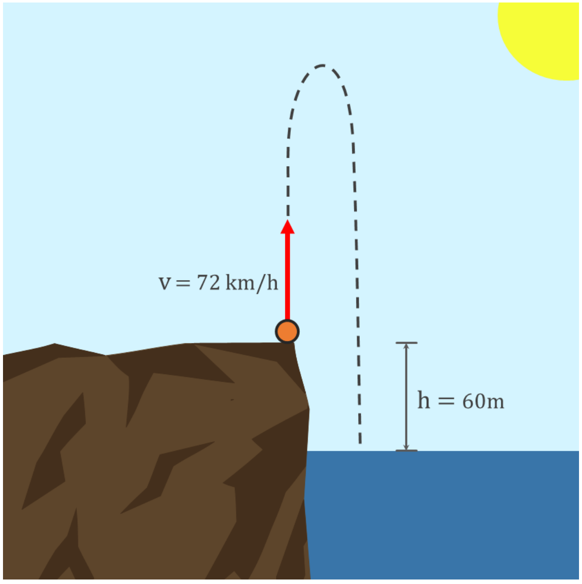
Ball Thrown off a Cliff



A ball is thrown up vertically with a initial velocity of 72 km/h at the edge of a 60 meter high cliff. What is the maximum height the ball reaches with respect to the cliff?

Neglect air resistance and take $\boldsymbol{g} = 10\ \mathrm{m/s^2}$.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (at + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (at + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

Given quantities:

Initial vertical velocity: $v_{y,0} = 72$ km/h $= 20$ m/s
Initial height of the ball (with respect to the cliff): $y_0 = s_{y,0} = 0$ m
Gravitational acceleration: $g = 10$ m/s^2

Solution:

For the vertical displacement in y-direction with the origin at cliff level, Equation (6) results in:

$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + s_{y,0} \tag{7}$$

At the instant the ball reaches its maximum height, $v_y = 0$ m/s. The vertical acceleration on the ball is the gravitational acceleration: $a_y = -g$. Combining this into Equation (3) yields an equation for the time t when the ball reaches it highest point:

$$v_y(t) = 0 = a_y t + v_{y,0} \quad \Rightarrow \quad 0 = -gt + v_{y,0} \tag{8}$$

Rewriting gives:

$$t = \frac{v_{y,0}}{g} = \frac{20}{10} = 2 \text{ s} \tag{9}$$

Inserting $t = 2$ s into Equation (7) yields the maximum height H with respect to the cliff:

$$H = y(2) = -\frac{1}{2} \cdot 10 \cdot 2^2 + 20 \cdot 2 = 20 \text{ m} \tag{10}$$