Solutions lecture 2

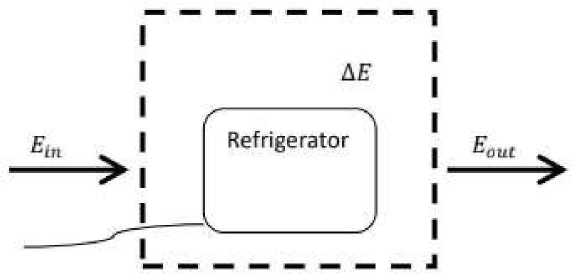
Starting with lecture 2, the approach presented in the assignment set is used to recognize and solve the problems.

2.1 Energy balance of a student room

a)

Analysis

We have two identical rooms with windows and doors closed. One room will contain a refrigerator. The question is whether the room with the refrigerator becomes warmer or cooler. In the following figure, a sketch is made of the problem.



In this sketch, the energy coming into the room, E_{in}, and the energy coming out of the room with the refrigerator, E_{out}, is drawn. Due to the conservation of energy, the difference must be stored in the room. This difference is denoted by ΔE. The refrigerator is connected to electricity.

Approach

Assumptions

We assume that there is no heat loss at the walls.

Route to solution

We can solve this problem by looking at the energy balance:

$$E_{in} - E_{out} = \Delta E$$

Since it is a closed system, no energy can leave the room, and $E_{out} = 0$. The incoming energy is in the form of electricity, so $E_{in} > 0$

Elaboration

The room including the refrigerator will have a larger rise in temperature compared to the one without a refrigerator. Both rooms are identical, so the temperature difference can be explained by focusing on the energy balance regarding the difference of both rooms:

Looking at the energy balance, $E_{out} = 0$ and $E_{in} > 0$. This means that $\Delta E > 0$. The total amount of energy is increasing, so the temperature is rising.

Evaluation

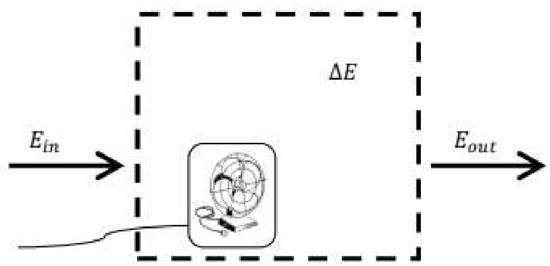
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

b)

Analysis

A fan is turned on in a room with doors and windows closed, and is turned on for 10 hours. The question asked is to determine the temperature of the room after 10 hours. In the following figure, a sketch is made of the problem.



In this sketch, the energy coming into the room, E_{in}, and the energy coming out of the room, E_{out}, is drawn. Due to the conservation of energy, the difference must be stored in the room. The difference is denoted by ΔE. The fan is connected to electricity. The dimensions of the room are given, which are 4m x 6m x 6m. The fan has a power of 150 W. The starting temperature is 15 °C.

Approach

Assumptions

We assume no heat loss at the walls. Specific heats can be taken around room temperature.

Route to solution

We start with the energy balance. Since no energy is leaving the room $E_{out} = 0$. The ingoing energy can be calculated with

$$E_{in} = P_{electric} \cdot t$$

Since $E_{in} > 0, \Delta E > 0$, so the total energy in the system is increasing, and the temperature is rising. This also means that $\Delta E = Q$. The temperature can be calculated with

$$Q = m \cdot c_v \cdot \Delta T \implies \Delta T = \frac{Q}{m \cdot c_v}$$

Because the room has a constant volume, c_v is used (c_p is used when the room is maintained at a constant pressure). This value can be taken at room temperature. Now, only the mass of the the air is unknown. Using the density of air at room temperature and calculating the volume of the room, the mass of the air in the system can be calculated. Substituting these results in the equation for ΔT gives the temperature rise.

