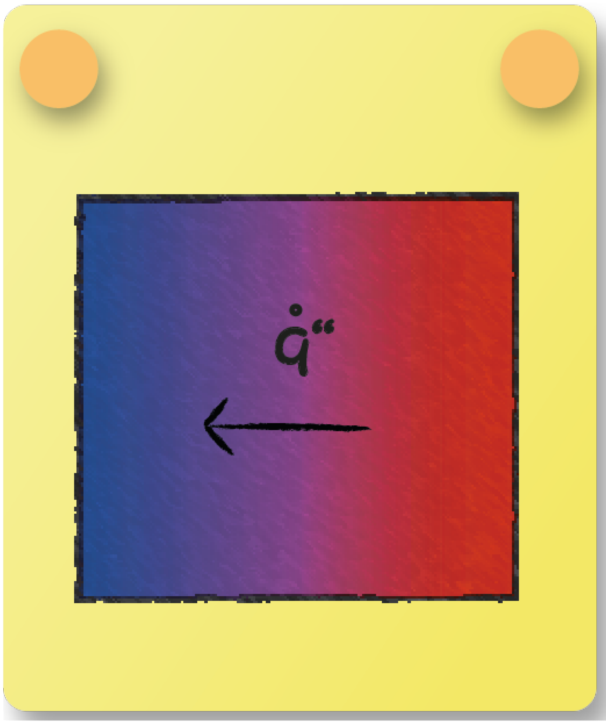
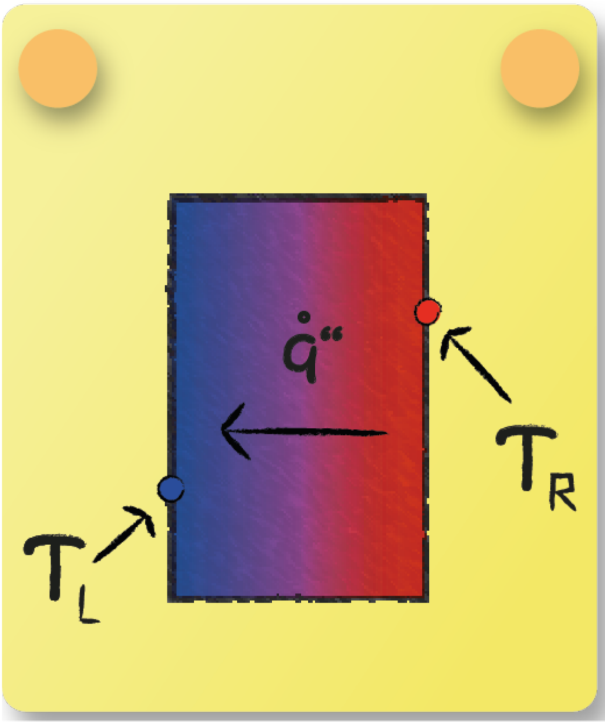
Lecture 1 - Question 1



What are the physical mechanisms of heat conduction?

Heat conduction is governed by different mechanisms depending on the materials state of aggregation. In solid bodies heat is conducted via lattice vibrations that transfer energy from one molecule to another. Within fluids intermolecular collisions lead to an exchange of kinetic energy. Since kinetic energy on a molecular basis is synonym with temperature, the mechanism is refered to as heat conduction.

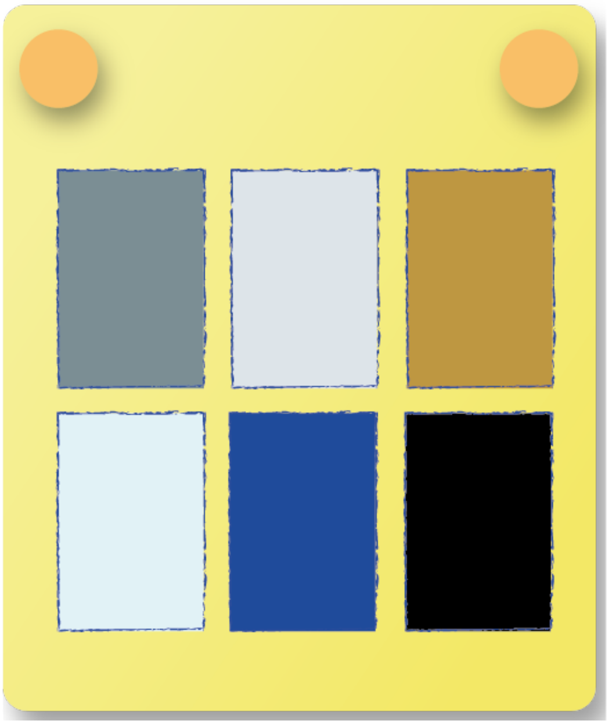
Lecture 1 - Question 2



What is the driving potential for heat conduction?

It is found that a non uniform temperature distribution within a body leads to heat transfer in such way, that the temperature balances out. That is heat is conducted towards the lower temperature. The resulting heat flux is proportional to the temperature gradient, long story short: temperature is the driving potential for heat conduction.

Lecture 1 - Question 3

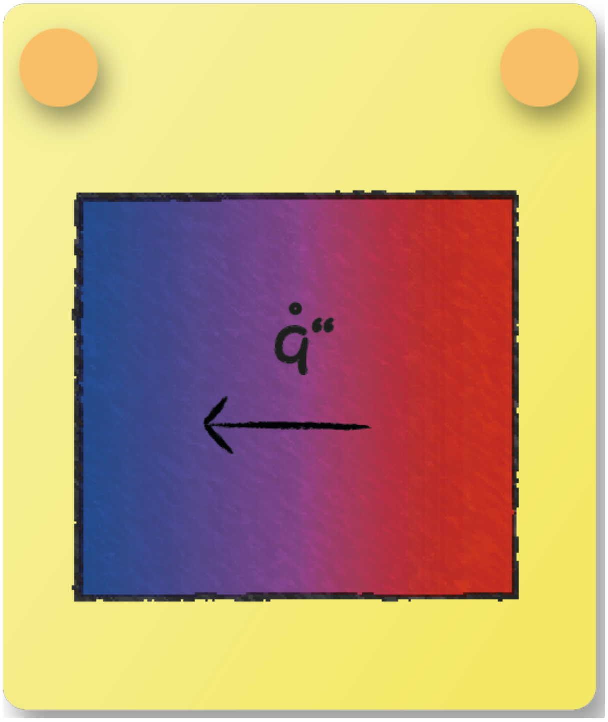


Select the answer in which the materials are listed in ascending order of thermal conductivity.

Air - Oil - Water - Stainless steel - Aluminium - Copper

A rough classification can be obtained by examining the material's state of aggregation. Besides few exceptions the thermal conductivity of liquids exceeds that of gases. Thermal conductivities of solids vary in a wide range from isolating to higly conductive. Highest thermal conductivities are usually observed in metals.

Lecture 1 - Question 4

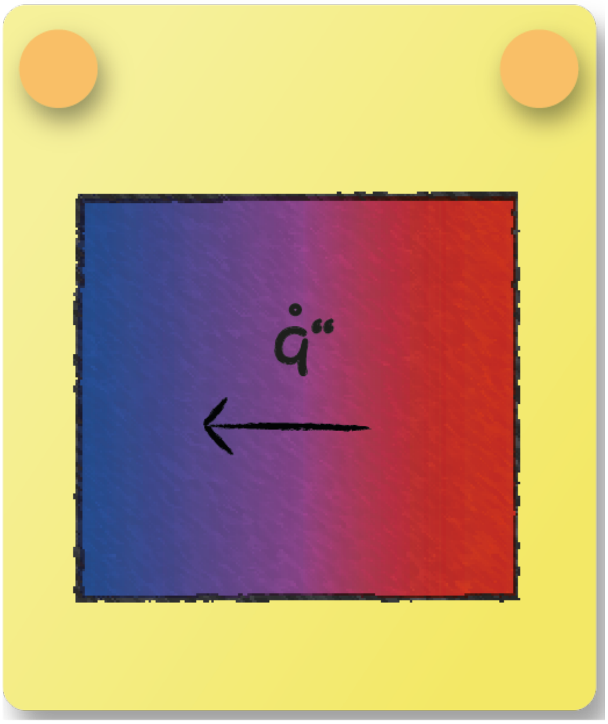


Select the correct equation for one-dimensional heat conduction.

$$\dot{Q} = -\,A\,\lambda\,\frac{\mathrm{d}T}{\mathrm{d}x}$$

Since temperature is the driving potential for heat conduction the heat flux is proportional to it's gradient $\frac{\mathrm{d}T}{\mathrm{d}x}$. The negative sign states that heat is conducted towards lower temperatures. Material properties are accounted for by the thermal conductivity λ as the geometry is considered by area A.

Lecture 1 - Question 5



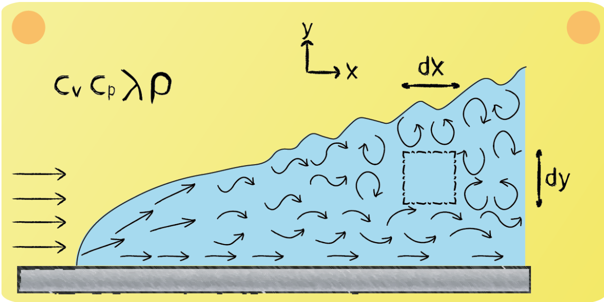
What is the alternative name for the equation for heat conduction?

The linear correlation between temperature gradient and area specific heat flux was discovered by Fourier in 1822 and consequently is refered to as Fourier's law. A general form is given by:

$$\dot{q}'' = -\lambda \frac{\partial T}{\partial n}$$

with $\frac{\partial T}{\partial n}$ representing the temperature gradient normal to a cross section of uniform temperature.

Lecture 1 - Question 10



Give the enthalpy balance to derive the first part of the energy equation. Assume two-dimensional steady state flow. Neglect kinetic and potential energy. Furthermore the control volume has the dimensions dx dy and dz and the velocities are described by u, v and w. Assume ρ, c_p, c_v, and λ to be constant.

Energy balance:

$$0 = \dot{H}_x(x) - \dot{H}_x(x+dx) + \dot{H}_y(y) - \dot{H}_y(y+dy)$$

Energy fluxes:

$$\dot{H}_x(x) = \rho \cdot c_p \cdot u \cdot T \cdot dy \cdot dz$$

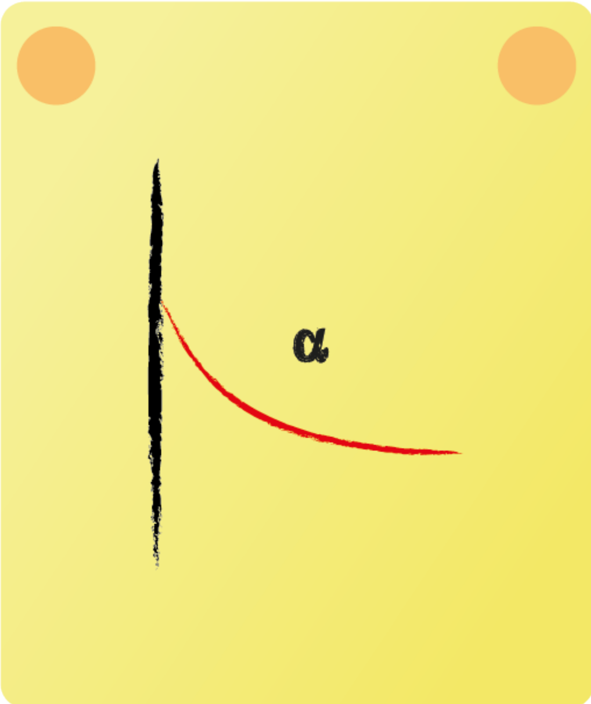
$$\dot{H}_x(x+dx) = \rho \cdot c_p \cdot u \cdot T \cdot dy \cdot dz + \rho \cdot c_p \cdot \frac{\partial u \cdot T}{\partial x} dx \cdot dy \cdot dz$$

$$\dot{H}_y(y) = \rho \cdot c_p \cdot v \cdot T \cdot dx \cdot dz$$

$$\dot{H}_y\,(y+dy) = \rho \cdot c_p \cdot v \cdot T \cdot dx \cdot dz + \rho \cdot c_p \cdot \frac{\partial v \cdot T}{\partial y} dx \cdot dy \cdot dz$$

The total energy of a flowing fluid stream is the sum of the enthalpy, kinetic energy and potential energy. Since the potential and kinetic energy are neglected, the total energy equals the enthalpy. Since ρ, c_p, η and λ are constant, the total energy and thus the enthalpy and can be described as $H = m \cdot c_p \cdot T$.

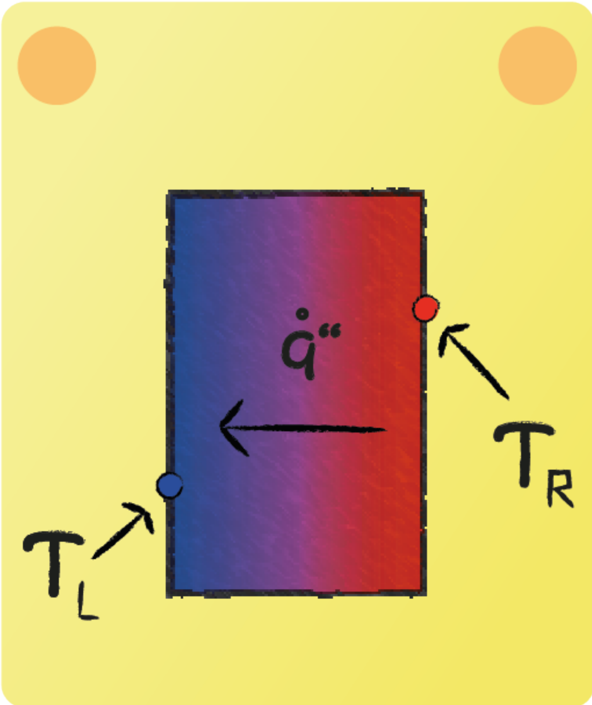
Lecture 1 - Question 6



Why can we stand it in the sauna at 100C but washing our hands at 60C water temperature hurts?

To investigate this problem it is suitable to compare convective heat transfer from air to skin (sauna) and water to skin (washing). Heat flux is proportional to temperature difference of body and fluid such as to the heat transfer coefficient α. Assuming a nusselt number of the same order of magnitude in each case the heat transfer coefficients differ due to a difference in thermal conductivity of the fluids. Since thermal conductivity of water exceeds the value of air to about a factor of 20, it is obvious that despite the smaller temperature difference still a greater amount of heat is transfered while washing our hands. This translates into a higher local temperature at our skin which eventually is recognized by our body and causes pain.

Lecture 2 - Question 1



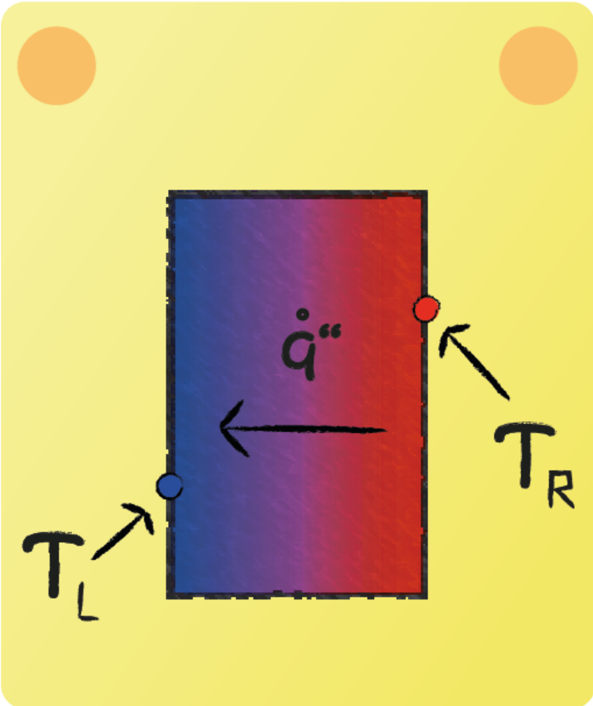
How many boundary conditions are needed to solve the steady one-dimensional conduction equation?

For steady and one dimensional problems the conduction equation simplifies to:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{\Phi}'''}{\lambda} = 0$$

A solution is obtained by integrating the equation twice yielding two unknowns. These integration constants can be specified by two boundary conditions. At least one is needed as a boundary temperature and a further either in form of a temperature gradient or temperature at a second position.

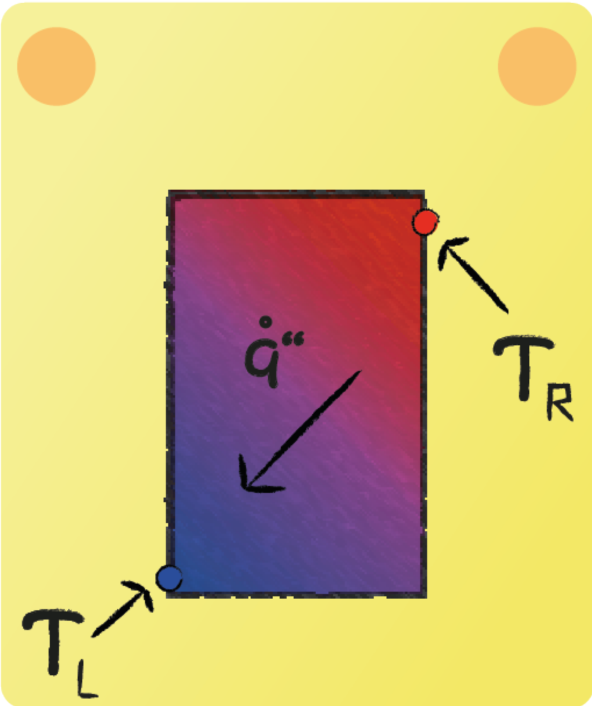
Lecture 2 - Question 2



What are important rules to derive the differential equation for the temperature field of heat conduction problems?

Energy conservation is only valid for absolute values and therefore volumetric expressions cannot be used for a derivation based on energy conservation.

Lecture 2 - Question 3



What statements are correct regarding the 2-D heat conduction equation with sources?

The general heat conduction equation derived for a control volume with constant thermal conductivity can be written as:

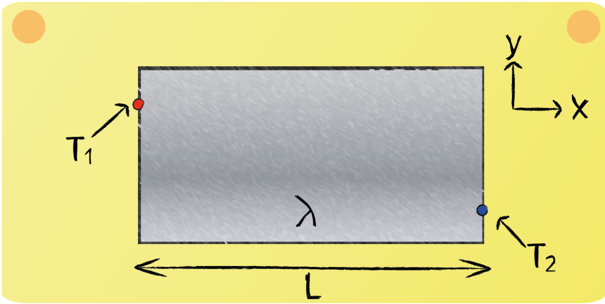
$$\frac{\rho c}{\lambda}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{\Phi}'''}{\lambda}$$

In case of a steady two-dimensional problem the equation simplifies to:

$$0 = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\dot{\Phi}'''}{\lambda}$$

This form is also known as the Poisson equation.

Lecture 2 - Question 4



Develop an energy balance to calculate the temperature profile inside the wall and give the boundary conditions. Assume one-dimensional steady-state heat transfer in x-direction.

Energy Balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} = 0$$

The sum of the in- and outgoing fluxes should equal zero, because of steady-state conditions.

Heat Fluxes:

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

The ingoing flux can be described by use of Fourier's law. The outgoing flux can be approximated by use of the Taylor series expansion.

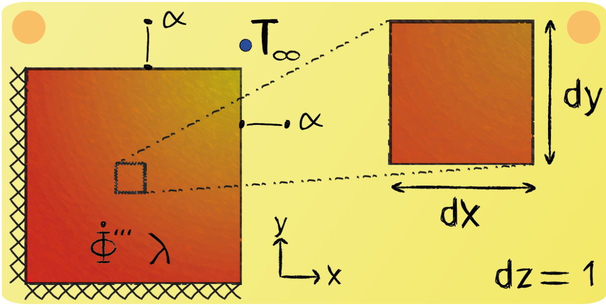
Boundary conditions:

$$T(x = 0) = T_1$$

$$T(x = L) = T_2$$

The boundary conditions above describe that the temperature of the wall equals T_1 on the left side and T_2 on the right side, as can be seen in the sketched situation.

Lecture 2 - Question 5



Provide the governing energy balance in the following infinitesimal 2D element in Cartesian coordinates to describe the heat conduction problem in the body with heat source $\dot{\Phi}'''$. Assume steady-state conditions.

Energy balance:

$$\dot{Q}_{x,in} + \dot{Q}_{y,in} - \dot{Q}_{x,out} - \dot{Q}_{y,out} + \dot{\Phi} \ = 0$$

It is denoted that we are dealing with a source. For that reason the term $\dot{\Phi}$ should be positive.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda dydz\frac{\partial T}{\partial x}$$

$$\dot{Q}_{y,in} = -\lambda dxdz\frac{\partial T}{\partial y}$$

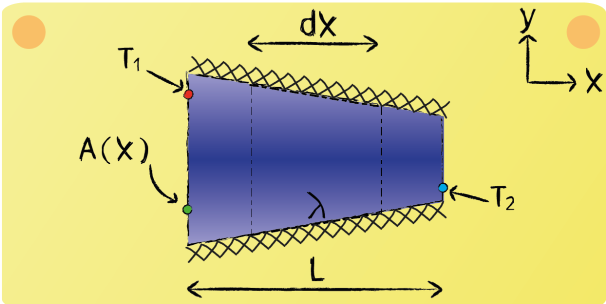
$$\dot{Q}_{x,out} = -\lambda dydz\frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x}dx$$

$$\dot{Q}_{y,out} = -\lambda dxdz\frac{\partial T}{\partial y} + \frac{\partial \dot{Q}_{y,in}}{\partial y}dy$$

$$\dot{\Phi} \ = \dot{\Phi}''' \cdot V = \dot{\Phi}''' \cdot dx \cdot dy \cdot dz$$

The heat fluxes are described by conductive heat transfer. The outgoing heat fluxes can be approximated by use of the Taylor series expansion.

Lecture 2 - Question 6



Develop an energy balance to calculate the temperature profile inside the truncated cone and give the boundary conditions. The sides are covered with an adiabatic wall. Assume for that reason one-dimensional steady-state conditions.

Energy Balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} = 0$$

Heat Fluxes:

$$\dot{Q}_{x,in} = -\lambda A(x) \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx = -\lambda A(x) \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(-\lambda A(x) \frac{\partial T}{\partial x} \right) dx$$

The in and outgoing flux should equal each other and are characterized by conductive heat transfer. The outgoing flux can be approximated by use of the Taylor series expansion. It should be noted that the cross sectional area is not constant along axial direction.

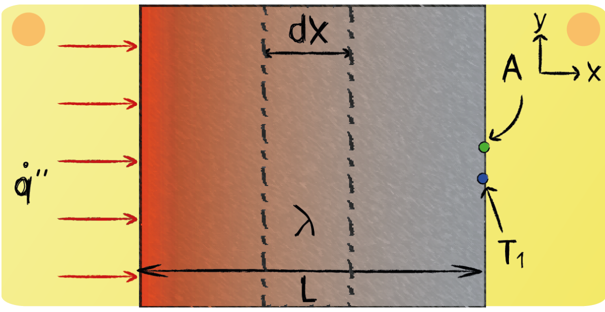
Boundary conditions:

$$T(x=0) = T_1$$

$$T(x=L) = T_2$$

The boundary conditions above describe that the temperature of the body equals T_1 on the left side and T_2 on the right side, as can be seen in the sketched situation.

Lecture 2 - Question 7



A constant heat flux is being transferred through a wall. Develop an energy balance to calculate the temperature profile inside the wall and give the boundary condition. Assume steady-state heat transfer in x-direction.

Energy Balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} = 0$$

Heat Fluxes:

$$\dot{Q}_{x,in} \;=\; -\lambda A \tfrac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} \;=\; -\lambda A \tfrac{\partial T}{\partial x} + \tfrac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

The in- and outgoing flux should equal each other. The ingoing flux can be described by use of Fourier's law and the outgoing flux can be described by use of the Taylor series expansion.

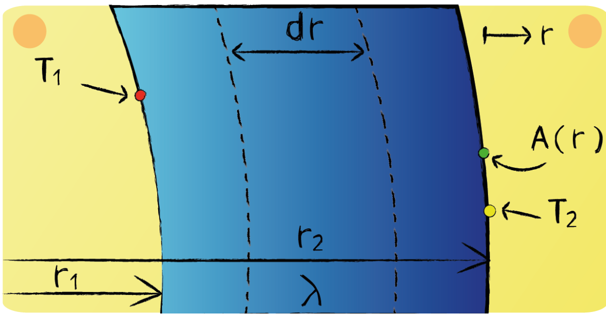
Boundary conditions:

$$\tfrac{\partial T(x=0)}{\partial x} \;=\; -\tfrac{\dot{q}''}{\lambda}$$

$$T(x = L) \;=\; T_1$$

The first boundary condition results from the fact that $\dot{Q}_{x=0} = -\lambda A \frac{\partial T(x=0)}{\partial x} = \dot{q}'' A$, the second boundary condition results from the fact that the temperature equals T_1 on the right side of the wall.

Lecture 2 - Question 8



Develop an energy balance to calculate the temperature profile inside the spherical wall and give the boundary conditions. Assume one-dimensional steady-state heat transfer without sources or sinks in radial direction.

Energy Balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

Since the type of heat transfer is steady-state, the sum of the in- and outgoing heat fluxes of the control volume should equal zero.

Heat Fluxes:

$$\dot{Q}_{r,in} \;=\; -\lambda A(r)\frac{\partial T}{\partial r} = -\lambda 4\pi r^2 \frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} \;= \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r}dr = -\lambda 4\pi r^2 \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}(-\lambda 4\pi r^2 \frac{\partial T}{\partial r})dr$$

The ingoing flux can be described by use of Fouriers law and the outgoing flux can be approximated by use of the Taylor series expansion.

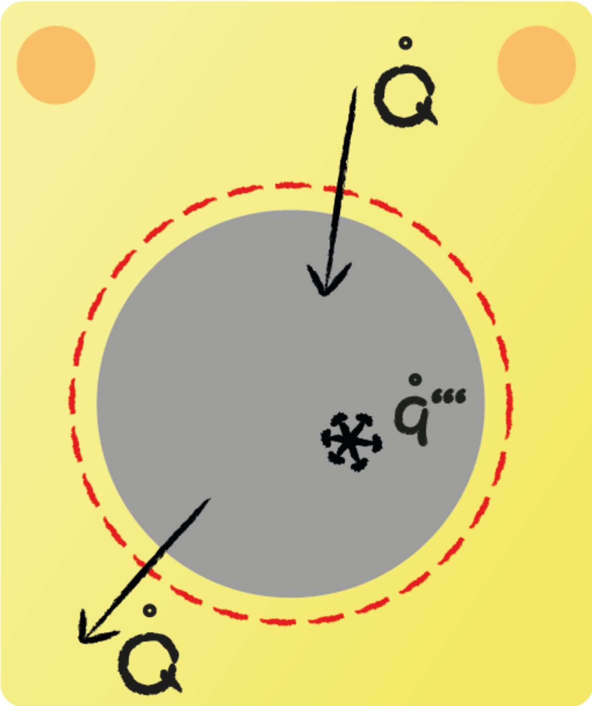
Boundary conditions:

$$T(r = r_1) = T_1$$

$$T(r = r_2) = T_2$$

The boundary conditions describe that on the left side of the cylindrical wall the temperature equals T_1 and on the right side T_2

Lecture 3 - Question 1

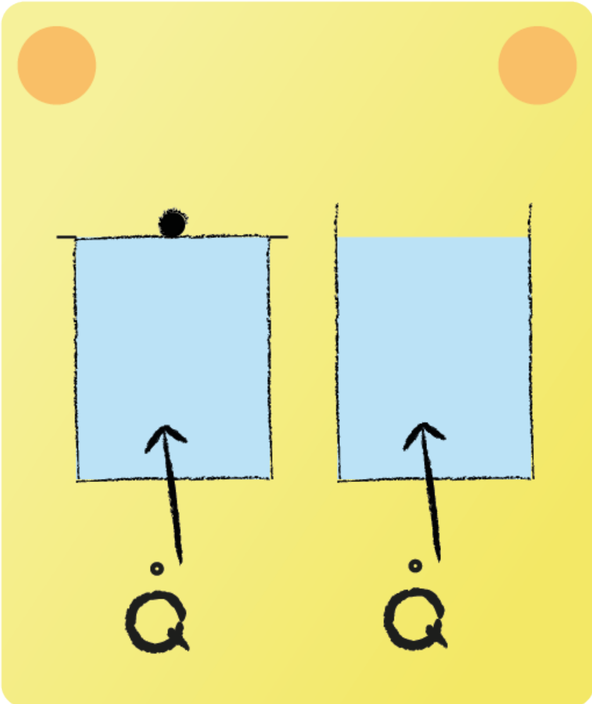


What is the recommended procedure for drawing up the energy balance?

