University of Twente
Faculty of Engineering Technology (ET)
Research group Thermal Engineering (TE)

Exam Energy & Heat Transfer (E&HT)

07 November 2022, 08:45 - 11:45

- Do not forget to write your name and student number on the provided answer sheets.
- This exam consists of 5 multiple-choice questions and 2 open questions.
- A total of 100 points can be earned:
- Read each question carefully. If you think you made a mistake in your calculations, please explain why you think it is wrong.
- Please start every new question on a new page.
- The use of a calculator, the lecture slides, your notes, and the books '*Heat and Mass Transfer: Fundamental & Application*' and '*Introduction to Heat Transfer*' are allowed.
- On the last page, a table can be found with air properties at a wide variety of temperatures.

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Industrial Design Engineering
202000198 Energy & Heat Transfer

UNIVERSITY OF TWENTE.

Approach

The approach below gives a guideline in how to solve the problems presented during this course. Correctly applying this approach will lead to a good understanding of the concepts presented in this course.

Analysis

1. Explain the problem: which physical phenomena are important in this problem?
2. Make a sketch of the problem
3. Give the known variables (with the appropriate units!)

Approach

1. Explain the assumptions you make to solve the problem
2. Show the solution method for solving the problem

Elaboration

1. Show the calculation steps and explain the equations you use
2. Give references if values are found online or in tables

Evaluation

1. Check the units of your solution
2. Is the answer realistic/expected?
3. Did you answer all the questions asked?
4. Iterate if this is required

Multiple choice questions (10 points)

Please **do not** circle your answers on this page. Instead, write your answers on the exam paper.

Question 1 (2 points)

Which of the following scenarios describes natural convection?

1) A cooked egg cooling down in normal air.
2) A cold can that is warming up over time.
3) A HVAC system in a train compartment.
4) An operating steam turbine.
5) A fan cooling a room.
6) A melting iceberg

Pick the correct numbers:

a) 1,2
b) 1,3,5
c) 1,2,6
d) 2,4,6
e) 3,5,6
f) 1,2,3
g) 1,2,4,6
h) 2,4
i) 3,4,6
j) 3,6

Question 2 (2 points)

A temperature profile as in Figure 1.1 is given.

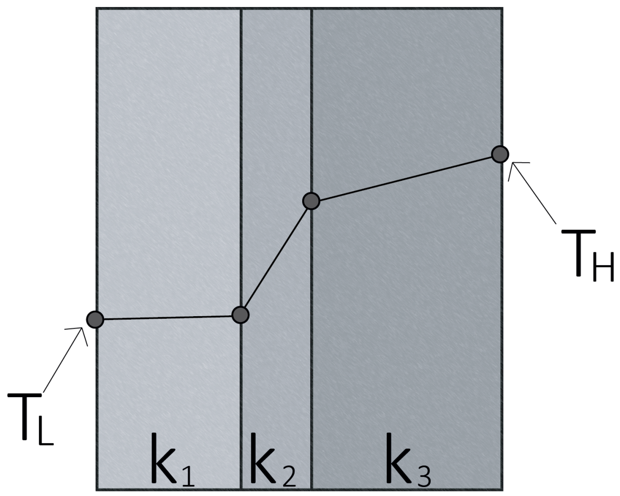


Figure 1.1: Temperature profile inside a multilayer plane wall

Select the correct statements with respect to the thermal conductivities.

a) $k_1 < k_2 < k_3$
b) $k_2 < k_1 < k_3$
c) $k_3 < k_1 < k_2$
d) $k_1 < k_3 < k_2$
e) $k_2 < k_3 < k_1$
f) $k_3 < k_2 < k_1$

Question 3 (2 points)

Three bodies with an identical shapes (but not the same size) are heating up. They are made of the same material and subjected to identical conditions. The development of their temperature over the course of time can be seen in Figure 1.2. the smallest?

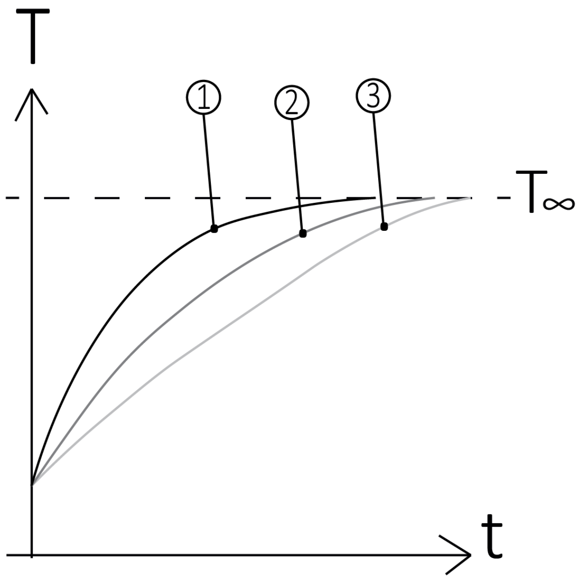


Figure 1.2: Temperature profile of three objects over time

For which of the sketched temperature profiles is the Biot number the smallest?

a) Temperature profile 1

b) Temperature profile 2

c) Temperature profile 3

Question 4 (2 points)

An optic made of a special diffused glass has the following spectral radiation properties (τ and ρ) as sketched in Figure 1.3. Select the correct course of the spectral absorption coefficient (α) of the glass from the selection below.

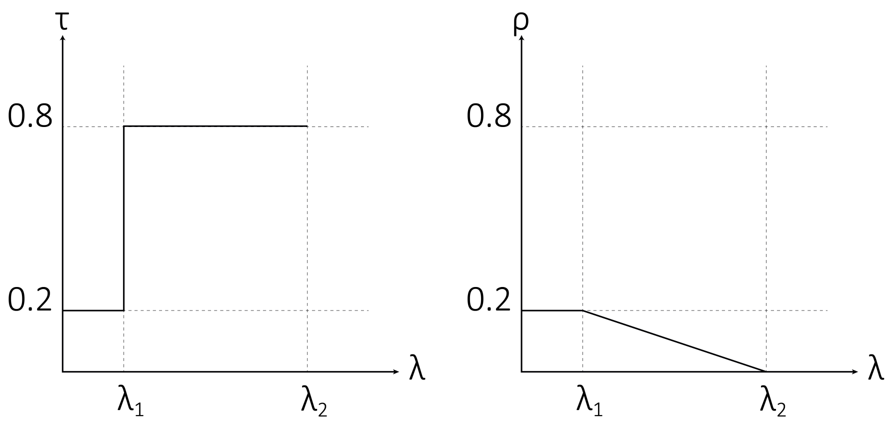
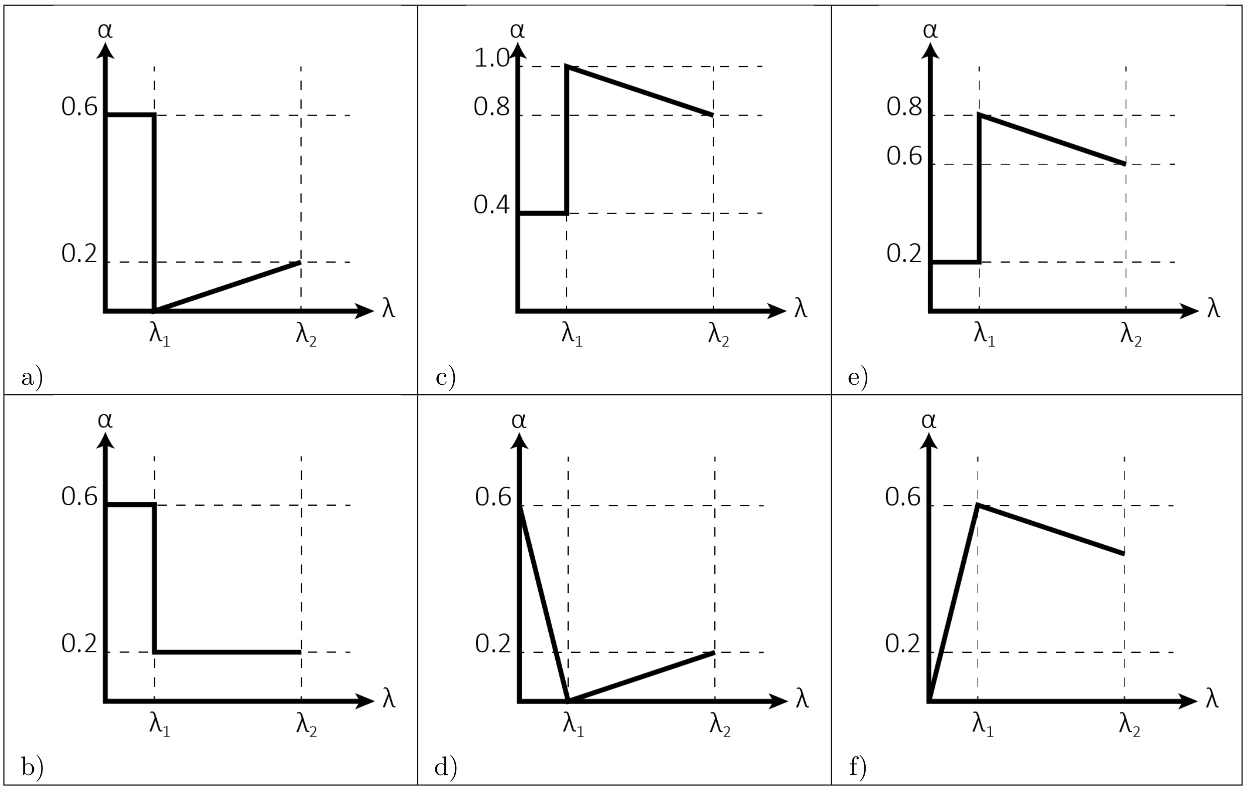


Figure 1.3: Spectral properties τ and ρ as a function λ



Question 5 (2 points)

Read the following statements.

1. When it comes to natural convection, the difference in density of a fluid is the driving force for convectional heat transfer.
2. When it comes to natural convection forced fluid flow results in convectional heat transfer.
3. When it comes to natural convection, the driving force is the temperature difference within a solid object.
4. When a fluid flows over a flat plate with a relatively low velocity, it is possible for the phenomena of natural convection to occur.

Pick the statement(s) that is/are true.

a) 1	d) 2	g) 3	i) 4
b) 1,2	e) 1,3	h) 1,4	j) 2,3
c) 2,4	f) 3,4		

Open question 1 (45 points)

On a hot summer day, the food truck designed by UT IDE students can be found near the sport center. The outside temperature is $T_i = 35\,°\mathrm{C}$, there is no wind, and the solar irradiation is $G = 1100\ \mathrm{W\,m^{-2}}$. In the initial design, as shown in Figure 2.1 (a), the roof of a food truck compartment is made of composite construction, consisting of a layer of insulation ($t_2 = 60$ mm, $k_2 = 0.036\ \mathrm{W\,m^{-1}\,K}$) sandwiched between a roof steel panel ($t_1 = 4$ mm, $k_1 = 130\ \mathrm{W\,m^{-1}\,K}$) with reflectivity of 0.9 and an inside plastic panel ($t_3 = 5$ mm, $k3 = 0.26$ $\mathrm{W\,m^{-1}\,K}$). The roof's length and width are $L = 3$ m and $W = 2$ m, respectively, and the inner plastic surface temperature is the same as the inside temperature $T_{s,in} = 22°\mathrm{C}$. Due to the use of special roof paint, radiation losses from the roof are negligible.

When writing down your answers, please keep in mind the following:

- Clearly state the assumptions you make.
- Clearly mention the values and their units.
- Clearly mention the relations you use and why you are using them.

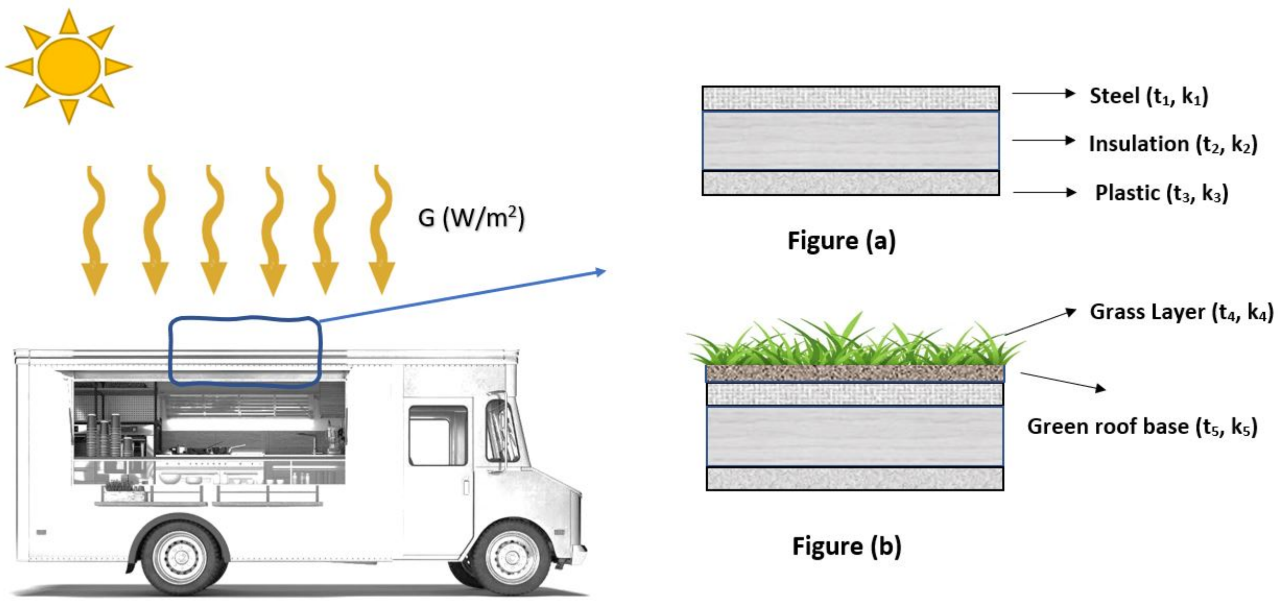


Figure 2.1: Cross-section of the food truck roof.

a) Determine the temperature of the roof surface as well as the temperatures of each layer, and then draw a temperature profile within the roof structure as well as outside air.

b) One of the group members suggested using a green roof for the food truck, as shown in Figure 2.1 (b), to save energy for cooling by adding grass layer ($t_4 = 10$mm, $k_4 = 0.038\ \mathrm{W\,m^{-1}\,K}$) with reflectivity of 0.95 and green roof base layer ($t_5 = 15$mm, $k_5 = 0.36\ \mathrm{W\,m^{-1}\,K}$). Determine the temperature of the roof surface as well as the temperatures of each layer, and then draw a temperature profile within the roof structure as well as outside air.

c) The heat generated by the cooking and other appliances inside the food truck is $45\ \mathrm{kJ\,min^{-1}}$. On this day, an AC is used to cool down the inside and keep the temperature constant, reaching equilibrium for 8 hours using electricity at a cost of 0.7 €/kW h. How much money would be saved by using the green roof for the food truck on this particular day? The losses from the truck's sides and bottom are insignificant.

