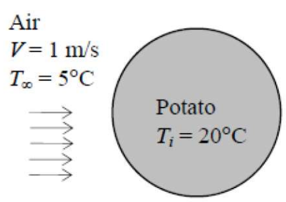
Solutions lecture 3

3.1 Cooling a potato

Analysis

In the assignment, several heat transfer coefficients are given for different airflow velocities. The heat transfer at the start of the cooling process should be determined. A sketch is presented below



Approach

Assumptions

- The conditions of the surroundings are constant.
- The potato has a spherical shape.
- The heat transfer coefficient is constant across the surface.

Route to solution

This exercise can be solved by substituting the known values into the Newton's cooling law:

$$\dot{Q} = hA(T_s - T_\infty)$$

where h is the heat transfer coefficient with an airflow of 1 m/s (19.1 $\mathrm{W\,m^{-2}\,K^{-1}}$), A is the surface area of the potato, T_s is the initial temperature, and T_∞ the temperature of the surroundings.

Elaboration

First, calculating the area of the sphere:

$$A = 4\pi R^2 = \pi D^2 = \pi(0.08)^2 = 0.2011\mathrm{m}^2$$

Substitution of all values in Newtons cooling law yields:

$$\dot{Q} = 19.1 \cdot 0.2011 \cdot (20 - 5) = 5.8\mathrm{W}$$

Evaluation

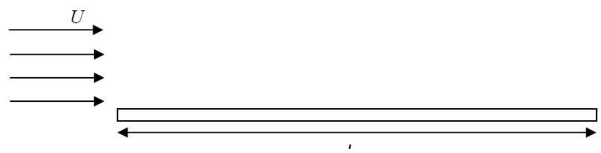
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

3.2 Heat loss through a wall

Analysis

We need to determine the rate of heat loss by convection. The wind is blowing parallel to the wall and thus it can be considered as an airflow across a flat plate, as can be seen in the figure below. Provided data is airflow velocity $U = 55\text{km}\,\text{h}^{-1}$, length of the wall $L = 10M$, surface temperature of the wall $T_s = 12°\text{C}$, and the outside air temperature $T_\infty = 5°\text{C}$



Approach

Assumptions

- The conditions of the surroundings are constant.
- The critical Reynolds number is $\text{Re} = 5 \cdot 10^5$.
- Heat transfer by radiation is negligible.
- Air is considered an ideal gas.

Route to solution

We need to determine $\dot{Q}$, so the solution route is:

1. Determine Re and Pr at the average temperature (use quantities like μ, ρ, k,Pr from tables).
2. Choose the right correlation based on geometry and Re.
3. Determine Nu.
4. Derive h.
5. Substitute into Newton's cooling law.

Elaboration

Firstly, we need to determine the Reynolds and Prandtl number at the average temperature.

$$T_f = \frac{T_s - T_\infty}{2} = \frac{12 + 5}{2} = 8.5°\text{C}$$

The closest temperature is 10 °C, so we take the table data at that temperature: $\rho = 1.246\ \text{kg}\,\text{m}^{-3}$, $\mu = 1.778 \cdot 10^{-5}\ \text{kg}\,\text{m}^{-1}\,\text{s}^{-1}$. The airflow is $U = \frac{55}{3.6} = 15.3\text{m}\,\text{s}^{-1}$. Substituting these values in the equation for the Reynolds number:

$$\text{Re} = \frac{\rho U L}{\mu} = \frac{1.246 \cdot 15.3 \cdot 10}{1.778 \cdot 10^{-5}} = 10.71 \cdot 10^6[-]$$

The Prandtl number at this temperature is also given, Pr=0.7336[-].

Now, we have to choose the right correlation based on the geometry and Reynolds number. Because the Reynolds number is larger than $5.0 \cdot 10^5$, so the flow is turbulent. For a turbulent flow across a flat plate, the needed coefficients are: a=0.037, b=0.8 and c=1/3.

