]^{1/4}}$	(9-26)

**Properties** The properties of air at the film temperature of  $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60^\circ\text{C}$  and 1 atm are (Table A-15)

$$k = 0.02808 \text{ W/m} \cdot ^\circ\text{C}$$

$$Pr = 0.7202$$

$$\nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{333 \text{ K}}$$

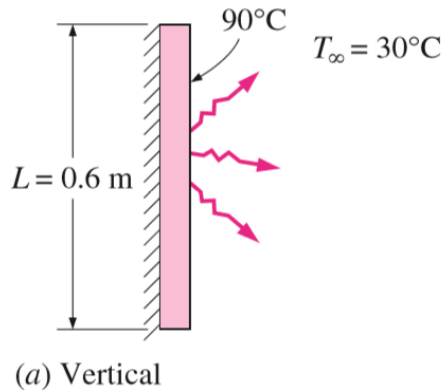
**Analysis** (a) *Vertical*. The characteristic length in this case is the height of the plate, which is  $L = 0.6 \text{ m}$ . The Rayleigh number is

$$\begin{aligned} Ra_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr \\ &= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.722) = 7.656 \times 10^8 \end{aligned}$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\begin{aligned} Nu &= \left\{ 0.825 + \frac{0.387 Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2 \\ &= \left\{ 0.825 + \frac{0.387(7.656 \times 10^8)^{1/4}}{1 + (0.492/0.7202)^{9/16}]^{8/27}} \right\}^2 = 113.4 \end{aligned}$$

# Example 1



Note that the simpler relation Eq. 9-19 would give  $Nu = 0.59 Ra_L^{1/4} = 98.14$ , which is 13 percent lower. Then,

$$h = \frac{k}{L} Nu = \frac{0.02808 \text{ W/m} \cdot ^\circ\text{C}}{0.6 \text{ m}} (113.4) = 5.306 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{115 \text{ W}}$$

Vertical plate:

$$Nu = 0,59 Ra_L^{1/4} \quad (10^4 < Ra_L < 10^9)$$

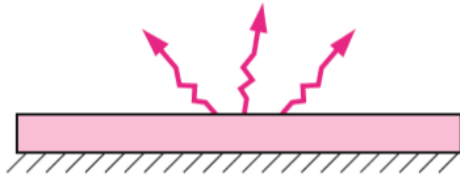
$$Nu = 0,1 Ra_L^{1/3} \quad (10^{10} < Ra_L < 10^{13})$$

TABLE 9-1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length $L_c$	Range of $Ra$	$Nu$
Vertical plate 	$L$	$10^4$ – $10^9$ $10^6$ – $10^{13}$ Entire range	$Nu = 0.59 Ra_L^{1/4}$ (9-19) $Nu = 0.1 Ra_L^{1/3}$ (9-20) $Nu = \left\{ 0.825 + \frac{0.387 Ra_L^{1/4}}{[1 + (0.492/Pr)^{3/4}]^{1/4}} \right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	$L$		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace $g$ by $g \cos \theta$ for $Ra < 10^9$
Horizontal plate (Surface area $A$ and perimeter $p$ ) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	$A_s/p$	$10^4$ – $10^7$ $10^7$ – $10^{11}$	$Nu = 0.54 Ra_L^{1/4}$ (9-22) $Nu = 0.15 Ra_L^{1/3}$ (9-23)
(b) Lower surface of a hot plate (or upper surface of a cold plate) 	$A_s/p$	$10^4$ – $10^{11}$	$Nu = 0.27 Ra_L^{1/4}$ (9-24)
Vertical cylinder 	$L$		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	$D$	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387 Ra_D^{1/4}}{[1 + (0.559/Pr)^{3/4}]^{1/4}} \right\}^2$ (9-25)
Sphere 	$D$	$Ra_D \leq 10^{11}$ ( $Pr \geq 0.7$ )	$Nu = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{3/4}]^{1/4}}$ (9-26)

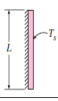
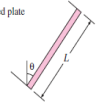

