HEATQUIZ

HEAT TRANSFER

Course reader - Radiation

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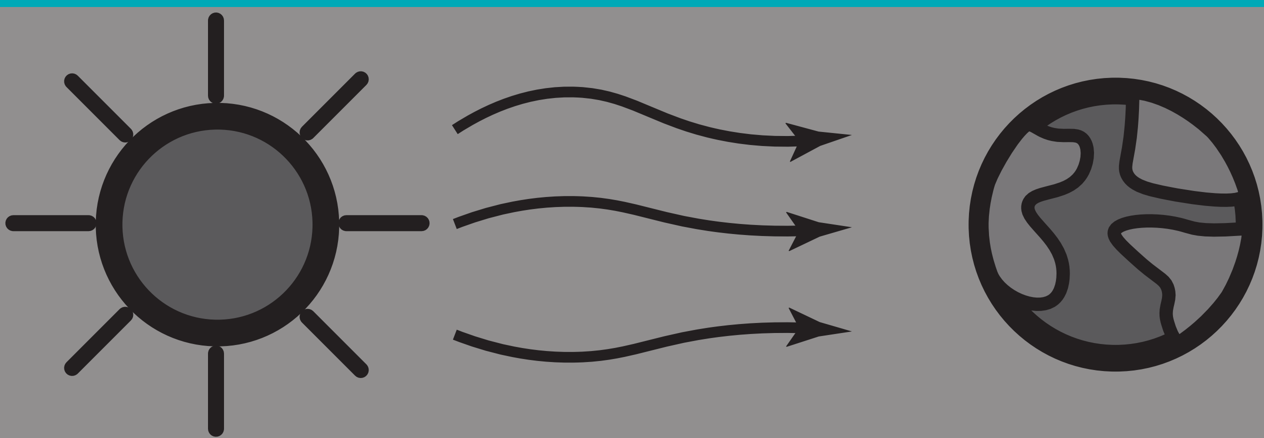

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Nomenclature

Symbol:	Physical constant:	Numerical value:	Unit:
c	Speed of light	299,792,458	$[\mathrm{m/s}]$
g	Gravitational constant	9.81	$[\mathrm{m/s^2}]$
h	Planck's constant	$6.6261 \cdot 10^{-34}$	$[\mathrm{Js}]$
σ	Stefan–Boltzmann constant	$5.6703 \cdot 10^{8}$	$[\mathrm{W/m^2K^4}]$

Symbol:	Description:	Unit:
A	Area	$[\mathrm{m^2}]$
C	Constant	$[-]$
E	Energy of a photon	$[\mathrm{J}]$
F	Force or radiation fraction	$[\mathrm{N}]$ or $[-]$
H	Enthalpy	$[\mathrm{J}]$
I	Momentum	$[\mathrm{Ns}]$
L	(Characteristic) length length or radiosity	$[\mathrm{m}]$ or $[\mathrm{W/m^2}]$
L_{th}	Thermal entrance length	$[\mathrm{m}]$
Q	Heat	$[\mathrm{J}]$
R	Thermal resistance	$[\mathrm{W/K}]$
T	Temperature	$[\mathrm{K}]$
U	Internal energy or perimeter	$[\mathrm{J}]$ or $[\mathrm{m}]$
V	Volume	$[\mathrm{J}]$
W	Work	$[\mathrm{J}]$
a	Thermal diffusivity	$[\mathrm{m^2/s}]$
c	Specific heat capacity or constant	$[\mathrm{J/kgK}]$ or $[-]$
d	Diameter	$[\mathrm{m}]$
dx	Infinitesimal distance in x-direction	$[\mathrm{m}]$
dy	Infinitesimal distance in y-direction	$[\mathrm{m}]$
dz	Infinitesimal distance in z-direction	$[\mathrm{m}]$
dr	Infinitesimal distance in r-direction	$[\mathrm{m}]$
f_{e}	Arrangement factor	$[-]$
h	Mass-specific enthalpy	$[\mathrm{J/kg}]$
k	Overall heat transfer coefficient	$[\mathrm{W/m^2K}]$
m	Mass or fin parameter	$[\mathrm{kg}]$ or $[1/\mathrm{m}]$
n	Total number of layers	$[-]$
p	Pressure	$[\mathrm{N/m^2}]$
$\dot{q}''$	Heat flux density	$[\mathrm{W/m^2}]$
r	Radius	$[\mathrm{m}]$
t	Time	$[\mathrm{s}]$
u	Velocity (in x-direction)	$[\mathrm{m/s}]$
v	Velocity (in y-direction)	$[\mathrm{m/s}]$
w	Velocity (in z-direction)	$[\mathrm{m/s}]$
x	Spatial coordinate	$[\mathrm{m}]$
y	Spatial coordinate	$[\mathrm{m}]$
z	Spatial coordinate	$[\mathrm{m}]$
Δ	Difference	$[-]$
Φ	View factor	$[-]$
$\dot{\Phi}$	Heat source	$[\mathrm{W}]$
Ω	Solid angle	$[\mathrm{Str}]$
α	Convective heat transfer coefficient or absorptivity	$[\mathrm{W/m^2K}]$ or $[-]$
β	Volumetric expansion coefficient	$[1/\mathrm{K}]$
δ	Wall thickness or penetration depth	$[\mathrm{m}]$
ϵ	Emissivity	$[-]$
δ_{T}	Thermal boundary layer thickness	$[\mathrm{m}]$
δ_{u}	Velocity boundary layer thickness	$[\mathrm{m}]$
η	Efficiency or wavenumber	$[-]$ or $\left[\frac{1}{\mathrm{m}}\right]$
θ	Dimensionless spatial temperature	$[-]$
θ^*	Dimensionless temporal temperature	$[-]$
λ	Thermal conductivity or wavelength	$[\mathrm{W/mK}]$ or $[\mathrm{m}]$
μ	Dynamic viscosity	$[\mathrm{kg/ms}]$
ν	Kinematic viscosity or frequency of radiation	$[\mathrm{m^2/s}]$ or $\left[\frac{1}{\mathrm{s}}\right]$
ϕ	Viewing angle	$[\mathrm{rad}]$
ρ	Density or reflectivity	$[\mathrm{kg/m^3}]$ or $[-]$
τ	Shear stress or transmissivity	$[\mathrm{N/m^2}]$ or $[-]$

Superscript:	Description:
x^*	Dimensionless
x'	Distance-related or variation
x''	Area-related
x'''	Volume-related
$\dot{x}$	Time derivated
$\overline{x}$	Average
$\vec{x}$	Vector

Subscript:	Description:
x_A	Ambient A
x_a	Ambient
x_B	Ambient B
x_b	Black body
x_c	Cross-section
x_crit	Critical
x_cond	Conduction
x_conv	Convection
x_d	Hydraulic diameter as characteristic length
x_eff	Effective
x_F	Fin
x_f	Fluid
x_fl	Fluid
x_h	Hydraulic
x_in	Inlet
x_kin	Kinetic
x_L	Length as characteristic length
x_{ij}	From i to j
$x_{i\rightarrow j}$	From i to j
$x_{i\rightleftarrows j}$	Net between i and j
x_m	Mean
x_max	Maximum
x_min	Minimum
x_out	Outlet
x_p	At constant pressure
x_prop	Property
x_rad	Radiation
x_s	Solid or distance as characteristic length or surface
x_t	Turbulent
x_th	Thermal
x_v	At constant volume
x_w	Wall
x_x	Local
x_0	Incident or initial
x_1	Reference 1
x_2	Reference 2
x_3	Reference 3
x_4	Reference 4
x_α	Absorbed
x_ϵ	Emitted
x_η	Wavenumber-specific
x_λ	Wavelength-specific
x_ρ	Reflected
x_τ	Transmitted
x_∞	Upstream

Symbol:	Dimensionless number:	
Ar	Archimedes number	$\equiv \frac{\text{Bouyance forces}}{\text{Inertia forces}}$
Bi	Biot number	$\equiv \frac{\text{Thermal resistance in body}}{\text{Convective thermal resistance at surface}}$
Fo	Fourier number	$\equiv \frac{\text{Rate of diffusivity}}{\text{Rate of storage}}$
Gr	Grashof number	$\equiv \frac{\text{Bouyancy forces}}{\text{Viscous forces}}$
Nu	Nusselt number	$\equiv$ Dimensionless heat transfer coefficient
Pe	Peclet number	$\equiv \frac{\text{Rate of advection}}{\text{Rate of diffusion}}$
Pr	Prandtl number	$\equiv \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}}$
Re	Reynolds number	$\equiv \frac{\text{Inertia forces}}{\text{Viscous forces}}$
c_f	Friction coefficient	$\equiv \frac{\text{frictional head loss}}{\text{dynamic pressure}}$

Chapter elements

Here an overview is presented clarifying the individual components that form the structural organization of this text. Readers will encounter specific sections.

Phenomena

The part on **phenomena** illustrates a principle of heat transfer through tangible, real-world examples that can be readily observed. Consider scenarios such as the cooling of a cup or the sensation of warmth from the sun.

Fundamental EQ

The **fundamental EQ** part presents equations derived or experimentally determined, providing a convenient means to calculate specific parameters within the framework of heat transfer theory. Consider well-known examples such as Fourier's law or Newton's law of cooling.

Definition

The **definition** part introduces the meanings of parameters utilized in the theory of heat transfer. Common definitions within this framework include terms such as rate of heat, internal heat source, and dimensionless numbers.

Derivation

The **derivation** part is dedicated to obtaining a particular theorem or defining a parameter. Common derivations discussed include those related to conservative equations within solids and fluids. □

Approach

The part delving into the **approach** presents a systematic framework for addressing a particular problem, outlining steps such as establishing and solving an energy balance equation, for instance.

Example

The **example** part furnishes a relatively straightforward illustration demonstrating the practical application of the recently derived theory. Consider, for instance, the derivation of the temperature profile within a plane wall.

HeatQuiz

The **HeatQuiz** part offers game-based learning tasks designed to assess the comprehension of the previously discussed theory and ascertain whether sufficient knowledge of the content has been acquired to apply the theory to practical examples.

Demonstration

The **demonstration** part presents a task previously evaluated in past course exams, accompanied by a QR code offering a video solution. Therefore, it is strongly advised to attempt the task independently before consulting the video solution for the best learning experience.

HTC

The **HTC** part presents a Nusselt correlation tailored for a specific application. These correlations, determined through numerical or experimental means, serve as tools for calculating the heat transfer coefficient in particular scenarios.

Criterion

The **criterion** part introduces a set of conditions that must be satisfied for a theory to be deemed applicable.

Exercise ($\star$):

> Exercises marked with a single star $\star$ serve as foundational exercises to reinforce your fundamental understanding. While not yet at the exam level, they function as crucial stepping stones, helping you build confidence and proficiency before tackling more advanced challenges.

Exercise ($\star\star$):

> For exercises denoted with two stars $\star\star$, you can expect tasks slightly below the anticipated difficulty level of the exam or smaller assignments with a few exam points at stake.

Exercise ($\star\star\star$):

> The exercises adorned with three stars $\star\star\star$ are those that have previously appeared on exams. These tasks, often carrying significant point values, reflect the kinds of challenges that demand a higher level of mastery and are critical for thorough exam preparation.

PART

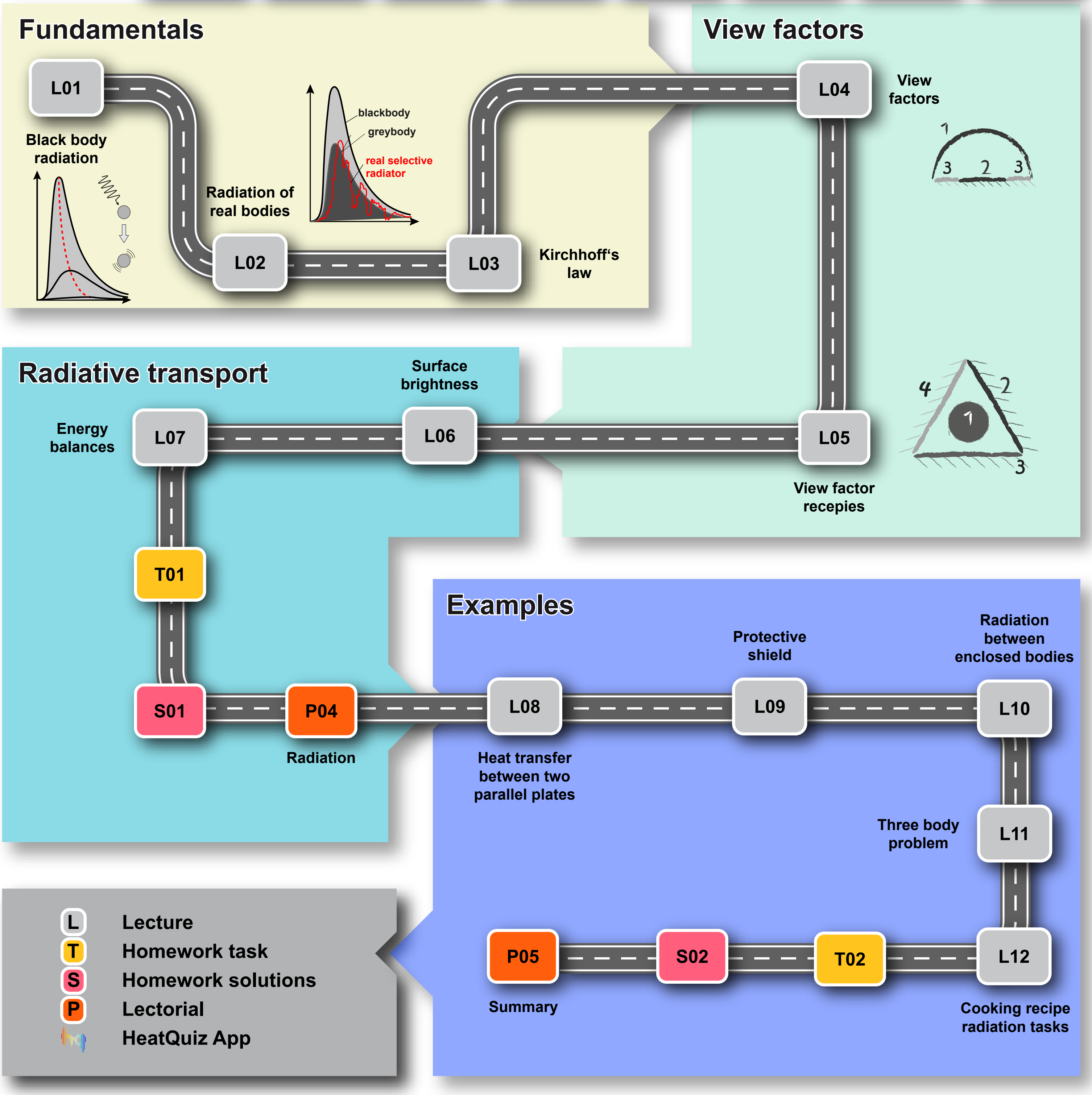
IV

Radiation

Heat transfer: Radiation
Learning path

HEATQUIZ

Book of formularies | Lecture notes | Video script | Basics | View factors | Balances | Exam preparation | Mobile version



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SECTION 17

Fundamentals

L01 - Black body radiation:

Learning goals:

- Achieving a comprehensive understanding of Wave-Quantum Duality.
- Describing the spectral radiation intensity of a Black Body according to Planck's theory.
- Developing a solution approach for the integration of Planck's Distribution Law.
- Applying the Stefan-Boltzmann Law for practical use.
- Understanding the relationship between temperature and the position of maximum spectral radiation intensity.

Comprehension questions:

- Define the term "Black Body" in the context of radiation.
- Specify the assumptions that hold true when calculating properties of "Black Bodies".
- Identify the law used to determine the wavelength at the intensity maximum of a "Black Body".
- Explain the methodology employed to derive the Stefan-Boltzmann constant.
- How is the radiation intensity calculated within a specific wavelength range $\lambda_1 - \lambda_2$?

L02 - Steady conservation equation:

Learning goals:

- Understanding the definition of and interpreting emissivity, absorptivity, transmissivity, and reflectivity.
- Understanding the behavior of real bodies in contrast to ideal bodies.
- Exploring the angular dependence of the radiation properties of real bodies.

Comprehension questions:

- In what proportions is incident radiation divided when it strikes a real body?
- Differentiate between black, grey, and real bodies, particularly concerning wavelength-related characteristics.

L03 - Kirchoff's law

Learning goals:

- Understanding the relationship between absorptivity and emissivity.
- Identifying the conditions under which $\alpha = \epsilon$ (wavelength independent) holds true.

Comprehension questions:

- Under what circumstances can it be assumed that both $\alpha(\lambda) = \epsilon(\lambda)$ and $\alpha = \epsilon$ are valid?
- Regarding radiation, to which portion does emissivity refer, and to which portion does absorptivity?
- When $\alpha(\lambda) = \epsilon(\lambda)$ holds, is the absorbed and emitted heat flux identical?

SUBSECTION 17.1
Introduction to radiation

Conduction and convection, as discussed in Parts **??** and **??**, are heat transfer mechanisms, where energy is transported through molecular processes respectively macroscopic movement of fluids, whereas heat radiation does not require any medium since this way of transferring heat is based on electromagnetic processes. The intensity and the sort of radiation, emitted from a gaseous, liquid, or solid body depends on the surface properties of the body and the body temperature, yet they are independent of the surroundings. If not only the emitted radiation is to be considered, but also the heat exchange between the body and the surroundings, the type, temperature, and geometrical orientation in space of the surrounding bodies must be taken into account.

