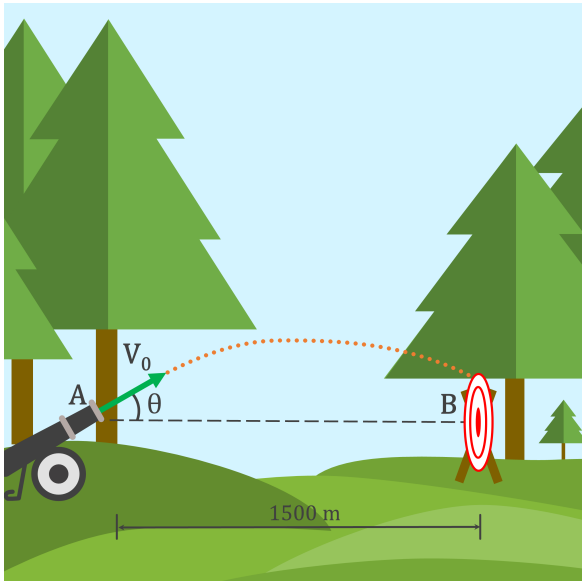
Bullet on Target



A cannon fires a bullet from A toward a target B. Find an expression for the time t it takes for the bullet to reach the target, in terms of d, v_0 and θ. The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is $d = 1500$ m. Neglect air resistances and assume that the bullet is directed along the vertical centreline of the target. Take g = 10 m/s^2.
t = . . .

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_{0}^{t} \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_{0}^{t} (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

Given:
Distance A-B: $d = 1500m$
Gravitational acceleration: $g = 10m/s^2$
Initial velocity: $v_0 = 900m/s$
Diameter target: $D = 2m$

Filling in Equation 6 gives an relation for the x-position with respect to time. Where $a = x_0 = 0$, since there is no acceleration in the x-direction and the coordinate system is chosen at the end of the cannon barrel.

$$x(t) = v_{0,x} \cdot t \Rightarrow x(t) = \cos\theta \cdot v_0 \cdot t = d \tag{7}$$

Rewriting gives a relation for the time t with respect to θ and v_0.

$$\cos\theta \cdot v_0 \cdot t = d \Rightarrow t = \frac{d}{\cos(\theta) \cdot v_0} \tag{8}$$