Tutorial T06 – Material behavior

December 12, 2024

Answer the following questions as they could come up in an exam.

Exercises 1 – 13 are given in tutorial T05

14 Materials beyond elasticity

Questions:
a) Sketch the stress-strain relation for a linear elastic material, and
b) add possible non-linear material behavior (with explanation/motivation).
c) Explain what happens for unloading of:
c1) a linear, elastic material, or
c2) an elastic-plastic material (for small AND for large strains).
d) Sketch the relation of shear-stress versus strain-rate, for (fluid) materials that behave:
d1) linear,
d2) shear-thickening,
d3) shear-thinning, or
d4) yield-stress fluid.

Answers:
a)
Stress-strain linear means a straight line, with slope E being the modulus, while elastic means the return path is identical (from point 2 in Fig. 1 Left), unlike after plastic deformation (from point 4 in Fig. 1 Left).

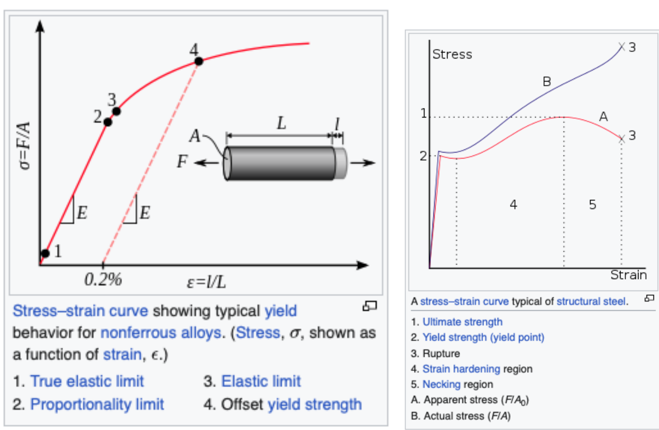

Figure 1: Plots taken from `https://en.wikipedia.org/wiki/Plasticity_(physics)`

b)
Many types of non-linear behavior are possible, see above Fig. 1, here only a few examples, linear (visco-)elastic, yield strength, strain-hardening, etc. (*note that hardening/softening for solids do not mean the same as for fluids*).
c1)
The load-unload curve is reversible in the elastic limit, and usually for small strains (see panel (a) in Fig. 3), whereas for a visco-elastic material one obtains hysteresis (red) in the stress-strain curve, which increases with the strain-rate (see panel (b) in Fig. 3), and actually its surface area (red) is the dissipated viscous energy.
c2)
for (very) small loads, the stress-strain path is typically reversible (c1) – but after a larger strain, the return (unloading) path is not identical to the loading path. Four possible stress strain relations for plastic material are given ...

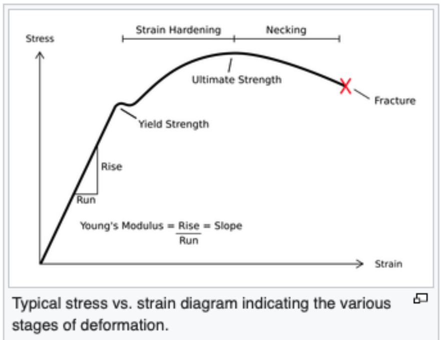

Figure 2: Plot taken from
`https://en.wikipedia.org/wiki/Deformation_(engineering)#Plastic_deformation`

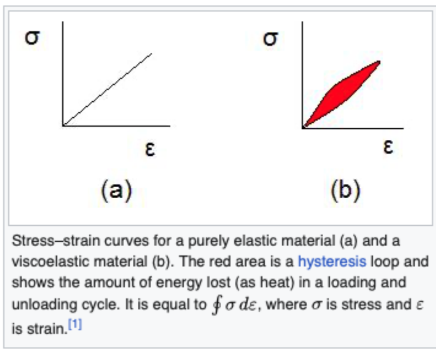

Figure 3: Plot taken from `https://en.wikipedia.org/wiki/Viscoelasticity`

d)
Note that while in solid mechanics, mostly stress and strain are considered, for fluids we typically plot shear-stress versus strain-rate (or shear-rate), with the following material behavior: d1=Newtonian, d2=dilatant, d3=pseudo-plastic, or d4=Bingham plastic (examples).