With this, the Nusselt number can be determined:

$$\text{Nu} = a \cdot \text{Re}^b \cdot \text{Pr}^c = 0.037 \cdot (10.71 \cdot 10^6)^{0.8} \cdot 0.7336^{\frac{1}{3}} = 14.0 \cdot 10^3[-]$$

The Nusselt number is defined as:

$$\text{Nu} = \frac{h \cdot L}{k}$$

At 10 °C, the value for the thermal conductivity is $k = 0.02439\ \text{W}\,\text{m}^{-1}\,\text{K}^{-1}$. Rewriting and substitution yields

$$h = \frac{\text{Nu} \cdot k}{L} = \frac{14.0 \cdot 10^3 \cdot 0.02439}{10} = 34.2\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$$

Substituting this in Newton's cooling law:

$$\dot{Q} = h \cdot A \cdot \Delta T$$

where $A = 10 \cdot 4 = 40\text{m}^2$ is the area, and $\Delta T = 12 - 5 = 7°\text{C}$ is the temperature difference. Substituting gives:

$$\dot{Q} = h \cdot A \cdot \Delta T = 34.2 \cdot 40 \cdot 7 = 9.58\text{kW}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

3.3 Flow over an airplane wing

a) A boundary layer is the layer between an object and the region in a flow that is close to a boundary surface, in which the influence of the surface is still noticeable regarding speed or temperature. There are two types of boundary layers; the velocity boundary layer and the thermal boundary layer. Both layers contain the transition of velocity and temperature, respectively, between the object in the flow and the undisturbed surroundings. A sketch of this phenomena is given below.

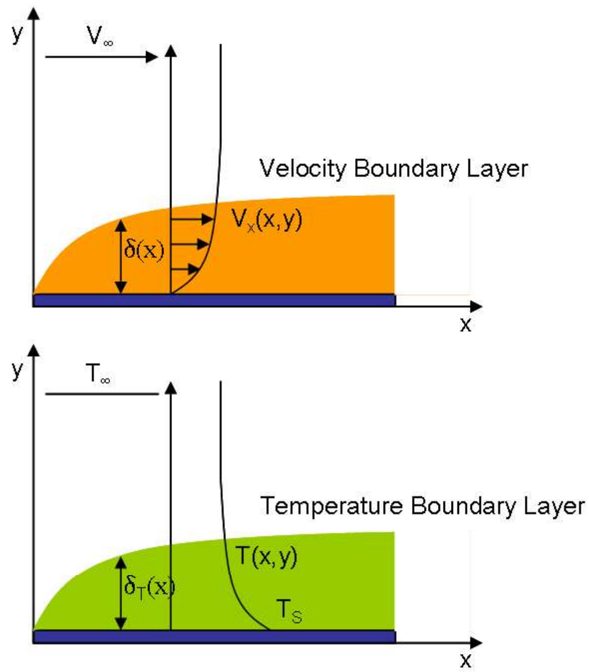


Figure 3.1: Thermal velocity of an airplane wing

b) The highest heat loss takes place where the temperature difference is maximal at a minimal distance (boundary layer thickness). $\dot{Q}$ is directly proportional to ΔT and h, given by the formula:

$$\dot{Q} = hA\Delta T$$

Heat transfer coefficient h gets larger with a decrease in boundary layer thickness, since the temperature gradient increases. Since the temperature difference between the wing surface and the boundary layer limit in this assignment is constant, the minimal thickness will determine where the heat loss is greatest. That is the case directly at the front edge of the wing.

c) In the laminar layer, the particles in the flow move in 'organized layers' across the surface, and will thus not promote heat transfer. In the turbulent layer, the particles do not only flow across the surface, but also swirl to- and from the surface, thus promoting heat transfer. The turbulent boundary layer will consequently have a much steeper temperature gradient just above the surface than the laminar boundary layer. The heat transfer will increase at the transition between the laminar to the turbulent flow.

3.4 Competition of soccer and tennis balls

a)

Analysis

We have a soccer ball and a tennis ball, with diameters $D_s = 0.22m$ and $D_t = 0.066m$. The soccer ball has a velocity of 58 km/h. What velocity should the tennis ball have to obtain a flow pattern similar to the soccer ball?

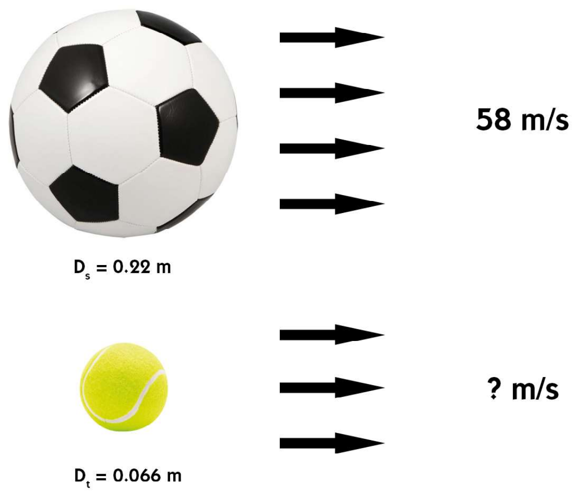


Figure 3.2: Comparison soccer and tennis ball

Approach

Assumptions

Assume that both balls have similar roughness patterns.

