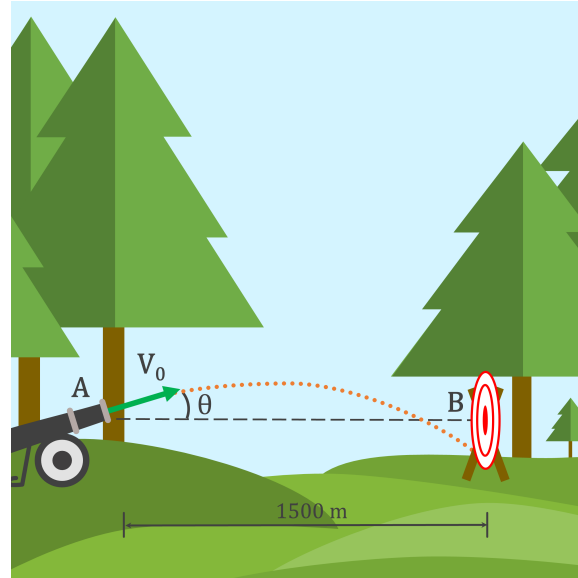


## Bullet on Target Bottom



A cannon fires a bullet from A toward a target B. Find an expression for the vertical displacement  $y(t)$  as a function of time,  $v_0$  and  $\theta$ , for which the round just hits the bottom of the target. The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is 1500 m. Neglect air resistances and assume that the bullet is directed along the vertical centreline of the target. Take  $g = 10 \text{ m/s}^2$ .

$y(t) = \dots = \dots$

*Using known expressions (for constant acceleration):*

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (v_0 + at) dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (v_0 + at) dt \quad (5)$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \quad (6)$$

*Given quantities:*

Distance A-B:  $s_{AB} = 1500$  m

Gravitational acceleration:  $g = 10$  m/s<sup>2</sup>

Initial velocity:  $v_0 = 900$  m/s

Target diameter:  $D = 2$  m

*Solution:*

Filling in Equation (6) gives an relation for the y-position with respect to time. Where  $a = -g$  and  $y_0 = 0$  m, since the cannon barrel is at the same altitude as the target centre.

$$y(t) = -\frac{1}{2}gt^2 + v_{0,y}t + y_0 = -\frac{1}{2}gt^2 + v_0t \sin \theta \quad (7)$$

The bottom of the target is at 1 m from the centreline thus  $y(t_{\text{end}}) = -1$  m. Furthermore, inserting  $g = 10$  m/s<sup>2</sup> result in.

$$y(t) = -\frac{1}{2} \cdot 10 \cdot t^2 + v_0t \sin \theta = 1 \quad \Rightarrow \quad -5 \cdot t^2 + v_0t \sin \theta = -1 \quad (8)$$