THERMOQUIZ

W2-4-1 Ideal gas 1

Consider an ideal gas with a temperature of $T_1 = 300$ K and a specific volume of $v_1 = 0.86$ m^3/kg. As a result of a disruption, the state of the gas changes to $T_2 = 302$ K and $v_2 = 0.87$ m^3/kg ($R = 0.287$ kJ/kgK).

Estimate the change of pressure of the ideal gas by using the total differential of a gas with a pressure of $P(T, v)$.

There is asked for the change in pressure of the gas due to the change in temperature and the volume. For this, it is convenient to use the total differential of the pressure P as a function of T and v:

$$\underset{\cdot}{\mathrm{P}} = \left(\frac{\partial P}{\partial T}\right)_v \underset{\cdot}{\mathrm{T}} + \left(\frac{\partial P}{\partial v}\right)_T \underset{\cdot}{\mathrm{v}}$$

Actually, the total differential is only valid for infinitesimal small changes of T and v. However, it can also be used with an acceptable accuracy for small changes. Since the changes in temperature and volume are small, it can be stated that $\underset{\cdot}{\mathrm{T}} \cong \Delta T = T_2 - T_1 = (302 - 300)$ K $=$ 2 K and $\underset{\cdot}{\mathrm{v}} \cong \Delta v = v_2 - v_1 = (0.87 - 0.86)$ m^3/kg $=$ 0.01 m^3/kg. The partial differentials follow from the ideal gas law, $Pv = RT$, where R is a constant. Then it follows for the total differential:

$$\underset{\cdot}{\mathrm{P}} = \left(\frac{R}{v}\right) \underset{\cdot}{\mathrm{T}} - \left(\frac{RT}{v^2}\right) \underset{\cdot}{\mathrm{v}} = R\left(\frac{1}{v}\right)\ dT - R\left(\frac{T}{v^2}\right) \underset{\cdot}{\mathrm{v}}$$

Filling in the numbers, with for v the average volume and T the average temperature, gives:

$$\underset{\cdot}{\mathrm{P}} = 0.287\left(\frac{2}{0.865} - \frac{301 \cdot 0.01}{0.865^2}\right) = 0.664 - 1.155 = -0.491 \text{ kPa}$$

Hence, due to the disturbance, the pressure reduces with 0.491 kPa.