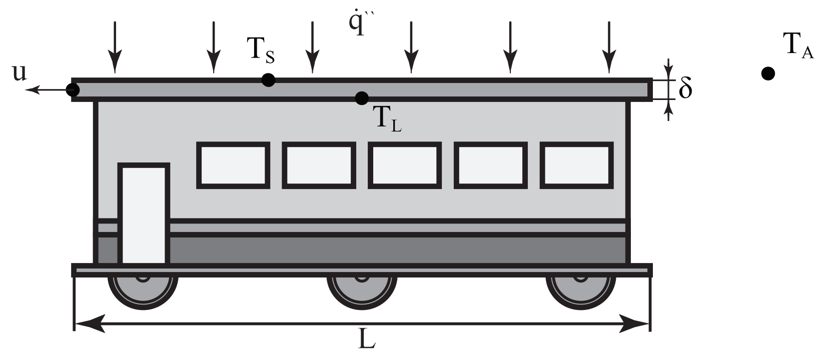
Exercise III.6: (Moving train ⋆)

Consider the roof surface of a passenger car on a moving train. This surface is exposed to solar radiation, with an incident heat flux denoted as $\dot{q}''$. The ambient air temperature is represented by T_A.



Given parameters:

- Velocity of the train: $u = 50$ km/h
- Length of the train roof: $L = 10$ m
- Width of the train roof: $W = 3$ m
- Thickness of the train roof: $\delta = 20$ cm
- Thermal conductivity of the train roof: $\lambda = 0.03$ W/mK
- Lower temperature of the train roof: $T_L = 16$ °C
- Ambient temperature: $T_A = 15$ °C
- Solar irradiation: $\dot{q}'' = 288$ W/m^2
- Properties of air:

T (°C)	ρ (kg/m^3)	c (kJ/kg·K)	λ (W/mK)	ν (m^2/s)	Pr (−)
0	1.275	1.006	$24.18 \cdot 10^{-3}$	$13.52 \cdot 10^{-6}$	0.7179
20	1.188	1.007	$25.69 \cdot 10^{-3}$	$15.35 \cdot 10^{-6}$	0.7148
40	1.112	1.007	$27.16 \cdot 10^{-3}$	$17.26 \cdot 10^{-6}$	0.7122
80	0.9859	1.008	$30.01 \cdot 10^{-3}$	$21.35 \cdot 10^{-6}$	0.7083
100	0.9329	1.009	$31.39 \cdot 10^{-3}$	$23.51 \cdot 10^{-6}$	0.7073

Hints:

- Assume steady-state heat transfer to be one-dimensional
- Neglect radiation heat exchange with the surroundings.

Tasks:

a) Determine the equilibrium temperature of the top surface T_s.

Solution III.6: (Moving train ⋆)

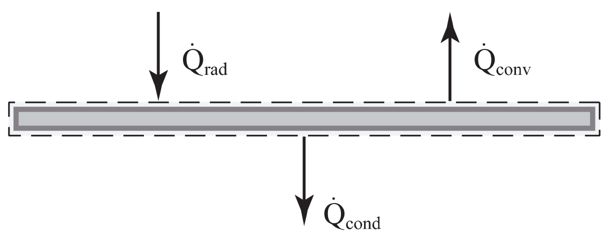
Task a)

The upper surface of the moving train's passenger car absorbs solar radiation while simultaneously losing heat through convection and conduction. As a result, the surface temperature reaches equilibrium. Determining this equilibrium temperature involves establishing an energy balance around this roof and assuming the top surface temperature. Throughout this process, it is key to evaluate the assumption. This process can lead to two potential outcomes:

- No further action is required if the assumed temperature for average fluid properties aligns with the calculated result.
- If the assumed temperature does not correspond to the calculated result, the process must be repeated, incorporating a new assumption.

1 **Setting up the balance:**

The roof of the train is subjected to solar radiation. A portion of this energy is conducted, while the remainder is transferred to the surroundings through convection.



