HEATQUIZ

Formulary Heat Transfer: Radiation

Version 1 from 2021

from 7th June 2021

Black body radiation

$$\dot{q}''_{\lambda,\mathrm{b}} = \frac{c_1 \lambda^{-5}}{\exp\left[c_2/(\lambda T)\right] - 1} \qquad \text{(Planck's distribution law)}$$

$$\dot{q}''_{\mathrm{b}} = \int\limits_{\lambda=0}^{\infty} \dot{q}''_{\lambda\mathrm{b}} d\lambda = \sigma T^4 \qquad \text{(Stefan-Boltzmann's law)}$$

$$\lambda_{\mathrm{max}} T = 2898\,\mu\mathrm{m\,K} \qquad \text{(Wien's law of displacement)}$$

with the constants

$$\sigma = 5.67\ 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2\,K^4}} \qquad \text{(Stefan-Boltzmann constant)}$$

$$c_1 = 3.741\ 10^{-16}\,\mathrm{W\,m^2}$$

$$c_2 = 1.439\ 10^{-2}\,\mathrm{m\,K}$$

λT in µm K	1000.0	1250.0	1500.0	1750.0	2000.0	2500.0
$F(\lambda)$	0.00031	0.00308	0.01283	0.03363	0.06663	0.16115
λT in µm K	3000.0	3500.0	4000.0	5000.0	6000.0	8000.0
$F(\lambda)$	0.27322	0.38250	0.48085	0.63315	0.73715	0.85556

Distribution of black body radiation: $F(\lambda) = \int_0^{\lambda} \dot{q}''_{\lambda\mathrm{b}} d\lambda \,/\, \sigma T^4$

Properties of radiating bodies

- spectral properties

$$\left.\begin{aligned} \rho\left(\lambda\right) &\equiv \frac{\dot{q}''_{\lambda\rho}}{\dot{q}''_{\lambda\mathrm{o}}} \\ \alpha\left(\lambda\right) &\equiv \frac{\dot{q}''_{\lambda\alpha}}{\dot{q}''_{\lambda\mathrm{o}}} \\ \tau\left(\lambda\right) &\equiv \frac{\dot{q}''_{\lambda\tau}}{\dot{q}''_{\lambda\mathrm{o}}} \end{aligned}\right\} \text{with } \rho(\lambda) + \alpha(\lambda) + \tau(\lambda) = 1$$

here: $\dot{q}''_{\lambda\mathrm{o}}$ impacting spectral heat flux

$$\varepsilon\left(\lambda\right) \equiv \frac{\dot{q}''_{\lambda\varepsilon}}{\dot{q}''_{\lambda\mathrm{b}}}$$

$$\alpha(\lambda) = \varepsilon(\lambda) \qquad \text{(Kirchhoff's law)}$$

- spectrally averaged

$$\varepsilon \equiv \frac{\dot{q}''_{\varepsilon}}{\dot{q}''_{\mathrm{b}}} \equiv \frac{\int\limits_0^\infty \dot{q}''_{\lambda\varepsilon}\, d\lambda}{\int\limits_0^\infty \dot{q}''_{\lambda\mathrm{b}}\, d\lambda} \qquad \alpha \equiv \frac{\dot{q}''_{\alpha}}{\dot{q}''_{\mathrm{o}}} \equiv \frac{\int\limits_0^\infty \dot{q}''_{\lambda\alpha}\, d\lambda}{\int\limits_0^\infty \dot{q}''_{\lambda\mathrm{o}}\, d\lambda}$$

$$\rho \equiv \frac{\dot{q}''_{\rho}}{\dot{q}''_{\mathrm{o}}} \equiv \frac{\int\limits_0^\infty \dot{q}''_{\lambda\rho}\, d\lambda}{\int\limits_0^\infty \dot{q}''_{\lambda\mathrm{o}}\, d\lambda} \qquad \tau \equiv \frac{\dot{q}''_{\tau}}{\dot{q}''_{\mathrm{o}}} \equiv \frac{\int\limits_0^\infty \dot{q}''_{\lambda\tau}\, d\lambda}{\int\limits_0^\infty \dot{q}''_{\lambda\mathrm{o}}\, d\lambda}$$

- special cases

Radiation properties independent of wavelength:

$$\rho + \alpha + \tau = 1 \quad \text{and} \quad \alpha = \varepsilon \qquad \text{(Grey body)}$$

$$\alpha = 1 \quad \text{and} \quad \alpha = \varepsilon = 1 \qquad \text{(Black body)}$$

