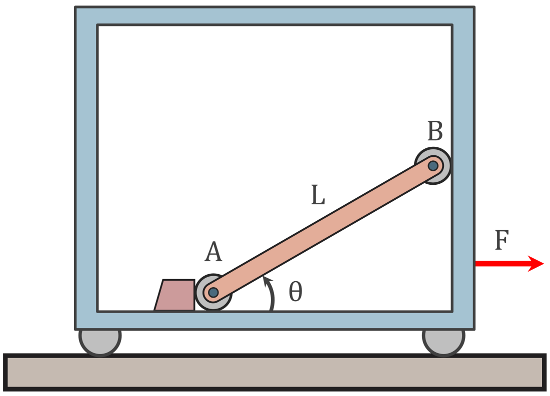
Hinged Bar under Acceleration



The uniform slender bar of mass m = 200 g and length L = 10 cm is held in the position shown by the stop at A. The mass of the crate is 800 g, the angle θ is 30° and the gravitational acceleration can be assumed to be 10 m/s^2. What is the minimum force F that needs to be applied to the crate for point B to start lifting up and off the wall?

Using known expressions:

$$\sum F_x = m \cdot a_x \tag{1}$$

$$\sum F_y = m \cdot a_y \tag{2}$$

$$\sum M_G = I_G \cdot \alpha \tag{3}$$

Given:
Mass bar: $m_b = 0.2$ kg
Mass crate: $m_c = 0.8$ kg
Angle of bar: $\theta = 30°$
Gravitational acceleration: $g = 10$ m/s^2

First a FBD of the isolated bar is made, this is shown in Figure 1. Since B comes from the wall there is no normal force present, only point A has a normal force action from the side, due to force F. As well as a normal force action from the bottom, due to the interaction between the wheel and the floor. From this we can set up the equations of motion as follows.

$$\sum F_{x,G} = m_b \cdot a_{x,G} = F_A \tag{4}$$

$$\sum F_{y,G} = m_b \cdot a_{y,G} = N_A - m_b \cdot g = 0 \tag{5}$$

$$\sum M_G = I_G \cdot \alpha = L \sin\theta \cdot F_A - L \cos\theta \cdot N_A = 0 \tag{6}$$

From Equation 5 it results that $N_A = m_b \cdot g$. Inserting this into Equation 6 gives a solution for F_A.

$$\sum M_G = L \sin\theta \cdot F_A - L \cos\theta \cdot N_A = 0 \quad \Rightarrow \quad F_A = \frac{L \cos\theta}{L \sin\theta} \cdot N_A = \frac{\cos\theta}{\sin\theta} \cdot m_b \cdot g \tag{7}$$

From Equation 4 it results that $a_{x,G} = \frac{F_A}{m_b}$. Inserting the solution found for F_A results in the needed acceleration of the bar to cause it to come off the wall.

$$a_{x,G} = \frac{F_A}{m_b} = \frac{\frac{\cos\theta}{\sin\theta} \cdot m_b \cdot g}{m_b} = \frac{\cos\theta}{\sin\theta} \cdot g \tag{8}$$

Now a FBD of the complete body is made, where the bar and crate are assumed to move as one body. This results in the following equation of motion in the x-direction.

$$\sum F_x = (m_b + m_c) \cdot a_{x,G2} = F \tag{9}$$

Since the bar and crate move as one body $a_{x,G2} = a_{x,G}$, thus the force becomes.

$$F = (m_b + m_c) \cdot a_{x,G} = (m_b + m_c) \cdot \frac{\cos\theta}{\sin\theta} \cdot g \quad \Rightarrow \tag{10}$$

$$F = (0.2 + 0.8) \cdot \frac{\cos(30)}{\sin(30)} \cdot 10 = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}} \cdot 10 = 10\sqrt{3}$$

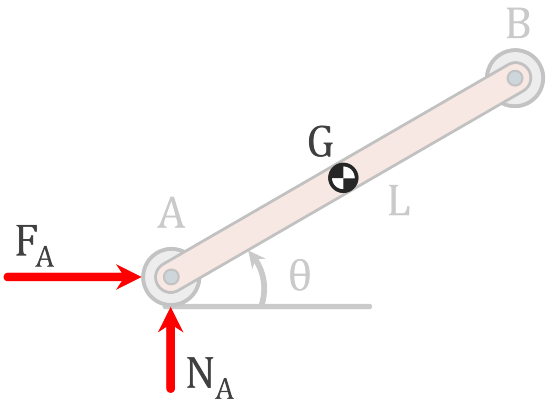


Figure 1: FBD isolated bar

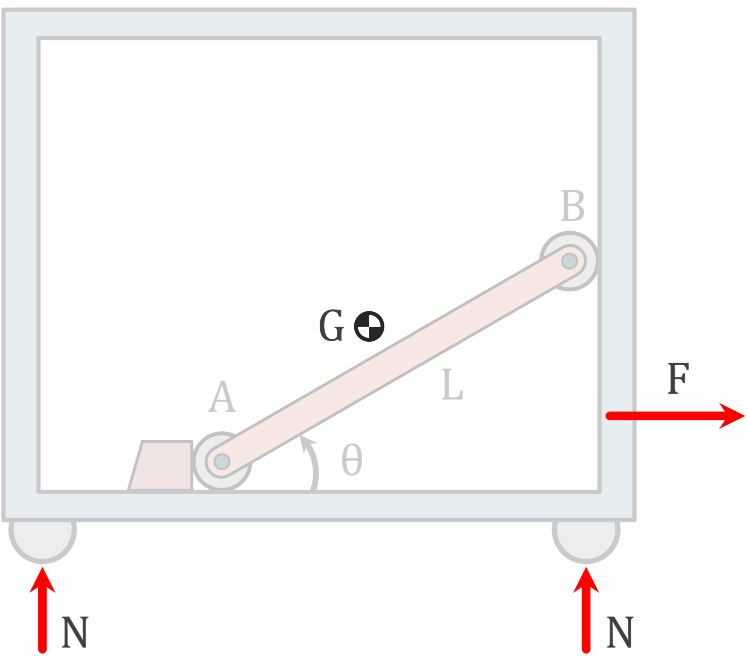


Figure 2: FBD crate with bar fixed to it