2.1 Walking speed ★

The first step in tackling the sub problems is defining the expression for the rate of heat loss. Afterwards, the unknown parameters within these expression can be determined and filled in, which will result in the final answer.

a) *Determine the rate of heat loss from an average person walking in air at 15 °C by convection in the case of walking in still air with a velocity of 1 m/s.*

1) Defining the rate of heat loss:

The rate of heat loss due to convection can be described by:

$$\boxed{\rightarrow \dot{Q} = \alpha A_{\mathrm{s}} \left(T_{\mathrm{s}} - T_{\infty}\right)} \tag{2.1}$$

2) Determining the heat transfer coefficient:

The relative velocity in the given problem is 1 m/s and heat transfer coefficient equals:

$$\boxed{\rightarrow \alpha = 8.2V^{0.49} = 8.2 \cdot 1^{0.49} = 8.6 \ [\mathrm{W/m^2K}]} \tag{2.2}$$

3) Filling in:

$$\dot{Q} = \alpha A_s (T_s - T_\infty) = 8.2 \ [\mathrm{W/m^2K}] \cdot 1.8 \ [\mathrm{m}] \cdot (30 - 15) \ [\mathrm{K}] \tag{2.3}$$

$$\boxed{\rightarrow \dot{Q} = 221.4 \ [\mathrm{W}]} \tag{2.4}$$

b) *Determine the rate of heat loss from an average person walking in air at 15 °C by convection in the case of standing still, while the wind blowing at a velocity of 1.5 m/s.*

1) Defining the rate of heat loss:

The rate of heat loss due to convection can be described by:

$$\boxed{\rightarrow \dot{Q} = \alpha A_{\mathrm{s}} \left(T_{\mathrm{s}} - T_{\infty}\right)} \tag{2.5}$$

2) Determining the heat transfer coefficient:

The relative velocity in the given problem is 1.5 m/s and heat transfer coefficient equals:

$$\boxed{\rightarrow \alpha = 8.2V^{0.49} = 8.2 \cdot 1.5^{0.49} = 10.0 \ [\mathrm{W/m^2K}]} \tag{2.6}$$

3) Filling in:

$$\dot{Q} = \alpha A_s (T_s - T_\infty) = 10.0 \ [\mathrm{W/m^2K}] \cdot 1.8 \ [\mathrm{m}] \cdot (30 - 15) \ [\mathrm{K}] \tag{2.7}$$

$$\boxed{\rightarrow \dot{Q} = 270.1 \ [\mathrm{W}]} \tag{2.8}$$

c) *Determine the rate of heat loss from an average person walking in air at 15 °C by convection in the case of walking along in the flow direction of the wind with a velocity of 2 m/s, while the wind blowing at a velocity of 1.5 m/s.*

1) Defining the rate of heat loss:

The rate of heat loss due to convection can be described by:

$$\boxed{\rightarrow \dot{Q} = \alpha A_\mathrm{s} \left(T_\mathrm{s} - T_\infty \right)} \tag{2.9}$$

2) Determining the heat transfer coefficient:

The relative velocity in the given problem is 0.5 m/s and heat transfer coefficient equals:

$$\boxed{\rightarrow \alpha = 8.2 V^{0.49} = 8.2 \cdot 0.5^{0.49} = 5.8 \ [\mathrm{W/m^2K}]} \tag{2.10}$$

3) Filling in:

$$\dot{Q} = \alpha A_s (T_s - T_\infty) = 5.8 \ [\mathrm{W/m^2K}] \cdot 1.8 \ [\mathrm{m}] \cdot (30 - 15) \ [\mathrm{K}] \tag{2.11}$$

$$\boxed{\rightarrow \dot{Q} = 157.6 \ [\mathrm{W}]} \tag{2.12}$$

2.2 Flat plate in a wind tunnel ★

The Reynolds is used to determine whether the fluid flow is laminar or turbulent. In both questions some information is given or is asked about the type of flow. With the definition of the Reynolds number both questions can be answered.

a) *What type of flow regime will the airflow experience in the boundary layer at 0.25 m from the leading edge?*

1) Setting up the definition of the Reynolds number:

The general expression for the Reynolds number at a specified location x is:

$$\boxed{\rightarrow \mathrm{Re_x} = \frac{U \cdot x}{\nu}} \tag{2.13}$$

2) Defining the characteristics:

The velocity U as well as the kinematic viscosity ν of the flow are given. Furthermore, the position x is specified in the task, which is 0.25 m.

