5.8 Exercises

Problem 5.1. *Incompressible viscous oil flows steadily between stationary parallel plates. The flow is laminar and fully developed. The total gap width between the plates is h. The oil viscosity is μ and the pressure drop over a distance L is Δp.*

(a) Derive an expression for the shear stress on the upper plate.
(b) Derive an expression for the volume flow rate through the channel over a width w.
(c) Compute the shear stress on the upper plate and the volume flow rate through the channel over a width w if $h = 5\,mm$, $\Delta p = -1000\,Pa$, $L = 1m$, $\mu = 0.5\,Ns/m^2$.

Hint: *first derive an expression for the velocity field starting from the reduced Navier-Stokes equations.*

Problem 5.2. *An incompressible fluid of density ρ flows steadily between two parallel plates. The flow is laminar and fully developed, the viscosity is μ, the mean velocity is U, and the distance between the plates is h. Divide the flow into two horizontal layers, with the divide located at a distance y above the lower plate. Derive an expression for the shear stress experienced by the lower layer as a function of y, and sketch this function.*

Problem 5.3. *An hydraulic jack (NL: krik) supports a load of mass M. The diameter of the piston is D, the radial clearance between the piston and the cylinder is d, the length of the piston is L, and the viscosity of the oil is μ.*

(a) Derive an expression for the pressure-drop in the gap between the piston and the cylinder.
(b) Derive a formula for the rate of leakage of hydraulic fluid past the piston. Compute the leakage rate when $M = 9000\,kg$, $D = 100\,mm$, $d = 0.05\,mm$, $L = 120\,mm$, and $\mu = 2 \times 10^{-1}\,Ns/m^2$.

Hint: *approximate the gap between the piston and the cylinder as the gap between two flat plates (why would this be a very good approximation?). First compute the vertical pressure derivative by assuming the piston to be in equilibrium (moves extremely slowly due to the leakage).*

Problem 5.4. *Consider the steadily falling water film along a vertical wall with thickness a. The flow is incompressible, laminar, and fully developed. At the wall the velocity is zero, whereas at the outer edge of the film the shear stress is zero.*

(a) Defend the approximation assumption of zero shear stress at the film boundary.
(b) Derive an expression for $\frac{\partial p}{\partial x}$.
(c) Derive an expression for $u(y)$.

Problem 5.5. *An incompressible fluid flows steadily between two parallel plates. The flow is laminar and fully developed. The total gap width between the plates is h. The upper plate moves to the right with speed U_2, the lower plate moves to the left with speed U_1. The pressure gradient in the direction of the flow is zero.*

(a) Derive an expression for the velocity distribution in the gap.
(b) Derive an expression for the volume flow rate per unit depth.

Problem 5.6. *An incompressible fluid flows steadily between two parallel plates. The flow is laminar and fully developed. The total gap width between the plates is h. The upper plate moves to the right with speed U, the lower plate is fixed.*

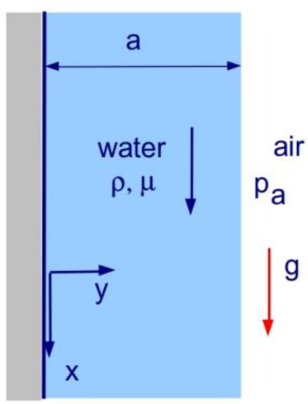
(a) Derive an expression for the pressure gradient at which the upper plate experiences zero shear stress.
(b) Derive an expression for the pressure gradient at which the lower plate experiences zero shear stress.

Problem 5.7. *The record-read head for a computer disk-drive memory storage system rides above the spinning disk on a very thin film of air (the film thickness is h). The head location is a from the disk centerline; the disk spins at angular velocity Ω. The surface area of the record-read head is A. Finally, the viscosity of air is μ and the density is ρ.*

(a) Derive an expression for the Reynolds number of the flow.
(b) Derive an expression for the shear stress.
(c) Derive an expression for the power required to overcome the viscous shear stress.
(d) Compute the values of the three expressions if $h = 0.5\,\mu m$, $a = 150\,mm$, $\Omega = 3600\,rpm$, and $A = 100\,mm^2$, $\mu = 18.0 \times 10^{-6}\,kg/ms$, and $\rho = 1.2\,kg/m^3$.

Problem 5.8. *Consider fully developed laminar incompressible flow in a pipe.*

(a) Derive an expression for the average velocity in a cross-section.
(b) Transform the previous expression to obtain a formula for the pressure gradient as a function of (amongst others) the average velocity.



