1.12 Solar collector ★

a) *Determine the rate of heat loss for* $\theta = 0^o$

1) Setup the definition of rate of heat loss:

The rate of heat loss by convection can be described by:

$$\boxed{\rightarrow \dot{Q} = \overline{\alpha} A_\mathrm{s} \left(T_\mathrm{A} - T_\mathrm{G}\right)} \tag{1.204}$$

2) Defining all required parameters:

The average heat transfer coefficient for natural internal convection can be expressed as:

$$\overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} \tag{1.205}$$

To find the expression for the Nusselt number, we first should know what the value for the Grashof number. Which can be expressed as:

$$\mathrm{Gr_s} = \frac{g\beta \left(T_\mathrm{A} - T_\mathrm{G}\right) s^3}{\nu^2} \tag{1.206}$$

The expansion coefficient β for an ideal gas can be approximated by the following relation: (where we take T_∞ to be the average temperature inside the collector.)

$$\boxed{\rightarrow \beta = \frac{1}{T_\infty} = \frac{1}{\left(\frac{40+80}{2} + 273\right)\ [\mathrm{K}]} = 0.0030\ [\mathrm{K}^{-1}]} \tag{1.207}$$

Filling in all numerical values into the expression of the Grashof number yields:

$$\mathrm{Gr_s} = \frac{9.81\ [\mathrm{m/s^2}] \cdot 0.0030\ [\mathrm{K^{-1}}] \left(80 - 40\right)\ [\mathrm{K}] \cdot 0.02^3\ [\mathrm{m^3}]}{\left(1.9305 \cdot 10^{-5}\right)^2\ [\mathrm{m^4/s^2}]} \tag{1.208}$$

$$\boxed{\rightarrow \mathrm{Gr_s} = 2.2584 \cdot 10^4} \tag{1.209}$$

With this we find that the correlation $\overline{\mathrm{Nu}}_\mathrm{s} = 0.21 \left(\mathrm{Gr_s Pr}\right)^{1/4}$ for the Nusselt number, as $\mathrm{Gr_s} > 2 \cdot 10^3$:

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_\mathrm{s} = 0.21 \left(\mathrm{Gr_s Pr}\right)^{1/4} = 0.21 \left(2.2584 \cdot 10^4 \cdot 0.7103\right)^{1/4} = 2.43} \tag{1.210}$$

Average heat transfer coefficient:

$$\boxed{\rightarrow \overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} = \frac{2.43 \cdot 0.0286\ [\mathrm{W/mK}]}{0.02\ [\mathrm{m}]} = 3.47\ [\mathrm{W/m^2K}]} \tag{1.211}$$

3) Inserting and rearranging:

Rate of heat transfer:

$$\dot{Q} = \overline{\alpha} \cdot A_\mathrm{s} \cdot (T_\mathrm{A} - T_\mathrm{G}) = 3.47\ [\mathrm{W/m^2K}] \cdot\ 0.8\ [\mathrm{m}] \cdot\ 3\ [\mathrm{m}] \cdot (80 - 40)\ [\mathrm{K}] \tag{1.212}$$

$$\boxed{\rightarrow \dot{Q} = 333.6\ [\mathrm{W}]} \tag{1.213}$$

b) *Determine the rate of heat loss for $\theta = 90^o$*

1) Setup the definition of rate of heat loss:

The rate of heat loss by convection can be described by:

$$\boxed{\rightarrow \dot{Q} = \overline{\alpha} A_\mathrm{s} \left(T_\mathrm{A} - T_\mathrm{G}\right)} \tag{1.214}$$

2) Defining all required parameters:

The average heat transfer coefficient for natural internal convection can be expressed as:

$$\overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} \tag{1.215}$$

To find the expression for the Nusselt number, we first should know the value for the Grashof number. Which can be expressed as:

$$\mathrm{Gr_s} = \frac{g\beta \left(T_\mathrm{A} - T_\mathrm{G}\right) s^3}{\nu^2} \tag{1.216}$$

Filling in all numerical values, as in question a), into the expression of the Grashof number yields:

$$\boxed{\rightarrow \mathrm{Gr_s} = 2.2584 \cdot 10^4} \tag{1.217}$$

We find that the correlation $\overline{\mathrm{Nu}}_\mathrm{s} = 0.20 \left(L/s\right)^{-1/9} \left(\mathrm{Gr_s Pr}\right)^{1/4}$ for the Nusselt number does meet the criteria $3.1 < L/s < 42.2$, but is does not meet the requirement $2 \cdot 10^3 < \mathrm{Gr_s} < 2 \cdot 10^4$. As the Grashof number just falls outside this range, we decide still to use it, but we have to keep in mind that our found rate of heat transfer will have some error in it!

$$\overline{\mathrm{Nu}}_\mathrm{s} = 0.20 \left(L/s\right)^{-1/9} \left(\mathrm{Gr_s Pr}\right)^{1/4} = 0.20 \left(0.8/0.02\right)^{-1/9} \left(2.2584 \cdot 10^4 \cdot 0.7103\right)^{1/4} \tag{1.218}$$