Open question 2 (45 points)

Plastic parts are formed in an injection mold and dropped (flat) onto a conveyor belt, see figure 3.1. The parts are disk-shaped with a thickness of t_h =2.0 mm and a diameter of D = 5 cm. The plastic has a thermal conductivity of k = 0.35 W/mK, a density of ρ = 1100 kg/m^3 and a specific heat capacity of c = 1900 J/kgK. The side of the part that faces the conveyor belt is adiabatic and the conveyor velocity is 1m/min. The top surface of the part is exposed to air at T_∞ = 20 °C with a convective heat transfer coefficient h_{conv} = 10 W/m^{2}K. The temperature of the part immediately after it is formed is T_{ini} = 180 °C. The part must be cooled to T_{max} before it can be stacked and packaged. The packaging system is positioned L = 15 m away from the molding machine.

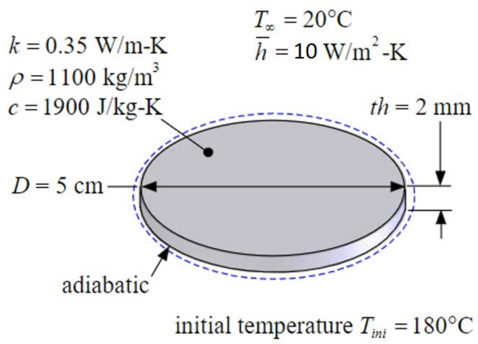


Figure 3.1: Hot plastic disk

a) What would be the maximum temperature of the parts arriving at the packaging station?

b) One of the engineers realized that in calculations for maximum temperature they should have taken in account the radiation losses from the plastic parts as well. He suggested to use average heat transfer coefficient: $h_{avg} = h_{conv} + h_{rad}$ while the h_{rad} is calculated at T_{avg} = 50 °C and parts treated as black body. What would be the maximum temperature of the parts arriving at the packaging station in this situation?

c) Another engineer suggested using forced air to speed up the cooling process and shorten the conveyer belt. He recalculated the convective heat transfer coefficient to be h_{conv} = 20 W/m^{2}K for this case. What would be the length of the conveyer belt with the same velocity to have T_{max} = 30 °C? Neglect the effects of radiation.

Tables

Below, the properties of air at atmospheric pressure have been listed for different temperatures.

Properties of air at 1 atm pressure

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m^2/s	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
−150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
−100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-5}	5.837×10^{-6}	0.7263
−50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
−40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
−30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
−20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
−10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946
300	0.6158	1044	0.04418	6.871×10^{-5}	2.934×10^{-5}	4.765×10^{-5}	0.6935

Multiple choice questions (10 points)

Question 1 (2 points)

Which of the following scenarios describes natural convection?

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Pick the correct numbers:

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d) 2,4,6
e) 3,5,6
f) 1,2,3
g) 1,2,4,6
h) 2,4
i) 3,4,6
j) 3,6

Question 2 (2 points)

A temperature profile as in Figure 1 is given.

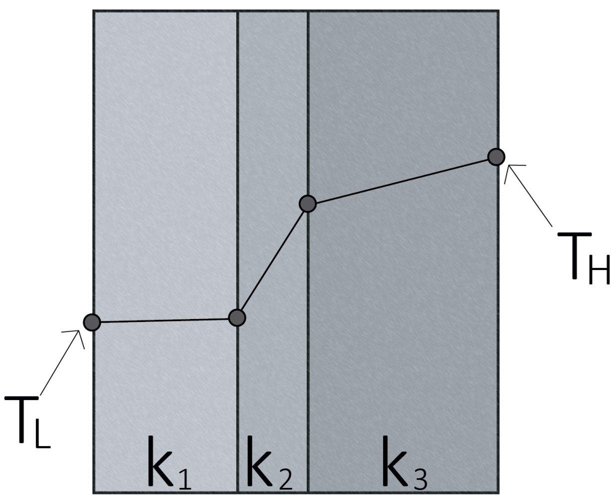


Figure 1: Temperature profile inside a multilayer plane wall

Select the correct statements with respect to the thermal conductivities.

a) $k_1 < k_2 < k_3$
b) $k_2 < k_1 < k_3$
c) $k_3 < k_1 < k_2$
d) $k_1 < k_3 < k_2$
e) $k_2 < k_3 < k_1$
f) $k_3 < k_2 < k_1$

Question 3 (2 points)

Three bodies with an identical shapes (but not the same size) are heating up. They are made of the same material and subjected to identical conditions. The development of their temperature over the course of time can be seen in Figure 2. the smallest?

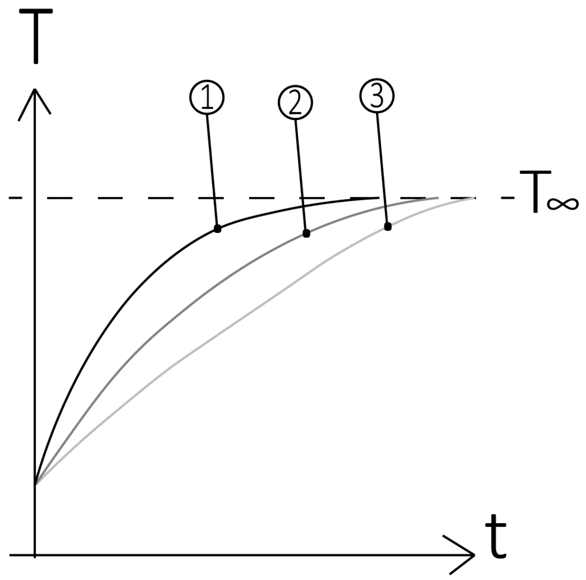


Figure 2: Temperature profile of three objects over time

For which of the sketched temperature profiles is the Biot number the smallest?

a) Temperature profile 1

b) Temperature profile 2

c) Temperature profile 3

Question 4 (2 points)

An optic made of a special diffused glass has the following spectral radiation properties (τ and ρ) as sketched in Figure 3. Select the correct course of the spectral absorption coefficient (α) of the glass from the selection below.

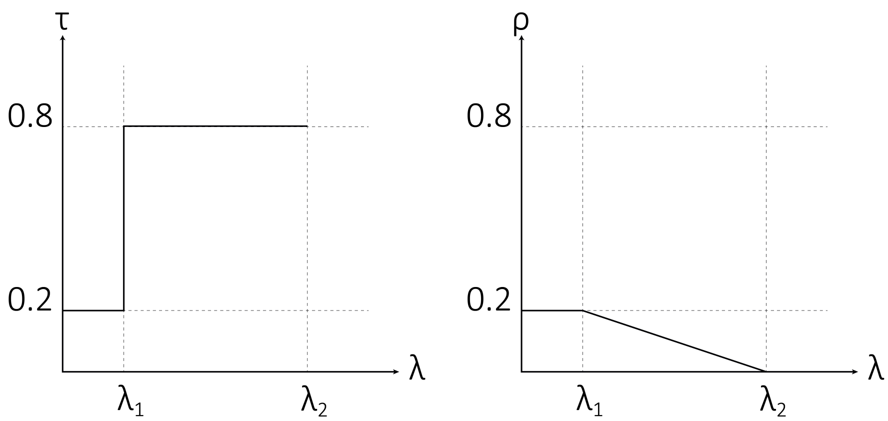
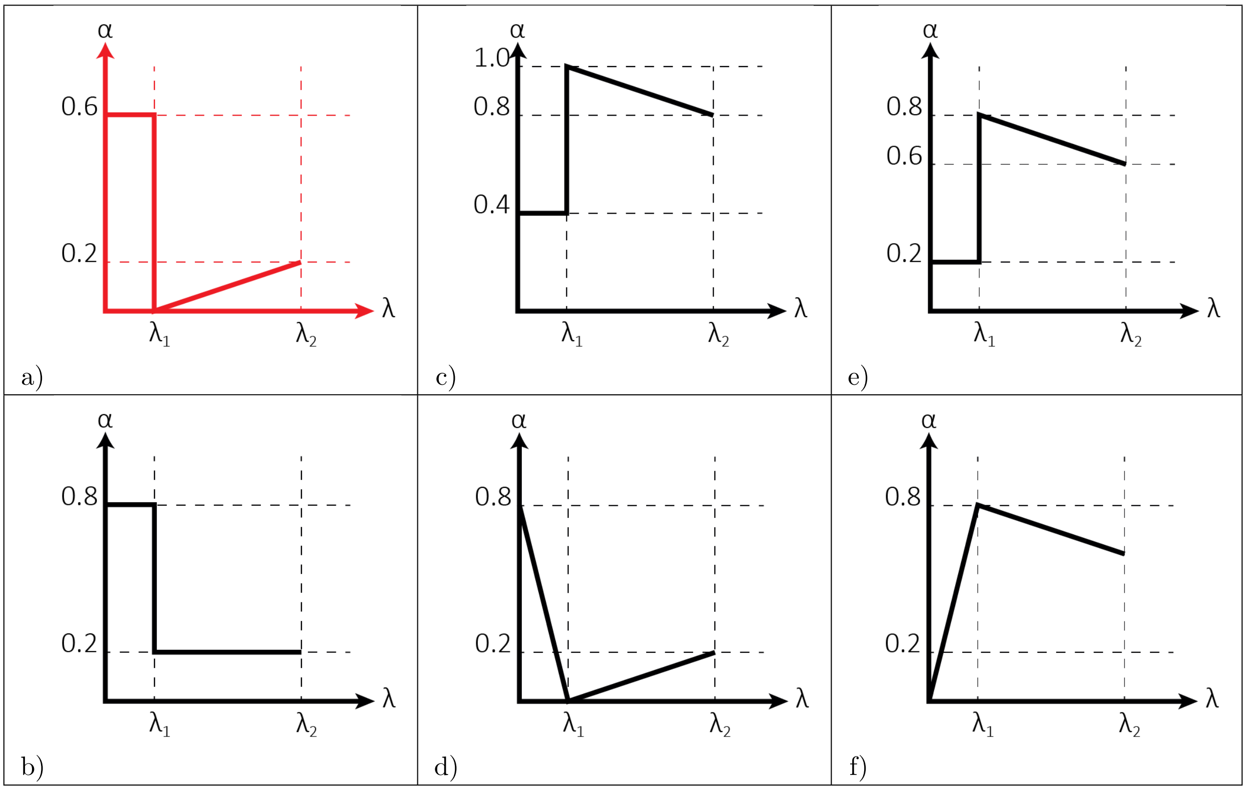


Figure 3: Spectral properties τ and ρ as a function λ



Question 5 (2 points)

Read the following statements.

1. When it comes to natural convection, the difference in density of a fluid is the driving force for convectional heat transfer.
2. When it comes to natural convection forced fluid flow results in convectional heat transfer.
3. When it comes to natural convection, the driving force is the temperature difference within a solid object.
4. When a fluid flows over a flat plate with a relatively low velocity, it is possible for the phenomena of natural convection to occur.

Pick the statement(s) that is/are true.

a) 1	d) 2	g) 3	i) 4
b) 1,2	e) 1,3	h) 1,4	j) 2,3
c) 2,4	f) 3,4		

Open question 1 (45 points)

a) [20 points in total]

The energy balance revolves around heat gained from solar irradiation, and heat lost from natural convection and conduction losses:

$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 0$$

The solar irradiance $\dot{Q}_{irr}$ is equal to:

$$\dot{Q}_{\text{irr}} = \alpha G A = (1 - 0.9) \cdot 1100 \cdot (3 \cdot 2) = 660\text{W}$$

The losses as a result of natural convection can be determined by first considering an assumed roof temperature. The actual roof temperature will be close to $T_s = 55$ °C. Relevant air properties at $T_f = 45$ °C from the provided table:

- $\nu = 1.750 \cdot 10^{-5}\ \frac{\text{m}}{\text{s}^2}$
- $\text{Pr} = 0.7241$
- $k = 0.02699\ \frac{\text{W}}{\text{m}\cdot\text{K}}$
- $\beta = \frac{2}{T_s + T_\infty} = \frac{2}{(55+35)+273.15} = 0.0031\ \text{K}^{-1}$

$$\begin{aligned} L_c &= \frac{A}{2L + 2W} = \frac{2 \cdot 3}{2 \cdot 3 + 2 \cdot 2} = 0.6\ \text{m} \\ \text{Gr} &= \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{9.81 \cdot 0.0031(55 - 35)0.6^3}{(1.750 \cdot 10^{-5})^2} = 4.3496 \cdot 10^8 \\ \text{Ra} &= \text{Gr} \cdot \text{Pr} = 4.3496 \cdot 10^8 \cdot 0.7241 = 3.1495 \cdot 10^8 \end{aligned}$$

At this Rayleigh number, the following Nusselt correlation should be used:

$$\begin{aligned} \text{Nu} &= 0.15\text{Ra}^{\frac{1}{3}} = 0.15 \cdot (3.1495 \cdot 10^8)^{\frac{1}{3}} = 102.06 \\ h &= \frac{k_{air}\text{Nu}}{L_c \cdot} = \frac{0.02699}{0.6 \cdot 102.06} = 4.59\ \text{W/m}^2\text{K} \\ \dot{Q}_{\text{conv}} &= hA(T_s - T_\infty) = 4.59 \cdot 6 \cdot (55 - 35) = 550.9\ \text{W} \end{aligned}$$

Finally, $\dot{Q}_{\text{cond}}$ can be determined by considering the sum of all thermal resistances the roof represents:

$$\begin{aligned} R_1 &= \frac{t_1}{k_1 \cdot A_c} = \frac{0.004}{130 \cdot (2 \cdot 3)} = 5.13 \cdot 10^{-6}\ \text{K/W} \\ R_2 &= \frac{t_2}{k_2 \cdot A_c} = \frac{0.06}{0.036 \cdot (2 \cdot 3)} = 2.78 \cdot 10^{-1}\ \text{K/W} \\ R_3 &= \frac{t_3}{k_3 \cdot A_c} = \frac{0.005}{0.26 \cdot (2 \cdot 3)} = 3.21 \cdot 10^{-3}\ \text{K/W} \end{aligned}$$

$$R_{\text{total}} = R_1 + R_2 + R_3 = 5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3} = 2.81 \cdot 10^{-1}\ \text{K/W}$$

$$\dot{Q}_{\text{cond}} = \frac{\Delta T}{R_{\text{total}}} = \frac{55 - 22}{2.81 \cdot 10^{-1}} = 117.4\ \text{W}$$

To validate whether the made assumption was correct, we should check the energy balance.

