Energy & Heat Transfer

Lecture 4

By: Mohammad Mehrali

Study Materials

- **Slides**
- **Book : (Y. A. Cengel& A. J. Ghajar. Heat and Mass Transfer: Fundamental & Application)**
- **Heat Quiz**
- **Learning activities**

Recap of lecture 3

Forced and natural/free convection:

$\dot{Q} = h A \Delta T$ $\dot{q} = h \Delta T$ Newton's cooling law

Nusselt Number: dimensionless variety of heat transfer coefficient *h*

$$\mathrm{Nu} = \frac{h L_c}{k}$$

With L_c a characteristic length for the considered geometry

Determining the Nusselt Number: (empirical) correlations

- Forced convection: Nu as function of Re, Pr : Nu = *f*(Re, Pr)
- Natural convection: This lecture

$\Rightarrow$ Relation Nu, Re, Pr dependent on geometry and flow regime (laminar/turbulent)

$\Rightarrow$ Nu, Re, Pr dimensionless numbers: reduction number of variables

Recap of lecture 3

Convection Resistance :

$$\dot{Q} = h\,A\,\Delta T = \frac{\Delta T}{R_{conv}} \quad \text{met} \quad R_{conv} = \frac{1}{hA} \quad \text{(K/W)}$$

Dimensionless Numbers :

Nusselt Number: $$\mathrm{Nu} = \frac{hL_c}{k}$$

Reynolds Number: $$\mathrm{Re} = \frac{\rho U L_c}{\mu}$$

Prandtl Number: $$\mathrm{Pr} = \frac{\mu c_p}{k}$$

+ Background on boundary layers

Recap of lecture 3

If $\dot{Q}$ must be found:

- Calculate at film temperature : $T_f = \dfrac{T_s + T_\infty}{2}$
- Pull out ingredients like *μ*, *ρ*, *k*, Pr from tables – like assignment bundle: air or given fluid) at $T_f = \dfrac{T_s + T_\infty}{2}$

- Calculate Re and choose appriopratе correlation based on geometry and Re
- Calculate Nu
- Derive *h* from it
- Fill out Newton's cooling

law: $\dot{Q} = hA\Delta T$

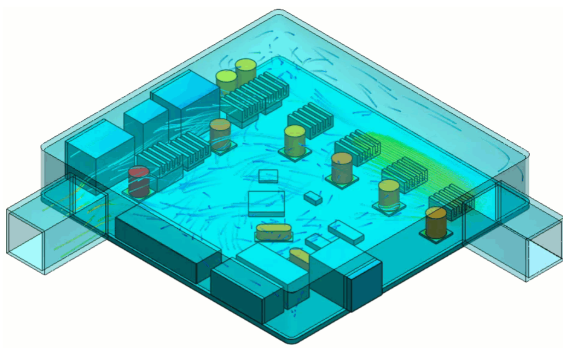
Learning objectives lecture 4

Heat transfer through natural convection

- **Natural Convection**
- Grashof Number
- Rayleigh Number
- Nusselt Number
- Using correlations for various configurations
- Calculating natural convection with step-by-step plan

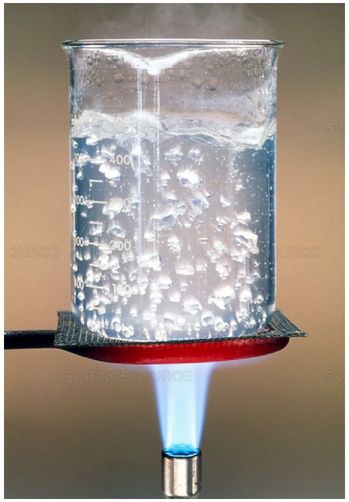
TYPES OF CONVECTION (FROM LECTURE 3)

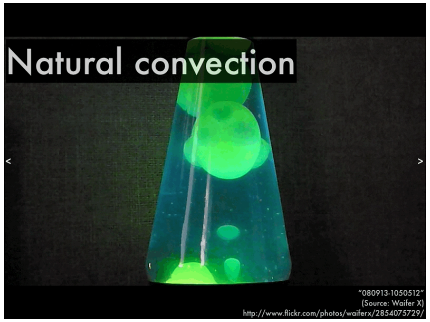
Forced convection



Imposed flow (by pump, fan, …)

Natural/free convection





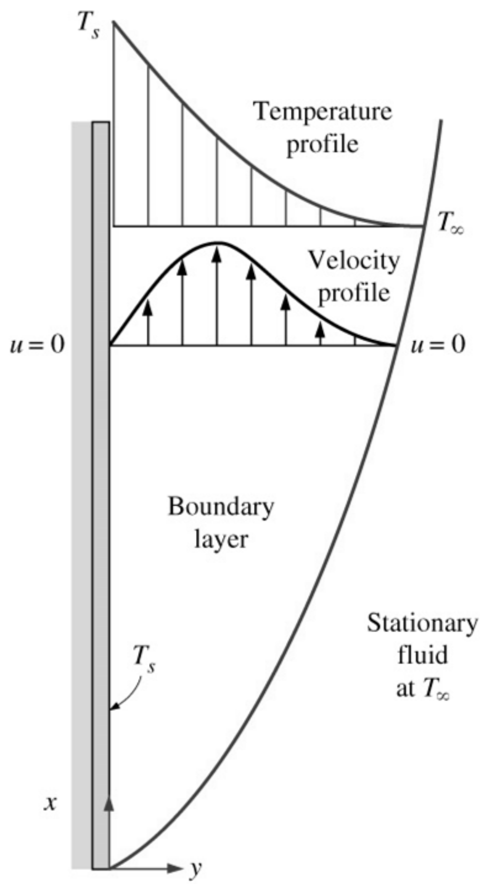
Temperature difference itself starts the flow

