Solutions lecture 5

5.1 Heat loss of a person by radiation

Analysis

We need to determine the rate of heat loss from a person by radiation.

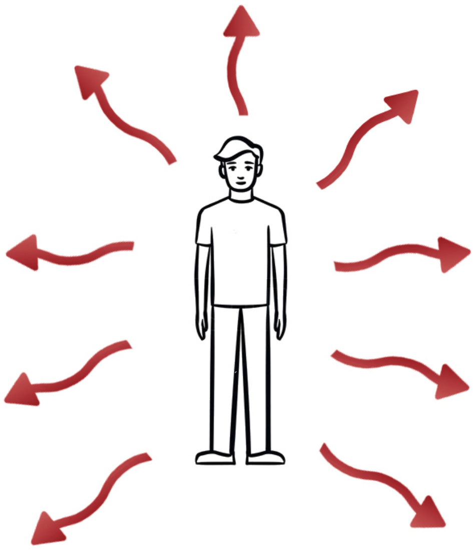


Figure 5.1: Radiation person

Given values are the surface area of the person, $A = 1.7\,\mathrm{m}^2$, the emissivity constant $\varepsilon = 0.7$, the temperature of the person, $T_s = 32\,°\mathrm{C}$, and the temperature of the environment (the wall temperature) of $T_\infty = 27\,°\mathrm{C}$

Approach

Assumptions

Route to solution

This question can be solved by substituting all values in the Stefan-Boltzmann law:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot (T^4 - T_\infty^4)$$

Where $\sigma = 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. Note that the temperatures are in **Kelvin**, not Celsius!

Elaboration

Substituting all values gives:

$$\dot{Q}_{rad} = 0.7 \cdot 5.67 \cdot 10^{-8} \cdot 1.7(305.15^4 - 300.15^4) = 37.4W$$

Evaluation

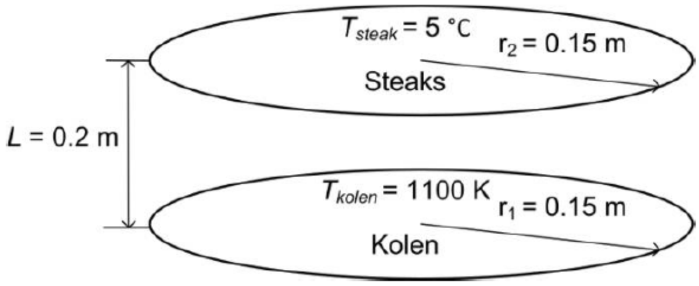
Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

5.2 The BBQ

Analysis

We need to determine the initial rate of radiation heat transfer from the coal bricks to the steaks,and the initial rate of radiation heat transfer to the stakes if the side opening of the grill is covered by aluminium foil, which can be approximated as a re-radiating surface. The diameter of the grill is 0.30m. The coal bricks have a temperature of 827 °C, the steaks have an initial temperature of 5 °C. The distance between the bricks and the steaks is 0.20 m. See the sketch below.



Approach

Assumptions

Both surfaces can be treated as blackbodies, we assume the grill to be completely covered with steaks.

Route to solution

This question can be solved by substituting all values in the Stefan-Boltzmann law:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot (T_{coal}^4 - T_{steak}^4)$$