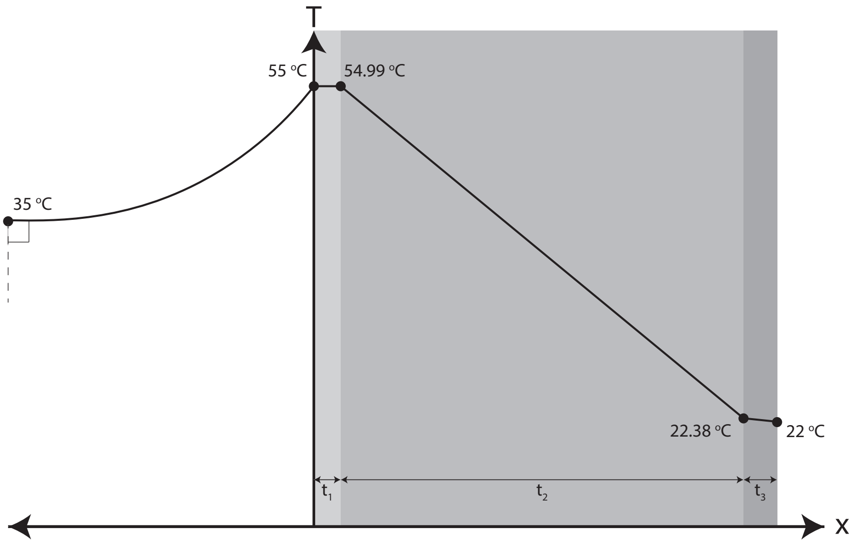
$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 660 - 550.9 - 117.4 = -8.3\ \text{W}$$

A small deviation from the balance of actual being in equilibrium is found ($\approx$ 1.2%), which is acceptable in this case. Furthermore, the temperatures in each layer can be found by use of the thermal resistance theorem, which results in:

$$T_{\text{plas-insul}} = R_3 \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = 3.21 \cdot 10^{-3} \cdot 117.4 + 22 = 22.38\ °\text{C}$$

$$T_{\text{insul-steel}} = (R_2 + R_3) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = \left(2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3}\right) \cdot 117.4 + 22 = 54.99\ °\text{C}$$

Where the temperature profile can be drawn as:



The following criteria should be clearly specified in the graph:

- $x \to -\infty$ the slope should approach zero.
- The temperature drop in the outside air should be curved with a decreasing slope when moving in the negative x-direction.
- The temperature drops in each solid layer should be straight.
- The slope of layer 2 should be the steepest and the slope of layer 1 the least.
- The temperatures should be correct.

b) [15 points in total]

The energy balance revolves around heat gained from solar irradiation, and heat lost from natural convection and conduction losses:

$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 0$$

The solar irradiance $\dot{Q}_{irr}$ is equal to:

$$\dot{Q}_{\text{irr}} = \alpha G A = (1 - 0.95) \cdot 1100 \cdot (3 \cdot 2) = 330\text{W}$$

The losses as a result of natural convection can be determined by first considering an assumed roof temperature. The actual roof temperature will be close to $T_s = 46$ °C. Relevant air properties at $T_f = 40$ °C from the provided table:

- $\nu = 1.702 \cdot 10^{-5}\ \frac{\text{m}}{\text{s}^2}$
- $\text{Pr} = 0.7255$
- $k = 0.02662\ \frac{\text{W}}{\text{m}\cdot\text{K}}$
- $\beta = \frac{2}{T_s + T_\infty} = \frac{2}{(55+273.15)+(35+273.15)} = 0.0031\ \text{K}^{-1}$

$$\begin{aligned} L_{\text{c}} &= \frac{A}{2L + 2W} = \frac{2 \cdot 3}{2 \cdot 3 + 2 \cdot 2} = 0.6\ \text{m} \\ \text{Gr} &= \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{9.81 \cdot 0.0032(46-35)0.6^3}{(1.702 \cdot 10^{-5})^2} = 2.5654 \cdot 10^8 \\ \text{Ra} &= \text{Gr} \cdot \text{Pr} = 2.5654 \cdot 10^8 \cdot 0.7255 = 1.8612 \cdot 10^8 \end{aligned}$$

At this Rayleigh number, the following Nusselt correlation should be used:

$$\begin{aligned} \text{Nu} &= 0.15\text{Ra}^{\frac{1}{3}} = 0.15 \cdot (1.8612 \cdot 10^8)^{\frac{1}{3}} = 85.64 \\ h &= \frac{k_{air}\text{Nu}}{L_{\text{c}}\cdot} = \frac{0.02662}{0.6 \cdot 85.64} = 3.80\ \text{W/m}^2\text{K} \\ \dot{Q}_{\text{conv}} &= hA(T_s - T_\infty) = 3.80 \cdot 6 \cdot (46 - 35) = 250.8\ \text{W} \end{aligned}$$

Finally, $\dot{Q}_{\text{cond}}$ can be determined by considering the sum of all thermal resistances the roof represents:

$$\begin{aligned} R_4 &= \frac{t_4}{k_4 \cdot A_{\text{c}}} = \frac{0.01}{0.038 \cdot (2 \cdot 3)} = 4.39 \cdot 10^{-2}\ \text{K/W} \\ R_5 &= \frac{t_5}{k_5 \cdot A_{\text{c}}} = \frac{0.015}{0.36 \cdot (2 \cdot 3)} = 6.94 \cdot 10^{-3}\ \text{K/W} \end{aligned}$$

$$\begin{aligned} R_{\text{total}} &= R_1 + R_2 + R_3 + R_4 + R_5 \\ &= 5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3} + 4.39 \cdot 10^{-2} + 6.94 \cdot 10^{-3} \\ &= 3.32 \cdot 10^{-1}\ \text{K/W} \end{aligned}$$

$$\dot{Q}_{\text{cond}} = \frac{\Delta T}{R_{\text{total}}} = \frac{46 - 22}{3.32 \cdot 10^{-1}} = 72.3\ \text{W}$$

To validate whether the made assumption was correct, we should check the energy balance.

$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 330 - 250.8 - 72.3 = 6.9\ \text{W}$$

A small deviation from the balance of actual being in equilibrium is found ($\approx 2.1\%$), which is acceptable in this case. Furthermore, the temperatures in each layer can be found by use of the thermal resistance theorem, which results in:

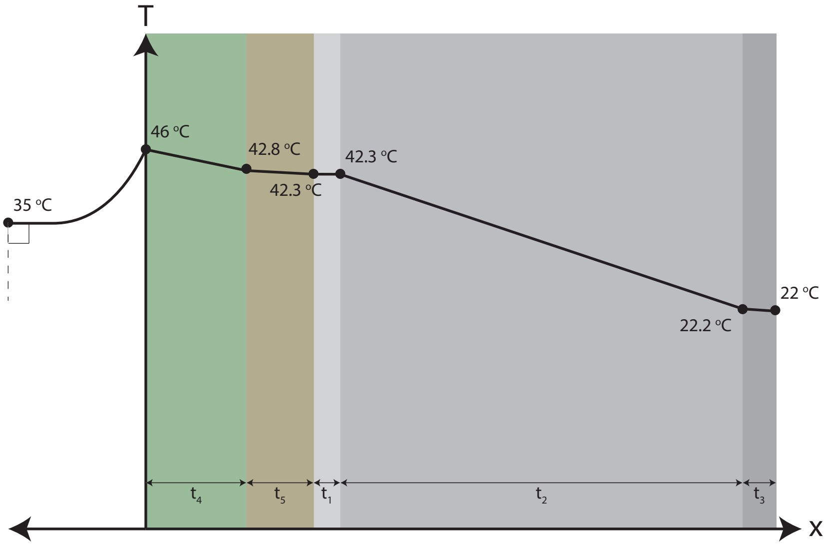
$$T_{\text{plas-insul}} = R_3 \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = 3.21 \cdot 10^{-3} \cdot 72.3 + 22 = 22.23\ °\text{C}$$

$$T_{\text{insul-steel}} = (R_2 + R_3) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = \left(2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3}\right) \cdot 72.3 + 22 = 42.32 \text{ °C}$$

$$T_{\text{steel-base}} = (R_1 + R_2 + R_3) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = \left(5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3}\right) \cdot 72.3 + 22 = 42.32 \text{ °C}$$

$$\begin{aligned} T_{\text{base-grass}} &= (R_1 + R_2 + R_3 + R_5) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} \\ &= \left(5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3} + 6.94 \cdot 10^{-3}\right) \cdot 72.3 + 22 = 42.82 \text{ °C} \end{aligned}$$

Where the temperature profile can be drawn as:



- $x \to -\infty$ the slope should approach zero.
- The temperature drop in the outside air should be curved with a decreasing slope when moving in the negative x-direction.
- The temperature drops in each solid layer should be straight.
- The slopes should be correct, sequence of largest slope to smallest is: 2-4-3-5-1
- The temperatures should be correct.

c) [10 points in total]

Scenario where no green roof is there:

$$\dot{Q}_{\mathrm{cond},1} = 117\ \mathrm{W}$$

$$\dot{Q}_{\mathrm{heat},1} = 45\ \mathrm{kJ/min} = 750\ \mathrm{W}$$

Total heat that should be extracted by use of the HVAC system in 8 hours of time:

$$E_1 = \left(\dot{Q}_{\mathrm{cond},1} + \dot{Q}_{\mathrm{heat},1}\right) \cdot \Delta t = (117 + 750) \cdot 8 = 6.94\ \mathrm{kWh}$$

In terms of price, this operation will cost:

$$\mathrm{Price}_1 = E_1 \cdot €_{\mathrm{kWh}} = 6.94 \cdot 0.7\ = €\ 4.86$$

Scenario where a green roof is there:

$$\dot{Q}_{\mathrm{cond},2} = 72\ \mathrm{W}$$

$$\dot{Q}_{\mathrm{heat},2} = 45\ \mathrm{kJ/min} = 750\ \mathrm{W}$$

Total heat that should be extracted by use of the AC system in 8 hours of time:

$$E_2 = \left(\dot{Q}_{\mathrm{cond},2} + \dot{Q}_{\mathrm{heat},2}\right) \cdot \Delta t = (72 + 750) \cdot 8 = 6.58\ \mathrm{kWh}$$

In terms of price, this operation will cost:

$$\mathrm{Price}_2 = E_2 \cdot €_{\mathrm{kWh}} = 6.58 \cdot 0.7\ = €\ 4.61$$

Savings at the specified day:

$$\mathrm{Savings} = \mathrm{Price}_1 - \mathrm{Price}_2 = 4.86 - 4.61 = €\ 0.25$$

Open question 2 (45 points)

a.) [20 points in total] What would be the maximum temperature of the parts arriving at the packaging station?

First the Biot number is calculated to check whether or not the lumped system assumption is valid:

$$L_c = \frac{V}{A} = \frac{\frac{1}{4}\pi D^2 t}{\frac{1}{4}\pi D^2} = t = 0.002\text{m} \tag{2.1}$$

$$Bi = h\frac{L_c}{k} = 10\frac{0.002}{0.35} = 0.0571 < 0.1 \tag{2.2}$$

So the lumped system assumption is valid.
The coefficient b is equal to:

$$b = \frac{hA}{\rho * V * c_p} = \frac{10 \cdot 0.002}{1100 \cdot 3.9270 \cdot 10^{-6} \cdot 1900} = 0.0024 \tag{2.3}$$

The time the parts spend on the conveyor belt can be calculated using the conveyor belt length and velocity:

$$n = \frac{L_{conv}}{V_{conv}} = \frac{15}{0.0167} = 900\text{s} \tag{2.4}$$

Then finally rewriting and substitution of the parameters gives:

$$\frac{T(n) - T_\infty}{T_{initial} - T_\infty} = e^{-bn} \tag{2.5}$$

$$T(900) = (T_{initial} - T_\infty)\, e^{-bn} + T_\infty = (180 - 20)\, e^{-0.0024 \cdot 900} + 20 = 38.58°\text{C} \tag{2.6}$$

b.) [20 points in total] One of the engineers realized that in calculations for maximum temperature they should have taken in account the radiation losses from the plastic parts as well. He suggested to use average heat transfer coefficient: $h_{avg} = h_{conv} + h_{rad}$ while the h_{rad} is calculated at $T_{avg} = 50$ °C and parts treated as black body. What would be the maximum temperature of the parts arriving at the packaging station in this situation?

