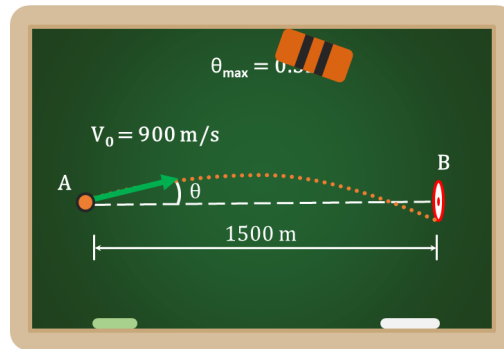# Bullet on Target Bottom

A person fires a round from **A** toward a target **B**. Find an expression for the vertical displacement **y(t)** as a function of time, v_0 and θ, for which the round just hits the bottom of the target. The target diameter is **2 m** and the target centre is at the same altitude as the end of the rifle barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between **A** and **B** is 1500 m. Neglect air resistances and assume that the round is directed along the vertical centreline of the target. Take **g = 10 m/s²**.
y(t) $= \cdots = \ldots$

Using known expressions:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v} \mathrm{d}v = a \int_{0}^{t} \mathrm{d}t \tag{2}$$

$$v(t) = a \cdot t + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_{0}^{t} (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}a \cdot t^2 + v_0 \cdot t + s_0 \tag{6}$$

Given:
Distance A-B: $s = 1500m$
Gravitational acceleration: $g = 10m/s^2$
Initial velocity: $v_0 = 900m/s$
Diameter target: $D = 2m$

Filling in Equation 6 gives an relation for the y-position with respect to time. Where $a = -g$ and $y_0 = 0m$, since the riffle barrel is at the same altitude as the target centre.

$$y(t) = -\frac{1}{2} \cdot g \cdot t^2 + v_{0,y} \cdot t + y_0 \Rightarrow y(t) = -\frac{1}{2} \cdot g \cdot t^2 + \sin\theta \cdot v_0 \cdot t \tag{7}$$

The bottom of the target is at $1m$ from the centreline thus $y(t_{end}) = -1m$. Furthermore, inserting $g = 10m/s^2$ result in.

$$y(t) = -\frac{1}{2} \cdot 10 \cdot t^2 + \sin\theta \cdot v_0 \cdot t = 1 \Rightarrow -5 \cdot t^2 + \sin\theta \cdot v_0 \cdot t = -1 \tag{8}$$