For reasons of clarity the following common structure should be used for derivation of an energy balance:

$$temporal\ derivative =$$
$$ingoing\ fluxes - outgoing\ fluxes + sources - sinks$$

Lecture 3 - Question 2



What is the difference between specific heat capacityc_{p}andc_{v}?

The specific heat capacity defines the relation of heat supplied to a matter and the corresponding change in it's temperature. An equation for specific heat capacity is given by:

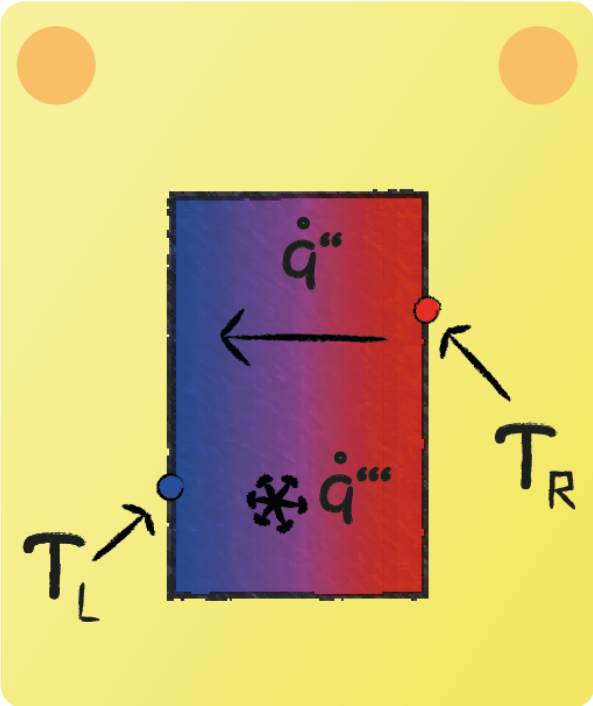
$$c = \frac{\Delta Q}{m\,\Delta T}$$

Depending on an isochor or isobaric change of state during the heat absorption the relation is given by c_v or c_p. In case of an isobaric change of state the fluid usually is expanding while it is heated such that the supplied heat partly accounts for the expansion energy. Hence a greater amount of heat is needed to obtain the same temperature difference, that is $c_p > c_v$. For liquids c_p and c_v are almost identical due to a weaker pressure and density coupling compared to gases.

Lecture 3 - Question 3

Which statements are true regarding the internal energy U?

Lecture 3 - Question 4



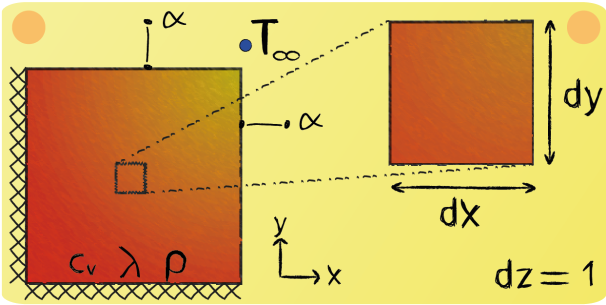
Which of the following equations describes the 3-dimensional transient heat diffusion process with sources?

The general heat conduction equation derived for a control volume with constant thermal conductivity can be written as:

$$\frac{\rho c}{\lambda}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{\Phi}'''}{\lambda}$$

It states that temporal change in internal energy (temperature) is composed of ingoing and outgoing conduction energy fluxes (divergence of the temperature field) and heat sources and sinks, respectively. Common notations for heat sources/sinks are $\dot{\Phi}'''$ just as $\dot{q}'''$ which represent a volume specific heat flux that is positive for sources and negative for sinks.

Lecture 3 - Question 5



Provide the governing energy balance in the following infinitesimal 2D element in Cartesian coordinates to describe the heat conduction problem in the body without a heat source. Assume the process to be isochoric with transient conditions.

Energy balance:

$$\frac{\partial U}{\partial t} = \dot{Q}_{x,in} - \dot{Q}_{x,out} + \dot{Q}_{y,in} - \dot{Q}_{y,out}$$

For unsteady heat transfer the internal energy will change over time and equals the sum of in- and outgoing heat fluxes.

Change of internal energy over time:

$$\frac{\partial U}{\partial t} = \rho \cdot c_v \cdot dx \cdot dy \cdot dz \cdot \frac{\partial T}{\partial t}$$

The internal energy of a constant volume can be described as: $U = m \cdot c_v \cdot T$

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda dydz \frac{\partial T}{\partial x}$$

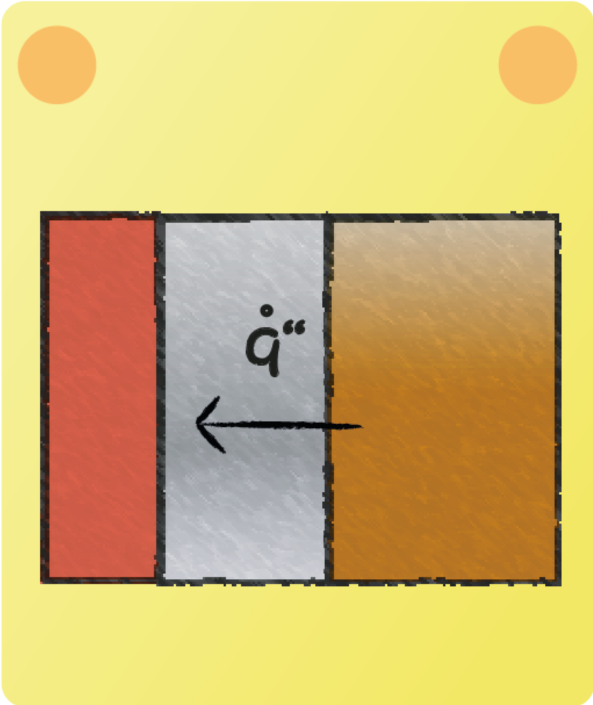
$$\dot{Q}_{y,in} = -\lambda dxdz \frac{\partial T}{\partial y}$$

$$\dot{Q}_{x,out} = -\lambda dydz \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

$$\dot{Q}_{y,out} = -\lambda dxdz \frac{\partial T}{\partial y} + \frac{\partial \dot{Q}_{y,in}}{\partial y} dy$$

The heat fluxes are described by conductive heat transfer. The outgoing heat fluxes can be approximated by use of the Taylor series expansion.

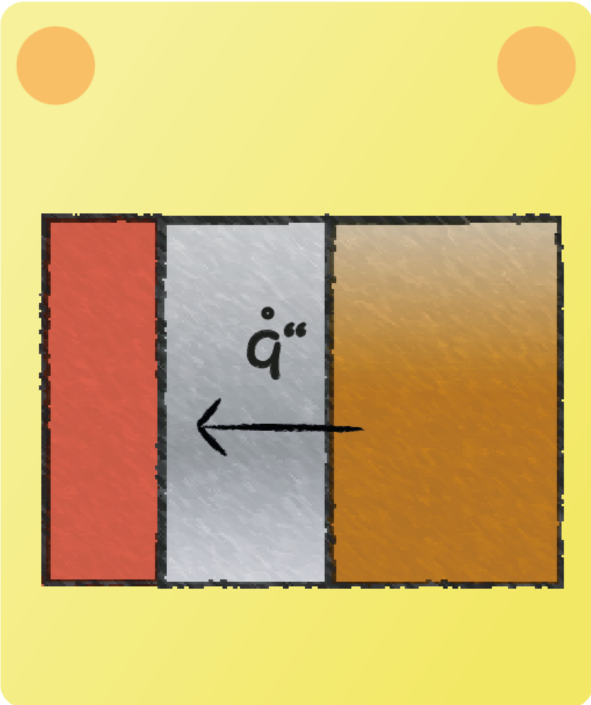
Lecture 4 - Question 1



Which statements are correct for the steady state heat transfer in a multi layered wall of constant cross sectional area without sources?

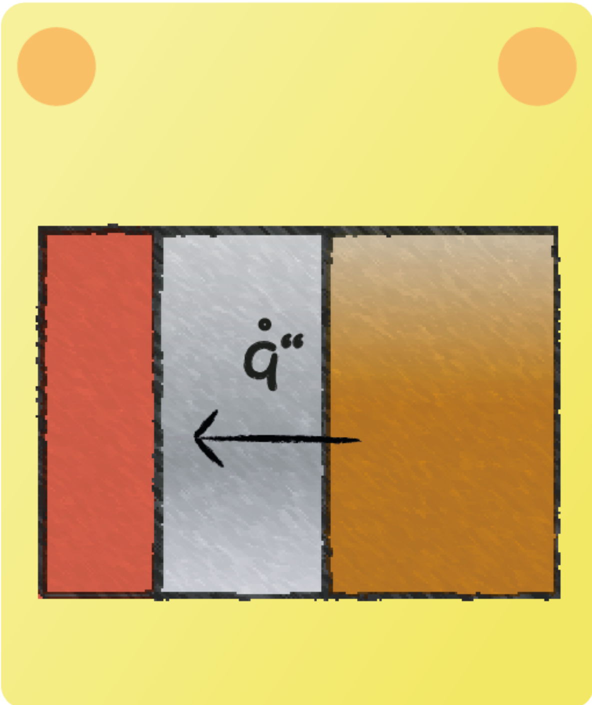
For a wall with no heat sources/sinks the energy conservation yields a constant heat flux $\dot{Q}$ for a steady case. Since it is a material property, thermal conductivity can be assumed to be constant within a layer. Together with a constant cross section area, this yields a linear temperature profile which is inversely proportional to the layer's thermal conductivity. From the definition of thermal conductivity it is obvious that thermal resistance is inversely proportional to this quantity. That is an increased thermal conductivity leads to an increased heat flux for a given temperature difference, hence the thermal resistance is decreased.

Lecture 4 - Question 2



Which statements are correct for the steady state heat transfer in a multi layered wall of constant cross sectional area without sources?

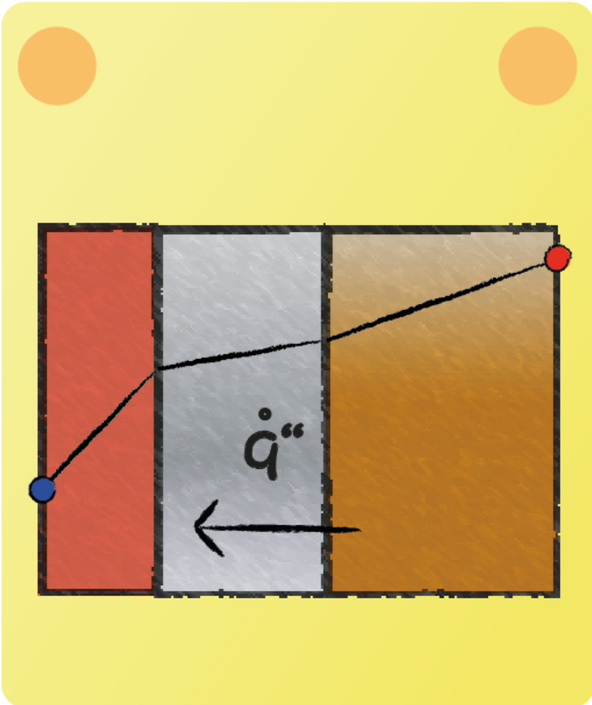
Lecture 4 - Question 3



Given a wall consisting of three layers. How many differential equations, coupling conditions and boundary conditions are required for a unique solution?

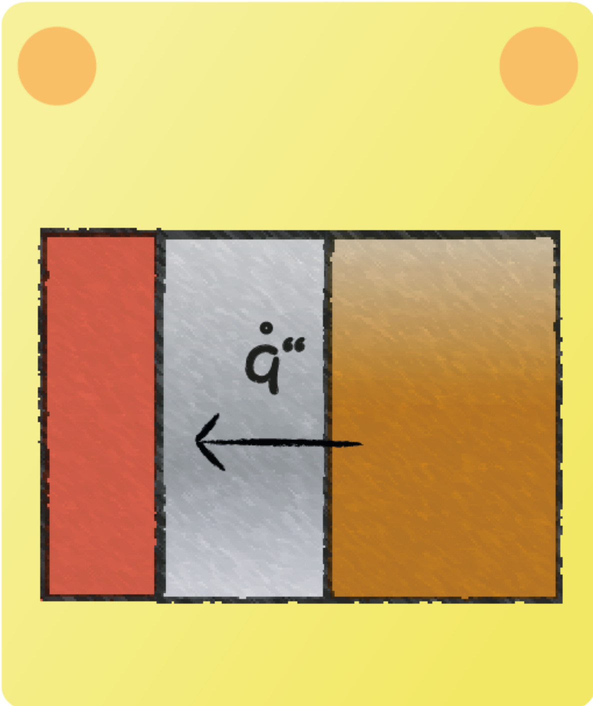
As thermal conductivities within the wall segments might differ from each other three differential equations are required. Boundaries are specified either as given temperatures or heat fluxes yielding two boundary conditions in total. At the interfaces temperature as well as the heat flux must be continuous, that is two coupling conditions are required for each interface.

Lecture 4 - Question 4



What is the physical reason for a kink in the temperature profile of a multi layered wall?

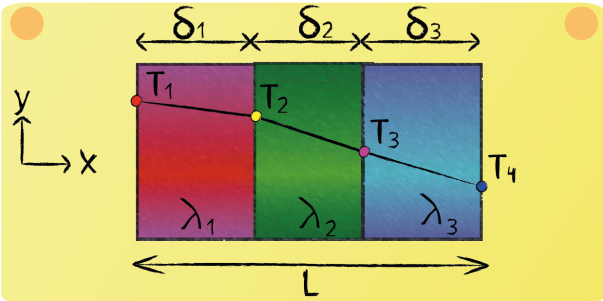
Lecture 4 - Question 5



What quantities within a multi layered wall remain constant in radial direction when no heat sources are present?

In a steady state the heat flux is constant due to energy conservation. Thermal conductivities can vary as they are properties of the corresponding materials. A heat flux goes along with a non zero temperature gradient, therefore temperature cannot be constant in this case. Gradients might change depending on thermal conductivities and cross section areas and therefore cannot be treated as constants in general.

Lecture 4 - Question 6



Give a description for the heat flux passing the multi-layer wall. Assume one-dimensional steady-state heat transfer in x-direction. Only use the given parameters.

Fourier's law:

$$\dot{Q} = -\lambda A \frac{\partial T}{\partial x}$$

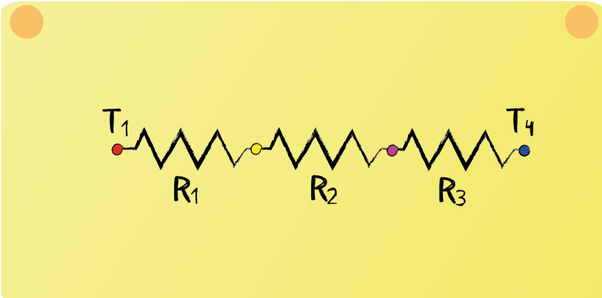
Using Fouriers law and keeping in mind that the heat passing each layer should equal each other:

$$\dot{Q} = -\lambda_1 A \frac{1}{\delta_1}(T_2 - T_1)$$

$$\dot{Q} = -\lambda_2 A \frac{1}{\delta_2}(T_3 - T_2)$$

$$\dot{Q} = -\lambda_3 A \frac{1}{\delta_3}(T_4 - T_3)$$

Lecture 4 - Question 7



Give a description for the heat flux passing the network. Assume one-dimensional steady-state heat transfer. Assume $T_1 > T_4$.

The heat flux passing a resistor network can be expressed as:

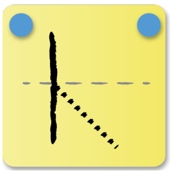
$$\dot{Q} \;=\; \tfrac{1}{R_{c,tot}}(T_1 - T_{n+1})$$

Using this equation, the heat passing the network will be:

$$\dot{Q} \;=\; (T_1 - T_4)/(R_1 + R_2 + R_3)$$

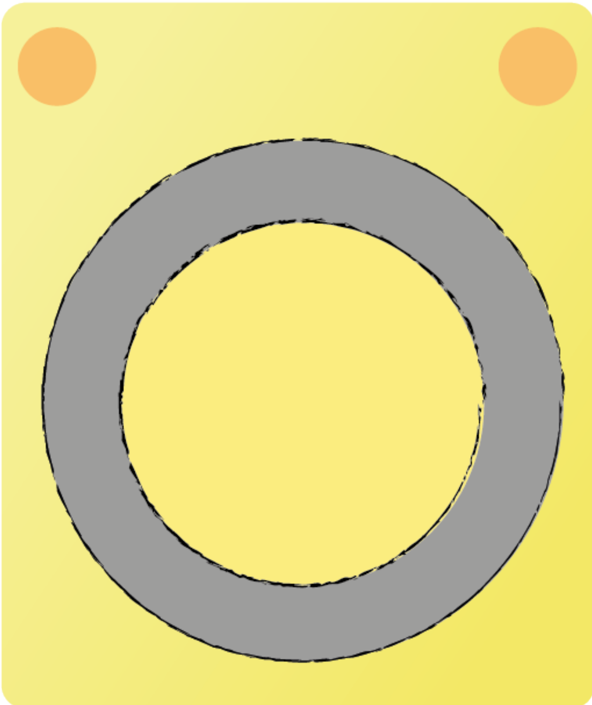
Lecture 4 - Question 8

Give temperature profile for the following case. Assume one-dimensional steady-state heat transfer.



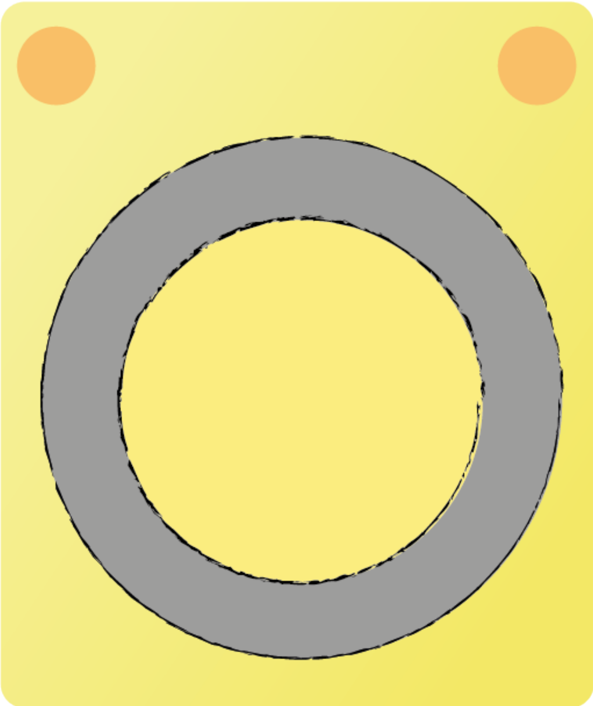
Since the highest temperature is given to be on the left, the temperature profile should decrease linearly from that point on, for a wall.

Lecture 5 - Question 1



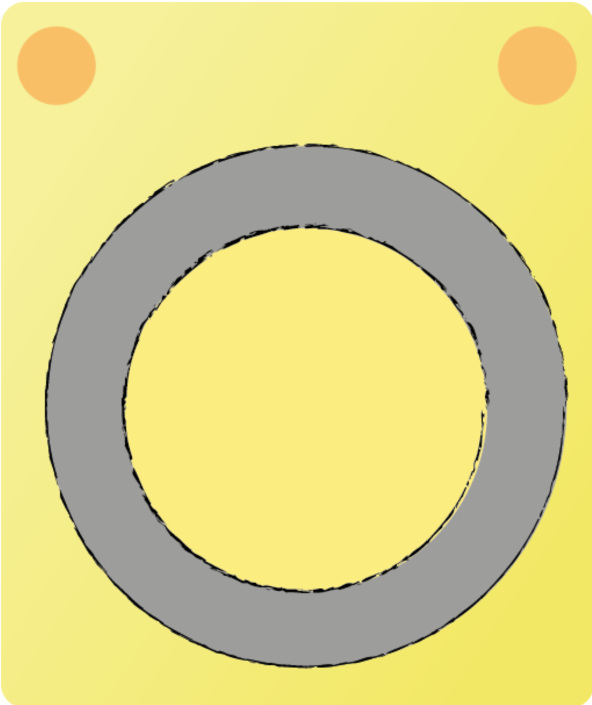
Which statements are true regarding the heat flow through a pipe's wall?

Lecture 5 - Question 2



Which statements are true regarding the heat flow through a pipe's wall?

Lecture 5 - Question 3



Can the curvature of a pipes wall and thus the change of area be neglected and if so under which conditions?

In case of a thin pipe and large radii the relative change of cross section area is small and can thus be neglected. That is a thin pipe can be treated as a plane wall. A more formal derivation of the simplification can be obtained by investigating the expression for heat flux or temperature profile in a pipe:

$$\dot{Q} = 2\pi\lambda L \frac{T_1 - T_2}{\ln\frac{r_2}{r_1}}$$

For values close to unity the logarithm can be simplified as

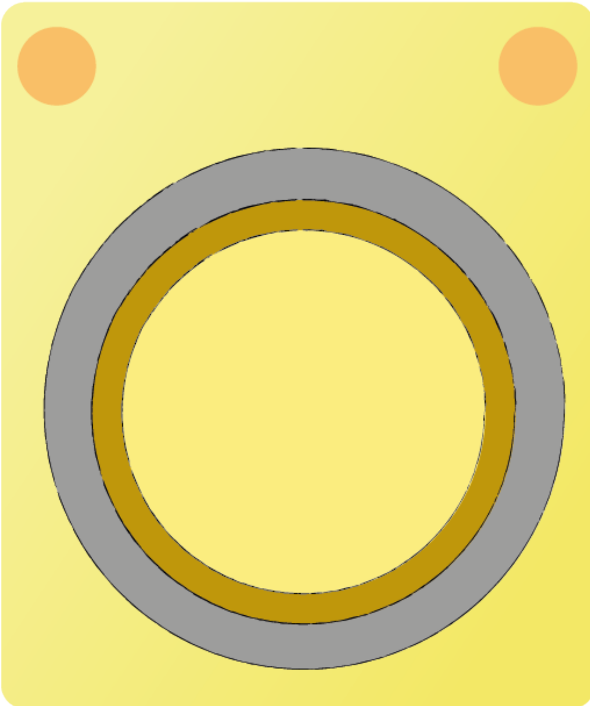
$$\ln x \approx x - 1$$

and such

$$\ln\frac{r_2}{r_1} \approx \frac{\delta}{r}$$

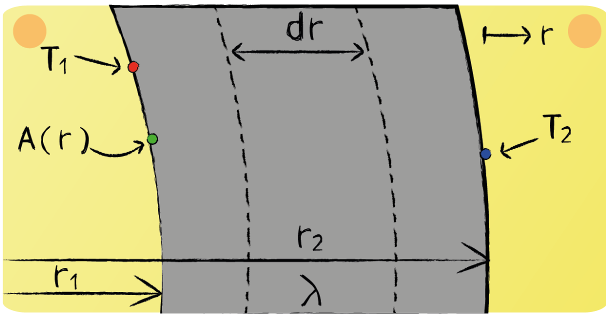
This yields the corresponding equation for a wall of area $2\pi L r$ and thickness δ.

Lecture 5 - Question 4



What quantities within a multi-layered pipe remain constant in radial direction when no heat sources are present?

Lecture 5 - Question 5



Develop an energy balance to calculate the temperature profile inside the pipe wall and give the boundary conditions. Assume one-dimensional steady-state conditions. The expansion of the pipe in axial directions is L.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

In order to obtain the steady-state energy balance, the sum of the heat fluxes entering and leaving the system should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda 2\pi r L \frac{\partial T}{\partial r}$$

$$\begin{aligned} \dot{Q}_{r,out} &= \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r} dr = \\ -\lambda 2\pi r L \frac{\partial T}{\partial r} &+ \frac{\partial}{\partial r}\left(-\lambda 2\pi r L \frac{\partial T}{\partial r}\right) dr \end{aligned}$$

The ingoing heat flux can be described by use of Fourier's law. The outgoing flux can be approximated by use of the Taylor series expansion.

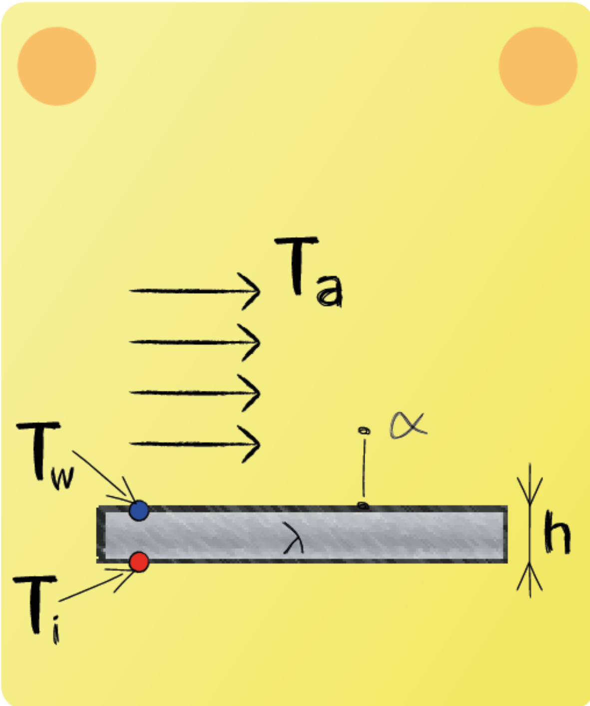
Boundary conditions:

$$T(r = r_1) = T_1$$

$$T(r = r_2) = T_2$$

The boundary conditions describe that the inner and outer surface temperature equal T_1 and T_2 respectively.

Lecture 6 - Question 1



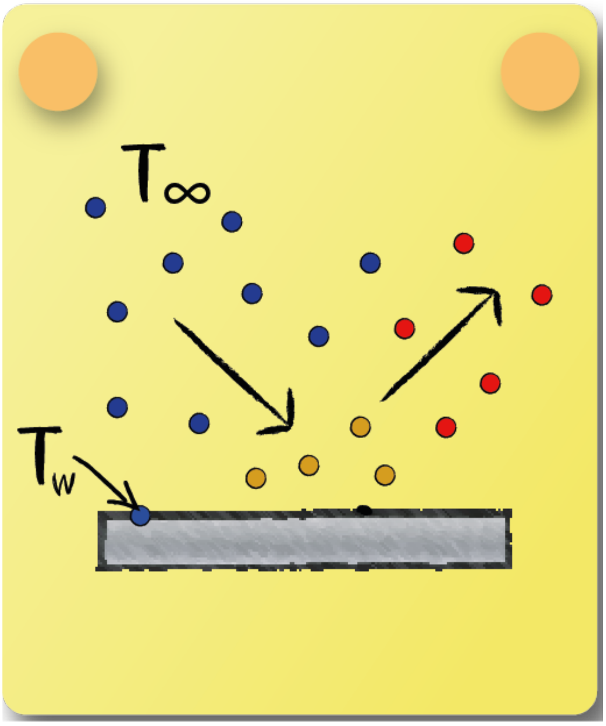
Which statement is true regarding the heat transfer for air blowing over a horizontal steel plate? Assume a steady state and one-dimensional heat transfer. The plates area is given to be A.

In case of a steady state conductive and convective heat fluxes must equal each other in order to satisfy energy conservation.

$$\dot{Q}_{\mathrm{cond}} = \dot{Q}_{\mathrm{conv}}$$

$$-\lambda A \frac{T_{\mathrm{w}} - T_{\mathrm{i}}}{h} = \alpha A (T_{\mathrm{w}} - T_{\mathrm{a}})$$

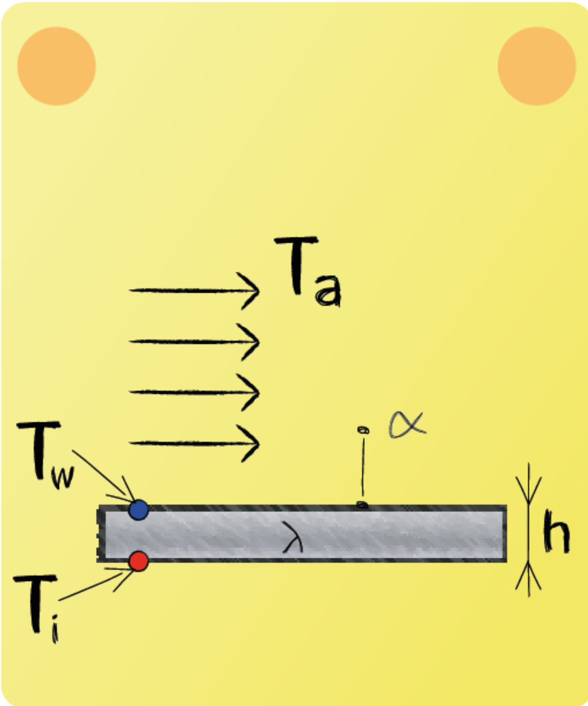
Lecture 6 - Question 2



Is the following statement true or false?
- Convection is the summed energy transfer of advection and conduction.

