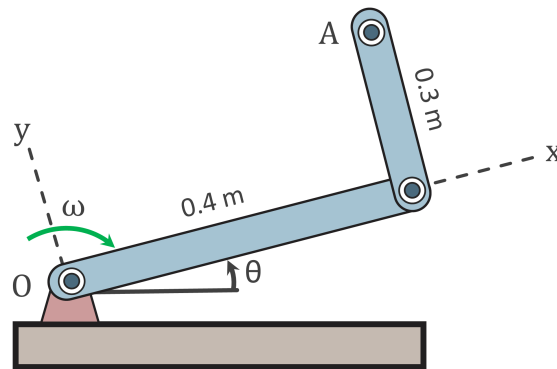# Acceleration at the End of a Right-Angled Bar

The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4 rad/s². Determine the vector expression for the acceleration of point A when $\omega = 2$ rad/s.

Using known expressions (for rigid bodies and a (relative) fixed point O):

$$\mathbf{a}_{A,x} = \boldsymbol{\omega}_{OA} \times (\boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O}) \tag{1}$$

$$\mathbf{a}_{A,y} = \boldsymbol{\alpha}_{OA} \times \mathbf{r}_{A/O} \tag{2}$$

$$\mathbf{a}_{A} = \mathbf{a}_{A,x} + \mathbf{a}_{A,y} \tag{3}$$

Given quantities:

Distance from O to A: $\mathbf{r}_{A/O} = 0.4\mathbf{i} + 0.3\mathbf{j}$
Absolute angular velocity: $\omega = 2$ rad/s
Angular velocity: $\boldsymbol{\omega}_{OA} = -\omega\mathbf{k}$
Absolute angular acceleration: $\alpha_{OA} = 4$ rad/s²
Angular acceleration: $\boldsymbol{\alpha}_{OA} = \alpha_{OA}\mathbf{k}$

Solution:

Since the angular velocity is clockwise, and in the coordinate system positive is defined as counterclockwise, the angular velocity becomes: $\omega_{OA} = -2$ rad/s. The angular acceleration is clockwise and decreasing at a rate of 4 rad/s², thus this becomes: $-1 \cdot -4 = 4$ rad/s² in the defined coordinate system.

Inserting this in Equation (1) and (2) gives:

$$\mathbf{a}_{\mathrm{A},x} = \boldsymbol{\omega}_{\mathrm{OA}} \times (\boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}}) = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 0.4 \\ 0.3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1.6 \\ -1.2 \\ 0 \end{pmatrix} \ \mathrm{m/s}^2 \quad (4)$$

$$\mathbf{a}_{\mathrm{A},y} = \boldsymbol{\alpha}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0.4 \\ 0.3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1.2 \\ 1.6 \\ 0 \end{pmatrix} \ \mathrm{m/s}^2 \quad (5)$$

Combining both gives the final answer for $\mathbf{a}_{\mathrm{A}}$:

$$\mathbf{a}_{\mathrm{A}} = \mathbf{a}_{\mathrm{A},x} + \mathbf{a}_{\mathrm{A},y} = \begin{pmatrix} -1.6 \\ -1.2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1.2 \\ 1.6 \\ 0 \end{pmatrix} = \begin{pmatrix} -2.8 \\ 0.4 \\ 0 \end{pmatrix} \ \mathrm{m/s}^2 \quad (6)$$