This question is very similar to question a.), the only difference being the use of an average or total heat transfer coefficient.
The radiative part of the total heat transfer coefficient can be calculated using:

$$h_rad = \epsilon\sigma(T_{avg}^2 + T_\infty^2)(T_{avg} + T_\infty) = 1 \cdot 5.67 \cdot 10^{-8}(2900)(70) = 6.6521 \tag{2.7}$$

Which gives for the total heat transfer coefficient:

$$h_{avg} = h_{rad} + h_{conv} = 10 + 6.6521 = 16.6521 \tag{2.8}$$

The Biot number needs to be calculated again to check whether or not the lumped system assumption is valid even for this higher heat transfer coefficient:

$$L_c = \frac{V}{A} = \frac{\frac{1}{4}\pi D^2 t}{\frac{1}{4}\pi D^2} = t = 0.002\text{m} \tag{2.9}$$

$$Bi = h_{avg}\frac{L_c}{k} = 16.6521\frac{0.002}{0.35} = 0.0952 < 0.1 \tag{2.10}$$

So the lumped system assumption is valid.
The coefficient b is equal to:

$$b = \frac{h_{avg}A}{\rho * V * c_p} = \frac{16.6521 \cdot 0.002}{1100 \cdot 3.9270 \cdot 10^{-6} \cdot 1900} = 0.0040 \tag{2.11}$$

The time the parts spend on the conveyor belt can be calculated using the conveyor belt length and velocity:

$$n = \frac{L_{conv}}{V_{conv}} = \frac{15}{0.0167} = 900\text{s} \tag{2.12}$$

Then finally rewriting and substitution of the parameters gives:

$$\frac{T(n) - T_\infty}{T_{initial} - T_\infty} = e^{-bn} \tag{2.13}$$

$$T(900) = (T_{initial} - T_\infty)\, e^{-bn} + T_\infty = (180 - 20)\, e^{-0.0040 \cdot 900} + 20 = 24.44°\text{C} \tag{2.14}$$

c.) [5 points in total] Another engineer suggested using forced air to speed up the cooling process and shorten the conveyor belt. He recalculated the convective heat transfer coefficient to be h_{conv} = 20 W/m^{2}K for this case. What would be the length of the conveyor belt with the same velocity to have T_{max} = 30 °C? Neglect the effects of radiation.

$h = 20$ gives:

$$Bi = \frac{hL_c}{k} = \frac{20 \cdot 0.002}{0.35} = 0.1143 > 0.1$$

The Biot number is larger than 0.1, which means the lumped element assumption does not hold.
In the case where the lumped element assumption is used as an approximation, the conveyor belt length will be:

$$b = \frac{hA}{\rho V c_p} = \frac{20 \cdot 0.002}{1100 \cdot 3.927 \cdot 10^{-6} \cdot 1900} = 0.0048$$

$$L_{required} = -\frac{v_{conv}}{b} \log\left(\frac{T_{final} - T_{inf}}{T_{int} - T_{inf}}\right) = -\frac{0.0167}{0.0048} \log\left(\frac{30 - 20}{180 - 20}\right) = 9.6579m$$

Deduction of points

Standard: Missing/wrong units: -0.5pt per time, -3 pt. total Missing/wrong assumptions: -0.5pt per time, -3 pt. total Missing/wrong conclusion/discussion: -0.5pt per time, -3 pt. total Missing/wrong description: -0.5pt per time, -3 pt. total

University of Twente
Faculty of Engineering Technology (ET)
Research group Thermal Engineering (TE)

Exam Energy & Heat Transfer (E&HT)

29 October 2021, 08:45 - 11:45

- Do not forget to write your name and student number on the provided answer sheets.
- This exam consists of 5 multiple choice questions and 2 open questions.
- A total of 100 points can be earned:
- Read each question carefully. If you think you made a mistake in your calculations, please provide an explanation why you think it is wrong.
- Please start every new question on a new page.
- The use of a calculator, the lecture slides, your notes and the books '*Heat and Mass Transfer: Fundamental & Application*' and '*Introduction to Heat Transfer*' are allowed.
- On the last page, a table can be found with air properties at a wide variety of temperatures.

Lecturer: dr. M. Mehrali

Industrial Design Engineering
202000198 Energy & Heat Transfer

UNIVERSITY OF TWENTE.

Approach

The approach below gives a guideline in how to solve the problems presented during this course. Correctly applying this approach will lead to a good understanding of the concepts presented in this course.

Analysis

1. Explain the problem: which physical phenomena are important in this problem?
2. Make a sketch of the problem
3. Give the known variables (with the appropriate units!)

Approach

1. Explain the assumptions you make to solve the problem
2. Show the solution method for solving the problem

Elaboration

1. Show the calculation steps and explain the equations you use
2. Give references if values are found online or in tables

Evaluation

1. Check the units of your solution
2. Is the answer realistic/expected?
3. Did you answer all the questions asked?
4. Iterate if this is required

Multiple choice questions (10 points)

Please **do not** circle your answers on this page. Instead, write your answers on the exam paper.

Question 1 (2 points)

Which of the following would lead to a reduction in thermal resistance?

a) In conduction, reduction in the thickness of the material and an increase in thermal conductivity.

b) In convection, stirring of the fluid and cleaning the heating surface.

c) In radiation, increasing the temperature and reducing the emissivity.

d) Only a) and b) are true.

e) Only a) and c) are true.

f) Only b) and c) are true.

g) Statements a), b), and c) are true.

Question 2 (2 points)

A steam pipe is to be lined with two layers of insulating materials of different thermal conductivities. The change in surface area is negligible. For the minimum heat transfer,

a) The better insulator must be put on the inside.

b) The better insulator must be put on the outside.

c) One could place either insulation on either side.

d) One should take into account the steam temperature before deciding as to which material is put where.

Question 3 (2 points)

What happens to the insulation ability of an insulation layer of a wall when moisture is present?

a) The insulation ability would increase.

b) The insulation ability would decrease.

c) The insulation ability would remain unaffected.

d) The insulation ability may increase/decrease depending on the temperature and thickness of the insulation.

Question 4 (2 points)

Thermal conductivity of a material may be defined as:

a) The quantity of heat flowing through a cube of material when opposite faces are maintained at a temperature difference of 1 K.

b) Quantity of heat flowing through a slab of the material of an area of one meter squared, with a thickness of 1 meter when it faces a difference in temperature of 1 K.

c) Heat conducted across an unit area through unit thickness when a temperature difference of unity is maintained between opposite faces.

d) All of the above.

e) None of the above.

Question 5 (2 points)

Consider weather conditions for which the prevailing wind blows past the penthouse tower on a tall building, which has been displayed in Figure 1.1. The flow over the first window is laminar, the flow over the last window is turbulent. Which of the following statements is true:

a) The first window has a higher heat transfer coefficient than the second window. The flow over the first window is still laminar, meaning it allows for better heat transfer as the path of flow is smooth.

b) The second window has a higher heat transfer coefficient than the first window. The flow over the second window is turbulent, meaning it allows for better heat transfer as the path of flow is chaotic.

c) The first window has a higher heat transfer coefficient than the second window. Both the boundary layer and thermal boundary layer are still small here, meaning the heat transfer coefficient is large as it is defined per unit of area.

d) The second window has a higher heat transfer coefficient than the first window. Both the boundary layer and thermal boundary layer are large here, meaning the heat transfer coefficient is large as it is defined per unit of area.

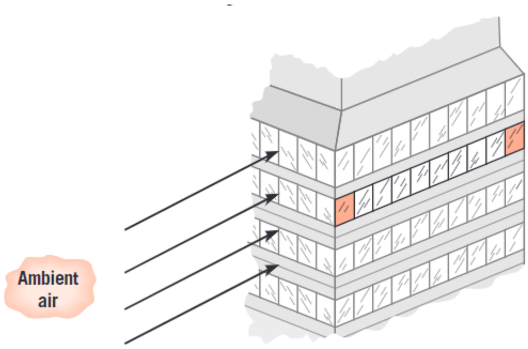


Figure 1.1: Wind blowing over windows of a penthouse

Open question 1 (45 points)

In the figure below a cross section of the wall of a house is given. The inside wall of the house has a temperature $T_i = 20$ °C. The temperature of the air outside is $T_\infty = -5$ °C. The wall is 2 m wide, and has heights $h_a = 1.50$ m, $h_b = 1$ m, and thicknesses t_a=30 mm and t_b=50 mm. The thermal conductivity of the layers of brick is $k_1 = k_4 = 0.72\ \mathrm{W\,m^{-1}\,K^{-1}}$. The thermal conductivity of the rockwool layer at 2 is $k_2 = 0.0350\ \mathrm{W\,m^{-1}\,K^{-1}}$, and that of the wooden layer at 3 is $k_3 = 0.0550\ \mathrm{W\,m^{-1}\,K^{-1}}$. We will analyse this problem on a day with no wind.

In this exercise, we want to know what the heat loss through the wall is. When writing down your answers, please keep in mind the following:

- Clearly state the assumptions you make.
- Clearly mention the values and their units.
- Clearly mention the relations you use and why you are using them.

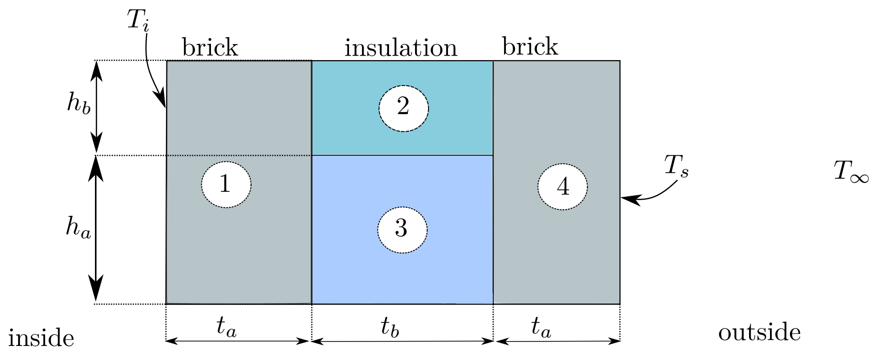


Figure 2.1: Cross-section of the wall of a house

a) Draw the resistance network of the wall and determine the formula for the total resistance R_{tot} inside of the wall.

b) Determine the total heat loss from the wall. If you need to iterate, do so until a precision of Q of 5 percent is achieved. What is the surface temperature of the wall?

c) Determine the temperatures after each layer and draw the temperature profile including the outside.

Assume the house has a total circumference of 40 meters, where the walls are made from the material above and has the exact same properties as above. With the current national gas shortage, the price of a cubic meter of gas is €0.79, and consists of 32 $\mathrm{MJ\,m^{-3}}$. During the entire month of December, the average temperature outside is -5°C.

d) How much energy does the house lose in the month December and how much does it cost to compensate this. You may assume an ideal energy conversion.

Open question 2 (45 points)

In this question, we will analyse the heat transfer of a thermocouple placed inside an oven. The goal is to calibrate a newly acquired oven using the thermocouple. The thermocouple works as follows:

> *A thermocouple is a device for measuring temperature. It consists of two dissimilar metallic wires joined together to form a junction. When the junction is heated or cooled, a small voltage is generated in the electrical circuit of the thermocouple which can be measured, and this corresponds to temperature.*

The oven is preheated at a temperature of $T_{oven} = 200$ °C. Both the air inside the oven and the sides of the oven are preheated. The inside of the oven and the thermocouple bead can be approximated as black bodies. The temperature within the oven is uniform. Once the oven has reached the designated temperature, we want to measure the temperature inside to confirm it is indeed 200 °C. Use the values from the figure below.

a) Determine the rate of heat transfer to the thermocouple bead. Make a sketch of the situation you are going to solve. Clearly state your assumptions!

Once you are done calibrating the oven, the oven is turned off and the door is opened. The air in the oven cools down immediately, resulting in $T_\infty = 25$ °C. You want to take the thermocouple out of the oven, but it is still too hot to touch. A safe temperature to touch the thermocouple is 40 °C. Radiation can be neglected for the following calculations.

b) Determine the time it takes the thermocouple to cool down from 200 °C to 40 °C.

The cooling time of the thermocouple is too long in your opinion, as you need to calibrate a few more ovens. To speed up this process, we use a fan to cool down the oven. The speed of the fan is 50 km/h.

c) Explain what will happen now that the fan is turned on. Calculate again the time it takes the thermocouple to cool down from 200 °C to 40 °C.

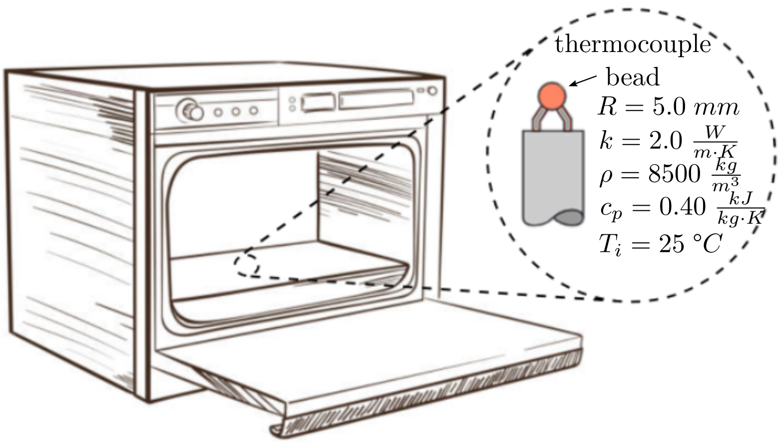


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Figure 3.1: Thermocouple inside an oven

Tables

Below, the properties of air at atmospheric pressure have been listed for different temperatures.

Properties of air at 1 atm pressure

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m^2/s	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
−150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
−100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-5}	5.837×10^{-6}	0.7263
−50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
−40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
−30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
−20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
−10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946
300	0.6158	1044	0.04418	6.871×10^{-5}	2.934×10^{-5}	4.765×10^{-5}	0.6935

Multiple choice questions (10 points)

Question 1 (2 points)

Which of the following would lead to a reduction in thermal resistance?

a) In conduction, reduction in the thickness of the material and an increase in thermal conductivity.
b) In convection, stirring of the fluid and cleaning the heating surface.
c) In radiation, increasing the temperature and reducing the emissivity.
d) Only a) and b) are true.
e) Only a) and c) are true.
f) Only b) and c) are true.
g) Statements a), b), and c) are true.

Question 2 (2 points)

A steam pipe is to be lined with two layers of insulating materials of different thermal conductivities. The change in surface area is negligible. For the minimum heat transfer,

a) The better insulator must be put on the inside.
b) The better insulator must be put on the outside.
c) One could place either insulation on either side.
d) One should take into account the steam temperature before deciding as to which material is put where.

Question 3 (2 points)

What happens to the insulation ability of an insulation layer of a wall when moisture is present?

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Thermal conductivity of a material may be defined as:

a) The quantity of heat flowing through a cube of material when opposite faces are maintained at a temperature difference of 1 K.
b) Quantity of heat flowing through a slab of the material of an area of one meter squared, with a thickness of 1 meter when it faces a difference in temperature of 1 K.
c) Heat conducted across an unit area through unit thickness when a temperature difference of unity is maintained between opposite faces.
d) All of the above.
e) None of the above.