3) Inserting and rewriting:

Inserting numerical values yields:

$$\boxed{\rightarrow \mathrm{Re_{x=0.25\ m}} = \frac{50\ [\mathrm{m/s}] \cdot 0.25\ [\mathrm{m}]}{15.35 \cdot 10^{6}\ [\mathrm{m^2/s}]}} = 8.14 \cdot 10^5 \tag{2.14}$$

It is therefore found that $\mathrm{Re_{x=0.25\ m}} > \mathrm{Re_{crit}}$, wich implies that at position x the flow has transited into a turbulent flow.

b) *Determine the position of the plate where the flow starts its transition from laminar to turbulent flow.*

1) Setting up the definition of the Reynolds number:

The general expression for the Reynolds number at a specified location x is:

$$\boxed{\rightarrow \mathrm{Re_x} = \frac{U \cdot x}{\nu}} \tag{2.15}$$

2) Defining the characteristics:

The velocity U as well as the kinematic viscosity ν of the flow are given. Furthermore, position x should be determined where the flows transits from laminar flow to turbulent flow. This is the case where $\mathrm{Re_x} = \mathrm{Re_{crit}}$.

3) Inserting and rewriting:

Inserting $\mathrm{Re_x} = \mathrm{Re_{crit}}$ yields:

$$\mathrm{Re_{crit}} = \frac{U \cdot x_{\mathrm{crit}}}{\nu} \tag{2.16}$$

Rewriting:

$$x_{\mathrm{crit}} = \frac{\mathrm{Re_{crit}} \cdot \nu}{U} \tag{2.17}$$

Inserting numerical values:

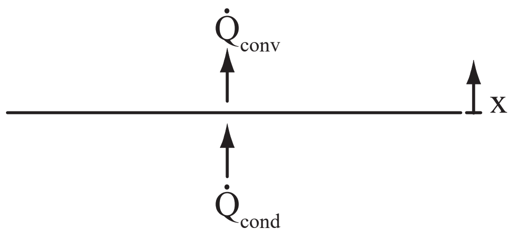
$$\boxed{\rightarrow x_{\mathrm{crit}} = \frac{5 \cdot 10^5 \cdot 15.35 \cdot 10^6 \ [\mathrm{m^2/s}]}{50 \ [\mathrm{m/s}]} = 0.15 \ [\mathrm{m}]} \tag{2.18}$$

2.3 Thick solid plate ⋆

The heat transfer coefficient should be determined. This can be derived from the fact that at the interface the rate of heat transfer by conduction is equal to the rate of heat transfer by convection. When having determined the heat transfer coefficient, this can be rewritten in terms of the temperature gradient at the interface.

a) *Determine the convection heat transfer coefficient in terms of the given variables.* First step to see that the conductive heat flux and the convective heat flux are equal to each other can be seen from deriving an energy balance.

