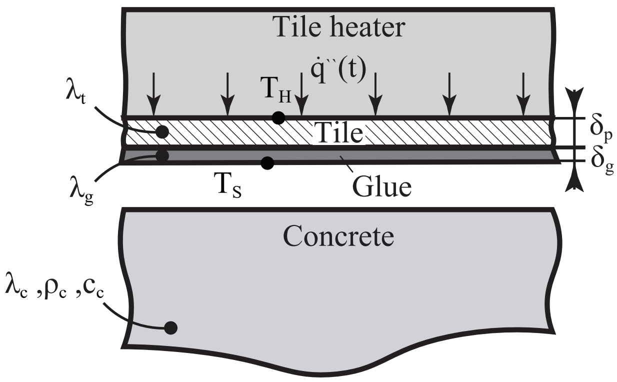
Exercise II.15: (Tile setting ⋆⋆)

A tile setter employs a modern technique for tile installation, involving preheating the tile and glue before affixing them to the concrete. The tile and glue are heated until they reach a steady-state condition, achieving a uniform heating temperature T_H and a constant heat flux $\dot{q}_0''$. Once these conditions are met, the tile setter places the heated tile and glue on the concrete, maintaining a constant temperature T_S throughout the process. After reaching a critical temperature T_{crit} at a distance δ_{crit} within the concrete, the heater is removed. Initially, the concrete used to be at a homogeneous temperature T_0



Given parameters:

- Steady-state heat flux: $\dot{q}_0''$ = 7.5 kW/m^2
- Thickness of the pile: δ_p = 10 mm
- Thickness of the glue: δ_g = 2 mm
- Conductivity of the pile: λ_p = 1.0 W/mK
- Conductivity of the glue: λ_g = 0.35 W/mK
- Conductivity of the concrete: λ_c = 2.3 W/mK
- Heat capacity of the concrete: c_c = 1,000 J/kgK
- Density of the concrete: ρ_c = 2,400 kg/m^3
- Initial temperature of the concrete: T_0 = 20 °C
- Heating temperature of the tile heater: T_H = 200 °C
- Critical temperature: T_{crit} = 35 °C
- Critical distance: δ_{crit} = 10 mm

Hints:

- Heat will never penetrate entirely through the concrete.

Tasks:

a) Derive the differential equation and establish the boundary and/or initial conditions to determine the temperature profile of the concrete. Based on your findings, identify the method that can be employed to determine the temperature at a particular position and time.

b) Determine the time t_{crit} at which the heater can be removed.

c) Illustrate the concrete's temperature profile that depicts both temporal and spatial variations.

Exercise II.16: (Heating and quenching of a sphere ⋆⋆⋆)

A sphere, initially at a homogeneous temperature of T_0, is put into an oven. The oven temperature remains constant at a homogeneous temperature of T_o.

Given parameters:

- Initial temperature of the sphere: $T_0 = 25\ °\mathrm{C}$
- Intermediate temperature of the sphere: $T_\mathrm{h} = 150\ °\mathrm{C}$
- Oven temperature: $T_\mathrm{o} = 200\ °\mathrm{C}$
- Quenching temperature: $T_\mathrm{q} = 30\ °\mathrm{C}$
- Heat transfer coefficient: $\alpha = 110\ \mathrm{W/m^2K}$
- Radius of the sphere: $r_1 = 1.5\ \mathrm{cm}$
- Thermal conductivity of the sphere: $\lambda = 1.52\ \mathrm{W/mK}$
- Density the sphere: $\rho = 1.45 \cdot 10^3\ \mathrm{kg/m^3}$
- Specific heat capacity the sphere: $c_\mathrm{p} = 0.88\ \mathrm{kJ/kg \cdot K}$

Hints:

- Heat radiation can be neglected.
- It always remains that $\mathrm{Fo} > 0.2$.

Tasks:

a) Derive the differential equation and establish the boundary and/or initial conditions to determine the temperature profile of the sphere. Based on your findings, identify the method that can be employed to determine the temperature at a particular position and time.

b) Determine the temperature of the center of the sphere after 3 minutes.

After some time the sphere has a hot homogeneous temperature T_h and is being quenched. During this process, the quenching temperature is constant at T_q. Further, in time, the center of the sphere has a temperature of 54 °C and the surface has a temperature of 44.4 °C.

c) Determine the time instant when the center of the sphere has a temperature of 54 °C and the surface has a temperature of 44.4 °C.

d) Determine the amount of heat dissipated at this time instant.