Solutions lecture 6

6.1 Warm coffee

Analysis

A student has made a nice warm cup of coffee in a mug that has a heating element in it to keep the coffee warm. The heating element is in the coffee and keeps the coffee at a constant temperature of 66 °C. The thermal conductivity of the mug is 3.8 W m^{-1} K^{-1}. All heat is lost through convection and there is no heat loss through the ground. The outside temperature is 20 °C. Calculate the power needed for the heating element to keep the coffee at the same temperature. You may assume that the flat plate assumption is true for the convection at the sides of the mug.

Approach

Assumptions

- Only natural convection
- Coffee mug can be modeled as a cylinder with a hot plate on top.

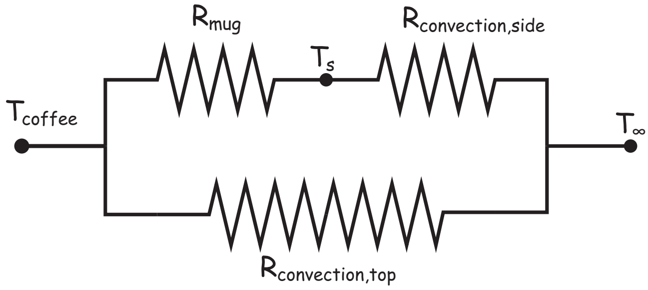
Route to solution

1. Determine the surface temperature of the mug
2. Determine the average temperatures
3. Determine the Grashof numbers
4. Determine the Rayleigh numbers
5. Choose the right correlation based on geometry and the Rayleigh numbers
6. Determine the Nusselt numbers
7. Derive the values of h
8. Substitute in Newton's cooling law
9. Add heat dissipation up to find the heating element requirement

Elaboration

It is important to see that there are two routes the heat can take to escape the environment. One flow is directly from the coffee to the environment and the other flow through the sides of the mug to the environment. As the sides of the mug add insulation, the temperatures at which the convection will take place are not equal, therefore.

To find the surface temperature of the mug, the thermal resistance network can be used. From it, it can be seen that the conductive heat flow through the mug sides must be equal to the convective heat flow at the sides.



$$\dot{Q}_{mug} = \dot{Q}_{convection,side}$$
$$\frac{T_{coffee} - T_s}{R_{mug}} = \frac{T_s - T_\infty}{R_{conv,side}}$$
$$\frac{T_{coffee} - T_s}{\frac{1}{2\pi \cdot k \cdot L} ln \frac{r_{out}}{r_{in}}} = \frac{T_s - T_\infty}{\frac{1}{\pi h \cdot D_{out} \cdot L}}$$

As h depends on the surface temperature, an estimation of T_s must be made to find the value h. There are many ways to confirm whether the right T_s value has been assumed. But one way could be to calculate the value of h for a certain assumption of T_s and then test whether:

$$\frac{T_{coffee} - T_s}{\frac{1}{2\pi \cdot k \cdot L} ln \frac{r_{out}}{r_{in}}} - \frac{T_s - T_\infty}{\frac{1}{\pi h \cdot D_{out} \cdot L}} \approx 0$$

The final answer should be close to 65.7 °C With this, the average temperature is:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{65.7 + 20}{2} = 42.85\,°\mathrm{C}$$

The Grashof number is defined as:

$$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

in which $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational constant, β is the thermal expansion coefficient:

$$\beta = \frac{1}{T_f} = \frac{2}{42.85 + 273.15} = 0.0032\,\mathrm{K^{-1}}$$

Note that the temperatures in β has the unit of Kelvin, not Celsius.

At 42.85 °C, air has the following properties: $k = 0.0268\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $\nu = 1.7250 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$. Substituting all these values gives:

$$\mathrm{Gr} = \frac{9.81 \cdot 0.0032 \cdot (65.7 - 20) \cdot 0.40^3}{(1.7250 \cdot 10^{-5})^2} = 3.0514 \cdot 10^8$$

At 42.85 °C, the Prandtl number is Pr = 0.7250. With this, the Rayleigh number can be determined to be:

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = 3.0514 \cdot 10^8 \cdot 0.7250 = 2.2123 \cdot 10^8$$

Now, it was given that the cylinder may be assumed as a flat plate. For the Rayleigh number in the magnitude of 10^4, the Nusselt number is:

$$\mathrm{Nu} = 0.59 \cdot \mathrm{Ra}^{1/4} = 0.59 \cdot (2.2123 \cdot 10^8)^{1/4} = 71.96$$

The Nusselt number is defined as

$$\mathrm{Nu} = \frac{hL_c}{k}$$

Substitution of all variables gives

$$h = \frac{Nu \cdot k}{L_c} = \frac{71.96 \cdot 0.0268}{0.40} = 4.82\,\mathrm{W\,m^{-2}\,K^{-1}}$$