Elaboration

We start with calculating the ingoing energy

$$E_{in} = P_{electric} \cdot t = 150 \cdot 10 \cdot 3600 = 5.4\text{MJ}$$

At room temperature (20°C), the values for the density of air and the specific heat at constant volume are [Engineering Toolbox]:

$$\rho = 1.2041\mathrm{kg\,m^{-3}}, \qquad c_v = 717\mathrm{J\,kg^{-1}\,K^{-1}}$$

The volume of the room is $4 \cdot 6 \cdot 6 = 144\mathrm{m}^3$. The mass of the air is then $m = V \cdot \rho = 144 \cdot 1.2041 = 173.4$ kg. Substitution of all these variables yields:

$$\Delta T = \frac{Q}{m \cdot c_v} = \frac{5.4 \cdot 10^6}{173.4 \cdot 717} = 43.4°\mathrm{C}$$

The final temperature of the room is then $15 + 43.4 = 58.4$ °C.

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.2 Shopping centre escalator

Analysis

An escalator in a shopping center is designed to move 30 persons with an average weight of 75 kg each, at a constant speed of 0.8 m/s at a 45 degree slope. The question is to determine the minimum power input needed to drive. The second question is what will happen to the answer when the velocity is doubled. A sketch is provided below

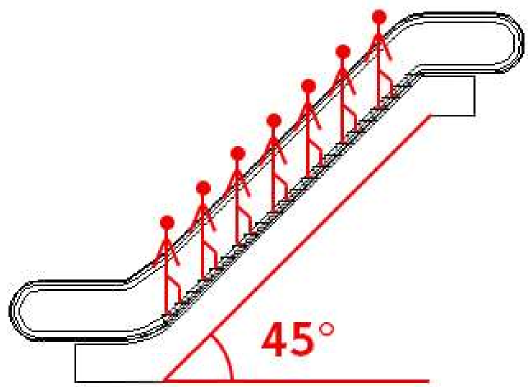


Figure 2.1: Shopping centre escalator

Approach

Assumptions

Since the minimum power input is asked, we assume all electrical energy is transferred to movement and that there is no friction.

Route to solution

The minimum power can be determined using

$$P = \frac{E}{t}$$

Here, the potential energy is the input energy E, where $E = m \cdot g \cdot h$. Substitution gives

$$P = \frac{m \cdot g \cdot h}{t}$$

in which h and t are still unknown. The height can be calculated using the velocity in y-direction, u_y. Looking at the sketch, $u_y = v \cdot \sin 45$. The height can be calculated using

$$h = u_y \cdot t$$

Substituting this gives the final expression for the power input

$$P = m \cdot g \cdot u_y$$

Elaboration

Determining the vertical velocity:

$$u_y = v \cdot \sin 45 = 0.8 \cdot \sin 45 = 0.57$$

Substitution of all known variables and calculated values in the expression for the power input:

$$P = 30 \cdot 75 \cdot 9.81 \cdot 0.57 = 12.6\text{kW}$$

It is clear from the expression of the power input that doubling the velocity will lead to a doubled vertical velocity, and hence a doubled power input.

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.3 Energy balance of a fitness centre

a)

Analysis

There are four treadmills plus 8 weightlifting machines in a fitness centre. The treadmills have a shaft output power of 2.5 hp. They operate on an average load factor f of 0.7 and an efficiency of $\eta = 77$ %. During peak hours, all machines are used and a total of 14 people are present, with an average rate of heat dissipation per person of 300 W. Determine the rate of heat gain of the exercise room.

Approach

Assumptions

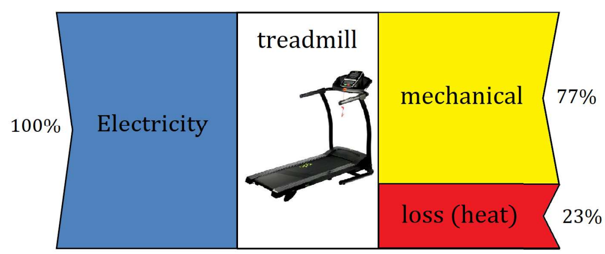
Two different assumptions can be made:

1. All electric energy provided to the treadmills is converted into heat.
2. The power loss of the treadmills is converted into heat.

Furthermore, the room is regarded as a closed system, meaning there is no heat loss at the walls.

