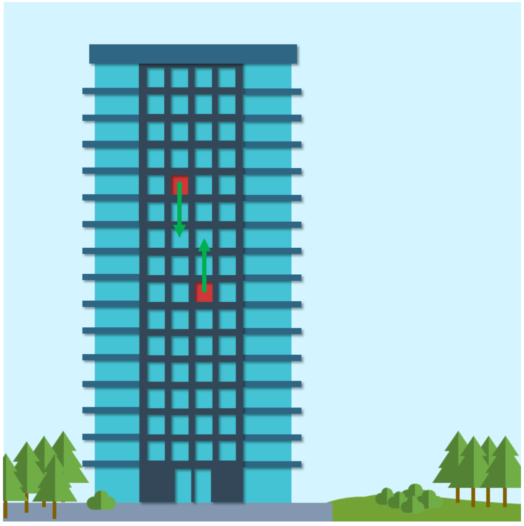
Elevator in Building



The elevator of a skyscraper rises 350 meters at a maximum speed of 22 km/h. Both the acceleration and deceleration have a constant magnitude of 0.25g during this rise. After reaching its maximum speed the elevator stops accelerating and keeps moving at this speed until it is time to decelerate to come to a full stop at the top floor. Determine the duration T of this elevator rise.

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

Given:

Distance: $s = 350m$

Velocity: $v = 22km/h = 6.11m/s$

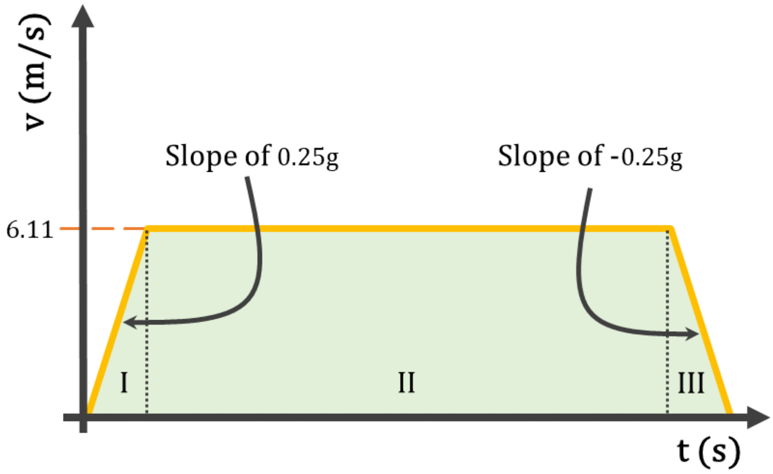


Figure 1: Visualization graph of changing velocity over time

Acceleration: $a = 0.25g = 2.4525m/s^2$

The easiest thing is to visualize the problem by drawing a graph of how the velocity changes over time (see Figure 1). The problem is divided in three parts, the first one is the acceleration of $a = 0.25g$ of the elevator from standstill ($v = 0$) to its maximum speed of $v = 6.11m/s$, then there is a time the elevator has a constant speed of 6.11 m/s and finally it decelerates with $a = -0.25g$ to standstill.

Using Equation 3, the time for the elevator to reach its maximum constant velocity of $6.11m/s$ from standstill ($v_0 = 0$) can be calculated.

$$t = \frac{v}{a} \Rightarrow t = \frac{6.11}{0.25 \cdot 9.81} = 2.49s \tag{7}$$

The travelled distance in this time is the area of section I of Figure 1. It can be directly seen from the figure or calculated using Equation 6, with $s_0 = v_0 = 0$.

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 = \frac{1}{2}a \cdot t^2 \Rightarrow s(2.49) = \frac{1}{2}0.25 \cdot g \cdot 2.49^2 = 7.61m \tag{8}$$

The elevator both accelerates and decelerates thus the total distance travelled during this motion is $2 \cdot 7.61 = 15.22m$ (area I and area II from the figure combined).
To calculate the time the velocity of the elevator is $6.11m/s$, we use the fact that the total length is $350m$ (total area under the curve). Thus the distance the elevator travels at $v = 6.11m/s$ (or area II under the curve) is equal to $350 - 15.22 = 334.8m$.

Since the velocity is constant the acceleration is 0 thus Equation 6 becomes:

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 = v_0 \cdot t \Rightarrow t = \frac{s}{v} = \frac{334.8}{6.11} = 54.79s \tag{9}$$

Thus the total time is $t = 2.49 + 54.79 + 2.49 = 59.77s$