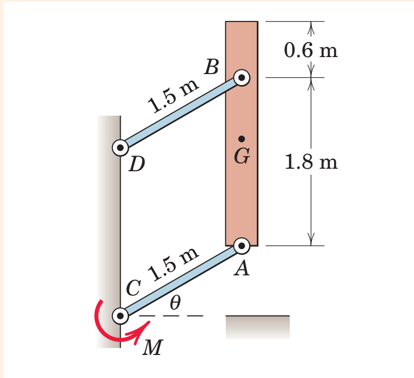
Sample Problem 6/2

The vertical bar AB has a mass of 150 kg with center of mass G midway between the ends. The bar is elevated from rest at $\theta = 0$ by means of the parallel links of negligible mass, with a constant couple $M = 5$ kN·m applied to the lower link at C. Determine the angular acceleration α of the links as a function of θ and find the force B in the link DB at the instant when $\theta = 30°$.



Solution. The motion of the bar is seen to be curvilinear translation since the bar itself does not rotate during the motion. With the circular motion of the mass center G, we choose n- and t-coordinates as the most convenient description. ① With negligible mass of the links, the tangential component A_t of the force at A is obtained from the free-body diagram of AC, where $\Sigma M_C \cong 0$ and ② $A_t = M/\overline{AC} = 5/1.5 = 3.33$ kN. The force at B is along the link. All applied forces are shown on the free-body diagram of the bar, and the kinetic diagram is also indicated, where the $m\bar{\mathbf{a}}$ resultant is shown in terms of its two components.

The sequence of solution is established by noting that A_n and B depend on the n-summation of forces and, hence, on $m\bar{r}\omega^2$ at $\theta = 30°$. The value of ω depends on the variation of $\alpha = \ddot{\theta}$ with θ. This dependency is established from a force summation in the t-direction for a general value of θ, where $\bar{a}_t = (\bar{a}_t)_A = \overline{AC}\alpha$. Thus, we begin with

$$[\Sigma F_t = m\bar{a}_t] \qquad 3.33 - 0.15(9.81)\cos\theta = 0.15(1.5\alpha)$$

$$\alpha = 14.81 - 6.54\cos\theta \text{ rad/s}^2 \qquad \textit{Ans.}$$

With α a known function of θ, the angular velocity ω of the links is obtained from

$$[\omega\, d\omega = \alpha\, d\theta] \qquad \int_0^\omega \omega\, d\omega = \int_0^\theta (14.81 - 6.54\cos\theta)\, d\theta$$

$$\omega^2 = 29.6\theta - 13.08\sin\theta$$

Substitution of $\theta = 30°$ gives

$$(\omega^2)_{30°} = 8.97 \text{ (rad/s)}^2 \qquad \alpha_{30°} = 9.15 \text{ rad/s}^2$$

and

$$m\bar{r}\omega^2 = 0.15(1.5)(8.97) = 2.02 \text{ kN}$$

$$m\bar{r}\alpha = 0.15(1.5)(9.15) = 2.06 \text{ kN}$$

The force B may be obtained by a moment summation about A, which eliminates A_n and A_t and the weight. Or a moment summation may be taken about the intersection of A_n and the line of action of $m\bar{r}\alpha$, which eliminates A_n and $m\bar{r}\alpha$. Using A as a moment center gives

$$[\Sigma M_A = m\bar{a}d] \qquad 1.8\cos 30°\, B = 2.02(1.2)\cos 30° + 2.06(0.6)$$

$$B = 2.14 \text{ kN} \qquad \textit{Ans.}$$

The component A_n could be obtained from a force summation in the n-direction or from a moment summation about G or about the intersection of B and the line of action of $m\bar{r}\alpha$.

Helpful Hints

① Generally speaking, the best choice of reference axes is to make them coincide with the directions in which the components of the mass-center acceleration are expressed. Examine the consequences of choosing horizontal and vertical axes.

② The force and moment equations for a body of negligible mass become the same as the equations of equilibrium. Link BD, therefore, acts as a two-force member in equilibrium.