SS2_E1: Voorbeelden

Given are three vectors: $\underline{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\underline{b} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, and $\underline{d} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$.

1. Compute the lengths of these vectors

2. Compute the scalar products $\underline{a \cdot b}$, $\underline{a \cdot d}$, and $\underline{b \cdot d}$
   and compute the angles between these vector pairs.

3. Compute the outer (vector) products $\underline{a * b}$, $\underline{a * d}$, and $\underline{b * d}$
   and from these compute the angles between these vector pairs.

   Do the angles from 2. and 3. agree?

4. We define the new basis: $\underline{\hat{e}'}_1 = \underline{d}/|\underline{d}|$, $\underline{\hat{e}'}_2 = \underline{b}/|\underline{b}|$, and $\underline{\hat{e}'}_3 = \underline{\hat{e}'}_1 * \underline{\hat{e}'}_2$.
   Compute the orthogonal (transformation) rotation matrix $R_{pi} = \underline{\hat{e}'}_p \cdot \underline{\hat{e}}_i$ and confirm its orthogonality …

5. Compute $R_{pi} a_i = a'_p$, $R_{pi} b_i = b'_p$, $R_{pi} d_i = d'_p$, and $R_{pi} \underline{\hat{e}_{1|2|3}}'_i = ?$, (1|2|3 means all).

NOTE for future use – the transformation behavior can be co- or contra-variant.
Basis vector behave/transform DIFFERENT from the vectors a, b, d.
Indices can in the general case be sub- or super-scripts, respectively. NOT HERE.

SS2_E2: Voorbeelden

Given is the stress tensor from Voorbeeld 1 pg.53: $\underline{\underline{\sigma}} = \begin{bmatrix} 10 & 0 & 20 \\ 0 & -10 & 10 \\ 20 & 10 & 0 \end{bmatrix}$.

6 Compute the eigenvalues and eigenvectors from the stress tensor. Build the rotation matrix from these eigenvectors (used as new basis) and the original basis – and rotate stress into this new system.

7 Assuming E=10GPa and ν=0.3 compute the strain assuming isotropic Hooke

8 Confirm that the strain eigenvectors are parallel to the stress eigenvectors

9a Compute isotropic (volumetric) contributions of stress and strain
… their eigenvalues and eigenvectors

9b Compute the deviatoric contributions of stress and strain
… their eigenvalues and eigenvectors

9c Compute the invariants of the tensors, volumetric tensors and deviatoric tensors

10 Compute the equivalent stresses according to Mises and Tresca.
Which of the two criteria is “safer”? Discuss.

ASSIGNMENT – TENSORS

1.a	Give examples for tensors of rank 0, 1, and 2.	2
1.b	How many independent components has a symmetric stress tensor in two/three dimensions ? – and how many eigenvalues ?	2
1.c	Under which conditions does one find no shear stress?	2
1.d	What physical quantity is described by the trace of the stress tensor ? What is the trace of the deformation tensor ? What is the meaning of the deviator ? What is the meaning of symmetric and anti-symmetric tensors – give examples.	3
2.	Given are the tensors (two-dimensional – plane-stress): $\underline{\underline{\sigma}} = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 1.3 \end{pmatrix} p$ and $\underline{\underline{\sigma}} = \begin{pmatrix} 1.0 & 0.6 \\ 0.6 & 1.0 \end{pmatrix} p$ Determine (mathematically and graphically) their eigenvalues, the traces, the deviators and the orientations of their major eigen-values with respect to the horizontal. Determine also the pressure and the maximum possible shear stress. Discuss the results – similarities and differences.	5