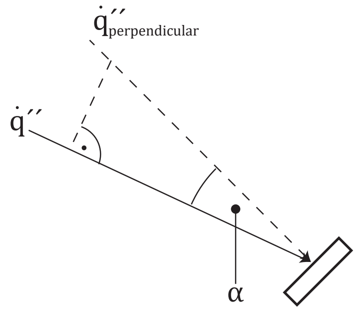
2.1 Solar Cell ★

a) *Determine the efficiency η of the thermal collector, and use the given numerical values to calculate the results.*

The efficiency of the solar collector describes the ratio of heat flow $\dot{Q}_{\text{Water}}$, which is added to the water for heating, and the radiant heat flux $\dot{Q}_{\text{rad}}$ on the solar collector:

$$\eta = \frac{\dot{Q}_{\text{Water}}}{\dot{Q}_{\text{rad}}} \tag{2.1}$$

Since the solar collector is inclined by the angle α relative to the direction of the solar radiation, to calculate the incident radiative heat flux $\dot{Q}_{\text{rad}}$ the fraction from the radiance $\dot{q}''$ that is projected on A of the collector can be calculated by use of trigonometry:



Where it results that:

$$\dot{q}''_{\text{perpendicular}} = \dot{q}'' \cos\alpha \tag{2.2}$$

And therefore:

$$\dot{Q}_{\text{rad}} = \dot{q}''_{\text{perpendicular}} \cdot A = \dot{q}'' \cos\alpha \cdot A \tag{2.3}$$

$$= 1367\ [\text{W/m}^2] \cdot \cos(20^\circ) \cdot 2\ [\text{m}^2] = 2.569\ \text{kW}$$

The heat absorbed by the water can be determined using a global energy balance around the water:

$$H'_{\text{Water}} - H''_{\text{Water}} + \dot{Q}_{\text{Water}} = 0 \tag{2.4}$$

$$\rightarrow \dot{Q}_{\text{Water}} = \dot{m} \cdot c_p \cdot (T'' - T') \tag{2.5}$$

$$= 0.0145\ [\text{kg/s}] \cdot 4180\ [\text{J/kgK}] \cdot (55 - 15)\ [\text{K}] = 2.424\ [\text{kW}]$$

Knowing these two heat fluxes, the efficiency can now be calculated:

$$\boxed{\eta = \frac{\dot{Q}_{\text{Water}}}{\dot{Q}_{\text{rad}}} = \frac{2.424}{2.569} = 0.943 = 94.3\,\%} \tag{2.6}$$

2.2 Solar power tower ★

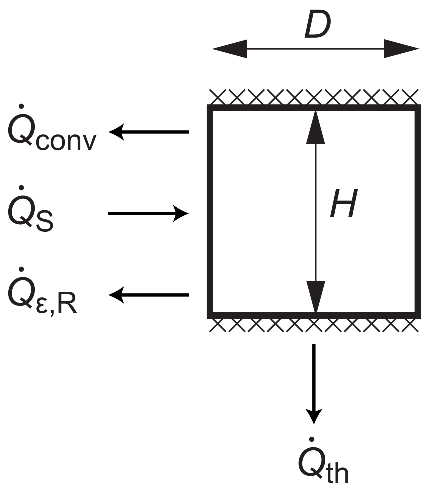
a) *From a balance around the receiver, determine the mean radiation density $\dot{q}''_\mathrm{S}$ as a function of the thermal load $\dot{Q}_\mathrm{th}$.*

To determine the mean radiation density $\dot{q}''_\mathrm{S}$ as a function of the thermal load $\dot{Q}_\mathrm{th}$, an energy balance around the receiver should be established.

The sun is radiating on the solar power tower ($\dot{Q}_\mathrm{S}$) with a heat flux density $\dot{q}''_\mathrm{S}$, but at the same time the receiver is loosing heat due to emitting radiation as a grey body ($\dot{Q}_{\epsilon,\mathrm{R}}$) and convection ($\dot{Q}_\mathrm{conv}$). The remaining thermal load ($\dot{Q}_\mathrm{th}$) is being transferred inside the solar power tower where it is converted into electrical power.