Substitution of this result into Newton's cooling law, where the surface is $A = \pi D_{out} \cdot L$:

$$\dot{Q}_{side} = hA\Delta T = 4.82 \cdot \pi \cdot 0.091 \cdot 0.40 \cdot (65.7 - 20) = 62.99W$$

With a very similar method, the convection at the top of the mug can be calculated; Here, the average temperature is a lot easier, it is just:

$$T_f = \frac{T_{coffee} + T_\infty}{2} = \frac{66+20}{2} = 43\,^\circ\mathrm{C}$$

$$\beta = \frac{1}{T_f} = \frac{2}{43+273.15} = 0.0032\,\mathrm{K}^{-1}$$

Note that the temperatures in β have the unit of Kelvin, not Celsius. The characteristic length is;

$$L_c = \frac{A_s}{p} = \frac{\pi r_{inner}^2}{2\pi r_{inner}} = \frac{\pi 0.0435^2}{2\pi 0.0435} = 0.0217$$

At 43 °C, air has the following properties: $k = 0.0268\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $\nu = 1.7250 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$. Substituting all these values gives:

$$\mathrm{Gr} = \frac{9.81 \cdot 0.0032 \cdot (66-20) \cdot 0.0217^3}{(1.7250 \cdot 10^{-5})^2} = 4.936 \cdot 10^4$$

At 43 °C, the Prandtl number is $\mathrm{Pr} = 0.7250$. With this, the Rayleigh number can be determined to be:

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \left(4.936 \cdot 10^4\right) \cdot 0.7250 = 3.578 \cdot 10^4$$

Now, for this Rayleigh number, the following Nusselt relationship is valid:

$$\mathrm{Nu} = 0.59 \cdot \mathrm{Ra}^{1/4} = 0.59 \cdot (3.578 \cdot 10^4)^{1/4} = 8.115$$

The Nusselt number is defined as

$$\mathrm{Nu} = \frac{hL_c}{k}[-]$$

Substitution of all variables gives

$$h = \frac{Nu \cdot k}{L_c} = \frac{8.115 \cdot 0.0268}{0.0217} = 9.9987\,\mathrm{W\,m^{-2}\,K^{-1}}$$

Substitution of this result into Newton's cooling law, where the surface is $A = \pi r_{in}^2$:

$$\dot{Q}_{top} = hA\Delta T = 9.9987 \cdot \pi \cdot 0.0435^2 \cdot (66-20) = 2.73W$$

With the heat flows calculated, the total heat flows can be calculated.

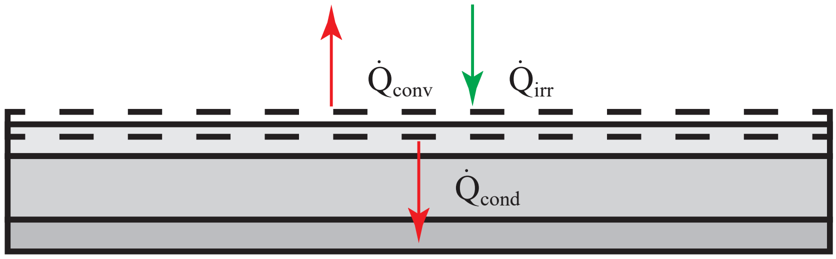
$$\dot{Q}_{loss} = \dot{Q}_{top} + \dot{Q}_{side} = 2.73 + 62.99 = 65.72W$$

As the temperature needs to remain constant, the heat loss should equal the power of the heating element, therefore, the heating element should have a power of 65.72 W

6.2 Food truck

a) Determine the roof surface's temperature and each layer's temperatures, and then draw a temperature profile within the roof structure and outside air.

The energy balance revolves around heat gained from solar irradiation, and heat lost from natural convection and conduction losses:



$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 0$$

The solar irradiance $\dot{Q}_{irr}$ is equal to:

$$\dot{Q}_{\text{irr}} = \alpha G A = (1 - 0.9) \cdot 1100 \cdot (3 \cdot 2) = 660\text{W}$$

The losses as a result of natural convection can be determined by first considering an assumed roof temperature. The outside roof temperature will be assumed to be close to $T_{\text{s,out}} = 55$ °C. Relevant air properties at $T_f = 45$ °C from the provided table are:

- $\nu = 1.750 \cdot 10^{-5}\ \text{m}\,\text{s}^{-2}$
- $\text{Pr} = 0.7241$
- $k = 0.02699\ \text{W}\,\text{m}^{-1}\,\text{K}^{-1}$
- $\beta = \frac{2}{T_{\text{s,out}} + T_\infty} = \frac{2}{(55+35)+273.15} = 0.0031\ \text{K}^{-1}$

The characteristic length yields to be:

$$L_{\text{c}} = \frac{A}{2L + 2W} = \frac{2 \times 3}{2 \times 3 + 2 \cdot 2} = 0.6\ \text{m}$$