(a) Problem 5.4

6.6 Exercises

Problem 6.1. *Determine the dimensions of force F, stress σ, power $\dot{W}$, dynamic viscosity μ and thermal conductivity k.*

Problem 6.2. *The variables which control the motion of a boat are the resistance force, F, speed V, length L, density of the liquid ρ and its viscosity μ, as well as gravity acceleration g. Obtain an expression for F using dimensional analysis.*

Problem 6.3. *It is believed that the power P of a fan depends upon the density of the liquid ρ, the volumetric flux Q, the diameter of the propeller D and the angular speed Ω. Using dimensional analysis, determine the dependence of P with respect to the other dimensionless variables.*

Problem 6.4. *In fuel injection systems, a jet of liquid breaks, forming small drops of fuel. The diameter of the resulting drops, d, supposedly depends upon the density of the liquid ρ, the viscosity μ, surface tension σ (force/length), and also upon the speed of the stream V and its diameter D. How many dimensionless parameters are required to characterize the process? Find them.*

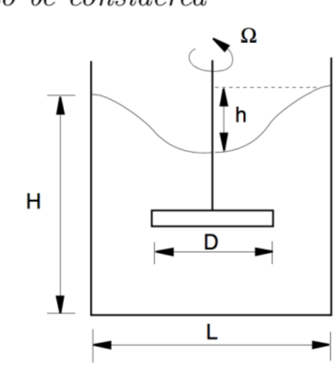
Problem 6.5. *A disc spins close to a fixed surface. The radius of the disc is R, and the space between the disk and the surface is filled with a liquid of viscosity μ. The distance between the disc and the surface is h and the disc spins at an angular velocity ω. Determine the functional relationship between the torque that acts upon the disc, T, and the other variables.*

Problem 6.6. *The drag force, F, experienced by a submarine that moves at a great depth from the surface of the water, is a function of the density ρ, viscosity μ, speed V and the transversal area of the submarine A. An expert suggests that the nondimensional relationship that allows the calculation of F is: $F = f(\frac{\rho V A}{\mu})\rho V^2 A$.*

(a) *Is the number of dimensionless parameters in the expression correct? Why?*
(b) *Are the parameters correct? If not, correct them.*
(c) *A geometrically similar model to that of the real submarine has been constructed, so that all the lengths of the model are 1/10 of those corresponding to the submarine. The model is tested in sea water. (1) The force of the real submarine when it moves at 5 m/s is to be de- termined. (2) At which speed should the model be tested?*

Problem 6.7. *An automobile must travel through standard air conditions at a speed of $100 km/h$. To determine the pressure distribution, a model at a scale of 1/5 of the length of the vehicle is tested in water. Find the speed of water to be used.*
$\mu_{water} = 10^{-3} kg/(ms)$, $\rho_{water} = 1000 kg/m^3$, $\mu_{air} = 1.8 \times 10^{-5} kg/(ms)$, $\rho_{air} = 1.2 kg/m^3$.

Problem 6.8. *The depth of the steady central vortex h in a large tank of oil being stirred by a propeller needs to be predicted. One way is to carry out a study using a reduced scale model. Determine the conditions under which the experiment should be conducted to be considered a valid predictive tool. Note: Consider h a function of g, H, D, L and Ω.*

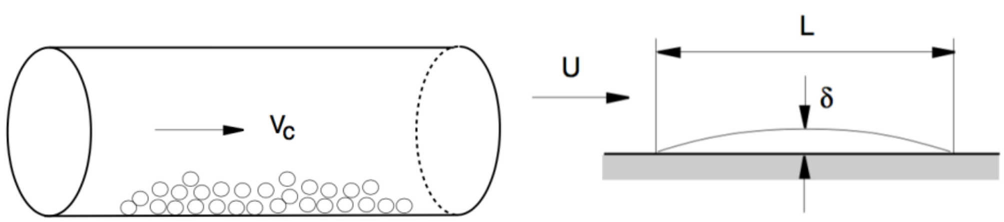


Problem 6.9. *A rectangular, thin, flat plate, with length h and width w is placed perpendicularly to a liquid flow. Imagine that the drag force D which the liquid has upon the plate is a function of w and h, the density ρ, the viscosity μ, and the speed V of the liquid coming towards the plate. Determine the set of dimensionless parameters to study the problem experimentally.*

Problem 6.10. *The Reynolds number is a very important parameter for studying transport phenomena and fluid mechanics. Estimate the Reynolds number that would be characteristic of the flow around a car traveling along the highway.*
Note: $\rho_{air} \approx 1.25\,kg/m^3$, $\mu_{air} \approx 1.8 \times 10^{-5} Pa\,s$.

Problem 6.11. *A thin layer of spherical particles are lying at the bottom of a horizontal tube, as indicated in the Figure. When an incompressible liquid flows along the tube, it can be seen that at a certain critical speed the particles move and are carried along the length of the tube. We wish to study the value of this critical speed V_c. Suppose that V_c is a function of the diameter of the tube D, the particle?s diameter D_p, the liquid density ρ, the viscosity of the liquid μ, the density of the particles ρ_p and the gravity acceleration g.*

(a) Using ρ, D and g as fundamental variables, obtain the dimensionless parameters of the problem.
(b) Repeat the first question using ρ, D and μ as fundamental variables.