Route to solution

Firstly, the electricity used by the treadmills needs to be determined. The shaft output power of 2.5 hp is the (mechanical) power, provided to the treadmill band. This can also be seen in the Sankey diagram below:



The total electricity used by the treadmills is thus dependant on the number of treadmills, the shaft power provided per treadmill, the load factor and the efficiency. Note that a horsepower is defined as 745.7 W. This means that the total power input can be determined with:

$$P_{treadmills,in} = 4 \cdot \frac{f(2.5 \cdot 745.7)}{\eta}$$

During peak hours, fourteen persons are present in total, with an average rate of heat dissipation of 300 W. With this, the total heat output from people can be determined:

$$\dot{Q}_{people} = 14 \cdot 300 = 4.2\text{kW}$$

Now depending on the assumption, the rate of heat gain can be calculated. For the first assumption:

$$\dot{Q}_{total} = \dot{Q}_{treadmills} + \dot{Q}_{people} = P_{treadmills,in} \cdot 1.0 + \dot{Q}_{people}$$

If the second assumption is used:

$$\dot{Q}_{total} = \dot{Q}_{treadmills} + \dot{Q}_{people} = P_{treadmills,in} \cdot 0.23 + \dot{Q}_{people}$$

Elaboration

First, calculating the total power input of the treadmills:

$$P_{treadmills,in} = 4 \cdot \frac{0.7(2.5 \cdot 745.7)}{0.77} = 6.8\text{kW}$$

The total rate of heat gain using the first assumption:

$$\dot{Q}_{total} = P_{treadmills,in} \cdot 1.0 + \dot{Q}_{people} = 6.8 + 4.2 = 11.0\text{kW}$$

The total rate of heat gain using the second assumption:

$$\dot{Q}_{total} = P_{treadmills,in} \cdot 0.23 + \dot{Q}_{people} = 1.6.4.2 = 5.8\text{kW}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

b)

Analysis

Peak hours run from 18:00 – 22.00. Temperature in the room at 18:00 is 20 °C. Estimate the room temperature at 22:00. The dimensions of the room are given (20x20x3 m). See the sketch below.

Approach

Assumptions

Assume no heat loss through walls and windows. Also, assume properties of air at 20 °C: $\rho = 1.2041\ \text{kg}\,\text{m}^{-3}$ and since we have a constant volume, $c_v = 717\text{J}\,\text{kg}^{-1}\,\text{K}$

Route to solution

The temperature increase can be determined using

$$Q = m \cdot c_v \cdot \Delta T \implies \Delta T = \frac{Q}{m \cdot c_v}$$

Q can be calculated by multiplying $\dot{Q}_{total}$ with time t. The mass can be determined by multiplying the density with the volume.

Elaboration

The volume of the fitness room is $20 \cdot 20 \cdot 3 = 1200\text{m}^3$. The mass of the air in the room is then $1200 \cdot 1.2041 = 1445\text{kg}$. For four hours, Q becomes $Q = 11 \cdot 10^3 \cdot 4 \cdot 3600 = 158\text{MJ}$ for the first assumption of a), and $Q = 5.8 \cdot 10^3 \cdot 4 \cdot 3600 = 83.5\text{MJ}$ for the second assumption in a).

Substituting these values in the equation for the temperature increase gives for the first assumption in a)

$$\Delta T = \frac{158 \cdot 10^6}{1445 \cdot 717} = 152.5°\text{C}$$

Hence, the end temperature is

$$T_{end} = T_{begin} + \Delta T = 20 + 152.5 = 172.5°\text{C}$$

and for the second assumption:

$$\Delta T = \frac{83.5 \cdot 10^6}{1445 \cdot 717} = 80.6°\text{C}$$

$$T_{end} = T_{begin} + \Delta T = 20 + 80.6 = 100.6°\text{C}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

c)

Analysis

We have to calculate the required cooling power (using an energy balance) if the room temperature rises by a maximum of 5 degrees Celsius.

Approach

Assumptions

We take all variables at a temperature of 20 degrees. As the room is open, the assumption of constant volume is not valid, therefore we use the assumption of constant pressure.

Route to solution

The energy balance is:

$$\dot{E}_{out} = \dot{E}_{in} - \Delta\dot{E}$$

Here, $\dot{E}_{in}$ depends on the result from question a), $\Delta\dot{E}$ is the allowed rate of heat gain during peak hours and E_{out} is the required cooling power. The following equation can be used to determine the maximum amount of energy that can be added to the system:

$$Q = m \cdot c_p \cdot \Delta T$$

Note that c_v can not be used as the room is now open, so c_p is used. At 20 degrees C, the value of $c_p = 1007\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$. The allowed rate of heat gain can then be calculated with

$$\Delta\dot{E} = \frac{Q}{t}$$

Now, the required cooling power can be calculated by simply filling in the energy balance.