The steady-state energy balance around the roof reads:

$$0 = \dot{Q}_{\text{rad}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}}.$$

2 **Defining the elements within the balance:**

The rate of heat transfer yielding from the sun is calculated as:

$$\begin{aligned} \dot{Q}_{\text{rad}} &= \dot{q}'' \left(W \times L\right) \\ &= 288 \left(\frac{\text{W}}{\text{m}^2}\right) \cdot (3 \times 10) \ \left(\text{m}^2\right) = 8.64 \ (\text{kW}). \end{aligned} \tag{III.6.1}$$

To determine the rate of heat loss by convection from the roof, the average heat transfer coefficient is required. At this point this parameter is undetermined. To determine this coefficient, the surface temperature is required to determine the average fluid property temperature.

To do so, the assumption has been made that the top surface is $T_{\text{s}} = 25$ °C.

The temperature to evaluate the fluid properties at yields from:

$$\begin{aligned} T_{\text{prop}} &= \frac{T_{\text{s}} + T_{\text{A}}}{2} \\ &= \frac{25 + 15 \ (^\circ\text{C})}{2} = 20 \ (^\circ\text{C}). \end{aligned} \tag{III.6.2}$$

The properties of air at 20 °C are:

$\lambda_\mathrm{f} = 25.69 \cdot 10^{-3}\ \frac{\mathrm{W}}{\mathrm{mK}}$,

$\nu = 15.35 \cdot 10^{-6}$ fracm^{2}s,

$\mathrm{Pr} = 0.7148.$

With this, the Reynolds number is determined, with the characteristic length L_c being the length of the train L for flow over a flat plate:

$$\begin{aligned}\mathrm{Re_L} &= \frac{uL}{\nu} \\ &= \frac{13.9\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot 10\ (\mathrm{m})}{15.35 \cdot 10^{-6}\ (\mathrm{fracm^2s})} = 9.0 \cdot 10^6\ \ (-)\,.\end{aligned} \tag{III.6.3}$$

Given the value of the Reynolds number HTC.6 can be applied, which describes the average Nusselt number for forced turbulent flow over a flat plate with an isothermal surface:

$$\begin{aligned}\overline{\mathrm{Nu}}_\mathrm{L} &= 0.036 \cdot \mathrm{Pr}^{0.43}(\mathrm{Re_L}^{0.8} - 9400) \\ &= 0.036 \cdot 0.7148^{0.43}\left[\left(9.0481 \cdot 10^6\right)^{0.8} - 9400\right] = 1.1 \cdot 10^4\ \ (-)\,.\end{aligned} \tag{III.6.4}$$

Rewriting the definition of the Nusselt number, the heat transfer coefficient is determined:

$$\begin{aligned}\overline{\alpha} &= \frac{\overline{\mathrm{Nu}}_\mathrm{L}\lambda_\mathrm{f}}{L} \\ &= \frac{1.1 \cdot 10^4\ \ (-) \cdot 25.69 \cdot 10^{-3}\ \left(\frac{\mathrm{W}}{\mathrm{mK}}\right)}{10\ (\mathrm{m})} = 28.7\ \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right).\end{aligned} \tag{III.6.5}$$

Which results in the convective rate of heat transfer:

$$\begin{aligned}\dot{Q}_\mathrm{conv} &= \overline{\alpha} \cdot (W \times L)\,(T_\mathrm{s} - T_\mathrm{A}) \\ &= 28.7\ \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right) \cdot (3 \times 10)\ \left(\mathrm{m^2}\right) \cdot (25 - 15)\ \ (^\circ\mathrm{C}) = 8.6\ (\mathrm{kW}).\end{aligned} \tag{III.6.6}$$

Lastly, the conductive rate of heat transfer is calculated as follows:

$$\begin{aligned}\dot{Q}_\mathrm{cond} &= \lambda\,(W \times L)\,\frac{T_\mathrm{s} - T_\mathrm{L}}{\delta} \\ &= 0.03\ \left(\frac{\mathrm{W}}{\mathrm{mK}}\right) \cdot (3 \times 10)\ \left(\mathrm{m^2}\right) \cdot \frac{(25 - 16)\ \ (^\circ\mathrm{C})}{0.2\ (\mathrm{m})} = 40.3\ (\mathrm{W}).\end{aligned} \tag{III.6.7}$$

3 **Inserting and rearranging:**

Inserting the numerical values into the energy balance:

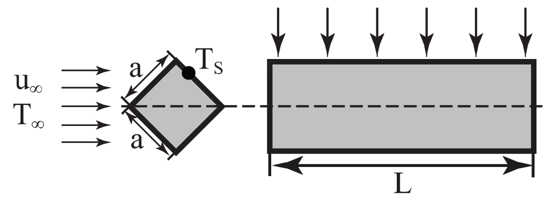
$$8{,}640\ (\mathrm{W}) - 8{,}600\ (\mathrm{W}) - 40.3\ (\mathrm{W}) = -0.3\ (\mathrm{W}). \tag{III.6.8}$$

Conclusion

This suggests that the initial assumption of a surface temperature of 25 °C was highly accurate, given that the balance deviates by only 0.003% from being precisely zero to the incoming rate of heat transfer.

