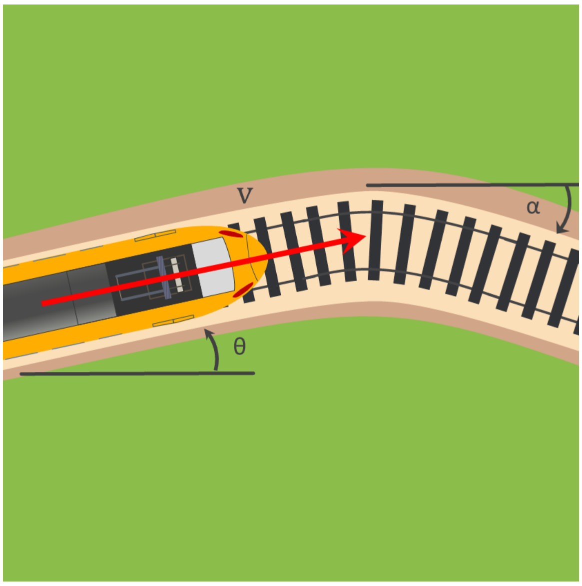
Train in a Curve



A train is taking a sudden curve as shown in the figure. Decompose the velocity into the new normal tangential coordinates for the train when it just took the turn. $\boldsymbol{\theta = 15°}$ and $\boldsymbol{\alpha = 10°}$.

Given quantities:

Angle: $\theta = 15°$
Angle: $\alpha = 10°$
Train speed: $v = |\vec{v}|$

Solution:

Firstly, the train trajectory is split into the corner and the prior and posterior straight sections. We decide to name the prior straight as section I and the posterior straight as section II. Since the trains is currently at section I, its velocity is tangent to this straight. Figure shows a schematic diagram of the situation. The normal and tangent coordinate system at section II is also included ($\vec{e}_{n,\mathrm{II}}$ and $\vec{e}_{t,\mathrm{II}}$).

