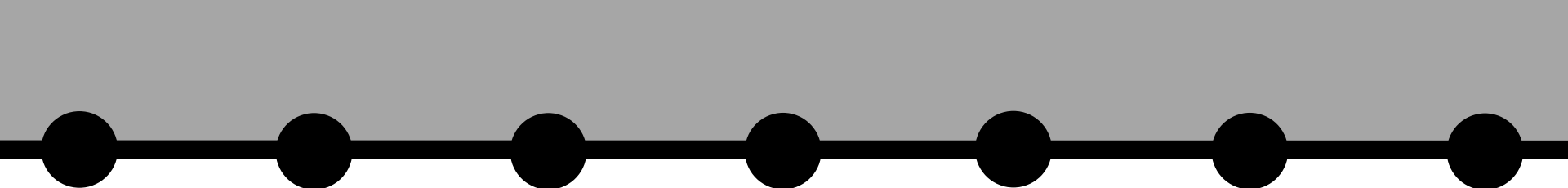


Lecture 8

By: Mohammad Mehrali

- 
- Allowed items in the exam :
 - Reference books
 - Slides and notes
 - Calculator

Please note that only table of air properties will be given to you.

- Q&A Session (To be announced)

GENERAL LEARNING OBJECTIVES



At the end of the E&HT course the student is expected to:

- **Classify definitions and basic concepts in the field of heat and energy**
- **Explain the physical principles involved in the energy and heat management of products**
- **Recognize the forms of energy (conversion) and heat transfer in a product**
- **Be able to make energy and heat management calculations/estimations for products using the provided equations**
- **Critically judge and compare data and results.**

WORK, ENERGY, POWER

- Work W , energy E , power P
- Units J, $W = \text{J/s}$, kWh, hp, ...
- Work W , Energy E in
- Power in $W = \text{J} / \text{s}$
- Units kWh, kcal, hp,
- Comparison / estimating / proportion

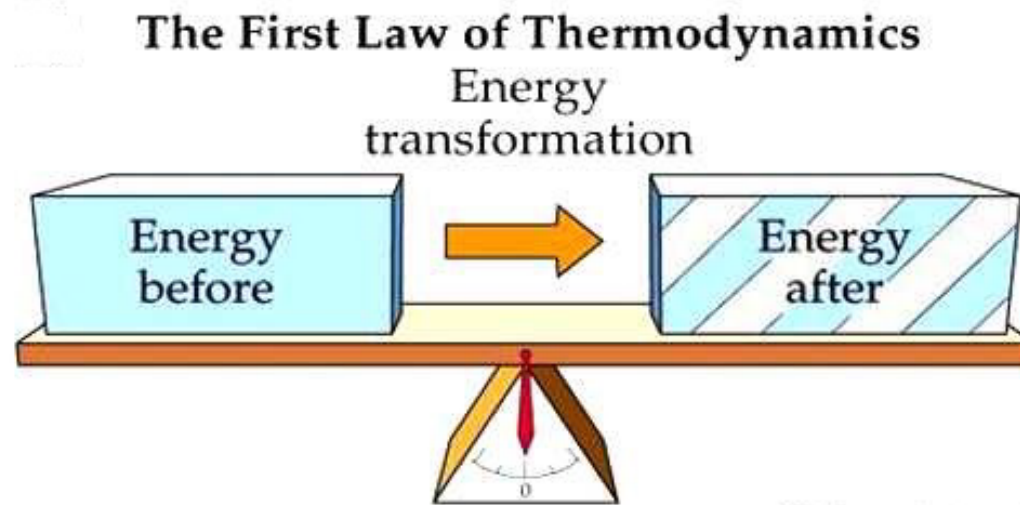
WORK, ENERGY, POWER

Symbol	Definition	Units
W	Work	kJ
w	Specific work = work per unit mass, $w = W/m$	kJ/kg
	Power = rate of work*	$\text{kW} (= \text{kJ/s})$
Q	Heat transfer	kJ
q	Specific heat transfer = heat transfer per unit mass, $q = Q/m$	kJ/kg
\dot{Q}	Rate of heat transfer*	$\text{kW} (= \text{kJ/s})$

- NOTE: Rates are denoted by a dot on top of the variable.

ENERGY BALANCE

- Energy is always conserved!
- First law of Thermodynamics:



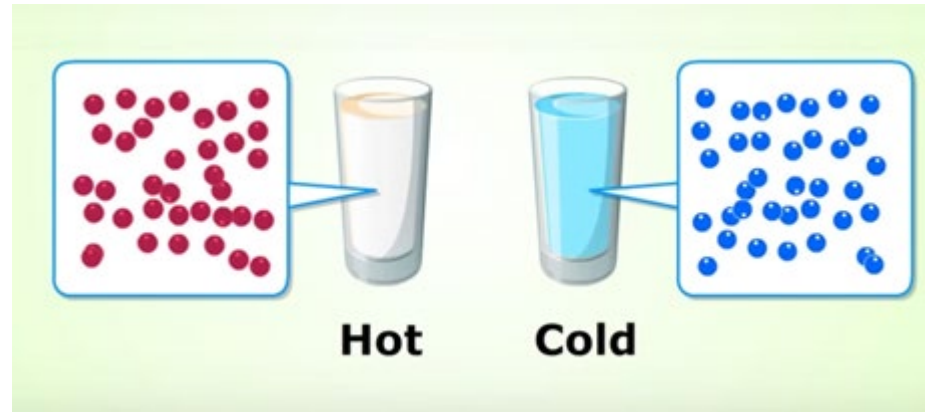
EFFICIENCY

- Efficiency = fraction of “useful” work/power used
 - What is defined as useful?
 - What is the reference?
- Efficiency $\eta = \frac{\text{useful work}}{\text{input energy}} = \frac{\text{useful power}}{\text{input power}} \quad (-)$
- Use a Sankey diagram!

THERMAL ENERGY

- Thermal energy: kinetic energy of molecules and atoms.

Heat Transfer: $\Delta E = Q$ [J]



m = mass of “system” (kg)

ΔT = temperature change of system during process (K)

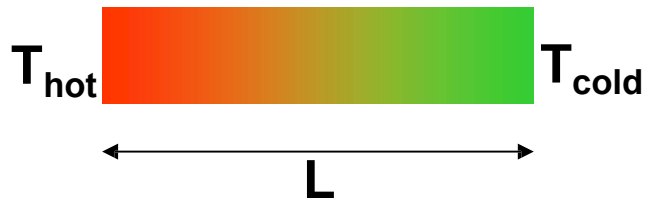
c = specific heat (J / (kg · K))

$$Q = m \cdot c \cdot \Delta T \text{ [J]}$$

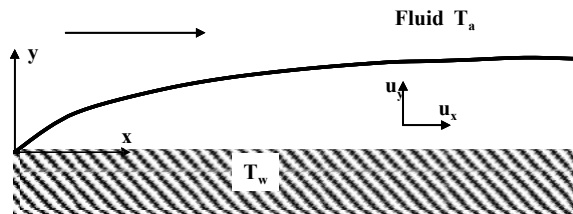
Recap of last lectures

Heat Transfer Modes

Conduction

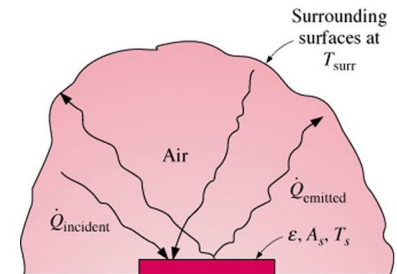


Convection

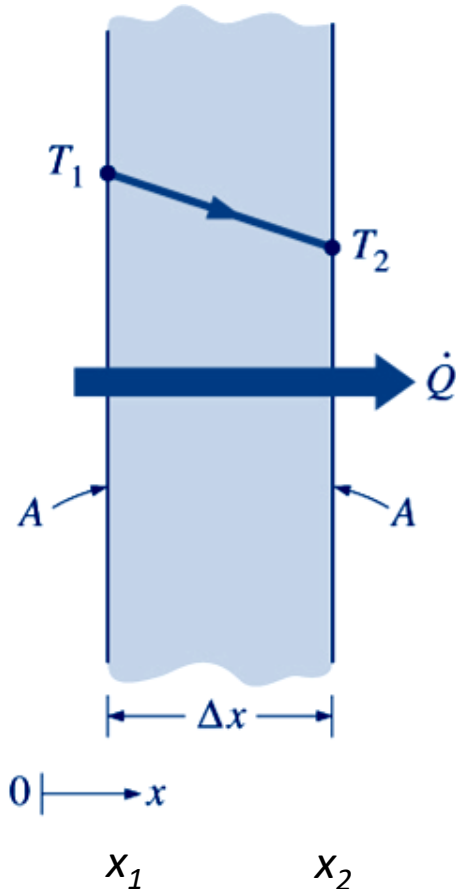


- Natural Convection
- Forced Convection

Radiation



CONDUCTION(FOURIER'S LAW)