Exercise III.7: (Transverse flow ⋆⋆)

Air flows transversely across a beam of length L, with a square cross-sectional area, as can be seen in the figure. The circles with crosses indicate streamlines that are moving away from the observer.



Given parameters:

- Beam geometrical dimensions: a,L
- Material properties of the air: η,ρ, $\Pr$,λ
- Velocity of the crossflow: u_∞
- Temperatures of the surface and ambient: T_s, T_∞

Hints:

- Assume steady-state conditions
- $10^4 \leq \mathrm{Re} \leq 10^5$
- Heat loss from the sides is negligible.

Tasks:

a) Provide an expression for the rate of heat loss in terms of the given parameters.

b) Determine the percentual change of the heat transfer coefficient if we had a similar-form rod with four times the crosswise width at double flow velocity.

Solution III.7: (Transverse flow ⋆⋆)

Task a)

The beam is subjected to a transverse flow, which cools the beam by convective heat transfer.

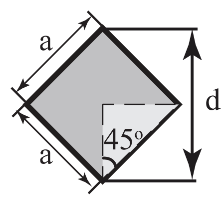
1 **Setting up the balance:**

The rate of heat loss by convection is determined from Newton's law of cooling:

$$\dot{Q} = \overline{\alpha} A_s \left(T_s - T_\infty \right). \tag{III.7.1}$$

2 **Defining the elements within the balance:**

The characteristic length for a square cylinder subjected to transverse flow equals d, in the figure below:



With trigonometry length d is derived:

$$\begin{aligned} \cos\left(45^{\circ}\right) &= \frac{\frac{1}{2}d}{\mathrm{a}} \\ \Rightarrow d &= \sqrt{2}\mathrm{a}. \end{aligned} \tag{III.7.2}$$

The Reynolds number is defined as follows:

$$\begin{aligned} \mathrm{Re} &= \frac{\rho u_\infty d}{\eta} \\ &= \frac{\rho u_\infty \sqrt{2}\mathrm{a}}{\eta}. \end{aligned} \tag{III.7.3}$$

Having determined the Reynolds number, the Nusselt number can be determined using HTC.9, which is valid for $10^4 \leq \mathrm{Re} \leq 10^5$. For this specific case $C = 0.246$, and $m = 0.588$.

$$\begin{aligned} \overline{\mathrm{Nu}}_\mathrm{d} &= C\, \mathrm{Re}_\mathrm{d}^{m}\, \mathrm{Pr}^{1/3} \\ &= 0.246 \cdot \mathrm{Re}_\mathrm{d}^{0.588}\, \mathrm{Pr}^{1/3}. \end{aligned} \tag{III.7.4}$$

Rewriting the definition of the Nusselt number results the heat transfer coefficient:

$$\begin{aligned} \overline{\alpha} &= \frac{\overline{\mathrm{Nu}}_\mathrm{d} \lambda}{d} \\ &= \frac{0.246 \cdot \lambda}{\sqrt{2} \cdot \mathrm{a}} \cdot \mathrm{Re}_\mathrm{d}^{0.588}\, \mathrm{Pr}^{1/3}. \end{aligned} \tag{III.7.5}$$

Lastly, only the surface area subjected to the air streams needs to be expressed:

$$A_\mathrm{s} = 4\mathrm{a}L. \tag{III.7.6}$$

Conclusion

3 **Inserting and rearranging:**

$$\dot{Q} = 0.8531 \cdot \lambda L \cdot \left(\frac{\rho u_\infty \mathrm{a}}{\eta}\right)^{0.588} \mathrm{Pr}^{1/3} \left(T_s - T_\infty\right). \tag{III.7.7}$$

Task b)