Spectral radiative properties

$$\rho(\lambda) + \alpha(\lambda) = 1 \qquad \text{(Solid body impermeable for radiation)}$$

$$\alpha(\lambda) + \tau(\lambda) = 1 \qquad \text{(Gas)}$$

Radiative heat exchange

$$\dot{Q}_{i\to j} = \dot{Q}_i \Phi_{ij} \qquad \text{(Radiative heat flow)}$$

$$\dot{Q}_i = \dot{q}_i'' A_i = \dot{Q}_{i,\mathrm{b}}\,\varepsilon_i + \underbrace{\sum_j \dot{Q}_{j\to i}\,\rho_i}_{\text{Reflection}} + \underbrace{\sum_k \dot{Q}_{k\to i}\,\tau_i}_{\text{Transmission}} \qquad \text{(Surface brightness)}$$

$$\text{with} \quad \dot{Q}_{i,\mathrm{b}} = \dot{q}_{i,\mathrm{b}}'' A_i \qquad \text{(Black body radiation)}$$

$$\Phi_{ij} = \frac{1}{A_i} \int_{A_j} \int_{A_i} \frac{\cos\varphi_i\ \cos\varphi_j}{\pi r^2}\, dA_i\, dA_j \qquad \text{(View factor)}$$

$$A_i \Phi_{ij} = A_j \Phi_{ji} \qquad \text{(Reciprocity relationship)}$$

$$\sum_j \Phi_{ij} = 1 \qquad \text{(Sum rule)}$$

$$\dot{Q}_{\mathrm{i,net}} = \dot{Q}_i - \sum_j \dot{Q}_{j\to i} \qquad \text{(Net radiative heat flow)}$$

$$\dot{Q}_{1\rightleftharpoons 2} = \dot{Q}_{1\to 2} - \dot{Q}_{2\to 1} \qquad \text{(Radiative heat exchange)}$$

$$\begin{aligned} \dot{Q}_{1\rightleftharpoons 2} &= A_1 \Phi_{12}\, \sigma \left[(T_1)^4 - (T_2)^4 \right] \qquad \text{(Between two black bodies)} \\ &= A_2 \Phi_{21}\, \sigma \left[(T_1)^4 - (T_2)^4 \right] \end{aligned}$$

$$\dot{q}_{1\rightleftharpoons 2}'' = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \sigma \left(T_1^4 - T_2^4 \right) \qquad \text{(Between two grey plates)}$$

- Plates are plane, parallel and infinitely long

$$\dot{Q}_{1\rightleftharpoons 2} = \frac{A_1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1 \right)} \sigma \left(T_1^4 - T_2^4 \right) \qquad \text{(Between two grey bodies)}$$

- Body 2 encloses body 1 ($A_2 > A_1$)
- Body 1 convex ($\Phi_{11} = 0$)

