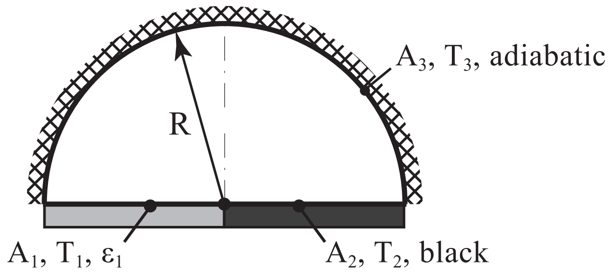
Exercise IV.5: (Cupola ⋆ ⋆ ⋆)

Both semi-circular slabs A_1 and A_2 of the geometric configuration depicted below are conditioned to maintain a constant temperature of and, respectively. Surface A_2 can be considered a black body, and the hemispherical surface A_3 above the slabs is adiabatic.



Given parameters:

- Temperature of slab 1: $T_1 = 150\,°C$
- Temperature of slab 2: $T_2 = 20\,°C$
- Emissivity of slab 1: $\varepsilon_1 = 0.6$
- Radius of the dome: $R = 3$ m

Hints:

- Surfaces A_1 and A_3 are grey bodies and emit diffuse radiation.
- The hemispherical volume is filled with a vacuum.
- All surfaces are opaque.

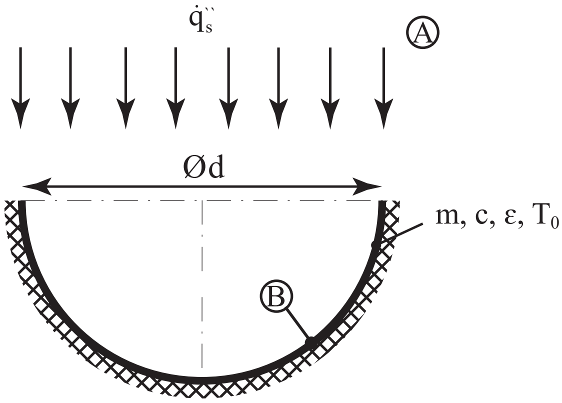
Tasks:

a) Compute the amount of heat transferred through radiation between the surfaces A_1 and A_2 (= net radiative flux to surface A_2).

b) Which temperature T_3 is obtained for surface A_3?

Exercise IV.6: (Pokè bowl ⋆⋆⋆)

An empty bowl, that is used for serving the typical traditional Hawaiian dish called pokè bowl, has the homogeneous temperature T_0 and is adiabatically insulated at its convex side. At the time t_0, the bowl is suddenly exposed to parallel radiation from the sun.



Given parameters:

- Mass of the bowl: m
- Specific heat capacity of the bowl: c
- Emissivity of the bowl: $\epsilon \approx 0.5$
- Starting temperature of the bowl: T_0
- Diameter of the bowl: d
- Heat flux of the solar radiation on the ground: $\dot{q}''_{\mathrm{S}}$
- View factor of the bowl to the ambient: Φ_{BA}
- View factor of the bowl to itself: Φ_{BB}

Hints:

- The bowl radiates grey and diffuse and has a homogeneous temperature at any time.
- Influences from the ambient or the atmosphere can be neglected.
- The sun is a black body.

Tasks:

a) Determine the surface brightness of the bowl $\dot{Q}_{\mathrm{B}}$.

   Hint: In a nonsteady state, the surface brightness of a grey, adiabatic body is not the same as the surface brightness of a black body.

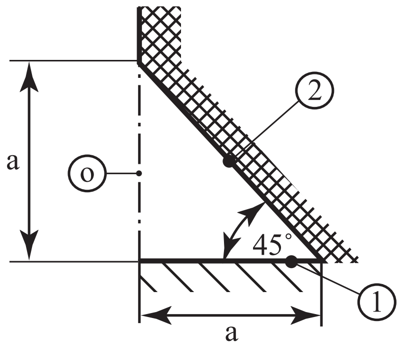
b) Derive the differential equation for the temperature as a function of time and the necessary initial condition to solve this differential equation.

c) Determine the steady-state final temperature T_{S} of the bowl.

d) Draw the temperature as a function of time qualitatively.

Exercise IV.7: (Radiation within a wedge-shaped opening ⋆⋆⋆)

Consider an infinitely long opening with a wedge-shaped cross-section as shown in the figure below.