1) Setting up an energy balance:



Where it can be seen that:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\mathrm{out}} \tag{2.19}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\mathrm{cond}} - \dot{Q}_{\mathrm{conv}}} \tag{2.20}$$

2) Defining the fluxes:

The rate of heat transfer by conduction can be described by use of Fourier's law:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{cond}} = -\lambda_{\mathrm{p}} \cdot A_{\mathrm{c}} \cdot \frac{dT}{dx} = \lambda_{\mathrm{p}} \cdot A_{\mathrm{c}} \cdot \frac{T_{\mathrm{L}} - T_{\mathrm{U}}}{t}} \tag{2.21}$$

$$\boxed{\rightarrow \dot{Q}_{\mathrm{conv}} = \alpha \cdot A_{\mathrm{s}} \cdot (T_{\mathrm{U}} - T_{\infty})} \tag{2.22}$$

3) Inserting and rewriting:

Inserting into the energy balance yields:

$$0 = \dot{Q}_{\mathrm{cond}} - \dot{Q}_{\mathrm{conv}} \tag{2.23}$$

$$0 = \lambda_\mathrm{p} \cdot A_\mathrm{c} \cdot \frac{T_\mathrm{L} - T_\mathrm{L}}{t} - \alpha \cdot A_\mathrm{s} \cdot (T_\mathrm{U} - T_\infty) \tag{2.24}$$

Note that $A_\mathrm{s} = A_\mathrm{c}$, therefore rewriting yields:

$$\boxed{\rightarrow \alpha = \frac{\lambda_\mathrm{p}}{t} \cdot \frac{T_\mathrm{L} - T_\mathrm{U}}{T_\mathrm{U} - T_\infty}} \tag{2.25}$$

b) *Determine the temperature gradient inside the water at the interface in terms of the given variables.* The heat transfer coefficient is defined as:

$$\alpha = \frac{-\left(\lambda_\mathrm{w} \frac{dT}{dx}|_\mathrm{water,w}\right)}{T_\mathrm{U} - T_\infty} \tag{2.26}$$

Equaling this expression to the found expression for α in task a) yields:

$$\frac{-\left(\lambda_\mathrm{w} \frac{dT}{dx}|_\mathrm{water,w}\right)}{T_\mathrm{U} - T_\infty} = \frac{\lambda_\mathrm{p}}{t} \cdot \frac{T_\mathrm{L} - T_\mathrm{U}}{T_\mathrm{U} - T_\infty} \tag{2.27}$$

And therefore rewriting gives us:

$$\boxed{\rightarrow \frac{dT}{dx}|_\mathrm{water,w} = -\frac{\lambda_\mathrm{p}}{\lambda_\mathrm{w} \cdot t} \cdot (T_\mathrm{L} - T_\mathrm{U})} \tag{2.28}$$

2.4 Moving Train ★★

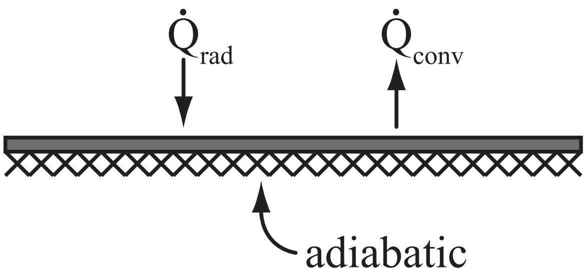
The top surface of the passenger car of a train in motion is absorbing solar radiation. But at the same time it is loosing heat due to convection. Therefore it's surface temperature remains in equilibrium. This equilibrium temperature can be determined by setting up an energy balance and making an assumption for the average properties. Based on the found the value for the surface temperature, the made assumption should be evaluated. This process has 2 possible outcomes:

1. The assumed temperature for the average fluid properties matches with the one that results from calculations. No steps should be taken.

2. The assumed temperature for the average fluid properties does not match with the one that results from calculations. The same process should be taken, with a new assumption for the temperature for the average fluid properties.

a) *Determine the equilibrium temperature of the top surface* T_s.

