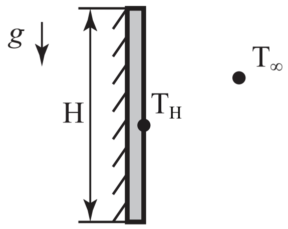
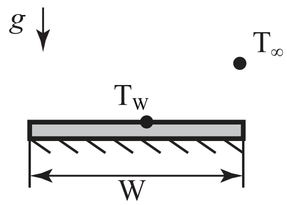
Exercise III.13 (Horizontal and vertical wall ★★):

Two heat-emitting surfaces (case 1: H height, case 2: W width) with the respective wall temperatures T_H and T_W are given. The quiescent environment has a temperature T_∞.



(a) Case 1



(b) Case 2

Given parameters:

- Prandtl Number: $\mathrm{Pr} = 1$
- Value range for laminar boundary layer: $1 \cdot 10^5 < \mathrm{Gr_L Pr} < 1 \cdot 10^6$
- Geometrical ratio: $W = 2 \cdot H$
- Length of both plates: L
- Temperatures: $T_H = 2 \cdot T_W = 4 \cdot T_\infty$

Hint:

- The difference in average fluid properties between both cases is negligible.
- $L \gg W$

Tasks:

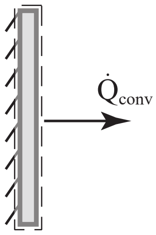
a) Determine the ratio of the convective losses from the surfaces.

Solution III.13 (Horizontal and vertical wall ⋆⋆):

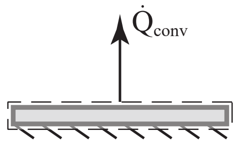
Task a)

The ratio of the convective losses from the surfaces can be calculated by determining their rate of heat loss.

1 **Setting up the balance:**



(a) Case 1



(b) Case 2

For case 1:

$$\begin{aligned}\dot{Q}_{\text{conv},1} &= \overline{\alpha}_1\,(H\times L)\,(T_\text{H}-T_\infty)\\ &= 3\overline{\alpha}_1\,(H\times L)\,T_\infty\end{aligned} \quad \text{(III.13.1)}$$

For case 2:

$$\begin{aligned}\dot{Q}_{\text{conv},2} &= \overline{\alpha}_2\,(W\times L)\,(T_\text{w}-T_\infty)\\ &= 2\overline{\alpha}_2\,(H\times L)\,T_\infty\end{aligned} \quad \text{(III.13.2)}$$

Thus the ratio of the convective losses from the surfaces can be written as:

$$\frac{\dot{Q}_{\text{conv},1}}{\dot{Q}_{\text{conv},2}} = \frac{3\overline{\alpha}_1}{2\overline{\alpha}_2} \quad \text{(III.13.3)}$$

2 **Defining the elements within the balance:**

The heat transfer coefficients for both cases can be determined by their respective Nusselt numbers.

To do so, first, the characteristic length for both cases needs to be determined. The characteristic length in case 1 is defined as:

$$L_1 = H \quad \text{(III.13.4)}$$

The characteristic length in case 2 yields from the ratio between the surface area and the perimeter of the plate:

$$\begin{aligned}L_2 &= \frac{A}{U} = \frac{W\times L}{2\,(W+L)}\\ &\approx \frac{W}{2} = H\end{aligned} \quad \text{(III.13.5)}$$

For case 1, with the characteristic length $L_1 = H$, HTC.17 is used to estimate the Nusselt number. For which $C = 0.516$:

$$\begin{aligned}\overline{\text{Nu}}_1 &= C\,(\text{Gr}_1\,\text{Pr})^{1/4}\\ &= 0.535\cdot(\text{Gr}_1\,\text{Pr})^{1/4}\end{aligned} \quad \text{(III.13.6)}$$

For case 2, with the characteristic length $L_2 = H$, HTC.22 is used to estimate the Nusselt number:

$$\overline{\mathrm{Nu}}_2 = 0.54 \cdot (\mathrm{Gr}_2 \cdot \mathrm{Pr})^{1/4} \tag{III.13.7}$$