Convection describes the transfer of energy by the advective motion of molecules and the conductive heat transfer between molecules by collisions. Thus the statement is true.

Lecture 6 - Question 3



Air blows over a horizontal steel hot plate. The plate surface is maintained at a constant temperature. Determine the heat transfer coefficient α. Take T_a= 20 $°C$, h = 0.1 m, T_w = 30 $°C$, T_i = 80 $°C$, λ = 2 W/mK.

In case of steady state heat transfer, the conductive and convective heat fluxes must equal each other in order to satisfy the energy conservation.

$$\dot{Q}_{cond} = \dot{Q}_{conv}$$

$$-\lambda \cdot A_s \cdot \frac{T_w - T_i}{h} = \alpha \cdot A_s \cdot (T_w - T_a)$$

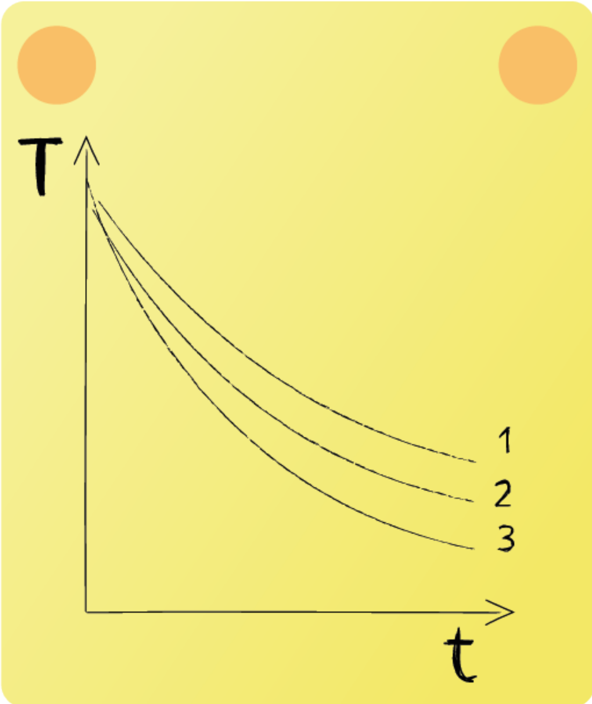
Rewriting yields:

$$\alpha = -\lambda \cdot \frac{T_w - T_i}{h} \frac{1}{(T_w - T_a)}$$

Filling in the numerical values:

$$\alpha = -2 \cdot \frac{30 - 80}{0.1} \frac{1}{(30 - 20)} = 100 \; W/m^2K$$

Lecture 6 - Question 4



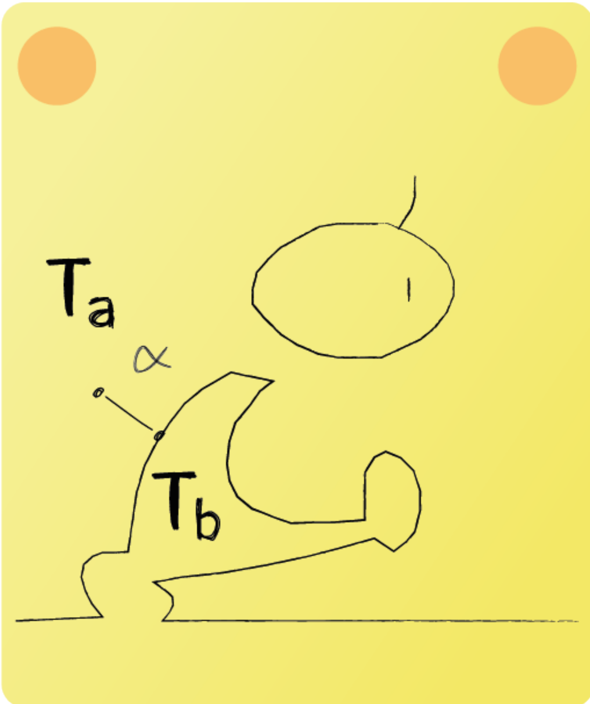
A sphere, a cube and a thin circular plate, all made of the same material and having the same volume are initially heated up above room temperature. Afterwards they are left at room temperature. Assume all three to have a homogeneous temperature distribution. Radiation can be neglected. Which temperature profile do you expect to belong to which object?

When having a homogeneous temperature distribution, the change in body temperature is determined by the rate of heat loss. Heat is lost due to convection.

$$\dot{Q}_{conv} = \alpha \cdot A_s \cdot (T_w - T_\infty)$$

. Since they are under the same environmental conditions $\alpha_{sphere} \approx \alpha_{plate} \approx \alpha_{cube}$. For the surface Since the bodies are of the same volume, the relation of their surface areas is only depending on their shapes, yielding: $A_{plate} > A_{cube} > A_{sphere}$. Which is the order of fastest to slowest cool down.

Lecture 6 - Question 5



A man is walking in still air. Determine the rate of heat loss. Radiation can be neglected. Assume steady-state heat transfer.Take $T_a = 10\ °C$, $T_b = 30\ °C$, $\alpha = 15\ W/m^2K$, $A_s = 2\ m^2$.

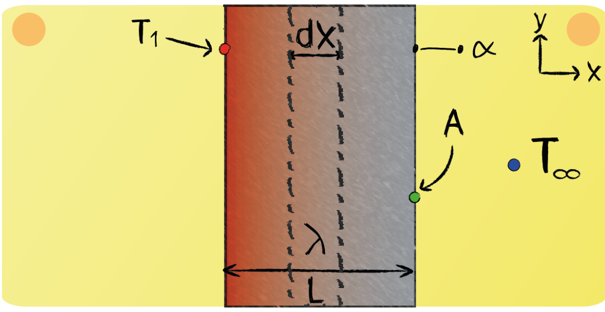
The rate of heat loss is characterized by convective heat transfer.

$$\dot{Q}_{conv} = \alpha \cdot A_s \cdot (T_b - T_a)$$

Filling in the numerical values:

$$\dot{Q}_{conv} = 15 \cdot 2 \cdot (30 - 10) = 600\ W$$

Lecture 6 - Question 6



A wall is subjected to convection. Develop an energy balance to calculate the temperature profile inside the wall and give the boundary conditions. Assume one-dimensional steady-state conditions.

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} = 0$$

The sum of the in- and outgoing heat fluxes of the control volume should equal zero, because of steady-state conditions.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

The heat flux entering the control volume can be described by use of Fouriers law. The outgoing heat flux can be approximated by use of the Taylor series expansion.

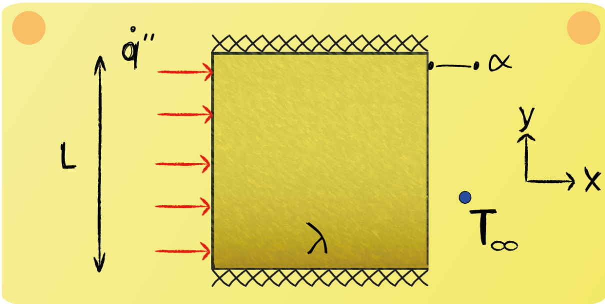
Boundary conditions:

$$T(x=0) = T_1$$

$$\frac{\partial T(x=L)}{\partial x} = -\frac{\alpha}{\lambda}\left(T(x=L) - T_\infty\right)$$

The first boundary condition describes that the temperature on the left side equals T_1. The second boundary conditions results from the fact that $\dot{Q}_{x=L} = -\lambda A \frac{\partial T(x=L)}{\partial x} = \alpha A \left(T(x=L) - T_\infty\right)$.

Lecture 6 - Question 7



A constant heat flux is entering a cube on the left side. At the same time is the cube losing heat on the right side due to convection. The other surfaces are fully adiabatic. Assume the process to be steady and the temperature inside the cube to be homogeneous. Develop an energy balance to calculate the cube temperature T_w.

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} = 0$$

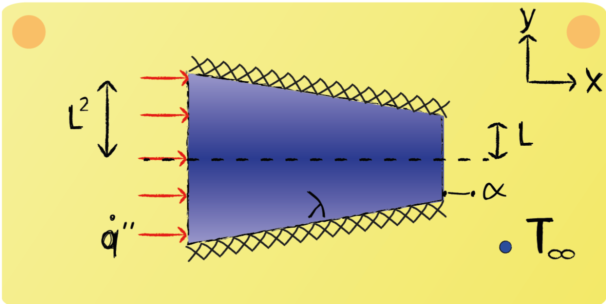
The sum of the in- and outgoing heat fluxes of the control volume should equal zero, because of steady-state conditions.

Heat fluxes:

$$\dot{Q}_{x,in} = \dot{q}'' L^2$$

$$\dot{Q}_{x,out} = \alpha L^2 \left(T_w - T_\infty\right)$$

Lecture 6 - Question 8



A constant heat flux is entering a truncated cone. At the same time it is loosing the same amount of heat due to convection. The sides are covered with an adiabatic wall. The temperature of the cone T_{w} can be assumed to be homogeneous. Derive an energy balance to determine the temperature of the cone T_{w}.

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} = 0$$

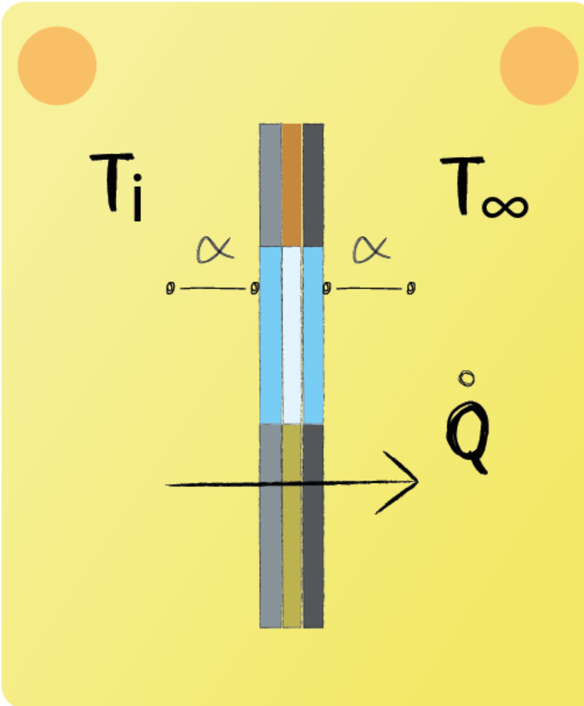
The sum of the in- and outgoing heat fluxes of the control volume should equal zero, because of steady-state conditions.

Heat fluxes:

$$\dot{Q}_{in} = \dot{q}''\pi L^4$$

$$\dot{Q}_{out} = \alpha\pi L^2 \left(T_w - T_\infty\right)$$

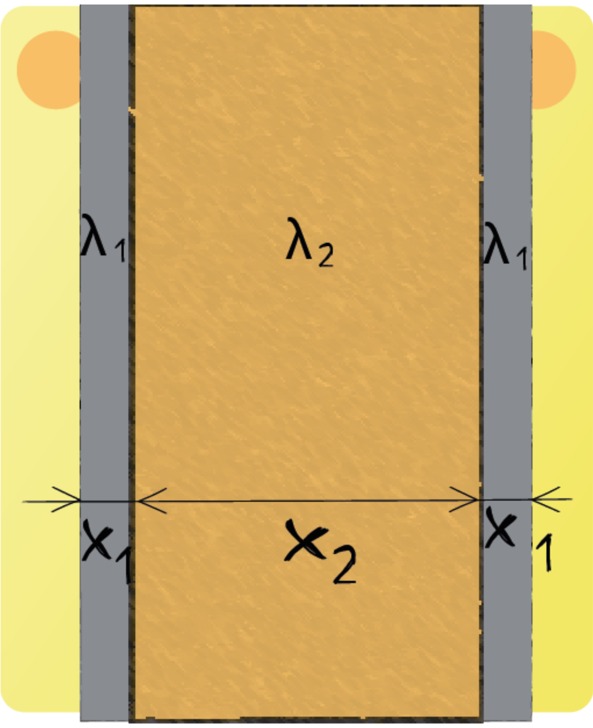
Lecture 7 - Question 1



A cross section of the wall of a train cabin can be seen in the figure. The wall consists out of a part that exists out of three layers and double insulation glass. The double glass has stagnant argon gas in between. The constant indoor and outdoor temperatures are T_i and T_∞. Assume one-dimensional steady-state heat transfer. When determining the rate of heat transfer, which network of resistors is correct for the situation described. Note that no simplifications have been made by simplifying series or parallel networks.

No parallel connected resistors.
From the figure it can be seen that the cross sectional area per layer remains constant, which is the height multiplied by the length of the cabin.
As there is only a temperature gradient in horizontal direction, one-dimensional heat transfer can be assumed.
The material properties of a train wall will not change over time.
The temperatures of the surrounding fluids remain constant, implying that the temperature gradient will not change. Since this is the driving force when it comes to heat transfer, steady-state heat transfer can be assumed.
The resistor network will exist out of parallel connected resistors. This because the serial chains of -aluminium (light grey)-foam (brown)-plastic (dark grey)- and -glass (blue)-argon (white)-glass(blue)- will be connected parallel.

Lecture 7 - Question 2



The wall of a refrigerator is constructed of fiberglass insulation sandwiched between two layers sheet metal. The refrigerated space and the kitchen are kept at constant temperature. Determine the thermal resistance of the fiberglass insulation. Take $A_s = 1\ m^2$, $\lambda_2 = 0.02$ W/mK, $x_2 = 10$ mm, $\lambda_1 = 15$ W/mK, $x_2 = 10$ mm.

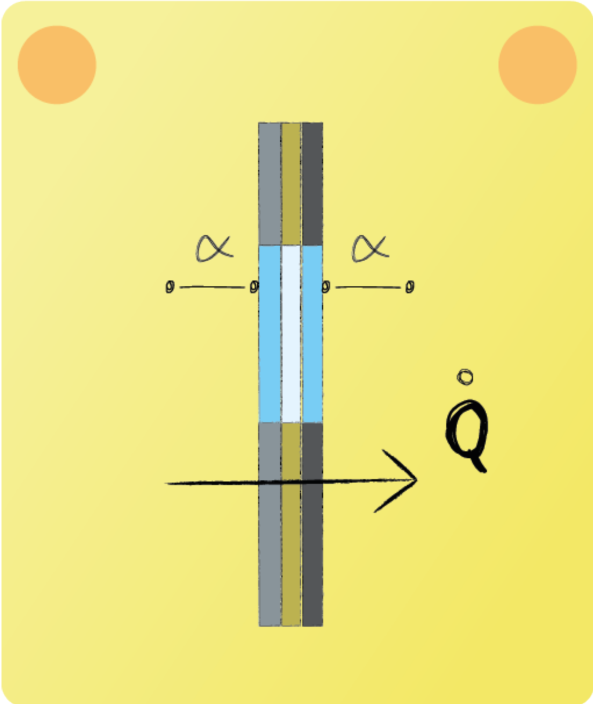
Conductive thermal resistance:

$$R_{fiberglass} = \frac{x_2}{\lambda_2 \cdot A_s}$$

Filling in the numerical values:

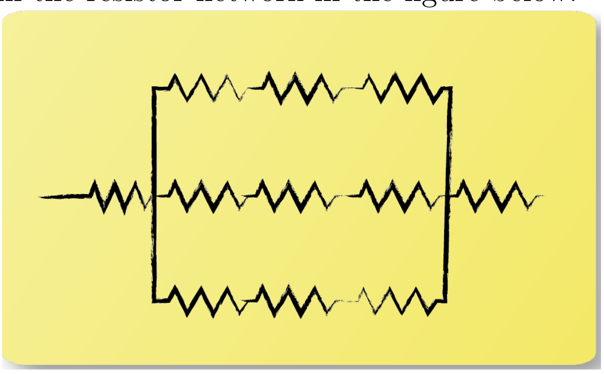
$$R_{fiberglass} = \frac{0.01}{0.02 \cdot 1} = 0.5\ °C/W$$

Lecture 7 - Question 3

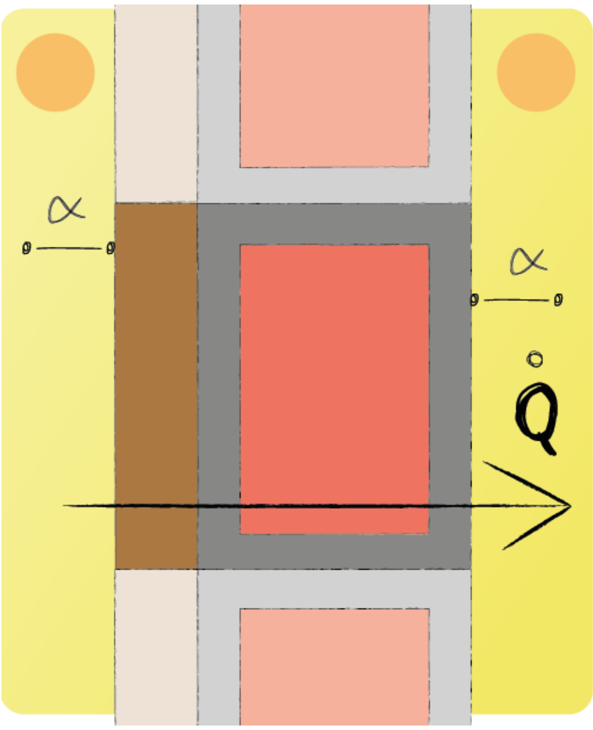


A cross section of the wall of a train cabin can be seen in the figure. The wall consists out of a part that exists out of three layers and double insulation glass. The double glass has stagnant argon gas in between. The constant indoor and outdoor temperatures are T_i and T_∞. Assume one-dimensional steady-state heat transfer. When determining the rate of heat transfer, which network of resistors is correct for the situation described. Note that no simplifications have been made by simplifying series or parallel networks.

Conduction will occur through the top part, the middle part (double glass) and the bottom part. The serial networks the top, middle and bottom parts will be connected in parallel. Since on the in- and outside convective heat transfer plays a role, the parallel network will be connected to two additional resistors. One on the right side and one on the left side. Eventually resulting in the resistor network in the figure below.

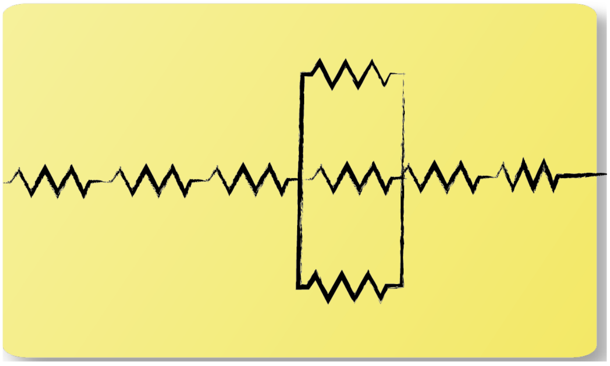


Lecture 7 - Question 4

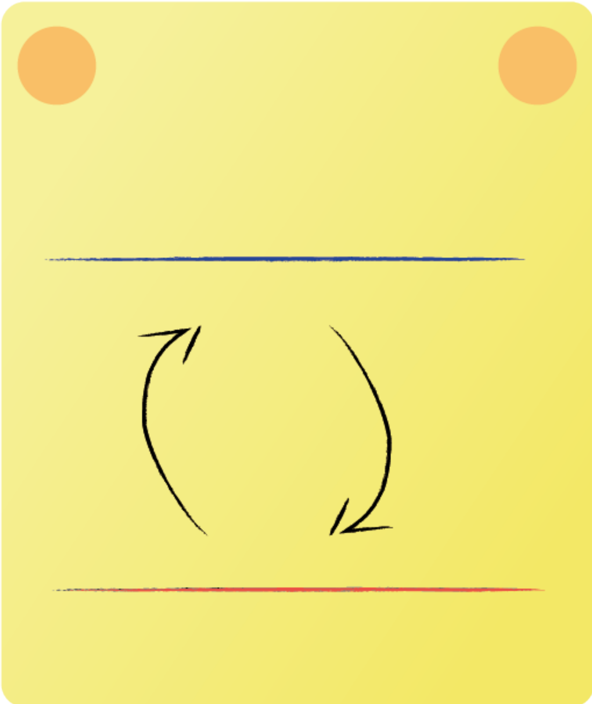


A wall consists out of bricks (red). The bricks have a thermal conductivity of λ_{brick}. The wall is surrounded by a plaster layer (grey) and has a thermal conductivity of $\lambda_{\mathrm{plaster}}$. On one side of the wall there is a foam (brown) with a thermal conductivity of λ_{foam}. The constant in- and outdoor temperatures are T_{i} and T_{o}. Assume one-dimensional heat transfer. When determining the rate of heat transfer, which network of resistors is correct for the situation described. Note that no simplifications have been made by simplifying series or parallel networks.

From left to right, first convective heat transfer plays a role. Then heat will be conducted through the foam. After passing through the foam, conduction through the plaster layer occurs. This results in a serial network of three resistors. At some point the heat can be conducted via three paths. The top layer of the plaster, the brick or the bottom layer of the plaster. Implying a parallel network of three resistors will be connected to the three serial resistors. After this parallel network, heat will be conducted through the next plaster layer and will eventually be transported by convective heat transfer at the right side. Implying two additional resistors connected to the parallel network on the right side. Resulting in the resistor network in the figure.



Lecture 7 - Question 5



Can we define the convection resistance for a unit surface area as the inverse of the convection heat transfer coefficient?

Convective resistance:

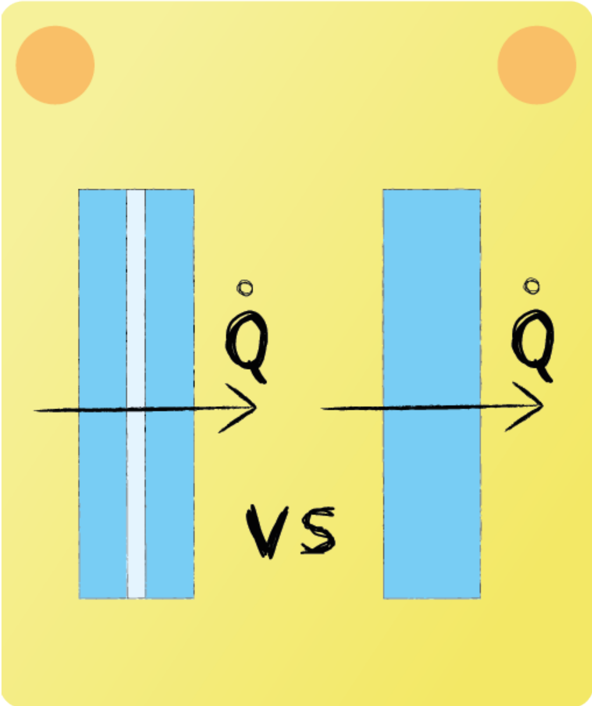
$$R_{conv} = \frac{1}{\alpha \cdot A}$$

For a unit surface $A = 1\ m^2$

$$R_{conv} = \frac{1}{\alpha} = \alpha^{-1}$$

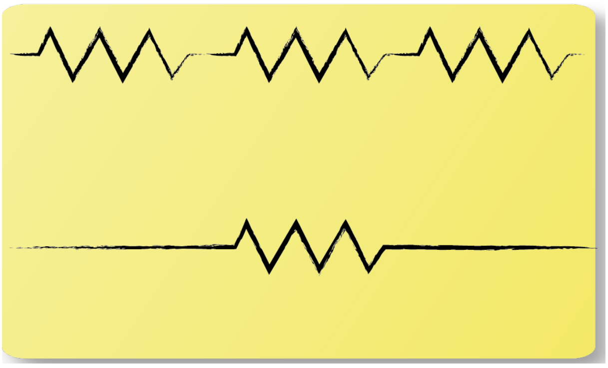
Thus the final answer is yes.

Lecture 7 - Question 6

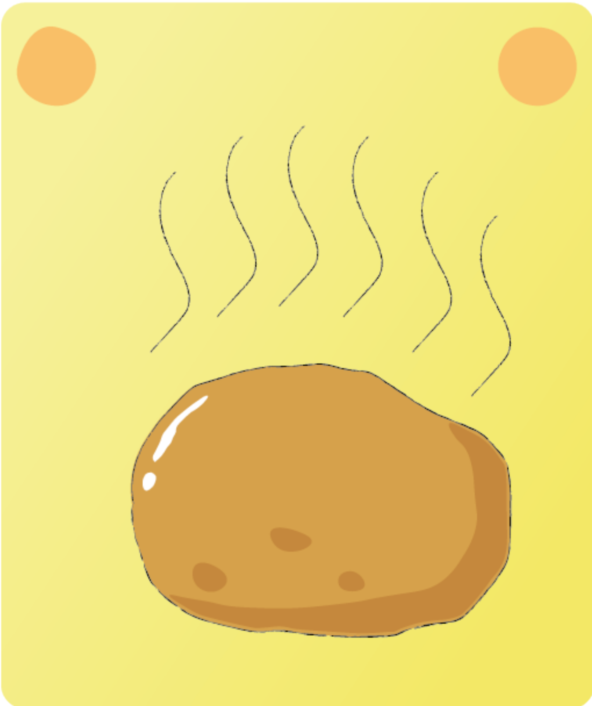


Consider a window glass consisting of two 4-mm thick glass sheets with a stagnant gas in between. Compare the heat transfer rate through this window with that one consisting of a single 8-mm-thick glass sheet under identical conditions.

More heat is being transferred through the single 8-mm-thick glass sheet, due to the fact that this has the lowest thermal resistance. In the figure below, the resistor networks for the two types of windows can be seen. The lower one is the single-8-mm-thick glass sheet. Where the upper one is for the double 4-mm-thick glass sheets. The magnitude of the sum of the outer two resistors for the upper one is identical to the magnitude of the single resistor below. The upper network has an additional resistor in between due to stagnant gas, implying that two-4mm-thick glass sheets with stagnant gas has a higher thermal resistance. Thus less heat will pass through two 4-mm-thick glass sheets compared to single 8-mm-thick glass sheet.



Lecture 7 - Question 7



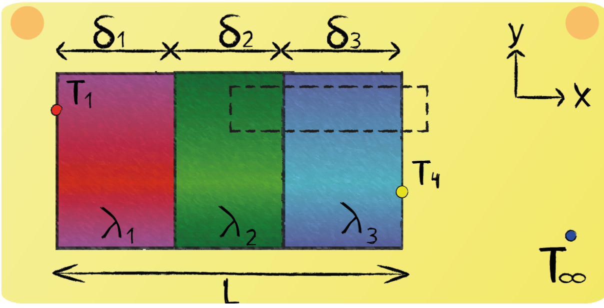
Consider a hot baked potato on a plate. The temperature of the potato is observed to drop by 4 °C during the first minute. Will the temperature drop during the second minute be less than, equal to or more than 4 °C? Why?

The drop will be less than. This due to a decreasing temperature gradient.The difference between surface temperature and environmental temperature is bigger at the beginning than after the first minute. It can be said that the temperature gradient has decreased. The rate of heat transfer is described as:

$$\dot{Q} = \frac{\Delta T}{R}$$

Thus less heat will leave during the second minute. The temperature drop is proportional to the amount of heat leaving. This is characterized by specific heat capacity. If less heat leaves the body, the temperature drop will thus be less as well.

Lecture 7 - Question 8



Determine the heat transfer passing the multi-layer wall. Take $\delta_1 = 0.8\ cm$, $\delta_2 = 1.6\ cm$, $\delta_3 = 1.2\ cm$, $\lambda_1 = 2\ W/mK$, $\lambda_2 = 4\ W/mK$, $\lambda_3 = 3\ W/mK$, $A = 1\ m^2$, $T_1 = 293\ K$, $\alpha = 30\ W/m^2K$ and $T_\infty = 273\ K$.

