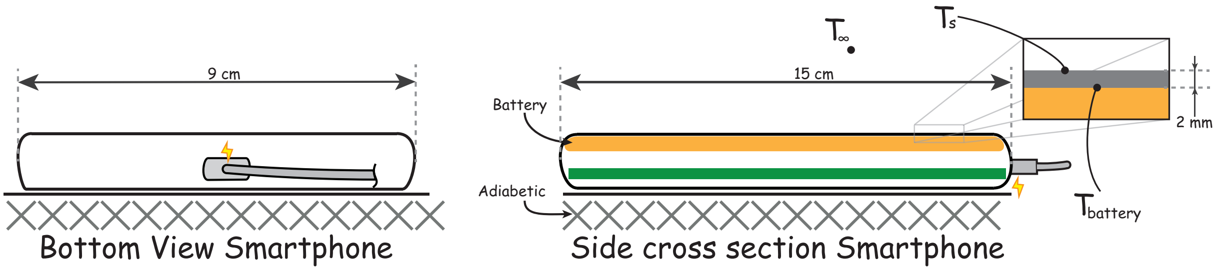
W04

The battery of a smartphone is being charged, while the device is laying outside during the day. It is a windstill day and the sun is shining. The dimensions of the phone are 15 cm x 9 cm and the smartphone is laying flat with the backside upwards towards the sky. The smartphone has a large battery that covers the entire backside of the device and between the battery is only covered by a thin 2 mm plastic backplate. The battery is not perfect and some part of the energy supplied by the charging cable is wasted into heat. The backplate is heated by the radiation of the sun and the warmth of the battery. It has a temperature of 42 K and only looses heat by convection. The outside temperature is 15 ^{o}C and the device absorbs an effective radiative heat flux of 79.6 W/m^2. Assume steady state-state heat (and energy) transfer.



a) Explain what the Grashof number physically represents.

b) Determine the convective heat transfer coefficient h of the backplate and calculate the convective heat flux to the environment $\dot{Q}_{\text{convection}}$

Most of the power supplied by the charging cable $\dot{E}_{cable}$ is stored in the battery. Therefore, the charging rate of the battery is $\dot{E}_{battery,stored}$. The rest of the electrical power supplied by the charging cable is wasted into heat $\dot{Q}_{battery,heat}$ and is lost fully by the backplate. The efficiency at which the battery is charged $\dot{\eta}_{battery} = \frac{\dot{E}_{battery,stored}}{\dot{E}_{cable}}$ is 95%.

c) Determine the conductive heat flux from the battery $\dot{Q}_{\text{battery,heat}}$
Hint:
$0 = \dot{Q}_{\text{battery,heat}} + \dot{Q}_{\text{radiation}} - \dot{Q}_{\text{convection}}$

d) Find the rate at which the charging cable $\dot{E}_{cable}$ supplies energy to the smartphone.
Hint:
The units of the electricity flows $\dot{E}$ and heat flows $\dot{Q}$ are the same; [W s^{-1}]!

e) Determine the temperature of the battery. The thermal conductivity of the plastic backplate is 0.12 W m^{-1} K^{-1}. It may be assumed that the temperature of the battery is constant.

The smartphone is accidentally nudged off the side off the table and now only hangs on the charging cable. The front of the smartphone is still adiabatic and the solar radiation on the backplate of the smartphone is still the same. Also, the charging rate has not changed. However, due to the now vertical position, the convective properties of the device have changed. The owner of the smartphone wants to know if this changes the temperature of the battery.

f) Recalculate the convective heat transfer coefficient h for this vertical scenario and compare it with the horizontal case. Argue, in words or a short additional calculation, how the difference in coefficients influence the temperature of the backplate and the battery of the smartphone. Will it be higher, lower or remain the same? You may neglect the change in the properties of the air.
Hint:
The surface temperature of the backplate is not known in the vertical case. You may - if you feel you need to - simplify the problem by neglecting this change when calculating the Grashof number aswell (and thus use the horizontal surface temperature).

Solution W04

1. *Explain what the Grashof number physically represents.*

The Grashof number is a dimensionless number in fluid dynamics and heat transfer which approximates the ratio of the buoyancy to viscous force acting on a fluid.

(1) correct definition

1. *Determine the convective heat transfer coefficient h of the backplate and calculate the convective heat flux to the environment*

Average fluid properties:

$$T_f = \frac{T_s + T_{infty}}{2} = \frac{15+42}{2}\ [^oC] = 28.5\ [^oC]$$

Properties of air at 28.5 [^{o}C]

$$\rho = 1.170\ [kg/m^3]$$

$$k = 0.0258\ [W/mK]$$

$$\nu = 1.5942 \cdot 10^{-5}\ [m^2/s]$$

$$\Pr = 0.7286$$

$$\beta = \frac{1}{28.5+273.15}\ [K^{-1}] = 0.0033\ [W/mK]$$

(0.5) properties at the correct avg. temperature

Characteristic length of the described situation:

$$L_c = \frac{A_s}{p} = \frac{0.15 \cdot 0.09}{2 \cdot 0.15 + 2 \cdot 0.09}\ [m] = 0.0281\ [m]$$

(0.5) correct characteristic length

Rayleigh number:

$$\mathrm{Ra}_L = \frac{g\beta\,(T_s - T_\infty)\,L_c^3}{\nu^2} \cdot \Pr = \frac{9.81\ [m/s^2] \cdot 0.0033\ [K^{-1}] \cdot (42-15)\ [K] \cdot 0.0281^3\ [m^3]}{\left(1.5942 \cdot 10^{-5}\right)^2\ [m^4/s^2]} \cdot 0.7286 = 5.60 \cdot 10^4$$

