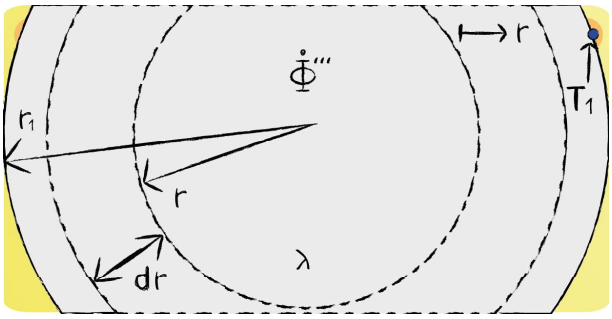
Lecture 13 - Question 6



Develop an energy balance to calculate the temperature profile inside the cylinder and give the boundary conditions. The cylinder is losing heat to the environment. Assume one-dimensional steady-state heat with a source.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} + d\dot{\Phi} = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda \cdot 2 \cdot \pi \cdot r \cdot L \cdot \frac{dT}{dr}$$

$$\dot{Q}_{r,out} = -\lambda \cdot 2 \cdot \pi \cdot r \cdot L \cdot \frac{dT}{dr} + \frac{d\dot{Q}}{dr} \cdot dr$$

$$d\dot{\Phi} = \dot{\Phi}''' \cdot 2 \cdot \pi \cdot r \cdot L \cdot dr$$

The heat entering the system is transferred from the centre of the cylinder by conductive heat transfer. In the centre of the control volume, heat is generated because of the source. $\dot{Q}_{\mathrm{r,out}}$ can be approximated by use of the Taylor series expansion.

Boundary Conditions:

$$\frac{\partial T(r=0)}{\partial r} = 0$$

$$T(r = r_1) = T_1$$

The first boundary condition describes that the temperature gradient in the center equals zero. This is because of symmetry. The second one describes that the temperature on the surface equals T_1.