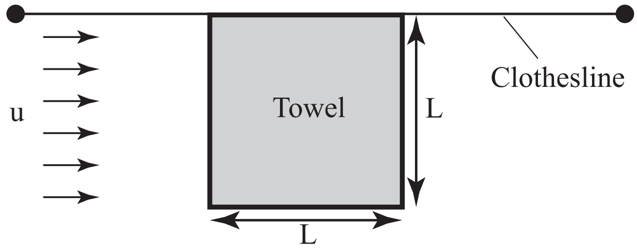
Exercise V.2: (Soaked towel ⋆⋆)

A square towel (edge length L) soaked with water is hung up on a clothesline. The wind is blowing against the towel from the side with a velocity of u. The air and the towel have the same temperature T that remains constant over time.



Given parameters:

- Temperature of towel and air: $T = 20$ °C
- Velocity of the wind: $u = 1$ m/s
- Edge length of the towel: $L = 0.5$ m

Hints:

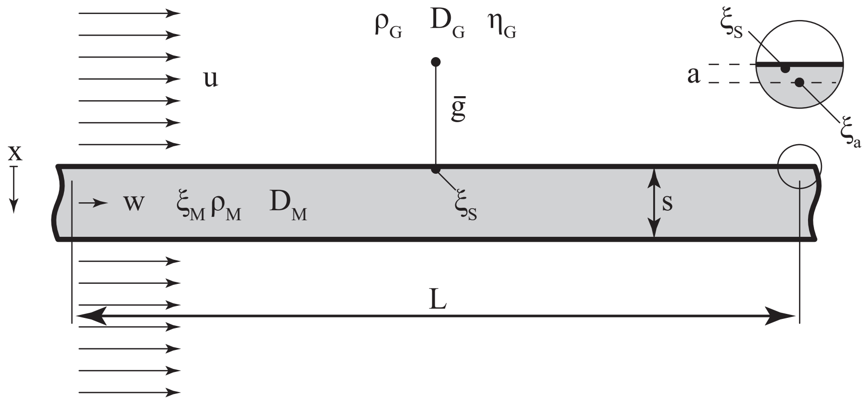
- The influence of radiation on the problem is to be neglected.
- The Lewis relation is valid.

Tasks:

a) Calculate the average mass transfer coefficient $\overline{g}$.

Exercise V.3: (Metal strap ⋆⋆⋆)

A metal strap (M) with a sheet-thickness s is pulled at a velocity w through an oven of length L. Thereby a CO_2-containing gas atmosphere (G) flows over the strap with a velocity u. On entering the furnace, the metal strap has a homogeneous fraction of carbon ξ_M. Under quasi-steady conditions, a constant fraction of carbon ξ_S exists just below the surface of the metal strip, while carbon constantly diffuses into its interior.



Given parameters:

- Diffusion coefficient of carbon in metal: $D_M = 5 \cdot 10^{-10}$ m²/s
- Diffusion coefficient of carbon in gas: $D_G = 1 \cdot 10^{-4}$ m²/s
- Combined density of metal: $\rho_M = 8 \cdot 10^3$ kg/m³
- Combined density of the gases: $\rho_G = 0.8$ kg/m³
- Combined viscosity of the gases: $\eta_G = 5.6 \cdot 10^{-5}$ Pa · s
- Flow velocity: $u = 10$ m/s
- Sheet velocity: $w = 2 \cdot 10^{-2}$ m/s
- Initial carbon fraction: $\xi_M = 0.2 \cdot 10^{-4}$
- Surface carbon fraction: $\xi_S = 1 \cdot 10^{-4}$
- Oven length: $L = 10$ m
- Strap thickness: $s = 50 \cdot 10^{-3}$ m
- Layer depth: $a = 0.2 \cdot 10^{-3}$ m

Hints:

- Use a co-moving coordinate system.
- Neglect diffusion in the direction of the sheet.
- The strap thickness is significantly greater than twice the depth of penetration of the carbon.

