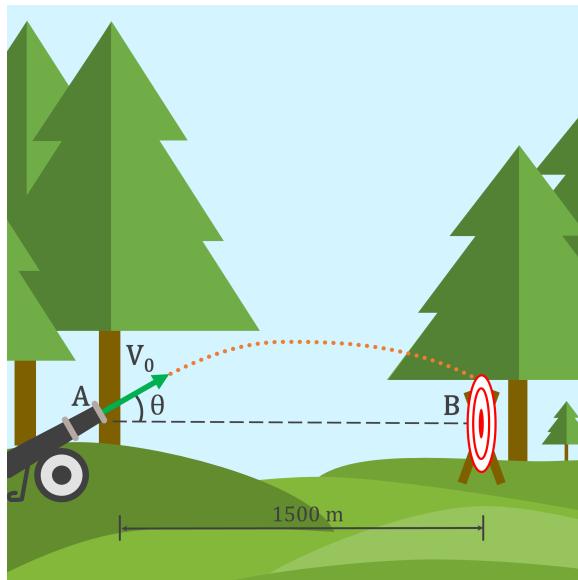


Bullet on Target



A cannon fires a bullet from A toward a target B. Find an expression for the time t_{end} it takes for the bullet to reach the target, in terms of d , v_0 and θ .

The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is $d = 1500$ m. Neglect all air resistances and assume that the bullet is directed along the vertical centreline of the target. Take $g = 10$ m/s²

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = adt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = vdt = (v_0 + at)dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (v_0 + at) dt \quad (5)$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \quad (6)$$

Given quantities:

Distance A-B: $d = 1500$ m

Gravitational acceleration: $g = 10$ m/s²

Initial velocity: $v_0 = 900$ m/s

Target diameter: $D = 2$ m

Solution:

Filling in Equation (6) gives an relation for the x -position with respect to time. Where $a = 0$ m/s² and $x_0 = 0$ m, since there is no acceleration in the x -direction and the coordinate system is chosen at the end of the cannon barrel.

$$x(t_{\text{end}}) = v_{0,x}t_{\text{end}} = d \quad \Rightarrow \quad v_0 t_{\text{end}} \cos \theta = d \quad (7)$$

Rewriting gives a relation for the time t_{end} with respect to θ and v_0 .

$$v_0 t_{\text{end}} \cos \theta = d \quad \Rightarrow \quad t_{\text{end}} = \frac{s_{AB}}{v_0 \cos \theta} \quad (8)$$