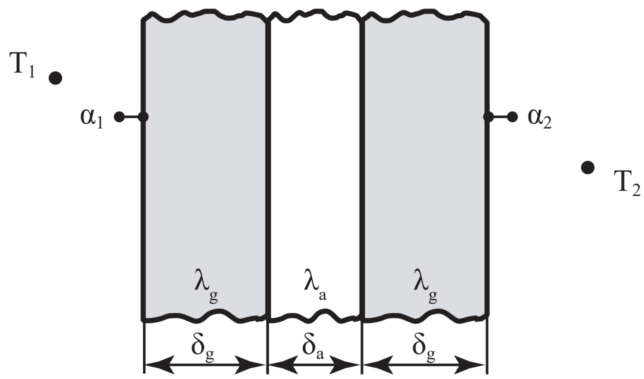
Exercise II.4: (Window insulation ⋆)

Consider a 1.2-m-height and 2-m-wide double-pane window consisting of two layers of glass separated by a stagnant air space. Convection occurs at the inside and outside of the pane window. Disregard any heat transfer by radiation.



Given parameters:

- Conductivity of glass: $\lambda_g = 0.78$ W/mK
- Conductivity of air: $\lambda_a = 0.026$ W/mK
- Thickness of glass layer: $\delta_g = 3$ mm
- Thickness of air layer: $\delta_a = 15$ mm
- Inside convection coefficient: $\alpha_1 = 10$ W/m^{2}K
- Outside convection coefficient: $\alpha_2 = 25$ W/m^{2}K
- Inside temperature: $T_1 = 22$ °C
- Outside temperature: $T_2 = -7$ °C

Tasks:

a) Determine the steady heat transfer rate through this double-pane window and the temperature of its inner surface.

b) Compare your results with a three-layer glass (3-mm-thickness) with two stagnant air spaces filled with krypton ($\delta_k = 8\,\text{mm}$, $\lambda_k = 0.00949\,\text{W/m K}$).

c) Discuss the reason for choosing a three-layer glass and scrutinize all assumptions made in tasks a) and b).

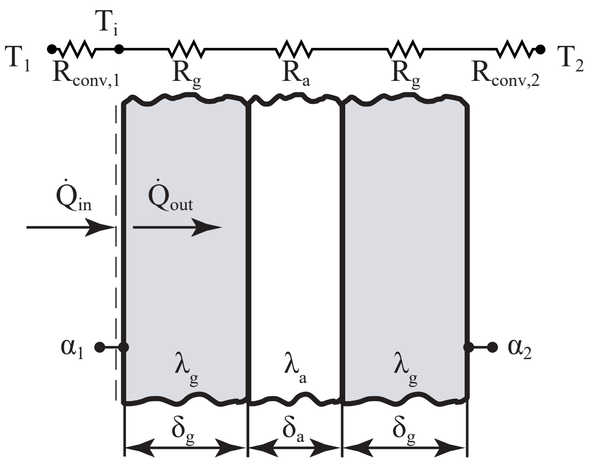
Solution II.4: (Window insulation ⋆)

Task a)

The problem is solved by establishing the energy balance of the system. This is done by defining the thermal resistance network for a multi-layer wall without any generation of heat. Subsequently, the thermal resistances are determined, allowing for the calculation of the rate of heat loss in both situations.

1 Setting up the balance:

Before performing calculations, it is essential to comprehend the thermal resistance network. In the provided scenario, five thermal resistances are interconnected in series.



The energy balance at the interface between the ambient and the glass on the left side reads:

$$0 = \dot{Q}_{\mathrm{in}} - \dot{Q}_{\mathrm{out}}. \tag{II.4.1}$$

2 Defining the elements within the balance:

The ingoing rate of heat transfer is calculated from:

$$\dot{Q}_{\mathrm{in}} = \frac{T_1 - T_{\mathrm{i}}}{R_{\mathrm{conv,1}}}, \tag{II.4.2}$$

and outgoing rate of heat transfer is defined as:

$$\dot{Q}_{\mathrm{out}} = \frac{T_{\mathrm{i}} - T_2}{2R_{\mathrm{g}} + R_{\mathrm{a}} + R_{\mathrm{conv,2}}}. \tag{II.4.3}$$

