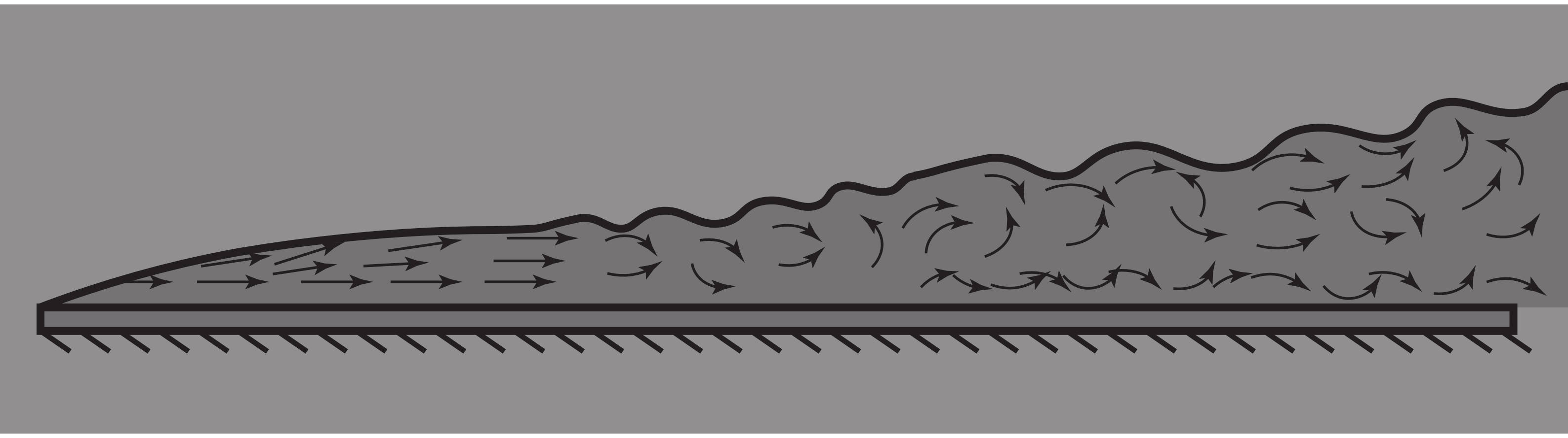
HEATQUIZ

HEAT TRANSFER

Book of Formularies



UNIVERSITY OF TWENTE. | RWTH AACHEN UNIVERSITY

Contents

.

SECTION D

Dimensionless numbers

D.1 Fluid mechanics

Definition

Reynolds number:

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho u L}{\eta} = \frac{uL}{\nu} \quad (-)\,, \tag{D.1}$$

where L is the characteristic length.

Definition

Grashof number:

$$\mathrm{Gr} = \frac{\text{Bouyancy forces}}{\text{Viscous forces}} = \frac{\beta g \rho^2 \left(T_\mathrm{W} - T_\infty\right) L^3}{\eta^2} = \frac{\beta g \left(T_\mathrm{W} - T_\infty\right) L^3}{\nu^2} \quad (-)\,, \tag{D.2}$$

where L is the characteristic length.

Definition

Archimedes number:

$$\mathrm{Ar} = \frac{\mathrm{Gr}}{\mathrm{Re}^2} = \frac{\text{Bouyance forces}}{\text{Inertia forces}} = \frac{g_\mathrm{x} \beta L (T_\mathrm{W} - T_\infty)}{u_\infty^2} \quad (-)\,, \tag{D.3}$$

where L is the characteristic length.

D.2 Heat transfer

Definition

Biot number:

$$\mathrm{Bi} = \frac{\text{Conductive thermal resistance in body}}{\text{Convective thermal resistance at surface}} = \frac{\alpha L}{\lambda_\mathrm{s}} \quad (-)\,, \tag{D.4}$$

where λ_s is the thermal conductivity of the solid and $L = V/A$.

Definition

Fourier number:

$$\mathrm{Fo} = \frac{\text{Rate of diffusivity}}{\text{Rate of storage}} = \frac{at}{L^2} \ (-), \tag{D.5}$$

where L is the characteristic length.

Definition

Nusselt number:

$$\mathrm{Nu} = \text{Dimensionless heat transfer coefficient} = \frac{\alpha L}{\lambda_\mathrm{f}} \ (-), \tag{D.6}$$

where λ_f is the thermal conductivity of the fluid and L is the characteristic length.

Definition

Prandtl number:

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\eta}{\lambda/c_p} \quad (-)\,. \tag{D.7}$$

Definition

Peclet number:

$$\mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = \frac{\text{Rate of advection}}{\text{Rate of diffusion}} = \frac{\rho c_p u L}{\lambda} \quad (-)\,, \tag{D.8}$$

where L is the characteristic length.

D.3 Mass transfer

Definition

Schmidt number:

$$\mathrm{Sc} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of mass}} = \frac{\eta}{\rho D} = \frac{\nu}{D} \quad (-)\,. \tag{D.9}$$

Definition

Sherwood number:

$$\mathrm{Sh} = \text{Dimensionless mass transfer coefficient} = \frac{gL}{\rho D} \ (-), \tag{D.10}$$

where L is the characteristic length.

Theorem

Lewis' law:

$$g = \frac{\alpha}{c_\mathrm{p}}, \tag{D.11}$$

for $\mathrm{Le} \approx 1$.

SECTION C

Conduction

C.1 Fundamentals

Fundamental EQ

Fourier's law:

$$\dot{q}'' = -\lambda \frac{\partial T}{\partial x}. \tag{C.1}$$

C.2 Conservation equations

Fundamental EQ

Equation of energy conservation for solids in Cartesian coordinates (x,y,z,t):

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \dot{\Phi}'''. \tag{C.2}$$

Fundamental EQ

Equation of energy conservation for solids in cylindrical coordinates (r,θ,z,t):

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\lambda \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \dot{\Phi}'''. \tag{C.3}$$

Fundamental EQ

Equation of energy conservation for solids in spherical coordinates (r,θ,ϕ,t):

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\lambda \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \theta}\left(\lambda \sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(\lambda \frac{\partial T}{\partial \phi}\right) + \dot{\Phi}'''. \tag{C.4}$$

Definition

Thermal diffusivity:

$$a = \frac{\lambda}{\rho c}\ \left(\frac{m^2}{s}\right). \tag{C.5}$$

C.3 Multi-layer walls

Fundamental EQ

Rate of heat transfer through a solid multi-layer wall without convection:

$$\dot{Q} = \frac{1}{\sum_{i=1}^{n} R_{\text{cond},i}}\left(T_1 - T_{n+1}\right). \tag{C.6}$$

Definition

Conductive resistance of a solid plane layer i:

$$R_{\text{cond},i} = \frac{\delta_i}{A\lambda_i}\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \tag{C.7}$$

Definition

Conductive resistance of a solid cylindrical layer i:

$$R_{\text{cond.},i} = \frac{1}{2\pi L\lambda_i}\ln\frac{r_{i+1}}{r_i}\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \tag{C.8}$$

Fundamental EQ

Rate of heat transfer through a solid multi-layer wall with convection:

$$\dot{Q} = \frac{1}{R_{\mathrm{conv},A} + \sum_{i=1}^{n} R_{\mathrm{cond},i} + R_{\mathrm{conv},B}} \left(T_{\mathrm{A}} - T_{\mathrm{B}}\right). \quad \text{(C.9)}$$

Definition

Convective resistance at a solid plane layer j:

$$R_{\mathrm{conv},j} = \frac{1}{A\alpha_j} \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \quad \text{(C.10)}$$

Definition

Convective resistance at a solid cylindrical layer j:

$$R_{\mathrm{conv},j} = \frac{1}{A_j\alpha_j} \left(\frac{\mathrm{K}}{\mathrm{W}}\right), \quad \text{(C.11)}$$

with $A_{\mathrm{A}} = 2\pi r_1 L$ for convection on the inside and $A_{\mathrm{B}} = 2\pi r_{\mathrm{n}+1} L$ for convection on the outside.

Fundamental EQ

Rate of heat transfer through a solid multi-layer wall with convection:

$$\dot{Q} = kA\left(T_{\mathrm{A}} - T_{\mathrm{B}}\right). \quad \text{(C.12)}$$

Definition

Overall heat transfer coefficient solid multi-layer plane wall system:

$$k = \left(\frac{1}{\alpha_{\mathrm{A}}} + \sum_{i=1}^{n} \frac{\delta_i}{\lambda_i} + \frac{1}{\alpha_{\mathrm{B}}}\right)^{-1} \left(\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}}\right). \quad \text{(C.13)}$$

Definition

Overall heat transfer coefficient solid multi-layer cylindrical wall system:

$$k = \left(\frac{1}{\alpha_{\mathrm{A}}}\frac{r}{r_1} + r\sum_{i=1}^{n} \frac{1}{\lambda_i}\ln\frac{r_{i+1}}{r_i} + \frac{1}{\alpha_{\mathrm{B}}}\frac{r}{r_{n+1}}\right)^{-1} \left(\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}}\right), \quad \text{(C.14)}$$

where any reference radius r may be used.

C.4 Fins

Definition

Fin parameter:

$$\mathrm{m} = \sqrt{\frac{\alpha U}{\lambda A_{\mathrm{c}}}} \left(\frac{1}{\mathrm{m}}\right), \quad \text{(C.15)}$$

where for a rod fin $\mathrm{m} = \sqrt{\frac{4\alpha}{\lambda d}}$, and for a plane fin $\mathrm{m} = \sqrt{\frac{2\alpha}{\lambda\delta}}$.

Fundamental EQ

Homogeneous fin equation:

$$\frac{d^2\theta}{dx^2} - \mathrm{m}^2\theta = 0, \quad \text{(C.16)}$$

where $\theta = T(x) - T_{\mathrm{a}}$.

Fundamental EQ

General solution of the fin equation:

$$\theta = A\, e^{\mathrm{mx}} + B\, e^{-\mathrm{mx}} = A^*\ \sinh(\mathrm{mx}) + B^* \cosh(\mathrm{mx}). \tag{C.17}$$

Fundamental EQ

Temperature profile of a fin with base temperature T_b and an adiabatic tip:

$$\theta = \theta_\mathrm{b} \frac{\cosh\left[\mathrm{m}\,(L - \mathrm{x})\right]}{\cosh\left(\mathrm{mL}\right)}. \tag{C.18}$$

Fundamental EQ

Rate of heat transfer for a fin with base temperature T_b and an adiabatic tip:

$$\dot{Q} = \lambda A_\mathrm{c} \mathrm{m} \theta_\mathrm{b} \tanh\left(\mathrm{mL}\right). \tag{C.19}$$

Definition

Fin efficiency:

$$\eta_\mathrm{F} = \frac{\text{transferred heat}}{\text{maximum transferable heat}} = \frac{\dot{Q}}{\dot{Q}_\mathrm{max}} \quad (-). \tag{C.20}$$

C.5 Unsteady heat conduction

Fundamental EQ

Lumped capacity model:

$$\theta^* = 1 - \exp\left(-\frac{\alpha}{\rho c}\frac{A}{V}t\right), \tag{C.21}$$

which is valid for $\mathrm{Bi} \ll 1$, and where $\theta^* = \frac{T-T_0}{T_\mathrm{a}-T_0}$.

Fundamental EQ

Temperature profile semi-infinite plate with negligible thermal surface resistance:

$$\theta^* = 1 - \mathrm{erf}\left(\frac{1}{\sqrt{4\mathrm{Fo}}}\right), \tag{C.22}$$

for $\mathrm{Bi} \gg 1$, where $\theta^* = \frac{T-T_0}{T_\mathrm{a}-T_0}$, and $\mathrm{Fo} = \frac{at}{x^2}$.

Fundamental EQ

Penetration depth semi-infinite plate with negligible thermal surface resistance:

$$\delta(t) = 3.6\sqrt{at}, \tag{C.23}$$

for $\mathrm{Bi} \gg 1$.

Fundamental EQ

Heat flux semi-infinite plate with negligible thermal surface resistance:

$$\dot{q}''\Big|_{x=0} = \frac{\lambda}{\sqrt{\pi a t}}\left(T_\mathrm{a} - T_0\right), \tag{C.24}$$

for $\mathrm{Bi} \gg 1$.

