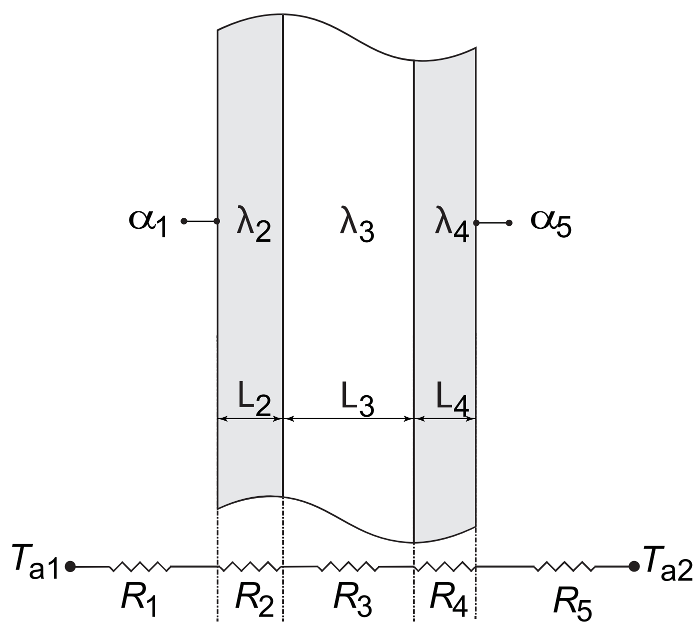
2.3 Window insulation

1. Problem type:
   Steady-state, one-dimensional heat conduction through a multi-layered wall without heat sources.

2. System:



a) Determining the rate of heat transfer $\dot{Q}$.

Area:

$$A = 1.2\ [\mathrm{m}] \cdot 2\ [\mathrm{m}] = 2.4\ [\mathrm{m}^2] \tag{2.17}$$

Thermal resistances:

$$R_1 = \frac{1}{\alpha_1 \cdot A} = \frac{1}{10\ [\mathrm{W/m^2K}] \cdot 2.4\ [\mathrm{m}^2]} = 0.04167\ [\mathrm{K/W}] \tag{2.18}$$

$$R_2 = R_4 = \frac{L_2}{\lambda_2 \cdot A} = \frac{0.003\ [\mathrm{m}]}{0.78\ [\mathrm{W/mK}] \cdot 2.4\ [\mathrm{m}^2]} = 0.00160\ [\mathrm{K/W}] \tag{2.19}$$

$$R_3 = \frac{L_3}{\lambda_3 \cdot A} = \frac{0.015\ [\mathrm{m}]}{0.026\ [\mathrm{W/mK}] \cdot 2.4\ [\mathrm{m}^2]} = 0.2404\ [\mathrm{K/W}] \tag{2.20}$$

$$R_5 = \frac{1}{\alpha_5 \cdot A} = \frac{1}{25\ [\mathrm{W/m^2K}] \cdot 2.4\ [\mathrm{m^2}]} = 0.01667\ [\mathrm{K/W}] \tag{2.21}$$

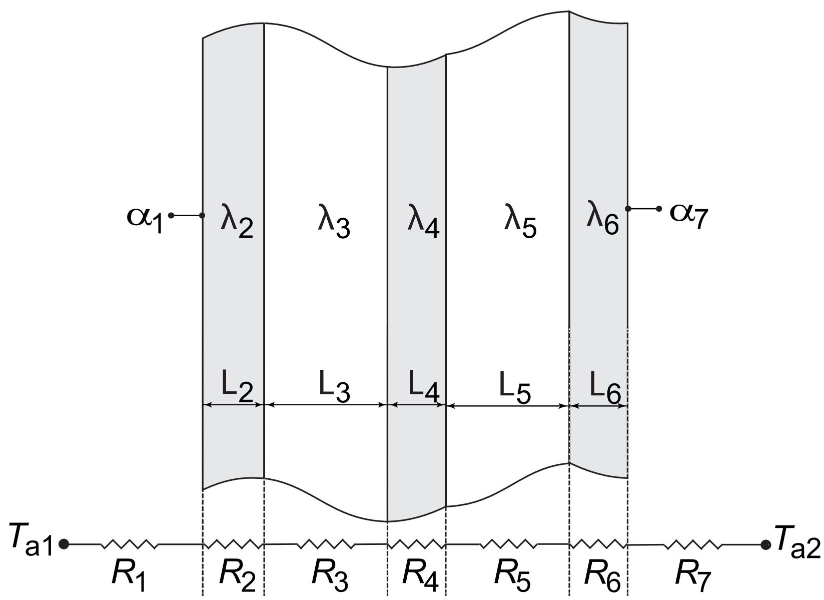
Total resistance:

$$R_{\text{total}} = R_1 + R_2 + R_3 + R_4 + R_5 = 0.30192\ [\mathrm{K/W}] \tag{2.22}$$

