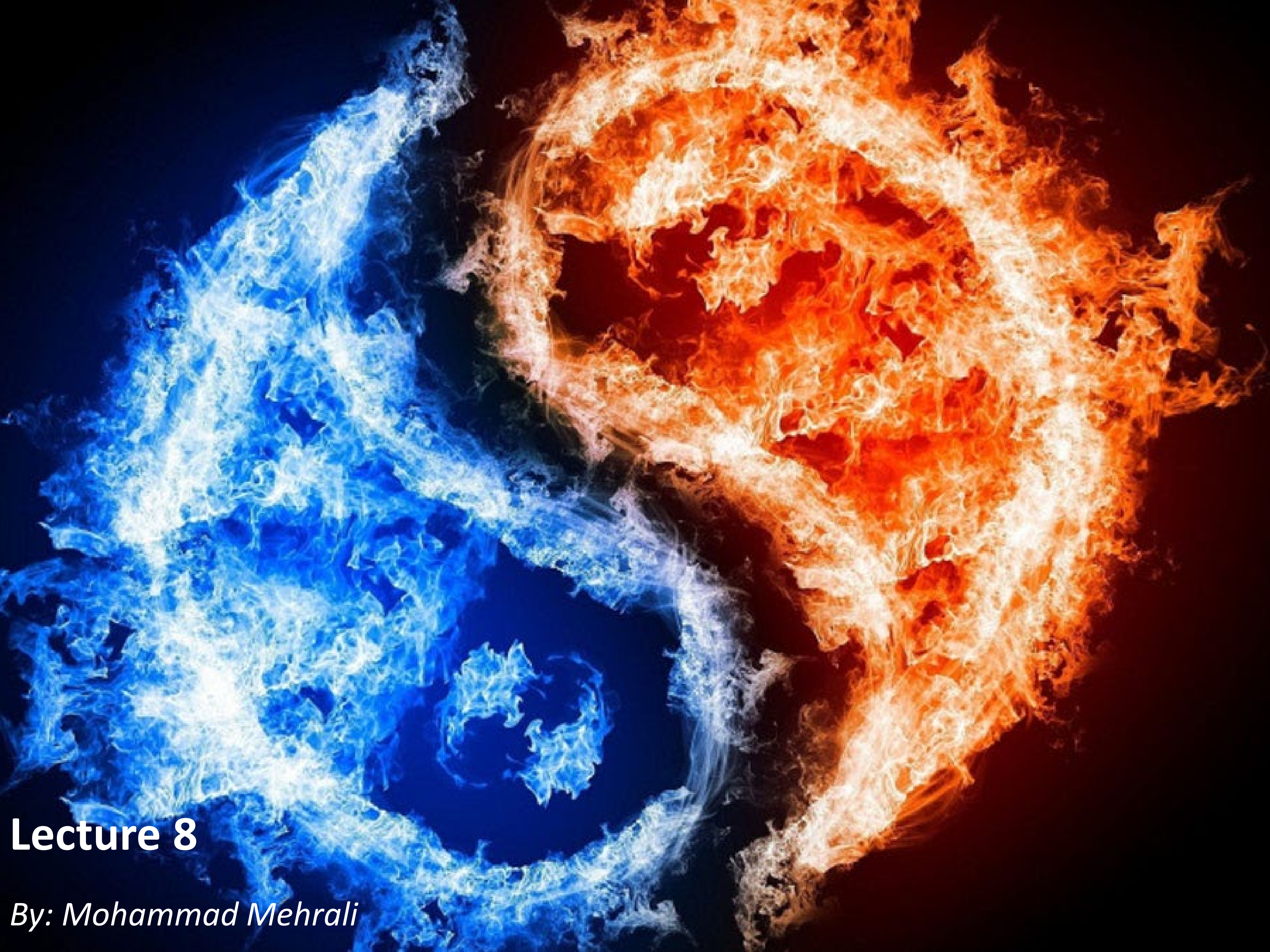


Lecture 8

By: Mohammad Mehrali

- Allowed items in the exam :
  - Reference books
  - Slides and notes
  - Calculator

Please note that only table of air properties will be given to you.

- Q&A Session (To be announced)

Work, energy, power

- Work W, energy E, power P
- Units J, W = J/s, kWh, hp, …
- Work *W*, Energy *E* in
- Power in W = J / s
- Units kWh, kcal, hp, ….
- Comparison / estimating / proportion

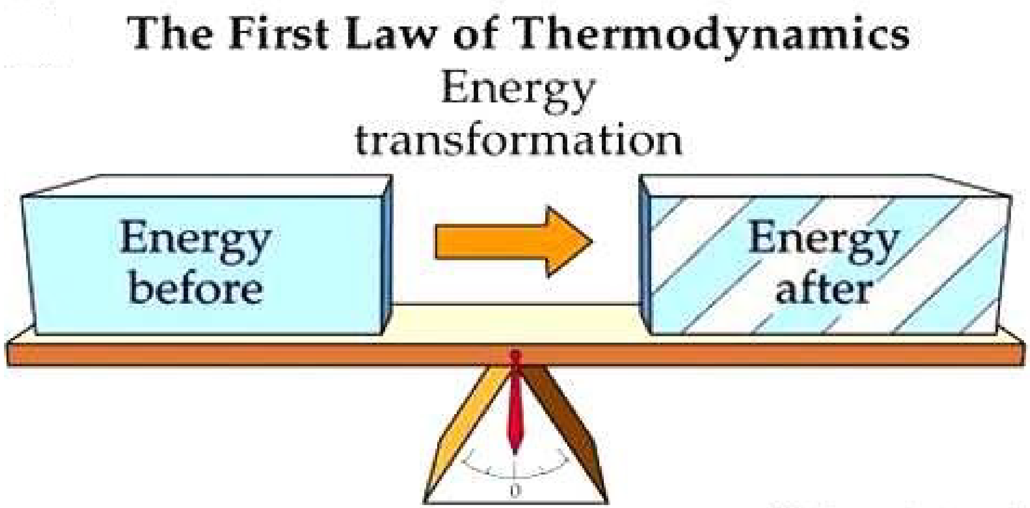
WORK, ENERGY, POWER

Symbol	Definition	Units
W	Work	kJ
w	Specific work = work per unit mass, w = W/m	kJ/kg
	Power = rate of work*	kW (= kJ/s)
Q	Heat transfer	kJ
q	Specific heat transfer = heat transfer per unit mass, q = Q/m	kJ/kg
$\dot{Q}$	Rate of heat transfer*	kW (= kJ/s)

- NOTE: Rates are denoted by a dot on top of the variable.

