

W2-3-4 The total differential of a function with two variables 4

Consider a cyclical process through the points (x, z) : $(1, 1) \rightarrow (1, 4) \rightarrow (3, 4) \rightarrow (3, 1) \rightarrow (1, 1)$. Calculate the cyclic integral around this path assuming the total differential $dy = 2axdx + 2bzdz$.

The contour integral is: $\oint dy = \oint (2axdx + 2bzdz) = \oint 2axdx + \oint 2bzdz = \int_1^1 2axdx + \int_1^3 2axdx + \int_3^3 2axdx + \int_3^1 2axdx + \int_1^4 2bzdz + \int_4^4 2bzdz + \int_4^1 2bzdz + \int_1^1 2bzdz = 0 + \int_1^3 2axdx + 0 - \int_1^3 2axdx + \int_1^4 2bzdz + 0 - \int_1^4 2bzdz + 0 = 0$. This is the expected answer, because y is a total differential.