

W4-6-2 Determining pressure and temperature 2

Determine an expression for the pressure of a system for which the fundamental relation is: $U = U(S, V) = \left(\frac{\nu_o \theta}{R^2}\right) \frac{S^3}{NV}$, in which ν_o , θ , N and R are constants.

Tip: remember that you know that:

$$\left(\frac{\partial U}{\partial V}\right)_S = -P \quad (1)$$

$U = \left(\frac{\nu_o \theta}{R^2}\right) \frac{S^3}{NV} = C \frac{S^3}{V}$, where $C = \frac{\nu_o \theta}{NR^2}$ is a constant.

Write $U = U(S, V)$ as the total differential: $dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$.

It is known that:

$$\left(\frac{\partial U}{\partial V}\right)_S = -P \quad (2)$$

This differential can be determined for the function:

$$\left(\frac{\partial U}{\partial V}\right)_S = \frac{\partial}{\partial V} \left(\frac{CS^3}{V}\right) = \frac{-CS^3}{V^2} = -P \quad \Rightarrow \quad P = \frac{\nu_o \theta}{R^2} \frac{S^3}{NV^2} \quad (3)$$