Cross section of part of a wall (almost flat)

Heat transfer rate : \dot{Q}

- **Steady state condition-1D**
- **Proportional** to area A
- **Proportional** to temp. Difference : $T_2 - T_1$
- **Inversely proportional** to thickness: $\Delta x = x_2 - x_1$
- **Dependent on material** \rightarrow thermal conductivity: k

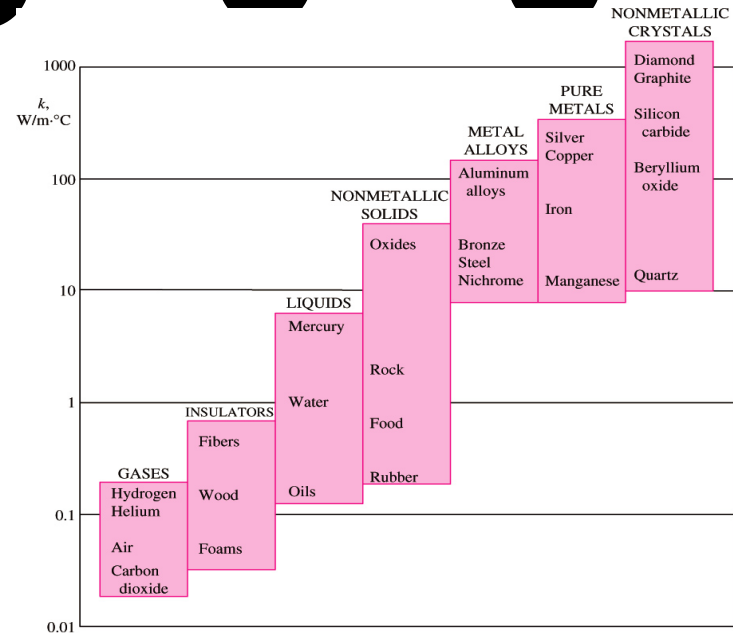
$$\dot{Q} = -k A \frac{T_2 - T_1}{x_2 - x_1} = -k A \frac{\Delta T}{\Delta x} \quad (\text{W})$$

Negative sign since **heat flow** is **positive** at a negative temperature gradient.

Thermal conductivity (k)

Unit: $\frac{\text{W}}{\text{m} \cdot \text{K}}$

⇒ The amount of power conducted through 1 m of material at a 1 K temperature difference



Heat transfer rate: $\dot{Q} = -k A \frac{\Delta T}{\Delta x}$ (W)

Heat flux: $\dot{q} = -k \frac{\Delta T}{\Delta x}$ (W / m²)

ANALOGY ELECTRICITY - HEAT

Heat conduction through plane wall

Fourier's law:

$$\begin{aligned}\dot{Q} &= -k A \frac{T_2 - T_1}{\Delta x} \\ &= +k A \frac{T_1 - T_2}{\Delta x} \\ &= \frac{T_1 - T_2}{\frac{\Delta x}{k A}}\end{aligned}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{cond}} \text{ with } R_{cond} = \frac{\Delta x}{kA} \text{ (K/W)}$$

Heat transfer in “Ohmic way”!

Electrical resistance

Ohm's law:

$$I = \frac{V_1 - V_2}{R}$$

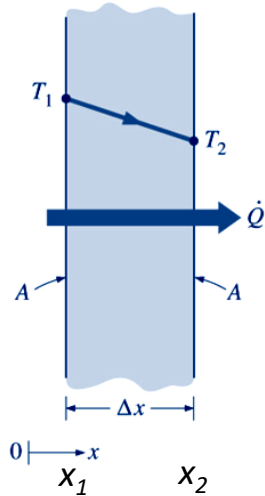
$$V_1 - V_2 \quad \leftrightarrow \quad T_1 - T_2$$

$$I \quad \leftrightarrow \quad \dot{Q}$$

$$R \quad \leftrightarrow \quad R_{cond}$$

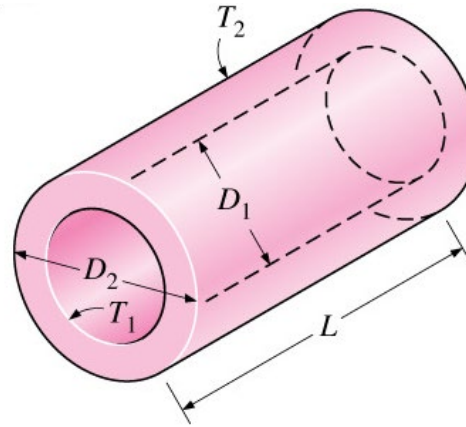
$$\Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{cond}}$$

VARIOUS CONDUCTION RESISTANCES



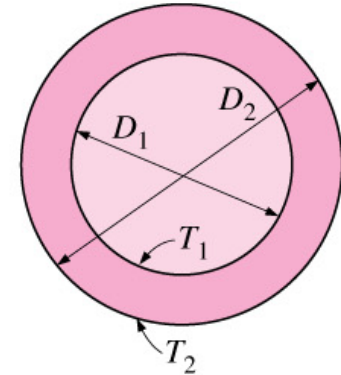
Plane wall

$$R = \frac{\Delta x}{kA}$$



Cilindrical pipe

$$R = \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi L k}$$



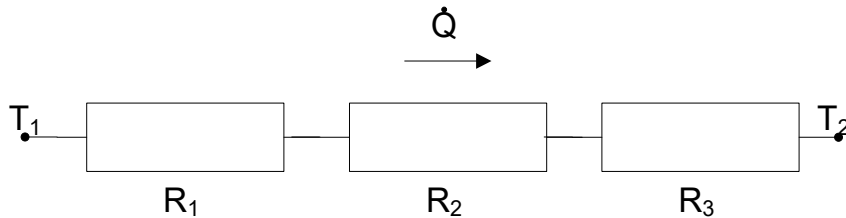
Spherical shell

$$R = \frac{D_2 - D_1}{2\pi k D_1 D_2}$$

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

THERMAL RESISTANCE NETWORKS

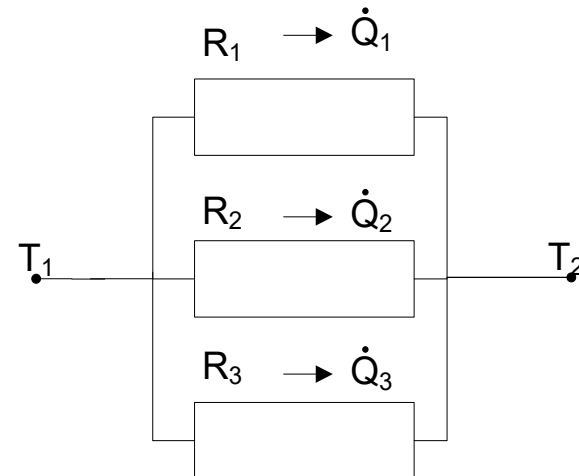
Series Resistors



$$R_{tot} = \sum_i R_i$$

(Add Resistors)

Parallel Resistors

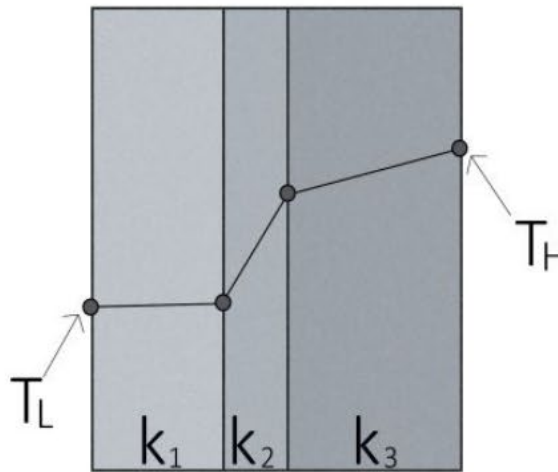


$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

(Add Heat Flows)

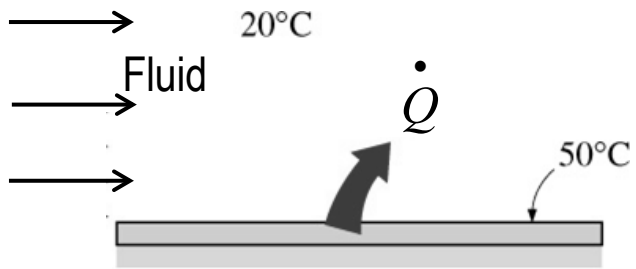
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Select the correct statements with respect to the thermal ...



