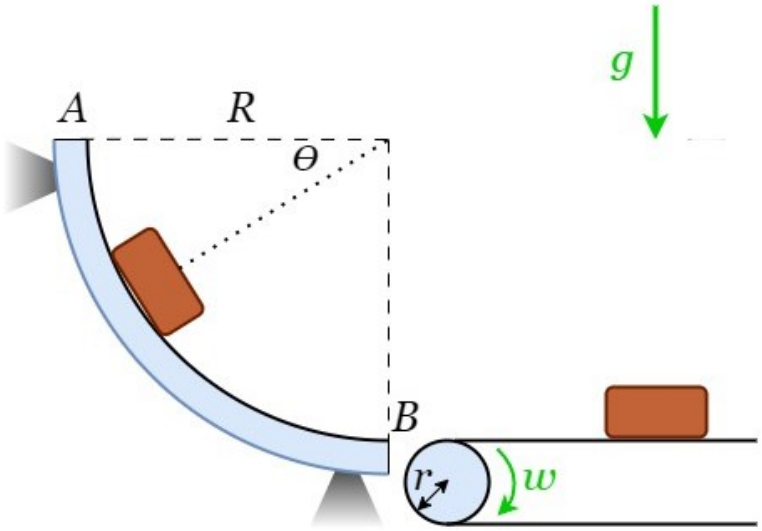
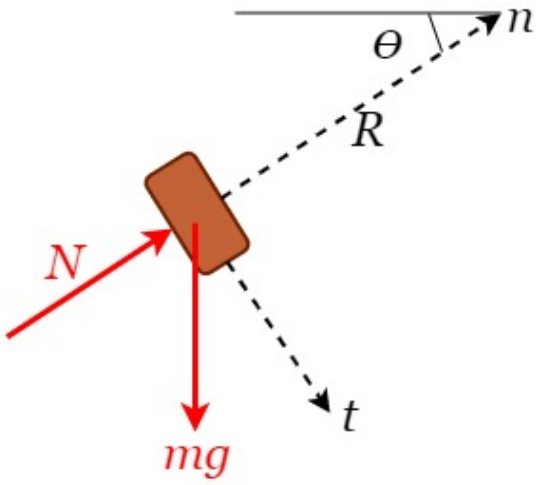
Block sliding down a circular surface



Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B. The gravitational acceleration is g.

1) Determine the expression for the tangential acceleration a_t of the object for any arbitrary position on the surface.
2) Provide an expression for the velocity SQUARED, for any general position.
Hint: $vdv = a_t ds$
3) Provide an expression for the normal force.
4) Specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.



The free body diagram includes the normal and tangential directions. The normal force N depends on the n-component of acceleration which, in turn, depends on the velocity. The velocity will be cumulative according to tangential acceleration a_t. We will first find a_t.

$$[\sum F_t = m \cdot a_t]$$

$$mg \cdot cos(\theta) = ma_t \quad \Rightarrow \quad a_t = g\ cos(\theta)$$

Now we can find the velocity by integrating:

$$[v\ dv = a_t\ ds]$$

We obtain the normal force by summing forces in the positive n-direction:

$$[\sum F_n = m \cdot a_n]$$

$$N - mg \cdot sin(\theta) = m\frac{v^2}{R} \quad \Rightarrow \quad N = 3mg\ sin(\theta)$$

The conveyor pulley must turn at the rate $v = r\omega$ for $\theta = \pi/2$, so that:

$$\omega = \frac{\sqrt{2gR}}{r}$$