2.5 Transverse flow ★★

a) *Provide an expression for the rate of heat loss in terms of the given parameters.*

1) Setup the definition of the rate of heat transfer:

The rate of heat transfer can be described by Newton's law of cooling:

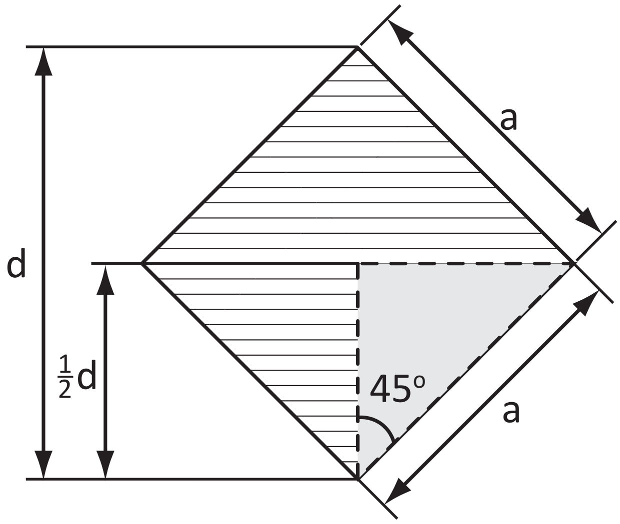
$$\boxed{\rightarrow \dot{Q} = \overline{\alpha} A_s \left(T_s - T_\infty\right)} \tag{2.42}$$

2) Defining all required parameters:

The Reynolds number is defined as follows:

$$\mathrm{Re} = \frac{\rho \cdot u_\infty \cdot L_\mathrm{c}}{\eta} \tag{2.43}$$

Where the characteristic length for a square subjected to transverse flow equals d, in the figure below:



With trigonometry d can be derived:

$$\cos\left(45^\circ\right) = \frac{\frac{1}{2}d}{a} \tag{2.44}$$

$$\boxed{\rightarrow d = 2a \cdot \cos\left(45^\circ\right) = \sqrt{2} \cdot a} \tag{2.45}$$

Therefore the Reynolds number can be expressed as:

$$\boxed{\rightarrow \mathrm{Re} = \frac{\rho \cdot u_\infty \cdot \sqrt{2} \cdot a}{\eta}} \tag{2.46}$$

Having determined Reynolds number, the Nusselt number can be determined:

$$\boxed{\rightarrow \overline{\mathrm{Nu}} = 0.246 \cdot \mathrm{Re}^{0.588} \cdot \mathrm{Pr}^{0.4}} \tag{2.47}$$

Rewriting definition of the Nusselt number yields the the heat transfer coefficient:

$$\boxed{\rightarrow \overline{\alpha} = \frac{\overline{\mathrm{Nu}} \cdot \lambda}{\sqrt{2} \cdot \mathrm{a}}} \tag{2.48}$$

Lastly, only the surface area which loses heat should be expressed, which is:

$$\boxed{\rightarrow A_{\mathrm{s}} = 4 \cdot a \cdot L} \tag{2.49}$$

3) Inserting and rearranging:

Inserting the found equations in the the expression for the rate of heat loss:

$$\dot{Q} = \overline{\alpha} A_s \left(T_s - T_\infty\right) \tag{2.50}$$

$$\dot{Q} = \frac{\overline{\mathrm{Nu}} \cdot \lambda}{\sqrt{2} \cdot \mathrm{a}} \cdot 4 \cdot a \cdot L \cdot \left(T_s - T_\infty\right) \tag{2.51}$$

$$\dot{Q} = \frac{0.246 \cdot \mathrm{Re}^{0.588} \cdot \mathrm{Pr}^{0.4} \cdot \lambda}{\sqrt{2} \cdot \mathrm{a}} \cdot 4 \cdot a \cdot L \cdot \left(T_s - T_\infty\right) \tag{2.52}$$

$$\boxed{\rightarrow \dot{Q} = 0.492 \cdot \sqrt{2} \cdot \left(\frac{\rho \cdot u_\infty \cdot \sqrt{2} \cdot a}{\eta}\right)^{0.588} \cdot \mathrm{Pr}^{0.4} \cdot \lambda \cdot L \cdot \left(T_s - T_\infty\right)} \tag{2.53}$$

b) *Determine the percentual change of the heat transfer coefficient if we had a similar-form rod with four times the crosswise width at double flow velocity.*