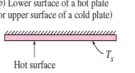

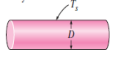

# Example 1



(b) Hot surface facing up

TABLE 9-1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length $L_c$	Range of Ra	Nu
Vertical plate 	$L$	$10^4$ – $10^9$ $10^4$ – $10^{13}$ Entire range	$Nu = 0.59Ra^{1/4}$ (9-19) $Nu = 0.1Ra^{1/3}$ (9-20) $Nu = \left\{ 0.825 + \frac{0.387Ra^{1/4}}{[1 + (0.492/Pr)^{1/4}]^{1/4}} \right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	$L$		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace $g$ by $g \cos \theta$ for $Ra < 10^9$
Horizontal plate (Surface area $A$ and perimeter $p$ ) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	$A_s/p$	$10^4$ – $10^7$ $10^7$ – $10^{11}$	$Nu = 0.54Ra^{1/4}$ (9-22) $Nu = 0.15Ra^{1/3}$ (9-23)
(b) Lower surface of a hot plate (or upper surface of a cold plate) 	$A_s/p$	$10^5$ – $10^{11}$	$Nu = 0.27Ra^{1/4}$ (9-24)
Vertical cylinder 	$L$		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr^{1/4}}$
Horizontal cylinder 	$D$	$Ra_0 \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra^{1/4}}{[1 + (0.559/Pr)^{1/4}]^{1/4}} \right\}^2$ (9-25)
Sphere 	$D$	$Ra_0 \leq 10^{11}$ ( $Pr \geq 0.7$ )	$Nu = 2 + \frac{0.589Ra^{1/4}}{[1 + (0.469/Pr)^{1/4}]^{1/4}}$ (9-26)

(b) Horizontal with hot surface facing up. The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6 \text{ m}}{4} = 0.15 \text{ m}$$

$$\begin{aligned} Ra_L &= \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr \\ &= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.15 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7202) = 1.196 \times 10^7 \end{aligned}$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$Nu = 0.54 Ra_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

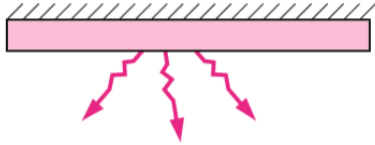
$$h = \frac{k}{L_c} Nu = \frac{0.0280 \text{ W/m} \cdot ^\circ\text{C}}{0.15 \text{ m}} (31.76) = 5.946 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{128 \text{ W}}$$

# Example 1










(c) Hot surface facing down

(c) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (b). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$\text{Nu} = 0.27 \text{Ra}_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

TABLE 9-1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length $L_c$	Range of Ra	Nu
Vertical plate 	$L$	$10^4$ – $10^9$ $10^5$ – $10^{13}$ Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20) $\text{Nu} = \left\{ 0.825 + \frac{0.387\text{Ra}_L^{1/4}}{[1 + (0.492/\text{Pr})^{1/4}]^{1/4}} \right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	$L$		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace $g$ by $g \cos \theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area $A_s$ and perimeter $p$ ) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	$A_s/p$	$10^4$ – $10^7$ $10^5$ – $10^{11}$	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)
Vertical cylinder 	$L$		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder 	$D$	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{ 0.6 + \frac{0.387\text{Ra}_D^{1/4}}{[1 + (0.559/\text{Pr})^{1/4}]^{1/4}} \right\}^2$ (9-25)
Sphere 	$D$	$\text{Ra}_D \leq 10^{11}$ ( $\text{Pr} \geq 0.7$ )	$\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{1/4}]^{1/4}}$ (9-26)

$$h = \frac{k}{L_c} \text{Nu} = \frac{0.02808 \text{ W/m} \cdot ^\circ\text{C}}{0.15 \text{ m}} (15.86) = 2.973 \text{ W/m}^2 \cdot ^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{64.2 \text{ W}}$$

# Step-by-step plan natural convection

If  $\dot{Q}$  must be found:

- Determine ingredients necessary for dimensionless no. (Pr,  $k$  and  $\nu$  at average temperature  $\frac{T_s + T_\infty}{2}$  )
- Determine Ra:  $Ra = Gr \cdot Pr = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} Pr$
- Choose appropriate correlation based on geometry and Ra
- Determine Nu
- Resolve  $h$  from it
- Fill out Newton's cooling law:  $\dot{Q} = hA\Delta T$