The conductive thermal resistances within the system are calculated as follows:

$$R_{\mathrm{g}} = \frac{\delta_{\mathrm{g}}}{\lambda_{\mathrm{g}} A} = \frac{0.003\ (\mathrm{m})}{0.78\ \left(\frac{\mathrm{W}}{\mathrm{mK}}\right) \cdot (1.2 \times 2)\ \ (\mathrm{m}^2)} = 0.002\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right), \tag{II.4.4}$$

and:

$$R_{\mathrm{a}} = \frac{\delta_{\mathrm{a}}}{\lambda_{\mathrm{a}} A} = \frac{0.015\ (\mathrm{m})}{0.026\ \left(\frac{\mathrm{W}}{\mathrm{mK}}\right) \cdot (1.2 \times 2)\ \ (\mathrm{m}^2)} = 0.240\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \tag{II.4.5}$$

The two convective thermal resistances are determined from:

$$R_{\text{conv},1} = \frac{1}{\alpha_1 A} = \frac{1}{10\ (\text{W/m}^2\text{K}) \cdot (1.2 \times 2)\ (\text{m}^2)} = 0.042 \left(\frac{\text{K}}{\text{W}}\right), \tag{II.4.6}$$

and:

$$R_{\text{conv},2} = \frac{1}{\alpha_2 A} = \frac{1}{25\ (\text{W/m}^2\text{K}) \cdot (1.2 \times 2)\ (\text{m}^2)} = 0.017 \left(\frac{\text{K}}{\text{W}}\right). \tag{II.4.7}$$

3 Inserting and rearranging:

$$T_\text{i} = \frac{T_1\left(2R_\text{g} + R_\text{a} + R_{\text{conv},2}\right) + T_2 R_{\text{conv},1}}{R_{\text{conv},1} + 2R_\text{g} + R_\text{a} + R_{\text{conv},2}} \tag{II.4.8}$$

$$= \frac{22\ (^\circ\text{C}) \cdot (2 \cdot 0.002 + 0.240 + 0.017)\ \left(\frac{\text{K}}{\text{W}}\right) + -7\ (^\circ\text{C}) \cdot 0.042\ \left(\frac{\text{K}}{\text{W}}\right)}{(0.042 + 2 \cdot 0.002 + 0.240 + 0.017)\ \left(\frac{\text{K}}{\text{W}}\right)} = 18\ (^\circ\text{C}).$$

Substitution of the interface temperature into the expression describing the ingoing rate of heat transfer yields the heat transfer rate:

$$\dot{Q}_\text{in} = \frac{T_1 - T_\text{i}}{R_{\text{conv},1}} = \frac{(22 - 18)\ (^\circ\text{C})}{0.042\ \left(\frac{\text{K}}{\text{W}}\right)} = 96\ (\text{W}). \tag{II.4.9}$$

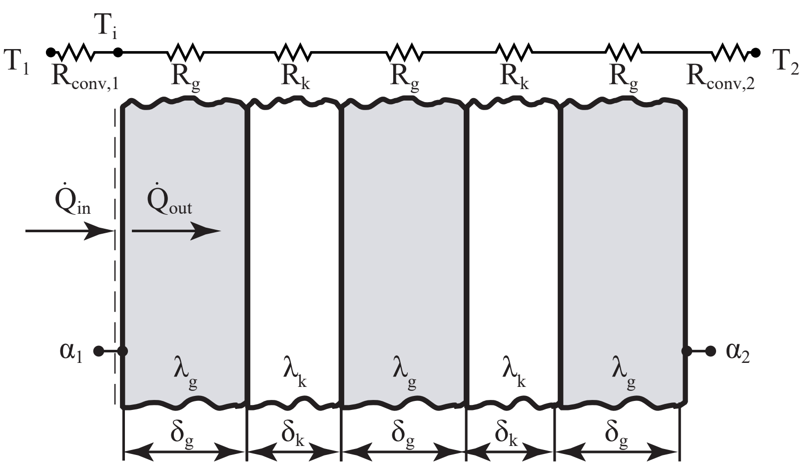
Conclusion

The heat transfer rate through the double-pane window is 96 W, and the temperature of the inner surface is 18 °C.

Task b)

1 Setting up the balance:

The same approach as in the previous task is applied. However, in this instance, there are three layers instead of two, and the enclosures are thinner, and filled with krypton. This results in a scenario where seven thermal resistances are arranged in series.