1) Setting up the definition of the heat transfer coefficient:

We know that:

$$\boxed{\rightarrow \overline{\alpha} = \frac{\overline{\mathrm{Nu}}_{\mathrm{d}} \cdot \lambda}{d}} \tag{2.54}$$

2) Defining all required parameters:

In question a) we already gave the definitions for the required parameters, which were:

The characteristic length:

$$d = \sqrt{2} \cdot a \tag{2.55}$$

The Reynolds number:

$$\mathrm{Re}_{\mathrm{d}} = \frac{\rho \cdot u_{\infty} \cdot \sqrt{2} \cdot a}{\eta} \tag{2.56}$$

The correlation of the Nusselt number (which is the same for both situations due to the given restriction that $10^4 < \mathrm{Re}_{\mathrm{d}} < 10^5$)

$$\overline{\mathrm{Nu}}_{\mathrm{d}} = 0.246 \cdot \mathrm{Re}_{\mathrm{d}}^{0.588} \cdot \mathrm{Pr}^{0.4} \tag{2.57}$$

For the initial situation, which we call situation 1, we can define the following parameters:

$$\boxed{\rightarrow d_1 = \sqrt{2} \cdot a_1} \tag{2.58}$$

$$\boxed{\rightarrow \mathrm{Re}_{\mathrm{d},1} = \frac{\rho \cdot u_1 \cdot \sqrt{2} \cdot a_1}{\eta}} \tag{2.59}$$

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_{\mathrm{d},1} = 0.246 \cdot \mathrm{Re}_{\mathrm{d},1}^{0.588} \cdot \mathrm{Pr}^{0.4} = 0.246 \cdot \left(\frac{\rho \cdot u_1 \cdot \sqrt{2} \cdot a_1}{\eta}\right)^{0.588} \cdot \mathrm{Pr}^{0.4}} \tag{2.60}$$

For the situation where the crosswise width is four times as big and the flow velocity doubles, which we call situation 2, we can define the following parameters:

$$\boxed{\rightarrow d_2 = \sqrt{2} \cdot a_2 = 4 \cdot \sqrt{2} \cdot a_1} \tag{2.61}$$

$$\boxed{\rightarrow \mathrm{Re}_{\mathrm{d},2} = \frac{\rho \cdot u_2 \cdot \sqrt{2} \cdot a_2}{\eta} = \frac{8 \cdot \rho \cdot u_1 \cdot \sqrt{2} \cdot a_1}{\eta}} \tag{2.62}$$

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_{\mathrm{d},2} = 0.246 \cdot \mathrm{Re}_{\mathrm{d},2}^{0.588} \cdot \mathrm{Pr}^{0.4} = 0.246 \cdot \left(\frac{8 \cdot \rho \cdot u_1 \cdot \sqrt{2} \cdot a_1}{\eta}\right)^{0.588} \cdot \mathrm{Pr}^{0.4}} \tag{2.63}$$

3) Inserting and rearranging:

This gives us the following heat transfer coefficients for both situations:

$$\overline{\alpha}_1 = \frac{0.246 \cdot \left(\frac{\rho \cdot u_1 \cdot \sqrt{2} \cdot a_1}{\eta}\right)^{0.588} \cdot \mathrm{Pr}^{0.4} \cdot \lambda}{\sqrt{2} \cdot a_1} \tag{2.64}$$

$$\overline{\alpha}_2 = \frac{0.246 \cdot \left(\frac{8 \cdot \rho \cdot u_1 \cdot \sqrt{2} \cdot a_1}{\eta}\right)^{0.588} \cdot \mathrm{Pr}^{0.4} \cdot \lambda}{4 \cdot \sqrt{2} \cdot a_1} \tag{2.65}$$

And therefore:

$$\boxed{\frac{\overline{\alpha}_2}{\overline{\alpha}_1} = \frac{1}{4} \left(8\right)^{0.588} = 0.85} \tag{2.66}$$

So the heat transfer coefficient reduces with 15%.