This allows us to calculate the Grashof and Rayleigh number:

$$\text{Gr} = \frac{g\beta(T_{\text{s,out}} - T_\infty)L_c^3}{\nu^2} = \frac{9.81 \cdot 0.0031(55 - 35)0.6^3}{(1.750 \cdot 10^{-5})^2} = 4.3496 \cdot 10^8$$
$$\text{Ra} = \text{Gr} \cdot \text{Pr} = 4.3496 \cdot 10^8 \cdot 0.7241 = 3.1495 \cdot 10^8$$

At this Rayleigh number, the following Nusselt correlation can be used:

$$\text{Nu} = 0.15\text{Ra}^{\frac{1}{3}} = 0.15 \cdot (3.1495 \cdot 10^8)^{\frac{1}{3}} = 102.06$$

Using the definition to determine the outside heat transfer coefficient:

$$h = \frac{k_{air}\text{Nu}}{L_{\text{c}}\cdot} = \frac{0.02699}{0.6 \cdot 102.06} = 4.59\ \text{W/m}^2\text{K}$$

By use of Newton's law of cooling, we can determine the rate of heat loss by natural convection to the outside:

$$\dot{Q}_{\text{conv}} = hA(T_{\text{s,out}} - T_\infty) = 4.59 \cdot 6 \cdot (55 - 35) = 550.9\ \text{W}$$

To determine $\dot{Q}_{\text{cond}}$ the sum of all thermal conductive resistances are required. All individual components can be calculated as:

$$R_1 = \frac{t_1}{k_1 \cdot A_c} = \frac{0.004}{130 \cdot (2 \times 3)} = 5.13 \cdot 10^{-6} \text{ K/W}$$
$$R_2 = \frac{t_2}{k_2 \cdot A_c} = \frac{0.06}{0.036 \cdot (2 \times 3)} = 2.78 \cdot 10^{-1} \text{ K/W}$$
$$R_3 = \frac{t_3}{k_3 \cdot A_c} = \frac{0.005}{0.26 \cdot (2 \times 3)} = 3.21 \cdot 10^{-3} \text{ K/W}$$

Where the sum of these resistances yields the total conductive thermal resistance in the roof:

$$R_{\text{total}} = R_1 + R_2 + R_3 = 5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3} = 2.81 \cdot 10^{-1} \text{ K/W}$$

Using the thermal resistance theorem one can determine the rate of heat loss from the roof due to conduction to the inside:

$$\dot{Q}_{\text{cond}} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{\text{s,out}} - T_{\text{s,in}}}{R_{\text{total}}} = \frac{55 - 22}{2.81 \cdot 10^{-1}} = 117.4 \text{ W}$$

To validate whether the made assumption for $T_{\text{s,out}}$ was correct, we should check the energy balance.

$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 660 - 550.9 - 117.4 = -8.3 \text{ W}$$

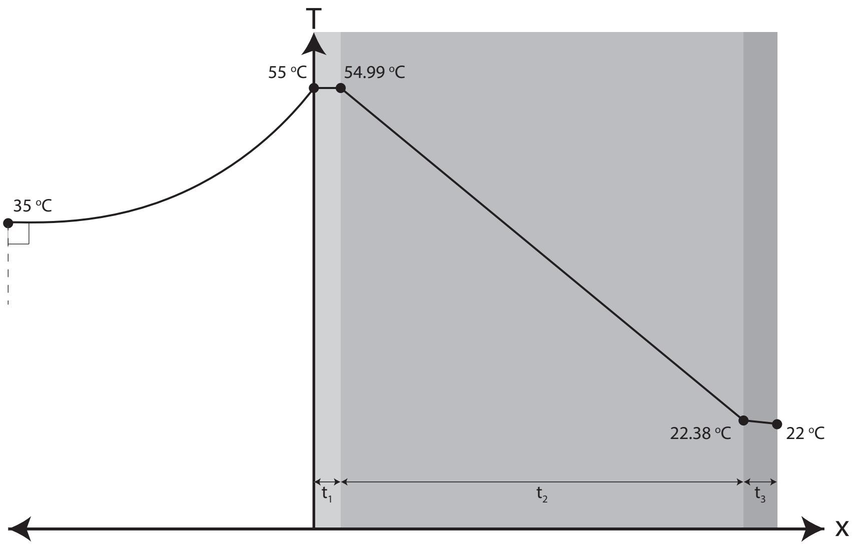
A small deviation from the balance of actual being in equilibrium is found ($\approx 1.2\%$), which is acceptable in this case.

