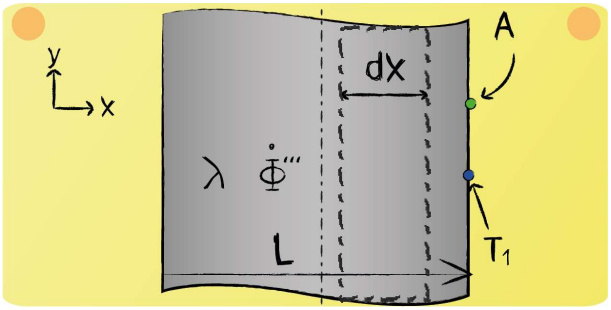
Lecture 13 - Question 7



Develop an energy balance to calculate the temperature profile inside the wall and give the boundary conditions. The plate is losing heat to the environment. Assume one-dimensional steady-state heat with a source. The temperature profile within the wall is symmetrical.

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} + d\dot{\Phi} = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

$$d\dot{\Phi} = \dot{\Phi}''' A dx$$

The outgoing flux can be approxmated by use of the Taylor series expansion.

Boundary Conditions:

$$\frac{\partial T(x=\frac{1}{2}L)}{\partial x} = 0$$

$$T(x = L) = T_1$$

The first boundary condition describes that the temperature gradient in the middle equals zero. This is because of symmetry. The second one describes that the temperature on the right side of the wall equals T_1.