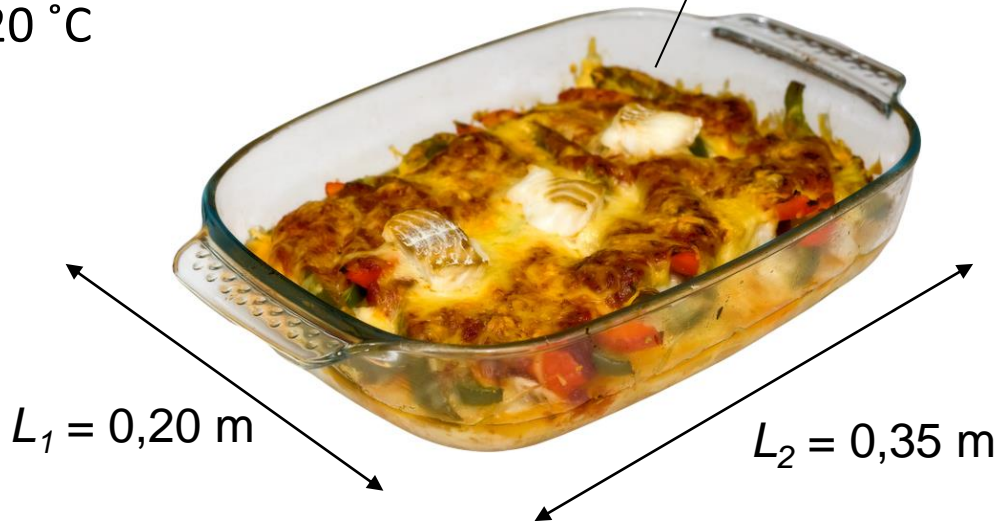
## Example 2

How large is the heat flow?

Surrounding temperature

$$T_{\infty} = 20\text{ }^{\circ}\text{C}$$

$$T_s = 220\text{ }^{\circ}\text{C}$$



*Natural convection*



# Example 2

$$L_1 = 0,20 \text{ m}$$

$$L_2 = 0,35 \text{ m}$$

$$T_s = 220 \text{ }^{\circ}\text{C}$$

$$T_{\infty} = 20 \text{ }^{\circ}\text{C}$$

Horizontal, flat, warm plate:

$$\text{Nu} = 0,54 \text{ Ra}^{1/4} \quad (10^4 < \text{Ra} < 10^7)$$

$$\text{Nu} = 0,15 \text{ Ra}^{1/3} \quad (10^7 < \text{Ra} < 10^{11})$$

Characteristic length  $L_c = A/p$

$$\text{Ra} = \text{Gr} \cdot \text{Pr} = \frac{g \beta (T_s - T_{\infty}) L_c^3}{\nu^2} \text{Pr}$$

Properties of air (in the back of assignment module):

Temp. $T, \text{ }^{\circ}\text{C}$	Density $\rho, \text{ kg/m}^3$	Specific Heat $c_p, \text{ J/kg} \cdot \text{K}$	Thermal Conductivity $k, \text{ W/m} \cdot \text{K}$	Thermal Diffusivity $\alpha, \text{ m}^2/\text{s}^2$	Dynamic Viscosity $\mu, \text{ kg/m} \cdot \text{s}$	Kinematic Viscosity $\nu, \text{ m}^2/\text{s}$	Prandtl Number Pr
20	1.204	1007	0.02514	$2.074 \times 10^{-5}$	$1.825 \times 10^{-5}$	$1.516 \times 10^{-5}$	0.7309
25	1.184	1007	0.02551	$2.141 \times 10^{-5}$	$1.849 \times 10^{-5}$	$1.562 \times 10^{-5}$	0.7296
30	1.164	1007	0.02588	$2.208 \times 10^{-5}$	$1.872 \times 10^{-5}$	$1.608 \times 10^{-5}$	0.7282
35	1.145	1007	0.02625	$2.277 \times 10^{-5}$	$1.895 \times 10^{-5}$	$1.655 \times 10^{-5}$	0.7268
40	1.127	1007	0.02662	$2.346 \times 10^{-5}$	$1.918 \times 10^{-5}$	$1.702 \times 10^{-5}$	0.7255
45	1.109	1007	0.02699	$2.416 \times 10^{-5}$	$1.941 \times 10^{-5}$	$1.750 \times 10^{-5}$	0.7241
50	1.092	1007	0.02735	$2.487 \times 10^{-5}$	$1.963 \times 10^{-5}$	$1.798 \times 10^{-5}$	0.7228
60	1.059	1007	0.02808	$2.632 \times 10^{-5}$	$2.008 \times 10^{-5}$	$1.896 \times 10^{-5}$	0.7202
70	1.028	1007	0.02881	$2.780 \times 10^{-5}$	$2.052 \times 10^{-5}$	$1.995 \times 10^{-5}$	0.7177
80	0.9994	1008	0.02953	$2.931 \times 10^{-5}$	$2.096 \times 10^{-5}$	$2.097 \times 10^{-5}$	0.7154
90	0.9718	1008	0.03024	$3.086 \times 10^{-5}$	$2.139 \times 10^{-5}$	$2.201 \times 10^{-5}$	0.7132
100	0.9458	1009	0.03095	$3.243 \times 10^{-5}$	$2.181 \times 10^{-5}$	$2.306 \times 10^{-5}$	0.7111
120	0.8977	1011	0.03235	$3.565 \times 10^{-5}$	$2.264 \times 10^{-5}$	$2.522 \times 10^{-5}$	0.7073
140	0.8542	1013	0.03374	$3.898 \times 10^{-5}$	$2.345 \times 10^{-5}$	$2.745 \times 10^{-5}$	0.7041
160	0.8148	1016	0.03511	$4.241 \times 10^{-5}$	$2.420 \times 10^{-5}$	$2.975 \times 10^{-5}$	0.7014
180	0.7788	1019	0.03646	$4.593 \times 10^{-5}$	$2.504 \times 10^{-5}$	$3.212 \times 10^{-5}$	0.6992
200	0.7459	1023	0.03779	$4.954 \times 10^{-5}$	$2.577 \times 10^{-5}$	$3.455 \times 10^{-5}$	0.6974
250	0.6746	1033	0.04104	$5.890 \times 10^{-5}$	$2.760 \times 10^{-5}$	$4.091 \times 10^{-5}$	0.6946