View factors of simple geometries

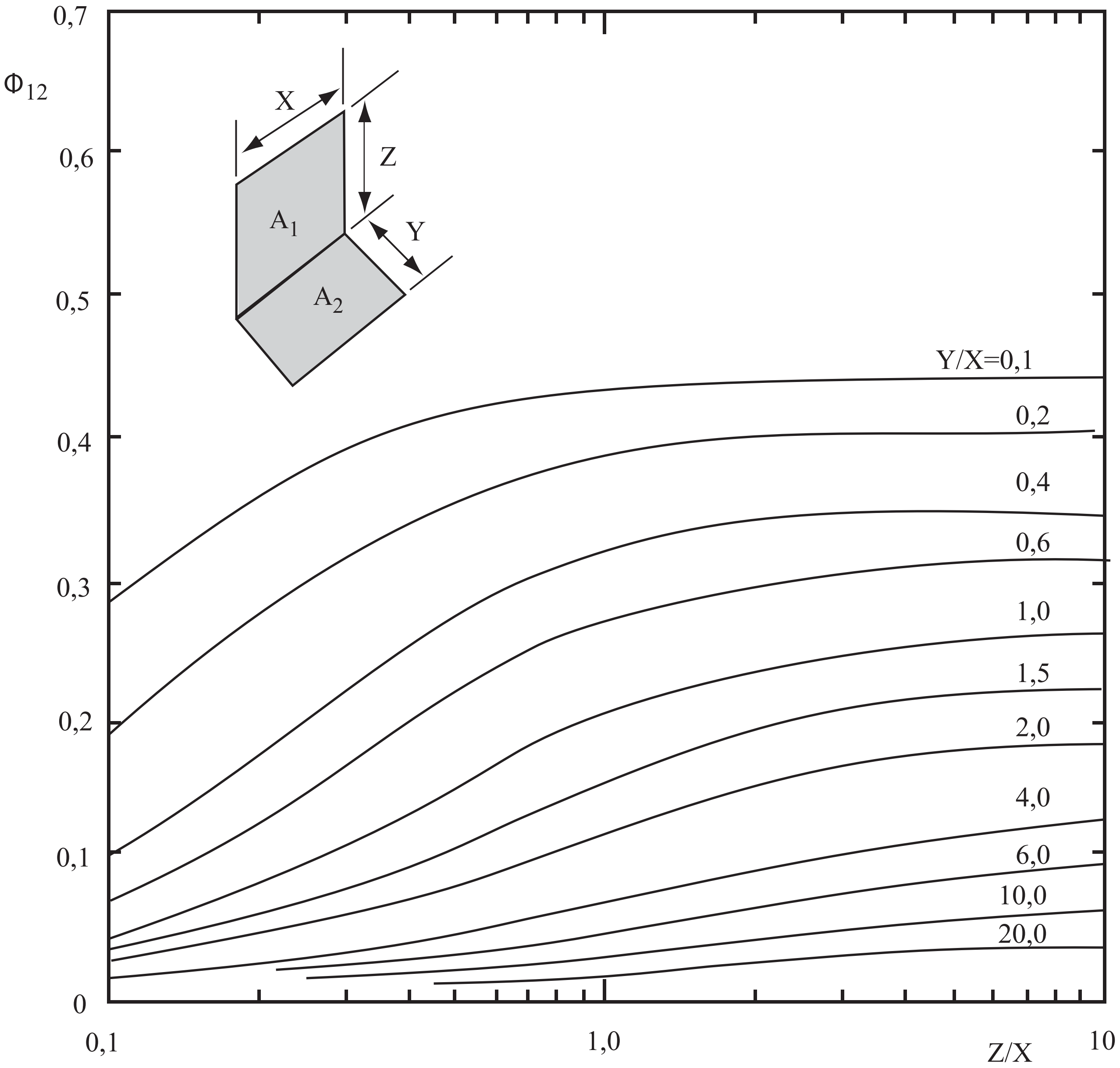

Diagramm 1: View factor of the radiation transfer between perpendicular, rectangular plates