The energy balance at the interface still reads the same:

$$0 = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}. \tag{II.4.10}$$

2 Defining the elements within the balance:

The ingoing rate of heat transfer is defined as:

$$\dot{Q}_{\text{in}} = \frac{T_1 - T_{\text{i}}}{R_{\text{conv},1}}. \tag{II.4.11}$$

The outgoing rate of heat transfer is written as:

$$\dot{Q}_{\text{out}} = \frac{T_{\text{i}} - T_2}{3R_{\text{g}} + 2R_{\text{k}} + R_{\text{conv},2}}. \tag{II.4.12}$$

The two convective thermal resistances were computed in the previous task. The conductive thermal resistances within the system yield from:

$$R_{\text{g}} = \frac{\delta_{\text{g}}}{\lambda_{\text{g}} A} = \frac{0.003\ (\text{m})}{0.78\ \left(\frac{\text{W}}{\text{mK}}\right) \cdot (1.2 \times 2)\ \ (\text{m}^2)} = 0.002 \left(\frac{\text{K}}{\text{W}}\right), \tag{II.4.13}$$

and:

$$R_{\text{k}} = \frac{\delta_{\text{k}}}{\lambda_{\text{k}} A} = \frac{0.008\ (\text{m})}{0.00949\ \left(\frac{\text{W}}{\text{mK}}\right) \cdot (1.2 \times 2)\ \ (\text{m}^2)} = 0.351 \left(\frac{\text{K}}{\text{W}}\right). \tag{II.4.14}$$

3 Inserting and rearranging:

$$\begin{aligned} T_{\text{i}} &= \frac{T_1 \left(3 \cdot R_{\text{g}} + 2 \cdot R_{\text{k}} + R_{\text{conv},2}\right) + T_2 R_{\text{conv},1}}{R_{\text{conv},1} + 3 \cdot R_{\text{g}} + 2 \cdot R_{\text{k}} + R_{\text{conv},2}} \\ &= \frac{22\ (^\circ\text{C})\,(3 \cdot 0.002 + 2 \cdot 0.351 + 0.017)\ \left(\frac{\text{K}}{\text{W}}\right) + -7\ (^\circ\text{C}) \cdot 0.042\ \left(\frac{\text{K}}{\text{W}}\right)}{(0.042 + 3 \cdot 0.002 + 2 \cdot 0.351 + 0.017)\ \left(\frac{\text{K}}{\text{W}}\right)} = 20\ (^\circ\text{C}). \end{aligned} \tag{II.4.15}$$

Substitution of the interface temperature into the definition of the ingoing rate of heat transfer results:

$$\dot{Q}_{\text{in}} = \frac{T_1 - T_{\text{i}}}{R_{\text{conv},1}} = \frac{(22 - 20)\ \ (^\circ\text{C})}{0.042\ \left(\frac{\text{K}}{\text{W}}\right)} = 38\ (\text{W}). \tag{II.4.16}$$

Conclusion

The heat transfer rate through the triple-pane window is 38 W, and the temperature of the inner surface is 20 °C. In comparison to the two-pane window, the heat loss has decreased by 60%, while the interface temperature has risen by 2 °C.

Task c)

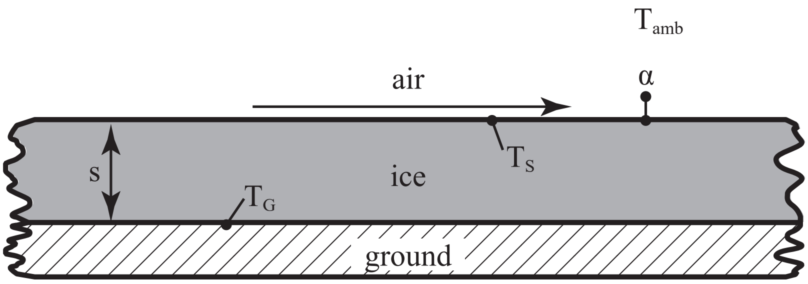
Conclusion

Selecting a three-layer glass configuration serves several purposes in thermal insulation. Firstly, the addition of an extra layer introduces an additional thermal barrier, further impeding heat transfer. This results in a reduction in the overall heat loss through the window. Secondly, the use of thinner enclosures filled with krypton is a deliberate choice to enhance thermal resistance. The lower thermal conductivity of krypton compared to air contributes to improved insulation performance.

> Calculations assume ideal behavior in terms of thermal conductivity and neglect potential non-idealities that may exist in real-world scenarios. Furthermore, the assumptions of steady-state conditions and uniform temperature distribution are inherent in the analysis.

Exercise II.5: (Ice layer ⋆⋆)

During a cold winter day, the ground is covered with an ice layer of thickness s. Air is flowing over the ice layer. The problem is one-dimensional and steady-state. No layer of water is forming on top of the ice.