2.6 Heating of a cylinder ★★

a) *Determine the average heat transfer coefficient $\overline{\alpha}$.*

1) Setting up the definition of the heat transfer coefficient:

We know that:

$$\boxed{\rightarrow \overline{\alpha} = \frac{\overline{\mathrm{Nu}} \cdot \lambda}{L_\mathrm{c}}} \tag{2.67}$$

2) Defining all required parameters:

The characteristic length for a cylinder subjected to transverse flow equals its diameter, and therefore:

$$\boxed{\rightarrow L_\mathrm{c} = D = 0.055 \text{ m}} \tag{2.68}$$

In order to determine the correlation applicable for the Nusselt number, first the Reynolds number should be determined. Which equals:

$$\boxed{\rightarrow \mathrm{Re} = \frac{u \cdot L_\mathrm{c}}{\nu_\mathrm{a}} = \frac{0.1\ [\mathrm{m/s}] \cdot 0.055\ [\mathrm{m}]}{15.35 \cdot 10^{-6}\ [\mathrm{m^2/s}]} = 358.31} \tag{2.69}$$

It can be found that for the correlation of the Nusselt number ($\overline{\mathrm{Nu}}_\mathrm{D} = C \cdot \mathrm{Re}_\mathrm{D}^\mathrm{m} \cdot \mathrm{Pr}^{0.4}$) $C = 0.683$ and $m = 0.466$. Which gives us:

$$\boxed{\rightarrow \overline{\mathrm{Nu}}_\mathrm{D} = 0.683 \cdot 358.31^{0.466} \cdot 0.71^{0.4} = 9.23} \tag{2.70}$$

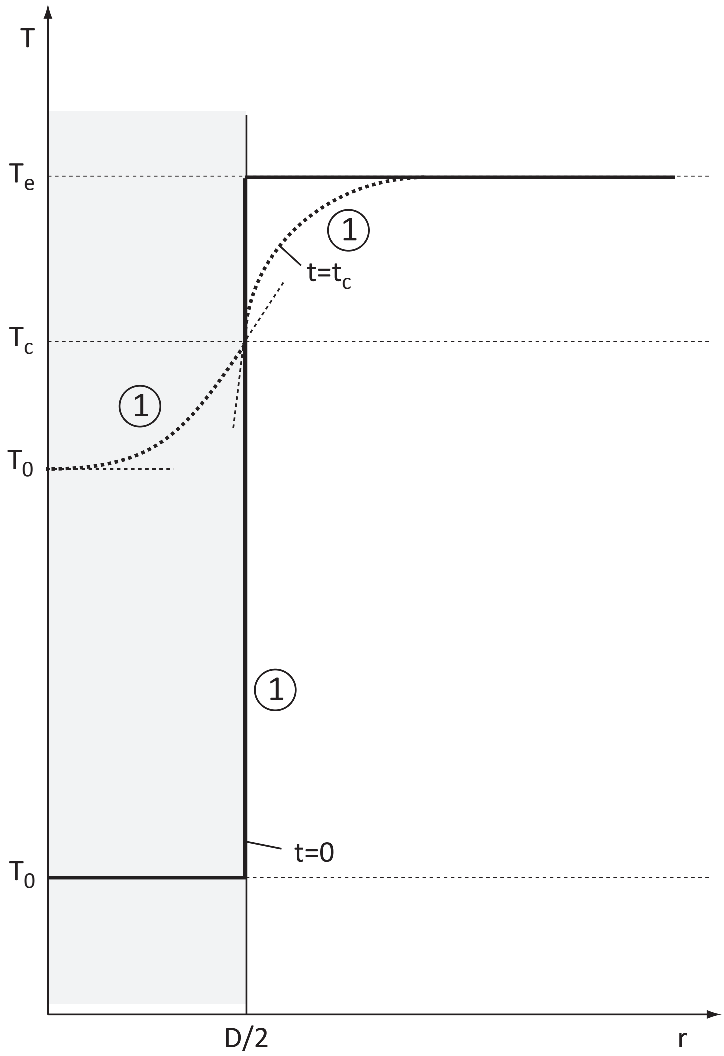
Note that also the correlation $\overline{\mathrm{Nu}}_\mathrm{D} = \left(0.4 \cdot \mathrm{Re}_\mathrm{D}^{1/2} + 0.06 \cdot \mathrm{Re}_\mathrm{D}^{2/3}\right) \cdot \mathrm{Pr}^{0.4} \cdot \left(\frac{\eta_\infty}{\eta_\mathrm{w}}\right)^{1/4}$ could have been used, which gives a similar result.