In spite of the fact that in most heat transfer problems energy is simultaneously transported through conduction and/or convection and radiation, henceforth only radiation is regarded, as far as possible.

To get a first understanding of radiation consider a scenario where a heated object hangs within a vacuum-sealed chamber, with the chamber walls maintained at room temperature. This situation is illustrated in Figure 17.1. Over time, the elevated object naturally cools down, ultimately attaining thermal equilibrium with the surroundings. In this process, heat is gradually released until the object's temperature aligns with that of the chamber walls. Notably, the transfer of heat between the object and the chamber cannot be attributed to conduction or convection, as these mechanisms are ineffective in a vacuum. Consequently, the heat transfer must have occurred through an alternative mechanism, specifically involving the emission of the object's internal energy—known as radiation.

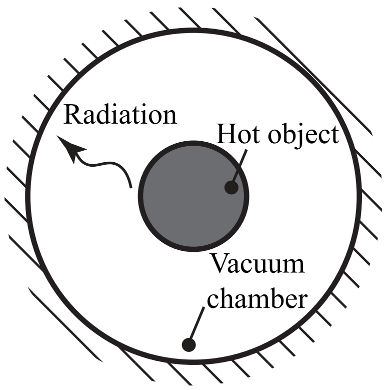


Figure 17.1. Hot object cooling down by radiation.

17.1.1 Physics of radiation

Physics teaches us that radiation can be explained using the wave and the quantum mechanics theory. This dual character has been impressively proven experimentally by the double slit experiment, as illustrated in Figure 17.2. In such an experiment, particles, such as electrons or photons, are directed towards a barrier with two closely spaced slits. When these particles pass through the slits and strike a photographic plate on the other side, an interference pattern emerges.

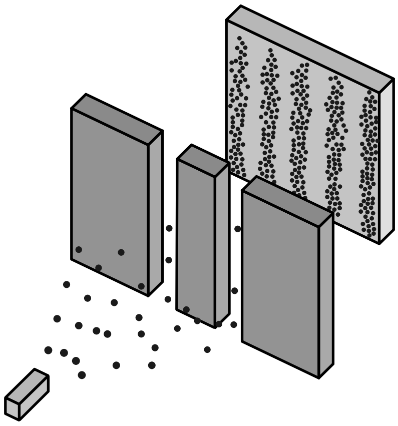
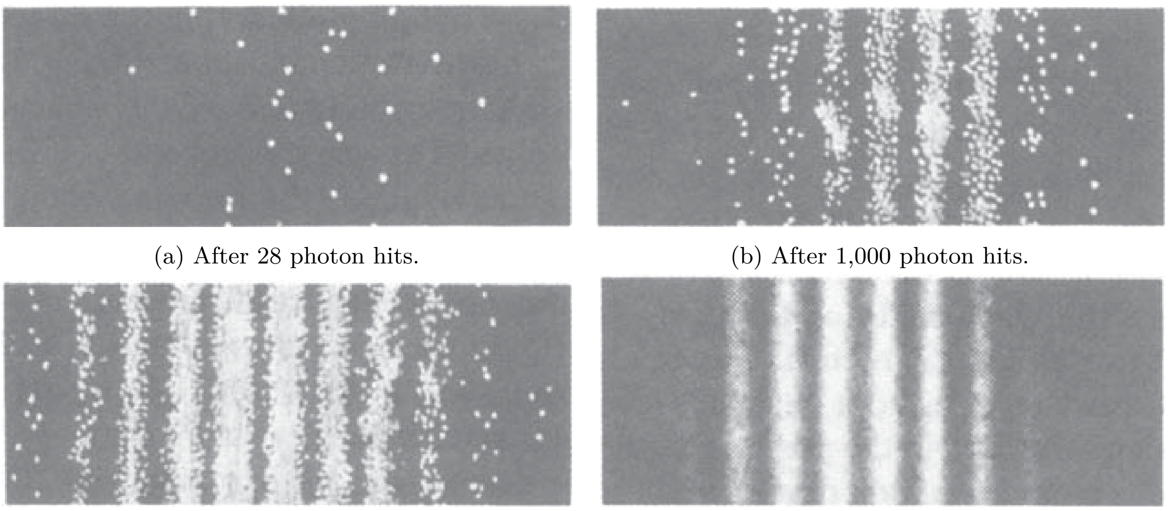


Figure 17.2. Double slit experiment.

After a very brief time interval, distinct light points are recognized, as illustrated in Figure 17.3a. These points indicate that individual light quanta, known as photons, are striking the plate. Over longer periods, these photons exhibit non-random behavior, forming a striped pattern as observed in Figures 17.3b and 17.3c. These stripes create an interference pattern, which can also be observed during the superposition of circular waves passing through a diffraction grating and striking a dark screen behind, as depicted in Figure 17.3d. This phenomenon underscores the wave-like nature of radiation and shows the character when the size of the "slits" through which the rays are refracted has the same order of magnitude as the wavelength of the radiation.



(a) After 28 photon hits.

(b) After 1,000 photon hits.

(c) After 10,000 photon hits.

(d) Double-slit interference pattern.

Figure 17.3. Double-slit experiment by E.R. Huggins, Physics I, W.A. Benjamin, Inc., Menlo Park, California, USA, 1968.

The wave-like nature of radiation, including light, is characterized by properties such as wavelength, frequency, interference, and diffraction. The relationship between wavelength λ, frequency ν, and speed of light c is a key expression that relates these characteristics and emphasizes the wave-like behavior of radiation.

Definition

Wavelength in a medium:

$$\lambda = \frac{c}{\nu}\ [\mathrm{m}] \tag{17.1}$$

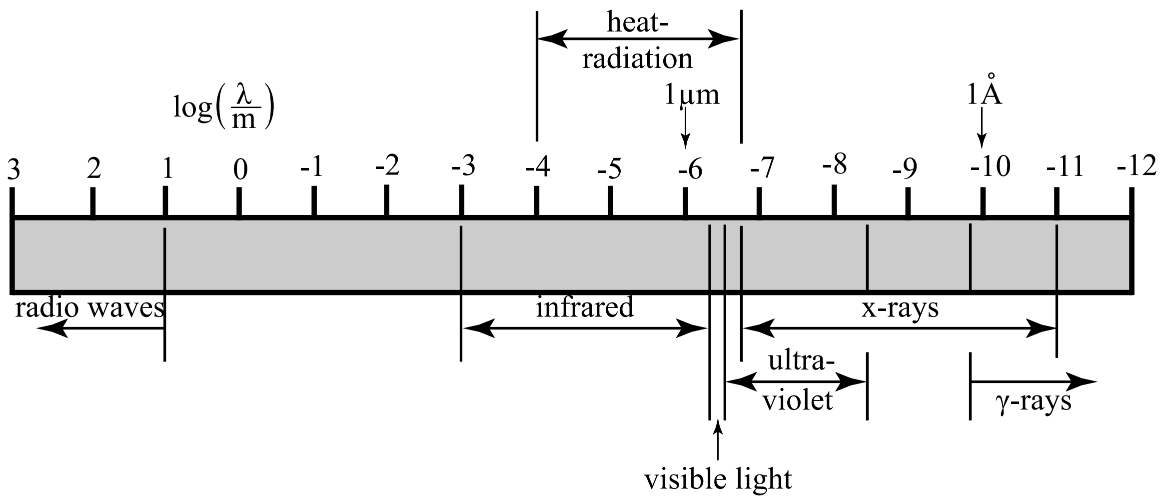


Figure 17.4. Electromagnetic spectrum.

In Figure 17.4 the electromagnetic spectrum and the wavelengths at which heat radiation takes place are observed. The wave-like nature of radiation becomes apparent when the size of the "slits" through which the rays are refracted have the same order of magnitude as the wavelength of the radiation. However, the spectrum clearly shows that the radiation wavelengths are much smaller than the size of a typical "slit". Thus, the quantum character dominates over the wave-like behavior during heat radiation.

Considering electromagnetic radiation as the transmission of discrete energy packets, termed photons or quanta, has proven valuable. This concept was introduced by Max Planck in 1900 as part of his quantum theory. In this perspective, each photon with a specific frequency possesses some photon energy, which is described as:

Fundamental EQ

Photon energy:

$$E = h\,\nu = \frac{hc}{\lambda}, \tag{17.2}$$

where Planck's constant is $h = 6.626\ 10^{-34}\ \mathrm{J \cdot s}$.

Equation (17.2) shows that the transported energy of each quantum relates inversely proportional to the wavelength λ. This relation leads to the definition of the wavenumber:

Definition

Wavenumber:

$$\eta = \frac{1}{\lambda}\left[\frac{1}{\mathrm{m}}\right] \tag{17.3}$$

In contradistinction to the wavelength, the wavenumber offers the advantage of maintaining a proportional relationship with the quantum's transported energy. This holds under the condition that the propagation velocity of the radiation remains constant, as indicated by equation (17.2). In instances where the velocity changes, such as when entering a medium with a disparate refraction index, both the wavelength and wavenumber are altered, as denoted by equation (17.3). As a result, when addressing issues involving distinct refraction indexes, considering the frequency ν is preferable, which is independent of the refraction index.

SUBSECTION 17.2

Black body radiation

An object at a thermodynamic temperature above zero emits radiation across a broad spectrum of wavelengths in all directions. The quantity of radiation energy emitted at a specific wavelength is contingent upon the material and condition of the surface, as well as the surface temperature. Consequently, surfaces at the same temperature may emit varying amounts of radiation per unit surface area. This prompts an inquiry into the maximum radiation emission possible from a surface at a given temperature. To address this inquiry, the concept of a black body is introduced as an idealized reference, representing a perfect emitter and absorber of radiation. The most important characteristics of a black body are:

- Absorption of all incident radiation, regardless of wavelength and direction.
- At a defined temperature and wavelength, no surface can surpass a black body in energy emission.
- A continuous and monotonic increase in radiation across all wavelengths with rising absolute temperature.
- A black body is characterized as a diffuse emitter, signifying independence from direction.

Max Planck derived from the theory of quantum mechanics a relationship for the distribution of radiation intensity over the wavelength of a black body. Planck's distribution law describes the emitted radiation intensity $\dot{q}''_{b\lambda}$ in an infinitesimal wavelength range $d\lambda$, the so-called monochrome or spectral emissive power. Planck's distribution law is defined as follows:

Fundamental EQ

Planck's distribution law:

$$\dot{q}''_{\mathrm{b}\lambda} = \frac{c_1 \lambda^{-5}}{\exp\left[\frac{c_2}{\lambda T}\right] - 1} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\frac{1}{\mathrm{m}}\right] \tag{17.4}$$

where b refers to it being a black body, and λ to it being wavelength-specific.

Furthermore $c_1 = 3.741 \cdot 10^{-16}$ Wm^2 and $c_2 = 1.439 \cdot 10^{-2}$ mK.

Equation (17.4) describes that the spectral emissive power is contingent upon the temperature of the emitting body. This characteristic is shown in Figure 17.5, which presents Planck's distribution law across various temperatures and radiation wavelengths.

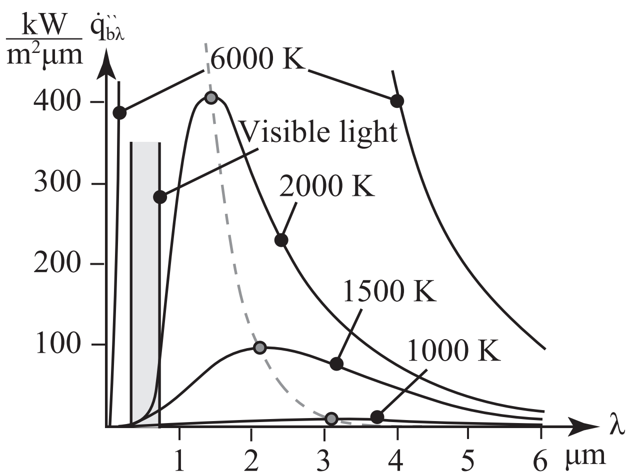


Figure 17.5. Spectral radiant flux density of a black body.

With the rising temperature, the peak of the curve depicted in Figure 17.5 moves towards shorter wavelengths. The derivative of equation (17.4) gives the position of the maxima of the curve. This is the so-called Wien's Law of displacement. The specific wavelength at which the peak occurs for a given temperature can determined by Wien's displacement law.

Fundamental EQ

Wien's law of displacement:

$$\lambda_{\max} = \frac{2898\ \mu\mathrm{mK}}{T} \tag{17.5}$$

To determine the entire emitted radiation power per unit area of a black body, Planck's distribution law, stated in equation (17.4), should be integrated over the entire wavelength range:

$$\dot{q}_b'' = \int_{\lambda=0}^{\infty} \dot{q}_{b\lambda}'' d\lambda = \int_{\lambda=0}^{\infty} \frac{c_1 \lambda^{-5}}{\exp\left[\frac{c_2}{\lambda T}\right] - 1} d\lambda$$

An illustrative example is given in Figure 17.6, with the area below the intensity curve being equal to the emitted radiation power. This integration leads to the Stefan-Boltzmann law. The Stefan Boltzmann law describes the radiation emitted per unit area of a black body and is described in equation (17.6).

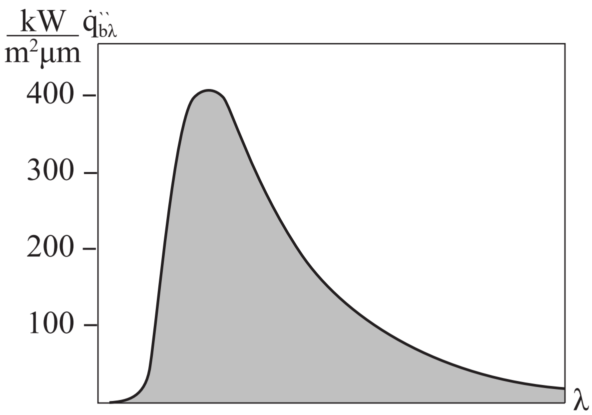


Figure 17.6. Entire emitted radiation, shown as the integral of the Planck distribution.

Fundamental EQ

Stefan-Boltzmann law:

$$\dot{q}_b'' = \sigma T^4, \tag{17.6}$$

where Stefan-Boltzmann constant σ has the value $\sigma = 5.67 \cdot 10^{-8} \left[\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}\right]$.

Note that While a black body would visually appear black, an idealized black body and a common black surface are not the same. Any surface that absorbs light, particularly the visible portion of radiation, would appear black to the eye, while a surface that fully reflects would appear white. Given that visible radiation spans a narrow spectrum from 0.4 to 0.76 mm, visual assessments alone cannot determine the blackness of a surface. For instance, snow and white paint may reflect light and appear white, but they effectively act as black surfaces for infrared radiation, as they strongly absorb long-wavelength radiation. Surfaces coated with lampblack paint exhibit behavior approaching that of an idealized black body.

17.2.1 Radiative emissions within a spectral range

Occasionally, instead of considering the radiation emissions of the entire spectrum, the emissions only in a specific range between the wavelengths 0 and λ, as illustrated in Figure 17.7a should be considered. The radiation density of this interval is determined by integration of the Planck distribution:

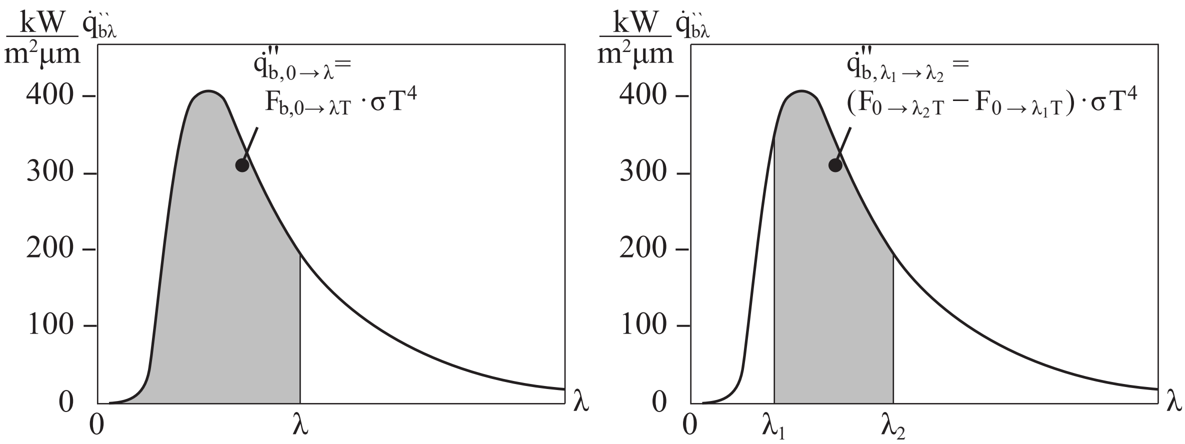
$$\dot{q}''_{b,0\to\lambda} = \int_0^\lambda \dot{q}''_{b\lambda} d\lambda$$

The solution of this integral is described by the following series approach:

$$F_{0\to\lambda T} = \frac{15}{\pi^4} \sum_{n=1}^{\infty} \left[\frac{e^{-n\xi}}{n} \left(\xi^3 + \frac{3\xi^2}{n} + \frac{6\xi}{n^2} + \frac{6}{n^3} \right) \right]$$

with

$$\xi = \frac{c_2}{\lambda T}$$



(a) Emitted radiation power between $0 \to \lambda$.

(b) Emitted radiation power between $\lambda_1 \to \lambda_2$.