Rate of heat transfer:

$$\dot{Q} = \frac{T_{\text{a1}} - T_{\text{a2}}}{R_{\text{total}}} = \frac{22\ [\mathrm{K}] - -7\ [\mathrm{K}]}{0.30192\ [\mathrm{K/W}]} = 96.05\ [\mathrm{W}] \tag{2.23}$$

3. New System:



b) Determining the rate of heat transfer $\dot{Q}$.

Thermal resistances:

$$R_1 = \frac{1}{\alpha_1 \cdot A} = \frac{1}{10\ [\mathrm{W/m^2K}] \cdot 2.4\ [\mathrm{m^2}]} = 0.04167\ [\mathrm{K/W}] \tag{2.24}$$

$$R_2 = R_4 = R_6 = \frac{L_2}{\lambda_2 \cdot A} = \frac{0.003\ [\mathrm{m}]}{0.78\ [\mathrm{W/mK}] \cdot 2.4\ [\mathrm{m^2}]} = 0.00160\ [\mathrm{K/W}] \tag{2.25}$$

$$R_3 = R_5 = \frac{L_3}{\lambda_3 \cdot A} = \frac{0.008\ [\mathrm{m}]}{0.00949\ [\mathrm{W/mK}] \cdot 2.4\ [\mathrm{m^2}]} = 0.3512\ [\mathrm{K/W}] \tag{2.26}$$

$$R_7 = \frac{1}{\alpha_7 \cdot A} = \frac{1}{25\ [\mathrm{W/m^2K}] \cdot 2.4\ [\mathrm{m^2}]} = 0.01667\ [\mathrm{K/W}] \tag{2.27}$$

Total resistance:

$$R_{\mathrm{total}} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 = 0.76563\ [\mathrm{K/W}] \tag{2.28}$$

Rate of heat transfer:

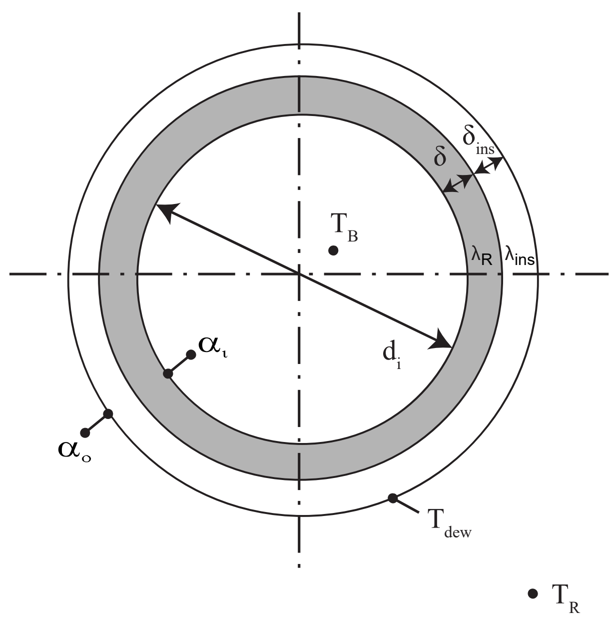
$$\dot{Q} = \frac{T_{\mathrm{a1}} - T_{\mathrm{a2}}}{R_{\mathrm{total}}} = \frac{22\ [\mathrm{K}] - -7\ [\mathrm{K}]}{0.76563\ [\mathrm{K/W}]} = 37.88\ [\mathrm{W}] \tag{2.29}$$

2.4 Brine pipeline

a. Determine the thickness of the insulation δ_{ins} such that no condensate is formed at the surface of the insulation is formed, even when the maximum dew point temperature of the surrounding air of $T_{\text{dew}} = 15\,°\text{C}$.

Problem type:
Steady-state one-dimensional heat transfer through a multi-layer pipe wall.