The temperatures in each layer can be found by rewriting the thermal resistance theorem, which yields:

$$T_{\text{plas-insul}} = R_3 \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = 3.21 \cdot 10^{-3} \cdot 117.4 + 22 = 22.38 \text{ °C}$$

$$T_{\text{insul-steel}} = (R_2 + R_3) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = \left(2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3}\right) \cdot 117.4 + 22 = 54.99 \text{ °C}$$

Where the temperature profile can be drawn as:

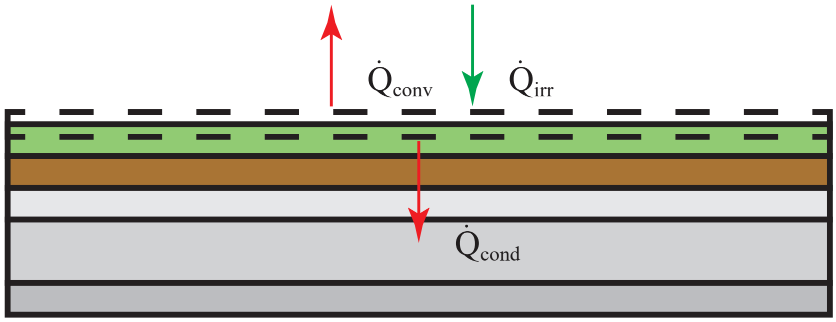


The following criteria should be clearly specified in the graph:

- $x \to -\infty$ the slope should approach zero.
- The temperature drop in the outside air should be curved with a decreasing slope when moving in the negative x-direction.
- The temperature drops in each solid layer should be straight.
- The slope of layer 2 should be the steepest and the slope of layer 1 the least.
- The temperatures should be correct.

b) One of the group members suggested using a green roof for the food truck to save energy for cooling by adding grass layer (t_4 = 10mm, k_4 = 0.038 $\mathrm{W\,m^{-1}\,K}$) with reflectivity of 0.95 and green roof base layer (t_5 = 15mm, k_5 = 0.36 $\mathrm{W\,m^{-1}\,K}$). Determine the roof surface's temperature and each layer's temperatures, and then draw a temperature profile within the roof structure and outside air.

The energy balance revolves around heat gained from solar irradiation, and heat lost from natural convection and conduction losses:



$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 0$$

The solar irradiance $\dot{Q}_{irr}$ is equal to:

$$\dot{Q}_{\text{irr}} = \alpha G A = (1 - 0.95) \cdot 1100 \cdot (3 \cdot 2) = 330\text{W}$$

The losses as a result of natural convection can be determined by first considering an assumed roof temperature. The assumed roof temperature will be close to $T_{\text{s,out}} = 46$ °C. Relevant air properties at $T_f = 40$ °C from the provided table:

- $\nu = 1.702 \cdot 10^{-5}\ \mathrm{m\,s^{-2}}$
- $\mathrm{Pr} = 0.7255$
- $k = 0.02662\ \mathrm{W\,m^{-1}\,K^{-1}}$
- $\beta = \frac{2}{T_s + T_\infty} = \frac{2}{(55+273.15)+(35+273.15)} = 0.0031\ \mathrm{K^{-1}}$

The characteristic length yields to be:

$$L_{\text{c}} = \frac{A}{2L + 2W} = \frac{2 \cdot 3}{2 \cdot 3 + 2 \cdot 2} = 0.6\ \text{m}$$

This allows us to calculate the Grashof and Rayleigh number:

$$\begin{aligned} \mathrm{Gr} &= \frac{g\beta(T_{\text{s,out}} - T_\infty)L_c^3}{\nu^2} = \frac{9.81 \cdot 0.0032(46 - 35)0.6^3}{(1.702 \cdot 10^{-5})^2} = 2.5654 \cdot 10^8 \\ \mathrm{Ra} &= \mathrm{Gr} \cdot \mathrm{Pr} = 2.5654 \cdot 10^8 \cdot 0.7255 = 1.8612 \cdot 10^8 \end{aligned}$$

At this Rayleigh number, the following Nusselt correlation can be used:

$$\mathrm{Nu} = 0.15\mathrm{Ra}^{\frac{1}{3}} = 0.15 \cdot (1.8612 \cdot 10^8)^{\frac{1}{3}} = 85.64$$

Using the definition to determine the outside heat transfer coefficient:

$$h = \frac{k_{air}\mathrm{Nu}}{L_{\text{c}}\cdot} = \frac{0.02662}{0.6 \cdot 85.64} = 3.80\ \mathrm{W/m^2K}$$

By use of Newton's law of cooling, we can determine the rate of heat loss by natural convection to the outside:

$$\dot{Q}_{\text{conv}} = hA(T_{\text{s,out}} - T_{\infty}) = 3.80 \cdot 6 \cdot (46 - 35) = 250.8 \text{ W}$$

To determine $\dot{Q}_{\text{cond}}$ the sum of all thermal conductive resistances are required. All individual components can be calculated as:

$$R_4 = \frac{t_4}{k_4 \cdot A_c} = \frac{0.01}{0.038 \cdot (2 \cdot 3)} = 4.39 \cdot 10^{-2} \text{ K/W}$$
$$R_5 = \frac{t_5}{k_5 \cdot A_c} = \frac{0.015}{0.36 \cdot (2 \cdot 3)} = 6.94 \cdot 10^{-3} \text{ K/W}$$