Elaboration

First, the value of Q is calculated:

$$Q = m \cdot c_p \Delta T = 1445 \cdot 1007 \cdot 5 = 7.28\text{MJ}$$

Now, the allowed rate of heat gain can be determined:

$$\Delta\dot{E} = \frac{7.28\text{MJ}}{4 * 3600} = 505W$$

For assumption 1, this leads to

$$\dot{E}_{out} = \dot{E}_{in} - \Delta\dot{E} = 11.0 - 0.505 = 10.5\text{kW}$$

and for assumption 2:

$$\dot{E}_{out} = \dot{E}_{in} - \Delta\dot{E} = 5.8 - 0.505 = 5.3\text{kW}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.4 Microwave

a) A possible definition for the efficiency η of a microwave is the ratio between the amount of thermal energy Q_{in} which is absorbed by the object which is supposed to be heated, and the amount of electrical energy $E_{electrical,in}$ used for that objective:

$$\eta = \frac{Q_{in}}{E_{electrical,in}}$$

In this definition, $Q_{in} = m \cdot c_p \cdot \Delta T$, where m is the mass of the object, c_p the specific heat value at constant pressure and ΔT the rise in temperature of the object. The electrical input $E_{electrical,in}$ can be determined by measuring the electrical power P with a power meter and the time Δt in which the power is delivered $E_{electrical,in} = P\Delta t$. This leads to:

$$\eta_{magnetron} = \frac{m \cdot c_p \cdot \Delta T}{P \cdot \Delta t}$$

Other definitions might be possible as well.

b)

Analysis

A 250 ml cup of milk at 20 °C is placed in a microwave. After two minutes it starts to boil. Determine the amount of energy transferred to the milk.

Approach

Assumptions

Considering the fact that milk consists largely of water, the following assumptions are valid without lowering the accuracy of the answer in a harmful way:

- Milk has the same density as water, $\rho = 1000\ \mathrm{kg\,m^{-3}}$
- Milk has the same specific heat value as water, $c_p = 4.184 \cdot 10^3 \mathrm{J\,kg^{-1}\,K^{-1}}$
- Milk has the same boiling point as water, at 100 °C.

Route to solution

The temperature difference between the boiling point and the intitial temperature can be determined quite easily. Substitution of this in

$$Q_{in} = m \cdot c_p \cdot \Delta T$$

gives the energy transferred to the milk.

Elaboration

The temperature difference is

$$\Delta T = T_1 - T_2 = 100 - 20 = 80°\mathrm{C}$$

Substitution of the temperature difference and the mass and specific heat value gives

$$Q_{in} = m \cdot c_p \cdot \Delta T = 0.250 \cdot 4.184 \cdot 10^3 \cdot 80 = 83.7\mathrm{kJ}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?

- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

c) With Q_{in} determined, only $E_{electrical,in}$ is still undetermined, when the efficiency in a) is used. The latter can be calculated by multiplying the power with the time

$$E_{electrical,in} = P_{electrical,in} \cdot \Delta t = 800 \cdot 2 \cdot 60 = 96.0\text{kJ}$$

Substitution of this and Q_{in} in the equation for the efficiency:

$$\eta = \frac{Q_{in}}{E_{electrical,in}} = \frac{83.7}{96.0} = 0.872 = 87.2\%$$

d) A possible definition for the efficiency of the power plant is the following:

$$\eta_{power\ plant} = \frac{E_{electrical,out}}{E_{coal}}$$

In this formula $E_{electrical,out}$ is the electrical energy provided to the grid by the power plant. The chemical energy bounded in the coal is the E_{coal} and this energy can be released by the combustion of coal. Usually this value is given as the LHV (Lower Heating Value).

e) The overall efficiency of the system can defined as the fraction of the original input energy (E_{coal}) that is consumed as useful energy, in this case (Q_{in}). This is the result of all efficiencies along the way. This leads to the following value:

$$\begin{aligned}\eta_{total} = \frac{Q_{in}}{E_{coal}} &= \frac{E_{electrical,out}}{E_{coal}} \cdot \eta_{electrical\ grid} \cdot \frac{Q_{in}}{E_{electrical,in}} \\ &= 0.50 \cdot 0.93 \cdot 0.872 = 0.405 = 40.5\%\end{aligned}$$