Energy balance

- Energy is **always conserved**!
- First law of Thermodynamics:



Efficiency

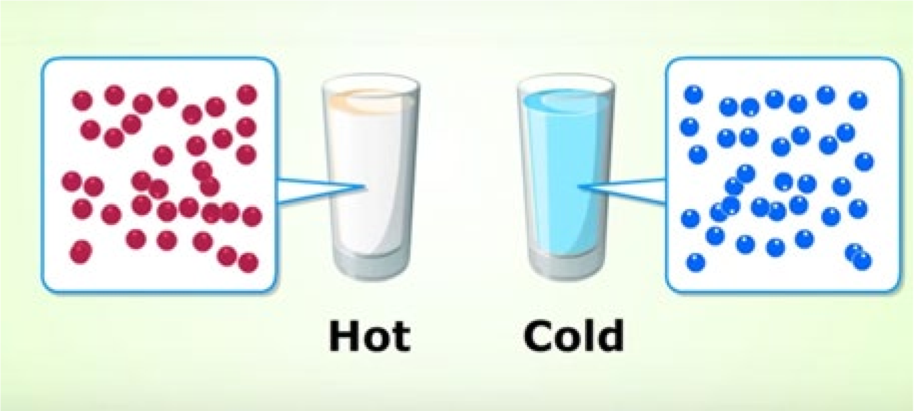
- Efficiency = fraction of "useful" work/power used
  - What is defined as useful?
  - What is the reference?

- Efficiency $\eta = \dfrac{\textbf{useful work}}{\textbf{input energy}} = \dfrac{\textbf{useful power}}{\textbf{input power}}$ (-)

- Use a Sankey diagram!

Thermal energy

- Thermal energy: kinetic energy of molecules and atoms.



Heat Transfer: $\Delta E = Q$ [J]

m = mass of "system" (kg)

ΔT = temperature change of system during process (K)

c = specific heat (J / (kg · K))

$$\boldsymbol{Q = m \cdot c \cdot \Delta T} \text{ [J]}$$

Recap of last lectures

Heat Transfer Modes

Conduction

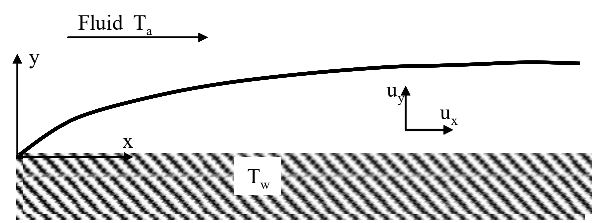
- **Fourier Law**

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x}\,[W]$$

Thermal Conductivity [W/m.K]
Material properties

Cross-Sectional Area $[\boldsymbol{m^2}]$

Convection



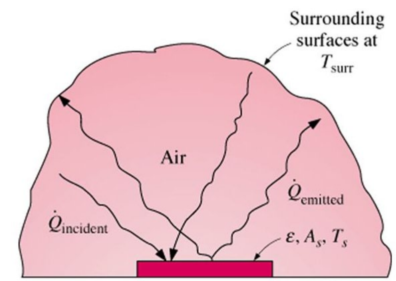
- **Newton's law of cooling**

$$\dot{Q} = hA(T_w - T_a)\,[W]$$

Convective Heat Transfer Coefficient [W/m²K]
Flow dependent

- **Natural Convection**
- **Forced Convection**

Radiation



- **Stefan-Boltzmann law**

$$\dot{Q} = \varepsilon\sigma A(T_s^4 - T_\infty^4)[W]$$

Emissivity

Stefan-Boltzmann constant

$$\sigma = 5.670 \times 10^{-8}\ \frac{W}{m^2K^4}$$

Step by Step Plan

Conduction:

1. Schematic (Steady state)

2. Negligible heat losses (insulation, adiabatic,..)

3. Geometry (flat plate, cylinder,…)

4. Arrangement of the layers

5. Resistance network

6. Calculation of resistances

7. Calculation of overall resistance

8. Calculation of heat flow

9. Calculation of temperatures for different layers

Radiation:

1. Schematic (Steady state)

2. Negligible Radiations

3. Surrounding Temperature

4. Calculation of heat flow

Convection:

1. Schematic

2. Force of natural convection?

3. Geometry

4. Finding the film temperature

5. Finding ingredients from Table

6. Calculation of dimensionless numbers

7. Calculation of characteristic length

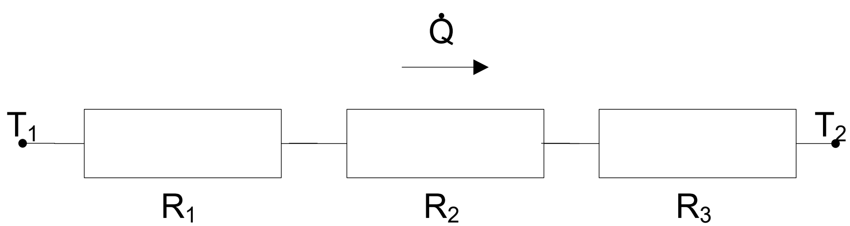
8. Finding correlations from tables to calculate Nu

9. Calculation of h

10. Calculation of heat flow

Thermal resistance networks

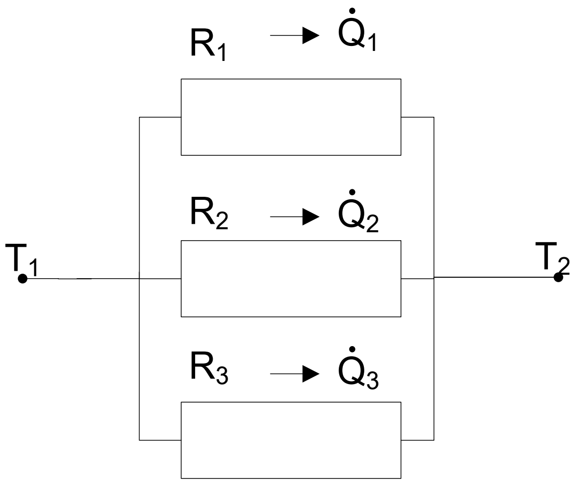
Series Resistors



$$R_{tot} = \sum_i R_i$$

(Add Resistors)

