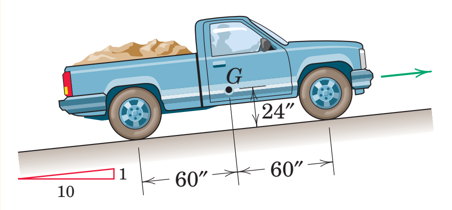
Sample Problem 6/1

The pickup truck weighs 3220 lb and reaches a speed of 30 mi/hr from rest in a distance of 200 ft up the 10-percent incline with constant acceleration. Calculate the normal force under each pair of wheels and the friction force under the rear driving wheels. The effective coefficient of friction between the tires and the road is known to be at least 0.80.

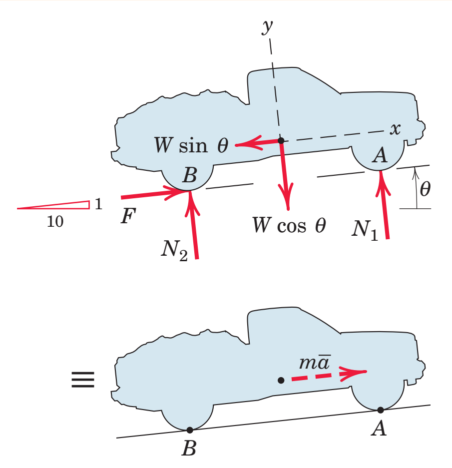


Solution. We will assume that the mass of the wheels is negligible compared
① with the total mass of the truck. The truck may now be simulated by a single rigid body in rectilinear translation with an acceleration of

② $[v^2 = 2as]$ $$\bar{a} = \frac{(44)^2}{2(200)} = 4.84 \text{ ft/sec}^2$$

The free-body diagram of the complete truck shows the normal forces N_1 and N_2, the friction force F in the direction to oppose the slipping of the driving wheels, and the weight W represented by its two components. With $\theta = \tan^{-1} 1/10 = 5.71°$, these components are $W \cos\theta = 3220 \cos 5.71° = 3200$ lb and $W \sin\theta = 3220 \sin 5.71° = 320$ lb. The kinetic diagram shows the resultant, which passes through the mass center and is in the direction of its acceleration. Its magnitude is

$$m\bar{a} = \frac{3220}{32.2}(4.84) = 484 \text{ lb}$$



Applying the three equations of motion, Eqs. 6/1, for the three unknowns gives

③ $[\Sigma F_x = m\bar{a}_x]$ $\quad F - 320 = 484 \qquad F = 804$ lb *Ans.*

$[\Sigma F_y = m\bar{a}_y = 0]$ $\quad N_1 + N_2 - 3200 = 0$ $\qquad (a)$

$[\Sigma M_G = \bar{I}\alpha = 0]$ $\quad 60N_1 + 804(24) - N_2(60) = 0$ $\qquad (b)$

Solving (a) and (b) simultaneously gives

$$N_1 = 1441 \text{ lb} \qquad N_2 = 1763 \text{ lb}$$ *Ans.*

In order to support a friction force of 804 lb, a coefficient of friction of at least $F/N_2 = 804/1763 = 0.46$ is required. Since our coefficient of friction is at least 0.80, the surfaces are rough enough to support the calculated value of F so that our result is correct.

Alternative Solution. From the kinetic diagram we see that N_1 and N_2 can be obtained independently of one another by writing separate moment equations about A and B.

$[\Sigma M_A = m\bar{a}d]$ $\quad 120N_2 - 60(3200) - 24(320) = 484(24)$

④ $N_2 = 1763$ lb *Ans.*

$[\Sigma M_B = m\bar{a}d]$ $\quad 3200(60) - 320(24) - 120N_1 = 484(24)$

$N_1 = 1441$ lb *Ans.*

Helpful Hints

① Without this assumption, we would be obliged to account for the relatively small additional forces which produce moments to give the wheels their angular acceleration.

② Recall that 30 mi/hr is 44 ft/sec.

③ We must be careful not to use the friction equation $F = \mu N$ here since we do not have a case of slipping or impending slipping. If the given coefficient of friction were less than 0.46, the friction force would be μN_2, and the car would be unable to attain the acceleration of 4.84 ft/sec². In this case, the unknowns would be N_1, N_2, and a.

④ The left-hand side of the equation is evaluated from the free-body diagram, and the right-hand side from the kinetic diagram. The positive sense for the moment sum is arbitrary but must be the same for both sides of the equation. In this problem, we have taken the clockwise sense as positive for the moment of the resultant force about B.