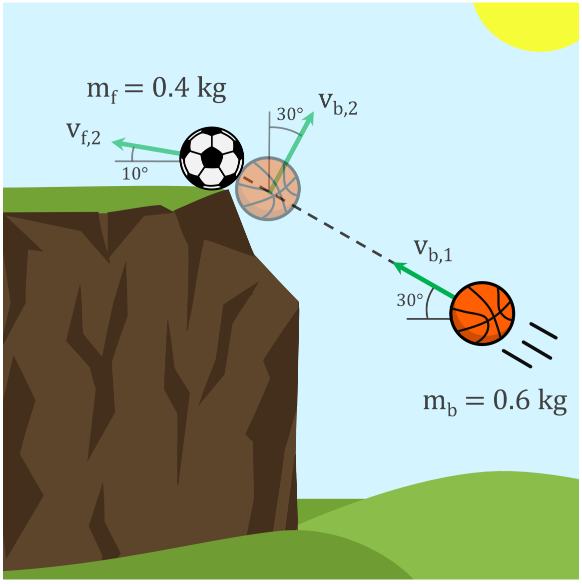
Basketball Hits Football



A basketball weighing 0.6 kg is thrown with a velocity of $v_{b,1} = 10$ m/s towards a football at rest on the edge of a cliff, at an angle of 30° with the horizontal. Determine the velocity $v_{f,2}$ in m/s of the football if the basketball has a velocity $v_{b,2}$, at an angle of 30° with the vertical, directly after impact. Assume the football weighing 0.4 kg and its velocity after impact makes an angle of 10° with the horizontal. Neglect friction. Round to the nearest integer.

Using known expressions:
Linear momentum:

$$\mathbf{G}_1 = m\mathbf{v} \tag{1}$$

Linear momentum conservation:

$$\mathbf{G}_1 = \mathbf{G}_2 \tag{2}$$

Given:

Mass basketball: $m_b = 0.6$ kg
Mass football: $m_f = 0.4$ kg
Initial velocity basketball: $v_{b,1} = 10$ m/s
Initial velocity football: $v_{f,1} = 0$ m/s

Angle $v_{b,1}$: 30°
Angle $v_{b,2}$: 30°
Angle $v_{f,1}$: 10°

Solution:

At the instant just before impact only the basketball has a linear momentum, since the football is at rest. At the instant just after the impact both the football and basketball have a linear momentum. Using the equation of conservation of linear momentum gives us:

$$\mathbf{G}_1 = \mathbf{G}_2 \quad \Rightarrow \quad m_b \mathbf{v}_{\mathrm{b},1} = m_b \mathbf{v}_{\mathrm{b},2} + m_f \mathbf{v}_{\mathrm{f},2} \tag{3}$$

Using geometry and a standard coordinate system, where the positive x-and y-direction are to the right and upwards respectively results in:

$$m_b \begin{pmatrix} -\cos(30°) \\ \sin(30°) \\ 0 \end{pmatrix} v_{b,1} = m_f \begin{pmatrix} -\cos(10°) \\ \sin(10°) \\ 0 \end{pmatrix} v_{f,2} + m_b \begin{pmatrix} \sin(30°) \\ \cos(30°) \\ 0 \end{pmatrix} v_{b,2} \tag{4}$$

Here we have two equations and two unknowns, which means this problem can be solved. First we write the first and second equation in terms of $v_{b,2}$.

$$v_{b,2} = \frac{-m_b \cos(30°) v_{b,1} + m_f \cos(10°) v_{f,2}}{m_b \sin(30°)} \tag{5}$$

$$v_{b,2} = \frac{m_b \sin(30°) v_{b,1} - m_f \sin(10°) v_{f,2}}{m_b \cos(30°)} \tag{6}$$

These two equations must be equal to each other, thus we can solve for $v_{f,2}$.

$$\frac{-m_b \cos(30°) v_{b,1} + m_f \cos(10°) v_{f,2}}{m_b \sin(30°)} = \frac{m_b \sin(30°) v_{b,1} - m_f \sin(10°) v_{f,2}}{m_b \cos(30°)} \tag{7}$$

Rewriting gives:

$$-m_b^2 \cos^2(30°) v_{b,1} + m_f m_b \cos(10°) \cos(30°) v_{f,2} =$$

$$m_b^2 \sin^2(30°) v_{b,1} - m_f m_b \sin(10°) \sin(30°) v_{f,2} \tag{8}$$

Bringing all terms with $v_{b,1}$ to the left side and all terms with $v_{f,2}$ to the right side results in:

$$-m_b^2 (\cos^2(30°) + \sin^2(30°)) v_{b,1} =$$

$$-m_f m_b(\sin(10°)\sin(30°)+\cos(10°)\cos(30°))v_{f,2} \tag{9}$$

Since $\cos^2(30°)+\sin^2(30°)=1$, we can write $v_{f,2}$ as follows.

$$v_{f,2}=\frac{-m_b^2 v_{b,1}}{-m_f m_b(\sin(10°)\sin(30°)+\cos(10°)\cos(30°))} \tag{10}$$

Inserting m_b, m_f and $v_{b,1}$ results in a final value for $v_{f,2}$

$$v_{f,2}=\frac{-0.6^2\cdot 10}{-0.6\cdot 0.4\cdot(\sin(10°)\sin(30°)+\cos(10°)\cos(30°))}=15.96\ \mathrm{m/s} \tag{11}$$

Rounding to the nearest integer gives: $v_{f,2}\approx 16$ m/s.