Fundamental EQ

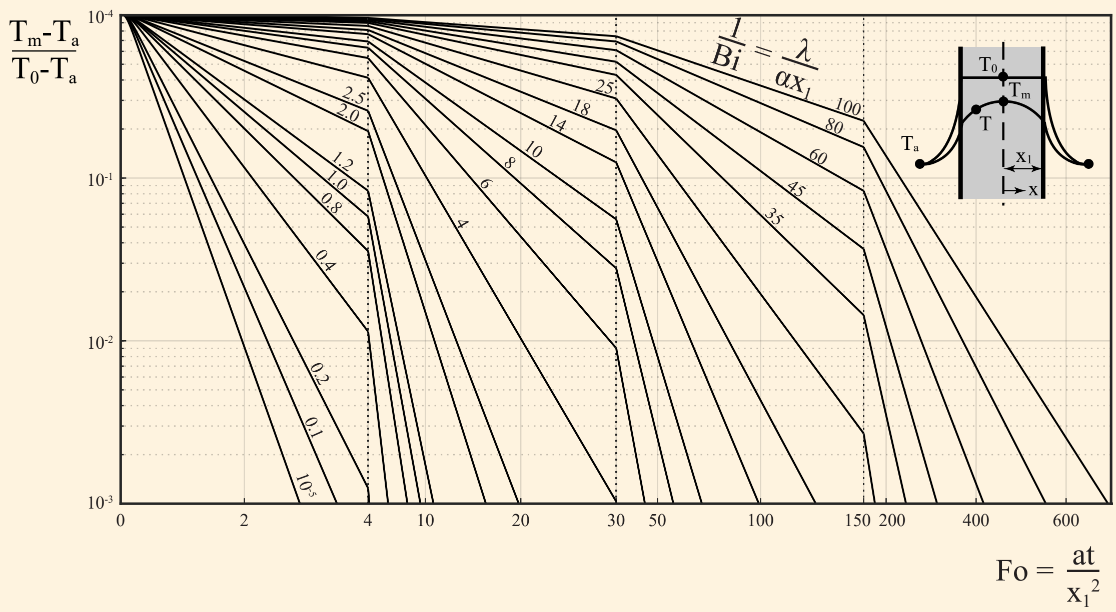
Temperature profile semi-infinite plate with non-negligible thermal surface resistance:

$$\theta^* = 1 - \operatorname{erf}\left(\frac{1}{\sqrt{4\mathrm{Fo}}}\right) - \left[\exp\left(\mathrm{Bi} + \mathrm{Fo}\,\mathrm{Bi}^2\right)\right]\left[1 - \operatorname{erf}\left(\frac{1}{\sqrt{4\mathrm{Fo}}} + \sqrt{\mathrm{Fo}}\cdot\mathrm{Bi}\right)\right] \tag{C.25}$$

where $\theta^* = \frac{T-T_0}{T_\mathrm{a}-T_0}$, $\mathrm{Fo} = \frac{at}{x^2}$, and $\mathrm{Bi} = \frac{\alpha x}{\lambda}$.

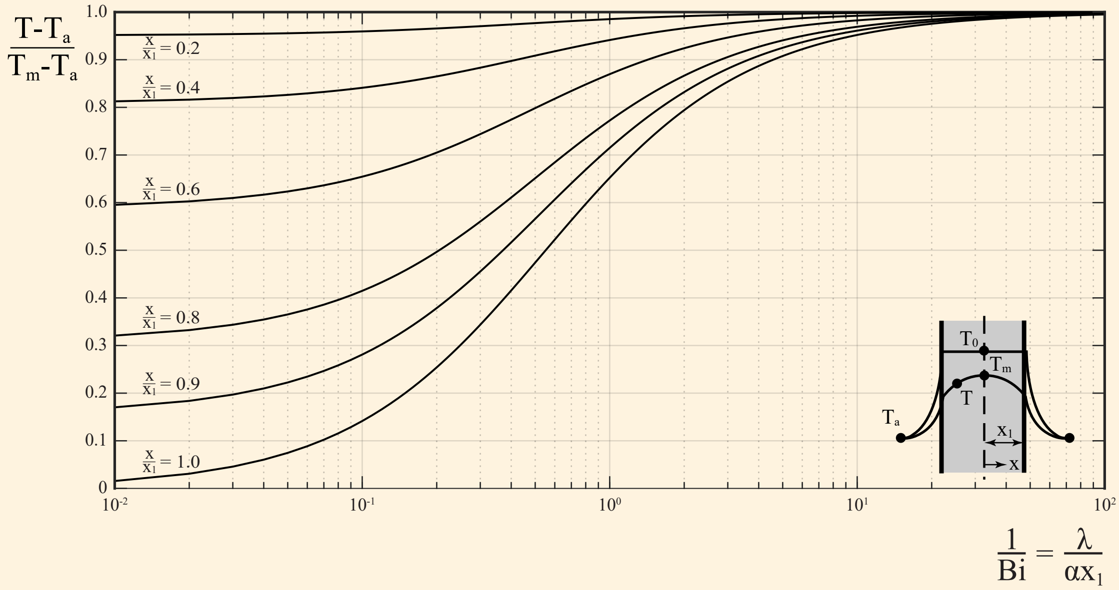
Fundamental EQ

Temperature profile mid-plane of a plate with thickness $2x_1$ [?]:



Fundamental EQ

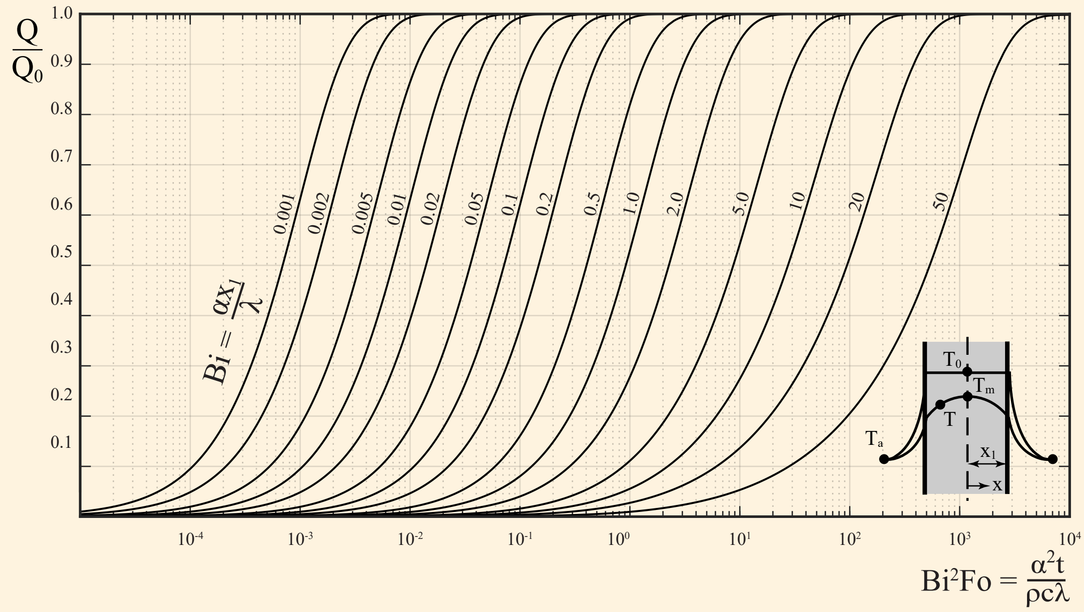
Temperature distribution in a plate with thickness $2x_1$ [?]:



which is for Fo > 0.2.

Fundamental EQ

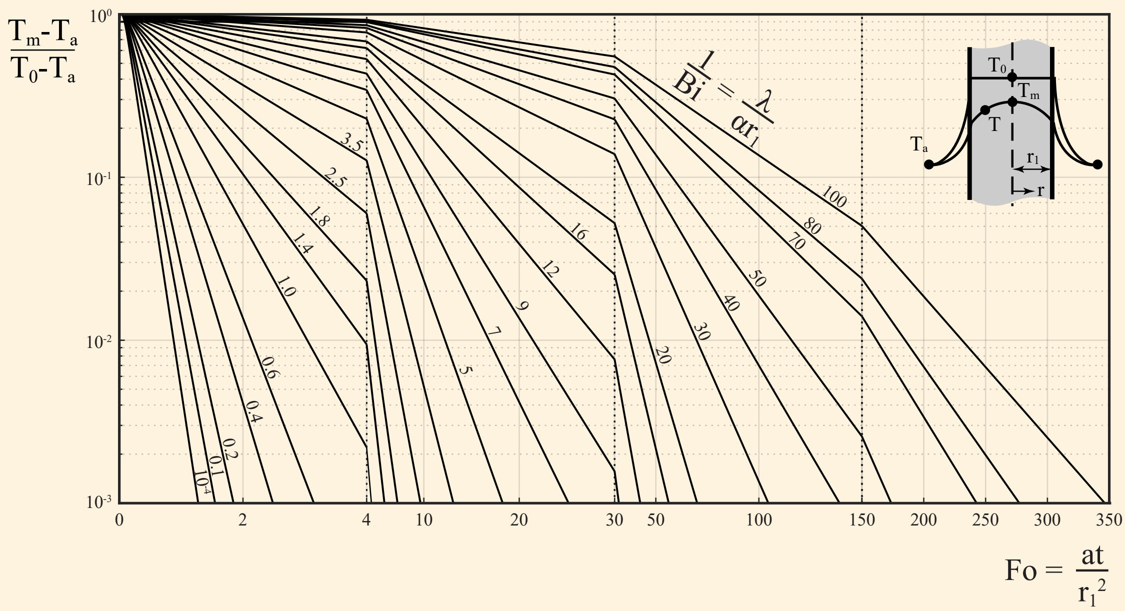
Heat loss of a plate [?]:



where $Q = mc\,(T(t) - T_\mathrm{a})$ and $Q_0 = mc\,(T_0 - T_\mathrm{a})$.

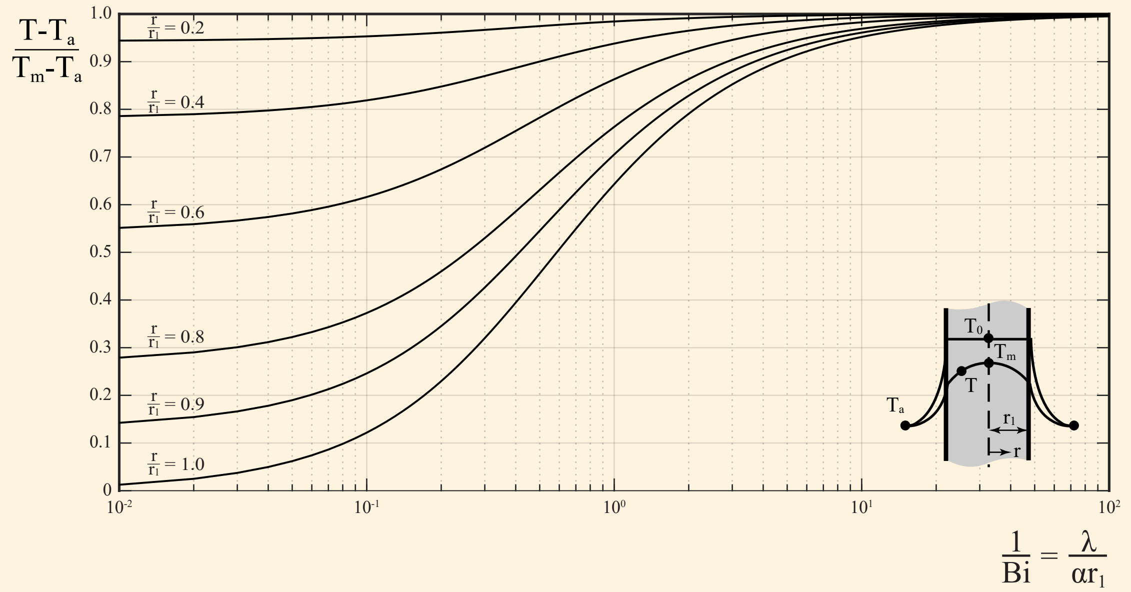
Fundamental EQ

Temperature along the axis of a cylinder with radius r_1 [?]:



Fundamental EQ

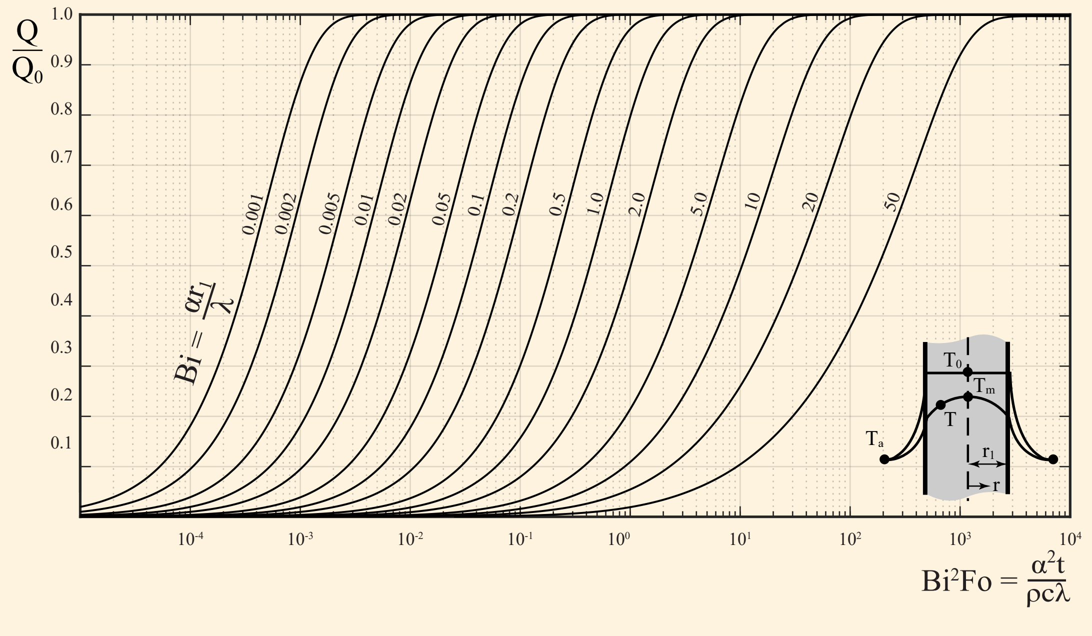
Temperature distribution in a cylinder with radius r_1 [?]:



which is for Fo > 0.2.