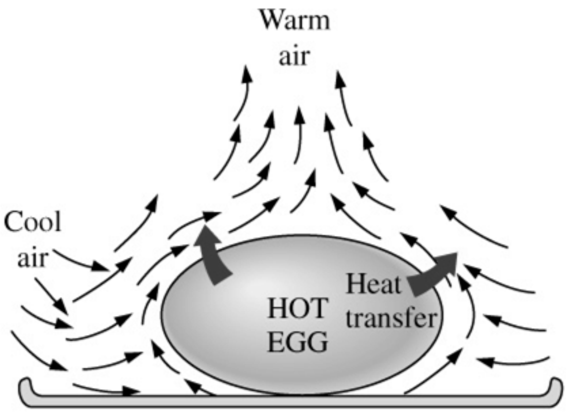
General: flow velocity and heat transfer rates are larger for forced convection

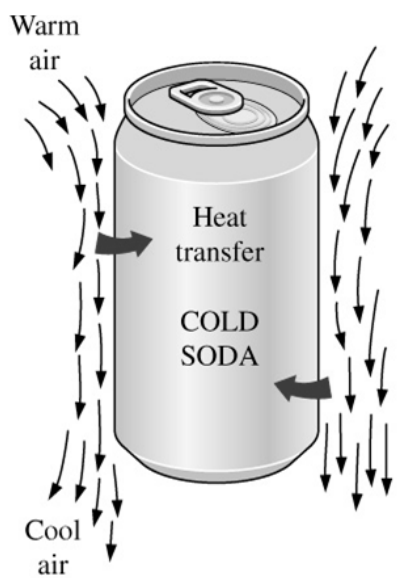
Natural Convection

Also for natural convection velocity and thermal boundary layers!

- Velocity boundary layer is only area in which flow occurs
- **Hot** surface: **Upward flow**
- **Cold** surface: **Downward flow**







Convection Currents

Learning objectives lecture 4

Heat transfer through natural convection

- Natural Convection
- **Grashof Number**
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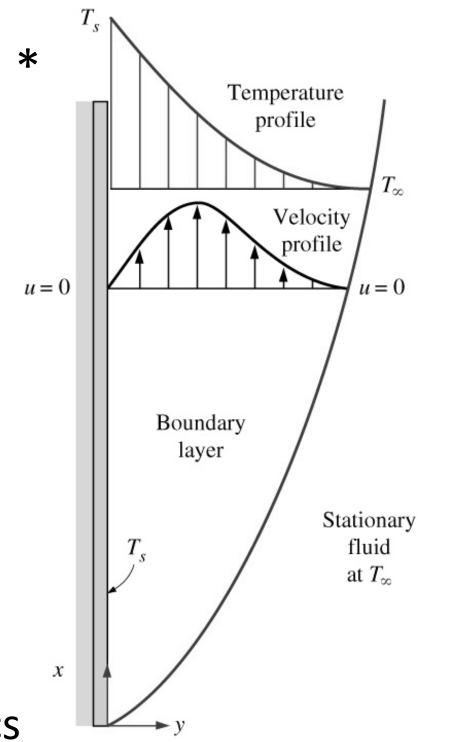
Grashof Number

Forced convection: velocity U imposed

Natural convection: velocity follows from temp.difference T_s - T_∞ *

$\Rightarrow$ Alternative for Reynolds number, with temp.diff. instead U:



Grashof number: $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (-)$

Greek letter "nu"

The **Grashof number (Gr)** is a **dimensionless** number in fluid dynamics and heat transfer which approximates the **ratio of** the **buoyancy** to **viscous force** acting on a fluid.

*Choose ΔT positive for convenience

Grashof Number

Grashof number: $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2} \quad (-)$

- Gravitational acceleration : **$g = 9{,}81\ m/s^2$**
- Volume expansion coefficient ***β* (K^{-1}); for most gases:** $\beta = \dfrac{2}{T_s + T_\infty}$

  (temperature in Kelvin; 0 ˚C = 273,15 K)

- Length **L_c** characteristic for geometry **(length *L* for plate, diameter *D* for sphere/cylinder)**
- Kinematic viscosity : $\nu = \dfrac{\mu}{\rho}\,(m^2/s)$ at avg. Temp: $T_f = \dfrac{T_s + T_\infty}{2}$

Learning objectives lecture 4

Heat transfer through natural convection

- Natural Convection
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- **Rayleigh Number**
- Nusselt Number
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Rayleigh Number

Laminar / turbulent:

↔

Forced convection:
Determined by Reynolds number : **Re**

Natural convection:
Determined by Grashof number **Gr**

Often combined with Prandtl number:
Rayleigh number **Ra = Gr · Pr**

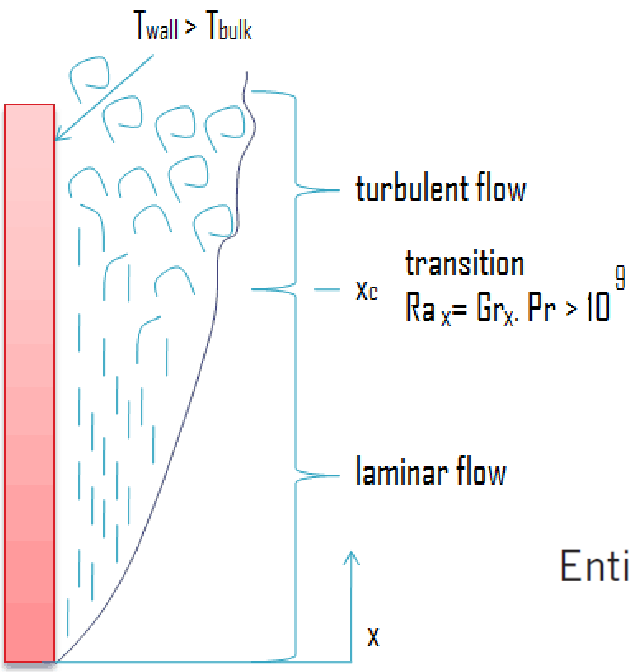
Rayleigh Number : $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$

Rayleigh Number is a dimensionless number associated with buoyancy-driven flow, also known as free or natural convection

Rayleigh Number

Rayleigh Number : $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$

Vertical, flat plate (ex. radiator)



$\mathrm{Nu} = 0{,}59\ \mathrm{Ra}_L^{\ 1/4}$ with $L_c = L$ $(10^4 < \mathrm{Ra}_L < 10^9)$

$\mathrm{Nu} = 0{,}1\ \mathrm{Ra}_L^{\ 1/3}$ with $L_c = L$ $(10^9 < \mathrm{Ra}_L < 10^{13})$

$10^9 < \mathrm{Ra}_L < 10^{10}$: find intermediate

Entire range ⟹ $\mathrm{Nu} = \left\{0.825 + \dfrac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$

(complex but more accurate)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Learning objectives lecture 4

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NUSSELT NUMBER

Forced Convection

$$\frac{hL}{k} = a\left(\frac{\rho U L}{\mu}\right)^b \left(\frac{\mu c_p}{k}\right)^c$$

$$\mathrm{Nu} = a \cdot \ \mathrm{Re}^b \cdot \ \mathrm{Pr}^c$$

Natural Convection

Constant coefficient

$$\mathrm{Nu} = C\,\mathrm{Ra}_L^n$$

Constant exponent

Nusselt number

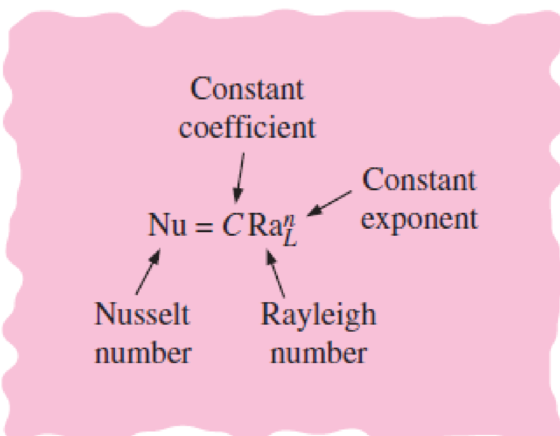
Rayleigh number

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$$

NUSSELT NUMBER

$$\text{Rayleigh Number} : \mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\;3}}{\nu^2}\mathrm{Pr}$$

Rayleigh number, which is the product of the Grashof and Prandtl numbers



The values of the constants C and n depend on the

- ✓ geometry of the surface
- ✓ the flow regime

The value of n is usually

- ✓ 0,25 for laminar flow
- ✓ 0,33 for turbulent flow.

The value of C

- ✓ is normally less than 1.

All fluid properties are to be evaluated at the film temperature Tf =$(T_s+T_{inv})/2$.

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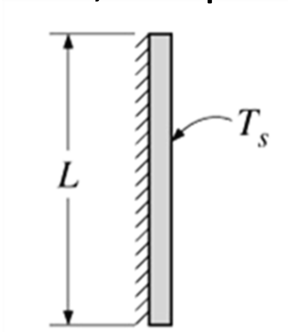
Free Convection Correlations

Vertical, flat plate

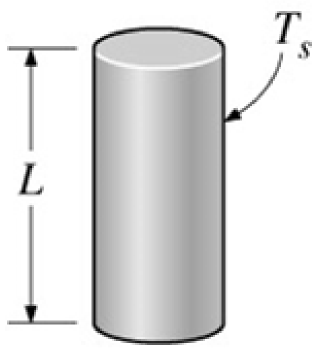


Nu = 0,59 $\mathrm{Ra}_L^{1/4}$ with $L_c = L$ ($10^4 < \mathrm{Ra}_L < 10^9$)

Nu = 0,1 $\mathrm{Ra}_L^{1/3}$ with $L_c = L$ ($10^{10} < \mathrm{Ra}_L < 10^{13}$)

Entire range ⟹ $$\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

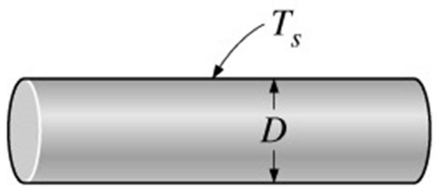
(complex but more accurate)



