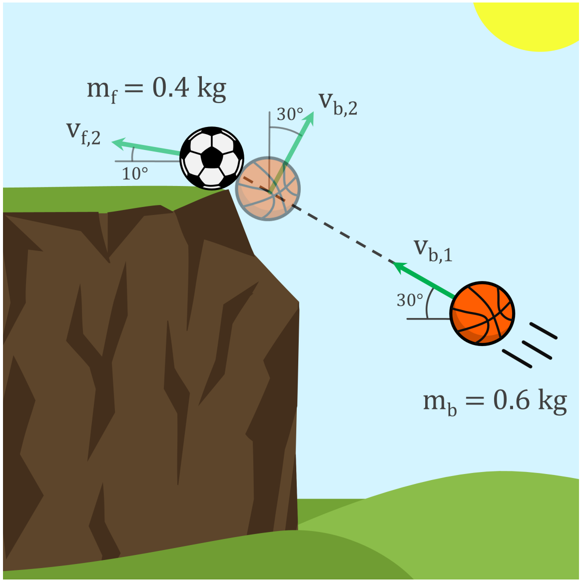
Basketball Hits Football



A basketball weighing 0.6 kg is thrown with a speed of $v_{b,1} = 10$ m/s towards a football at rest on the edge of a cliff, at an angle of 30° with the horizontal. Determine the speed $v_{f,2}$ in m/s of the football if the basketball has a speed $v_{b,2}$, at an angle of 30° with the vertical, directly after impact.

Assume the football weighing 0.4 kg and its velocity after impact makes an angle of 10° with the horizontal. Neglect friction and round to the nearest integer.

Using known expressions:

Linear momentum:

$$\Sigma \mathbf{G}_1 = m_b \mathbf{v}_{b,1} + m_f \mathbf{v}_{f,1} \tag{1}$$

$$\Sigma \mathbf{G}_2 = m_b \mathbf{v}_{b,2} + m_f \mathbf{v}_{f,2} \tag{2}$$

Linear momentum conservation:

$$\Sigma \mathbf{G}_1 = \Sigma \mathbf{G}_2 \tag{3}$$

Given quantities:

Mass basketball: $m_b = 0.6$ kg
Mass football: $m_f = 0.4$ kg
Initial speed basketball: $v_{b,1} = 10$ m/s
Initial speed football: $v_{f,1} = 0$ m/s
Angle of $\mathbf{v}_{b,1}$ to the horizontal: $30°$
Angle of $\mathbf{v}_{b,2}$ to the vertical: $30°$
Angle of $\mathbf{v}_{f,2}$ to the horizontal: $10°$

Solution:

At the instant just before impact only the basketball has linear momentum, since the football is at rest. At the instant just after the impact both the football and basketball have a linear momentum. The total linear momentum of a system is conserved in a collision, so using the equation of the conservation of linear momentum gives us:

$$\Sigma \mathbf{G}_1 = \Sigma \mathbf{G}_2 \quad \Rightarrow \quad m_b \mathbf{v}_{\mathrm{b},1} = m_b \mathbf{v}_{\mathrm{b},2} + m_f \mathbf{v}_2 \tag{4}$$

Using geometry and a fixed standard coordinate system, where the positive x-and y-direction are to the right and upwards direction respectively results in:

$$m_b v_{b,1} \begin{pmatrix} -\cos(30°) \\ \sin(30°) \\ 0 \end{pmatrix} = m_f v_{f,2} \begin{pmatrix} -\cos(10°) \\ \sin(10°) \\ 0 \end{pmatrix} + m_b v_{b,2} \begin{pmatrix} \sin(30°) \\ \cos(30°) \\ 0 \end{pmatrix} \tag{5}$$

Here we have a system of two equations with two unknowns. To solve this, we write the first and second equation in terms of $v_{b,2}$.

$$\begin{cases} -m_b v_{b,1} \cos(30°) = -m_f v_{f,2} \cos(10°) + m_b v_{b,2} \sin(30°) \\ m_b v_{b,1} \sin(30°) = m_f v_{f,2} \sin(10°) + m_b v_{b,2} \cos(30°) \end{cases} \tag{6}$$

$$\begin{cases} v_{b,2} = \dfrac{-m_b v_{b,1} \cos(30°) + m_f v_{f,2} \cos(10°)}{m_b \sin(30°)} \\ v_{b,2} = \dfrac{m_b v_{b,1} \sin(30°) - m_f v_{f,2} \sin(10°)}{m_b \cos(30°)} \end{cases} \tag{7}$$

These two equations must be equal to each other, thus we can solve for $v_{f,2}$.

$$\frac{-m_b v_{b,1} \cos(30°) + m_f v_{f,2} \cos(10°)}{m_b \sin(30°)} = \frac{m_b v_{b,1} \sin(30°) - m_f v_{f,2} \sin(10°)}{m_b \cos(30°)} \tag{8}$$

Rewriting gives:

$$-m_b^2 v_{b,1} \cos^2(30^\circ) + m_f m_b v_{f,2} \cos(10^\circ)\cos(30^\circ) =$$
$$m_b^2 v_{b,1} \sin^2(30^\circ) - m_f m_b v_{f,2} \sin(10^\circ)\sin(30^\circ) \tag{9}$$

Bringing all terms with $v_{b,1}$ to the left side and all terms with $v_{f,2}$ to the right side results in:

$$m_b^2 v_{b,1}(\cos^2(30^\circ) + \sin^2(30^\circ)) = m_f m_b v_{f,2}(\sin(10^\circ)\sin(30^\circ) + \cos(10^\circ)\cos(30^\circ)) \tag{10}$$

Since $\cos^2(30^\circ) + \sin^2(30^\circ) = 1$, we can write $v_{f,2}$ as follows:

$$v_{f,2} = \frac{m_b^2 v_{b,1}}{m_f m_b(\sin(10^\circ)\sin(30^\circ) + \cos(10^\circ)\cos(30^\circ))} \tag{11}$$

Inserting m_b, m_f and $v_{b,1}$ results in a final value for $v_{f,2}$:

$$v_{f,2} = \frac{0.6^2 \cdot 10}{0.6 \cdot 0.4 \cdot (\sin(10^\circ)\sin(30^\circ) + \cos(10^\circ)\cos(30^\circ))} = 15.96 \text{ m/s} \tag{12}$$

Rounding to the nearest integer gives: $v_{f,2} \approx 16$ m/s.