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_\mathrm{s} = 1.5369} \tag{1.219}$$

Average heat transfer coefficient:

$$\boxed{\rightarrow \overline{\alpha} = \frac{\overline{\mathrm{Nu}}_\mathrm{s} \cdot \lambda}{s} = \frac{1.5369 \cdot 0.0286\ [\mathrm{W/mK}]}{0.02\ [\mathrm{m}]} = 2.20\ [\mathrm{W/m^2K}]} \tag{1.220}$$

3) Inserting and rearranging:

Rate of heat transfer:

$$\dot{Q} = \overline{\alpha} \cdot A_\mathrm{s} \cdot (T_\mathrm{a} - T_\mathrm{g}) = 2.20\ [\mathrm{W/m^2K}] \cdot\ 0.8\ [\mathrm{m}] \cdot\ 3\ [\mathrm{m}] \cdot (80 - 40)\ \ [\mathrm{K}] \tag{1.221}$$

$$\boxed{\rightarrow \dot{Q} = 210.9\ [\mathrm{W}]} \tag{1.222}$$

1.13 Horizontal & vertical wall ★★

a) *Determine the ratio of the two heat transfer coefficients* α_{H} *and* α_{H} *of the surfaces.*

1) Setup the definition of the heat transfer coefficient:

The heat transfer coefficient can be described by:

$$\boxed{\rightarrow \alpha = \frac{\overline{\mathrm{Nu}}_{\mathrm{L_c}} \lambda_\infty}{L_c}} \tag{1.223}$$

2) Defining all required parameters:

The characteristic length in case 1 is defined as:

$$\boxed{\rightarrow L_{\mathrm{c}} = H} \tag{1.224}$$

The characteristic length in case 2 is defined as:

$$\boxed{\rightarrow L_{\mathrm{c}} = W = 2H} \tag{1.225}$$

For case 1, with the characteristic length $L_{\mathrm{c}} = H$,the correlation of the Nusselt number that is valid is:

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_{\mathrm{H}} = 0.535 \cdot (\mathrm{Gr}_{\mathrm{H}} \cdot \mathrm{Pr})^{1/4}} \tag{1.226}$$

For case 2, with the characteristic length $L_{\mathrm{c}} = W = 2H$,the correlation of the Nusselt number that is valid is:

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_{\mathrm{W}} = 0.54 \cdot (\mathrm{Gr}_{\mathrm{W}} \cdot \mathrm{Pr})^{1/4}} \tag{1.227}$$

Furthermore, the Grashof number for case 1 can be described as:

$$\boxed{\rightarrow \mathrm{Gr}_{\mathrm{H}} = \frac{\beta g \left(T_{\mathrm{H}} - T_\infty\right) H^3}{\nu^2} = \frac{3\beta g T_\infty H^3}{\nu^2}} \tag{1.228}$$

The Grashof number for case 2 can be described as:

$$\boxed{\rightarrow \mathrm{Gr}_{\mathrm{W}} = \frac{\beta g \left(T_{\mathrm{W}} - T_\infty\right) W^3}{\nu^2} = \frac{8\beta g T_\infty H^3}{\nu^2}} \tag{1.229}$$

3) Inserting and rearranging:

Inserting the found expressions of the characteristic length into the definition of the heat transfer coefficient yields:

$$\alpha_{\mathrm{H}} = \frac{\overline{\mathrm{Nu}}_{\mathrm{H}} \lambda_\infty}{H} \tag{1.230}$$

$$\alpha_{\mathrm{W}} = \frac{\overline{\mathrm{Nu}}_{\mathrm{W}} \lambda_{\infty}}{2H} \tag{1.231}$$

Which gives us the following ratio:

$$\frac{\alpha_{\mathrm{W}}}{\alpha_{\mathrm{H}}} = \frac{\frac{\overline{\mathrm{Nu}}_{\mathrm{W}} \lambda_{\infty}}{2H}}{\frac{\overline{\mathrm{Nu}}_{\mathrm{H}} \lambda_{\infty}}{H}} = \frac{\overline{\mathrm{Nu}}_{\mathrm{W}}}{2\overline{\mathrm{Nu}}_{\mathrm{H}}} \tag{1.232}$$

Inserting the definitions of the Nusselt and Grashof numbers results:

$$\frac{\alpha_{\mathrm{W}}}{\alpha_{\mathrm{H}}} = \frac{0.54 \cdot \left(\frac{8\beta g T_{\infty} H^3}{\nu^2} \cdot \mathrm{Pr}\right)^{1/4}}{2 \cdot 0.535 \cdot \left(\frac{3\beta g T_{\infty} H^3}{\nu^2} \cdot \mathrm{Pr}\right)^{1/4}} \tag{1.233}$$

The hint is given that all material properties remain constant, which is why all fluid properties do cancel out.

Rewriting:

$$\boxed{\rightarrow \frac{\alpha_{\mathrm{W}}}{\alpha_{\mathrm{H}}} = \frac{0.54 \cdot 8^{1/4}}{2 \cdot 0.535 \cdot 3^{1/4}} = 0.645} \tag{1.234}$$