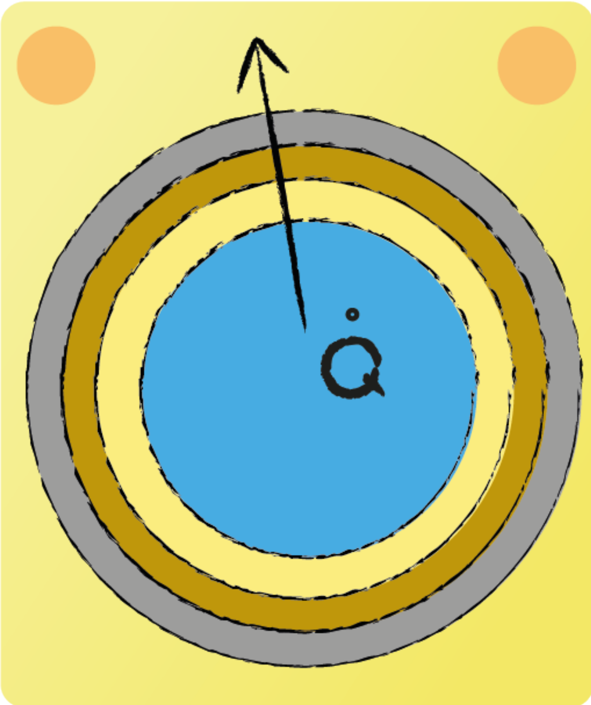
Using the following equation:

$$\dot{Q} = \frac{1}{R_{c,tot}}(T_1 - T_{n+1})$$

Results in:

$$\dot{Q} = \frac{T_1 - T_\infty}{\frac{\delta_1}{\lambda_1 A} + \frac{\delta_2}{\lambda_2 A} + \frac{\delta_3}{\lambda_3 A} + \frac{1}{\alpha A}} = \frac{20}{0.004+0.004+0.004+0.0333} = 441\ W$$

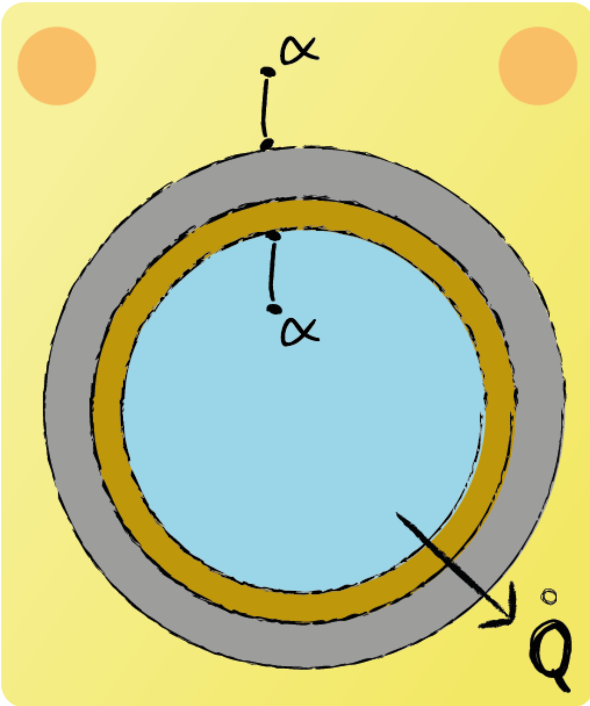
Lecture 8 - Question 1



Which of the following assumptions is/are **not** true when performing calculations with on a multi-layer pipe wall containing a fluid at constant temperature T_i on the inside and being surrounded by a fluid at constant temperature T_∞.

Constant cross section area along an increasing radius per layer.
As the radius increases the perimeter of the cross section increases and thus so does the cross section area.
Heat transfer occurs along the direction of the temperature gradient. Since there is only a temperature gradient in the direction of r, the rate of heat transfer can be characterized to be one-dimensional.
The material properties of a multi-layer pipe wall will remain constant per layer of material.
Since the fluid temperatures remain constant, so will the temperature gradient. For this reason we can speak of steady-state heat transfer.

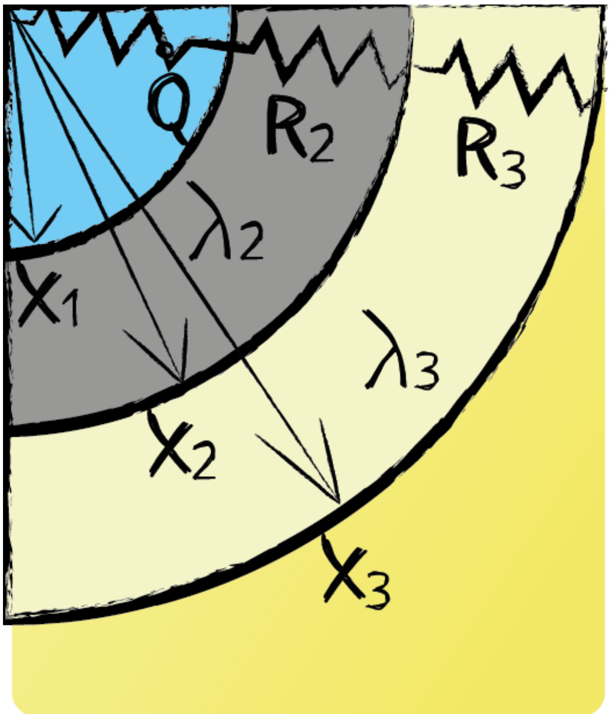
Lecture 8 - Question 2



Which of the following statements is/are **not** true when performing steady-state heat transfer calculations with on a multi-layer pipe wall?

$A_{in} = A_{out}$
$\dot{q}''_{in} = \dot{q}''_{out}$
$A_{in} = A_{out}$ states that the cross section remains constant. But along r the perimeter of a multi-layer pipe wall will increase and so will the cross section area. Thus this statement is not true.
$\dot{Q}_{in} = \dot{Q}_{out}$ states that the heat entering the multi-layer pipe wall equals the heat leaving the system. Since there is steady-state heat transfer this should be fulfilled in order to satisfy the energy equation. This statement is true.
$\dot{q}''_{in} = \dot{q}''_{out}$ states that the area related heat transfer rates entering and leaving the system are equal to each other. As described above the heat fluxes entering and leaving do equal each other. But as the cross sectional area is not constant along r, nor will the area related heat transfer rates be. This statement is thus not true.

Lecture 8 - Question 3



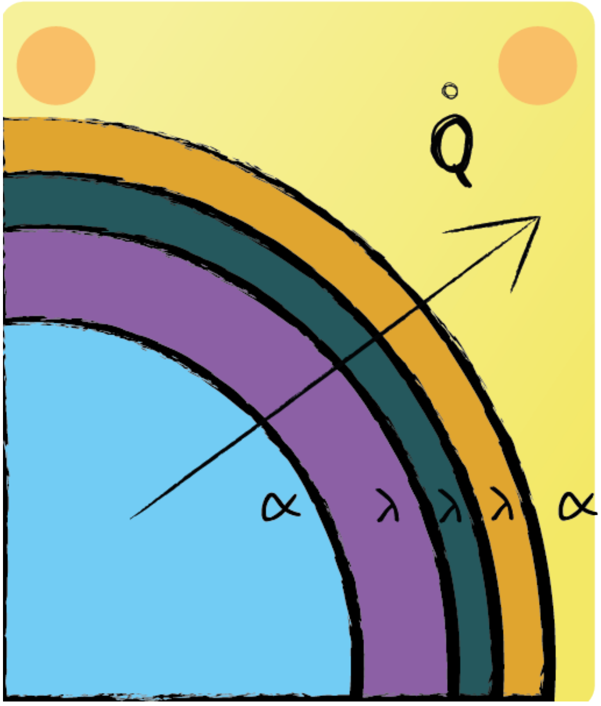
Consider the case as in the figure for a multi-layer pipe wall, where r_1 = 10 mm , r_2 = 20 mm, λ_2 = 0.2 W/mK, r_3 = 40 mm,λ_3 = 0.1 W/mK and L = 1 m. Which statement is true?

$$R_2 = \frac{1}{\lambda_2} \cdot \frac{1}{2\pi L} \cdot ln\left(\frac{r_2}{r_1}\right) \quad = \quad 0.5516\ °C/W$$

$$R_3 = \frac{1}{\lambda_3} \cdot \frac{1}{2\pi L} \cdot ln\left(\frac{r_3}{r_2}\right) \quad = \quad 1.1032\ °C/W$$

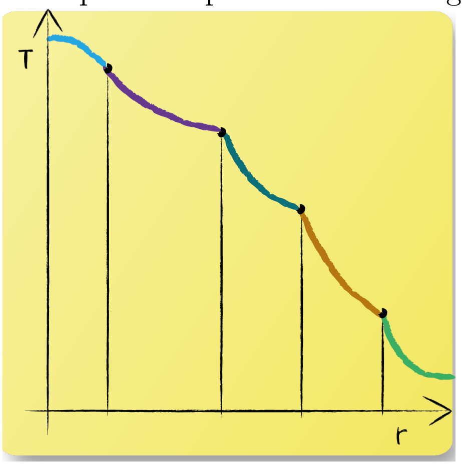
Thus $R_2 \ < R_3$

Lecture 8 - Question 4

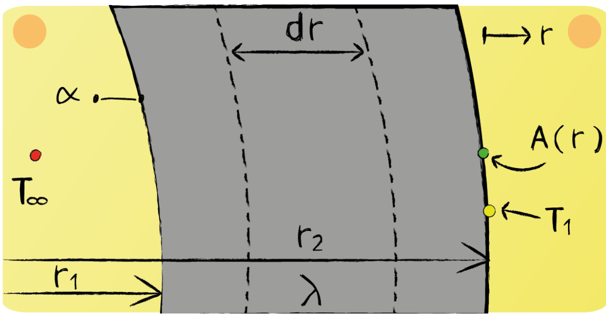


Consider steady-state heat transfer for a multi-layered pipe wall as in the figure. A fluid is flowing through the pipe. The center and the environmental temperatures remain constant. Which of the following temperature profiles is correct?

At the center r=0 the temperature gradient equals zero due to symmetry. After passing the first layer the heat flux is constant, for which the temperature gradient needs to decrease. This because the thermal resistance increases due to an increase of surface area. This is similar for the next layers. After passing the latter layer, when $r \to \infty$ the temperature gradient should equal zero again. This because of the fact that the temperature approaches the environmental temperature. Resulting in a temperature profile as in the figure below.



Lecture 8 - Question 5



Develop an energy balance to calculate the temperature profile inside the pipe wall and give the boundary conditions. Assume one-dimensional steady-state heat transfer in radial direction. The expansion of the pipe in axial direction is L.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

Since the type of heat transfer is steady-state, the sum of the in- and outgoing heat fluxes of the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda A(r)\frac{\partial T}{\partial r} = -\lambda 2\pi r L\frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} = \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r}dr = -\lambda 2\pi r L\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}(-\lambda 2\pi r L\frac{\partial T}{\partial r})dr$$

The ingoing flux can be described by use of Fouriers law and the outgoing flux can be approximated by use of the Taylor series expansion.

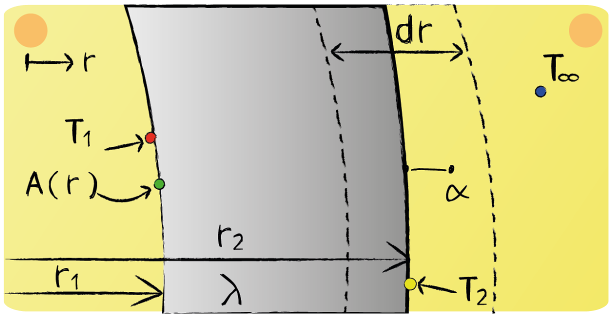
Boundary conditions:

$$\frac{\partial T(r=r_1)}{\partial r} = -\frac{\alpha}{\lambda}(T(r=r_1) - T_\infty)$$

$$T(r=r_2) = T_1$$

The first boundary condition results from the fact that $\dot{Q}_{r=r_1} = -\lambda A(r)\frac{\partial T(r=r_1)}{\partial r} = \alpha\ A(r)(T(r=r_1) - T_\infty)$, the second one describes that the temperature at the surface equals T_1.

Lecture 8 - Question 6



Develop an energy balance for the infinitesimal element and give the numerical values for the used elements. Take $T_1 = 20\ ^oC$, $T_2 = 10\ ^oC$, $\lambda = 0.01\ W/mK$, $L = 10\ m$, $r_1 = 13\ cm$, $r_2 = 15\ cm$ and $\alpha = 20\ W/m^2K$. Assume steady-state conditions in radial direction.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

Since the type of heat transfer is steady-state, the sum of the in- and outgoing heat fluxes of the control volume should equal zero.

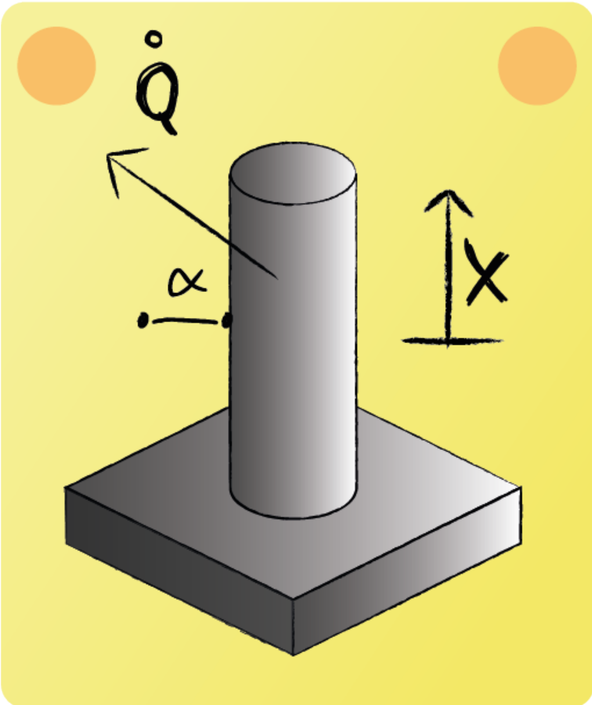
Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda 2\pi L \left(T_2 - T_1\right) \frac{1}{ln\frac{r_2}{r_1}} = -0.01 \cdot 2\pi \cdot 10 \left(20 - 10\right) \frac{1}{ln\left(\frac{0.15}{0.13}\right)} = 44\ W$$

$$\dot{Q}_{r,out} = -\lambda 2\pi L \left(T_2 - T_1\right) \frac{1}{ln\frac{r_2}{r_1}} = 44\ W$$

The heat fluxes can be described in terms of conductive heat transfer.

Lecture 9 - Question 1

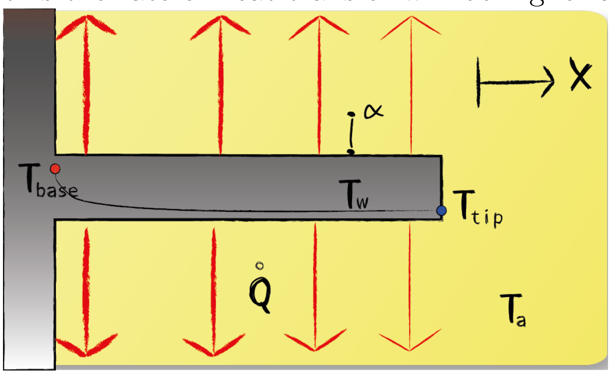


Consider a fin exchanging heat with a fluid due to convective heat transfer. Note that the tip has a lower temperature than the base. This heat transfer is characterized by a constant heat transfer coefficient α. Which of the following statements is true?

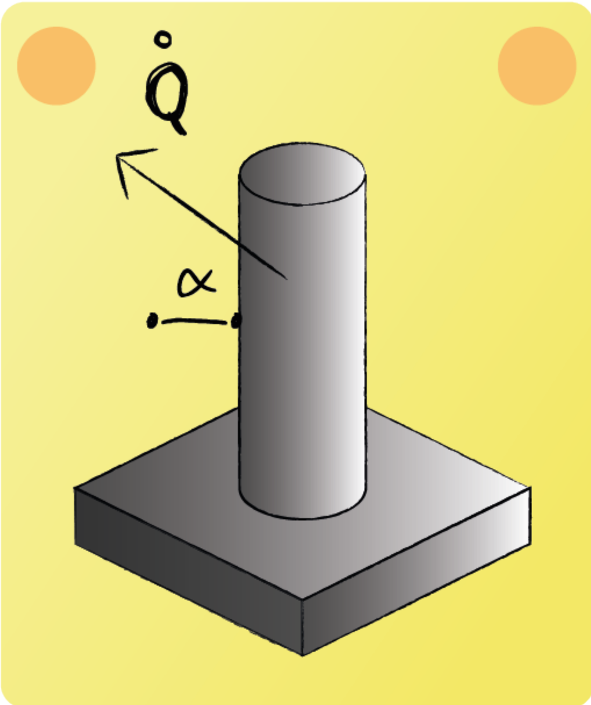
The rate of heat transfer at the base is higher than the rate of heat transfer at the tip. Heat transfer is described by:

$$\dot{Q} = \alpha \cdot A_s \cdot (T_\mathrm{w} - T_\mathrm{a})$$

T_w will be higher at the base than at the tip, because of this the rate of heat transfer will be higher at the base.



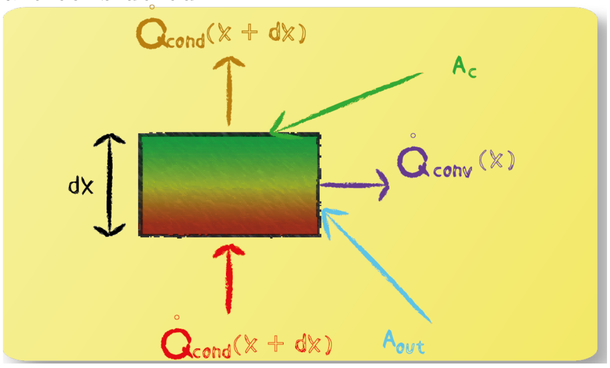
Lecture 9 - Question 2



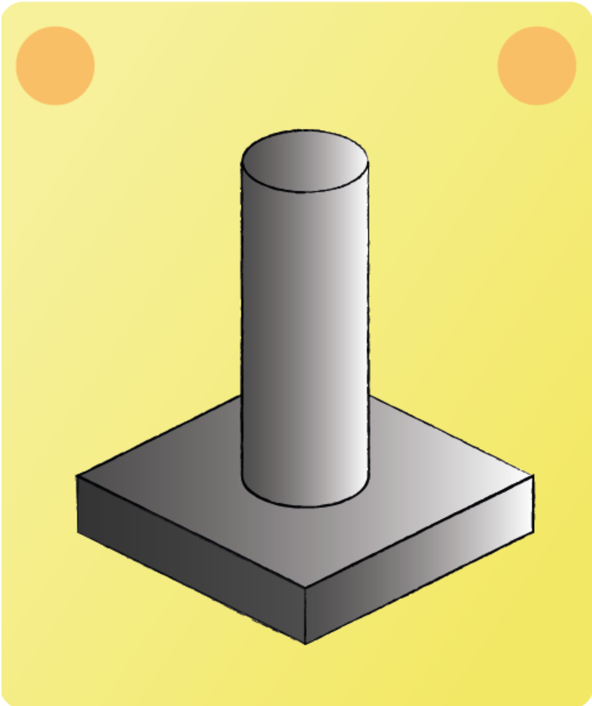
Consider a fin exchanging heat with a fluid. When performing calculations regarding this heat exchange, which types of heat exchange are of relevance when using the fin differential equation?

$$\lambda \cdot A_c \cdot \frac{\partial^2 T}{\partial x^2} = \alpha \cdot U \cdot (T(x) - T_A)$$

In the study of heat transfer, fins are surfaces that extend from an object to increase the rate of heat transfer to or from the environment by convection. The heat that is transferred by convection is first conducted from the base to the fin. For the derivation of the fin differential equation by use of the energy balance only heat fluxes entering and leaving due to conduction and convection are considered.



Lecture 9 - Question 3



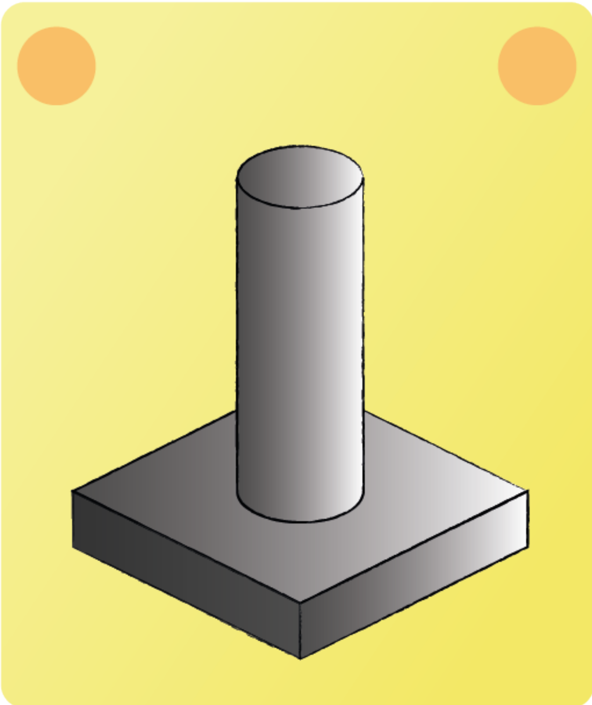
What are the disadvantages of fins?

Higher material consumption
Increased pressure loss
Additional weight and volume
When fins are used an increase of the rate of heat transfer is desired. When using fins the surface area increases. The rate of heat transfer by convection is described as:

$$\dot{Q} = \alpha \cdot A_s \cdot (T_{\mathrm{w}} - T_{\mathrm{a}})$$

Thus heat is better transferred. Making the higher surface area to be an advantage indirectly causing the higher rate of heat transfer.
This additional surface area causes the weight and volume of a design to increase as well, which is undesirable. Due to the increase in volume the material consumption will rise and so will the production costs, which is a disadvantage.
The increased pressure loss is a disadvantage because it is a loss of energy, which can not be recovered. Due to the energy lost, more energy will be required to move the same volume of fluid when no fins are used.

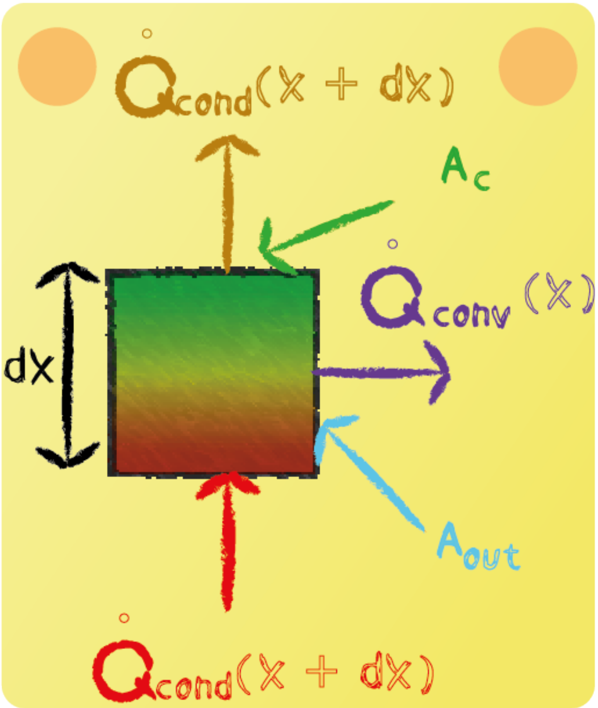
Lecture 9 - Question 4



Which definition describes a fin the best? A fin

is a surface that extends from an object to increase the rate of heat transfer to or from the environment by increasing convection.
A fin is a part of a machine or equipment that has the purpose to increase the surface area, so that the heat transfer increases. This heat transfer is usually between air and a heat generating device like an engine , a processor or a heat exchanger for example.

Lecture 9 - Question 5



For the derivation of the fin differential equation the energy balance of in- and outgoing fluxes are used. What is the unit of the fin differential equation, check whether the right and left side have the same unit.

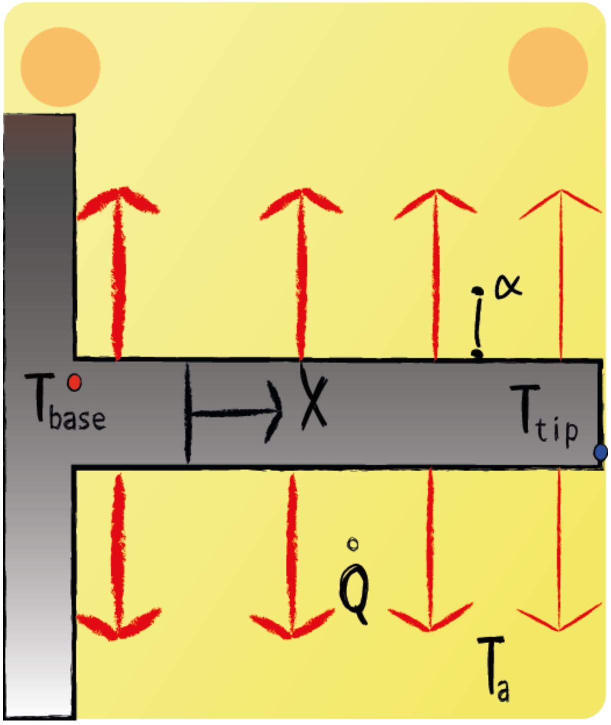
$$\lambda \cdot A_C \cdot \frac{\partial^2 T}{\partial x^2} = \alpha \cdot U \cdot (T(x) - T_A)$$

$$\left[\lambda \cdot A_C \cdot \frac{\partial^2 T}{\partial x^2}\right] = (W^1 m^{-1} K^{-1}) \cdot (W^0 m^2 K^0) \cdot (W^0 m^{-2} K^1) = W^1 m^{-1} = J^1 s^{-1} m^{-1}$$

$$[\alpha \cdot U \cdot (T(x) - T_A)] = (W^1 m^{-2} K^{-1}) \cdot (W^0 m^1 K^0) \cdot (W^0 m^0 K^1) = J^1 s^{-1} m^{-1}$$

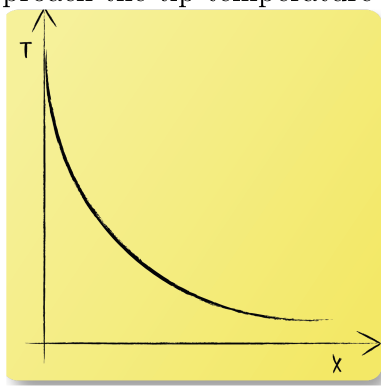
So they have the same units.

Lecture 9 - Question 6

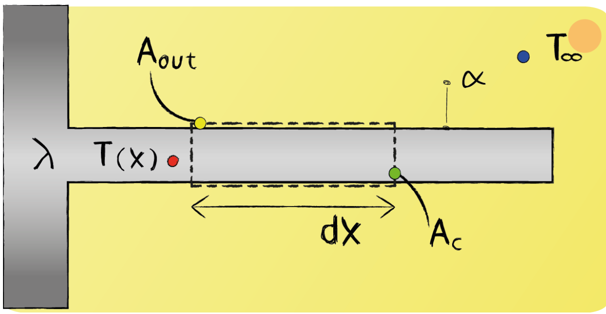


Consider a fin exchanging heat. Which of the following temperature profiles is correct?

At the base the rate of heat transfer is the biggest. This is due to a large temperature difference. Here the temperature should be at its maximum as well as the slope of the temperature profile.
At the tip the rate of heat transfer is the smallest and because of this the slope will be relatively small. This is due to a small difference between the ambient and tip temperature. Thus the temperature profile will approach the tip temperature with a small slope.



Lecture 9 - Question 7



Give the energy balance to derive the fin equation. Assume one-dimensional steady-state heat transfer in x-direction. The surface area temperature of the fin T(x) changes in axial direction.

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} - \dot{Q}_{conv}(x) = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

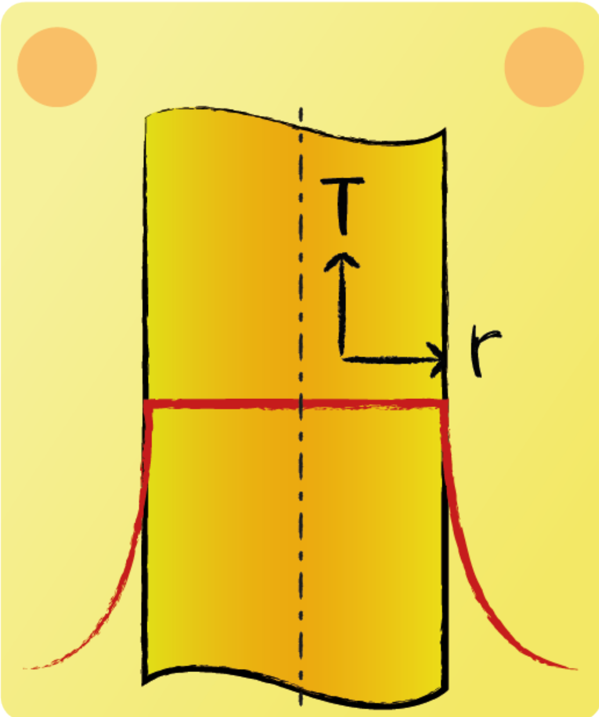
$$\dot{Q}_{x,in} = -\lambda \cdot A_c \cdot \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda \cdot A_c \cdot \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx$$

$$\dot{Q}_{conv}(x) = \alpha \cdot A_{out} \left(T(x) - T_\infty\right)$$

The heat entering the system is transferred from the base by conductive heat transfer. This heat flux is distributed over a convective and conductive heat flux. $\dot{Q}_{conv}(x)$ can be described by Newton's law of cooling, $\dot{Q}_{x,out}$ can be approximated by use of the Taylor series expansion.