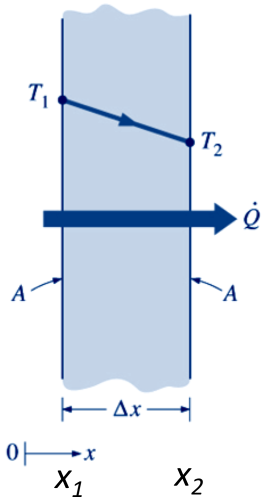
Parallel Resistors



$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

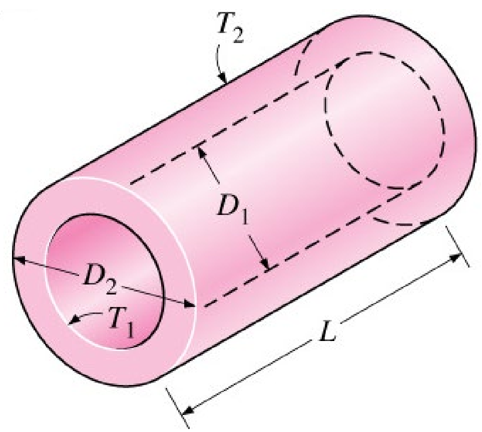
(Add Heat Flows)

VARIOUS CONDUCTION RESISTANCES



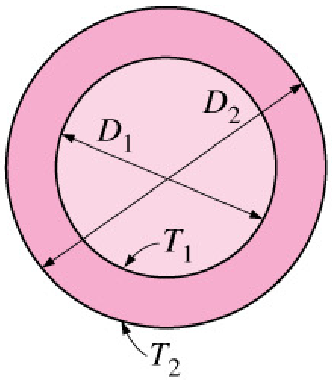
Plane wall

$$R = \frac{\Delta x}{kA}$$



Cilindrical pipe

$$R = \frac{\ln(\frac{D_2}{D_1})}{2\pi L k}$$



Spherical shell

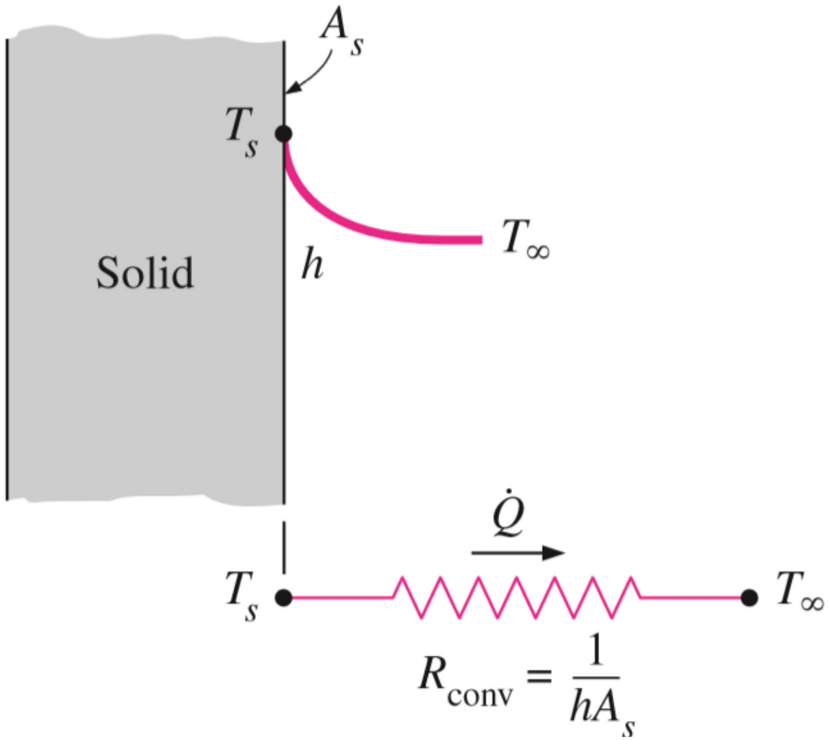
$$R = \frac{D_2 - D_1}{2\pi k D_1 D_2}$$

$$\boxed{\dot{Q} = \frac{T_1 - T_2}{R}}$$

CONVECTION REISTANCE

$$\dot{Q} = hA\Delta T = \frac{\Delta T}{\frac{1}{hA}} \text{ with } \Delta T = T_S - T_\infty$$

$$\Rightarrow \dot{Q} = \frac{\Delta T}{R_{conv}}$$



Where **convection resistance**:

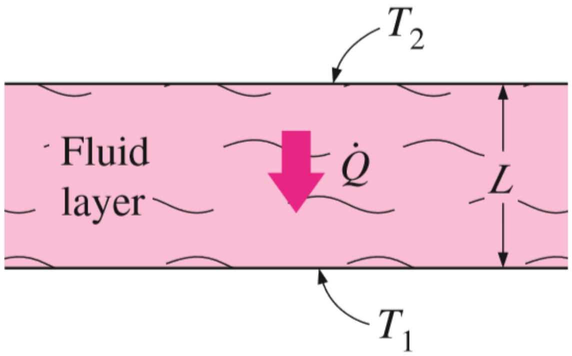
$$R_{conv} = \frac{1}{hA} \left(\frac{\mathrm{K}}{\mathrm{W}}\right)$$

Remember $\quad R_{Cond}, plane = \frac{\Delta x}{k\,A} \quad \left(\frac{\mathrm{K}}{\mathrm{W}}\right)$

