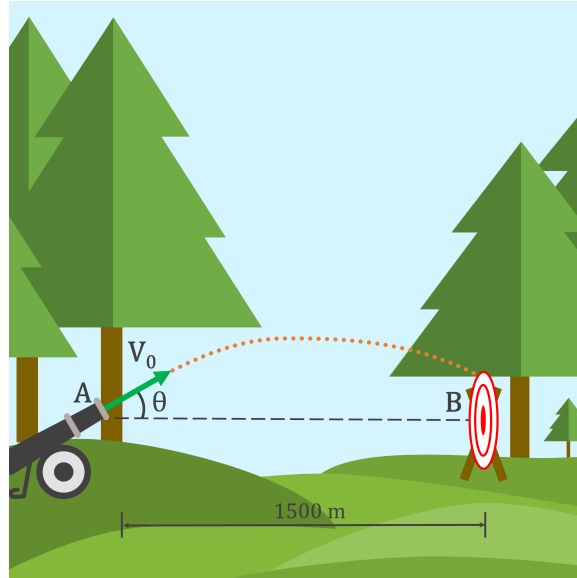


Bullet on Target Top



A cannon fires a bullet from A toward a target B. Find the equation which includes the vertical displacement $y(t)$ as a function of time, v_0 and θ , for which the bullet just hits the top of the target. The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is 1500 m. Neglect air resistances and assume that the bullet is directed along the vertical centreline of the target.

Take $g = 10 \text{ m/s}^2$.

$y(t) = \dots = \dots$

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (v_0 + at) dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (v_0 + at) dt \quad (5)$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \quad (6)$$

Given quantities:

Distance A-B: $s_{AB} = 1500$ m

Gravitational acceleration: $g = 10$ m/s²

Initial velocity: $v_0 = 900$ m/s

Target diameter: $D = 2$ m

Solution:

Filling in Equation (6) gives an relation for the y -position with respect to time. Where $a = -g$ and $y_0 = 0$ m, since the cannon barrel is at the same altitude as the target centre.

$$y(t) = -\frac{1}{2}gt^2 + v_{0,y}t + y_0 = -\frac{1}{2}gt^2 + v_0t \sin \theta \quad (7)$$

The top of the target is at 1 m from the centreline thus $y(t_{\text{end}}) = 1$ m. Furthermore, inserting $g = 10$ m/s² result in.

$$y(t) = -\frac{1}{2} \cdot 10 \cdot t^2 + v_0t \sin \theta = 1 \quad \Rightarrow \quad -5 \cdot t^2 + v_0t \sin \theta = 1 \quad (8)$$