Given parameters:

- Conductivity of ice: $\lambda = 2.2$ W/mK
- Heat transfer coefficient at the ice surface: $\alpha = 10$ W/m^{2}K
- Temperature of the air: $T_{\text{amb}} = 5$ °C
- Temperature of the ice at the surface: $T_{\text{S}} = -3$ °C
- Temperature of the ice at the ground: $T_{\text{G}} = -10$ °C
- Temperature of the air: $T_{\text{amb}} = 5$ °C

Tasks:

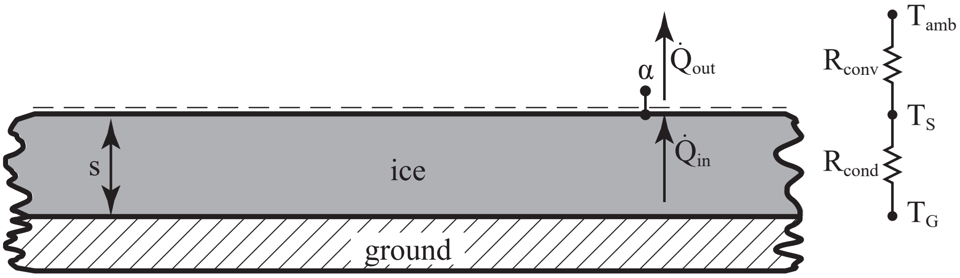
a) Determine the thickness s of the ice layer.

Solution II.5: (Ice layer ★★)

Task a)

1 Setting up the balance:

The problem can be considered to be a multi-layer wall problem. In the given scenario, two resistors are connected in series.



The problem involves one-dimensional steady-state heat transfer. Therefore, the energy balance at the interface with the ambient reads:

$$0 = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}. \tag{II.5.1}$$

2 Defining the elements within the balance:

The ingoing rate of heat transfer is described by use of the conductive resistance:

$$\dot{Q}_{\text{in}} = \frac{T_{\text{S}} - T_{\text{G}}}{R_{\text{cond}}}, \tag{II.5.2}$$

and the outgoing rate of heat transfer using the thermal convective resistance:

$$\dot{Q}_{\text{out}} = \frac{T_{\text{amb}} - T_{\text{S}}}{R_{\text{conv}}}. \tag{II.5.3}$$

The conductive resistance is written as:

$$R_{\text{cond}} = \frac{s}{\lambda A}, \tag{II.5.4}$$

and the convective resistance as:

$$R_{\text{conv}} = \frac{1}{\alpha A}. \tag{II.5.5}$$

3 Inserting and rearranging:

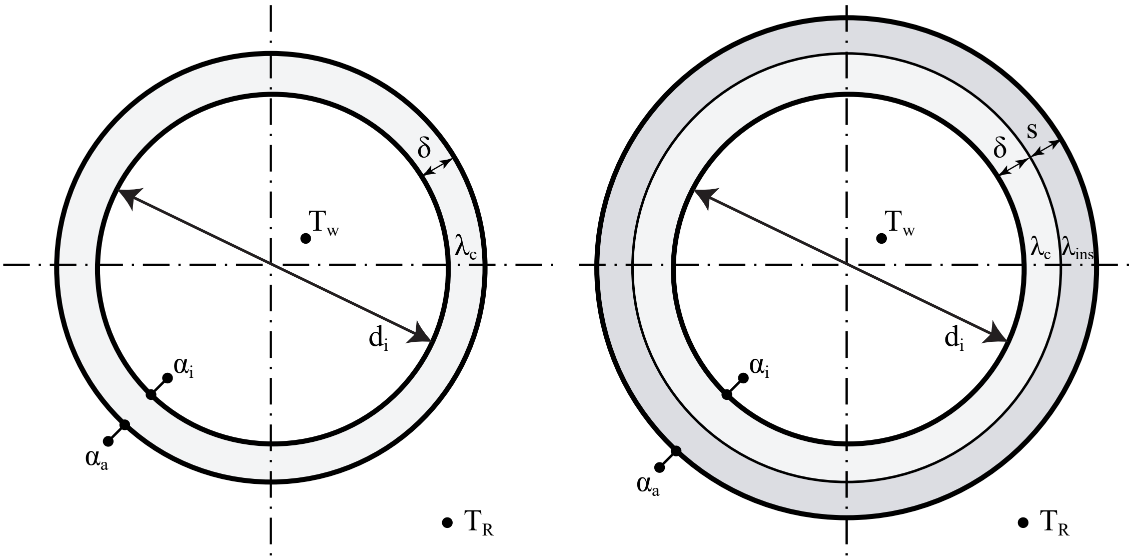
$$\begin{aligned} s &= \frac{\lambda}{\alpha} \cdot \frac{T_{\text{S}} - T_{\text{G}}}{T_{\text{amb}} - T_{\text{S}}} \qquad \text{(II.5.6)} \\ &= \frac{2.2\ \left(\frac{\text{W}}{\text{mK}}\right)}{10\ \left(\frac{\text{W}}{\text{m}^2\text{K}}\right)} \cdot \frac{(-3+10)\ (^\circ\text{C})}{(5+3)\ (^\circ\text{C})} = 0.19\ (\text{m}). \end{aligned}$$

Conclusion

The thickness of the ice layer is 19 cm.

