Tutorial T06 – Material behavior

Answer the following questions as they could come up in an exam.

Exercises 1 – 13 are given in tutorial T05

14 Materials beyond elasticity

Questions:
a) Sketch the stress-strain relation for a linear elastic material, and
b) add possible non-linear material behavior (with explanation/motivation).
c) Explain what happens for unloading of:
c1) a linear, elastic material, or
c2) an elastic-plastic material (for small AND for large strains).
d) Sketch the relation of shear-stress versus strain-rate, for (fluid) materials that behave:
d1) linear,
d2) shear-thickening,
d3) shear-thinning, or
d4) yield-stress fluid.

<u>**Answers:**</u>
a)
Stress-strain linear means a straight line, with slope E being the modulus, while elastic means the return path is identical (from point 2 in Fig. 1 Left), unlike after plastic deformation (from point 4 in Fig. 1 Left).

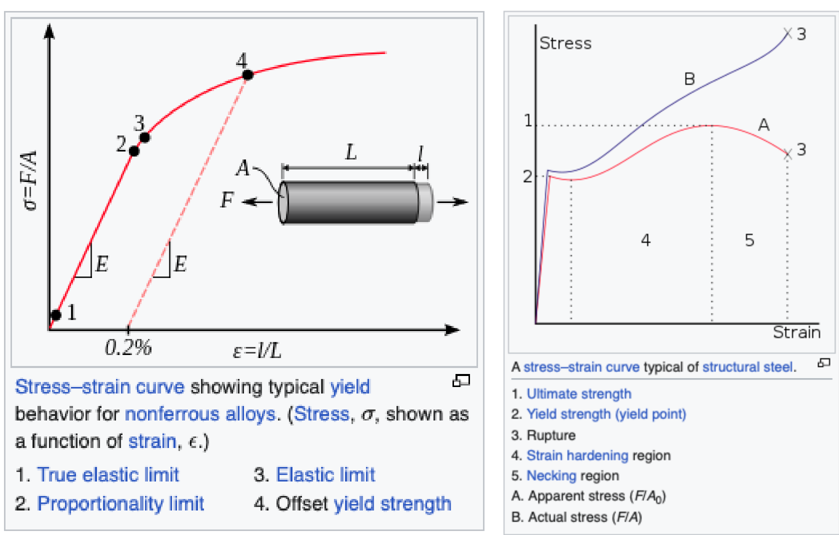
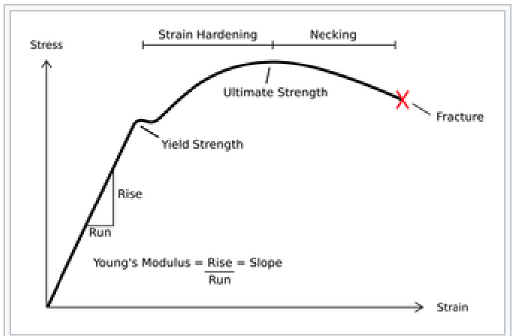


Figure 1: Plots taken from `https://en.wikipedia.org/wiki/Plasticity_(physics)`

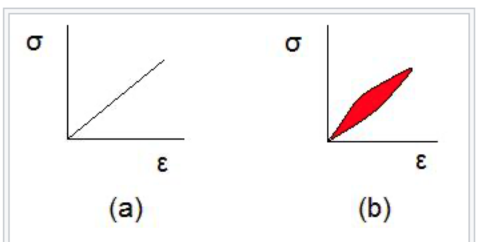
b)
Many types of non-linear behavior are possible, see above Fig. 1, here only a few examples, linear (visco-)elastic, yield strength, strain-hardening, etc. (*note that hardening/softening for solids do not mean the same as for fluids*).
c1)



Typical stress vs. strain diagram indicating the various stages of deformation.

Figure 2: Plot taken from
`https://en.wikipedia.org/wiki/Deformation_(engineering)#Plastic_deformation`

The load-unload curve is reversible in the elastic limit, and usually for small strains (see panel (a) in Fig. 3), whereas for a visco-elastic material one obtains hysteresis (red) in the stress-strain curve, which increases with the strain-rate (see panel (b) in Fig. 3), and actually its surface area (red) is the dissipated viscous energy.