System:



From the resistance theorem we know that:

$$\dot{Q} = \frac{T_1 - T_{n+1}}{R_{\text{total}}} \tag{2.30}$$

Since heat is transferred from the outside to the inside of the pipeline, the equation can be rewritten (where for this case there is accounted for the fact that heat transfer takes place from outer to inner):

$$\Rightarrow \Delta T = (T_{\text{R}} - T_{\text{B}}) = R_{\text{total}} \cdot \dot{Q} \tag{2.31}$$

We can calculate the total heat flux taking into account the temperature at the outside of the pipe, the room temperature and the convective resistance between the two locations.

$$\dot{Q} = \alpha_{\mathrm{o}} \cdot \pi \left(d_{\mathrm{i}} + 2\delta + 2\delta_{\mathrm{ins}}\right) \cdot L \cdot \left(T_{\mathrm{R}} - T_{\mathrm{dew}}\right) \tag{2.32}$$

We can express the sum of the resistances using the following expression:

$$R_{\mathrm{total}} = R_{\mathrm{conv,i}} + R_{\mathrm{R}} + R_{\mathrm{ins}} + R_{\mathrm{conv,o}} \tag{2.33}$$

Resistance convection on the inside:

$$R_{\mathrm{conv,i}} = \frac{1}{\alpha_{\mathrm{i}} \cdot \pi \cdot d_{\mathrm{i}} \cdot L} \tag{2.34}$$

Resistance steel layer:

$$R_{\mathrm{R}} = \frac{\ln\left[\left(d_i + 2\delta\right)/d_i\right]}{2 \cdot \pi \cdot L \cdot \lambda_{\mathrm{R}}} \tag{2.35}$$

Resistance insulation layer:

$$R_{\mathrm{ins}} = \frac{\ln\left[\left(d_i + 2\delta + 2\delta_{\mathrm{ins}}\right)/\left(d_i + 2\delta\right)\right]}{2 \cdot \pi \cdot L \cdot \lambda_{\mathrm{ins}}} \tag{2.36}$$

Resistance convection on the outside:

$$R_{\mathrm{conv,o}} = \frac{1}{\alpha_{\mathrm{o}} \cdot \pi \cdot \left(d_{\mathrm{i}} + 2\delta + 2\delta_{\mathrm{ins}}\right) \cdot L} \tag{2.37}$$

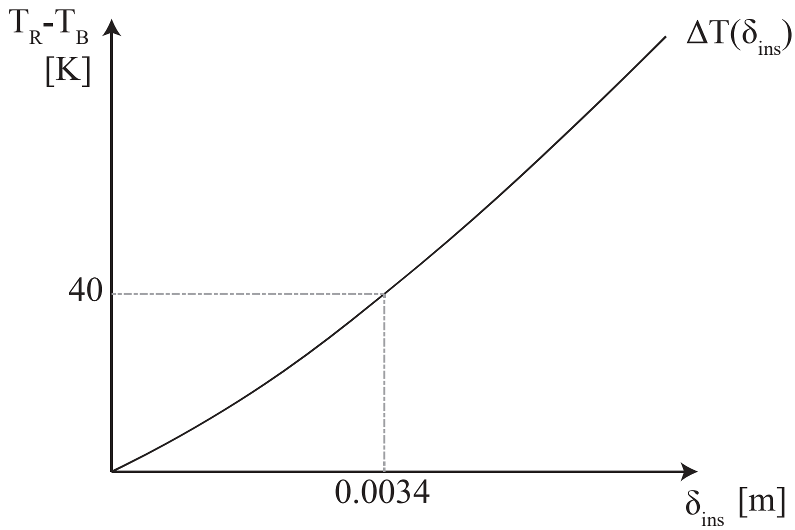
Inserting into the equation of the resistance theorem:

$$\begin{aligned} \Delta T\left(\delta_{\mathrm{ins}}\right) &= \dot{Q}\left(\delta_{\mathrm{ins}}\right) \cdot R_{\mathrm{total}}\left(\delta_{\mathrm{ins}}\right) = \\ &\alpha_{\mathrm{o}} \cdot \pi \left(d_{\mathrm{i}} + 2\delta + 2\delta_{\mathrm{ins}}\right) \cdot \left(T_{\mathrm{R}} - T_{\mathrm{dew}}\right) \cdot \Big(\frac{1}{\alpha_{\mathrm{i}} \cdot \pi \cdot d_{\mathrm{i}}} + \frac{\ln\left[\left(d_i + 2\delta\right)/d_i\right]}{2 \cdot \pi \cdot \lambda_{\mathrm{R}}} + \ldots \\ &\ldots \frac{\ln\left[\left(d_i + 2\delta + 2\delta_{\mathrm{ins}}\right)/\left(d_i + 2\delta\right)\right]}{2 \cdot \pi \cdot \lambda_{\mathrm{ins}}} + \frac{1}{\alpha_{\mathrm{o}} \cdot \pi \cdot \left(d_{\mathrm{i}} + 2\delta + 2\delta_{\mathrm{ins}}\right)}\Big) \end{aligned} \tag{2.38}$$