Exercise II.6: (Warm-water pipe ⋆ ⋆ ⋆)

In a room, a copper warm-water pipe is utilized to contain water. This copper pipe features an inner diameter of d_i and a wall thickness denoted as δ. During a chilly winter day, insulation measures are taken, involving the addition of an extra insulation layer with a thickness of s.



Given parameters:

- Heat transfer coefficient at the inner side of the pipe: $\alpha_\mathrm{i} = 2300\ \mathrm{W/m^2K}$
- Heat transfer coefficient at the outer side of the pipe: $\alpha_\mathrm{a} = 6\ \mathrm{W/m^2K}$
- Temperature of the room: $T_\mathrm{R} = 20\ °\mathrm{C}$
- Temperature of the water: $T_\mathrm{W} = 80\ °\mathrm{C}$
- Conductivity of copper: $\lambda_\mathrm{c} = 372\ \mathrm{W/mK}$
- Conductivity of insulation material: $\lambda_\mathrm{ins} = 0.042\ \mathrm{W/mK}$
- Inner diameter of the copper pipe: $d_\mathrm{i} = 6\ \mathrm{mm}$
- Thickness of the copper pipe: $\delta = 1\ \mathrm{mm}$
- Thickness of the insulation layer: $s = 4\ \mathrm{mm}$

Hints:

- Changes to the heat transfer coefficient at the outer side of the pipe as a function of the diameter are disregarded.

Tasks:

a) Calculate the heat transferred per unit length of the pipe, denoted as $\dot{q}'$, for both an uninsulated pipe and an insulated pipe. What noteworthy observations can be made from your findings?

b) Qualitatively sketch the heat emission profile $\dot{q}'$ as a function of the insulation thickness for different thermal conductivities of the insulation material. Explain the underlying physical principles.

c) Calculate the required thermal conductivity for the insulating material to always achieve a reduction in heat loss, regardless of the thickness of the insulation.

Solution II.6: (Warm-water pipe ⋆ ⋆ ⋆)

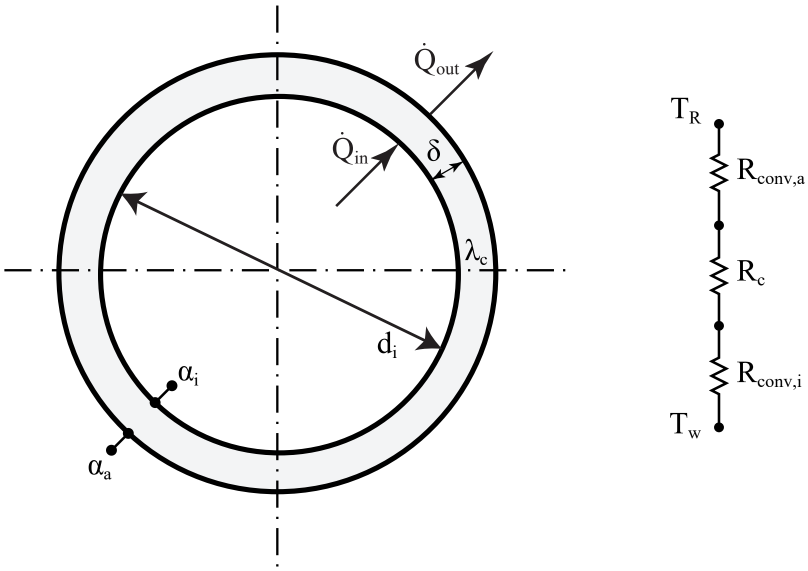
Task a)

To calculate the heat transferred per unit length, denoted as $\dot{q}'$, consider the length of the pipe to be precisely one unit, i.e., $L = 1$ m. For this instance, $\dot{Q} = \dot{q}'$.

System without insulation:

1 Setting up the balance:

Before the calculations, the thermal resistance network must be understood. In the given uninsulated scenario, three resistors are connected in series.



