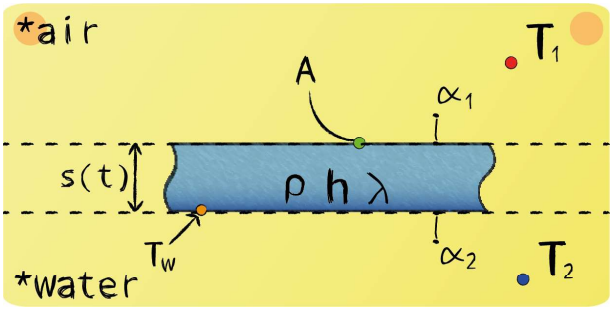
Lecture 14 - Question 10



Ice is continuously increasing in thickness. Convection occurs on both sides. Derive the differential equation to describe the ice layer thickness $s(t)$. The enthalpy of fusion is h. Take $T_2 > T_w > T_1$. The cooling of the ice, contrary to freezing, is energetically negligible, $c_i \cdot \Delta T << h$.

Energy balance:

$$\frac{dU}{dt} = \dot{Q}_{water} - \dot{Q}_{air}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{dU}{dt} = -\rho A h \frac{ds}{dt}$$

The internal energy can be described by use of the enthalpy of fusion. This is denoted with an negative sign, since solidification of water costs energy.

Heat fluxes:

$$\dot{Q}_{water} = \alpha_2 A (T_2 - T_w)$$

$$\dot{Q}_{air} = (T_w - T_1) A \frac{1}{\frac{1}{\alpha_1} + \frac{s(t)}{\lambda}}$$