NUSSELT NUMBER

$$\dot{q}_{\text{conv}} = h\Delta T$$

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L}$$



$\Delta T = T_2 - T_1$

Taking their ratio gives

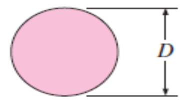
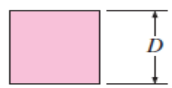
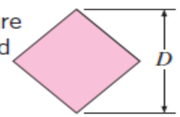
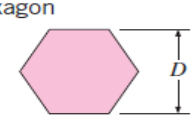
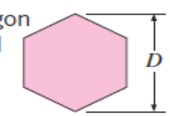
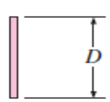
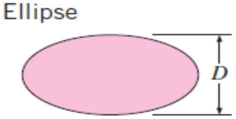
$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

- **The larger the Nusselt number, the more effective the convection.**
- **A Nusselt number of Nu=1 for a fluid layer represents heat transfer across the layer by pure conduction.**

CORRELATIONS FOR h – FORCED CONVECTION

TABLE 7-1

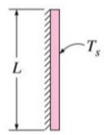
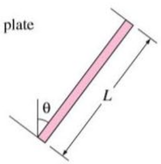
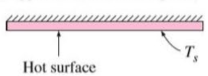
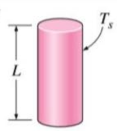
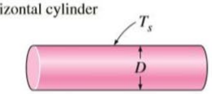
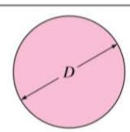
Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

Free Convection Correlations

TABLE 9–1

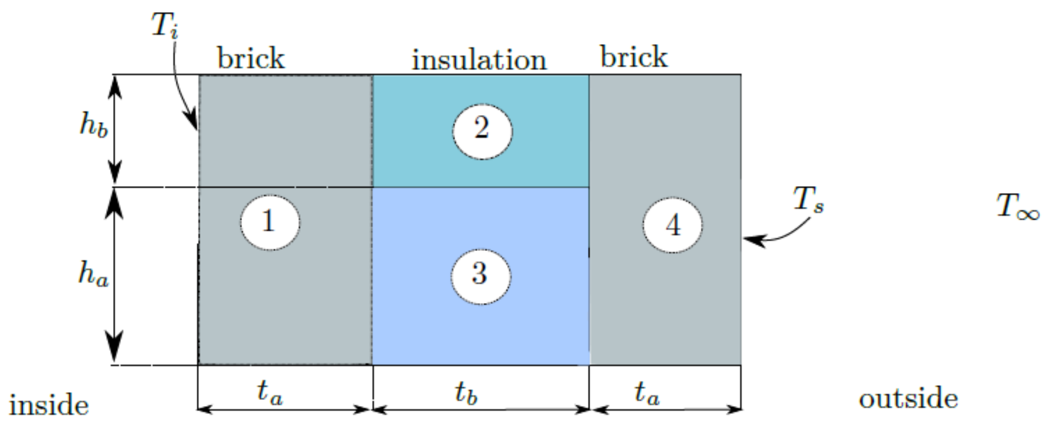
Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20) $\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for Ra < 10^9
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ ($\text{Pr} \geq 0.7$)	$\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)

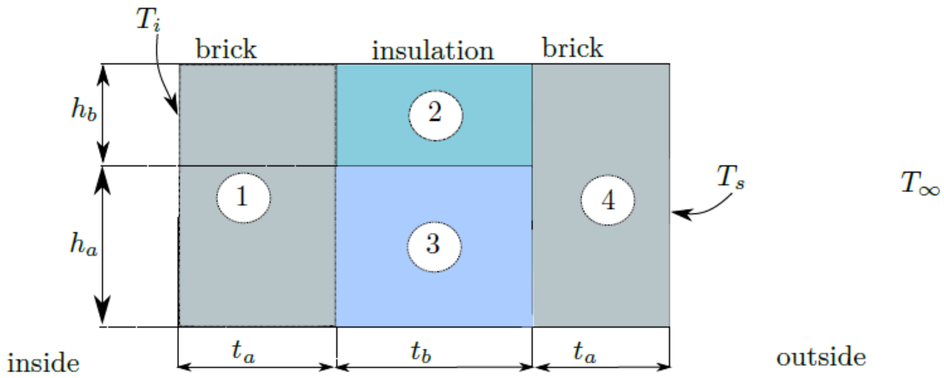
In the figure below a cross section of the wall of a house is given. The inside wall of the house has a temperature $T_i = 20$ °C. The temperature of the air outside is $T_\infty = -5$ °C. The wall is 2 m wide, and has heights $h_a = 1.50$ m, $h_b = 1$ m, and thicknesses t_a=30 mm and t_b=50 mm. The thermal conductivity of the layers of brick is $k_1 = k_4 = 0.72\ \mathrm{W\,m^{-1}\,K^{-1}}$. The thermal conductivity of the rockwool layer at 2 is $k_2 = 0.0350\ \mathrm{W\,m^{-1}\,K^{-1}}$, and that of the wooden layer at 3 is $k_3 = 0.0550\ \mathrm{W\,m^{-1}\,K^{-1}}$. We will analyse this problem on a day with no wind.

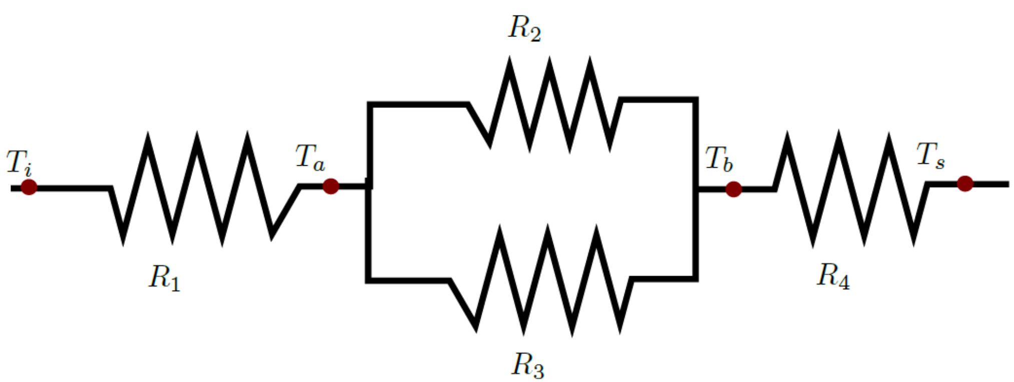
In this exercise, we want to know what the heat loss through the wall is. When writing down your answers, please keep in mind the following:

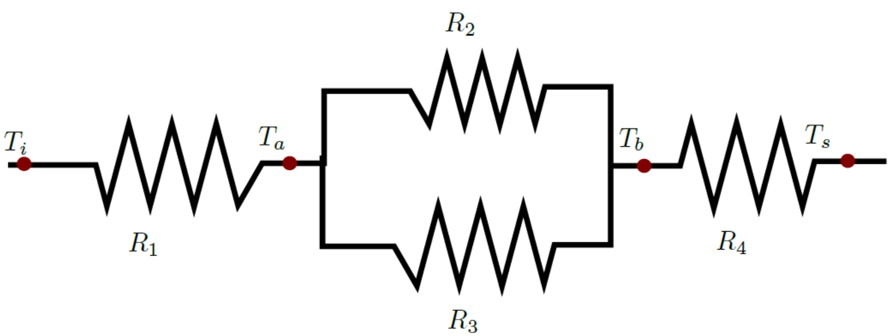
- Clearly state the assumptions you make.
- Clearly mention the values and their units.
- Clearly mention the relations you use and why you are using them.



a) Draw the resistance network of the wall and determine the formula for the total resistance R_{tot} inside of the wall.







Correct network - 3 pt.

The equations for the resistances are:

$$R_1 = \frac{t_a}{k_1 A_1} \qquad R_2 = \frac{t_b}{k_2 A_2} \qquad R_3 = \frac{t_b}{k_3 A_3} \qquad R_4 = \frac{t_a}{k4 A_4} \tag{1.1}$$

Correct equations for the separate resistances- 1 pt.

The resistance of the parallel part can be determined using

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \implies R_{23} = \frac{R_2 R_3}{R_2 + R_3} \tag{1.2}$$

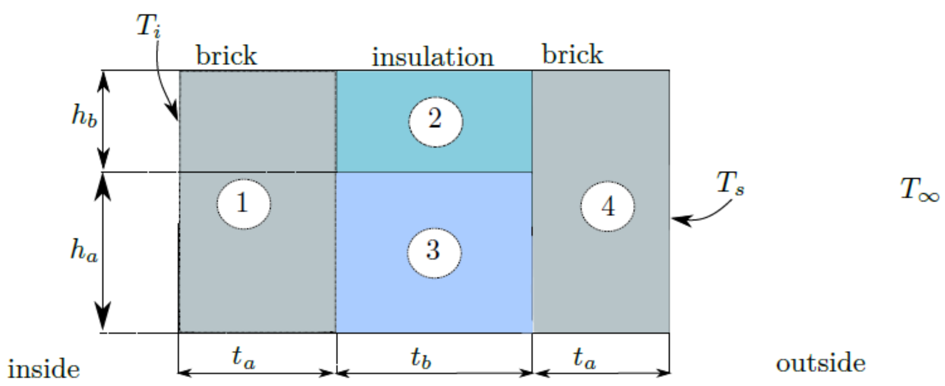
Correct R_{23} - 2 pt.

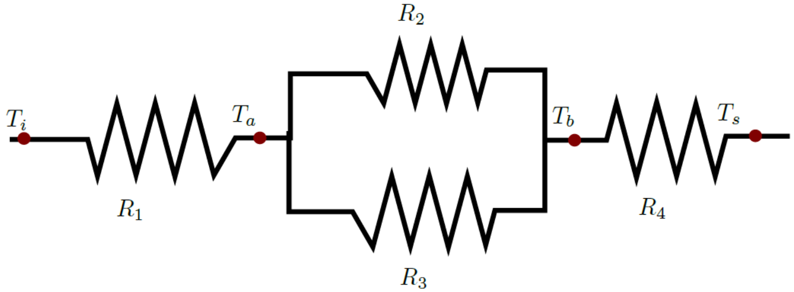
The total resistance can then be determined using:

$$R_{tot} = R_1 + R_{23} + R_4 \tag{1.3}$$

Correct R_{tot} - 1 pt.

b) Determine the total heat loss from the wall. If you need to iterate, do so until a precision of Q of 5 percent is achieved. What is the surface temperature of the wall?





In order to solve this, we need the following energy balance:

$$Q_{conduction} = Q_{naturalconvection} \tag{1.4}$$

Correct energy balance - 2 pt.

$\dot{Q}$ Known -Ts unknown

Problem: T_s unknown → starting values for step-by-step plan unknown (Pr, k, ν, …)

Ingredients dimensionless nos.
↓
Rayleigh number
↓
Correlation → Nusselt number
↓
h
↓
$\dot{Q}$

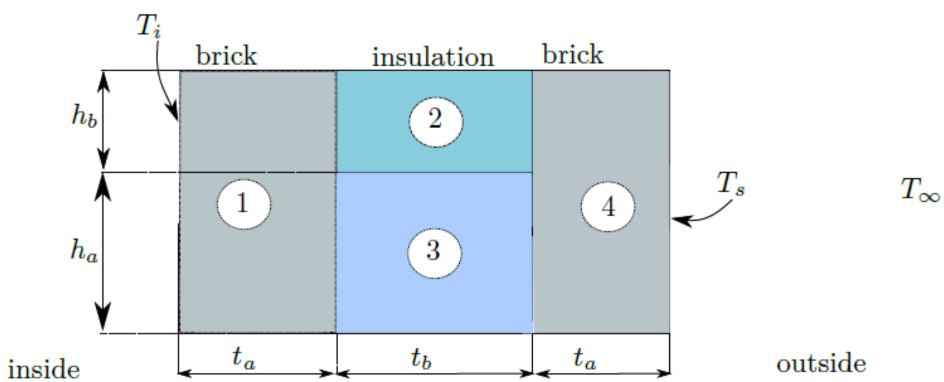
At what avg. temp? $\dfrac{T_s + T_\infty}{2}$?