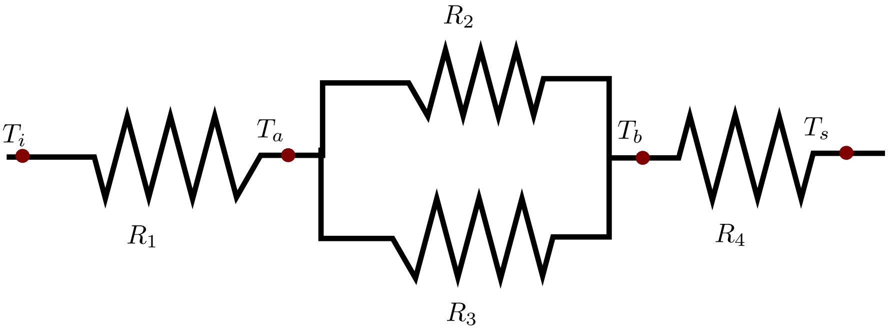
Question 5 (2 points)

Consider weather conditions for which the prevailing wind blows past the penthouse tower on a tall building, which has been displayed in (figure given in exam itself). The flow over the first window is laminar, the flow over the last window is turbulent. Which of the following statements is true:

a) The first window has a higher heat transfer coefficient than the second window. The flow over the first window is still laminar, meaning it allows for better heat transfer as the path of flow is smooth.
b) The second window has a higher heat transfer coefficient than the first window. The flow over the second window is turbulent, meaning it allows for better heat transfer as the path of flow is chaotic.
c) The first window has a higher heat transfer coefficient than the second window. Both the boundary layer and thermal boundary layer are still small here, meaning the heat transfer coefficient is large as it is defined per unit of area.
d) The second window has a higher heat transfer coefficient than the first window. Both the boundary layer and thermal boundary layer are large here, meaning the heat transfer coefficient is large as it is defined per unit of area.

Open question 1 (45 points)

a) 7 points in total The resistance network of this problem will look like the following: The equations for the



resistances are:

$$R_1 = \frac{t_a}{k_1 A_1} \qquad R_2 = \frac{t_b}{k_2 A_2} \qquad R_3 = \frac{t_b}{k_3 A_3} \qquad R_4 = \frac{t_a}{k4 A_4} \tag{1.1}$$

The resistance of the parallel part can be determined using

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \implies R_{23} = \frac{R_2 R_3}{R_2 + R_3} \tag{1.2}$$

The total resistance can then be determined using:

$$R_{tot} = R_1 + R_{23} + R_4 \tag{1.3}$$

b) 26 points in total
In order to solve this, we need the following energy balance:

$$Q_{conduction} = Q_{naturalconvection} \tag{1.4}$$

We start with determining the conduction part by looking at the resistances
Firstly we start with determining the area's:

$$A_1 = A_4 = (h_a + h_b) \cdot B = (1.5 + 1) \cdot 2 = 5\text{m}^2 \tag{1.5}$$

$$A_2 = h_b \cdot B = 1 \cdot 2 = 2\text{m}^2 \tag{1.6}$$

$$A_3 = h_a \cdot B = 1.5 \cdot 2 = 3\text{m}^2 \tag{1.7}$$

Substituting all known values in the equations for the resistances:

$$\begin{aligned} R_1 &= \frac{t_a}{k_1 A_1} = \frac{0.03}{0.72 \cdot 5} = 0.00833\ \text{K}\,\text{W}^{-1} \\ R_2 &= \frac{t_b}{k_2 A_2} = \frac{0.05}{0.035 \cdot 3} = 0.7143\ \text{K}\,\text{W}^{-1} \\ R_3 &= \frac{t_b}{k_3 A_3} = \frac{0.05}{0.055 \cdot 2} = 0.3030\ \text{K}\,\text{W}^{-1} \\ R_4 &= \frac{t_a}{k4 A_4} = \frac{0.03}{0.72 \cdot 5} = 0.00833\ \text{K}\,\text{W}^{-1} \end{aligned} \tag{1.8}$$

This gives:

$$R_{23} = 0.2182\ \text{K}\,\text{W}^{-1} \tag{1.9}$$

Such that the total resistance is

$$R_{tot} = 0.00833 + 0.2182 + 0.0083 = 0.2294\mathrm{K\,W^{-1}} \tag{1.10}$$

Convection

Answers based on simple Nusselt

Iterations show:

$$T_s = 1.75°\mathrm{C} \tag{1.11}$$

$$T_f = \frac{T_s + T_\infty}{2} = -1.625°\mathrm{C} \tag{1.12}$$

This means we take values for air at 0 degrees, being:

$$\begin{aligned} \nu &= 1.338 \cdot 10^{-5}\mathrm{m^2\,s^{-1}} \\ k &= 0.02364\mathrm{W\,m^{-1}\,K^{-1}} \\ \mathrm{Pr} &= 0.7362 \end{aligned} \tag{1.13}$$

Calculation for β

$$\beta = \frac{1}{T_f + 273.15} = 0.00368\mathrm{K^{-1}} \tag{1.14}$$

Characteristic length for vertical flat plate

$$L_c = 2.5m \tag{1.15}$$

Answer based on complex Nusselt

Iterations show:

$$T_s = 1.0°\mathrm{C} \tag{1.16}$$

$$T_f = \frac{T_s + T_\infty}{2} = -2.0°\mathrm{C} \tag{1.17}$$

This means we take values for air at 0 degrees, being:

$$\begin{aligned} \nu &= 1.338 \cdot 10^{-5}\mathrm{m^2\,s^{-1}} \\ k &= 0.02364\mathrm{W\,m^{-1}\,K^{-1}} \\ \mathrm{Pr} &= 0.7362 \end{aligned} \tag{1.18}$$

Calculation for β

$$\beta = \frac{1}{T_f + 273.15} = 0.00368\mathrm{K^{-1}} \tag{1.19}$$

Characteristic length for vertical flat plate

$$L_c = 2.5m \tag{1.20}$$

The Grashof number:

$$\begin{aligned} \mathrm{Gr} &= \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \\ \mathrm{Gr} &= \frac{9.81 \cdot 0.00368 \cdot (6.175) \cdot 2.5^3}{(1.338 \cdot 10^{-5})^2} \\ \mathrm{Gr} &= 2.1285 \cdot 10^{10} \end{aligned} \tag{1.21}$$

Determining the Rayleigh number

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 2.1285 \cdot 10^{10} \cdot 0.7362 = 1.576 \cdot 10^{10} \tag{1.22}$$

Because this is between $10^9 - 10^{13}$ we can use the following relation:

$$\mathrm{Nu} = 0.1 \cdot \mathrm{Ra}^{1/3} \tag{1.23}$$

$$\mathrm{Nu} = 250.239 \tag{1.24}$$

We can now find h using

$$\mathrm{Nu} = \frac{hL}{k} \tag{1.25}$$

$$h = \frac{\mathrm{Nu} \cdot k}{L} = 2.3663 \mathrm{W\,m^{-2}\,K^{-1}} \tag{1.26}$$

Using $T_s = 1.75°\mathrm{C}$

$$\begin{aligned} Q_{cond} &= \frac{T_s - T_\infty}{R_{tot}} \\ Q_{cond} &= \frac{6.75}{0.2294} \\ Q_{cond} &= 79.544\mathrm{W} \end{aligned} \tag{1.27}$$

And for convection:

$$\begin{aligned} Q_{conv} &= hA(T_s - T_\infty) \\ Q_{conv} &= 2.3663 \cdot 5 \cdot (6.75) \\ Q_{conv} &= 79.861\mathrm{W} \end{aligned} \tag{1.28}$$

The total rate of heat loss is $\approx$ 80 W. The wall has a surface temperature of $T_s = 1.75°\mathrm{C}$

The Grashof number:

$$\begin{aligned} \mathrm{Gr} &= \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \\ \mathrm{Gr} &= \frac{9.81 \cdot 0.00368 \cdot (6) \cdot 2.5^3}{(1.338 \cdot 10^{-5})^2} \\ \mathrm{Gr} &= 1.8946 \cdot 10^{10} \end{aligned} \tag{1.29}$$

Determining the Rayleigh number

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 1.8946 \cdot 10^{10} \cdot 0.7362 = 1.3948 \cdot 10^{10} \tag{1.30}$$

Because this is between $10^9 - 10^{13}$ we can use the following relation:

$$\mathrm{Nu} = \left(0.825 \frac{0.387\mathrm{Ra}^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right)^2 \tag{1.31}$$

$$\mathrm{Nu} = 287.1 \tag{1.32}$$

We can now find h using

$$\mathrm{Nu} = \frac{hL}{k} \tag{1.33}$$

$$h = \frac{\mathrm{Nu} \cdot k}{L} = 2.7148 \mathrm{W\,m^{-2}\,K^{-1}} \tag{1.34}$$

Using $T_s = 1.0°\mathrm{C}$

$$\begin{aligned} Q_{cond} &= \frac{T_s - T_\infty}{R_{tot}} \\ Q_{cond} &= \frac{6}{0.2294} \\ Q_{cond} &= 82.82\mathrm{W} \end{aligned} \tag{1.35}$$

And for convection:

$$\begin{aligned} Q_{conv} &= hA(T_s - T_\infty) \\ Q_{conv} &= 2.7148 \cdot 5 \cdot (6) \\ Q_{conv} &= 81.44\mathrm{W} \end{aligned} \tag{1.36}$$

The total rate of heat loss is $\approx$ 82 W. The wall has a surface temperature of $T_s = 1.0°\mathrm{C}$

c) 7 points in total Two possibilities, based on the used Nusselt relation

Using simple Nusselt relation	**Using complex Nusselt relation**
$T_a = T_i - Q_{cond} \cdot R_1 = 20 - 79.544 \cdot 0.0083 = 19.33°C \quad (1.37)$	$T_a = T_i - Q_{cond} \cdot R_1 = 20 - 82.82 \cdot 0.0083 = 19.30°C \quad (1.39)$
$T_b = T_i - Q_{cond} \cdot R_{23} = 20 - 79.544 \cdot 0.2128 = 2.42°C \quad (1.38)$	$T_b = T_i - Q_{cond} \cdot R_{23} = 20 - 82.82 \cdot 0.2128 = 1.69°C \quad (1.40)$

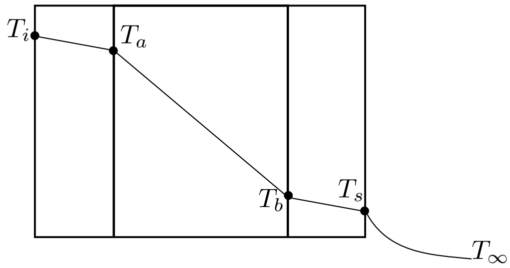


Figure 1.1: Temperature profile

d) 5 points in total
Formula for determining the total amount of energy needed:

$$E_{tot} = (Q_{cond} \cdot 20) \cdot 3600 \cdot 31 \tag{1.41}$$

To determine amount of money spend:

$$\text{costs} = \frac{E_{tot}}{32 \cdot 10^6} \cdot 0.79 \tag{1.42}$$

	Using simple Nusselt	Using complex Nusselt
E_{tot}	4.26 GJ	4.43 GJ
Costs	€105,19	€109,51

Deduction of points

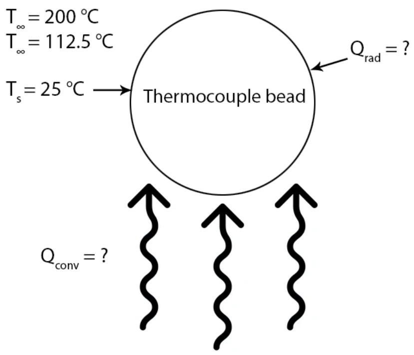
Standard:
Missing/wrong units: -0.5pt per time, -3 pt. total
Missing/wrong assumptions: -0.5pt per time, -3 pt. total
Missing/wrong conclusion/discussion: -0.5pt per time, -3 pt. total
Missing/wrong description: -0.5pt per time, -3 pt. total

Open question 2 (45 points)

a.) [20 points in total] Determine the rate of heat transfer to the thermocouple bead:

Assumptions:

- The thermocouple bead can be modelled as a sphere.
- The given temperatures are constant.
- The only heat transfer types are natural convection and radiation.



The rate of heat transfer to the thermocouple bead is a combination of radiation and natural convection:

$$\dot{Q}_{tot} = \dot{Q}_{rad} + \dot{Q}_{conv} \tag{2.1}$$

Convection part:
The film temperature should be used:

$$T_f = \frac{T_\infty + T_i}{2} = \frac{200 + 25}{2} = 112.5°\mathrm{C} = 385.65\ \mathrm{K} \tag{2.2}$$

This gives the following values from the table:

$$\nu = 2.522 \cdot 10^{-5}\ \mathrm{kg\,m^{-1}\,s^{-1}} \tag{2.3}$$

$$k = 0.03235\ \mathrm{W\,m^{-1}\,K^{-1}} \tag{2.4}$$

$$\mathrm{Pr} = 0.7073 \tag{2.5}$$

The characteristic length of a sphere should be used in this case. For natural convection, this is equal to the diameter.

$$L_c = D = 10\mathrm{mm} = 0.01\mathrm{m} \tag{2.6}$$

The Grashof number and Rayleigh number can be calculated next. For this, β should be determined first:

$$\beta = \frac{1}{T_f} = \frac{1}{385.65} = 0.0026\ \mathrm{K^{-1}} \tag{2.7}$$

Using the known values:

$$\mathrm{Gr} = \frac{g * \beta * (T_\infty - T_i) * L_c^3}{\nu^2} = \frac{9.81 * 0.0026 * (200 - 25) * 0.01^3}{(2.522 \cdot 10^{-5})^2} = 6.999 \cdot 10^3 \tag{2.8}$$

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 6.9988 \cdot 10^3 \cdot 0.7073 = 4.950 \cdot 10^3 \tag{2.9}$$

Because the thermocouple bead is approximated as a sphere, the Nusselt relation which can be used is:

$$\text{Nu} = 2 + \frac{0.589 * \text{Ra}^{1/4}}{\left(1 + \left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right)^{4/9}} = 5.8107 \tag{2.10}$$

The Nusselt number can be used to calculate the heat transfer coefficient:

$$h = \frac{k \cdot Nu}{L_c} = \frac{0.03235 \cdot 5.8107}{0.01} = 18.7975\ \text{W}\,\text{m}^{-2}\,\text{K}^{-1} \tag{2.11}$$

The heat transfer can be used to calculate the rate of heat transfer by convection:

$$\dot{Q} = h \cdot A \cdot (T_\infty - T_i) = 18.7975 \cdot 4\pi \cdot 0.005^2 \cdot (200 - 25) = 1.0301\ \text{W} \tag{2.12}$$

Radiation part: The formula for radiation is given by:

$$\dot{Q}_{rad} = \sigma \cdot A \cdot ((T_\infty + 273.15)^4 - (T_i + 273.15)^4) = \tag{2.13}$$

$$5.670 \cdot 10^{-8} \cdot 4\pi \cdot 0.005^2 \cdot ((200 + 273.15)^4 - (25 + 273.15)^4) = 0.7520\ \text{W} \tag{2.14}$$

So the total rate of heat transfer equals:

$$\dot{Q}_{tot} = \dot{Q}_{rad} + \dot{Q}_{conv} = 1.0301 + 0.7520 = 1.7854\ \text{W} \tag{2.15}$$

So the total heat transfer rate equals 1.7854 W.

b) [12 points in total] Determine the time it takes the thermocouple to cool down from 200 °C to 40 °C.