The Grashof number for case 1 can be written as:

$$\begin{aligned} \mathrm{Gr}_1 &= \frac{\beta g \,(T_\mathrm{H} - T_\infty)\, L_1^3}{\nu^2} \\ &= \frac{3\beta g T_\infty H^3}{\nu^2} \end{aligned} \tag{III.13.8}$$

The Grashof number for case 2 can be described as:

$$\begin{aligned} \mathrm{Gr}_2 &= \frac{\beta g \,(T_\mathrm{W} - T_\infty)\, L_2^3}{\nu^2} \\ &= \frac{\beta g T_\infty H^3}{\nu^2} \end{aligned} \tag{III.13.9}$$

Lastly, rewriting the definition of the Nusselt number yields the heat transfer coefficient for case 1:

$$\begin{aligned} \overline{\alpha}_1 &= \frac{\overline{\mathrm{Nu}}_1 \lambda}{L_1} \\ & \frac{0.535\lambda}{H} \cdot \left(\frac{3\beta g T_\infty H^3}{\nu^2} \,\mathrm{Pr} \right)^{1/4}, \end{aligned} \tag{III.13.10}$$

and for case 2:

$$\begin{aligned} \overline{\alpha}_2 &= \frac{\overline{\mathrm{Nu}}_2 \lambda}{L_2} \\ & \frac{0.54\lambda}{H} \cdot \left(\frac{\beta g T_\infty H^3}{\nu^2} \,\mathrm{Pr} \right)^{1/4} \end{aligned} \tag{III.13.11}$$

3 **Inserting and rearranging:**

Which yields the ratio of the convective losses from the surfaces:

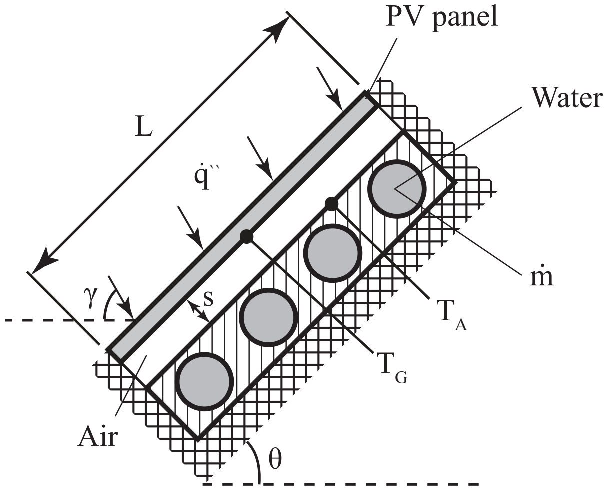
$$\begin{aligned} \frac{\dot{Q}_{\mathrm{conv},1}}{\dot{Q}_{\mathrm{conv},2}} &= \frac{1.605 \left(\frac{3\beta g T_\infty H^3}{\nu^2} \,\mathrm{Pr} \right)^{1/4}}{1.08 \cdot \left(\frac{\beta g T_\infty H^3}{\nu^2} \,\mathrm{Pr} \right)^{1/4}} \\ &= 2.0 \;\; [-] \end{aligned} \tag{III.13.12}$$

Conclusion

The convective loss for case 1 is almost twice as big as the convective loss for case 2.

Exercise III.14 (PV-T Panel ⋆⋆):

PV-T panels, generating thermal and electrical power, are frequently inclined towards the sun to enhance their efficiency. The tilt angle, denoted as θ, plays a crucial role in determining the effectiveness of the solar panel. Radiation is incident upon a PV-T collector at an angle γ, and it possesses a constant heat density represented by $\dot{q}''$. Within the tube collectors, water flows in, entering at a temperature T_{in}, and exits at a temperature T_{out}.