Where the sum of these resistances yields the total conductive thermal resistance in the roof:

$$\begin{aligned} R_{\text{total}} &= R_1 + R_2 + R_3 + R_4 + R_5 \\ &= 5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3} + 4.39 \cdot 10^{-2} + 6.94 \cdot 10^{-3} \\ &= 3.32 \cdot 10^{-1} \text{ K/W} \end{aligned}$$

Using the thermal resistance theorem, one can determine the rate of heat loss from the roof due to conduction to the inside:

$$\dot{Q}_{\text{cond}} = \frac{\Delta T}{R_{\text{total}}} = \frac{46 - 22}{3.32 \cdot 10^{-1}} = 72.3 \text{ W}$$

To validate whether the made assumption for $T_{\text{s,out}}$ was correct, we should check the energy balance:

$$\sum \dot{Q} = \dot{Q}_{\text{irr}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{cond}} = 330 - 250.8 - 72.3 = 6.9 \text{ W}$$

A small deviation from the balance of actual being in equilibrium is found ($\approx 2.1\%$), which is acceptable in this case.

The temperatures in each layer can be found by use of the thermal resistance theorem, which yields:

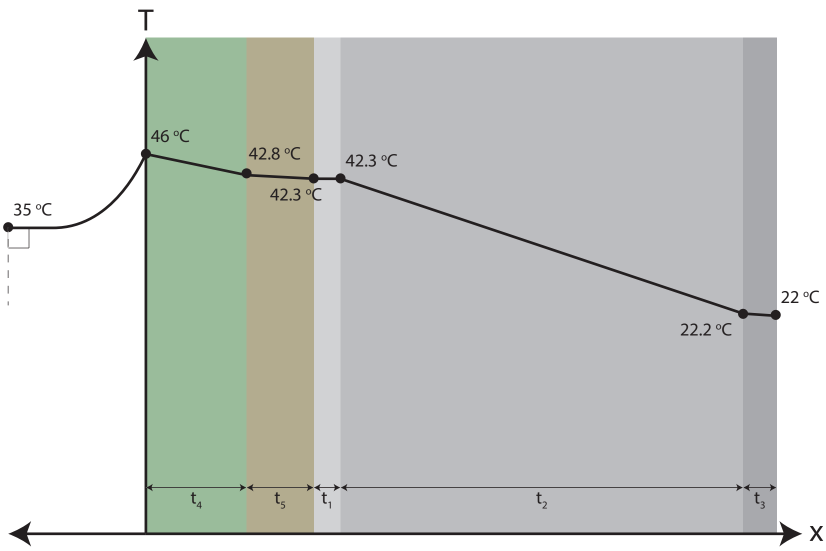
$$T_{\text{plas-insul}} = R_3 \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = 3.21 \cdot 10^{-3} \cdot 72.3 + 22 = 22.23 \text{ °C}$$

$$T_{\text{insul-steel}} = (R_2 + R_3) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = \left(2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3}\right) \cdot 72.3 + 22 = 42.32 \text{ °C}$$

$$T_{\text{steel-base}} = (R_1 + R_2 + R_3) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} = \left(5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3}\right) \cdot 72.3 + 22 = 42.32 \text{ °C}$$

$$\begin{aligned} T_{\text{base-grass}} &= (R_1 + R_2 + R_3 + R_5) \cdot \dot{Q}_{\text{cond}} + T_{\text{s,in}} \\ &= \left(5.13 \cdot 10^{-6} + 2.78 \cdot 10^{-1} + 3.21 \cdot 10^{-3} + 6.94 \cdot 10^{-3}\right) \cdot 72.3 + 22 = 42.82 \text{ °C} \end{aligned}$$

Where the temperature profile can be drawn as:



- $x \to -\infty$ the slope should approach zero.
- The temperature drop in the outside air should be curved with a decreasing slope when moving in the negative x-direction.
- The temperature drops in each solid layer should be straight.
- The slopes should be correct, the sequence of largest slope to smallest is: 2-4-3-5-1
- The temperatures should be correct.

c) The heat generated by the cooking and other appliances inside the food truck is 45 $\mathrm{kJ\,min^{-1}}$. On this day, an AC is used to cool down the inside and keep the temperature constant, reaching equilibrium for 8 hours using electricity at a cost of 0.7 €/kW h. How much money would be saved by using the green roof for the food truck on this particular day? The losses from the truck's sides and bottom are insignificant.