Finding the point of intersection:

$$\Delta T\left(\delta_{\mathrm{ins}}\right) = T_{\mathrm{R}} - T_{\mathrm{B}} = 40 \tag{2.39}$$

Note that the condition given in Equation 2.39 is rather hard to solve analytically, given Equation 2.38. This because we can not obtain an explicit equation for δ_{ins} Therefore this has to be solved numerically or by an iterative approach.



$$\Rightarrow \delta_{\mathrm{ins}} \approx 34\ \mathrm{mm} \tag{2.40}$$

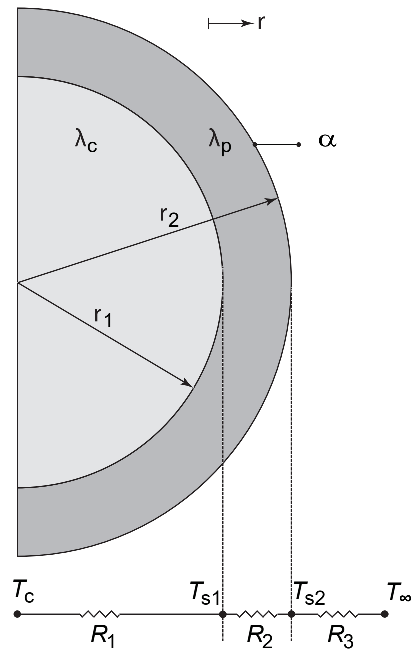
b. Determine the amount of heat $\dot{q}'$ absorbed by the brine per unit pipe length and time under the conditions given above.

$$\begin{aligned} \dot{q}' = \frac{\dot{Q}}{L} &= \frac{\alpha_{\mathrm{o}} \cdot \pi \left(d_{\mathrm{i}} + 2\delta + 2\delta_{\mathrm{ins}}\right) \cdot L \cdot \left(T_{\mathrm{R}} - T_{\mathrm{dew}}\right)}{L} \\ &= 6\ [\mathrm{W/m^2K}] \cdot \pi \left(0.05 + 0.01 + 0.068\right)\ [\mathrm{m}] \cdot (20 - \ 15)\ [\mathrm{K}] = 12\ [\mathrm{W/m}] \end{aligned} \tag{2.41}$$

2.5 Electrical wire

1. Problem type:
   Steady-state, one-dimensional heat conduction through a multi-layered wall with a heat source.

2. System:



a) Determining the temperature at the interface of the wire and the plastic cover T_{s1}.

Rate of heat loss:

$$\dot{Q} = V \cdot I = 8 \text{ [V]} \cdot 13 \text{ [A]} = 104 \text{ [W]} \tag{2.42}$$

Thermal resistances:

$$R_2 = \frac{\ln(r_2/r_1)}{2 \cdot \pi \cdot \lambda_p \cdot L} = \frac{\ln(2.1 \text{ [mm]}/1.1 \text{ [mm]})}{2 \cdot \pi \cdot 0.15 \text{ [W/mK]} \cdot 10 \text{ [m]}} = 0.0686 \text{ [K/W]} \tag{2.43}$$

$$R_3 = \frac{1}{\alpha \cdot 2 \cdot \pi \cdot r_2 \cdot L} = \frac{1}{24 \text{ [W/m}^2\text{K]} \cdot 2 \cdot \pi \cdot 0.0021 \text{ [m]} \cdot 10 \text{ [m]}} = 0.3158 \text{ [K/W]} \tag{2.44}$$

Summation of the resistances:

$$R_{(2+3)} = R_2 + R_3 = 0.3844 \text{ [K/W]} \tag{2.45}$$

And thus:

$$\dot{Q} = \frac{T_{s1} - T_\infty}{R_{(2+3)}} \rightarrow T_{s1} = \dot{Q} \cdot R_{(2+3)} + T_\infty = 104\,[\mathrm{W}] \cdot 0.3844\,[\mathrm{K/W}] + 303.15\,[\mathrm{K}] = 343.15\,[\mathrm{K}] \tag{2.46}$$