Tasks:

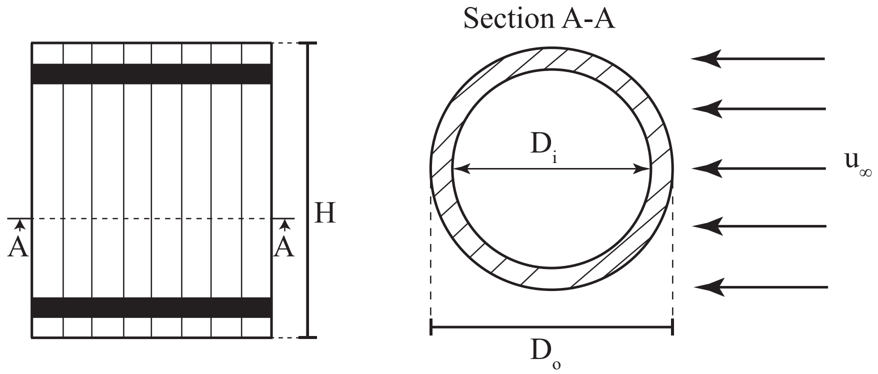
a) Calculate the mean mass transfer coefficient $\bar{g}$ on the strip's surface. Show that the assumption of a constant carbon content of ξ_S just below the surface is valid.

b) Provide the differential equation for the carbon transport in the strap as well as the appropriate initial and boundary conditions. For this, provide a suitable equation for the carbon content in the strap $\xi(x,t)$.

c) Calculate the carbon content of ξ_a which is established at the end of the furnace at a depth a from the strap's surface. Show that the assumption of a low penetration depth is justified.

Exercise V.4: (Whiskey barrel ⋆ ⋆ ⋆)

Deep in the cellars of a distillery, Whisky is stored in wooden barrels for its maturation. The whisky is composed of an ethanol-water mixture with an ethanol mass fraction $\xi_{\mathrm{EW,t0}}$ at the initial time t_0 (at the beginning of the storage period). The ventilation system of the cellar provides a continuous flow of fresh air (free of ethanol, $\xi_{\mathrm{EA},\infty} = 0$) to the barrels. During the storage period, the concentration of ethanol in the Whisky falls due to mass transport processes down to a value of $\xi_{\mathrm{EW,t1}}$ (time t_1, is the end of the storage).



Hint: Use the following nomenclature for the indexes:

ξ_{EW} : Ethanol mass fraction in Whisky
$\xi_{\mathrm{EWo,i}}$: Ethanol mass fraction in Wood, inner side of the barrel
$\xi_{\mathrm{EWo,o}}$: Ethanol mass fraction in Wood, outer surface of the barrel
$\xi_{\mathrm{EA,o}}$: Ethanol mass fraction in air, at the outer surface of the barrel
$\xi_{\mathrm{EA},\infty}$: Ethanol mass fraction in air, at the free flow

Given parameters:

- Initial time of the barrel storage: t_0
- End time of the barrel storage: t_1
- Length of the storage time: $\Delta t = 3$ years
- Constant: $Z = 4.34 \cdot 10^{-7}$ kg/s
- Outer diameter: $D_{\mathrm{o}} = 0.56$ m
- Inner diameter: $D_{\mathrm{i}} = 0.50$ m
- Height: $H = 0.9$ m
- Density of the wood: $\rho_{\mathrm{Wo}} = 500$ kg/m^3
- Diffusion coefficient of ethanol in wood: $D_{\mathrm{EWo}} = 1 \cdot 10^{-11}$ m^2/s
- Density of the whisky: $\rho_{\mathrm{W}} = 870$ kg/m^3
- Dynamic viscosity of the whisky: $\eta_{\mathrm{W}} = 1 \cdot 10^{-3}$ Pa · s
- Mass fraction of ethanol at initial time t_0: $\xi_{\mathrm{EW,t0}} = 0.54$
- Flow velocity: $u_\infty = 0.1$ m/s

- Density of the air: $\rho_{\mathrm{A}} = 1.18\ \mathrm{kg/m^3}$
- Diffusion coefficient of ethanol in air: $D_{\mathrm{EA}} = 1.47 \cdot 10^{-5}\ \mathrm{m^2/s}$
- Dynamic viscosity of the air: $\eta_{\mathrm{A}} = 1.81 \cdot 10^{-5}\ \mathrm{Pa \cdot s}$

Hints:

- Due to the high relative humidity in the cellar, Whisky only loses ethanol.
- There is no mass transport resistance at the inner surface of the barrel.
- Ignore the Stefan flow.
- The overall density of the Whisky, the wood and the air do **not** change due to mass transport processes.
- The Whisky barrels are cylindrical and ethanol diffuses **only** through the lateral surfaces, neglecting the effect of the metal rings around the barrel.
- The Whisky is ideally mixed inside the barrel.
- All given material properties are constant.
- Jumps in concentrations are only due to density differences.