Route to solution

The flow profile depends on the Reynolds number, and the Reynolds number depends on the density of the flow medium, the flow velocity, the diameter of the sphere and the surface roughness, described by the formula:

$$\mathrm{Re} = \frac{\rho U D}{\mu}$$

The density of the flow medium and the surface roughness is the same for the soccer- and tennis ball. The difference in diameter will thus have to be compensated by a difference in velocity to keep the Reynolds number (and thus the flow profile) the same.

$$\mathrm{Re}_{tennis} = \mathrm{Re}_{soccer}$$
$$\frac{\rho U_{tennis} D_{tennis}}{\mu} = \frac{\rho U_{soccer} D_{soccer}}{\mu}$$

The densities and roughnesses cancel, and with some rewriting the velocity of the tennis ball can be obtained:

$$U_{tennis} D_{tennis} = U_{soccer} D_{soccer}$$
$$U_{tennis} = \frac{U_{soccer} D_{soccer}}{D_{tennis}}$$

Elaboration

The diameters are known, only the velocity of the soccer ball must me in the correct units

$$U_{soccer} = \frac{58}{3.6} = 16.11 \mathrm{m\,s^{-1}}$$

Substitution of the values in the equation for the velocity of the tennis ball:

$$U_{tennis} = \frac{U_{soccer} D_{soccer}}{D_{tennis}} = \frac{16.11 \cdot 0.22}{0.066} = 53.7 \mathrm{m\,s^{-1}}$$

This is equivalent to $53.7 \cdot 3.6 = 193 \mathrm{km\,h^{-1}}$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

b) If the Reynolds number of the soccer ball and the tennis ball are the same, then also the Nusselt number is the same. The formula for the Nusselt number is:

$$\mathrm{Nu} = \frac{hD}{k}$$
$$\mathrm{Nu}_{soccer} = \mathrm{Nu}_{tennis}$$
$$\frac{h_{soccer} D_{soccer}}{k} = \frac{h_{tennis} D_{tennis}}{k}$$

Both balls are present in the same air, so the k is the same.

Because the diameter of the soccer ball is greater than the diameter of the tennis ball, the heat transfer coefficient of the soccer ball will be smaller than the coefficient of the tennis ball, in order to satisfy the above balance. In other terms: the ball diameter is inversely proportional to the heat transfer coefficient ($h \propto D^{-1}$). For the heat loss the ball surface is important. The heat loss $\dot{Q}$is calculated by:

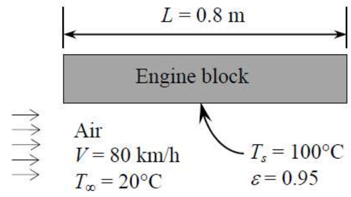
$$\dot{Q} = hA\Delta T$$

The temperature difference is the same for the two balls, so $\dot{Q}$ is directly proportional to hA ($\dot{Q} \propto hA$). The sphere surface area is directly proportional to D^2 and h is inversely proportional to D. Consequently, $\dot{Q}$ is directly proportional to ($D^2 \cdot D^{-1} = D$) the diameter. The soccer ball has the greatest diameter and will thus have the greatest heat loss.

3.5 Cooling of an engine

Analysis

We need to determine the rate of heat transfer from the bottom surface of the engine block by convection, at a velocity of 80 km/h. See the figure below:



The height of the block is 0.50m, the width 0.40m, the length 0.80m.

Approach

Assumptions

Conditions of the surroundings are constant, and the critical Reynoldsnumber is $5.0{\cdot}10^5$.

Route to solution

We need to determine $\dot{Q}$, so the solution route is:

1. Determine Re and Pr at the average temperature (use quantities like μ, ρ, k,Pr from tables).
2. Choose the right correlation based on geometry and Re.
3. Determine Nu.
4. Derive h.
5. Substitute into Newton's cooling law.

Elaboration

We start with determining the average temperature:

$$T_f = \frac{T_s - T_\infty}{2} = \frac{100+20}{2} = 60^\circ\mathrm{C}$$

At 60 °C, values for the density and viscosity are: $\rho = 1.059\mathrm{kg\,m^{-3}}$ and $\mu = 2.008{\cdot}10^{-5}\mathrm{kg\,m^{-1}\,s^{-1}}$. Substituting these values in the formula for the Reynolds number:

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{1.059 \cdot 22.21 \cdot 0.8}{2.008 \cdot 10^{-5}} = 937.6 \cdot 10^3[-]$$

At 60 °C, the Prandtl number is $\mathrm{Pr} = 0.7202[-]$.