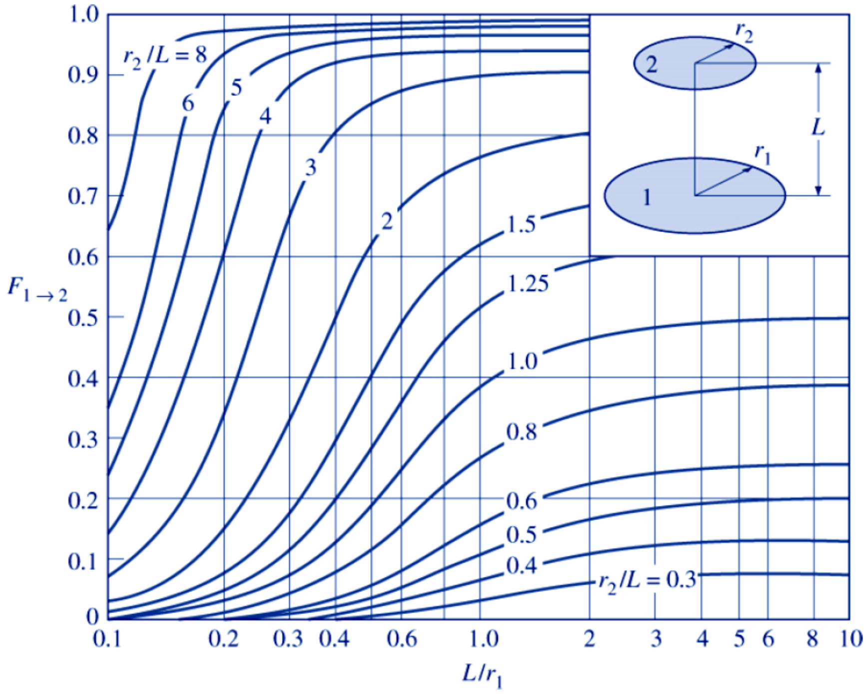
Where $\sigma = 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. Note that the temperatures are in **Kelvin**, not Celsius! However, not all radiation will reach the steaks, since there is a fair distance between the surfaces. Consequently, the 'visibility factor' $F_{1\to2}$ needs to be calculated. This factor gives the ratio between the radiation coming from surface 1 reaching surface 2. Since both bodies can be treated as black bodies, they both have an emissivity of $\varepsilon = 1$.

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot F_{1\to2}(T_{coal}^4 - T_{steak}^4)$$

Determining the visibility factor $F_{1\to2}$ can be done using the diagram given in lecture 5, also given below.

Substituting all values, in Stefan Boltzmann's law will give the initial rate of radiation heat transfer.

For the second part, we assume that the aluminum foil is an ideal reflector, such that all radiation coming from the coals reaches the steaks. In other words, $F_{1\to2} = 1$. Substituting all values will give the initial rate of radiation heat transfer for this case.



Elaboration

We first need to determine the visibility factor from the figure. The following ratios are needed:

L/r_{coal} (along the x-axis) $\implies$ in this situation; $L/r_{coal} = 0.2/0.15 = 1.33$ (x-axis)

r_{steak}/L (along the lines) $\implies$ in this situation; $r_{steak}/L = 0.15\ /\ 0.2 = 0.75$ (line)

The line 0.75 is not depicted in the diagram, so the answer needs to be extrapolated between the 0.6 and 0.8 line at the 1.33 point on the x-axis. This gives a visibility factor of about 0.28. All parameters are now determined and can be substituted into the formula:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot F_{1\to2}(T^4_{coal} - T^4_{steak}) = 1 \cdot 5.67 \cdot 10^{-8} \cdot \pi \cdot 0.15^2 \cdot 0.28 \cdot (1100^4 - 278^4) = 1637\text{W}$$

And for the case when the sides are covered with aluminum foil:

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A \cdot F_{1\to2}(T^4_{coal} - T^4_{steak}) = 1 \cdot 5.67 \cdot 10^{-8} \cdot \pi \cdot 0.15^2 \cdot 1 \cdot (1100^4 - 278^4) = 5845\text{W}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

5.3 Radiation of heat from a coffee machine

Analysis

We look at the coffee machine from exercise 4.2. We need to determine the emissivity of the heater plate surface, and find the total thermal resistance between the heater surface and the surroundings, as well as the total heat transfer coefficient, including convection and radiation.

Approach

Assumptions

Route to solution

The total power is 90 W. From that, $90 - 42.9 = 47.1$ W is radiation. Equating this to Stefan Boltzmanns law, will give ε.

For the second question, $h_{convection}$ was determined in 4.2. The radiation part can be determined with

$$h_{rad} = \varepsilon \cdot \sigma(T_s^2 + T_\infty^2) \cdot (T_s + T_\infty)$$

The total heat transfer coefficient is defined by

$$h_{tot} = h_{convection} + h_{rad}$$

The thermal resistance from the convection part is :

$$R_{conv} = \frac{1}{h_{convection} A}$$

And the thermal resistance for the radiation part:

$$R_{rad} = \frac{1}{h_{rad} \cdot A} = \frac{1}{\varepsilon \cdot \sigma \cdot (T_s^2 + T_\infty^2) \cdot (T_s + T_\infty) \cdot A}$$

The total thermal resistance is then

$$\frac{1}{R_{tot}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$

Elaboration

For the first question, we can substitute all known values in Stefan Boltzmann's law.