Figure 17.7. Radiative emissions within a spectral range.

For simple calculations the series approach, listed above, is not practical; therefore tables with exemplary values for the factor $F_{0\to\lambda T}$ are used. The series approach is more common for computer-based calculations. Even with a small number of links n, results of sufficient accuracy are obtained.

The factor $F_{0\to\lambda T}$ describes the ratio between the entire emitted radiation intensity and the emitted radiation intensity in the spectral range between 0 and λ. By using Stefan-Boltzmann law the actual emitted radiation intensity, shown in Figure 17.7a is calculated:

Fundamental EQ

Black body radiation in the spectral range between 0 and λ:

$$\dot{q}''_{b,0\to\lambda} = F_{0\to\lambda T} \cdot \sigma T^4, \tag{17.7}$$

where:

λT	[μmK]	1000	1250	1500	1750	2000	2500
$F_{0\to\lambda T}$	[–]	0.00031	0.00308	0.01283	0.03363	0.06663	0.16115
λT	[μmK]	3000	3500	4000	5000	6000	8000
$F_{0\to\lambda T}$	[–]	0.27322	0.38250	0.48085	0.63315	0.73715	0.85556

In some cases, the emitted radiation power between $\lambda_1 \to \lambda_2$ is of relevance as well, as illustrated in Figure 17.7b. This yields from the difference in emitted radiation power between $0 \to \lambda_2$ and $0 \to \lambda_1$:

Fundamental EQ

Black body radiation in the spectral range between λ_1 and λ_2:

$$\dot{q}''_{b,\lambda_1\to\lambda_2} = \left(F_{0\to\lambda_2 T} - F_{0\to\lambda_1 T}\right) \cdot \sigma T^4, \tag{17.8}$$

where:

λT	[μmK]	1000	1250	1500	1750	2000	2500
$F_{0\to\lambda T}$	[–]	0.00031	0.00308	0.01283	0.03363	0.06663	0.16115
λT	[μmK]	3000	3500	4000	5000	6000	8000
$F_{0\to\lambda T}$	[–]	0.27322	0.38250	0.48085	0.63315	0.73715	0.85556

SUBSECTION 17.3

Radiation of real bodies

Real materials deviate from black body behavior, requiring measurements to quantify the deviation from black body behavior and define the spectral, directional, and temperature characteristics of actual surfaces about those of a black body. The notation conventionally employed to represent a spectrally dependent property is denoted as x_λ. These subscript symbols are excluded when the property under consideration has been averaged over the entire wavelength spectrum.

When radiation of a specific wavelength falls upon the surface of a real nonblack body, this radiation is either reflected off the surface, absorbed by the body, or transmitted, whereas, in the case of reflection, there can be distinguished between fully reflective (angle of incidence is equal to the angle of reflection) or diffuse, in which case the radiation is equally distributed in all directions.

The reflected component of the radiation at wavelength λ is called reflectivity ρ_λ, the absorbed portion absorptivity α_λ, and the transmitted part of radiation transmissivity τ_λ, whereas their definitions are given here:

Definition

Spectral reflectivity:

$$\rho_\lambda = \frac{\text{Radiation reflected at wavelength } \lambda}{\text{Incident radiation at wavelength } \lambda} = \frac{\dot{q}''_{\lambda\rho}}{\dot{q}''_{\lambda 0}} \; [-] \tag{17.9}$$

Definition

Spectral absorptivity:

$$\alpha_\lambda = \frac{\text{Radiation absorbed at wavelength } \lambda}{\text{Incident radiation at wavelength } \lambda} = \frac{\dot{q}''_{\lambda\alpha}}{\dot{q}''_{\lambda 0}} \; [-] \tag{17.10}$$

Definition

Spectral transmissivity:

$$\tau_\lambda = \frac{\text{Radiation transmitted at wavelength } \lambda}{\text{Incident radiation at wavelength } \lambda} = \frac{\dot{q}''_{\lambda\tau}}{\dot{q}''_{\lambda 0}} \; [-] \tag{17.11}$$

Figure 17.8 shows the experimental results of measurements of the spectral absorptivity of a polished and anodized aluminum plate for wavelengths in the range of heat radiation. The reflectivity, absorptivity, and transmissivity of radiation for a real body are inherently wavelength-specific due to the material properties and molecular structure of the substance involved, and therefore a function of λ. Different materials interact with electromagnetic radiation in unique ways at various wavelengths. These properties vary with wavelength because the energy levels and resonant frequencies of the material's molecular structures dictate the absorption and reflection characteristics at different parts of the electromagnetic spectrum. Thus, the wavelength specificity of reflectivity, absorptivity, and transmissivity highlights the intricate relationship between electromagnetic radiation and the molecular composition of real bodies.

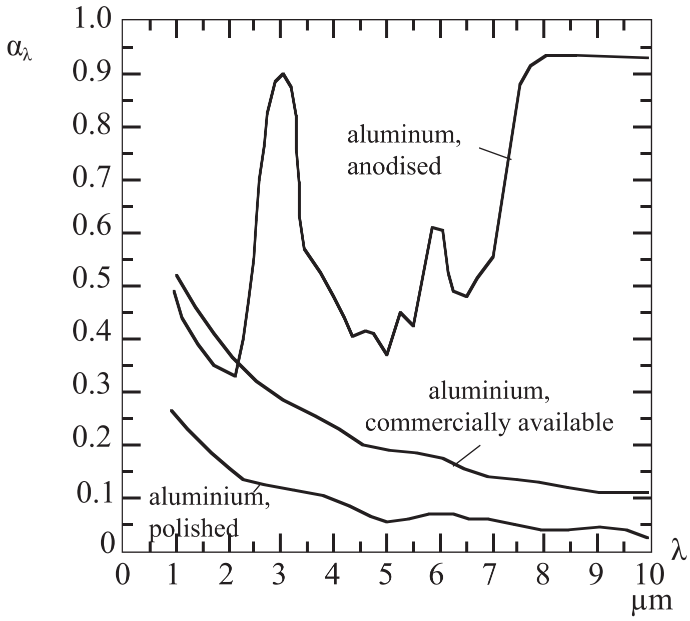


Figure 17.8. Absorptivity of a polished and anodized aluminum plate. Source: Whitaker, 1977. [?] at 0°C.

Derivation

If an energy balance is set up around a real body subjected to incident radiation, a relationship between the reflected, absorbed, and transmitted radiation is found.

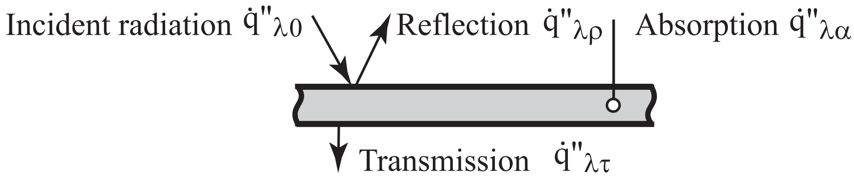


Figure 17.9. Contributions to radiation transport.

1 **Setting up the balance:**

The first law of thermodynamics, yields that the sum of the absorbed, reflected, and transmitted radiation needs to be equal to the incident radiation:

$$0 = \dot{Q}_0 - \dot{Q}_\rho - \dot{Q}_\alpha - \dot{Q}_\tau$$

2 **Defining the elements within the balance:**

The incident radiation yields from the product of the radiative flux and the incident area:

$$\dot{Q}_{\lambda 0} = \dot{q}''_{\lambda 0}\ A$$

The reflected, transmitted, and absorbed radiation is written as:

$$\dot{Q}_{\lambda\rho} = \rho_\lambda\ \dot{q}''_{\lambda 0}\ A \qquad \dot{Q}_{\lambda\alpha} = \tau_\lambda\ \dot{q}''_{\lambda 0}\ A \qquad \dot{Q}_{\lambda\rho} = \tau_\lambda\ \dot{q}''_{\lambda\tau}\ A$$

3 **Inserting and rearranging:**

Inserting the definitions, and cancelling the incident radiation heat flux and area yields:

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$$

□

Thus, the following relationship between the reflectivity, absorptivity, and transmissivity for real bodies has been found:

Fundamental EQ

Relation between ρ_λ, α_λ, and τ_λ for a real body:

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \tag{17.12}$$

Moreover, considering special cases within the context of radiation interaction with solids is important. In many instances, solids exhibit opacity to radiation, where the incoming radiation undergoes a dual transformation—partly reflected and partly absorbed. This phenomenon occurs within thin layers, ranging from a few micrometers for electric conductors to up to 2 millimeters for electric insulators. Consequently:

Fundamental EQ

Relation between ρ_λ, α_λ, and τ_λ for an opaque body:

$$\rho_\lambda + \alpha_\lambda = 1 \tag{17.13}$$

with $\tau_\lambda = 0$

Gases typically exhibit minimal reflection of radiation due to their sparse molecular structure, lacking well-defined interfaces for reflection.

Fundamental EQ

Relation between ρ_λ, α_λ, and τ_λ for a gasses:

$$\alpha_\lambda + \tau_\lambda = 1 \tag{17.14}$$

with $\rho_\lambda = 0$

Solids that completely absorb all incident radiation are termed "black bodies" (and thus $\alpha_\lambda = 1$). The designation of black bodies stems from their unique characteristic of absorbing radiation across all wavelengths without any wavelength-dependent variations (so $\alpha_\lambda = \alpha$).

Fundamental EQ

Relation between ρ, α, and τ for a black body:

$$\alpha = 1 \tag{17.15}$$

with $\rho = 0$, and $\tau = 0$

"Grey bodies" refer to materials with radiation properties that remain wavelength-independent (so $x_\lambda = x$), exhibiting consistent behavior across all wavelengths. In essence, the term "grey" denotes the uniformity of their radiative properties, making them distinct from materials with wavelength-dependent characteristics.

Fundamental EQ

Relation between ρ, α, and τ for a grey body:

$$\rho + \alpha + \tau = 1 \tag{17.16}$$

17.3.1 Total reflection, absorption, and transmission

Typically, limited information is available regarding the wavelength dependence of radiation properties, and determining these dependencies is generally challenging. In practical applications, constant mean values are often employed. Illustrated in Figure 17.8, both polished and commercially available aluminum exhibit a decrease in absorptivity with increasing wavelengths, which is a characteristic trend for all-electric conductors. Conversely, insulators, exemplified by the anodized aluminum surface layer, display an opposite pattern. Due to the complexity of establishing wavelength dependencies for all materials, total values are often used. Total reflectivity, absorptivity, and transmissivity are defined as the averages across all wavelengths:

Definition

Total reflectivity:

$$\rho = \frac{\text{Total radiation reflected}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda\rho} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \frac{\int_0^\infty \rho_\lambda \dot{q}''_{\lambda 0} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} \; [-] \tag{17.17}$$

Definition

Total absorptivity:

$$\alpha = \frac{\text{Total radiation absorbed}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda\alpha} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \frac{\int_0^\infty \alpha_\lambda \dot{q}''_{\lambda 0} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} \; [-] \tag{17.18}$$

Definition

Total transmissivity:

$$\tau = \frac{\text{Total radiation transmitted}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda\tau} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \frac{\int_0^\infty \tau_\lambda \dot{q}''_{\lambda 0} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} \; [-] \tag{17.19}$$

where still the relationship between (total) reflectivity, absorptivity, and transmissivity remains to be valid:

Fundamental EQ

Relation between ρ, α, and τ for bodies:

$$\rho + \alpha + \tau = 1. \tag{17.20}$$

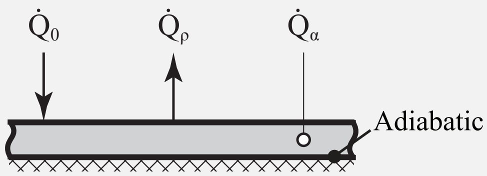
This approach is more practical as total values simplify the effect of variations introduced by the emitting body of the incident radiation. Unlike spectral values, total values provide a more accessible and better representation of the overall radiative behavior of materials.

Example 17.1 Some scientists are conducting groundbreaking experiments on a newly discovered material. This extraordinary substance exhibits unique properties under specific conditions, particularly when exposed to controlled spectra of blackbody radiation. They have developed a device capable of subjecting the material to a precisely tuned partial spectrum of blackbody radiation.

Determine the material's absorptivity, reflectivity, and transmissivity for this spectrum. Assume that the blackbody source is operating at a temperature of 727 °C, and the emitted spectrum spans from 1.5 to 2.5 μm. The specimen's incident area is 10×10 mm, and experimental measurements indicate that the reflected radiation amounts to 8 kW/m^2.

Hint:

- Assume steady-state conditions.
- Conductive and convective effects are negligible.
- Due to the material's inherent properties the back can be considered to be adiabatic.



1 Setting up the balance:

Because the material's back is adiabatic no radiation is transmitted, so $\dot{Q}_\tau = 0$ and thus $\tau = 0$. This yields the following energy balance:

$$0 = \dot{Q}_0 - \dot{Q}_\alpha - \dot{Q}_\rho$$

2 Defining the elements within the balance:

The incident radiation is defined as:

$$\dot{Q}_0 = \left(F_{0\to\lambda_2 T} - F_{0\to\lambda_1 T}\right) \cdot A \cdot \sigma T^4$$

Note that the T^4 requires the temperature to be in Kelvins! Thus $T = 1000$ K.

Using equation (17.8), $\lambda_1 = 1.5\ \mu$m and $\lambda_2 = 2.5\ \mu$m results:

$$\lambda_1 T = 1500\ \mu\text{K} \quad \to F_{0\to\lambda_1 T} = 0.01283 \qquad \text{and} \qquad \lambda_2 T = 2500\ \mu\text{K} \quad \to F_{0\to\lambda_2 T} = 0.16115$$

And so:

$$\dot{Q}_0 = \left(F_{0\to\lambda_2 T} - F_{0\to\lambda_1 T}\right) \cdot A \cdot \sigma T^4 = 0.841\ \text{W}$$

Furthermore, the reflected radiation yields from:

$$\dot{Q}_\rho = \dot{q}''_\rho A = 0.8\ \text{W}$$

Lastly, the absorbed radiation is written as:

$$\dot{Q}_\alpha = \alpha \dot{Q}_0$$

3 Inserting and rearranging:

Inserting all expressions and rewriting yields:

$$\alpha = \frac{\dot{Q}_0 - \dot{Q}_\rho}{\dot{Q}_0} = 0.049$$

Furthermore, the relationship between ρ, α, and τ for bodies yields:

$$\rho = 1 - \alpha = 0.951$$

SUBSECTION 17.4

Kirchhoff's law

As explained, a body emits radiation as a result of thermal energy. According to Planck's law, all objects with a temperature above absolute zero emit electromagnetic radiation. This emitted radiation is in the form of photons, the intensity and spectrum depend on the temperature of the body. The Stefan-Boltzmann law describes the total power radiated per unit surface area, stating that the radiative power is proportional to the fourth power of the absolute temperature. The surface properties influence how efficiently the body emits radiation. Essentially, the higher the temperature of a body, the larger the amount and intensity of radiation being emitted, which contributes to the overall energy exchange in thermal processes.

Kirchhoff's law states the relationship between the absorbed and the emitted radiation of a body. As shown in the following experiment, illustrated in Figure 17.10, a small particle has been located within a large enclosure, where both the particle and the enclosure are constructed of different materials. Furthermore, the enclosure is adiabatic to the outside. The particle is small relative to the enclosure and has a negligible influence on the radiation field, which is due to the cumulative effect of emission and reflection by the enclosure surface. Recall that, regardless of radiative properties, such a surface forms a blackbody cavity. Accordingly, regardless of orientation, the irradiation experienced by the particle in the cavity is diffuse and equal to emission from a blackbody.

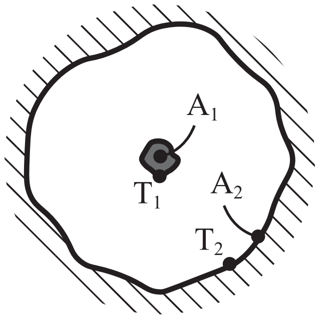


Figure 17.10. Radiation exchange between a small particle within a large enclosure.

Derivation

In thermal equilibrium, according to the 2nd law of thermodynamics, both bodies have the same temperature $T_1 = T_2 = T$.

Consider the cavity to be within a large isothermal enclosure. Because the cavity is much larger than the particle, all radiation within the cavity is diffuse and equal to emission from a blackbody. Section 20.3 explains why this assumption is justified.

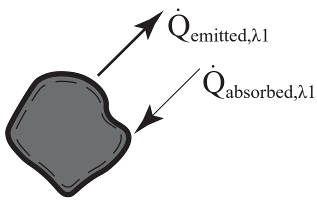


Figure 17.11. Energy balance around the particle.

1 **Setting up the balance:**

Setting up an inner energy balance around the particle would yield:

$$0 = \dot{Q}_{\text{absorbed},\lambda 1} - \dot{Q}_{\text{emitted},\lambda 1}$$

where $\dot{Q}_{\text{absorbed},\lambda 1}$ describes the absorbed radiation at wavelength λ by the particle, and $\dot{Q}_{\text{emitted},1}$ described the radiation at wavelength λ emitted by cavity 1.