The energy balance through the pipe wall reads:

$$0 = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}. \tag{II.6.1}$$

2 Defining the elements within the balance:

The energy balance states that $\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}}$. Both are expressed using the total thermal resistance between the two specified reference temperatures. The rate of heat transfer reads:

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}} = \frac{T_{\text{W}} - T_{\text{R}}}{R_{\text{conv,i}} + R_{\text{c}} + R_{\text{conv,a}}}. \tag{II.6.2}$$

Calculation of the thermal resistance due to convection on the inside:

$$\begin{aligned} R_{\text{conv,i}} &= \frac{1}{\alpha_{\text{i}} \pi d_{\text{i}} L} \qquad \text{(II.6.3)} \\ &= \frac{1}{2300 \left(\frac{\text{W}}{\text{m}^2\text{K}}\right) \cdot \pi \cdot 0.006 \text{ (m)} \cdot 1 \text{ (m)}} = 0.02 \left(\frac{\text{K}}{\text{W}}\right). \end{aligned}$$

Thermal resistance of the copper layer:

$$R_{\mathrm{c}} = \frac{1}{2\pi L\lambda_{\mathrm{c}}}\ln\left(\frac{d_{\mathrm{i}}+2\delta}{d_{\mathrm{i}}}\right)$$
$$= \frac{1}{2\cdot\pi\cdot 1\ (\mathrm{m})\cdot 372\ \left(\frac{\mathrm{W}}{\mathrm{mK}}\right)}\ln\left(\frac{(0.006+2\cdot 0.001)\ \ (\mathrm{m})}{0.006\ (\mathrm{m})}\right) = 0.0001\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \tag{II.6.4}$$

Thermal resistance convection on the outside:

$$R_{\mathrm{conv,a}} = \frac{1}{\alpha_{\mathrm{a}}\pi\,(d_{\mathrm{i}}+2\delta)\cdot L}$$
$$= \frac{1}{6\ \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right)\cdot\pi\cdot(0.006+2\cdot 0.001)\ \ (\mathrm{m})\cdot 1\ (\mathrm{m})} = 6.63\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \tag{II.6.5}$$

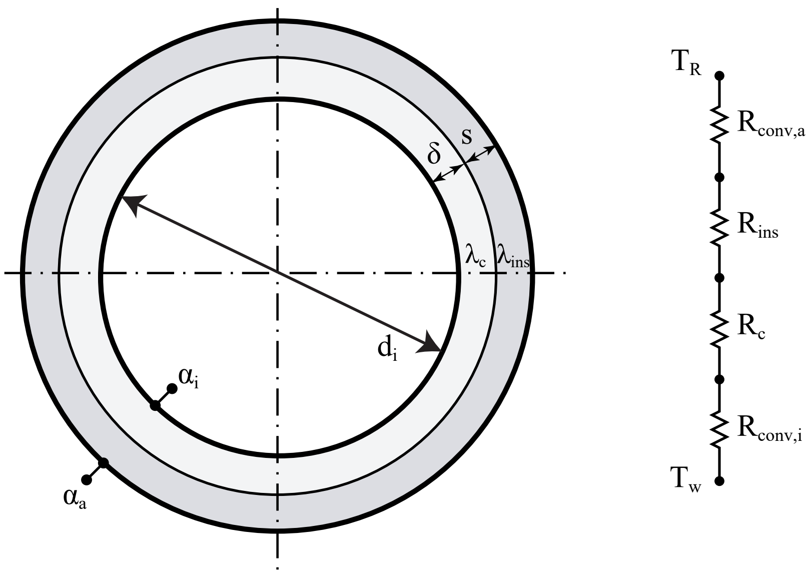
3 **Inserting and rearranging:**

$$\dot{Q} = \frac{(80-20)\ \ (^{\circ}\mathrm{C})}{(0.02+0.0001+6.63)\ \ \left(\frac{\mathrm{K}}{\mathrm{W}}\right)} = 9\ (\mathrm{W}). \tag{II.6.6}$$

System with insulation:

1 **Setting up the balance:**

First, an understanding of the thermal resistance network is essential. In the described situation four resistances are connected in series.



The energy balance through the network reads:

$$0 = \dot{Q}_{\mathrm{in}} - \dot{Q}_{\mathrm{out}}. \tag{II.6.7}$$

2 **Defining the elements within the balance:**

The energy balance states that $\dot{Q}_{\mathrm{in}} = \dot{Q}_{\mathrm{out}}$. Both are written using the total thermal resistance between the two specified reference temperatures. The rate of heat transfer reads:

$$\dot{Q}_{\mathrm{in}} = \dot{Q}_{\mathrm{out}} = \frac{T_{\mathrm{W}} - T_{\mathrm{R}}}{R_{\mathrm{conv,i}} + R_{\mathrm{c}} + R_{\mathrm{ins}} + R_{\mathrm{conv,a}}}. \tag{II.6.8}$$

