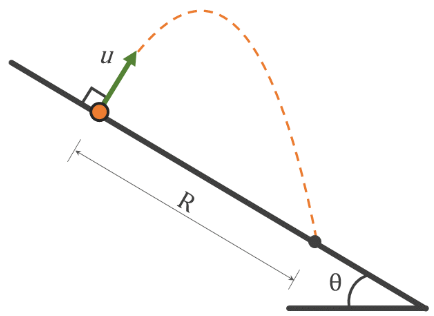
Projectile at Right Angle



A projectile is fired with a velocity u at a right angle to the slope, which is inclined at an angle θ with the horizontal. Which of the following is the correct expression for the distance R to the point of impact.

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (a \cdot t + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t (a \cdot t + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2} a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

Given:
Initial velocity: $v_0 = u$
Displacement to point of impact: R

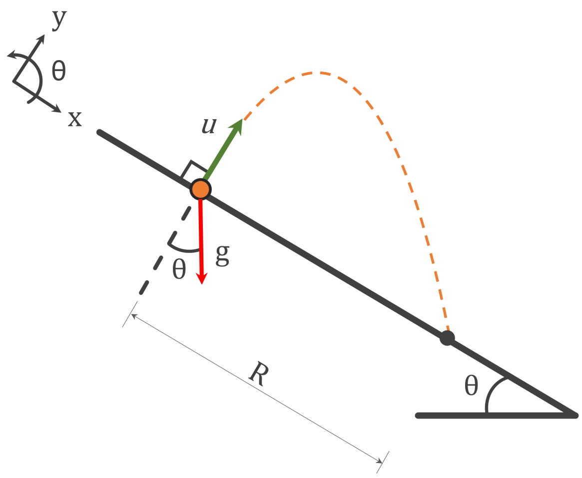


Figure 1: Projectile at Right Angle, with coordinate system

The coordinate system is defined such that y is defined upwards positive in the same direction of u. Figure 1 shows the coordinate system, together with the gravitational constant g. From Figure 1 the following two relations of x and y can be established.

$$x(t) = \frac{1}{2}a_x \cdot t^2 + v_{x,0} \cdot t + x_0 \tag{7}$$

$$y(t) = \frac{1}{2}a_y \cdot t^2 + v_{y,0} \cdot t + y_0 \tag{8}$$

Where $v_{x,0} = 0$, $v_{y,0} = u$ and $x_0 = y_0 = 0$. In addition, the x-and y-displacements after the projectile has landed on the slope are equal to R adn 0 respectively. The acceleration in x-direction is $a_x = \sin\theta \cdot g$ and in the y-direction is $a_x = -\cos\theta \cdot g$.

$$x(t) = \frac{1}{2}a_x \cdot t^2 + v_{x,0} \cdot t + x_0 \quad \Rightarrow \quad x(t) = \frac{1}{2}\sin\theta \cdot g \cdot t^2 = R \tag{9}$$

$$y(t) = \frac{1}{2}a_y \cdot t^2 + v_{y,0} \cdot t + y_0 \quad \Rightarrow \quad y(t) = -\frac{1}{2}\cos\theta \cdot g \cdot t^2 + u \cdot t = 0 \tag{10}$$

From Equation 10, time t can be rewritten as follows.

$$u = \frac{1}{2} \cdot g \cdot t \quad \Rightarrow \quad t = \frac{2 \cdot u}{\cos\theta \cdot g} \tag{11}$$

Inserting Equation 11 into Equation 9 gives.

$$R = \frac{1}{2}\sin\theta \cdot g \cdot \left(\frac{2 \cdot u}{\cos\theta \cdot g}\right)^2 = \frac{2 \cdot u^2}{g} \cdot \frac{\tan\theta}{\cos\theta} \tag{12}$$