2 Defining the elements within the balance:

As said, all radiation within the cavity is diffuse and equal to emission from a blackbody, and therefore the particle absorbs a specific portion of this blackbody radiation:

$$\dot{Q}_{\text{absorbed},\lambda 1} = \alpha_{\lambda 1} \dot{q}''_{\lambda \text{b}} A_1 = \alpha_{\lambda 1} A_1 \sigma T^4$$

Because $\alpha \leq 1$ for all cases, a body can never emit more radiation than a black body would at the same temperature. Therefore, only a specific portion of the black body spectrum at that portion is emitted, which is described by the material property called the emissivity ϵ:

$$\dot{Q}_{\text{emitted},\lambda 1} = \epsilon_{\lambda 1} \dot{q}''_{\lambda \text{b}} A_1 = \epsilon_{\lambda 1} A_1 \sigma T^4$$

3 Inserting and rearranging:

Substitution of all terms yields:

$$0 = \alpha_{\lambda 1} A_1 \sigma T^4 - \epsilon_{\lambda 1} A_1 \sigma T^4$$

Where rewriting yields Kirchoff's law:

$$\alpha_{\lambda 1} = \epsilon_{\lambda 1}$$

□

The definition of the introduced term called emissivity reads as follows:

Definition

Spectral emissivity:

$$\epsilon_\lambda = \frac{\text{Radiation emitted at wavelength } \lambda}{\text{Blackbody radiation at wavelength } \lambda} = \frac{\dot{q}''_{\lambda \epsilon}}{\dot{q}''_{\lambda b}} \; [-] \qquad (17.21)$$

Moreover, as proven, the spectral absorptivity of a body equals spectral emissivity:

Fundamental EQ

Kirchoff's law of thermal radiation:

$$\alpha_\lambda = \epsilon_\lambda \qquad (17.22)$$

Yet, for many practical purposes Kirchoff's law for the total absorptivity and emissivity must be employed to avoid unnecessary complex computations or also because of the absence of spectral data. The requirements for the validity of Kirchhoff's law for total absorptivity and emissivity are discussed next. For this purpose, the total emissivity ϵ has to be determined by integrating the spectral radiation power.

Definition

Total emissivity:

$$\epsilon = \frac{\text{Total radiation emitted}}{\text{Total blackbody radiation}} = \frac{\int_0^\infty \dot{q}''_{\lambda \epsilon} d\lambda}{\int_0^\infty \dot{q}''_{\lambda b} d} = \frac{\int_0^\infty \epsilon_\lambda \dot{q}''_{\lambda b} d\lambda}{\int_0^\infty \dot{q}''_{\lambda b} d\lambda} = [-] \qquad (17.23)$$

Recall the definition of the total absorptivity and emissivity:

$$\alpha = \frac{\int_0^\infty \dot{q}''_{\lambda\alpha} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \frac{\int_0^\infty \alpha_\lambda \dot{q}''_{\lambda 0} d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0} d\lambda} = \alpha\left(T_{\text{body}}, T_{\text{rad}}\right)$$

$$\epsilon = \frac{\int_0^\infty \dot{q}''_{\lambda\epsilon} d\lambda}{\int_0^\infty \dot{q}''_{\lambda b} d} = \frac{\int_0^\infty \epsilon_\lambda \dot{q}''_{\lambda b} d\lambda}{\int_0^\infty \dot{q}''_{\lambda b} d\lambda} = \epsilon\left(T_{\text{body}}\right)$$

Its definition shows that the total absorptivity is contingent on the characteristics of both radiating bodies—the one emitting the incident radiation and the one absorbing, along with their respective temperatures T_{body} and T_{rad}.

Comparing the definitions of total emissivity and absorptivity highlights that Kirchhoff's law for total emissivity ϵ and total absorptivity α, may not universally hold. This is because the dependence on temperature and spectral distribution is not necessarily equal for incident and emitted radiation.

Two special cases warrant mention in which Kirchhoff's law holds for the **total** radiation properties. These cases are deduced from the definitions of total emissivity and absorptivity:

- The radiating body is a black or grey body whose temperature is equal to that of the investigated body, $T_{\text{rad}} = T_{\text{body}}$
- The surfaces of the body are grey, i.e. their absorptivity is independent of the wavelength.

Fundamental EQ

Kirchoff's law of thermal radiation for total absorptivity and emissivity:

$$\alpha = \epsilon, \tag{17.24}$$

if $T_{\text{rad}} = T_{\text{body}}$ or the surfaces of the body are grey.

Especially critical is the consideration of grey bodies, as this simplification applies to numerous practical scenarios. In applications where a high level of precision is required, having accurate knowledge of monochromatic radiation properties becomes indispensable. Nevertheless, employing the approximation of bodies behaving as grey bodies can be a convenient approach. To elucidate this concept further, Figure 17.12 depicts the radiant flux density of an actual real body at $T = 2000\text{K}$. The radiant power of this real body is compared to that of a black body and a grey body. This illustration underscores that such a straightforward assumption may still provide a first satisfactory estimation.

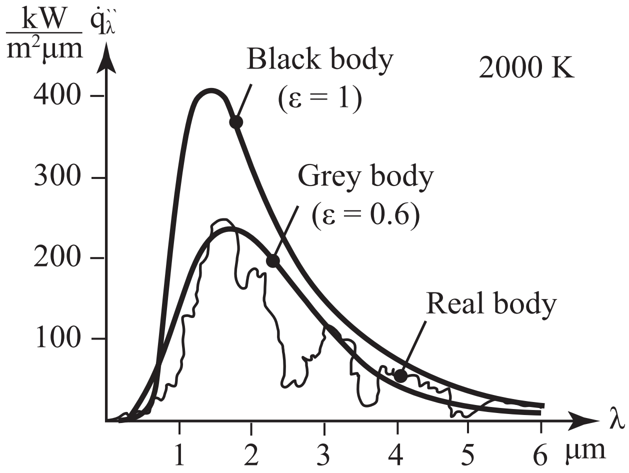


Figure 17.12. Spectral radiant flux density of a black, grey, and a real body surface.

17.4.1 Radiation from a diffuse surface and direction-dependent radiation

Emissivity data for different materials, see e.g. tables in Appendix, are usually determined by measurements in the entire hemisphere. Most of the real surfaces used in practice have a direction-dependent emissivity, so only ideal surfaces such as that of the black or grey body are considered to be diffuse, i.e. independent of the direction.

Measurement results, as depicted in Figure 17.13, show that for example, electric conductors have direction-dependent emissivity, which increases with increasing viewing angle φ. On the other hand, insulators, as depicted in Figure 17.14, possess nearly constant and relatively large emissivity over a wide range of viewing angles.

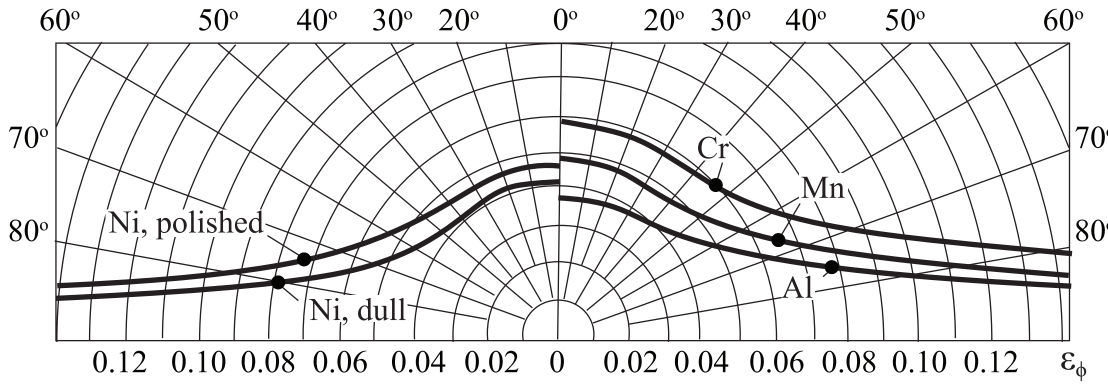


Figure 17.13. Electric conductors.

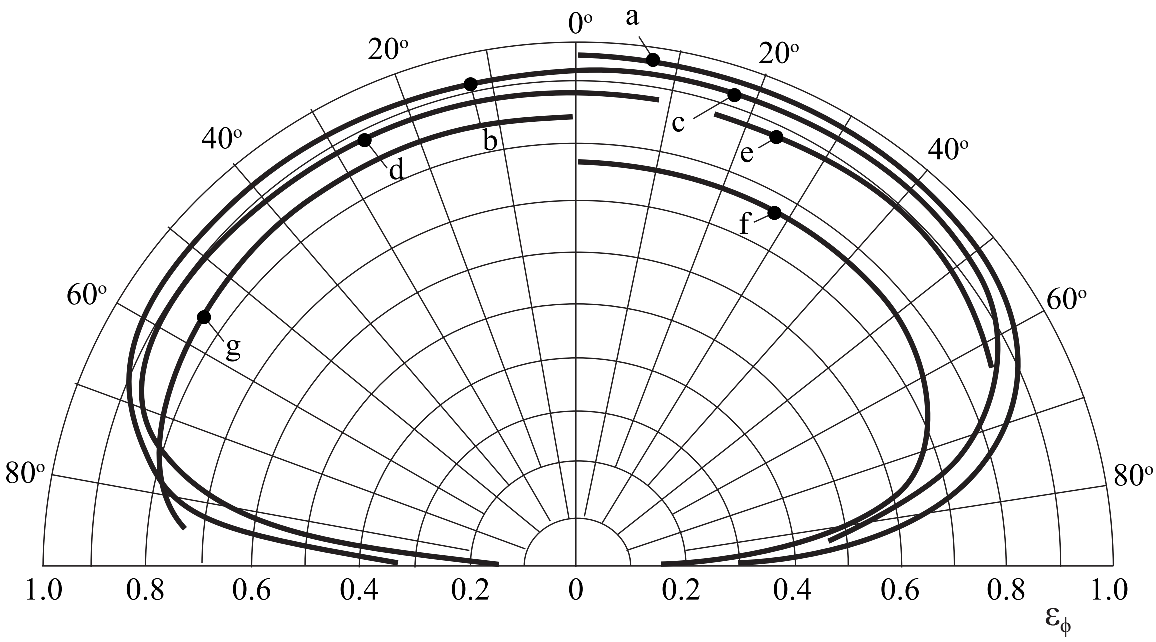


Figure 17.14. Isolators: (a) Wet ice, (b) wood, (c) glass, (d) paper, (e) chalk, (f) copper oxide, (g) aluminium oxide at room temperature.

SECTION 18
View factors

L04 - View factors:

Learning goals:

- Comprehending the ratio of radiated to incident radiation.
- Understanding the distribution of radiation irradiating from a surface within an enclosing hemisphere.
- Developing the ability to determine view factors between two surfaces at specified angles.

Comprehension questions:

- What parameters of radiation emerging from a surface are encompassed or described by the view factor concept?
- Is the calculation of radiation exchange using view factors valid when the bodies radiate directionally?
- In general, what factors do view factors depend on?

L05 - View factor recipies:

Learning goals:

- Understanding that the sum of view factors for one object is equal to 1.
- Acquiring the skill to determine view factors by examining the opposite surface or object.
- Employing symmetry conditions in problem-solving.
- Recognizing and utilizing meaningful auxiliary planes for calculating view factors.

Comprehension questions:

- What rules are employed for the determination of view factors?
- In the context of body shapes, for which shapes must Φ_{ii} be taken into consideration?

Corresponding tutorial exercises:

- Exercise IV.1 Infinite pipe segment

SUBSECTION 18.1
Principle of view factors

The radiation properties necessary to describe the heat flow from a body have been discussed so far. If not the total emitted heat flow from a body, but the heat exchange between the body and a nearby object is regarded, the exact radiation amount in the direction of the object as well as the radiation absorbed by this object needs to be known, which introduces the topic of view factors.

The principle of view factors is illustrated by a 2D example, using two bodies: body 1, a small plate emitting diffuse radiation, and body 2, a larger plate positioned further away, as depicted in Figure 18.1.

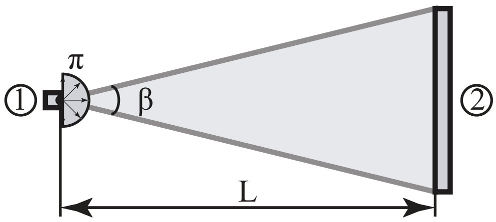


Figure 18.1. Small plate radiating diffuse radiation on a larger big plate.

Given that body 1 is significantly smaller in length than body 2, body 1 is treated as a point source emitting radiation in the form distributed over a semi-circle with an angle of π. However, only a fraction of this radiation reaches body 2. This fraction is represented by the angle β within the semi-circle, where the total angle is π. Therefore the fraction radiated from body 1 onto body 2 is defined as:

$$\Phi_{12} = \frac{\beta}{\pi}$$

where Φ_{12} is defined as the view factor from body 1 to body 2.

Definition

View factor:

$$\Phi_{ij} = \frac{\text{Radiation intercepted by body } j}{\text{Radiation leaving body } i} \quad [-] \tag{18.1}$$

This factor, denoted by Φ, provides insights into how radiation is distributed between surfaces. The specific portion being radiated depends on the distance L and thus the geometric arrangement of the system.

Subsection 18.2
Radiosity

To better comprehend the impact of a system's geometrical arrangement, the concept of radiosity must be understood. The radiosity is the rate at which radiation energy leaves a unit area of a surface in all directions.

Definition

Radiosity or radiation density:

$$L = \frac{\text{Power of radiator}}{\text{Projected area} \times \text{Solid angle}} = \frac{\dot{q}''}{\Omega} \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \tag{18.2}$$

The following derivation details how the definition of radiosity is established.

Derivation

Imagine a hemisphere of radius r over a small surface element dA, positioned at the center of the hemisphere. In this case, the entire heat flow emitted from the surface element is emitted on the hemisphere.

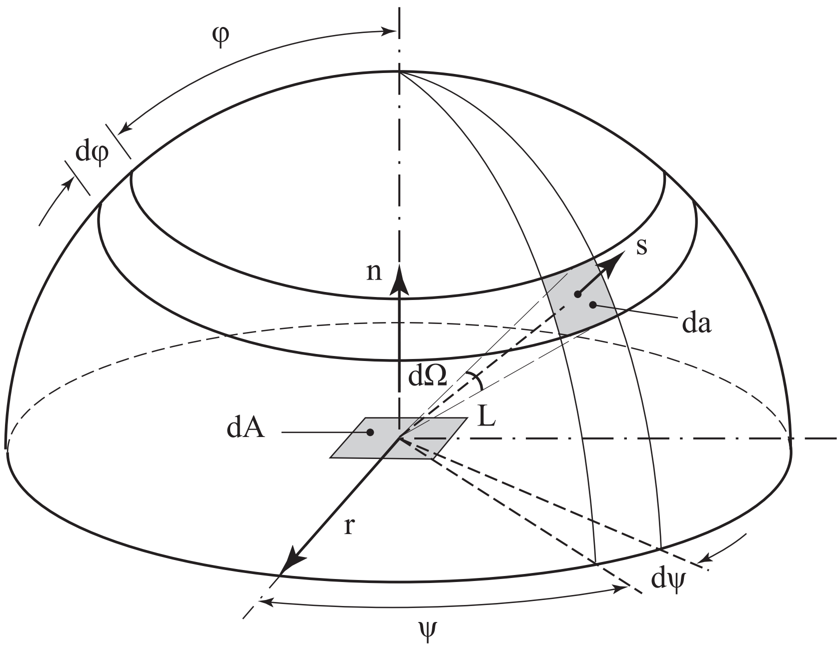


Figure 18.2. Radiation between a surface element and a hemisphere.

1 **Setting up the definition of the radiative power:**

Considering a surface element on the hemisphere da, the energy flow from dA to da is defined by the radiation density or radiosity L.

Diffuse surfaces, i.e. surfaces whose radiation is direction-independent, such as a grey or black body, have constant radiosity L. The energy flux $d\dot{Q}_{dA\rightarrow da}(\varphi,\psi)$ is thus proportional to the projected area $dA\cos\varphi$. The radiation density L refers to the radiation density of a surface, perceptible to the eye. Notably, for diffusely radiating bodies, this quantity remains independent of the viewing angle.

$$d\dot{Q}_{dA\rightarrow da}(\varphi,\psi) = \underbrace{L}_{\text{Radiosity}} \; d\Omega \; \underbrace{dA\,\cos\varphi}_{\text{In radiation direction}} \quad ,$$

$dA\,\cos\varphi$ is in the direction of the radiation per solid angle $d\Omega$.

2 Defining the required elements:

The area da is expressed in terms of the radius r, and angles φ, $d\Psi$, $d\varphi$:

$$da = (r \cdot \sin\varphi\, d\Psi) \cdot (r \cdot d\varphi)$$

The solid angle $d\Omega$ is measured in units of "steradian" [sr] and describes the ratio of the surface element on the hemisphere to the radius of the sphere squared

$$d\Omega = \frac{da\,(\varphi,\psi)}{r^2} = \frac{(r\cdot\sin\varphi\ \cdot d\Psi)\cdot(r\cdot d\varphi)}{r^2} = \frac{r\ \sin\varphi\, d\psi r d\varphi}{r^2} = \sin\varphi\ \ d\varphi\, d\psi$$

3 Inserting and rearranging:

Inserting the definition of the area da and the solid angle $d\Omega$ yields:

$$d\dot{Q}_{dA\rightarrow da}(\varphi,\psi) = L\ \sin\varphi\ \cos\varphi\, d\phi\, d\psi\, dA$$

Rewriting yields:

$$\underbrace{\int \frac{d\dot{Q}_{dA\rightarrow da}(\varphi,\psi)}{dA}}_{\dot{q}''} = L \underbrace{\int_{\Psi=0}^{2\pi}\int_{\varphi=0}^{\pi/2} \sin\varphi\ \cos\varphi\, d\phi\, d\psi}_{\pi}$$

After integration over the hemisphere, the total radiative power from the surface to the hemisphere is found. This showcases the relationship between the radiation density and the emissive power of the surface element

$$\dot{q}'' = \pi L$$

Consequently, the total emissive power originating from a surface is determined by multiplying the surface area by the radiosity and the solid angle, which yields a quantity known as surface brightness. Further details on surface brightness are explored in Section 19.1. □

Subsection 18.3

Radiation transfer bodies

Analyzing radiation within an enclosure with n surfaces involves assessing n^2 view factors, and this assessment is likely the most time-intensive aspect of radiation analysis. However, directly evaluating all view factors is neither practical nor essential. Once a satisfactory number of view factors are obtained, the remaining ones are determined by employing fundamental relations for view factors, as explained in the following discussion.