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot F_{1 \to \text{surface}} \cdot A \cdot (T_s^4 - T_\infty^4) = 47.1\text{W}$$

With a visibility factor of 1, we get

$$\dot{Q}_{rad} = \varepsilon \cdot 5.670 \cdot 10^{-8} \cdot 1 \cdot \pi \left(\frac{0.16}{2}\right)^2 \cdot ((220 + 273)^4 - (20 + 273)^4) = 47.1\text{W}$$

This gives $\varepsilon = 0.799$.

Now for the second question, substituting the known values into the equation for the radiation heat transfer coefficient:

$$h_{rad} = \varepsilon \cdot \sigma(T_s^2 + T_\infty^2) \cdot (T_s + T_\infty) = 11.7\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$$

With $h_{convection} = 10.7\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$, the total heat transfer coefficient becomes

$$h_{tot} = h_{convection} + h_{rad} = 10.7 + 11.7 = 22.4\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$$

The convection resistance is

$$R_{conv} = \frac{1}{h_{convection} A} = \frac{1}{10.7 \cdot \pi \cdot \left(\frac{0.16}{2}\right)^2} = 4.65\,\text{K}\,\text{W}^{-1}$$

The radiation resistance:

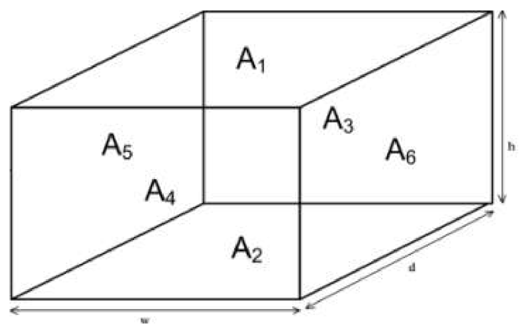
$$R_{rad} = \frac{1}{h_{rad} \cdot A} = \frac{1}{\varepsilon \cdot \sigma \cdot (T_s^2 + T_\infty^2) \cdot (T_s + T_\infty) \cdot A} = 4.10\,\mathrm{K\,W^{-1}}$$

The total thermal resistance is then

$$\frac{1}{R_{tot}} = \frac{1}{4.65} + \frac{1}{4.10} = 2.22\,\mathrm{K\,W^{-1}}$$

5.4 Heating a meal in an oven

a) The oven can be seen as a box with 6 faces, of which 1 radiates and the other 5 absorbs faces. It can be assumed that all faces are blackbodies ($\epsilon = 1$)



$F_{1\to2}$:

$$\frac{L_2}{D} = \frac{w}{h} = \frac{0.45}{0.30} = 1.5$$
$$\frac{L_1}{D} = \frac{d}{h} = \frac{0.30}{0.30} = 1$$
$$\Rightarrow F_{1\to2} = 0.26$$

$F_{1\to3} = F_{1\to4}$:

$$\frac{L_2}{W} = \frac{h}{w} = \frac{0.30}{0.45} = 0.67$$
$$\frac{L_1}{W} = \frac{d}{w} = \frac{0.30}{0.45} = 0.67$$
$$\Rightarrow F_{1\to3} = F_{1\to4} = 0.23$$

$F_{1\to5} = F_{1\to6}$:

$$\frac{L_2}{W} = \frac{h}{d} = \frac{0.30}{0.30} = 1$$
$$\frac{L_1}{W} = \frac{w}{d} = \frac{0.45}{0.30} = 1.5$$
$$\Rightarrow F_{1\to5} = F_{1\to6} = 0.15$$

b) Sum of the view factors:

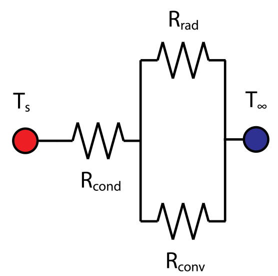
$$\sum F_{1\to\dots} = 1.02$$

It should have been 1. The difference is caused by reading errors.

