

## 2.13 Cooling of a copper rod

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a) Determine the time it takes for the copper rod to cool to an average temperature of 25 °C:

The could possibly be solved by use of the lumped capacity model in the case of  $Bi \ll 1$ .

Therefore checking whether the lumped capacity model can be used:

$$Bi = \frac{\alpha \cdot V}{\lambda \cdot A_s} = \frac{\alpha \cdot d}{4 \cdot 399 \text{ [W/mK]}} = 0.025 \quad (2.239)$$

$$\rightarrow Bi \ll 1 \quad (2.240)$$

The lumped capacity model can be found in the slides, or it can be derived by use of an energy balance (steps 1-5).

### 1) Setting up an energy balance:

Energy balance around the copper rod:

$$\frac{\partial U}{\partial t} = \sum \dot{Q}_{in} - \sum \dot{Q}_{out} \quad (2.241)$$

$$\rightarrow \frac{\partial U}{\partial t} = -\dot{Q}_{conv} \quad (2.242)$$

### 2) Defining the fluxes:

Where the change of internal energy over the course of time can be expressed as:

$$\rightarrow \frac{\partial U}{\partial t} = V \cdot \rho \cdot c_p \cdot \frac{\partial T}{\partial t} \quad (2.243)$$

And the rate of heat loss by convection

$$\rightarrow \dot{Q}_{conv} = \alpha \cdot A_s \cdot (T(t) - T_{\infty}) \quad (2.244)$$

### 3) Inserting and rearranging:

Inserting yields:

$$\frac{\partial U}{\partial t} = -\dot{Q}_{conv} \quad (2.245)$$

$$\rightarrow V \cdot \rho \cdot c_p \cdot \frac{\partial T}{\partial t} = -\alpha \cdot A_s \cdot (T(t) - T_{\infty}) \quad (2.246)$$

#### 4) Defining the initial condition:

In order to solve the equation, an initial condition should be given. This can be described in terms of the initial temperature of the copper rod, which is:

$$\boxed{\rightarrow T(t=0) = T_0} \quad (2.247)$$

#### 5) Solving the equation:

In order to solve the equation, an homogenized temperature should be introduced, which is:

$$\Theta^* = \frac{T(t) - T_0}{T_\infty - T_0} \quad (2.248)$$

And:

$$\frac{\partial \Theta^*}{\partial t} = \frac{\partial}{\partial t} \left( \frac{T(t) - T_0}{T_\infty - T_0} \right) = \frac{1}{T_\infty - T_0} \frac{\partial T}{\partial t} \quad (2.249)$$

And therefore the energy balance can be written to:

$$V \cdot \rho \cdot c_p \cdot \frac{\partial T}{\partial t} = -\alpha \cdot A_s \cdot (T(t) - T_\infty) \quad (2.250)$$

$$V \cdot \rho \cdot c_p \cdot \frac{1}{T_\infty - T_0} \cdot \frac{\partial T}{\partial t} = -\alpha \cdot A_s \cdot \frac{T(t) - T_\infty}{T_\infty - T_0} \quad (2.251)$$

$$\frac{\partial \Theta^*}{\partial t} = -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot (\Theta^* - 1) \quad (2.252)$$

Rewriting yields:

$$\frac{1}{\Theta^* - 1} \frac{\partial \Theta^*}{\partial t} = -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \quad (2.253)$$

Integration:

$$\ln |\Theta^* - 1| = -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t + c_1 \quad (2.254)$$

$$\Theta^* = C_1 \cdot \exp \left( -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t \right) + 1 \quad (2.255)$$

At  $T(t=0) = T_0$ , thus:

$$\Theta^*(t=0) = \frac{T_0 - T_0}{T_\infty - T_0} = 0 \quad (2.256)$$

And therefore:

$$C_1 = -1 \quad (2.257)$$

So:

$$\boxed{\rightarrow \Theta^*(t) = -\exp \left( -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t \right) + 1} \quad (2.258)$$

**6) Determining the time where the temperature of the rod is equal to 25 °C:**

At  $t = t_1$ , the temperature of the rod is equal to 25 °C. Thus:

$$\Theta^*(t = t_1) = -\exp\left(-\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t_1\right) + 1 = \frac{T(t = t_1) - T_0}{T_\infty - T_0} = 0.9375 \quad (2.259)$$

Rewriting yields:

$$t_1 = -\ln(0.0625) \frac{V \cdot \rho \cdot c_p}{\alpha \cdot A_s} \quad (2.260)$$

$$t_1 = -\ln(0.0625) \frac{d \cdot \rho \cdot c_p}{4 \cdot \alpha} = -\ln(0.0625) \frac{0.02 \text{ [m]} \cdot 8930 \text{ [kg/m}^3\text{]} \cdot 382 \text{ [J/kgK]}}{4 \cdot 200 \text{ [W/m}^2\text{K]}} \quad (2.261)$$

$$\boxed{\rightarrow t_1 = 236.45 \text{ [s]}} \quad (2.262)$$

## 2.14 The temperature delay

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a) Determine  $\Delta t$ :

1) Setting up an energy balance:

Energy balance:

$$\frac{\partial U}{\partial t} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \quad (2.263)$$

$$\rightarrow \frac{dU}{dt} = \dot{Q}_{\text{conv}} \quad (2.264)$$

2) Defining the fluxes:

Where:

$$\rightarrow \frac{dU}{dt} = m \cdot c_p \cdot \frac{dT}{dt} \quad (2.265)$$

$$\rightarrow \dot{Q}_{\text{conv}} = \alpha \cdot A_s \cdot \Delta T \quad (2.266)$$

3) Inserting and rearranging:

Inserting yields:

$$\frac{dU}{dt} = \dot{Q}_{\text{conv}} \quad (2.267)$$

$$\rightarrow m \cdot c_p \cdot \frac{dT}{dt} = \alpha \cdot A_s \cdot \Delta T \quad (2.268)$$

For  $t \rightarrow \infty$  the temperature increases linearly and therefore we can say:  $\frac{dT}{dt} = \frac{\Delta T}{\Delta t}$   
Substitution of  $\frac{dT}{dt}$  results in:

$$m \cdot c_p \cdot \frac{\Delta T}{\Delta t} = \alpha \cdot A_s \cdot \Delta T \quad (2.269)$$

And for that reason:

$$\rightarrow \Delta t = \frac{m \cdot c_p}{\alpha \cdot A_s} \quad (2.270)$$