

Energy & Heat Transfer



Lecture 2

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RECAP OF LECTURE 1

- Work (W), energy (E), power (P)
- Work (W), Energy (E) in $J = N \cdot m = \frac{kg \cdot m}{s^2} \cdot m$
- Power : $P = \frac{\Delta E}{\Delta t}$ in Watt (W) = J / s
- Units kWh, kcal, hp,

RECAP OF LECTURE 1



- Potential energy : $E = F \cdot h$
 - Thermal energy : $Q = m \cdot c \cdot \Delta T$
 - c is specific heat capacity in J / (kg · K)
 - Electrical power: $P = U \cdot I$
 - Electrical energy: $E = P \cdot t = U \cdot I \cdot t$
 - Ohm's law: $U = I \cdot R$
- $$P = I^2 \cdot R$$
$$E = I^2 \cdot R \cdot t$$

LEARNING OBJECTIVES LECTURE 2



- Energy in general**
 - Using an energy balance
 - Determine efficiency
 - Calculate losses for electric resistors

- Conductive heat transfer**
 - Explaining conduction principles*
 - Calculate conductive heat transfer*
 - Using thermal resistances and insulation values*

LEARNING OBJECTIVES LECTURE 2



● Energy in general

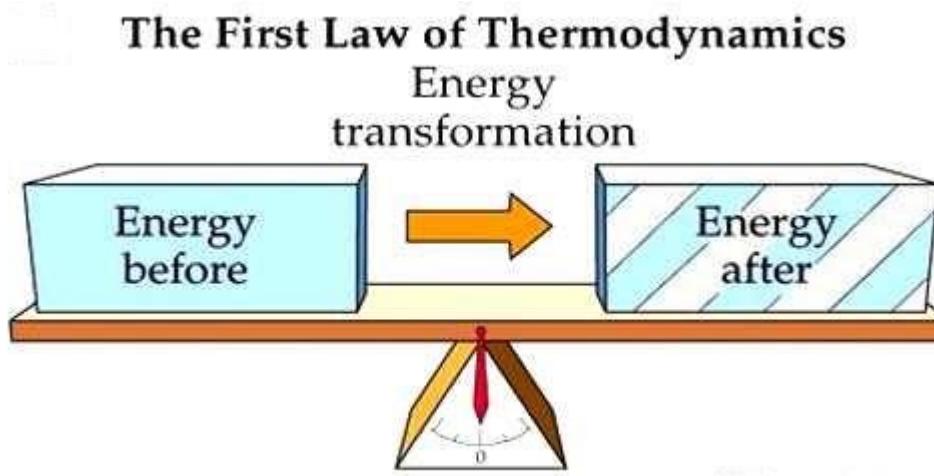
- Using an energy balance
- Determine efficiency
- Calculate losses for electric resistors

○ Conductive heat transfer

- *Explaining* conduction principles
- *Calculate* conductive heat transfer
- *Using thermal resistances* and insulation values

ENERGY BALANCE

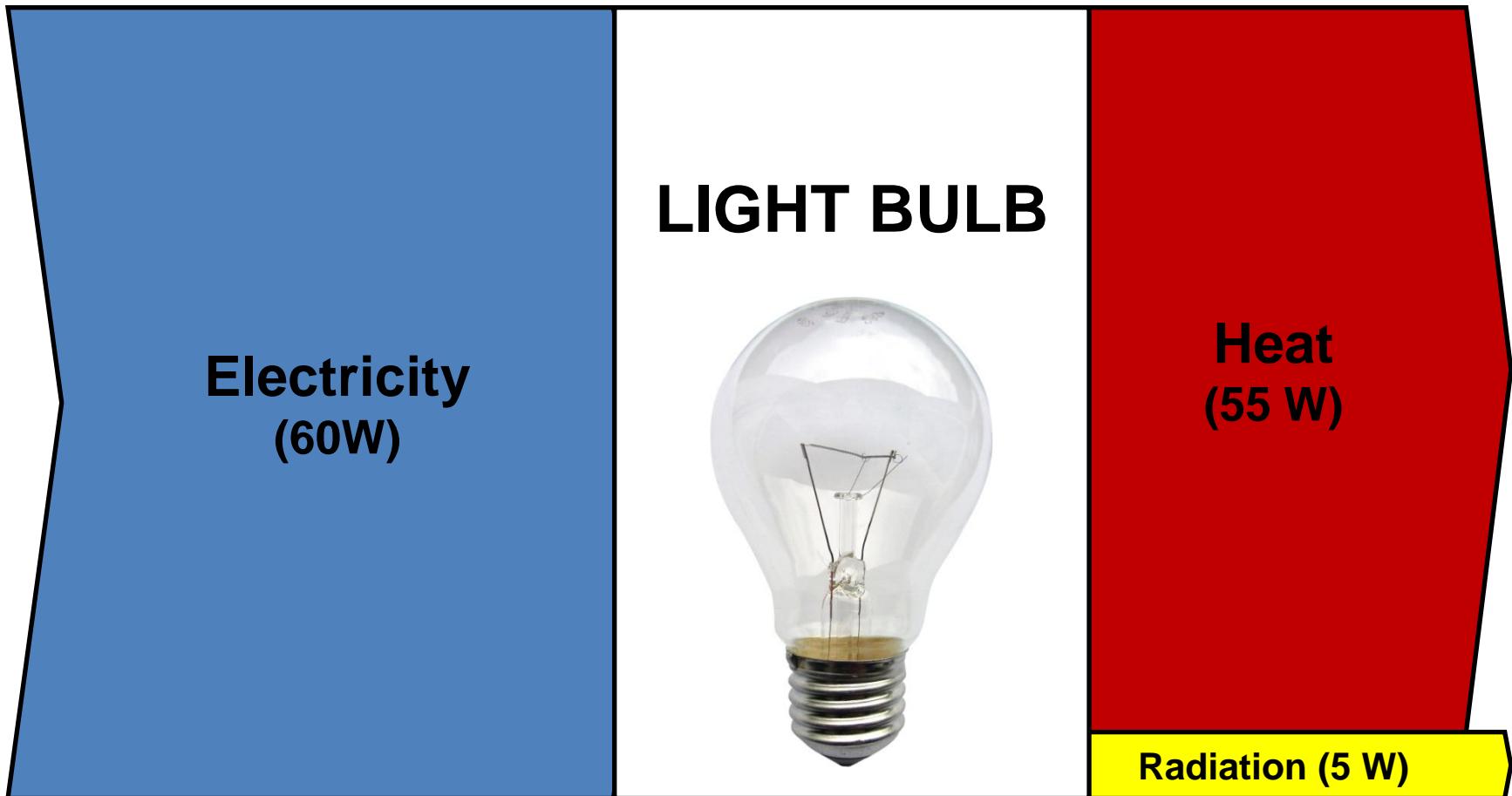
- Energy is always conserved!
- First law of Thermodynamics:



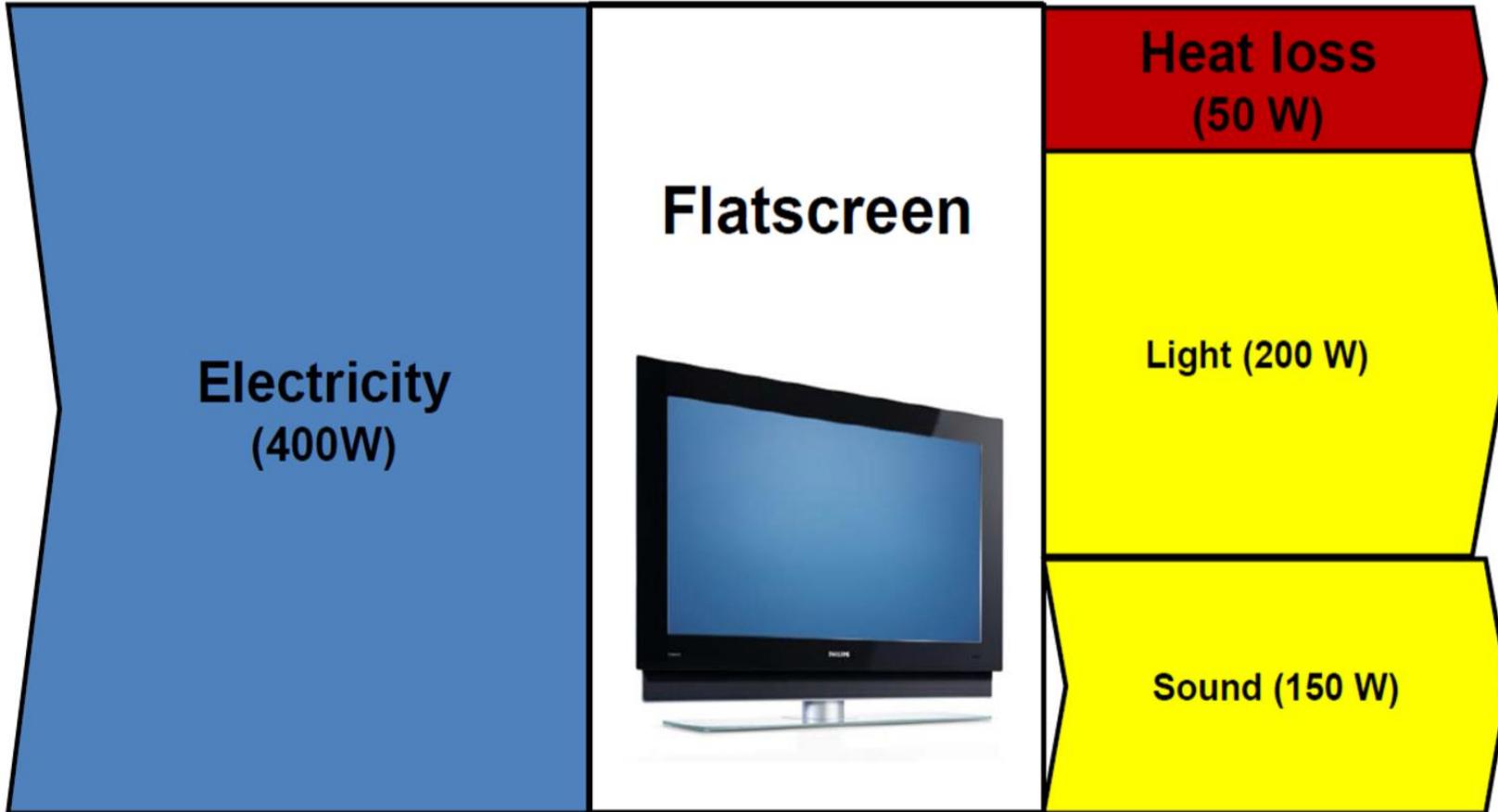
- https://www.youtube.com/watch?v=kP7q28wQ2P8&ab_channel=BBCEarthLab

ENERGY BALANCE

- Sankey diagram light bulb



ENERGY BALANCE



ENERGY BALANCE



LEARNING OBJECTIVES LECTURE 2



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- **Conductive heat transfer**
 - *Explaining conduction principles*
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EFFICIENCY

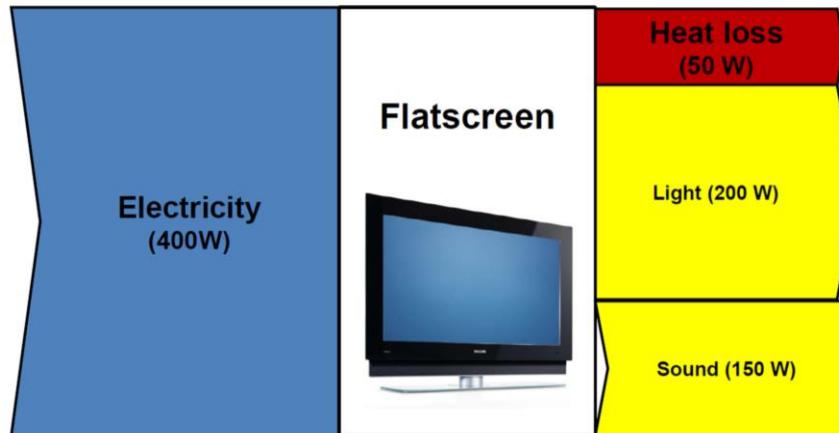


- Efficiency = fraction of “useful” work/power used
 - What is defined as useful?
 - What is the reference?
- Efficiency $\eta = \frac{\text{useful work}}{\text{input energy}} = \frac{\text{useful power}}{\text{input power}}$ (-)
- Use a Sankey diagram!

EFFICIENCY

Efficiency definition not clear:

- What is useful? What is the input? Give the reasons for the choices made



Flatscreen: useful spent power?

So always explain how you have defined efficiency!

EFFICIENCY – PRACTICAL VALUES



Product	Efficiency	Remark
Solar panel	12%	Part of sun's radiation converted to electricity
Vacuum cleaner	20%	Part of electrical power converted to suction power
Petrol engine	35%	Part of chemical energy converted to kinetic energy
Electric kettle	84%	Part of electrical energy converted to useful heat in the water

Always state the used definition when talking about efficiency

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RESISTORS

Electric resistors: barrier in electric circuit
→ heat generation



Heat: purpose

Heat: “byproduct”

RESISTORS

Schematic symbols:



Resistance R determined by:

- Length of the wire (L)
- Through-flow area (A)
- Material (resistivity ρ)

Pouillet's law:

$$R = \frac{\rho L}{A}$$

Not density:
 $\Omega \cdot m \neq kg/m^3$



Compare: flow
through straw

RESISTOR AS HEAT SOURCE



Resistor converts electric energy into thermal energy:

$$E = I^2 R \Delta t = Q \quad [\text{J}]$$

So electric power becomes thermal power:

Notation: \dot{Q} (“Heat-flow” or heat transfer rate)

$$P = I^2 R = \dot{Q} \quad [\text{J/s}] = [\text{W}]$$

SIDESTEP



Dot sign means: per unit time (time derivative)

$$\text{Power} = \frac{\text{Energy change}}{\text{Time}}$$

$$P = \frac{dE}{dt} \approx \frac{\Delta E}{\Delta t}$$

$$\dot{Q} = \frac{dQ}{dt} \approx \frac{\Delta Q}{\Delta t}$$

$$(\text{Velocity } v = \frac{dx}{dt} = \dot{x} \approx \frac{\text{displacement } \Delta x}{\Delta t})$$

$$(\text{Acceleration } a = \frac{dv}{dt} = \ddot{v} = \ddot{x} \approx \frac{\text{velocity change } \Delta v}{\Delta t})$$



RESISTIVITY

$$R = \rho \frac{L}{A}$$

Material	$\rho (\Omega \cdot \text{m}) \text{ at } 20^\circ \text{ C}$
Copper	$1,67 \cdot 10^{-8}$
Tungsten	$5,5 \cdot 10^{-8}$
Aluminum	$2,65 \cdot 10^{-8}$
Glass	$1 \cdot 10^{12}$
PVC	$1 \cdot 10^{14}$
Rubber	$1 \cdot 10^{15}$

} Conductors
} Insulators

CONDUCTORS / INSULATORS

Also thermal!