In the previous task, the average heat transfer coefficient for the first case was found:

$$\overline{\alpha}_1 = \frac{0.2133 \cdot \lambda}{\mathrm{a}_1^{0.412}} \cdot \left(\frac{\rho u_1}{\eta}\right)^{0.588} \mathrm{Pr}^{1/3}. \tag{III.7.8}$$

For the situation where the crosswise width is four times as big and the flow velocity doubles, the characteristic length is written as:

$$\begin{aligned} d_2 &= \sqrt{2}\mathrm{a}_2 \\ &= 4 \cdot \sqrt{2}\mathrm{a}_1, \end{aligned} \tag{III.7.9}$$

and the Reynolds number is written as:

$$\begin{aligned} \mathrm{Re}_2 &= \frac{\rho u_2 \sqrt{2}\mathrm{a}_2}{\eta} \\ &= \frac{8\sqrt{2}\rho u_1 \mathrm{a}_1}{\eta}. \end{aligned} \tag{III.7.10}$$

Having determined the Reynolds number, the Nusselt number can be determined from HTC.9, which is valid for $10^4 \leq \mathrm{Re} \leq 10^5$. For this specific case $C = 0.246$, and $m = 0.588$.

$$\begin{aligned} \overline{\mathrm{Nu}}_{d,2} &= C\, \mathrm{Re}_2^m\, \mathrm{Pr}^{1/3} \\ &= 0.246 \cdot \mathrm{Re}_2^{0.588}\, \mathrm{Pr}^{1/3}. \end{aligned} \tag{III.7.11}$$

Rewriting the definition of the Nusselt number gives the heat transfer coefficient:

$$\begin{aligned} \overline{\alpha}_2 &= \frac{0.246 \cdot \lambda}{\sqrt{2} \cdot \mathrm{a}_2} \cdot \mathrm{Re}_2^{0.588}\, \mathrm{Pr}^{1/3} \\ &= \frac{0.1811 \cdot \lambda}{\mathrm{a}_1^{0.412}} \cdot \left(\frac{\rho u_1}{\eta}\right)^{0.588} \mathrm{Pr}^{1/3}, \end{aligned} \tag{III.7.12}$$

and therefore:

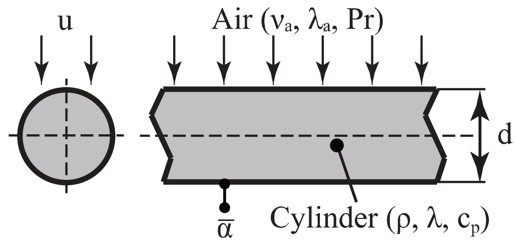
$$\frac{\overline{\alpha}_2}{\overline{\alpha}_1} = \frac{0.1811}{0.2133} = 0.85. \tag{III.7.13}$$

Conclusion

So the heat transfer coefficient reduces with 15%.

Exercise III.8: (Heating of a cylinder ★★)

A long cylinder is kept at a homogeneous temperature T_0. This cylinder is suddenly, at time $t = 0$, exposed to a warm air flow with temperature T_a and a transverse velocity of u.



Given parameters:

- Diameter of the cylinder: $d = 0.055$ m
- Density of the cylinder: $\rho = 1500$ kg/m^3
- Thermal conductivity of the cylinder: $\lambda = 0.119$ W/mK
- Heat capacity of the cylinder: $c_\mathrm{p} = 1000$ J/kgK
- Initial homogenous temperature of the cylinder: $T_0 = 10$ °C
- Critical temperature of the cylinder: $T_\mathrm{c} = 38$ °C
- Temperature of the air flow: $T_\mathrm{a} = 40$ °C
- Velocity of the air: $u = 0.1$ m/s
- Thermal conductivity of the air: $\lambda_\mathrm{a} = 25.7 \cdot 10^{-3}$ W/mK
- Kinematic viscosity of the air: $\nu_\mathrm{a} = 15.35 \cdot 10^{-6}$ m^2/s
- Prandtl number of the air: Pr = 0.71

Hints:

- The average heat transfer coefficient $\overline{\alpha}$ is steady in time.
- The material properties can be taken as constant.