18.3.1 Reciprocity rule

If radiation transfer between two diffusely radiating surfaces of two bodies with different temperatures occurs, the hotter body emits more heat than the colder one, so that the net heat flow is from the hotter to the colder body. In an arbitrary orientation in space, heat $d\dot{Q}_{1\to 2}$ flows from the surface area dA_1 of body 1 to the surface area dA_2 of body 2.

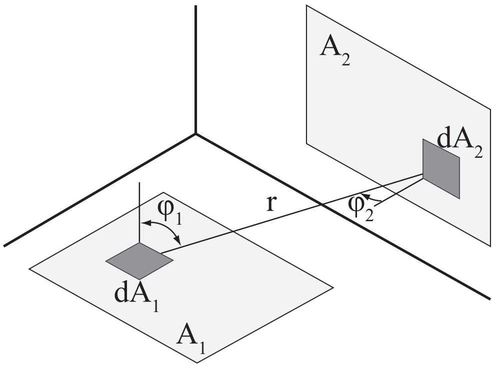


Figure 18.3. Radiation transfer between two surfaces.

Derivation

1 **Setting up the balance:**
The energy balance around surface 2 would read:

$$0 = d\dot{Q}_{1\to 2} + d\dot{Q}_{1\rightleftharpoons 2} - d\dot{Q}_{2\to 1}$$

Where $d\dot{Q}_{1\rightleftharpoons 2}$ describes the net rate of heat transfer from surface 1 to 2.

2 **Defining the elements within the balance:**

The rate of heat transfer from body 1 to 2 reads:

$$d\dot{Q}_{1\to 2} = L_1 \cos\varphi_1 \, d\Omega_1 \, dA_1$$

Since the surface element dA_2 is not perpendicular to the connecting line r, only the projected portions of the area dA_2 should be used to determine the solid angle, i.e.

$$d\Omega_1 = \frac{dA_2 \cos\varphi_2}{r^2}$$

Which yields

$$d\dot{Q}_{1\to 2} = L_1 \int\int \frac{\cos\varphi_1 \cos\varphi_2}{r^2} dA_1 dA_2$$

Replacing the radiosity L by the radiation power $\dot{q}''$ using equation (18.2):

$$d\dot{Q}_{1\to 2} = \dot{q}_1'' \underbrace{\int\int \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2}_{\Phi_{12} A_1}$$

A similar relationship is obtained for the heat $d\dot{Q}2 \to 1$ from the surface element $dA2$ radiated to the surface element dA_1.

$$d\dot{Q}_{2\to 1} = \dot{q}_2'' \underbrace{\int\int \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2}_{\Phi_{21} A_2}$$

3 **Inserting and rearranging:**

Hence, the net radiation transfer between the two bodies is

$$d\dot{Q}_{1\rightleftharpoons 2} = \dot{q}_1'' \underbrace{\int\int \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2}_{\Phi_{12} A_1} - \dot{q}_2'' \underbrace{\int\int \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2}_{\Phi_{21} A_2}$$

The double integration over the surfaces of the two bodies gives an expression that contains only geometrical quantities and none of the radiation properties, but on the other hand, such integration is normally too complex and is usually replaced by numerical, graphic, or photographic methods. The result of integration, which is the product of the view factor and the area, is readily available for several typical geometrical orientations in space and can be found in other literature. Usually, these results are given as the so-called view factors.

Integrating yields a relationship for the view factor Φ_{12}, which depends only on the geometric values:

$$\Phi_{12} = \frac{1}{A_1} \int_{A_2}\int_{A_1} \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2$$

and for Φ_{21}

$$\Phi_{21} = \frac{1}{A_2} \int_{A_2}\int_{A_1} \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2$$

Since the double integrals stated in the two equations above are equal, the following relationship called **the reciprocity rule** is found:

$$A_1 \Phi_{12} = A_2 \Phi_{21}$$

Finally, using the definition of the view factors, the net rate of radiation yields:

$$\begin{aligned}\dot{Q}_{1\rightleftharpoons 2} &= A_1 \Phi_{12} \left(\dot{q}_1'' - \dot{q}_2''\right) \\ &= A_2 \Phi_{21} \left(\dot{q}_1'' - \dot{q}_2''\right)\end{aligned}$$

If only black bodies are involved, the emissive power consists only of the emitted radiation, for which the Stefan-Boltzmann law, stated in equation (17.6), is valid. Hence

$$\begin{aligned}\dot{Q}_{1\rightleftharpoons 2} &= A_1 \Phi_{12} \sigma \left[(T_1)^4 - (T_2)^4\right] \\ &= A_2 \Phi_{21} \sigma \left[(T_1)^4 - (T_2)^4\right]\end{aligned}$$

□

The derivation stated above, showed that the product of surface A_1 and view factor Φ_{12} is identical to the product of surface A_2 and view factor Φ_{21}, as they both describe the same integral containing geometrical quantities. This relationship is called the reciprocity rule and is expressed in a more general form as:

Fundamental EQ

Reciprocity rule:

$$A_i \Phi_{ij} = A_j \Phi_{ji} \tag{18.3}$$

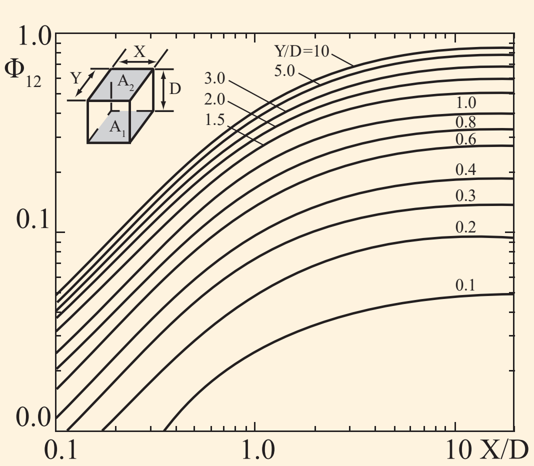
18.3.2 View factor diagrams

As said, view factors are expressions that contain only geometrical quantities and no radiation properties. Often integration is too complex and is usually replaced by numerical, graphic, or photographic methods.

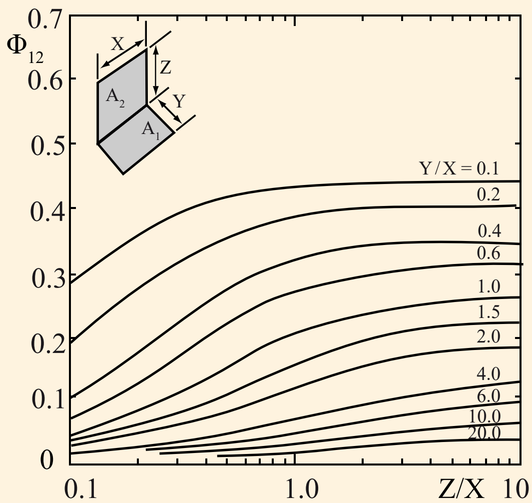
As an example, the view factor diagrams, depicted below, list how the view factors can be determined for radiation transfer between parallel or perpendicular plates [**?**]. This may be used for example, in the determination of direct radiation transfer between two walls of a room.

Fundamental EQ

View factor diagrams:



(a) Radiation transfer between parallel, rectangular plates.



(b) Radiation transfer between perpendicular, rectangular plates.

18.3.3 Summation rule

The summation rule of view factors stems from the conservation of energy during thermal radiation exchange among surfaces. This rule states that the total sum of view factors from a specific surface, denoted as i, to all other surfaces within the system equals 1. This conservation principle arises from fully considering all the radiation leaving a surface. Consequently, the sum of the fractions of radiation directed toward every other surface in the system must be equivalent to 1.

Derivation

The principle of the summation rule is depicted in Figure 18.5, where a semicylinder is radiating on itself and the neighboring bodies.

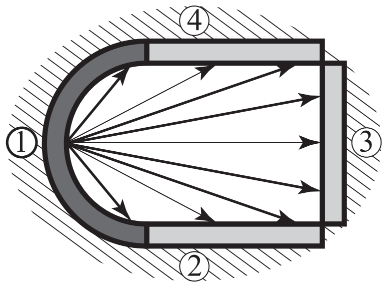


Figure 18.5. Radiative heat exchange within and between adjacent bodies.

1 Setting up the definition of the emitted heat:

Drawing upon the principles of thermodynamics, which assert the conservation of energy, all thermal radiation must be emitted to the surrounding bodies. Therefore the emitted radiation by body 1 is defined as:

$$\dot{Q}_1 = \dot{Q}_{1\to1} + \dot{Q}_{1\to2} + \dot{Q}_{1\to3} + \dot{Q}_{1\to4}$$

2 Defining the required elements:

Where:

$$\dot{Q}_{1\to1} = \Phi_{11}\dot{Q}_1 \qquad \dot{Q}_{1\to2} = \Phi_{12}\dot{Q}_1 \qquad \dot{Q}_{1\to3} = \Phi_{13}\dot{Q}_1 \qquad \dot{Q}_{1\to4} = \Phi_{14}\dot{Q}_1$$

3 Inserting and rearranging:

Inserting gives:

$$\dot{Q}_1 = (\Phi_{11} + \Phi_{12} + \Phi_{13} + \Phi_{14}) \cdot \dot{Q}_1$$

Cancelling $\dot{Q}_1$ yields:

$$\Phi_{11} + \Phi_{12} + \Phi_{13} + \Phi_{14} = 1$$

Which describes the summation rule. □

This summation rule is written in a more general form:

Fundamental EQ

Summation rule:

$$\sum_{j=1}^{n} \Phi_{ij} = \Phi_{i1} + \Phi_{i2} + \Phi_{i3} + \ldots + \Phi_{in} = 1 \tag{18.4}$$

18.3.4 Symmetry

Determining view factors becomes more achievable when the geometry involved exhibits some form of symmetry. Hence, there must be checked for symmetry in a problem before directly calculating view factors. Surfaces that are identical and oriented in the same way concerning another surface intercept equivalent amounts of radiation from that surface. Therefore, the symmetry rule can be stated as two (or more) surfaces displaying symmetry about a third surface, they have identical view factors from that surface.

In the scenario illustrated in Figure 18.5, bodies 2 and 4 exhibit symmetry relative to body 1. Consequently:

$$\Phi_{12} = \Phi_{14}$$

This is written in a more general way:

Fundamental EQ

Symmetry rule:

$$\Phi_{ij} = \Phi_{ik}, \tag{18.5}$$

if two or more surfaces display symmetry about a third surface, they will have identical view factors from that surface.

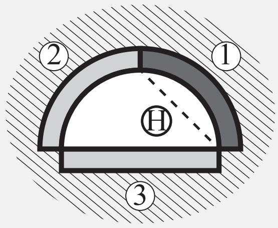
18.3.5 Auxiliary planes

Simplifying problems becomes more achievable when the geometry makes use of auxiliary planes. Smart selection of auxiliary planes helps in dividing a more complicated geometry into simplified geometries.

Example 18.1

The principle of the auxiliary planes can be illustrated best by the following example. Ta aim is to determine the view factor Φ_{11} for the following case:

(a) Body 1 emitting.



(b) Introduction of the auxiliary plane "H".

To determine the view factor, the auxiliary plane "H" is introduced, as depicted in the right figure above. The auxiliary plane cannot see itself and thus does not emit radiation onto itself. Therefore:

$$\Phi_{\mathrm{HH}} = 0$$

The summation rule around plane H ($\Phi_{\mathrm{HH}} + \Phi_{\mathrm{H1}} = 1$) yields:

$$\Phi_{\mathrm{H1}} = 1$$

Using the reciprocity rule ($\Phi_{\mathrm{1H}} A_1 = \Phi_{\mathrm{H1}} A_{\mathrm{H}}$) results:

$$\Phi_{\mathrm{1H}} = \frac{A_{\mathrm{H}}}{A_1} = \frac{D/\sqrt{2}}{\pi D/4} = \frac{\sqrt{8}}{\pi}$$

The summation rule for body 1 ($\Phi_{11} + \Phi_{\mathrm{1H}} = 1$) gives:

$$\boxed{\Phi_{11} = 1 - \Phi_{\mathrm{1H}} = 1 - \frac{\sqrt{8}}{\pi}}$$

The determination of Φ_{11} would not be possible without the presence of the auxiliary plane. This underscores the effectiveness of using this method. Thoughtful selection of auxiliary planes facilitates the subdivision of complex problems into simplified forms.

HeatQuiz 18.1

View factors:

Section 19

Radiative transport

L06 - Surface brightness:

Learning goals:

- Comprehending the concept of surface brightness and its significance.
- Acquiring the knowledge and proficiency to formulate the surface brightness of individual bodies and systems of bodies.

Comprehension questions:

- What is the physical interpretation of surface brightness?
- What principles should be considered when establishing surface brightness?
- Why is the infrared measurement of surface temperatures challenging, and which aspect of surface brightness conveys this information?

L07 - Energy balances:

Learning goals:

- Comprehending the concept of energy balances in radiative heat transfer.
- Developing the ability to construct energy balances around a body.
- Understanding both internal and external energy balances.

Comprehension questions:

- What events contribute to a temporal change in thermal energy within the control volume?
- Which terms are additionally considered in the outer energy balance? How can inner and outer energy balances be interconnected?
- In which applications is an inner or external energy balance more advantageous?

Corresponding tutorial exercises:

- Exercise IV.2 Solar power tower
- Exercise IV.3 Hemispherical dome
- Exercise IV.4 Light bulb
- Exercise IV.5 Cupola

Subsection 19.1

Surface brightness

Examining the transfer of radiation comprising black surfaces is relatively straightforward. However, practical cases commonly feature non-black surfaces, permitting reflection and transmission. The radiation of such cases becomes highly intricate unless certain simplifying assumptions are applied.

To still facilitate analysis, often assumptions about the surfaces of bodies are made. These assumptions include considering the surfaces as diffuse and gray. In other words, the surfaces function as diffuse emitters reflectors, and transmittors, and their radiation properties remain constant across all wavelengths. Additionally, surfaces need to be assumed to be isothermal.

Before working with radiative energy balances, the topic of surface brightness should be understood. Surfaces play a role in emitting, transmitting, and reflecting radiation. When computing the heat transfer of radiation between surfaces, the focus is on the overall radiation energy coming from a surface, without considering the source. The quantity representing the total rate of radiation energy leaving a surface is known as surface brightness.

Take into account an object receiving radiation on both the top and back, as illustrated in Figure 19.1. A portion of the radiation received on the top is reflected. Simultaneously, some of the radiation received on the bottom transmits through the top surface. Due to the object's temperature, also radiation is emitted. The combination of these three heat fluxes—reflection, transmission, and emission—contributes to the surface brightness of the object.

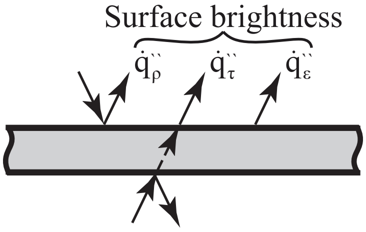


Figure 19.1. Surface reflecting, transmitting, and emitting radiation.

Definition

Surface brightness:

$$\dot{Q} = \dot{Q}_\rho + \dot{Q}_\tau + \dot{Q}_\epsilon \text{ [W]} \tag{19.1}$$

The definition stated in equation (19.1), yields that for an opaque body, that does not transmit any radiation, the surface brightness only consist of the reflection and emission term:

$$\dot{Q} = \dot{Q}_\rho + \dot{Q}_\epsilon$$

Or for a black body, which does not transmit, nor reflect radiation:

$$\dot{Q} = \dot{Q}_\epsilon$$

Note that surface brightness is contingent on the specific characteristics of the surface. For the plate depicted in Figure 19.1, the surface brightness for the top and bottom surfaces is not identical.

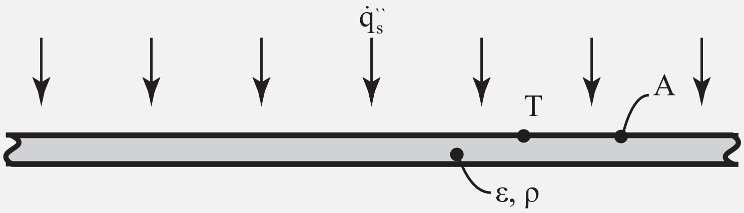
HeatQuiz 19.1

Surface brightness:

The following example clearly illustrates why the top and bottom of the plate do not have the same surface brightness.

Example 19.1

A grey flat plate (ϵ, ρ, A, T) is subjected to solar irradiation $\dot{q}''_s$ from the top. The plate does not receive any radiation on the bottom. Determine the surface brightness on the top and the bottom of the plate.



