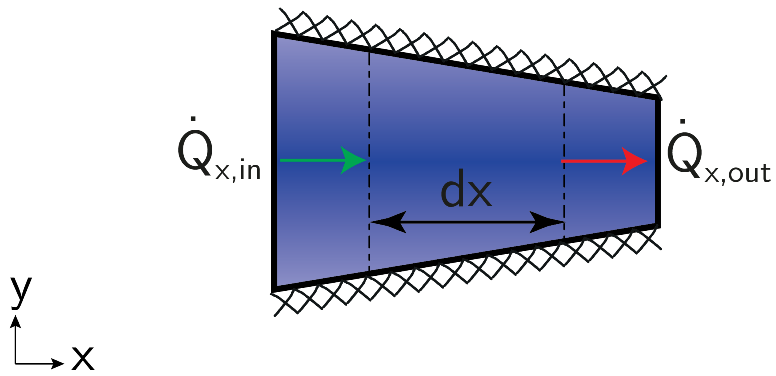
EB - Cond. - IE 4

Derive the energy balance and boundary conditions required to calculate the one-dimensional steady-state temperature profile inside the cone.

1 Setting up the balance:

To derive the one-dimensional steady-state temperature profile, an energy balance around an infinitesimal element is needed. Heat is conducted in and out of the element.



Hence, the steady-state balance reads:

$$\dot{Q}_{\mathrm{x,in}} - \dot{Q}_{\mathrm{x,out}} = 0,$$

the sum of the in- and outgoing fluxes should equal zero, because of steady-state conditions.

2 Defining the elements within the balance:

The ingoing flux described by use of Fourier's law:

$$\dot{Q}_{\mathrm{x,in}} = -\lambda A(x)\frac{\partial T}{\partial x},$$

and the outgoing flux is approximated by the use of the Taylor series expansion.

$$\begin{aligned}\dot{Q}_{\mathrm{x,out}} &= \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{\mathrm{x,in}}}{\partial x}\cdot dx\\ &= -\lambda A\frac{\partial T}{\partial x} - \lambda A(x)\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-\lambda A(x)\frac{\partial T}{\partial x}\right)\cdot dx.\end{aligned}$$

3 Inserting and rearranging:

$$-\lambda A\frac{\partial T}{\partial x} + \lambda A\frac{\partial T}{\partial x} - \frac{\partial}{\partial x}\left(-\lambda A\frac{\partial T}{\partial x}\right)dx = 0,$$

λ is constant, hence cancels out:

$$A(x)\frac{\partial^2 T}{\partial x^2} = 0.$$

4 Defining the boundary and/or initial conditions:

The temperature on the left side of the wall, at $x = 0$, is given by:

$$T(x = 0) \;=\; T_1,$$

and

$$T(x = L) \;=\; T_2.$$