Fundamental EQ

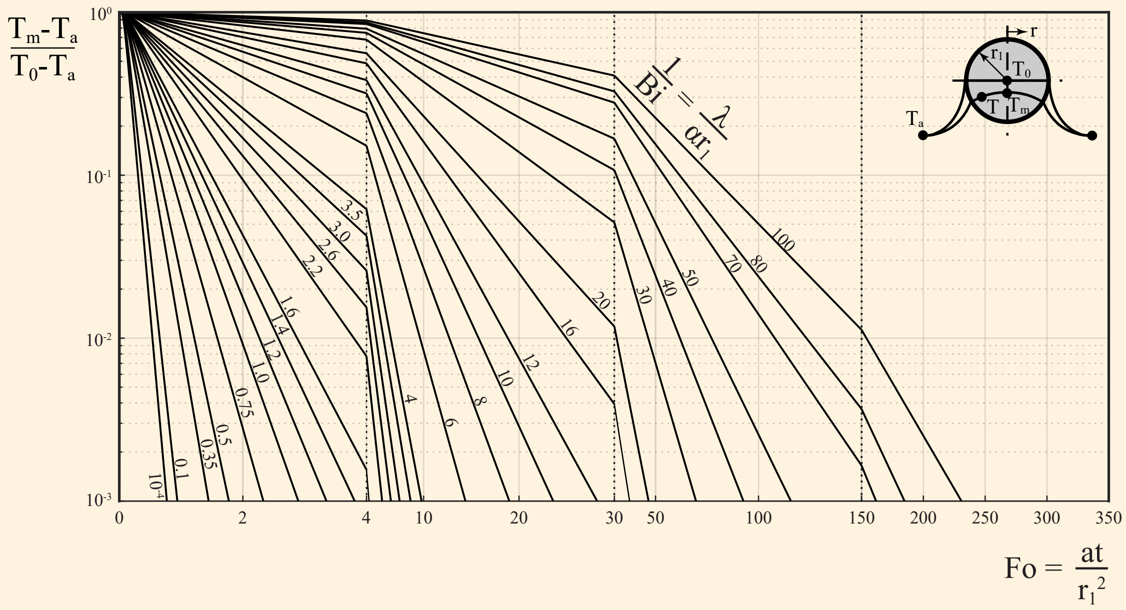
Heat loss of a cylinder [?]:



where $Q = mc\,(T(t) - T_\mathrm{a})$ and $Q_0 = mc\,(T_0 - T_\mathrm{a})$.

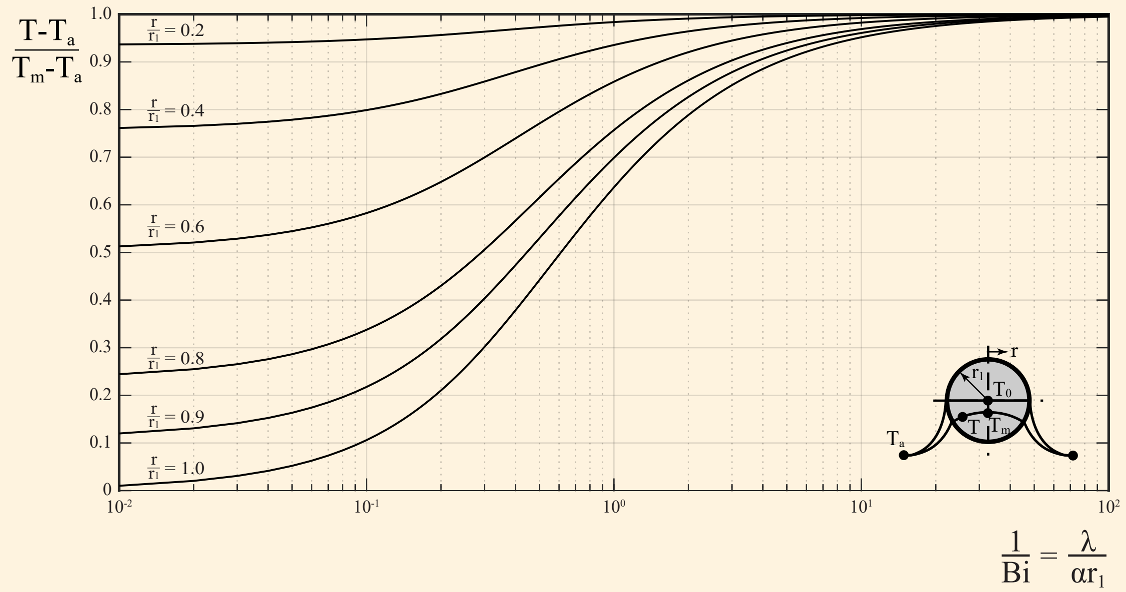
Fundamental EQ

Temperature in the centre of a sphere with radius r_1 [?]:



Fundamental EQ

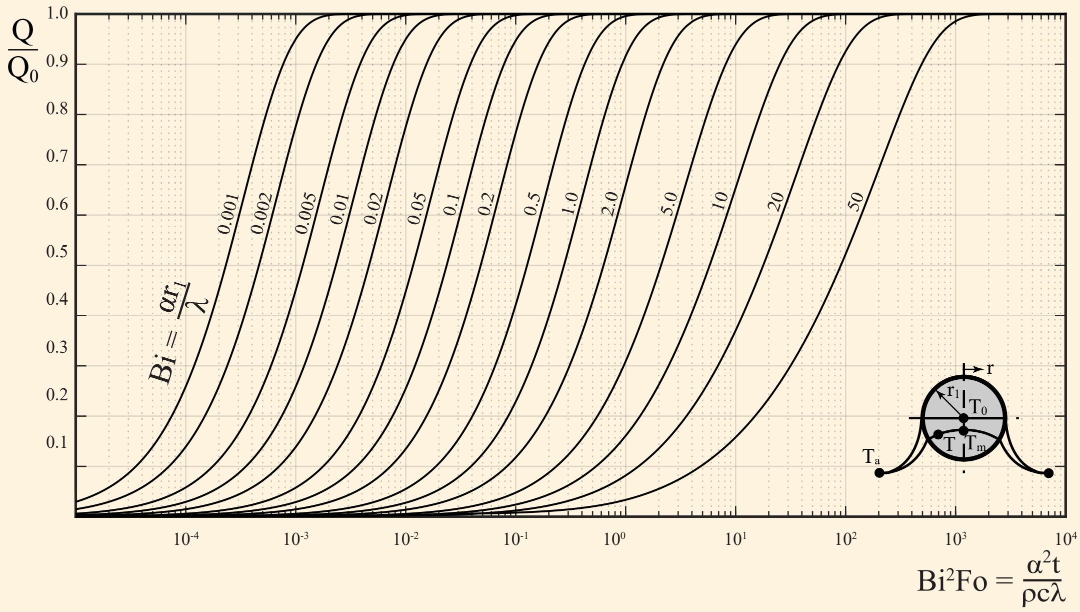
Temperature distribution in a sphere with radius r_1 [?]:



which is for Fo > 0.2.

Fundamental EQ

Heat loss of a sphere [?]:



where $Q = mc\,(T(t) - T_\mathrm{a})$ and $Q_0 = mc\,(T_0 - T_\mathrm{a})$.

SECTION K
Convection

K.1 Fundamentals

Fundamental EQ

Newton's law of cooling:

$$\dot{Q} = \alpha A \left(T_\mathrm{W} - T_\mathrm{A}\right). \tag{K.1}$$

Definition

Convective heat transfer coefficient:

$$\alpha = \frac{-\left(\lambda_\mathrm{f} \frac{\partial T_\mathrm{f}}{\partial y}\right)_\mathrm{W}}{T_\mathrm{W} - T_\mathrm{A}} = \frac{-\left(\lambda_\mathrm{s} \frac{\partial T_\mathrm{s}}{\partial y}\right)_\mathrm{W}}{T_\mathrm{W} - T_\mathrm{A}} \left(\frac{\mathrm{W}}{\mathrm{m^2 K}}\right). \tag{K.2}$$

Definition

Volumetric expansion coefficient:

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = \frac{\rho_\infty - \rho}{\rho\left(T - T_\infty\right)} \left(\frac{1}{\mathrm{K}}\right). \tag{K.3}$$

Definition

Volumetric expansion coefficient for ideal gases:

$$\beta = \frac{1}{T} \left(\frac{1}{\mathrm{K}}\right). \tag{K.4}$$

K.2 Conservation equations

Fundamental EQ

Equation of continuity:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0. \tag{K.5}$$

Fundamental EQ

Equations of momentum:

$$\rho\left(\vec{u} \cdot \nabla\right)\vec{u} = -\nabla p + \eta \nabla^2 \vec{u} + \rho \vec{g}\,. \tag{K.6}$$

Fundamental EQ

Equation of energy conservation:

$$\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = \frac{\lambda}{c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{\Phi}'''. \tag{K.7}$$

K.3 Boundary layer equations laminar flow

Fundamental EQ

Velocity boundary layer thickness for a linear velocity profile:

$$\frac{\delta_\mathrm{u}}{x} \approx \sqrt{\frac{12}{\mathrm{Re_x}}}, \tag{K.8}$$

for $\mathrm{Re_x} < 2 \cdot 10^5$.

Fundamental EQ

Velocity boundary layer thickness for a linear velocity profile:

$$\frac{\delta_\mathrm{T}}{x} \approx \left(\frac{\lambda}{\eta c_p}\right)^{1/3} \sqrt{\frac{12\eta}{\rho \mathrm{u}_\infty \mathrm{x}}} = \frac{1}{\mathrm{Pr}^{1/3}} \sqrt{\frac{12}{\mathrm{Re_x}}}, \tag{K.9}$$

for $\mathrm{Re_x} < 2 \cdot 10^5$, and $0.6 < \mathrm{Pr} < 10$.

Fundamental EQ

Velocity boundary layer thickness for flow over a flat plate:

$$\frac{\delta_\mathrm{u}}{x} = \frac{4.91}{\sqrt{\mathrm{Re_x}}}, \tag{K.10}$$

for $\mathrm{Re_x} < 2 \cdot 10^5$.

Fundamental EQ

Thermal boundary layer thickness for flow over a flat plate:

$$\frac{\delta_\mathrm{T}}{x} = \frac{4.91}{\sqrt{\mathrm{Re_x}}\mathrm{Pr}^{1/3}}, \tag{K.11}$$

for $\mathrm{Re_x} < 2 \cdot 10^5$, and $0.6 < \mathrm{Pr} < 10$.

K.4 Internal forced convection

Fundamental EQ

Heat transfer rate in ducts:

$$\dot{Q} = \overline{\alpha} A \Delta T_{\mathrm{m}}, \tag{K.12}$$

where ΔT_{m} is a representative temperature difference between the temperature at the wall T_{W} and the energetically averaged caloric mean temperature of the fluid T_{fl}.