The thermal resistance insulation layer is calculated as follows:

$$\begin{aligned} R_{\mathrm{ins}} &= \frac{1}{2\pi L \lambda_{\mathrm{ins}}} \ln\left(\frac{d_{\mathrm{i}} + 2\delta + 2s}{d_{\mathrm{i}} + 2\delta}\right) \\ &= \frac{1}{2 \cdot \pi \cdot 1\ (\mathrm{m}) \cdot 0.042\ \left(\frac{\mathrm{W}}{\mathrm{mK}}\right)} \ln\left[\frac{(0.006 + 2 \cdot 0.001 + 2 \cdot 0.004)\ \ (\mathrm{m})}{(0.006 + 2 \cdot 0.001)\ \ (\mathrm{m})}\right] = 2.63\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \end{aligned} \tag{II.6.9}$$

With the increased thickness of the pipe, there is a corresponding increase in the surface area of the outer wall. Consequently, the thermal resistance of the convection layer on the exterior has also changed. The thermal resistance of convection on the outside is computed as follows:

$$\begin{aligned} R_{\mathrm{conv,a}} &= \frac{1}{\alpha_{\mathrm{a}} \pi \left(d_{\mathrm{i}} + 2\delta + 2s\right) \cdot L} \\ &= \frac{1}{6\ \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right) \cdot \pi \cdot (0.006 + 2 \cdot 0.001 + 2 \cdot 0.004)\ \ (\mathrm{m}) \cdot 1\ (\mathrm{m})} = 3.32\ \left(\frac{\mathrm{K}}{\mathrm{W}}\right). \end{aligned} \tag{II.6.10}$$

3 Inserting and rearranging:

$$\dot{Q} = \frac{(80 - 20)\ \ (^{\circ}\mathrm{C})}{(0.02 + 0.0001 + 2.63 + 3.32)\ \ \left(\frac{\mathrm{K}}{\mathrm{W}}\right)} = 10\ (\mathrm{W}). \tag{II.6.11}$$

Conclusion

Evident is that in the uninsulated scenario, the heat transferred per unit length is 9 W/m, while for the insulated scenario 10 W/m. This might initially seem counterintuitive, anticipating a decrease in the heat transfer rate for the insulated pipe, given the expectation of reduced heat losses.

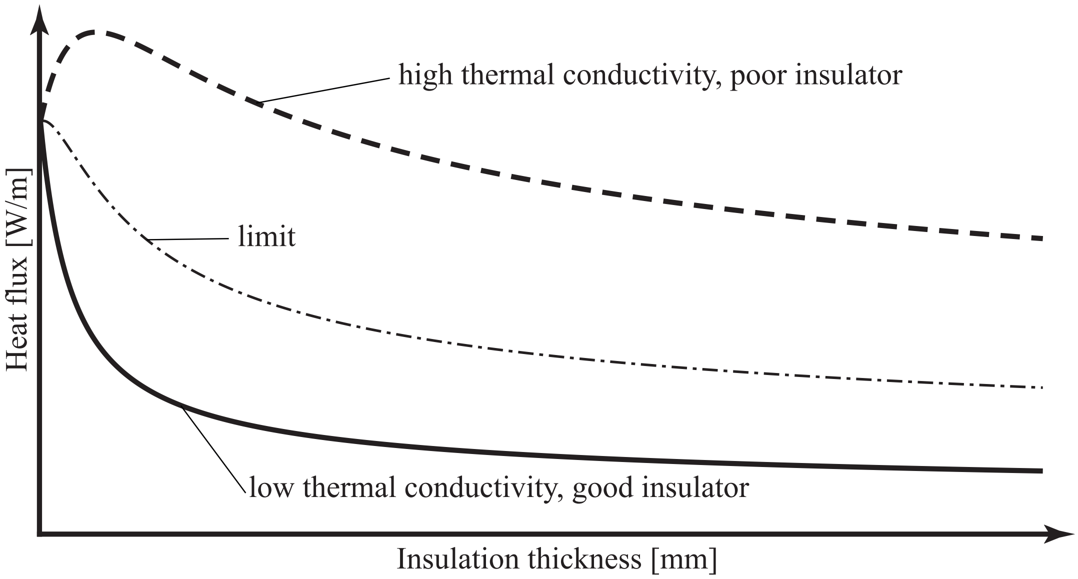
However, this is explained by considering that the additional resistance introduced by insulation serves to further diminish the convective thermal resistance on the outside. Consequently, the overall thermal resistance of the system is reduced due to this modification, resulting in a larger heat loss for the insulated pipe.

