Faculty of Engineering Technology (ET)

Solution Set
Energy & Heat Transfer

Industrial Design Engineering
202000198 Energy & Heat Transfer

Lecturer:
dr. M. Mehrali

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UNIVERSITY OF TWENTE.

Contents

Solutions lecture 1

1.1 Joule's test setup

a) A load of $50kg$ is displaced over a distance of $20m$. All the mechanical energy is converted into thermal energy, which increases the temperature of $5.0L$ water. First, the energy gained by this displacement of the load can be calculated:

$$\begin{aligned} \Delta E_{mech} &= F \cdot \Delta h = m \cdot g \cdot \Delta h \\ &= 50 \cdot 9.81 \cdot 20 \\ &= 9810J \end{aligned} \tag{1.1}$$

All the mechanical energy is converted into thermal energy:

$$\Delta E_{mech} = Q = m_{water} \cdot c_v \cdot \Delta T \tag{1.2}$$

From this, the temperature increase can be calculated, as the mass of the water ($\approx 5.0kg$) and the specific heat capacity ($= 4186J \cdot kg^{-1} \cdot K^{-1}$) are known:

$$\Delta T = \frac{\Delta E_{mech}}{m_{water} \cdot c_p} = \frac{9810}{5.0 \cdot 4186} = 0.47°C \tag{1.3}$$

b) First, some data from diesel has to be found online. The values that should be found online are the density of diesel and the price per liter: $0.85\ \mathrm{kg\,L^{-1}}$ and €1.21 respectively.

With a budget of €1, the amount of liters diesel you can buy is equal to:

$$V_{diesel} = \frac{1}{1.21} = 0.8264L \tag{1.4}$$

The amount of energy in this diesel volume is equal to:

$$E_{diesel} = V_{diesel} \cdot \rho_{diesel} \cdot \hat{E}_{diesel} = 0.8264 \cdot 0.85 \cdot 45.5 = 31.96\mathrm{MJ} \tag{1.5}$$

In the equation, $\hat{E}$ is the given specific calorific value of (or chemical energy in) diesel. Now, we can calculate the number of times the load can be lifted:

$$N_{lifts} = \frac{31.96 \cdot 10^6}{9.810 \cdot 10^3} = 3258\ times \tag{1.6}$$

So, the load can be lifted 3258 times.

c) First, the power is converted to W ($1hp = 746W$):

$$P = 2.5 \cdot 746 = 1865W \tag{1.7}$$

Now, the time it takes to lift this load using the maximum amount of power can be calculated:

$$t = \frac{\Delta E_{mech}}{P} = \frac{9810}{1865} = 5.26s \tag{1.8}$$

It takes longer than 5 seconds to lift the load, which means that the engine is not suitable.

d) The energy in a chocolate bar can be found online, which is equal to approximately $530kcal$ per $100g$. One kcal is equal to $4184J$, so the total energy in 900 g chocolate can be calculated:

$$E_{chocolate} = m_{choc} \cdot \hat{E}_{choc} = 0.9 \cdot 5300 \cdot 4184 = 19.96MJ \tag{1.9}$$

The energy required per lift was already calculated: $9810J$. The number of days that the engine can run on chocolate then becomes:

$$t = \frac{19.96 \cdot 10^6}{9810 \cdot 120} = 16.96 \; days \tag{1.10}$$

The engine can run for almost 17 days on the chocolate.

e) The electricity costs are given: €0.17 per kWh. One kWh is equal to $3.6MJ$. Now, we can calculate how much energy we can get with a budget of €1:

$$E = \frac{1}{0.17} \cdot 3.6 = 21.18MJ \tag{1.11}$$

Comparing this answer to the answer obtained in question 1b, we see that the amount of energy from diesel is higher with the same budget. This means that the diesel engine is the cheaper option.

1.2 Transport by car and bus

a) In both cases the distance to be travelled is $2 \cdot 75 = 150$ km. The car has a consumption of 7.2 liters per 100 kilometers, so the fuel used per car is

$$150\text{km} \cdot \frac{7.2\text{L}}{100\text{km}} = 10.8\text{L}$$

Thus, for six cars a total of $6 \cdot 10.8 = 64.8$L of fuel is needed.

The consumption of the bus is 35 liters per 100 kilometers, so

$$150\text{km} \cdot \frac{35\text{L}}{100\text{km}} = 52.5\text{L}$$

So for the bus, 52.5 L of fuel is needed.

b) The total amount of energy E_{total} is the mass of the fuel m multiplied by the caloric value (chemical energy released when combusted) of the fuel. The mass of the fuel needed is the needed amount of liters V multiplied by the specific mass m_{spec} of the fuel:

$$E_{total} = m \cdot E = m_{spec} \cdot V \cdot E$$

For the car, the above equation becomes:

$$E_{total,car} = 64.8\text{L} \cdot 0.70\text{kg}\,\text{L}^{-1} \cdot 42\text{MJ}\,\text{kg}^{-1} = 1905\text{MJ}$$

Per person this comes down to 79.4 MJ. For the bus:

$$E_{total,bus} = 52.5\text{L} \cdot 0.85\text{kg}\,\text{L}^{-1} \cdot 43\text{MJ}\,\text{kg}^{-1} = 1919\text{MJ}$$

Per person this comes down to 79.9 MJ.

c) For the car, the fuel costs are

$$\text{Price fuel car} = 64.8 \cdot €1.49 = €96.55$$

For the bus, the fuel costs are:

$$\text{Price fuel bus} = 52.5 \cdot €1.14 = €59.85$$

The total costs for the bus is €59.85+€200=€259.85. This means that the going by car is the cheapest option.

d) The minimum amount of kilometers can be calculated by equating the costs of the cars per kilometer to the costs of the bus per kilometer:

$$\text{Costs car} = \text{Costs bus}$$

In order to do so, we need to calculate the costs per kilometer. For the car:

$$\text{Costs per kilometer car} = \frac{96.55}{150} = €0.64$$

And for the bus:

$$\text{Costs per kilometer bus} = \frac{59.85}{150} = €0.40$$

Equating the costs of the car to the costs of the bus:

$$€0.64 \cdot x = €0.40 \cdot x + €200$$

where x is the distance in kilometers. Solving this gives a distance of 833 km.

e) In the lecture sheets, you can find that one hour of cycling at 15 km/h costs a person 1600 kJ of energy. The time needed to travel will be 150 km/15 km/h=10 hours. The amount of energy needed per person is thus $10\text{h} \cdot 1600\text{kJ}\,\text{h}^{-1} = 16\text{MJ}$. Note that this is much less than by car or bus!

The lecture slides also give the definition of the Calvé: 1 Calvé = 200 kcal= $200 \cdot 4.184\text{kJ} = 0.8368\text{MJ}$. For the 150 kilometers of cycling, all students needs

$$\frac{16\text{MJ}}{0.8368\text{MJCalvé}^{-1}} = 19,1\text{Calvé}$$

f) Which travel option is the most efficient, depends on the energy conversions included in the analysis. While the cars use less energy than the bus when transporting 24 people, the production of the bus fuel might be more efficient than the production of the car fuel. The same principle applies to the cycling; the energy used while cycling may be five times less than using cars, but to produce, distribute and digest the sandwiches with peanut butter also costs a considerable amount of energy. Using only the energy conversions in the assignment, cycling is the most efficient, followed by the cars and then the bus. In reality, factors including the environment, the travel time and costs play an important role.

1.3 Boiling water - Hand in

a) In the first lecture material, the definition of one kilocalory is given: one kilocalory is the amount of energy needed to heat 1 kg of water 1 °C; 1 kcal = 4.184 kJ. In this exercise 1 L of water with a mass of approximately 1 kg needs to be heated $\Delta T = 100 - 20 = 80$ °C. This will cost:

$$80 \text{ kcal} = 80 \cdot 4.184 \text{ kJ} = 334.7 \text{ kJ}$$

b) First determine the total amount of energy that needs to be delivered. The 334.7 kJ from answer a is the useful 40% of this total amount.

$$E_{\text{total}} = \frac{E_{\text{useful}}}{\eta} = \frac{334.7 \text{ kJ}}{0.40} = 836 \text{ kJ}$$

With η being the efficiency of 40%

Next, determine the amount of mass needed:

$$m_{\text{gas}} = \frac{E_{\text{total}}}{E_{\text{gas}}} = \frac{836 \cdot 10^5 \text{ J}}{50 \cdot 10^6 \text{ J/kg}} = 0.017 \text{ kg}$$

With E_{gas} being the calorific value of the gas.

Using the density of the gas $\rho_{\text{gas}} = 0.7$ kg/m^3, calculate the gas volume needed:

$$\rho_{\text{gas}} = \frac{m_{\text{gas}}}{V_{\text{gas}}} \Rightarrow V_{\text{gas}} = \frac{m_{\text{gas}}}{\rho_{\text{gas}}} = \frac{0.017 \text{ kg}}{0.7 \text{ kg/m}^3} = 0.024 \text{ m}^3 = 24 \text{ L}$$

Conclusion: we need 24 litres of gas to heat 1 liter of water from 20 °C to 100 °C.

c) An amount of 2000 W = 2000 J/s of power P is added to the water. The total energy change ΔE of the water during heating is 334.7 kJ (consider the answer at a). The amount of time Δt needed can be estimated as follows:

$$\Delta t = \frac{\Delta E}{P} = \frac{334.7 \cdot 10^3 \text{ J}}{2000 \text{ J/s}} = 167 \text{ s} = 2 \text{ min } 47 \text{ s}$$

d) See the lecture notes: 1 kWh = 3.6 MJ. An amount of 334.7 kJ is needed, so this is equal to:

$$\frac{334.7 \cdot 10^3 \text{ J}}{3.6 \cdot 10^6 \text{ J/kWh}} = 0.093 \text{ kWh}$$

e) Heating with gas costs more energy than heating with electricity. This conclusion could have been made straight away, since the energy use of the gas has an efficiency of only 40%, while the efficiency of electricity is 100%.
For an exhaustive and fair comparison, all the energy conversions of the gas and the electricity should be taken into consideration. An electricity plant of course also uses (fossil)fuels, and it is reasonable to consider whether these conversions and the transportation across the electricity network have a greater energy loss than the transportation of the gas.

f) Costs of gas:

$$0.024 \text{ m}^3 \cdot €0.67 \cdot m^{-3} = €0.016$$

Costs of electricity:

$$0.093 \text{ kWh} \cdot €0.22 \cdot kWh^{-1} = €0.020$$

Conclusion: heating with gas is the cheaper solution, but it costs more energy (disregarding energy conversions outside the household).

Solutions lecture 2

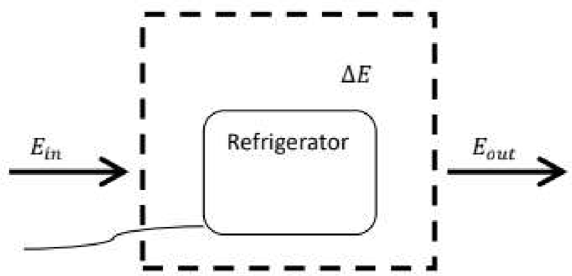
Starting with lecture 2, the approach presented in the assignment set is used to recognize and solve the problems.

2.1 Energy balance of a student room

a)

Analysis

We have two identical rooms with windows and doors closed. One room will contain a refrigerator. The question is whether the room with the refrigerator becomes warmer or cooler. In the following figure, a sketch is made of the problem.



In this sketch, the energy coming into the room, E_{in}, and the energy coming out of the room with the refrigerator, E_{out}, is drawn. Due to the conservation of energy, the difference must be stored in the room. This difference is denoted by ΔE. The refrigerator is connected to electricity.

Approach

Assumptions

We assume that there is no heat loss at the walls.

Route to solution

We can solve this problem by looking at the energy balance:

$$E_{in} - E_{out} = \Delta E$$

Since it is a closed system, no energy can leave the room, and $E_{out} = 0$. The incoming energy is in the form of electricity, so $E_{in} > 0$

Elaboration

The room including the refrigerator will have a larger rise in temperature compared to the one without a refrigerator. Both rooms are identical, so the temperature difference can be explained by focusing on the energy balance regarding the difference of both rooms:

Looking at the energy balance, $E_{out} = 0$ and $E_{in} > 0$. This means that $\Delta E > 0$. The total amount of energy is increasing, so the temperature is rising.

Evaluation

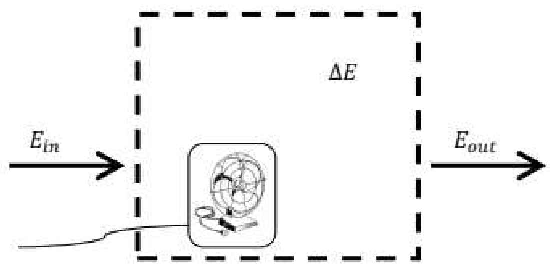
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

b)

Analysis

A fan is turned on in a room with doors and windows closed, and is turned on for 10 hours. The question asked is to determine the temperature of the room after 10 hours. In the following figure, a sketch is made of the problem.



In this sketch, the energy coming into the room, E_{in}, and the energy coming out of the room, E_{out}, is drawn. Due to the conservation of energy, the difference must be stored in the room. The difference is denoted by ΔE. The fan is connected to electricity. The dimensions of the room are given, which are 4m x 6m x 6m. The fan has a power of 150 W. The starting temperature is 15 °C.

Approach

Assumptions

We assume no heat loss at the walls. Specific heats can be taken around room temperature.