- | | |
|---|---|
| <div>1</div> <div>$k_1 < k_2 < k_3$</div> <div>0%</div> <div>0</div> <div></div> | <div>4</div> <div>$k_1 < k_3 < k_2$</div> <div>41%</div> <div>9</div> <div></div> |
| <div>2</div> <div>$k_2 < k_1 < k_3$</div> <div>5%</div> <div>1</div> <div></div> | <div>5</div> <div>$k_2 < k_3 < k_1$</div> <div>55%</div> <div>12</div> <div></div> |
| <div>3</div> <div>$k_3 < k_1 < k_2$</div> <div>0%</div> <div>0</div> <div></div> | <div>6</div> <div>$k_3 < k_2 < k_1$</div> <div>0%</div> <div>0</div> <div></div> |

HEAT TRANSFER RATE IN CONVECTION



Newton's Law:

$$\dot{Q} = h \cdot A \cdot \Delta T (W)$$

In the case of Convection

\dot{Q}

Depends on :

- 1) Temperature Difference
- 2) convection heat transfer coefficient
- 3) Surface area of the object

h is the “convection heat transfer coefficient” which basically takes care of various effects of fluid properties and flow properties

Unit: $\frac{W}{m^2 \cdot K}$

CONVECTION REISTANCE

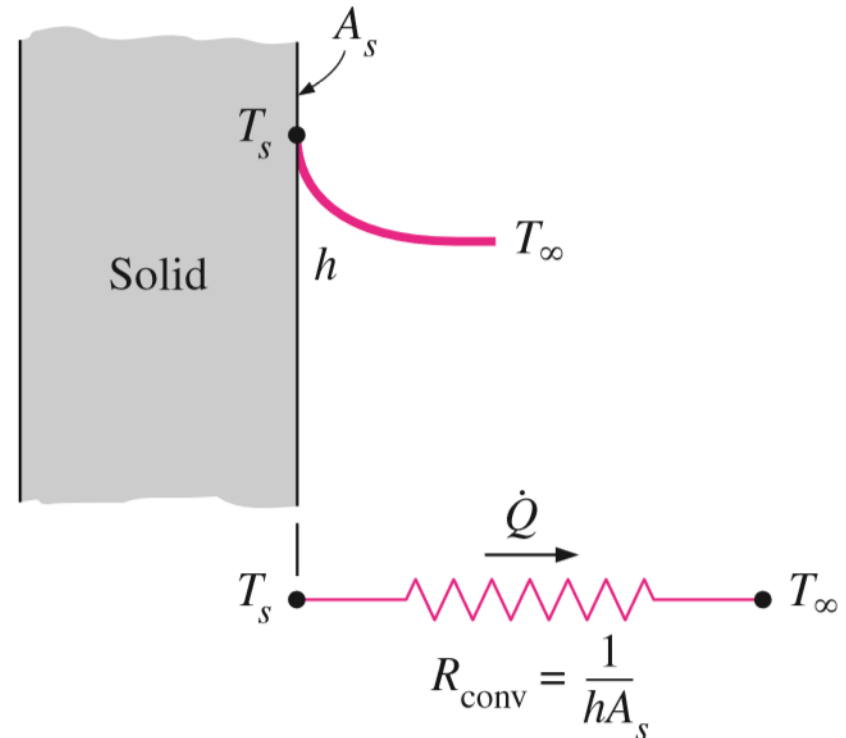
$$\dot{Q} = hA\Delta T = \frac{\Delta T}{\frac{1}{hA}} \text{ with } \Delta T = T_s - T_\infty$$

$$\Rightarrow \dot{Q} = \frac{\Delta T}{R_{conv}}$$

Where **convection resistance**:

$$R_{conv} = \frac{1}{hA} \left(\frac{\text{K}}{\text{W}} \right)$$

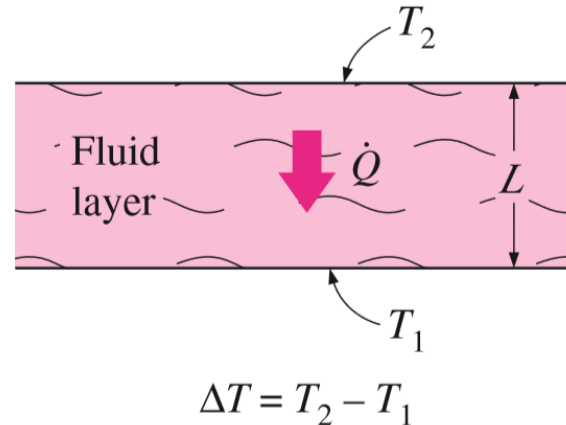
Remember $R_{Cond, plane} = \frac{\Delta x}{kA} \left(\frac{\text{K}}{\text{W}} \right)$



NUSSELT NUMBER

$$\dot{q}_{\text{conv}} = h\Delta T$$

$$\dot{q}_{\text{cond}} = k \frac{\Delta T}{L}$$



Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

- The larger the Nusselt number, the more effective the convection.
- A Nusselt number of **Nu=1** for a fluid layer represents heat transfer across the layer by pure conduction.

CONCLUSION FORCED CONVECTION

General (also natural convection):

$$\dot{Q} = h A \Delta T \quad (\text{W}) \quad \text{Newton's cooling law} \quad \dot{q} = h \Delta T \quad (\text{W/m}^2)$$

“Supporting” equations for h (*Forced Convection*):

$$\text{Nu} = a \cdot \text{Re}^b \text{Pr}^c \quad a, b, c \text{ dependent on geometry and flow regime (laminar / turbulent)}$$

Nusselt Number $\text{Nu}_L = \frac{hL}{k}; \text{Nu}_D = \frac{hD}{k} \quad (-)$

Reynolds Number $\text{Re}_L = \frac{\rho U L}{\mu}; \text{Re}_D = \frac{\rho U D}{\mu} \quad (-)$

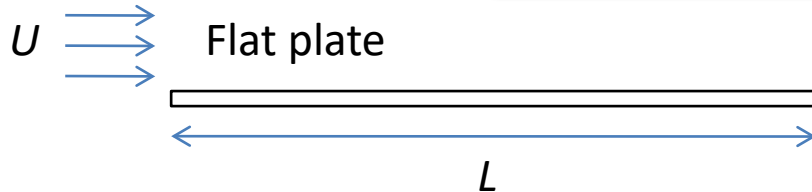
Prandtl Number $\text{Pr} = \frac{\mu c_p}{k}$

Dimensionless numbers
make similar shaped
situations comparable;
“universal” parameters

CORRELATIONS FOR h – FORCED CONVECTION

External flow

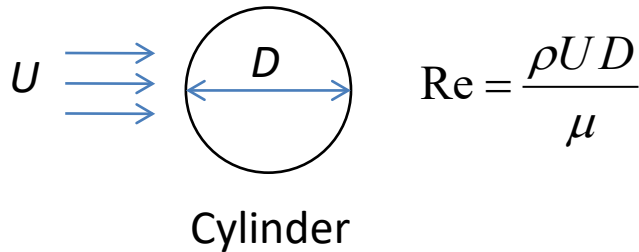
$$Nu = a \cdot Re^b Pr^c$$



$$a = 0,664; b = 0,5; c = 1/3 \quad (Re < 5 \cdot 10^5)$$

$$a = 0,037; b = 0,8; c = 1/3 \quad (Re > 5 \cdot 10^5)$$

$$Re = \frac{\rho U L}{\mu}$$

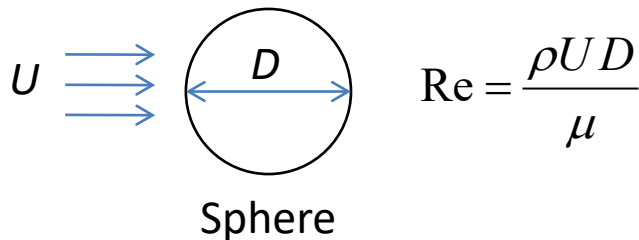


$$Re = \frac{\rho U D}{\mu}$$

$$a = 0,193; b = 0,618; c = 1/3 \quad (4000 < Re < 40.000)$$

$$a = 0,027; b = 0,805; c = 1/3 \quad (40.000 < Re < 400.000)$$

$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re}{282,000} \right)^{5/8} \right]^{4/5}$$



$$Re = \frac{\rho U D}{\mu}$$



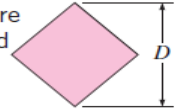
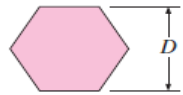
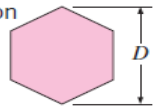
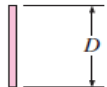
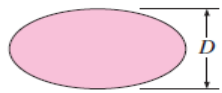
$$Nu \approx 2 + [0,4Re^{1/2} + 0,06Re^{2/3}] Pr^{0,4}$$