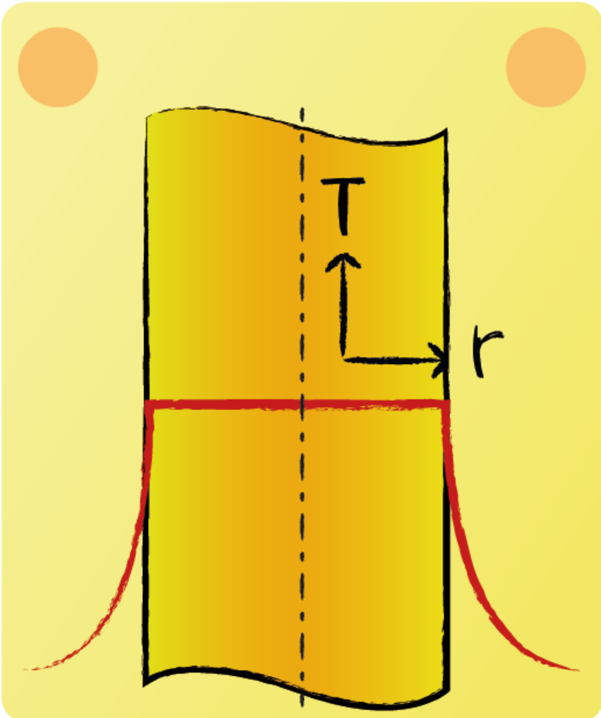
Lecture 10 - Question 1



Give the definition of the Biot number

It is the resistance of **conduction inside** the body **divided** by the resistance of **convection outside** the body.
The Biot number is a dimensionless number. It gives a simple index of the ratio of the thermal conductive resistance inside of a body and of the thermal convective resistance outside the body. This ratio helps determining whether or not the temperatures inside a body will vary significantly, while the body heats or cools over time.

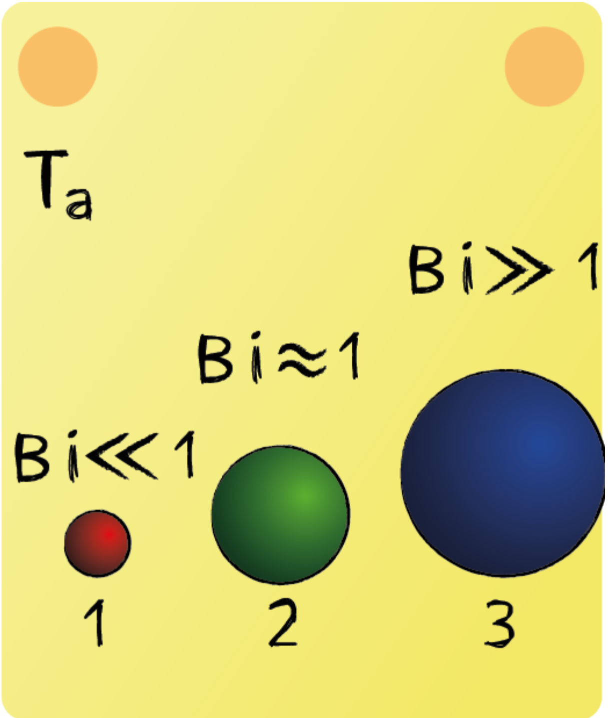
Lecture 10 - Question 2



Consider the situation as in the figure. Which statement is true?

The conductive resistance inside the body is much smaller relative to the convective resistance outside the body.
The temperature inside the body does not vary significantly, while the body is cooling down. Implying that $Bi << 1$. When $Bi << 1$, $R_\lambda << R_\alpha$.

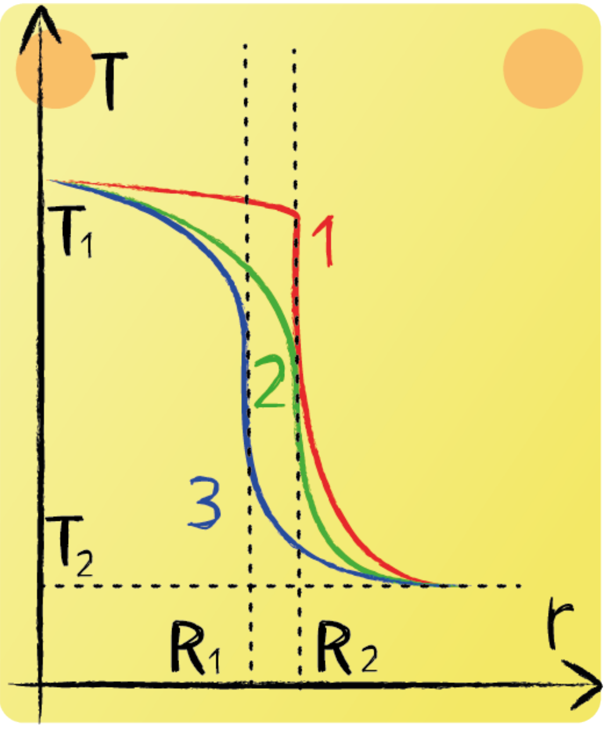
Lecture 10 - Question 3



Consider three spheres cooling down. For sphere 1 Bi << 1, for sphere 2 Bi ≈ 1 and for sphere 3 Bi >> 3. The center temperature is denoted by $T_{\mathrm{r=0}}$ and the surface temperature by $T_{\mathrm{r=R}}$ for all the three spheres. Which answer is correct?

The difference between $T_{\mathrm{r=0}}$ and $T_{\mathrm{r=R}}$ is the smallest for sphere **1** and the biggest for sphere **3**.
For sphere 1 it is known that $Bi << 1$, implying that the temperatures inside the body will not vary significantly and thus the difference between $T_{\mathrm{r=0}}$ and $T_{\mathrm{r=R}}$ is the smallest for this sphere.
For sphere 3 it is known that $Bi >> 1$, implying that the temperatures inside the body will vary significantly and thus the difference between $T_{\mathrm{r=0}}$ and $T_{\mathrm{r=R}}$ is the biggest for this sphere.

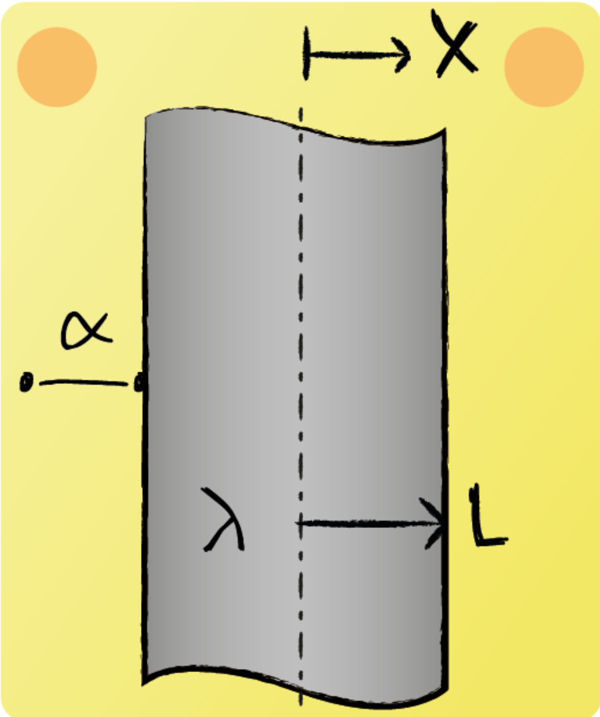
Lecture 10 - Question 4



Consider the following temperature profiles in radial direction for three type of rods. Which of the rods 1, 2 and 3 has the smallest Biot number?

Rod 1 has the smallest Biot number. The body temperature of rod 1 varies the least compared to the other two. For rod 1 and 2 this is in the range of $r = 0$ to $r = R_2$ and for rod 3 this is in the range of $r = 0$ to $r = R_1$. The smaller the Biot number, the less the temperature inside the body will vary. For this reason rod 1 has the smallest Biot number.

Lecture 10 - Question 5



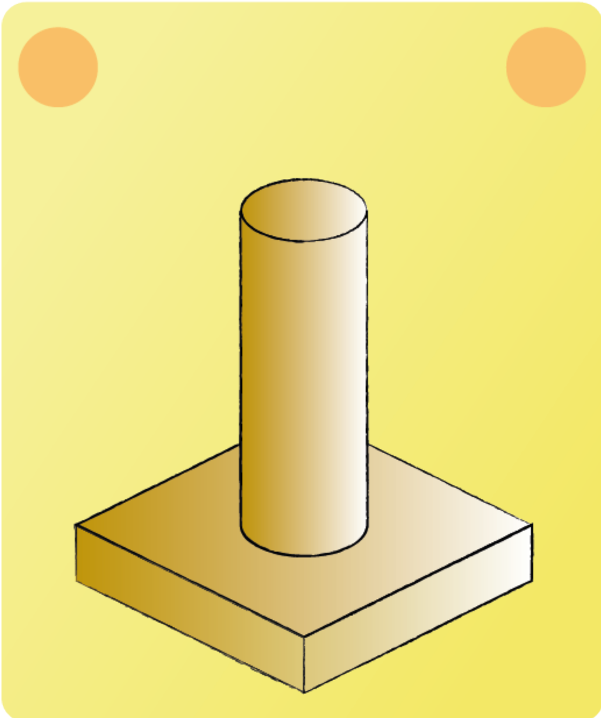
A sheet made out of aluminum is cooling down. It has a thermal conductivity of $\lambda = 200$ W/mK. Besides it has a characteristic length of L = 0.01 m and a convective heat transfer coefficient of $\alpha = 20\ W/m^2K$. Which statement is true?

Since Bi << 1, as x increases the drop of the temperature profile inside the body is negligible relative to the drop of the temperature profile outside the body.

$$Bi = \frac{\alpha \cdot L}{\lambda} = \frac{20 \cdot 0.01}{200} = 1 \cdot 10^{-3}$$

$Bi << 1$, implying that the temperatures inside the body will not vary significantly relative to the temperature profile outside the body.

Lecture 11 - Question 1



Which of the following equations represents the inhomogeneous differential equation for fins?

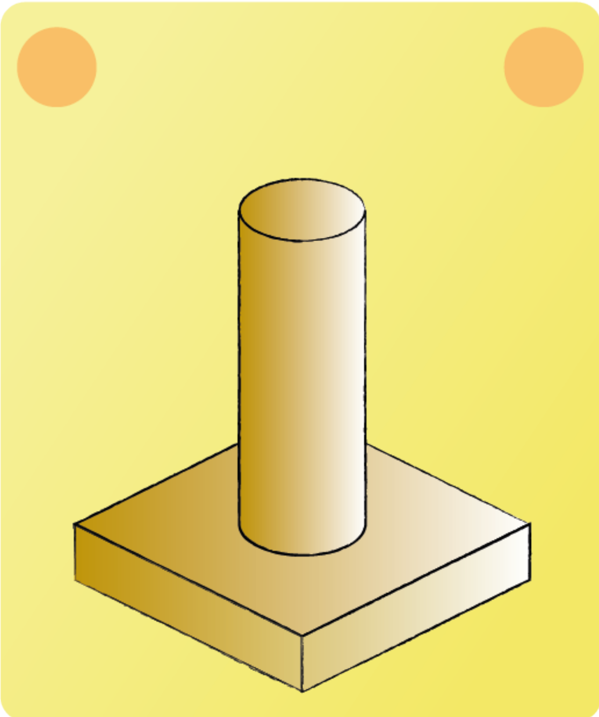
$$\lambda \cdot A_c \cdot \frac{\partial^2 T}{\partial x^2} = \alpha \cdot U\left(T\left(x\right) - T_A\right)$$

The only other differential equation for fins given is:

$$\frac{\partial^2 \theta}{\partial x^2} - m^2 \theta\left(x\right) = 0$$

But this one is homogeneous and is for that reason not correct.

Lecture 11 - Question 2



Remember the inhomogeneous differential equation for fins:

$\lambda \cdot A_c \cdot \frac{\partial^2 T}{\partial x^2} = \alpha \cdot U \left(T(x) - T_A\right)$

Which of the following terms can be used to simplify it to a 2nd order homogeneous differential equation?

$$\theta(x) \; = \; T(x) - T_A$$

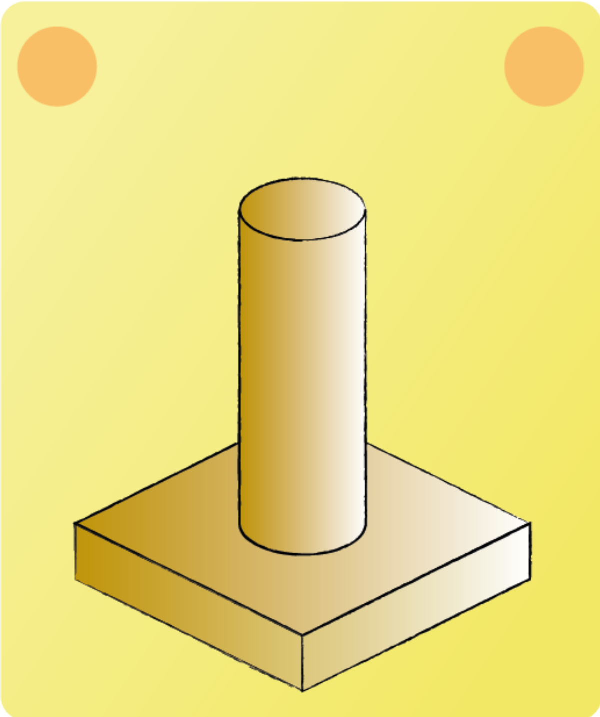
Substitution of: $\theta(x) \; = \; T(x) - T_A$ and thus $\frac{\partial^2 \theta}{\partial x^2} \; = \; \frac{\partial^2 T}{\partial x^2}$ into $\lambda \cdot A_c \cdot \frac{\partial^2 T}{\partial x^2} = \alpha \cdot U \left(T(x) - T_A\right)$ leads to:

$$\lambda \cdot A_c \cdot \frac{\partial^2 \theta}{\partial x^2} = \alpha \cdot U \cdot \theta(x)$$

Rearranging results in the 2nd order homogeneous differential equation:

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{\alpha \cdot U}{\lambda \cdot A_c} \cdot \theta(x) \; = \; 0$$

Lecture 11 - Question 3



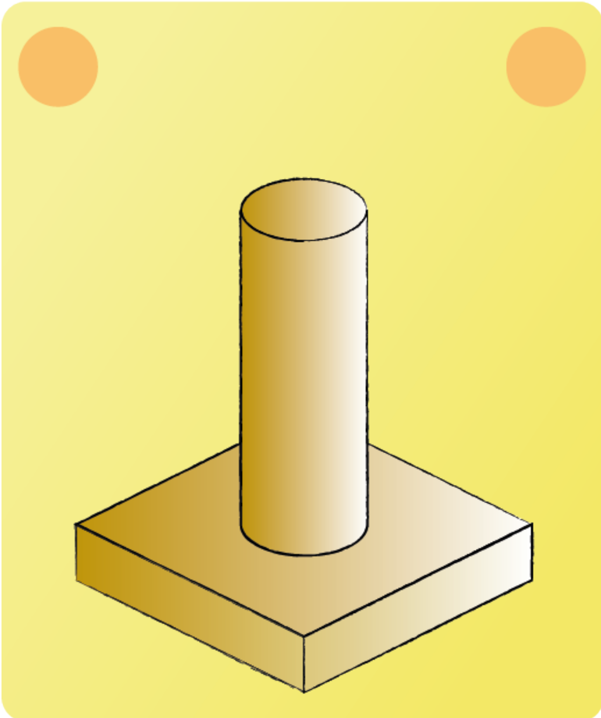
After substitution of $\theta\left(x\right) = T\left(x\right) - T_A$ into $\lambda \cdot A_c \cdot \frac{\partial^2 T}{\partial x^2} = \alpha \cdot U \left(T\left(x\right) - T_A\right)$, the equation becomes: $\frac{\partial^2 \theta}{\partial x^2} - \frac{\alpha \cdot U}{\lambda \cdot A_c} \cdot \theta \ = \ 0$. Which of the following options can be used to simplify the equation even further?

$$m^2 = \frac{\alpha \cdot U}{\lambda \cdot A_c}$$

Substitution leads to:

$$\frac{\partial^2 \theta}{\partial x^2} - m^2 \cdot \theta \ = \ 0$$

Lecture 11 - Question 4



After simplifications, the result is the following 2nd order homogeneous differential equation for fins: $\frac{\partial^2\theta}{\partial x^2} - m^2 \cdot \theta = 0$ Which of the following options describe possible solutions for the homogeneous differential equation?

$$\theta\left(x\right)=A\cdot sinh(m\cdot x)+B\cdot cosh(m\cdot x)$$

$$\theta\left(x\right)=C\cdot e^{mx}\ +D\cdot e^{-mx}$$

See the derivation:

$$Try:\ \theta(x)\ =\ e^{sx}\ \ \Rightarrow\frac{\partial^2\theta}{\partial x^2}=s^2e^{sx}$$

$$s^2e^{sx}{-}m^2e^{sx}=0$$

$$(s^2{-}m^2)e^{sx}=0,\ note\ that\ e^{sx}\neq 0,\ \ for\ x\in R$$

$$s_{1,2}\ =\ \pm\sqrt{m^2}$$

$$\rightarrow\theta(x)\ =\ Ce^{s_1x}+De^{s_2x}\ =\ Ce^{mx}+De^{-mx}$$

Remember the mathematical transformations:

$$\sinh\left(x\right)=\tfrac{1}{2}\ \left(e^{x}-e^{-x}\right)$$

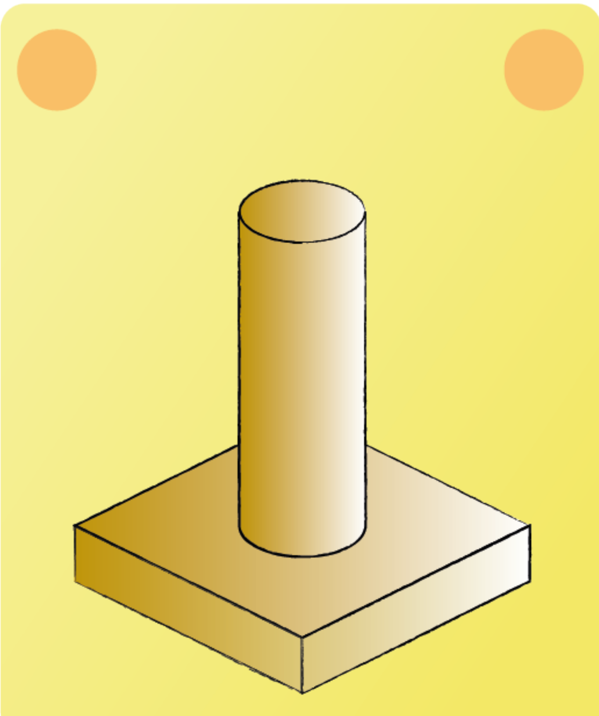
$$\cosh\left(x\right)=\tfrac{1}{2}\ \left(e^{x}+e^{-x}\right)$$

$$C=\tfrac{A+B}{2},\qquad D=\tfrac{B-A}{2}$$

Resulting in a different form:

$$\theta\left(x\right)=A\cdot\sinh\left(m\cdot x\right)+B\cosh\left(m\cdot x\right)$$

Lecture 11 - Question 5



One of the possible solutions for the homogeneous differential equations for fins is

$$\theta\left(x\right)=A\cdot sinh(m\cdot x)+B\cdot cosh(m\cdot x)$$

Consider an sufficiently long fin with no heat exchange at the tip. Futhermore, the base temperature T_{B} and the environmental temperature T_{A} are known. Which two boundary conditions can be **directly** used for determining coefficients A and B?

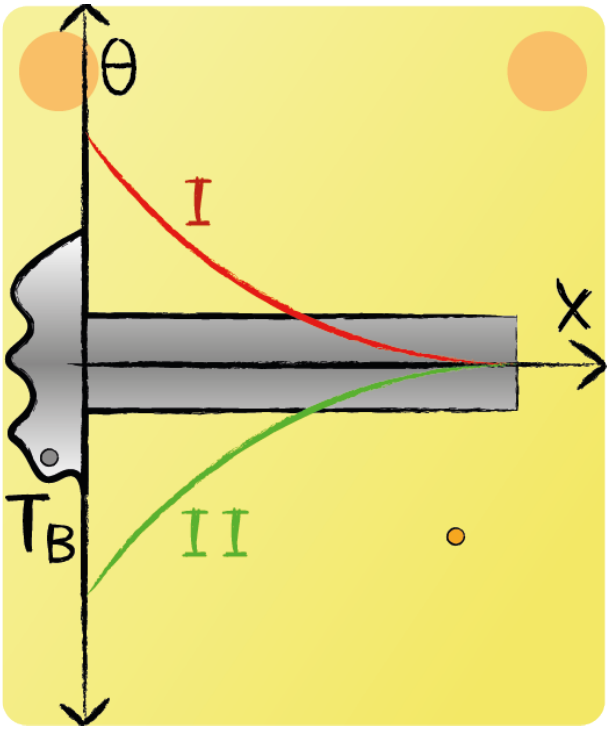
$$\theta(0) = T_B - T_A$$

Describes directly that the temperature difference between the surface temperature T(x) at x=0 and the ambient temperature T_A is the difference between the base temperature T_B and ambient temperature T_A. Which is a result from the fact that the surface temperature at x=0 equals the base temperature $T(0) = T_\mathrm{B}$.

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

Describes directly that, due to the fact that the fin is infinitely long, no heat exchange will take place at x=L. Therefore the gradient of the temperature difference between the surface temperature T(x) at x=L and the ambient temperature T_A is zero. Which is a result from the fact that the surface temperature gradient at x=L approaches zero $\left.\frac{dT}{dx}\right|_{x=L} = 0$

Lecture 11 - Question 6



In the figure there is a fin with a base temperature T_B and a surrounding fluid with temperature T_A. Consider the following temperature profiles for a fin. Fill in the correct answers.

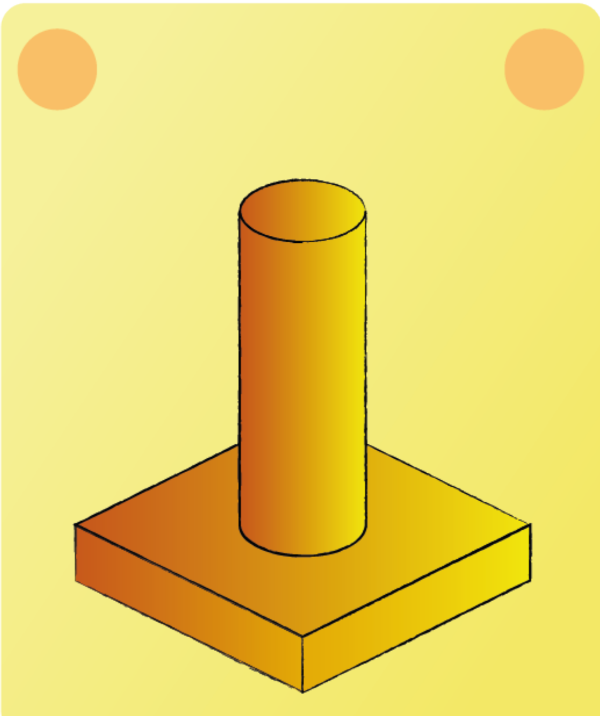
Profile I applies when $T_{\mathrm{B}} > T_{\mathrm{A}}$ and describes **heating** of the surrounding fluid.
Profile II applies when $T_{\mathrm{B}} < T_{\mathrm{A}}$ and describes **cooling** of the surrounding fluid.
Profile I as well as II will **never equal** $\theta = 0$ at x=L in practice.
θ will be positive when $T_{\mathrm{B}} > T_{\mathrm{A}}$ and negative when $T_{\mathrm{B}} < T_{\mathrm{A}}$. Where a positive value for θ will describe heating of the fluid, because heat is transferred from the fin to the fluid. So will a negative value for θ describe heat transfer to the fin and thus cooling of the fluid.
Even with $\dot{Q}_{head} = 0$ the temperature at the tip is always above the ambient temperature and θ will only approach zero.
With the given boundary conditions: $\theta(0) = \theta_B$ and $\frac{d\theta}{dx}\Big|_{x=L} = 0$

$$\rightarrow \theta(x) = \theta_B \cdot \left(\frac{e^{m(L-x)}+e^{-m(L-x)}}{e^{mL}+e^{-mL}}\right)$$

and thus

$$\theta(L) = \theta_B \cdot \left(\frac{e^0+e^0}{e^{mL}+e^{-mL}}\right) = \theta_B \cdot \left(\frac{2}{e^{mL}+e^{-mL}}\right) \neq 0$$

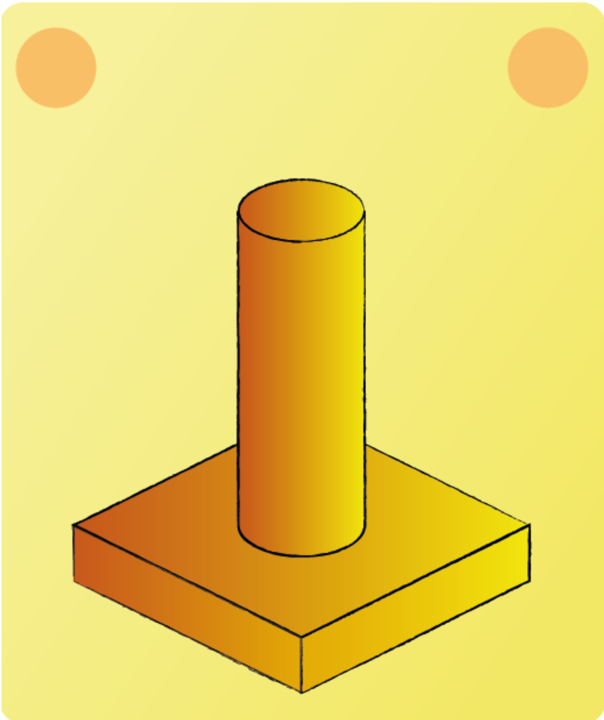
Lecture 12 - Question 1



Which of the statements is true regarding fin efficiency?

As the fin efficiency increases, the transferred heat approaches the transferable heat.
The efficiency tells us something about the state or quality of being efficient and not about the capacity of heat that is or can be transferred.

Lecture 12 - Question 2



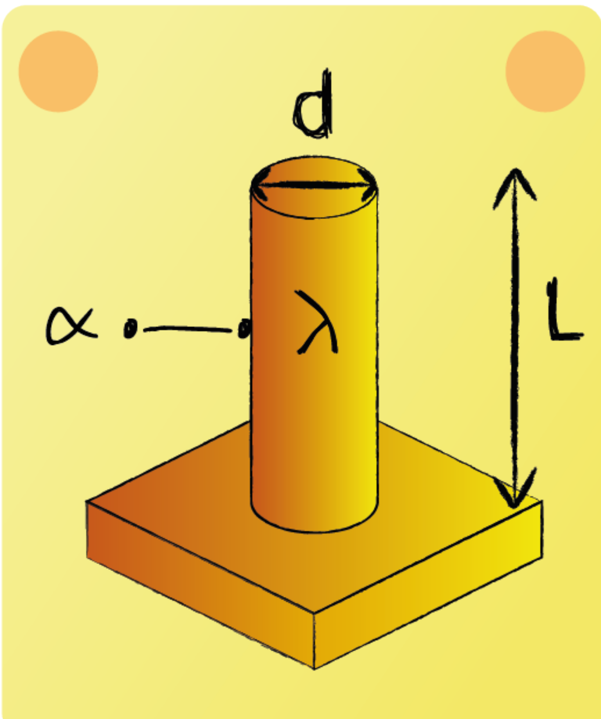
Which of the statements is/are true regarding an ideal fin?