Tasks:

Hint: For the sub-problems a) to c) the system is in a (quasi-) stationary state and the concentration ξ_{EW} is known at each point in time

a) Sketch (qualitatively) the radial concentration profile of ethanol ξ_{EW} in the Whisky at any time t within the time interval $t_0 < t < t_1$.

b) Determine an expression for the ethanol mass flow rate taking place due to mass diffusion through the wood barrel ($\dot{m}_{\mathrm{D}}$) and another for the mass flow produced by the convection process ($\dot{m}_{\mathrm{K}}$) occuring at the external surface of the barrel. Additionally, calculate the mass transfer coefficient g. To do this use the known ethanol concentrations at the interfaces.

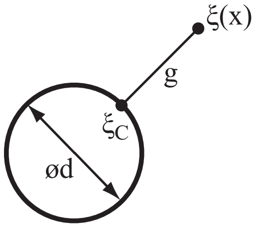
c) Determine the unknown ethanol concentrations $\xi_{\mathrm{EWo,i}}$, $\xi_{\mathrm{EWo,o}}$, $\xi_{\mathrm{EA,o}}$ and $\xi_{\mathrm{EA,\infty}}$, and give and expression for the ethanol mass flow rate $\dot{m}_E$.

d) By means of an instationary mass balance upon the whisky, calculate the ethanol concentration $\xi_{\mathrm{EW,t1}}$ in the barrel, which is attained after a storage time of $\Delta t = 3$ years in the barrel. Use the simplified expression of the ethanol mass flow given by $\dot{m}_{\mathrm{E}} = Z \cdot \xi_{\mathrm{EW}}$ with $Z = 4.34 \cdot 10^{-7}\,\frac{\mathrm{kg}}{\mathrm{s}}$

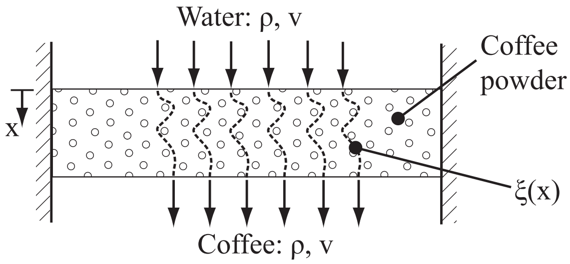
Exercise V.5: (Coffee brewing ⋆ ⋆ ⋆)

Coffee powder consists of many coffee grains, forming a highly concentrated coffee solution during brewing. The flavors come out of the coffee grains (ξ_C) through diffusion into the passing water flow ($\xi(x)$). This diffusion process is well described by the mass-transfer coefficient g.

a) Idealized coffee grain (sphere)



b) Coffee preparation



Given parameters:

- Diffusion coefficient of the flavors in water: D
- Water flow velocity: v
- Total density of flavored water: ρ
- Concentration of highly concentrated coffee solution: ξ_C
- Mass transfer coefficient: g
- Diameter of a coffee grain: d
- Packing density in the coffee powder: p

Hints:

- The problem is stationary and the Stefan flow is negligible.
- All coffee grains have the same diameter and are homogeneously distributed in the powder.
- If you cannot solve part a), you may use $\dot{m}'''_C = \frac{g}{d} \cdot (\xi_C - \xi(x))$.

Tasks:

a) Determine the mass flow out of the grains per unit volume $\dot{m}'''_C$, which is transferred from the coffee grains to the passing water flow.

b) Derive the differential equation for the mass concentration of flavors $\xi(x)$ in the passing water flow. Consider the packing density p of the coffee grains within the ground coffee powder in the source term:$\dot{m}''' = p \cdot \dot{m}'''_C$.

c) Write down the required differential equation and the boundary conditions, when diffusion is negligible in the flow direction, and determine the concentration profile of $\xi(x)$ for this case.

d) Draw the concentration profiles $\xi(x)$ for two different grain diameters $d_1 < d_2$ over the length of the ground coffee powder in the given chart and mark the curves clearly.