

EXAMINATION  
 FLUID MECHANICS I  
 (WB MODULE-7, 202000138)  
 31 MAY 2022: 8.45 - 11.45 (12.30)

- Electronic devices or books/notes are not allowed to be present on your desk.
  - You are not allowed to write with a red pencil or pen, but you can use one in a drawing.
  - Check the dimensions and signs of your answers!
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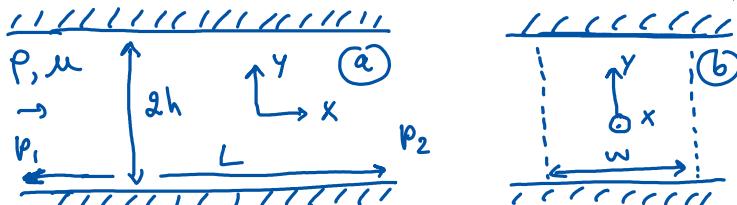
**Problem 1 [2 POINTS.]**

A two-dimensional velocity field is given by  $u = ax^2$  and  $v = -2axy$ , where  $a$  is a positive constant and  $x$  and  $y$  are the cartesian coordinates. A very small particle follows the fluid exactly along the trajectory  $(x_p(t), y_p(t))^T$ .

- Compute the divergence of the velocity field. What does the answer tell you?
  - Compute  $\frac{Dv}{Dt}$  as a function of  $x$  and  $y$ .
  - Compute  $\frac{dy_p}{dt}$  and  $\frac{d^2y_p}{dt^2}$  as functions of  $x_p(t)$  and  $y_p(t)$ , and explain the relation between  $\frac{Dv}{Dt}$  and  $\frac{d^2y_p}{dt^2}$ .
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**Problem 2 [3 POINTS.]**

An incompressible fluid with density  $\rho$  and viscosity  $\mu$  flows steadily through a two-dimensional channel of height  $2h$  and length  $L$ , see figure (a). The  $x$ -axis points in the streamwise direction, the  $y$ -axis points in the height direction, with  $y = 0$  in the middle of the channel. At the entrance the pressure is  $p_1$  and at the exit the pressure is  $p_2$ . We consider a portion of the channel of width  $w$ , see figure (b) where the  $x$ -axis points towards the observer. Gravity can be neglected. The flow is fully developed and the velocity distribution is  $u(y) = V \left(1 - \left(\frac{y}{h}\right)^2\right)$ ,



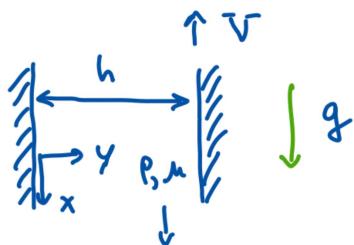
with  $V$  a known constant.

- Compute the ratio of the average speed and the maximum speed.
- What is the meaning of the integral  $\int_A \sigma_j n_j dA$  if  $A$  is the surface of a blob of fluid?
- Compute the force  $F$  by the fluid on the walls of the portion of the channel depicted in the two figures without using the integral-formulation of momentum conservation.

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**Problem 3 [3 POINTS.]**

An incompressible viscous fluid flows steadily between two vertical parallel plates. The left plate is stationary and the right plate moves upward with velocity  $V$ . The flow is laminar and fully developed. The total gap width between the plates is  $h$ , the width ( $z$ -direction) is  $w$ . The  $x$ -axis points downstream and the  $y$ -axis starts at the left plate. The fluid viscosity is  $\mu$  and the pressure satisfies  $p(x, y) = ax + b$ , with  $a$  and  $b$  constant. Gravity points downward.



- Derive an expression for the velocity field.
  - Derive an expression for the shear stress by the fluid on the right plate.
  - Give formulas for the average velocity  $U$ , the volume flow rate  $Q$ , and the mass flow rate  $\dot{m}$ . If your answers contain an integral, you don't have to calculate the integral, just give the formula's.
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**Problem 4 [2 POINTS.]**

Consider steady flow of a perfect gas around a sphere of diameter  $D$ . The gas constant is  $R$  and the ratio of specific heats is  $\gamma$ . Viscosity, heat conduction, and gravity can be neglected. The temperature and velocity far upstream are  $T_\infty$  and  $U_\infty$ , respectively. The stagnation temperature is unknown.