# Example 2

## (Intermediate) answers

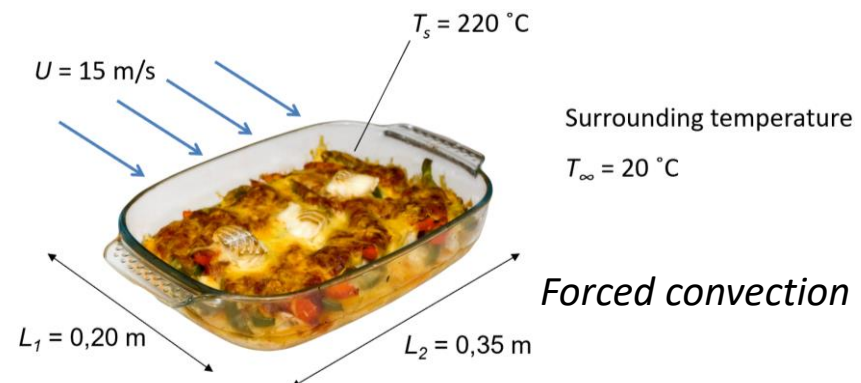
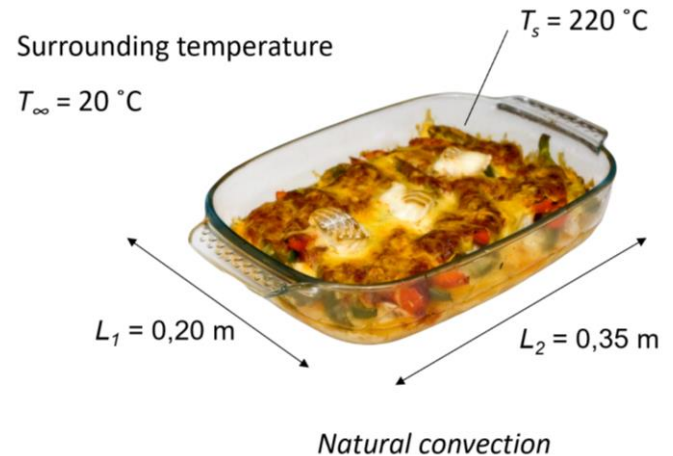
- Average temperature: 120 °C
- $Ra = 1,45 \cdot 10^6$
- $Nu = 18,7$
- $h = 9,48 \text{ W}/(\text{m}^2 \cdot \text{K})$
- Heat flow 133 W (around 1/3 forced convection: 462 W)

## Blowing from long side

- $h = 33 \text{ W}/(\text{m}^2 \cdot \text{K})$
- Heat flow 462 W

## Blowing from short side:

- $h = 24,9 \text{ W}/(\text{m}^2 \cdot \text{K})$
- Heat flow 349 W



## Example 3

Imagine: 25 W light bulb with diameter  $D = 0,08$  m releases 22,5 W of heat. Determine  $T_s$  for  $T_\infty = 20$  °C