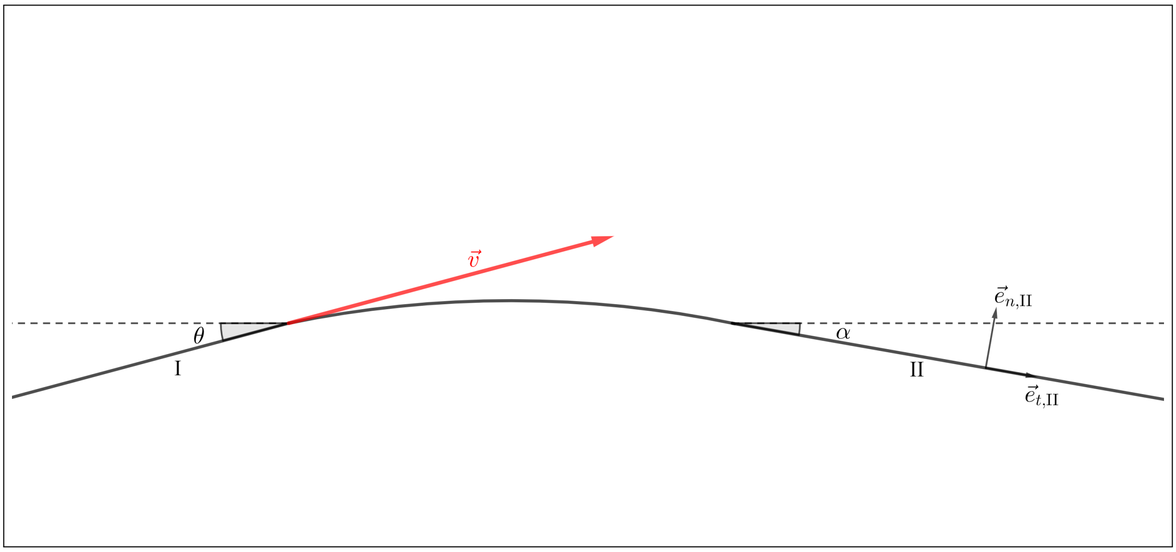


Figure 1: Schematic diagram of the trajectory of the train

We then decompose $\vec{v}$ into the normal and tangential coordinate system at section II in Figure . This is done by drawing a straight line tangent to $\vec{e}_{n,\mathrm{II}}$ and through the point of the velocity and a straight line tangent to $\vec{e}_{n,\mathrm{II}}$ through the point of the velocity. These straight lines are perpendicular to each other. Decomposition is now done via simple projection of $\vec{v}$ onto both lines, respectively to obtain $\vec{v}_{n,\mathrm{II}}$ and $\vec{v}_{t,\mathrm{II}}$.

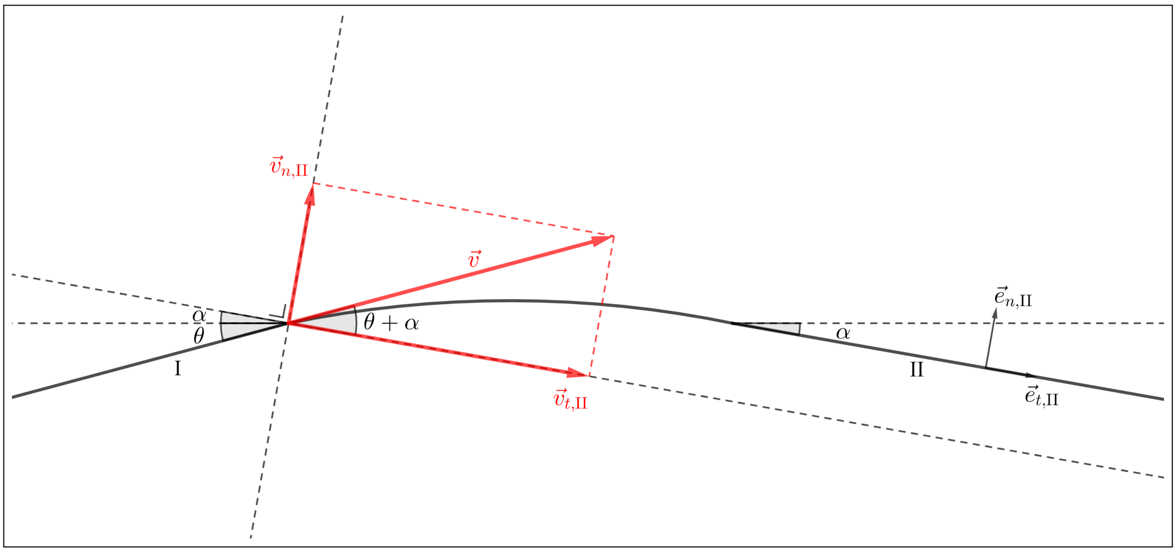


Figure 2: Decomposition of $\vec{v}$ into tangent-normal coordinate system of section II

Notice $\vec{v}_{n,\mathrm{II}}$ and $\vec{v}_{t,\mathrm{II}}$ span a perfect rectangle. Because $\vec{v}_{n,\mathrm{II}}$ and $\vec{v}_{t,\mathrm{II}}$ are perpendicular, standard trigonometry rules for a right triangle can be applied, giving the final expressions for both velocity components:

$$v_{t,\mathrm{II}} = |\vec{v}_{t,\mathrm{II}}| = |\vec{v}| \cos(\theta + \alpha) = v \cos(15^\circ + 10^\circ) = v \cos(25^\circ) \tag{1}$$

$$v_{n,\mathrm{II}} = |\vec{v}_{n,\mathrm{II}}| = |\vec{v}| \sin(\theta + \alpha) = v \sin(15^\circ + 10^\circ) = v \sin(25^\circ) \tag{2}$$