Definition

LMTD for flow along ducts with isothermal surface:

$$\Delta T_{\mathrm{m}} = \Delta T_{\mathrm{ln}} = \frac{\Delta T_{\mathrm{in}} - \Delta T_{\mathrm{out}}}{\ln \frac{\Delta T_{\mathrm{in}}}{\Delta T_{\mathrm{out}}}} \ (\mathrm{K}), \tag{K.13}$$

where $\Delta T_{\mathrm{in}} = T_{\mathrm{in}} - T_{\mathrm{w}}$, and $\Delta T_{\mathrm{out}} = T_{\mathrm{out}} - T_{\mathrm{w}}$.

Definition

Mean temperature difference for flow along ducts with a constant impressed heat flux:

$$\Delta T_{\mathrm{m}} = \left(T_{\mathrm{W}} - T_{\mathrm{fl}}\right)_{\mathrm{m}} \ (\mathrm{K}). \tag{K.14}$$

Definition

Average fluid velocity:

$$u_{\mathrm{m}} = \frac{\int\limits_{A_{\mathrm{c}}} \rho u \, dA_{\mathrm{c}}}{\int\limits_{A_{\mathrm{c}}} \rho \, dA_{\mathrm{c}}} \left(\frac{\mathrm{m}}{\mathrm{s}}\right). \tag{K.15}$$

Definition

Caloric mean temperature:

$$T_{\mathrm{m}} = \frac{\int\limits_{A_{\mathrm{c}}} \rho u c_{\mathrm{p}} T \, dA_{\mathrm{c}}}{\int\limits_{A_{\mathrm{c}}} \rho u c_{\mathrm{p}} dA_{\mathrm{c}}} \ (\mathrm{K}). \tag{K.16}$$

Fundamental EQ

Laminar hydrodynamic entry length:

$$L_{\mathrm{h}} \approx 0.05 \, \mathrm{Re_d} \, d. \tag{K.17}$$

Fundamental EQ

Laminar thermodynamic entry length:

$$L_{\mathrm{th}} \approx 0.05 \, \mathrm{Re_d Pr} \, d. \tag{K.18}$$

Fundamental EQ

Turbulent hydrodynamic entry length:

$$L_{\mathrm{h}} \approx 10 \, d. \tag{K.19}$$

Fundamental EQ

Turbulent thermodynamic entry length:

$$L_{\mathrm{th}} \approx 10 \, d. \tag{K.20}$$

Section H

Heat transfer correlations

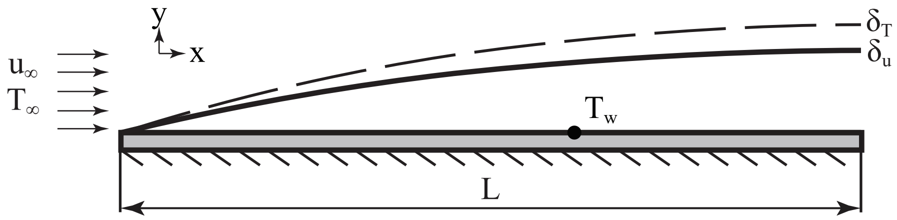
H.1 External forced convection

Definition

Fluid property temperature external forced convection:

$$T_{\text{prop}} = \frac{T_{\text{W}} + T_{\infty}}{2} \ \text{(K)}, \tag{H.1}$$

if not stated otherwise.



HTC

Local Nusselt number for forced laminar flow over a flat plate with isothermal surface:

$$\text{Nu}_{\text{x}} = 0.332 \text{Re}_{\text{x}}^{\frac{1}{2}} \text{Pr}^{\frac{1}{3}}, \tag{HTC.1}$$

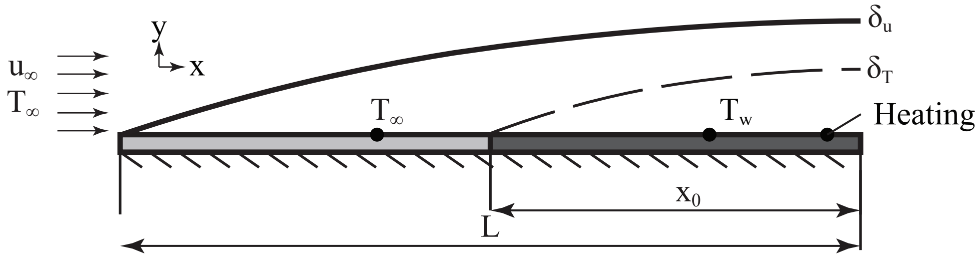
for $\text{Re}_{\text{x}} < 2 \cdot 10^5$, $0.6 < \text{Pr} < 10$, and where $\text{Re}_{\text{x,crit}} \approx 2 \cdot 10^5$.

HTC

Average Nusselt number for forced laminar flow over a flat plate with isothermal surface:

$$\overline{\text{Nu}}_{\text{L}} = 0.664\, \text{Re}_{\text{L}}^{\frac{1}{2}} \text{Pr}^{\frac{1}{3}}, \tag{HTC.2}$$

for $\text{Re}_{\text{L}} < 2 \cdot 10^5$, $0.6 < \text{Pr} < 10$, and where $\text{Re}_{\text{x,crit}} \approx 2 \cdot 10^5$.



HTC

Local Nusselt number for forced laminar flow over a flat plate with isothermal surface and first hydrodynamic inflow:

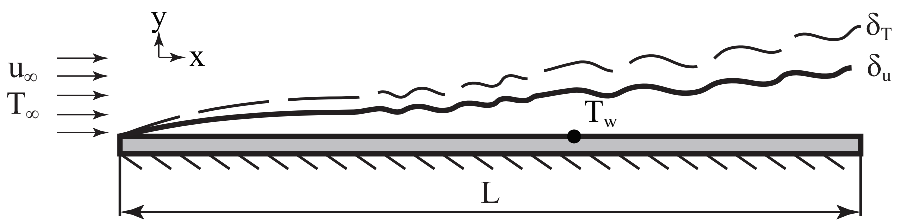
$$\text{Nu}_{\text{x}} = 0.332\, \text{Re}_{\text{x}}^{\frac{1}{2}} \text{Pr}^{\frac{1}{3}} \left[1 - \left(\frac{x_0}{x}\right)^{\frac{3}{4}}\right]^{-\frac{1}{3}}, \tag{HTC.3}$$

for $\text{Re}_{\text{x}} < 2 \cdot 10^5$, $0.6 < \text{Pr} < 10$, and where $\text{Re}_{\text{x,crit}} \approx 2 \cdot 10^5$.

HTC **Average Nusselt number for forced laminar flow over a flat plate with isothermal surface and first hydrodynamic inflow:**

$$\overline{\mathrm{Nu}}_{\mathrm{L}} = \frac{L}{L - x_0} \frac{1}{\lambda} \int_{x_0}^{L} \alpha(x)\, dx = 0.664\, \mathrm{Re_L}^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{3}} \frac{\left[1 - \left(\frac{x_0}{L}\right)^{\frac{3}{4}}\right]^{\frac{2}{3}}}{\left[1 - \frac{x_0}{L}\right]}, \tag{HTC.4}$$

for $\mathrm{Re_L} < 2 \cdot 10^5$, $0.6 < \mathrm{Pr} < 10$, and where $\mathrm{Re_{x,crit}} \approx 2 \cdot 10^5$.



HTC **Local Nusselt number for forced turbulent flow over a flat plate with isothermal surface:**

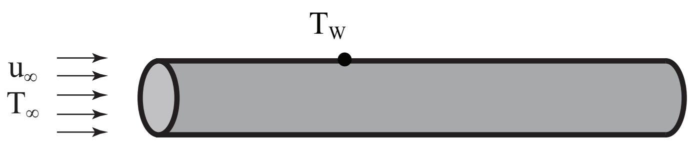
$$\mathrm{Nu_x} = 0.0296\, \mathrm{Re_x}^{0.8}\, \mathrm{Pr}^{0.43}, \tag{HTC.5}$$

for $5 \cdot 10^5 < \mathrm{Re_x} < 10^7$, where $\mathrm{Re_{x,crit}} \approx 2 \cdot 10^5$.

HTC **Average Nusselt number for forced turbulent flow over a flat plate with isothermal surface:**

$$\overline{\mathrm{Nu}}_{\mathrm{L}} \approx 0.036\, \mathrm{Pr}^{0.43} \left(\mathrm{Re_L}^{0.8} - 9400\right), \tag{HTC.6}$$

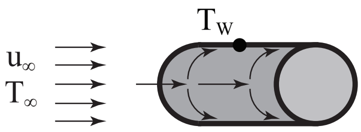
for $5 \cdot 10^5 < \mathrm{Re_L} < 10^7$, where $\mathrm{Re_{x,crit}} \approx 2 \cdot 10^5$.



Criterion **Criterion for approximating parallel flow along a cylinder's longitudinal axis as flow over a flat plate:**

$$d \gg \delta.$$

If so, HTC.1 to HTC.6 are applicable.



HTC **Average Nusselt number for forced flow perpendicular to the longitudinal axis of a circular cylinder with isothermal surface:**

$$\overline{\mathrm{Nu}}_\mathrm{d} = C\,\mathrm{Re_d}^m \mathrm{Pr}^{1/3}, \tag{HTC.7}$$

for $\mathrm{Pr} > 0.7$, where:

$\mathrm{Re_d}$	C	m
0.4 - 4	0.989	0.330
4 - 40	0.911	0.385
40 - 4000	0.683	0.466
4000 - 40,000	0.193	0.618
40,000 - 400,000	0.0266	0.805

HTC **Average Nusselt number for forced flow perpendicular to the longitudinal axis of a circular cylinder with isothermal surface:**

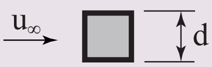
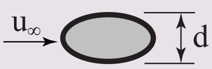
$$\overline{\mathrm{Nu}}_\mathrm{d} = \left(0.4\,\mathrm{Re_d}^{\frac{1}{2}} + 0.06\,\mathrm{Re_d}^{\frac{2}{3}}\right)\mathrm{Pr}^{0.4}\left(\frac{\eta_\infty}{\eta_\mathrm{w}}\right)^{\frac{1}{4}}, \tag{HTC.8}$$

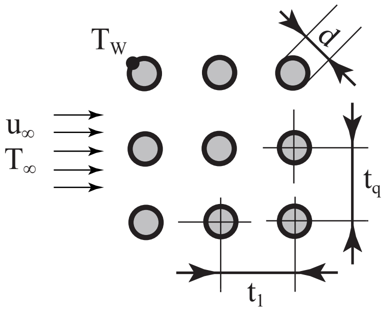
for $1.0 < \mathrm{Re}\cdot 10^5$, $0.67 < \mathrm{Pr} < 300$, $0.25 < \frac{\eta_\infty}{\eta_\mathrm{w}} < 5.2$, and where $T_\mathrm{prop} = T_\infty$.

