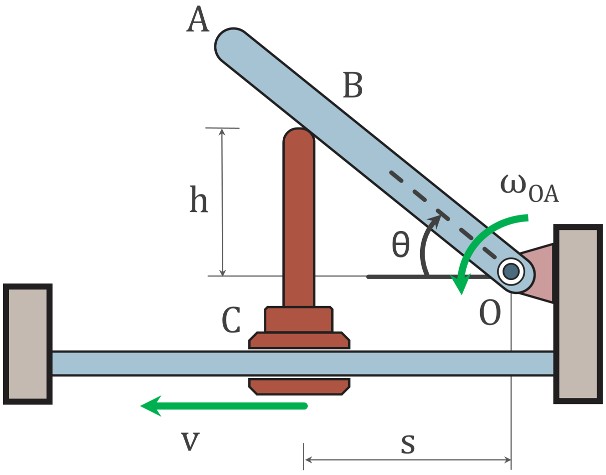
Collar induces Angular Velocity



The collar C moves to the left on a fixed guide with speed v. Determine the magnitude of the angular velocity ω_{OA} as a function of v, the collar position s, and the height h.

Using known expressions (for rigid bodies and a (relative) fixed point O*)*:

$$\mathbf{v}_{\mathrm{B}} = \boldsymbol{\omega}_{\mathrm{OB}} \times \mathbf{r}_{\mathrm{B/O}} \tag{1}$$

Given:

Speed of collar C: v
Velocity of collar C: $\mathbf{v} = -v\mathbf{i}$
Absolute angular velocity: ω_{OA}
Angular velocity: $\boldsymbol{\omega}_{\mathrm{OA}} = \omega_{\mathrm{OA}}\mathbf{k}$
Horizontal displacement of collar C: s
Height of contact point B with respect to point O: h

Solution:

Figure 1 shows the kinematic diagram of the situation, including geometric relations.

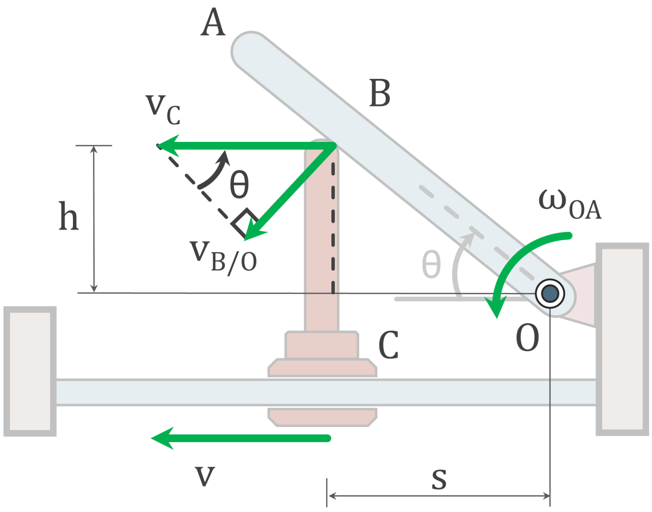


Figure 1: Kinematic diagram of the collar C and rotating bar AO.

The absolute angular velocity can be determined using the speed v_B and the distance L_OB with the relation from Equation 1. This results in:

$$v_\text{B} = |\mathbf{v}_\text{B}| = |\boldsymbol{\omega}_\text{OB}||\mathbf{r}_\text{B/O}| = \omega_\text{OB} L_\text{OB} \tag{2}$$

Where $L_\text{OB} = \sqrt{h^2 + s^2}$.

For the time instant that $\theta = 90°$, this results in a horizontal velocity that must be equal to $v_\text{B} = v$.

$$v_\text{B} = \omega_\text{OB} h = v \tag{3}$$

These two equations can be manipulated to get an equation for ω_OB. First, a relation to write v_B in terms of v_C must be found. From the geometry of Figure 1 it follows:

$$v_\text{C} = \frac{\sqrt{h^2 + s^2}}{h} v_\text{B} \tag{4}$$

Inserting Equation (3) into Equation (4) gives:

$$v_{\mathrm{C}} = \frac{\sqrt{h^2 + s^2}}{h} v_{\mathrm{B}} = \frac{\sqrt{h^2 + s^2}}{h} \sqrt{h^2 + s^2} \cdot \omega_{\mathrm{OB}} = \frac{h^2 + s^2}{h} \omega_{\mathrm{OB}} = v \tag{5}$$

Rewriting give the following relation for ω_{OB}:

$$\omega_{\mathrm{OB}} = \frac{vh}{h^2 + s^2} \tag{6}$$