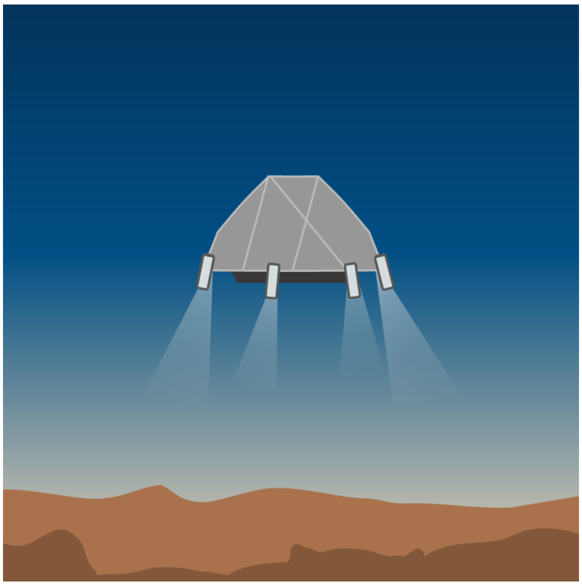
Mars Rover Landing



In the final stages of a landing on Mars, the rover descends under retrothrust of its descent engine to within a distance of 5 m above the mars surface where it has a downward velocity of 2 m/s. Compute the impact velocity of the rover with the surface it the engine is shut down at this point. Gravity on Mars is 0.38 times earth's gravity, take g = 10 m/s^2.

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

Given:
Gravitational acceleration on Earth: $g = 10m/s^2$

Gravitational acceleration on Mars: $g_{mars} = 0.38 \cdot g = 3.8m/s^2$
Initial downward velocity: $v_0 = -2m/s$
Distance above surface: $h = s_0 = 5m$

The time it takes to reach the surface can be calculated using Equation 6, with g_{mars} is pointing downward, and take the origin of the coordinate system at the ground (hence $s(t_{end}) = 0m$).

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \Rightarrow \frac{1}{2} \cdot -3.8 \cdot t^2 + -2 \cdot t + 5 = 0 \tag{7}$$

From this t can be calculated using:

$$t = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot \frac{-3.8}{2} \cdot 5}}{-3.8} = -2.23s \quad \vee \quad 1.18s \tag{8}$$

Inserting $t = 1.18s$ in Equation 3 gives the velocity at the surface.

$$v(t) = a \cdot t + v_0 \Rightarrow v(1.18) = -3.8 \cdot 1.18 - 2 = -6.48m/s \tag{9}$$

Thus the impact velocity at the surface is $v = 6.48m/s$ pointing in the downward direction.