HTC **Average Nusselt number for forced flow perpendicular to the longitudinal axis of a non-circular cylinder with isothermal surface:**

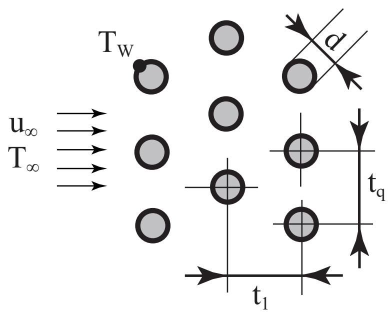
$$\overline{\mathrm{Nu}}_\mathrm{d} = C\,\mathrm{Re_d}^m \mathrm{Pr}^{1/3}, \tag{HTC.9}$$

where:

Cross-section	$\mathrm{Re_d}$	C	m	Medium
	5000-100,000	0.246	0.588	Gas
	5000-100,000	0.102	0.675	Gas
	5000-19,500 19,500-100,000	0.160 0.0385	0.638 0.782	Gas
	5000-100,000	0.153	0.638	Gas
	4000-15,000	0.228	0.731	Gas
	2500-15,000	0.248	0.612	Gas



(a) In-line arrangement



(b) Staggered arrangement

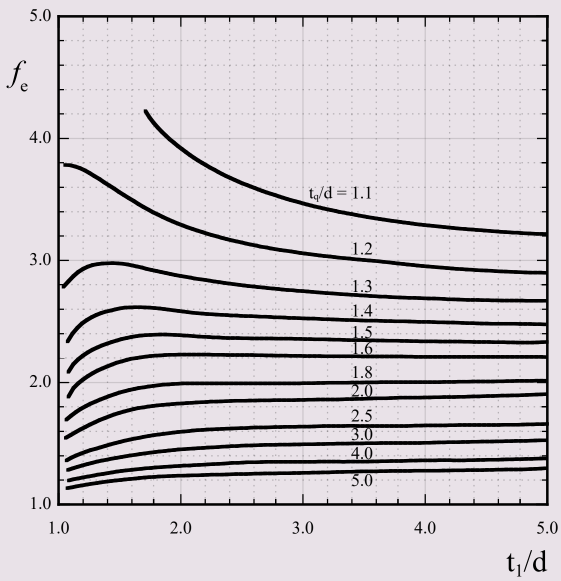
HTC

Average Nusselt number for forced flow perpendicular to the longitudinal axis of a bundle of smooth tubes:

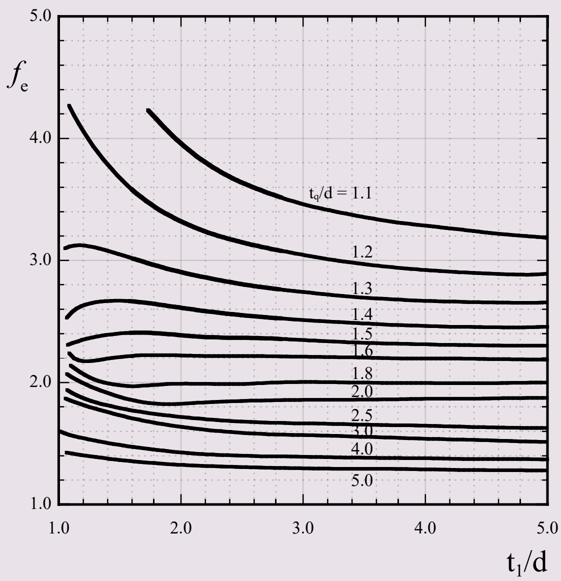
$$\overline{\mathrm{Nu}}_{\mathrm{d}} = 0.287\,\mathrm{Re}_{\mathrm{d}}^{0.6}\mathrm{Pr}^{0.36} \cdot f_{\mathrm{e}}, \qquad \text{(HTC.10)}$$

where $T_{\text{prop}} = \frac{T_{\text{out}}+T_{\text{in}}}{2}$.

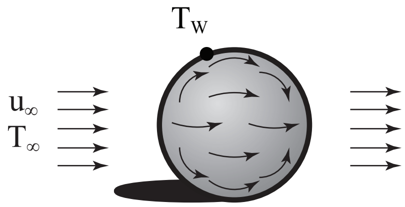
The tube arrangement factor f_{e} depends on the relative longitudinal distance/diameter ratio t_{l}/d, the relative transverse distance/diameter ratio t_{q}/d of the tubes, as well as the tube arrangement. This factor can be retrieved from:



(a) In-line arrangement



(b) Staggered arrangement.



HTC

Average Nusselt number for forced flow around a sphere with isothermal surface:

$$\overline{\mathrm{Nu}}_\mathrm{d} = 2 + \left(0.4\,\mathrm{Re_d}^{1/2} + 0.06\,\mathrm{Re_d}^{2/3}\right)\mathrm{Pr}^{0.4}\left(\frac{\eta_\infty}{\eta_\mathrm{w}}\right)^{1/4}, \qquad \text{(HTC.11)}$$

for $3.5 < \mathrm{Re_d} < 7.6 \cdot 10^4$, $0.7 < \mathrm{Pr} < 380$, $1.0 < \frac{\eta_\infty}{\eta_\mathrm{w}} < 3.2$, and where $T_\mathrm{prop} = T_\infty$.

H.2 Internal forced convection

Definition

Fluid property temperature internal forced convection:

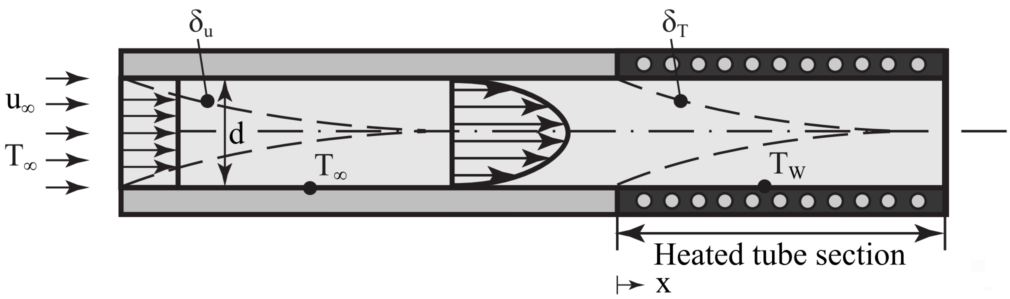
$$T_\mathrm{prop} = \frac{T_\mathrm{out} + T_\mathrm{in}}{2}\ (\mathrm{K}), \qquad \text{(H.2)}$$

if not stated otherwise.

Definition

Characteristic length of pipes:

$$d_\mathrm{h} = 4\,\frac{\text{cross-section area}}{\text{wetted perimeter}} = 4\,\frac{A_\mathrm{c}}{U}\ (\mathrm{m}). \qquad \text{(H.3)}$$

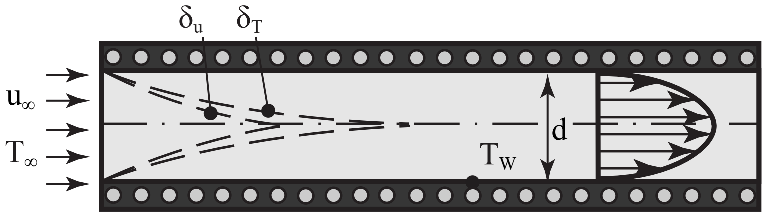


HTC

Average Nusselt number for forced laminar flow within a circular pipe being hydrodynamic developed at the start of the isothermal heated or cooled section:

$$\overline{\mathrm{Nu}}_\mathrm{d} = \left(3.66 + \frac{0.19\left(\mathrm{Re_d Pr}\frac{d}{L}\right)^{0.8}}{1 + 0.117\left(\mathrm{Re_d Pr}\frac{d}{L}\right)^{0.467}}\right)\left(\frac{\eta}{\eta_\mathrm{W}}\right)^{0.14}, \qquad \text{(HTC.12)}$$

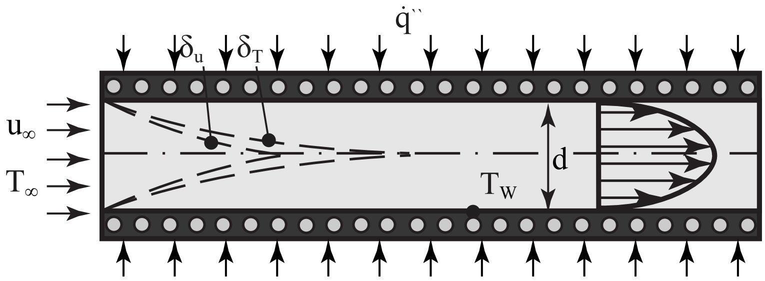
for $\mathrm{Re_d} < 2300$, where $\mathrm{Re_{d,crit}} \approx 2300$.



HTC **Average Nusselt number for forced laminar flow within an isothermal circular pipe with simultaneous hydrodynamic and thermal start:**

$$\overline{\mathrm{Nu}}_{\mathrm{d}} = \left(3.66 + \frac{0.0677\left(\mathrm{Re_d}Pr\frac{\mathrm{d}}{\mathrm{L}}\right)^{1.33}}{1+0.1Pr\left(\mathrm{Re_d}\frac{\mathrm{d}}{\mathrm{L}}\right)^{0.83}}\right)\left(\frac{\eta}{\eta_{\mathrm{W}}}\right)^{0.14}, \quad \text{(HTC.13a)}$$

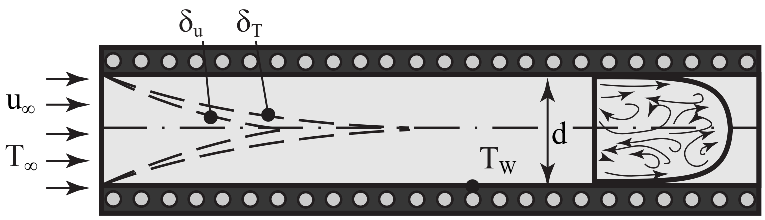
for $\mathrm{Re_d} < 2300$, where $\mathrm{Re_{d,crit}} \approx 2300$.



HTC **Average Nusselt number for forced laminar flow within a circular pipe being fully developed with impressed heat flow:**

$$\overline{\mathrm{Nu}}_{\mathrm{d}} = 4.36, \quad \text{(HTC.13b)}$$

for $\mathrm{Re_d} < 2300$ and $L_{\mathrm{th}} \ll L$, where $\mathrm{Re_{d,crit}} \approx 2300$.



HTC **Average Nusselt number for forced turbulent flow within a pipe with simultaneous hydrodynamic and thermal start:**

$$\overline{\mathrm{Nu}}_{\mathrm{d}} = 0.0235\left(\mathrm{Re_d^{0.8}} - 230\right)\left(1.8\mathrm{Pr}^{0.3} - 0.8\right)\left(1 + \left(\frac{\mathrm{d}}{\mathrm{L}}\right)^{\frac{2}{3}}\right)\left(\frac{\eta}{\eta_{\mathrm{W}}}\right)^{0.14}, \quad \text{(HTC.14)}$$

for $\mathrm{Re_d} > 2300$, $0.6 < \mathrm{Pr} < 500$ and $\frac{\mathrm{L}}{\mathrm{d}} > 1$, where $\mathrm{Re_{d,crit}} \approx 2300$.

HTC

Average Nusselt number for forced turbulent flow within a pipe being fully developed:

$$\overline{\mathrm{Nu}}_\mathrm{d} = 0.027\mathrm{Re}_\mathrm{d}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\eta}{\eta_\mathrm{W}}\right)^{0.14}, \tag{HTC.15}$$

for $3000 < \mathrm{Re}_\mathrm{d} < 10^5$ and $\frac{\mathrm{L}}{\mathrm{d}} > 40$, where $\mathrm{Re}_{\mathrm{d,crit}} \approx 2300$.

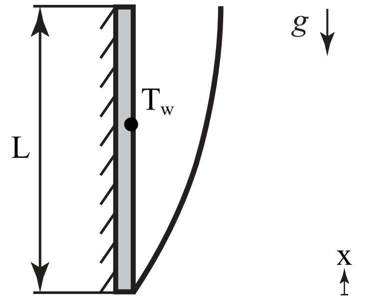
H.3 External natural convection

Definition

Fluid property temperature external natural convection:

$$T_\mathrm{prop} = \frac{T_\mathrm{W} + T_\infty}{2}\ (\mathrm{K}), \tag{H.4}$$

if not stated otherwise.



HTC

Local Nusselt number for natural laminar flow along a vertical plate with isothermal surface:

$$\mathrm{Nu}_x = 0.508\left(\frac{\mathrm{Pr}}{0.952+\mathrm{Pr}}\right)^{\frac{1}{4}}(\mathrm{Gr}_\mathrm{x}\mathrm{Pr})^{\frac{1}{4}}, \tag{HTC.16}$$

for $\mathrm{Gr}_\mathrm{x} \cdot \mathrm{Pr} < 4 \cdot 10^9$.

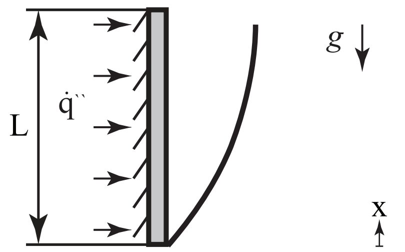
HTC

Average Nusselt number for natural laminar flow along a vertical plate with isothermal surface:

$$\overline{\mathrm{Nu}}_\mathrm{L} = C\,(\mathrm{Gr}_\mathrm{L}\mathrm{Pr})^{\frac{1}{4}}, \tag{HTC.17}$$

for $\mathrm{Gr}_\mathrm{L} \cdot \mathrm{Pr} < 4 \cdot 10^9$, and:

Pr	0.003	0.01	0.03	0.72	1	2	10	100	1000	∞
C	0.182	0.242	0.305	0.516	0.535	0.568	0.620	0.653	0.665	0.670



Definition

Modified Grashof number for natural flow along a vertical plate with an impressed heat flux:

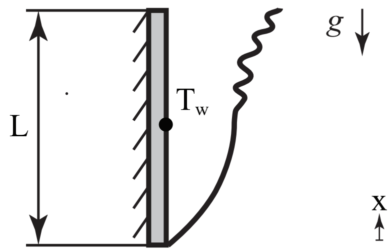
$$\mathrm{Gr}_\mathrm{x}^* = \mathrm{Gr}_\mathrm{x}\mathrm{Nu}_\mathrm{x} = \frac{\rho^2 g \beta \dot{q}_\mathrm{W}'' x^4}{\lambda \eta^2} \quad (-) \tag{H.5}$$

HTC

Local Nusselt number for natural laminar flow along a vertical plate with an impressed heat flux:

$$\mathrm{Nu}_\mathrm{x} = 0.60\left(\mathrm{Gr}_\mathrm{x}^*\mathrm{Pr}\right)^{\frac{1}{5}}, \tag{HTC.18}$$

for $10^5 < \mathrm{Gr}_\mathrm{x}^* < 10^{11}$.

