

The Maxwell relation $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$ results from:

$$da = \left(\frac{\partial a}{\partial T}\right)_v dT + \left(\frac{\partial a}{\partial v}\right)_T dv = -s dT - P dv$$

$$\text{where } -s = \left(\frac{\partial a}{\partial T}\right)_v \quad \text{and} \quad -P = \left(\frac{\partial a}{\partial v}\right)_T$$

$$\text{Mixed partial derivatives: } \frac{\partial^2 a}{\partial v \partial T} = \left(\frac{\partial}{\partial v} \left(\frac{\partial a}{\partial T}\right)_v\right)_T = -\left(\frac{\partial s}{\partial v}\right)_T$$

$$\text{and } \frac{\partial^2 a}{\partial T \partial v} = \left(\frac{\partial}{\partial T} \left(\frac{\partial a}{\partial v}\right)_T\right)_v = -\left(\frac{\partial P}{\partial T}\right)_v$$

The order of differentiation does not matter and the following applies: $\frac{\partial^2 a}{\partial v \partial T} = \frac{\partial^2 a}{\partial T \partial v}$ and thus $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$