Tasks:

a) Determine the initial rate of heat transfer per unit length of the cylinder.

b) At $t = t_\mathrm{c}$ the cylinder reaches the critical temperature T_c at its hottest point. Sketch qualitatively the temperature distribution at time-points $t = 0$ and $t = t_\mathrm{c}$.

c) Determine the time t_c, until the cylinder reaches the critical temperature T_c at its hottest point.

Solution III.8: (Heating of a cylinder ⋆⋆)

Task a)

The cylinder is heated by a stream that crosses the cylinder's longitudinal axis.

1 **Setting up the balance:**

The initial rate of heat transfer is described by Newton's law of cooling:

$$\dot{Q}(t=0) = \overline{\alpha} A_s \left(T_\mathrm{a} - T_0\right), \tag{III.8.1}$$

where the surface area is taken for a unit length $L = 1$ m.

2 **Defining the elements within the balance:**

To determine the initial rate of heat transfer, the average heat transfer coefficient needs to be determined. To determine the correlation applicable to the Nusselt number, first, the Reynolds number must be determined. Which is calculated as:

$$\begin{aligned} \mathrm{Re_d} &= \frac{ud}{\nu_\mathrm{a}} \\ &= \frac{0.1\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right)\cdot 0.055\ (\mathrm{m})}{15.35\cdot 10^{-6}\ \left(\frac{\mathrm{m}^2}{\mathrm{s}}\right)} = 358\ (-)\,. \end{aligned} \tag{III.8.2}$$

HTC.7, which provides the average Nusselt number for forced perpendicular flow perpendicular to the longitudinal axis of a circular cylinder with an isothermal surface. For the given Reynolds number $C = 0.683$ and $m = 0.466$:

$$\begin{aligned} \overline{\mathrm{Nu}}_\mathrm{d} &= C\,\mathrm{Re}_\mathrm{d}^m\,\mathrm{Pr}^{0.4} \\ &= 0.683\cdot 358^{0.466}\cdot 0.71^{0.4} = 9.2\ (-) \end{aligned} \tag{III.8.3}$$

Note that also the HTC.8 $\left(\overline{\mathrm{Nu}}_\mathrm{d} = \left(0.4\mathrm{Re}_\mathrm{d}^{1/2} + 0.06\mathrm{Re}_\mathrm{d}^{2/3}\right)\mathrm{Pr}^{0.4}\cdot\left(\frac{\eta_\infty}{\eta_\mathrm{w}}\right)^{1/4}\right)$ could have been used, which gives a similar result.

Rewriting the definition of the Nusselt number gives the heat transfer coefficient:

$$\begin{aligned} \overline{\alpha} &= \frac{\overline{\mathrm{Nu}}_\mathrm{d}\lambda}{d} \\ &= \frac{9.2\ (-)\cdot 0.0257\ \left(\frac{\mathrm{W}}{\mathrm{mK}}\right)}{0.055\ (\mathrm{m})} = 4.3\ \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right) \end{aligned} \tag{III.8.4}$$

Lastly, the surface area for a unit length is found from:

$$\begin{aligned} A_\mathrm{s} &= \pi d L \\ &= \pi\cdot 0.055\ (\mathrm{m})\cdot 1\ (\mathrm{m}) = 0.17\ \left(\mathrm{m}^2\right) \end{aligned} \tag{III.8.5}$$

3 **Inserting and rearranging:**

$$\begin{aligned} \dot{Q}(t=0) &= \overline{\alpha} A_s \left(T_\mathrm{a} - T_0\right) \\ &= 4.3\ \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right)\cdot 0.17\ \left(\mathrm{m}^2\right)(40-10)\ (^\circ\mathrm{C}) = 22.1\ (\mathrm{W}) \end{aligned} \tag{III.8.6}$$

Conclusion

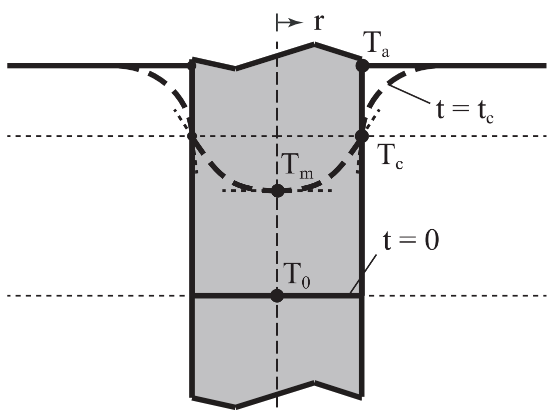
The initial rate of heat transfer from the ambient to the cylinder per unit length is thus 22.1 W.

