W07c

One way of preparing an chicken egg is boiling it. Before these can be enjoyed, they have to cool down to a comfortable temperature.

(a) Boiled eggs

(b) Potato

1. Based on which criteria can the lumped capacity model be applied?
2. For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why you come to this conclusion. Are there differences in properties?

A egg (k = 3 W/mK, ρ = 1150 kg/m^3, c_p = 3600 J/kgK) with a diameter of 3 cm, which initially has a surface temperature of 95 ^{o}C, is located outside with an ambient temperature of 20 ^{o}C. It cools down due to convection (h = 40 W/m^{2}K).

1. Using the lumped capacity model, determine the time that it takes for the egg to cool down to 35 ^{o}C.

1. Evaluate the accuracy of the found answer in c).

1. Determine the amount of energy that the egg has lost, when cooled down from 95 ^{o}C to 35 ^{o}C.

1. Determine the maximum diameter of the egg, for which the lumped capacity model is still valid.

1. Provide a sketch of the temperature profile as a function of time, in the case that we would have let the potato cool down in the room for a very long time.

Note: clearly indicate the temperatures for t=0 and t→ ∞

Solution W07c

1. *Based on which criteria can the lumped capacity model be applied?*

$$\mathrm{Bi} \leq 0.1 \text{ or } \mathrm{Bi} \rightarrow 0$$

(1) correct criteria

1. *For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why you come to this conclusion. Are there differences in properties?*

We know:

$$\mathrm{Bi} = \frac{hL_\mathrm{c}}{k}$$

Simplifying both objects to spheres, L_c will be bigger for the potato. A egg and a potato have roughly similar thermal conductivity as can be seen on `https://www.engineeringtoolbox.com/food-thermal-conductivity-d_2177.html`. Therefore it may be assumed that the lumped capacity model is more applicable for the egg, as the characteristic length is smaller. However, a potato has a more uniform temperature distribution as its properties are more constant throughout its body. A egg contains parts such the shell, egg white and yolk, where other properties for thermal conductivity will be found. Therefore, it might also be argued that the lumped model would be more suitable for the potato.

(1) correct explanation

1. *Using the lumped capacity model, determine the time that it takes for the egg to cool down to 35 $^\mathrm{o}C$*

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p}t}$$
$$\rightarrow t = \frac{\rho V c_p}{hA_s} \ln\left(\frac{T_i - T_\infty}{T(t) - T_\infty}\right)$$

(0.5) correct definition for t

Where

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 0.015^3 \ \left[\mathrm{m}^3\right] = 1.414 \cdot 10^{-5} \ \left[\mathrm{m}^3\right]$$
$$A_s = 4\pi r^2 = 4\pi 0.015^2 \ \left[\mathrm{m}^3\right] = 0.0023 \ \left[\mathrm{m}^2\right]$$

(0.5) correct definitions for V and $\mathrm{A_s}$

So:

$$\rightarrow t = \frac{1150 \ \left[\mathrm{kg/m^3}\right] \cdot 1.414 \cdot 10^{-5} \ \left[\mathrm{m}^3\right] \cdot 3600 \ \left[\mathrm{J/kgK}\right]}{40 \ \left[\mathrm{W/m^2K}\right] \cdot 0.0023 \ \left[\mathrm{m}^2\right]} \ln\left(\frac{95-20}{35-20}\right) = 797.1 \ [\mathrm{s}] = 13.3 \ [\mathrm{min}]$$

(0.5) correct final answer for t

1. *Evaluate the accuracy of the found answer in c).*

$$\mathrm{Bi} = \frac{hL_\mathrm{c}}{k} = \frac{hV}{kA_s} = \frac{40 \ \left[\mathrm{W/m^2K}\right] \cdot 1.414 \cdot 10^{-5} \ \left[\mathrm{m}^3\right]}{3 \ [\mathrm{W/mK}] \cdot 0.0023 \ \left[\mathrm{m}^2\right]} = 0.0820 < 0.1$$

(0.5) correct evaluation

Therefore the lumped capacity model provides a high accuracy.

