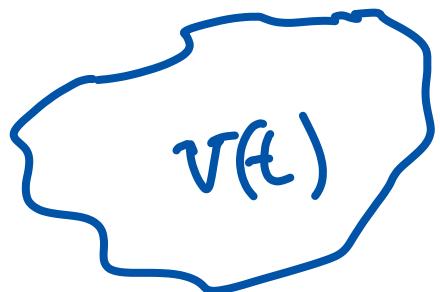


# Lecture # 10

# Energy Conservation



$$\text{Energy}(t) \equiv \int P E dV$$

$V(t)$

$$E = e + \frac{1}{2} u_k u_h$$

↑  
 total (J/kg)  
 ↑  
 internal (thermal)  
 { kinetic

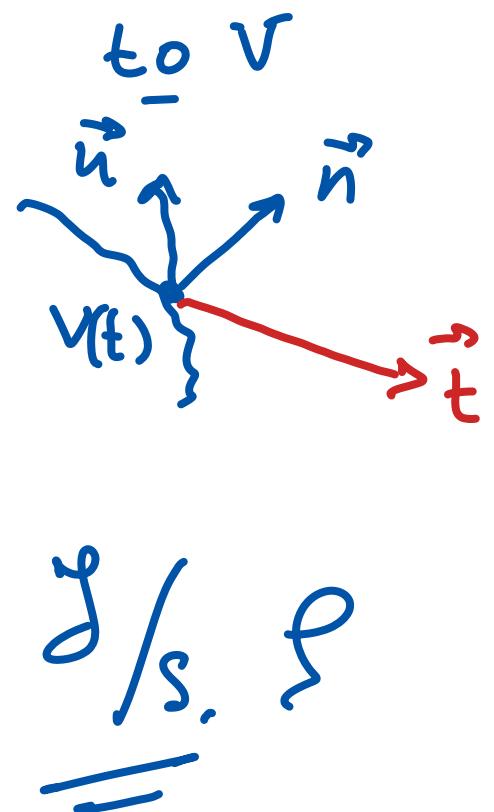
## 1<sup>st</sup> law of thermodynamics:

$$\frac{d}{dt} (\text{Energy}) = \underbrace{\frac{d}{dt} (\text{Work})}_{\text{on } V} + \underbrace{\frac{d}{dt} (\text{Heat})}_{\text{to } V}$$

$$\frac{d}{dt}(\text{Work}) = \int t_i u_i \, du_i$$

$S(t)$

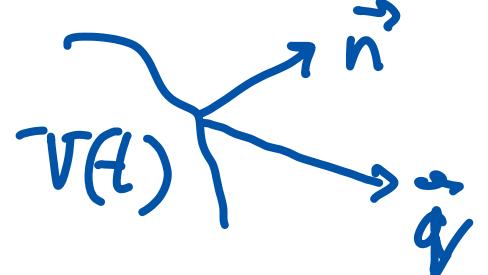
$$+ \int \rho g_i \cdot u_i \, dV$$



$$[t_i; u_i; dS] = \frac{N}{m^2} \frac{m}{S} \cancel{u^2} = \frac{Nm}{S} = \frac{g}{S}$$

if  $\vec{t} \perp \vec{u}$  : Work done is zero

$$\frac{d}{dt}(\text{Heat}) = - \int_{S(t)} q_i \cdot n_i dS$$



$$\frac{d}{dt} \text{Energy} = \frac{d}{dt} \int_{V(t)} \rho E dV$$

$$= \int_{V(t)} \frac{\partial}{\partial t} (\rho E) dV + \int_{S(t)} \rho E u_j n_j dS$$

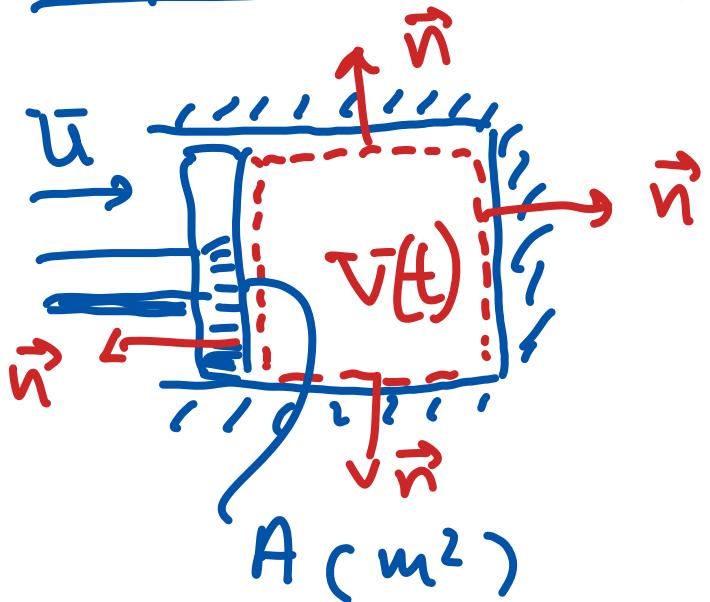
$$\Rightarrow \int_{V(t)} \frac{\partial}{\partial t} (\rho E) dV + \int_{S(t)} \rho E u_j n_j dS$$