For this question, the lumped systems analysis method should be used. The characteristic length for a sphere is different for the lumped system analysis:

$$L_c = \frac{R}{3} = \frac{0.005}{3} = 0.0017\text{m} \tag{2.16}$$

To be able to use this method, the Biot number should be smaller than 0.1. Note that the same heat transfer coefficient from 2a can be used here.

$$\text{Bi} = \frac{hL_c}{k} = \frac{18.7975 \cdot 0.0017}{2} = 0.0157 < 0.1 \tag{2.17}$$

Which is smaller than 0.1, so the lumped system analysis can be used. The equation states the following:

$$\frac{T(t) - T_\infty}{T_{bead} - T_\infty} = e^{-bt} \tag{2.18}$$

Where b is defined as:

$$b = \frac{hA}{\rho V c_p} = \frac{h}{\rho L_c c_p} = \frac{18.7975}{8500 \cdot 0.0017 \cdot 400} = 0.0033\text{s}^{-1} \tag{2.19}$$

Substituting and solving gives:

$$e^{-bt} = \frac{T(t) - T_\infty}{T_{bead} - T_\infty} \tag{2.20}$$

$$-bt = ln\left(\frac{T(t) - T_\infty}{T_{bead} - T_\infty}\right) \tag{2.21}$$

$$t = -\frac{ln\left(\frac{T(t) - T_\infty}{T_{bead} - T_\infty}\right)}{b} - \frac{ln\left(\frac{40 - 25}{200 - 25}\right)}{0.0033} = 740\ \text{s} \tag{2.22}$$

So it will take 12 minutes and 20 seconds for the sphere to cool down.

c) [13 points in total] Explain what will happen now that the fan is turned on. Calculate again the time it takes the thermocouple to cool down from 200 °C to 40 °C.

Now that the fan is turned on, the type of heat transfer changes from natural to forced convection. This changes the heat transfer coefficient, meaning the Biot number changes.

The speed of the fan is 50 km/h = 13.89 m/s. A new heat transfer coefficient should be calculated, as the flow is now turbulent instead of laminar:

$$\mathrm{Re} = \frac{UL_c}{\nu} = \frac{13.89 \cdot 0.01}{2.5220 \cdot 10^{-5}} = 5507.1 \tag{2.23}$$

The Nusselt relation for a sphere is:

$$\mathrm{Nu} = 2 + (0.4 * \mathrm{Re}^{1/2} + 0.06 * \mathrm{Re}^{2/3}) * \mathrm{Pr}^{0.4} = 44.1349 \tag{2.24}$$

The heat transfer coefficient is given by:

$$h = \frac{k\mathrm{Nu}}{D} = \frac{0.0324. * 44.1349}{0.01} = 142.7765\ \mathrm{W\,m^{-2}\,K^{-1}} \tag{2.25}$$

With this new heat transfer coefficient, we can calculate the Biot number again:

$$\mathrm{Bi} = \frac{h * L_c}{k_{bead}} = \frac{142.7765 * 0.0017}{2} = 0.1190 > 0.1 \tag{2.26}$$

This means that the lumped system analysis cannot be used! This question is thus not solvable with the know techniques.

If a student would state that they approximate the solution and continue, the following solution is retrieved:

$$b = \frac{h}{\rho L_c c_p} = \frac{142.7765}{8500 \cdot 0.0017 \cdot 400} = 0.0252\ \mathrm{s^{-1}} \tag{2.27}$$

$$t = -\frac{ln\left(\frac{T(t) - T_\infty}{T_{bead} - T_\infty}\right)}{b} = -\frac{ln\left(\frac{40 - 25}{200 - 25}\right)}{0.0252} = 97.5056\ \mathrm{s} \tag{2.28}$$

Meaning it takes about 98 seconds to cool down to 40 °C.

Deduction of points

Standard:
Missing/wrong units: -0.5pt per time, -3 pt. total
Missing/wrong assumptions: -0.5pt per time, -3 pt. total
Missing/wrong conclusion/discussion: -0.5pt per time, -3 pt. total
Missing/wrong description: -0.5pt per time, -3 pt. total

University of Twente
Faculty of Engineering Technology (ET)
Research group Thermal Engineering (TE)

Exam Energy & Heat Transfer (E&HT)

22 October 2020, 18:15 - 21:15

- Do not forget to write your name and student number on the provided answer sheets.
- This exam consists of 5 multiple choice questions and 3 open questions.
- A total of 100 points can be earned:
  - 10 points can be earned for the multiple choice questions (2 points each).
  - 25 points can be earned for open question 1.
  - 40 points can be earned for open question 2.
  - 25 points can be earned for open question 3.
- Read each question carefully. If you think you made a mistake in your calculations, please provide an explanation why you think it is wrong.
- The use of a calculator, the lecture slides, your notes and the books '*Heat and Mass Transfer: Fundamental & Application*' and '*Introduction to Heat Transfer*' are allowed.
- On the last page, a table can be found with air properties at different temperatures.

Lecturer: dr. M. Mehrali

Industrial Design Engineering
202000198 Energy & Heat Transfer

UNIVERSITY OF TWENTE.

Multiple choice questions (10 points)

Question 1 (2 points)

You are standing next to a huge camp fire, which makes your body warmer. What will be the dominant heat transfer mechanism in this situation?

a) Conduction

b) Convection

c) Radiation

Question 2 (2 points)

In this course, problems are often solved by using dimensionless numbers, like the Reynolds number. The following two statements are made:

I) Dimensionless numbers are used to reduce the number of variables that describe your system.

II) The Reynolds number gives an indication of the type of flow, which is laminar if $\text{Re} < 5 \cdot 10^5$.

Which statements are correct?

a) I and II are both true

b) I is true, II is false

c) I is false, II is true

d) I and II are both false

Question 3 (2 points)

Another dimensionless number is the Nusselt number. Which of the following three statements is true for the Nusselt number in a stagnant (non-moving) fluid?

a) $\text{Nu} = 0$. There is no convection in a stagnant fluid, which indicates pure conduction.

b) $\text{Nu} = 1$. This indicates pure conduction, where the heat transfer due to convection is equal to the heat transfer with only conduction.

c) $\text{Nu} = \infty$. Conduction happens in solid bodies, without a solid body the conduction is equal to 0, giving pure convection.

Question 4 (2 points)

A flat copper plate which dimensions are 1 meter x 1 meter (1 m^2) is subjected to a cold wind flow over its top. The wind flow has a speed of 10 m/s. The temperature of the small plate is 25 °C, whilst the temperature of the wind is 5 °C. Another flat copper plate is placed parallel to the first plate with respect to the wind direction. Its dimensions are 10 meter x 10 meter (100 m^2). Its temperature is also 25 °C. What is the ratio between the small plate's and the large plate's heat transfer coefficient?

a) The heat transfer coefficient of the large plate is about 10 times smaller than the small plate.

b) The heat transfer coefficients are in the same order of magnitude.

c) The heat transfer coefficient of the large plate is about 10 times larger than the small plate.

d) The heat transfer coefficient of the large plate is about 100 times larger than the small plate.

Question 5 (2 points)

When the Biot number is smaller than 0.1, a lumped system can be assumed. What does a lumped system mean in this case?

a) A lumped system means that the system's density is high and that the system lacks a definite or regular shape.

b) Assuming a lumped system means that the convection, conduction and radiation are in the same order of magnitude, meaning they cannot be neglected.

c) A lumped system means that a perfect system can be assumed, for example a completely smooth and round sphere in an ideal air flow.

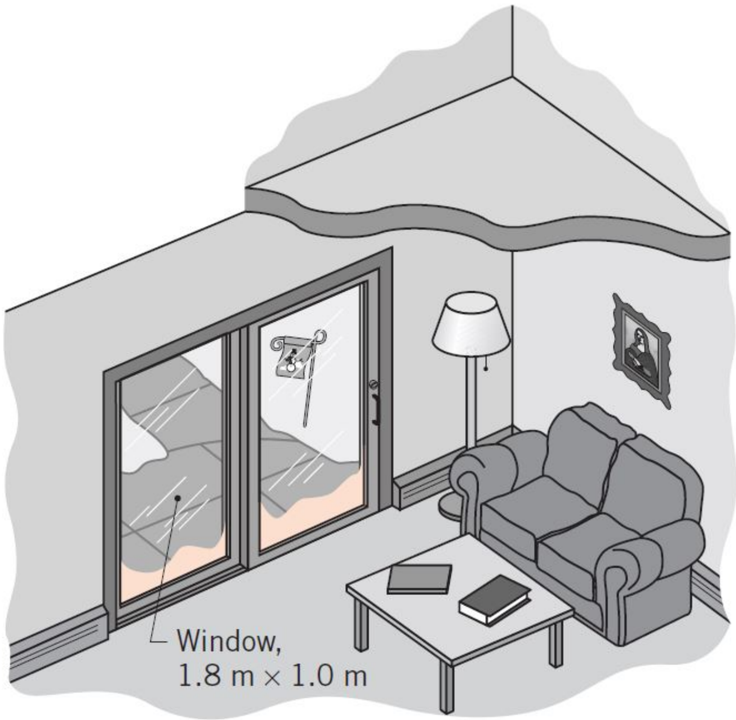
d) A lumped system has a near uniform temperature due to the ratio between the involved heat transfer mechanisms.

Open question 1 (25 points)

During a winter day, the window of a patio door with a height of 1.8 m and width of 1.0 m shows a frost line near its base. The room wall and air temperatures are 15 °C.

a) Explain why the window would show a frost layer at the base rather than at the top.

b) Estimate the heat loss through the window due to free convection and radiation. Assume the window has a uniform temperature of 0 °C and the emissivity of the glass surface is 0.94. If the room has electric baseboard heating, estimate the corresponding daily cost of the window heat loss for a utility rate of €0.18 per kWh.



Open question 2 (40 points)

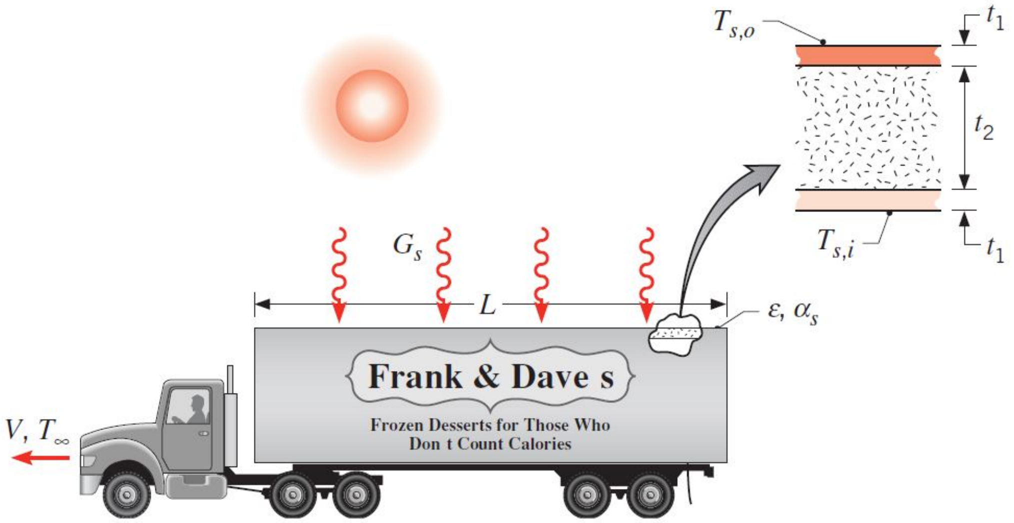
The roof of a refrigerated truck compartment is of composite construction, consisting of a layer of foamed urethane insulation (t_2 = 50.00 mm, $k_i = 0.0260\ W/m \cdot K$) sandwiched between aluminum alloy panels (t_1 = 5.00mm, $k_p = 180.00\ W/m{\cdot}K$). The length and width of the roof are L = 10.00 m and W = 3.50 m, respectively, and the temperature of the inner surface is $T_{s,i}$ = -10.00°C. Consider conditions for which the truck is moving at a speed of V = 105.00 km/h, the air temperature is T_∞ = 32.00°C, and the solar irradiation is G_S = 750.00 $\mathrm{W\,m^{-2}}$. Turbulent flow may be assumed over the entire length of the roof. Hint: you may assume that the irradiation from the sky is negligible. To solve the equilibrium equation, you can use that $T_{s,o}^4 = T_{s,o}^2 * T_\infty^2$

a) For equivalent values of the solar absorptivity and the emissivity of the outer surface ($\alpha_S = \varepsilon = 0.500$), estimate the average temperature $T_{s,o}$ of the outer surface. What is the corresponding heat load imposed on the refrigeration system? The refrigeration has to compensate for this heat load to keep the system in equilibrium.

b) A special finish ($\alpha_S = 0.150, \varepsilon = 0.800$) may be applied to the outer surface. What effect would such an application have on the surface temperature and the heat load?

c) If, with $\alpha_S = \varepsilon = 0.500$, the roof is not insulated ($t_2 = 0$), what are the corresponding values of the surface temperature and the heat load?

