

2.13 Heat generation in an electrical wire

- Problem type

Steady-state heat transfer with a source.

a) Determining the heat transfer coefficient α .

Energy balance:

$$\dot{Q}_{\text{conv}} + \dot{\Phi} = 0 \quad (2.151)$$

Source:

$$\dot{\Phi} = V \cdot I \quad (2.152)$$

Convection:

$$\dot{Q}_{\text{conv}} = \alpha \cdot \pi \cdot d \cdot L \cdot (T_s - T_a) \quad (2.153)$$

Rearranging and filling in:

$$\begin{aligned} \rightarrow \alpha &= \frac{V \cdot I}{\pi \cdot d \cdot L \cdot (T_s - T_a)} \\ &= \frac{110 \text{ [V]} \cdot 3 \text{ [A]}}{\pi \cdot 0.002 \text{ [m]} \cdot 2.1 \text{ [m]} \cdot (453.15 \text{ [K]} - 293.15 \text{ [K]})} = 156.31 \text{ [Wm}^{-2}\text{K}^{-1}] \end{aligned} \quad (2.154)$$

2.14 Resistance wire

- Problem type

1-dimensional steady-state heat transfer with a source.

a) Determining the temperature at $r = 3.5\text{mm}$.

1-dimensional heat transportation equation with source cylindrical coordinates:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda \frac{\partial T}{\partial r} \right) + \Phi''' = 0 \quad (2.155)$$

Integration:

$$\frac{\partial T}{\partial r} = -\frac{\Phi'''}{2\lambda} r + C_1 \quad (2.156)$$

$$T(r) = -\frac{\Phi'''}{4\lambda} r^2 + C_1 r + C_2 \quad (2.157)$$

Boundary conditions:

$$\frac{\partial T}{\partial r}(r = 0) = 0 \quad (2.158)$$

$$T(r = r_0) = T_s \quad (2.159)$$

Using the boundary conditions results in:

$$\rightarrow C_1 = 0 \quad (2.160)$$

$$\rightarrow C_2 = T_s + \frac{\Phi'''}{4\lambda} r_0^2 \quad (2.161)$$

Giving the temperature equation:

$$T(r) = -\frac{\Phi'''}{4\lambda} r^2 + T_s + \frac{\Phi'''}{4\lambda} r_0^2 \quad (2.162)$$

And thus:

$$\begin{aligned} T(r = 0.0035 \text{ m}) &= \\ &= -\frac{5 \cdot 10^7 \text{ [Wm}^{-3}\text{]}}{4 \cdot 6 \text{ [Wm}^{-1}\text{K}^{-1}\text{]}} \cdot 0.0035^2 \text{ [m}^2\text{]} + 453.15 \text{ [K]} + \frac{5 \cdot 10^7 \text{ [Wm}^{-3}\text{]}}{4 \cdot 66 \text{ [Wm}^{-1}\text{K}^{-1}\text{]}} \cdot 0.005^2 \text{ [m}^2\text{]} \\ &= 478.71 \text{ [K]} \quad (2.163) \end{aligned}$$

2.15 Transistors

- Problem type

Steady-state heat transfer with a source.

a) Determining the surface temperature T_s

Energy balance:

$$\dot{Q}_{\text{conv}} + \dot{\Phi} = 0 \quad (2.164)$$

Heat generation:

$$\dot{\Phi} = 4 \cdot \dot{\Phi}_{\text{transistor}} \quad (2.165)$$

Convective heat transfer:

$$\dot{Q}_{\text{conv}} = 2 \cdot \alpha \cdot A \cdot (T_s - T_a) \quad (2.166)$$

Rearranging and inserting:

$$\begin{aligned} \rightarrow T_s &= \frac{\dot{\Phi}}{2 \cdot \alpha \cdot A} + T_a \\ &= \frac{60 \text{ [W]}}{2 \cdot 25 \text{ [Wm}^{-2}\text{K}^{-1}] \cdot 0.0576 \text{ [m}^2\text{]}} + 291.15 \text{ [K]} = 311.98 \text{ [K]} \end{aligned} \quad (2.167)$$