- Which physical dimensions are present in this problem?
- The dimensionless ratio  $T_o/T_\infty$  depends on dimensionless parameters. Use dimension analysis to derive these dimensionless parameters.

Hint: the unit of  $R$  is  $\frac{J}{kgK}$

EXAMINATION  
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 11 APRIL 2022: 8.45 - 11.45 (12.30)

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- You are *not* allowed to use a red pencil or pen
- Success!

**Problem 1 [2 POINTS.]**

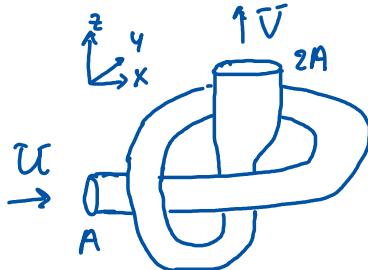
The lift force  $L$  on the Concorde airplane at high speeds primarily depends on air-density  $\rho$ , air-speed  $U$ , air-temperature  $T$ , angle-of-attack  $\alpha$ , and finally on the specific gas-constant of air,  $R$  ( $J\ kg^{-1}\ K^{-1}$ ). The size of the airplane is characterized by the wingspan  $b$ .



- How many independent physical dimensions are present in this problem?
- The lift coefficient,  $C_L \equiv L/(\frac{1}{2}\rho U^2 b^2)$ , depends on two dimensionless parameters. Determine these two parameters.

**Problem 2 [3 POINTS.]**

An incompressible fluid flows steadily through the tube depicted in the figure. At the entrance and exit viscosity can be neglected, and the velocity fields and pressures are uniform. At the entrance the fluid enters at speed  $U$  in the  $x$ -direction, and at the exit it leaves at speed  $V$  in the  $z$ -direction. The cross-sectional areas of the entrance and exit are  $A$  and  $2A$ , respectively. Gravity can be neglected.



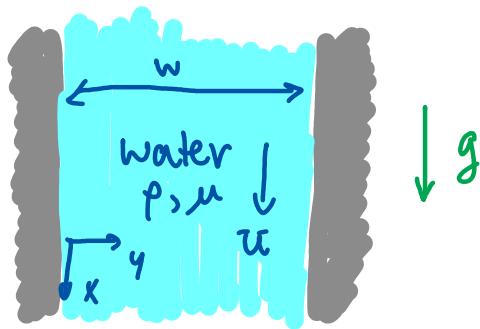
- Give the velocity vectors and outward unit normal vectors at the entrance and at the exit in three components.
- Compute  $u_j n_j$  at the entrance and at the exit.
- Compute the horizontal force  $F$  by the fluid on the tube.

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**Problem 3 [3 POINTS.]**

Water of constant density  $\rho$  and viscosity  $\mu$  flows steadily and fully developed through a vertical gap of width  $w$  under the influence of gravity. Pressure is constant throughout the fluid.

- (a) Determine the velocity field  $u(y)$ .
- (b) Compute the average speed of the fluid.
- (c) Compute the shear stress by the fluid on the right wall.



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**Problem 4 [2 POINTS.]**

Consider the steady flow of a perfect gas (with gas-constant  $R$  and constant ratio of specific heats  $\gamma$ ) around a sphere. Neglect viscosity, heat conduction, and gravity. Under these conditions the conservation equations of mass and energy reduce to:

$$\frac{\partial}{\partial x_j} (\rho u_j) = 0, \quad \frac{\partial}{\partial x_j} (\rho u_j H) = 0.$$

The temperature, pressure, and Mach number far upstream are given:  $T_\infty$ ,  $p_\infty$  and  $M_\infty$ .



- (a) Show that  $H$  is constant along streamlines.
- (b) Compute the density  $\rho_o$  in the stagnation point of the sphere.

**EXAMINATION**  
**FLUID MECHANICS I**  
(WB MODULE-7 201700127)  
7 APRIL 2020: 8.45 - 12.30 (OR 12.45, SEE PAGE -1-)  
(you are not allowed use a red pencil or pen)

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**Problem 1 [2 POINTS.]**

A two-dimensional velocity field is given by  $u = -by$  and  $v = bx$ , where  $x$  and  $y$  are the cartesian coordinates.