Insulators?



Conductors?

LEARNING OBJECTIVES LECTURE 2



● Energy in general

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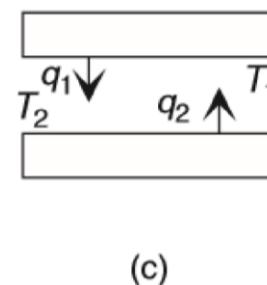
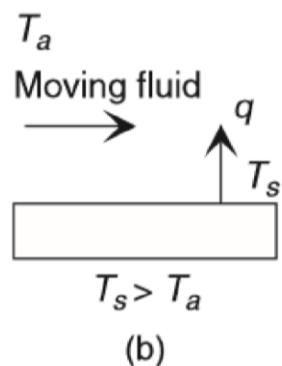
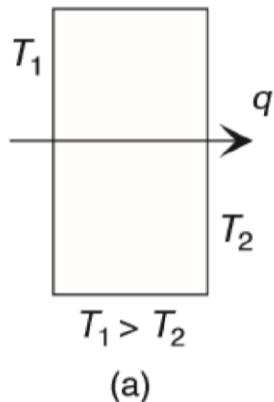
● Conductive heat transfer

- *Explaining conduction principles*
- *Calculate conductive heat transfer*
- *Using thermal resistances and insulation values*

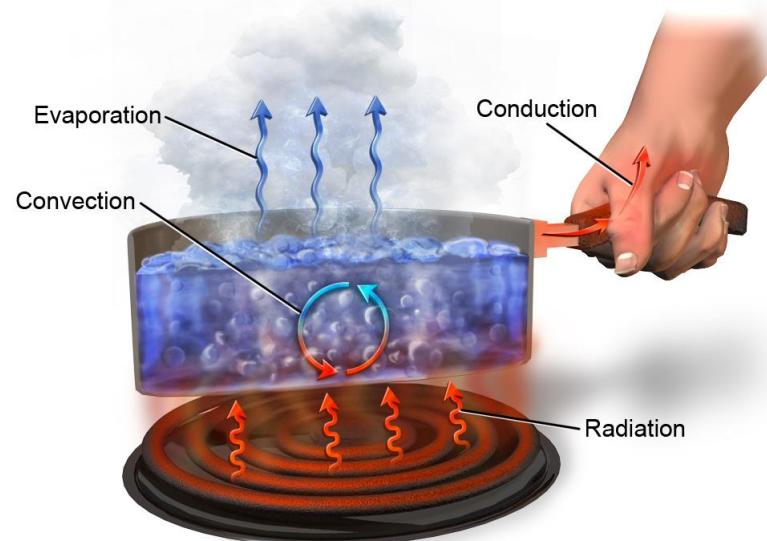
HEAT TRANSFER MECHANISMS

Representations of heat transfer modes:

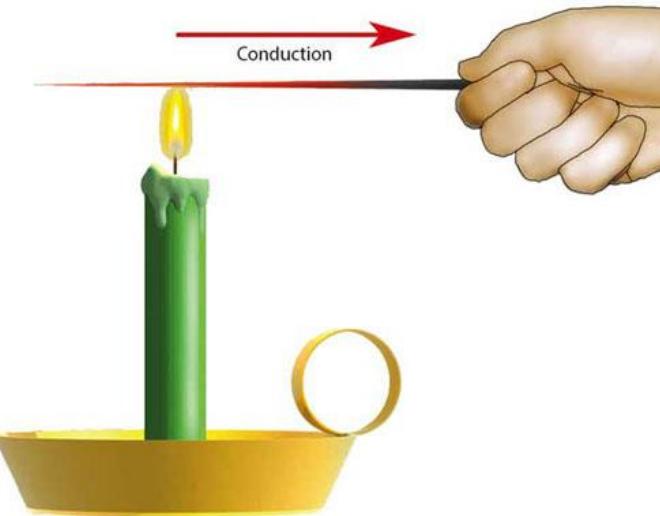
- (a) **conduction** through a solid, liquid or gas,
- (b) **convection** from a surface to a moving fluid,
- (c) **radiation** between two surfaces



Mechanisms of Heat Transfer



HEAT TRANSFER MECHANISMS



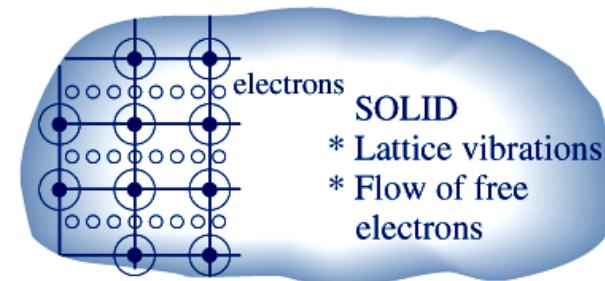
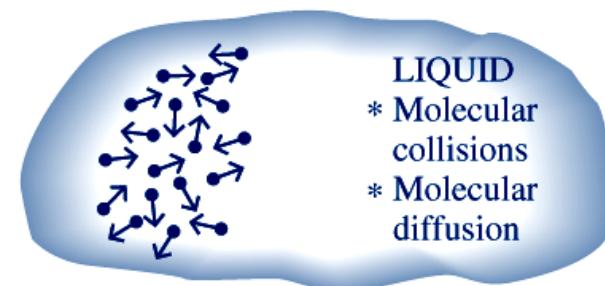
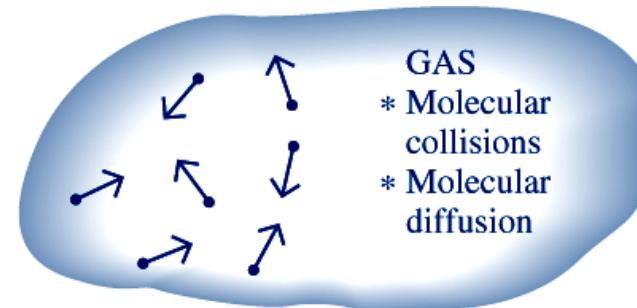
- Conduction
- Convection
- Radiation

CONDUCTION

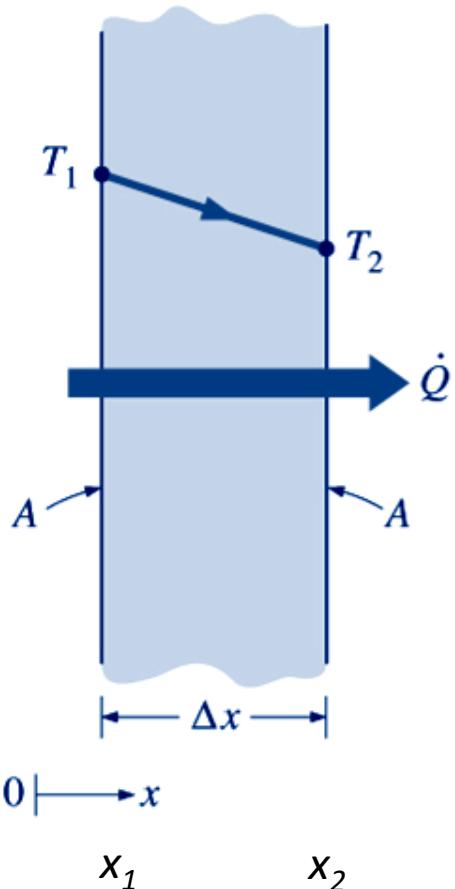
Conduction:

Transfer of thermal energy between (non flowing) molecules

- Always **from** more energetic (**warm**) to less energetic (**cold**) particles!



