

The minus 1 rule

There are some useful relations between partial derivatives. Which one is called the minus 1 rule (or the cyclic relation or triple product rule).

Reciprocity relation: $\left(\frac{\partial x}{\partial f}\right)_y = \frac{1}{(\partial f/\partial x)_y}$

Chain rule: $\left(\frac{\partial x}{\partial f}\right)_y = \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial f}\right)_y$

Minus 1 rule: $\left(\frac{\partial f}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial f}\right)_x = -1$

For exact functions is for mixed derivatives the order of differentiation not important:

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$