V.14

a) Sketch de stress-strain relation for a linear, elastic material, and
b) add possible non-linear material behavior (with explanation/motivation).
c) Explain what happens for unloading of c1) a linear, elastic material, of c2) a elastic-plastic material (for small AND for large strains).
d) Sketch the relation of shear-stress versus strain- or deformation-rate, for d1) a lineair, d2) a shear-thickening, d3) a shear-thinning, or d4) a yield-stress-fluid.

V.15

Given is a wire (length *L*=0.1m, cross-section *HW,* volume *V=LHW*) for a homogeneous, elastic, isotropic, rubber-like material. What is the work necessary to quickly (or very slowly) stretch the wirefrom stress 0 to length 3*L*. Which strain-rate is needed for making the elastic and the viscous contribution equaly important?

Material-properties:
Kevin-Voigt viscoelastic solid (http://en.wikipedia.org/wiki/Viscoelasticity):

relation for stress = function of strain and strain-rate:
$\sigma = E\varepsilon + \eta\dot{\varepsilon}$ with modulus of Young E=0.02 MPa and viscosity η = 10 Pa s.

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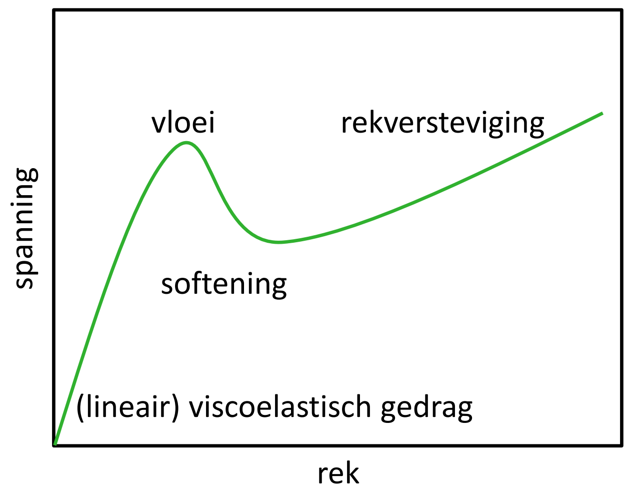
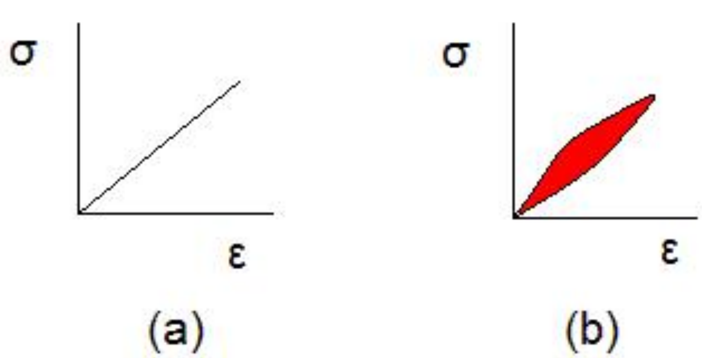
Solutions:
a) Stress-strain linear means a straight line, while elastic means the return path is identical.

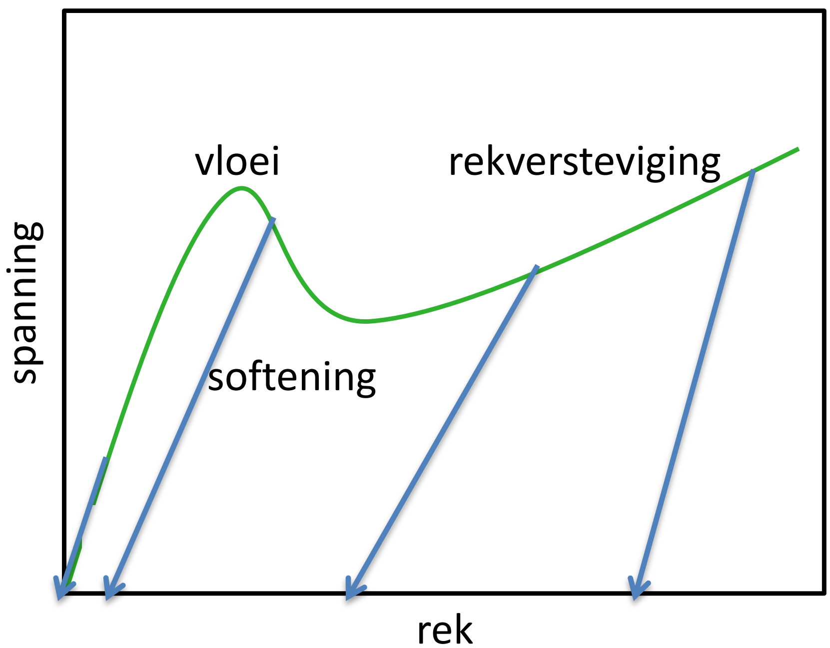
b) many types of non-linear behavior are possible, here is only one example, with explanations in dutch for stress-strain on the axis, linear (visco) elastic, flowing, softening, and strain-hardening behavior (not that hardening and softening for solids are not meaning the same as for fluids):