Task b)

At $t = 0$, the temperature within the cylinder is initially uniform and constant in the radial direction. Similarly, the air temperature exhibits homogeneity.

When reaching the time $t = t_c$, at the center $r = 0$, the temperature gradient is horizontal, as a consequence of symmetry. Additionally, the interface temperature between the solid and ambient corresponds to T_c. Notably, a distinct change in slope occurs between the temperature profiles inside and outside the cylinder, with the steeper gradient located externally, attributed to the effects of convection. Lastly, as r approaches infinity, the temperature outside will asymptotically approach the ambient temperature with a gradient slope of zero.

Conclusion



Task c)

The time t_c necessary to attain the critical temperature T_c is established through the utilization of Heissler diagrams. However, before this, the temperature at the center of the cylinder T_m must be determined first at the same instance. To do so, the inverse of the Biot number must be determined:

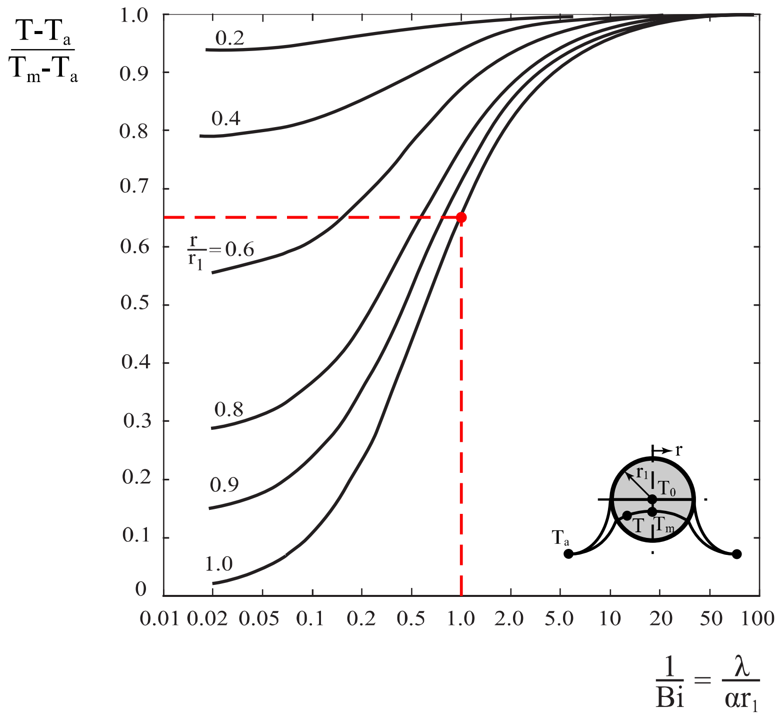
$$\frac{1}{\mathrm{Bi}} = \frac{\lambda}{\bar{\alpha} L} = \frac{\lambda}{\bar{\alpha} \cdot \frac{d}{2}} \tag{III.8.7}$$

$$= \frac{0.119 \left(\frac{\mathrm{W}}{\mathrm{mK}}\right)}{4.31 \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right) \cdot \frac{0.055}{2} \ (\mathrm{m})} = 1 \ (-)$$

and the ratio between the outer radius and the radius at which T_c is observed, which is at the surface of the cylinder. This gives:

$$\frac{r}{r_1} = 1 \ (-) \tag{III.8.8}$$

The value of the center temperature is derived by consulting the corresponding Heissler diagram as follows:



Reading the chart:

$$\frac{T_\mathrm{c} - T_\mathrm{a}}{T_\mathrm{m} - T_\mathrm{a}} \approx 0.65 \ \ (-) \tag{III.8.9}$$