<u>Scenario where no green roof is there:</u>

$$\dot{Q}_{\mathrm{cond},1} = 117 \text{ W}$$

$$\dot{Q}_{\mathrm{heat},1} = 45 \text{ kJ/min} = 750 \text{ W}$$

Total heat that should be extracted by use of the HVAC system in 8 hours of time:

$$E_1 = \left(\dot{Q}_{\mathrm{cond},1} + \dot{Q}_{\mathrm{heat},1}\right) \cdot \Delta t = (117 + 750) \cdot 8 = 6.94 \text{ kWh}$$

In terms of price, this operation will cost:

$$\text{Price}_1 = E_1 \cdot \text{€}_{\mathrm{kWh}} = 6.94 \cdot 0.7 \ = \text{€ } 4.86$$

<u>Scenario where a green roof is there:</u>

$$\dot{Q}_{\mathrm{cond},2} = 72 \text{ W}$$

$$\dot{Q}_{\mathrm{heat},2} = 45 \text{ kJ/min} = 750 \text{ W}$$

Total heat that should be extracted by use of the AC system in 8 hours of time:

$$E_2 = \left(\dot{Q}_{\mathrm{cond},2} + \dot{Q}_{\mathrm{heat},2}\right) \cdot \Delta t = (72 + 750) \cdot 8 = 6.58 \text{ kWh}$$

In terms of price, this operation will cost:

$$\text{Price}_2 = E_2 \cdot \text{€}_{\mathrm{kWh}} = 6.58 \cdot 0.7 \ = \text{€ } 4.61$$

<u>Savings at the specified day:</u>

$$\text{Savings} = \text{Price}_1 - \text{Price}_2 = 4.86 - 4.61 = \text{€ } 0.25$$

6.3 Pizza oven - Hand in

a) Sketch the thermal resistance network for the complete oven and give a clear description for all components.

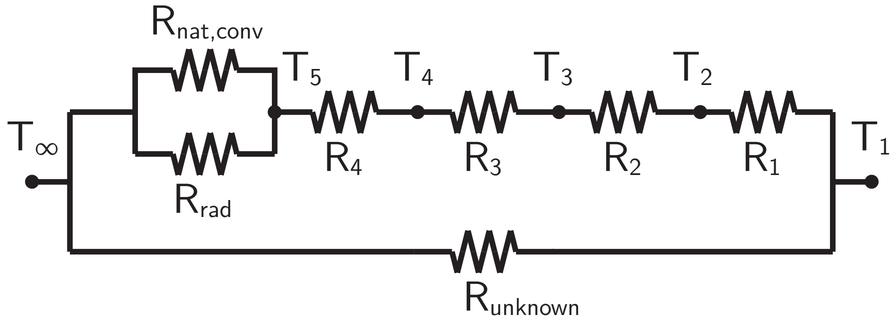


Figure 6.1: Resistance network

- $R_{\text{nat,conv}}$: Convective resistance due to natural convection from the oven to the ambient.
- R_{rad}: Radiative resistance due to emission from the aluminum sheet to the ambient.
- R_4: Conductive resistance due to the aluminum layer.
- R_3: Conductive resistance due to the rockwool layer.
- R_2: Conductive resistance due to the clay layer.
- R_1: Convective resistance due to convection from the hot air in the oven.
- R_{unknown}: Sum of the total resistance for the top of the oven, including similar resistances as for the sides due to radiation, convection, and conduction.
- T_∞: Ambient temperature.
- T_5, T_4, T_3, T_2: Interface temperatures.
- T_1: Inside oven air temperature.

b) Sketch the temperature profile starting from the air on the inside all the way to the outside, through all the layers of the cylinder wall.

