Tutorial T03 – Elasticity Stress

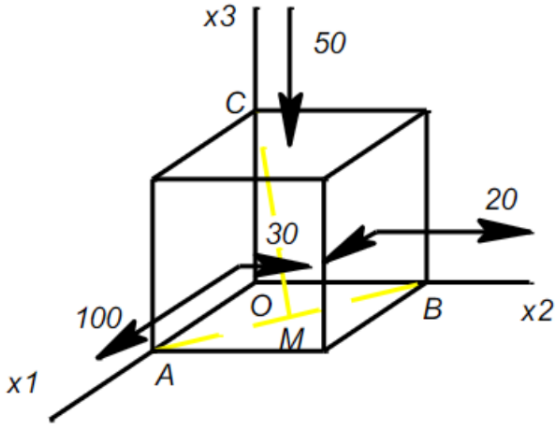
Answer the following questions as they could come up in an exam.

1 Stress basics (geometry)

(Exercise V1 in old material before 2022)

Given:
$E = 200\,\text{GPa}$, $\upsilon = 0.25$
$OA = OB = a$ and $OC = \frac{1}{2}\sqrt{2} \cdot a$
In this stress-state, the maximal principal stress must not be larger than: 150 MPa.

Questions:
a) Find σ_{ABC} and τ_{ABC}



2 Stress and transformation

(Exercise V? in old material before 2022)

Given:
$E = 2 \cdot 10^{11}$ Pa, $\upsilon = 0.25$

Stress-state in point P: $\sigma = \begin{bmatrix} 19 & -5 & -\sqrt{6} \\ -5 & 19 & -\sqrt{6} \\ -\sqrt{6} & -\sqrt{6} & 10 \end{bmatrix}$ MPa

Questions:
a) Show that the principal stresses are 8, 16 and 24 MPa.
Compute the directional cosines (transformation matrix entries) of the smallest eigen-stress.

3 Stress equilibrium

(Exercise V12 in old material before 2022)

In a linear elastic ($E = 2 \cdot 10^5\,\mathrm{MPa}$, $\upsilon = 0.25$) body under load, the stress-field is given (with four free parameters), with respect to the Cartesian $x_1 - x_2 - x_3$ coordinate system as:

$$\sigma_{11}(x_1, x_2, x_3) = \sigma_0 \left[20 + \alpha_1(\tfrac{x_1}{L}) - 10(\tfrac{x_2}{L}) + \alpha_2(\tfrac{x_1}{L})^2\right]$$

$$\sigma_{22}(x_1, x_2, x_3) = \sigma_0 \left[10 + 8(\tfrac{x_1}{L}) + \beta_1(\tfrac{x_2}{L}) + \beta_2(\tfrac{x_2}{L})^2\right]$$

$$\sigma_{12}(x_1, x_2, x_3) = \sigma_0 \left[12 - 10(\tfrac{x_1}{L}) + 7(\tfrac{x_2}{L}) - 8(\tfrac{x_1}{L})(\tfrac{x_2}{L})\right]$$

$$\sigma_{13}(x_1, x_2, x_3) = \sigma_{23}(x_1, x_2, x_3) = \sigma_{33}(x_1, x_2, x_3) = 0$$

with reference stress $\sigma_0 = 1\,\mathrm{MPa}$ and reference length $L = 1\,\mathrm{m}$.
Note that all stresses are independent on x_3 and that the calculation in question (a) below is general with variables x_1, x_2, and x_3; from question (b) on, use the point $P(x_1 = 0, x_2 = 0, x_3 = 0)$.

Questions:
a) Does the stress field agree with the stress-equilibrium equations in absence of volume-forces? Which relations have to be valid for the free parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$ due to stress equilibrium.
b) Compute the eigen-stresses in point P using linear algebra, mathematics -– not the circle of Mohr.
Describe and name the state of stress in point P (and in all other points in the body).
c) Compute the eigen-direction of the major eigen-stress.