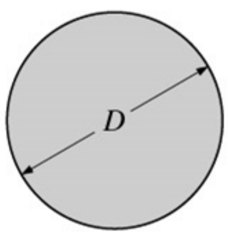
Vertical cylinder (e.g. can)

Treat a vertical plate with height L and surface πDL, if diameter D sufficiently large with respect to L ($D \geq \frac{35L}{Gr_L^{1/4}}$)

Horizontal cylinder (e.g. pipe)



$$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$



$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$$

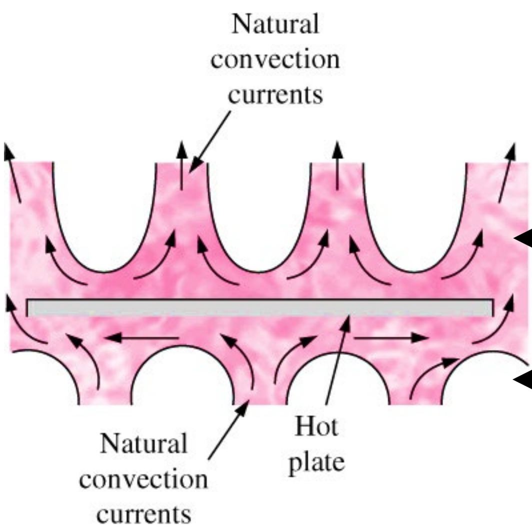
Sphere (e.g. light bulb)

Notation:
Ra_L is Ra with $L_c = L$;
Ra_D is Ra with $L_c = D$

Free Convection Correlations

Horizontal plate: upward / downward flow perpendicular to surface→ plumes instead of adjacent boundary layer

Hot surface



Fluid away from surface

Effective heat transfer

$Nu = 0{,}54\ Ra^{1/4}$ $(10^4 < Ra < 10^7)$

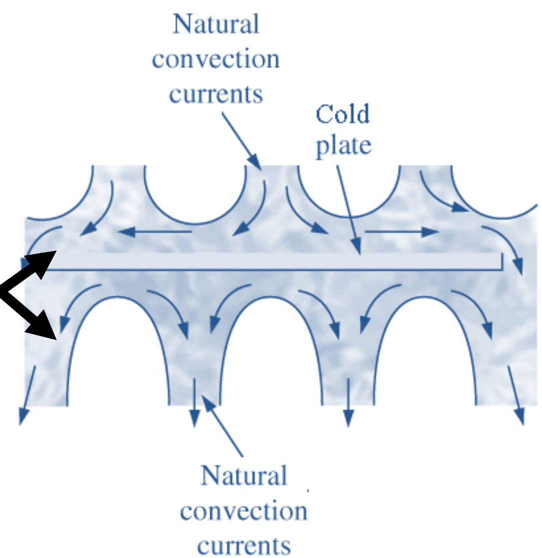
$Nu = 0{,}15\ Ra^{1/3}$ $(10^7 < Ra < 10^{11})$

Fluid towards surface

Reduced heat transfer

$Nu = 0{,}27\ Ra^{1/4}$ $(10^5 < Ra < 10^{11})$

Cold surface



N.B.: Characteristic length

So actually: $Ra_{A/p}$

$$L_c = \frac{\text{Area}}{\text{Perimeter}} = \frac{A}{p}$$

Free Convection Correlations

Plate at angle $\boldsymbol{\theta}$ with vertical axis

($\boldsymbol{\theta}$ < 60º)

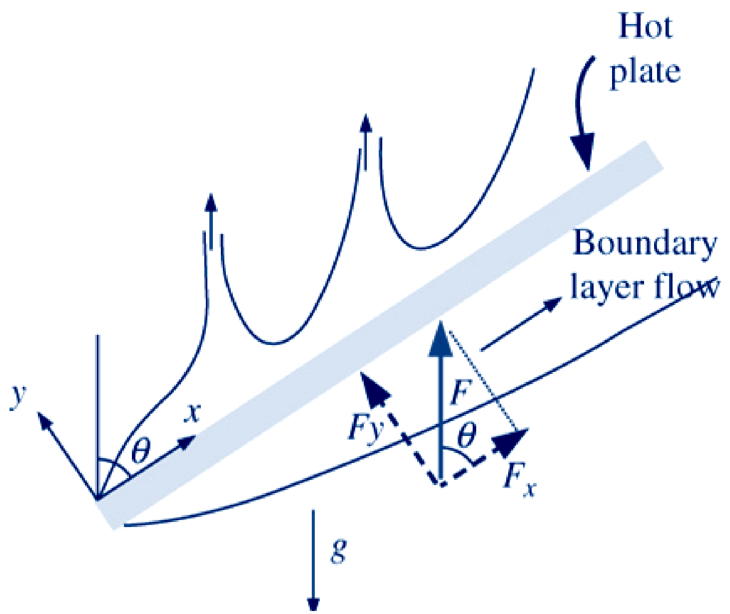


Plate at angle θ (ex. radiator in attic room)

<u>Hot surface:</u>

- Top side: advanced methods
- Bottom side: same as vertical plate, except for using g cos θ instead of g:

$$\mathrm{Ra}_L = \mathrm{Gr}_L \cdot \mathrm{Pr} = \frac{g\cos\theta\,\beta(T_s - T_\infty)L^3}{\nu^2}\cdot \mathrm{Pr}$$

<u>Cold surface:</u> everything upside down

- Bottom side: advanced methods
- Top side: same as vertical plate, except for using g cos θ instead of g