An ideal fin is characterized by a fin efficiency of 1. Looking at the fin efficiency equation:

$$\eta_R = \frac{\textit{Heat transferred from a fin}}{\textit{Maximum amount of transferable heat}}$$

It can be noted that the heat transferred should equal the maximum amount of transferable heat. For that reason an ideal fin does transfer the maximum amount of heat.
The effectiveness does not tell something about the state or quality of being efficient. For that reason an ideal fin does not have to have a effectiveness of 1.
Maximum transferable heat flow is achieved when the temperature remains equal to the base temperature along the entire length of the fin. For that reason the temperature along an ideal fin equals the base temperature.
An ideal fin does not always transfer more heat than a non-ideal fin. The word 'ideal' gives information regarding the efficiency and not regarding the capacity.
As mentioned before an ideal fin will always have an efficiency of 1.

Lecture 12 - Question 3



Determine the fin efficiency. Take d = 10 mm, $\lambda = 240$ W/mK, $\alpha = 5$ W/m^{2}K and L = 50 mm.

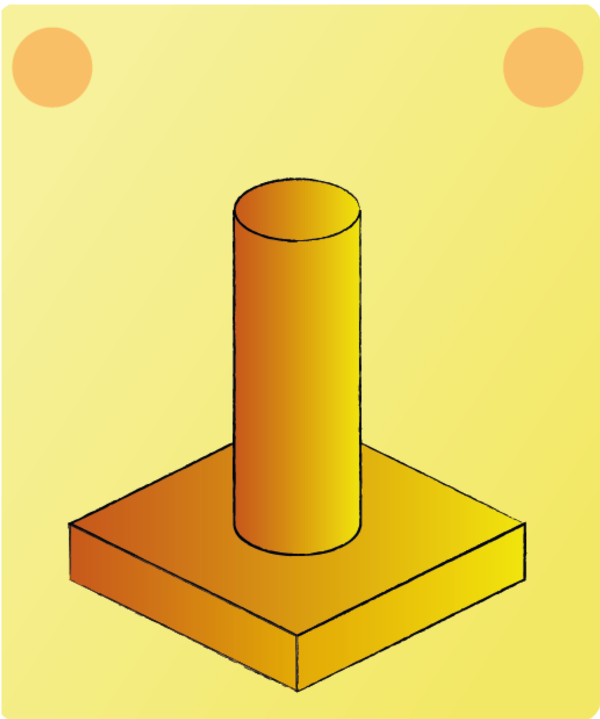
$$A_c = \frac{1}{4}\pi d^2 = 7.854 \cdot 10^{-5}\ m^2$$

$$U = \pi d = 0.0314\ m$$

$$m = \left(\frac{\alpha U}{\lambda A_c}\right)^{1/2} = 2.8868\ m^{-1}$$

$$\eta_R = \frac{tanh(mL)}{mL} = 0.9931$$

Lecture 12 - Question 4



Give the unit of the fin efficiency, by checking the unit of:

$$\eta_R = \frac{tanh(mL)}{mL}$$

$$m = \left(\frac{(\alpha)(U)}{(\lambda)(A_c)}\right)^{1/2}$$

$$[m] = \left(\frac{(W^1m^{-2}K^{-1})(W^0m^1K^0)}{(W^1m^{-1}K^{-1})(W^0m^2K^0)}\right)^{1/2} = m^{-1}$$

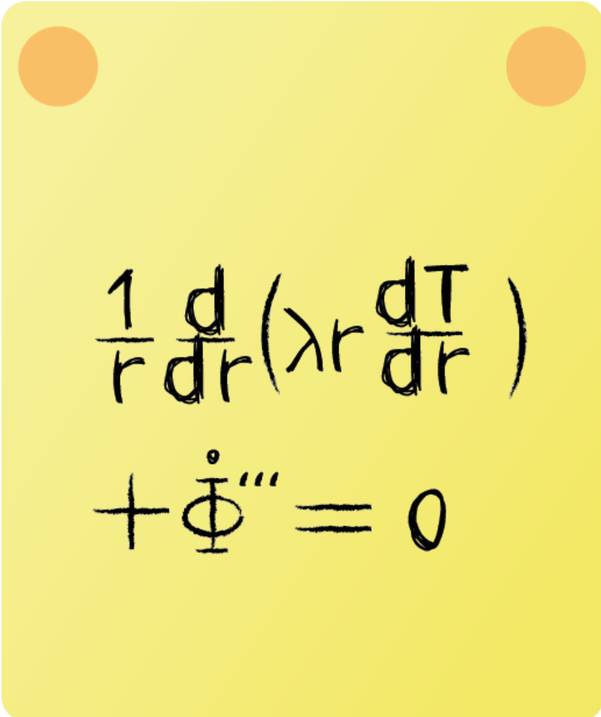
$$[L] = m^1$$

$$\eta_R = \frac{tanh(mL)}{mL}$$

$$[\eta_R] = \left(\left(\frac{(m^{-1})(m^1)}{(m^{-1})(m^1)}\right)\right) = 1 = [-]$$

Thus the fin efficiency is a dimensionless number.

Lecture 13 - Question 1



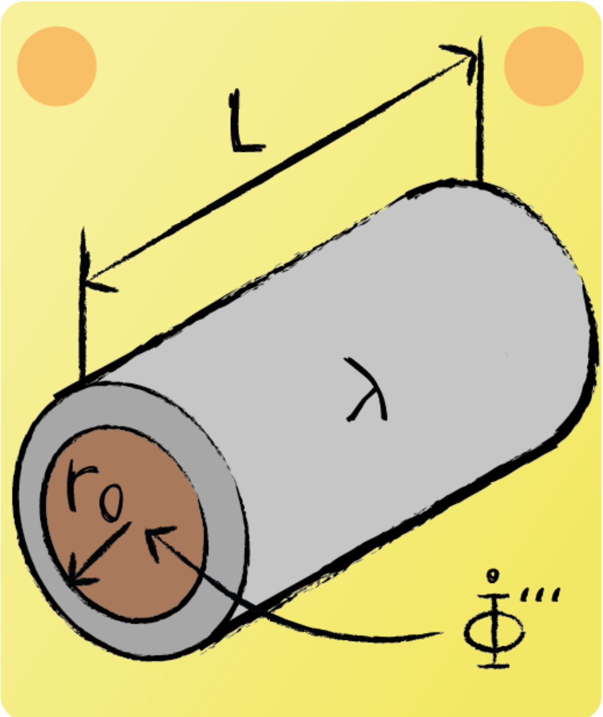
Consider a medium in which the heat conduction equation is given in its simplest form as:

$$\frac{1}{r}\frac{d}{dr}\left(\lambda r \frac{dT}{dr}\right) + \dot{\Phi}''' = 0$$

Indicate whether:
The heat transfer is **steady** / **transient**
The heat transfer is **one-**/ **two-**/ **three-**dimensional.
There is **heat generation** / **no heat generation** in the medium.

From the equation it can be seen that the heat transfer is **steady**, as $\frac{\partial}{\partial t} = 0$.
The heat transfer is **one**-dimensional as $\frac{\partial}{\partial r} = \frac{d}{dr} \neq 0$.
$\dot{\Phi}'''$ represents a volume-based heat source. Implying that **heat generation** takes place.

Lecture 13 - Question 2

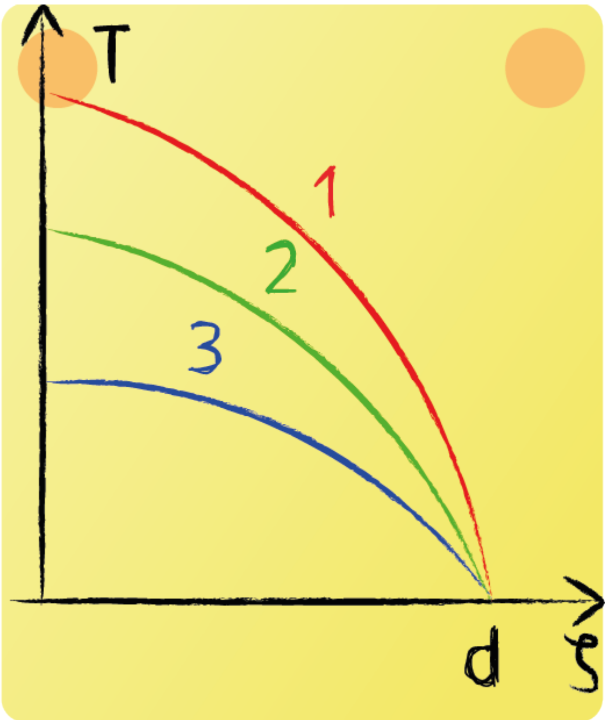


Heat is generated in a long wire of radius r_0 covered with a plastic insulation layer at a constant rate of $\dot{\Phi}'''$. Express the heat flux density $\dot{q}$" at the interface using parameters π, r_0, L or $\dot{\Phi}'''$.

$$\dot{Q} = V \cdot \dot{\Phi}''' = \pi {r_0}^2 L \cdot \dot{\Phi}'''$$

$$\dot{q}" = \frac{\dot{Q}}{A_s} = \frac{\pi {r_0}^2 L \cdot \dot{\Phi}'''}{2\pi r_0 L} = \frac{r_0 \cdot \dot{\Phi}'''}{2} = \frac{1}{2} r_0 \cdot \dot{\Phi}'''$$

Lecture 13 - Question 3



A plate, a sphere and a cylinder are surrounded by a fluid with temperature T_A and convection coefficient α. All three object are made out of the same material with thermal conductivity λ. Heat is generated at the center of the objects at a constant rate $\dot{\Phi}'''$. For the plate ð= d and for the cylinder and sphere r_1 = d. Assign the temperature profiles to the corresponding objects.

1. Plate, 2. Cylinder and 3. Sphere.
Looking at the general temperature profile for a plate, cylindrical or spherical geometry and symmetry with source:

$$T(\zeta) \;=\; T_A + \frac{s^{\mathbf{2}} \cdot \Phi'''}{\mathbf{2(}n\mathbf{+1)} \cdot \lambda} \left[1 + \frac{2 \cdot \lambda}{\alpha \cdot s} - \left(\frac{\zeta}{s}\right)^2 \right]$$

The only difference is that for a plane n=0, for the cylinder n=1 and for the sphere n=2. At $\zeta = 0$ their temperatures will be respectively:

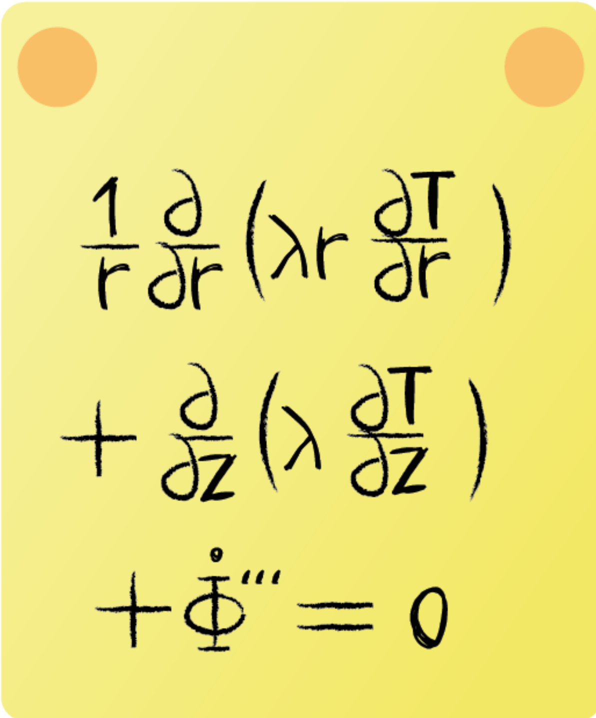
$$T_{plate}(0) \;=\; T_A + \frac{d^{\mathbf{2}} \cdot \Phi'''}{\mathbf{2} \cdot \lambda} \left[1 + \frac{2 \cdot \lambda}{\alpha \cdot d} \right]$$

$$T_{cylinder}(0) = T_A + \frac{d^{\mathbf{2}} \cdot \Phi'''}{\mathbf{4} \cdot \lambda} \left[1 + \frac{2 \cdot \lambda}{\alpha \cdot d} \right]$$

$$T_{sphere}(0) \;=\; T_A + \frac{d^{\mathbf{2}} \cdot \Phi'''}{6 \cdot \lambda} \left[1 + \frac{2 \cdot \lambda}{\alpha \cdot d} \right]$$

It can be seen that T for $\zeta = 0$ will be the biggest for the plate, and the smallest for the sphere.

Lecture 13 - Question 4



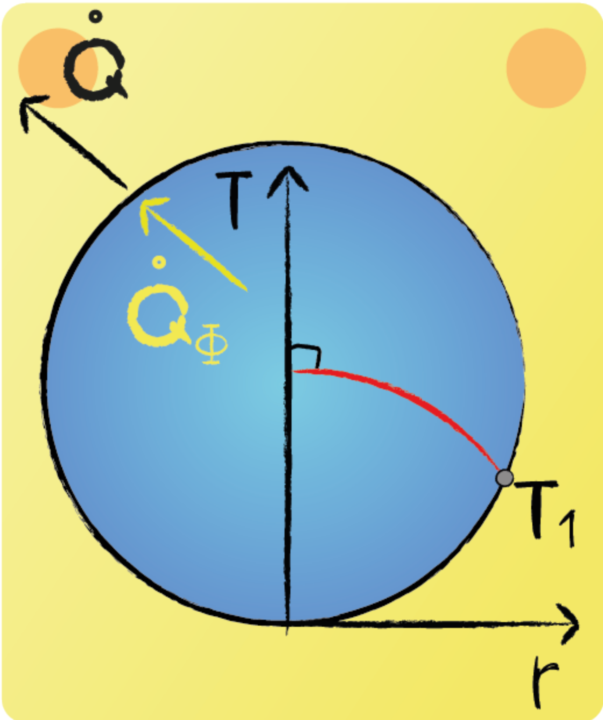
Consider a medium in which the heat conduction equation is given in its simplest form as:

$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\dot{\Phi}'''=0$

How many boundary conditions are needed to solve the equation?

Four boundary conditions
From the heat conduction equation it can be seen that it is a two-dimensional problem. One knows that for a one-dimensional problem two boundary conditions are required. Thus four boundary conditions are required.

Lecture 13 - Question 5



Consider a cylinder in a medium in which the heat conduction equation is given in its simplest form as:

$$\frac{1}{r}\frac{d}{dr}\left(\lambda r \frac{dT}{dr}\right) + \dot{\Phi}''' = 0$$

Which boundary conditions are applicable for solving this problem.

$$2\pi r_1 L\alpha\,(T_1 - T_A) = V \cdot \dot{\Phi}'''$$

Is applicable. It describes that the heat leaving equals the heat that is generated.

$$T(0) = T_0$$

Is not applicable, since the gradient at r=0 will not be zero, as it should.

$$\left(\frac{dT}{dr}\right)_{r=0} = 0$$

Is applicable. It described that, due to the symmetry, at r=0 the gradient of the temperature profile will be zero.

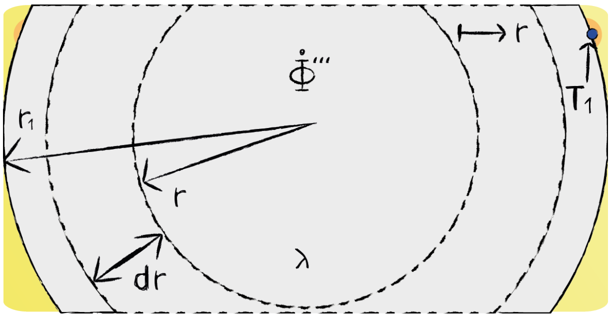
$$T(r_1) = T_1$$

Is applicable. It describes that the temperature profile should equal T_1 at the boundary.

$$\left(\frac{dT}{dr}\right)_{r=r_1} = 0$$

Is not applicable. It describes that the temperature profile has a zero gradient at the boundary. As can be seen in the figure, this is not the case.

Lecture 13 - Question 6



Develop an energy balance to calculate the temperature profile inside the cylinder and give the boundary conditions. The cylinder is losing heat to the environment. Assume one-dimensional steady-state heat with a source.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} + d\dot{\Phi} = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} \;= -\lambda \cdot 2 \cdot \pi \cdot r \cdot L \cdot \frac{\partial T}{}$$

$$\dot{Q}_{r,out} = -\lambda \cdot 2 \cdot \pi \cdot r \cdot L \cdot \frac{\partial T}{\partial r} + \frac{\partial \dot{Q}_{r,in}}{\partial r} \cdot dr$$

$$d\dot{\Phi} \;= \dot{\Phi}''' \cdot 2 \cdot \pi \cdot r \cdot L \cdot dr$$

The heat entering the system is transferred from the centre of the cylinder by conductive heat transfer. Heat is generated because of the source. $\dot{Q}_{\mathrm{r,out}}$ can be approximated by use of the Taylor series expansion.

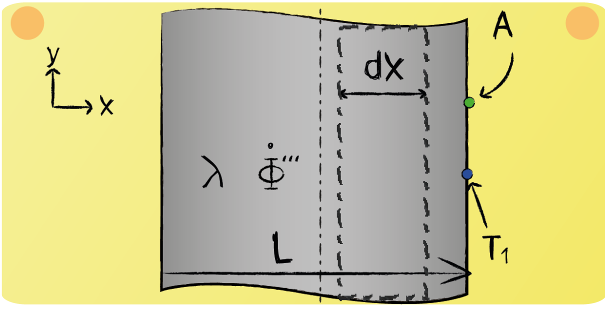
Boundary Conditions:

$$\frac{\partial T(r=0)}{\partial r} = 0$$

$$T(r = r_1) = T_1$$

The first boundary condition describes that the temperature gradient in the center equals zero. This is because of symmetry. The second one describes that the temperature on the surface equals T_1.

Lecture 13 - Question 7



Develop an energy balance to calculate the temperature profile inside the wall and give the boundary conditions. The plate is losing heat to the environment. Assume one-dimensional steady-state heat with a source. The temperature profile within the wall is symmetrical.

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} + d\dot{\Phi} = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

$$d\dot{\Phi} = \dot{\Phi}''' A dx$$

The outgoing flux can be approxmated by use of the Taylor series expansion.

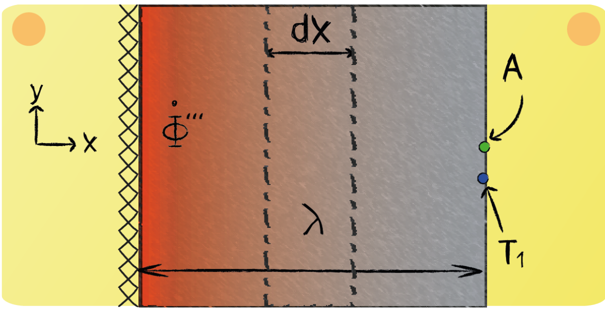
Boundary Conditions:

$$\frac{\partial T(x = \frac{1}{2}L)}{\partial x} = 0$$

$$T(x = L) = T_1$$

The first boundary condition describes that the temperature gradient in the middle equals zero. This is because of symmetry. The second one describes that the temperature on the right side of the wall equals T_1.

Lecture 13 - Question 8



A wall is adiabatic on the left side. Heat is transferred steadily from left to rights with a source. Develop an energy balance to calculate the temperature profile inside the wall and give the boundary conditions.

Energy Balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} + d\dot{\Phi} = 0$$

Heat Fluxes:

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

$$d\dot{\Phi} = d\dot{\Phi} = \dot{\Phi}''' A dx$$

The in and outgoing flux should equal each other and are characterized by conductive heat transfer. The outgoing flux can be approximated by use of the Taylor series expansion.

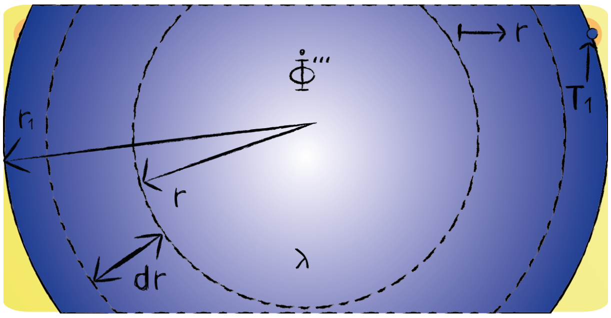
Boundary conditions:

$$\frac{\partial T(x=0)}{\partial x} = 0$$

$$T(x = L) = T_1$$

The first boundary condition describes that the temperature gradient on the left should be zero. This due to the fact that heat transfer to the environment at the left side is zero, because of the insulation. The second describes that the temperature of the wall equals T_1 on the right side.

Lecture 13 - Question 9



Develop an energy balance to calculate the temperature profile inside the sphere and give the boundary conditions. The sphere is losing heat to the environment. Assume one-dimensional steady-state heat with a source.

Energy Balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} + d\dot{\Phi} = 0$$

Heat Fluxes:

$$\dot{Q}_{r,in} \;=\; -\lambda A(r)\frac{\partial T}{\partial r} = -\lambda 4\pi r^2 \frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} \;= \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r}dr = -\lambda 4\pi r^2 \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}(-\lambda 4\pi r^2 \frac{\partial T}{\partial r})dr$$

$$d\dot{\Phi} \;= dV \cdot \dot{\Phi}''' = 4 \cdot \pi \cdot r^2 dr \cdot \dot{\Phi}'''$$

The ingoing flux can be described by use of Fourier's law and the outgoing flux can be approximated by use of the Taylor series expansion. Note that for the energy generation, the infinitesimal volume can be approximated by removing higher order terms for dr. This will have a neglectable influence on the volume.

$$dV \;=\; \frac{4}{3} \cdot \pi \cdot (r + dr)^3 - \frac{4}{3} \cdot \pi \cdot r^3 =$$
$$\frac{4}{3} \cdot \pi \cdot \left(3r^2 dr + 3r dr^2 + \mathrm{dr}^3\right) \approx 4 \cdot \pi \cdot r^2 dr$$

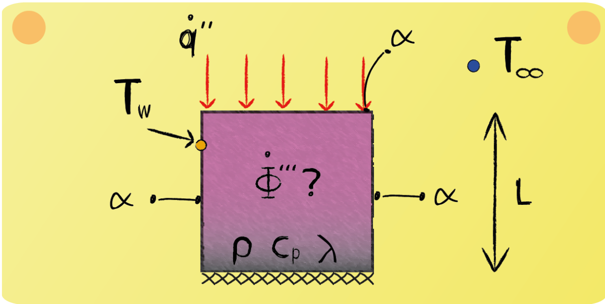
Boundary conditions:

$$\frac{\partial T(r=0)}{\partial r} = 0$$

$$T(r = r_1) = T_1$$

The first boundary condition describes that the temperature gradient in the center equals zero. This is because of symmetry. The second one describes that the temperature on the surface equals T_1.

Lecture 13 - Question 10



A steel cube is exposed to a heat flux from the upper side. The bottom is adiabatic. Furthermore all free surfaces are subjected to convection, the remaining heat is extracted by a sink. Derive an energy balance to determine the rate of heat being extracted per unit volume. Assume the temperature to be homogeneous and neglect radiation.

Energy Balance:

$$\sum \dot{Q}_{in} - \sum \dot{Q}_{out} - \dot{\Phi} = 0$$

From the context it can be noted that we are dealing with a steady-state problem. For that reason the sum of the in- and outgoing fluxes should equal zero.

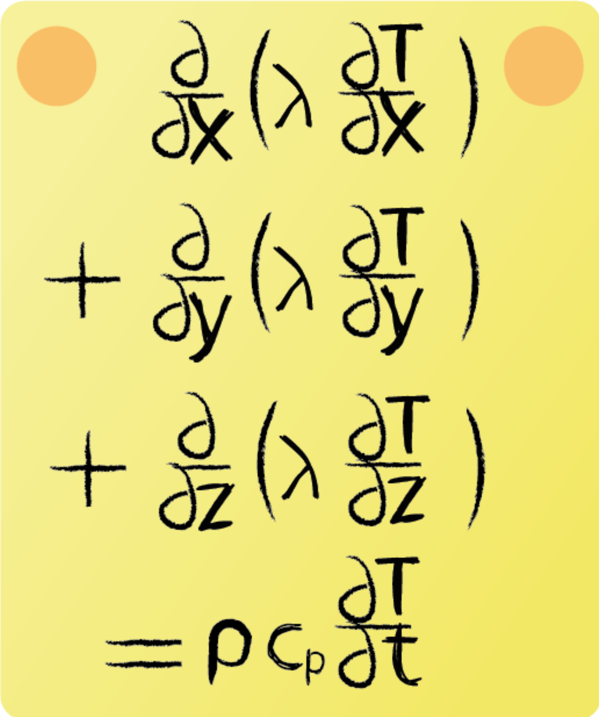
Heat Fluxes:

$$\sum \dot{Q}_{in} = \dot{q}'' L^2$$

$$\sum \dot{Q}_{out} = 5\alpha \left(T_w - T_\infty\right) L^2$$

$$\dot{\Phi} = \dot{\Phi}''' L^3$$

Lecture 14 - Question 1



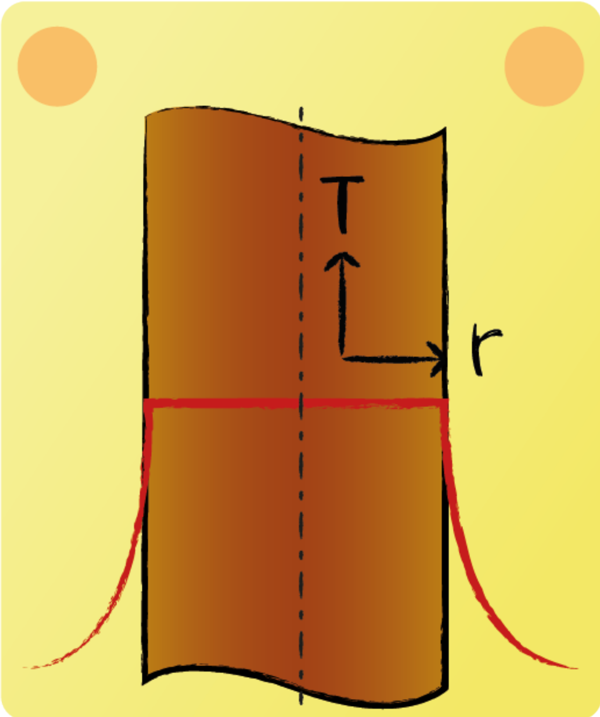
Consider a medium in which the heat conduction equation is given in its simplest form as:

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)=\rho c_p\frac{\partial T}{\partial t}$$

Indicate whether:
The heat transfer is **steady** / **transient**.
The heat transfer is **one-**/ **two-**/ **three-**dimensional.
There is **heat generation** / **no heat generation** in the medium.

From the equation it can be seen that the heat transfer is **transient**, as $\frac{\partial}{\partial t} \neq 0$.
The heat transfer is **three**-dimensional as there are three partial derivatives with respect to x, y and z
No $\dot{\Phi}'''$ is given. Implying that **no heat generation** takes place.

Lecture 14 - Question 2



Which condition for the Biot number must be met when using the lumped capacity model?

$\theta^* = 1 - e^{-\frac{\alpha A}{\rho c_p V}t} = 1 - e^{-[Bi \cdot Fo]}$

$Bi << 1$

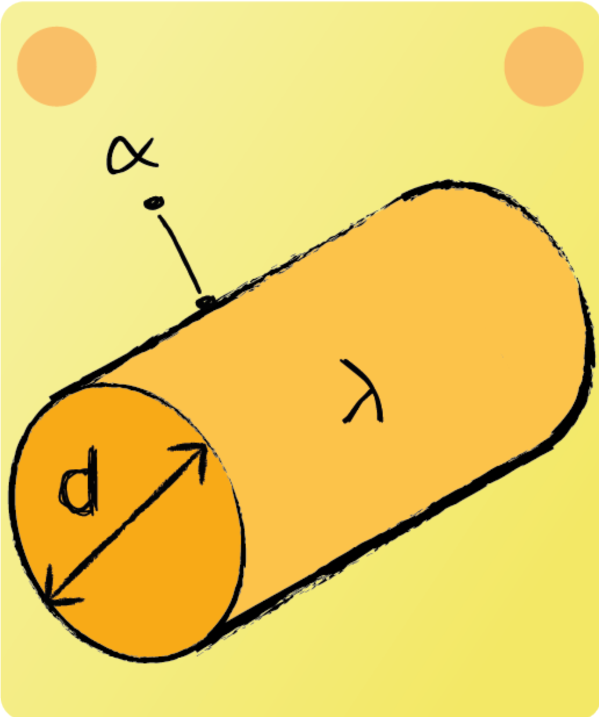
The lumped capacity model is a common approximation in transient conduction, which may be used whenever heat conduction within an object is much faster than heat transfer across the boundary of the object. Which is the case for $Bi << 1$.