CONDUCTION(FOURIER's LAW)



Cross section of part of a wall (almost flat)

Heat transfer rate : \dot{Q}

- Steady state condition-1D
- Proportional to area A
- Proportional to temp. Difference : $T_2 - T_1$
- Inversely proportional to thickness: $\Delta x = x_2 - x_1$
- Dependent on material \rightarrow thermal conductivity: k

$$\dot{Q} = -k A \frac{T_2 - T_1}{x_2 - x_1} = -k A \frac{\Delta T}{\Delta x} \quad (\text{W})$$

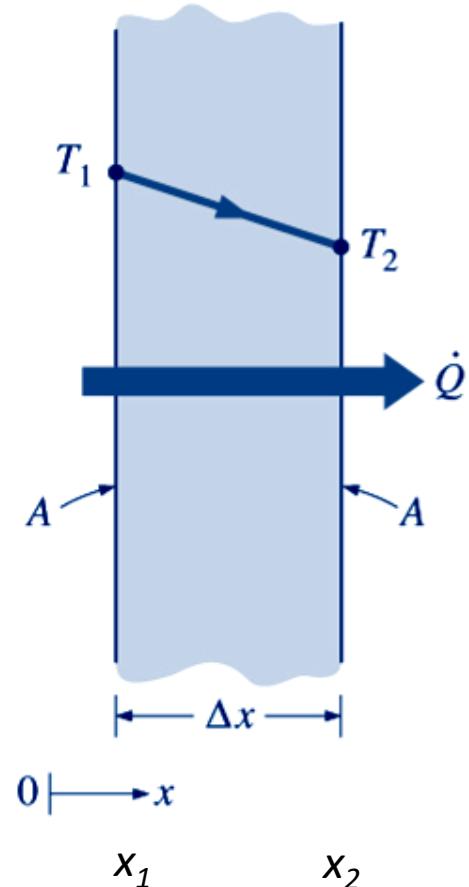
Negative sign since **heat flow is positive** at a negative temperature gradient.

CONDUCTION(FOURIER's LAW)

$$\dot{Q} = -kA \frac{T_2 - T_1}{x_2 - x_1} = -kA \frac{\Delta T}{\Delta x} (\text{W})$$

- Fourier's law simplified for **plane surface**
- Per m^2 : **Heat flux**

$$\dot{q} = \frac{\dot{Q}}{A} = -k \frac{T_2 - T_1}{x_2 - x_1} = -k \frac{\Delta T}{\Delta x} (\text{W/m}^2)$$



Thermal conductivity (k)

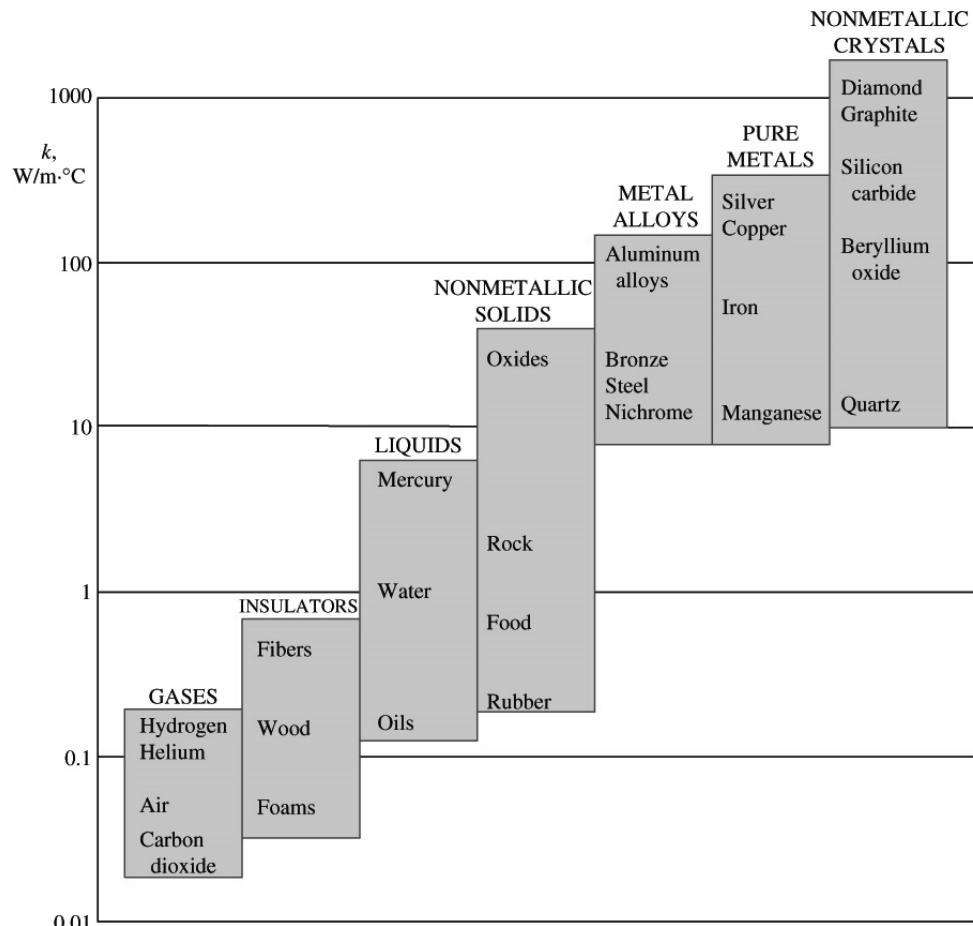
Unit:

$$\frac{\text{W}}{\text{m} \cdot \text{K}}$$

→ The amount of power conducted through 1 m of material at a 1 K temperature difference

High k : Conductor

Low k : Insulator



LEARNING OBJECTIVES LECTURE 2



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ASSIGNMENT: ELECTRIC KETTLE

What is the **efficiency** of an electric kettle when the water is almost boiling ($T=100\text{ }^{\circ}\text{C}$)?