Compare the different cases to each other and report your results.



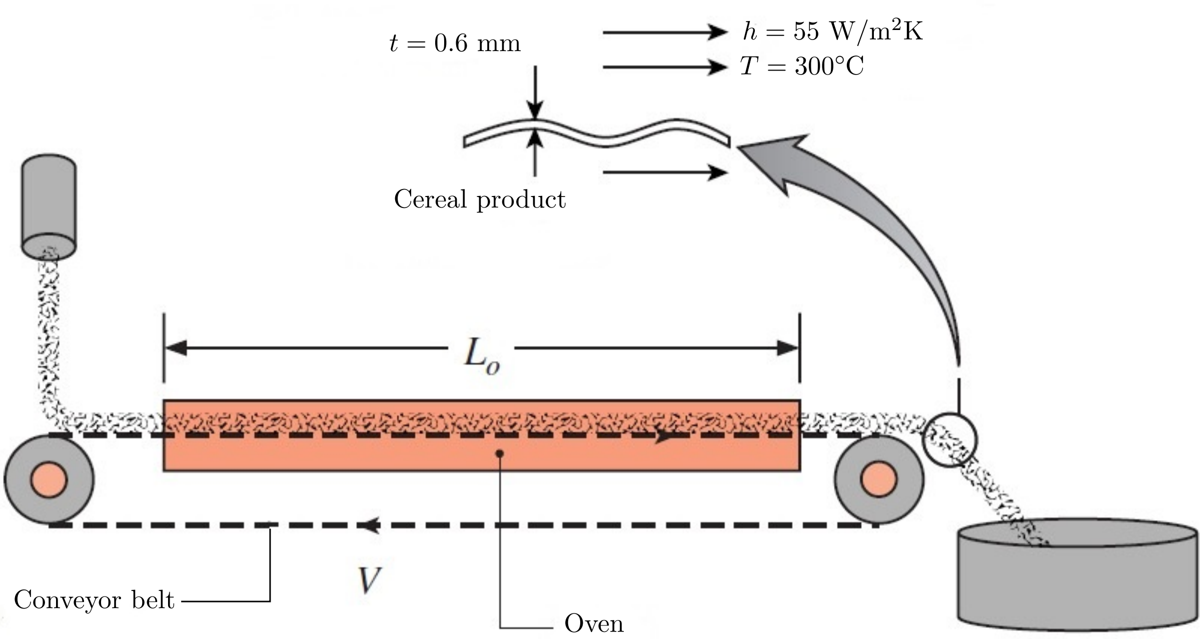
Open question 3 (25 points)

A flaked cereal is of thickness $t = 0.6$ mm. The density, specific heat and thermal conductivity of the flake are $\rho = 700\ \mathrm{kg\,m^{-3}}$, $c_p = 2400\ \mathrm{J\,kg^{-1}\,K^{-1}}$ and $k = 0.34\ \mathrm{W\,m^{-1}\,K^{-1}}$ respectively. The product is to be baked by increasing its temperature from $T_i = 20$ °C to $T_f = 220$ °C in a convection oven, through which the product is carried on a conveyor. The oven is $L_0 = 3$ m long and the convection heat transfer coefficient at the product surface and oven air temperature are $h = 55\ \mathrm{W\,m^{-2}\,K^{-1}}$ and $T_\infty = 300$ °C respectively.

a) Determine the required conveyor velocity V.

An engineer suggests that the productivity can be increased if the flake thickness is reduced to $t = 0.4$ mm.

b) Determine the required conveyor velocity and the percentual change in productivity when a flake with a reduced thickness is used. Is the engineer correct?



Tables

Below, the properties of air at atmospheric pressure have been listed for different temperatures.

Properties of air at 1 atm pressure

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg · K	Thermal Conductivity k, W/m · K	Thermal Diffusivity α, m^2/s	Dynamic Viscosity μ, kg/m · s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
−150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
−100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-5}	5.837×10^{-6}	0.7263
−50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
−40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
−30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
−20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
−10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946
300	0.6158	1044	0.04418	6.871×10^{-5}	2.934×10^{-5}	4.765×10^{-5}	0.6935

University of Twente
Faculty of Engineering Technology (ET)
Research group Thermal Engineering (TE)

Exam Correction Energy & Heat Transfer (E&HT)

22 October 2020, 18:15 - 21:15

- Do not forget to write your name and student number on the provided answer sheets.
- This exam consists of 5 multiple choice questions and 3 open questions.
- A total of 100 points can be earned:
  - 10 points can be earned for the multiple choice questions (2 points each).
  - 25 points can be earned for open question 1.
  - 40 points can be earned for open question 2.
  - 25 points can be earned for open question 3.
- Read each question carefully. If you think you made a mistake in your calculations, please provide an explanation why you think it is wrong.
- The use of a calculator, the lecture slides, your notes and the books '*Heat and Mass Transfer: Fundamental & Application*' and '*Introduction to Heat Transfer*' are allowed.
- On the last page, a table can be found with air properties at different temperatures.

Lecturer: dr. M. Mehrali

Industrial Design Engineering
202000198 Energy & Heat Transfer

UNIVERSITY OF TWENTE.

Multiple choice questions (10 points)

Question 1 (2 points)

You are standing next to a huge camp fire, which makes your body warmer. What will be the dominant heat transfer mechanism in this situation?

a) Conduction

b) Convection

c) Radiation

Question 2 (2 points)

In this course, problems are often solved by using dimensionless numbers, like the Reynolds number. The following two statements are made:

I) Dimensionless numbers are used to reduce the number of variables that describe your system.

II) The Reynolds number gives an indication of the type of flow, which is laminar if $\mathrm{Re} < 5 \cdot 10^5$.

Which statements are correct?

a) I and II are both true

b) I is true, II is false

c) I is false, II is true

d) I and II are both false

Question 3 (2 points)

Another dimensionless number is the Nusselt number. Which of the following three statements is true for the Nusselt number in a stagnant (non-moving) fluid?

a) $\mathrm{Nu} = 0$. There is no convection in a stagnant fluid, which indicates pure conduction.

b) $\mathrm{Nu} = 1$. This indicates pure conduction, where the heat transfer due to convection is equal to the heat transfer with only conduction.

c) $\mathrm{Nu} = \infty$. Conduction happens in solid bodies, without a solid body the conduction is equal to 0, giving pure convection.

Question 4 (2 points)

A flat copper plate which dimensions are 1 meter x 1 meter (1 m^2) is subjected to a cold wind flow over its top. The wind flow has a speed of 10 m/s. The temperature of the small plate is 25 °C, whilst the temperature of the wind is 5 °C. Another flat copper plate is placed parallel to the first plate with respect to the wind direction. Its dimensions are 10 meter x 10 meter (100 m^2). Its temperature is also 25 °C. What is the ratio between the small plate's and the large plate's heat transfer coefficient?

a) The heat transfer coefficient of the large plate is about 10 times smaller than the small plate.

b) The heat transfer coefficients are in the same order of magnitude.

c) The heat transfer coefficient of the large plate is about 10 times larger than the small plate.

d) The heat transfer coefficient of the large plate is about 100 times larger than the small plate.

Question 5 (2 points)

When the Biot number is smaller than 0.1, a lumped system can be assumed. What does a lumped system mean in this case?

a) A lumped system means that the system's density is high and that the system lacks a definite or regular shape.

b) Assuming a lumped system means that the convection, conduction and radiation are in the same order of magnitude, meaning they cannot be neglected.

c) A lumped system means that a perfect system can be assumed, for example a completely smooth and round sphere in an ideal air flow.

d) A lumped system has a near uniform temperature due to the ratio between the involved heat transfer mechanisms.

Open question 1 (25 points)

a) total points=4, b) total points=21

a.) For these winter conditions, a frost line could appear and it would be at the bottom of the window. The boundary layer is thicker at the top of the window, and hence the heat flux from the warmer room is greater than compared to that at the bottom portion of the window where the boundary layer is thinner. Also, the air in the room may be stratified and cooler near the floor compared to near the ceiling.

b.) The heat loss from the room to the window, having a temperature $T_s = 0°\text{C}$ by convection and radiation:

$$\begin{aligned} Q_{loss} &= Q_{conv} + Q_{rad} \\ Q_{loss} &= Ah(T_\infty - T_s) + A\epsilon\sigma(T_\infty^4 - T_4^s) \end{aligned} \tag{2.1}$$

Calculating T_f:

$$T_f = \frac{T_s + T\infty}{2} = 7.5°\text{C} \tag{2.2}$$

Values taken at 5 degrees C

Taking values at 5°C:

$$\begin{aligned} \nu &= 1.382 \cdot 10^{-5}\text{m}^2\,\text{s}^{-1} \\ k &= 0.02401\text{W}\,\text{m}^{-1}\,\text{K}^{-1} \\ \alpha &= 1.880 \cdot 10^{-5}\text{m}^2\,\text{s}^{-1} \\ Pr &= 0.7350 \end{aligned} \tag{2.3}$$

Substituting in the Grasshoff equation

$$Gr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where

$$\beta = \frac{2}{T_s + T_\infty} = \frac{1}{T_f} = 0.00356 \tag{2.4}$$

$$\text{Gr} = \frac{9.81 \cdot 0.00356 \cdot (15)1.8^3}{(1.382 \cdot 10^{-5})^2} = 1.6018 \cdot 10^{10}$$

The Grashoff and Prandtl number can be substituted for the Rayleigh number

$$\text{Ra} = \text{Gr} \cdot \text{Pr} = 1.1773 \cdot 10^{10} \tag{2.5}$$

There are now two possibilities for the Nusselt number.

Using the complex Nusselt relation

$$\text{Nu} = \left[0.825 + \frac{0.387 \cdot \text{Ra}^{1/6}}{\left(1 + (0.492/\text{Pr})^{9/16}\right)^{8/27}}\right] = 241.5 \tag{2.6}$$

We then get:

$$h = \frac{\text{Nu}_{large} \cdot k}{L} = 3.23\text{W}\,\text{m}^{-2}\,\text{K} \tag{2.7}$$

$$Q_{conv} = A \cdot h_{large} \cdot (T_s - T_\infty) = 87.083\text{W} \tag{2.8}$$

$$Q_{rad} = A\epsilon\sigma(T_\infty^4 - T_s^4) = 127.2\text{W} \tag{2.9}$$

So

$$Q_{loss} = 214.212\text{W} \tag{2.10}$$

Now, the daily costs can be calculated. The equation is

$$\text{costs} = Q_{loss} \cdot 10^{-3} \cdot 0.18 \cdot 24h \tag{2.11}$$

' Which gives

$$\text{costs} = 0.925\,\text{euro}$$

Using the simplified Nusselt relation

$$\mathrm{Nu} = 0.1 \cdot \mathrm{Ra}^{1/3} = 227.5 \tag{2.12}$$

$$h = \frac{\mathrm{Nu}_{small} \cdot k}{L} = 3.038\mathrm{W}\,\mathrm{m}^{-2}\,\mathrm{K} \tag{2.13}$$

$$Q_{conv} = A \cdot h_{short} \cdot (T_s - T_\infty) = 82.0347\mathrm{W} \tag{2.14}$$

$$Q_{rad} = A\epsilon\sigma(T_\infty^4 - T_s^4) = 127.2\mathrm{W} \tag{2.15}$$

So the Q is :

$$Q_{loss} = 209.163 \tag{2.16}$$

Now, the daily costs can be calculated. The equation is

$$\mathrm{costs} = Q_{loss} \cdot 10^{-3} \cdot 0.18 \cdot 24h \tag{2.17}$$

Which gives

$$\mathrm{costs} = 0.903\,\mathrm{euro}$$

Values taken at 10 degrees C

Taking values at 10°C:

$$\begin{aligned}\nu &= 1.426 \cdot 10^{-5}\mathrm{m}^2\,\mathrm{s}^{-1}\\ k &= 0.02439\mathrm{W\,m^{-1}\,K^{-1}}\\ \alpha &= 1.994 \cdot 10^{-5}\mathrm{m}^2\,\mathrm{s}^{-1}\\ Pr &= 0.7336\end{aligned} \tag{2.18}$$

Substituting in the Grasshoff equation

$$Gr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where

$$\beta = \frac{2}{T_s + T_\infty} = \frac{1}{T_f} = 0.00356\mathrm{K}^{-1} \tag{2.19}$$

$$\mathrm{Gr} = \frac{9.81 \cdot 0.00356 \cdot (15)1.8^3}{(1.462 \cdot 10^{-5})^2} = 1.504 \cdot 10^{10}$$

The Grashoff and Prandtl number can be substituted for the Rayleigh number

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 1.10374 \cdot 10^{10} \tag{2.20}$$

There are now two possibilities for the Nusselt number.