Given parameters:

- Temperature of surface 1: $T_1 = 1000\,\text{K}$
- Temperature of space surrounding: $T_o = 0\,\text{K}$
- Emissivity of surface 1: $\varepsilon_1 = 1$
- Width: $a = 30$ cm

Hints:

- Surface 2 is a grey body and adiabatically insulated at the back.
- The space surrounding the opening can be considered to be a black body.
- Influences due to convection shall be disregarded.

Tasks:

a) Determine all view factors.

b) Determine the energy through the opening $\dot{q}'_{o,\text{loss}}$ for a unit length of the opening.

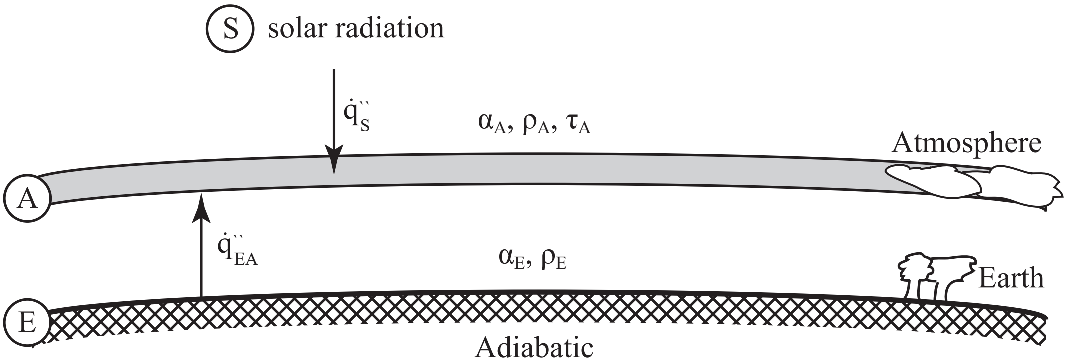
c) Determine the temperature T_2 of surface 2.

Exercise IV.8: (Earth's atmosphere ⋆ ⋆ ⋆)

The climate on Earth is influenced by the atmosphere to a great extent. To describe heat transfer between Earth and space, it is assumed that the atmosphere surrounds the Earth as a thin, distinct layer.

When balancing radiative heat flows, long-wave and short-wave radiation must be distinguished (indices LW and SW). Earth and atmosphere (indices E and A) have specific absorption, reflection, and transmission coefficients (α, ρ, τ) for long-wave and short-wave radiation each. The spectrum of solar radiation ($\dot{q}''_S$) is assumed to be in the short-wave range only, whereas emission from earth and atmosphere is in the long-wave range only.

Additionally to the radiative heat fluxes, a net heat flux $\dot{q}''_{EA}$ is carried from the earth into the atmosphere, which leads back to convective heat transfer and vaporization.



Given parameters:

- Short-wave solar radiation: $\dot{q}''_{S,SW} = 341\ \mathrm{W/m^2}$
- Long-wave solar radiation: $\dot{q}''_{S,LW} = 0\ \mathrm{W/m^2}$
- Convection and vaporization: $\dot{q}''_{EA} = 101\ \mathrm{W/m^2}$

	Short-wave	**Long-wave**
Atmosphere	$\rho_{A,SW} = 0.23$	$\rho_{A,LW} = 0.34$
	$\tau_{A,SW} = 0.54$	$\tau_{A,LW} = 0.10$
	$\alpha_{A,SW} = 0.23$	$\alpha_{A,LW} = 0.56$
	emission negligible	emission
Earth	$\rho_{E,SW} = 0.16$	$\alpha_{E,LW} = 1.00$
	$\alpha_{E,SW} = 0.84$	
	emission negligible	emission
Solar radiation	emission	emission negligible

Hints:

- Curvature is negligible, i.e. earth and atmosphere have the same surface area and the atmosphere does not radiate onto itself.
- The atmosphere emits equally in both directions.
- The given heat fluxes are averaged across the entire earth and over multiple years. Do not distinguish between the light and dark hemispheres.

- Assume steady-state conditions.

Tasks:

a) Determine the flux of short-wave radiation which hits onto the earth's surface $\dot{q}''_{\mathrm{SW\ to\ E}}$.

b) Give all energy balances and surface brightnesses necessary to determine the temperature at the earth's surface. You may assume that the spectrum of black body radiation is completely within the long-wave range for that temperature.