

## Learning activities before - Lecture 2

The learning objective of the session are:

### Energy in general

- Using an energy balance
- Determine efficiency
- Calculate losses for electric resistors

### Conductive heat transfer

- Explaining conduction principles
- Calculate conductive heat transfer
- Using thermal resistances and insulation values

Please prepare yourself in the following way:

Question to make you think:

**Why are energy labels put on appliances? How do they calculate the efficiency ?**

**Why is a microwave oven much more efficient at cooking than using a regular oven or gas cooker?**

**What are the different Modes of Heat Transfer?**

### What is Efficiency ?

In general, efficiency is a measurable concept, quantitatively determined by the ratio (Links to an external site.) of useful output to total input. Effectiveness is the simpler concept of being able to achieve a desired result, which can be expressed quantitatively but does not usually require more complicated mathematics than addition. Efficiency can often be expressed as a percentage of the result that could ideally be expected, for example if no energy were lost due to friction or other causes, in which case 100% of fuel or other input would be used to produce the desired result.

No machine is 100% efficient – some energy is always 'lost' in the process. This is why we can never have a 'Perpetual Motion' machine.

Formula to calculate how efficient a machine is:

$$\text{Percentage Efficiency} = (\text{Useful Power Out}) / (\text{Power In}) \times 100$$

## What are the different Modes of Heat Transfer?

In our everyday life, it has been observed that when a pan full of water is boiled on a flame, its temperature increases, but when the flame is turned off, it slowly cools down.

This is because of the phenomenon of heat transfer taking place between the pan full of water and the flame. It has been established that heat transfer takes place from hotter objects to colder objects.



When there are objects which are at different temperatures or there is an object at a different temperature from the surroundings, then the transfer of heat takes place so that the object and the surrounding, both reach an equilibrium temperature.

There are three modes of heat transfer.

1. Conduction
2. Convection
3. Radiation

### Conduction of Heat

Heat conduction is a process in which heat is transferred from the hotter part to the colder part in a body without involving any actual movement of the molecules of the body. Heat transfer takes place from one molecule to another molecule as a result of the vibratory motion of the molecules. Heat transfer through the process of conduction occurs in substances which are in direct contact with each other. It generally takes place in solids.

**Conduction example:** When frying vegetables in a pan. Heat transfer takes place from flame to the pan and then to the vegetables.

Based on the conductivity of heat, substances can be classified as conductors and insulators. Substances that conduct heat easily are known as conductors and those that do not conduct heat are known as insulators.

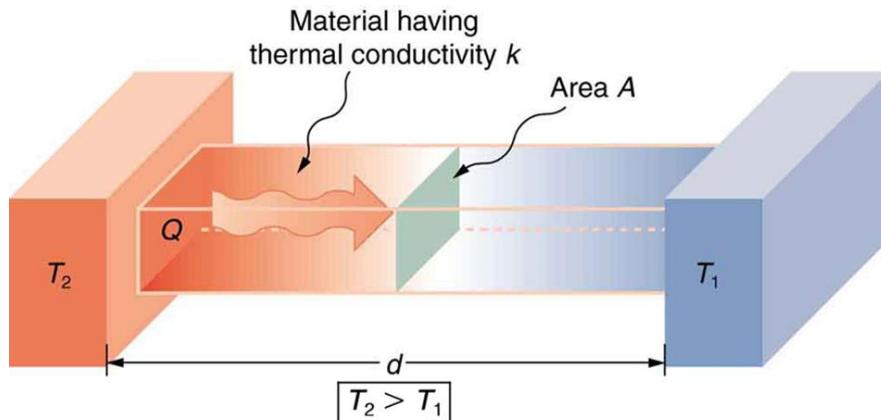
## What is Thermal Conductivity?

Thermal conductivity refers to the ability of a given material to conduct/transfer heat. It is generally denoted by the symbol 'k' but can also be denoted by ' $\lambda$ ' and ' $k'$ . The reciprocal of this quantity is known as thermal resistivity. Materials with high thermal conductivity are used in heat sinks whereas materials with low values of  $\lambda$  are used as thermal insulators.