- (a) Is this field representing the flow of an incompressible fluid?
- (b) Compute  $\frac{Du}{Dt}$  as a function of  $x$  and  $y$ .
- (c) Compute  $\frac{d}{dt}(r_p^2)$ , with  $r_p(t)^2 \equiv x_p(t)^2 + y_p(t)^2$ , and  $x_p(t), y_p(t)$  the coordinates of a particle moving with the flow.

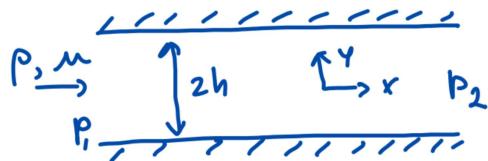
$$\text{Hint: } x_p \frac{dx_p}{dt} + y_p \frac{dy_p}{dt} = \frac{1}{2} \frac{dx_p^2}{dt} + \frac{1}{2} \frac{dy_p^2}{dt} = \frac{1}{2} \frac{dr_p^2}{dt}.$$


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**Problem 2 [3 POINTS.]**

An incompressible fluid with density  $\rho$  and viscosity  $\mu$  flows steadily through a two-dimensional channel of height  $2h$ , width  $w$ , and length  $L$ . The  $x$ -axis points in the streamwise direction, the  $y$ -axis points in the height direction, with  $y = 0$  in the middle of the channel. At the entrance the pressure is  $p_1$  and at the exit the pressure is  $p_2$ . Gravity can be neglected. The flow is fully developed and the velocity distribution is:

$$u(y) = V \left( 1 - \left( \frac{y}{h} \right)^2 \right), \quad V \text{ is known.}$$



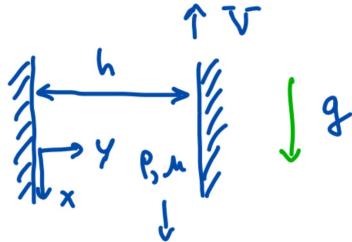
- (a) What is the meaning of  $V$ ?
- (b) Compute the average velocity  $U$  as a factor times  $V$ .
- (c) Derive an expression for the force by the fluid on the walls in  $x$ -direction by using the integral-formulation of momentum conservation.

$$\text{Hint: } \int_{-h}^h \left( 1 - \left( \frac{y}{h} \right)^2 \right)^2 dy = \frac{16}{15} h.$$

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**Problem 3 [3 POINTS.]**

An incompressible viscous fluid flows steadily between two vertical parallel plates. The left plate is stationary and the right plate moves upwards with velocity  $V$ . The flow is laminar and fully developed. The total gap width between the plates is  $h$ , the width ( $z$ -direction) is  $w$ . The  $x$ -axis points downstream and the  $y$ -axis starts at the left plate. The fluid viscosity is  $\mu$  and the pressure derivative is zero:  $\frac{\partial p}{\partial x} = 0$ . Gravity points downward.



- Derive an expression for the velocity field.
  - Derive an expression for the shear stress by the fluid on the right plate.
  - Give a formula for the average velocity, the volume flow rate, and the mass flow rate. If your answers contain an integral, you don't have to calculate the integral.
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**Problem 4 [2 POINTS.]**

Consider steady flow of a perfect gas (with gas constant  $R$  and ratio of specific heats  $\gamma$ ) around a sphere of diameter  $D$ , and neglect viscosity, heat conduction, and gravity. The temperature and velocity far upstream are given:  $T_\infty$  and  $U_\infty$ , respectively.



- Is mass (kg) one of the physical dimensions that is present in this problem?
- Use dimension analysis to derive the dimensionless parameters on which the stagnation temperature  $T_o$  depends.
- Derive an exact expression for the stagnation temperature  $T_o$  and compare it to the answer of (b).

Hint: the unit of  $R$  is  $\frac{J}{kg K} = \frac{m^2}{K s^2}$ .

EXAM  
FLUID MECHANICS I  
(WB MODULE-7 201700127)

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**Problem 1 [2 POINTS.]** *The ideal flow around a corner placed at the origin is given by*

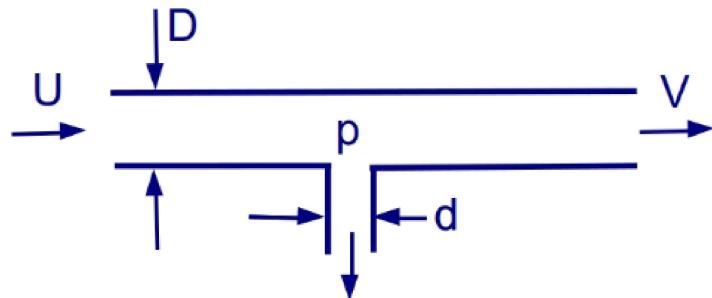
$$u = ax, \quad v = -ay,$$

*with  $a > 0$  a constant.*

- (a) Determine the trajectories and draw the trajectory that passes the point  $(x_o, y_o)$  at time  $t = 0$  and indicate the flow direction.
- (b) Calculate the material derivative of the velocity vector.