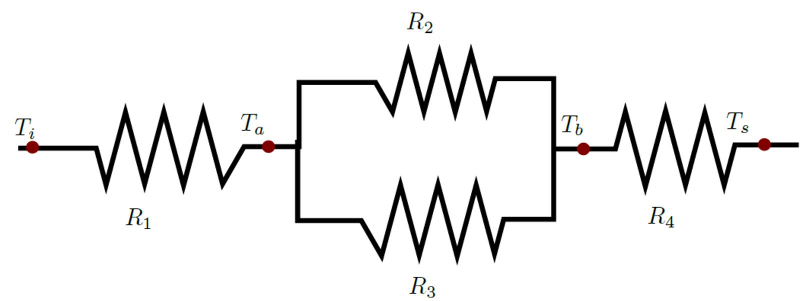
Unknown

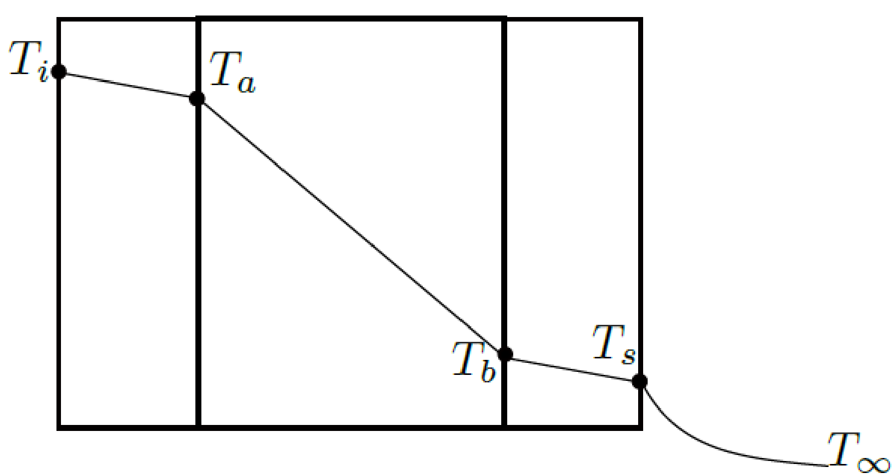
Approach: iterative solving

- Choose an estimated T_s to start with
- Follow steps and check whether T_s yields the right heat flow
- Resulting heat flow ≠ Given heat flow ?
- Adjust estimated T_s and repeat all steps till you find a T_s that (approximately) yields Given heat flow

c) Determine the temperatures after each layer and draw the temperature profile including the outside.







Radiation Heat Transfer

To calculate the heat transfer rate by radiation, we must include terms for power output and energy received from the surroundings.

$$\boxed{\varepsilon\sigma A T_s^4}$$

$$\boxed{\sigma\alpha A T_\infty^4}$$

Making the usual assumption that $\varepsilon = \alpha$, and multiplying by area yields:

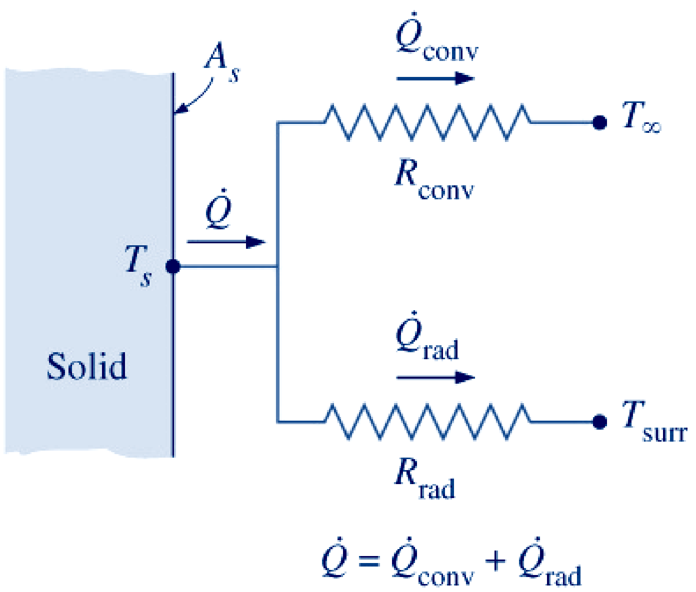
$$\boxed{\dot{Q} = \varepsilon\sigma A(T_s^4 - T_\infty^4)}$$

This is the expression for an object totally enclosed by surroundings at T_∞.

Radiation and convection

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$= h_{conv} A \,\Delta T + h_{rad} A \,\Delta T$$
$$= (h_{conv} + h_{rad}) A \,\Delta T$$

$$\Rightarrow \dot{Q} = h_{tot} A \,\Delta T \quad \text{with} \quad h_{tot} = h_{conv} + h_{rad}$$



$$h_{rad} = \varepsilon \cdot \sigma \cdot (T_s^2 + T_\infty^2) \cdot (T_s + T_\infty)$$

$$\text{with} \;\; R_{rad} = \frac{1}{h_{rad} A} \quad \Rightarrow \quad \frac{1}{R_{tot}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$

An optic made of a special diffused glass has the following spectral radiation properties (τ and ρ) as sketched in Figure 1.3. Select the correct course of the spectral absorption coefficient (α) of the glass from the selection below.

