1.5 Greenhouse gas effect

a) *Calculate the unknown temperatures* $T_{\mathrm{A}} and T_{\mathrm{E}}$.

Balance around area A:

$$-2\sigma T_{\mathrm{A}}^4 + \sigma T_{\mathrm{E}}^4 = 0 \tag{1.31}$$

Balance around area E:

$$\sigma T_{\mathrm{A}}^4 - \sigma T_{\mathrm{E}}^4 + (1 - \rho_{\mathrm{E,K}})\dot{q}_S'' = 0 \tag{1.32}$$

Subtract the balances:

$$T_{\mathrm{A}} = \left(\frac{(1 - \rho_{\mathrm{E,K}})\dot{q}_S''}{\sigma}\right)^{1/4} = \left(\frac{0,7 \cdot 343,25\,\mathrm{W\ m^{-2}}}{5,76\,10^{-8}\,\mathrm{W\ m^{-2}\ K^{-1}}}\right)^{1/4} = 255,14\,\mathrm{K} \tag{1.33}$$

$$T_{\mathrm{E}} = \left(2\,T_{\mathrm{A}}^4\right)^{1/4} = 303,4\mathrm{K} \tag{1.34}$$

1.6 Grilling pig

a) *Determine the surface temperature of the pig T_P in function of the given variables.*

The outer energy balance around the pig provides:

$$\dot{Q}_\mathrm{P} = \dot{Q}_\mathrm{C}\Phi_\mathrm{CP} \tag{1.35}$$

The pig acts as a grey body, which gives the following surface brightness:

$$\dot{Q}_\mathrm{P} = \varepsilon_\mathrm{P}\sigma A_\mathrm{P}T_\mathrm{P}^4 + (1-\varepsilon_\mathrm{P})\left(\dot{Q}_\mathrm{C}\Phi_\mathrm{CP}\right) \tag{1.36}$$

The coal is a black radiator:

$$\dot{Q}_\mathrm{C} = \sigma A_\mathrm{C}T_\mathrm{C}^4 \tag{1.37}$$

Inserting the surface brightness yields:

$$\varepsilon_\mathrm{P}\sigma A_\mathrm{P}T_\mathrm{P}^4 + (1-\varepsilon_\mathrm{P})\left(\dot{Q}_\mathrm{C}\Phi_\mathrm{CP}\right) = \Phi_\mathrm{CP}\dot{Q}_\mathrm{C} \tag{1.38}$$

The equation corresponds to a inner balance, which can established directly.

$$\Longleftrightarrow \sigma A_\mathrm{P}\varepsilon_\mathrm{P}T_\mathrm{P}^4 = \sigma A_\mathrm{C}\varepsilon_\mathrm{C}T_\mathrm{C}^4\Phi_\mathrm{CP} \Longleftrightarrow T_\mathrm{P}^4 = \frac{A_\mathrm{C}\Phi_\mathrm{CP}}{A_\mathrm{P}}T_\mathrm{C}^4 \tag{1.39}$$

Using the reciprocity gives:

$$T_\mathrm{P} = T_\mathrm{C}\Phi_\mathrm{PC}^{\frac{1}{4}} \tag{1.40}$$

1.7 Radiation within a wedge-shaped opening

a) *Determine the view factors* $\Phi_{1,2}$,$\Phi_{2,1}$,$\Phi_{1,o}$,$\Phi_{2,o}$.

Sum rule (where $\Phi_{1,1}$=$\Phi_{2,2}$=$\Phi_{o,o}$=0):

$$\Phi_{1,2} + \Phi_{1,\ddot{o}} = 1 \tag{1.41}$$

$$\Phi_{2,2} + \Phi_{2,\ddot{o}} = 1 \tag{1.42}$$

$$\Phi_{\ddot{o},1} + \Phi_{\ddot{o},2} = 1 \tag{1.43}$$

Reciprocity rule:

$$a \cdot \Phi_{1,2} = a\sqrt{2} \cdot \Phi_{2,1} \tag{1.44}$$

$$a \cdot \Phi_{1,\ddot{o}} = a \cdot \Phi_{\ddot{o},1} \tag{1.45}$$

$$a \cdot \Phi_{\ddot{o},2} = a\sqrt{2} \cdot \Phi_{2,\ddot{o}} \tag{1.46}$$

Given Equations 1.41 - 1.46, we have 6 unknowns and 6 equations. Rearranging, substituting and filling in numerical values results in:

$$\Phi_{1,2} = \frac{1}{2} \cdot \sqrt{2} \tag{1.47}$$

$$\Phi_{1,\ddot{o}} = 1 - \frac{1}{2} \cdot \sqrt{2} \tag{1.48}$$

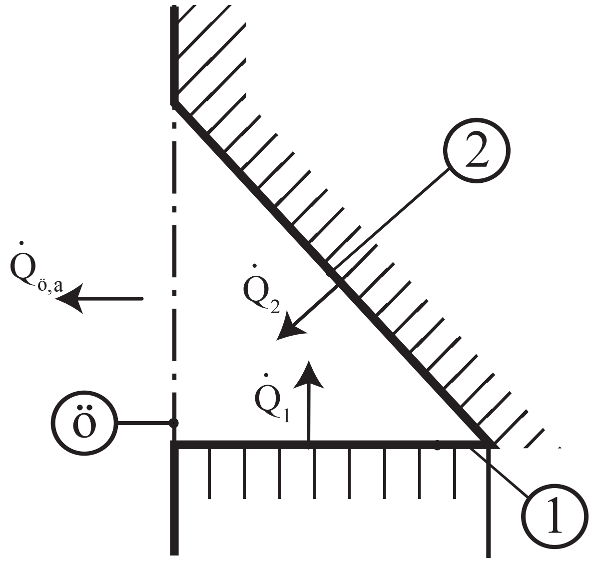
$$\Phi_{2,1} = \frac{1}{2} \tag{1.49}$$

$$\Phi_{2,\ddot{o}} = \frac{1}{2} \tag{1.50}$$

$$\Phi_{\ddot{o},1} = 1 - \frac{1}{2} \cdot \sqrt{2} \tag{1.51}$$

$$\Phi_{\ddot{o},2} = \frac{1}{2} \cdot \sqrt{2} \tag{1.52}$$

b) *Determine the energy loss of surface (1)* $\dot{q}'_{1,V}$ *and the opening*$\dot{q}'_{\ddot{o},V}$ *for a unit length of the opening.*



For the opening:

$$\tau_{\ddot{o}} = 1 \Rightarrow \epsilon_{\ddot{o}} = \rho_{\ddot{o}} = o \tag{1.53}$$

Surface brightness of the opening:

$$\dot{Q}_{\ddot{o},a} = \dot{Q}_1 \Phi_{1,\ddot{o}} \cancelto{1}{\Phi_{\ddot{o}}} + \dot{Q}_2 \Phi_{2,\ddot{o}} \cancelto{1}{\Phi_{\ddot{o}}} \tag{1.54}$$

For body 1 (black body):

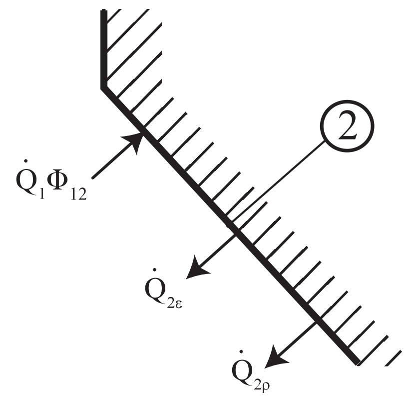
$$\tau_1 = 0 \text{ and } \epsilon_1 = \alpha_1 = 1 \tag{1.55}$$

Surface brightness of body 1:

$$\dot{Q}_1 = A_1 \sigma T_1^4 \cancelto{1}{\epsilon_1} \tag{1.56}$$

Surface brightness of body 2:

$$\dot{Q}_2 = A_2 \sigma T_2^4 \epsilon_2 + \dot{Q}_1 \Phi_{12} \rho_2 \tag{1.57}$$



Energy balance around body 2:

$$\dot{Q}_1\Phi_{12} + \cancelto{0}{\dot{Q}_{\ddot{o}}\Phi_{\ddot{o}2}} - \dot{Q}_2 = 0 \tag{1.58}$$

Rate of heat transfer from the opening to the environment:

$$\dot{Q}_{\ddot{o},a} = \dot{Q}_1\Phi_{1\ddot{o}} + \dot{Q}_2\Phi_{2\ddot{o}} \tag{1.59}$$

Substitution of $\dot{Q}_2$ (Equation 1.58) into Equation 1.59 and dividing by L results in:

$$\begin{gathered} \dot{q}'_{\ddot{o},a} = \dot{q}'_1\left(\Phi_{1\ddot{o}} + \Phi_{12}\Phi_{2\ddot{o}}\right) = a\sigma T_1^4\left(\Phi_{1\ddot{o}} + \Phi_{12}\Phi_{2\ddot{o}}\right) = \\ 0.3\cdot 5.67\ [\mathrm{m}]\cdot 10^{-8}\ [\mathrm{W/m^2K^4}]1000^4\ [\mathrm{K}]\left(1 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4}\right) = 11\ [\mathrm{kW/m}] \end{gathered} \tag{1.60}$$

c) *Determine the temperature T_2 of surface (2)*

Substitution of Equation 1.57 into 1.58 results in:

$$\dot{Q}_1\Phi_{12} = A_2\sigma T_2^4\epsilon_2 + \dot{Q}_1\Phi_{12}\rho_2 \tag{1.61}$$

Rearranging (note that from Kirchhoff and the fact that we are dealing with a grey body it results that $(1-\rho_2) = \alpha_2 = \epsilon_2$):

$$T_2 = \sqrt[4]{\frac{\dot{Q}_1\Phi_{12}\left(1-\rho_2\right)}{A_2\sigma\epsilon_2}} = \sqrt[4]{\frac{T_1^4\Phi_{12}}{\sqrt{2}}} = \sqrt[4]{\frac{1000^4}{2}} = 840.9\ [\mathrm{K}] \tag{1.62}$$