1) Setting up the energy balance:

With this given, the inner energy balance around the solar receiver is:



$$\cancelto{0}{\frac{\partial U}{\partial t}} = \alpha \sum \dot{Q}_\mathrm{in} - \sum \dot{Q}_\mathrm{emitted} \tag{2.7}$$

Note that for a grey body $\alpha = \epsilon$. So:

$$\boxed{\rightarrow 0 = \epsilon \dot{Q}_\mathrm{S} - \dot{Q}_\mathrm{conv} - \dot{Q}_{\epsilon,\mathrm{R}} - \dot{Q}_\mathrm{th}} \tag{2.8}$$

2) Defining the fluxes:

The solar radiation emitted on the receiver ($\dot{Q}_{\mathrm{S}}$) can be expressed as:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{S}} = \dot{q}''_{\mathrm{S}} A_{\mathrm{S}} = \dot{q}''_{\mathrm{S}} \pi D H} \tag{2.9}$$

And the rate of heat loss due to convection ($\dot{Q}_{\mathrm{conv}}$):

$$\boxed{\rightarrow \dot{Q}_{\mathrm{conv}} = \alpha_{\mathrm{conv}} A_{\mathrm{s}} \left(T_{\mathrm{R}} - T_{\mathrm{A}}\right) = \alpha \pi D H \left(T_{\mathrm{R}} - T_{\mathrm{A}}\right)} \tag{2.10}$$

Furthermore, the emission of the solar receiver ($\dot{Q}_{\epsilon,\mathrm{R}}$) can be expressed in terms of a grey body radiator:

$$\boxed{\rightarrow \dot{Q}_{\epsilon,\mathrm{R}} = \epsilon \sigma A_{\mathrm{s}} T_{\mathrm{R}}^4 = \epsilon \sigma \pi D H T_{\mathrm{R}}^4} \tag{2.11}$$

3) Inserting and rearranging:

Inserting the expressions of the fluxes into the inner energy balance results:

$$0 = \epsilon \dot{Q}_{\mathrm{S}} - \dot{Q}_{\mathrm{conv}} - \dot{Q}_{\epsilon,\mathrm{R}} - \dot{Q}_{\mathrm{th}} \tag{2.12}$$

$$0 = \epsilon \dot{q}''_{\mathrm{S}} \pi D H - \alpha_{\mathrm{conv}} \pi D H \left(T_{\mathrm{R}} - T_{\mathrm{A}}\right) - \epsilon \sigma \pi D H T_{\mathrm{R}}^4 - \dot{Q}_{\mathrm{th}} \tag{2.13}$$

Rewriting yields:

$$\boxed{\rightarrow \dot{q}''_{\mathrm{S}} = \frac{\alpha_{\mathrm{conv}} \pi D H \left(T_{\mathrm{R}} - T_{\mathrm{A}}\right) + \epsilon \sigma \pi D H T_{\mathrm{R}}^4 + \dot{Q}_{\mathrm{th}}}{\epsilon \pi D H}} \tag{2.14}$$

2.3 Infinite pipe segment ★★

Tasks:

a) Specify the view factors Φ_{12}, Φ_{31} and Φ_{33} as a function of Φ_{13}.

From the figure some information regarding the view factors can already be found:

$$\Phi_{11} = \Phi_{22} = 0 \tag{2.15}$$

With this given, Φ_{12} can be expressed in terms of Φ_{13} by making use of the summation rule:

$$\cancelto{0}{\Phi_{11}} + \Phi_{12} + \Phi_{13} = 1 \tag{2.16}$$

$$\boxed{\rightarrow \Phi_{12} = 1 - \Phi_{13}} \tag{2.17}$$

Furthermore, Φ_{31} can be expressed in terms of Φ_{13} by making use of the summation rule (Assuming the length of the pipe to be L):

$$\Phi_{31} A_3 = \Phi_{13} A_1 \tag{2.18}$$

$$\boxed{\rightarrow \Phi_{31} = \Phi_{13} \frac{A_1}{A_3} = \Phi_{13} \cdot \frac{R \cdot L}{\frac{2\pi \cdot R \cdot L}{4}} = \frac{2}{\pi} \cdot \Phi_{13}} \tag{2.19}$$

From the figure it can be seen that due to symmetry Φ_{31} equals Φ_{32}.