1. *Determine the amount of energy that the egg has lost, when cooled down from 95 $^\mathrm{o}C$ to 35 $^\mathrm{o}C$.*

Heat lost:

$$Q = V\rho c_p (T_i - T(t)) = 1.414 \cdot 10^{-5} \ \left[\mathrm{m}^3\right] \cdot 1150 \ \left[\mathrm{kg/m^3}\right] \cdot 3600 \ \left[\mathrm{J/kgK}\right] (95-35) \ [\mathrm{K}] = 3512 \ [\mathrm{J}]$$

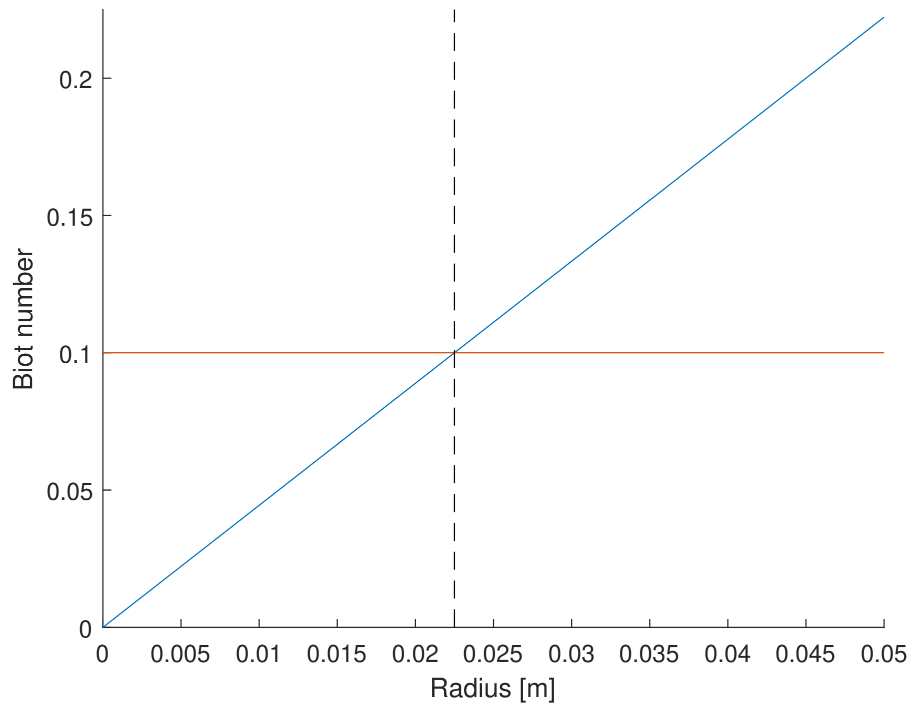
(1) correct determination of Q

1. *Determine the maximum diameter of the egg, for which the lumped capacity model is still valid.*

$$\mathrm{Bi}(r) = \frac{hr}{3k}$$

(1) correct definition for Bi(r)

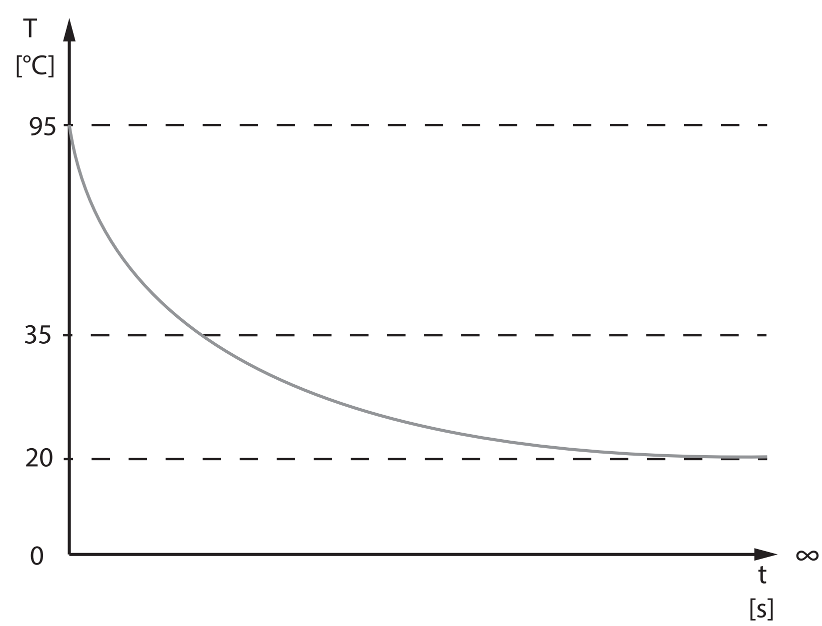
Plotting:



Intersection is at r = 2.25 cm (D=4.5 cm)

(1) correct point of intersection

1. *Provide a sketch of the temperature profile as a function of time, in the case that we would have let the egg cool down in the room for a very long time.*



- At t=0 T = 95 ^{o}C.
- T = 20 ^{o}C should be clearly defined.
- For t→ ∞ the temperature should approach the ambient temperature with a zero gradient slope.

(2) for all correct criteria, (-1) per missing criteria, (0) as minimum