(optimal for $Re < 80.000$)

CORRELATIONS FOR h – FORCED CONVECTION

TABLE 7-1

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330} Pr^{1/3}$ $Nu = 0.911Re^{0.385} Pr^{1/3}$ $Nu = 0.683Re^{0.466} Pr^{1/3}$ $Nu = 0.193Re^{0.618} Pr^{1/3}$ $Nu = 0.027Re^{0.805} Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675} Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588} Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638} Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638} Pr^{1/3}$ $Nu = 0.0385Re^{0.782} Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731} Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612} Pr^{1/3}$

STEP-BY-STEP PLAN FORCED CONVECTION

If \dot{Q} must be found:

- Calculate at film temperature : $T_f = \frac{T_s + T_\infty}{2}$
- Pull out ingredients like μ, ρ, k, Pr from tables – like assignment bundle:
air or given fluid) at $T_f = \frac{T_s + T_\infty}{2}$
- Calculate Re and choose appropriate correlation based on geometry and Re
- Calculate Nu
- Derive h from it
- Fill out Newton's cooling law: $\dot{Q} = hA\Delta T$

Natural convection

Forced convection: velocity U imposed

Natural convection: velocity follows from temp.difference $T_s - T_\infty$ *

⇒ Alternative for Reynolds number, with temp.diff. instead U :

Grashof number
$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (-)$$

Greek letter “nu”

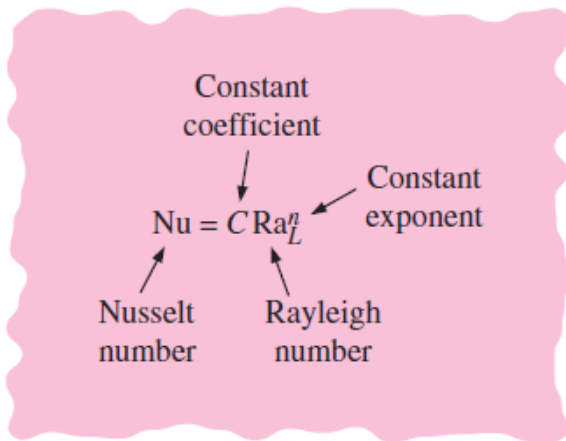
- Gravitational acceleration $g = 9,81 \text{ m/s}^2$
- Volume expansion coefficient $\beta \text{ (K}^{-1}\text{)}$; most gases: $\beta = \frac{2}{T_s + T_\infty}$
(temperature in **Kelvin**; $0^\circ\text{C} = 273,15 \text{ K}$)
- Length L_c characteristic for geometry (length L for plate, diameter D for sphere/cylinder)
- Kinematic viscosity $\nu = \frac{\mu}{\rho} \text{ (m}^2\text{/s)}$
- at average temperature $T_f = \frac{T_s + T_\infty}{2}$

*) Choose ΔT positive for convenience

NUSSELT NUMBER

$$\text{Rayleigh Number : } Ra = Gr \cdot Pr = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} Pr$$

Rayleigh number, which is the product of the Grashof and Prandtl numbers



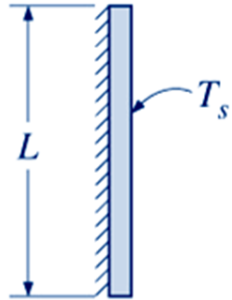
The values of the constants C and n depend on the

- ✓ geometry of the surface
- ✓ the flow regime

All fluid properties are to be evaluated at the film temperature $T_f = (T_s + T_{\text{inv}})/2$.

FREE CONVECTION CORRELATIONS

Vertical, flat plate



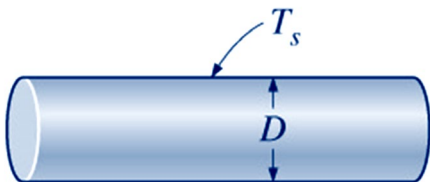
$$Nu = 0,59 Ra_L^{1/4} \text{ with } L_c = L \quad (10^4 < Ra_L < 10^9)$$

$$Nu = 0,1 Ra_L^{1/3} \text{ with } L_c = L \quad (10^{10} < Ra_L < 10^{13})$$

Entire range \rightarrow
$$Nu = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$$

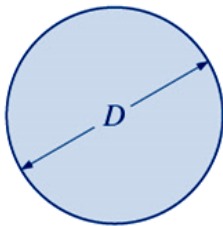
Horizontal cylinder (e.g. pipe)

(complex but more accurate)



$$Nu = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$$

Sphere (e.g. light bulb)



$$Nu = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$$

Notation:

Ra_L is Ra with

$L_c = L$;

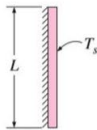
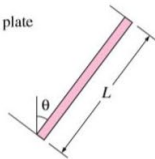


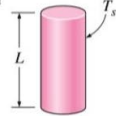
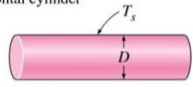
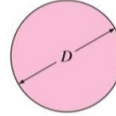
Ra_D is Ra with

$L_c = D$

FREE CONVECTION CORRELATIONS

TABLE 9-1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4 – 10^9 10^9 – 10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{ 0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos \theta$ for $Ra < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	A_s/p	10^4 – 10^7 10^7 – 10^{11}	$Nu = 0.54Ra_L^{1/4}$ (9-22) $Nu = 0.15Ra_L^{1/3}$ (9-23)
(b) Lower surface of a hot plate (or upper surface of a cold plate) 		10^5 – 10^{11}	$Nu = 0.27Ra_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)

Step-by-step plan natural convection

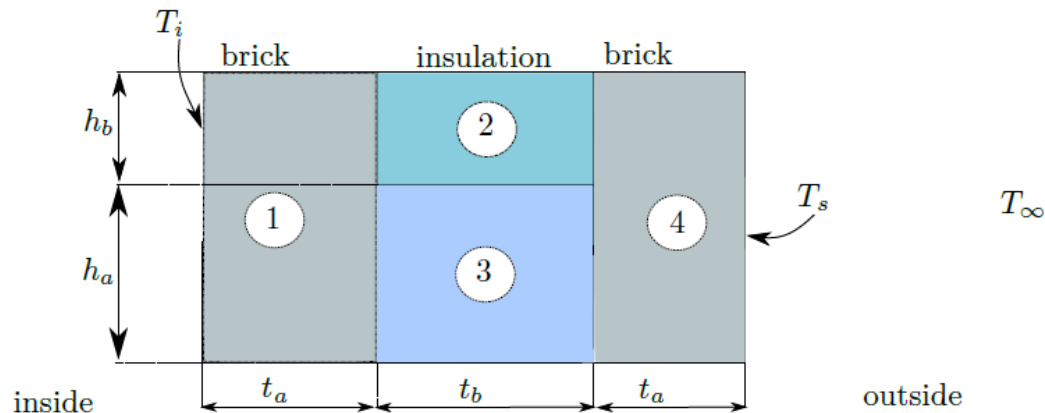
If \dot{Q} must be found:

- Determine ingredients necessary for dimensionless no. (Pr, k and ν at average temperature $\frac{T_s + T_\infty}{2}$)
- Determine Ra: $Ra = Gr \cdot Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$
- Choose appropriate correlation based on geometry and Ra
- Determine Nu
- Resolve h from it
- Fill out Newton's cooling law: $\dot{Q} = hA\Delta T$

