# EB - Cond. - IE 10

Derive the energy balance required to calculate the one-dimensional steady-state temperature profile inside the fin element. The fin is subjected to convection (α). The diameter d of the fin is known.

1 Setting up the balance:

To derive the one-dimensional steady-state temperature profile, an energy balance around a one-dimensional infinitesimal element is needed. Heat is conducted in and out of the element, while heat is also lost through convection.
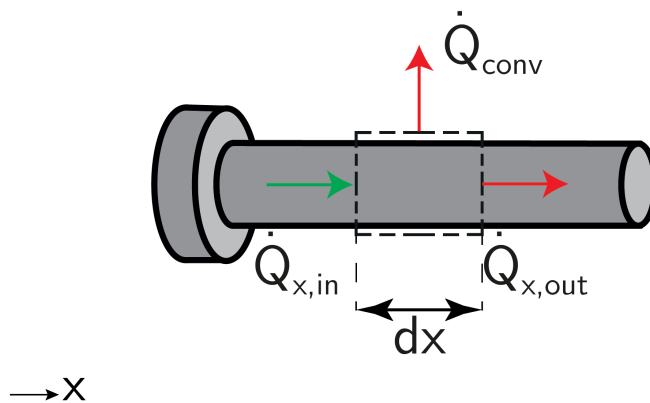
Hence, the steady-state balance reads:

$$\dot{Q}_{\text{x,in}} - \dot{Q}_{\text{x,out}} - d\dot{Q}_{\text{conv}} = 0,$$

the sum of the in- and outgoing fluxes should equal zero, because of steady-state conditions.

2 Defining the elements within the balance:

The ingoing flux described by use of Fourier's law:

$$\dot{Q}_{\text{x,in}} = -\lambda \frac{\pi d^2}{4} \frac{\partial T}{\partial x},$$

and the outgoing flux is approximated by the use of the Taylor series expansion.

$$\begin{aligned}
\dot{Q}_{\text{x,out}} &= \dot{Q}_{\text{x,in}} + \frac{\partial \dot{Q}_{\text{x,in}}}{\partial x} \cdot dx \\
&= -\lambda \frac{\pi d^2}{4} \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-\lambda \frac{\pi d^2}{4} \frac{\partial T}{\partial x}\right) \cdot dx.
\end{aligned}$$

The convective term is described by:

$$d\dot{Q}_{\text{conv}} = \alpha \cdot \pi d \cdot dx \cdot (T(x) - T_\infty)$$

3 Inserting and rearranging:

$$0 = \lambda \frac{d}{4} \frac{\partial^2 T}{\partial x^2} - \alpha \left(T(x) - T_\infty\right)$$