Route to solution

We start with the energy balance. Since no energy is leaving the room $E_{out} = 0$. The ingoing energy can be calculated with

$$E_{in} = P_{electric} \cdot t$$

Since $E_{in} > 0, \Delta E > 0$, so the total energy in the system is increasing, and the temperature is rising. This also means that $\Delta E = Q$. The temperature can be calculated with

$$Q = m \cdot c_v \cdot \Delta T \implies \Delta T = \frac{Q}{m \cdot c_v}$$

Because the room has a constant volume, c_v is used (c_p is used when the room is maintained at a constant pressure). This value can be taken at room temperature. Now, only the mass of the the air is unknown. Using the density of air at room temperature and calculating the volume of the room, the mass of the air in the system can be calculated. Substituting these results in the equation for ΔT gives the temperature rise.

Elaboration

We start with calculating the ingoing energy

$$E_{in} = P_{electric} \cdot t = 150 \cdot 10 \cdot 3600 = 5.4\text{MJ}$$

At room temperature (20°C), the values for the density of air and the specific heat at constant volume are [Engineering Toolbox]:

$$\rho = 1.2041 \mathrm{kg\,m^{-3}}, \qquad c_v = 717 \mathrm{J\,kg^{-1}\,K^{-1}}$$

The volume of the room is $4 \cdot 6 \cdot 6 = 144\mathrm{m}^3$. The mass of the air is then $m = V \cdot \rho = 144 \cdot 1.2041 = 173.4$ kg. Substitution of all these variables yields:

$$\Delta T = \frac{Q}{m \cdot c_v} = \frac{5.4 \cdot 10^6}{173.4 \cdot 717} = 43.4°\mathrm{C}$$

The final temperature of the room is then $15 + 43.4 = 58.4$ °C.

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.2 Shopping centre escalator

Analysis

An escalator in a shopping center is designed to move 30 persons with an average weight of 75 kg each, at a constant speed of 0.8 m/s at a 45 degree slope. The question is to determine the minimum power input needed to drive. The second question is what will happen to the answer when the velocity is doubled. A sketch is provided below

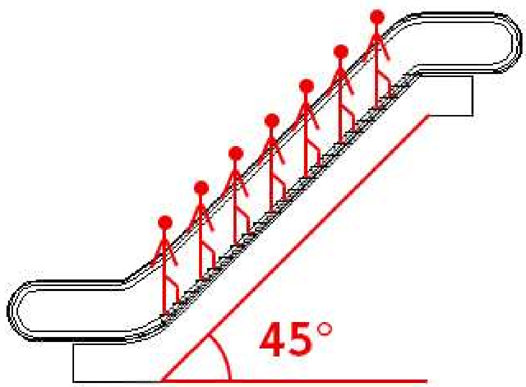


Figure 2.1: Shopping centre escalator

Approach

Assumptions

Since the minimum power input is asked, we assume all electrical energy is transferred to movement and that there is no friction.

Route to solution

The minimum power can be determined using

$$P = \frac{E}{t}$$

Here, the potential energy is the input energy E, where $E = m \cdot g \cdot h$. Substitution gives

$$P = \frac{m \cdot g \cdot h}{t}$$

in which h and t are still unknown. The height can be calculated using the velocity in y-direction, u_y. Looking at the sketch, $u_y = v \cdot \sin 45$. The height can be calculated using

$$h = u_y \cdot t$$

Substituting this gives the final expression for the power input

$$P = m \cdot g \cdot u_y$$

Elaboration

Determining the vertical velocity:

$$u_y = v \cdot \sin 45 = 0.8 \cdot \sin 45 = 0.57$$

Substitution of all known variables and calculated values in the expression for the power input:

$$P = 30 \cdot 75 \cdot 9.81 \cdot 0.57 = 12.6\text{kW}$$

It is clear from the expression of the power input that doubling the velocity will lead to a doubled vertical velocity, and hence a doubled power input.

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.3 Energy balance of a fitness centre

a)

Analysis

There are four treadmills plus 8 weightlifting machines in a fitness centre. The treadmills have a shaft output power of 2.5 hp. They operate on an average load factor f of 0.7 and an efficiency of $\eta = 77$ %. During peak hours, all machines are used and a total of 14 people are present, with an average rate of heat dissipation per person of 300 W. Determine the rate of heat gain of the exercise room.

Approach

Assumptions

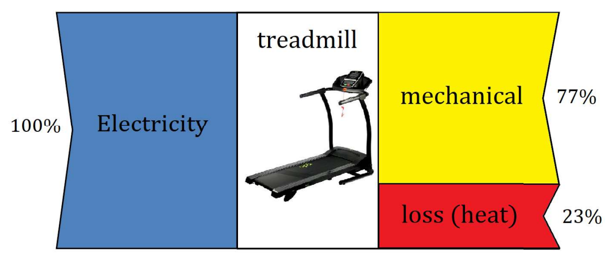
Two different assumptions can be made:

1. All electric energy provided to the treadmills is converted into heat.
2. The power loss of the treadmills is converted into heat.

Furthermore, the room is regarded as a closed system, meaning there is no heat loss at the walls.

Route to solution

Firstly, the electricity used by the treadmills needs to be determined. The shaft output power of 2.5 hp is the (mechanical) power, provided to the treadmill band. This can also be seen in the Sankey diagram below:



The total electricity used by the treadmills is thus dependant on the number of treadmills, the shaft power provided per treadmill, the load factor and the efficiency. Note that a horsepower is defined as 745.7 W. This means that the total power input can be determined with:

$$P_{treadmills,in} = 4 \cdot \frac{f(2.5 \cdot 745.7)}{\eta}$$

During peak hours, fourteen persons are present in total, with an average rate of heat dissipation of 300 W. With this, the total heat output from people can be determined:

$$\dot{Q}_{people} = 14 \cdot 300 = 4.2\text{kW}$$

Now depending on the assumption, the rate of heat gain can be calculated. For the first assumption:

$$\dot{Q}_{total} = \dot{Q}_{treadmills} + \dot{Q}_{people} = P_{treadmills,in} \cdot 1.0 + \dot{Q}_{people}$$

If the second assumption is used:

$$\dot{Q}_{total} = \dot{Q}_{treadmills} + \dot{Q}_{people} = P_{treadmills,in} \cdot 0.23 + \dot{Q}_{people}$$

Elaboration

First, calculating the total power input of the treadmills:

$$P_{treadmills,in} = 4 \cdot \frac{0.7(2.5 \cdot 745.7)}{0.77} = 6.8\text{kW}$$

The total rate of heat gain using the first assumption:

$$\dot{Q}_{total} = P_{treadmills,in} \cdot 1.0 + \dot{Q}_{people} = 6.8 + 4.2 = 11.0\text{kW}$$

The total rate of heat gain using the second assumption:

$$\dot{Q}_{total} = P_{treadmills,in} \cdot 0.23 + \dot{Q}_{people} = 1.6.4.2 = 5.8\text{kW}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

b)

Analysis

Peak hours run from 18:00 – 22.00. Temperature in the room at 18:00 is 20 °C. Estimate the room temperature at 22:00. The dimensions of the room are given (20x20x3 m). See the sketch below.

Approach

Assumptions

Assume no heat loss through walls and windows. Also, assume properties of air at 20 °C: $\rho = 1.2041\ \text{kg}\,\text{m}^{-3}$ and since we have a constant volume, $c_v = 717\text{J}\,\text{kg}^{-1}\,\text{K}$

Route to solution

The temperature increase can be determined using

$$Q = m \cdot c_v \cdot \Delta T \implies \Delta T = \frac{Q}{m \cdot c_v}$$

Q can be calculated by multiplying $\dot{Q}_{total}$ with time t. The mass can be determined by multiplying the density with the volume.

Elaboration

The volume of the fitness room is $20 \cdot 20 \cdot 3 = 1200\text{m}^3$. The mass of the air in the room is then $1200 \cdot 1.2041 = 1445\text{kg}$. For four hours, Q becomes $Q = 11 \cdot 10^3 \cdot 4 \cdot 3600 = 158\text{MJ}$ for the first assumption of a), and $Q = 5.8 \cdot 10^3 \cdot 4 \cdot 3600 = 83.5\text{MJ}$ for the second assumption in a).

Substituting these values in the equation for the temperature increase gives for the first assumption in a)

$$\Delta T = \frac{158 \cdot 10^6}{1445 \cdot 717} = 152.5°\text{C}$$

Hence, the end temperature is

$$T_{end} = T_{begin} + \Delta T = 20 + 152.5 = 172.5°\text{C}$$

and for the second assumption:

$$\Delta T = \frac{83.5 \cdot 10^6}{1445 \cdot 717} = 80.6°\text{C}$$

$$T_{end} = T_{begin} + \Delta T = 20 + 80.6 = 100.6°\text{C}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

c)

Analysis

We have to calculate the required cooling power (using an energy balance) if the room temperature rises by a maximum of 5 degrees Celsius.

Approach

Assumptions

We take all variables at a temperature of 20 degrees. As the room is open, the assumption of constant volume is not valid, therefore we use the assumption of constant pressure.

Route to solution

The energy balance is:

$$\dot{E}_{out} = \dot{E}_{in} - \Delta\dot{E}$$

Here, $\dot{E}_{in}$ depends on the result from question a), $\Delta\dot{E}$ is the allowed rate of heat gain during peak hours and E_{out} is the required cooling power. The following equation can be used to determine the maximum amount of energy that can be added to the system:

$$Q = m \cdot c_p \cdot \Delta T$$

Note that c_v can not be used as the room is now open, so c_p is used. At 20 degrees C, the value of $c_p = 1007\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$. The allowed rate of heat gain can then be calculated with

$$\Delta\dot{E} = \frac{Q}{t}$$

Now, the required cooling power can be calculated by simply filling in the energy balance.

Elaboration

First, the value of Q is calculated:

$$Q = m \cdot c_p \Delta T = 1445 \cdot 1007 \cdot 5 = 7.28\text{MJ}$$

Now, the allowed rate of heat gain can be determined:

$$\Delta\dot{E} = \frac{7.28\text{MJ}}{4 * 3600} = 505W$$

For assumption 1, this leads to

$$\dot{E}_{out} = \dot{E}_{in} - \Delta\dot{E} = 11.0 - 0.505 = 10.5\text{kW}$$

and for assumption 2:

$$\dot{E}_{out} = \dot{E}_{in} - \Delta\dot{E} = 5.8 - 0.505 = 5.3\text{kW}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.4 Microwave

a) A possible definition for the efficiency η of a microwave is the ratio between the amount of thermal energy Q_{in} which is absorbed by the object which is supposed to be heated, and the amount of electrical energy $E_{electrical,in}$ used for that objective:

$$\eta = \frac{Q_{in}}{E_{electrical,in}}$$

In this definition, $Q_{in} = m \cdot c_p \cdot \Delta T$, where m is the mass of the object, c_p the specific heat value at constant pressure and ΔT the rise in temperature of the object. The electrical input $E_{electrical,in}$ can be determined by measuring the electrical power P with a power meter and the time Δt in which the power is delivered $E_{electrical,in} = P\Delta t$. This leads to:

$$\eta_{magnetron} = \frac{m \cdot c_p \cdot \Delta T}{P \cdot \Delta t}$$

Other definitions might be possible as well.

b)

Analysis

A 250 ml cup of milk at 20 °C is placed in a microwave. After two minutes it starts to boil. Determine the amount of energy transferred to the milk.

Approach

Assumptions

Considering the fact that milk consists largely of water, the following assumptions are valid without lowering the accuracy of the answer in a harmful way:

- Milk has the same density as water, $\rho = 1000\ \mathrm{kg\,m^{-3}}$
- Milk has the same specific heat value as water, $c_p = 4.184 \cdot 10^3 \mathrm{J\,kg^{-1}\,K^{-1}}$
- Milk has the same boiling point as water, at 100 °C.

Route to solution

The temperature difference between the boiling point and the intitial temperature can be determined quite easily. Substitution of this in

$$Q_{in} = m \cdot c_p \cdot \Delta T$$

gives the energy transferred to the milk.

Elaboration

The temperature difference is

$$\Delta T = T_1 - T_2 = 100 - 20 = 80^\circ\mathrm{C}$$

Substitution of the temperature difference and the mass and specific heat value gives

$$Q_{in} = m \cdot c_p \cdot \Delta T = 0.250 \cdot 4.184 \cdot 10^3 \cdot 80 = 83.7\mathrm{kJ}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?

- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

c) With Q_{in} determined, only $E_{electrical,in}$ is still undetermined, when the efficiency in a) is used. The latter can be calculated by multiplying the power with the time

$$E_{electrical,in} = P_{electrical,in} \cdot \Delta t = 800 \cdot 2 \cdot 60 = 96.0\text{kJ}$$

Substitution of this and Q_{in} in the equation for the efficiency:

$$\eta = \frac{Q_{in}}{E_{electrical,in}} = \frac{83.7}{96.0} = 0.872 = 87.2\%$$

d) A possible definition for the efficiency of the power plant is the following:

$$\eta_{power\ plant} = \frac{E_{electrical,out}}{E_{coal}}$$

In this formula $E_{electrical,out}$ is the electrical energy provided to the grid by the power plant. The chemical energy bounded in the coal is the E_{coal} and this energy can be released by the combustion of coal. Usually this value is given as the LHV (Lower Heating Value).

e) The overall efficiency of the system can defined as the fraction of the original input energy (E_{coal}) that is consumed as useful energy, in this case (Q_{in}). This is the result of all efficiencies along the way. This leads to the following value:

$$\begin{aligned}\eta_{total} = \frac{Q_{in}}{E_{coal}} &= \frac{E_{electrical,out}}{E_{coal}} \cdot \eta_{electrical\ grid} \cdot \frac{Q_{in}}{E_{electrical,in}} \\ &= 0.50 \cdot 0.93 \cdot 0.872 = 0.405 = 40.5\%\end{aligned}$$

This means that only 40.5% of the original amount is transferred to the milk. Moreover, in this example is is considered that no other losses occur, while in real life the coal needs to be processes and transported. In any case, you need to remember:

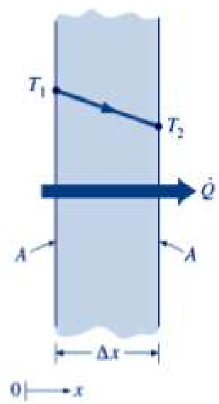
- Efficiencies can be multiplied.
- An efficiency can be expressed as a ratio ($0 \leq \eta \leq 1$) or as a percentage ($0\% \leq \eta \leq 100\%$). Both expressions mean physically the same thing.

f) It expresses the energy dissipation along the way. Only 40.5% of the energy in the coal is being transferred to the to be heated milk. The remaining 59.5% is lost as heat to the environment in the power plant, the grid, and the microwave itself.