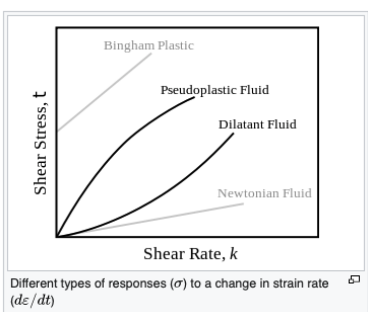

Figure 4: Plot taken from `https://en.wikipedia.org/wiki/Viscoelasticity`

15 Visco-elastic material behavior

Given is a rectangular shaped wire (length $L_0 = 0.1\,\mathrm{m}$, cross-section $A = HW$, volume $V = L_0 A = L_0 HW$) for a homogeneous, isotropic, visco-elastic, rubber-like material.

Questions:
a) What is the work necessary to quickly (or slowly) stretch the wire from stress 0 to length $3L_0$.
b) Which strain-rate is makes the elastic and viscous contributions equally important?
c) Estimate the corresponding speed at which elastic and viscous energies are similar.

Material-properties:
Kevin-Voigt viscoelastic solid (http://en.wikipedia.org/wiki/Viscoelasticity),
relation for stress = function of strain and strain-rate:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

with modulus of Young $E = 0.02\,\mathrm{MPa}$ and viscosity $\eta = 10\,\mathrm{Pa\,s}$.

Answers:
a)
Zero stress: length $L = L_0$, stretch to length $L_1 = 3L \rightarrow \varepsilon = \varepsilon_{11} = (L_1 - L_0)/L_0 = 2\,.$

Assume that the cross-section is not changing, i.e., H and W are constant.
Estimate the strain-rate: $\dot{\varepsilon} \approx$ const. $= (d/dt)\varepsilon = \frac{\text{length-change}}{\text{length*time}} = 2L_0/(L_0\Delta t) = 2/\Delta t\,$, with unspecified time Δt used to stretch.
The specific work is thus:
$a = \int d\varepsilon\sigma = \int d\varepsilon\,(E\varepsilon + \eta\dot{\varepsilon}) = \frac{1}{2}E\varepsilon^2\big|_0^2 + \eta\dot{\varepsilon}\varepsilon\big|_0^2 = 2\,(E + \eta\dot{\varepsilon}) = 2\left(E + \eta\frac{2}{\Delta t}\right)\;.$
The work in the total volume is:
$A = \int dV a = 2VE + 4V\eta/\Delta t$

b)
The strain rate at which viscous and elastic contributions are equal to each other:
$E \sim \eta\dot{\varepsilon}_{ve}$ (from integration above), so that the (constant) strain-rate is: $\dot{\varepsilon}_{ve} = E/\eta = 2 \times 10^3\,\mathrm{s}$.

c)
To estimate the speed at which viscous and elastic contributions to work equal each other, use:
$E \sim \eta\dot{\varepsilon}_{ve}$, so that (initially, engineering strain) the speed is: $v_0 = L_0\dot{\varepsilon}_{ve} = 2L_0/\Delta t$,
using: $\dot{\varepsilon} = \dot{\varepsilon}_{ve} = E/\eta$ (from b), or $\Delta t = 2\eta/E$ (from a), results in:

$$v_0 = L_0\dot{\varepsilon}_{ve} \rightarrow v_0 = 0.1 \times 2 \times 10^4/10 = 2 \times 10^2\,\mathrm{m/s},$$

thus a bit smaller than sound-speed, in some rubber material.