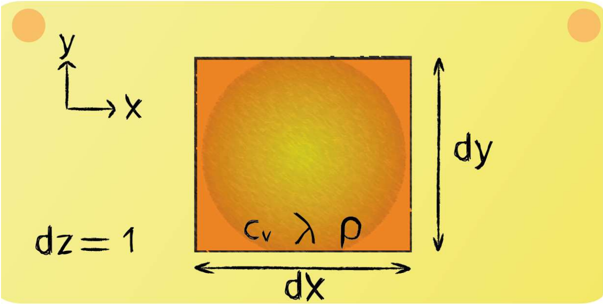
Lecture 3 - Question 5



Give the energy balance to derive the heat conduction equation. Assume two-dimensional transient heat conduction without sources or sinks.

Energy balance:

$$\frac{\partial U}{\partial t} = \dot{Q}_{x,in} - \dot{Q}_{x,out} + \dot{Q}_{y,in} - \dot{Q}_{y,out}$$

For unsteady heat transfer the internal energy will change over time and equals the sum of in- and outgoing heat fluxes.

Change of internal energy over time:

$$\frac{\partial U}{\partial t} = \rho \cdot c_v \cdot dx \cdot A \cdot \frac{\partial T}{\partial t} \;\; = \;\; \rho \cdot c_v \cdot dy \cdot A \cdot \frac{\partial T}{\partial t}$$

The internal energy of a constant volume can be described as: $U = m \cdot c_v \cdot T$

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda dydz \frac{\partial T}{\partial x}$$

$$\dot{Q}_{y,in} = -\lambda dxdz \frac{\partial T}{\partial y}$$

$$\dot{Q}_{x,out} = -\lambda dydz \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

$$\dot{Q}_{y,out} = -\lambda dxdz \frac{\partial T}{\partial y} + \frac{\partial \dot{Q}_{y,in}}{\partial y} dy$$

The heat fluxes are described by conductive heat transfer. The outgoing heat fluxes can be approximated by use of the Taylor series expansion.