<u>Results after doubling the thickness:</u>

Thermal resistances:

$$R_2 = \frac{\ln(r_2/r_1)}{2 \cdot \pi \cdot \lambda_\mathrm{p} \cdot L} = \frac{\ln(3.1\,[\mathrm{mm}]/1.1\,[\mathrm{mm}])}{2 \cdot \pi \cdot 0.15\,[\mathrm{W/mK}] \cdot 10\,[\mathrm{m}]} = 0.1099\,[\mathrm{K/W}] \tag{2.47}$$

$$R_3 = \frac{1}{\alpha \cdot 2 \cdot \pi \cdot r_2 \cdot L} = \frac{1}{24\,[\mathrm{W/m^2K}] \cdot 2 \cdot \pi \cdot 0.0031\,[\mathrm{m}] \cdot 10\,[\mathrm{m}]} = 0.2139\,[\mathrm{K/W}] \tag{2.48}$$

Summation of the resistances:

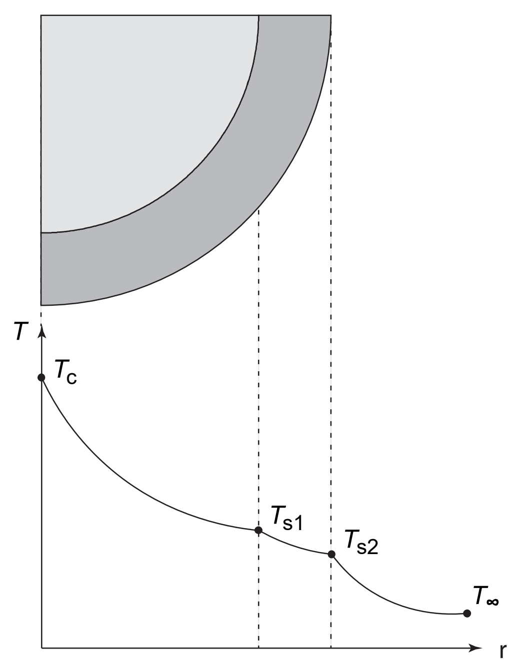
$$R_{(2+3)} = R_2 + R_3 = 0.3239\,[\mathrm{K/W}] \tag{2.49}$$

And thus:

$$\dot{Q} = \frac{T_{s1} - T_\infty}{R_{(2+3)}} \rightarrow T_{s1} = \dot{Q} \cdot R_{(2+3)} + T_\infty = 104\,[\mathrm{W}] \cdot 0.3239\,[\mathrm{K/W}] + 303.15\,[\mathrm{K}] = 336.83\,[\mathrm{K}] \tag{2.50}$$

So doubling the thickness results in a decrease of the interface temperature T_{s1}.

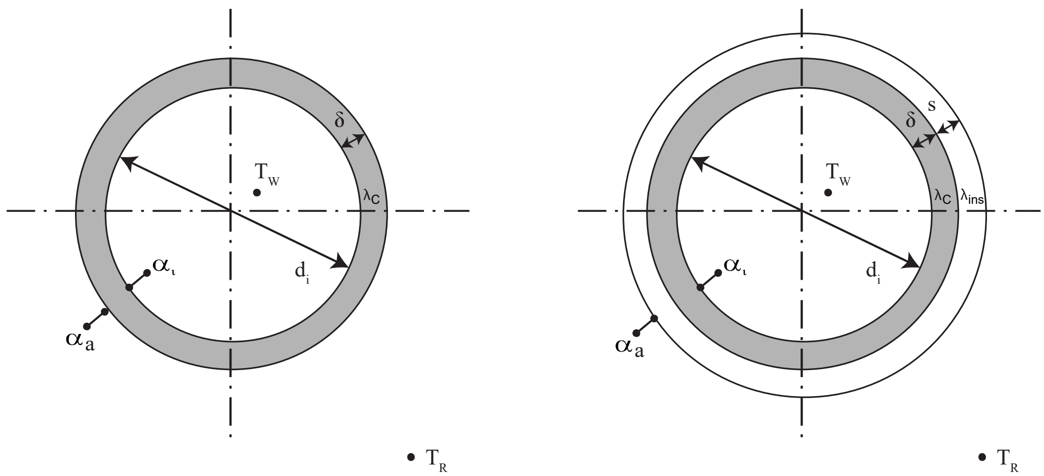
b) Draw the temperature profile.



