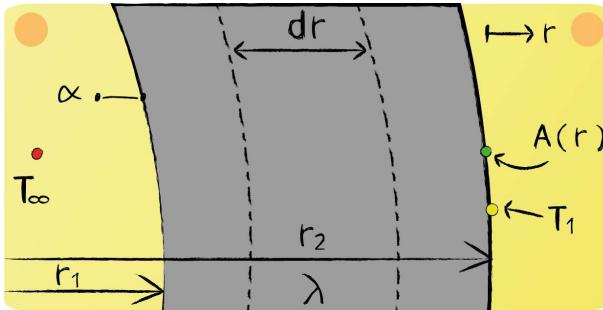


Lecture 8 - Question 5



Hot water flows through a long pipe of length L . The water temperature and external surface temperature of the pipe are constant and equal to T_∞ and T_1 respectively. Set up the energy balance for radial heat conduction in the pipe wall and give the appropriate boundary conditions.

Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

Since the type of heat transfer is steady-state, the sum of the in- and outgoing heat fluxes of the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda A(r) \frac{\partial T}{\partial r} = -\lambda 2\pi r L \frac{\partial T}{\partial r}$$

$$\begin{aligned}\dot{Q}_{r,out} &= \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r} dr = \\ &= -\lambda 2\pi r L \frac{\partial T}{\partial r} + \frac{\partial}{\partial r} (-\lambda 2\pi r L \frac{\partial T}{\partial r}) dr\end{aligned}$$



The ingoing flux can be described by use of Fourier's law and the outgoing flux can be approximated by use of the Taylor series expansion.

Boundary conditions:

$$\frac{\partial T(r=r_1)}{\partial r} = -\frac{\alpha}{\lambda} (T_\infty - T(r=r_1))$$

$$T(r=r_2) = T_1$$

The first boundary condition results from the fact that $\dot{Q}_{r=r_1} = -\lambda A(r) \frac{\partial T(r=r_1)}{\partial r} = \alpha A(r)(T_\infty - T(r=r_1))$, the second one describes that the temperature at the surface equals T_1 .