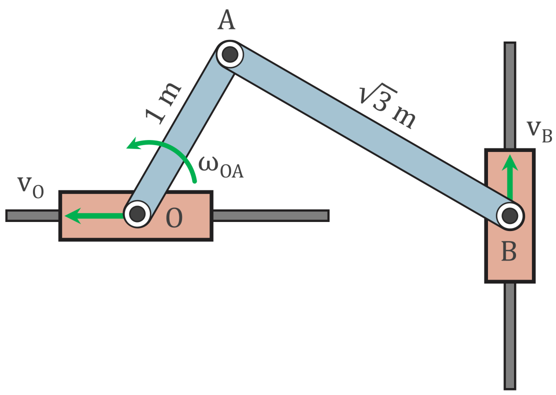
Two DOF Planar Manipulator



The two degree of freedom manipulator is driven by two linear actuators (O and B). The lines from O to A and from B to A are perpendicular to each other in the given situation. The length of the arms OA and AB are 1 m and $\sqrt{3}$ m respectively. Calculate the velocity of actuator B in m/s , if the velocity of actuator O is known to be $v_O = \frac{3}{2}\sqrt{3}$ m/s and the angular velocity of arm OA is $\omega_{OA} = \frac{5}{4}$ rad/s.

Using known expressions:

$$\mathbf{v}_{\mathrm{A}} = \mathbf{v}_{\mathrm{O}} + \boldsymbol{\omega}_{\mathrm{OA}} \times \mathbf{r}_{\mathrm{A/O}} + \mathbf{v}_{\mathrm{rel}} \tag{1}$$

$$\mathbf{v}_{\mathrm{A}} = \mathbf{v}_{\mathrm{B}} + \boldsymbol{\omega}_{\mathrm{AB}} \times \mathbf{r}_{\mathrm{A/B}} + \mathbf{v}_{\mathrm{rel}} \tag{2}$$

Given:

Length of OA: $L_{OA} = 1$ m
Length of AB: $L_{AB} = \sqrt{3}$ m
Velocity of O: $v_O = \frac{3}{2}\sqrt{3}$ m/s
Angular velocity arm OA: $\omega_{OA} = \frac{5}{4}$ rad/s
Angle $\angle$ OAB: 90°

Solution:

Using trigonometry it can be seen that ∠BOA = 60°, since it is given that $L_{OA} = 1$, $L_{AB} = \sqrt{3}$ and ∠ OAB: 90° (triangle △ OAB is a 30 − 60 − 90 triangle). For the derivations we use θ for the ∠BOA. Inserting the known values (with $v_{rel} = 0$) in Equation 1 gives:

$$\mathbf{v}_\mathrm{A} = \begin{pmatrix} -v_O \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{OA} \end{pmatrix} \times \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} L_{OA} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{v}_\mathrm{A} = \begin{pmatrix} -v_O \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} L_{OA} \cdot \omega_{OA} = \begin{pmatrix} -v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA} \\ \cos\theta \cdot L_{OA} \cdot \omega_{OA} \\ 0 \end{pmatrix} \tag{4}$$

Inserting the known values (with $v_{rel} = 0$) in Equation 2 gives:

$$\mathbf{v}_\mathrm{A} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} L_{AB} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\mathbf{v}_\mathrm{A} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} + \begin{pmatrix} -\sin\theta \\ -\cos\theta \\ 0 \end{pmatrix} \cdot L_{AB} \cdot \omega_{AB} = \begin{pmatrix} -\cos\theta \cdot L_{AB} \cdot \omega_{AB} \\ v_B - \sin\theta \cdot L_{AB} \cdot \omega_{AB} \\ 0 \end{pmatrix} \tag{6}$$

We now have two equations (Equations 4 and 6) for v_A that must be equal to each other. This follows.

$$\begin{pmatrix} -v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA} \\ \cos\theta \cdot L_{OA} \cdot \omega_{OA} \\ 0 \end{pmatrix} = \begin{pmatrix} -\cos\theta \cdot L_{AB} \cdot \omega_{AB} \\ v_B - \sin\theta \cdot L_{AB} \cdot \omega_{AB} \\ 0 \end{pmatrix} \tag{7}$$

This gives us two equations with two unknowns, ω_{AB} and v_B, thus it is possible to solve for v_B. First we solve for $-\mathrm{L}_{AB} \cdot \omega_{AB}$ using the top equation, as we can use this in the second equation to solve for v_B:

$$-v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA} = -\cos\theta \cdot L_{AB} \cdot \omega_{AB} \tag{8}$$

$$-L_{AB} \cdot \omega_{AB} = \frac{-v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA}}{\cos\theta} \tag{9}$$

Inserting this in the second equation gives us a solution for v_B.

$$\cos\theta \cdot L_{OA} \cdot \omega_{OA} = v_B - \sin\theta \cdot L_{AB} \cdot \omega_{AB} \tag{10}$$

$$\cos\theta \cdot L_{OA} \cdot \omega_{OA} = v_B + \sin\theta \cdot (-L_{AB} \cdot \omega_{AB}) \tag{11}$$

$$\cos\theta \cdot L_{OA} \cdot \omega_{OA} = v_B + \sin\theta \cdot \left(\frac{-v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA}}{\cos\theta}\right) \tag{12}$$

$$v_B = \cos\theta \cdot L_{OA} \cdot \omega_{OA} - \sin\theta \cdot \left(\frac{-v_O - \sin\theta \cdot L_{OA} \cdot \omega_{OA}}{\cos\theta}\right) \tag{13}$$

$$v_B = \cos\theta \cdot L_{OA} \cdot \omega_{OA} + \sin\theta \cdot \left(\frac{v_O + \sin\theta \cdot L_{OA} \cdot \omega_{OA}}{\cos\theta}\right) \tag{14}$$

$$v_B = \cos\theta \cdot L_{OA} \cdot \omega_{OA} + (v_O + \sin\theta \cdot L_{OA} \cdot \omega_{OA}) \cdot \frac{\sin\theta}{\cos\theta} \tag{15}$$

$$v_B = \cos\theta \cdot L_{OA} \cdot \omega_{OA} + (v_O + \sin\theta \cdot L_{OA} \cdot \omega_{OA}) \cdot \tan\theta \tag{16}$$

Inserting the values for $\theta, L_{OA}, \omega_{OA}, v_O$ gives:

$$v_B = \cos(60°) \cdot 1 \cdot \frac{5}{4} + \left(\frac{3}{2}\sqrt{3} + \sin(60°) \cdot 1 \cdot \frac{5}{4}\right) \cdot \tan(60°) \tag{17}$$

$$v_B = \frac{1}{2} \cdot \frac{5}{4} + \left(\frac{3}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} \cdot \frac{5}{4}\right) \cdot \sqrt{3} = \frac{5}{8} + \frac{9}{2} + \frac{15}{8} = 7 \text{ m/s} \tag{18}$$