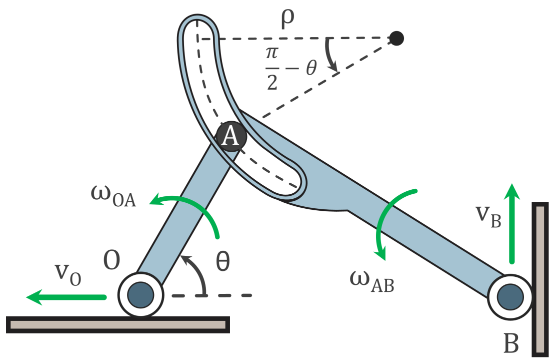
# Sliding Bar in Slot




**Bar OA slides over a horizontal platform and bar AB slides over a vertical platform. Both bars are connect through a slot in bar AB where point A slides through. The following properties of the frame structure are valid for this time-instant. Triangle OBA is a $1-2-\sqrt{3}$ triangle, where $\theta = \frac{\pi}{3}$ rad. The distance between O and A is 1 m. Furthermore, $\rho = 3\sqrt{3}$ m, $v_O = \frac{3}{2}\sqrt{3}$ m/s, $v_B = 4$ m/s and $\omega_{OA} = \frac{1}{2}$ rad/s. Calculate the angular velocity $\omega_{AB}$ in rad/s.**

*Using known expressions*:

$$\mathbf{v}_{\mathrm{A}} = \mathbf{v}_{\mathrm{O}} + \boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} + \mathbf{v}_{\mathrm{rel}} \tag{1}$$

$$\mathbf{v}_{\mathrm{A}} = \mathbf{v}_{\mathrm{B}} + \boldsymbol{\omega}_{\mathrm{AB}} \times \mathbf{r}_{\mathrm{A/B}} + \mathbf{v}_{\mathrm{rel}} \tag{2}$$

*Given*:

Length of OA: $L_{OA} = 1$ m
Angle: $\theta = \frac{\pi}{3}$ rad
Velocity of O: $v_O = \frac{3}{2}\sqrt{3}$ m/s
Angular velocity OA: $\omega_{OA} = \frac{1}{2}$ rad/s

Velocity of B: $v_B = 4$ m/s
Radius of curvature of A: $\rho = 3\sqrt{3}$ m
Angle $\angle$ OAB: 90°

*Solution*:

Using the fact that $\angle$ OAB $= 90°$, together with the angle $\theta = \frac{\pi}{3}$ rad and the length of OA is 1 m, the length of AB is calculated to be $L_{AB} = \sqrt{3}$ m. (Triangle $\triangle$ OAB is a $30-60-90$ triangle, thus the lengths of OA, AB and OB are 1, $\sqrt{3}$ and 2 m respectively). Inserting the known values in Equation 1 gives:

$$\mathbf{v}_{\mathrm{A}} = \begin{pmatrix} -v_O \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{OA} \end{pmatrix} \times \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} L_{OA} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{v}_{\mathrm{A}} = \begin{pmatrix} -v_O \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} L_{OA} \cdot \omega_{OA} = \begin{pmatrix} -v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA} \\ \cos\theta \cdot L_{OA} \cdot \omega_{OA} \\ 0 \end{pmatrix} \tag{4}$$

Inserting the known values in Equation 2 gives:

$$\mathbf{v}_{\mathrm{A}} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} L_{AB} + \begin{pmatrix} -\cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \cdot v_{rel} \tag{5}$$

$$\mathbf{v}_{\mathrm{A}} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} + \begin{pmatrix} -\sin\theta \\ -\cos\theta \\ 0 \end{pmatrix} \cdot L_{AB} \cdot \omega_{AB} + \begin{pmatrix} -\cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \cdot v_{rel} \tag{6}$$

$$\mathbf{v}_{\mathrm{A}} = \begin{pmatrix} -(L_{AB} \cdot \omega_{AB} + v_{rel}) \cdot \cos\theta \\ v_B + (v_{rel} - L_{AB} \cdot \omega_{AB}) \cdot \sin\theta \\ 0 \end{pmatrix} \tag{7}$$

We now have two equations (Equations 4 and 7) for $v_A$ that must be equal to each other. This follows.

$$\begin{pmatrix} -v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA} \\ \cos\theta \cdot L_{OA} \cdot \omega_{OA} \\ 0 \end{pmatrix} = \begin{pmatrix} -(L_{AB} \cdot \omega_{AB} + v_{rel}) \cdot \cos\theta \\ v_B + (v_{rel} - L_{AB} \cdot \omega_{AB}) \cdot \sin\theta \\ 0 \end{pmatrix} \tag{8}$$

This gives us two equations with two unknowns, $v_{rel}$ and $\omega_{AB}$, thus it is possible to solve for $\omega_{AB}$. First we solve for $v_{rel}$ using the top equation, as we can use this in the second equation to solve for $\omega_{AB}$:

$$-v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA} = -(L_{AB} \cdot \omega_{AB} + v_{rel}) \cdot \cos\theta \tag{9}$$

$$v_{rel} = \frac{v_O + \sin\theta \cdot \omega_{OA} \cdot L_{OA}}{\cos\theta} - L_{AB} \cdot \omega_{AB} \tag{10}$$

Inserting this in the second equation gives us a solution for $\omega_{AB}$.

$$\cos\theta \cdot L_{OA} \cdot \omega_{OA} = v_B + (v_{rel} - L_{AB} \cdot \omega_{AB}) \cdot \sin\theta \tag{11}$$

$$\cos\theta \cdot L_{OA} \cdot \omega_{OA} = v_B + \left(\frac{v_O + \sin\theta \cdot \omega_{OA} \cdot L_{OA}}{\cos\theta} - 2L_{AB} \cdot \omega_{AB}\right) \cdot \sin\theta \tag{12}$$

$$-2L_{AB} \cdot \omega_{AB} = \frac{\cos\theta \cdot L_{OA} \cdot \omega_{OA} - v_B}{\sin\theta} - \frac{v_O + \sin\theta \cdot \omega_{OA} \cdot L_{OA}}{\cos\theta} \tag{13}$$

$$\omega_{AB} = \frac{-\cos\theta \cdot L_{OA} \cdot \omega_{OA} + v_B}{2L_{AB} \cdot \sin\theta} + \frac{v_O + \sin\theta \cdot \omega_{OA} \cdot L_{OA}}{2L_{AB} \cdot \cos\theta} \tag{14}$$

Inserting the values for $\theta, L_{OA}, L_{AB}, \omega_{OA}, v_O$ and $v_B$ gives:

$$\omega_{AB} = \frac{-\cos\left(\frac{\pi}{3}\right) \cdot 1 \cdot \frac{1}{2} + 4}{2\sqrt{3} \cdot \sin\left(\frac{\pi}{3}\right)} + \frac{\frac{3}{2}\sqrt{3} + \sin\left(\frac{\pi}{3}\right) \cdot \frac{1}{2} \cdot 1}{2\sqrt{3} \cdot \cos\left(\frac{\pi}{3}\right)} \tag{15}$$

$$\omega_{AB} = \frac{-\frac{1}{2} \cdot \frac{1}{2} + 4}{2\sqrt{3} \cdot \frac{1}{2}\sqrt{3}} + \frac{\frac{3}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2} \cdot 1}{2\sqrt{3} \cdot \frac{1}{2}} \tag{16}$$

$$\omega_{AB} = \frac{\frac{15}{4}}{3} + \frac{\frac{7}{4}\sqrt{3}}{\sqrt{3}} = \frac{5}{4} + \frac{7}{4} = 3 \quad \text{rad/s} \tag{17}$$