

1.15 Heating and quenching of a sphere

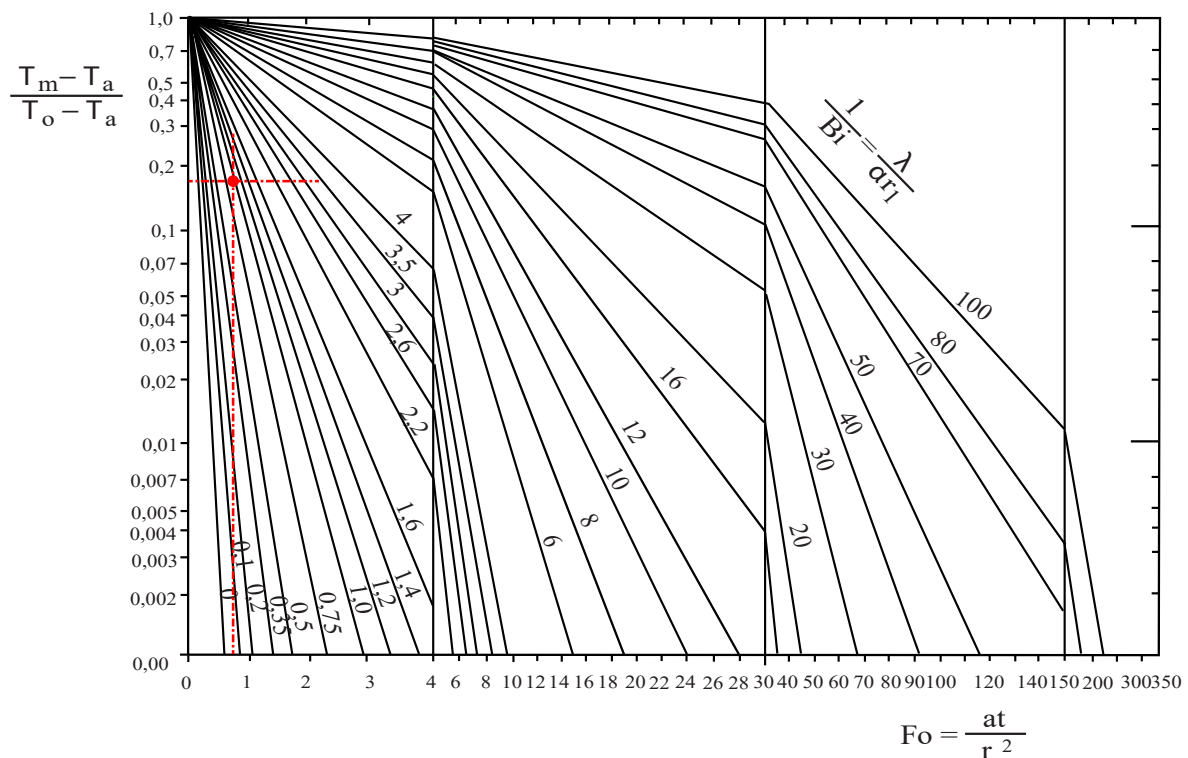
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a) Determine T_m after 3 minutes:

$$\frac{1}{Bi} = \frac{\lambda}{\alpha \cdot r_1} = \frac{1.52 \text{ [Wm}^{-1}\text{K}^{-1}]}{110 \text{ [Wm}^{-2}\text{K}^{-1}] \cdot 0.015 \text{ [m]}} = 0.9212 \quad (1.271)$$

$$Fo = \frac{a \cdot t}{r_1^2} = \frac{9.5 \cdot 10^7 \text{ [m}^2\text{s}^{-1}] \cdot 180 \text{ [s]}}{0.015^2 \text{ [m}^2]} = 0.76 \quad (1.272)$$

Using the Heisler diagram for the temperature in the centre of a sphere:



Results in:

$$\frac{T_m - T_a}{T_0 - T_a} \approx 0.18 \quad (1.273)$$

$$T_m = 0.18 (T_0 - T_a) + T_a = 0.18 (25 \text{ [}^\circ\text{C]} - 200 \text{ [}^\circ\text{C]}) + 200 \text{ [}^\circ\text{C]} \quad (1.274)$$

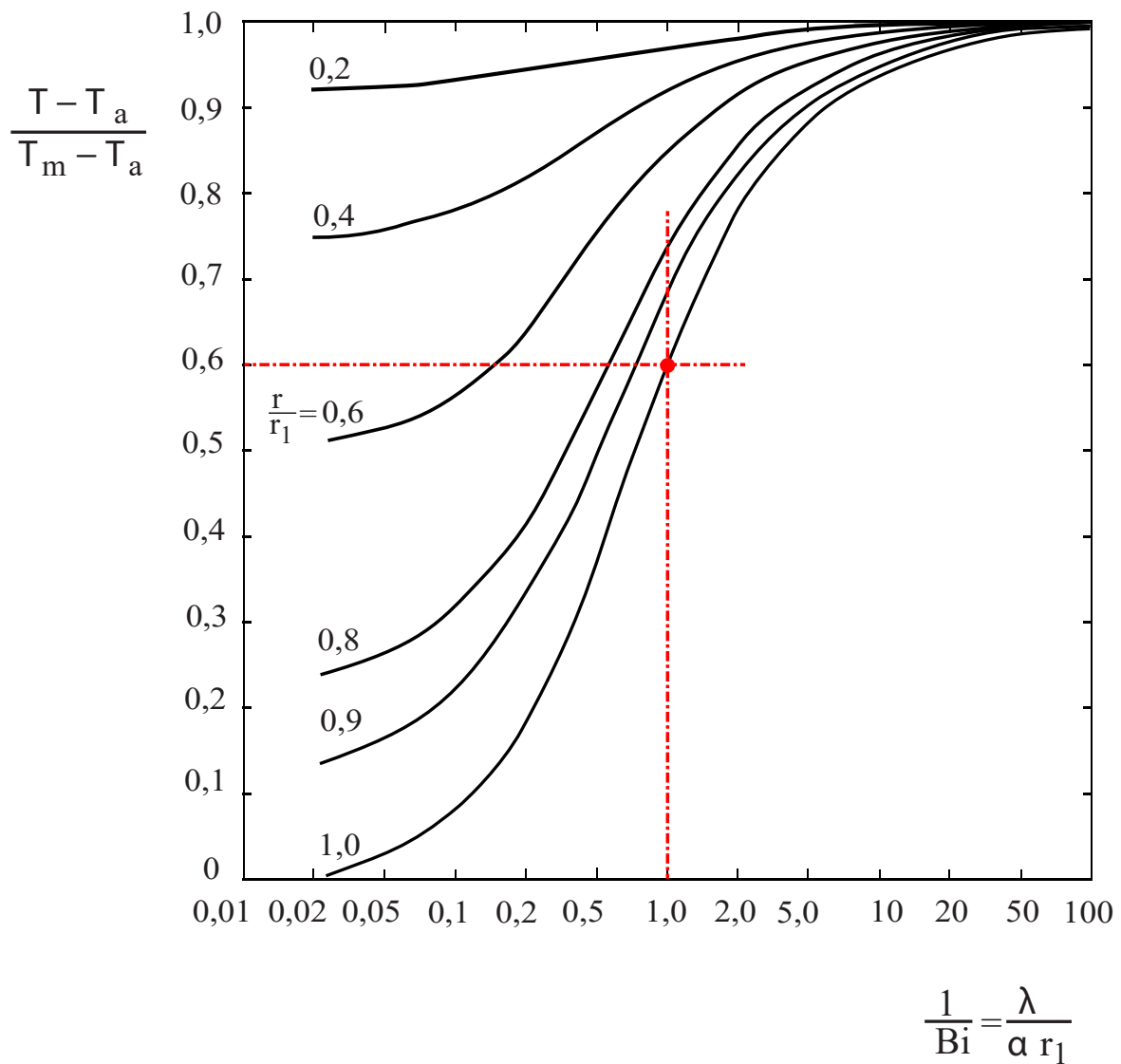
$$\rightarrow T_m = 168.5 \text{ [}^\circ\text{C]} \quad (1.275)$$

