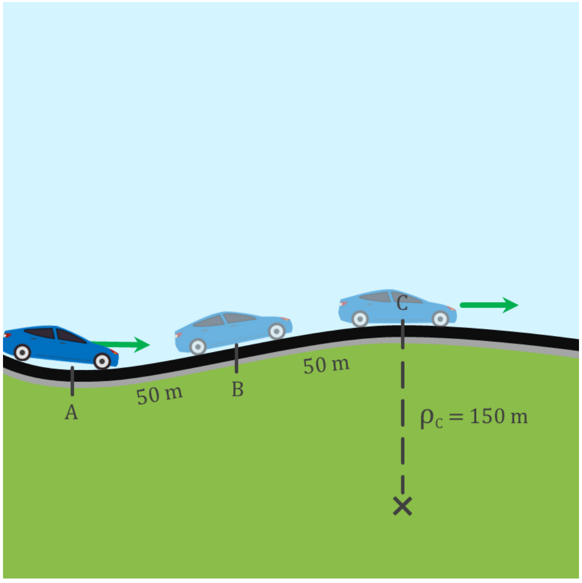
# Dip and Hump in the Road




**The driver of the car applies the brakes to produce a uniform deceleration, to go over the dip and hump in the road. The speed at the bottom A of the dip is 90 km/h and 54 km/h at the top C of the hump, which is 100m along the road from A. If the passengers experience a total acceleration of 3 m/s$^2$ at A and if the radius of curvature of the hump at C is 150m, determine the radius of curvature $\rho$ in meters at A. Round to the nearest integer (no decimals).**

*Using known expressions*:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}v}{a} \tag{1}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}s}{v} \tag{2}$$

Combining both expressions results in:

$$\frac{\mathrm{d}v}{a} = \frac{\mathrm{d}s}{v} \tag{3}$$

$$a\mathrm{d}s = v\mathrm{d}v \tag{4}$$

Finding expressions for $a\mathrm{d}s$ and $v\mathrm{d}v$ gives:

$$a\mathrm{d}s = a\int_0^s \mathrm{d}s = a\cdot(s-0) = a\cdot s \tag{5}$$

$$v\mathrm{d}v = \int_{v_A}^{v_C} v\,\mathrm{d}v = \frac{1}{2}({v_C}^2 - {v_A}^2) \tag{6}$$

Combining Equations 5 and 6 gives:

$$a\cdot s = \frac{1}{2}({v_C}^2 - {v_A}^2) \quad\Rightarrow\quad a = \frac{{v_C}^2 - {v_A}^2}{2\cdot s} \tag{7}$$

Furthermore, we know:

$$a_n = \frac{{v_A}^2}{\rho} \quad\Rightarrow\quad \rho = \frac{{v_A}^2}{a_n} \tag{8}$$

*Given*:
Velocity at A: $v_A = 90km/h = 25m/s$
Velocity at C: $v_C = 54km/h = 15m/s$
Acceleration at A: $a_A = 3m/s^2$
Distance: $s = 100m$

The acceleration calculated using Equation 7 is the average acceleration, in this case deceleration, over the path the car travels. Since the deceleration is uniform, the acceleration is the same as the tangential acceleration $a_t$ and is constant over the path.

$$a = a_{t,A} = \frac{{v_C}^2 - {v_A}^2}{2\cdot s} = \frac{15^2 - 25^2}{2\cdot 100} = -2m/s^2 \tag{9}$$

To calculate the radius of curvature $\rho$, we need to find the normal acceleration $a_n$. From Pythagoras it follows that:

$${a_A}^2 = {a_{t,A}}^2 + {a_{n,A}}^2 \quad\Rightarrow\quad a_{n,A} = \sqrt{{a_A}^2 - {a_{t,A}}^2} \tag{10}$$

This is true, since $a_t$ and $a_n$ are perpendicular to each other.

$$a_{n,A} = \sqrt{{a_A}^2 - {a_{t,A}}^2} = \sqrt{3^2 - (-2)^2} = 2.236m/s^2 \tag{11}$$

Inserting this in Equation 8 gives:

$$\rho_A = \frac{{v_A}^2}{a_{n,A}} = \frac{25^2}{2.236} = 279.5m \approx 280m \tag{12}$$