3) Inserting and rearranging:

Inserting yields

$$\boxed{\rightarrow \overline{\alpha} = \frac{9.23 \cdot 0.0257\ [\mathrm{W/mK}]}{0.055\ [\mathrm{m}]} = 4.32\ [\mathrm{W/m^2K}]} \tag{2.71}$$

b) *At $t = t_c$ the cylinder reaches the critical temperature T_c at its hottest point. Sketch qualitatively the temperature distribution at time-points $t = 0$ and $t = t_c$.*

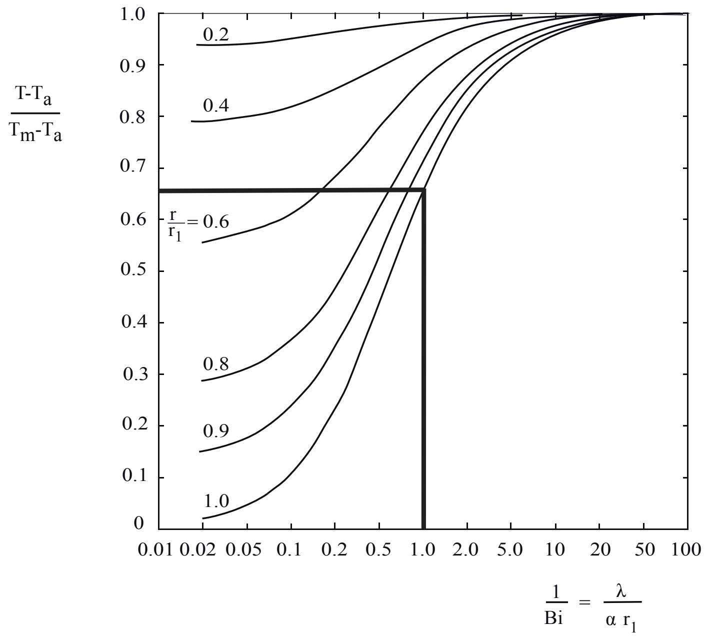


At $t = 0$ the temperature inside the cylinder is still homogeneous, and therefore constant when moving in radial direction. Similarly the air temperature is homogeneous.

At time $t = t_c$ at the center $r = 0$ the temperature is zero due to symmetry. Furthermore the surface temperature equals T_c, and there is a kink in the slope between the temperature profiles in- and outside the cylinder, where the steeper slope is located outside due to convection. Lastly, the temperature outside should approach the ambient temperature with a zero gradient slope for $r \to \infty$.

c) *Determine the time t_c, until the cylinder reaches the critical temperature T_c at its hottest point.*

The time t_c which is required to reach the critical temperature T_c can be determined using the Heissler diagrams. But first the temperature in the middle of the cylinder T_m has to be given for the same point in time. Which can be determined using the following Heissler diagram:



It has to be understood that the critical temperature is initially at the surface of the cylinder at $r = r_1$. This yields:

$$\frac{r}{r_1} = 1.0 \tag{2.72}$$

The inverse of the Biot number can be determined with the known thermal conductivity, heat transfer coefficient and radius:

$$\frac{1}{\text{Bi}} = \frac{\lambda}{\overline{\alpha} \cdot \left(\frac{D}{2}\right)} = \frac{0.119\ [\text{W/mK}]}{4.31\ [\text{W/m}^2\text{K}] \cdot \frac{0.055}{2}\ [\text{m}]} = 1 \tag{2.73}$$