(0.5) correct determination of Ra

Nusselt number:

$$\mathrm{Nu}_L = 0.59 \cdot {\mathrm{Ra}_L}^{1/3} = 0.59 \cdot \left(5.60 \cdot 10^4\right)^{1/4} = 9.076$$

(0.5) correct determination of Nu

Heat transfer coefficient:

$$h = \frac{\mathrm{Nu}_L \cdot k}{L_c} = \frac{9.076 \cdot 0.0258\ [W/mK]}{0.0281\ [m]} = 8.316\ [W/m^2K]$$

(0.5) correct determination of h with the correct properties

Rate of heat loss by convection:

$$\dot{Q}_{convection} = h \cdot A_s \cdot (T_s - T_\infty) = 8.316\ [W/m^2K] \cdot (2 \cdot 0.0135)\ [m^2] \cdot (42-15)\ [K] = 3.03\ [W]$$

(0.5) correct determination of $\dot{Q}_{convection}$

1. *Determine the conductive heat flux from the battery $\dot{Q}_{battery,heat}$*

Rate of heat transfer towards the backplate by incident radiation:

$$\dot{Q}_{incident} = \dot{q}_{incident} \cdot A = 79.6\ [W/m^2] \cdot (0.15 \cdot 0.09)\ [m^2] = 1.075\ [W]$$

(1) correct determination of $\dot{Q}$

Rate of heat transfer from the battery to the outside of the backplate:

$$\dot{Q}_{battery,heat} = \dot{Q}_{convection} - \dot{Q}_{incident} = 3.03\ [W] - 1.075\ [W] = 1.96\ [W]$$

(0.5) correct determination of $\dot{Q}_{battery,heat}$

1. *Find the rate at which the charging cable $\dot{E}_{cable}$ supplies energy to the smartphone.*

Finding relationship of the efficiency and the energy fluxes.

$$\eta_{battery} = \frac{\dot{E}_{battery,stored}}{\dot{E}_{cable}} = 0.95$$

And;

$$\dot{E}_{cable} = \dot{E}_{battery,stored} + \dot{Q}_{battery,heat}$$

(0.5) Finding relationship of $\dot{E}_{cable}$

So ;

$$\dot{E}_{battery,stored} = \dot{E}_{cable} - \dot{Q}_{battery,heat}$$

Substituting;

$$\eta_{battery} = \frac{\dot{E}_{cable} - \dot{Q}_{battery,heat}}{\dot{E}_{cable}} = 1 - \frac{\dot{Q}_{battery,heat}}{\dot{E}_{cable}}$$

Finally;

$$\dot{E}_{cable} = \frac{\dot{Q}_{battery,heat}}{1 - \eta_{battery}} = \frac{1.96}{1 - 0.95} = 39.1$$

(1) correct determination of $\dot{E}_{cable}$

1. *Determine the temperature of the battery. The thermal conductivity of the plastic backplate is 0.12 W m^{-1} K^{-1}. It may be assumed that the temperature of the battery is constant.*

Thermal resistance of the backplate:

$$R = \frac{t}{k \cdot A_s} = \frac{0.002}{0.12 \cdot 0.0135} = 1.234$$

Conduction of heat trough the backplate;

$$\dot{Q}_{battery,heat} = \frac{T_{battery} - T_s}{R}$$

Rewritten to $T_{battery}$;

$$T_{battery} = \dot{Q}_{battery,heat} \cdot R + T_s = 1.96 * 1.234 + 42 = 44.4^{o}\text{C}$$

(1) correct determination of $T_{battery}$

1. *Recalculate the convective heat transfer coefficient h for this vertical scenario and compare it with the horizontal case. Argue, in words or a short additional calculation, how the difference in coefficients influence the temperature of the backplate and the battery of the smartphone. Will it be higher, lower or remain the same? You may neglect the change in the properties of the air.*

The Rayleigh number can be calculated with or without the $(T_s - T_{infty})$ term neglected. Neglecting this term will not influence the qualitative answer. It is most important to see that the characteristic length will change.

$$L_c = L = 0.15[m]$$

Then the the Rayleigh number is;

$$\text{Ra}_\text{L} = \frac{g\beta L_\text{c}^3}{\nu^2} \cdot \text{Pr} \cdot (T_\text{s} - T_\infty) = \frac{9.81\ [\text{m/s}^2] \cdot 0.0033\ [\text{K}^{-1}] \cdot 0.15^3\ [\text{m}^3]}{\left(1.5942 \cdot 10^{-5}\right)^2\ [\text{m}^4/\text{s}^2]} \cdot 0.7286\,(T_\text{s} - T_\infty)\,[\text{K}] = 3.15 \cdot 10^5\,(T_\text{s} - T_\infty)$$

Or when change in T_s is neglected in Grashof;

$$\text{Ra}_\text{L} = 8.50 \cdot 10^6$$

(1) correct determination of Ra_L

In both cases it is clear that the Nusselt relationship remains the same;

$$Nu = 0.59 * Ra_L^{1/4}$$

(1) correct determination of Nu

Giving;

$$h = \frac{Nu \cdot k}{L_c} = \frac{13.97 \cdot (T_s - T_{infty})^{1/4} \cdot 0.258}{0.15} = 2.40 \cdot (T_s - T_{infty})^{1/4} \vee h = 5.47$$

(1) correct determination of h

There are multiple ways to show that the convective heat transfer coefficient has decreased. The conclusion should be that the temperature of the device should have increased to still have the same energy flow.

(1) Right conclusion that the device has an increased temperature