This means that only 40.5% of the original amount is transferred to the milk. Moreover, in this example is is considered that no other losses occur, while in real life the coal needs to be processes and transported. In any case, you need to remember:

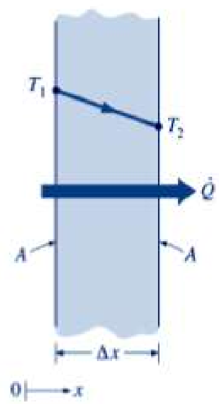
- Efficiencies can be multiplied.
- An efficiency can be expressed as a ratio ($0 \leq \eta \leq 1$) or as a percentage ($0\% \leq \eta \leq 100\%$). Both expressions mean physically the same thing.

f) It expresses the energy dissipation along the way. Only 40.5% of the energy in the coal is being transferred to the to be heated milk. The remaining 59.5% is lost as heat to the environment in the power plant, the grid, and the microwave itself.

2.5 Heat loss from an oven

Analysis

The inside of an oven door is measured at 180 °C, the outside at 50 °C. The oven door is 0.25 m in height and 0.15 m in width, and has a thickness of 5 mm. The glass has a thermal conductivity of 0.70 $\mathrm{W\,m^{-1}\,K^{-1}}$. We need to determine the rate of heat loss through the glass panel of the oven door. A schematic is presented below:



Here, $T_1 = 180°\mathrm{C}$ and $T_2 = 50°\mathrm{C}$

Approach

Assumptions

Route to solution

The governing equation for this problem is

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x}$$

where k is the thermal conductivity, A the area of the glass oven door, ΔT the temperature difference and Δx is the thickness of the oven door. Note that $\Delta T = T_2 - T_1$ gives a negative value.

Elaboration

We start with calculating the area:

$$A = 0.25 \cdot 0.15 = 0.0375\mathrm{m}^2$$

Subsequently the temperature difference

$$\Delta T = T_2 - T_1 = 50 - 180 = -130°\mathrm{C}$$

Substitution in the equation:

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x} = -0.70 \cdot 0.0375 \cdot \frac{-130}{0.005} = 683W$$

Note that the equation for $\dot{Q}$ has a minus sign. This is present here because heat flow is positive at a negative temperature gradient. As the temperature gradient is negative, and $\dot{Q}$ must then be positive, a minus sign is placed. Keep this in mind, it is a very expensive mistake on your exam!

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.6 Heat conduction through the bottom of a pan

Analysis

We are comparing two different pans, a aluminium one with a thickness of 4 mm and a conductivity k_{alu} of 237 $\mathrm{W\,m^{-1}\,K^{-1}}$, and a pan consisting of a 3 mm thick copper layer sandwiched between two 1 mm thick aluminium layers. The thermal conductivity. The thermal conductivity of copper k_{copper} is 390 $\mathrm{W\,m^{-1}\,K^{-1}}$.

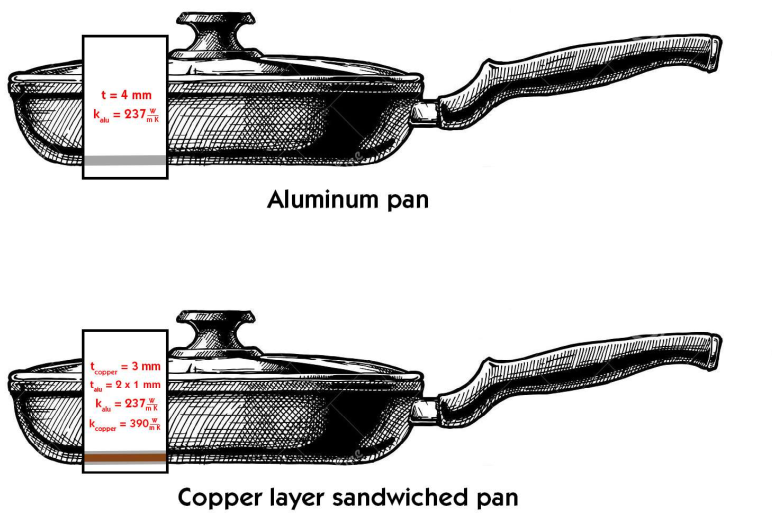


Figure 2.2: Pans with different bottoms

The questions are:

1. Does the pan with the sandwiched copper conduct better than the pan consisting solely of aluminium
2. What is the maximum temperature of the copper layer when the pan bottom is 200 °C at its lower surface and 175 °C at its upper surface.