Vertical plate:

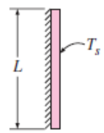
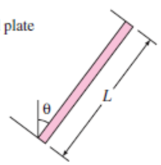
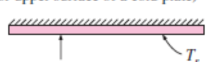
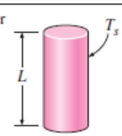
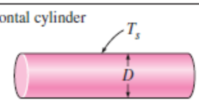
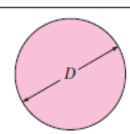
Nu = 0,59 $\mathrm{Ra}_L^{1/4}$ (10^4 < RaL < 10^9)

Nu = 0,1 $\mathrm{Ra}_L^{1/3}$ (10^{10} < RaL < 10^{13})

Free Convection Correlations

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20) $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $\text{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \dfrac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \dfrac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)

Learning objectives lecture 4

Heat transfer through natural convection

- Natural Convection
- Grashof Number
- Rayleigh Number
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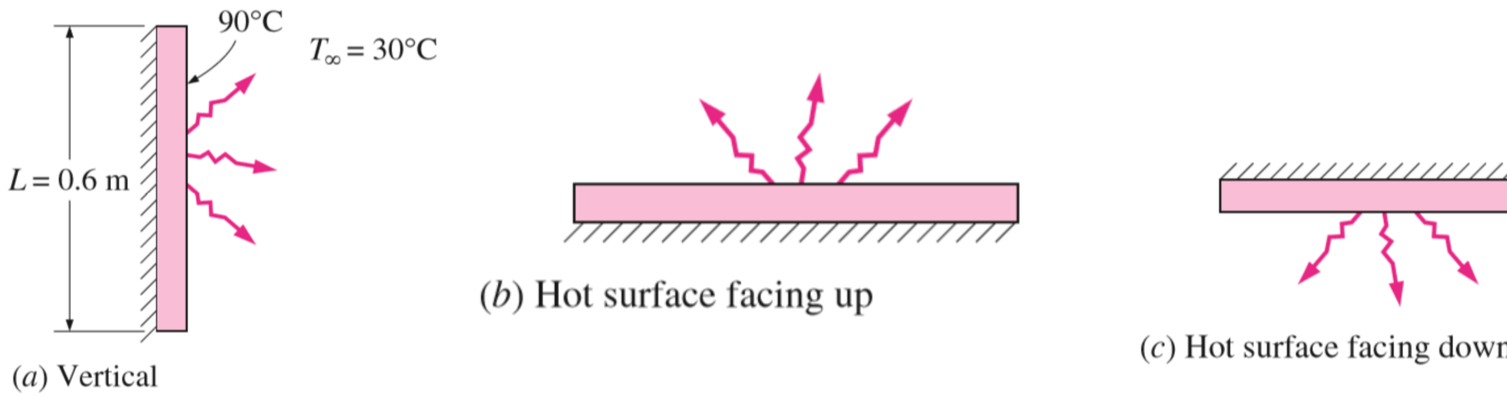
- **Calculating natural convection with step-by-step plan**

Example 1

Cooling of a Plate in Different Orientations

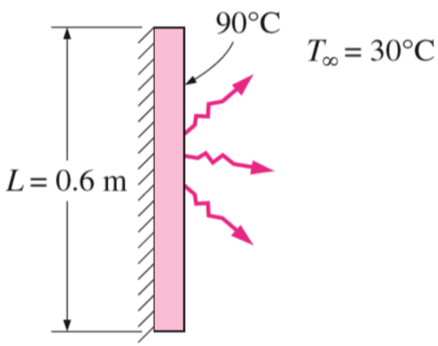


- A: 0.6-m x 0.6-m thin square plate
- room temperature: 30˚C.
- One side of the plate is maintained at a temperature of 90˚C, while the other side is insulated
- Determine the rate of heat transfer from the plate by natural convection if the plate is
  - ➢ (a) vertical,
  - ➢ (b) horizontal with hot surface facing up, and
  - ➢ (c) horizontal with hot surface facing down.

Properties of air (in the back of assigment bundle):

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m^2/s^2	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946

Example 1



(*a*) Vertical

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate (L, T_s)	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	(9-19) (9-20) (9-21)
Inclined plate (θ, L)	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $Ra < 10^9$	
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate) Hot surface, T_s (*b*) Lower surface of a hot plate (or upper surface of a cold plate) Hot surface, T_s	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$ $Nu = 0.27Ra_L^{1/4}$	(9-22) (9-23) (9-24)
Vertical cylinder (L, T_s)	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$	
Horizontal cylinder (T_s, D)	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$	(9-25)
Sphere (D)	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$	(9-26)

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°C$ and 1 atm are (Table A-15)

$$k = 0.02808 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7202$$

$$\nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333 \text{ K}}$$

Analysis (*a*) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr}$$

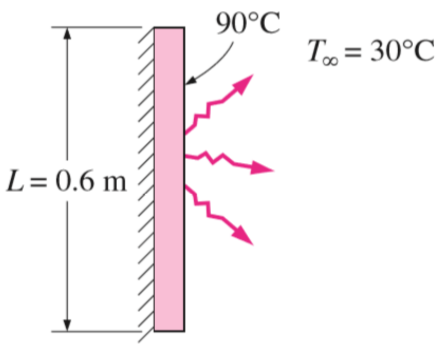
$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.722) = 7.656 \times 10^8$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\text{Nu} = \left\{0.825 + \frac{0.387 \text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4$$