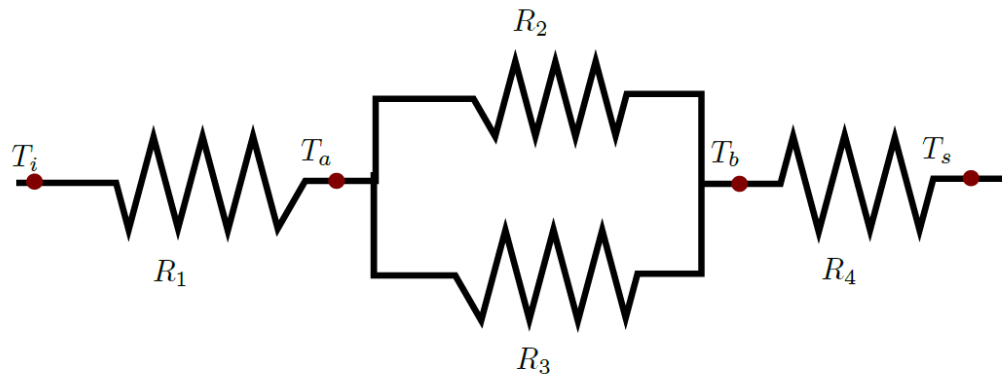
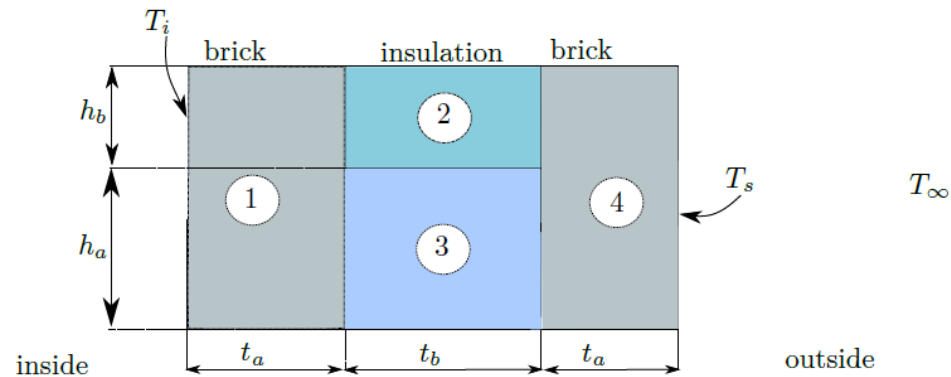
In the figure below a cross section of the wall of a house is given. The inside wall of the house has a temperature $T_i = 20\text{ }^{\circ}\text{C}$. The temperature of the air outside is $T_{\infty} = -5\text{ }^{\circ}\text{C}$. The wall is 2 m wide, and has heights $h_a = 1.50\text{ m}$, $h_b = 1\text{ m}$, and thicknesses $t_a = 30\text{ mm}$ and $t_b = 50\text{ mm}$. The thermal conductivity of the layers of brick is $k_1 = k_4 = 0.72\text{ W m}^{-1}\text{ K}^{-1}$. The thermal conductivity of the rockwool layer at 2 is $k_2 = 0.0350\text{ W m}^{-1}\text{ K}^{-1}$, and that of the wooden layer at 3 is $k_3 = 0.0550\text{ W m}^{-1}\text{ K}^{-1}$. We will analyse this problem on a day with no wind.

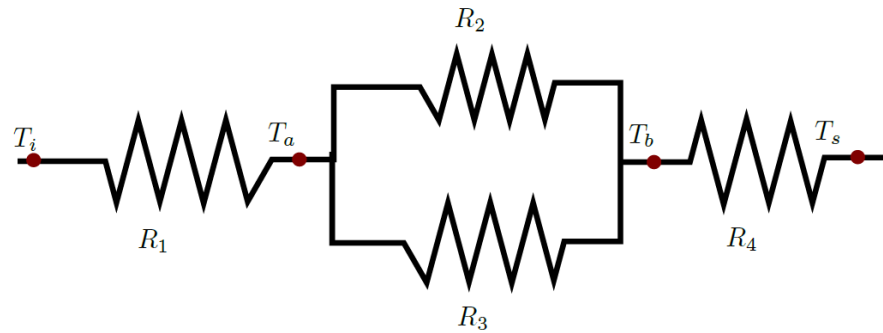
In this exercise, we want to know what the heat loss through the wall is. When writing down your answers, please keep in mind the following:

- Clearly state the assumptions you make.
- Clearly mention the values and their units.
- Clearly mention the relations you use and why you are using them.



- a) Draw the resistance network of the wall and determine the formula for the total resistance R_{tot} inside of the wall.





Correct network - 3 pt.

The equations for the resistances are:

$$R_1 = \frac{t_a}{k_1 A_1} \quad R_2 = \frac{t_b}{k_2 A_2} \quad R_3 = \frac{t_b}{k_3 A_3} \quad R_4 = \frac{t_a}{k_4 A_4} \quad (1.1)$$

Correct equations for the separate resistances- 1 pt.

The resistance of the parallel part can be determined using

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \Rightarrow R_{23} = \frac{R_2 R_3}{R_2 + R_3} \quad (1.2)$$

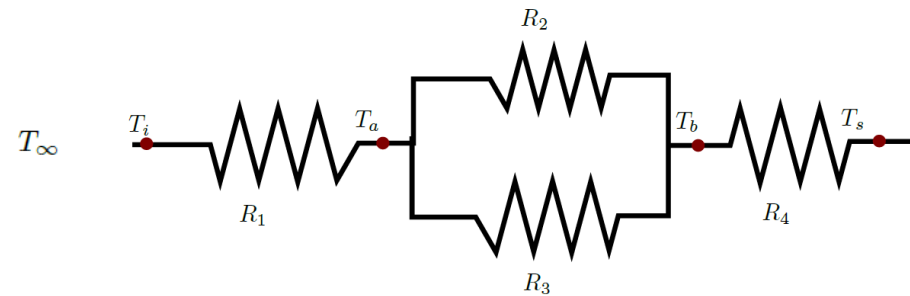
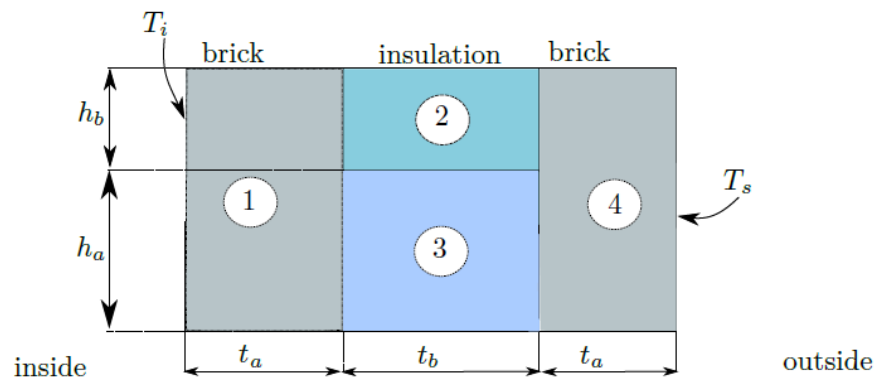
Correct R_{23} - 2 pt.

The total resistance can then be determined using:

$$R_{tot} = R_1 + R_{23} + R_4 \quad (1.3)$$

Correct R_{tot} - 1 pt.

- b) Determine the total heat loss from the wall. If you need to iterate, do so until a precision of Q of 5 percent is achieved. What is the surface temperature of the wall?



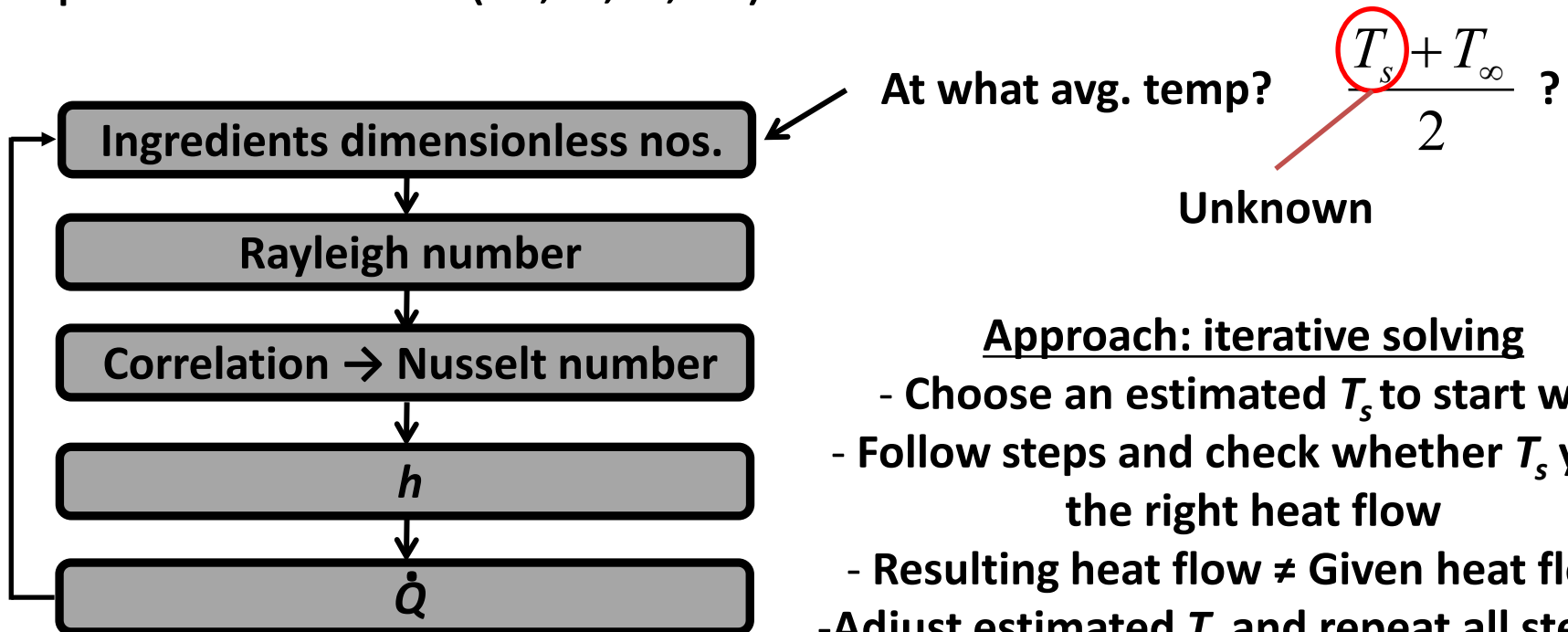
In order to solve this, we need the following energy balance:

$$Q_{conduction} = Q_{naturalconvection} \quad (1.4)$$

Correct energy balance - 2 pt.

\dot{Q} Known - T_s unknown

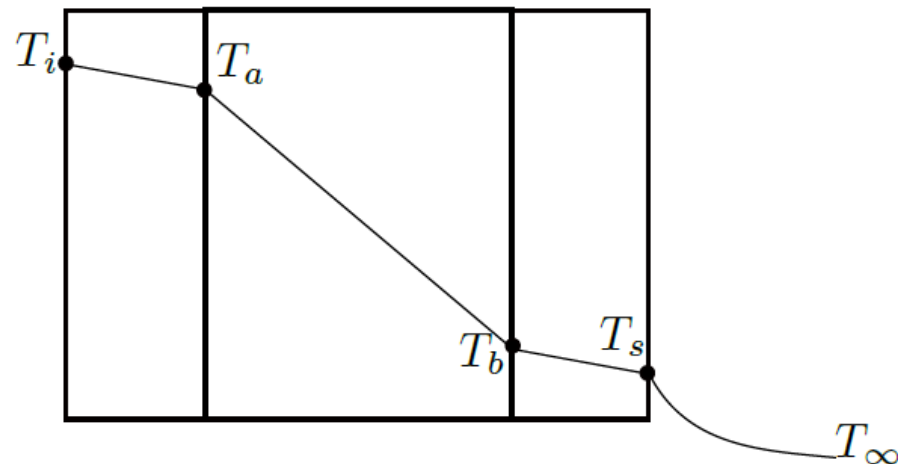
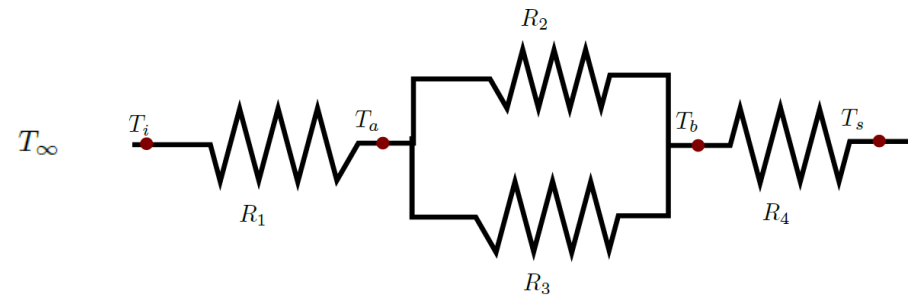
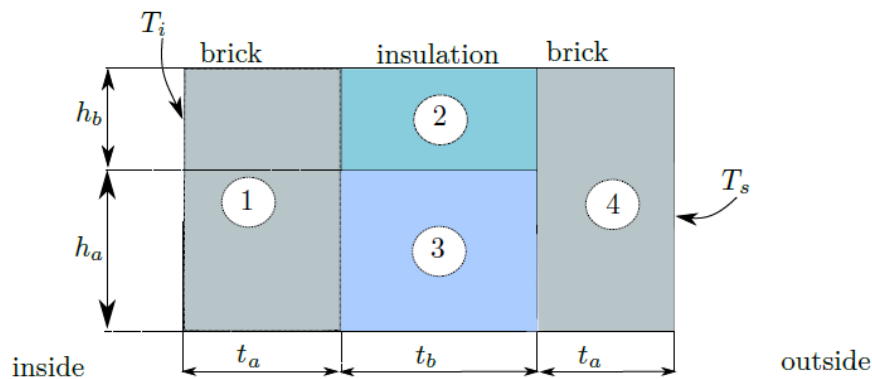
Problem: T_s unknown \rightarrow starting values for step-by-step plan unknown (Pr, k, ν, \dots)



Approach: iterative solving

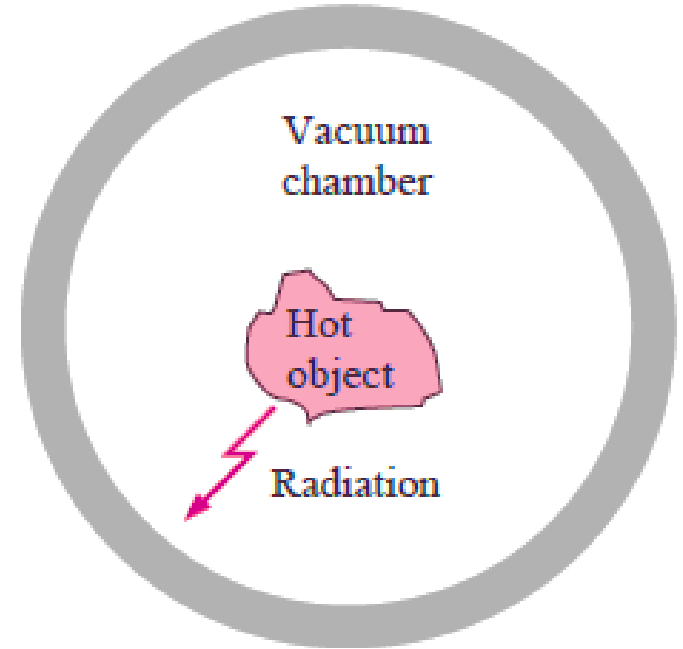
- Choose an estimated T_s to start with
- Follow steps and check whether T_s yields the right heat flow
- Resulting heat flow \neq Given heat flow ?
- Adjust estimated T_s and repeat all steps till you find a T_s that (approximately) yields Given heat flow

c) Determine the temperatures after each layer and draw the temperature profile including the outside.



CONCEPT OF RADIATION

- ♣ Consider a hot object that is suspended in an evacuated chamber whose walls are at room temperature.
- ♣ The Hot object will cool down and reach equilibrium with the chamber walls due to change in **internal energy**.



But How is this heat (internal energy) being transported without any medium ? ?

Electromagnetic Waves or Electromagnetic Radiation
(no medium required!)

Heat transfer in the form of electromagnetic waves due to change in internal energy.