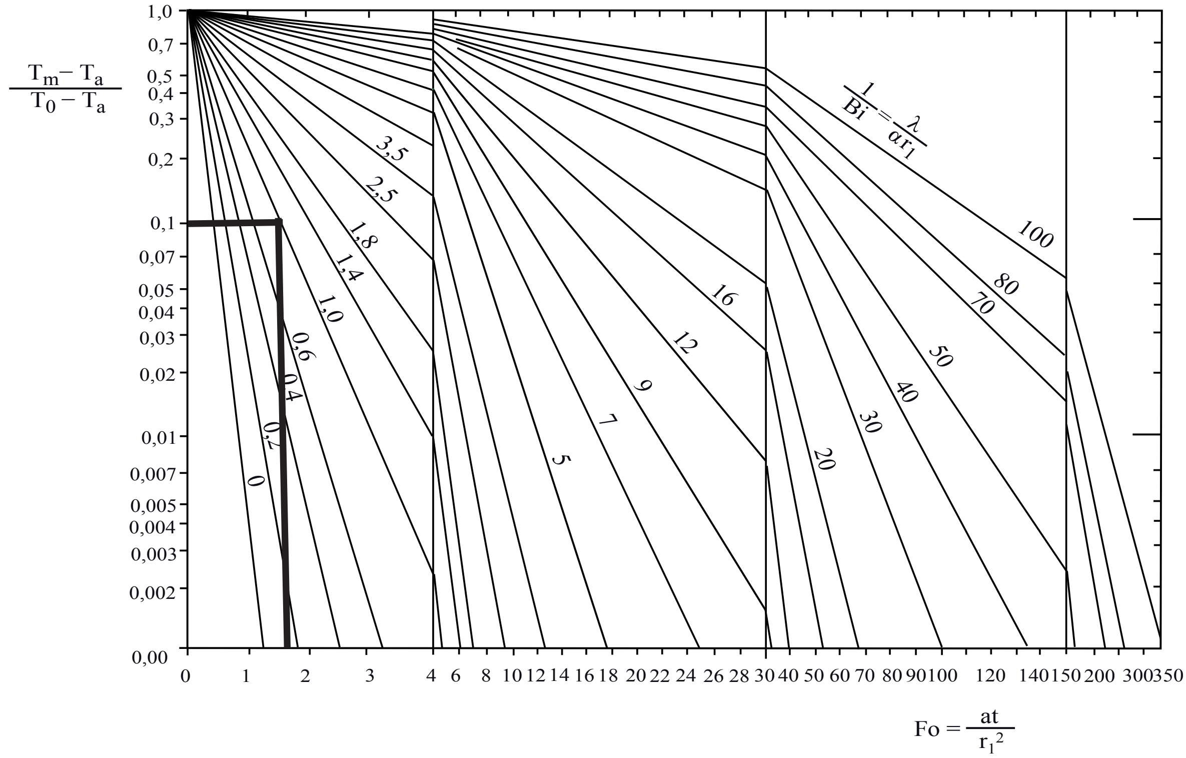
This yields in:

$$\frac{T_c - T_a}{T_m - T_a} \approx 0.65 \tag{2.74}$$

Which gives us:

$$T_m = \frac{T_c - T_a}{0.65} + T_a = \frac{38\ [^\circ\text{C}] - 40\ [^\circ\text{C}]}{0.65} + 40\ [^\circ\text{C}] = 36.92\ [^\circ\text{C}] \tag{2.75}$$

Now diagram the following Heissler diagram can be put to use.



First the temperature difference needs to be determined:

$$\frac{T_\mathrm{m} - T_\mathrm{a}}{T_0 - T_\mathrm{a}} = \frac{36.92\ [^\circ\mathrm{C}] - 40\ [^\circ\mathrm{C}]}{10\ [^\circ\mathrm{C}] - 40\ [^\circ\mathrm{C}]} = 0.103 \tag{2.76}$$

The intersection of the temperature difference with our determined $\frac{1}{\mathrm{Bi}}$ yields in the following Fourier number:

$$\mathrm{Fo} \approx 1.5 \tag{2.77}$$

The time can be taken from the Fourier number.