*Problem:*

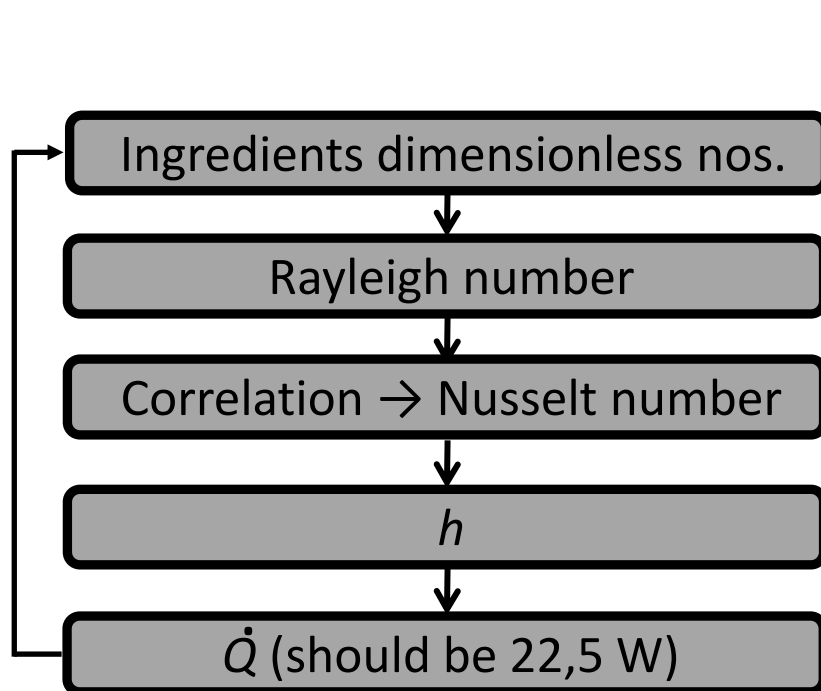
To calculate  $T_s$ ,  $h$  is necessary ( $\dot{Q} = hA(T_s - T_\infty)$ ) but  $h$  depends on  $T_s$  (because Nu, Ra, Pr are dependent on temperature) and that is the one we are looking for...



$$\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$$

## Example 3

Problem:  $T_s$  unknown  $\rightarrow$  starting values for step-by-step plan unknown ( $Pr, k, \nu, \dots$ )



At what avg. temp?

$$\frac{T_s + T_\infty}{2} ?$$

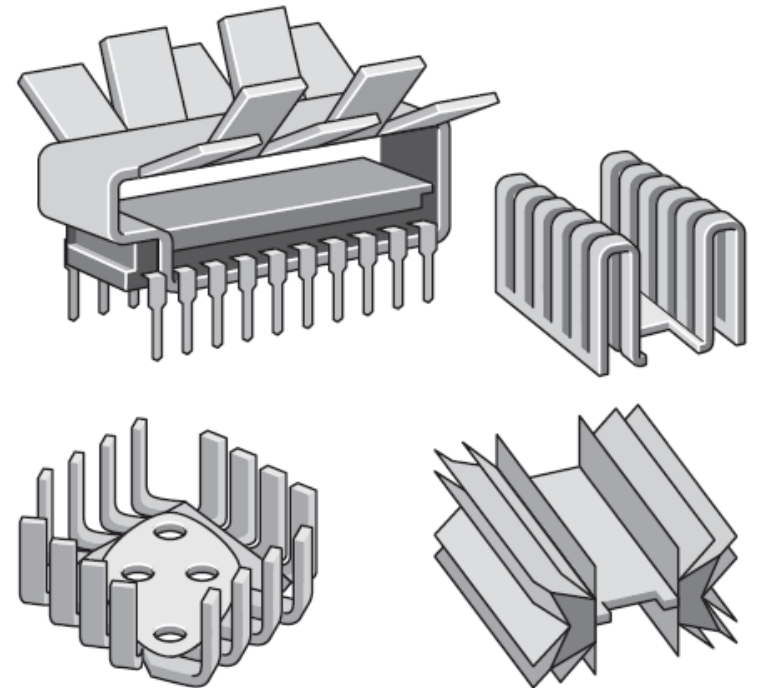
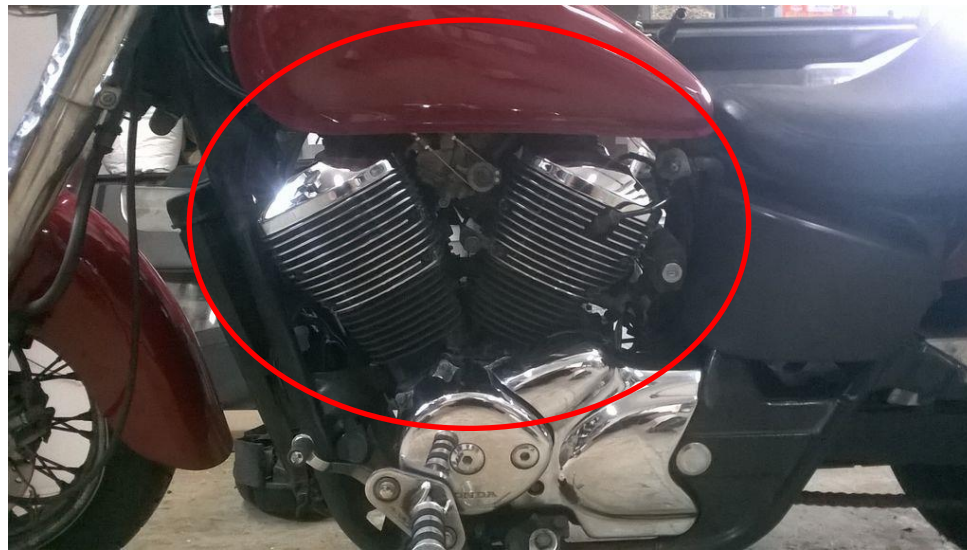
Unknown

