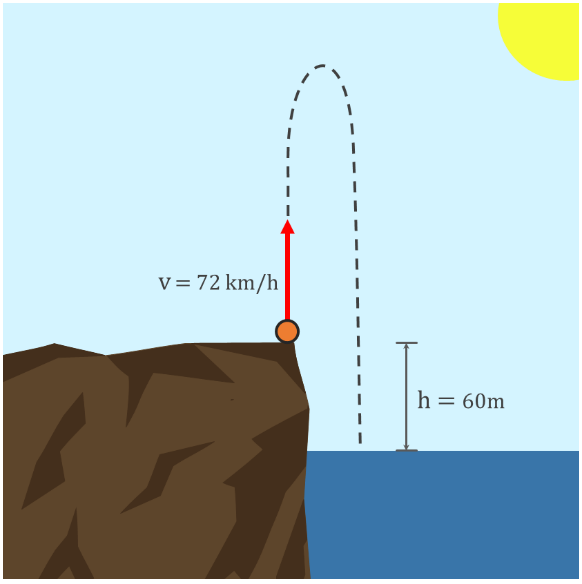
Ball Thrown from Cliff



A ball is thrown vertically up with a velocity of 72 km/h at the edge of a 60 meter high cliff. What is the maximum height the ball reaches with respect to the cliff? Neglect air resistance and take g = 10 m/s^2.
Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a\int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (a \cdot t + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t (a \cdot t + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

For the vertical displacement in y-direction, this results in:

$$y(t) = \frac{1}{2}a_y \cdot t^2 + v_{y,0} \cdot t + s_{y,0} \tag{7}$$

Given:
Initial velocity in y-direction: $v_{y,0} = 72km/h = 20m/s$
Initial height of the ball (with respect to the cliff): $H_0 = s_{y,0} = 0m$
Gravitational constant: $g = 10m/s^2$

At the instant the ball reaches its maximum height its velocity is zero. The acceleration on the ball is the gravitational acceleration: $a_y = -g$. Combining this into Equation 3 yields an equation for the times until the ball reaches it maximum point:

$$v_y(t) = a_y \cdot t + v_{y,0} \quad \Rightarrow \quad 0 = -g \cdot t + v_{y,0} \tag{8}$$

Rewriting gives:

$$t = \frac{-v_{y,0}}{-g} = \frac{20}{10} = 2s \tag{9}$$

Inserting t = 2s into Equation 7 yields:

$$H_{max} = y(2) = -\frac{1}{2} \cdot 10 \cdot 2^2 + 20 \cdot 2 = 20m \tag{10}$$