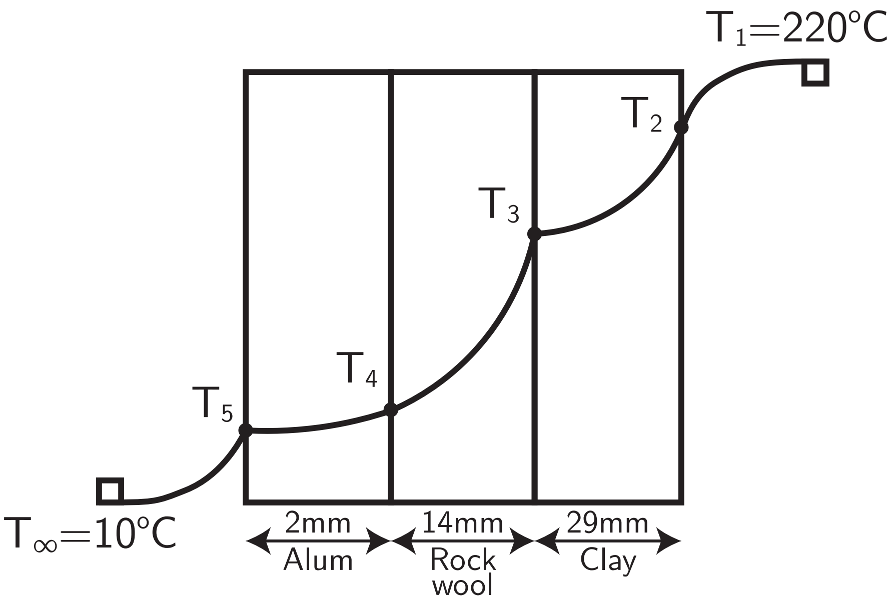


Figure 6.2: Temperature profile

- The temperature drop in the outside and inside air should be curved with a decreasing slope when moving away from the solid layers.
- The temperature drops in each solid layer should be curved.
- The slopes should be correct, with different slopes at the interfaces (also at the alum-rockwool interface). The order of largest to smallest temperature drop is Rock wool - Clay - Aluminium.
- The only known temperatures at this point are the given air temperatures. As such, indicating values in the figure is not strictly necessary.
- For each wrong layer, subtract 2 points.

c) Check with calculations that the outside aluminum wall of the oven will not get hotter than 40 °C. Define all parameters with clear distinctive symbols.

The thermal conductive resistances can be calculated as:

$$
\begin{aligned}
R_1 &= \frac{1}{h_{in}A} = \frac{1}{h_{in}\pi D_1 L} = \frac{1}{11 \cdot \pi \cdot 0.32 \cdot 0.5} = 0.1809 \mathrm{K\,W^{-1}} \\
R_2 &= \frac{ln\left(\frac{D_2}{D_1}\right)}{2\pi L k_{clay}} = \frac{ln\left(\frac{378}{320}\right)}{2\pi 0.5 \cdot 0.1} = 0.5302 \mathrm{K\,W^{-1}} \\
R_3 &= \frac{ln\left(\frac{D_3}{D_2}\right)}{2\pi L k_{rw}} = \frac{ln\left(\frac{406}{378}\right)}{2\pi 0.5 \cdot 0.03} = 0.7582 \mathrm{K\,W^{-1}} \\
R_4 &= \frac{ln\left(\frac{D_4}{D_3}\right)}{2\pi L k_{al}} = \frac{ln\left(\frac{410}{406}\right)}{2\pi 0.5 \cdot 237} = 1.32 \cdot 10^{-4} \mathrm{K\,W^{-1}}
\end{aligned}
$$

Note that the thermal conductivity of aluminum is not given. As aluminum has high thermal conductivity and the sheet is relatively thin, the assumption of neglecting this resistance is justified.

$$\rightarrow R_{tot,1} = R_1 + R_2 + R_3 + R_4 = 1.4694 \mathrm{K\,W^{-1}}$$

A surface temperature should be guessed. The actual temperature will be close to 32.7 °C:

$$T_f = \frac{T_S + T_\infty}{2} = \frac{42.7}{2} = 21.35 \approx 20°\mathrm{C}$$

Air properties at this film temperature are:

$$
\begin{aligned}
\rho_{air} &= 1.204 \mathrm{kg\,m^{-3}} \\
k_{air} &= 0.02514 \mathrm{W\,m^{-1}\,K^{-1}} \\
\nu_{air} &= 1.516 \cdot 10^{-5} \mathrm{m^2\,s^{-1}} \\
Pr_{air} &= 0.7309 \\
\beta &= \frac{2}{T_S + T_\infty} = \frac{2}{(273.15 + 32.7) + 283.15} = 0.0034
\end{aligned}
$$

The characteristic length in this case equals the height of the oven.

$$L_c = h = 0.5\mathrm{m}$$

The Grashof number can be calculated as:

$$Gr_L = \frac{9.81 \cdot 0.0034 \cdot (32.7 - 10) \cdot 0.5^3}{(1.516 \cdot 10^{-5})^2} = 4.118 \cdot 10^8$$

From the Grashof and Prandtl numbers, the Rayleigh number follows:

$$Ra_L = Gr_L Pr = 4.118 \cdot 10^8 \cdot 0.7309 = 3.0098 \cdot 10^8$$

Next up a correlation for the Nusselt number needs to be chosen.

We find that:

$$D_4 \geq \frac{35L_c}{(Gr_L)^{\frac{1}{4}}} \rightarrow 0.41\text{m} \geq \frac{35 \cdot 0.5}{(4.118 \cdot 10^8)^{\frac{1}{4}}} \rightarrow 0.41\text{m} \geq 0.1228\text{m}$$

which means the cylinder may be treated as a flat plate.