2.6 Warm-water pipe

Problem type:
Steady-state one-dimensional heat transfer through a single- and multi-layer pipe wall.

System without and with insulation:



a. Determine the heat transfered per unit pipe length for $\dot{q}'$.

Resistance theorem:

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} \tag{2.51}$$

Where the temperature difference is:

$$\Delta T = T_{\text{W}} - T_{\text{R}} \tag{2.52}$$

Calculations for the system without insulation:

The total thermal resistance:

$$R_{\text{total,wo,ins}} = R_{\text{conv,i}} + R_{\text{C}} + R_{\text{conv,a,wo}} \tag{2.53}$$

Thermal resistance convection on the inside:

$$R_{\text{conv,i}} = \frac{1}{\alpha_{\text{i}} \cdot \pi \cdot d_{\text{i}} \cdot L} \tag{2.54}$$

Thermal resistance copper layer:

$$R_{\mathrm{C}} = \frac{\ln\left[\left(d_i + 2\delta\right)/d_i\right]}{2 \cdot \pi \cdot L \cdot \lambda_{\mathrm{C}}} \tag{2.55}$$

Thermal resistance convection on the outside:

$$R_{\mathrm{conv,a,wo}} = \frac{1}{\alpha_{\mathrm{a}} \cdot \pi \cdot (d_{\mathrm{i}} + 2\delta) \cdot L} \tag{2.56}$$

Filling in:

$$\dot{q}' = \frac{\dot{Q}}{L} = \frac{T_{\mathrm{W}} - T_{\mathrm{R}}}{\frac{1}{\alpha_{\mathrm{i}} \cdot \pi \cdot d_{\mathrm{i}}} + \frac{\ln\left[\left(d_i+2\delta\right)/d_i\right]}{2 \cdot \pi \cdot \lambda_{\mathrm{C}}} + \frac{1}{\alpha_{\mathrm{a}} \cdot \pi \cdot (d_{\mathrm{i}}+2\delta)}} = \dots$$

$$\dots \frac{60\ [\mathrm{K}]}{(0.0231 + 0.0001 + 6.6315)\ \left[\frac{\mathrm{K \cdot m}}{\mathrm{W}}\right]} = 9\ [\mathrm{W/m}] \tag{2.57}$$

<u>Calculations for the system with insulation:</u>

The total thermal resistance:

$$R_{\mathrm{total,w,ins}} = R_{\mathrm{conv,i}} + R_{\mathrm{C}} + R_{\mathrm{ins}} + R_{\mathrm{conv,a,wo}} \tag{2.58}$$

Thermal resistance insulation layer:

$$R_{\mathrm{ins}} = \frac{\ln\left[\left(d_i + 2\delta + 2s\right)/\left(d_i + 2\delta\right)\right]}{2 \cdot \pi \cdot L \cdot \lambda_{\mathrm{ins}}} \tag{2.59}$$

Thermal resistance convection on the outside:

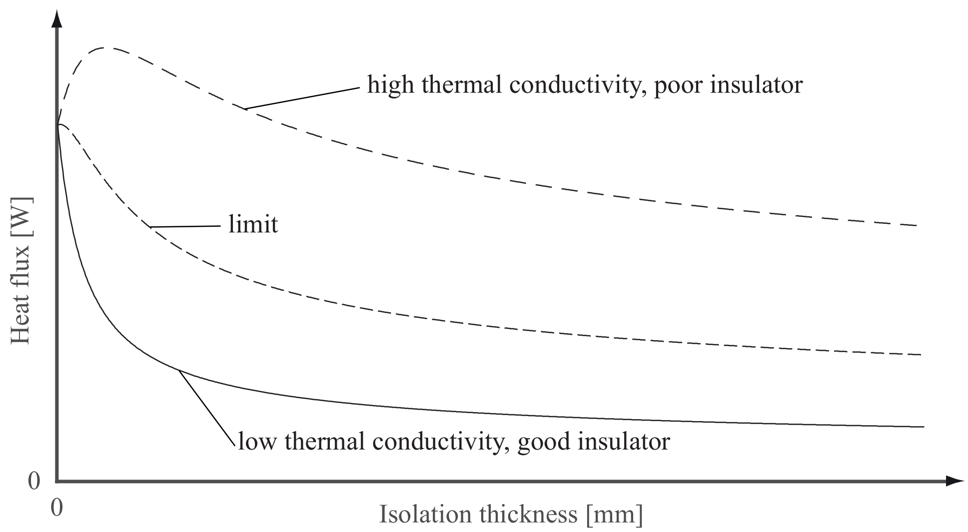
$$R_{\mathrm{conv,a,w}} = \frac{1}{\alpha_{\mathrm{a}} \cdot \pi \cdot (d_{\mathrm{i}} + 2\delta + 2s) \cdot L} \tag{2.60}$$