Data/assumptions/simplifications:

- Wall temperature outside $50\text{ }^{\circ}\text{C}$
- Diameter 10 cm
- Height 20 cm
- Plastic wall:
 - Thickness 2,7 mm
 - $k = 0,20 \text{ W} / (\text{m} \cdot \text{K})$
- Neglect losses through top and bottom
- It can be treated as plane wall
- Electric power $P_{el} = 930 \text{ W}$



$$\dot{Q} = -k A \frac{\Delta T}{\Delta x}$$

CONDUCTION FLAT WALL- SUMMARIZED



- Fourier's law for plane surface:

– Temperature difference	$\Delta T = T_2 - T_1$ (K)
– Thickness	$\Delta x = x_2 - x_1$ (m)
– Surface area	A (m^2)
– Thermal conductivity	k (W / (m · K))

Heat transfer rate: $\dot{Q} = -k A \frac{\Delta T}{\Delta x}$ (W)

Heat flux: $\dot{q} = -k \frac{\Delta T}{\Delta x}$ (W / m²)

**Next: model heat flow as thermal resistance network
(Make it easier for calculations)**

LEARNING OBJECTIVES LECTURE 2



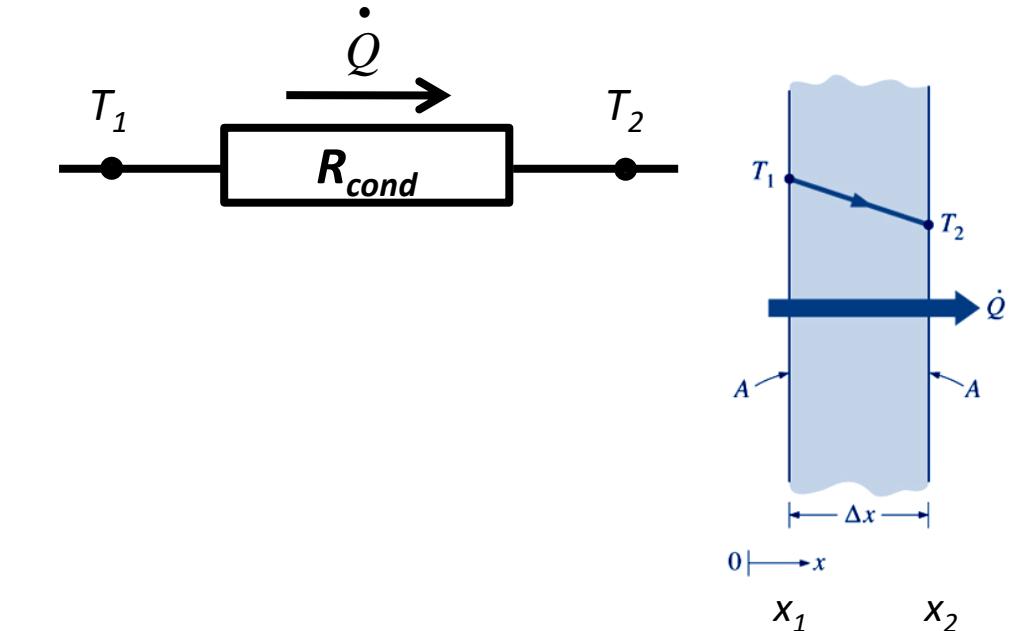
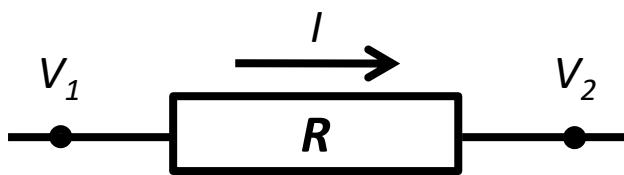
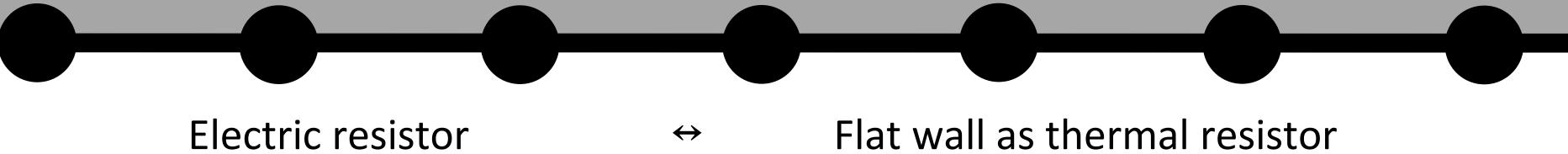
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ANALOGY ELECTRICITY - HEAT



Voltage difference

$$V_1 - V_2 \leftrightarrow$$

Current flow

$$I \leftrightarrow$$

Electrical resistance

$$R \leftrightarrow$$

unit: Ω

Temperature difference

Heat flow

Thermal resistance

$$T_1 - T_2$$

$$\dot{Q}$$

$$R_{cond}$$

unit: K/W

ANALOGY ELECTRICITY - HEAT

Heat conduction through plane wall

Fourier's law:

$$\dot{Q} = -k A \frac{T_2 - T_1}{\Delta x}$$

$$= +k A \frac{T_1 - T_2}{\Delta x}$$

$$= \frac{T_1 - T_2}{\frac{\Delta x}{k A}}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{cond}} \text{ with } R_{cond} = \frac{\Delta x}{kA} \text{ (K/W)}$$

Heat transfer in “Ohmic way”!

Electrical resistance

Ohm's law:

$$I = \frac{V_1 - V_2}{R}$$

$$V_1 - V_2 \leftrightarrow T_1 - T_2$$

$$I \leftrightarrow \dot{Q}$$

$$R \leftrightarrow R_{cond}$$

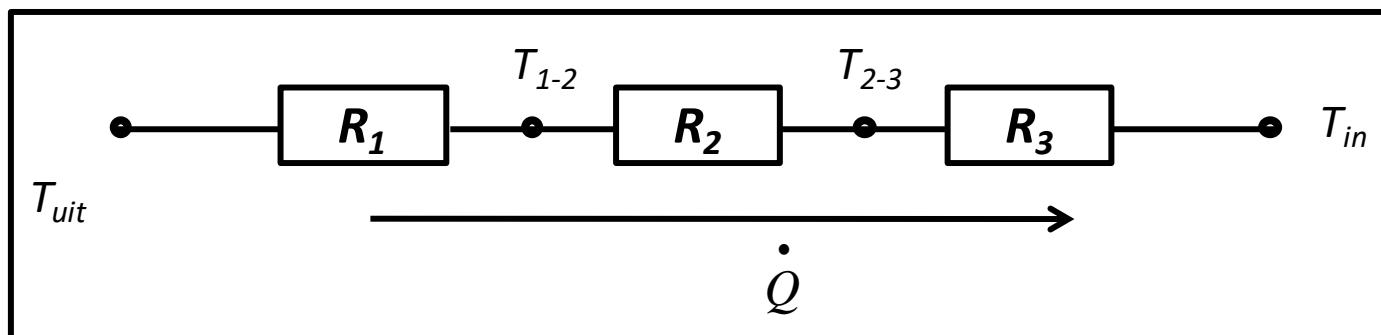
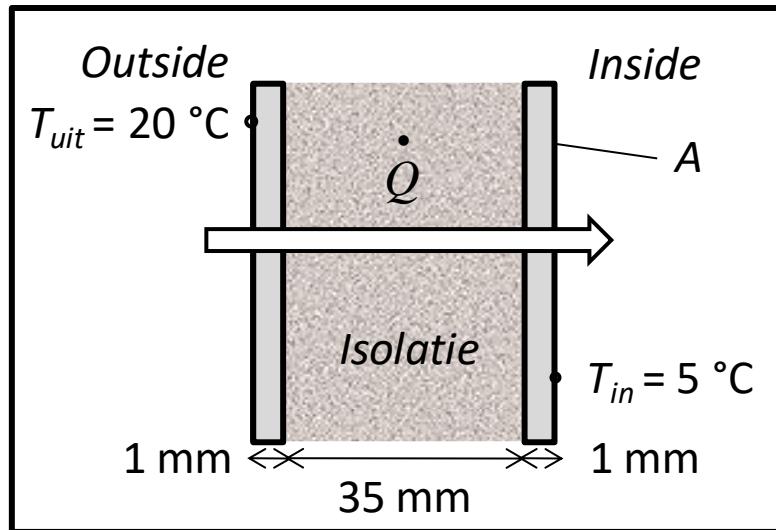
$$\Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{cond}}$$