Example 1



(*a*) Vertical

Note that the simpler relation Eq. 9-19 would give $Nu = 0.59\ Ra_L^{1/4} = 98.14$, which is 13 percent lower. Then,

$$h = \frac{k}{L} Nu = \frac{0.02808\ \text{W/m} \cdot °\text{C}}{0.6\ \text{m}} (113.4) = 5.306\ \text{W/m}^2 \cdot °\text{C}$$

$$A_s = L^2 = (0.6\ \text{m})^2 = 0.36\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306\ \text{W/m}^2 \cdot °\text{C})(0.36\ \text{m}^2)(90 - 30)°\text{C} = \mathbf{115\ W}$$

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

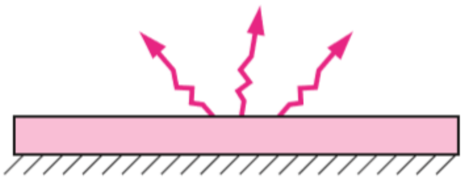
Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	(9-19) (9-20) (9-21)
Inclined plate	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $Ra < 10^9$	
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate) (*b*) Lower surface of a hot plate (or upper surface of a cold plate)	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$ $Nu = 0.27Ra_L^{1/4}$	(9-22) (9-23) (9-24)
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$	
Horizontal cylinder	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$	(9-25)
Sphere	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$	(9-26)

Vertical plate:

$Nu = 0{,}59\ Ra_L^{1/4}$ $(10^4 < Ra_L < 10^9)$

$Nu = 0{,}1\ Ra_L^{1/3}$ $(10^9 < Ra_L < 10^{13})$

Example 1



(b) Hot surface facing up

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate (L, T_s)	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	(9-19) (9-20) (9-21)
Inclined plate (θ, L)	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$	
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) (Hot surface, T_s) (b) Lower surface of a hot plate (or upper surface of a cold plate) (Hot surface, T_s)	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ $\text{Nu} = 0.27\text{Ra}_L^{1/4}$	(9-22) (9-23) (9-24)
Vertical cylinder (L, T_s)	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$	
Horizontal cylinder (T_s, D)	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \dfrac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$	(9-25)
Sphere (D)	D	$\text{Ra}_D \leq 10^{11}$ ($\text{Pr} \geq 0.7$)	$\text{Nu} = 2 + \dfrac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$	(9-26)

(b) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\ \text{m}}{4} = 0.15\ \text{m}$$

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\text{Pr}$$

$$= \frac{(9.81\ \text{m/s}^2)[1/(333\ \text{K})](90 - 30\ \text{K})(0.15\ \text{m})^3}{(1.896 \times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$\text{Nu} = 0.54\ \text{Ra}_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

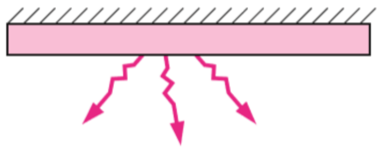
$$h = \frac{k}{L_c}\text{Nu} = \frac{0.0280\ \text{W/m} \cdot {}^\circ\text{C}}{0.15\ \text{m}}(31.76) = 5.946\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

$$A_s = L^2 = (0.6\ \text{m})^2 = 0.36\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{128\ W}$$

Example 1



(c) Hot surface facing down

(c) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (b). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$\mathrm{Nu} = 0.27\,\mathrm{Ra}_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate	L	10^4–10^9 10^9–10^{13} Entire range	$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$ (9-19) $\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3}$ (9-20) $\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\mathrm{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) (b) Lower surface of a hot plate (or upper surface of a cold plate)	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\mathrm{Nu} = 0.54\mathrm{Ra}_L^{1/4}$ (9-22) $\mathrm{Nu} = 0.15\mathrm{Ra}_L^{1/3}$ (9-23) $\mathrm{Nu} = 0.27\mathrm{Ra}_L^{1/4}$ (9-24)
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\mathrm{Gr}_L^{1/4}}$
Horizontal cylinder	D	$\mathrm{Ra}_D \leq 10^{12}$	$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere	D	$\mathrm{Ra}_D \leq 10^{11}$ ($\mathrm{Pr} \geq 0.7$)	$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1+(0.469/\mathrm{Pr})^{9/16}]^{4/9}}$ (9-26)

$$h = \frac{k}{L_c}\mathrm{Nu} = \frac{0.02808\ \mathrm{W/m\cdot{}^\circ C}}{0.15\ \mathrm{m}}(15.86) = 2.973\ \mathrm{W/m^2\cdot{}^\circ C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973\ \mathrm{W/m^2\cdot{}^\circ C})(0.36\ \mathrm{m^2})(90-30)^\circ\mathrm{C} = \mathbf{64.2\ W}$$

Step-by-step plan natural convection

If $\dot{Q}$ must be found:

- Determine ingredients necessary for dimensionless no. (Pr, *k* and *v* at average temperature $\frac{T_s + T_\infty}{2}$)
- Determine Ra: $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr}$
- Choose appropriate correlation based on geometry and Ra
- Determine Nu
- Resolve *h* from it
- Fill out Newton's cooling law: $\dot{Q} = hA\Delta T$

Example 2

How large is the heat flow?