Filling in:

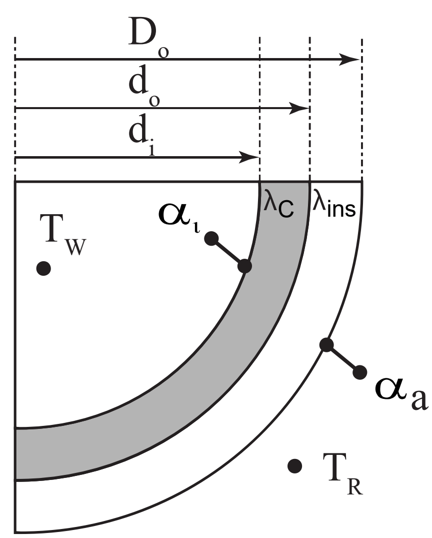
$$\dot{q}' = \frac{\dot{Q}}{L} = \frac{T_{\mathrm{W}} - T_{\mathrm{R}}}{\frac{1}{\alpha_{\mathrm{i}} \cdot \pi \cdot d_{\mathrm{i}}} + \frac{\ln\left[\left(d_i+2\delta\right)/d_i\right]}{2 \cdot \pi \cdot \lambda_{\mathrm{C}}} + \frac{\ln\left[\left(d_i+2\delta+2s\right)/\left(d_i+2\delta\right)\right]}{2 \cdot \pi \cdot \lambda_{\mathrm{ins}}} + \frac{1}{\alpha_{\mathrm{a}} \cdot \pi \cdot (d_{\mathrm{i}}+2\delta+2s)}} = \dots$$

$$\dots \frac{60\ [\mathrm{K}]}{(0.0231 + 0.0001 + 2.6266 + 3.3157)\ \left[\frac{\mathrm{K \cdot m}}{\mathrm{W}}\right]} = 10\ [\mathrm{W/m}] \tag{2.61}$$

b. Qualitatively sketch the heat emission profile $\dot{q}'$ as a function of the insulation thickness for different thermal conductivities of the insulation material. Explain the underlying physical principles.



c. Determine the necessary thermal conductivity λ_{ins} for the insulating material to obtain a general reduction in heat loss.



Insulation only has a positive effect when the rate of heat transfer $\dot{q}'$ decreases. The rate of heat transfer $\dot{q}'$ starts decreasing from a certain point where the insulation layer thickness has reached a critical value. The rate of heat transfer $\dot{q}'$ is maximum, when R_{total} is minimum.
Differentiating with respect to the outer diameter D_{o}:

$$\frac{d}{dD_{\text{o}}}\left[R_{\text{conv,i}} + R_{\text{C}} + R_{\text{ins}} + R_{\text{conv,a,wo}}\right] = 0 \tag{2.62}$$

$$\frac{d}{dD_{\text{o}}}\left[R_{\text{ins}} + R_{\text{conv,a,wo}}\right] = 0 \tag{2.63}$$

Inserting simplifying:

$$\frac{d}{dD_{\text{o}}}\left[\frac{\ln\left[D_{\text{o}}/d_{\text{o}}\right]}{2 \cdot \lambda_{\text{ins}}} + \frac{1}{\alpha_{\text{a}} \cdot D_{\text{o}}}\right] = 0 \tag{2.64}$$

$$\frac{1}{2 \cdot \lambda_{\text{iso}}} \cdot \frac{1}{D_{\text{o}}} - \frac{1}{\alpha_{\text{a}} D_{\text{o}}^2} = 0 \tag{2.65}$$

$$\Rightarrow D_{\text{o,max}} = \frac{2 \cdot \lambda_{\text{iso}}}{\alpha_{\text{a}}} \tag{2.66}$$

Equation 2.66 tells us that maximum heat transfer would occur if $D_{\text{o}} = \frac{2 \cdot \lambda_{\text{iso}}}{\alpha_{\text{a}}}$.
Rearranging this equation will result in

$$\rightarrow \frac{2 \cdot \lambda_{\text{iso}}^*}{\alpha_{\text{a}}} = d_{\text{o}} \tag{2.67}$$

Equation 2.67 tells us that for a specific value of λ_{iso}^*, isolation has no effect.
With this given, we can rearrange and insert the numeric values for the known parameters.

