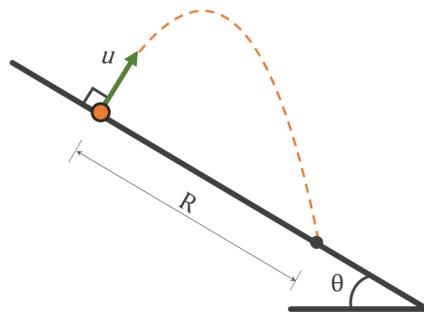


Projectile at Right Angle



A projectile is fired with a velocity u at a right angle to the slope, which is inclined at an angle θ with the horizontal. Which of the following is the correct expression for the distance R to the point of impact.

Using known expressions (for constant acceleration):

$$a = \frac{dv}{dt} \Rightarrow dv = adt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = vdt = (at + v_0)dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (at + v_0) dt \quad (5)$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \quad (6)$$

Given:

Initial velocity: $v_0 = u$

Displacement to point of impact: R

Solution:

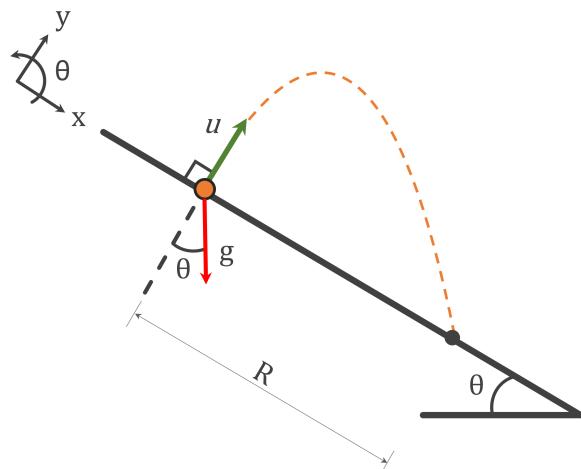


Figure 1: Projectile at Right Angle, with coordinate system

The coordinate system is defined such that y is defined positive upwards in the same direction of \vec{u} . Figure 1 shows the coordinate system, together with the gravitational acceleration \vec{g} . From Figure 1 the following two relations of x and y can be established.

$$\begin{cases} x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + x_0 \\ y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0 \end{cases} \quad (7)$$

Where $v_{x,0} = 0$ m/s, $v_{y,0} = u$ and $x_0 = y_0 = 0$ m. In addition, the x -and y -displacements after the projectile has landed on the slope are equal to R and 0 respectively. The acceleration in x -direction is $a_x = g \sin \theta$ and the acceleration in the y -direction is $a_y = -g \cos \theta$.

$$\begin{cases} x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + x_0 \\ y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0 \end{cases} \Rightarrow \begin{cases} x(t) = \frac{1}{2}gt^2 \sin \theta = R \\ y(t) = -\frac{1}{2}gt^2 \cos \theta + ut = 0 \end{cases} \quad (8)$$

From Equation (8), the time t can be rewritten as follows. The $t = 0$ solution is not considered.

$$t \neq 0 \Rightarrow u = \frac{1}{2}gt \cos \theta \Rightarrow t = \frac{2u}{g \cos \theta} \quad (9)$$

Inserting Equation (9) into Equation (8) for t gives the final expression for R .

$$R = \frac{1}{2}g \sin \theta \cdot \left(\frac{2u}{g \cos \theta} \right)^2 = \frac{2u^2 \tan \theta}{g \cos \theta} \quad (10)$$