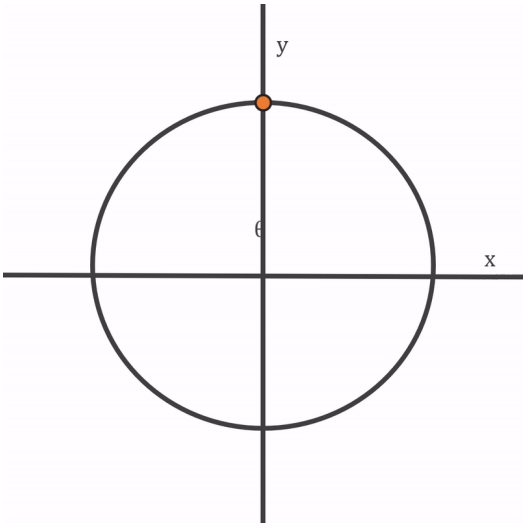
Ball in Circular Motion



Which of the following equations gives the correct relation for the acceleration vector $\vec{a}$ in cartesian coordinates for this ball following a circular motion with radius R?

Solution:

The transformation to x- and y-coordinates is the following (the angle is measured with respect to the vertical):

$$\begin{cases} x = R\sin\theta \\ y = R\cos\theta \end{cases} \tag{1}$$

Take the first and second derivatives of both components to get relations for the velocity and acceleration. The chain rule is applied.

$$\begin{cases} \dot{x} = R\dot{\theta}\cos\theta \\ \dot{y} = -R\dot{\theta}\sin\theta \end{cases} \tag{2}$$

$$\begin{cases} \ddot{x} = R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta \\ \ddot{y} = -R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos\theta \end{cases} \tag{3}$$

Combining both results in an acceleration vector $\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$:

$$\vec{a} = (R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta)\hat{\imath} + (-R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos\theta)\hat{\jmath} \tag{4}$$