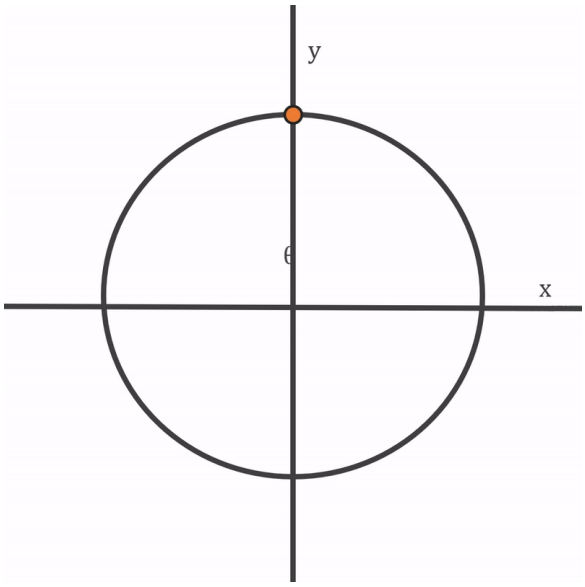
Particle in Circular Motion



Which of the following equations gives the correct relation for the velocity vector $\mathbf{v}$ in Cartesian coordinates for this particle following a circular motion with radius R?

Hint: Pay close attention to the definition of θ.

Solution:

The standard formulas of polar coordinates cannot be directly applied, because θ is defined with respect to the vertical. The actual relations will be obtained ourselves.

Using the Figure, the transformation to x- and y-coordinates is the following:

$$\begin{cases} x = R\sin\theta \\ y = R\cos\theta \end{cases} \tag{1}$$

Take the first derivative of both components to get relations for the velocity and acceleration. The chain rule is applied.

$$\begin{cases} \dot{x} = R\dot{\theta}\cos\theta \\ \dot{y} = -R\dot{\theta}\sin\theta \end{cases} \tag{2}$$

Combining both components results in the velocity vector $\mathbf{v} = \langle \dot{x}, \dot{y} \rangle$:

$$\mathbf{v} = \left\langle R\dot{\theta}\cos\theta, -R\dot{\theta}\sin\theta \right\rangle \tag{3}$$