① **Setting up the definition of the surface brightness:**

$$\dot{Q}_{\text{top}} = \dot{Q}_{\text{top},\rho} + \dot{Q}_{\text{top},\tau} + \dot{Q}_{\text{top},\epsilon}$$

$$\dot{Q}_{\text{bottom}} = \dot{Q}_{\text{bottom},\rho} + \dot{Q}_{\text{bottom},\tau} + \dot{Q}_{\text{bottom},\epsilon}$$

② **Defining the required elements:**

The reflected radiation on the top yields the portion of the solar radiation being reflected:

$$\dot{Q}_{\text{top},\rho} = \rho\, \dot{q}''_s\, A$$

The bottom does not receive any radiation, and thus:

$$\dot{Q}_{\text{bottom},\rho} = 0$$

Furthermore, the transmissivity yields from Kirchoff's law $\epsilon = \alpha$, and the relationship for the spectral properties $\tau + \alpha + \rho = 1$:

$$\tau = 1 - \rho - \epsilon$$

Since the bottom does not receive any radiation, no radiation is transmitted through the top surface:

$$\dot{Q}_{\text{top},\tau} = 0$$

The solar radiation received on the top is transmitted through the bottom surface:

$$\dot{Q}_{\text{bottom},\tau} = (1 - \rho - \epsilon)\ \dot{q}''_s\ A$$

Lastly, the emission on the top and on the bottom surface are the same and is expressed as:

$$\dot{Q}_{\text{top},\epsilon} = \dot{Q}_{\text{bottom},\epsilon} = \epsilon\, A\, \sigma\, T^4$$

3 Inserting and rearranging:

Inserting yields the surface brightness of the top and bottom of the plate:

$$\dot{Q}_{\text{top}} = \rho\,\dot{q}_{\text{s}}''\,A + \epsilon\,A\,\sigma\,T^4$$

$$\dot{Q}_{\text{bottom}} = (1 - \rho - \epsilon)\;\dot{q}_{\text{s}}''\,A + \epsilon\,A\,\sigma\,T^4$$

This illustration highlights the importance of exercising caution when discussing surface brightness in the context of a single body, such as a flat plate. Despite the plate having a uniform temperature distribution, this does not mean that the surface brightness are identical on both the top and bottom surfaces.

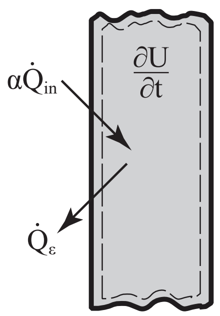
Subsection 19.2
Energy balances

Now understanding both surface brightness, representing the total radiation energy leaving a surface per unit of time, and view factors, denoting the fraction of radiation intercepted by body j from the total radiation leaving body i, these concepts can be applied to evaluate energy balances related to thermal radiation.

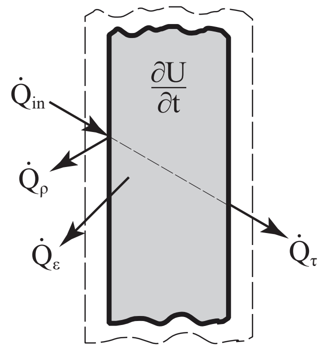
The general definition of energy balances had been stated before in equation (**??**) and is listed here again:

Temporal change in inner energy = ingoing fluxes − outgoing fluxes + sources − sinks

In the context of radiative transport, the definition of the boundary over which the energy balance is applied is crucial. Take as an example a solid body exposed to thermal radiation, as illustrated in Figure 19.2. The energy balance involves terms such as "temporal change in inner energy," "ingoing fluxes," and "outgoing fluxes." However, the definitions of "ingoing fluxes" and "outgoing fluxes" vary depending on how one defines the boundary of the control volume.



(a) Inner energy balance.



(b) Outer energy balance.

Figure 19.2. Body exposed to thermal radiation.

Consider another body positioned far away from the body, possessing a specific surface brightness. A fraction of this surface brightness, determined by the view factors, impinges upon the body, forming the basis for establishing the energy balance.

Suppose the control volume is established to encompass the volume just within the solid body, as depicted in Figure 19.2a. In that case, the boundary of this control volume will not experience any reflected or transmitted radiation. The energy balance solely considers the changes in inner energy, derived from the difference between the absorbed incoming flux and the radiation emitted due to the body's temperature. In the absence of sources or heat sinks, the inner energy balance would be expressed as:

Fundamental EQ

Inner energy balance closed system without heat generation:

$$\frac{\partial U}{\partial t} = \alpha \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\epsilon} \qquad (19.2)$$

But now, suppose the control volume is set just outside the volume of the solid body, as depicted in Figure 19.2b. In that case, the boundary of this control volume encounters reflected or transmitted radiation. In this scenario, the temporal change in inner energy is determined by the disparity between the incoming radiation flux and the reflected, emitted, and transmitted fluxes. In this case, the outgoing fluxes arise from the surface brightness of the body and not only the thermal emission due to the temperature. In the absence of sources or heat sinks, the outer energy balance would be expressed as follows:

Fundamental EQ

Outer energy balance closed system without heat generation:

$$\frac{\partial U}{\partial t} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{19.3}$$

When comparing the inner and outer energy balances, they both delineate the same temporal change in inner energy. The inner energy balance exclusively deals with the absorbed portion of the incoming surface brightness, disregarding the roles of reflection and transmission. On the other hand, the outer energy balance incorporates both incoming and outgoing surface brightness, accounting for the influences of reflection and transmission in the process. Noteworthy is that the balances are equivalent.

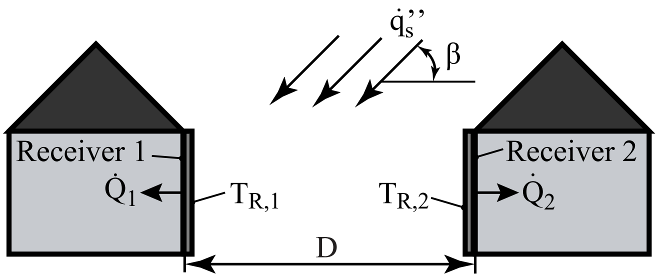
In certain instances, utilizing the inner energy balance may be more convenient due to the fewer terms. Yet, the outer balance, derived directly from surface brightness calculations, can be more straightforward.

HeatQuiz 19.2

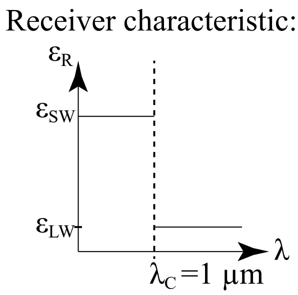
Energy balances:

Demonstration 19.1

Solar radiation $\dot{q}''_S$ is to be converted into a dissipated heat flux using a receiver. The receiver is mounted vertically on a house's wall and is irradiated at an angle β by the sun. The receiver surface has wavelength-specific radiation properties with the cut-off wavelength λ_C. The characteristics are shown in the graph below.



(a) Two houses with a solar receiver installed on its side.



(b) Receiver emissivity.

Given parameters:

- Heat flux density of the sun: $\dot{q}''_S$
- Irradiation angle of the sun: β
- Temperature of the sun: $T_S = 6000$ K
- Length of receiver area: $L = 5$ m

- Width of receiver area: $W = 3$ m
- Distance between both receivers: $D = 10$ m
- Area of receiver: $A_{\mathrm{R}} = L \times W = 15\ \mathrm{m}^2$
- Cut of wavelength receiver: $\lambda_{\mathrm{C}} = 1\ \mu\mathrm{m}$
- Emissivity short wavelength receiver: $0 < \epsilon_{\mathrm{SW}} < 1$
- Emissivity long wavelength receiver: $0 < \epsilon_{\mathrm{LW}} < 1$
- Transmissivity all wavelengths receiver: $\tau = 0$
- Temperature of receiver 1: $T_{\mathrm{R},1} = 400$ K
- Temperature of receiver 2: $T_{\mathrm{R},2} < T_{\mathrm{R},1}$

Hints:

- All surfaces radiate diffusely.
- The receivers are no grey radiators.
- Heat conduction and convection are negligible.
- Disregard radiant heat flows from the surroundings.

Tasks:

a) Determine the heat flux $\dot{Q}_1$ dissipated by receiver 1. To this end, neglect the influence of receiver 2 in this subproblem.

To utilize the radiation reflected by receiver 1 the neighboring house, too, installs an identical receiver (receiver 2).

b) Determine the view factor Φ_{12} between the two identical receivers.

c) Determine the surface brightness in the short and long wavelength range for the surface of receiver 2. In this subproblem assume that the surface brightness value of receiver 1 in the short $\dot{q}''_{1,\mathrm{SW}}$ and long wavelength $\dot{q}''_{1,\mathrm{LW}}$ range are known. Due to the orientation, no sunlight directly impinges on receiver 2.

Video solution:

Section 20

Radiative examples

L08 - Heat transfer between two parallel plates:

Learning goals:

- Understanding the calculation of the radiation transfer between two surfaces using radiation tracking.
- Developing the ability to describe the radiation exchange using surface brightness.

Comprehension questions:

- In which case is radiation tracking a reasonable method for calculation?
- Why is the use of surface brightness the more elegant method for calculating radiation transfer?

L09 - Protective shield:

Learning goals:

- How well can radiation be shielded and which properties make a good radiation protective shield in the case of two parallel plates?

Comprehension questions:

- Why is the radiation exchange reduced, even in the case of the shield being a black body?
- What happens if the three plates have identical radiation properties?

L10 - Radiation between enclosed bodies:

Learning goals:

- Learn to calculate the radiation exchange for enclosed bodies.
- Use the approach for solving radiation tasks.

Comprehension questions:

- For self-enclosed grey bodies, which properties may contribute to increased radiative exchange?
- Which marginal cases exist and what is their meaning?

L11 - Three body problem:

Learning goals:

- Expansion of the balances from two-body to multi-body problems.
- Learning Approaches to solve Radiation tasks using the example of a three-body problem.

Comprehension questions:

- Why is a calculation of radiation transfer much more complicated when a third object is added?
- If several bodies are involved in the radiation transfer, can certain bodies be combined? In which case can bodies be combined?

L12 - Cooking recipe radiation tasks:

Learning goals:

- Developing the ability to solve radiation problems through a systematic approach.

Comprehension questions:

- What are the most important points that need to be clarified before calculating radiation tasks?

Corresponding tutorial exercises:

- Exercise IV.6 Pokè bowl
- Exercise IV.7 Radiation within a wedge-shaped opening
- Exercise IV.8 Earth's atmosphere

SUBSECTION 20.1

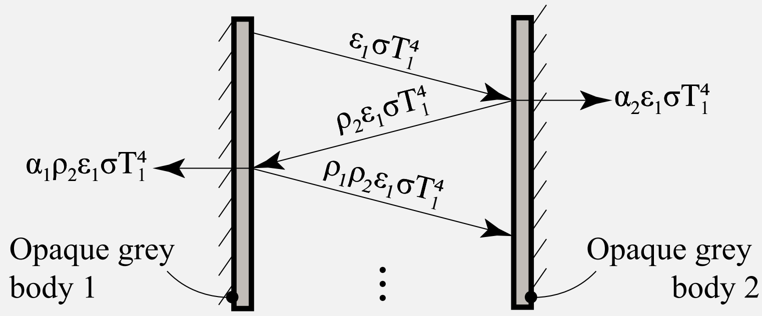
Radiation transfer between two parallel plates

This section delves into two methods for calculating the radiation exchange between two bodies: the radiation tracking method and surface brightnesses. To illustrate these methods, two infinitely large parallel plates facing each other are considered, acting as opaque grey bodies. The infinite size of these plates simplifies the problem, eliminating the need to consider view factors.

20.1.1 Radiation tracking

The radiation tracking method is based on ray tracing to determine the net rate of heat transfer between bodies 1 and 2.

Example 20.1 To determine the net rate of heat transfer between body 1 and 2, the trajectory of the ray emitted by body 1, characterized as $\epsilon_1 \sigma T_1^4$, is followed as the ray is reflected back and forth between the plates.



1 **Setting up the balance:**

The net heat transfer rate arises from the disparity between the heat transfer rate from body 1 to body 2 and the heat transfer rate from body 2 to body 1:

$$\dot{Q}_{1 \rightleftarrows 2} = \dot{Q}_{1\to2} - \dot{Q}_{2\to1}$$

2 **Defining the elements within the balance:**

To define the rate of heat transfer from body 1 to body 2, a ray of emission denoted by $\epsilon_1 \sigma T_1^4$ is traced. When the ray reaches body 2, a portion is absorbed, expressed as $\alpha_2 \epsilon_1 \sigma T_1^4$, while another portion is reflected to body 1, represented as $\rho_2 \epsilon_1 \sigma T_1^4$.

This reflected radiation gets back to body 1. When reaching body 1, the ray is partly absorbed by body 1, defined as $\alpha_1 \rho_2 \epsilon_1 \sigma T_1^4$, and partly reflected body 2, known as $\rho_1 \rho_2 \epsilon_1 \sigma T_1^4$, and so on.

This iterative procedure is reiterated, and ultimately, aggregating all radiation absorbed by body 2 and multiplying by the surface area determines the rate of heat transfer from body 1 to body 2:

$$\dot{Q}_{1\to2} = \epsilon_1 \sigma A T_1^4 \left[\alpha_2 + \alpha_2 \rho_1 \rho_2 + \alpha_2 \rho_1^2 \rho_2^2 + \alpha_2 \rho_1^3 \rho_2^3 + ...\right]$$

Similarly, the same process is applied to determine the rate of heat transfer from body 2 to body 1, which would yield:

$$\dot{Q}_{2\to1} = \epsilon_2 \sigma A T_2^4 \left[\alpha_1 + \alpha_1 \rho_1 \rho_2 + \alpha_1 \rho_1^2 \rho_2^2 + \alpha_1 \rho_1^3 \rho_2^3 + ...\right]$$

③ **Inserting and rearranging:**

Whereas the net rate of heat transfer between body 1 and 2 yields from:

$$\dot{Q}_{1\rightleftarrows 2} = \epsilon_1\epsilon_2\sigma A\left(T_1^4 - T_2^4\right)\left[1 + \rho_1\rho_2 + \rho_1^2\rho_2^2 + ...\right]$$

Eventually, the net rate of heat transfer would yield from summation with an infinite number of reflections, which yield to be:

$$\dot{Q}_{1\rightleftarrows 2} = \epsilon_1\epsilon_2\sigma A\left(T_1^4 - T_2^4\right)\frac{1}{1-\rho_1\rho_2}$$

Since grey opaque bodies are being dealt with, the property $(\rho = 1 - \epsilon)$ is utilized, and thus:

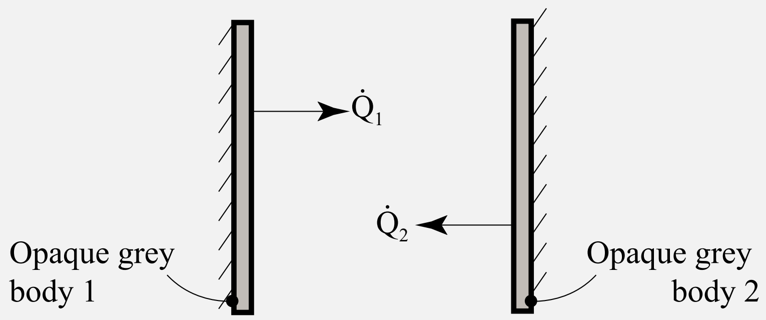
$$\dot{Q}_{1\rightleftarrows 2} = \epsilon_1\epsilon_2\sigma A\left(T_1^4 - T_2^4\right)\frac{1}{1-(1-\epsilon_1)(1-\epsilon_2)}$$

Be aware that this method involves a considerable number of steps due to the complexity. However, an alternative and more straightforward approach involves calculating the net rate of heat transfer using surface brightness, which is explained in the following section.

20.1.2 Surface brightness

Now, the net rate of heat transfer is determined by the use of surface brightness and the corresponding view factors.

Example 20.2 Both bodies radiate a specific amount of heat from their surfaces.



① **Setting up the balance:**

The net heat transfer rate arises from the disparity between the heat transfer rate from body 1 to body 2 and the heat transfer rate from body 2 to body 1:

$$\dot{Q}_{1\rightleftarrows 2} = \dot{Q}_{1\to 2} - \dot{Q}_{2\to 1} = \overset{1}{\cancel{\Phi_{12}}}\dot{Q}_1 - \overset{1}{\cancel{\Phi_{21}}}\dot{Q}_2$$

② **Defining the elements within the balance:**

The surface brightness of body 1 is defined as:

$$\dot{Q}_1 = \dot{Q}_{1,\rho} + \overset{0\text{ - opaque}}{\cancel{\dot{Q}_{1,\tau}}} + \dot{Q}_{1,\epsilon}$$

Where the reflected radiation from body 1 yields from the incident radiation on the surface received from body 2:

$$\dot{Q}_{1,\rho} = \rho_1 \cancelto{1}{\Phi_{21}} \dot{Q}_2$$

And the emitted radiation due to the temperature of the body:

$$\dot{Q}_{1,\epsilon} = \epsilon_1 \sigma A T_1^4$$

Substitution of all terms yields the surface brightness of body 1:

$$\dot{Q}_1 = \left(\rho_1 \epsilon_2 \sigma A T_2^4 + \epsilon_1 \sigma A T_1^4\right) \frac{1}{1 - \rho_1 \rho_2}$$

Similarly, the same process is applied to determine the surface brightness of body 2, which eventually yields:

$$\dot{Q}_2 = \left(\rho_2 \epsilon_1 \sigma A T_1^4 + \epsilon_2 \sigma A T_2^4\right) \frac{1}{1 - \rho_1 \rho_2}$$

3 **Inserting and rearranging:**

Inserting the definition of both surface brightnesses yields:

$$\dot{Q}_{1 \rightleftarrows 2} = \left(\left(1 - \rho_2\right) \epsilon_1 \sigma A T_1^4 - \left(1 - \rho_1\right) \epsilon_2 \sigma A T_2^4\right) \frac{1}{1 - \rho_1 \rho_2}$$

Since there is dealt with grey opaque bodies, the property $(\rho = 1 - \epsilon)$ is utilized, and thus:

$$\dot{Q}_{1 \rightleftarrows 2} = \epsilon_1 \epsilon_2 \sigma A \left(T_1^4 - T_2^4\right) \frac{1}{1 - \left(1 - \epsilon_1\right)\left(1 - \epsilon_2\right)}$$

This method produces identical results to ray tracing. Nonetheless, this procedure demands significantly less effort and is more straightforward. The challenge arises from the interdependence of surface brightness between both bodies.