Given parameters:

- Collector height: $L = 0.8$ m
- Collector width: $W = 3$ m
- Space between absorber plate and glass cover: $s = 2$ cm
- Heat flux density: $\dot{q}'' = 1000$ W/m^2
- Heat flux angle: $\gamma = 60^{\circ}$
- Glass cover temperature: $T_{\text{G}} = 40$ °C
- Absorber plate temperature: $T_{\text{A}} = 100$ °C
- Air average density: $\rho = 1.05$ kg/m^3
- Air average thermal conductivity: $\lambda = 0.029$ W/mK
- Air average kinematic viscosity: $\nu = 1.9 \cdot 10^{-5}$ m^2/s
- Air average Prandtl number: $\text{Pr} = 0.71$
- Water inlet temperature: $T_{\text{in}} = 10$ °C
- Water mass flow rate: $\dot{m} = 0.01$ kg/s
- Water average specific heat capacity: $c_{\text{p}} = 4.2$ kJ/kgK
- Water outlet temperature:

$$T_{\text{out}} = T_{\text{in}} \left(2 + 2 \cdot \sin\left(2\gamma - \theta\right)\right)$$

Hints:

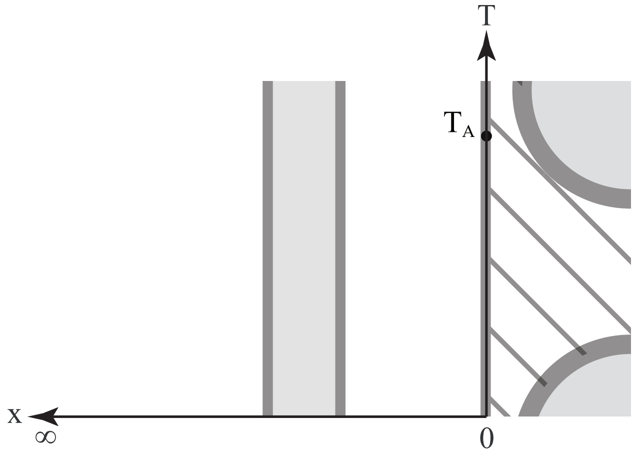
- All incident radiation is absorbed by the collector.
- The back side of the absorber is heavily insulated.
- The process can be assumed to be steady-state.

Tasks:

a) Determine the overall efficiency of the PV-T panel for $\theta = 0^o$.

b) Reason what you expect to be the optimal tilt angle for the PV-T panel to have the highest generation of useful energy.

c) Draw the temperature profile for the domain $0 \leq x \leq \infty$.

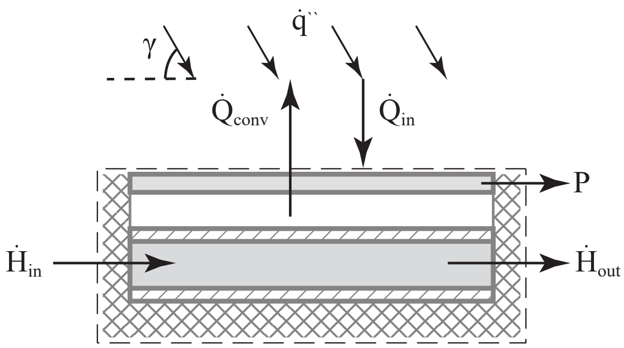


Solution III.14 (PV-T Panel ⋆):

Task a)

To determine the overall efficiency of the PV-T panel, the energy that is used in an efficient manner should be determined.