2.5 Heat loss from an oven

Analysis

The inside of an oven door is measured at 180 °C, the outside at 50 °C. The oven door is 0.25 m in height and 0.15 m in width, and has a thickness of 5 mm. The glass has a thermal conductivity of 0.70 $\mathrm{W\,m^{-1}\,K^{-1}}$. We need to determine the rate of heat loss through the glass panel of the oven door. A schematic is presented below:



Here, $T_1 = 180°\mathrm{C}$ and $T_2 = 50°\mathrm{C}$

Approach

Assumptions

Route to solution

The governing equation for this problem is

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x}$$

where k is the thermal conductivity, A the area of the glass oven door, ΔT the temperature difference and Δx is the thickness of the oven door. Note that $\Delta T = T_2 - T_1$ gives a negative value.

Elaboration

We start with calculating the area:

$$A = 0.25 \cdot 0.15 = 0.0375\mathrm{m}^2$$

Subsequently the temperature difference

$$\Delta T = T_2 - T_1 = 50 - 180 = -130°\mathrm{C}$$

Substitution in the equation:

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x} = -0.70 \cdot 0.0375 \cdot \frac{-130}{0.005} = 683W$$

Note that the equation for $\dot{Q}$ has a minus sign. This is present here because heat flow is positive at a negative temperature gradient. As the temperature gradient is negative, and $\dot{Q}$ must then be positive, a minus sign is placed. Keep this in mind, it is a very expensive mistake on your exam!

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.6 Heat conduction through the bottom of a pan

Analysis

We are comparing two different pans, a aluminium one with a thickness of 4 mm and a conductivity k_{alu} of 237 $\mathrm{W\,m^{-1}\,K^{-1}}$, and a pan consisting of a 3 mm thick copper layer sandwiched between two 1 mm thick aluminium layers. The thermal conductivity. The thermal conductivity of copper k_{copper} is 390 $\mathrm{W\,m^{-1}\,K^{-1}}$.

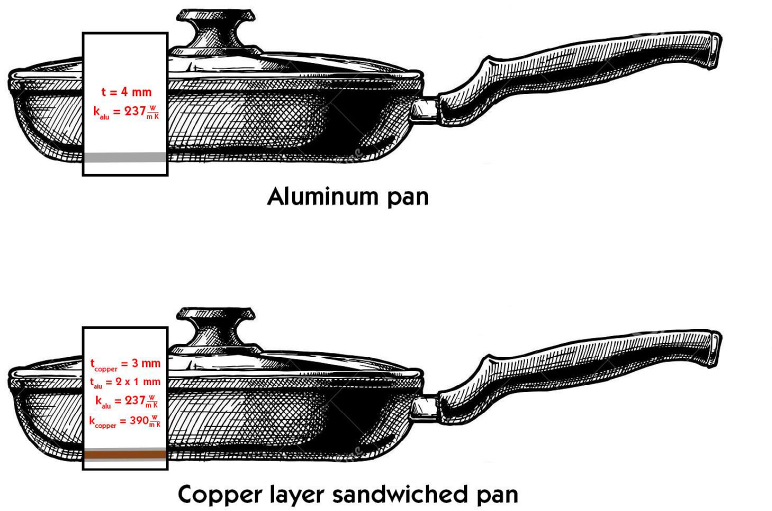


Figure 2.2: Pans with different bottoms

The questions are:

1. Does the pan with the sandwiched copper conduct better than the pan consisting solely of aluminium
2. What is the maximum temperature of the copper layer when the pan bottom is 200 °C at its lower surface and 175 °C at its upper surface.

Approach

Assumptions

Route to solution

Firstly, the heat resistance of the old design (solely aluminium) needs to be determined. This can be done with

$$R = \frac{\Delta x}{k_{alu} A}$$

The new design consists of three conduction resistance values in series for a flat plate design. The heat flow runs into these resistances one after another, resulting the total heat resistance to be the sum of all three separate resistances:

$$R_{tot} = 2 \cdot R_{alu} + R_{copper} = 2 \cdot \frac{\Delta x_{alu}}{k_{alu} A} + \frac{\Delta x_{copper}}{k_{copper} A}$$

Comparing these values will give an answer to the first question.

Now for the second question:

Using logic, the maximum temperature of the copper layer will be at the contact point with the lower aluminum layer. This temperature will be called $T_{a/k}$. The heat flow has just passed the heat resistance of the aluminum layer, as stated in the formula:

$$\dot{Q} = \frac{\Delta T_{alu}}{R_{alu}}$$

In this formula, the $\Delta T_{alu} = T_{lower} - T_{a/k}$, as the temperature difference has already been defined. The same heat flow can also be defined using the total heat resistance instead of the aluminum part. After all, the heat flow has to have the same value while flowing through all the subsequent layers.

$$\dot{Q} = \frac{\Delta T_{tot}}{R_{tot}}$$

In this equation, the total temperature difference ΔT_{tot} is 25 °C (200-175). Using both equations, a substitution of variables can be used to botain the correct answer:

$$\frac{\Delta T_{alu}}{R_{alu}} = \frac{\Delta T_{tot}}{R_{tot}}$$
$$\frac{T_{lower} - T_{a/k}}{\left(\frac{\Delta x_{alu}}{k_{alu}A}\right)} = \frac{T_{lower} - T_{upper}}{\left(2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A}\right)}$$

With some rewriting, the unkown $T_{a/k}$ can be determined:

$$T_{a/k} = T_{lower} - \left[\frac{T_{lower} - T_{upper}}{\left(2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A}\right)}\right] \cdot \left(\frac{\Delta x_{alu}}{k_{alu}A}\right)$$

Elaboration

We start with calculating the conductive resistances of both types of pans. For the solely aluminium pan:

$$R = \frac{\Delta x}{k_{alu}A} = \frac{4 \cdot 10^{-3}}{237 \cdot A} = \frac{1.69 \cdot 10^{-5}}{A} \mathrm{K\,W^{-1}}$$

For the pan with a strip of copper sandwiched between aluminium:

$$R_{tot} = 2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A} = 2 \cdot \frac{1 \cdot 10^{-3}}{237 \cdot A} + \frac{3 \cdot 10^{-3}}{390 \cdot A} = \frac{1.61 \cdot 10^{-5}}{A} \mathrm{K\,W^{-1}}$$

If both values are compared, it is obvious that the new design has a lower heat resistance at the same used area. In other words, it has a better thermal conduction, despite the larger thickness. Notice that the surface area A is not needed as a comparison is made for which the area is not a changing variable. Only if the exact solution of the heat resistance is needed, the surface area might be needed.

For the second question, substituting all values:

$$\begin{aligned} T_{a/k} &= T_{lower} - \left[\frac{T_{lower} - T_{upper}}{\left(2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A}\right)}\right] \cdot \left(\frac{\Delta x_{alu}}{k_{alu}A}\right) \\ &= 200 - \left[\frac{200 - 175}{\left(\frac{1.61 \cdot 10^{-5}}{A}\right)}\right] \cdot \left(\frac{1 \cdot 10^{-3}}{237 \cdot A}\right) = 200 - 6.54 = 193.5°\mathrm{C} \end{aligned}$$

The maximum temperature of the copper value, at the contact point with the lower aluminum layer, will be 193.5 °C. Notice that the surface area A is not relevant for the obtained value.

Evaluation

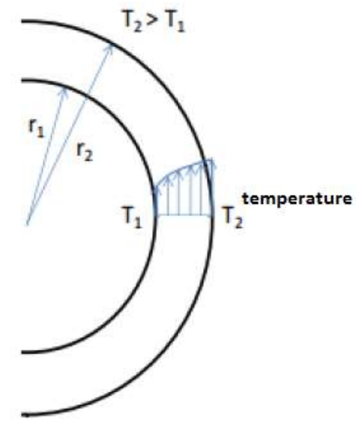
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.7 Temperature profiles

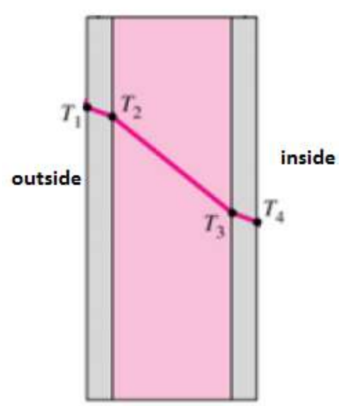
a) The temperature profile through a tube with cold water in a warmer environment is presented below. As the cross-sectional area on the inside is quite small, the resistance value will be higher. By comparison, the area on the outside will be large, leading to a lower resistance value. Therefore, the temperature gradient is steeper at the inside, gradually diminishing to the outside boundary. This also follows from the Fourier law:

$$\dot{Q} = -kA\frac{dT}{dx}$$



Concluding, the temperature at the inner boundary will have a sharp rise and afterwards rise more and more slowly towards the outer boundary. The profile will have a convex shape.

b)The temperature profile through a refrigerator door with an insulation layer will be like the following:



Both thin metal plates will have a lower heat resistance value, therefore leading to a small decrease in temperature. However, the insulating material will have a large heat resistance value (as this is its purpose) and therefore take into account for the largest part of the temperature decline. The temperature gradient needs to be significantly larger in the insulating material section as metals are quite good conductors while the insulating material is not (remember the formula below).

$$R = \frac{\Delta x}{kA}$$

2.8 Ski jacket - Hand in

a) Required characteristics of the sketch:

- Straight lines
- No jumps
- Bigger temperature drop in air-layer, smaller in the jacket, but identical in similar layers.
- Clearly indicated layers.
- Temperatures on the in- and outside.

b)

$$\dot{Q} = \frac{\Delta T_{\text{total}}}{R_{\text{total}}}$$

where:

$$R_{\text{total}} = R_{\text{synth}} + R_{\text{air}} + R_{\text{synth}} + R_{\text{air}} + R_{\text{synth}} + R_{\text{air}} + R_{\text{synth}} + R_{\text{air}} + R_{\text{synth}}$$

Resistance in the air layer:

$$R_{\text{air}} = \frac{\Delta x_{\text{air}}}{k_{\text{air}} A} = \frac{1.5 \cdot 10^{-3}}{0.026 \cdot 1.1} = 0.052 \text{ K/W}$$

$$R_{\text{synth}} = \frac{\Delta x_{\text{synth}}}{k_{\text{synth}} A} = \frac{0.1 \cdot 10^{-3}}{0.13 \cdot 1.1} = 0.0007 \text{ K/W}$$

$$\Rightarrow R_{\text{total}} = 5 \cdot R_{\text{synth}} + 4 \cdot R_{\text{synth}} = 0.21 \text{ K/W}$$

Rate of heat loss:

$$\dot{Q} = \frac{\Delta T_{\text{total}}}{R_{\text{total}}} = \frac{28 - (-5)}{0.21} = 155 \text{ W}$$

c)

$$R_{0,5 \text{ mm } synth} = \frac{0.5 \cdot 10^{-3}}{0.13 \cdot 1.1} = 0.0035 \text{ K/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{0,5 \text{ mm synth}}} = \frac{33}{0.0035} = 9438 \text{ W}$$

d) To have the same rate of heat transfer:

$$R_{\text{wool}} = R_{\text{total}}$$

$$R_{\text{wool}} = \frac{\Delta x_{\text{wool}}}{k_{\text{wool}} A} \rightarrow \Delta x_{\text{wool}} = R_{\text{total}} \cdot k_{\text{wool}} \cdot A = 0.21 \cdot 0.035 \cdot 1.1 = 8.2 \text{ mm}$$

e) Firstly, determine the temperature T_{surf} at the back surface of the first layer (where the first air layer starts):

$$\dot{Q} = \frac{\Delta T}{R} = \frac{T_{\text{surf}} - T_{\text{out}}}{R_{\text{synth}}} \Rightarrow T_{\text{surf}} = \dot{Q} R_{\text{synth}} + T_{\text{out}} = 155 \cdot 0.0007 + (-5) = -4.89 \text{ °C}$$

Then determine the temperature T_{air} at the end of the first air layer:

$$\dot{Q} = \frac{\Delta T}{R} = \frac{T_{\text{surf}} - T_{\text{out}}}{R_{\text{synth}} + R_{\text{air}}} \Rightarrow T_{\text{surf}} = \dot{Q}\left(R_{\text{synth}} + R_{\text{air}}\right) + T_{\text{out}} = 155 \cdot 0.053 + (-5) = 3.22 \text{ °C}$$

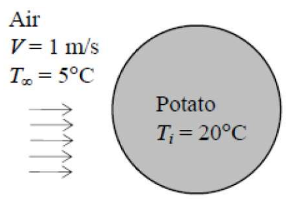
So the temperature has increased from below zero to above zero through the <u>first air layer</u> (seen from the outside of the jacket).

