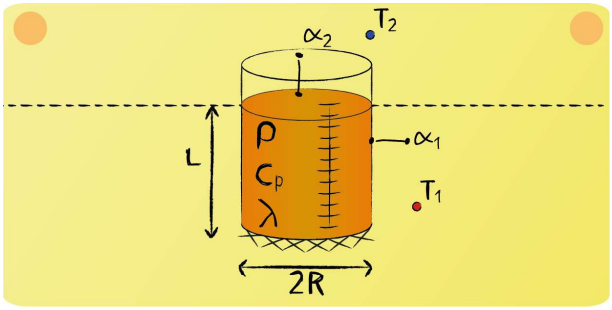
Lecture 14 - Question 13



A mixture in a cup is being heated in a water bath, but is losing heat at the same time. The bottom surface is adiabatic. Specify the energy balance to obtain the differential equation that expresses the change in temperature T_w of the mixture over the course of time. Assume the process to be isobaric and the temperatures to be homogeneous. Neglect the thickness of the cup.

Energy balance:

$$\frac{dU}{dt} = \dot{Q}_{in} - \dot{Q}_{out}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{dU}{dt} = \rho \cdot c_p \cdot \pi \cdot R^2 \cdot L \cdot \frac{dT_w}{dt}$$

The internal energy of the control volume can be described as: $U = m \cdot c_p \cdot T$.

Heat fluxes:

$$\dot{Q}_{in} = \alpha_1 2\pi RL \left(T_w - T_1\right)$$

$$\dot{Q}_{out} = \alpha_2 \pi R^2 \left(T_w - T_2\right)$$