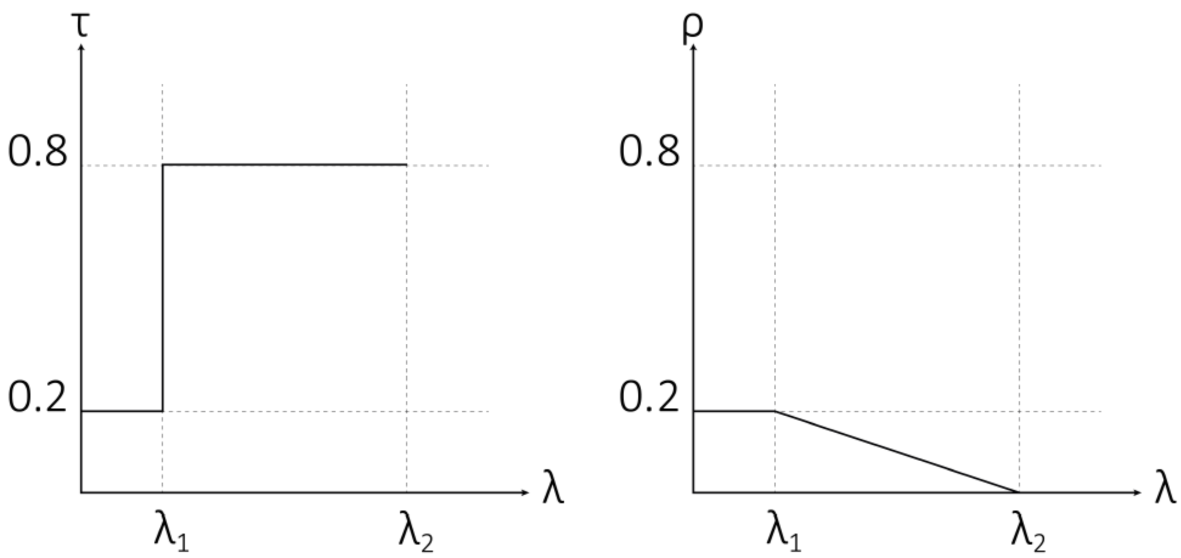
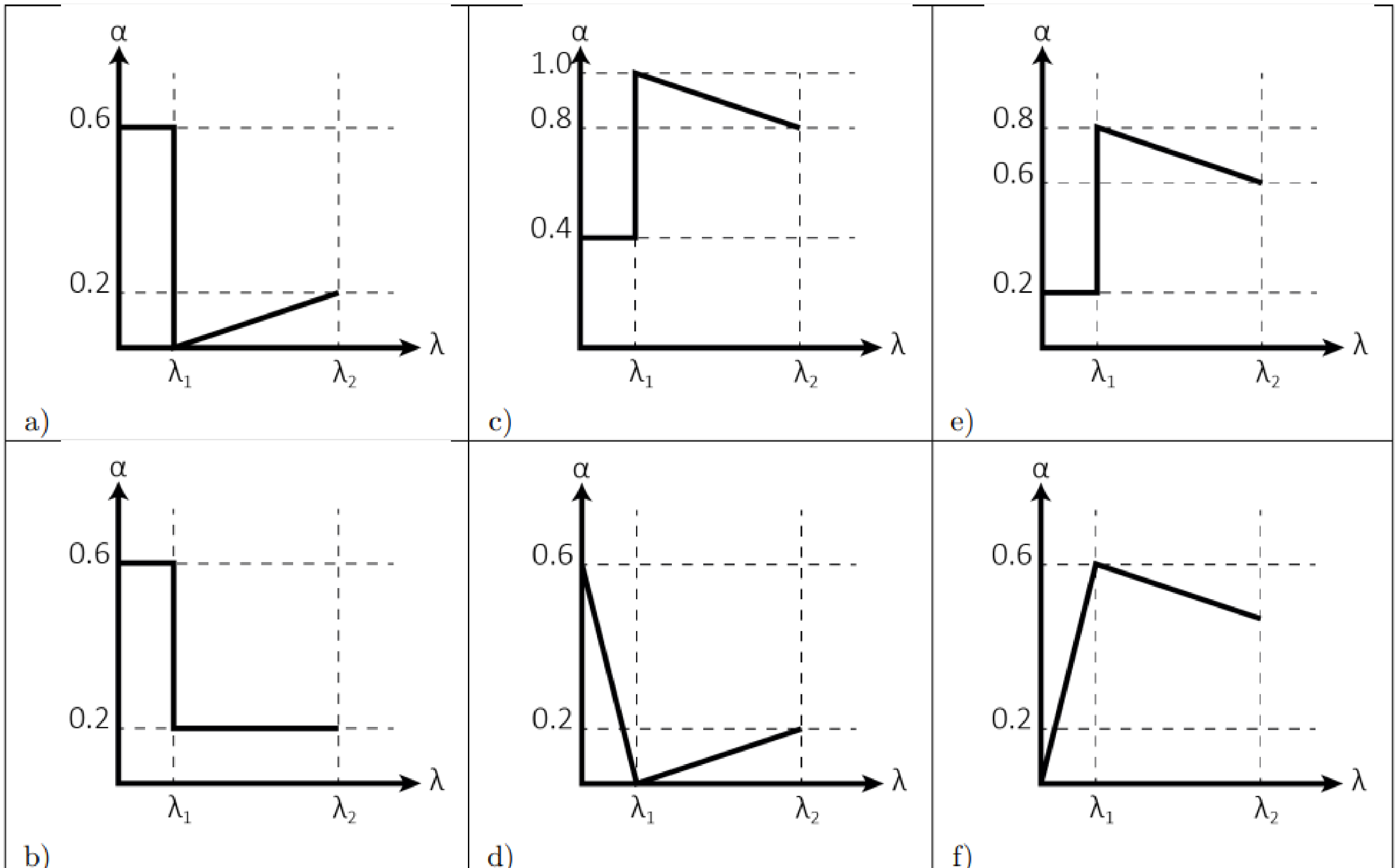


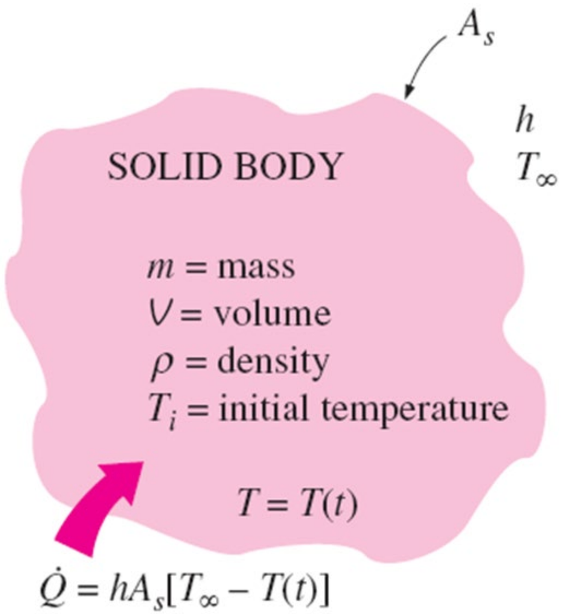
Figure 1.3: Spectral properties τ and ρ as a function λ



α
0.6
0.2
λ
λ1
λ2
a)
α
1.0
0.8
0.4
λ
λ1
λ2
c)
α
0.8
0.6
0.2
λ
λ1
λ2
e)
α
0.6
0.2
λ
λ1
λ2
b)
α
0.6
0.2
λ
λ1
λ2
d)
α
0.6
0.2
λ
λ1
λ2
f)

