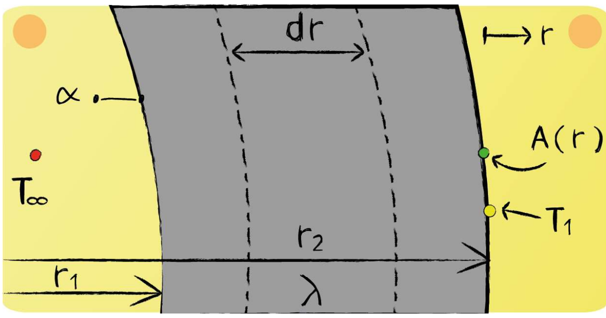
Lecture 8 - Question 5



Develop an energy balance to calculate the temperature profile inside the pipe wall and give the boundary conditions. Assume one-dimensional steady-state heat transfer in radial direction. The expansion of the pipe in axial direction is L.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

Since the type of heat transfer is steady-state, the sum of the in- and outgoing heat fluxes of the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda A(r)\frac{\partial T}{\partial r} = -\lambda 2\pi r L\frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} = \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r}dr = -\lambda 2\pi r L\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}(-\lambda 2\pi r L\frac{\partial T}{\partial r})dr$$

The ingoing flux can be described by use of Fouriers law and the outgoing flux can be approximated by use of the Taylor series expansion.

Boundary conditions:

$$\frac{\partial T(r=0)}{\partial r} = -\frac{\alpha}{\lambda}(T(r=0) - T_\infty)$$

$$T(r=r_2) = T_1$$

The first boundary condition results from the fact that $\dot{Q}_{r=0} = -\lambda A(r)\frac{\partial T(r=0)}{\partial r} = \alpha\ A(r)(T(r=0) - T_\infty)$, the second one describes that the temperature at the surface equals T_1.