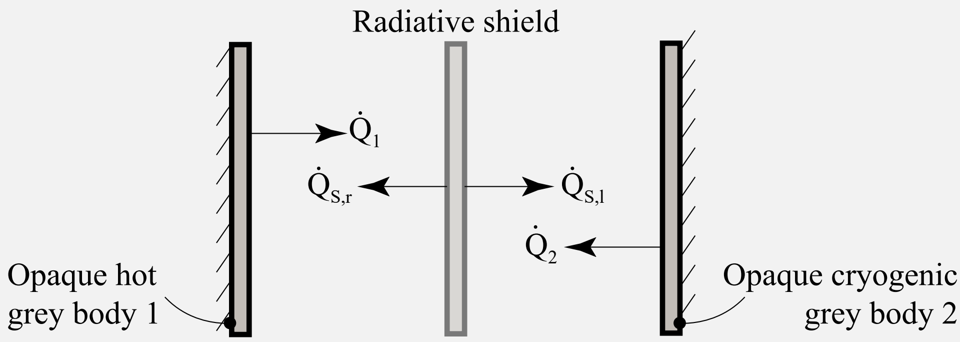
SUBSECTION 20.2

Protective shield

Radiative shielding is a technique employed to control heat transfer through electromagnetic radiation. This method has found applicability in various fields, especially in cryogenics, where maintaining extremely low temperatures is essential. One notable example is in scanning electron microscope analysis. In such cases, a sample holder must be kept at cryogenic temperatures to preserve the integrity of the sample. To achieve this, a radiative shield is strategically placed between the sample holder and any radiation-emitting source. This shield acts as a barrier, preventing excessive heat absorption and ensuring that the sample holder remains at the desired low temperature. By effectively minimizing radiative heat transfer, radiative shielding plays an important role in maintaining the thermal stability required for accurate and reliable cryogenic applications.

Example 20.3

To understand the role of a protective shield on the net rate of heat transfer, the example of the two parallel plates is considered again, but this time a shield is positioned in between. Plate 1 is held at a constant isothermal temperature T_1, and plate 2 at T_2.



(1) **Setting up the balance:**

Body 2 receives all the heat from the radiative shield positioned in between. The net heat transfer rate results from the difference between the heat transfer rate from the shield to body 2 and the heat transfer rate from body 2 to the shield:

$$\dot{Q}_{\mathrm{S}\rightleftarrows 2} = \cancelto{1}{\Phi_{\mathrm{S}2}}\dot{Q}_{\mathrm{S,r}} - \cancelto{1}{\Phi_{2\mathrm{S}}}\dot{Q}_2$$

However, the expression above includes the unknown plate temperature T_S. Therefore, the need arises to eliminate this temperature. This is achieved by the use of another energy balance. The net rate of heat transfer between the shield and body 1 is employed for this purpose.

$$\dot{Q}_{1\rightleftarrows \mathrm{S}} = \cancelto{1}{\Phi_{1\mathrm{S}}}\dot{Q}_2 - \cancelto{1}{\Phi_{\mathrm{S}1}}\dot{Q}_{\mathrm{S,l}}$$

Lastly, all the net rates of heat transfers from the bodies to each other are coupled.

$$\dot{Q}_{1\rightleftarrows 2} = \dot{Q}_{1\rightleftarrows \mathrm{S}} = \dot{Q}_{\mathrm{S}\rightleftarrows 2}$$

2 **Defining the elements within the balance:**

The surface brightness of the right side of the radiative shield is defined as:

$$\begin{aligned}\dot{Q}_{\mathrm{S,r}} &= \dot{Q}_{\mathrm{S,r},\rho} + \dot{Q}_{\mathrm{S,r},\tau} + \dot{Q}_{\mathrm{S,r},\epsilon} \\ &= \rho_{\mathrm{S}}\dot{Q}_2 + \tau_{\mathrm{S}}\dot{Q}_1 + \epsilon_{\mathrm{S}}\sigma A T_{\mathrm{S}}^4\end{aligned}$$

The surface brightness of the left side of the radiative shield is defined as:

$$\begin{aligned}\dot{Q}_{\mathrm{S,l}} &= \dot{Q}_{\mathrm{S,l},\rho} + \dot{Q}_{\mathrm{S,l},\tau} + \dot{Q}_{\mathrm{S,l},\epsilon} \\ &= \rho_{\mathrm{S}}\dot{Q}_1 + \tau_{\mathrm{S}}\dot{Q}_2 + \epsilon_{\mathrm{S}}\sigma A T_{\mathrm{S}}^4\end{aligned}$$

Similarly, the same process is applied to determine the surface brightness of body 1, which eventually yields:

$$\begin{aligned}\dot{Q}_1 &= \dot{Q}_{1,\rho} + \overset{0}{\cancel{\dot{Q}_{1,\tau}}} + \dot{Q}_{1,\epsilon} \\ &= \rho_1\dot{Q}_{\mathrm{S,l}} + \epsilon_1\sigma A T_1^4\end{aligned}$$

The surface brightness of body 2 is defined as:

$$\begin{aligned}\dot{Q}_2 &= \dot{Q}_{2,\rho} + \overset{0}{\cancel{\dot{Q}_{2,\tau}}} + \dot{Q}_{2,\epsilon} \\ &= \rho_2\dot{Q}_{\mathrm{S,r}} + \epsilon_2\sigma A T_2^4\end{aligned}$$

3 **Inserting and rearranging:**

Where inserting and rewriting yields:

$$\dot{Q}_{1\rightleftarrows \mathrm{S}}\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_{\mathrm{S}}} - 1\right) = \sigma A\left(T_1^4 - T_{\mathrm{S}}^4\right)$$

$$\dot{Q}_{\mathrm{S}\rightleftarrows 2}\left(\frac{1}{\epsilon_{\mathrm{S}}} + \frac{1}{\epsilon_2} - 1\right) = \sigma A\left(T_{\mathrm{S}}^4 - T_2^4\right)$$

Summing the two expressions above yields:

$$\dot{Q}_{1\rightleftarrows \mathrm{S}}\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_{\mathrm{S}}} - 1\right) + \dot{Q}_{\mathrm{S}\rightleftarrows 2}\left(\frac{1}{\epsilon_{\mathrm{S}}} + \frac{1}{\epsilon_2} - 1\right) = \sigma A\left(T_1^4 - T_2^4\right)$$

Substituting the relationship yielding from coupling $\dot{Q}_{1\rightleftarrows \mathrm{S}} = \dot{Q}_{1\rightleftarrows 2}$ and $\dot{Q}_{\mathrm{S}\rightleftarrows 2} = \dot{Q}_{1\rightleftarrows 2}$, and some rearranging yields:

$$\dot{Q}_{1\rightleftarrows 2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 + \underbrace{\frac{2}{\epsilon_{\mathrm{S}}} - 1}_{\text{Effect of shield}}}\sigma A\left(T_1^4 - T_2^4\right)$$

The expression above shows that even if the shield is a black body, the net rate of heat transfer is smaller than if no effect of the shield is present.

However, the optimum is found for an as small as possible emissivity of the shield.

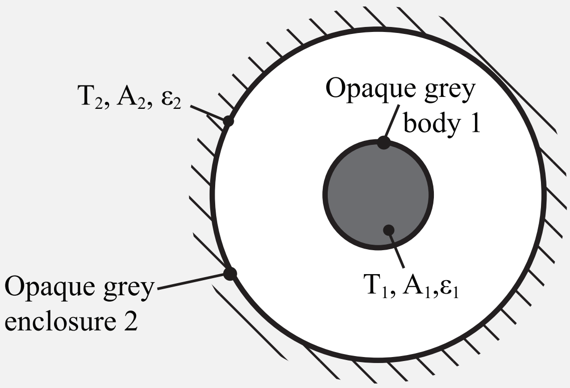
This example shows how the shield restricts the rate of heat exchange between the hot and cryogenic bodies. Thus, if the shield has a lower emissivity, the ultimate need for cooling power to sustain the cryogenic body at the designated temperature is reduced.

Subsection 20.3
Radiation between enclosed bodies

Radiative heat transfer within enclosed bodies plays an important role in heat transfer. Electromagnetic waves propagate through a confined space, exchanging energy between surfaces through radiation.

A real-world example of radiative heat transfer within an enclosed space is the design of spacecraft and satellites. In the vacuum of outer space, where there is no medium for conduction or convection, radiative heat transfer becomes a predominant mechanism. Spacecraft and satellites are equipped with various surfaces that interact with solar radiation, cosmic background radiation, and other thermal sources. The design of thermal control systems in these enclosed structures must carefully consider radiative heat transfer to prevent overheating or excessive cooling. Specialized coatings and materials with specific emissivity and absorptivity properties are employed to manage radiative heat exchange, ensuring that critical components remain within specified temperature ranges to guarantee proper functionality and longevity of the space mission.

Example 20.4 Consider an opaque body within an enclosure and ascertain the net rate of heat transfer between the two bodies. The properties of body 1, namely $T_1,\ A_1,\ \epsilon_1$, are known, while those of body 2, namely $T_2,\ A_2,\ \epsilon_2$, are also provided. Determine the net rate of heat transfer from body 1 to body 2.



1 **Setting up the balance:**

The net rate of heat transfer from body 1 to body 2 reads:

$$\dot{Q}_{1\rightleftarrows 2} = \Phi_{12}\dot{Q}_1 - \Phi_{21}\dot{Q}_2$$

2 **Defining the elements within the balance:**

Body 1 cannot see itself, and thus $\Phi_{11} = 0$, thus from the summation rule ($\Phi_{11} + \Phi_{12} = 1$) results in:

$$\Phi_{12} = 1$$

The reciprocity rule ($\Phi_{12}A_1 = \Phi_{21}A_2$) gives:

$$\Phi_{21} = \frac{A_1}{A_2}$$

And the summation rule around body 2 ($\Phi_{21} + \Phi_{22} = 1$) yields:

$$\Phi_{22} = 1 - \frac{A_1}{A_2}$$

The surface brightness of body 1 reads:

$$\dot{Q}_1 = \dot{Q}_{1,\rho} + \cancelto{0}{\dot{Q}_{1,\tau}} + \dot{Q}_{1,\epsilon}$$
$$= \rho_1 \Phi_{21} \dot{Q}_2 + \epsilon_1 \sigma A_1 T_1^4$$

The surface brightness of body 2 reads:

$$\dot{Q}_2 = \dot{Q}_{2,\rho} + \cancelto{0}{\dot{Q}_{2,\tau}} + \dot{Q}_{2,\epsilon}$$
$$= \rho_2 \left(\Phi_{12} \dot{Q}_1 + \Phi_{22} \dot{Q}_2 \right) + \epsilon_2 \sigma A_2 T_2^4$$

Furthermore, for a grey opaque body $\epsilon = \alpha$ and $\alpha + \rho = 1$, thus:

$$\rho = 1 - \epsilon$$

Substituting the definition of surface brightness 1 into 2 and rewriting yields that:

$$\dot{Q}_1 = \left(\rho_1 \rho_2 \Phi_{12} \Phi_{21} \epsilon_1 \sigma A_1 T_1^4 + \rho_1 \Phi_{21} \epsilon_2 \sigma A_2 T_2^4 \right) \frac{1}{1 - \rho_1 \rho_2 \Phi_{12} \Phi_{21} - \rho_2 \Phi_{22}} + \epsilon_1 \sigma A_1 T_1^4$$

and:

$$\dot{Q}_2 = \left(\rho_2 \Phi_{12} \epsilon_1 \sigma A_1 T_1^4 + \epsilon_2 \sigma A_2 T_2^4 \right) \frac{1}{1 - \rho_1 \rho_2 \Phi_{12} \Phi_{21} - \rho_2 \Phi_{22}}$$

3 **Inserting and rearranging:**

Inserting and rewriting yields:

$$\dot{Q}_{1 \rightleftarrows 2} = \frac{A_1 \sigma \left(T_1^4 - T_2^4 \right)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)}$$

The derived expression shows that when body 2 functions as a black body $\left(\left(\frac{1}{\epsilon_2} - 1 \right) = 0 \right)$, the resulting net rate of heat transfer mirrors that of a grey body situated within a black environment.

Similarly, when body 1 is significantly smaller than body 2 $\left(\frac{A_1}{A_2} \approx 0 \right)$, the net rate of heat transfer aligns with the heat exchange between a grey body in a black space, confirming the equivalence stated earlier.

This establishes the validity of the assumption made in the derivation of Kirchhoff's law of thermal radiation, as discussed in Section 17.4. In that section, the argument was made that the enclosure operates as a black diffuse emitter, supported by the condition that the particle within the enclosure is significantly smaller than the enclosure itself.

Lastly, when $\frac{A_1}{A_2} \approx 1$, the curvature of both bodies becomes insignificant, resulting in view factors $\Phi_{11} = 0$, $\Phi_{22} = 0$, $\Phi_{12} = 1$, and $\Phi_{21} = 1$. This simplifies the scenario to that of two plates standing in parallel, where, as discussed in the preceding sections, view factors do not play a role.

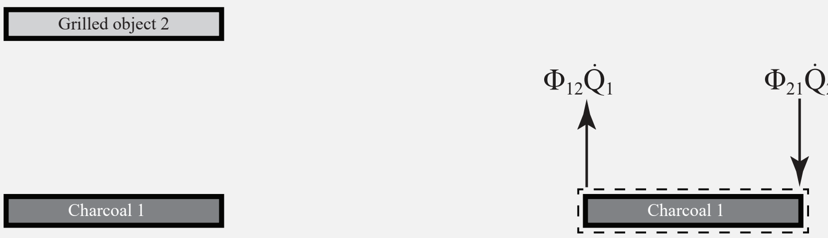
Subsection 20.4

Three body problem

Moving beyond the focus on two-body heat transfer scenarios, the dynamics of radiative heat transfer involving three bodies are explored. While previous discussions centered around the interplay between pairs of bodies, this broader consideration now introduces a triadic interaction. A common and illustrative example of this phenomenon is encountered in a barbecue grill. In this setup, not only is energy radiated towards the object being grilled from the heat source (such as burning coal), but the grill itself, particularly the walls, becomes an integral participant in the overall heat transfer process. Understanding the nuanced relationships between these three bodies is essential for optimizing heat distribution, and ensuring efficient cooking, and exemplifies the complexity that arises when additional elements come into play in radiative heat exchange scenarios.

Example 20.5 Consider a grilled object receiving heat from charcoal underneath. Determine the net rate of heat transfer from the charcoal to the grilled object.

Assuming bodies radiate as a black body and T_1 and T_2 are known. Furthermore, given the distance between the coal and the grilled object, all view factors can be determined using Figure 18.4a and are assumed to be known.



(a) General problem.

(b) Net rate of heat transfer from the charcoal to the wall and object.

1 Setting up the balance:

The net rate of heat transfer from the charcoal reads:

$$\dot{Q}_{1\rightleftarrows 2} = \Phi_{12}\dot{Q}_1 - \Phi_{21}\dot{Q}_2$$

2 Defining the elements within the balance:

Since all surfaces act as black bodies, their surface brightness is expressed as:

$$\dot{Q}_1 = \sigma A_1 T_1^4$$

and:

$$\dot{Q}_2 = \sigma A_2 T_2^4$$

3 Inserting and rearranging:

Inserting and rewriting the found definitions into the expression obtained for the net rate of heat transfer yields:

$$\dot{Q}_{1\rightleftarrows 2} = \Phi_{12}\sigma A_1 T_1^4 - \underbrace{\Phi_{21}\sigma A_2}_{\Phi_{12}\sigma A_1} T_2^4$$

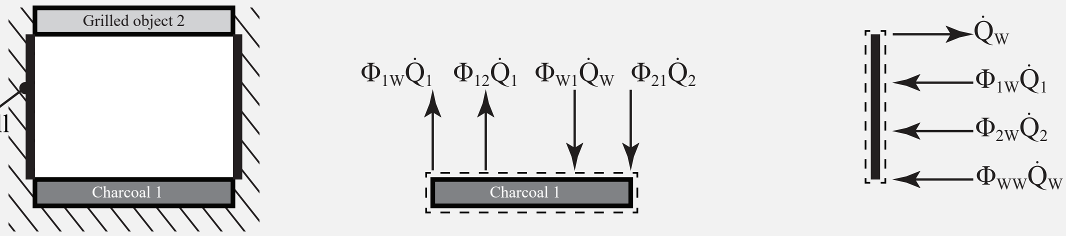
And thus:

$$\dot{Q}_{1\rightleftarrows 2} = \Phi_{12}\sigma A_1 \left(T_1^4 - T_2^4\right)$$

However, in reality, the charcoal and grilled objects are enclosed by the walls of the grill. These walls affect the net rate of heat transfer from the charcoal. Therefore, in the following example, the walls are included, and the effect on the net rate of heat transfer is observed.