UNSTEADY HEAT TRANSFER LUMPED SYSTEM ANALYSIS

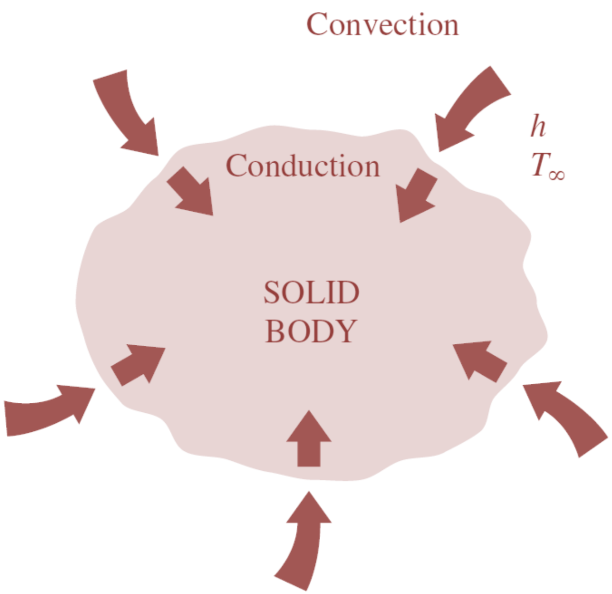
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$ **Where** $b = \frac{hA_s}{\rho V c_p}$



- ***h :*** **heat transfer coefficient around object ($W \cdot m^{-2} \cdot K^{-1}$**
- ***A_s*: external surface area (m^2)**
- ***ρ :*** **density of object ($kg \cdot m^{-3}$)**
- ***c_p*** **: specific heat of object ($J \cdot kg^{-1} \cdot K^{-1}$)**
- ***V*** **: volume of object, (m^3)**

Only for lumped system analysis

Biot number (Bi)



$$\text{Bi} = \frac{\text{heat convection}}{\text{heat conduction}}$$

$$\text{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

or

$$\text{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

$\text{Bi} = \dfrac{hL_c}{k}$ **When characteristic length is :** $L_c = \dfrac{V}{A_s}$

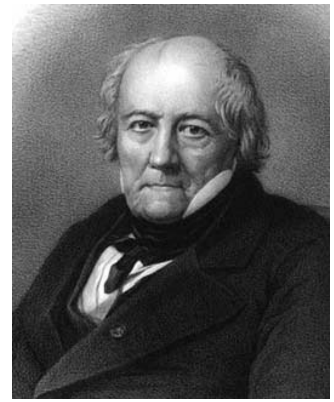
almost uniform temperature for Bi ≤ 0,1
"lumped system"

Nusselt vs. Biot

Nusselt number

$$\mathrm{Nu} = \frac{hL_c}{k}$$

$\neq$



Biot number

$$\mathrm{Bi} = \frac{hL_c}{k}$$

Dimensionless measure for convection so increase of heat transfer due to flow

k of fluid!

Dimensionless measure for degree of temperature distribution within body

k of surrounded object!

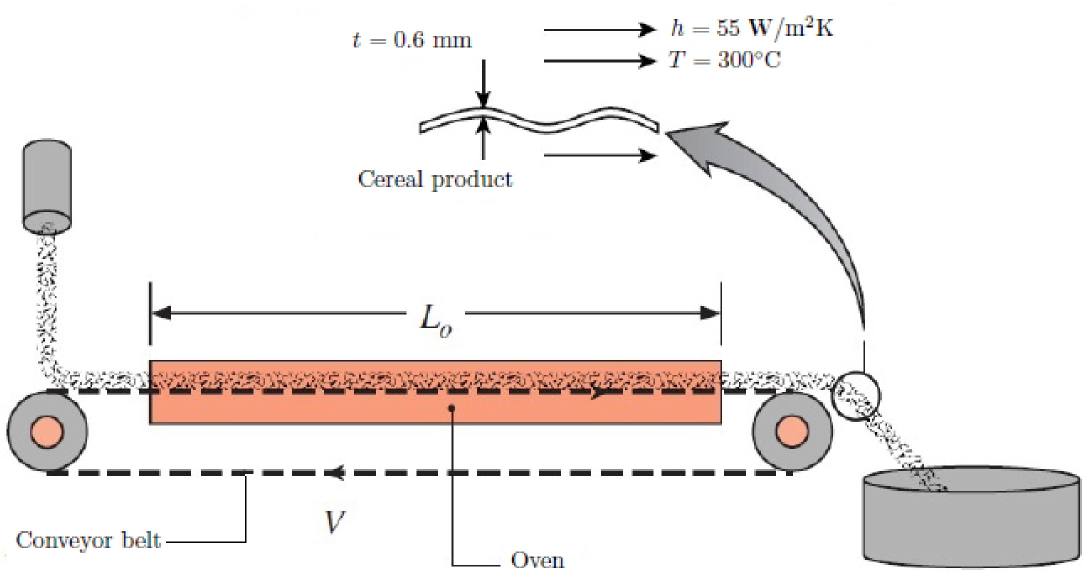
Definitions seem similar but are substantially different!

A flaked cereal is of thickness $t = 0.6$ mm. The density, specific heat and thermal conductivity of the flake are $\rho = 700\ \mathrm{kg\,m^{-3}}$, $c_p = 2400\ \mathrm{J\,kg^{-1}\,K^{-1}}$ and $k = 0.34\ \mathrm{W\,m^{-1}\,K^{-1}}$ respectively. The product is to be baked by increasing its temperature from $T_i = 20$ °C to $T_f = 220$ °C in a convection oven, through which the product is carried on a conveyor. The oven is $L_O = 3$ m long and the convection heat transfer coefficient at the product surface and oven air temperature are $h = 55\ \mathrm{W\,m^{-2}\,K^{-1}}$ and $T_\infty = 300$ °C respectively.

a) Determine the required conveyor velocity V.

An engineer suggests that the productivity can be increased if the flake thickness is reduced to $t = 0.4$ mm.

b) Determine the required conveyor velocity and the percentual change in productivity when a flake with a reduced thickness is used. Is the engineer correct?



Thank you for your attention