Solutions lecture 3

3.1 Cooling a potato

Analysis

In the assignment, several heat transfer coefficients are given for different airflow velocities. The heat transfer at the start of the cooling process should be determined. A sketch is presented below



Approach

Assumptions

- The conditions of the surroundings are constant.
- The potato has a spherical shape.
- The heat transfer coefficient is constant across the surface.

Route to solution

This exercise can be solved by substituting the known values into the Newton's cooling law:

$$\dot{Q} = hA(T_s - T_\infty)$$

where h is the heat transfer coefficient with an airflow of 1 m/s (19.1 $\mathrm{W\,m^{-2}\,K^{-1}}$), A is the surface area of the potato, T_s is the initial temperature, and T_∞ the temperature of the surroundings.

Elaboration

First, calculating the area of the sphere:

$$A = 4\pi R^2 = \pi D^2 = \pi(0.08)^2 = 0.2011\text{m}^2$$

Substitution of all values in Newtons cooling law yields:

$$\dot{Q} = 19.1 \cdot 0.2011 \cdot (20 - 5) = 5.8\text{W}$$

Evaluation

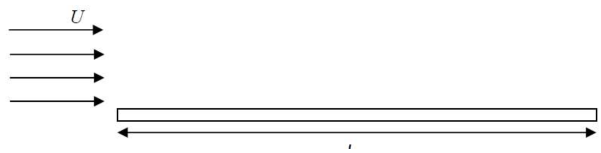
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

3.2 Heat loss through a wall

Analysis

We need to determine the rate of heat loss by convection. The wind is blowing parallel to the wall and thus it can be considered as an airflow across a flat plate, as can be seen in the figure below. Provided data is airflow velocity $U = 55\text{km}\,\text{h}^{-1}$, length of the wall $L = 10M$, surface temperature of the wall $T_s = 12°\text{C}$, and the outside air temperature $T_\infty = 5°\text{C}$



Approach

Assumptions

- The conditions of the surroundings are constant.
- The critical Reynolds number is $\text{Re} = 5 \cdot 10^5$.
- Heat transfer by radiation is negligible.
- Air is considered an ideal gas.

Route to solution

We need to determine $\dot{Q}$, so the solution route is:

1. Determine Re and Pr at the average temperature (use quantities like μ, ρ, k,Pr from tables).
2. Choose the right correlation based on geometry and Re.
3. Determine Nu.
4. Derive h.
5. Substitute into Newton's cooling law.

Elaboration

Firstly, we need to determine the Reynolds and Prandtl number at the average temperature.

$$T_f = \frac{T_s - T_\infty}{2} = \frac{12 + 5}{2} = 8.5°\text{C}$$

The closest temperature is 10 °C, so we take the table data at that temperature: $\rho = 1.246\ \text{kg}\,\text{m}^{-3}$, $\mu = 1.778 \cdot 10^{-5}\ \text{kg}\,\text{m}^{-1}\,\text{s}^{-1}$. The airflow is $U = \frac{55}{3.6} = 15.3\text{m}\,\text{s}^{-1}$. Substituting these values in the equation for the Reynolds number:

$$\text{Re} = \frac{\rho U L}{\mu} = \frac{1.246 \cdot 15.3 \cdot 10}{1.778 \cdot 10^{-5}} = 10.71 \cdot 10^6[-]$$

The Prandtl number at this temperature is also given, Pr=0.7336[-].

Now, we have to choose the right correlation based on the geometry and Reynolds number. Because the Reynolds number is larger than $5.0 \cdot 10^5$, so the flow is turbulent. For a turbulent flow across a flat plate, the needed coefficients are: a=0.037, b=0.8 and c=1/3.

With this, the Nusselt number can be determined:

$$\text{Nu} = a \cdot \text{Re}^b \cdot \text{Pr}^c = 0.037 \cdot (10.71 \cdot 10^6)^{0.8} \cdot 0.7336^{\frac{1}{3}} = 14.0 \cdot 10^3[-]$$

The Nusselt number is defined as:

$$\mathrm{Nu} = \frac{h \cdot L}{k}$$

At 10 °C, the value for the thermal conductivity is $k = 0.02439\ \mathrm{W\,m^{-1}\,K^{-1}}$. Rewriting and substitution yields

$$h = \frac{\mathrm{Nu} \cdot k}{L} = \frac{14.0 \cdot 10^3 \cdot 0.02439}{10} = 34.2 \mathrm{W\,m^{-2}\,K^{-1}}$$

Substituting this in Newton's cooling law:

$$\dot{Q} = h \cdot A \cdot \Delta T$$

where $A = 10 \cdot 4 = 40\mathrm{m}^2$ is the area, and $\Delta T = 12 - 5 = 7°\mathrm{C}$ is the temperature difference. Substituting gives:

$$\dot{Q} = h \cdot A \cdot \Delta T = 34.2 \cdot 40 \cdot 7 = 9.58\mathrm{kW}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

3.3 Flow over an airplane wing

a) A boundary layer is the layer between an object and the region in a flow that is close to a boundary surface, in which the influence of the surface is still noticeable regarding speed or temperature. There are two types of boundary layers; the velocity boundary layer and the thermal boundary layer. Both layers contain the transition of velocity and temperature, respectively, between the object in the flow and the undisturbed surroundings. A sketch of this phenomena is given below.

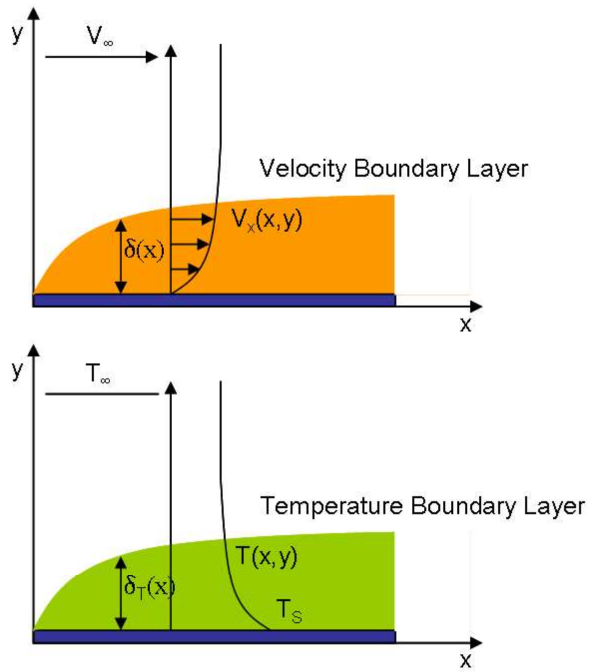


Figure 3.1: Thermal velocity of an airplane wing

b) The highest heat loss takes place where the temperature difference is maximal at a minimal distance (boundary layer thickness). $\dot{Q}$ is directly proportional to ΔT and h, given by the formula:

$$\dot{Q} = hA\Delta T$$

Heat transfer coefficient h gets larger with a decrease in boundary layer thickness, since the temperature gradient increases. Since the temperature difference between the wing surface and the boundary layer limit in this assignment is constant, the minimal thickness will determine where the heat loss is greatest. That is the case directly at the front edge of the wing.

c) In the laminar layer, the particles in the flow move in 'organized layers' across the surface, and will thus not promote heat transfer. In the turbulent layer, the particles do not only flow across the surface, but also swirl to- and from the surface, thus promoting heat transfer. The turbulent boundary layer will consequently have a much steeper temperature gradient just above the surface than the laminar boundary layer. The heat transfer will increase at the transition between the laminar to the turbulent flow.

3.4 Competition of soccer and tennis balls

a)

Analysis

We have a soccer ball and a tennis ball, with diameters $D_s = 0.22m$ and $D_t = 0.066m$. The soccer ball has a velocity of 58 km/h. What velocity should the tennis ball have to obtain a flow pattern similar to the soccer ball?

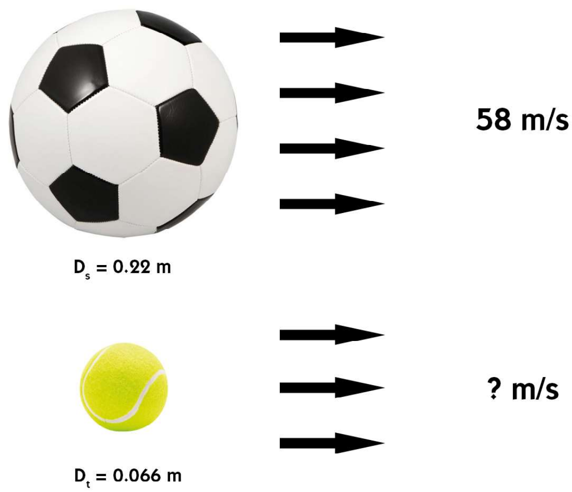


Figure 3.2: Comparison soccer and tennis ball

Approach

Assumptions

Assume that both balls have similar roughness patterns.

Route to solution

The flow profile depends on the Reynolds number, and the Reynolds number depends on the density of the flow medium, the flow velocity, the diameter of the sphere and the surface roughness, described by the formula:

$$\mathrm{Re} = \frac{\rho U D}{\mu}$$

The density of the flow medium and the surface roughness is the same for the soccer- and tennis ball. The difference in diameter will thus have to be compensated by a difference in velocity to keep the Reynolds number (and thus the flow profile) the same.

$$\mathrm{Re}_{tennis} = \mathrm{Re}_{soccer}$$

$$\frac{\rho U_{tennis} D_{tennis}}{\mu} = \frac{\rho U_{soccer} D_{soccer}}{\mu}$$

The densities and roughnesses cancel, and with some rewriting the velocity of the tennis ball can be obtained:

$$U_{tennis}D_{tennis} = U_{soccer}D_{soccer}$$
$$U_{tennis} = \frac{U_{soccer}D_{soccer}}{D_{tennis}}$$

Elaboration

The diameters are known, only the velocity of the soccer ball must me in the correct units

$$U_{soccer} = \frac{58}{3.6} = 16.11\,\mathrm{m\,s^{-1}}$$

Substitution of the values in the equation for the velocity of the tennis ball:

$$U_{tennis} = \frac{U_{soccer}D_{soccer}}{D_{tennis}} = \frac{16.11 \cdot 0.22}{0.066} = 53.7\,\mathrm{m\,s^{-1}}$$

This is equivalent to $53.7 \cdot 3.6 = 193\mathrm{km\,h^{-1}}$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

b) If the Reynolds number of the soccer ball and the tennis ball are the same, then also the Nusselt number is the same. The formula for the Nusselt number is:

$$\mathrm{Nu} = \frac{hD}{k}$$
$$\mathrm{Nu}_{soccer} = \mathrm{Nu}_{tennis}$$
$$\frac{h_{soccer}D_{soccer}}{k} = \frac{h_{tennis}D_{tennis}}{k}$$

Both balls are present in the same air, so the k is the same.

Because the diameter of the soccer ball is greater than the diameter of the tennis ball, the heat transfer coefficient of the soccer ball will be smaller than the coefficient of the tennis ball, in order to satisfy the above balance. In other terms: the ball diameter is inversely proportional to the heat transfer coefficient ($h \propto D^{-1}$). For the heat loss the ball surface is important. The heat loss $\dot{Q}$is calculated by:

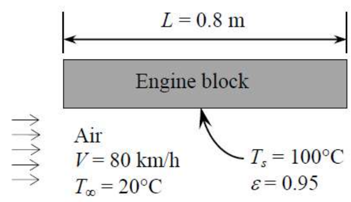
$$\dot{Q} = hA\Delta T$$

The temperature difference is the same for the two balls, so $\dot{Q}$ is directly proportional to hA ($\dot{Q} \propto hA$). The sphere surface area is directly proportional to D^2 and h is inversely proportional to D. Consequently, $\dot{Q}$ is directly proportional to ($D^2 \cdot D^{-1} = D$) the diameter. The soccer ball has the greatest diameter and will thus have the greatest heat loss.

3.5 Cooling of an engine

Analysis

We need to determine the rate of heat transfer from the bottom surface of the engine block by convection, at a velocity of 80 km/h. See the figure below:



The height of the block is 0.50m, the width 0.40m, the length 0.80m.

Approach

Assumptions

Conditions of the surroundings are constant, and the critical Reynoldsnumber is $5.0 \cdot 10^5$.

Route to solution

We need to determine $\dot{Q}$, so the solution route is:

1. Determine Re and Pr at the average temperature (use quantities like μ, ρ, k,Pr from tables).
2. Choose the right correlation based on geometry and Re.
3. Determine Nu.
4. Derive h.
5. Substitute into Newton's cooling law.

Elaboration

We start with determining the average temperature:

$$T_f = \frac{T_s - T_\infty}{2} = \frac{100 + 20}{2} = 60°\mathrm{C}$$

At 60 °C, values for the density and viscosity are: $\rho = 1.059\mathrm{kg\,m^{-3}}$ and $\mu = 2.008 \cdot 10^{-5}\mathrm{kg\,m^{-1}\,s^{-1}}$. Substituting these values in the formula for the Reynolds number:

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{1.059 \cdot 22.21 \cdot 0.8}{2.008 \cdot 10^{-5}} = 937.6 \cdot 10^3[-]$$

At 60 °C, the Prandtl number is $\mathrm{Pr} = 0.7202[-]$.

