

Appendix A

Formulas available during the Exam

A.1 Fluid kinematics, particle trajectories

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}(\mathbf{x}_p(t), t) \quad (\text{A.1})$$

A.2 Mass Conservation

A.2.1 Integral form

$$\int_{V(t)} \frac{\partial \rho}{\partial t} dV + \int_{A(t)} \rho (u_j n_j) dA = 0. \quad (\text{A.2})$$

A.2.2 Differential form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0. \quad (\text{A.3})$$

A.3 Momentum Conservation

A.3.1 Integral form

$$\int_{V(t)} \frac{\partial}{\partial t} (\rho u_i) dV + \int_{A(t)} \rho u_i (u_j n_j) dA = \int_{A(t)} \sigma_{ij} n_j dA + \int_{V(t)} \rho g_i dV, \quad i = 1, 2, 3. \quad (\text{A.4})$$

A.3.2 Stress tensor

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \quad (\text{A.5})$$

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \delta_{ij} \frac{\partial u_k}{\partial x_k}, \quad \delta_{ij} = \begin{cases} 0, & i \neq j, \\ 1, & i = j. \end{cases} \quad (\text{A.6})$$

A.3.3 Cauchy equation

Tension vector \mathbf{t} by medium A on medium B, \mathbf{n} pointing to A:

$$t_i = \sigma_{ij}n_j, \quad i = 1, 2, 3. \quad (\text{A.7})$$

A.3.4 Differential form (Navier-Stokes)

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j - \sigma_{ij}) = \rho g_i. \quad (\text{A.8})$$

A.3.5 Reduced Navier-Stokes

$$\frac{\partial p}{\partial x} - \mu \frac{\partial^2 u}{\partial y^2} = \rho g_1, \quad \frac{\partial p}{\partial y} = \rho g_2. \quad (\text{A.9})$$

A.3.6 Euler equations

Momentum conservation with $\mu = 0$:

$$\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \rho g_i, \quad i = 1, 2, 3. \quad (\text{A.10})$$

A.3.7 Material derivative

Time derivative while traveling with the flow:

$$\frac{Df}{Dt} \equiv \frac{\partial f}{\partial t} + u_j \frac{\partial f}{\partial x_j}, \quad \text{for any function } f(x, y, z, t) \quad (\text{A.11})$$

A.3.8 Bernoulli equation

(you have to remember the validity conditions yourself!)

$$p + \frac{1}{2}\rho V^2 + \rho g\zeta = \text{constant along streamlines}, \quad V^2 = u^2 + v^2 + w^2. \quad (\text{A.12})$$

A.4 Energy Conservation

A.4.1 Integral form

$$\int_{V(t)} \frac{\partial}{\partial t}(\rho E) dV + \int_{A(t)} (\rho E u_j n_j - \sigma_{ij} u_i n_j + q_j n_j) dA = \int_{V(t)} \rho g_j u_j dV, \quad i = 1, 2, 3. \quad (\text{A.13})$$

Total energy:

$$E \equiv e + \frac{1}{2}U^2, \quad U^2 = u^2 + v^2 + w^2. \quad (\text{A.14})$$

Enthalpy and total enthalpy:

$$h \equiv e + \frac{p}{\rho}, \quad H \equiv E + \frac{p}{\rho} \quad (\text{A.15})$$

Thermodynamics of a perfect gas:

$$p = \rho RT, \quad e = C_v T, \quad C_p - C_v = R, \quad \gamma \equiv C_p / C_v \quad (\text{A.16})$$

Speed of sound and Mach number:

$$a = \sqrt{\gamma RT}, \quad M \equiv U/a. \quad (\text{A.17})$$

A.4.2 Fourier's law (heat flux)

$$q_i = -k \frac{\partial T}{\partial x_i}, \quad i = 1, 2, 3. \quad (\text{A.18})$$

A.4.3 Compressor equation

$$\dot{m} (H_2 - H_1) = P + \dot{Q}. \quad (\text{A.19})$$

A.4.4 Differential form

$$\frac{\partial}{\partial t} \rho E + \frac{\partial}{\partial x_j} (\rho u_j E - \sigma_{ij} u_i + q_j) = \rho g_j u_j. \quad (\text{A.20})$$

A.4.5 Total temperature, -pressure and -density

$$T_t = T(1 + \frac{\gamma - 1}{2} M^2), \quad p_t = p(1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma - 1}}, \quad \rho_t = \rho(1 + \frac{\gamma - 1}{2} M^2)^{\frac{1}{\gamma - 1}}. \quad (\text{A.21})$$

A.5 Convection and diffusion

A.5.1 Convection equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = 0. \quad (\text{A.22})$$

A.5.2 Diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}. \quad (\text{A.23})$$

A.5.3 Convection-diffusion equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2}. \quad (\text{A.24})$$