[Fourier's law of thermal conduction](#) (Links to an external site.) (also known as the law of heat conduction) states that the rate at which heat is transferred through a material is proportional to the negative of the temperature gradient and is also proportional to the area through which the heat flows. The differential form of this law can be expressed through the following equation:

$$q_o = -k \cdot \nabla T$$

Where  $\nabla T$  refers to the temperature gradient,  $q$  denotes the thermal flux or heat flux, and  $k$  refers to the thermal conductivity of the material in question.



An illustration describing the thermal conductivity of a material in terms of the flow of heat through it is provided above. In this example, Temperature<sub>1</sub> is greater than Temperature<sub>2</sub>. Therefore, the thermal conductivity can be obtained via the following equation:

$$K = (Qd)/(A\Delta T)$$

Where,

- K is the thermal conductivity in W/m.K
- Q is the amount of heat transferred through the material in Joules/second or Watts
- d is the distance between the two isothermal planes
- A is the area of the surface in square meters
- $\Delta T$  is the difference in temperature in Kelvin

To understand more about the heat conduction watch the video below :

[https://www.youtube.com/watch?v=9joLYfayee8&feature=emb\\_imp\\_woyt](https://www.youtube.com/watch?v=9joLYfayee8&feature=emb_imp_woyt)

## Steady-State Techniques

- These methods involve measurements where the temperature of the material in question does not change over a period of time.
- An advantage of these techniques is that the analysis is relatively straightforward since the temperature is constant.
- An important disadvantage of steady-state techniques is that they generally require a very well-engineered setup to perform the experiments.
- Examples of these techniques are the Searle's bar method for measuring the thermal conductivity of a good conductor and Lee's disc method.

## Conduction Rate Equation - Fourier's Law

Fourier's Law is well explained in this video :

[https://www.youtube.com/watch?v=\\_Fh7dkTDTFE&feature=emb\\_imp\\_woyt](https://www.youtube.com/watch?v=_Fh7dkTDTFE&feature=emb_imp_woyt)

Do you want to see interesting demonstrations on heat conduction? please watch the below video :

<https://www.youtube.com/watch?v=COAeBrqKwVU>

## Notes – Lecture 2:

### Steady Heat Conduction

In thermodynamics, we considered the amount of heat transfer as a system undergoes a process from one equilibrium state to another. Thermodynamics gives no indication of how long the process takes. In heat transfer, we are more concerned about the rate of heat transfer.

The basic requirement for heat transfer is the presence of a temperature difference. The temperature difference is the driving force for heat transfer, just as voltage difference for electrical current. The total amount of heat transfer  $Q$  during a time interval can be determined from:

$$Q = \int_0^{\Delta t} Q^\bullet dt \quad (kJ)$$

The rate of heat transfer per unit area is called heat flux, and the average heat flux on a surface is expressed as

$$q^\bullet = \frac{Q^\bullet}{A} \quad (W/m^2)$$

### Steady Heat Conduction in Plane Walls

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as result of interactions between the particles.

Consider steady conduction through a large plane wall of thickness  $\Delta x = L$  and surface area  $A$ . The temperature difference across the wall is  $\Delta T = T_2 - T_1$ .

Note that heat transfer is the only energy interaction; the energy balance for the wall can be expressed:

$$Q_{in}^\bullet - Q_{out}^\bullet = \frac{dE_{wall}}{dt}$$

For steady-state operation,

$$Q_{in}^\bullet = Q_{out}^\bullet = \text{const.}$$

It has been *experimentally* observed that the rate of heat conduction through a layer is proportional to the temperature difference across the layer and the heat transfer area, but it is inversely proportional to the thickness of the layer.

