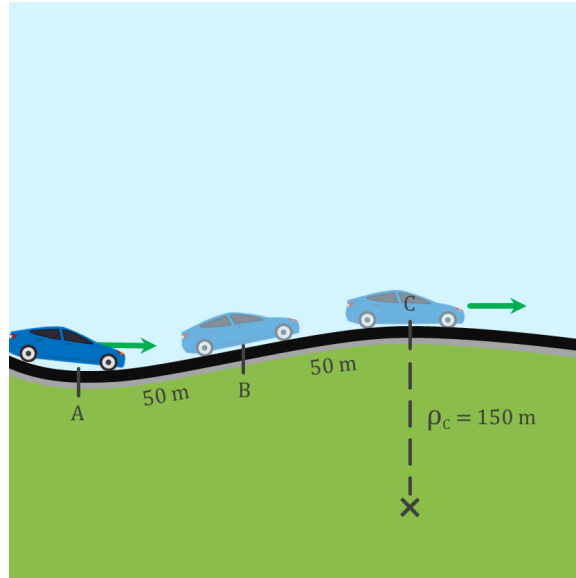# Dip and Hump in the Road



The driver of the car applies the brakes to produce a uniform deceleration, to go over the dip and hump in the road. The speed at the bottom A of the dip is 90 km/h and 54 km/h at the top C of the hump, which is 100m along the road from A. If the passengers experience a total acceleration of 3 m/s$^2$ at A and if the radius of curvature of the hump at C is 150m, determine the total acceleration at C.

*Using known expressions*:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}v}{a} \tag{1}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}s}{v} \tag{2}$$

Combining both expressions results in:

$$\frac{\mathrm{d}v}{a} = \frac{\mathrm{d}s}{v} \tag{3}$$

$$a\mathrm{d}s = v\mathrm{d}v \tag{4}$$

Finding expressions for $a\mathrm{d}s$ and $v\mathrm{d}v$ gives:

$$a\mathrm{d}s = a \int_0^s \mathrm{d}s = a \cdot (s - 0) = a \cdot s \tag{5}$$

$$vdv = \int_{v_A}^{v_C} v\,dv = \frac{1}{2}(v_C{}^2 - v_A{}^2) \tag{6}$$

Combining Equations 5 and 6 gives:

$$a \cdot s = \frac{1}{2}(v_C{}^2 - v_A{}^2) \quad \Rightarrow \quad a = \frac{v_C{}^2 - v_A{}^2}{2 \cdot s} \tag{7}$$

Furthermore, we know:

$$a_{n,C} = \frac{v_C{}^2}{\rho_C} \tag{8}$$

*Given*:
Velocity at A: $v_A = 90km/h = 25m/s$
Velocity at C: $v_C = 54km/h = 15m/s$
Acceleration at A: $a_A = 3m/s^2$
Distance: $s = 100m$

The acceleration calculated using Equation 7 is the average acceleration, in this case deceleration, over the path the car travels. Since the deceleration is uniform, the acceleration is the same as the tangential acceleration $a_t$ and is constant over the path.

$$a = a_{t,C} = \frac{v_C{}^2 - v_A{}^2}{2 \cdot s} = \frac{15^2 - 25^2}{2 \cdot 100} = -2m/s^2 \tag{9}$$

The normal acceleration $a_{n,C}$ can be calculated using Equation 10. This results in:

$$a_{n,C} = \frac{v_C{}^2}{\rho} \quad \Rightarrow \quad a_{n,C} = \frac{15^2}{150} = 1.5m/s^2 \tag{10}$$

To calculate the total acceleration at point C, we need to find the normal acceleration $a_{n,C}$. From Pythagoras it follows that:

$$a_C{}^2 = a_{t,C}{}^2 + a_{n,C}{}^2 \quad \Rightarrow \quad a_C = \sqrt{a_{t,C}{}^2 + a_{n,C}{}^2} \tag{11}$$

This is true, since $a_t$ and $a_n$ are perpendicular to each other. Inserting $a_{t,C}$ and $a_{n,C}$ gives:

$$a_C = \sqrt{a_{t,C}{}^2 + a_{n,C}{}^2} \quad \Rightarrow \quad a_C = \sqrt{(-2)^2 + 1.5^2} = 2.5m/s^2 \tag{12}$$