**Problem 2 [3 POINTS.]** *An incompressible fluid flows steadily into a T-junction of diameter  $D$  at uniform velocity  $U$ , at the opposite outlet the fluid leaves at uniform velocity  $V$ . At the lateral exit the flow leaves at unknown uniform velocity  $W$ . The pressure in the T-junction is uniform:  $p$ . Neglect viscosity and gravity.*

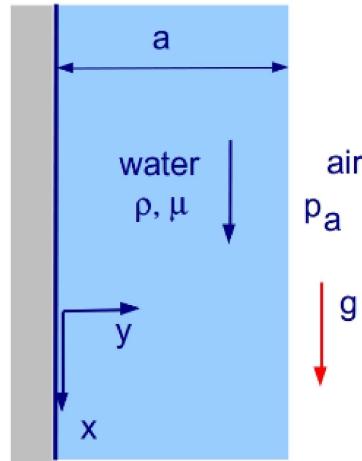
- (a) Compute the unknown velocity  $W$ .
- (b) Compute the force (in all directions) by the fluid on the pipe.
- (c) Compute the mass flow rate through the vertical exit.



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**Problem 3 [3 POINTS.]** Consider the steadily falling water film along a vertical wall with thickness  $a$ . The flow is incompressible, laminar, and fully developed. At the wall the velocity is zero, whereas at the outer edge of the film the shear stress is zero.

- (a) Defend the approximation assumption of zero shear stress at the film boundary.
- (b) Derive an expression for  $\frac{\partial p}{\partial x}$ .
- (c) Derive an expression for  $u(y)$ .



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**Problem 4 [2 POINTS.]** For steady flow with  $\mu = 0$  and  $k = 0$  explain that the pressure along a streamline can not exceed the total pressure along that streamline.

EXAM  
FLUID MECHANICS I  
(WB MODULE-7 201700127)

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**Problem 1 [2 POINTS.]**

The drag force,  $F$ , experienced by a submarine that moves at a great depth from the surface of the water, is a function of the density  $\rho$ , viscosity  $\mu$ , speed  $V$  and the transversal area of the submarine  $A$ . An expert suggests that the nondimensional function  $\tilde{f}$  that allows the calculation of  $F$  is:  $\tilde{f}\left(\frac{\rho V A}{\mu}\right) = \frac{F}{2\rho V A}$ .

- (a) Is this expression correct? Why or why not?
  - (b) If not, correct it.
- 

**Problem 2 [3 POINTS.]**

An incompressible fluid flows steadily in the entrance region of a two-dimensional channel of height  $2h$  and width  $w$ . The  $x$ -axis points in the streamwise direction, the  $y$ -axis points in the height direction, with  $y = 0$  in the middle of the channel. At the entrance the pressure is  $p_1$  and the uniform velocity is  $U$ . At the exit the pressure is  $p_2$  and the velocity distribution is:

$$\frac{u}{V} = 1 - \left(\frac{y}{h}\right)^2. \quad (1)$$

- (a) Derive an expression for  $V$ .
- (b) Derive an expression for the force by the fluid on the walls in  $x$ -direction, neglecting gravity, and neglecting viscosity at entrance and exit.

Hint:  $\int_{-h}^h \left(1 - \left(\frac{y}{h}\right)^2\right)^2 dy = \frac{16}{15}h$

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**Problem 3 [3 POINTS.]**

An incompressible viscous fluid flows steadily between two parallel plates. The bottom plate is stationary and the top plate moves to the right with velocity  $V$ . The flow is laminar and fully developed. The total gap width between the plates is  $h$ . The  $x$ -axis points downstream and the  $y$ -axis starts at the bottom plate. Gravity can be neglected, the viscosity is  $\mu$  and the given pressure derivative is constant:  $\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_o$ .