Lecture 14 - Question 3

Which statement is applicable to the Fourier number?

It describes the duration of a thermal process in relation to the duration of heat transport. In other words it is a measure of heat penetration depth.

Lecture 14 - Question 4



Consider the following thin cylinder. Determine whether the lumped capacity model can be applied. Take $\alpha = 20$ W/m^{2}K, $\lambda = 0.01$ W/mK and d = 0.1 m.

No

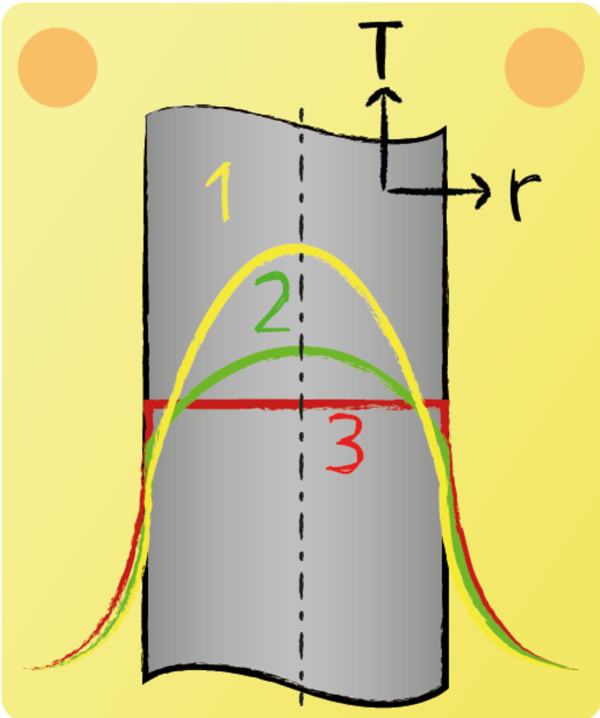
$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4}d^2L}{2\cdot\frac{\pi}{4}d^2+\pi dL}$$

$$Thin\ cylinder\ approximation:\ L_c \approx \frac{\frac{\pi}{4}d^2L}{\pi dL} = \frac{1}{4}d = 0.025\ m$$

$$Bi = \frac{\alpha \cdot L_c}{\lambda} = 50$$

No, since Bi » 1.

Lecture 14 - Question 5



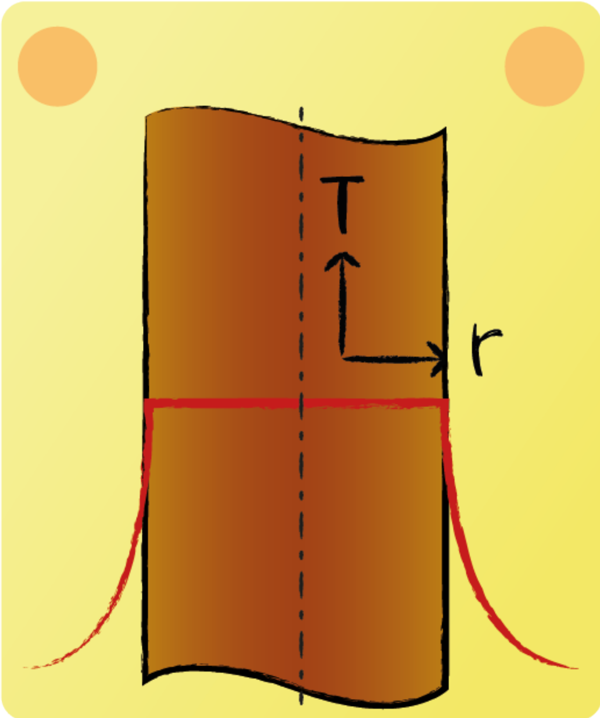
For which of these temperature profiles is the lumped capacity model best used?

Temperature profile 3

The lumped capacity model is a common approximation in transient conduction, which may be used whenever heat conduction within an object is much faster than heat transfer across the boundary of the object.

For temperature profile 3, the difference between the surface and center temperature is negligible compared to the difference between the surface and the environmental temperature. This is due to the fact that heat conduction within the body is much faster. For that reason temperature profile 3 will give the most accurate results when using the lumped capacity model.

Lecture 14 - Question 6

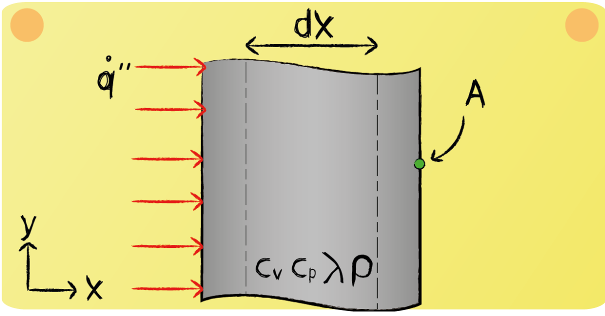


Select the terms that apply to the lumped capacity model.

$$\theta^* = 1 - e^{-\frac{\alpha A}{\rho c_p V} t}$$

Body and environment dependent - The convection coefficient α depends on the properties of the body as well as on the environmental properties.
Transient - The temperature of the body is changing over time. For that the heat transfer can be characterized to be transient.
Homogeneous body temperature - The lumped capacity model is applicable to cases where $Bi << 1$, here the difference of the temperature within a body is so small, that it is negligible. For that reason a homogeneous temperature can be assumed.

Lecture 14 - Question 7



Give the energy balance to do derive the heat conduction equation. Assume one-dimensional transient conditions in x-direction at constant atmospheric pressure.

Energy balance:

$$\frac{\partial U}{\partial t} = \dot{Q}_{x,in} - \dot{Q}_{x,out}$$

For unsteady heat transfer the internal energy will change over time and equals the sum of the in- and outgoing heat fluxes.

Change of internal energy over time:

$$\frac{\partial U}{\partial t} = \rho \cdot c_p \cdot dx \cdot A \cdot \frac{\partial T}{\partial t}$$

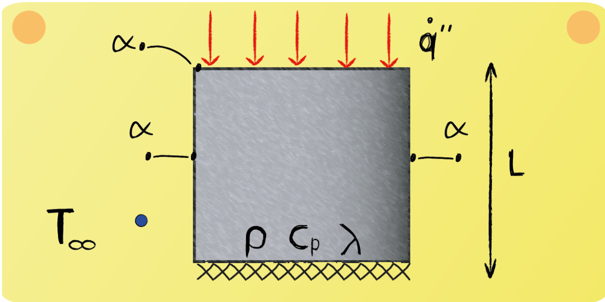
The internal energy of a constant volume can be described as: $U = m \cdot c_p \cdot T$. It should be noted that the control volume is at a constant pressure and for that reason c_p has to be used.

Heat fluxes:

$$\begin{aligned} \dot{Q}_{x,in} &= -\lambda A \frac{\partial T}{\partial x} \\ \dot{Q}_{x,out} &= -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx \end{aligned}$$

The ingoing flux can be described by use of Fourier's equation. The outgoing flux can be approximated by use of the Taylor series expansion.

Lecture 14 - Question 8



A steel cube is exposed to a heat flux from the upper side. The bottom is adiabatic. Furthermore all free surfaces are subjected to convection. Derive the differential equation that expresses the change in temperature of the cube over the course of time. Assume the temperature to be homogeneous and neglect radiation.

Energy balance:

$$\frac{dU}{dt} = \sum \dot{Q}_{in} - \sum \dot{Q}_{out}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{dU}{dt} = \rho \cdot c_p \cdot L^3 \cdot \frac{dT_w}{dt}$$

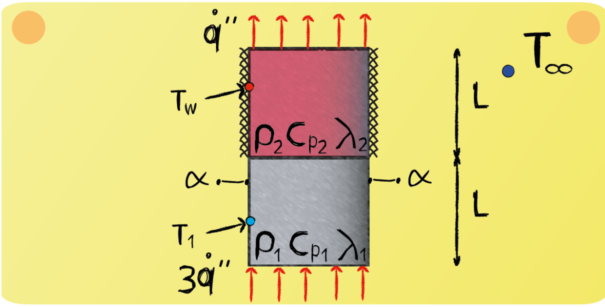
The internal energy of the control volume can be described as: $U = m \cdot c_p \cdot T$.

Heat fluxes:

$$\sum \dot{Q}_{in} = \dot{q}'' L^2$$

$$\sum \dot{Q}_{out} = 5\alpha \left(T_w - T_\infty\right) L^2$$

Lecture 14 - Question 9



Two cubes are placed on top of each other. The grey cube is experiencing steady-state heat transfer, which has a constant heat flux entering, but is losing heat at the side surfaces at a constant rate as well. The pink cube is losing heat at a constant rate at the top surface, but gaining heat by conduction from the grey cube. The side surfaces of this cube are fully adiabatic. Derive the differential equation that expresses the change in temperature of the pink cube over the course of time. Assume the temperature to be homogeneous and neglect radiation.

Energy balance:

$$\frac{dU}{dt} = \sum \dot{Q}_{in} \; - \sum \dot{Q}_{out}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{dU}{dt} = \rho_2 \cdot c_{p2} \cdot L^3 \cdot \frac{dT_w}{dt}$$

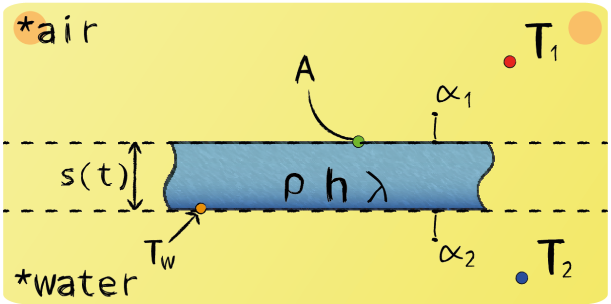
The internal energy of the control volume can be described as: $U = m \cdot c_{\mathrm{p}} \cdot T$.

Heat fluxes:

$$\sum \dot{Q}_{in} \;\; = \;\; 3\dot{q}''L^2 - 4\alpha L^2 \left(T_1 - T_\infty\right)$$

$$\sum \dot{Q}_{out} \;\; = \dot{q}''L^2$$

Lecture 14 - Question 10



Ice is continuously increasing in thickness. Convection occurs on both sides. Derive the differential equation to describe the ice layer thickness $s(t)$. The enthalpy of fusion is h. Take $T_2 > T_w > T_1$. The cooling of the ice, contrary to freezing, is energetically negligible, $c_\text{i} \cdot \Delta T << h$.

Energy balance:

$$\frac{dU}{dt} = \dot{Q}_{water} - \dot{Q}_{air}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{dU}{dt} = -\rho A h \frac{ds}{dt}$$

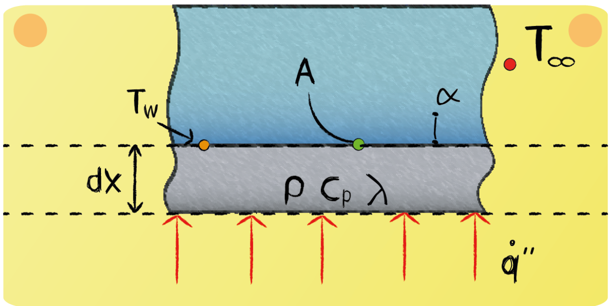
The internal energy can be described by use of the enthalpy of fusion. This is denoted with an negative sign, since solidification of water costs energy.

Heat fluxes:

$$\dot{Q}_{water} = \alpha_2 A (T_2 - T_w)$$

$$\dot{Q}_{air} = (T_w - T_1) A \frac{1}{\frac{1}{\alpha_1} + \frac{s(t)}{\lambda}}$$

Lecture 14 - Question 11



A pan of water is heated by means of a conduction plate, which provides a constant heat flux. Heat from the pan towards the water is transferred by means of convection. Derive the differential equation that expresses the change in temperature of the pan over the course of time. Assume the pan temperature to be homogeneous and neglect radiation.

Energy balance:

$$\frac{dU}{dt} = \dot{Q}_{in} - \dot{Q}_{out}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{dU}{dt} = \rho \cdot c_p \cdot Adx \cdot \frac{dT_w}{dt}$$

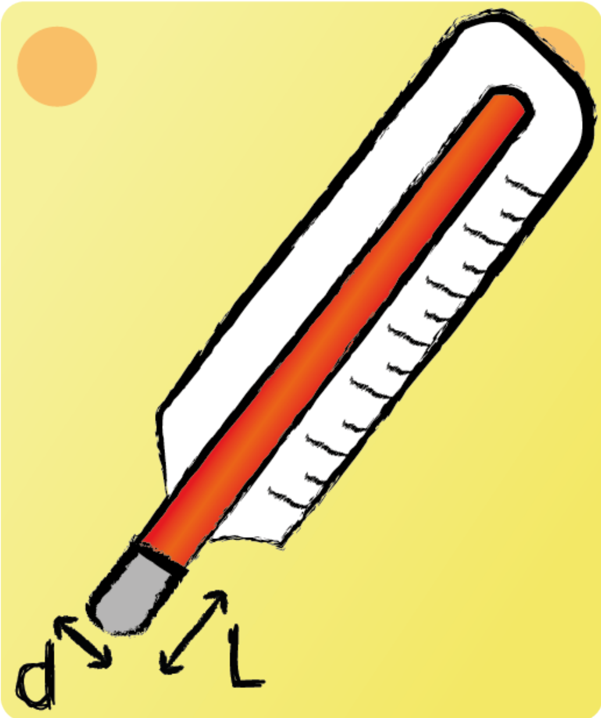
The internal energy of the control volume can be described as: $U = m \cdot c_\mathrm{p} \cdot T$.

Heat fluxes:

$$\dot{Q}_{in} = \dot{q}''A$$

$$\dot{Q}_{out} = \alpha A \left(T_w - T_\infty\right)$$

Lecture 15 - Question 1

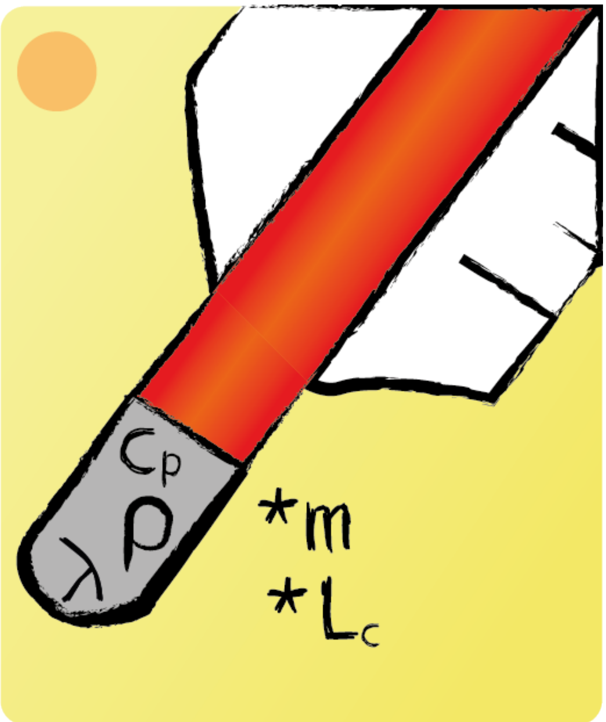


Consider the following fever thermometer. The tip can be assumed to be a thin cylinder. Determine the characteristic length L_c. Take $L = 40$ mm and $d = 5$ mm. Select the proper range.

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4}d^2 L}{2 \cdot \frac{\pi}{4}d^2 + \pi d L}$$

$$Thin\ cylinder\ approximation:\ L_c \approx \frac{\frac{\pi}{4}d^2 L}{\pi d L} = \frac{1}{4}d = 1.25\ mm$$

Lecture 15 - Question 2



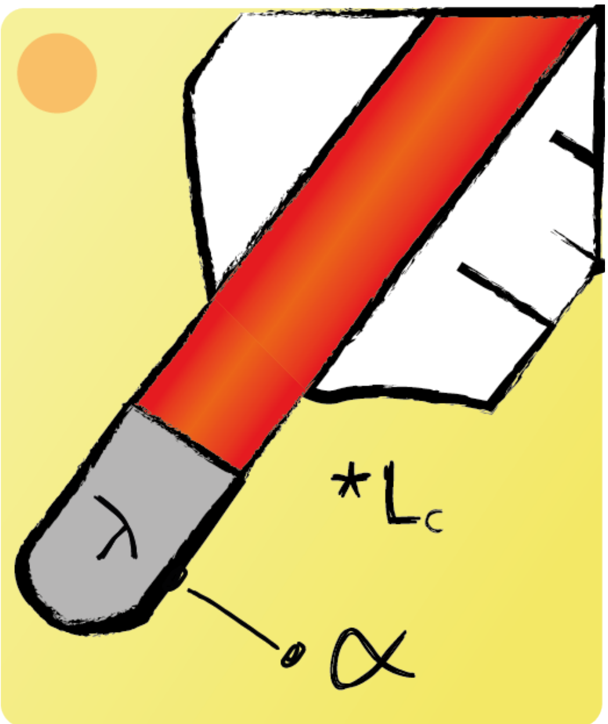
Remember, from the lumped capacity model, parameter m:

$$m = \frac{\alpha A}{\rho c_p V} = \frac{\alpha}{\rho c_p \cdot L_c}$$

Consider the thermometer as in the figure. Take $m = 0.03$, $\rho = 15000$ kg/m^3, $c_\mathrm{p} = 140$ J/kg·K and $L_\mathrm{c} = 1.25$ mm. Determine the heat transfer coefficient α. Select the proper range.

$$\alpha = \frac{m \cdot \rho \cdot c_p \cdot d}{4} = 78.75\ W/m^2K$$

Lecture 15 - Question 3

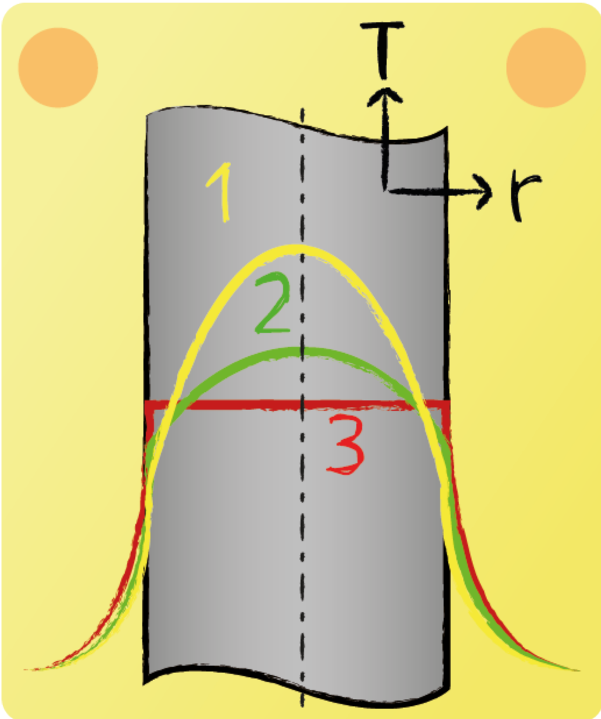


Check whether the lumped capacity model is applicable for the described situation. Take $\alpha = 78.75$ W/m^{2}K, $L = 1.25$ mm $\lambda = 10$ W/mK.

$$Bi = \frac{\alpha \cdot L_C}{\lambda} = 0.0098\ W/m^2K$$

The lumped capacity model is applicable, since Bi « 1.

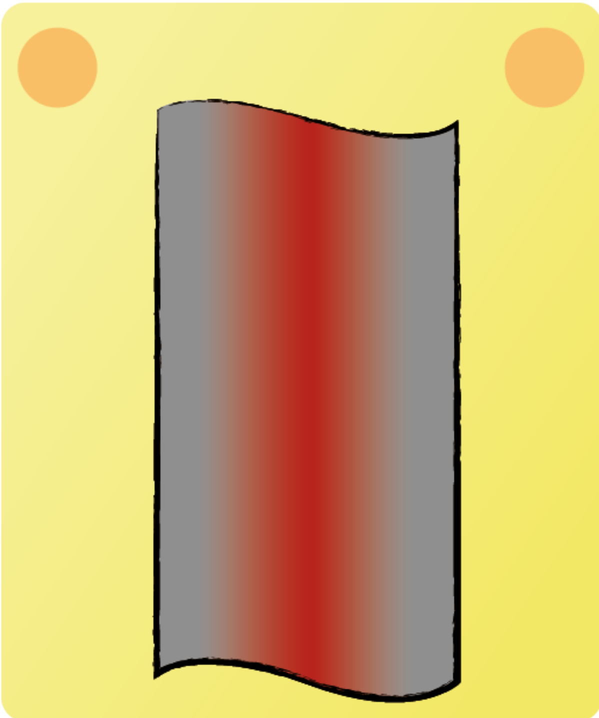
Lecture 15 - Question 4



Which of the following temperature profiles is most closely to the temperature profile of the thermometer?

Temperature profile 3.
The lumped capacity model was applicable, since Bi « 1. When Bi « 1 the difference of the temperature within a body is so small, that it is negligible. For that reason a homogeneous temperature can be assumed.

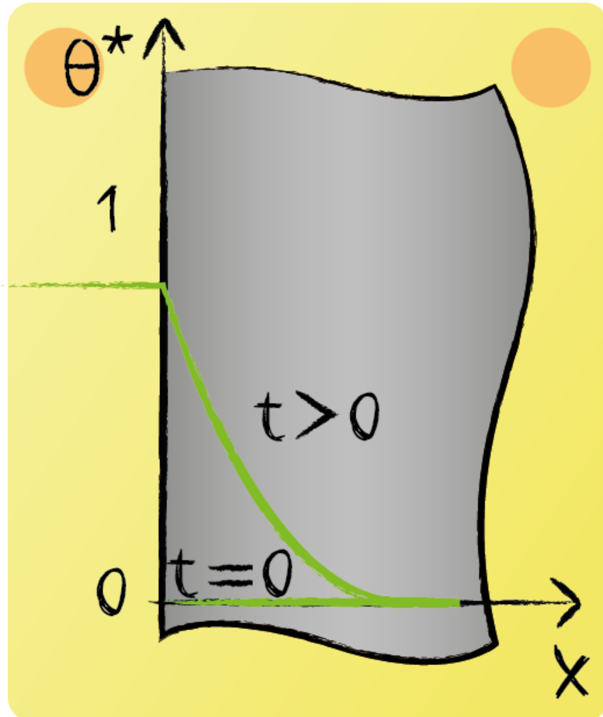
Lecture 16 - Question 1



What is the definition of the thermal diffusivity a?

Measurement of the rate of transfer of heat in a material from the hot end to the cold end, while considering the thermal inertia.. Thermal diffusivity is the thermal conductivity divided by density and specific heat capacity at constant pressure. It measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy.

Lecture 16 - Question 2



Consider the following semi-infinite body where heat transfer on the outside is neglected. Which boundary conditions are applicable?

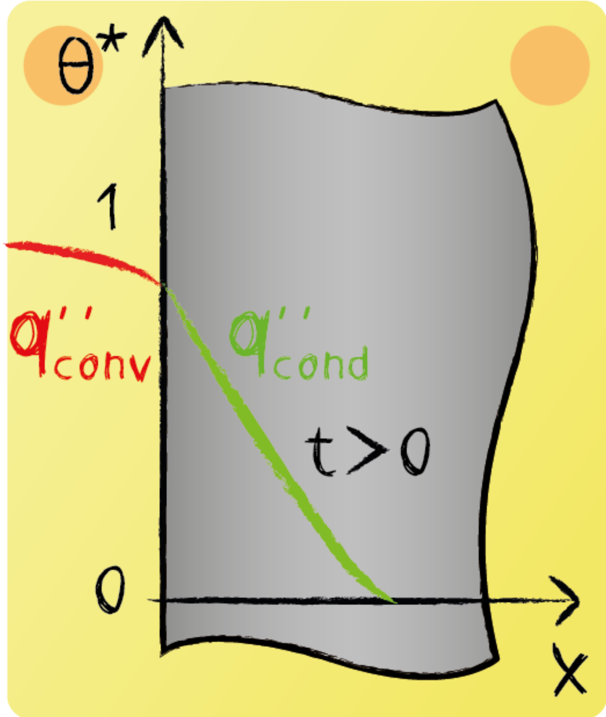
$$\left.\begin{array}{r} t > 0 \\ x \to \infty \end{array}\right\} T = T_0$$

States that the body temperature for x→∞ equals the initial body temperature. This can be seen from the fact that for x→∞, $\theta^* = \frac{T-T_0}{T_A-T_0} = 0$

$$\left.\begin{array}{r} t > 0 \\ x = 0 \end{array}\right\} T = T_A$$

States that the body temperature for x = 0 equals the ambient temperature. This can be seen from the fact that for x = 0, $\theta^* = \frac{T-T_0}{T_A-T_0} = 1$

Lecture 16 - Question 3



Consider the following semi-infinite body where heat transfer on the outside is not neglected. Which boundary conditions are applicable?

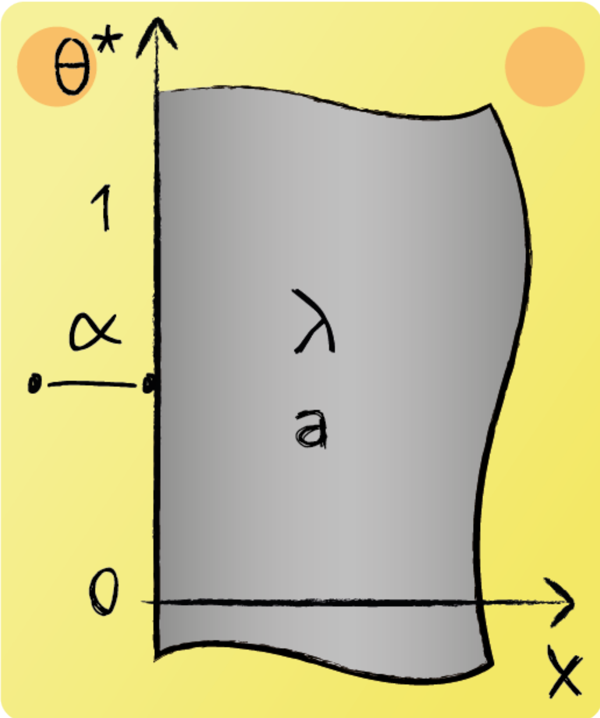
$$\left.\begin{matrix} t > 0 \\ x \to \infty \end{matrix}\right\} T \;=\; T_0$$

States that the body temperature for x→∞ equals the initial body temperature. This can be seen from the fact that for x→∞, $\theta^* = \frac{T-T_0}{T_A-T_0} = 0$

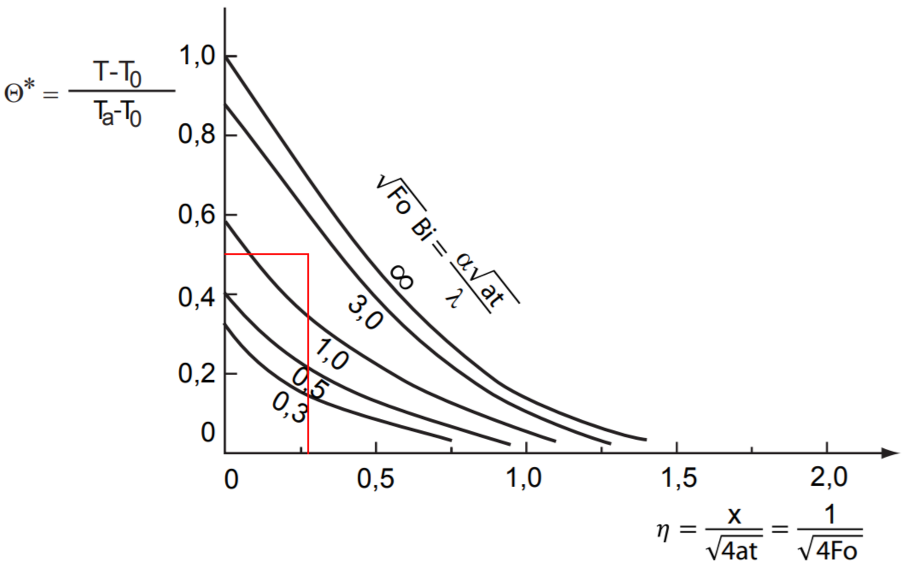
$$\left.\begin{matrix} t > 0 \\ x = 0 \end{matrix}\right\} \left.\frac{\partial T}{\partial x}\right|_{x=0} \;=\; \frac{\alpha}{\lambda}\left(T_{x=0} - T_A\right)$$

Results from the fact that $q''_{conv} = q''_{cond}$

Lecture 16 - Question 4



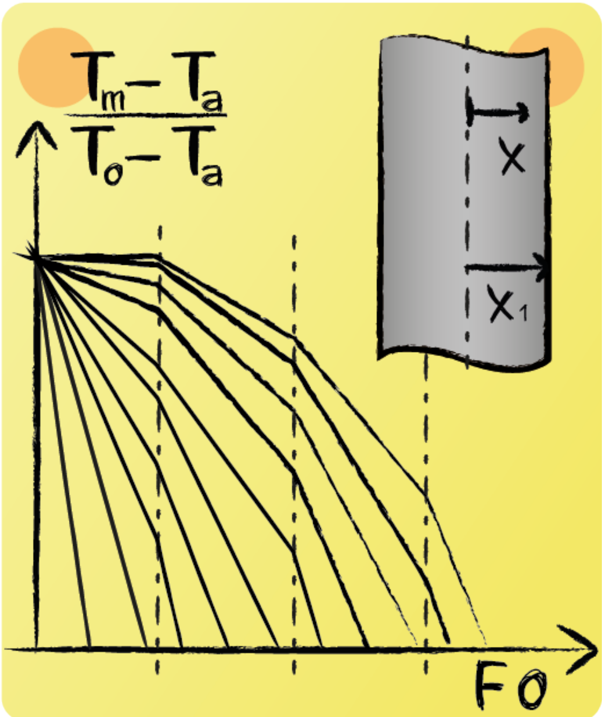
Determine for a plate for which position x is $\theta^*(\eta) = 0.5$ reached after t=10 s? Take $\alpha = 3000$ W/m^{2}K, $\lambda = 40$ W/mK and a $= 100{\cdot}10^{-6}$ m^2/s. Use Figure 5.6 from the Lecture Notes.



$$\sqrt{Fo} \cdot Bi = \frac{\alpha\sqrt{a \cdot t}}{\lambda} = 2.3717 \rightarrow \eta = 0.275$$

$$x = 2\eta\sqrt{at} = 0.0174\ m$$

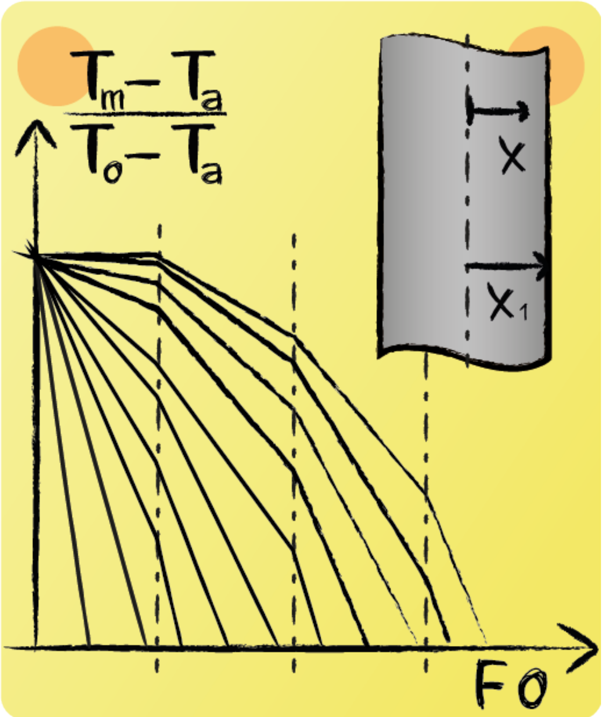
Lecture 17- Question 1



Remember the Heisler diagram for a plate as sketched in the figure. Which statement is in line with the definition of T_m?