$$\Phi_{32} = \Phi_{31} = \frac{2}{\pi} \cdot \Phi_{13} \tag{2.20}$$

And lastly, with Φ_{31} and Φ_{32} known, again the summation rule can be used to determine Φ_{33}

$$\Phi_{31} + \Phi_{32} + \Phi_{33} = 1 \tag{2.21}$$

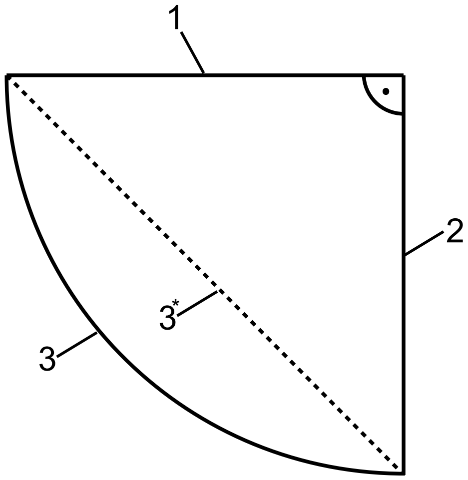
$$\boxed{\rightarrow \Phi_{33} = 1 - \Phi_{31} - \Phi_{32} = 1 - \frac{4}{\pi} \cdot \Phi_{13}} \tag{2.22}$$

b) Determine Φ_{13}.

To determine Φ_{13} we need a virtual surface. This is because, without surface, we have 9 unknown view factors and we only can establish 8 equations:

$$\begin{array}{lll} \Phi_{11} = 0 & \Phi_{11} + \Phi_{12} + \Phi_{13} = 1 & \Phi_{12} A_1 = \Phi_{21} A_2 \\ \Phi_{22} = 0 & \Phi_{21} + \Phi_{22} + \Phi_{23} = 1 & \Phi_{13} A_1 = \Phi_{31} A_3 \\ & \Phi_{31} + \Phi_{32} + \Phi_{33} = 1 & \Phi_{23} A_2 = \Phi_{32} A_3 \end{array}$$

Therefore the following virtual surface 3^* is introduced:



From symmetry it can be seen that:

$$\Phi_{3^*1} = \frac{1}{2}$$

Now the reciprocity rule can be used to find the numerical value for Φ_{13^*}:

$$\Phi_{13^*} A_1 = \Phi_{3^*1} A_{3^*} \tag{2.23}$$

$$\Phi_{13^*} = \Phi_{3^*1} \frac{A_{3^*}}{A_1} = \Phi_{3^*1} \frac{\sqrt{2} A_1}{A_1} = \frac{\sqrt{2}}{2} \tag{2.24}$$

And from the figure it can be seen that $\Phi_{3^*3} = 1$, therefore all radiation passing surface 3^* from body 1 is transferred towards body 3.

Thus:

$$\boxed{\rightarrow \Phi_{13} = \Phi_{13^*} = \frac{\sqrt{2}}{2}} \tag{2.25}$$

2.4 Hemispherical dome ★★

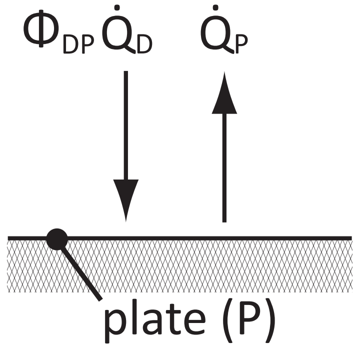
Derive an expression for the temperature of the plate T_P.

The temperature of the plate T_P can be determined by setting up an energy balance around the plate.

Partially the dome is radiating its surface brightness on the plate. Besides, the plate radiates its own surface brightness as well.

1) Setting up the energy balance:

Resulting in the following outer energy balance:



$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_\mathrm{in} - \sum \dot{Q}_\mathrm{out} \tag{2.26}$$

$$\boxed{\rightarrow 0 = \Phi_\mathrm{DP}\dot{Q}_\mathrm{D} - \dot{Q}_\mathrm{P}} \tag{2.27}$$

2) Defining the fluxes:

The plate acts as a black body radiator and therefore its surface brightness can be expressed as:

$$\boxed{\rightarrow \dot{Q}_\mathrm{P} = \sigma A_\mathrm{P} T_\mathrm{P}^4} \tag{2.28}$$

The dome emits radiation as a grey body, but it also transmits some of the ambient radiation. Therefore its surface brightness can be expressed as:

$$\dot{Q}_\mathrm{D} = \dot{Q}_{\mathrm{D},\epsilon} + \dot{Q}_{\mathrm{D},\tau} \tag{2.29}$$

Where:

$$\dot{Q}_{\mathrm{D},\epsilon} = \epsilon_\mathrm{D}\sigma A_\mathrm{D} T_\mathrm{D}^4 \tag{2.30}$$

And:

$$\dot{Q}_{D,\tau} = \tau_D \dot{q}''_{amb} A_D \tag{2.31}$$

So the surface brightness of the plate equals:

$$\boxed{\rightarrow \dot{Q}_D = \epsilon_D \sigma A_D T_D^4 + \tau_D \dot{q}''_{amb}} \tag{2.32}$$