- (a) Derive an expression for the velocity field.
  - (b) Derive an expression for the shear stress on the top plate.
  - (c) Compute the value of  $V$  when the shear stress on the top plate is zero and make a sketch of the velocity field for that case.
- 

**Problem 4 [2 POINTS.]**

Consider steady parallel flow of a perfect gas (with gas constant  $R$  and ratio of specific heats  $\gamma$ ) around a sphere and neglect viscosity, heat conduction, and gravity. The temperature and Mach number far upstream are given:  $T_\infty$  and  $M_\infty$ , respectively.

$$\begin{array}{l} \rightarrow \\ \rightarrow M_\infty \\ \rightarrow \\ \rightarrow T_\infty \end{array}$$



- (a) The temperature of the gas at the top of the sphere,  $T_1$ , is measured. Compute the Mach number at the top of the sphere.
- (b) Derive the value of the velocity at the bottom of the sphere.

EXAMPLE EXAMINATION  
FLUID MECHANICS I  
MODULE 7, 201500321, 2015

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**Problem 1 [2 POINTS.]**

*Given is the flow field*

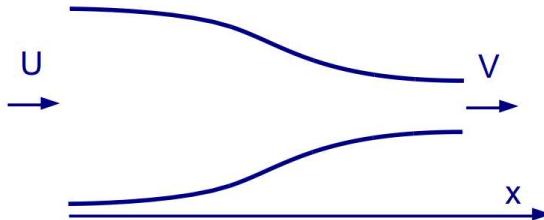
$$u(x, y, z, t) = \frac{1+t}{e^x}, \quad v(x, y, z, t) = \frac{\sin(t)}{y^2}, \quad w(x, y, z, t) = zt, \quad (1)$$

*where  $u$ ,  $v$  and  $w$  are the velocity components in  $x$ ,  $y$  and  $z$  direction respectively.*

- Compute the trajectory  $x_p(t)$ ,  $y_p(t)$ ,  $z_p(t)$  of a dust particle that is located in the point  $x = 0$ ,  $y = 1$ ,  $z = 1$  at  $t = 0$ .
- 

**Problem 2 [2 POINTS.]**

*An incompressible fluid with density  $\rho$  enters a construction through an opening of area size  $A$  at uniform velocity  $U$  and uniform pressure  $p_{in}$ , and leaves the construction through an opening of area size  $\frac{1}{2}A$  at uniform velocity  $V = 2U$  and uniform pressure  $p_{out}$ . Viscosity and gravity can be neglected and the flow is steady.*

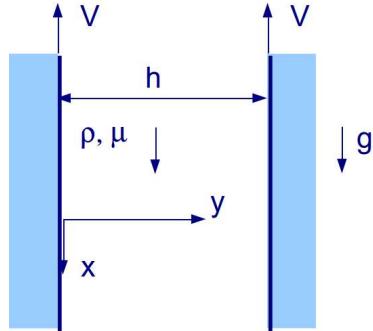


- Compute the force in  $x$ -direction by the fluid on the pipe.

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**Problem 3 [2 POINTS.]**

Incompressible viscous water with density  $\rho$  falls steadily between two vertical parallel plates, both plates move upward with speed  $V$ . The flow is laminar and fully developed. The gap width between the plates is  $h$ , gravity acceleration is  $g$ , the viscosity is  $\mu$  and the pressure is constant:  $p_0$ .



- Derive an expression for the velocity profile  $u(y)$ , starting from the reduced Navier-Stokes equations, with  $y = 0$  on the left plate.
  - Derive an expression for the shear stress by the flow on the left plate.
- 

**Problem 4 [1 POINT.]**

Consider the following temperature distribution:

$$T(x, t) = [a \cos(\lambda x) - b \sin(\lambda x)] \exp(-\beta t).$$

Determine the parameter  $\beta$  such that  $T(x, t)$  satisfies the following diffusion equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}.$$

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**Problem 5 [2 POINTS.]**

In the steady, inviscid and compressible flow around an object of an ideal (perfect) gas with given  $\gamma$  and gas constant  $R$ , and given temperature  $T_\infty$  and Mach number  $M_\infty$  at infinity, the velocity is measured at some point not far from the object:  $u$ .

- Compute the temperature  $T$  at that point.
- Show that the total enthalpy is constant along streamlines.