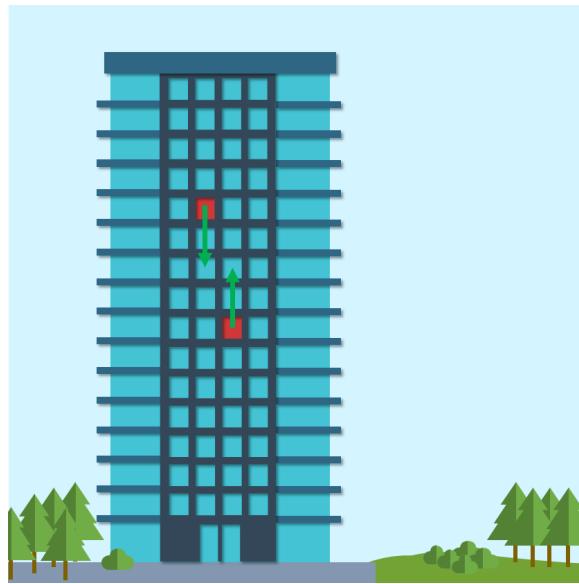


Elevator in a Skyscraper



The elevator of a skyscraper rises 350 meters at a maximum speed of 22 km/h. Both the acceleration and deceleration have a constant magnitude of $0.25g$ during this rise. After reaching its maximum speed, the elevator maintains this speed until it decelerates to perfectly come to a full stop at the top. Determine the duration T of this full elevator rise.

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = adt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = vdt = (v_0 + at)dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (v_0 + at) dt \quad (5)$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \quad (6)$$

Given quantities:

Distance: $s = 350 \text{ m}$

Acceleration: $a = 0.25g$

Velocity: $v = 22 \text{ km/h} \approx 6.11 \text{ m/s}$

Solution:

The easiest thing is to visualize the problem by drawing a graph of how the velocity changes over time (see Figure 1). The problem is divided in three parts; the first one is the acceleration of $a_1 = 0.25g$ of the elevator from standstill ($v_0 = 0 \text{ m/s}$) to its maximum speed $v_{\max} = 6.11 \text{ m/s}$, then there is a time the elevator has a constant speed of 6.11 m/s ($a_{II} = 0 \text{ m/s}^2$) and finally it decelerates with $a_{III} = -0.25g$ to a standstill.

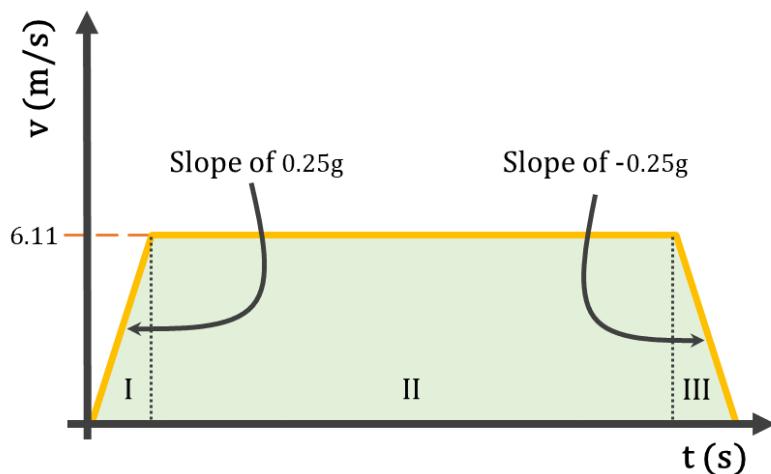


Figure 1: Visualization graph of changing velocity over time

Using Equation (3), the time t for the elevator to reach its maximum constant velocity of 6.11 m/s from standstill ($v_0 = 0 \text{ m/s}$) can be calculated.

$$t = \frac{v}{a} \quad \Rightarrow \quad t = \frac{6.11}{0.25 \cdot 9.81} \approx 2.49 \text{ s} \quad (7)$$

The traveled distance in this time is the area of section I of Figure 1. It can be directly seen from the figure or calculated using Equation 6, with $s_0 = 0 \text{ m}$ and $v_0 = 0 \text{ m/s}$.

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 = \frac{1}{2}at^2 = \frac{0.25}{2}gt^2 \quad \Rightarrow \quad s(2.49) = \frac{0.25}{2} \cdot 9.81 \cdot 2.49^2 \approx 7.61 \quad (8)$$

The elevator both accelerates and decelerates, thus the total distance traveled during both combined is $2 \cdot 7.61 = 15.22$ m (area I and area II from the figure combined).

To calculate the time that the elevators travels at 6.11 m/s, we use the fact that the total length is 350 m (intended total area under the curve). Thus the distance the elevator travels at $v_{\max} = 6.11$ m/s (or area II under the curve) is equal to $350 - 15.22 = 334.8$ m. Since the velocity is constant, the acceleration is 0 thus Equation 6 becomes:

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 = v_{\max}t \quad \Rightarrow \quad t = \frac{s(t)}{v_{\max}} = \frac{334.8}{6.11} \approx 54.79 \text{ s} \quad (9)$$

Thus the total time becomes $T = 2t + 54.79 = 2 \cdot 2.49 + 54.79 = 59.77$ s.