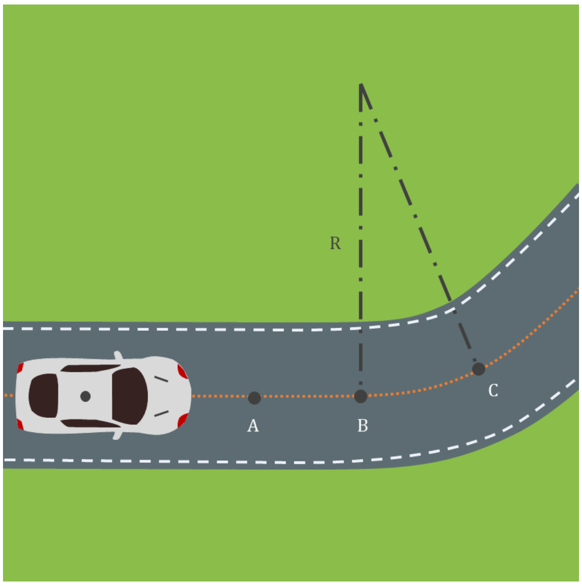
Car Brakes for Corner



A car traveling at speed v_A applies his brakes at point A and reduces his speed at a uniform rate to v_C at point C in a distance s_{AC}. Which of the following equations gives a relation for the tangential acceleration a_t of the car between point A and C?

Using known expressions (for arbitrary acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}v}{a} \tag{1}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}t = \frac{\mathrm{d}s}{v} \tag{2}$$

$$\mathrm{d}t = \frac{\mathrm{d}v}{a} = \frac{\mathrm{d}s}{v} \quad \Rightarrow \quad v\mathrm{d}v = a\mathrm{d}s \tag{3}$$

$$\int_{v_0}^{v_1} v\,\mathrm{d}v = \int_{s_0}^{s_1} a\,\mathrm{d}s \tag{4}$$

Solution:

Lets consider the distance s traveled by the car to measure along the trajectory of the car. Using Equation (5) for a constant acceleration a results in:

$$\int_{v_\mathrm{A}}^{v_\mathrm{C}} v \, \mathrm{d}v = a \int_{s_\mathrm{A}}^{s_\mathrm{C}} \mathrm{d}s \tag{5}$$

$$\frac{1}{2} v^2 \Big|_{v=v_\mathrm{A}}^{v_\mathrm{C}} = as|_{s=s_\mathrm{A}}^{s_\mathrm{C}} \tag{6}$$

$$\frac{1}{2}(v_\mathrm{C}^2 - v_\mathrm{A}^2) = a(s_\mathrm{C} - s_\mathrm{A}) = as_\mathrm{AC} \tag{7}$$

During the traversal of the corner by the car, the acceleration vector can be decomposed into two orthogonal components, the tangential acceleration and the normal acceleration. The tangential acceleration always runs tangent to the path (fully contributes to a) and the normal accelerations always runs normal to the path (does not contribute to a). Since the acceleration is constant, the tangential acceleration equals a. By rewriting equation (7), the tangential acceleration a_t of the car becomes:

$$a_t = a = \frac{v_\mathrm{C}^2 - v_\mathrm{A}^2}{2s_\mathrm{AC}} \tag{8}$$