RADIATION LAWS

Stefan-Boltzmann law :

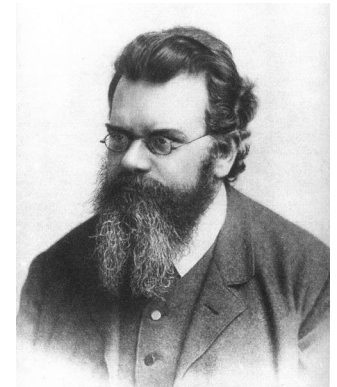
❖ Estimates the Total power emitted by blackbody

$$\dot{Q}_b = \sigma \cdot A \cdot T^4 [W]$$

$$\sigma = 5.670 \times 10^{-8} \frac{W}{m^2 K^4} \quad \text{Stefan-Boltzmann constant}$$



Josef Stefan



Ludwig Boltzmann

NON - IDEAL RADIATION

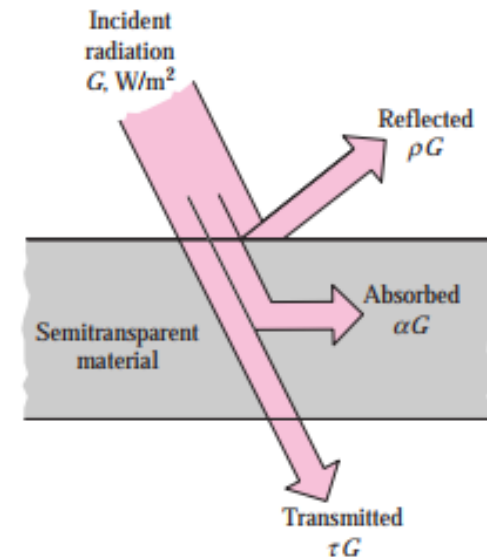
$$\text{Absorptivity } \alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}}$$

$$\text{Reflectivity } \rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}}$$

$$\text{Transmissivity } \tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}}$$

How are these properties related ?

$$\alpha + \tau + \rho = 1$$



NON – IDEAL RADIATION

EMISSIVITY

- In reality all materials: non ideal emitter
- Less power emitted than blackbody

Adjusted Stefan-Boltzmann Law for emitted radiation from object:

$$\dot{Q}_b = \sigma \cdot A \cdot T^4 \text{ (W)} \text{ so } \dot{Q} = \varepsilon \cdot \sigma \cdot A \cdot T^4 \text{ (W)}$$

Emissivity ε ($0 \leq \varepsilon \leq 1$) of a surface (material property):

Measure for emitted radiation (\dot{Q}) in comparison to radiation (\dot{Q}_b) emitted by a blackbody ($\varepsilon = 1$) at same temperature.

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_b}$$

Radiation Heat Transfer

To calculate the heat transfer rate by radiation, we must include terms for power output and energy received from the surroundings.

$$\varepsilon\sigma AT_s^4$$

$$\sigma\alpha AT_\infty^4$$

Making the usual assumption that $\varepsilon = \alpha$, and multiplying by area yields:

$$\dot{Q} = \varepsilon\sigma A(T_s^4 - T_\infty^4)$$

This is the expression for an object totally enclosed by surroundings at T_∞ .

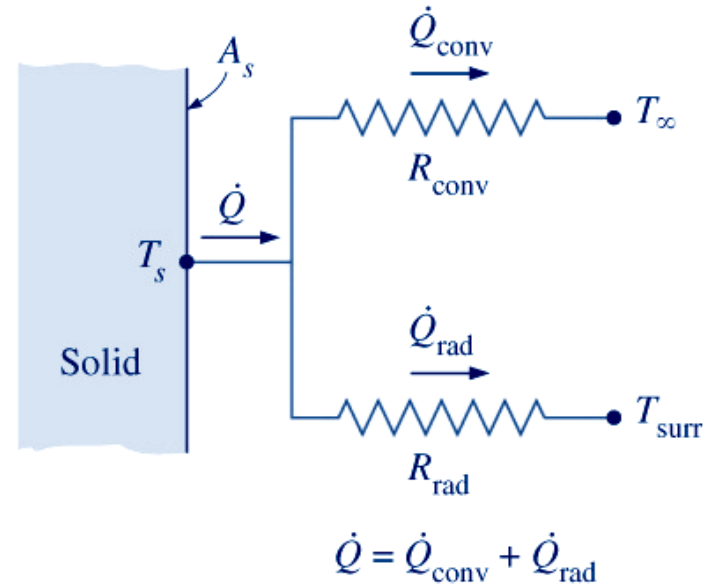
Radiation and convection

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

$$= h_{conv} A \Delta T + h_{rad} A \Delta T$$

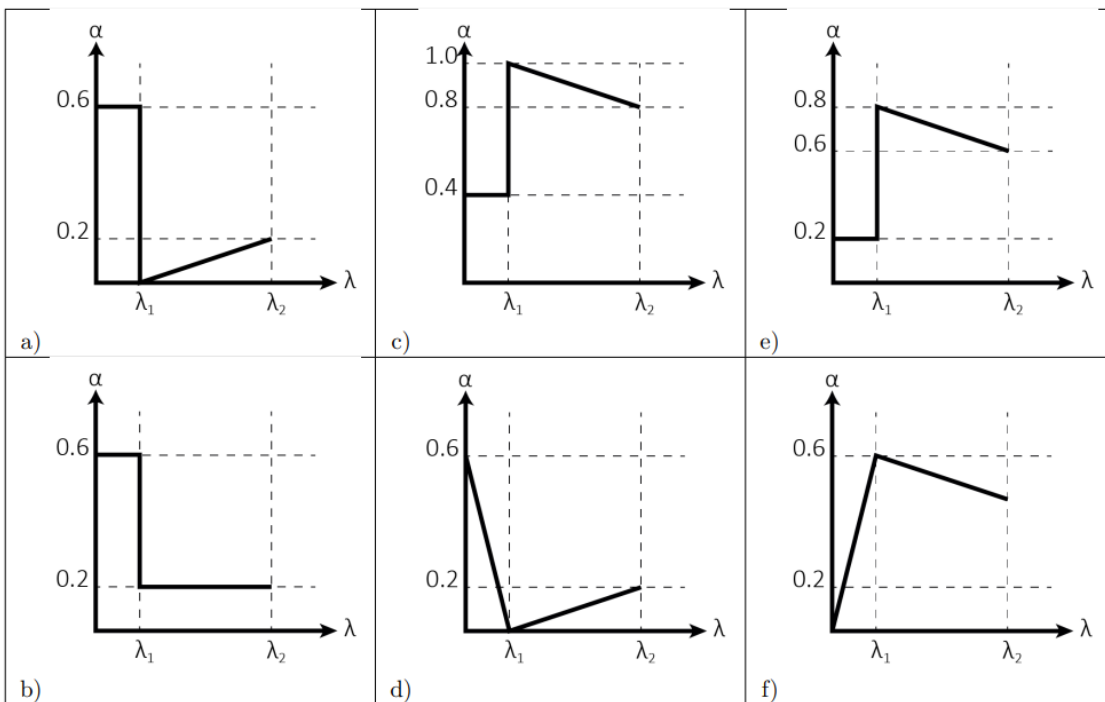
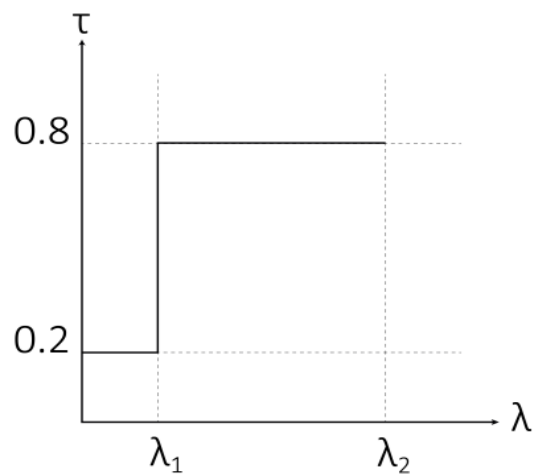
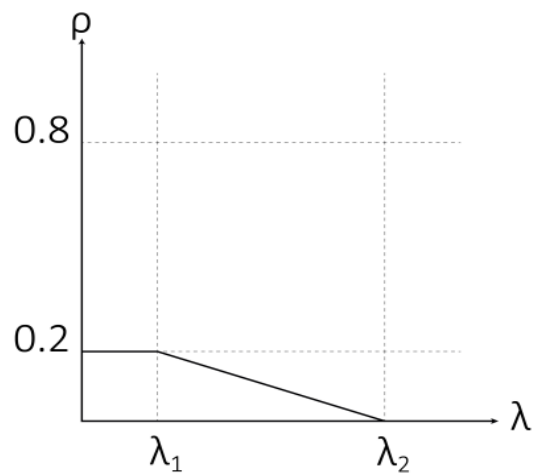
$$= (h_{conv} + h_{rad}) A \Delta T$$

$$\Rightarrow \dot{Q} = h_{tot} A \Delta T \quad \text{with} \quad h_{tot} = h_{conv} + h_{rad}$$

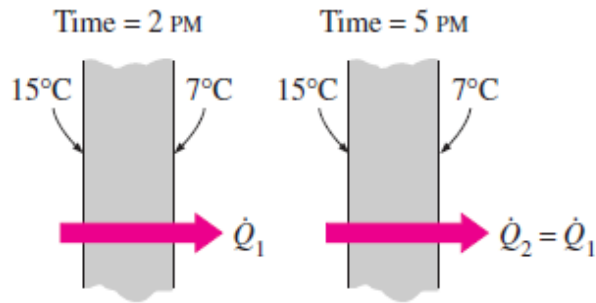


$$h_{rad} = \varepsilon \cdot \sigma \cdot (T_s^2 + T_\infty^2) \cdot (T_s + T_\infty)$$

