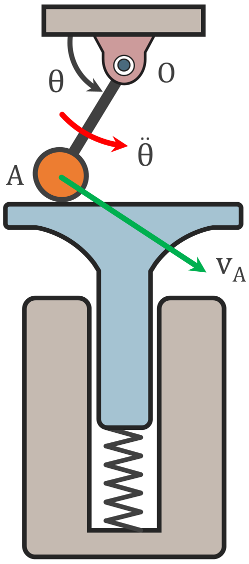
# Acceleration of Crankshaft




**Determine the acceleration of the blue shaft B if the crank OA has a angular acceleration of $\ddot{\theta} = 12$ rad/s$^2$ and ball A has a velocity of $v_A = 3$ m/s at this position. The spring maintains contact between the roller and the surface of the plunger.**

**Take $L_{OA} = 0.5$ m and $\theta = 60°$.**

*Using known expressions (for rigid bodies and a fixed point* O*):*

$$\mathbf{v}_{\mathrm{A}} = \boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} \tag{1}$$

$$\mathbf{a}_{\mathrm{A}} = \mathbf{a}_{\mathrm{A},n} + \mathbf{a}_{\mathrm{A},t} \tag{2}$$

$$\mathbf{a}_{\mathrm{A},n} = \boldsymbol{\omega}_{\mathrm{OA}} \times (\boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}}) \tag{3}$$

$$\mathbf{a}_{\mathrm{A},t} = \boldsymbol{\alpha}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} \tag{4}$$

*Given quantities*:

Angle: $\theta = 60°$
Angular acceleration: $\alpha_{\mathrm{OA}} = \ddot{\theta} = 12$ rad/s$^2$
Velocity at A: $v_{\mathrm{A}} = 3$ m/s
Distance from O to A: $L_{\mathrm{OA}} = 0.5$ m.

*Solution*:

The blue shaft B can only move up and down, thus it has a zero acceleration in the $x$-direction. A kinematic diagram of the situation is drawn in Figure 1. The total acceleration in the $y$-direction of the ball must be equal to the acceleration in the $y$-direction of the shaft, since they remain in contact. Both the normal and tangential acceleration can be decomposed in an acceleration in the $x$-and $y$-direction.

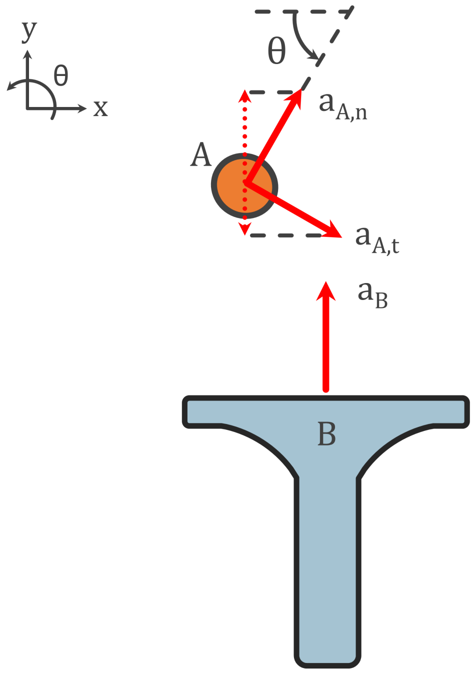



Figure 1: Kinematic diagram of the ball and the plunger.

As can be seen in Equation (3), the angular velocity of the crankshaft is needed. Using Equation (1) the angular velocity of the crankshaft can be calculated as follows:

$$\mathbf{v}_{\mathrm{A}} = \boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} \tag{5}$$

$$\Rightarrow \quad \begin{pmatrix} 12\sin\theta \\ -12\cos\theta \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \begin{pmatrix} -0.5\cos\theta \\ -0.5\sin\theta \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\omega_{\mathrm{OA}}\sin\theta \\ -0.5\omega_{\mathrm{OA}}\cos\theta \\ 0 \end{pmatrix}$$

Thus $\omega_{\mathrm{OA}} = 6$ rad/s.

Using Equation (3) the normal acceleration of ball A becomes:

$$\mathbf{a}_{\mathrm{A},n} = \boldsymbol{\omega}_{\mathrm{OA}} \times (\boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}}) \quad \Rightarrow \tag{6}$$

$$\mathbf{a}_{\mathrm{A},n} = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \left( \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \begin{pmatrix} -0.5\cos\theta \\ -0.5\sin\theta \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \times \begin{pmatrix} 0.5\omega_{\mathrm{OA}}\sin\theta \\ -0.5\omega_{\mathrm{OA}}\cos\theta \\ 0 \end{pmatrix} \quad \Rightarrow$$

$$\mathbf{a}_{\mathrm{A},n} = \begin{pmatrix} 0.5\omega_{\mathrm{OA}}^2\cos\theta \\ 0.5\omega_{\mathrm{OA}}^2\sin\theta \\ 0 \end{pmatrix}$$

Thus $a_{\mathrm{A},n,y} = 0.5\omega_{\mathrm{OA}}^2\sin\theta$.

Using Equation (7) the tangential acceleration of ball A becomes:

$$\mathbf{a}_{\mathrm{A},t} = \boldsymbol{\alpha}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{\mathrm{OA}} \end{pmatrix} \times \begin{pmatrix} -0.5\cos\theta \\ -0.5\sin\theta \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\alpha_{\mathrm{OA}}\sin\theta \\ -0.5\alpha_{\mathrm{OA}}\cos\theta \\ 0 \end{pmatrix} \tag{7}$$

Thus $a_{\mathrm{A},t,y} = -0.5\alpha_{\mathrm{OA}}\cos\theta$.

The total acceleration in $y$-direction of ball A is equal to the total acceleration of the blue shaft B, and this is equal to:

$$a_{\mathrm{B}} = a_{\mathrm{A},n,y} + a_{\mathrm{A},t,y} = 0.5\omega_{\mathrm{OA}}^2\sin\theta - 0.5\alpha_{\mathrm{OA}}\cos\theta = 9\sqrt{3} - 3 \approx 12.6\ \mathrm{m/s}^2 \tag{8}$$