$$= \int_{S(t)} \sigma_{ij} u_i n_j dS + \int_{V(t)} \rho g_i u_i dV$$

$$+ \int_{S(t)} k \frac{\partial T}{\partial x_i} n_i dS$$

Energy conservation in integral form.

## Application Example



Slow:  $\frac{1}{2} u_k u_h \ll c$   
 $\Rightarrow E \approx e$

Uniform:  $T, c, \rho, p$   
 $\sim \text{uniform}$ .

" $\mu = 0$ " neglect shear stress  
 "  $k = 0$ " " heat conducta  
 "  $g = 0$ " " gravity.

Assume: ideal gas:

$$p = pRT$$

$R$ : specific gas constant  
 ↳ depends on the gas.  $\text{J/kg, K}$

also:  $e = c_v T$

↳ specific heat at  
 constant volume  
 $(\gamma \text{ J/kg, K})$

$$c_v \equiv \left( \frac{\partial e}{\partial T} \right)_v$$

$$c_p \equiv \left( \frac{\partial h}{\partial T} \right)_p$$

$$c_p - c_v = R \quad c_p/c_v \equiv \gamma$$

$c_v, c_p, R$  constant.

Apply Energy conservation:

$$\int_{V(t)} \underbrace{\frac{\partial}{\partial t}(\rho E)}_{\text{uniform.}} d\bar{V} \approx \frac{\partial}{\partial t}(\rho E) \cdot \int_{V} d\bar{V}$$
$$= \frac{\partial}{\partial t}(\rho e) V$$
$$= \frac{\partial}{\partial t}(\rho c_v T) V$$

$$\int_{S(t)} \rho E u_i n_i dS \approx \rho E \int_{S(t)} u_i n_i dS \approx \rho e \frac{dV}{dt}$$
$$= \rho c_v T \frac{dV}{dt}$$

$$\int_{S(t)} \sigma_{ij} u_i n_j dS \approx \int_{S(t)} -\rho \sigma_{ij} u_i n_j dS$$

$$= - \int_{S(t)} \rho u_j n_j dS \approx -\rho \frac{dV}{dt}$$

$$\int_{V(t)} \rho g_i u_i d\bar{V} \approx 0 \quad "g=0" \quad \text{uniform.}$$

$$\int_{S(t)} k \frac{\partial T}{\partial x_i} n_i dS \approx 0 \quad "k=0", \frac{\partial T}{\partial x_i} = 0$$

$$V \frac{\partial}{\partial t} (\rho c_v T) + \rho c_v T \frac{dV}{dt} = \frac{d}{dt} (\rho c_v T V)$$

mass conservation:  $\rho V = \text{constant}$

$$\Rightarrow \frac{d}{dt} (\rho c_v T V) = \rho V c_v \frac{dT}{dt}.$$

$$\text{Energy conv: } \rho V c_v \frac{dT}{dt} = -P \frac{dV}{dt} = -\rho R T \frac{dV}{dt}$$

$$\frac{1}{\rho} : \quad c_v \cdot V \frac{dT}{dt} = -R \cdot T \frac{dV}{dt}$$

$$\frac{1}{V T} : \quad c_v \frac{1}{T} \frac{dT}{dt} = -R \frac{1}{V} \frac{dV}{dt}$$