Approach: iterative solving

- Choose an estimated  $T_s$  to start with
- Follow steps and check whether  $T_s$  yields the right heat flow
- Resulting heat flow  $\neq 22,5$  W? Adjust estimated  $T_s$  and repeat all steps till you find a  $T_s$  that (approximately) yields  $Q = 22,5$  W

# Fins

What is Purpose of the Fins on Engine Surfaces ? Why design such a thing ?



Fins are extended surfaces and they increase the surface area leading to increase in heat transfer rate. Useful in cooling applications

# Summary natural convection

General (same as for forced convection)

$$\dot{Q} = hA\Delta T \quad (\text{W}) \quad \text{Newton's cooling law}$$

“Supporting” equations for  $h$ :

Nusselt number  $Nu$  as function of Rayleigh number  $Ra = Gr \cdot Pr$

Grashof number  $Gr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$

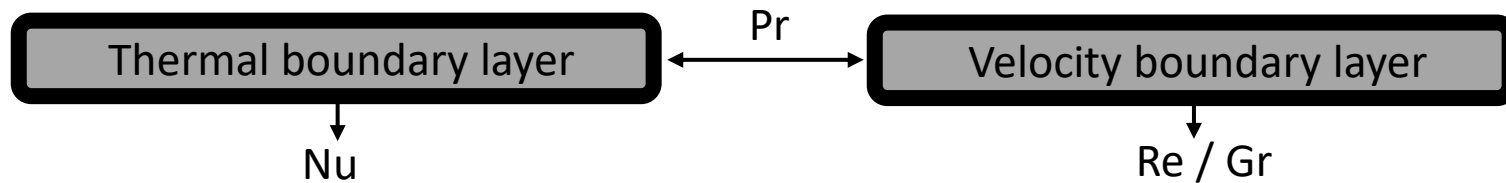
- Forced convection:  $Re \leftrightarrow$  natural convection:  $Gr$
- $Ra$  and  $Nu$  just like  $Re$  based upon characteristic length of geometry (indicate using subscript)

# General conclusion convection

- Calculate heat flow using Newton's cooling law
  - Determine heat transfer coefficient  $h$  in this using correlations between  $Nu$  ( $\rightarrow h$ ) and other dimensionless nos.
- $\Rightarrow$  step-by-step plans (+ iterative solving)

Other learning objectives convection: *flow phenomena*

- Being able to sum up what parameter are influencing  $h$
- Knowing differences between laminar and turbulent ,
- Explaining how velocity and thermal boundary layers form
- Predicting/reasoning the temperature development



# REFERENCE



Chapter 9 of Heat Trasnfer, Cenegel.



# Feedback Session

- Next Lectorial : Forced Convection
- Questions? Feel free to ask!

