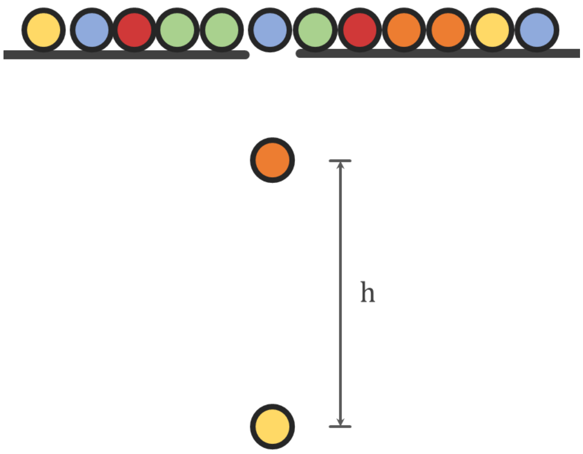
Falling Marbles



Marbles fall from rest through an opening at a steady rate of three per second. Determine the vertical separation h of two consecutive marbles when the lower one has dropped 7 meters.

Neglect air resistance and take $g = 10$ m/s^2.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Lowest drop distance: 7 m

Gravitational acceleration: $g = 10$ m/s^2

Solution:

The marbles fall from rest, thus $v_0 = 0$ m/s. Seen from the top $s_0 = 0$ m/s, thus Equation (6) simplifies to:

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \quad \Rightarrow \quad s(t) = \frac{1}{2}at^2 \tag{7}$$

Where $a = -g = -10$ m/s^2 (since it points in the negative y-direction). The time it takes for the ball to drop 7 m ($s = -7$ m) can be calculated using:

$$s(t) = \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2 \cdot s}{a}} = \sqrt{\frac{2 \cdot -7}{-10}} \approx 1.18 \text{ s} \tag{8}$$

Three balls are dropped every second at a steady rate, thus one ball is dropped every $\frac{1}{3}$ s, a ball is dropped. It took 1.18 seconds for the first ball to reach 7 m, and after $\frac{1}{3}$ s, a second ball was dropped. This second ball has dropped for $\frac{1}{3}$ s less time than the first. The distance the second ball traveled is thus:

$$s(t) = \frac{1}{2}at^2 \quad \Rightarrow \quad s\left(1.18 - \frac{1}{3}\right) = \frac{1}{2} \cdot -10 \cdot \left(1.18 - \frac{1}{3}\right)^2 \approx -3.61 \text{ m} \tag{9}$$

Thus the distance between two consecutive marbles is $h = -3.61 - -7 = 3.39$ m.