THERMAL RESISTANCE NETWORKS

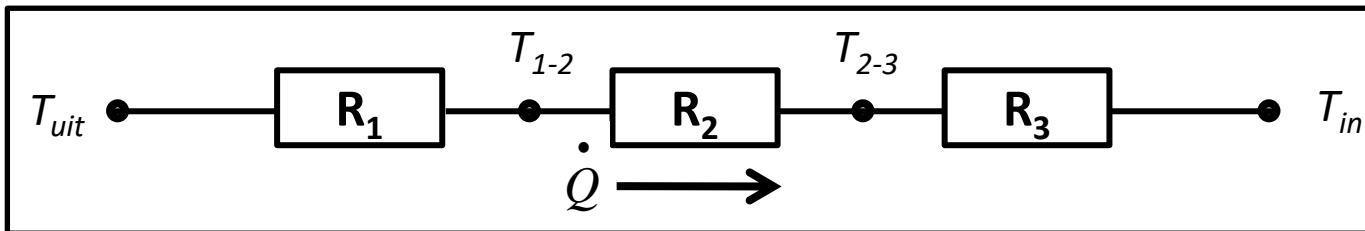


Fridge wall: heat flow \dot{Q} through three resistors

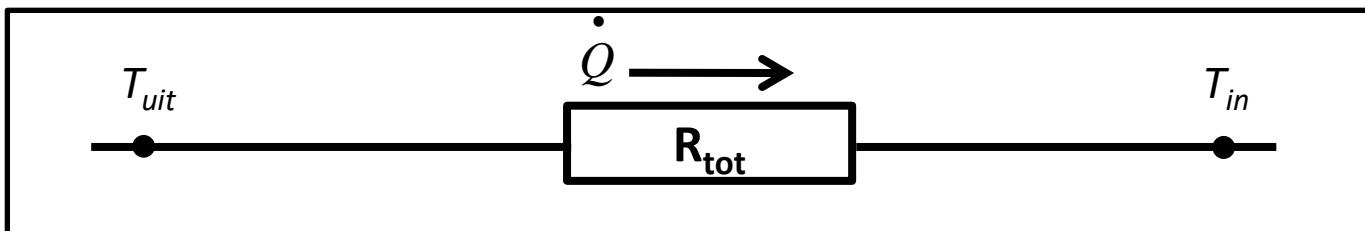
- Outer wall (R_1)
- Insulation (R_2)
- Inner wall (R_3)



THERMAL RESISTANCE NETWORKS



Total resistance $R_{tot} = R_1 + R_2 + R_3$ (series)

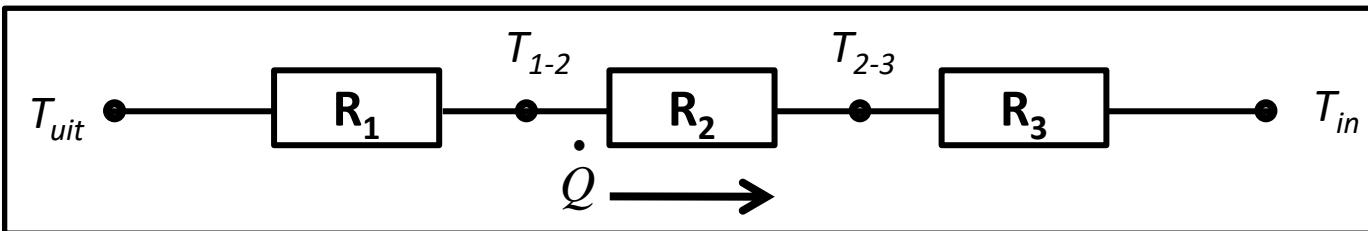


⇒ heat flow

$$\dot{Q} = \frac{T_{uit} - T_{in}}{R_{tot}}$$

Resistors in series: total resistance equals the sum of the individual resistances

THERMAL RESISTANCE NETWORKS



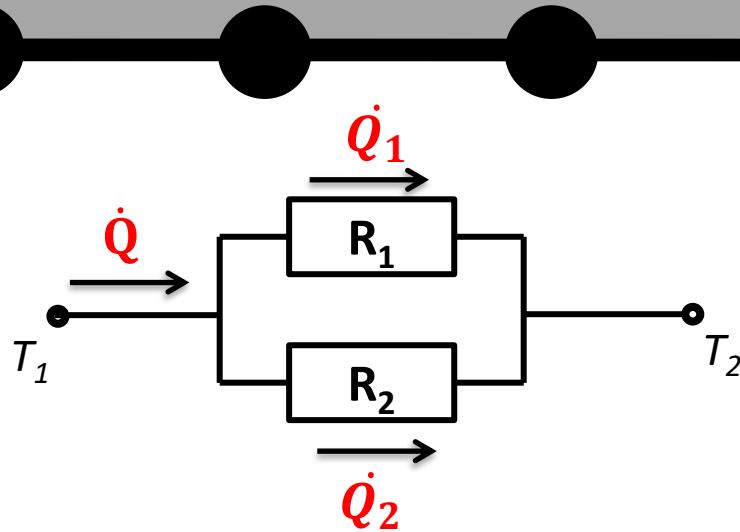
$T_{1-2}, T_{2-3} ?$ Use: $\dot{Q} = \text{constant}$ throughout the resistors

$$\begin{aligned}\dot{Q} &= \frac{T_{uit} - T_{in}}{R_{tot}} \\ &= \frac{T_{uit} - T_{1-2}}{R_1} = \frac{T_{1-2} - T_{2-3}}{R_2} = \frac{T_{2-3} - T_{in}}{R_3} = \frac{T_{uit} - T_{2-3}}{R_1 + R_2} = \frac{T_{1-2} - T_{in}}{R_2 + R_3}\end{aligned}$$

⇒ Solve all unknown temperatures by comparing total and partial resistors in series, similar to voltage in an electrical circuit

⇒ **Next: parallel thermal resistors**

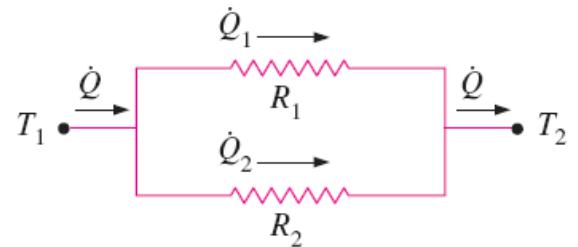
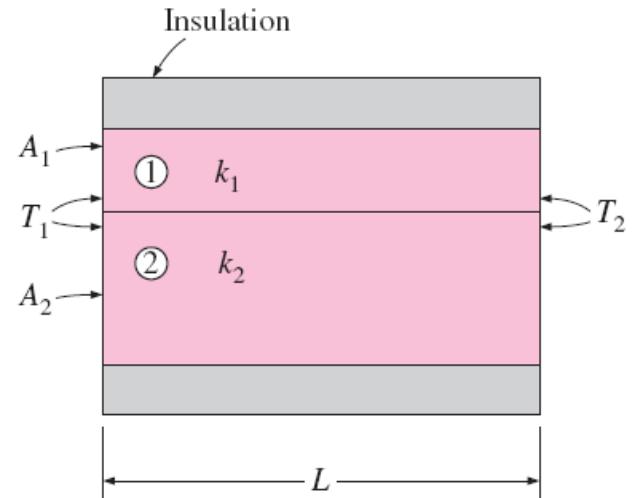
THERMAL RESISTANCE NETWORKS



