

1 Exercise V-1a

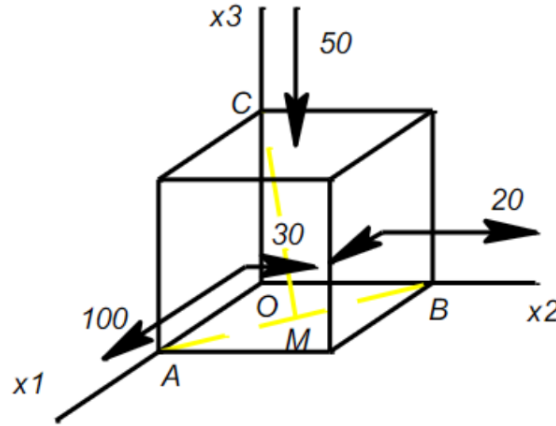
Given: $E = 200 \text{ GPa}$, $\nu = 0.25$

$OA = OB = a$ and $OC = \frac{1}{2}\sqrt{2} \cdot a$

In this stress-state, the maximal principal stress must not be larger than: 150 MPa.

Questions:

a) Find σ_{ABC} and τ_{ABC}



2 Exercise V.4

Given: $E = 2 \cdot 10^{11} \text{ Pa}$, $\nu = 0.25$

Stress-state in point P: $\sigma = \begin{bmatrix} 19 & -5 & -\sqrt{6} \\ -5 & 19 & -\sqrt{6} \\ -\sqrt{6} & -\sqrt{6} & 10 \end{bmatrix} \text{ MPa}$

Questions:

- Show that the principal stresses are 8, 16 and 24 MPa. Compute the directional cosines (transformation matrix entries) of the smallest eigen-stress.
- Compute the volumetric (isotropic) strain.
- What is the largest angle-change (not shear-strain) in P?
- Which material property is implicitly used in Hookes law?

3 Exercise V.12abc

In a linear elastic ($E = 2 \cdot 10^5 MPa$, $\nu = 0.25$) body under load, the stress-field is given (with four free parameters), with respect to the Cartesian $x_1 - x_2 - x_3$ coordinate system as:

$$\sigma_{11}(x_1, x_2, x_3) = \sigma_0[20 + \alpha_1(\frac{x_1}{L}) - 10(\frac{x_2}{L}) + \alpha_2(\frac{x_1}{L})^2]$$

$$\sigma_{22}(x_1, x_2, x_3) = \sigma_0[10 + 8(\frac{x_1}{L}) + \beta_1(\frac{x_2}{L}) + \beta_2(\frac{x_2}{L})^2]$$

$$\sigma_{12}(x_1, x_2, x_3) = \sigma_0[12 - 10(\frac{x_1}{L}) + 7(\frac{x_2}{L}) - 8(\frac{x_1}{L})(\frac{x_2}{L})]$$

$$\sigma_{13}(x_1, x_2, x_3) = \sigma_{23}(x_1, x_2, x_3) = \sigma_{33}(x_1, x_2, x_3) = 0$$

with reference stress $\sigma_0 = 1MPa$ and reference length $L = 1m$. Note that all stresses are independent on x_3 and that the calculation in question (a) below is general with variables x_1, x_2 , and x_3 ; from question (b) on, use the point $P(x_1 = 0, x_2 = 0, x_3 = 0)$.

Questions:

a) Does the stress field agree with the stress-equilibrium equations in absence of volume-forces? Which relations have to be valid for the four free parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$ due to stress equilibrium.

b) Compute the eigen-stresses in point P using linear algebra mathematics – not the circle of Mohr.

Describe and name the state of stress in point P (and in all other points in the body).

c) Compute the eigen-direction of the major eigen-stress.