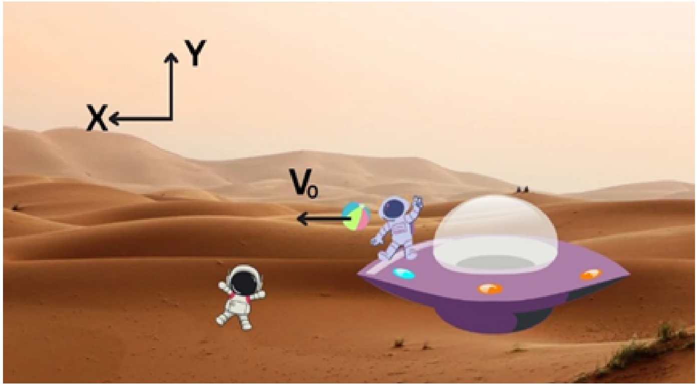
Mars Astronauts



Determine the height h of the spaceship if the ball is in the air for 6 seconds. Take $g_{\text{mars}} = 4\ \text{m/s}^2$ and assume that the ball is released at height h above the Martian surface.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Gravitational acceleration on Mars: $g_{\text{mars}} = 4\ \text{m/s}^2$
Airtime: $t_{\text{end}} = 6\ \text{s}$

Solution:

The time it takes to reach the surface can be calculated using Equation (6), where the gravity on Mars is pointing downwards. The initial velocity in y-direction is equal to zero, hence $v_0 = 0$. Take the origin of the coordinate system at the spaceship (hence $y(0) = 0$ m $\Rightarrow s_0 = 0$ m). Thus Equation (6) becomes:

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \quad \Rightarrow \quad y(t) = -\frac{1}{2}g_{\mathrm{mars}}t^2 \tag{7}$$

Inserting $t = \mathrm{end} = 6$ s and $g_{\mathrm{mars}} = 4$ m/s^2 gives:

$$y(t_{\mathrm{end}}) = -\frac{1}{2}g_{\mathrm{mars}}t_{\mathrm{end}}^2 \Rightarrow y(6) = -\frac{1}{2} \cdot 4 \cdot 6^2 = -72 \text{ m} \tag{8}$$

Thus the ball traveled 72 m downwards, which means that the height of the spaceship is $h = 72$ m.