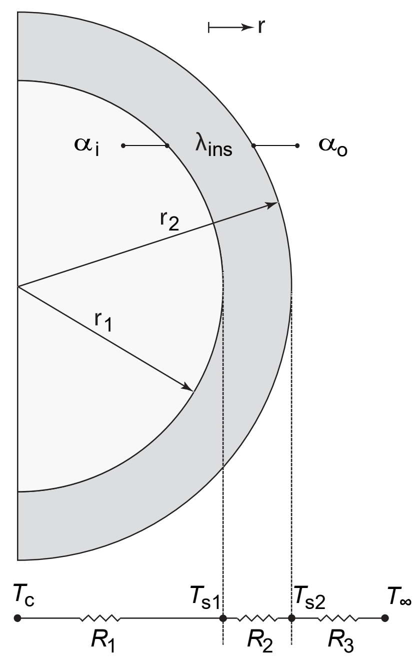
$$\lambda_{\text{iso}}^* = \alpha_{\text{a}} \cdot \frac{d_{\text{o}}}{2} = 6\ [\text{W}^2\text{K}] \cdot \frac{0.008\ [\text{m}]}{2} = 0.024\ [\text{W/mK}] \tag{2.68}$$

$$\Rightarrow \lambda_{\text{iso}}^* \geq 0.024\ [\text{W/mK}] \tag{2.69}$$

2.7 Spherical vessel

1. Problem type:
   Steady-state, one-dimensional heat conduction through a multi-layered wall without sources/sinks.

2. System:



a) Calculate all thermal resistances.

Thermal resistances:

$$R_1 = \frac{1}{\alpha_\mathrm{i} \cdot 4 \cdot \pi \cdot r_1^2} = \frac{1}{40\ [\mathrm{W/m^2K}] \cdot 4 \cdot \pi \cdot 1.5^2\ [\mathrm{m^2}]} = 8.84 \cdot 10^{-4}\ [\mathrm{K/W}] \quad (2.70)$$

$$R_2 = \frac{r_2 - r_1}{4 \cdot \pi \cdot r_1 \cdot r_2 \cdot \lambda_\mathrm{ins}} = \frac{1.55\ [\mathrm{m}] - 1.5\ [\mathrm{m}]}{4 \cdot \pi \cdot 1.5\ [\mathrm{m}] \cdot 1.55\ [\mathrm{m}] \cdot 0.2\ [\mathrm{w/mK}]} = 8.56 \cdot 10^{-3}\ [\mathrm{K/W}] \quad (2.71)$$

$$R_3 = \frac{1}{\alpha_\mathrm{o} \cdot 4 \cdot \pi \cdot r_1^2} = \frac{1}{10\ [\mathrm{W/m^2K}] \cdot 4 \cdot \pi \cdot 1.55^2\ [\mathrm{m^2}]} = 3.31 \cdot 10^{-3}\ [\mathrm{K/W}] \quad (2.72)$$

b) Calculate the rate of heat loss.

Rate of heat loss:

$$\dot{Q} = \frac{T_{\mathrm{c}} - T_{\infty}}{R_1 + R_2 + R_3} = \frac{(22-0)\ \ [\mathrm{K}]}{(8.84 \cdot 10^{-4} + 8.56 \cdot 10^{-3} + 3.31 \cdot 10^{-3})\ \ [\mathrm{K/W}]} = 1725\ [\mathrm{W}] \tag{2.73}$$

c) Calculate the temperature difference across the insulation layer.

Temperature difference across the insulation layer:

$$\dot{Q} = \frac{\Delta T_{\mathrm{ins}}}{R_2} \rightarrow \Delta T_{\mathrm{ins}} = \dot{Q} \cdot R_2 = 1725\ [\mathrm{W}] \cdot 8.56 \cdot 10^{-3}\ [\mathrm{K/W}] = 14.8\ [\mathrm{K}] \tag{2.74}$$