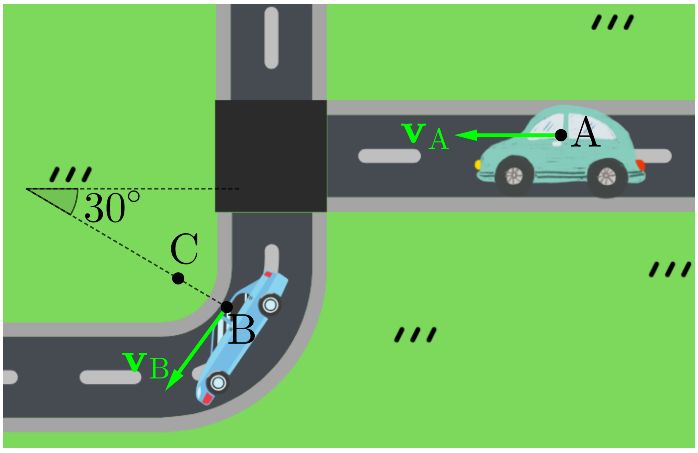
Tilted Road



Two cars are shown in the figure. Car A is driving to the left with a speed of 50 km/h. Car B is rounding through a circular curve (center of curvature at point C) with a speed of 36 km/h.

Determine the speed $v_{B/A}$ of car B with respect to an observer in car A.

Using known expressions (general):

$$\mathbf{v}_{\mathrm{B}} = \mathbf{v}_{\mathrm{A}} + \mathbf{v}_{\mathrm{B/A}} \quad \Rightarrow \quad \mathbf{v}_{\mathrm{B/A}} = \mathbf{v}_{\mathrm{B}} - \mathbf{v}_{\mathrm{A}} \tag{1}$$

Given quantities:

Speed of point A: $v_{\mathrm{A}} = 50$ km/h $= 13\frac{8}{9}$ m/s
Speed of point B: $v_{\mathrm{B}} = 36$ km/h $= 10$ m/s
Angle $\mathbf{v}_{\mathrm{B}}$ with horizontal: $\theta = 30°$

Solution:

Looking at the schematic in Figure 1, using basic trigonometry gives the velocity vectors $\mathbf{v}_{\mathrm{A}}$,$\mathbf{v}_{\mathrm{B}}$ of the two respective cars in the standard Cartesian cooordinate system as:

$$\mathbf{v}_{\mathrm{B}} = \begin{pmatrix} -v_{\mathrm{B}}\sin(30°) \\ -v_{\mathrm{B}}\cos(30°) \\ 0 \end{pmatrix} \qquad \mathbf{v}_{\mathrm{A}} = \begin{pmatrix} -v_{\mathrm{A}} \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

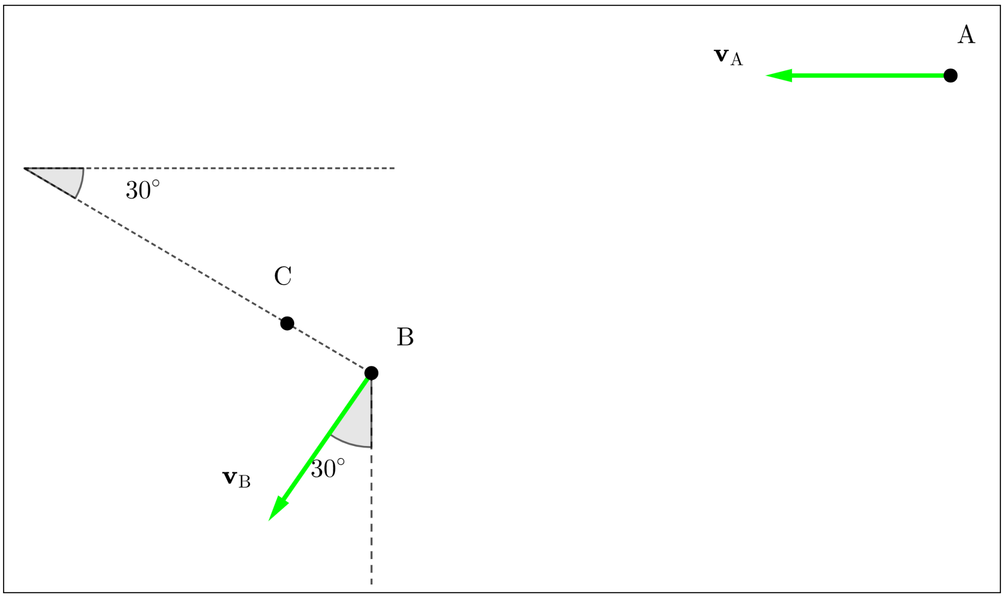


Figure 1: Schematic of the situation

This computes to:

$$\mathbf{v}_{\mathrm{B}} = \begin{pmatrix} -5 \\ -5\sqrt{5} \\ 0 \end{pmatrix} \qquad \mathbf{v}_{\mathrm{A}} = \begin{pmatrix} -13\frac{8}{9} \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Inserting both expressions into Equation (1) results in:

$$\mathbf{v}_{\mathrm{B/A}} = \mathbf{v}_{\mathrm{B}} - \mathbf{v}_{\mathrm{A}} = \begin{pmatrix} -5 \\ -5\sqrt{3} \\ 0 \end{pmatrix} - \begin{pmatrix} -13\frac{8}{9} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 8\frac{8}{9} \\ -5\sqrt{3} \\ 0 \end{pmatrix} \tag{4}$$

The speed $v_{\mathrm{B/A}}$ can now be calculated by using the Pythagorean Theorem:

$$v_{\mathrm{B/A}} = |\mathbf{v}_{\mathrm{B/A}}| = \sqrt{\left(13\frac{8}{9}\right)^2 + \left(-5\sqrt{3}\right)^2} \approx 12.4 \text{ m/s} \tag{5}$$