$T_\mathrm{m}(t)$ describes the temperature at the center of the plate as a function of time.

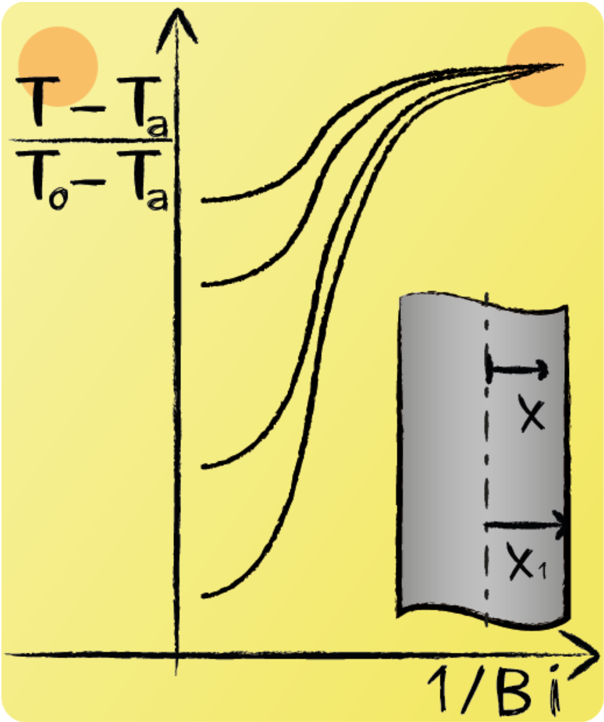
Lecture 17- Question 2



Remember the Heisler diagram for a plate as sketched in the figure. A flat plate surrounded by a fluid, with the known parameters λ, a, α, T_a and x_1, has at time instant t_0 temperature T_0. After some time the temperature at the center has cooled down to T_1. Which of the following parameters can **only** be determined with use of the Heisler diagram?

The time needed to cool down t_1.
The other parameters are given or can be calculated by their known formulas.
The surface temperature $T(x = x_1, t)$ at time instant t_1 cannot be determined when using this Heisler diagram.

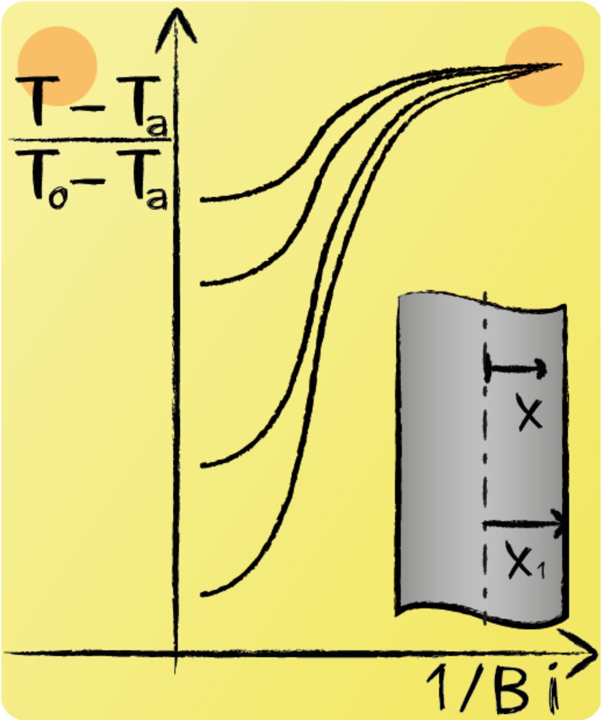
Lecture 17- Question 3



Remember the Heisler diagram for a plate as sketched in the figure. Which statement is in line with the definition of T?

$T(x, t)$ describes the temperature profile along the plate as a function of position and time.

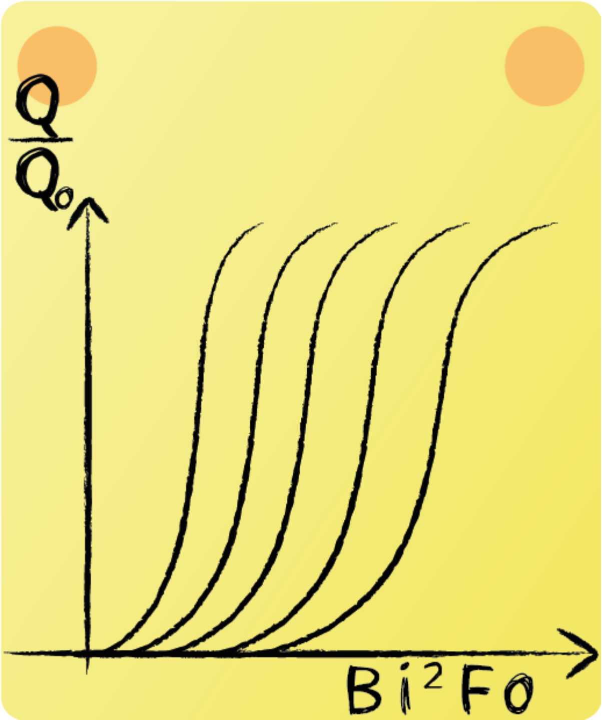
Lecture 17- Question 4



Remember the Heisler diagram for a plate as sketched in the figure. A flat plate surrounded by a fluid, with the known parameters λ, a, α, T_a and x_1, has at time instant t_0 temperature T_0. After some time the temperature at the center has cooled down to T_1 at time instant t_1. Which of the following parameters can **only** be determined with use of the Heisler diagram?

The surface temperature $T(x = x_1, t)$ at time instant t_1. The other parameters are given or can be calculated by their known formulas.

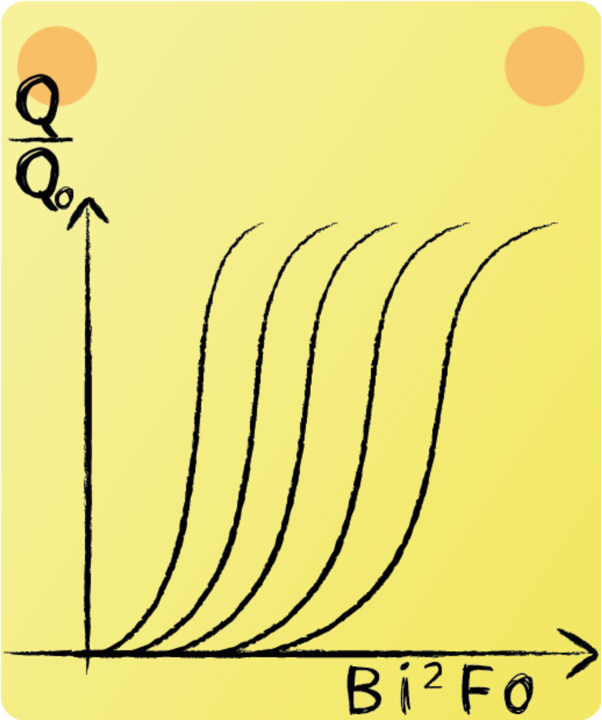
Lecture 17- Question 5



Remember the following Heisler diagram for a plate as sketched in the figure. Which statement is in line with the definition of Q?

Q describes the dissipated heat after heat dissipation.

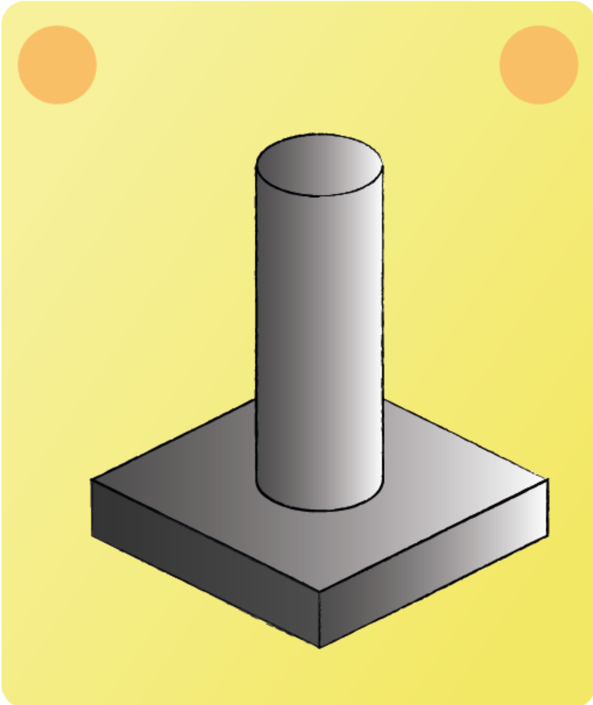
Lecture 17- Question 6



Remember the Heisler diagram for a plate as sketched in the figure. A flat plate surrounded by a fluid, with the known parameters λ, a, α, T_a, m and x_1, has at time instant t_0 temperature T_0. After some time the temperature at the center has cooled down to T_1 at time instant t_1. Which of the following parameters can **only** be determined with use of the Heisler diagram?

The dissipated heat after heat dissipation Q and the remaining heat after heat dissipation Q_t.
After determination of the dissipated heat Q by the Heisler diagram and after and calculation of total heat stored in the object $Q_\mathrm{o} = m \cdot c_\mathrm{p} \cdot (T_0 \cdot T_\mathrm{a})$ the remaining heat after heat dissipation $Q_\mathrm{t} = Q_\mathrm{o} - Q$ can be determined. This would not be possible without use of the Heisler diagram.
The other parameters are given or can be calculated by their known formulas.

Lecture Conduction Biot Number 1



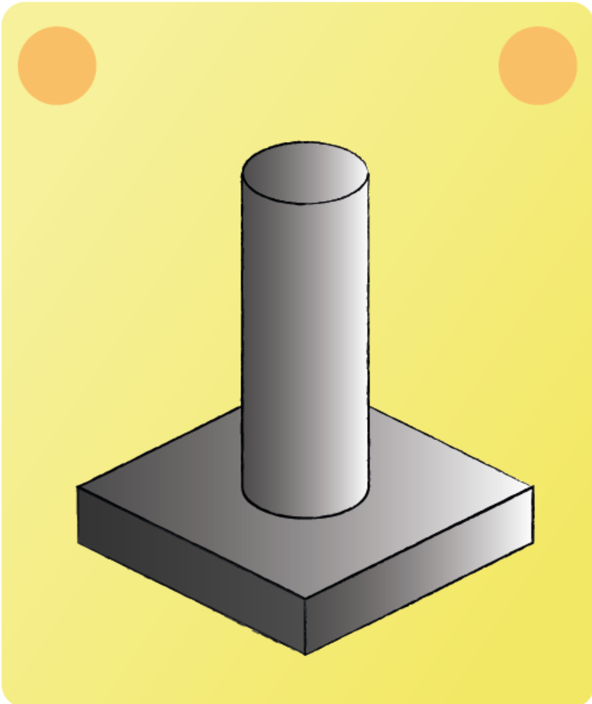
Why is the Biot number important to the solution of fin type problems?

The Biot Number is defined as the ratio of thermal resistances inside a body and the body's surface:

$$\mathrm{Bi} = \frac{W_{\text{conductive}}}{W_{\text{convective}}} = \frac{\alpha L}{\lambda}$$

With L representing a characteristic length of the body. In case of a pin fin the only direction where both mechanisms take part in the heat transfer is radial and hence, the characteristic length is chosen to be the fin's radius. The one-dimensional fin equation requires the temperature profile to be constant in radial direction within the fin. This assumption is valid, if thermal resistance inside the body is small compared to thermal resistance at the body's surface, which implies a small Biot number: $\mathrm{Bi} \ll 1$.

Lecture Conduction Biot Number 2



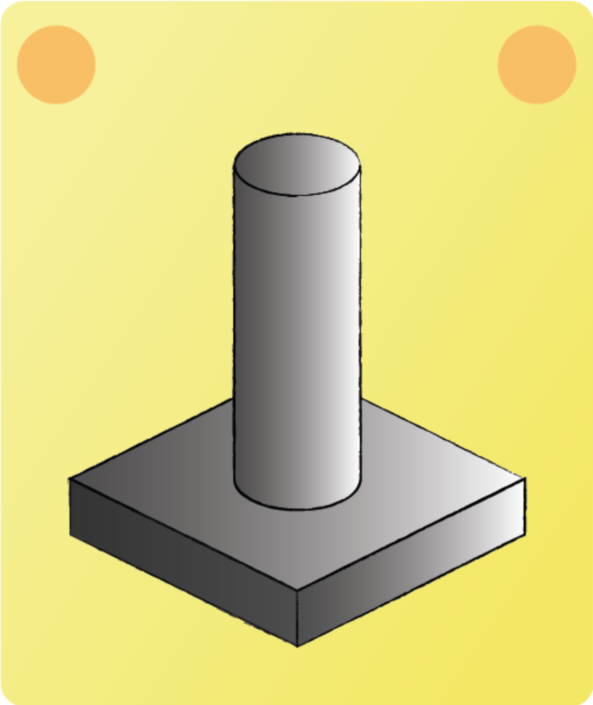
Specify the parameter's influence on a fin's Biot number.

The Biot Number is defined as the ratio of thermal resistances inside a body and the body's surface:

$$\mathrm{Bi} = \frac{W_{\text{conductive}}}{W_{\text{convective}}} = \frac{\alpha L}{\lambda}$$

With L representing a characteristic length of the body. In case of a pin fin the only direction where both mechanisms take part in the heat transfer is radial and hence, the characteristic length is chosen to be the fin's radius. By the definition the primary parameters that influence the Biot number are given. As the heat transfer for example depends on the fluid's velocity, those impacts must also be taken into account.

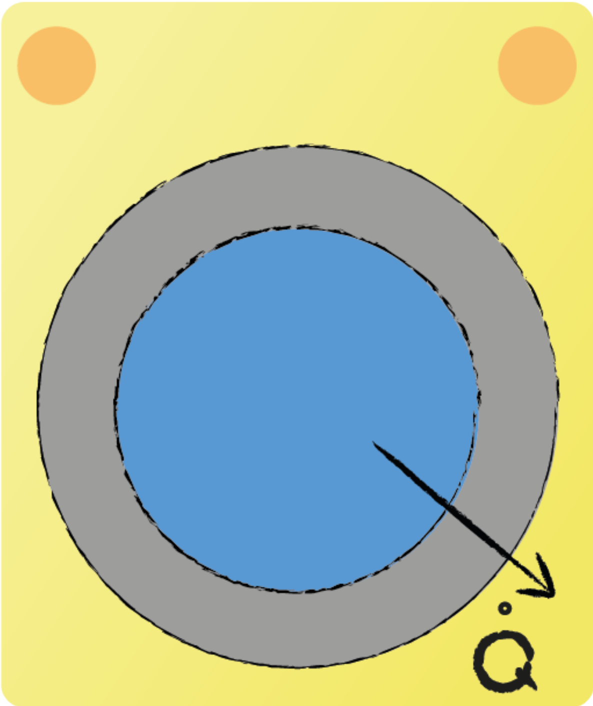
Lecture Fins Question 1



Which statements are correct regarding fins?

The purpose of fins is to decrease thermal resistance at a body's surface, due to an increased surface area the convective heat transfer is enhanced. Greater surface area goes along with increased friction forces, such that a higher pressure drop is observed within the fluid flow, which is an undesirable side effect in most applications. Since fins reduce thermal resistance at the convective surface they are most beneficial, when convective transfer is the bottle neck of heat transfer. That is the advantage of fins increases for decreased heat transfer coefficients.

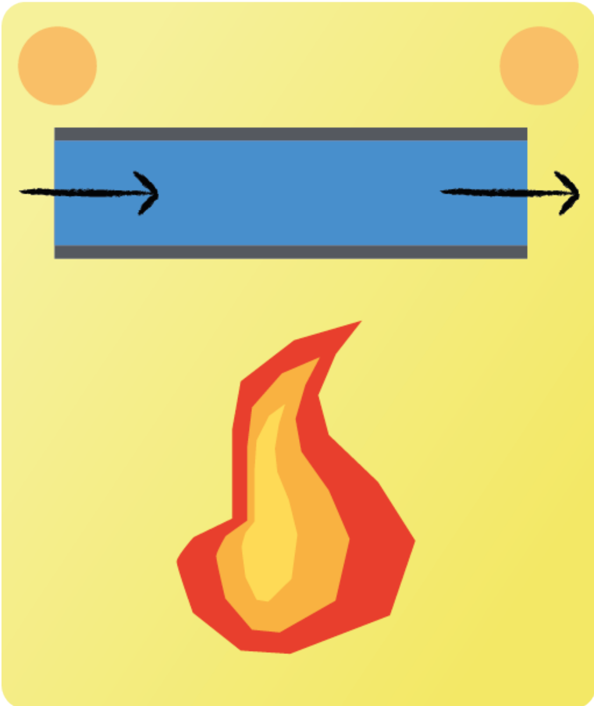
Lecture Fins Question 2



Water is flowing through a thin walled pipe. For cooling purposes heat shall be transfered to the surrounding air. In which reasonable way can fins be used to increase the cooling performance?

Heat transfer coefficient of a water flow is usually significantly greater than that of an air flow. Therefore fins are more advantageous at the outer surface, decreasing the convective thermal resistance. Never the less fins can also provide improvements on heat transfer at the inner surface. In this case a higher pressure drop within the water has to be considered.

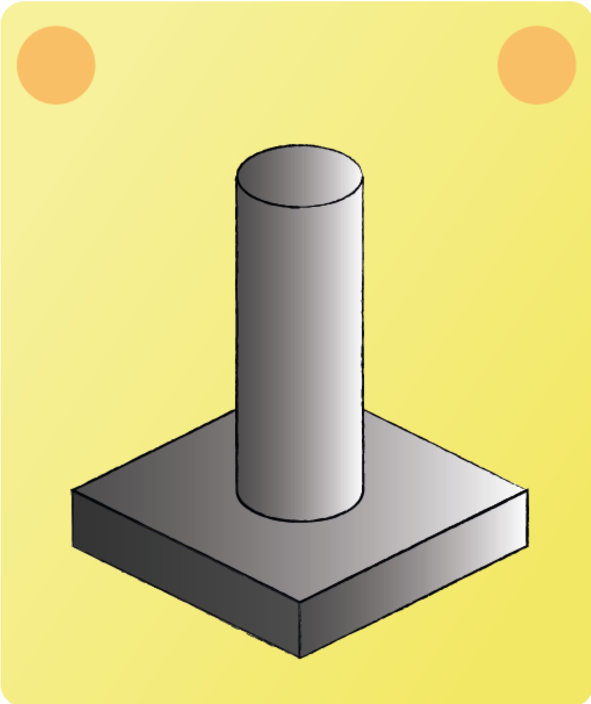
Lecture Fins Question 3



A fluid within a pipe shall be heated by a gas fired burner. On which side of the pipe are fins to be installed for an enhanced heat transfer?

While heat transfer from fluid to pipe is governed by convection, radiative heat transfer is dominant in the vicinity of the flame. Since fins are designed to enhance convective heat transfer the inner side is a reasonable placement.

Lecture Fins Question 4



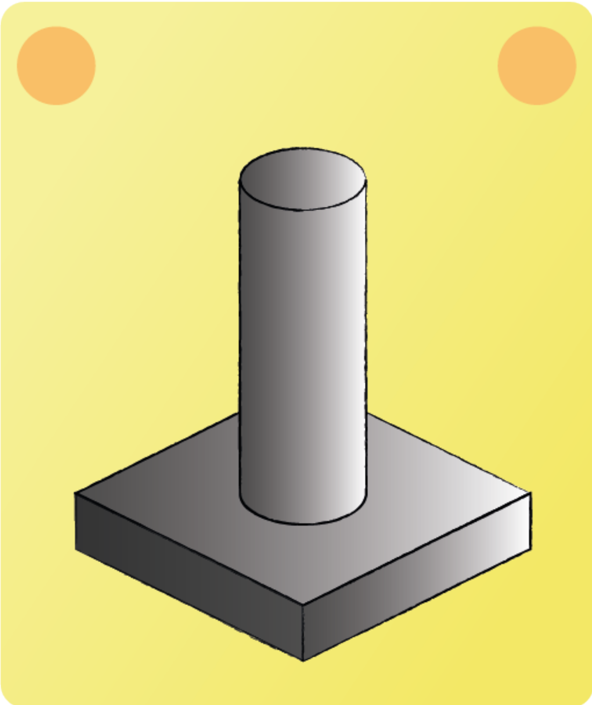
Which quantities are essential for the derivation of the one-dimensional fin equation using an infinitesimal energy balance?

The steady energy balance for an infinitesimal slice of fin in axial direction is given as:

$$0 = \dot{Q}_{\mathrm{cond}}(x) - \dot{Q}_{\mathrm{cond}}(x + \mathrm{d}x) - \dot{Q}_{\mathrm{conv}}(x)$$

Conductive fluxes are expressed using the cross section area, as the convective flux depends on the infinitesimal surface area.

Lecture Fins Question 5



Which of the following equations describe the fin problem?

The steady energy balance for an infinitesimal slice of fin in axial direction is given as:

$$0 = \dot{Q}_{\mathrm{cond}}(x) - \dot{Q}_{\mathrm{cond}}(x + \mathrm{d}x) - \dot{Q}_{\mathrm{conv}}(x)$$

Expressing the energy fluxes in terms of temperature and temperature gradient, respectively yields:

$$0 = -A_{\mathrm{c}}\lambda\frac{\partial T}{\partial x} + A_{\mathrm{c}}\lambda(\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2}\mathrm{d}x) - \alpha U[T(x) - T_{\mathrm{A}}]\mathrm{d}x$$

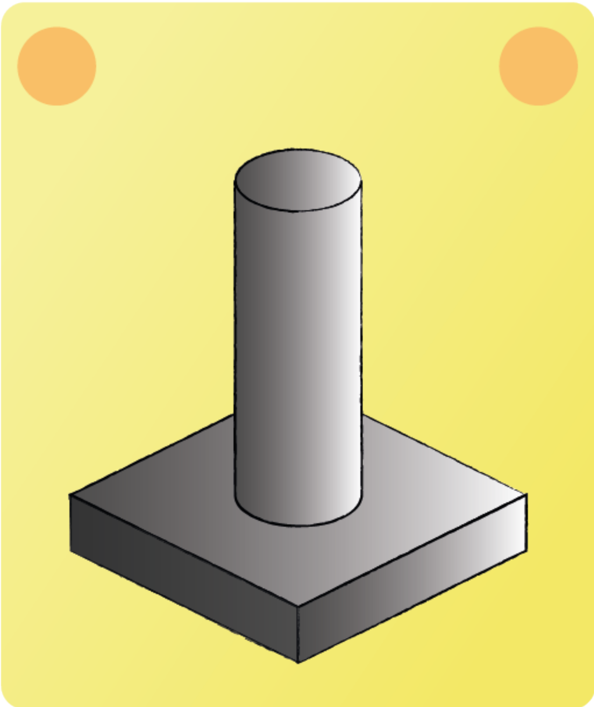
and eventually:

$$\frac{\partial^2 T}{\partial x^2} = \frac{\alpha U}{\lambda A_{\mathrm{C}}}[T(x) - T_{\mathrm{A}}]$$

A common simplification of the equation is to introduce the temperature difference $\Theta(x) = T(x) - T_{\mathrm{A}}$ and just write:

$$\frac{\partial^2 \Theta}{\partial x^2} = \frac{\alpha U}{\lambda A_{\mathrm{C}}}\Theta(x)$$

Lecture Fins Question 6



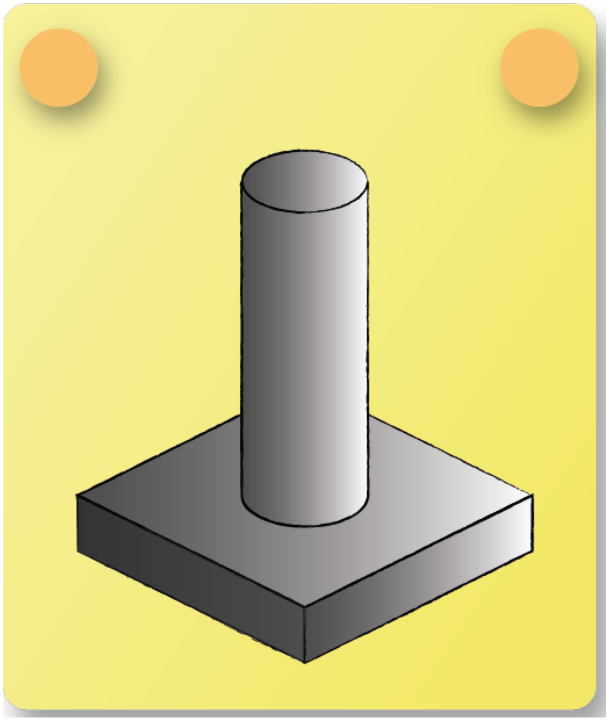
How many boundary conditions are required to solve the one-dimensional fin equation?

The one-dimensional fin equation is given to be:

$$\frac{\partial^2 T}{\partial x^2} = \frac{\alpha U}{\lambda A_\mathrm{C}}[T(x) - T_\mathrm{A}]$$

As it appears as a second order differential equation, two integration constants need to be determined in order to obtain a unique solution.

Lecture Fins Question 7



Which combination of boundary conditions is usually applied to the fin problem?