For this scenario, the Nusselt correlation is:

$$Nu_L = 0.59Ra_L^{\frac{1}{4}} = 0.59 \cdot (3.0098 \cdot 10^8)^{\frac{1}{4}} = 77.7117$$

Where rewriting the definition of the Nusselt number yields the heat transfer coefficient:

$$h_{nat.conv.} = \frac{Nu_L k_{air}}{L_c} = \frac{77.7117 \cdot 0.02514}{0.5} = 3.908\text{W m}^{-2}\text{K}^{-1}$$

The thermal resistance then be calculated as:

$$R_{nat.conv.} = \frac{1}{hA} = \frac{1}{3.908 \cdot \pi \cdot 0.41 \cdot 0.5} = 0.3973\text{K W}^{-1}$$

The radiative heat transfer coefficient can be calculated as follows:

$$h_{rad} = \varepsilon\sigma(T_5^2+T_\infty^2)(T_5+T_\infty) = 0.83 \cdot 5.67 \cdot 10^{-8}((32.7+273.15)^2+283.15^2)((32.7+273.15)+283.15) = 4.8153\text{W m}^{-2}\text{K}^{-1}$$

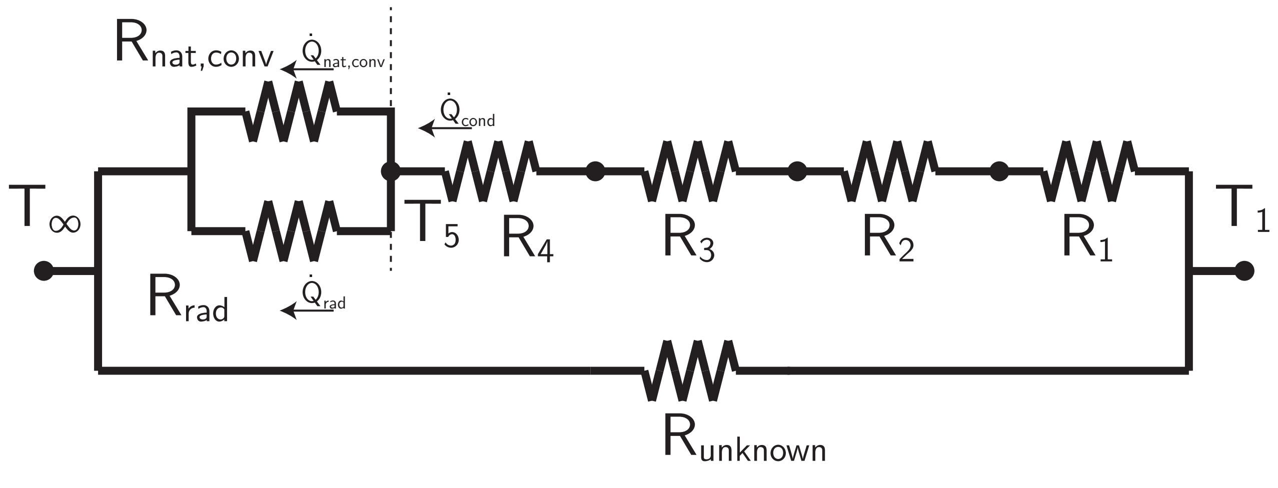
Which yields a thermal resistance of:

$$R_{rad} = \frac{1}{h_{rad}A} = \frac{1}{4.8153 \cdot \pi \cdot 0.41 \cdot 0.5} = 0.3225\text{K W}^{-1}$$

The two parallel resistances can be rewritten to an equivalent resistance that stands in series with the other conductive resistances:

$$\frac{1}{R_{tot,2}} = \frac{1}{R_{nat.conv.}} + \frac{1}{R_{radiation}} = \frac{1}{0.3973} + \frac{1}{0.3225} = 5.6178 \rightarrow R_{tot,2} = 0.1780\text{K W}^{-1}$$

The relevant energy balance is:



$$\dot{Q}_{cond.} = \dot{Q}_{nat.conv.} + \dot{Q}_{rad.} \rightarrow \frac{T_1 - T_5}{R_{tot,1}} = \frac{T_5 - T_\infty}{R_{tot,2}}$$

Rewriting for T_5:

$$T_5 = \frac{R_{tot,2}T_1 + R_{tot,1}T_\infty}{R_{tot,1} + R_{tot,2}} = \frac{0.1780 \cdot (273.15 + 220) + 1.4694(273.15 + 10)}{0.1780 + 1.4694} = 305.8403\text{K} \rightarrow 305.8403\text{K} = 32.69°\text{C}$$

So the surface temperature of the aluminum sheet is indeed 32.7 °C.

d) Henk claims: “to keep the outside of the oven even cooler, I could paint the aluminum using radiator paint. This way the oven can give off its heat even better and as a result, it will stay cooler this way.” Ingrid agrees with Henk that the emissivity of radiator paint is higher than that of brushed aluminum, but does not think the wall will become cooler after the oven is painted. Who is correct?

Increasing the emissivity would result in a larger value for h_{rad}, which in turn would lead to a lower value for $R_{tot,2}$. Considering just this lowered value would indeed give a lower surface temperature, meaning Henk would be right. However, the paint layer would represent additional thermal resistance. Depending on the thickness and thermal conductivity of the paint layer, the surface temperature could potentially be the same or higher as in the case without radiator paint, although paint layers are usually rather thin.