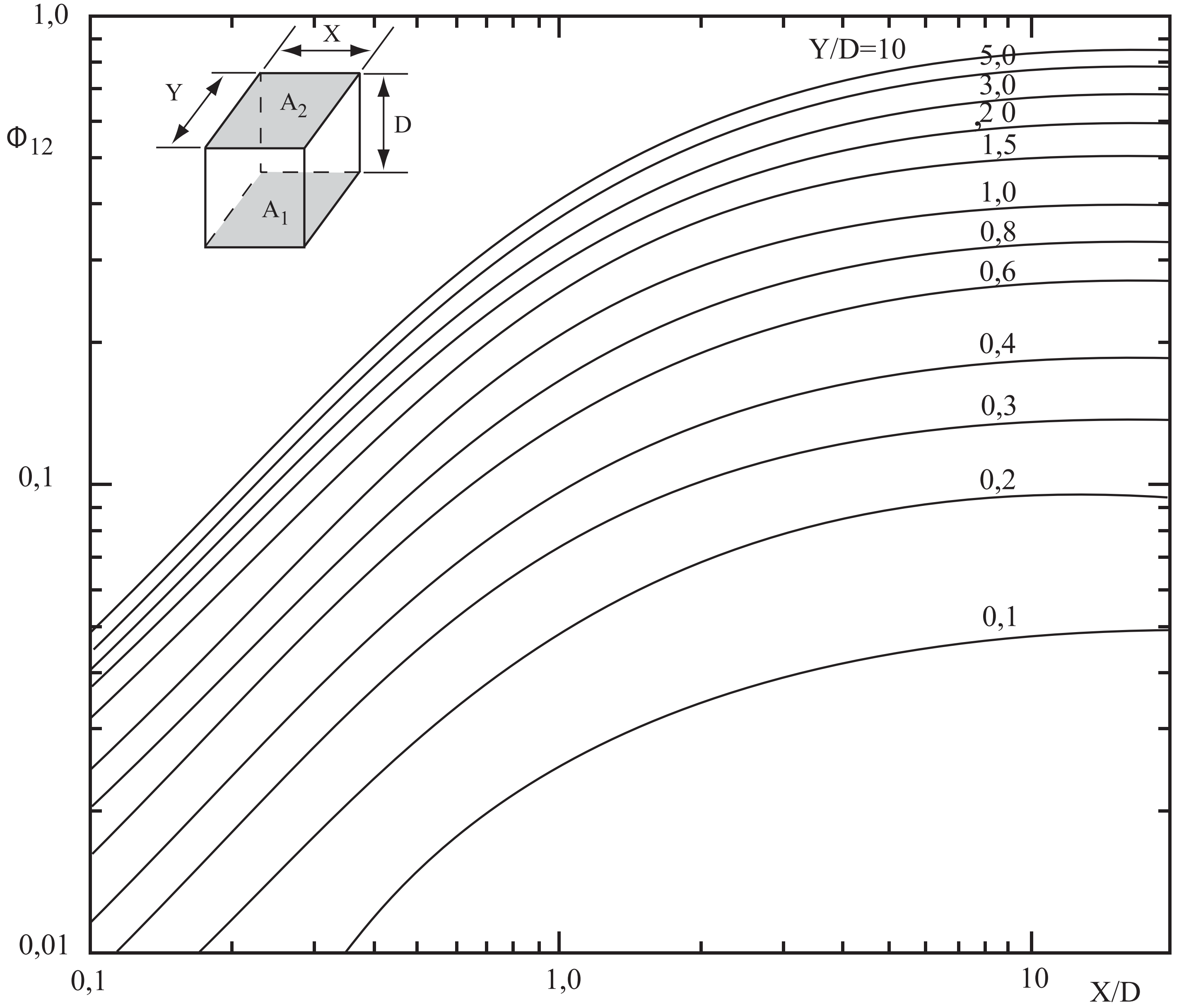

Diagramm 2: View factor of the radiation transfer between parallel, rectangular plates

Appendix A – Properties of various materials

Tabelle 1: Emissivity of various solids (Total emissivity ε, Emissivity in normal direction of the surface ε_n)

Surface	T K	ε_n	ε	**Surface**	T K	ε_n	ε
Metals				Zinc, highly polished poliert	500		0,045
Alumium, plain	443	0,039	0,049		600		0,055
... polished	373	0,095		Iron plate, galvanized			
... heavily oxidized	366	0,2		... plain	301	0,228	
	777	0,31		... grey oxidized	297	0,276	
Aluminum oxide	550	0,63		Tin, non oxidized	298		0,043
	1100	0,26			373	0,05	
	1089	0,052		**Non-Metals**			
Chromium, polished	423	423	423	Asbestos, paper	296	0,96	
				... Papier	311	0,93	
Gold. highly polished	500	0,018			644	0,94	
	900	900		Concrete, rough	273 – 366		0,94
Copper, polished	293	0,03		Roofing felt	294	0,91	
... struck	293	0,037		Gips	293	0,8 – 0,9	
... black oxidized	293	0,78		Glas	293	0,94	
... oxidized	403	0,76		Quartz (7 mm thick)	555	0,93	
Inconel, rolled	1089		0,69		1111	0,47	
... sandblasted	1089		0,79	Rubber	293	0,92	
Iron and steel,				Wood			
... highly polished	450	0,052		Oak, planed	273 – 366		0,9
... polished	700	0,144		Beech	343	0,94	0,91
	1300	0,377		Ceramics			
... sanded	293	0,242		White Al_2O_3	366		0,9
Cast iron, polished	473	0,21		Carbon			
Cast steel, polished	1044	0,52		... not oxidized	298		0,81
	1311	0,56			773		0,79
Iron sheet				... Fibers	533		0,95
... heavy rusty	292	0,685		... Graphite	373		0,76
... rolled	294	0,657		Corundum, rough	353	0,85	0,84
Cast iron,				Coating, colors:			
... oxidized at 866 K	472	0,64		Oil paint black	366		0,92
	872	0,78		... green	366		0,95
Steel,				... red	366		0,97
... oxidized at 866 K	472	0,79		... white	373		0,94
	872	0,79		Coating. white	373	0,925	
Brass, not oxidized	298	0,035		... flat black	353		0,97
	373	0,035		Bakelite coating	353	0,935	
... oxidized	473	0,61		Mennig color	373	0,93	
	873	0,59		Radiator (acc. to VDI-74)	373	0,925	
	1673	0,17		Enamel, white on iron	292	0,897	
Nickel, not oxidized	298	0,045		Marble			
	373	0,06		light grey. polished	273 – 366		0,9
	873	0,478		Paper	273		0,92
... oxidized	473	0,37			366		0,94
Platinum	422	0,022		Porcelain, white	295		0,924
	1089	0,123		Clay, glassy	298		0,9
Mercury,				... flat	298		0,93
... not oxidized	298	0,1		Water	273	0,95	
	373	0,12			373	0,96	
Silver, polished	311	0,022		Ice, smooth with water	273	0,966	0,92
	644	0,031		... rough surface	273	0,985	
Wolfram	298		0,024	Bricks, red	273 – 366		0,93
	1273		0,15				
	1773		0,23				