Now, the Reynoldsnumber is larger than $5 \cdot 10^5$, so the flow is turbulent. Based on this and the geometry, the coefficients are a=0.037, b=0.8 and c=1/3. With these coefficients, the Nusselt number can be determined:

$$\mathrm{Nu} = a \cdot \mathrm{Re}^b \cdot \mathrm{Pr}^c = 0.037 \cdot (937.6 \cdot 10^3)^{0.8} \cdot 0.7202^{\frac{1}{3}} = 1.99 \cdot 10^3[-]$$

The Nusselt number is defined as:

$$\mathrm{Nu} = \frac{h \cdot L}{k}$$

At 60 °C, the value for the thermal conductivity is $k = 0.02808 \text{ W m}^{-1}\text{ K}^{-1}$. Rewriting and substitution yields

$$h = \frac{\text{Nu} \cdot k}{L} = \frac{1.99 \cdot 10^3 \cdot 0.02808}{0.8} = 69.8\text{W m}^{-2}\text{ K}^{-1}$$

Substituting this in Newton's cooling law:

$$\dot{Q} = h \cdot A \cdot \Delta T$$

where $A = 0.8 \cdot 0.4 = 0.32\text{m}^2$ is the area, and $\Delta T = 100 - 20 = 80°\text{C}$ is the temperature difference. Substituting gives:

$$\dot{Q} = h \cdot A \cdot \Delta T = 69.8 \cdot 0.32 \cdot 80 = 1.79\text{kW}$$

3.6 Roof of a train - Hand in

a) In this assignment the temperature must be found at which the amount of heat absorbed by radiation is equal to the amount of heat emitted by convection.

Assumptions:

- The environmental conditions are constant.
- The critical Reynolds number is $\mathrm{Re}_{\mathrm{crit}} = 5 \cdot 10^5$
- Heat loss due to radiation from the roof of the train to the environment is negligible

Energy balance of the roof:

$$\dot{q}_{\mathrm{rad}} - \dot{q}_{\mathrm{conv}} = 0$$

Where $\dot{q}_{\mathrm{conv}}$ equals:

$$\dot{q}_{\mathrm{conv}} = h \cdot (T_{\mathrm{s}} - T_{\infty})$$

In order to determine h, the Nusselt number should be determined.

First step in doing so is making an assumption for the average fluid temperature T_{f}, which is assumed to be 30 °C.

Average fluid properties of air at 30 °C:

$$\rho = 1.164 \text{ kg/m}^3$$

$$\mu = 1.872 \cdot 10^{-5} \text{ kg/m} \cdot \text{s}$$

$$\mathrm{Pr} = 0.7282$$

$$k = 0.02588 \cdot 10^{-5} \text{ W/mK}$$

Now the Reynolds number Re can be determined (characteristic length for the flat plate equals its length):

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{1.164 \cdot 19.4 \cdot 8}{1.872 \cdot 10^{-5}} = 967 \cdot 10^6$$

After having determined the Reynolds number Re and Prandtl number Pr, an applicable correlation for the Nusselt number Nu can be found and used in order to determine the Nusselt number Nu.

$$\mathrm{Nu} = 0.037 \cdot \mathrm{Re}^{0.8} \cdot \mathrm{Pr}^{1/3}$$

$$= 0.037 \cdot \left(9.67 \cdot 10^6\right)^{0.8} \cdot 0.7282^{1/3} = 12.9 \cdot 10^3$$

And now h can be determined:

$$h = \frac{\mathrm{Nu} \cdot k}{L} = \frac{12.9 \cdot 10^3 \cdot 0.02588}{8} = 41.7 \ \cdot \mathrm{W/m^2K}$$

Rewriting the energy balance:

$$\dot{q}_{\mathrm{rad}} - h \cdot (T_{\mathrm{s}} - T_{\infty}) = 0$$

$$\rightarrow T_{\mathrm{s}} = T_{\infty} + \frac{\dot{q}_{\mathrm{conv}}}{h} = 30 + \frac{200}{41.7} = 34.8°C$$

Finally, it must be checked whether at this temperature found the h would not be appreciably different than at the assumed temperature of 30 °C. If so, the whole approach must be repeated with a new estimate. If not, the answer can be regarded as converged. The latter is the case here, but it is not further demonstrated here. This is desirable in submitted work.