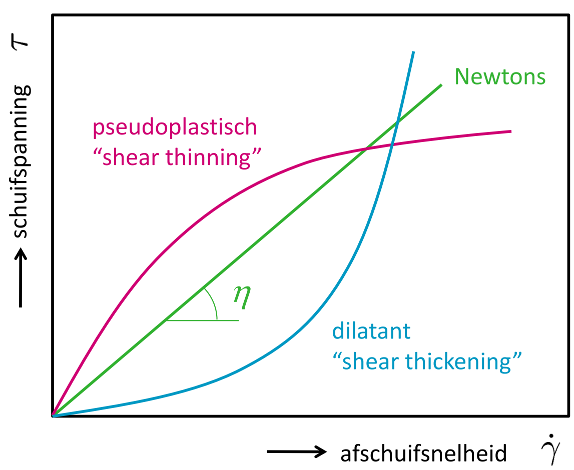
c1) the load-unload curce is reversible in the elastic limit, and usually for small strains (a), where as for a viscoelastic material (red) one obtains hysteresis in the stress-strain curve, which increases with the strain-rate (b), and actually is related to the disspated viscous energy.

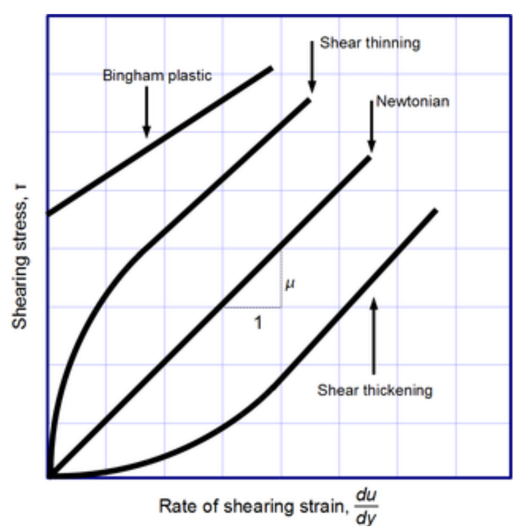


c2) for (very) small loads, the stress-strain path is typically reversible (c1) – but after a larger strain, the return (unloading) path is not indentical to the loading path. Four possible stress strain relations for plastic material are given …



d) Note that we typically plot shear-stress versus strain-rate for fluids with the following material behaviour: d1=Newtonian, d2=dilatant, d3=pseudo-plastic, and d4=Bingham plastic (examples)





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Stress 0 => length L, to length $3L$ => $\varepsilon = \varepsilon_{11}$ = normal strain = $(3L-L)/L$
Estimate: strain-rate = const. = (d/dt) ε = length-change / (length*time) = $2L/(L\Delta t) = 2/\Delta t$

specific work *a:*

$$a = \int d\varepsilon\, \sigma = \int d\varepsilon \left(E\varepsilon + \eta\dot{\varepsilon}\right) = \tfrac{1}{2} E\varepsilon^2 \Big| + \eta\dot{\varepsilon}\varepsilon \Big|_{\varepsilon=0}^{\varepsilon=2} = 2\left(E + \eta\frac{2}{\Delta t}\right)$$

work in the total volume:

$$A = \int dV\; a = 2V\left(E + \eta\tfrac{2}{\Delta t}\right)$$

Estimate the speed at which viscous and elastic forces equal each other:

$$E = \eta\tfrac{2}{\Delta t} = \eta\dot{\varepsilon} \quad \Rightarrow \quad \dot{\varepsilon}_E = \frac{E}{\eta} = 2.10^3\,\mathrm{s}^{-1}$$

$$v = L\dot{\varepsilon}_E = 2.10^2\,\mathrm{ms}^{-1}$$

thus a bit smaller than sound-speed.