Rewriting gives

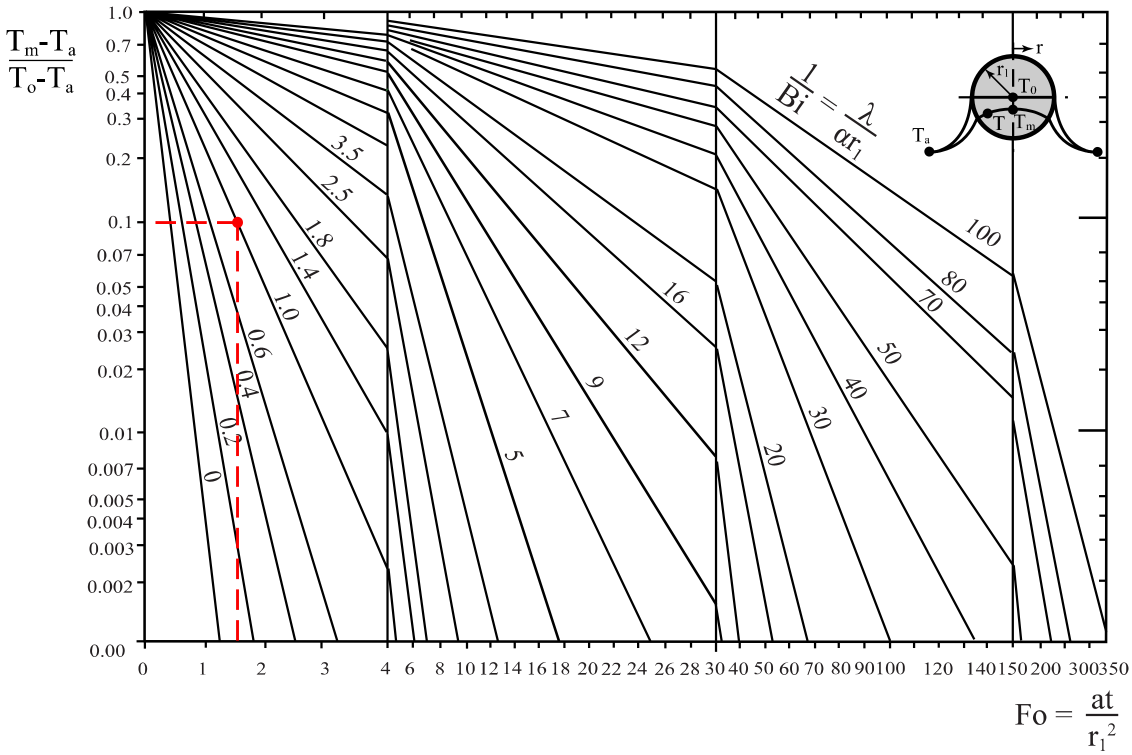
$$\begin{aligned} T_\mathrm{m} &= \frac{T_\mathrm{c} - T_\mathrm{a}}{0.65} + T_\mathrm{a} \ \ (-) \\ &= \frac{(38 - 40) \ \ (^\circ\mathrm{C})}{0.65} + 40 \ (^\circ\mathrm{C}) = 36.9 \ (^\circ\mathrm{C}) \end{aligned} \tag{III.8.10}$$

Now the Heissler diagram describing the center temperature as a function of time must be used to determine the time at which the center temperature reaches 36.9 °C.

First, the temperature difference needs to be determined:

$$\frac{T_\mathrm{m} - T_\mathrm{a}}{T_0 - T_\mathrm{a}} = \frac{(36.9 - 40) \ \ (^\circ\mathrm{C})}{(10 - 40) \ \ (^\circ\mathrm{C})} = 0.1 \ \ (-) \tag{III.8.11}$$

Reading the chart:



Which gives:

$$\text{Fo} \approx 1.5 \ (-) \tag{III.8.12}$$

Rewriting the definition of the Fourier number, where $a = \frac{\lambda}{\rho c_p}$, yields:

$$t_c = \text{Fo} \cdot \left(\frac{D}{2}\right)^2 \frac{\rho \cdot c_p}{\lambda} \tag{III.8.13}$$

$$= 1.5 \ (-) \cdot \left(\frac{0.055}{2}\right)^2 \ \left(\text{m}^2\right) \cdot \frac{1500 \ \left(\frac{\text{kg}}{\text{m}^3}\right) \cdot 1000 \ \left(\frac{\text{J}}{\text{kgK}}\right)}{0.119 \ \left(\frac{\text{W}}{\text{mK}}\right)} \approx 14{,}300 \ (\text{s})$$

Conclusion

After 4 hours the cylinder reaches the critical temperature T_c.