Now, the Reynoldsnumber is larger than $5 \cdot 10^5$, so the flow is turbulent. Based on this and the geometry, the coefficients are a=0.037, b=0.8 and c=1/3. With these coefficients, the Nusselt number can be determined:

$$\mathrm{Nu} = a \cdot \mathrm{Re}^b \cdot \mathrm{Pr}^c = 0.037 \cdot (937.6 \cdot 10^3)^{0.8} \cdot 0.7202^{\frac{1}{3}} = 1.99 \cdot 10^3[-]$$

The Nusselt number is defined as:

$$\mathrm{Nu} = \frac{h \cdot L}{k}$$

At 60 °C, the value for the thermal conductivity is $k = 0.02808\ \mathrm{W\,m^{-1}\,K^{-1}}$. Rewriting and substitution yields

$$h = \frac{\mathrm{Nu} \cdot k}{L} = \frac{1.99 \cdot 10^3 \cdot 0.02808}{0.8} = 69.8\mathrm{W\,m^{-2}\,K^{-1}}$$

Substituting this in Newton's cooling law:

$$\dot{Q} = h \cdot A \cdot \Delta T$$

where $A = 0.8 \cdot 0.4 = 0.32\mathrm{m}^2$ is the area, and $\Delta T = 100 - 20 = 80°\mathrm{C}$ is the temperature difference. Substituting gives:

$$\dot{Q} = h \cdot A \cdot \Delta T = 69.8 \cdot 0.32 \cdot 80 = 1.79\mathrm{kW}$$

3.6 Roof of a train - Hand in

a) In this assignment the temperature must be found at which the amount of heat absorbed by radiation is equal to the amount of heat emitted by convection.

Assumptions:

- The environmental conditions are constant.
- The critical Reynolds number is $\mathrm{Re}_{\mathrm{crit}} = 5 \cdot 10^5$
- Heat loss due to radiation from the roof of the train to the environment is negligible

Energy balance of the roof:

$$\dot{q}_{\mathrm{rad}} - \dot{q}_{\mathrm{conv}} = 0$$

Where $\dot{q}_{\mathrm{conv}}$ equals:

$$\dot{q}_{\mathrm{conv}} = h \cdot (T_{\mathrm{s}} - T_\infty)$$

In order to determine h, the Nusselt number should be determined.

First step in doing so is making an assumption for the average fluid temperature T_{f}, which is assumed to be 30 °C.

Average fluid properties of air at 30 °C:

$$\rho = 1.164 \text{ kg/m}^3$$

$$\mu = 1.872 \cdot 10^{-5} \text{ kg/m} \cdot \text{s}$$

$$\mathrm{Pr} = 0.7282$$

$$k = 0.02588 \cdot 10^{-5} \text{ W/mK}$$

Now the Reynolds number Re can be determined (characteristic length for the flat plate equals its length):

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{1.164 \cdot 19.4 \cdot 8}{1.872 \cdot 10^{-5}} = 967 \cdot 10^6$$

After having determined the Reynolds number Re and Prandtl number Pr, an applicable correlation for the Nusselt number Nu can be found and used in order to determine the Nusselt number Nu.

$$\mathrm{Nu} = 0.037 \cdot \mathrm{Re}^{0.8} \cdot \mathrm{Pr}^{1/3}$$

$$= 0.037 \cdot \left(9.67 \cdot 10^6\right)^{0.8} \cdot 0.7282^{1/3} = 12.9 \cdot 10^3$$

And now h can be determined:

$$h = \frac{\mathrm{Nu} \cdot k}{L} = \frac{12.9 \cdot 10^3 \cdot 0.02588}{8} = 41.7 \ \cdot \text{W/m}^2\text{K}$$

Rewriting the energy balance:

$$\dot{q}_{\mathrm{rad}} - h \cdot (T_{\mathrm{s}} - T_\infty) = 0$$

$$\rightarrow T_{\mathrm{s}} = T_\infty + \frac{\dot{q}_{\mathrm{conv}}}{h} = 30 + \frac{200}{41.7} = 34.8°C$$

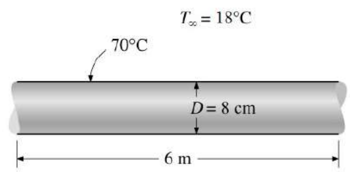
Finally, it must be checked whether at this temperature found the h would not be appreciably different than at the assumed temperature of 30 °C. If so, the whole approach must be repeated with a new estimate. If not, the answer can be regarded as converged. The latter is the case here, but it is not further demonstrated here. This is desirable in submitted work.

Solutions lecture 4

4.1 Cooling of a hot water pipe

Analysis

In this situation, natural convection will occur. Given data are: $T_s = 70\,°\mathrm{C}$, $T_\infty = 18\,°\mathrm{C}$, length is 6 metres and the diameter 8.0 cm. A sketch is presented below:



Approach

Assumptions

Route to solution

This natural convection problem can be solved by using the following steps:

1. Determine the average temperature.
2. Determine the Grashof number.
3. Determine the Rayleigh number.
4. Choose the right correlation based on geometry and the Rayleigh number.
5. Determine the Nusselt number.
6. Derive the value of h.
7. Substitute in Newton's cooling law.

Elaboration

We start with determining the average temperature:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{70 + 18}{2} = \frac{88}{2} = 44\,°\mathrm{C}$$

For natural convection, the Grasshof number needs to be calculated:

$$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where $9.81\,\mathrm{m\,s^{-2}}$ is the gravitational constant, β is the thermal expansion coefficient:

$$\beta = \frac{2}{T_s + T_\infty} = \frac{2}{342.15 + 291.15} = 3.15 \times 10^{-3}\,\mathrm{K^{-1}}$$

Note that the temperatures need to be substituted in **Kelvin**, not Celsius. Furthermore, D is the aforementioned diameter, and ν is the kinematic viscosity. The value for this is taken at 45 °C, to be $1.750 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$. Substituting these values:

$$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{9.81 \cdot (3.15 \cdot 10^{-3}) \cdot (70 - 18) \cdot 0.08^3}{(1.750 \cdot 10^{-5})^2} = 2.67 \cdot 10^6[-]$$

At 45 °C, the Prandtl number is $\mathrm{Pr} = 0.7241[-]$. With this, the Rayleigh number can be determined:

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 2.67 \cdot 10^6 \cdot 0.7241 = 1.94 \cdot 10^6[-]$$

Now, since the pipe is horizontal and cylindrical, we can use the following correlation:

$$\mathrm{Nu} = \left(0.6 + \frac{0.387 \cdot \mathrm{Ra}_D^{\frac{1}{6}}}{\left(1 + \left(\frac{0.559}{\mathrm{Pr}}\right)^{\frac{9}{16}}\right)^{\frac{8}{27}}} \right)^2$$

Substitution of all variables:

$$\mathrm{Nu} = \left(0.6 + \frac{0.387 \cdot (1.94 \cdot 10^6)^{\frac{1}{6}}}{\left(1 + \left(\frac{0.559}{0.7241}\right)^{\frac{9}{16}}\right)^{\frac{8}{27}}} \right)^2 = 17.58[-]$$

The Nusselt number is defined as:

$$\mathrm{Nu} = \frac{hD}{k}$$

At 45 °C, the value of k is $k = 0.026\,99\,\mathrm{W\,m^{-1}\,K^{-1}}$. Rewriting and substituting gives:

$$h = \frac{\mathrm{Nu}_D k}{D} = \frac{17.58 \cdot 0.02699}{0.08} = 5.93\,\mathrm{W\,m^{-2}\,K^{-1}}$$

We can now substitute all values in Newton's cooling law:

$$\dot{Q} = hA\Delta T$$

where the area is $A = \pi D L = \pi \cdot 0.08 \cdot 6 = 1.51\,\mathrm{m^2}$

$$\dot{Q} = h \cdot A(T_s - T\infty) = 5.39 \cdot 1.51 \cdot (70 - 18) = 466\,\mathrm{W}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

4.2 Convection of heat from a coffee machine

Analysis

This plate loses 90 W of heat, of which is 52.4% due to radiation and the other 47.6% due to natural convection. The situation is sketched below.

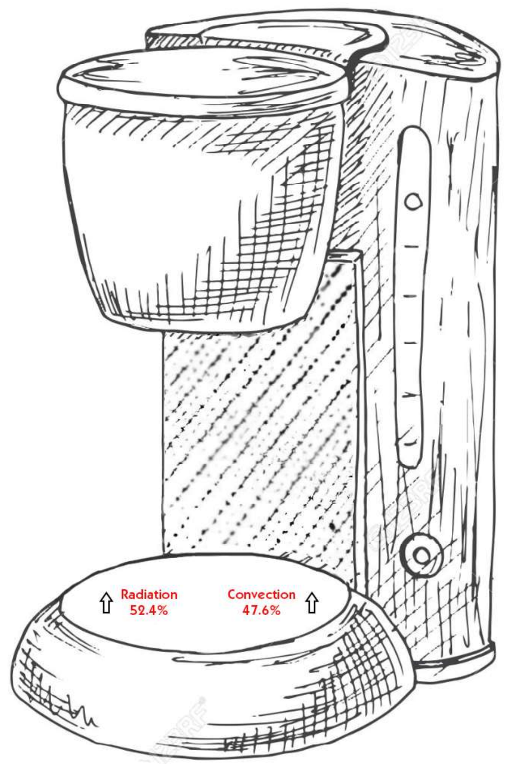


Figure 4.1: Heat loss of a coffee machine plate

This gives a total of $0.476 \cdot 90 = 42.84$ W of natural convection. For natural convection, the following relation is valid:

$$\dot{Q} = hA\Delta T$$

However, h is dependent on the average temperature, which is subsequently dependent on the temperature T_s of the plate, considering the fact that his is codependent for the film temperature of which the value will be based

$$T_f = \frac{T_s + T_\infty}{2}$$

We need to determine the temperature by means of iteration. The maximum temperature of the plate is given, 250 °C. A first initial guess for the temperature of the plate is 180 °C.

Approach

Assumptions

Route to solution

1. Determine the average temperature

2. Determine the Grashof number
3. Determine the Rayleigh number
4. Choose the right correlation based on geometry and the Rayleigh number
5. Determine the Nusselt number
6. Derive the value of h
7. Substitute in Newton's cooling law
8. If necessary, iterate

Elaboration

The average temperature is:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{180 + 20}{2} = 100\,^\circ\mathrm{C}$$

The Grashof number is defined as:

$$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

in which $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational constant, β is the thermal expansion coefficient:

$$\beta = \frac{1}{T_f} = \frac{1}{100 + 273} = 2.68 \times 10^{-3}\,\mathrm{K^{-1}}$$

Note that the temperatures in β has the unit of Kelvin, not Celsius. Furthermore L_c is the characteristic length, which, for a horizontal cylindrical flat plate with diameter D, is

$$L_c = \frac{A_s}{p} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4} = 0.04\mathrm{m}$$

At 100 °C, air has the following properties: $k = 0.030\,95\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $\nu = 2.306 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$. Substituting all these values give:

$$\mathrm{Gr} = \frac{9.81 \cdot (5.38 \cdot 10^{-3} \cdot (180 - 20) \cdot 0.04^3)}{(2.306 \cdot 10^{-5})^2} = 5.063 \cdot 10^5[-]$$

At 100 °C, the Prandtl number is $\mathrm{Pr} = 0.7111[-]$. With this, the Rayleigh number can be determined to be:

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 5.063 \cdot 10^5 \cdot 0.7111 = 3.60 \cdot 10^5[-]$$

Now, the situation concerns an upper part of a horizontal hot flat plate with $10^4 < \mathrm{Ra} < 10^7$. For this, the Nusselt number is

$$\mathrm{Nu} = 0.54\mathrm{Ra}^{\frac{1}{4}} = 0.54 \cdot (3.60 \cdot 10^5)^{\frac{1}{4}} = 13.2[-]$$

The Nusselt number is defined as

$$\mathrm{Nu} = \frac{hL_c}{k}$$

Substitution of all variables gives

$$h = \frac{Nu \cdot k}{L_c} = \frac{13.2 \cdot 0.03095}{0.04} = 10.2\,\mathrm{W\,m^{-2}\,K^{-1}}$$

Substitution of this result into Newton's cooling law, where the surface is $A = \frac{\pi}{4}D^2$:

$$\dot{Q} = hA\Delta T = 12.1 \cdot \frac{\pi}{4} \cdot 0.16^2 \cdot (180 - 20) = 32.8\,\mathrm{W}$$

As stated earlier, this 38.9 W is not in accordance with the 42.84 W we calculated earlier. This means that the plate will be hotter than 180 °C. This means we need to reiterate.

Let's say that the plate is 220 °C. The average temperature is then:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{220 + 20}{2} = 120\,^\circ\mathrm{C}$$

The Grashof number is defined as:

$$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

in which $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational constant, β is the thermal expansion coefficient:

$$\beta = \frac{1}{T_f} = \frac{1}{120+273} = 2.54 \times 10^{-3}\,\mathrm{K^{-1}}$$

At 120 °C, air has the following properties: $k = 0.032\,35\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $\nu = 2.522 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$. Substituting all these values give:

$$\mathrm{Gr} = \frac{9.81 \cdot (2.54 \cdot 10^{-3} \cdot (220-20) \cdot 0.04^3)}{(2.522 \cdot 10^{-5})^2} = 5.014 \cdot 10^5[-]$$

At 120 °C, the Prandtl number is $\mathrm{Pr} = 0.7073[-]$. With this, the Rayleigh number can be determined to be:

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 5.014 \cdot 10^5 \cdot 0.7073 = 3.55 \cdot 10^5[-]$$

Now, the situation concerns an upper part of a horizontal hot flat plate with $10^4 < \mathrm{Ra} < 10^7$. For this, the Nusselt number is

$$\mathrm{Nu} = 0.54\mathrm{Ra}^{\frac{1}{4}} = 0.54 \cdot (3.55 \cdot 10^5)^{\frac{1}{4}} = 13.2[-]$$

The Nusselt number is defined as

$$\mathrm{Nu} = \frac{hL_c}{k}$$

Substitution of all variables gives

$$h = \frac{Nu \cdot k}{L_c} = \frac{13.2 \cdot 0.03235}{0.04} = 10.7\,\mathrm{W\,m^{-2}\,K^{-1}}$$

Substitution of this result into Newton's cooling law:

$$\dot{Q} = hA\Delta T = 10.7 \cdot \frac{\pi}{4} \cdot 0.16^2 \cdot (220-20) = 42.9\,\mathrm{W}$$

This value is quite in accordance with the earlier presented 42.84 W. This indicates that the plate will be around 220 °C.