3) Inserting and rewriting:

Inserting the definitions of the surface brightness of all bodies into the energy balance yields:

$$\Phi_{DP}\epsilon_D \sigma A_D T_D^4 + \Phi_{DP}\tau_D \dot{q}''_{amb} A_D - \sigma A_P T_P^4 = 0 \tag{2.33}$$

Rewriting T_P only on the left hand side:

$$\boxed{\rightarrow T_P = \sqrt[4]{\frac{\Phi_{DP}\epsilon_D \sigma A_D T_D^4 + \Phi_{DP}\tau_D \dot{q}''_{amb} A_D}{\sigma A_P}}} \tag{2.34}$$

<u>**An alternative route**</u> is by use of the inner energy balance:

1) Setting up the energy balance:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \alpha \sum \dot{Q}_{in} - \sum \dot{Q}_{emitted} \tag{2.35}$$

$$0 = \cancelto{1}{\alpha_P} \Phi_{DP} \dot{Q}_D - \dot{Q}_{\epsilon,P} \tag{2.36}$$

$$\boxed{\rightarrow 0 = \Phi_{DP} \dot{Q}_D - \dot{Q}_{\epsilon,P}} \tag{2.37}$$

2) Defining the fluxes:

Where the radiation emitted by the plate can be described as the radiation emitted by a black body:

$$\boxed{\rightarrow \dot{Q}_{\epsilon,P} = \sigma A_P T_P^4}$$

3) Inserting and rewriting:

And therefore resulting in exactly the same equation as found for the outer energy balance:

$$\Phi_{DP}\epsilon_D \sigma A_D T_D^4 + \Phi_{DP}\tau_D \dot{q}''_{amb} A_D - \sigma A_P T_P^4 = 0 \tag{2.38}$$

$$\boxed{\rightarrow T_P = \sqrt[4]{\frac{\Phi_{DP}\epsilon_D \sigma A_D T_D^4 + \Phi_{DP}\tau_D \dot{q}''_{amb} A_D}{\sigma A_P}}} \tag{2.39}$$

2.5 Light bulb ★★

a) *Provide the energy balance in terms of given variables for determining the glass temperature T_G, while neglecting radiation from the environment.*

Since the surface of the filament in comparison to the glass body is small, it can be assumed that:

$$\Phi_{GG,inner} = 1 \tag{2.40}$$

Therefore all reflected and emitted radiation on the inside of the glass ball (surface brightness $\dot{Q}_{G,i}$) interacts only with the glass body.

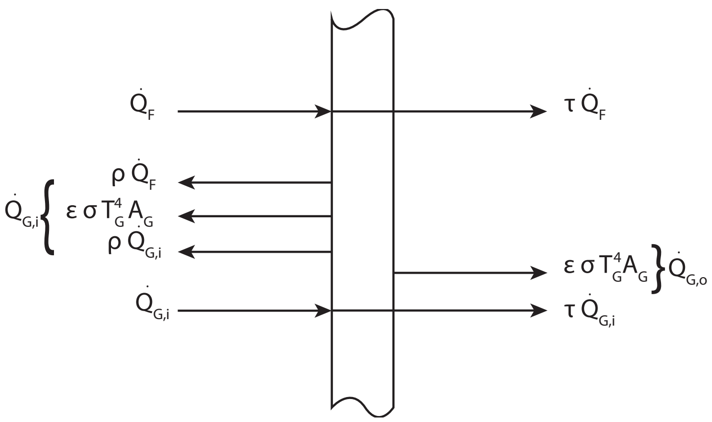
A possible way of deriving the glass temperature T_G is by setting up the outer energy balance for the glass.

On the inside of the glass bulb (left side in the figure) both the filament and the inner-side of the glass bulb radiate their surface brightness on the glass bulb. Partly this radiation is reflected by the glass bulb. Lastly, the inner side emits radiation as a grey body radiator.

On the outside (right side in the figure), partly the radiated surface brightness of the inner-side of the glass bulb and the filament are transmitted. Besides, the outer-side of the glass bulb emits radiation as a grey body radiator.

