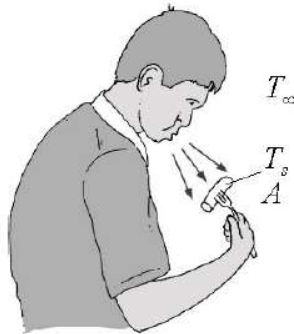# Solutions lecture 6

6.1   Blowing man with a carrot

Analysis

We need to determine the temperature of the carrot after one minute of blowing. Furthermore we need to determine how long the man should blow to lower the temperature of the carrot to 80 °C. The initial temperature is 100 °C, with an ambient temperature of 30 °C. The heat transfer coefficient, thermal conductivity, density and specific heat of the carrot are given.

Approach

Assumptions

We assume that the carrot is a perfect cylinder with a length of 7 cm and a diameter of 2 cm.

Route to solution

With the assumption that the carrot has a uniform temperature, we need to determine if the carrot can be considered a lumped system. To do this, first the characteristic length needs to be determined. This can be done with

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4} \cdot D^2 \cdot l}{\pi \cdot D \cdot L + 2 \cdot \left(\frac{\pi}{4} \cdot D^2 \right)}$$

This value can then be substituted in the formula for the Biot number

$$\text{Bi} = \frac{h L_c}{k}$$

If Bi< 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

For objects with a uniform temperature distribution, the constant c is defined as

$$c = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta$$

Substituting the known values and $t = 60$, the temperature is found.

For the second question, we again use the last-mentioned equation, and check for which t the right-hand side equals 80.

Elaboration

We start with determining the characteristic length:

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4} \cdot D^2 \cdot l}{\pi \cdot D \cdot L + 2 \cdot \left(\frac{\pi}{4} \cdot D^2\right)} = 0.004\,375\,\text{m}$$

Substituting the characteristic length in the formula for the Biot number

$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \cdot 0.004375}{0.8} = 0.082[-]$$

Because Bi< 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

The constant c is now:

$$c = \frac{hA}{\rho c_p V} = \frac{15 \cdot \left(\pi \cdot 0.02 \cdot 0.07 + 2\left(\frac{\pi}{4} \cdot 0.02^2\right)\right)}{1100 \cdot 3.60 \cdot 10^3 \cdot \frac{\pi}{4} \cdot 0.02 \cdot 0.07} = 0.866 \times 10^{-3}\,\text{s}^{-1}$$

Rewriting and substituting the values:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta = (100 - 20) \cdot e^{-0.866 \cdot 10^{-3} \cdot 60} + 20 = 95.96\,^\circ\text{C}$$

For the second part, as the temperature has decreased from 100 °C to 80 °C:

$$80 = (100 - 20) \cdot e^{-0.866 \cdot 10^{-3} \cdot t} + 20$$
$$60 = 80 e^{-0.866 \cdot 10^{-3} \cdot t}$$
$$0.75 = e^{-0.866 \cdot 10^{-3} \cdot t}$$
$$\ln(0.75) = -0.866 \cdot 10^{-3} \cdot t$$
$$-02877 = -0.866 \cdot 10^{-3} \cdot t$$
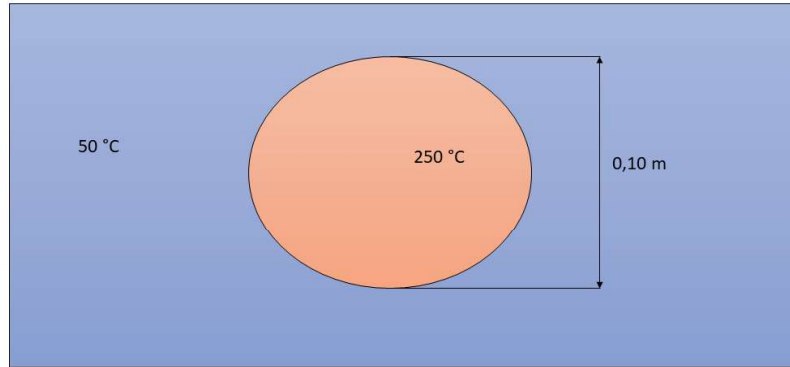$$t = 332\,\text{s}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

6.2 Cooling a copper sphere

Analysis

We need to determine the temperature of the copper block after it is immersed in a cold fluid, 5 minutes after immersion. The density, specific heat, and thermal conductivity of the copper sphere are given, as well as the diameter and the initial temperature. The heat transfer coefficient and the temperature of the fluid are also provided.

Approach

Assumptions

Route to solution

We start with determining the characteristic length of the copper sphere. This can be done with the following equation:

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6}$$

This value can be substituted in the equation for the Biot number:

$$\text{Bi} = \frac{hL_c}{k}$$

When Bi < 0.1, the system is considered lumped, and hence the lump capacitance method may be applied for the solution:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

For objects with a uniform temperature distribution, the constant c is defined as

$$c = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta$$

Substituting the known values and $t = 300$, the temperature is found.