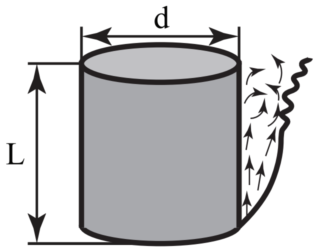


HTC

Average Nusselt number for natural turbulent flow along a vertical plate with isothermal surface:

$$\overline{\mathrm{Nu}}_\mathrm{L} = 0.13\left(\mathrm{Gr}_\mathrm{L}\mathrm{Pr}\right)^{\frac{1}{3}}, \tag{HTC.19}$$

for $10^9 < \mathrm{Gr}_\mathrm{L} \cdot \mathrm{Pr} < 10^{12}$

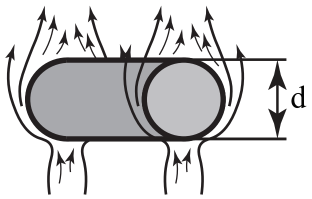


Criterion

Criterion for approximating a vertical cylinder as a vertical plate:

$$\frac{\mathrm{d}}{\mathrm{L}} > 35 \cdot \mathrm{Gr}_\mathrm{L}^{-\frac{1}{4}}.$$

If so, HTC.16 to HTC.19 are applicable.



HTC

Average Nusselt number for natural laminar flow around a horizontal cylinder with isothermal surface:

$$\overline{\mathrm{Nu}}_{\mathrm{d}} = 0.53\,(\mathrm{Gr_d Pr})^{\frac{1}{4}}\,, \tag{HTC.20}$$

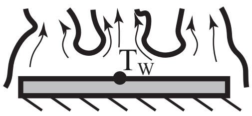
for $10^4 < \mathrm{Gr_d Pr} < 10^9$.

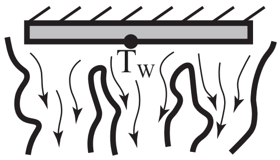
HTC

Average Nusselt number for natural turbulent flow around a horizontal cylinder with isothermal surface:

$$\overline{\mathrm{Nu}}_{\mathrm{d}} = 0.13\,(\mathrm{Gr_d Pr})^{\frac{1}{3}}\,, \tag{HTC.21}$$

for $10^9 < \mathrm{Gr_d Pr} < 10^{12}$.





Definition

Characteristic length horizontal plates:

$$L = \frac{\text{Surface area}}{\text{Perimeter}} = \frac{A}{U}\ (\mathrm{m}). \tag{H.6}$$

HTC

Average Nusselt number for natural laminar flow over a horizontal plate with a heated upper or cooled lower:

Isothermal surface:

$$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.54\,(\mathrm{Gr_L Pr})^{\frac{1}{4}}\,, \tag{HTC.22a}$$

for $2 \cdot 10^4 < \mathrm{Gr_L Pr} < 8 \cdot 10^6$.

Impressed heat flow:

$$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.13\,(\mathrm{Gr_L Pr})^{\frac{1}{3}}\,, \tag{HTC.22b}$$

for $\mathrm{Gr_L Pr} < 2 \cdot 10^8$.

HTC

Average Nusselt number for natural turbulent flow over a horizontal plate with a heated upper or cooled lower:

Isothermal surface:

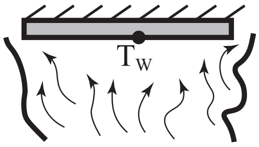
$$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.15\,(\mathrm{Gr_L Pr})^{\frac{1}{3}}\,, \tag{HTC.23a}$$

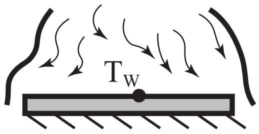
for $8 \cdot 10^6 < \mathrm{Gr_L Pr} < 10^{11}$.

Impressed heat flow:

$$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.16\,(\mathrm{Gr_L Pr})^{\frac{1}{3}}\,, \tag{HTC.23b}$$

for $2 \cdot 10^8 < \mathrm{Gr_L Pr} < 10^{11}$.





HTC

Average Nusselt number for natural laminar flow over a horizontal plate with a cooled upper or heated lower:

Isothermal surface:

$$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.27\,(\mathrm{Gr_L Pr})^{\frac{1}{4}}\,, \tag{HTC.24a}$$

for $10^5 < \mathrm{Gr_L Pr} < 10^{10}$.

Impressed heat flow:

$$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.58\,(\mathrm{Gr_L Pr})^{\frac{1}{5}}\,, \tag{HTC.24b}$$

for $10^6 < \mathrm{Gr_L Pr} < 10^{11}$.

H.4 Internal natural convection

Definition

Fluid property temperature internal natural convection:

$$T_{\mathrm{prop}} = \frac{T_1 + T_2}{2}\ (\mathrm{K}), \tag{H.7}$$

if not stated otherwise.

Definition

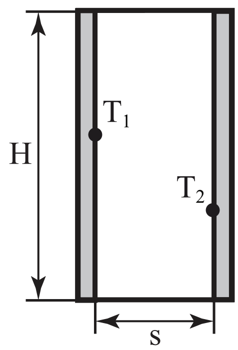
Characteristic length enclosures:

$$L = s\ (\mathrm{m}). \tag{H.8}$$

Definition

Grashof number enclosures:

$$\mathrm{Gr_s} = \frac{\beta g \rho^2\,(T_{\max} - T_{\min})\,s^3}{\eta^2}\ (-)\,. \tag{H.9}$$



HTC

Average Nusselt number for natural laminar flow within a vertical enclosure with isothermal surfaces:

<u>Negligible flow:</u>

$$\overline{\mathrm{Nu}}_\mathrm{s} = 1, \tag{HTC.25a}$$

for $\mathrm{Gr_s} < 2 \cdot 10^3$, and $3.1 < H/s < 42.2$.

<u>Laminar flow:</u>

$$\overline{\mathrm{Nu}}_\mathrm{s} = 0.20 \left(\frac{H}{s}\right)^{-\frac{1}{9}} (\mathrm{Gr_s Pr})^{\frac{1}{4}}, \tag{HTC.25b}$$

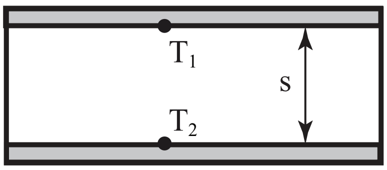
for $2 \cdot 10^3 < \mathrm{Gr_s} < 2 \cdot 10^4$, and $3.1 < H/s < 42.2$.

HTC

Average Nusselt number for natural turbulent flow within a vertical enclosure with isothermal surfaces:

$$\overline{\mathrm{Nu}}_\mathrm{s} = 0.071 \left(\frac{H}{s}\right)^{-\frac{1}{9}} (\mathrm{Gr_s Pr})^{\frac{1}{3}}, \tag{HTC.26}$$

for $2 \cdot 10^5 < \mathrm{Gr_s} < 10^7$, and $3.1 < H/s < 42.2$.



HTC

Average Nusselt number for natural laminar flow within a horizontal enclosure with isothermal surfaces:

<u>Negligible flow:</u>

$$\mathrm{Nu}_\mathrm{s} = 1, \tag{HTC.27a}$$

for $\mathrm{Gr_s} < 2 \cdot 10^3$, or $T_1 > T_2$.

<u>Laminar flow:</u>

$$\overline{\mathrm{Nu}}_\mathrm{s} = 0.21 (\mathrm{Gr_s Pr})^{\frac{1}{4}}, \tag{HTC.27b}$$

for $10^4 < \mathrm{Gr_s} < 3.2 \cdot 10^5$.

HTC

Average Nusselt number for natural turbulent flow within a horizontal enclosure with isothermal surfaces:

$$\overline{\mathrm{Nu}}_\mathrm{s} = 0.075 (\mathrm{Gr_s Pr})^{\frac{1}{3}}, \tag{HTC.28}$$

for $3.2 \cdot 10^5 < \mathrm{Gr_s} < 10^7$.

Section R

Radiation

R.1 Fundamentals

Definition

Wavelength in a medium:

$$\lambda = \frac{c}{\nu} \ (\mathrm{m}). \tag{R.1}$$

Fundamental EQ

Photon energy:

$$E = h\,\nu = \frac{hc}{\lambda}, \tag{R.2}$$

where Planck's constant $h = 6.626\ 10^{-34}\ \mathrm{J \cdot s}$.

Definition

Wavenumber:

$$\eta = \frac{1}{\lambda}\left(\frac{1}{\mathrm{m}}\right). \tag{R.3}$$

Fundamental EQ

Planck's distribution law:

$$\dot{q}''_{\mathrm{b}\lambda} = \frac{c_1 \lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1}\left(\frac{\mathrm{W}}{\mathrm{m}^2}\frac{1}{\mathrm{m}}\right), \tag{R.4}$$

where b refers to being a black body, and λ to being wavelength-specific. Besides, $c_1 = 3.741 \cdot 10^{-16}\ \mathrm{Wm}^2$ and $c_2 = 1.439 \cdot 10^{-2}\ \mathrm{mK}$.

Fundamental EQ

Wien's law of displacement:

$$\lambda_{\max} = \frac{2898\ \mu\mathrm{mK}}{T}. \tag{R.5}$$

Fundamental EQ

Stefan-Boltzmann law:

$$\dot{q}''_b = \int_0^\infty \dot{q}''_{b\lambda}\, d\lambda = \sigma T^4, \tag{R.6}$$

where the Stefan-Boltzmann constant $\sigma = 5.67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}$.

Fundamental EQ

Black body radiation in the spectral range between 0 and λ:

$$\dot{q}''_{b,0\to\lambda} = F_{0\to\lambda T} \cdot \sigma T^4, \tag{R.7}$$

where:

λT	(μmK)	1000	1250	1500	1750	2000	2500
$F_{0\to\lambda T}$	(–)	0.00031	0.00308	0.01283	0.03363	0.06663	0.16115
λT	(μmK)	3000	3500	4000	5000	6000	8000
$F_{0\to\lambda T}$	(–)	0.27322	0.38250	0.48085	0.63315	0.73715	0.85556

Fundamental EQ

Black body radiation in the spectral range between λ_1 and λ_2:

$$\dot{q}''_{b,\lambda_1\to\lambda_2} = \left(F_{0\to\lambda_2 T} - F_{0\to\lambda_1 T}\right)\cdot\sigma T^4. \tag{R.8}$$

Definition

Total reflectivity:

$$\rho = \frac{\text{Total radiation reflected}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda\rho} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \frac{\int_0^\infty \rho_\lambda \dot{q}''_{\lambda 0} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda}\ (-). \tag{R.9}$$

Definition

Total absorptivity:

$$\alpha = \frac{\text{Total radiation absorbed}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda\alpha} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \frac{\int_0^\infty \alpha_\lambda \dot{q}''_{\lambda 0} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda}\ (-). \tag{R.10}$$

Definition

Total transmissivity:

$$\tau = \frac{\text{Total radiation transmitted}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda\tau} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \frac{\int_0^\infty \tau_\lambda \dot{q}''_{\lambda 0} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda}\ (-). \tag{R.11}$$

Definition

Total emissivity:

$$\varepsilon = \frac{\text{Total radiation emitted}}{\text{Total blackbody radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda\varepsilon} d\lambda}{\int_0^\infty \dot{q}''_{\lambda b} d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda \dot{q}''_{\lambda b} d\lambda}{\int_0^\infty \dot{q}''_{\lambda b} d\lambda}\ (-). \tag{R.12}$$

Fundamental EQ

Relation between ρ_λ, α_λ, and τ_λ for a real body:

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1. \tag{R.13}$$

Fundamental EQ

Relation between ρ, α, and τ for bodies:

$$\rho + \alpha + \tau = 1. \tag{R.14}$$

Fundamental EQ

Kirchoff's law of thermal radiation:

$$\alpha_\lambda = \varepsilon_\lambda. \tag{R.15}$$

Fundamental EQ

Kirchoff's law of thermal radiation for total absorptivity and emissivity:

$$\alpha = \varepsilon, \tag{R.16}$$

if $T_{\text{rad}} = T_{\text{body}}$ or the surfaces of the body are grey.

