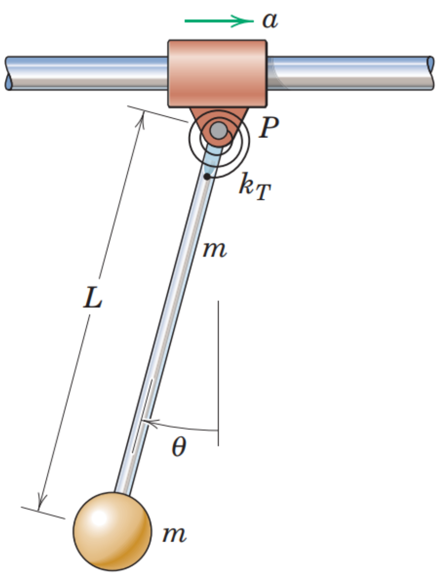
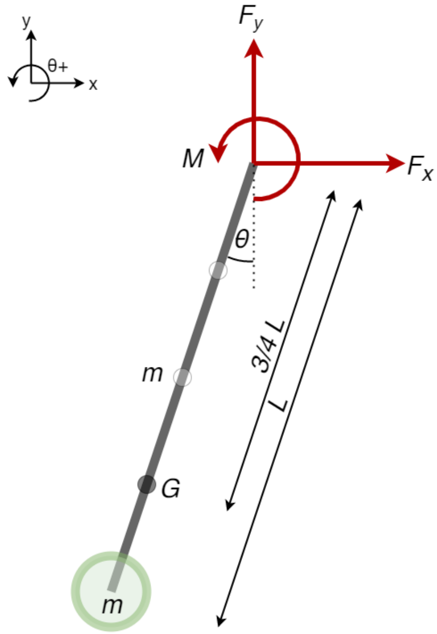


(a) Given problem



(b) FBD

If the collar P is given a constant acceleration $a = 3g$ to the right, the pendulum assumes a steady-state deflection $\theta = 30°$. The masses of the ball and the slender bar connecting element are both equal to m. The torsional spring is undeformed when the pendulum is in the vertical position. Determine the stiffness k_T of the torsional spring.

We start off by making an FBD which includes the bar and ball, as can be seen in Figure (b). The centre of mass of the bar is located at $1/2\ L$ from the hinge, and the ball is a point mass at L. Since both masses are equal, the centre of mass for this system is located a distance of $3/4\ L$ from the hinge. We can now write the 3 equations of motion, taking the moment balance about the centre of mass:

$$\sum F_X = 2m \cdot a_x = 6mg = F_x$$

$$\sum F_Y = 0 = F_y - mg \quad\quad \Rightarrow \quad\quad F_y = mg$$

$$\sum M_G = 0 = k_T\theta + F_y \cdot \frac{3}{4}Lsin(\theta) - F_x \cdot \frac{3}{4}Lcos(\theta)$$

F_x and F_y can now be plugged into the Moment equation:

$$k_T\theta + mg \cdot \frac{3}{4}Lsin(\theta) - 6mg \cdot \frac{3}{4}Lcos(\theta) = 0$$

We know that $\theta = 30° = \frac{1}{6}\pi\ [rad]$. Rearranging and solving for k_T we obtain:

$$k_T = \frac{\frac{9\sqrt{3}-3}{4}mgL}{\frac{1}{6}\pi} = 6.01mgL$$