Stress–strain curves for a purely elastic material (a) and a viscoelastic material (b). The red area is a hysteresis loop and shows the amount of energy lost (as heat) in a loading and unloading cycle. It is equal to $\oint \sigma \, d\varepsilon$, where σ is stress and ε is strain.[1]

Figure 3: Plot taken from `https://en.wikipedia.org/wiki/Viscoelasticity`

c2)
for (very) small loads, the stress-strain path is typically reversible (c1) – but after a larger strain, the return (unloading) path is not indentical to the loading path. Four possible stress strain relations for plastic material are given ...

d)
Note that while in solid mechanics, mostly stress and strain are considered, for fluids we typically plot shear-stress versus strain-rate (or shear-rate), with the following material behavior: d1=Newtonian, d2=dilatant, d3=pseudo-plastic, or d4=Bingham plastic (examples).

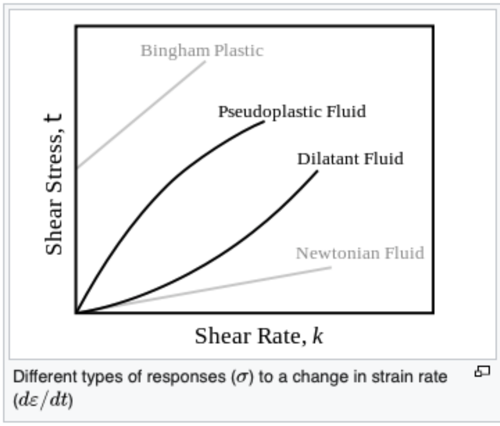


Figure 4: Plot taken from `https://en.wikipedia.org/wiki/Viscoelasticity`

15 Visco-elastic material behavior

Given is a rectangular shaped wire (length $L_0 = 0.1\,\mathrm{m}$, cross-section $A = HW$, volume $V = L_0 A = L_0 HW$) for a homogeneous, isotropic, visco-elastic, rubber-like material.

Questions:
a) What is the work necessary to quickly (or slowly) stretch the wire from stress 0 to length $3L_0$.
b) Which strain-rate is needed for making the elastic and the viscous contributions to work equally important?

Material-properties:
Kevin-Voigt viscoelastic solid (http://en.wikipedia.org/wiki/Viscoelasticity),
relation for stress = function of strain and strain-rate:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

with modulus of Young $E = 0.02\,\mathrm{MPa}$ and viscosity $\eta = 10\,\mathrm{Pa\,s}$.

Answers:
a)
Zero stress: length $L = L_0$, stretch to length $L_1 = 3L \to \varepsilon = \varepsilon_{11} = (L_1 - L_0)/L_0 = 2\,.$

Assume that the cross-section is not changing, i.e., H and W are constant.
Estimate the strain-rate: $\dot{\varepsilon} \approx \text{const.} = (d/dt)\varepsilon = \frac{\text{length-change}}{\text{length*time}} = 2L_0/(L_0\Delta t) = 2/\Delta t\,.$
The specific work is thus:
$a = \int d\varepsilon\sigma = \int d\varepsilon\,(E\varepsilon + \eta\dot{\varepsilon}) = \frac{1}{2}E\varepsilon^2\big|_0^2 + \eta\dot{\varepsilon}\varepsilon\big|_0^2 = 2\,(E + \eta\dot{\varepsilon}) = 2\left(E + \eta\frac{2}{\Delta t}\right)\;,$
The work in the total volume is:
$A = \int dVa = 2VE + 4V\eta/\Delta t$

b)
Estimate the speed at which viscous and elastic contributions to work equal each other:
$E \sim \eta\dot{\varepsilon}$ (after integration), so that (initially) the speed is: $v_0 = L_0\dot{\varepsilon} = 2L_0/\Delta t$,
using: $\dot{\varepsilon} = \dot{\varepsilon}_{ve} = E/\eta$ (from a), and $\Delta t = 2\eta/E$,
so that: $v_0 = L_0\dot{\varepsilon}_{ve} \to v_0 = 0.1 \times 2 \times 10^4/10 = 2 \times 10^2\,\mathrm{m/s}$,
thus a bit smaller than sound-speed.