$$\text{rate of heat transfer} \propto \frac{(\text{surface area})(\text{temperature difference})}{\text{thickness}}$$

$$Q_{Cond}^\bullet = kA \frac{\Delta T}{\Delta x} \quad (W)$$

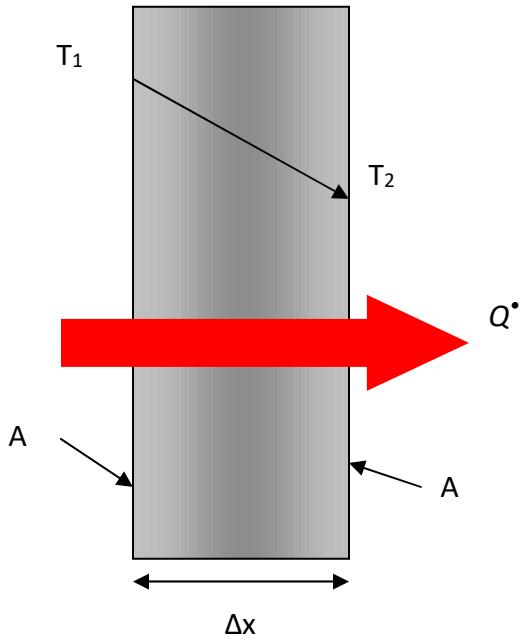


Fig. 1: Heat conduction through a large plane wall.

The constant proportionality  $k$  is the *thermal conductivity* of the material. In the limiting case where  $\Delta x \rightarrow 0$ , the equation above reduces to the differential form:

$$Q_{Cond}^{\bullet} = -kA \frac{dT}{dx} \quad (W)$$

which is called Fourier's law of heat conduction. The term  $dT/dx$  is called the temperature gradient, which is the slope of the temperature curve (the rate of change of temperature  $T$  with length  $x$ ).

## Thermal Conductivity

Thermal conductivity  $k$  [W/mK] is a measure of a material's ability to conduct heat. The thermal conductivity is defined as the rate of heat transfer through a unit thickness of material per unit area per unit temperature difference.

Thermal conductivity changes with temperature and is determined through experiments.

The thermal conductivity of certain materials show a dramatic change at temperatures near absolute zero, when these solids become *superconductors*.

An *isotropic* material is a material that has uniform properties in all directions.

*Insulators* are materials used primarily to provide resistance to heat flow. They have low thermal conductivity.

## The Thermal Resistance Concept

The Fourier equation, for steady conduction through a constant area plane wall, can be written:

$$Q^{\bullet}_{Cond} = -kA \frac{dT}{dx} = kA \frac{T_1 - T_2}{L}$$

This can be re-arranged as:

$$Q^{\bullet}_{Cond} = \frac{T_2 - T_1}{R_{wall}} \quad (W)$$

$$R_{wall} = \frac{L}{kA} \quad (\text{C/W})$$

$R_{wall}$  is the *thermal resistance* of the wall against heat conduction or simply the *conduction resistance* of the wall.

## Thermal Resistances in Parallel

The thermal resistance concept can be used to solve steady state heat transfer problem in parallel layers or combined series-parallel arrangements.

It should be noted that these problems are often two- or three dimensional, but approximate solutions can be obtained by assuming one dimensional heat transfer (using thermal resistance network).