b) Determine t_1 :

$$\frac{T - T_a}{T_m - T_a} = \frac{44.4 \text{ [}^\circ\text{C]} - 30 \text{ [}^\circ\text{C]}}{54 \text{ [}^\circ\text{C]} - 30 \text{ [}^\circ\text{C]}} = 0.6 \quad (1.276)$$

$$\frac{r}{r_1} = 1 \quad (1.277)$$

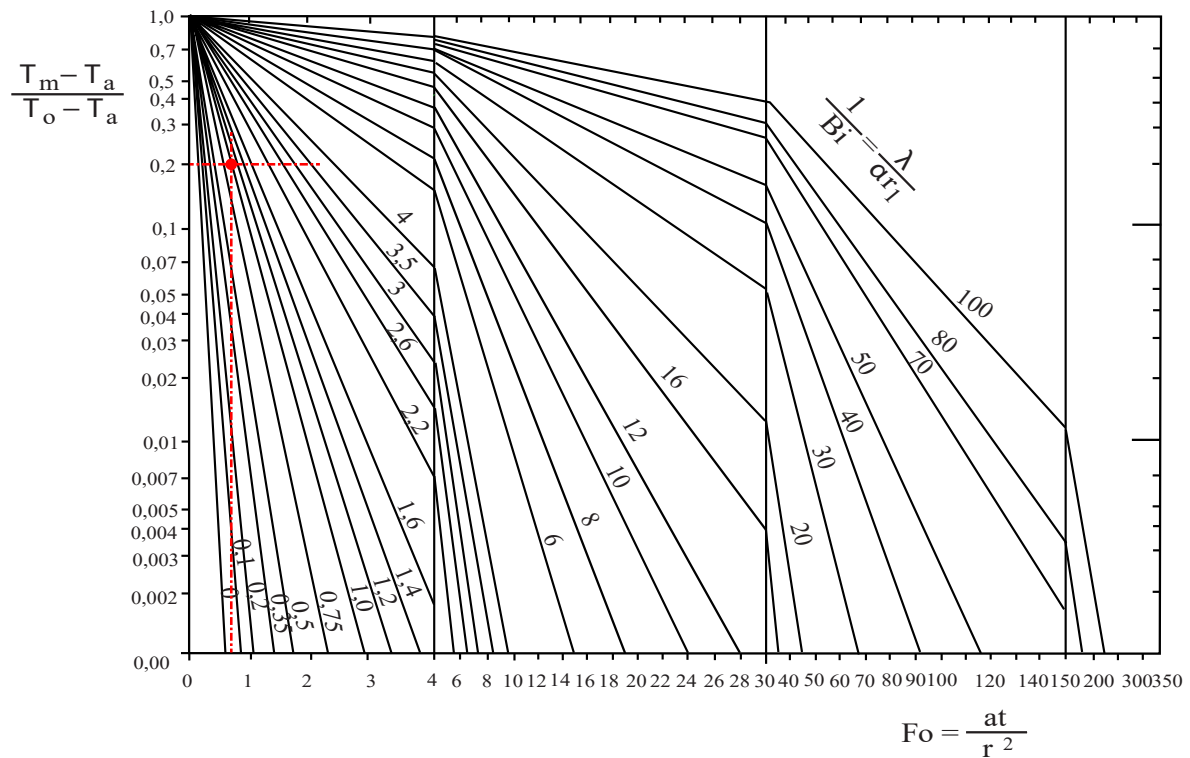
Using the Heisler diagram for the temperature distribution in a sphere to determine $\frac{1}{Bi}$: (Note that this diagram is only valid when $Fo > 0.2$)



$$\rightarrow \frac{1}{Bi} \approx 1 \quad (1.278)$$

$$\frac{T_m - T_a}{T_0 - T_a} = \frac{54 [^\circ\text{C}] - 30 [^\circ\text{C}]}{150 [^\circ\text{C}] - 30 [^\circ\text{C}]} = 0.2 \quad (1.279)$$