Using the complex Nusselt relation

$$\mathrm{Nu} = \left[0.825 + \frac{0.387 \cdot \mathrm{Ra}^{1/6}}{\left(1 + (0.492/\mathrm{Pr})^{9/16}\right)^{8/27}}\right]^2 = 236.3 \tag{2.21}$$

We then get:

$$h = \frac{\mathrm{Nu}_{large} \cdot k}{L} = 3.20\mathrm{W\,m^{-2}\,K} \tag{2.22}$$

$$Q_{conv} = A \cdot h_{large} \cdot (T_s - T_\infty) = 86.45\mathrm{W} \tag{2.23}$$

$$Q_{rad} = A\epsilon\sigma(T_\infty^4 - T_s^4) = 127.2\mathrm{W} \tag{2.24}$$

So

$$Q_{loss} = 213.5\mathrm{W} \tag{2.25}$$

Now, the daily costs can be calculated. The equation is

$$\mathrm{costs} = Q_{loss} \cdot 10^{-3} \cdot 0.18 \cdot 24h \tag{2.26}$$

Which gives

$$\mathrm{costs} = 0.922\,\mathrm{euro}$$

Using the simplified Nusselt relation

$$\mathrm{Nu} = 0.1 \cdot \mathrm{Ra}^{1/3} = 222.6 \tag{2.27}$$

$$h = \frac{\mathrm{Nu}_{small} \cdot k}{L} = 3.017\mathrm{W\,m^{-2}\,K} \tag{2.28}$$

$$Q_{conv} = A \cdot h_{short} \cdot (T_s - T_\infty) = 81.45\mathrm{W} \tag{2.29}$$

$$Q_{rad} = A\epsilon\sigma(T_\infty^4 - T_s^4) = 127.2\mathrm{W} \tag{2.30}$$

So the Q is :

$$Q_{loss} = 208.58W \tag{2.31}$$

Now, the daily costs can be calculated. The equation is

$$\mathrm{costs} = Q_{loss} \cdot 10^{-3} \cdot 0.18 \cdot 24h \tag{2.32}$$

Which gives

$$\mathrm{costs} = 0.901\,\mathrm{euro}$$

Deduction of points

Specific: Not using T_f : -8 pt. Using wrong value for β: - 5 pt. Using wrong characteristic length: -5 pt. Using negative dimensionless numbers: -3 pt. Standard: Missing/wrong units: -0.5pt per time, -3 pt. total Missing/wrong assumptions: -0.5pt per time, -3 pt. total Missing/wrong conclusion/discussion: -0.5pt per time, -3 pt. total Missing/wrong description: -0.5pt per time, -3 pt. total

Open question 2 (40 points)

26 points can be earned for part a). 8 points can be earned for part b). 6 points can be earned for part c).

a.) Estimate the average temperature $T_{s,o}$ of the outer surface. What is the corresponding heat load imposed on the refrigeration system? :

The physical phenomena which are important are conduction, convection and radiation. It is a steady state problem, resulting in the following equilibrium equation:

$$q''_{radiation} + q''_{convection} - E_{loss} = q''_{conduction} \tag{3.1}$$

The sketch below displays the situation, with different values for cases a, b and c.

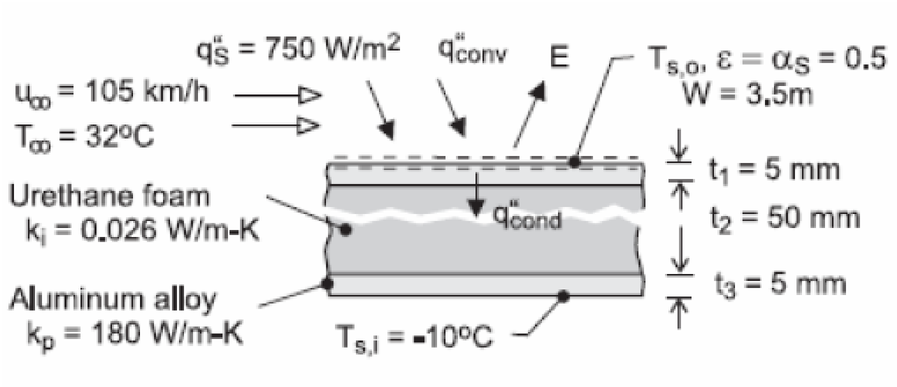


Figure 3.1: Sketch of the problem

Layer of foamed urethane insulation: t_2 = 50.00 mm, k_i = 0.0260 $W/m \cdot K$
Aluminum alloy panels: t_1 = 5.00 mm, k_p = 180.00 $W/m \cdot K$
Length roof = L = 10.00 m
Width roof = W = 3.50 m
Temperature of the inner surface = $T_{s,i}$ = -10°C = 263.00 K
Speed truck = V = 105.00 km/h = 29.20 m/s
Air temperature = T_∞ = 32.00°C = 305.00 K
Solar irradiation = G_S = 750.00 W m^{-2}
Stefan-Boltzmann constant = σ = 5.670 $* 10^{-8} W * m^{-2} * K^{-4}$
Solar absorptivity = α_S = 0.500
Emissivity = ε = 0.500

From tables:
Film temperature = $T_f \approx 300K$
Dynamic viscosity = ν = 15.89 $*10^{-6} m^2 s^{-1}$
Thermal conductivity = k_{air} = 0.0263 $W m^{-1} K^{-1}$

Assumptions:

- Irradiation from the sky is negligible
- The flow is turbulent over the entire outer surface
- The average convection coefficient may be used to estimate the average surface temperature
- The properties which are given are constant
- To translate Celsius to Kelvin, 273K is added

The equation which has to be solved is:

$$q^{''}_{radiation} + q^{''}_{convection} - E_{loss} = q^{''}_{conduction} = \frac{T_{s,o} - T_{s,i}}{R^{''}_{tot}} \tag{3.2}$$

Start by solving components of this equation. R_{tot} and $q_{radiation}$ are given by:

$$\begin{aligned}
R^{''}_{tot} &= 2R_p + R_i && (3.3)\\
&= 2\frac{t_1}{k_p} + \frac{t_2}{k_i} && (3.4)\\
&= 2 * 2.78 * 10^{-5} + 1.923 && (3.5)\\
&= 1.923\frac{m^2K}{W} && (3.6)\\
&= 0.0549\frac{K}{W} && (3.7)
\end{aligned}$$

$$\begin{aligned}
q^{''}_{radiation} &= \alpha_S G_S && (3.8)\\
&= 0.500 * 750.00 && (3.9)\\
&= 375.00\frac{W}{m^2K} && (3.10)\\
&= 13125\frac{W}{K} && (3.11)
\end{aligned}$$

$q_{convection}$ is given by the following formula, where h can be determined by using the Reynolds, Prandtl and Nusselt number.

$$\begin{aligned}
Re &= \frac{u_\infty L}{\nu} && (3.12)\\
&= \frac{29.2 * 10}{15.89 * 10^{-6}} && (3.13)\\
&= 1.84 * 10^7 && (3.14)\\
Pr &= 0.707 && (3.15)\\
& && (3.16)
\end{aligned}$$

$$\begin{aligned}
Nu &= 0.037 * Re^{4/5} * Pr^{1/3} && (3.17)\\
&= 0.037 * (1.84 * 10^7)^{4/5} * 0.707^{1/3} && (3.18)\\
&= 2.1373 * 10^4 && (3.19)\\
& && (3.20)
\end{aligned}$$

$$h = \frac{Nu * k_{air}}{L} \tag{3.21}$$

$$= \frac{2.1373 * 10^4 * 0.0263}{10} \tag{3.22}$$

$$= 56.2 \frac{W}{m^2 K} \tag{3.23}$$

The radiation loss, denoted by E_{loss} and the convection loss are given by:

$$E_{loss} = \epsilon \sigma T_{s,o}^4 \tag{3.24}$$

$$= 0.5 * 5.670 * 10^{-8} * T_{s,o}^4 \tag{3.25}$$

$$\tag{3.26}$$

$$q''_{convection} = \bar{h}(T_\infty - T_{s,o}) \tag{3.27}$$

$$= 56.2 * (305 - T_{s,o}) \tag{3.28}$$

Substituting in the total formula gives one equation with one unknown, which can be solved:

$$375 + 56.2 * (305 - T_{s,o}) - 0.500 * 5.670 * 10^{-8} * T_{s,o}^4 = \frac{T_{s,o} - 263.00}{1.923} \tag{3.29}$$

Solving by using the correlation $T_{s,o}^4 = T_{s,o}^2 * T_\infty^2$ gives:

$$T_{s,o} = 306.9K = 33.9°\text{C} \tag{3.30}$$

Note that this temperature is higher than the outer air due to radiation. The heat loss of the refrigeration system is given by the conduction loss of the total area. It is given by:

$$Q_{loss} = q''_{conduction} * A \tag{3.31}$$

$$= \frac{\Delta T}{R_{tot}} * W * L \tag{3.32}$$

$$= \frac{33.9 - -10}{1.923} * 3.5 * 10 = 799W \tag{3.33}$$

b.) Estimate the average temperature $T_{s,o}$ of the outer surface and its corresponding heat load for the changed values. :

The following values have changed:
Solar absorptivity = $\alpha_S = 0.15$
Emissivity = $\varepsilon = 0.8$
When looking at the equilibrium state:

$$q''_{radiation} + q''_{convection} - E_{loss} = q''_{conduction} = \frac{T_{s,o} - T_{s,i}}{R''_{tot}} \tag{3.34}$$

The values for $q''_{radiation}$ and E_{loss} will change.
Solving $q''_{radiation}$ and E_{loss} gives:

$$\begin{align} q''_{radiation} &= \alpha_S G_S \tag{3.35}\\ &= 0.15 * 750 \tag{3.36}\\ &= 112.5\frac{W}{m^2 K} \tag{3.37}\\ \tag{3.38} \end{align}$$

$$\begin{align} E_{loss} &= \epsilon\sigma T_{s,o}^4 \tag{3.39}\\ &= 0.8 * 5.670 * 10^{-8} * T_{s,o}^4 \tag{3.40}\\ \tag{3.41} \end{align}$$

Substituting the new values in the equilibrium equation gives:

$$112.5 + 56.2 * (305 - T_{s,o}) - 0.800 * 5.670 * 10^{-8} * T_{s,o}^4 = \frac{T_{s,o} - 263.00}{1.923} \tag{3.42}$$

And solving for $T_{s,o}$ by using the correlation $T_{s,o}^4 = T_{s,o}^2 * T_\infty^2$ gives:

$$T_{s,o} = 299.9K = 26.9°\text{C} \tag{3.43}$$

The heat loss is again given by the conduction over the total area, which is given by:

$$\begin{align} Q_{loss} &= q''_{conduction} * A \tag{3.44}\\ &= \frac{\Delta T}{R_{tot}} * W * L \tag{3.45}\\ &= \frac{27.1 - -10.00}{1.923} * 3.50 * 10.00 = 675W \tag{3.46} \end{align}$$

c.) Estimate the average temperature $T_{s,o}$ of the outer surface and its corresponding heat load for the changed values. :

The following values have changed:
Solar absorptivity = $\alpha_S = 0.500$
Emissivity = $\varepsilon = 0.500$
Thickness isolation = $t_2 = 0m$
When looking at the equilibrium state:

$$q''_{radiation} + q''_{convection} - E_{loss} = q''_{conduction} = \frac{T_{s,o} - T_{s,i}}{R''_{tot}} \tag{3.47}$$

The situation is the same as a), but with a different R''_{tot}:

$$R''_{tot} = 2R_p + R_i \tag{3.48}$$
$$= 2\frac{t_1}{k_p} + \frac{t_2}{k_i} \tag{3.49}$$
$$= 2 * 2.78 * 10^{-5} + 0 \tag{3.50}$$
$$= 5.56 * 10^{-5}\frac{m^2K}{W} \tag{3.51}$$
$$= 1.59 * 10^{-6}\frac{K}{W} \tag{3.52}$$

Substituting this in the total equilibrium equation gives:
Substituting in the total formula gives one equation with one unknown, which can be solved:

$$375 + 56.2 * (305.00 - T_{s,o}) - 0.500 * 5.670 * 10^{-8} * T^4_{s,o} = \frac{T_{s,o} - 263.00}{5.56 * 10^{-5}} \tag{3.53}$$

Solving by using the correlation $T^4_{s,o} = T^2_{s,o} * T^2_{\infty}$ gives:

$$T_{s,o} = 263.14K = -9.86°\text{C} \tag{3.54}$$

The heat loss of the refrigeration system is given by the conduction loss of the total area. It is given by:

$$Q_{loss} = q''_{conduction} * A \tag{3.55}$$
$$= \frac{\Delta T}{R_{tot}} * W * L \tag{3.56}$$
$$= \frac{-9.86 - -10.00}{5.56 * 10^{-5}} * 3.50 * 10.00 = 8.81 * 10^4 W \tag{3.57}$$

Open question 3 (25 points)

a.) Conveyor belt velocity:
The characteristic length of the flake cereal can be calculated by using that convection only takes place at the top surface:

$$L_c = \frac{V}{A_s} = \frac{t * A_s}{A_s} = t = 0.6\text{mm} \tag{4.1}$$

This characteristic length gives a Biot number of:

$$Bi = \frac{hL_c}{k} = \frac{55 * 0.6 * 10^{-3}}{0.34} = 0.097 < 0.1 \tag{4.2}$$

So we can assume a lumped system.
Therefore the lumped capacitance approximation is valid and the time will be:

$$\begin{aligned} t &= \frac{\rho V c_p}{h A_s} \ln\left(\frac{\theta_i}{\theta}\right) \\ &= \frac{\rho L_c c_p}{h} \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right) \\ &= \frac{700 * 0.0006 * 2400}{55} \ln\left(\frac{20 - 300}{220 - 300}\right) \\ &= 22.96\text{s} \end{aligned} \tag{4.3}$$

The conveyor velocity then becomes:

$$V = \frac{L_o}{t} = \frac{3}{22.96} = 0.13 m/s \tag{4.4}$$

b.) Conveyor belt velocity for reduced flake and productivity:
With a flake thickness of 0.4 mm, the conveyor velocity becomes:

$$L_c = 0.4\text{mm} \tag{4.5}$$

$$Bi = \frac{hL_c}{k} = \frac{55 * 0.4 * 10^{-3}}{0.34} = 0.065 < 0.1 \tag{4.6}$$

$$\begin{aligned} t &= \frac{\rho V c_p}{h A_s} \ln\left(\frac{\theta_i}{\theta}\right) \\ &= \frac{\rho L_c c_p}{h} \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right) \\ &= \frac{700 * 0.0004 * 2400}{55} \ln\left(\frac{20 - 300}{220 - 300}\right) \\ &= 15.31\text{s} \end{aligned} \tag{4.7}$$

$$V = \frac{L_o}{t} = \frac{3}{15.31} = 0.20 m/s \tag{4.8}$$

The percentual increase in productivity is:

$$\eta = \frac{V_{0.4} - V_{0.6}}{V_{0.6}} \cdot 100\% = \frac{0.20 - 0.13}{0.13} \cdot 100\% = 54\% \tag{4.9}$$

So the engineer is right, the productivity is increased by 54%.

Deduction of points

Standard: Missing/wrong units: -0.5pt per time, -3 pt. total Missing/wrong assumptions: -0.5pt per time, -3 pt. total Missing/wrong conclusion/discussion: -0.5pt per time, -3 pt. total Missing/wrong description: -0.5pt per time, -3 pt. total