1) Setting up an energy balance:



$$\frac{\partial U}{\partial t}^{\nearrow 0} = \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\mathrm{out}} \tag{2.29}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\mathrm{rad}} - \dot{Q}_{\mathrm{conv}}} \tag{2.30}$$

2) Defining the fluxes and heat transfer coefficient:

The rate heat transfer towards the roof due to solar radiation can be expressed as:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{rad}} = \dot{q}_{\mathrm{s}}'' A_{\mathrm{s}}} \tag{2.31}$$

Where A_s is the surface area of the roof:

$$A_s = W \cdot L = 3\ [\mathrm{m}] \cdot 10\ [\mathrm{m}] = 30\ [\mathrm{m}^2] \tag{2.32}$$

The rate of heat loss by convection from the roof can be expressed as:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{conv}} = \alpha \cdot A_s \cdot (T_s - T_A)} \tag{2.33}$$

This expression can be used to determine T_s, but at this point the heat transfer coefficient α is undetermined.

To do so, the assumption has been made that the average fluid temperature $T_f = 20$ °C.

The properties of air at 20 °C are:

$\lambda = 25.69 \cdot 10^{-3}$ W/mK

$\nu = 15.35 \cdot 10^{-6}$ m²/s

$\mathrm{Pr} = 0.7148$

With this the Reynolds number can be determined, with the characteristic length L_c being the length of the train L for flow over a flat plate:

$$\mathrm{Re_L} = \frac{U L_c}{\nu} = \frac{\frac{50 \cdot 1000}{3600}\ [\mathrm{m/s}] \cdot 10\ [\mathrm{m}]}{15.35 \cdot 10^{-6}\ [\mathrm{m^2/s}]} = 9.0481 \cdot 10^6 \tag{2.34}$$

From that it results that the following correlation is applicable:

$$\overline{\mathrm{Nu}}_\mathrm{L} = 0.036 \cdot \mathrm{Pr}^{0.43} (\mathrm{Re_L}^{0.8} - 9400) \tag{2.35}$$

$$\overline{\mathrm{Nu}}_\mathrm{L} = 0.036 \cdot 0.7148^{0.43} ((9.0481 \cdot 10^6)^{0.8} - 9400) = 1.1158 \cdot 10^4 \tag{2.36}$$

With the definition of the Nusselt number, the heat transfer coefficient can be determined:

$$\overline{\mathrm{Nu}}_\mathrm{L} = \frac{\alpha L}{\lambda} \rightarrow \alpha = \frac{\lambda \overline{\mathrm{Nu}}_\mathrm{L}}{L} \tag{2.37}$$

$$\boxed{\rightarrow \alpha = \frac{\lambda \overline{\mathrm{Nu}}_\mathrm{L}}{L} = \frac{25.69 \cdot 10^{-3}\ [\mathrm{W/mK}] \cdot 1.1158 \cdot 10^4}{10\ [\mathrm{m}]} = 28.67\ [\mathrm{W/m^2K}]} \tag{2.38}$$

3) Inserting and rearranging

As now only T_s is the only unknown quantity, the energy balance can be rearranged to determine this temperature:

$$0 = \dot{Q}_{\mathrm{rad}} - \dot{Q}_{\mathrm{conv}} \tag{2.39}$$

$$0 = \dot{q}_{\mathrm{s}}'' A_{\mathrm{s}} - \alpha \cdot A_{\mathrm{s}} \cdot (T_{\mathrm{s}} - T_{\mathrm{A}}) \tag{2.40}$$

$$\boxed{\rightarrow T_{\mathrm{s}} = \frac{\dot{q}_{\mathrm{s}}''}{\alpha} + T_{\mathrm{A}} = \frac{250}{28.67} [^{\circ}\mathrm{C}] + 20\ [^{\circ}\mathrm{C}] = 28.72\ [^{\circ}\mathrm{C}]} \tag{2.41}$$

With $T_{\mathrm{s}} = 28.72\ [^{\circ}\mathrm{C}]$, the average fluid temperature becomes $T_{\mathrm{f}} = 24.36\ [^{\circ}\mathrm{C}]$, which is relatively close to the assumed value of $T_{\mathrm{f}} = 20\ [^{\circ}\mathrm{C}]$. If a higher accuracy is required, the similar calculations with could be performed at $T_{\mathrm{f}} = 24.36\ [^{\circ}\mathrm{C}]$.