Approach

Assumptions

Route to solution

Firstly, the heat resistance of the old design (solely aluminium) needs to be determined. This can be done with

$$R = \frac{\Delta x}{k_{alu} A}$$

The new design consists of three conduction resistance values in series for a flat plate design. The heat flow runs into these resistances one after another, resulting the total heat resistance to be the sum of all three separate resistances:

$$R_{tot} = 2 \cdot R_{alu} + R_{copper} = 2 \cdot \frac{\Delta x_{alu}}{k_{alu} A} + \frac{\Delta x_{copper}}{k_{copper} A}$$

Comparing these values will give an answer to the first question.

Now for the second question:

Using logic, the maximum temperature of the copper layer will be at the contact point with the lower aluminum layer. This temperature will be called $T_{a/k}$. The heat flow has just passed the heat resistance of the aluminum layer, as stated in the formula:

$$\dot{Q} = \frac{\Delta T_{alu}}{R_{alu}}$$

In this formula, the $\Delta T_{alu} = T_{lower} - T_{a/k}$, as the temperature difference has already been defined. The same heat flow can also be defined using the total heat resistance instead of the aluminum part. After all, the heat flow has to have the same value while flowing through all the subsequent layers.

$$\dot{Q} = \frac{\Delta T_{tot}}{R_{tot}}$$

In this equation, the total temperature difference ΔT_{tot} is 25 °C (200-175). Using both equations, a substitution of variables can be used to botain the correct answer:

$$\frac{\Delta T_{alu}}{R_{alu}} = \frac{\Delta T_{tot}}{R_{tot}}$$
$$\frac{T_{lower} - T_{a/k}}{\left(\frac{\Delta x_{alu}}{k_{alu}A}\right)} = \frac{T_{lower} - T_{upper}}{\left(2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A}\right)}$$

With some rewriting, the unkown $T_{a/k}$ can be determined:

$$T_{a/k} = T_{lower} - \left[\frac{T_{lower} - T_{upper}}{\left(2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A}\right)}\right] \cdot \left(\frac{\Delta x_{alu}}{k_{alu}A}\right)$$

Elaboration

We start with calculating the conductive resistances of both types of pans. For the solely aluminium pan:

$$R = \frac{\Delta x}{k_{alu}A} = \frac{4 \cdot 10^{-3}}{237 \cdot A} = \frac{1.69 \cdot 10^{-5}}{A} \mathrm{K\,W^{-1}}$$

For the pan with a strip of copper sandwiched between aluminium:

$$R_{tot} = 2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A} = 2 \cdot \frac{1 \cdot 10^{-3}}{237 \cdot A} + \frac{3 \cdot 10^{-3}}{390 \cdot A} = \frac{1.61 \cdot 10^{-5}}{A} \mathrm{K\,W^{-1}}$$

If both values are compared, it is obvious that the new design has a lower heat resistance at the same used area. In other words, it has a better thermal conduction, despite the larger thickness. Notice that the surface area A is not needed as a comparison is made for which the area is not a changing variable. Only if the exact solution of the heat resistance is needed, the surface area might be needed.

For the second question, substituting all values:

$$\begin{aligned} T_{a/k} &= T_{lower} - \left[\frac{T_{lower} - T_{upper}}{\left(2 \cdot \frac{\Delta x_{alu}}{k_{alu}A} + \frac{\Delta x_{copper}}{k_{copper}A}\right)}\right] \cdot \left(\frac{\Delta x_{alu}}{k_{alu}A}\right) \\ &= 200 - \left[\frac{200 - 175}{\left(\frac{1.61 \cdot 10^{-5}}{A}\right)}\right] \cdot \left(\frac{1 \cdot 10^{-3}}{237 \cdot A}\right) = 200 - 6.54 = 193.5°\mathrm{C} \end{aligned}$$

The maximum temperature of the copper value, at the contact point with the lower aluminum layer, will be 193.5 °C. Notice that the surface area A is not relevant for the obtained value.