Elaboration

We start with determining the characteristic length:

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6} = 0.0167\,\text{m}$$

Substituting the characteristic length in the formula for the Biot number

$$\text{Bi} = \frac{hL_c}{k} = \frac{200 \cdot 0.01667}{386} = 8.64 \cdot 10^{-3}\,[-]$$

Because Bi< 0.1, the carrot may be considered a lumped system, and the following equations are then valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

The constant c is now:

$$c = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200}{8954 \cdot 0.01667 \cdot 383} = 0.0035\,\text{s}^{-1}$$

Rewriting and substituting the values:

$$T(t) = (T(0) - T_\infty) \cdot \theta(t) + T_\theta = (250 - 50) \cdot e^{-0.0035 \cdot 300} + 50 = 120\,^\circ\text{C}$$
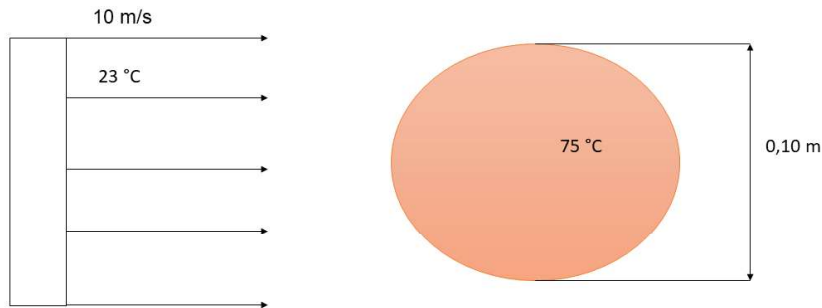
Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?

6.3 Cooling a copper sphere under forced convection conditions

Analysis

A copper sphere is wrapped in a plastic film and placed in an oven at 75 °C. After removal from the oven, the sphere is exposed to an air stream at 23 °C and 10 m/s. Estimate the time taken to cool the sphere to 35 °C. The following information is given:

- Copper:
  - $\rho = 8933 \ \mathrm{kg\,m^{-3}}$, $k = 400 \ \mathrm{W\,m^{-1}\,K^{-1}}$, $c_p = 380 \ \mathrm{J\,kg^{-1}\,K^{-1}}$

- For air at 23 °C
  - $\mu = 18.16 \cdot 10^{-6} \ \mathrm{N\,s\,m^{-2}}$, $\nu = 15.36 \cdot 10^{-6} \ \mathrm{m^2\,s^{-1}}$
  - $k = 0.0258 \ \mathrm{W\,m^{-1}\,K}$, Pr=0.709
  - $\mu_s = 19.78 \cdot 10^{-6} \ \mathrm{N\,s\,m^{-2}}$ at 35 °C

The diameter in the figure above is wrong!

Approach

Assumptions

Assume the sphere is a lumped system.

Route to solution

A relation for the Nusselt number is provided. The only unknown in this is the Reynolds number, so we first need to determine this:

$$\mathrm{Re} = \frac{\rho U D}{\mu} = \frac{U D}{\nu}$$

Substitute all the variables in the correlation for the Nusselt number. Now, the Nusselt number is defined as

$$\mathrm{Nu} = \frac{hD}{k} \implies h = \frac{k}{D} \cdot \mathrm{Nu}$$

With h known, and the system being lumped, the following equations are valid:

$$\theta(t) = e^{-c \cdot t}$$

$$\theta(t) = \frac{\Delta T(t)}{\Delta T(0)} = \frac{T(t) - T_\infty}{T(0) - T_\infty}$$

where c is defined as

$$c = \frac{hA}{\rho c_p V}$$

If we now rewrite the formula for $\theta(t)$ above to:

$$\frac{T(t) - T_\infty}{T(0) - T_\infty} = e^{-c \cdot t}$$

Substituting the known values and rewriting will give an expression for t.

Elaboration

We start with determining the Reynolds number:

$$\mathrm{Re} = \frac{UD}{\nu} = \frac{10 \cdot 0.01}{15.36 \cdot 10^{-6}} = 6510 [-]$$

Substitution of all variables in the correlation for the Nusselt number:

$$\mathrm{Nu} = 2 + \left[0.4 (6510)^{0.5} + 0.06 (6510)^{2/3} \right] (0.709)^{0.4} \left(\frac{18.16 \cdot 10^{-6}}{19.75 \cdot 10^{-6}} \right)^{0.25}$$

$$= 2 + [32.27 + 20.92] \cdot 0.87 \cdot 0.979 = 47.3 [-]$$

Rewriting the definition of the Nusselt number and substituting:

$$h = \frac{k}{D} \cdot \mathrm{Nu} = \frac{0.0258}{0.01} \cdot 47.3 = 122 \, \mathrm{W \, m^{-2} \, K^{-1}}$$

Now, calculating constant c:

$$c = \frac{h A_s}{\rho V c_p} = \frac{122 \cdot 4 \pi R^2}{\rho \cdot \dfrac{4}{3} \pi R^3 \cdot c_p} = \frac{122 \cdot 3}{8933 \cdot 0.005 \cdot 380} = 0.021\,56 \, \mathrm{s^{-1}}$$

Substituting c and the temperatures into the rewritten equation for $\theta(t)$

$$\frac{T(t) - T_\infty}{T(0) - T_\infty} = e^{-c \cdot t}$$

$$\frac{35 - 23}{75 - 23} = e^{-0.02156 \cdot t}$$

$$0.2308 = e^{-0.02156 \cdot t}$$

$$\ln(0.2308) = -0.02156 \cdot t$$

$$1.466 = 0.02156 t$$

$$t \approx 68 \mathrm{s}$$

Evaluation

Check your answer:

- Does the answer have the correct dimensions?
- Is the answer in the right order of magnitude?
- Did you answer all the questions that were asked?