c) Elaboration depends on the values found for aluminum foil. A possible solution gives:

It is assumed that 1 layer of 0.014 mm thick aluminum foil is added with an emissivity of 0.07:

Thermal resistance network describing the rate of heat transfer from the food towards the environment:



$$R_{\text{cond}} = \frac{\Delta x}{kA} = \frac{1.4 \cdot 10^{-5}}{237 \cdot 0.085} = 7 \cdot 10^{-7}\ \text{K/W}$$

After heat is conducted it is transported by radiation and convection:

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A}$$

Where:

$$h_{\text{rad}} = \epsilon\sigma \cdot \left(T_{\text{s}}^2 + T_{\infty}^2\right) \cdot \left(T_{\text{s}} + T_{\infty}\right) = 0.070 \cdot 5.67 \cdot 10^{-8} \cdot \left(453^2 + 293^2\right) \cdot (453 + 293) = 0.86 \text{ W/m}^2\text{K}$$

So:

$$R_{\text{rad}} = \frac{1}{0.86 \cdot 0.085} = 13.7 \text{ K/W}$$

And the convective resistance:

$$R_{\text{conv}} = \frac{1}{h_{\text{conv}}} = \frac{1}{10 \cdot 0.085} = 1.17 \text{ K/W}$$

Parallel resistance:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_{\text{rad}}} + \frac{1}{R_{\text{conv}}} \Rightarrow R_{\text{parallel}} = 1.07 \text{ K/W}$$

It can be concluded that $R_{\text{cond}} << R_{\text{parallel}}$ and therefore it is reasonable to assume the same surface temperature in the scenarios with and without aluminum foil.

d) Conduction through the aluminum foil may be neglected as it is a good conductor en it is very thin. Furthermore, it is assumed that the foil is packed tightly around the meal and does not affect the geometry for convection.

$$\dot{Q}_{\text{conv}} = hA\left(T_{\text{s}} - T_{\infty}\right) = 10 \cdot 0.085\,(180 - 25) = 131.75 \text{ W}$$

$$\dot{Q}_{\text{rad,no foil}} = \epsilon\sigma F_{1\to\text{surr}} A\left(T_{\text{s}}^4 - T_{\infty}^4\right) = 0.95 \cdot 5.67 \cdot 10^{8} \cdot 1 \cdot 0.085 \left((180 + 273)^4 - (25 + 273)^4\right) 156.7 \text{ W}$$

$$\dot{Q}_{\text{rad,with foil}} = \epsilon\sigma F_{1\to\text{surr}} A\left(T_{\text{s}}^4 - T_{\infty}^4\right) = 0.07 \cdot 5.67 \cdot 10^{8} \cdot 1 \cdot 0.085 \left((180 + 273)^4 - (25 + 273)^4\right) 11.5 \text{ W}$$

$$\dot{Q}_{\text{no foil}} = \dot{Q}_{\text{rad,no foil}} + \dot{Q}_{\text{conv}} = 156.7 + 131.75 = 288.45 \text{ W}$$

$$\dot{Q}_{\text{with foil}} = \dot{Q}_{\text{rad,with foil}} + \dot{Q}_{\text{conv}} = 11.5 + 131.75 = 143.25 \text{ W}$$

$$\eta = \frac{\dot{Q}_{\text{no foil}} - \dot{Q}_{\text{with foil}}}{\dot{Q}_{\text{no foil}}} \cdot 100\% = \frac{288.45 - 143.25}{288.45} \cdot 100\% = 50.3\%$$

e) Wavelength at maximum radiant power.
Maximum power at:

$$\lambda \cdot T = 2897.8\ \mu\text{m} \cdot \text{K}$$

$$\Rightarrow \lambda = \frac{2897.8}{180 + 273} = 6.397\ \mu\text{m}$$

f) Is it visible? Visible wavelengths range from 0.4 to 0.76 μm
$\lambda \cdot T$ for the highest wavelength:

$$0.76 \cdot (180 + 273) = 344.28\ \mu\text{m} \cdot \text{K}$$

Looking up in the table gives an $f_\lambda = 0$. Everything below this wavelength apparently has a negligible radiation fraction, so a wavelength range within this is certainly the case. The radiation is therefore not visible.