1 **Setting up the balance:**



The energy balance around the entire PV-T panel reads:

$$0 = \dot{Q}_{\text{in}} + \dot{H}_{\text{in}} - \dot{H}_{\text{out}} - P - \dot{Q}_{\text{conv}}, \tag{III.14.1}$$

where the overall efficiency stems from the sum of the energy extracted from the fluid and the power generated relative to the incident radiation:

$$\eta_{\text{overall}} = \frac{\dot{H}_{\text{out}} - \dot{H}_{\text{in}} + P}{\dot{Q}_{\text{in}}} \tag{III.14.2}$$

2 **Defining the elements within the balance:**

Before doing the calculations, the fluid outlet temperature should be determined, which yields from:

$$\begin{aligned} T_{\text{out}} &= T_{\text{in}}\left(2 + 2 \cdot \sin\left(2\gamma - \theta\right)\right) \\ &= 10\ [^{\circ}\text{C}] \cdot \left(2 + 2 \cdot \sin\left(2 \cdot 60^{\circ}\right)\right)\ [-] = 37.3\ [^{\circ}\text{C}] \end{aligned} \tag{III.14.3}$$

The incident radiation on the PV-T panel yields from trigonometry:

$$\begin{aligned} \dot{Q}_{\text{in}} &= \dot{q}''_{\text{perpendicular}}\left(W \times L\right) = \dot{q}'' \sin\left(\gamma\right)\left(W \times L\right) \\ &= 1000\ \left[\text{W/m}^2\right] \sin\left(60^{\circ}\right)\ [-]\ (3 \times 0.8) = 2{,}078.5\ [\text{W}] \end{aligned} \tag{III.14.4}$$

The rate of heat transport entering due to the motion of the fluid can be stated as:

$$\begin{aligned} \dot{H}_{\text{in}} &= \dot{m} c_{\text{p}} T_{\text{in}} \\ &= 0.01\ [\text{kg/s}] \cdot 4200\ [\text{J/kgK}] \cdot 10\ [^{\circ}\text{C}] = 420\ [\text{W}], \end{aligned} \tag{III.14.5}$$

and the rate of heat transport leaving due to the motion of the fluid can be written as:

$$\begin{aligned} \dot{H}_{\text{out}} &= \dot{m} c_{\text{p}} T_{\text{out}} \\ &= 0.01\ [\text{kg/s}] \cdot 4200\ [\text{J/kgK}] \cdot 37.3\ [^{\circ}\text{C}] = 1{,}567.5\ [\text{W}] \end{aligned} \tag{III.14.6}$$

Convective losses can be estimated by multiplying the heat transfer coefficient by the surface area of the collector and the temperature difference between the absorber plate and the glass cover.

The heat transfer coefficient can be determined using Nusselt correlations that are valid for natural convection within horizontal enclosures. To determine the appropriate correlation, the value of the Grashof number must first be determined. To calculate this value, first, the average fluid property temperature should be determined:

$$\begin{aligned} T_{prop} &= \frac{T_{\mathrm{G}} + T_{\mathrm{A}}}{2} \\ &= \frac{(40 + 100)\ [^{\circ}\mathrm{C}]}{2} = 70\ [^{\circ}\mathrm{C}] \end{aligned} \tag{III.14.7}$$

From this temperature, assuming the air to behave as an ideal gas, the volumetric expansion coefficient can be determined:

$$\begin{aligned} \beta &= \frac{1}{T_{prop}} \\ &= \frac{1}{(70 + 273)} \left[\mathrm{K}^{-1}\right] = 2.9 \cdot 10^{-3} \left[\mathrm{K}^{-1}\right] \end{aligned} \tag{III.14.8}$$

The Grashof number is calculated as follows:

$$\begin{aligned} \mathrm{Gr_s} &= \frac{g\beta\,(T_{\mathrm{A}} - T_{\mathrm{G}})\,s^3}{\nu^2} \\ &= \frac{9.81 \left[\mathrm{m/s^2}\right] \cdot 2.9 \cdot 10^{-3} \left[\mathrm{K^{-1}}\right] \cdot (100 - 40)\ [^{\circ}\mathrm{C}] \cdot 0.02^3 \left[\mathrm{m^3}\right]}{\left(1.9 \cdot 10^{-5}\right)^2\ \left[\mathrm{m^4/s^2}\right]} = 3.8 \cdot 10^4\ [-] \end{aligned} \tag{III.14.9}$$