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

4.3 Heat convection parameters

For forced convection, it was provided in lecture 3, that Nu is a function of Re and Pr:

$$\mathrm{Nu} = f(\mathrm{Re}, \mathrm{Pr})$$
$$\frac{hL_c}{k} = f\left(\frac{\rho U L_c}{\mu}, \mathrm{Pr}\right)$$

This means that the group of parameters in which the heat transfer coefficient h is processed, is a function of ρ, Y, L_c, μ and Pr. We can rewrite this to:

$$h = f(\rho, U, L_c, \mu, \mathrm{Pr}, k)$$

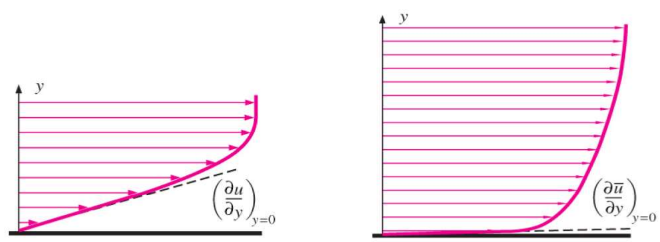
These variables will be discussed from here:

- Density ρ: Combined with the velocity U, the density forms the impulse of the fluid. The impulse can be seen as an indicator for the impact force of the oncoming particles. Is this value high, the fluid particles just above the surface will be hardly affected by the surface they are flowing above and by the particles close to the surface area. The particles close to the surface will stagnate by the surface and by the viscosity and will affect particles further away from the surface.

  Having a high density will result in a high impulse and therefore the boundary layer will grow less fast and remain thinner. A small boundary layer means a small distance for the heat to cover and subsequently an easier heat transfer. h will increase if the density increases.

  At a certain point the impulse will be that much larger than the viscous effects that the laminar flow will transform into turbulent flow. In that case the h will increase drastically as the gradient close to the surface will be a lot larger.

  For a turbulent flow, looking very close to the surface a very small boundary layer will appear, meaning a large value of h. It can be concluded that a turbulent flow results in a larger h.



Laminar flow   Turbulent flow

- Velocity U : Just like the density, the velocity is determining the impulse (ρU). So all statements for above are valid as well if ρ is replaced by U.

- Characteristic length L_c: the larger the surface area, the more the boundary layer has opportunity to grow. At the end of the characteristic length, the boundary layer will be thick with locally low h values, especially compared with the leading edge. The larger the L_c, the smaller the average value of h.

- Dynamic viscosity μ: the dynamic viscosity ensures the no slip boundary condition at the surface is felt in the fluid above the surface. The higher the viscosity, the larger the influence and the more thick the boundary layer will grow. The viscosity has an inverted effect compared to the density and the velocity. This is also quite obvious if the definition of the Reynolds number is considered.

  If the dynamic viscosity is high enough, the impulse will be dominated by the viscous effects and this will result in a laminar flow. The Reynolds number will be under the critical value for that specific geometry.

- Prandtl number Pr: the Prandtl number indicates how, given a certain velocity boundary layer (as well present without thermal effects), the thermal boundary layer will look like as a temperature difference is present between the surface and the free flow. A thermal boundary layer larger than (Pr ¡ 1), thinner than (Pr ¿ 1) or as thick as (Pr = 1) the velocity boundary layer.

  If in a flow the Prandtl number is enhanced, without altering velocity profile, the thermal boundary layer will get thinner. Using the statement at the density variable, the value of h will increase.

- Thermal conductivity k: If a fluid conducts heat better, the value of k will be higher. Convection can only be present if the particles are able to conduct heat. Convection can be stated as conduction being enhanced by the continuous supply of fresh particles and the take away of old particles. Therefore, if you calculate the convective heat transfer, the conductive heat transfer is already included and there is no need to calculate the conductive heat transfer independently.

For natural convection, the following correlations are valid:

$$\mathrm{Nu} = f(\mathrm{Ra}) = g(\mathrm{Gr}, \mathrm{Pr})$$

$$\frac{hL_c}{k} = f(g, \beta, (T_s - T_\infty), L_c, \nu, \mathrm{Pr}, k)$$

Giving for h:

$$h = f(g, \beta, (T_s - T_\infty), L_c, \nu, \mathrm{Pr}, k)$$

The gravitational constant g is always the same, and therefore has no influence on h. L_c, Pr and k have the same effect as in the case of forced convection. The temperature difference can be seen as the driver behind the phenomenon and therefore fulfills the same role as U in the case of forced convection. A higher value will result in a higher value for h.

- Thermal expansion coefficient β: This coefficient is larger the more a fluid expands with a certain increase in temperature. The larger the expansion, the more the decrease in density and therefore a larger flow velocity and a higher value for h.
- Kinematic viscosity ν: the kinematic viscosity is the dynamic viscosity divided by the density. Therefore it has the same effect as the effects of dynamic viscosity and density.

A last important remark is the influence of the geometry on the value of h. The geometry determines how the flow pattern will look like and which correlation is valid. Also, the roughness of the surface plays a large role: the rougher the surface the quicker the transition from laminar to turbulent flow.

4.4 Light bulb temperature

Analysis

Consider a 25 W lightbulb with a light-efficiency of 10 %. The lightbulb has a diameter of 8.0 cm, and an outside temperature of 25 °C. When assuming all heat is lost due to natural convection, determine the surface temperature of the lightbulb.

Approach

Assumptions

- Only natural convection
- Light bulb can be modelled as a sphere

Route to solution

1. Determine the average temperature
2. Determine the Grashof number
3. Determine the Rayleigh number
4. Choose the right correlation based on geometry and the Rayleigh number
5. Determine the Nusselt number
6. Derive the value of h
7. Substitute in Newton's cooling law
8. If necessary, iterate

Elaboration

We take an initial guess for the surface temperature, $T_s = 100$ °C. The average temperature is:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{100 + 25}{2} = 62.5\,°\mathrm{C}$$

The Grashof number is defined as:

$$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

in which $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational constant, β is the thermal expansion coefficient:

$$\beta = \frac{2}{T_f} = \frac{2}{298 + 373} = 2.98 \times 10^{-3}\,\mathrm{K^{-1}}$$

Note that the temperatures in β has the unit of Kelvin, not Celsius.

At 60 °C, air has the following properties: $k = 0.028\,08\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $\nu = 1.896 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$. Substituting all these values give:

$$\mathrm{Gr} = \frac{9.81 \cdot (2.98 \cdot 10^{-3} \cdot (100 - 25) \cdot 0.08^3}{(1.896 \cdot 10^{-5})^2} = 3.12 \cdot 10^6[-]$$

At 60 °C, the Prandtl number is $\mathrm{Pr} = 0.7202[-]$. With this, the Rayleigh number can be determined to be:

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 3.12 \cdot 10^6 \cdot 0.7202 = 2.25 \cdot 10^6[-]$$

Now, the situation concerns a sphere with $\mathrm{Ra}_D \leq 10^{11}$, where $Pr \geq 0.7$. For this, the Nusselt number is:

$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}} = 19.6$$

The Nusselt number is defined as

$$\mathrm{Nu} = \frac{hL_c}{k}[-]$$

Substitution of all variables gives

$$h = \frac{Nu \cdot k}{D} = \frac{19.6 \cdot 0.02808}{0.08} = 6.89\,\mathrm{W\,m^{-2}\,K^{-1}}$$

Substitution of this result into Newton's cooling law, where the surface is $A = \pi D^2$:

$$\dot{Q} = hA\Delta T = 6.89 \cdot \pi \cdot D^2 \cdot (100 - 25) = 10.4W$$

As stated earlier, this 10.4 W is not in accordance with the 22.5 W we need to have earlier. This means that the surface is hotter than 100°C. Reverse calculation shows a ΔT of 162 °C. After some iterations, a value of $T_s = 168°C$ is found. With this value, the following results are obtained:

$$T_f = 96.5\,°\mathrm{C} \approx 100\,°\mathrm{C}$$
$$\beta = 2.7 \times 10^{-3}\,\mathrm{K^{-1}}$$

At 100 °C, air has the following properties: $k = 0.030\,95\,\mathrm{W\,m^{-1}\,K^{-1}}$, $\nu = 2.306 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$ and $\mathrm{Pr} = 0.7202[-]$.

$$\mathrm{Gr} = 3.66 \cdot 10^6[-]$$
$$\mathrm{Ra} = 2.60 \cdot 10^6[-]$$
$$\mathrm{Nu} = 20.25[-]$$
$$h = 7.84\,\mathrm{W\,m^{-2}\,K^{-1}}$$
$$\dot{Q} = 22.53\,\mathrm{W}$$
$$T = 167.8\,°\mathrm{C}$$

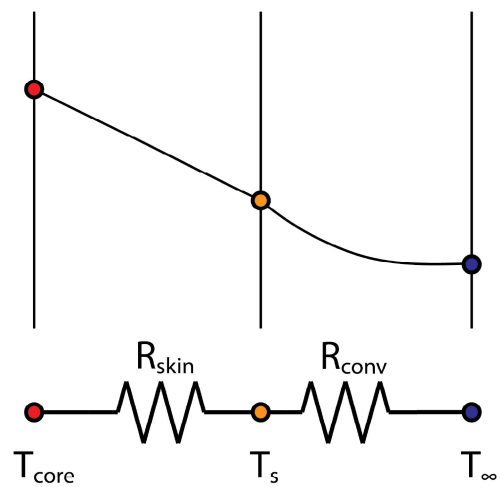
4.5 Cooling of a human head - Hand in

a) The skin temperature and heat loss must be calculated. The point of here is to find the state of equilibrium, in which the temperature of the main surface (T_s) is such that the heat flow through conduction in the skull is equal to the heat flow through convection around the skull. The law of conservation of energy is then fulfilled: one continuous heat flow 'moves' first via conduction and then via convection and on that route there can be no energy is lost or generated.

The procedure is therefore to choose a T_s and to calculate the heat flows through conduction and convection. If they do not match, then the calculation will have to be repeated with a newly estimated T_s in such a way that the largest heat flow of both will become smaller (and automatically the smaller larger).

This explanation of the approach should be clearly reflected in the elaborations and is at least as important as the elaboration itself. The exact way in which the sum is worked out may differ. A fairly mathematical criterion is formulated below. It is also conceivable (and perhaps more transparent) that the two heat flows are determined completely independently of each other and then compared. It is also possible to work with heat resistances and then it is possible, for example, that not both partial heat flows are compared, but, for example, the total heat flow with one of the two partial heat flows (by convection or conduction, one of the two is sufficient).

In short, several routes are possible and they are usually not necessarily right or wrong; what is important is that it is clearly explained what exactly is happening and what the thinking behind it is.



The figure above gives an idea of what the thermal resistance network looks like.
Important is that the rate of heat transfer through each resistor is equal to each other:

$$\dot{Q}_{\text{skin}} = \dot{Q}_{\text{conv}}$$

$$\frac{T_{\text{core}} - T_{\text{s}}}{R_{\text{skin}}} = \frac{T_{\text{s}} - T_{\infty}}{R_{\text{conv}}}$$

$$\frac{T_{\text{core}} - T_{\text{s}}}{\frac{D_2 - D_1}{2\pi k_{\text{skin}} D_1 D_2}} = \frac{T_{\text{s}} - T_{\infty}}{\frac{1}{h\pi D_2^2}}$$

An assumption for T_{s} should be made in order to determine h.

The final surface temperature should close to T_{s} =35.59 °C.

The average fluid properties at a temperature of 17.795 °C are:

$$\rho = 1.2133 \text{ kg/m}^3$$

$$\nu = 1.4957 \cdot 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7193$$

$$k = 0.025 \text{ W/mK}$$

$$\beta = \frac{1}{17.795 + 273.15} = 0.0034 \text{ K}^{-1}$$

With this given, the Grashof number Gr can be determined, where the characteristic length equals the diameter of the head.

$$\text{Gr} = \frac{g \cdot \beta \cdot (T_\text{s} - T_\infty) \cdot D_2^3}{\nu^2} = \frac{9.81 \cdot 0.0034 \cdot (35.59 - 0) \cdot 0.18^3}{(1.4957 \cdot 10^{-5})^2} = 3.1283 \cdot 10^7$$

And the Rayleigh number Ra:

$$\text{Ra} = \text{Gr} \cdot \text{Pr} = 3.1283 \cdot 10^7 \cdot 0.7193 = 2.2501 \cdot 10^7$$

As the Rayleigh number Ra has been determined, an applicable correlation for the Nusselt number Nu can be used to calculate this number:

$$\text{Nu} = 2 + \frac{0.589 \cdot \text{Ra}^{1/4}}{\left[1 + (0.469/\text{Pr})^{9/16}\right]^{4/9}} = 2 + \frac{0.589 \cdot \left(2.2501 \cdot 10^7\right)^{1/4}}{\left[1 + (0.469/0.7193)^{9/16}\right]^{4/9}} = 33.35$$

Having determined the Nusselt number Nu, the heat transfer coefficient h can be determined:

$$h = \frac{\text{Nu}k}{D_2} = \frac{33.35 \cdot 0.025}{0.18} = 4.63 \text{ W/m}^2\text{K}$$

Having determined the heat transfer coefficient, rate of heat transfer conducted through the skin and due to convection can be calculated:

$$\dot{Q}_\text{skin} = \frac{T_\text{core} - T_\text{s}}{\frac{D_2 - D_1}{2\pi k_\text{skin} D_1 D_2}} = \frac{37 - 35.59}{\frac{0.18 - 0.175}{2\pi \cdot 0.3 \cdot 0.175 \cdot 0.18}} = 16.74 \text{ W}$$

$$\dot{Q}_\text{conv} = \frac{35.59 - 0}{\frac{1}{4.63 \cdot \pi \cdot 0.18^2}} = 16.76 \text{ W}$$

Finally, it must be checked whether at this temperature found the h would not be appreciably different than at the assumed temperature. If so, the whole approach must be repeated with a new estimate. If not, the answer can be regarded as converged. The latter is the case here, but it is not further demonstrated here. This is desirable in submitted work.