R.2 View factors

Definition

View factor:

$$\Phi_{ij} = \frac{\text{Radiation leaving body } i \text{ intercepted by body } j}{\text{Radiation leaving body } i} = \frac{\dot{Q}_{ij}}{\dot{Q}_i} \ (-)\,. \tag{R.17}$$

Definition

Radiosity or radiation density:

$$L = \frac{\text{Power of radiator}}{\text{Projected area} \times \text{Solid angle}} = \frac{\dot{q}''}{\Omega} \left(\frac{\text{W}}{\text{m}^2}\right). \tag{R.18}$$

Fundamental EQ

Reciprocity rule:

$$A_i \Phi_{ij} = A_j \Phi_{ji}. \tag{R.19}$$

Fundamental EQ

Summation rule:

$$\sum_{j=1}^{n} \Phi_{ij} = \Phi_{i1} + \Phi_{i2} + \Phi_{i3} + \ldots + \Phi_{in} = 1. \tag{R.20}$$

Fundamental EQ

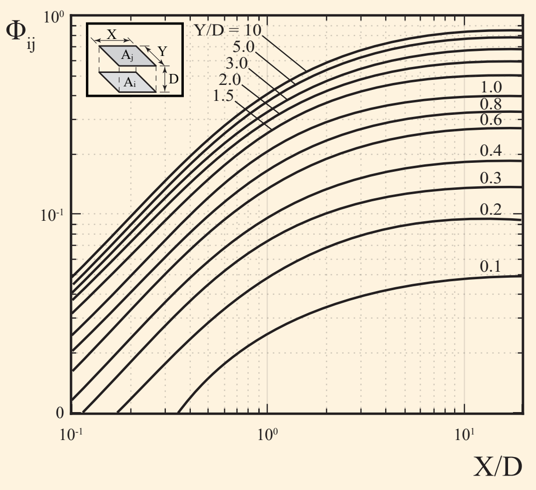
Symmetry rule:

$$\Phi_{ij} = \Phi_{ik}, \tag{R.21}$$

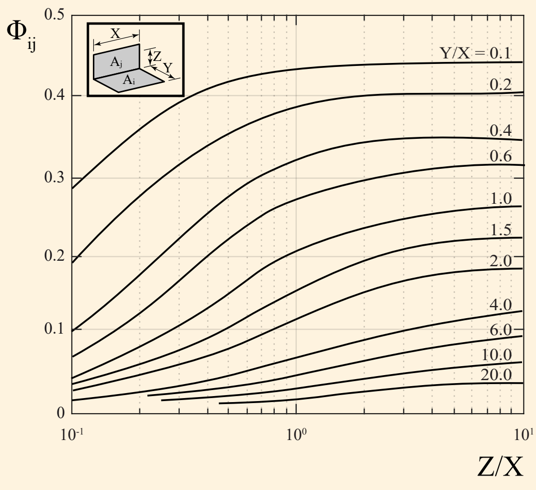
if two or more surfaces display symmetry about a third surface, they will have identical view factors from that surface.

Fundamental EQ

View factor diagrams:



(a) Radiation transfer between parallel, rectangular plates.



(b) Radiation transfer between perpendicular, rectangular plates.

R.3 Radiative transport

Definition

Surface brightness of surface i:

$$\dot{Q} = \dot{Q}_{i,\rho} + \dot{Q}_{i,\tau} + \dot{Q}_{i,\varepsilon} \text{ (W)}. \tag{R.22}$$

Fundamental EQ

Inner energy balance closed system without heat generation:

$$\frac{\partial U}{\partial t} = \alpha \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\varepsilon}. \tag{R.23}$$

Fundamental EQ

Outer energy balance closed system without heat generation:

$$\frac{\partial U}{\partial t} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}}. \tag{R.24}$$

Fundamental EQ

Net radiative heat flow from body i to n bodies:

$$\dot{Q}_{i,\text{net}} = \dot{Q}_{\text{i}} - \sum_{j=1}^{n} \dot{Q}_{j\to i}. \tag{R.25}$$

Fundamental EQ

Radiative heat exchange between bodies i and j:

$$\dot{Q}_{i\leftrightharpoons j} = \dot{Q}_{i\to j} - \dot{Q}_{j\to i}. \tag{R.26}$$

Fundamental EQ

Radiative heat exchange between two black bodies i and j:

$$\dot{Q}_{i\leftrightharpoons j} = A_i \Phi_{ij} \sigma \left(T_i^4 - T_j^4\right) = A_j \Phi_{ji} \sigma \left(T_i^4 - T_j^4\right). \tag{R.27}$$

Fundamental EQ

Radiative heat exchange between two grey plates i and j:

$$\dot{q}''_{i\leftrightharpoons j} = \frac{1}{\frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_j} - 1} \sigma \left(T_i^4 - T_j^4\right), \tag{R.28}$$

when the plates are plane, parallel, and infinitely long.

Fundamental EQ

Radiative heat exchange between two grey bodies i and j:

$$\dot{Q}_{i\leftrightharpoons j} = \frac{A_i}{\frac{1}{\varepsilon_i} + \frac{A_i}{A_j}\left(\frac{1}{\varepsilon_j} - 1\right)} \sigma \left(T_i^4 - T_j^4\right), \tag{R.29}$$

when body j encloses body i ($A_j > A_i$) and body i does not see itself ($\Phi_{ii} = 0$).

SECTION M
Mass Transfer

M.1 Fundamentals

Definition
Partial density of substance i:

$$\rho_i = \frac{\text{Total mass of substance } i}{\text{Volume of the medium}} = \frac{m_i}{V} \left(\frac{\text{kg}}{\text{m}^3}\right). \tag{M.1}$$

Definition
Total density:

$$\rho = \frac{\text{Total mass of the mixture } i}{\text{Volume of the medium}} = \frac{\sum_{i=1}^{n} m_i}{V} \left(\frac{\text{kg}}{\text{m}^3}\right). \tag{M.2}$$

Definition
Mass fraction of substance i:

$$\xi_\text{i} = \frac{\text{Total mass of substance } i}{\text{Total mass of the medium}} = \frac{m_i}{\sum_{j=1}^{n} m_j} = \frac{\rho_i \cdot V}{\sum_{j=1}^{n} \rho_j \cdot V} \quad (-)\,. \tag{M.3}$$

Definition
Number of moles of substance i:

$$n_i = \frac{\text{Number of entities } i}{\text{Avogadro constant}} = \frac{N_i}{N_\text{A}} \quad (\text{mol})\,, \tag{M.4}$$

where the the Avogadro constant $N_\text{A} = 6.022 \cdot 10^{23} \left(\frac{1}{\text{mol}}\right)$.

Definition
Molar concentration of substance i:

$$C_\text{i} = \frac{\text{Number of moles of substance } i}{\text{Unit volume}} = \frac{n_i}{V} \left(\frac{\text{mol}}{\text{m}^3}\right). \tag{M.5}$$

Definition
Total molar concentration:

$$C = \frac{\text{Number of moles of all mixture components}}{\text{Unit volume}} = \frac{\sum_{i=1}^{n} n_i}{V} \left(\frac{\text{mol}}{\text{m}^3}\right). \tag{M.6}$$

Definition
Mole fraction of substance i:

$$\psi_\text{i} = \frac{\text{Number of moles of substance } i}{\text{Total number of moles of the medium}} = \frac{n_i}{\sum_{j=1}^{n} n_j} = \frac{C_i}{C} \quad (-)\,. \tag{M.7}$$

Definition
Molar mass of substance i:

$$M_\text{i} = \frac{\text{Total mass of substance } i}{\text{Number of moles of substance } i} = \frac{m_i}{n_\text{i}} \left(\frac{\text{kg}}{\text{mol}}\right). \tag{M.8}$$

Definition

Mean molar mass:

$$\overline{M} = \frac{\text{Total mass of the mixture}}{\text{Total number of moles of the mixture}} = \frac{m_1 + m_2}{n_1 + n_2} = \frac{\rho}{C}\left(\frac{\text{kg}}{\text{mol}}\right). \tag{M.9}$$

Definition

Number of moles of substance i:

$$n_i = \frac{\text{Number of entities } i}{\text{Avogadro constant}} = \frac{N_i}{N_\text{A}} \;(\text{mol})\,, \tag{M.10}$$

where the the Avogadro constant $N_\text{A} = 6.022 \cdot 10^{23}\ \left(\frac{1}{\text{mol}}\right)$.

Definition

Relationship between the molar and mass fraction:

$$\psi_1 = \frac{\frac{\rho_1}{M_1}\frac{1}{\rho}}{\left(\frac{\rho_1}{M_1} + \frac{\rho_2}{M_2}\right)\frac{1}{\rho}} = \frac{\frac{\xi_1}{M_1}}{\frac{\xi_1}{M_1} + \frac{\xi_2}{M_2}}\ (-)\,. \tag{M.11}$$

Theorem

Dalton's Law of Partial Pressures:

$$p = \sum_{i=1}^{n} p_i = p_1 + p_2 + p_3 + \ldots + p_n. \tag{M.12}$$

Definition

Partial pressure of substance i:

$$p_i = \text{Mole fraction of substance } i \times \text{Total pressure} = \psi_i \cdot p\ \ (\text{Pa})\,. \tag{M.13}$$

Theorem

Ideal Gas Law:

$$p = R_\text{m} C T. \tag{M.14}$$

Theorem

Antoine equation:

$$\log p^* = A - \frac{B}{T + C}, \tag{M.15}$$

with p^* in mbar and T in °C.

M.2 Diffusive mass transfer

Fundamental EQ

Molar-based Fick's law:

$$\dot{n}''_\text{A} = -C D_\text{AB} \frac{\partial \psi_\text{A}}{\partial x}. \tag{M.16}$$

Fundamental EQ

Mass-based Fick's law:

$$j''_\text{A} = -\rho D_\text{AB} \frac{\partial \xi_\text{A}}{\partial x}. \tag{M.17}$$

Definition

Diffusion coefficient derived from the kinetic gas theory:

$$D_\text{AB} = 2\eta\,(\overline{u} l)\left(\frac{\text{m}^2}{\text{s}}\right). \tag{M.18}$$

Definition

Diffusion coefficient derived from the Stokes-Einstein equation:

$$D = \frac{k_\mathrm{B} T}{6\pi\eta r} \left(\frac{\mathrm{m}^2}{\mathrm{s}}\right). \tag{M.19}$$

Theorem

Transport equation without advective mass transfer:

$$\rho \frac{\partial \xi_i}{\partial t} = \rho D \left(\frac{\partial^2 \xi_i}{\partial x^2} + \frac{\partial^2 \xi_i}{\partial y^2} + \frac{\partial^2 \xi_i}{\partial z^2} \right) + \dot{m}_i'''. \tag{M.20}$$

M.3 Advective mass transfer

Theorem

Transport equation:

$$\rho \frac{\partial \xi_i}{\partial t} + \rho u \frac{\partial \xi_i}{\partial x} + \rho v \frac{\partial \xi_i}{\partial y} + \rho w \frac{\partial \xi_i}{\partial z} = \rho D \left(\frac{\partial^2 \xi_i}{\partial x^2} + \frac{\partial^2 \xi_i}{\partial y^2} + \frac{\partial^2 \xi_i}{\partial z^2} \right) + \dot{m}_i'''.$$

Theorem

Rate of convective mass transfer:

$$j_\mathrm{A}'' = g \left(\xi_\mathrm{W} - \xi_\infty \right). \tag{M.21}$$

Definition

Convective mass transfer coefficient:

$$g = \frac{-\left(\rho D \frac{\partial \xi_\mathrm{f}}{\partial y}\right)_\mathrm{W}}{\xi_\mathrm{W} - \xi_\mathrm{A}} \left(\frac{\mathrm{kg}}{\mathrm{m}^2\mathrm{s}}\right). \tag{M.22}$$

Theorem

Lewis' law:

$$g = \frac{\alpha}{c_\mathrm{p}}, \tag{M.23}$$

for $\mathrm{Le} \approx 1$.

