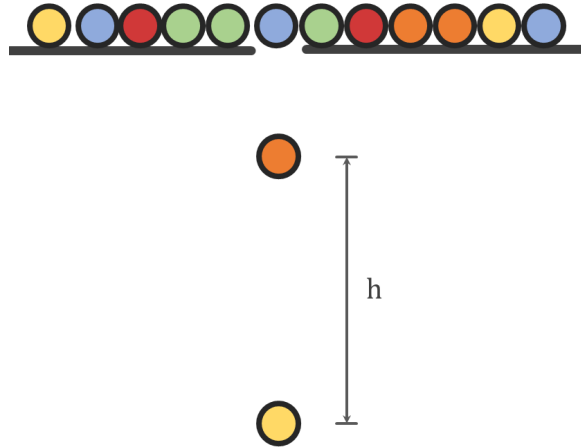


Falling Marbles



Marbles fall from rest through an opening at a steady rate of three per second. Determine the vertical separation h of two consecutive marbles when the lower one has dropped 7 meters.

Neglect air resistance and take $g = 10 \text{ m/s}^2$.

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (v_0 + at) dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (v_0 + at) dt \quad (5)$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \quad (6)$$

Given quantities:

Lowest drop distance: 7 m

Gravitational acceleration: $g = 10 \text{ m/s}^2$

Solution:

The marbles fall from rest, thus $v_0 = 0 \text{ m/s}$. Seen from the top $s_0 = 0 \text{ m/s}$, thus Equation (6) simplifies to:

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \Rightarrow s(t) = \frac{1}{2}at^2 \quad (7)$$

Where $a = -g = -10 \text{ m/s}^2$ (since it points in the negative y -direction). The time it takes for the ball to drop 7 m ($s = -7 \text{ m}$) can be calculated using:

$$s(t) = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2 \cdot s}{a}} = \sqrt{\frac{2 \cdot -7}{-10}} \approx 1.18 \text{ s} \quad (8)$$

Three balls are dropped every second at a steady rate, thus one ball is dropped every $\frac{1}{3} \text{ s}$, a ball is dropped. It took 1.18 seconds for the first ball to reach 7 m, and after $\frac{1}{3} \text{ s}$, a second ball was dropped. This second ball has dropped for $\frac{1}{3} \text{ s}$ less time than the first. The distance the second ball traveled is thus:

$$s(t) = \frac{1}{2}at^2 \Rightarrow s\left(1.18 - \frac{1}{3}\right) = \frac{1}{2} \cdot -10 \cdot \left(1.18 - \frac{1}{3}\right)^2 = -3.61 \text{ m} \quad (9)$$

Thus the distance between two consecutive marbles is $h = -3.61 - -7 = 3.39 \text{ m}$.