Solutions lecture 5

5.1 Heat loss of a person by radiation

Analysis

We need to determine the rate of heat loss from a person by radiation.

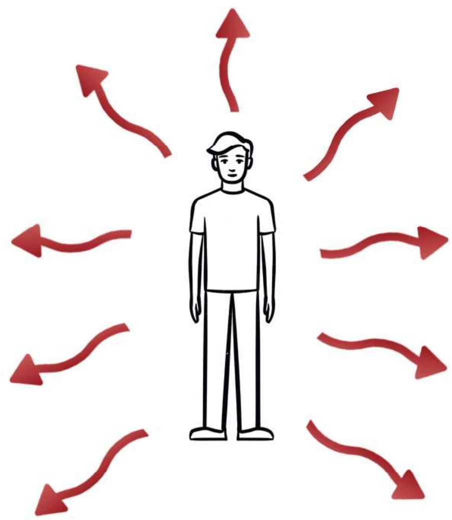


Figure 5.1: Radiation person

Given values are the surface area of the person, $A = 1.7\,\mathrm{m}^2$, the emissivity constant $\varepsilon = 0.7$, the temperature of the person, $T_s = 32\,°\mathrm{C}$, and the temperature of the environment (the wall temperature) of $T_\infty = 27\,°\mathrm{C}$

Approach

Assumptions

Route to solution

This question can be solved by substituting all values in the Stefan-Boltzmann law:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot (T^4 - T_\infty^4)$$

Where $\sigma = 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. Note that the temperatures are in **Kelvin**, not Celsius!

Elaboration

Substituting all values gives:

$$\dot{Q}_{rad} = 0.7 \cdot 5.67 \cdot 10^{-8} \cdot 1.7(305.15^4 - 300.15^4) = 37.4W$$

Evaluation

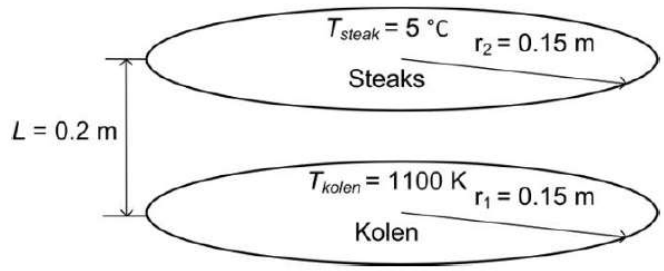
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

5.2 The BBQ

Analysis

We need to determine the initial rate of radiation heat transfer from the coal bricks to the steaks,and the initial rate of radiation heat transfer to the stakes if the side opening of the grill is covered by aluminium foil, which can be approximated as a re-radiating surface. The diameter of the grill is 0.30m. The coal bricks have a temperature of 827 °C, the steaks have an initial temperature of 5 °C. The distance between the bricks and the steaks is 0.20 m. See the sketch below.



Approach

Assumptions

Both surfaces can be treated as blackbodies, we assume the grill to be completely covered with steaks.

Route to solution

This question can be solved by substituting all values in the Stefan-Boltzmann law:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot (T_{coal}^4 - T_{steak}^4)$$

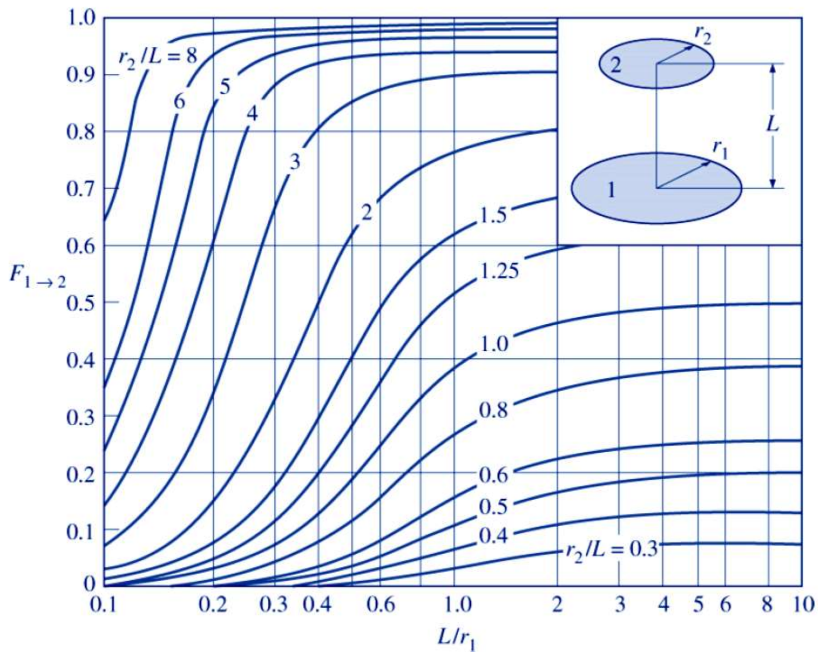
Where $\sigma = 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. Note that the temperatures are in **Kelvin**, not Celsius! However, not all radiation will reach the steaks, since there is a fair distance between the surfaces. Consequently, the 'visibility factor' $F_{1\to2}$ needs to be calculated. This factor gives the ratio between the radiation coming from surface 1 reaching surface 2. Since both bodies can be treated as black bodies, they both have an emissivity of $\varepsilon = 1$.

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot F_{1\to2}(T_{coal}^4 - T_{steak}^4)$$

Determining the visibility factor $F_{1\to2}$ can be done using the diagram given in lecture 5, also given below.

Substituting all values, in Stefan Boltzmann's law will give the initial rate of radiation heat transfer.

For the second part, we assume that the aluminum foil is an ideal reflector, such that all radiation coming from the coals reaches the steaks. In other words, $F_{1\to2} = 1$. Substituting all values will give the initial rate of radiation heat transfer for this case.



Elaboration

We first need to determine the visibility factor from the figure. The following ratios are needed:

L/r_{coal} (along the x-axis) $\implies$ in this situation; $L/r_{coal} = 0.2/0.15 = 1.33$ (x-axis)

r_{steak}/L (along the lines) $\implies$ in this situation; r_{steak}/L = 0.15 / 0.2 = 0.75 (line)

The line 0.75 is not depicted in the diagram, so the answer needs to be extrapolated between the 0.6 and 0.8 line at the 1.33 point on the x-axis. This gives a visibility factor of about 0.28. All parameters are now determined and can be substituted into the formula:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot F_{1\rightarrow 2}(T^4_{coal} - T^4_{steak}) = 1 \cdot 5.67 \cdot 10^{-8} \cdot \pi \cdot 0.15^2 \cdot 0.28 \cdot (1100^4 - 278^4) = 1637\text{W}$$

And for the case when the sides are covered with aluminium foil:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot F_{1\rightarrow 2}(T^4_{coal} - T^4_{steak}) = 1 \cdot 5.67 \cdot 10^{-8} \cdot \pi \cdot 0.15^2 \cdot 1 \cdot (1100^4 - 278^4) = 5845\text{W}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

5.3 Radiation of heat from a coffee machine

Analysis

We look at the coffee machine from exercise 4.2. We need to determine the emissivity of the heater plate surface, and find the total thermal resistance between the heater surface and the surrounding, as well as the total heat transfer coefficient, including convection and radiation.

Approach

Assumptions

Route to solution

The total power is 90 W. From that, $90 - 42.9 = 47.1$ W is radiation. Equating this to Stefan Boltzmanns law, will give ε.

For the second question, $h_{convection}$ was determined in 4.2. the radiation part can be determined with

$$h_{rad} = \varepsilon \cdot \sigma(T_s^2 + T_\infty^2) \cdot (T_s + T_\infty)$$

The total heat transfer coefficient is defined by

$$h_{tot} = h_{convection} + h_{rad}$$

The thermal resistance from the convection part is :

$$R_{conv} = \frac{1}{h_{convection} A}$$

And the thermal resistance for the radiation part:

$$R_{rad} = \frac{1}{h_{rad} \cdot A} = \frac{1}{\varepsilon \cdot \sigma \cdot (T_s^2 + T_\infty^2) \cdot (T_s + T_\infty) \cdot A}$$

The total thermal resistance is then

$$\frac{1}{R_{tot}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$

Elaboration

For the first question, we can substitute all known values in Stefan Boltzmann's law.

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot F_{1\rightarrow\text{surface}} \cdot A \cdot (T_s^4 - T_\infty^4) = 47.1\text{W}$$

With a visibility factor of 1, we get

$$\dot{Q}_{rad} = \varepsilon \cdot 5.670 \cdot 10^{-8} \cdot 1 \cdot \pi \left(\frac{0.16}{2}\right)^2 \cdot ((220 + 273)^4 - (20 + 273)^4) = 47.1\text{W}$$

This gives $\varepsilon = 0.799$.

Now for the second question, substituting the known values into the equation for the radiation heat transfer coefficient:

$$h_{rad} = \varepsilon \cdot \sigma(T_s^2 + T_\infty^2) \cdot (T_s + T_\infty) = 11.7\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$$

With $h_{convection} = 10.7\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$, the total heat transfer coefficient becomes

$$h_{tot} = h_{convection} + h_{rad} = 10.7 + 11.7 = 22.4\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$$

The convection resistance is

$$R_{conv} = \frac{1}{h_{convection} A} = \frac{1}{10.7 \cdot \pi \cdot \left(\frac{0.16}{2}\right)^2} = 4.65\,\text{K}\,\text{W}^{-1}$$

The radiation resistance:

$$R_{rad} = \frac{1}{h_{rad} \cdot A} = \frac{1}{\varepsilon \cdot \sigma \cdot (T_s^2 + T_\infty^2) \cdot (T_s + T_\infty) \cdot A} = 4.10\,\mathrm{K\,W^{-1}}$$

The total thermal resistance is then

$$\frac{1}{R_{tot}} = \frac{1}{4.65} + \frac{1}{4.10} = 2.22\,\mathrm{K\,W^{-1}}$$

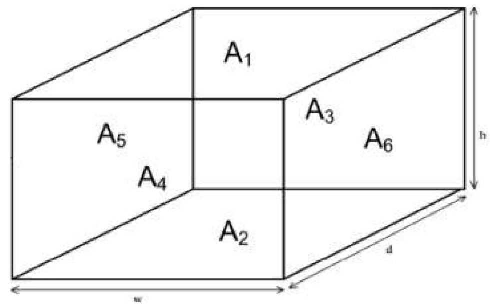
Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

5.4 Heating a meal in an oven - Hand in

a) The oven can be seen as a box with 6 faces, of which 1 radiates and the other 5 absorbs faces. It can be assumed that all faces are blackbodies ($\epsilon = 1$)



$F_{1\to 2}$:

$$\frac{L_2}{D} = \frac{w}{h} = \frac{0.45}{0.30} = 1.5$$
$$\frac{L_1}{D} = \frac{d}{h} = \frac{0.30}{0.30} = 1$$
$$\Rightarrow F_{1\to 2} = 0.26$$

$F_{1\to 3} = F_{1\to 4}$:

$$\frac{L_2}{W} = \frac{h}{w} = \frac{0.30}{0.45} = 0.67$$
$$\frac{L_1}{W} = \frac{d}{w} = \frac{0.30}{0.45} = 0.67$$
$$\Rightarrow F_{1\to 3} = F_{1\to 4} = 0.23$$

$F_{1\to 5} = F_{1\to 6}$:

$$\frac{L_2}{W} = \frac{h}{d} = \frac{0.30}{0.30} = 1$$
$$\frac{L_1}{W} = \frac{w}{d} = \frac{0.45}{0.30} = 1.5$$
$$\Rightarrow F_{1\to 5} = F_{1\to 6} = 0.15$$

b) Sum of the view factors:

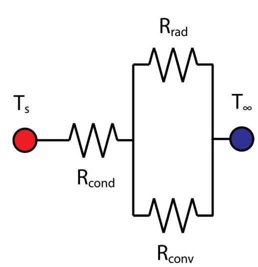
$$\sum F_{1\to \dots} = 1.02$$

It should have been 1. The difference is caused by reading errors.

c) Elaboration depends on the values found for aluminum foil. A possible solution gives:

It is assumed that 1 layer of 0.014 mm thick aluminum foil is added with an emissivity of 0.07:

Thermal resistance network describing the rate of heat transfer from the food towards the environment:



$$R_{\text{cond}} = \frac{\Delta x}{kA} = \frac{1.4 \cdot 10^{-5}}{237 \cdot 0.085} = 7 \cdot 10^{-7}\ \text{K/W}$$

After heat is conducted it is transported by radiation and convection:

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A}$$

Where:

$$h_{\text{rad}} = \epsilon\sigma \cdot \left(T_{\text{s}}^2 + T_{\infty}^2\right) \cdot \left(T_{\text{s}} + T_{\infty}\right) = 0.070 \cdot 5.67 \cdot 10^{-8} \cdot \left(453^2 + 293^2\right) \cdot (453 + 293) = 0.86 \text{ W/m}^2\text{K}$$