Example 20.6 Consider a grilled object receiving heat from charcoal underneath. The charcoal and grilled objects are enclosed by walls, which now are taken into account. Determine the net rate of heat transfer from the charcoal.

Assuming that all bodies radiate as a black body, and T_1 and T_2 are known. Furthermore, given the dimensions of the grill, all view factors can be determined using Figure 18.4a and are assumed to be known.



(a) General problem.

(b) Net rate of heat transfer from the charcoal to the wall and object.

(c) Outer energy balance around the wall.

1 Setting up the balance:

The net rate of heat transfer from the charcoal reads:

$$\dot{Q}_{1\rightleftarrows 2,W} = \dot{Q}_{1\rightleftarrows 2} + \dot{Q}_{1\rightleftarrows W} = \Phi_{12}\dot{Q}_1 - \Phi_{21}\dot{Q}_2 + \Phi_{1W}\dot{Q}_1 - \Phi_{W1}\dot{Q}_W$$

However, the surface brightness of the wall is included. But the temperature of the wall is not known. To determine this temperature, an additional energy balance is required. Therefore, also the outer energy balance around the wall is set:

$$0 = \Phi_{1W}\dot{Q}_1 + \Phi_{2W}\dot{Q}_2 + \Phi_{WW}\dot{Q}_W - \dot{Q}_W$$

2 Defining the elements within the balance:

Since all surfaces act as black bodies, their surface brightnesses are expressed as:

$$\dot{Q}_1 = \sigma A_1 T_1^4,$$

$$\dot{Q}_2 = \sigma A_2 T_2^4,$$

and:

$$\dot{Q}_W = \sigma A_W T_W^4$$

3 Inserting and rearranging:

Inserting and rewriting the outer energy balance of the wall yields the wall temperature:

$$T_W^4 = \frac{\Phi_{1W} A_1 T_1^4 + \Phi_{2W} A_2 T_2^4}{\underbrace{(1 - \Phi_{WW})}_{\Phi_{W1}+\Phi_{W2}} A_W}$$

The reciprocity rule yields that $\Phi_{W1}A_W = \Phi_{1W}A_1$ and $\Phi_{W2}A_W = \Phi_{2W}A_2$, which yields a expression for the temperature of the wall which is just expressed in a slightly modified manner:

$$T_W^4 = \frac{\Phi_{1W}A_1T_1^4 + \Phi_{2W}A_2T_2^4}{\Phi_{1W}A_1 + \Phi_{2W}A_2}$$

Inserting the expression of the surface brightnesses and the temperature of the wall into the definition of the net rate of heat transfer from body 1 yields:

$$\dot{Q}_{1\rightleftarrows 2,W} = \Phi_{12}\sigma A_1T_1^4 - \underbrace{\Phi_{21}\sigma A_2}_{\Phi_{12}\sigma A_1}T_2^4 + \Phi_{1W}\sigma A_1T_1^4 - \underbrace{\Phi_{W1}\sigma A_W}_{\Phi_{1W}\sigma A_1}\frac{\Phi_{1W}A_1T_1^4 + \Phi_{2W}A_2T_2^4}{\Phi_{1W}A_1 + \Phi_{2W}A_2}$$

Some further rewriting yields:

$$\dot{Q}_{1\rightleftarrows 2,W} = \sigma A_1\left(T_1^4 - T_2^4\right)\left(\Phi_{12} + \frac{1}{\frac{A_1}{\Phi_{2W}A_2} + \frac{1}{\Phi_{1W}}}\right)$$

The obtained expression for the net rate of heat transfer highlights the impact of considering the walls as a third body, along with evaluating the net heat transfer rate from the charcoal to the grilled object. Taking the walls into consideration introduces an additional body that contributes to the heat transfer to the grilled object, a factor that would be overlooked without acknowledging the influence of the walls. Hence, in certain scenarios, including a third object may enhance the accuracy of assessing radiative heat transfer.

PART

VI

Exercises

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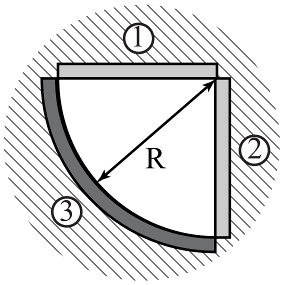
Section IV

Radiation exercises

Exercise IV.1 (Infinite pipe segment ⋆⋆)**:**

Consider an infinite long pipe segment as in the figure.



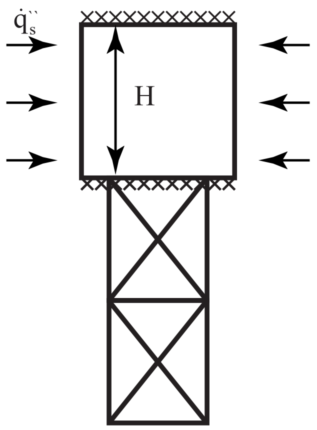
Tasks:

a) Specify the view factors Φ_{12}, Φ_{31} and Φ_{33} as a function of Φ_{13}.

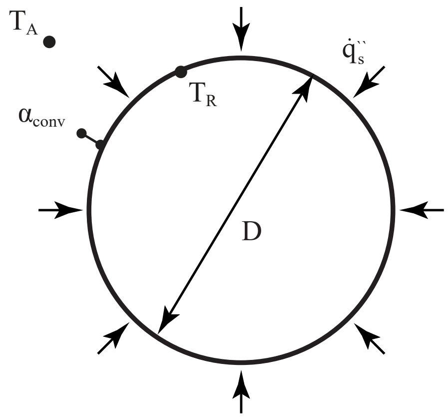
b) Determine Φ_{13}.

Exercise IV.2 (Solar power tower ⋆):

Solar radiation is uniformly and radially redirected toward a central cylindrical receiver in a solar tower plant by a surrounding mirror field (radiation density $\dot{q}''_S$). Consequently, the surface of the receiver is heated to a temperature of T_R, and the thermal power output of the plant is $\dot{Q}_{th}$.



(a) Side view



(b) Top view

Given parameters:

- Receiver height: H
- Receiver outer diameter: D
- Receiver surface temperature: T_R
- Receiver emissivity of the surface: ϵ
- Heat transfer coefficient: α_{conv}
- Ambient temperature: T_A

Hints:

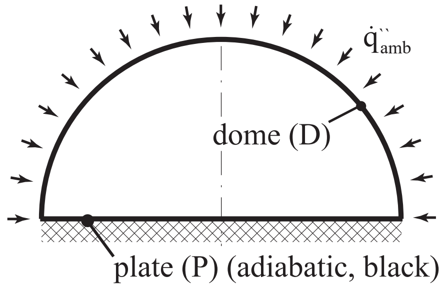
- Heat losses in the interior of the receiver as well as at its ends can be neglected.
- Radiation from the ambient can be neglected.
- The receiver can be considered as a grey body.

Tasks:

a) From a balance around the receiver, determine the mean radiation density $\dot{q}''_S$ as a function of the thermal load $\dot{Q}_{th}$.

Exercise IV.3 (Hemispherical dome ⋆⋆):

A thin circular plate (P) is covered by a hemispherical, transparent, grey dome (D). A radiative heat flux from the ambient $\dot{q}''_{\text{amb}}$ is uniformly falling on the dome.



Given parameters:

- Temperature of the dome: T_D
- Surfaces of the plate and dome: $A_\text{P},\ A_\text{D}$
- Radiative heat flux: $\dot{q}''_{\text{amb}}$
- View factor: Φ_{DP}
- Absorptivity of the plate: $\alpha_\text{P} = 1$
- Reflectivity of the dome: $\rho_\text{D} = 0$
- Transmissivity of the dome: τ_D
- Emissivity of the dome: ϵ_D

Hints:

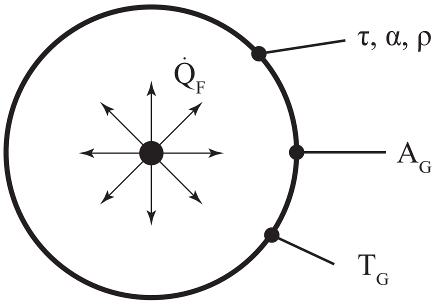
- Conduction and convection are to be neglected.
- All surfaces are radiating diffusely.

Tasks:

a) Derive an expression for the temperature of the plate T_P.

Exercise IV.4 (Light bulb ⋆⋆)**:**

The filament of a light bulb emits diffuse radiation $\dot{Q}_\mathrm{F}$. The glass of the bulb is thin, spherical, and acts as a gray body. The surface of the filament is small in comparison to the glass body and the problem is steady in time.



Given parameters:

- Power consumption of the filament: $\dot{Q}_\mathrm{F}$
- Glass properties: τ, α, ρ
- Surface of the glass sphere: A_G

Hints:

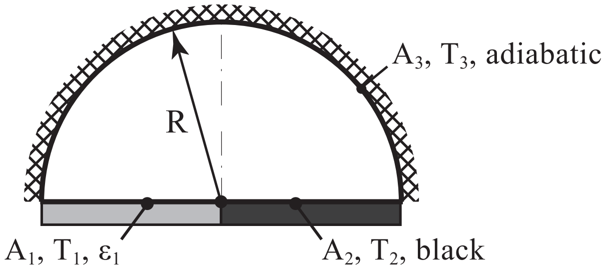
- The surface of the filament in comparison to the glass body is small.

Tasks:

a) Provide the energy balance in terms of given variables for determining the glass temperature T_G, while neglecting radiation from the environment.

Exercise IV.5 (Cupola ⋆ ⋆ ⋆):

Both semi-circular slabs A_1 and A_2 of the geometric configuration depicted below are conditioned to maintain a constant temperature of and, respectively. Surface A_2 can be considered a black body, and the hemispherical surface A_3 above the slabs is adiabatic.



Given parameters:

- Temperature of slab 1: $T_1 = 150\,°\mathrm{C}$
- Temperature of slab 2: $T_2 = 20\,°\mathrm{C}$
- Emissivity of slab 1: $\varepsilon_1 = 0.6$
- Radius of the dome: $R = 3$ m

Hints:

- Surfaces A_1 and A_3 are grey bodies and emit diffuse radiation.
- The hemispherical volume is filled with a vacuum.
- All surfaces are opaque.

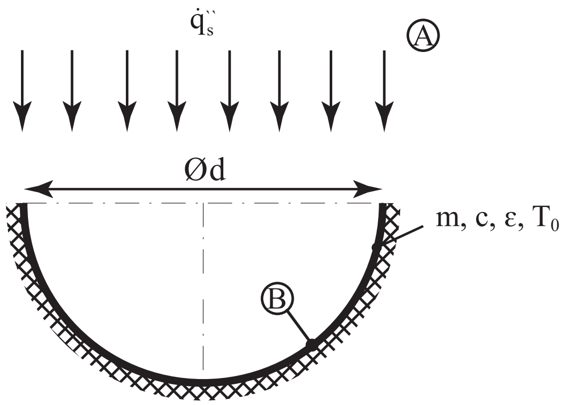
Tasks:

a) Compute the amount of heat transferred through radiation between the surfaces A_1 and A_2 (= net radiative flux to surface A_2).

b) Which temperature T_3 is obtained for surface A_3?

Exercise IV.6 (Pokè bowl ⋆⋆⋆):

An empty bowl, that is used for serving the typical traditional Hawaiian dish called pokè bowl, has the homogeneous temperature T_0 and is adiabatically insulated at its convex side. At the time t_0, the bowl is suddenly exposed to parallel radiation from the sun.



Given parameters:

- Mass of the bowl: m
- Specific heat capacity of the bowl: c
- Emissivity of the bowl: $\epsilon \approx 0.5$
- Starting temperature of the bowl: T_0
- Diameter of the bowl: d
- Heat flux of the solar radiation on the ground: $\dot{q}_S''$
- View factor of the bowl to the ambient: Φ_{BA}
- View factor of the bowl to itself: Φ_{BB}

Hints:

- The bowl radiates grey and diffuse and has a homogeneous temperature at any time.
- Influences from the ambient or the atmosphere can be neglected.
- The sun is a black body.

Tasks:

a) Determine the surface brightness of the bowl $\dot{Q}_B$.

Hint: In a nonsteady state, the surface brightness of a grey, adiabatic body is not the same as the surface brightness of a black body.

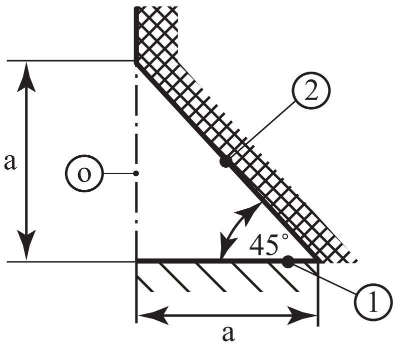
b) Derive the differential equation for the temperature as a function of time and the necessary initial condition to solve this differential equation.

c) Determine the steady-state final temperature T_S of the bowl.

d) Draw the temperature as a function of time qualitatively.

Exercise IV.7 (Radiation within a wedge-shaped opening ⋆ ⋆ ⋆):

Consider an infinitely long opening with a wedge-shaped cross-section as shown in the figure below.



Given parameters:

- Temperature of surface 1: $T_1 = 1000\,\mathrm{K}$
- Temperature of space surrounding: $T_o = 0\,\mathrm{K}$
- Emissivity of surface 1: $\varepsilon_1 = 1$
- Width: $a = 30$ cm

Hints:

- Surface 2 is a grey body and adiabatically insulated at the back.
- The space surrounding the opening can be considered to be a black body.
- Influences due to convection shall be disregarded.

Tasks:

a) Determine all view factors.

b) Determine the energy through the opening $\dot{q}'_{o,\mathrm{loss}}$ for a unit length of the opening.

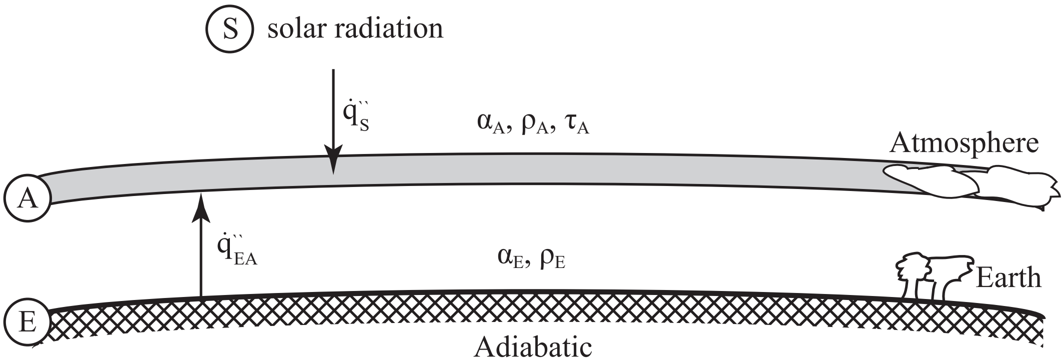
c) Determine the temperature T_2 of surface 2.

Exercise IV.8 (Earth's atmosphere ⋆⋆⋆):

The climate on Earth is influenced by the atmosphere to a great extent. To describe heat transfer between Earth and space, it is assumed that the atmosphere surrounds the Earth as a thin, distinct layer.

When balancing radiative heat flows, long-wave and short-wave radiation must be distinguished (indices LW and SW). Earth and atmosphere (indices E and A) have specific absorption, reflection, and transmission coefficients (α, ρ, τ) for long-wave and short-wave radiation each. The spectrum of solar radiation ($\dot{q}''_S$) is assumed to be in the short-wave range only, whereas emission from earth and atmosphere is in the long-wave range only.

Additionally to the radiative heat fluxes, a net heat flux $\dot{q}''_{EA}$ is carried from the earth into the atmosphere, which leads back to convective heat transfer and vaporization.



Given parameters:

- Short-wave solar radiation: $\dot{q}''_{S,SW} = 341$ W/m^2
- Long-wave solar radiation: $\dot{q}''_{S,LW} = 0$ W/m^2
- Convection and vaporization: $\dot{q}''_{EA} = 101$ W/m^2

	Short-wave	**Long-wave**
Atmosphere	$\rho_{A,SW} = 0.23$	$\rho_{A,LW} = 0.34$
	$\tau_{A,SW} = 0.54$	$\tau_{A,LW} = 0.10$
	$\alpha_{A,SW} = 0.23$	$\alpha_{A,LW} = 0.56$
	emission negligible	emission
Earth	$\rho_{E,SW} = 0.16$	$\alpha_{E,LW} = 1.00$
	$\alpha_{E,SW} = 0.84$	
	emission negligible	emission
Solar radiation	emission	emission negligible

Hints:

- Curvature is negligible, i.e. earth and atmosphere have the same surface area and the atmosphere does not radiate onto itself.
- The atmosphere emits equally in both directions.
- The given heat fluxes are averaged across the entire earth and over multiple years. Do not distinguish between the light and dark hemispheres.

- Assume steady state.

Tasks:

a) Determine the flux of short-wave radiation which hits onto the earth's surface $\dot{q}''_{\mathrm{SW\ to\ E}}$.

b) Give all energy balances and surface brightnesses necessary to determine the temperature at the earth's surface. You may assume that the spectrum of black body radiation is completely within the long-wave range for that temperature.