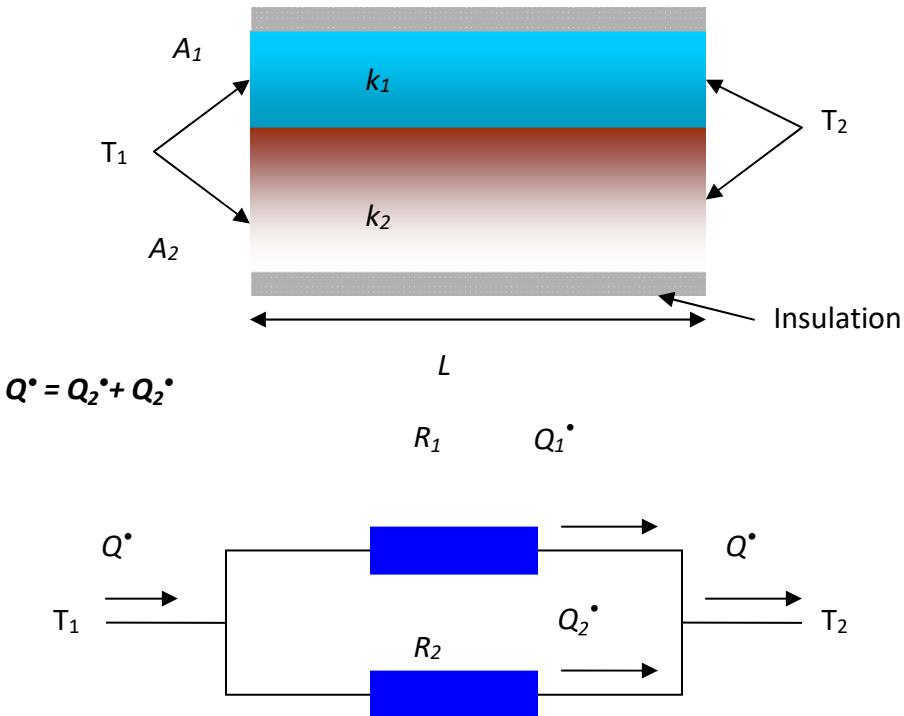


Fig. 3: Parallel resistances.

$$Q^* = Q_1^* + Q_2^* = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$Q^* = \frac{T_1 - T_2}{R_{total}}$$

$$\frac{1}{R_{total}} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow \frac{1}{R_{total}} = \frac{R_1 R_2}{R_1 + R_2}$$

### Example 1: Thermal Resistance Network

Consider the combined series-parallel arrangement shown in figure below. Assuming one-dimensional heat transfer, determine the rate of heat transfer.

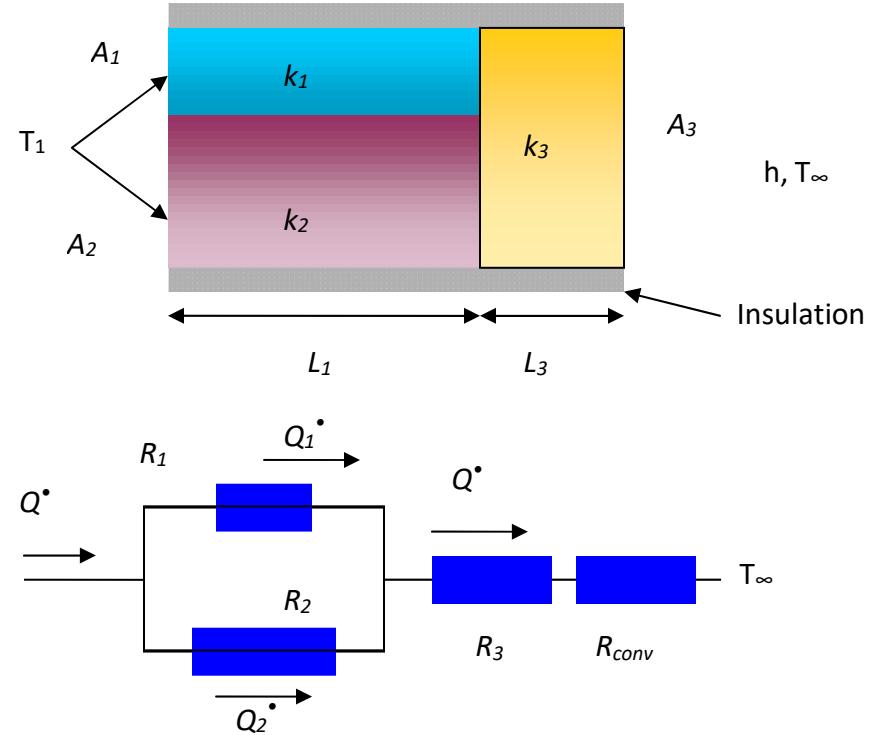


Fig. 4: Schematic for example 1.

Solution:

The rate of heat transfer through this composite system can be expressed as:

$$Q^* = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

Two approximations commonly used in solving complex multi-dimensional heat transfer problems by transfer problems by treating them as one dimensional, using the thermal resistance network:

1- Assume any plane wall normal to the x-axis to be isothermal, i.e. temperature to vary in one direction only  $T = T(x)$

2- Assume any plane parallel to the x-axis to be adiabatic, i.e. heat transfer occurs in the x-direction only.