$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\rightarrow \frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$

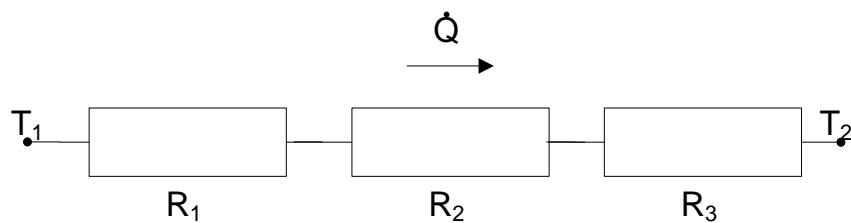


$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

THERMAL RESISTANCE NETWORKS



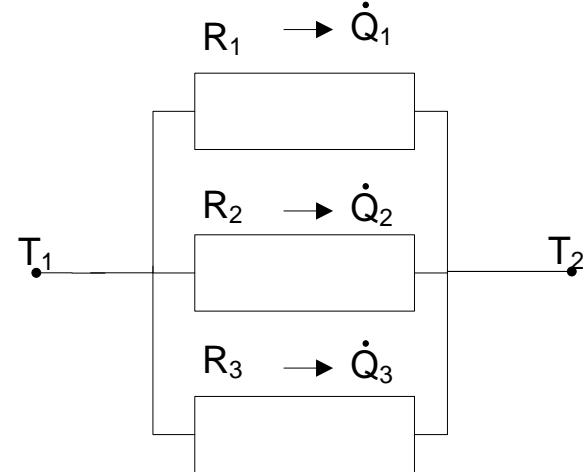
Series Resistors



$$R_{tot} = \sum_i R_i$$

(Add Resistors)

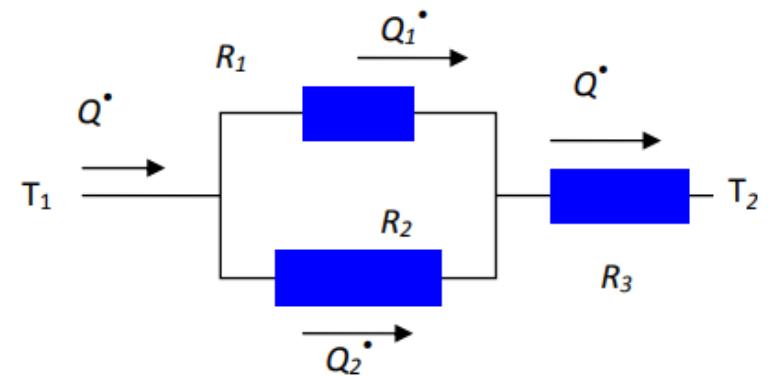
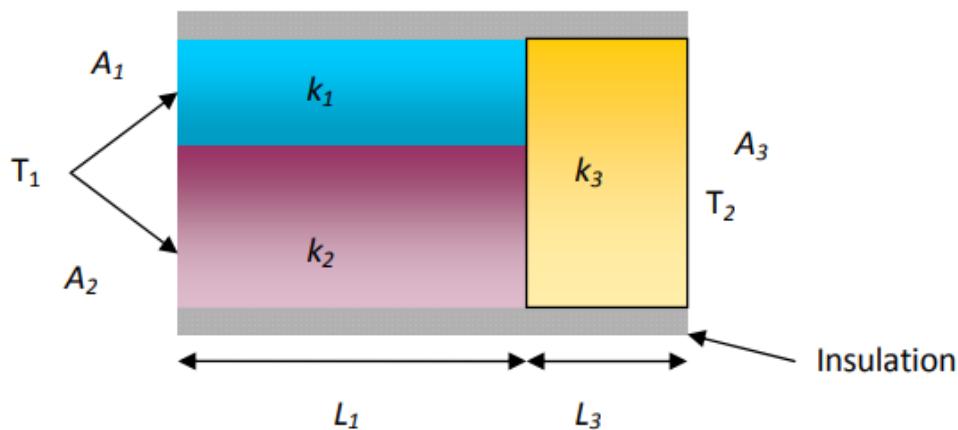
Parallel Resistors



$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

(Add Heat Flows)

THERMAL RESISTANCE NETWORKS



$$Q^\bullet = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

RESISTANCE AND INSULATION

Thermal resistance plane surface:

$$R = \frac{\Delta x}{kA} \quad \left(\frac{K}{W} \right)$$

Alternative concept: insulation value
(building materials):

$$R-value = \frac{\Delta x}{k} = R \cdot A \quad \left(\frac{m^2 \cdot K}{W} \right)$$

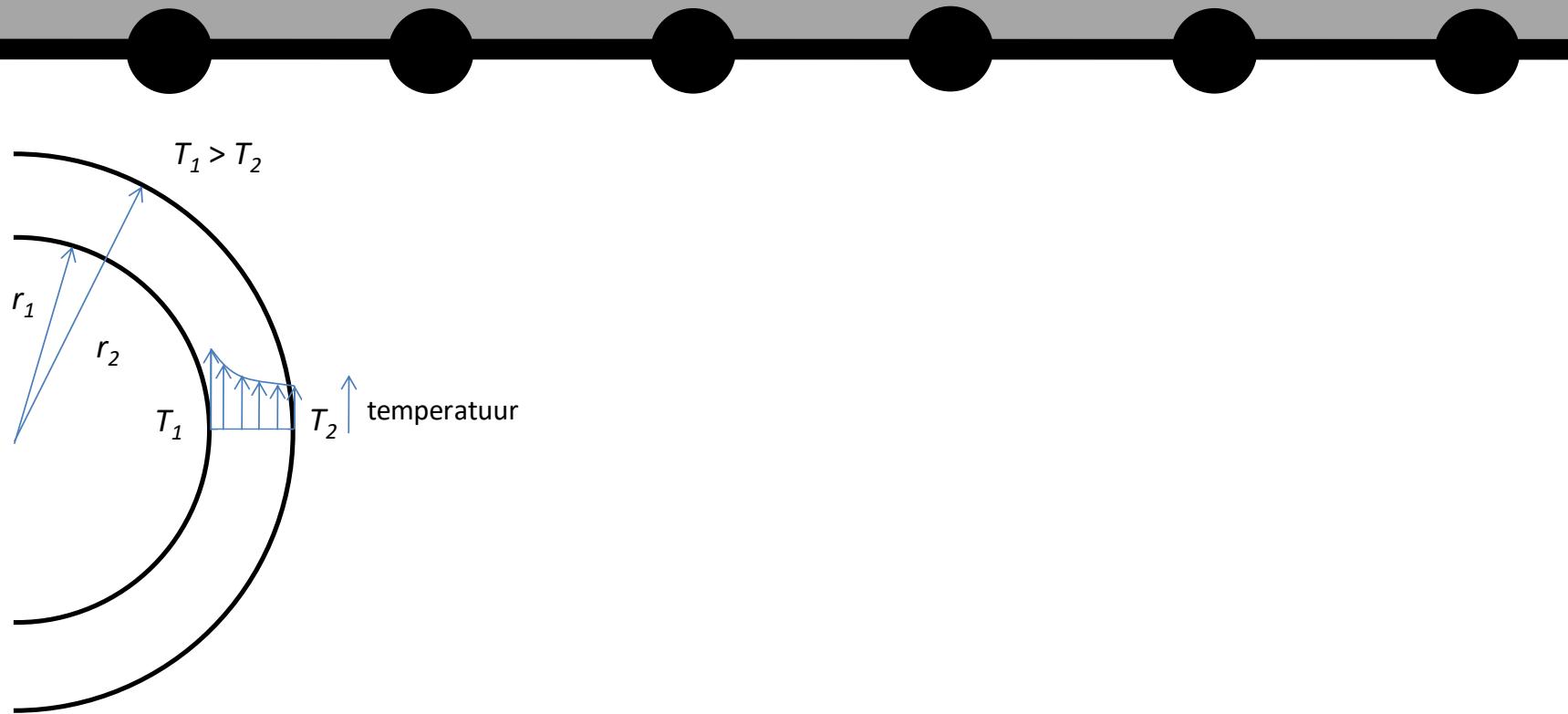
⇒ Insulation value is the heat resistance of
1 m² of material



Material	R-value (m ² · K / W)
Bricks (wall)	0,26
Rockwool 5 cm thick	1,45
Glasswool 11 cm thick	2,50

NB: Often the symbol *R* is used for both concepts. Look at the units!