1) Setting up the energy balance:

With this given, the outer energy balance for the glass bulb is:



$$\overset{0}{\cancel{\frac{\partial U}{\partial t}}} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{2.41}$$

$$0 = \dot{Q}_{\mathrm{F}} + \dot{Q}_{\mathrm{G,i}} - \dot{Q}_{\mathrm{G,i}} - \dot{Q}_{\mathrm{G,o}} - \tau\left(\dot{Q}_{\mathrm{F}} + \dot{Q}_{\mathrm{G,i}}\right) \tag{2.42}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\mathrm{F}} - \dot{Q}_{\mathrm{G,o}} - \tau\left(\dot{Q}_{\mathrm{F}} + \dot{Q}_{\mathrm{G,i}}\right)} \tag{2.43}$$

2) Definition of the fluxes:

Where the surface brightness of the inside of the glass bulb can be described as:

$$\dot{Q}_{\mathrm{G,i}} = \dot{Q}_{\epsilon,\mathrm{G,i}} + \dot{Q}_{\rho,\mathrm{G,i}} + \dot{Q}_{\tau,\mathrm{G,i}} \tag{2.44}$$

Each individual term can be described by:

$$\dot{Q}_{\epsilon,\mathrm{G,i}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} \tag{2.45}$$

$$\dot{Q}_{\rho,\mathrm{G,i}} = \rho\left(\dot{Q}_{\mathrm{F}} + \dot{Q}_{\mathrm{G,i}}\right) \tag{2.46}$$

$$\dot{Q}_{\tau,\mathrm{G,i}} = 0 \tag{2.47}$$

Plugging into the expression for the surface brightness yields:

$$\dot{Q}_{\mathrm{G,i}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\left(\dot{Q}_{\mathrm{F}} + \dot{Q}_{\mathrm{G,i}}\right) \tag{2.48}$$

Rewriting $\dot{Q}_{\mathrm{G,i}}$ to the left side yields:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{G,i}} = \frac{\epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho}} \tag{2.49}$$

Furthermore, the surface brightness of the outside of the glass bulb can be described as:

$$\dot{Q}_{\mathrm{G,o}} = \dot{Q}_{\epsilon,\mathrm{G,o}} + \cancelto{0}{\dot{Q}_{\rho,\mathrm{G,o}}} + \cancelto{0}{\dot{Q}_{\tau,\mathrm{G,o}}} \tag{2.50}$$

$$\boxed{\rightarrow \dot{Q}_{\mathrm{G,o}} \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}}} \tag{2.51}$$

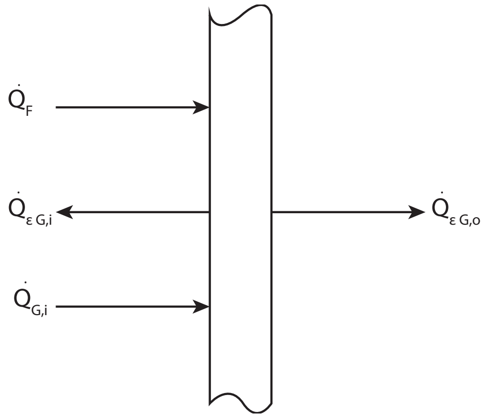
3) Inserting and rewriting:

Plugging the definition of $\dot{Q}_{\mathrm{G,i}}$ and $\dot{Q}_{\mathrm{G,o}}$ into the outer energy balance:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{F}} - \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} - \tau\left(\dot{Q}_{\mathrm{F}} + \frac{\epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho}\right) = 0} \tag{2.52}$$

An alternative solution would have been obtained by setting up the inner energy balance for the glass. This would result in:

1) Setting up the energy balance:



$$\overset{0}{\cancel{\frac{\partial U}{\partial t}}} = \alpha \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\mathrm{emitted}} = \tag{2.53}$$

$$\boxed{\rightarrow 0 = \alpha\left(\dot{Q}_{\mathrm{F}} + \dot{Q}_{\mathrm{G,i}}\right) - \dot{Q}_{\epsilon,\mathrm{G,i}} - \dot{Q}_{\epsilon,\mathrm{G,o}}} \tag{2.54}$$

2) Defining the fluxes:

Where the emitted radiation by the in- and outside of the glass bulb can be described as:

$$\boxed{\rightarrow \dot{Q}_{\epsilon,\mathrm{G,i}} = \dot{Q}_{\epsilon,\mathrm{G,o}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}}} \tag{2.55}$$

3) Inserting and rewriting:

Inserting the definitions of $\dot{Q}_{\epsilon,\mathrm{G}}$, $\dot{Q}_{\epsilon,\mathrm{G,i}}$ and $\dot{Q}_{\epsilon,\mathrm{G,o}}$ yields (note $\epsilon = \alpha$):

$$\boxed{\rightarrow \epsilon\left(\dot{Q}_{\mathrm{F}} + \frac{\epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho} - 2\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}}\right) = 0} \tag{2.56}$$