These two assumptions result in different networks (different results). The actual result lies between these two results.

## Heat Conduction in Cylinders and Spheres

Steady state heat transfer through pipes is in the normal direction to the wall surface (no significant heat transfer occurs in other directions). Therefore, the heat transfer can be

modeled as steady-state and one-dimensional, and the temperature of the pipe will depend only on the radial direction,  $T = T(r)$ .

Since, there is no heat generation in the layer and thermal conductivity is constant, the Fourier law becomes:

$$Q_{cond,cyl}^{\bullet} = -kA \frac{dT}{dr} \quad (W)$$

$$A = 2\pi rL$$

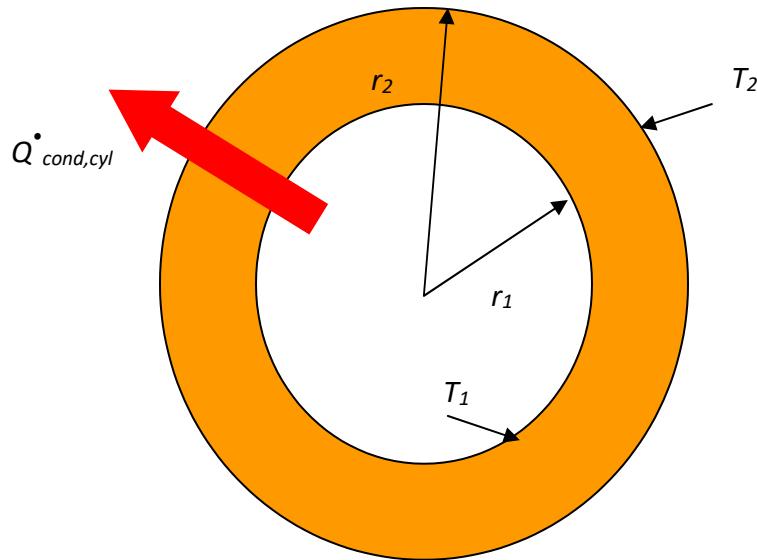


Fig. 5: Steady, one-dimensional heat conduction in a cylindrical layer.

After integration:

$$\int_{r_1}^{r_2} \frac{Q^{\bullet}_{cond,cyl}}{A} dr = - \int_{T_1}^{T_2} k dT \quad A = 2\pi r L$$

$$Q^{\bullet}_{cond,cyl} = 2\pi k L \frac{T_1 - T_2}{\ln(r_2 / r_1)}$$

$$Q^{\bullet}_{cond,cyl} = \frac{T_1 - T_2}{R}$$

$$R_{cyl} = \frac{\ln(r_2 / r_1)}{2\pi k L}$$

where  $R_{cyl}$  is the conduction resistance of the cylinder layer.

Following the analysis above, the conduction resistance for the spherical layer can be found:

$$Q_{cond,sph}^{\bullet}=\frac{T_1-T_2}{R}$$

$$^{sph}$$

$$R_{sph} = \frac{r_2 - r_1}{4\pi~r_1r_2k}$$

## Learning activities after - Lecture 2

You have joined the second Lecture of our E&H course. Below you will find some activities that will help you to bring your learning into practice and/or to learn more about the topics that were addressed.

### Efficiency and Heat conduction Questions

Try to think and find out the answers for below questions :

1. What is the relationship of input and output power with efficiency ?
2. Can we have the machine with 100% efficiency ?
3. Which part of the output power should be considered for efficiency calculation?
4. What are the heat transfer mechanisms?
5. What is thermal conductivity and its unit?
6. Could you explain the Fourier's Law?
7. How do you calculate the rate of conduction heat flow ?
8. What is thermal resistance network?
9. Could you define the different thermal resistance networks arrangements (series and parallel) and their equations?

Please watch below video for better understanding of the thermal resistance network for heat conduction and how to solve the problems associated with that :

[https://www.youtube.com/watch?v=eqoL788i3ns&feature=emb\\_imp\\_woyt](https://www.youtube.com/watch?v=eqoL788i3ns&feature=emb_imp_woyt)