CONDUCTION IN NON-PLANAR SURFACE

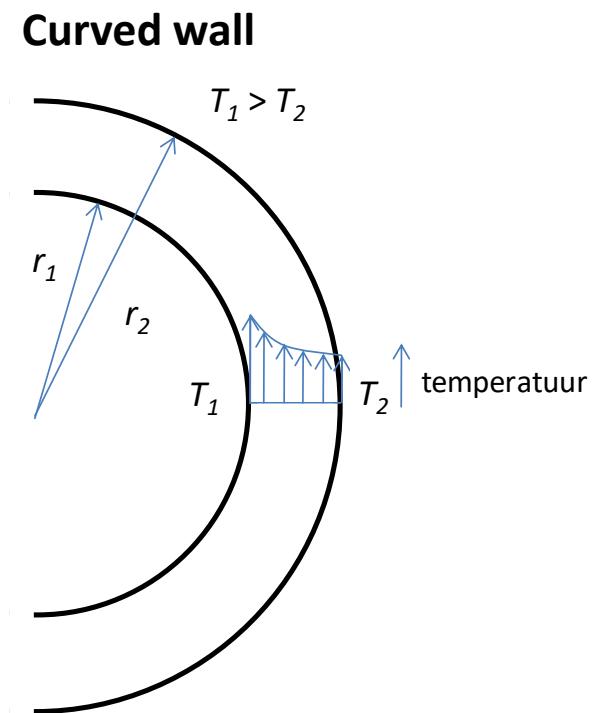
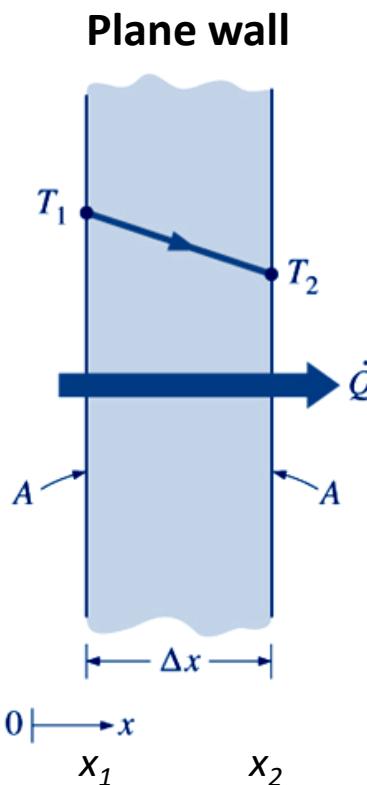


Small radius: small flow-through surface \rightarrow high resistance \rightarrow large temperature *gradient*

(In areas with a high resistance the temperature decreases more in the direction of the heat flow)

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

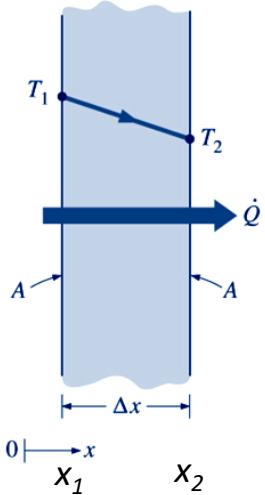
CONDUCTION IN NON-PLANAR SURFACE



- Surface area constant over x
- Temperature profile linearly decreasing (Fourier's law)
- Temperature gradient (= 'slope') constant

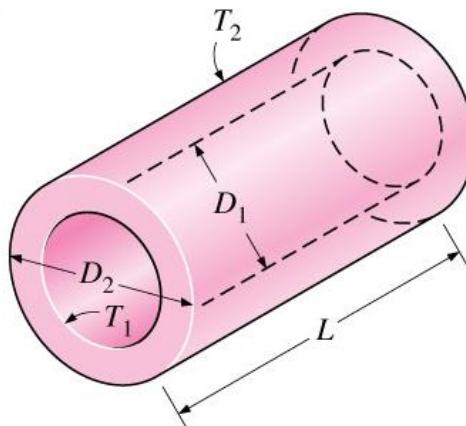
- Surface area increases with r
- Temperature profile decreasing concave up
- Temperature gradient smaller with increasing radius r

VARIOUS CONDUCTION RESISTANCES



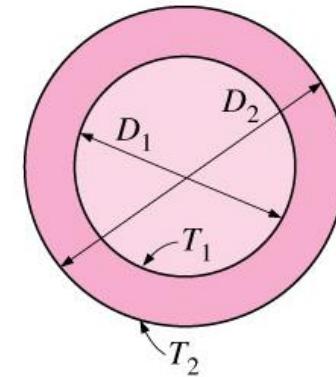
Plane wall

$$R = \frac{\Delta x}{kA}$$



Cylindrical pipe

$$R = \frac{\ln(\frac{D_2}{D_1})}{2\pi L k}$$



Spherical shell

$$R = \frac{D_2 - D_1}{2\pi k D_1 D_2}$$

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

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SUMMARY LECTURE 2 (1/2)

- Efficiency

$$\eta = \frac{\text{useful work}}{\text{inputted energy}} = \frac{\text{useful power}}{\text{inputted power}}$$

Justify the applied definition!

- Heat transfer rate \dot{Q} (W); Heat flux: $\dot{q} = \dot{Q}/A$ (W/m^2)

↑
“Q-flux”

- Heat dissipation electric resistor

$$\dot{Q} = I^2 R \quad (\text{W}) \quad \text{with} \quad R = \rho \frac{L}{A} \quad (\Omega)$$

SUMMARY LECTURE 2 (2/2)



- Conduction: heat transfer between molecules
- Different appearances of Fourier's law

- Plane surface: $\dot{Q} = -k A \frac{T_2 - T_1}{x_2 - x_1} = \frac{T_1 - T_2}{R}$ with $R = \frac{\Delta x}{kA}$ ($\frac{\text{K}}{\text{W}}$)

- Cylindrical tube: $\dot{Q} = \frac{T_1 - T_2}{R}$ with $R = \frac{\ln(\frac{D_2}{D_1})}{2\pi L k}$

- Spherical shell: $\dot{Q} = \frac{T_1 - T_2}{R}$ with $R = \frac{D_2 - D_1}{2\pi k D_1 D_2}$

- Thermal resistors: for ease of calculation
- Insulation value: $R\text{-value} = R \cdot A$ ($\frac{\text{m}^2 \cdot \text{K}}{\text{W}}$)