$$\Rightarrow c_v \frac{d}{dt} \ln T = -R \frac{d}{dt} \ln V$$

$$\Rightarrow \frac{d}{dt} \ln T + \frac{R}{c_v} \frac{d}{dt} \ln V = 0$$

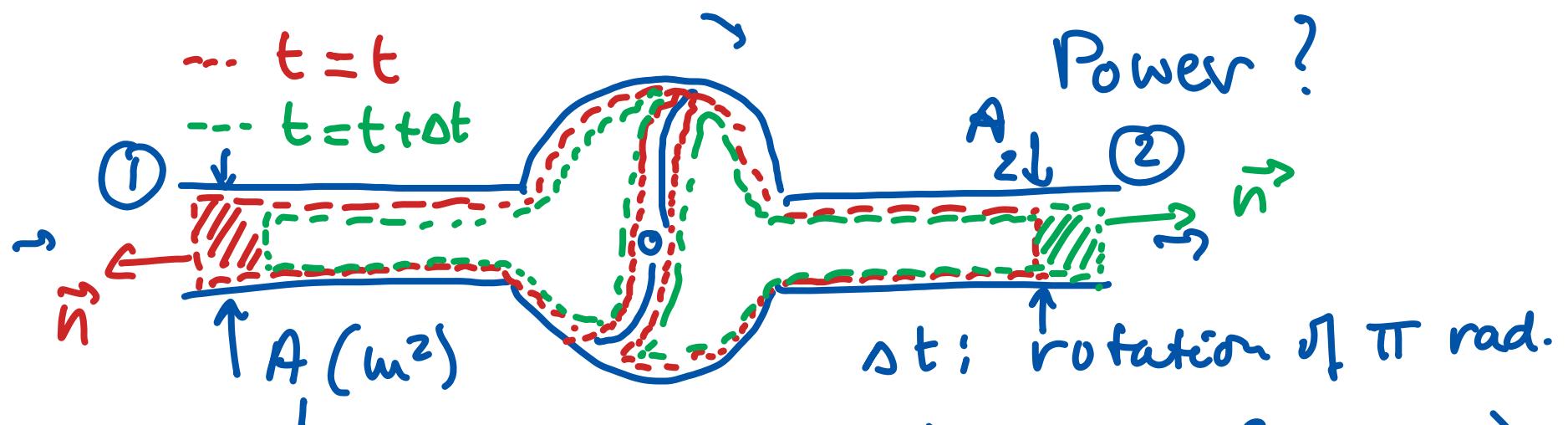
$$\frac{R}{c_v} = \gamma - 1 \Rightarrow \frac{d}{dt} \left( \ln T + \ln V^{\gamma-1} \right) = 0$$

$$\Rightarrow \frac{d}{dt} \ln (T V^{\gamma-1}) = 0$$

$$\Rightarrow \frac{d}{dt} ( ) = 0 \Rightarrow \boxed{T V^{\gamma-1} = \text{constant}}$$

entropy is constant  $\Rightarrow$  reversible.

## Application Example



$$\frac{d}{dt} \int_V \rho E dV \approx \frac{\left( \int_V \rho E dV \right)_{t+\Delta t} - \left( \int_V \rho E dV \right)_t}{\Delta t}$$

exact when  $\Delta t \rightarrow 0$ .

$$= \frac{1}{\Delta t} \left\{ \left( \rho E A \dot{\mu} \right)_2 \Delta t - \left( \rho E A \dot{\mu} \right)_1 \Delta t \right\}$$

$$= \dot{m} (E_2 - E_1), \quad \dot{m} \equiv \rho A \dot{\mu} \left( \frac{\text{kg}}{\text{s.}} \right)$$

$\dot{\mu} = 0$  at ①, ②

$$\int_{S(t)} \sigma_{ij} u_i \cdot n_j dS \approx \int_{S_1} -\rho u_i \cdot n_i dS + \int_{S_1} -\rho u_i \cdot n_i dS$$

$\downarrow$

$$+ \underbrace{\int_{\text{blade}} \sigma_{ij} u_i \cdot n_j dS}_{=0} + \underbrace{\int_{\text{wall}} \sigma_{ij} u_i \cdot n_j dS}_{=0}$$

$\dot{W} \equiv \text{Work by the blade on } \bar{V}$

$$\text{Similar } \dot{Q} \equiv \int_{S(t)} -k \frac{\partial T}{\partial x_i} n_i dS \text{ to } \bar{V}$$

neglect gravity.

note that  $\int_{S_1} -p u_i n_i dS \stackrel{\text{uniform}}{\downarrow} = (-p - \bar{u} \cdot A)$ ,

$$= \left( \frac{p}{\bar{p}} \rho \bar{u} A \right)_1 = \left( \frac{p}{\bar{p}} \right)_1 \cdot \dot{m}$$

at  $S_2$ :  $- \left( \frac{p}{\bar{p}} \right)_2 \cdot \dot{m}$   $\Delta() \equiv ()_2 - ()_1$

$$\Rightarrow \dot{m} \Delta E + \dot{m} \Delta \left( \frac{p}{\bar{p}} \right) = \dot{w} + \dot{Q}$$

$$\Rightarrow \boxed{\dot{m} (H_2 - H_1) = \dot{w} + \dot{Q}}$$
 Compressor equation.  
$$H \equiv E + \frac{p}{\rho}$$
 Total enthalpy