So:

$$R_{\text{rad}} = \frac{1}{0.86 \cdot 0.085} = 13.7 \text{ K/W}$$

And the convective resistance:

$$R_{\text{conv}} = \frac{1}{h_{\text{conv}}} = \frac{1}{10 \cdot 0.085} = 1.17 \text{ K/W}$$

Parallel resistance:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_{\text{rad}}} + \frac{1}{R_{\text{conv}}} \Rightarrow R_{\text{parallel}} = 1.07 \text{ K/W}$$

It can be concluded that $R_{\text{cond}} << R_{\text{parallel}}$ and therefore it is reasonable to assume the same surface temperature in the scenarios with and without aluminum foil.

d) Conduction through the aluminum foil may be neglected as it is a good conductor en it is very thin. Furthermore, it is assumed that the foil is packed tightly around the meal and does not affect the geometry for convection.

$$\dot{Q}_{\text{conv}} = hA \left(T_{\text{s}} - T_{\infty}\right) = 10 \cdot 0.085 \left(180 - 25\right) = 131.75 \text{ W}$$

$$\dot{Q}_{\text{rad,no foil}} = \epsilon\sigma F_{1\to\text{surr}} A \left(T_{\text{s}}^4 - T_{\infty}^4\right) = 0.95 \cdot 5.67 \cdot 10^{8} \cdot 1 \cdot 0.085 \left((180 + 273)^4 - (25 + 273)^4\right) 156.7 \text{ W}$$

$$\dot{Q}_{\text{rad,with foil}} = \epsilon\sigma F_{1\to\text{surr}} A \left(T_{\text{s}}^4 - T_{\infty}^4\right) = 0.07 \cdot 5.67 \cdot 10^{8} \cdot 1 \cdot 0.085 \left((180 + 273)^4 - (25 + 273)^4\right) 11.5 \text{ W}$$

$$\dot{Q}_{\text{no foil}} = \dot{Q}_{\text{rad,no foil}} + \dot{Q}_{\text{conv}} = 156.7 + 131.75 = 288.45 \text{ W}$$

$$\dot{Q}_{\text{with foil}} = \dot{Q}_{\text{rad,with foil}} + \dot{Q}_{\text{conv}} = 11.5 + 131.75 = 143.25 \text{ W}$$

$$\eta = \frac{\dot{Q}_{\text{no foil}} - \dot{Q}_{\text{with foil}}}{\dot{Q}_{\text{no foil}}} \cdot 100\% = \frac{288.45 - 143.25}{288.45} \cdot 100\% = 50.3\%$$

e) Wavelength at maximum radiant power.
Maximum power at:

$$\lambda \cdot T = 2897.8 \ \mu\text{m} \cdot \text{K}$$

$$\Rightarrow \lambda = \frac{2897.8}{180 + 273} = 6.397 \ \mu\text{m}$$

f) Is it visible? Visible wavelengths range from 0.4 to 0.76 μm
$\lambda \cdot T$ for the highest wavelength:

$$0.76 \cdot (180 + 273) = 344.28 \ \mu\text{m} \cdot \text{K}$$

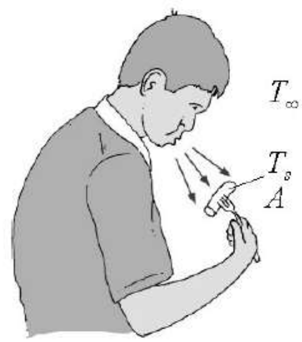
Looking up in the table gives an $f_\lambda = 0$. Everything below this wavelength apparently has a negligible radiation fraction, so a wavelength range within this is certainly the case. The radiation is therefore not visible.

Solutions lecture 6

6.1 Blowing man with a carrot

Analysis

We need to determine the temperature of the carrot after one minute of blowing. Furthermore we need to determine how long the man should blow to lower the temperature of the carrot to 80 °C. The initial temperature is 100 °C, with an ambient temperature of 30 °C. The heat transfer coefficient, thermal conductivity, density and specific heat of the carrot are given.



Approach

Assumptions

We assume that the carrot is a perfect cylinder with a length of 7 cm and a diameter of 2 cm.

Route to solution

With the assumption that the carrot has a uniform temperature, we need to determine if the carrot can be considered a lumped system. To do this, first the characteristic length needs to be determined. This can be done with

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4} \cdot D^2 \cdot l}{\pi \cdot D \cdot L + 2 \cdot \left(\frac{\pi}{4} \cdot D^2\right)}$$

This value can then be substituted in the formula for the Biot number

$$\mathrm{Bi} = \frac{hL_c}{k}$$

If Bi< 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

For objects with a uniform temperature distribution, the constant c is defined as

$$c = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta$$

Substituting the known values and $t = 60$, the temperature is found.

For the second question, we again use the last-mentioned equation, and check for which t the right-hand side equals 80.

Elaboration

We start with determining the characteristic length:

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4} \cdot D^2 \cdot l}{\pi \cdot D \cdot L + 2 \cdot \left(\frac{\pi}{4} \cdot D^2\right)} = 0.004\,375\,\mathrm{m}$$

Substituting the characteristic length in the formula for the Biot number

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{15 \cdot 0.004375}{0.8} = 0.082[-]$$

Because Bi< 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

The constant c is now:

$$c = \frac{hA}{\rho c_p V} = \frac{15 \cdot \left(\pi \cdot 0.02 \cdot 0.07 + 2\left(\frac{\pi}{4} \cdot 0.02^2\right)\right)}{1100 \cdot 3.60 \cdot 10^3 \cdot \frac{\pi}{4} \cdot 0.02^{\cdot} 0.07} = 0.866 \times 10^{-3}\,\mathrm{s}^{-1}$$

Rewriting and substituting the values:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta = (100 - 20) \cdot e^{-0.866 \cdot 10^{-3} \cdot 60} + 20 = 95.96\,^\circ\mathrm{C}$$

For the second part, as the temperature has decreased from 100 °C to 80 °C:

$$\begin{aligned}
80 &= (100 - 20) \cdot e^{-0.866 \cdot 10^{-3} \cdot t} + 20 \\
60 &= 80e^{-0.866 \cdot 10^{-3} \cdot t} \\
0.75 &= e^{-0.866 \cdot 10^{-3} \cdot t} \\
\ln(0.75) &= -0.866 \cdot 10^{-3} \cdot t \\
-02877 &= -0.866 \cdot 10^{-3} \cdot t \\
t &= 332\,\mathrm{s}
\end{aligned}$$

Evaluation

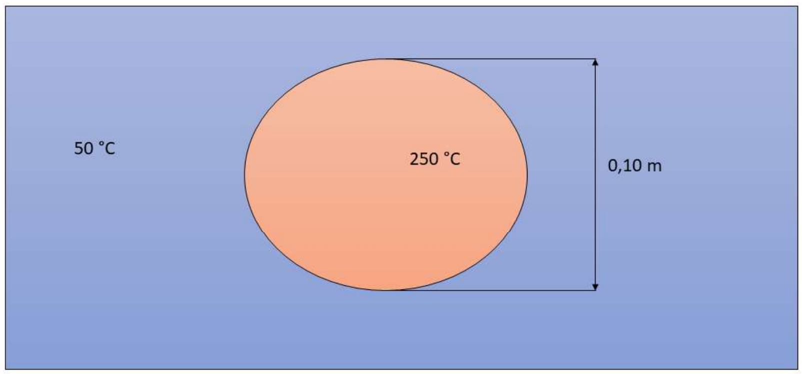
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

6.2 Cooling a copper sphere

Analysis

We need to determine the temperature of the copper block after it is immersed in a cold fluid, 5 minutes after immersion. The density, specific heat, and thermal conductivity of the copper sphere are given, as well as the diameter and the initial temperature. The heat transfer coefficient and the temperature of the fluid are also provided.



Approach

Assumptions

Route to solution

We start with determining the characteristic length of the copper sphere. This can be done with the following equation:

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6}$$

This value can be substituted in the equation for the Biot number:

$$\mathrm{Bi} = \frac{hL_c}{k}$$

When Bi < 0.1, the system is considered lumped, and hence the lump capacitance method may be applied for the solution:

$$\theta(t) = e^{-c\cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

For objects with a uniform temperature distribution, the constant c is defined as

$$c = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$T(t) = (T(0) - T_\infty)\cdot\theta(t) + T_\theta$$

Substituting the known values and $t = 300$, the temperature is found.

Elaboration

We start with determining the characteristic length:

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6} = 0.0167\,\mathrm{m}$$

Substituting the characteristic length in the formula for the Biot number

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{200 \cdot 0.01667}{386} = 8.64 \cdot 10^{-3}[-]$$

Because Bi$<$ 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

The constant c is now:

$$c = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200}{8954 \cdot 0.01667 \cdot 383} = 0.0035\,\mathrm{s}^{-1}$$

Rewriting and substituting the values:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta = (250 - 50) \cdot e^{-0.0035 \cdot 300} + 50 = 120\,^\circ\mathrm{C}$$

Evaluation

Check your answer:

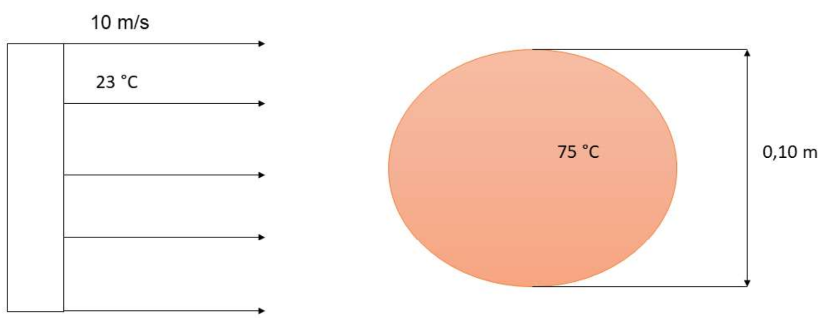
- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

6.3 Cooling a copper sphere under forced convection conditions

Analysis

A copper sphere is wrapped in a plastic film and placed in an oven at 75 °C. After removal from the oven, the sphere is exposed to an air stream at 23 °C and 10 m/s. Estimate the time taken to cool the sphere to 35 °C. The following information is given:

- Copper:
  - $\rho = 8933\ \mathrm{kg\,m^{-3}}$, $k = 400\ \mathrm{W\,m^{-1}\,K^{-1}}$, $c_p = 380\ \mathrm{J\,kg^{-1}\,K^{-1}}$
- For air at 23 °C
  - $\mu = 18.16 \cdot 10^{-6}\ \mathrm{N\,s\,m^{-2}}$, $\nu = 15.36 \cdot 10^{-6}\ \mathrm{m^2\,s^{-1}}$
  - $k = 0.0258\ \mathrm{W\,m^{-1}\,K}$, Pr=0.709
  - $\mu_s = 19.78 \cdot 10^{-6}\ \mathrm{N\,s\,m^{-2}}$ at 35 °C



The diameter in the figure above is wrong!

Approach

Assumptions

Assume the sphere is a lumped system.

Route to solution

A relation for the Nusselt number is provided. The only unknown in this is the Reynolds number, so we first need to determine this:

$$\mathrm{Re} = \frac{\rho U D}{\mu} = \frac{UD}{\nu}$$

Substitute all the variables in the correlation for the Nusselt number. Now, the Nusselt number is defined as

$$\mathrm{Nu} = \frac{hD}{k} \implies h = \frac{k}{D} \cdot \mathrm{Nu}$$

With h known, and the system being lumped, the following equations are valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

where c is defined as

$$c = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$\frac{T(t) - T_\infty}{T(0) - T_\infty} = e^{-c \cdot t}$$

Substituting the known values and rewriting will give an expression for t.

Elaboration

We start with determining the Reynolds number:

$$\mathrm{Re} = \frac{UD}{\nu} = \frac{10 \cdot 0.01}{15.36 \cdot 10^{-6}} = 6510[-]$$

Substitution of all variables in the correlation for the Nusselt number:

$$\begin{aligned}\mathrm{Nu} &= 2 + \left[0.4(6510)^{0.5} + 0.06(6510)^{2/3}\right](0.709)^{0.4}\left(\frac{18.16 \cdot 10^{-6}}{19.75 \cdot 10^{-6}}\right)^{0.25} \\ &= 2 + [32.27 + 20.92] \cdot 0.87 \cdot 0.979 = 47.3[-]\end{aligned}$$

Rewriting the definition of the Nusselt number and substituting:

$$h = \frac{k}{D} \cdot \mathrm{Nu} = \frac{0.0258}{0.01} \cdot 47.3 = 122\,\mathrm{W\,m^{-2}\,K^{-1}}$$

Now, calculating constant c:

$$c = \frac{hA_s}{\rho V c_p} = \frac{122 \cdot 4\pi R^2}{\rho \cdot \frac{4}{3}\pi R^3 \cdot c_p} = \frac{122 \cdot 3}{8933 \cdot 0.005 \cdot 380} = 0.021\,56\,\mathrm{s^{-1}}$$

Substituting c and the temperatures into the rewritten equation for $\theta(t)$

$$\begin{aligned}\frac{T(t) - T_\infty}{T(0) - T_\infty} &= e^{-c \cdot t} \\ \frac{35 - 23}{75 - 23} &= e^{-0.02156 \cdot t} \\ 0.2308 &= e^{-0.02156 \cdot t} \\ \ln(0.2308) &= -0.02156 \cdot t \\ 1.466 &= 0.02156t \\ t &\approx 68\mathrm{s}\end{aligned}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?