Evaluation

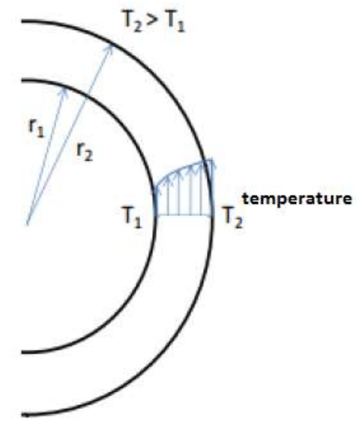
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

2.7 Temperature profiles

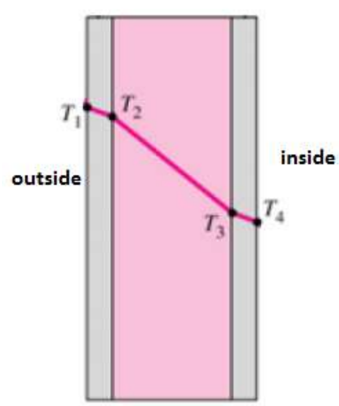
a) The temperature profile through a tube with cold water in a warmer environment is presented below. As the cross-sectional area on the inside is quite small, the resistance value will be higher. By comparison, the area on the outside will be large, leading to a lower resistance value. Therefore, the temperature gradient is steeper at the inside, gradually diminishing to the outside boundary. This also follows from the Fourier law:

$$\dot{Q} = -kA\frac{dT}{dx}$$



Concluding, the temperature at the inner boundary will have a sharp rise and afterward rise more and more slowly towards the outer boundary. The profile will have a convex shape.

b)The temperature profile through a refrigerator door with an insulation layer will be like the following:

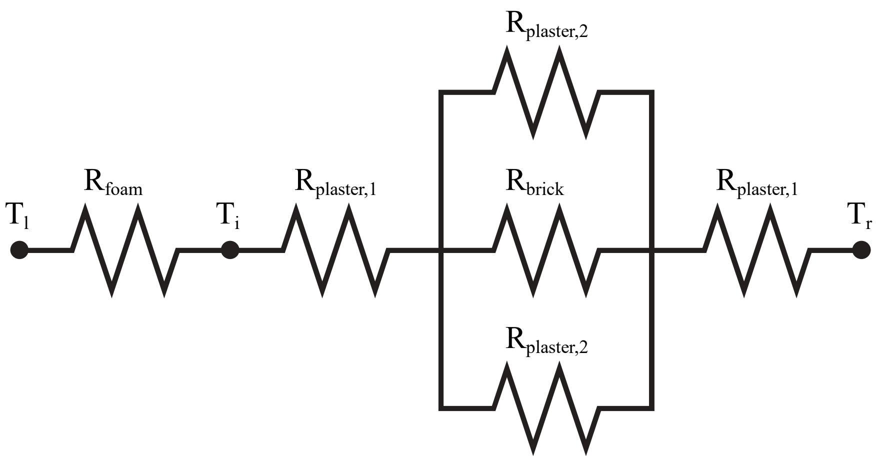


Both thin metal plates will have a lower heat resistance value, therefore leading to a small decrease in temperature. However, the insulating material will have a large heat resistance value (as this is its purpose) and therefore take into account for the largest part of the temperature decline. The temperature gradient needs to be significantly larger in the insulating material section as metals are quite good conductors while the insulating material is not (remember the formula below).