(a) Exercise 6.11 (b) Exercise 6.12

Problem 6.12. *During the drying process of a fine layer of liquid on a surface, the liquid evaporates and the vapor is transported in the air above the surface. We are interested in knowing the dependence of the drying time t upon the rest of the variables of the problem (length L, thickness of the layer δ, the liquid's vapor pressure p_v, air speed U, viscosity μ and air density ρ).*

(a) Obtain a set of dimensionless variables related to the drying time t with the rest of the variables.
(b) We wish to set up a laboratory experiment to determine the drying time of a soccerfield where $p_v = 2000\,Pa$, $L = 100\,m$, $\delta = 0.01\,m$ and $U = 2\,m/s$. In the experiment, the viscosity and the density of the air will be the same as that of the soccer field, but L will be $20\,m$ (we don't have a larger laboratory available). Calculate the values of U, δ and p_v in the experiment so that complete similarity exists with the real flow.
(c) If in the experiment the average drying time is $t = 10\,min$, calculate the drying time of the soccer field.

7.4 Exercises

Problem 7.1. *Show that* $\frac{\partial \tilde{\Psi}}{\partial \tilde{y}} = \tilde{u}$ *by using the definition of the streamfunction* $\tilde{\Psi}$*, and the definition of its partial derivative:*

$$\frac{\partial \tilde{\Psi}}{\partial \tilde{y}} = \tilde{u} \equiv \lim_{\Delta \tilde{y} \to 0} \frac{\tilde{\Psi}(\tilde{x}, \tilde{y} + \Delta \tilde{y}) - \tilde{\Psi}(\tilde{x}, \tilde{y})}{\Delta \tilde{y}}.$$

Problem 7.2. *Show that* $\frac{\partial \tilde{\Psi}}{\partial \tilde{x}} = -\tilde{v}$ *by using mass conservation and the definition of the streamfunction* $\tilde{\Psi}$

Problem 7.3. *For sufficiently small values of* η*, Blasius' solution can be approximated by a truncated Taylor series:*

$$f(\eta) = \sum_{n=0}^{5} a_n \eta^n$$

(a) Using the boundary conditions at $\eta = 0$*, show that* $a_o = a_1 = 0$*.*

(b) Using Blasius's equation, show that $a_3 = a_4 = 0$ *and that*

$$a_5 = -\frac{1}{60} a_2^2.$$

(c) Compute a_2 *and* a_5 *given that* $f''(0) \approx 0.322$*.*

(d) Compute $f(2)$ *and* $f'(2)$*.*

(e) Plot these values in Fig. (7.4).

Problem 7.4. *Make a sketch of the skin-friction coefficient* C_f *as a function of* $\tilde{x}$*.*

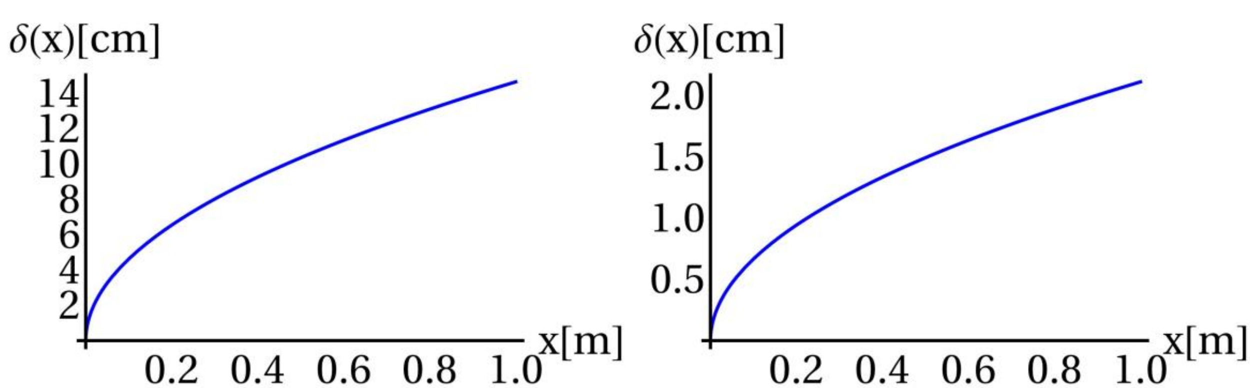


Figure 7.4: Flat plate boundary layer thickness based on Blasius' solution for water (left, $\nu = 894 \times 10^{-6}\, Pa\, s$*) and air (right,* $\nu = 18.6 \times 10^{-6}\, Pa\, s$*) with* $U = 1\, m/s$*, confirming that* $\delta \sim \sqrt{\nu}$*. [Note: the solution for air does not hold beyond a few centimeters in downstream direction because turbulence sets in and the boundary layer would be much thicker.]*