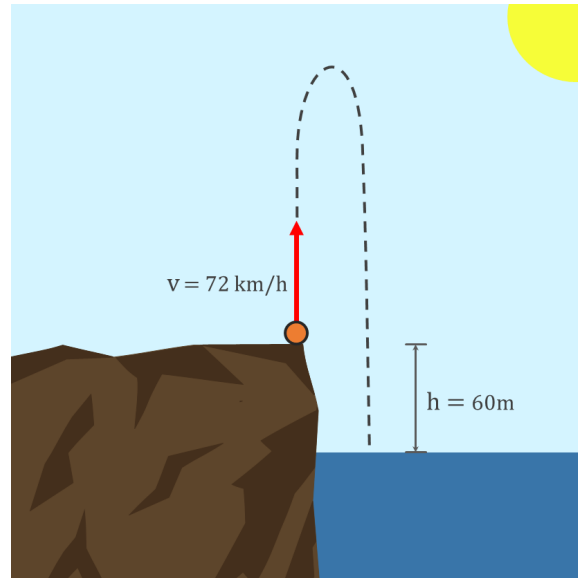


Ball Thrown from Cliff



A ball is thrown up vertically with a initial velocity of 72 km/h at the edge of a 60 meter high cliff. What is the maximum height the ball reaches with respect to the cliff?

Neglect air resistance and take $g = 10 \text{ m/s}^2$.

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (at + v_0) dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (at + v_0) dt \quad (5)$$

$$s(t) = \frac{1}{2} at^2 + v_0 t + s_0 \quad (6)$$

Given quantities:

Initial vertical velocity: $v_{y,0} = 72 \text{ km/h} = 20 \text{ m/s}$

Initial height of the ball (with respect to the cliff): $y_0 = s_{y,0} = 0 \text{ m}$

Gravitational acceleration: $g = 10 \text{ m/s}^2$

Solution:

For the vertical displacement in y -direction with the origin at cliff level, Equation (6) results in:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \quad (7)$$

At the instant the ball reaches its maximum height, $v_y = 0 \text{ m/s}$. The vertical acceleration on the ball is the gravitational acceleration: $a_y = -g$. Combining this into Equation (3) yields an equation for the time t when the ball reaches its highest point:

$$v_y(t) = 0 = a_y t + v_{y,0} \quad \Rightarrow \quad 0 = -gt + v_{y,0} \quad (8)$$

Rewriting gives:

$$t = \frac{v_{y,0}}{g} = \frac{20}{10} = 2 \text{ s} \quad (9)$$

Inserting $t = 2 \text{ s}$ into Equation (7) yields the maximum height H with respect to the cliff:

$$H = y(2) = -\frac{1}{2} \cdot 10 \cdot 2^2 + 20 \cdot 2 = 20 \text{ m} \quad (10)$$