$$R = \frac{\Delta x}{kA}$$

2.8 Multi-layer wall - Hand-in

a)



b) The rate of heat transfer can be determined by use of the correlation between thermal resistance, temperature difference, and rate of heat transfer:

$$\dot{Q} = \frac{\Delta T}{R}$$

The temperature at the left and right-hand sides are known, which yields the temperature difference:

$$\Delta T = T_\mathrm{l} - T_\mathrm{r} = 21 - (-5) = 26\ ^\circ\mathrm{C}$$

The resistances for each layer can be calculated by the following relationship:

$$R_\mathrm{cond} = \frac{\Delta x}{k \cdot A}$$

Which yields:

$$R_\mathrm{foam} = \frac{\Delta x_\mathrm{foam}}{k_\mathrm{foam} \cdot A_\mathrm{foam}} = \frac{0.02}{0.026 \cdot (0.34 \times 6)} = 0.3771\ \mathrm{K/W}$$

$$R_\mathrm{plaster,1} = \frac{\Delta x_\mathrm{plaster,1}}{k_\mathrm{plaster} \cdot A_\mathrm{plaster,1}} = \frac{0.02}{0.22 \cdot (0.34 \times 6)} = 0.0446\ \mathrm{K/W}$$

$$R_\mathrm{plaster,2} = \frac{\Delta x_\mathrm{plaster,2}}{k_\mathrm{plaster} \cdot A_\mathrm{plaster,2}} = \frac{0.18}{0.22 \cdot (0.02 \times 6)} = 6.8182\ \mathrm{K/W}$$

$$R_\mathrm{brick} = \frac{\Delta x_\mathrm{brick}}{k_\mathrm{brick} \cdot A_\mathrm{brick}} = \frac{0.18}{0.72 \cdot (0.3 \times 6)} = 0.1389\ \mathrm{K/W}$$

The three resistances in parallel can be simplified to a single resistance standing in series between the two plaster resistances:

$$\frac{1}{R_\mathrm{parallel}} = \frac{1}{R_\mathrm{plaster,2}} + \frac{1}{R_\mathrm{brick}} + \frac{1}{R_\mathrm{plaster,2}}$$

$$\rightarrow R_\mathrm{parallel} = \left(\frac{1}{6.8182} + \frac{1}{0.1389} + \frac{1}{6.8182}\right)^{-1} = 0.1335\ \mathrm{K/W}$$

The thermal resistance between the left and right side temperatures for the used section can be calculated as:

$$R_\mathrm{section} = R_\mathrm{foam} + R_\mathrm{plaster,1} + R_\mathrm{parallel} + R_\mathrm{plaster,1} = 0.3771 + 0.0446 + 0.1335 + 0.0446 = 0.5997\ \mathrm{K/W}$$

The rate of heat transfer through the section yields:

$$\dot{Q}_\mathrm{section} = \frac{\Delta T}{R_\mathrm{section}} = \frac{26}{0.6000} = 43.36\ \mathrm{W}$$

The thermal resistance of the entire wall can be calculated by scaling by use of the ratio between the area of the taken section and the entire wall:

$$R_{\text{total}} = \frac{A_{\text{section}}}{A_{\text{total}}} R_{\text{section}} = \frac{0.34 \times 6}{4 \times 6} \cdot 0.5997 = 0.0510 \text{ K/W}$$

The total rate of heat transfer through the entire wall yields:

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{26}{0.0510} = 510.10 \text{ W}$$

c) The temperature T_{i} can be found by using the correlation between the rate of heat transfer, temperature difference, and thermal resistance:

$$\dot{Q} = \frac{\Delta T}{R} = \frac{T_{\text{l}} - T_{\text{i}}}{R_{\text{foam}}}$$

Where rewriting yields:

$$T_{\text{i}} = T_{\text{l}} - R_{\text{foam}} \cdot \dot{Q}$$

The rate of heat transfer through each layer remains constant, so:

$$T_{\text{i}} = 25 - 0.3771 \cdot 43.36 = 4.6507 \text{ °C}$$

d) When considering the scenario described by Henk, we have two layers: the foam layer and the additional aluminum layer. The thermal resistance of each layer can be calculated as the ratio of its thickness to its thermal conductivity.

According to Henk, the thermal conductivity of aluminum is higher than that of foam. This implies that the thermal resistance of the aluminum layer is lower than that of the foam layer. In other words, heat can flow more easily through the aluminum layer than through the foam layer.

But, by adding the aluminum layer in front of the foam layer, the path for heat to travel from the hot inside to the cold outside surroundings becomes more difficult as it has to pass one more additional layer and so the total thermal resistance is increased.

A higher total thermal resistance means that heat can be transferred more difficult through the system. Therefore, Henk's claim is incorrect. By adding an additional layer of aluminum in front of the foam layer, the rate of heat transfer would decrease due to the higher overall thermal resistance.