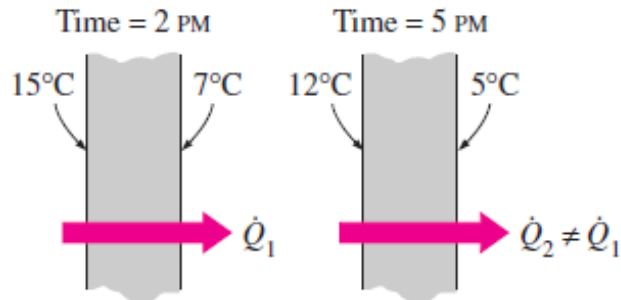
$$\text{with } R_{rad} = \frac{1}{h_{rad} A} \quad \Rightarrow \quad \frac{1}{R_{tot}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$



UNSTEADY HEAT TRANSFER CONCEPT



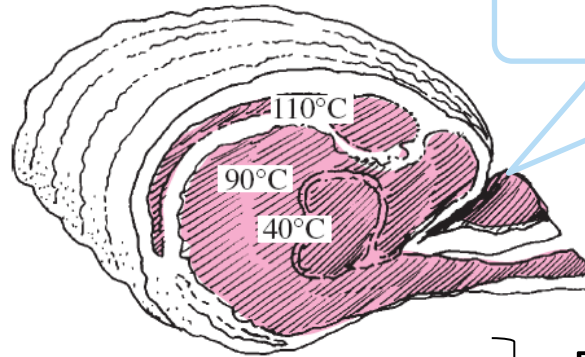
Steady State



Unsteady State

- In **steady state heat transfer**, the **temperature** at any particular point in the system remains **constant** after equilibrium is attained.
- The **amount of heat entering** any section is then **equal** to the **amount of heat exiting** the section, because the driving force (temperature difference) is constant.
- In **unsteady state**, the **temperature** within an object itself keeps **changing with time**.
- The **heat entering a section** thus might **not be the same** as the **heat exiting the section**, as the temperature difference across the section keeps changing with time.

UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS



Temperature is a function of time and space

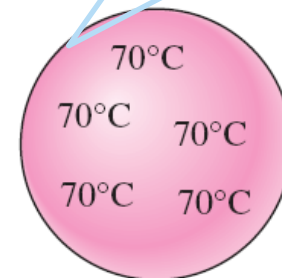
Convection: heat from outer layer

Conduction: heat transferred from outer layer to core

Factors:
 h , k ,
geometry

- **Interior temperature** of some bodies remains essentially **uniform** at **all times** during a heat transfer process.
- The **temperature** of such bodies can be taken to be a function of time only, $T(t)$.
- Heat transfer analysis that utilizes this idealization is known as **lumped system analysis**.

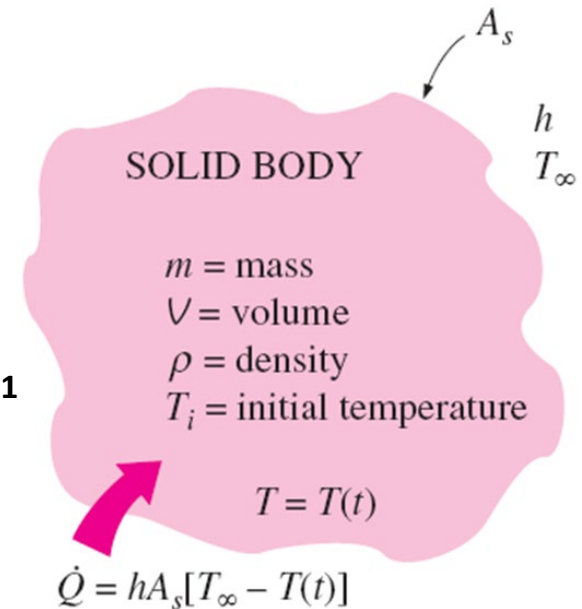
Temperature is only function of Time



UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

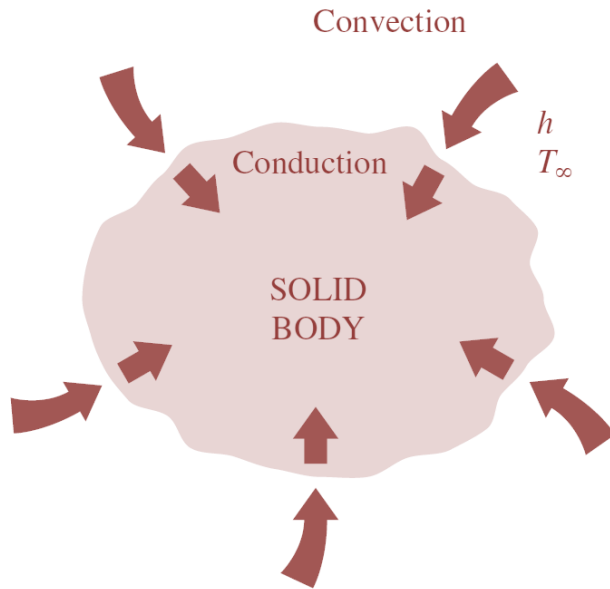
$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} \quad \text{Where } b = \frac{hA_s}{\rho V c_p}$$

- h : heat transfer coefficient around object ($\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$)
- A_s : external surface area (m^2)
- ρ : density of object ($\text{kg} \cdot \text{m}^{-3}$)
- c_p : specific heat of object ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$)
- V : volume of object, (m^3)



Only for lumped system analysis

BIOT NUMBER (Bi)



$$Bi = \frac{\text{heat convection}}{\text{heat conduction}}$$

$$Bi = \frac{h}{k/L_c} \frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

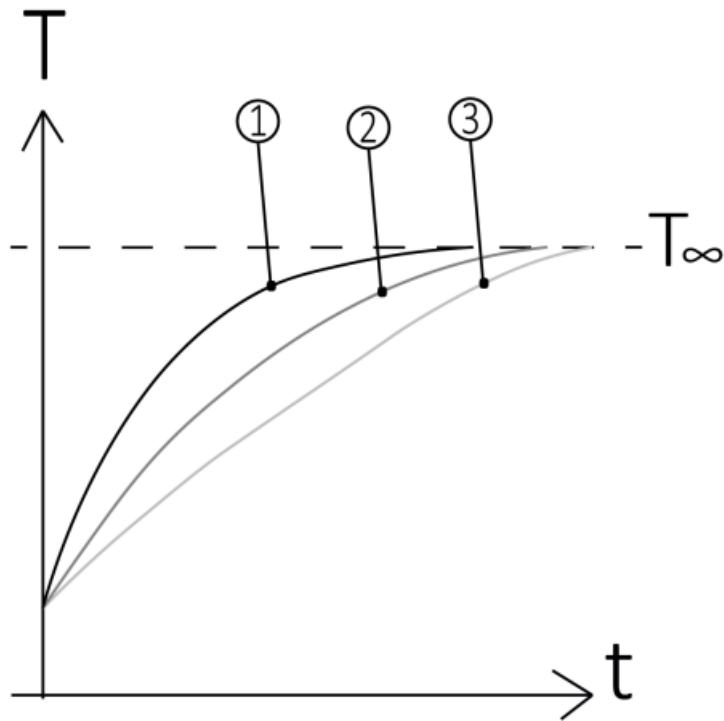
or

$$Bi = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$$Bi = \frac{hL_c}{k} \quad \text{When characteristic length is : } L_c = \frac{V}{A_s}$$

almost uniform temperature for $Bi \leq 0,1$
“lumped system”

Three bodies with an identical shapes (but not the same size) are heating up. They are made of the same material and subjected to identical conditions. The development of their temperature over the course of time can be seen in Figure.



For which of the sketched temperature profiles is the Biot number the smallest?

- a) Temperature profile 1
- b) Temperature profile 2
- c) Temperature profile 3

Nusselt vs. Biot

Nusselt number

$$Nu = \frac{hL_c}{k}$$



≠



Biot number

$$Bi = \frac{hL_c}{k}$$

Dimensionless measure for convection so increase of heat transfer due to flow

k of fluid!

Dimensionless measure for degree of temperature distribution within body

k of surrounded object!

Definitions seem similar but are substantially different!



**Thank you for your
attention**