Surrounding temperature

$T_\infty = 20\ ^\circ C$

$T_s = 220\ ^\circ C$

$L_1 = 0{,}20$ m

$L_2 = 0{,}35$ m

Natural convection

Example 2

L_1 = 0,20 m
L_2 = 0,35 m
T_s = 220 ˚C
T_∞ = 20 ˚C

Horizontal, flat, warm plate:

Nu = 0,54 $Ra^{1/4}$ (10^4 < Ra < 10^7)

Nu = 0,15 $Ra^{1/3}$ (10^7 < Ra < 10^{11})

Characteristic length $L_c = A/p$

$$\mathrm{Ra} = \mathrm{Gr}\cdot\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$$

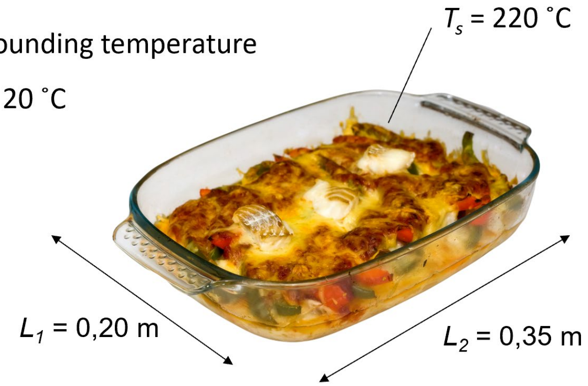
Properties of air (in the back of assignment module):

Temp. T, °C	Density ρ, kg/m³	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m²/s²	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m²/s	Prandtl Number Pr
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946

Example 2

(Intermediate) answers

- Average temperature: 120 ˚C
- Ra = 1,45·10^6
- Nu = 18,7
- h = 9,48 W/(m^2·K)
- Heat flow 133 W (around 1/3 forced convection: 462 W)

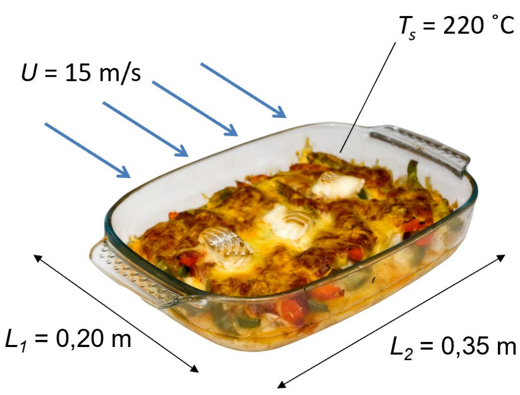


Blowing from long side

- h = 33 W/(m^2·K)
- Heat flow 462 W

Blowing from short side:

- h = 24,9 W/(m^2·K)
- Heat flow 349 W



Example 3

Imagine: 25 W light bulb with diameter D = 0,08 m releases 22,5 W of heat. Determine T_s for T_∞ = 20 ˚C

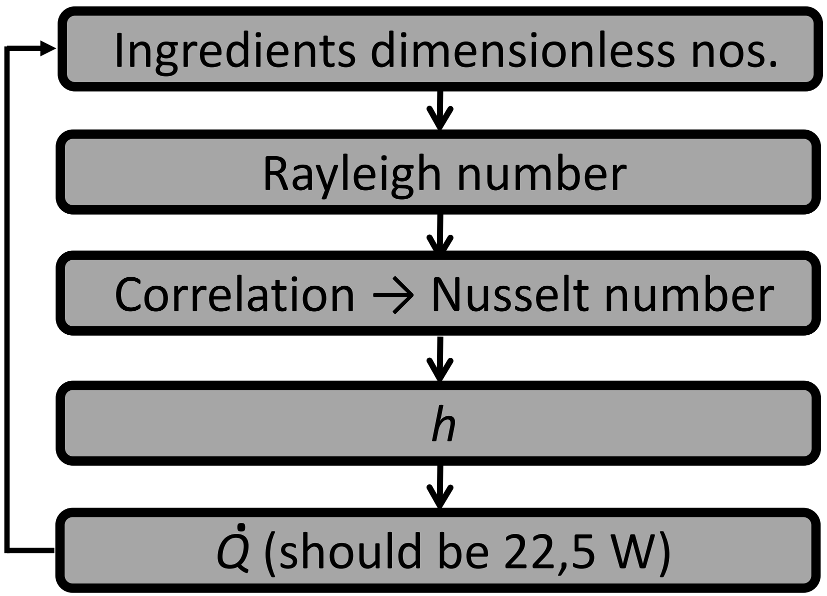
Problem:

To calculate T_s ,h is necessary ($\dot{Q} = hA\left(T_s - T_\infty\right)$) but h depends on T_s (because Nu, Ra, Pr are dependent on temperature) and that is the one we are looking for…

$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$$

Example 3

Problem: T_s unknown → starting values for step-by-step plan unknown (Pr, k, ν, …)



At what avg. temp? $\frac{T_s + T_\infty}{2}$?