Using the Heisler diagram for the temperature in the centre of a sphere to determine Fo :

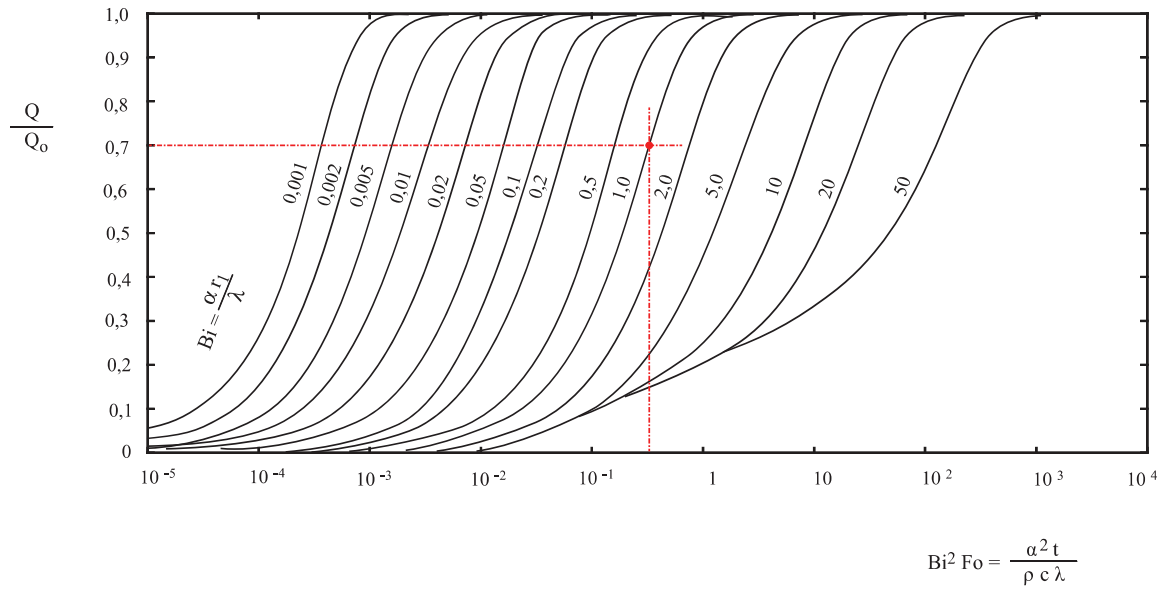


$$\rightarrow Fo \approx 0.7 \rightarrow t_1 \approx 165.79 \text{ [s]} \quad (1.280)$$

c) Determine the dissipated heat Q at time instant t_1 .

$$Bi^2 \cdot Fo = 0.7 \quad (1.281)$$

Using the Heisler diagram for the heat loss of a sphere:



$$\frac{Q}{Q_o} \approx 0.92 \quad (1.282)$$

$$Q \approx 0.92 \cdot \left(\rho \cdot \frac{4}{3} \cdot \pi \cdot r_1^2 \cdot c_p \cdot (T_0 - T_a) \right) = \quad (1.283)$$

$$Q = 0.92 \cdot \left(1.45 \cdot 10^3 \text{ [kg/m}^3\text{]} \cdot \frac{4}{3} \cdot \pi \cdot 0.015^2 \text{ [m}^2\text{]} \cdot 880 \text{ [J/kgK]} \cdot (150 - 30) \text{ [K]} \right) \quad (1.284)$$

$$\rightarrow Q = 132.76 \text{ [kJ]} \quad (1.285)$$