Thus, it is determined that HTC.27, which is applicable for natural laminar flow within a horizontal enclosure with isothermal surfaces, can be utilized to estimate the average Nusselt number:

$$\begin{aligned} \overline{\mathrm{Nu}}_{\mathrm{s}} &= 0.21\,(\mathrm{Gr_s\,Pr})^{1/4} \\ &= 0.21\left(3.8 \cdot 10^4\ [-] \cdot 0.71\ [-]\right)^{1/4} = 2.7\ [-] \end{aligned} \tag{III.14.10}$$

Rewriting the definition of the Nusselt number yields:

$$\begin{aligned} \overline{\alpha} &= \frac{\overline{\mathrm{Nu}}_{\mathrm{s}}\lambda}{s} \\ & \frac{2.7\ [-] \cdot 0.029\ [\mathrm{W/mK}]}{0.02\ [\mathrm{m}]} = 3.9 \left[\mathrm{W/m^2K}\right] \end{aligned} \tag{III.14.11}$$

Which yields the convective rate of heat transfer:

$$\begin{aligned} \dot{Q}_{\mathrm{conv}} &= \overline{\alpha}\,(W \times L)\,(T_{\mathrm{A}} - T_{\mathrm{G}}) \\ &= 3.9 \left[\mathrm{W/m^2K}\right] \cdot (3 \times 8)\ \left[\mathrm{m^2}\right] \cdot (100 - 40)\ [^{\circ}\mathrm{C}] = 562.1\ [\mathrm{W}] \end{aligned} \tag{III.14.12}$$

3 Inserting and rearranging:

Inserting the found expressions for the fluxes and rewriting the energy balance yields the generated power by the PV-T panel:

$$\begin{aligned} P &= \dot{Q}_{\text{in}} + \dot{H}_{\text{in}} - \dot{H}_{\text{out}} - \dot{Q}_{\text{conv}} \\ &= (2{,}078.5 + 420 - 1{,}567.5 - 562.1)\ [\text{W}] = 368.9\ [\text{W}], \end{aligned} \tag{III.14.13}$$

and thus the overall efficiency of the PV-T panel is calculated as:

$$\begin{aligned} \eta_{\text{overall}} &= \frac{\dot{H}_{\text{out}} - \dot{H}_{\text{in}} + P}{\dot{Q}_{\text{in}}} \\ &= \frac{(1{,}567.5 - 420 + 368.9)\ [\text{W}]}{2{,}078.5\ [\text{W}]} = 0.73 \end{aligned} \tag{III.14.14}$$

Conclusion

The overall efficiency of the PV-T panel for $\theta = 0^{\circ}$ is thus about 73%.

Task b)

Conclusion

The incoming heat from the sun is maximized when the PV-T panel stands exactly perpendicular to the angle of the sun. This is for a tilt angle of 40 °C. Many locations in the mid-latitudes (around 30° to 40° latitude), make use of tilt angles of approximately 30° to 40°.

Task c)

At $x = 0$, where the heat transport is solely via conduction, the temperature gradient near the wall is steep due to the high thermal resistance. As distance from the wall increases, fluid velocity rises, augmenting heat transport through advection and consequently reducing the temperature gradient.

When moving toward the glass plate, conduction becomes the predominant mode of heat transfer again. The temperature gradient near the glass plate intensifies. At the interface, a kink within the temperature profile is observed. This kink at the glass-fluid interface reflects the disparity in thermal conductivities between the glass and air with the glass being the better conducting, yielding a less steep gradient.

Within the glass plate, heat conduction through a plane wall results in a uniform temperature gradient, maintaining a consistent profile.

At the interface between the glass plate and the ambient environment, another kink emerges as heat is conducted back into the air, characterized by its lower thermal conductivity compared to glass. The steeper temperature gradient at this interface within the fluid is evident. Moving away from the fluid, the gradient diminishes as fluid velocity increases, enhancing heat transport by advection.

This progression continues until the temperature of the ambient environment is reached, with a gradient eventually approaching zero.

Conclusion