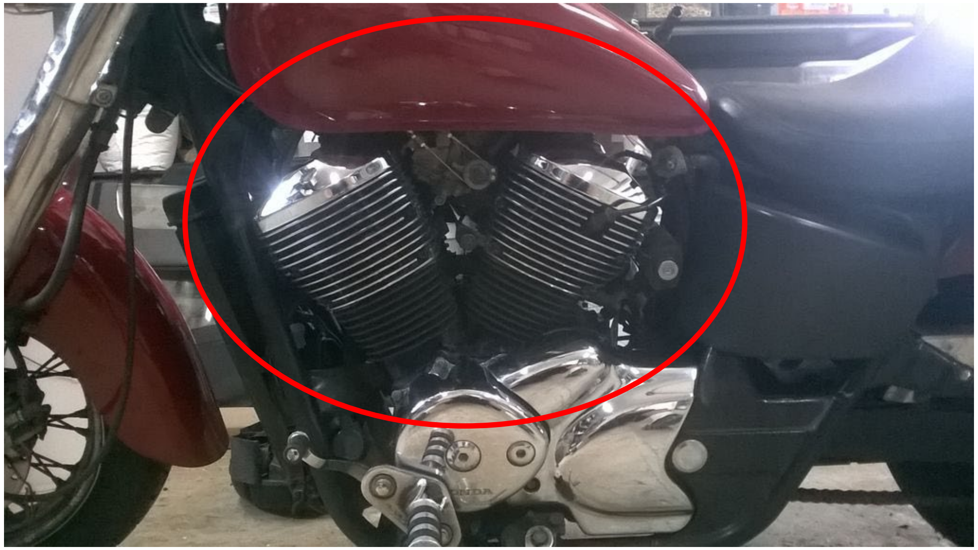
Unknown

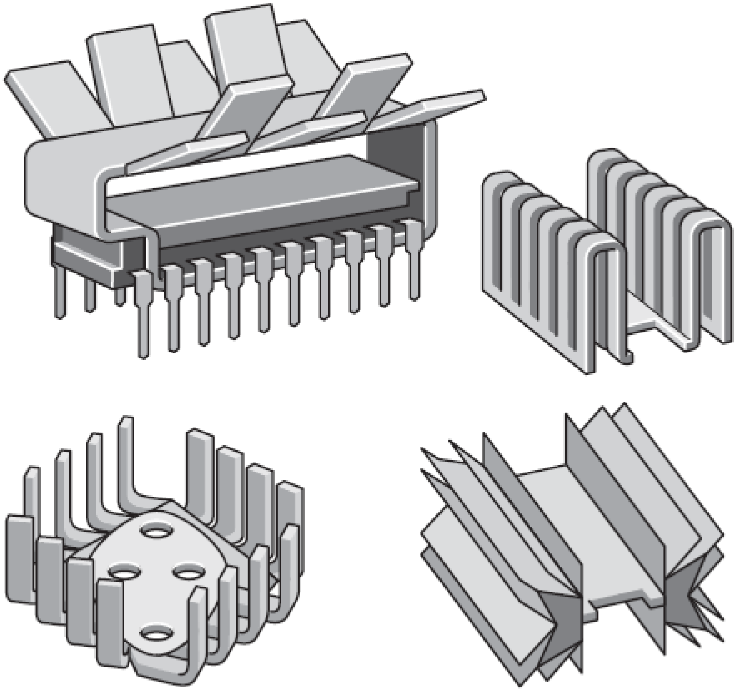
<u>Approach: iterative solving</u>

- Choose an estimated T_s to start with
- Follow steps and check whether T_s yields the right heat flow
- Resulting heat flow ≠ 22,5 W?Adjust estimated T_s and repeat all steps till you find a T_s that (approximately) yields $\dot{Q}$ = 22,5 W

Fins

What is Purpose of the Fins on Engine Surfaces ? Why design such a thing ?





Fins are extended surfaces and they increase the surface area leading to increase in heat transfer rate. Useful in cooling applications

Summary natural convection

General (same as for forced convection)

$$\dot{Q} = hA\Delta T \quad \text{(W)}$$

Newton's cooling law

"Supporting" equations for *h*:

Nusselt number Nu as function of Rayleigh number Ra = Gr · Pr

Grashof number $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\;3}}{\nu^2}$

- Forced convection: Re ↔ natural convection: Gr
- Ra and Nu just like Re based upon characteristic length of geometry (indicate using subscript)

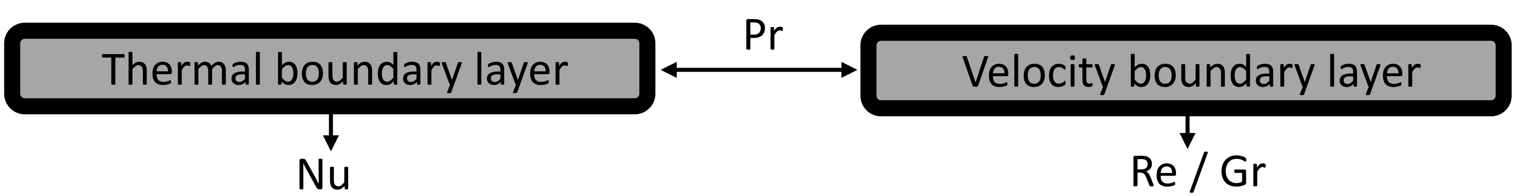
General conclusion convection

- Calculate heat flow using Newton's cooling law
- Determine heat transfer coefficient *h* in this using correlations between Nu (→ *h*) and other dimensionless nos.

⇒ step-by-step plans (+ iterative solving)

Other learning objectives convection: *flow phenomena*

- Being able to sum up what parameter are influencing *h*
- Knowing differences between laminar and turbulent ,
- Explaining how velocity and thermal boundary layers form
- Predicting/reasoning the temperature development



Feedback Session

- Next Lectorial : Natural Convection
- Questions? Feel free to ask!