Task b)

As noted in the previous task, the heat flux increased despite the addition of an insulating layer. This phenomenon was attributed to the reduction in convective thermal resistance on the outside being more substantial than the increase in thermal resistance resulting from the insulation. This behavior persists until a critical insulating thickness is reached, beyond which the rise in thermal resistance from the insulator consistently outweighs the decrease in convective thermal resistance on the outside. Notably, this behavior is characteristic of "poor insulators" which have relatively high thermal conductivity.

If the thermal conductivity of the insulator is low enough, there is "good insulation," and in such a case, the added thermal resistance from the insulator is consistently larger than the reduction in convective resistance on the outside. Consequently, for all insulation thicknesses, the addition of an insulator results in a reduction in heat flux.

Conclusion



Task c)

Insulation is beneficial only when the rate of heat transfer, denoted as $\dot{q}'$, experiences a decline. As previously discussed, in certain scenarios, the addition of an insulation layer may lead to an increase in the rate of heat transfer instead of a decrease. The rate of heat transfer, $\dot{q}'$, begins to decrease beyond a critical point where the thickness of the insulation layer reaches a crucial value. This critical value, where the rate of heat transfer $\dot{q}'$ is maximum, corresponds to the minima of the total thermal resistance's gradient. Following this point, the rate of heat transfer consistently decreases, regardless of the insulation thickness.

By taking the derivative of the total thermal resistance with respect to the outer diameter D_o and equalling to zero, the critical condition where R_{total} is minimized is determined:

$$\frac{d}{dD_o}\left[R_{conv,i} + R_c + R_{ins} + R_{conv,a}\right] = 0. \tag{II.6.12}$$

Setting the derivatives of $R_{conv,i}$ and R_c equal to zero, as both resistances are independent of the outer diameter:

$$\frac{d}{dD_o}\left[R_{ins} + R_{conv,a}\right] = 0. \tag{II.6.13}$$

Inserting the definitions of the thermal resistances yields:

$$\frac{d}{dD_o}\left[\frac{1}{2\pi L\lambda_{ins}}\ln\left(\frac{d_i + 2\delta + 2s}{d_i + 2\delta}\right) + \frac{1}{\alpha_a\pi\left(d_i + 2\delta + 2s\right)\cdot L}\right] = 0. \tag{II.6.14}$$

For simplicity, $D_o = d_i + 2\delta + 2s$ and $d_o = d_i + 2\delta$ is substituted. Besides,the constants π, and L are eliminated:

$$\frac{d}{dD_o}\left[\frac{1}{2\lambda_{ins}}\ln\left(\frac{D_o}{d_o}\right) + \frac{1}{\alpha_a D_o}\right] = 0. \tag{II.6.15}$$

Differentiating everything in between brackets with respect to D_o gives:

$$\frac{1}{2\cdot\lambda_{ins}}\cdot\frac{1}{D_o} - \frac{1}{\alpha_a D_o^2} = 0. \tag{II.6.16}$$

This provides the critical diameter beyond which the heat flux always decreases:

$$D_{\mathrm{o,crit}} = \frac{2 \cdot \lambda_{\mathrm{ins}}}{\alpha_{\mathrm{a}}}. \tag{II.6.17}$$

This expression is reformulated to derive an expression for the thermal conductivity as a function of the outer diameter:

$$\lambda_{\mathrm{ins}} = \frac{\alpha_{\mathrm{a}} \cdot D_{\mathrm{o,crit}}}{2}. \tag{II.6.18}$$

In this context, the aim is to equal the current outer diameter without any insulation to the critical diameter. Hence, $D_{\mathrm{o,crit}} = d_{\mathrm{i}} + 2\delta$. Substituting and filling in provides the minimum required thermal conductivity of the insulation layer to consistently achieve a reduction in heat loss, regardless of the thickness of the insulation:

$$\begin{aligned} \lambda_{\mathrm{ins}} &= \frac{\alpha_{\mathrm{a}} \cdot (d_{\mathrm{i}} + 2\delta)}{2} \\ &= \frac{6 \left(\frac{\mathrm{W}}{\mathrm{m^2K}}\right) \cdot (0.006 + 2 \cdot 0.001)\ \ (\mathrm{m})}{2} = 0.024 \left(\frac{\mathrm{W}}{\mathrm{mK}}\right). \end{aligned} \tag{II.6.19}$$

Conclusion

Hence, for the insulating material to always achieve a reduction in heat loss, irrespective of the insulation thickness, the thermal conductivity must not exceed 0.024 $\frac{\mathrm{W}}{\mathrm{mK}}$. Otherwise, any addition of insulation material increases the rate of heat loss, and only after reaching the critical outer diameter heat losses are reduced.