Consider a system that can exchange energy with the environment through magnetic work in addition to the regular heat flows and volume work. The magnetic work is represented by the term Bdm, where m is the magnetization and B the applied magnetic field. Adding this term to the differential equation for internal energy results in: $du = Tds - Pdv + Bdm$.

Derive the Maxwell equation that follows from the differential expression for internal energy in case that the process is isentropic. Clearly indicate with formulas /describe with words all steps that you do (if only the answer is given, it is considered wrong).

Isentropic: $ds = 0 \rightarrow du = -Pdv + Bdm$

$$du = \left(\frac{\partial u}{\partial m}\right)_v dm + \left(\frac{\partial u}{\partial v}\right)_m dv = Bdm - Pdv$$

$$B = \left(\frac{\partial u}{\partial m}\right)_v \quad \text{and} \quad -P = \left(\frac{\partial u}{\partial v}\right)_m$$

$$\frac{\partial^2 u}{\partial v \partial m} = \left(\frac{\partial}{\partial v}\left(\frac{\partial u}{\partial m}\right)_v\right)_m = \left(\frac{\partial B}{\partial v}\right)_m$$

$$\frac{\partial^2 u}{\partial m \partial v} = \left(\frac{\partial}{\partial m}\left(\frac{\partial u}{\partial v}\right)_m\right)_v = -\left(\frac{\partial P}{\partial m}\right)_v$$

$$\frac{\partial^2 u}{\partial v \partial m} = \frac{\partial^2 u}{\partial m \partial v} \rightarrow \left(\frac{\partial B}{\partial v}\right)_m = -\left(\frac{\partial P}{\partial m}\right)_v$$