M.4 Interphase mass transfer

Definition

Molar based Henry coefficient:

$$\mathrm{H}_i^{\mathrm{cc}} = \frac{C_{i,\mathrm{liq}}}{C_{i,\mathrm{gas}}} \ (-). \tag{M.24}$$

Definition

Mass based Henry coefficient:

$$\mathrm{H}_i^{*} = \mathrm{H}_i^{\mathrm{cc}} \frac{\rho_{\mathrm{tot,gas}}}{\rho_{\mathrm{tot,liq}}} = \frac{\xi_{i,\mathrm{liq}}}{\xi_{i,\mathrm{gas}}} \ (-). \tag{M.25}$$

Definition

Nernst coefficient (mass notation):

$$\mathrm{K_N} = \frac{\xi_{i,1}}{\xi_{i,2}} \ (-). \tag{M.26}$$

Theorem

Stefan flow mass transfer rate:

$$\dot{m}''_{\mathrm{A}} = g \cdot \frac{\xi_{\mathrm{A,S}} - \xi_{\mathrm{A},\infty}}{1 - \xi_{\mathrm{A,S}}}. \tag{M.27}$$

SECTION P

Properties of materials

P.1 Thermophysical - metals

Metals at 20°C.

	ρ 10^3 kg/m^3	c kJ/kg K	λ W/m K	a 10^{-6} m^2/s
Aluminum	2.70	0.888	237	98.80
Lead	11.34	0.129	35	23.90
Chromium	6.92	0.440	91	29.90
Iron	7.86	0.452	81	22.80
Gold	19.26	0.129	316	127.20
Copper	8.93	0.382	399	117.00
Magnesium	1.74	1.020	156	87.90
Manganese	7.42	0.473	21	6.00
Molybdenum	10.20	0.251	138	53.90
Sodium	9.71	1.220	133	11.20
Nickel	8.85	0.448	91	23.00
Platinum	21.37	0.133	71	25.00
Silver	10.50	0.235	427	173.00
Titanium	4.50	0.522	22	9.40
Wolfram	19.00	0.134	173	67.90
Zinc	7.10	0.387	121	44.00
Tin, white	7.29	0.225	67	40.80
Bronze	8.80	0.377	62	18.70
Cast iron	7.80	0.540	42...50	10...12
Carbon steel (<0.4% C)	7.85	0.465	42...50	12...15
Cr-Ni-steel (X12CrNi 18.8)	7.80	0.500	15	3.80

P.2 Thermophysical - non-metal solids

Non-metal solids at 20°C.

	ρ 10^3 kg/m^3	c kJ/kg K	λ W/m K	a 10^{-6} m^2/s
Acryl glass	1.18	1.44	0.184	0.108
Asphalt	2.12	0.92	0.7	0.36
Concrete	2.1	0.88	1	0.54
Ice (water 0 C)	0.917	2.04	2.25	1.203
Soil coarse gravel	2.04	1.84	0.52	0.14
Sand, dry	1.65	0.8	0.27	0.2
Sand, wet	1.75	1	0.58	0.33
Clay	1.45	0.88	1.28	1
Glass.				
window	2.48	0.7	0.87	0.5
mirror	2.7	0.8	0.76	0.35
quarz	2.21	0.73	1.4	0.87
Glass wool	1.2	0.66	0.046	0.58
Gypsum	1	1.09	0.51	0.47
Granite	2.75	0.89	2.9	1.18
Cork	0.19	1.88	0.041	0.115
Marble	2.6	0.8	2.8	1.35
Mortar	1.9	0.8	0.93	0.61
Paper	0.7	1.2	0.12	0.14
Polyethylene	0.92	2.3	0.35	0.17
Polytetrafluorethylene	2.2	1.04	0.23	0.1
PVC	1.38	0.96	0.15	0.11
Porcelain (95 C)	2.4	1.08	1.03	0.4
Hard coal	1.35	1.26	0.26	0.15
Fir wood (radial)	0.415	2.72	0.14	0.12
Plaster	1.69	0.8	0.79	0.58
Bricks	1.6...1.8	0.84	0.38...0.52	0.28...0.34

P.3 Thermophysical - liquids

Liquids at 1 bar.

	T	ρ	c	λ	ν	a	Pr
	°C	10^3 kg/m^3	kJ/kg K	W/m K	10^{-6} m^2/s	10^{-6} m^2/s	1
Nitrogen	-190	0.861	1.988	0.161	0.321	0.0939	3.42
Water	0	0.9998	4.218	0.561	1.793	0.133	13.48
	20	0.9982	4.181	0.598	1.004	0.1434	7.001
	40	0.9922	4.177	0.631	0.658	0.1521	4.3280
	60	0.9832	4.184	0.654	0.475	0.1591	2.983
	80	0.9718	4.197	0.67	0.365	0.1643	2.221
	99.63	0.9586	4.216	0.679	0.295	0.168	1.757
Aqueous non-organic solution							
21% NaCl	-10	1.187	3.312	0.528	4.02	0.136	29.5
Benzene	20	0.879	1.738	0.154	0.74	0.101	7.33
Methanol	20	0.792	2.495	0.22	0.737	0.111	6.57
Fuel oil	20	0.819	2	0.116	1.82	0.0709	25.7
	100	0.766	2.38	0.104	0.711	0.0572	12.4
Mercury	20	13.55	0.139	9.3	0.115	4.9	0.023

P.4 Thermophysical - gases

Gases at 1 bar.

	T	ρ	c	λ	ν	a	Pr
	°C	kg/m^3	kJ/kg K	10^{-3} W/m K	10^{-6} m^2/s	10^{-6} m^2/s	1
Air	-200	5.106	1.186	6.886	0.979	1.137	0.8606
	-100	2.019	1.011	16.2	5.829	7.851	0.7423
	0	1.275	1.006	24.18	13.52	18.83	0.7179
	20	1.188	1.007	25.69	15.35	21.47	0.7148
	40	1.112	1.007	27.16	17.26	24.24	0.7122
	80	0.9859	1.01	30.01	21.35	30.14	0.7083
	100	0.9329	1.012	31.39	23.51	33.26	0.707
	200	0.7356	1.026	37.95	35.47	50.3	0.7051
	400	0.517	1.069	49.96	64.51	90.38	0.7137
	600	0.3986	1.116	61.14	99.63	137.5	0.7247
	800	0.3243	1.155	71.54	140.2	191	0.7342
	1000	0.2734	1.185	80.77	185.9	249.2	0.7458
Steam	100	0.5896	2.042	25.08	20.81	20.83	0.999
	200	0.4604	1.975	33.28	35.14	36.6	0.96
	400	0.3223	2.07	54.76	75.86	82.07	0.9243
	600	0.2483	2.203	79.89	131.4	146.1	0.8993
	800	0.2019	2.343	107.3	199.9	226.8	0.8816
	1000	0.1702	2.478	163.3	280	323.2	0.8665
Hydrogen	0	0.0886	14.24	176	95	139	0.68
	50	0.0748	14.36	202	126	188	0.67
	100	0.0649	14.44	229	159	244	0.65
Carbon dioxide	0	1.95	0.829	14.3	7.1	8.86	0.8
	50	1.648	0.875	17.8	9.8	12.3	0.8
	100	1.428	0.925	21.3	12.4	16.1	0.8
Helium	27	0.1625	5.193	155.7	122.6	184.5	0.655

P.5 Emissivity - solids

Emissivity of various solids (Total emissivity ε, Emissivity in normal direction of the surface ε_n).

Surface	T K	ε_n	ε	**Surface**	T K	ε_n	ε
Metals				Zinc, highly polished	500		0.045
Aluminum, plain	443	0.039	0.049		600		0.055
...polished	373	0.095		Iron plate, galvanized			
...heavily oxidized	366	0.2		...plain	301	0.228	
	777	0.31		...grey oxidized	297	0.276	
Aluminum oxide	550	0.63		Tin, non oxidized	298		0.043
	1100	0.26			373	0.05	
	1089	0.052		**Non-Metals**			
Chromium, polished	423	423	423	Asbestos, paper	296	0.96	
				...Papier	311	0.93	
Gold. highly polished	500	0.018			644	0.94	
	900	900		Concrete, rough	273 – 366		0.94
Copper, polished	293	0.03		Roofing felt	294	0.91	
...struck	293	0.037		Gips	293	0.8 – 0.9	
...black oxidized	293	0.78		Glas	293	0.94	
...oxidized	403	0.76		Quartz (7ṁm thick)	555	0.93	
Inconel, rolled	1089		0.69		1111	0.47	
...sandblasted	1089		0.79	Rubber	293	0.92	
Iron and steel,				Wood			
...highly polished	450	0.052		Oak, planed	273 – 366		0.9
...polished	700	0.144		Beech	343	0.94	0.91
	1300	0.377		Ceramics			
...sanded	293	0.242		White Al_2O_3	366		0.9
Cast iron, polished	473	0.21		Carbon			
Cast steel, polished	1044	0.52		...not oxidized	298		0.81
	1311	0.56			773		0.79
Iron sheet				...Fibers	533		0.95
...heavy rusty	292	0.685		...Graphite	373		0.76
...rolled	294	0.657		Corundum, rough	353	0.85	0.84
Cast iron.				Coating, colors:			
...oxidized at 866 K	472	0.64		Oil paint black	366		0.92
	872	0.78		...green	366		0.95
Steel,				...red	366		0.97
...oxidized at 866 K	472	0.79		...white	373		0.94
	872	0.79		Coating. white	373	0.925	
Brass, not oxidized	298	0.035		...flat black	353		0.97
	373	0.035		Bakelite coating	353	0.935	
...oxidized	473	0.61		Mennig color	373	0.93	
	873	0.59		Radiator (acc. to VDI-74)	373	0.925	
	1673	0.17		Enamel, white on iron	292	0.897	
Nickel, not oxidized	298	0.045		Marble			
	373	0.06		light grey. polished	273 – 366		0.9
	873	0.478		Paper	273		0.92
...oxidized	473	0.37			366		0.94
Platinum	422	0.022		Porcelain, white	295		0.924
	1089	0.123		Clay, glassy	298		0.9
Mercury,				...flat	298		0.93
...not oxidized	298	0.1		Water	273	0.95	
	373	0.12			373	0.96	
Silver, polished	311	0.022		Ice, smooth with water	273	0.966	0.92
	644	0.031		...rough surface	273	0.985	
Wolfram	298		0.024	Bricks, red	273 – 366		0.93
	1273		0.15				
	1773		0.23				

SECTION X

Mathematical function

X.1 Error functions

Definition

Error function

$$\mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_{\xi=0}^{\xi=\eta} \exp\left(-\xi^2\right) d\xi, \tag{X.1}$$

which has the characteristic $\mathrm{erf}(\infty) = 1 \quad \mathrm{erf}(-\eta) = -\mathrm{erf}(\eta) \quad \frac{d}{d\eta}[\mathrm{erf}(\eta)] = \frac{2}{\sqrt{\pi}} \exp\left(-\eta^2\right)$.

Definition

Complementary error function

$$\mathrm{erfc}(\eta) = 1 - \mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_{\xi=\eta}^{\xi=\infty} \exp\left(-\xi^2\right) d\xi. \tag{X.2}$$

Evaluation of the error function.

η	$\mathrm{erf}(\eta)$	$\mathrm{erfc}(\eta)$	$^2/_{\sqrt{\pi}} \exp\left(-\eta^2\right)$
0	0	1	1.128
0.05	0.056	0.944	1.126
0.1	0.112	0.888	1.117
0.15	0.168	0.832	1.103
0.2	0.223	0.777	1.084
0.25	0.276	0.724	1.060
0.3	0.329	0.671	1.031
0.35	0.379	0.621	0.998
0.4	0.428	0.572	0.962
0.45	0.475	0.525	0.922
0.5	0.520	0.480	0.879
0.55	0.563	0.437	0.834
0.6	0.604	0.396	0.787
0.65	0.642	0.378	0.740
0.7	0.678	0.322	0.691
0.75	0.711	0.289	0.643
0.8	0.742	0.258	0.595
0.85	0.771	0.229	0.548
0.9	0.797	0.203	0.502
0.95	0.821	0.179	0.458
1	0.843	0.157	0.415
1.1	0.880	0.120	0.337
1.2	0.910	0.090	0.267
1.3	0.934	0.066	0.208
1.4	0.952	0.048	0.159
1.5	0.966	0.034	0.119
1.6	0.976	0.024	0.087
1.7	0.984	0.016	0.063
1.8	0.989	0.011	0.044
1.9	0.993	0.007	0.030
2	0.995	0.005	0.021