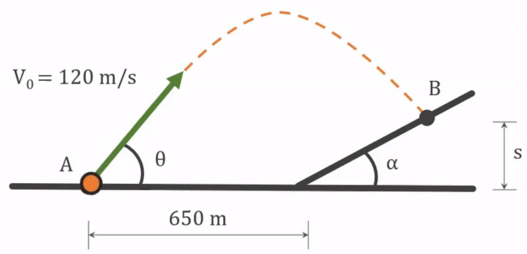
Trajectory of a Ball



A ball is launched from point A with the initial conditions shown. Find an expression for the vertical displacement $y(t)$.

Neglect all air resistances.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (at + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (at + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Solution:

For the vertical displacement in y-direction, equation (6) results in:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \tag{7}$$

Since $s_{y,0} = 0$ m and $a_y = -g$, the resulting equation reduces to:

$$y(t) = -\frac{1}{2}gt^2 + v_{y,0}t \tag{8}$$

Where $v_{y,0} = v_0 \sin\theta$. Substituting for $v_{y,0}$ gives the final expression:

$$y(t) = -\frac{1}{2}gt^2 + v_0 t \sin\theta \tag{9}$$

Or written alternatively as:

$$y(t) = \frac{1}{2}a_y t^2 + v_0 t \sin\theta \tag{10}$$