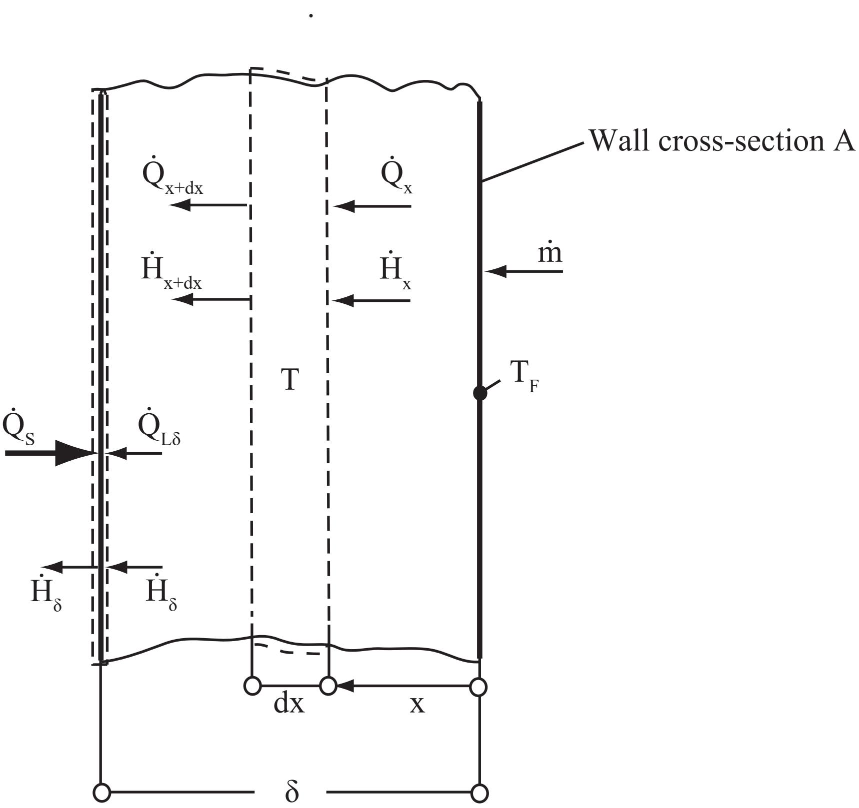
$$\boxed{\rightarrow t_\mathrm{c} = \frac{\mathrm{Fo} \cdot \left(\frac{D}{2}\right)^2}{\frac{\lambda}{\rho \cdot c_\mathrm{p}}} = \frac{1.5 \cdot \left(\frac{0.055}{2}\right)^2\ [\mathrm{m}^2]}{\frac{0.119\ [\mathrm{W/mK}]}{1500\ [\mathrm{kg/m}^3] \cdot 1000\ [\mathrm{J/kgK}]}} \approx 4\ [\mathrm{h}]} \tag{2.78}$$

2.7 Absorption in a porous wall ★★★

a) *Determine the temperature profile $T(x)$ for the porous wall.*

We are dealing with as steady-state, one-dimensional heat transfer problem through conduction and convection. In order to derive the temperature profile $T(x)$ inside the wall, we should solve its differential equation that can be obtained by setting up an energy balance around an infinitesimal element.

1) Setting up an energy balance:



The energy balance for the steady-state infinitesimal element can be described as:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{2.79}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\text{x}} + \dot{H}_{\text{x}} - \dot{Q}_{\text{x+dx}} - \dot{H}_{\text{x+dx}}} \tag{2.80}$$

2) Defining the fluxes:

Ingoing conductive flux in the wall can be described by:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{x}} = -\lambda \cdot A \cdot \frac{\partial T}{\partial x}} \tag{2.81}$$

Ingoing enthalpy flux in the wall can be described by:

$$\boxed{\rightarrow \dot{H}_{\mathrm{x}} = A \cdot \dot{m}'' \cdot c \cdot T} \tag{2.82}$$

Where the outgoing conductive heat flow and enthalpy flow for an infinitesimal element can be approximated by use of Taylor series expansion:

$$\dot{Q}_{\mathrm{x+dx}} = \dot{Q}_{\mathrm{x}} + \frac{\partial \dot{Q}_{\mathrm{x}}}{\partial x} dx \tag{2.83}$$

$$\boxed{\rightarrow \dot{Q}_{\mathrm{x+dx}} = -\lambda \cdot A \cdot \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-\lambda \cdot A \cdot \frac{\partial T}{\partial x}\right) dx} \tag{2.84}$$

$$\dot{H}_{\mathrm{x+dx}} = \dot{H}_{\mathrm{x}} + \frac{\partial \dot{H}_{\mathrm{x}}}{\partial x} dx \tag{2.85}$$

$$\boxed{\rightarrow \dot{H}_{\mathrm{x+dx}} = A \cdot \dot{m}'' \cdot c \cdot T + \frac{\partial}{\partial x}\left(A \cdot \dot{m}'' \cdot c \cdot T\right) dx} \tag{2.86}$$

3) Inserting and rearranging:

Inserting the found expressions for the fluxes into the energy balance yields:

$$0 = \dot{Q}_{\mathrm{x}} + \dot{H}_{\mathrm{x}} - \dot{Q}_{\mathrm{x+dx}} - \dot{H}_{\mathrm{x+dx}} \tag{2.87}$$

$$0 = \lambda \cdot A \cdot \frac{\partial^2 T}{\partial x^2} dx - A \cdot \dot{m}'' \cdot c \cdot \frac{\partial T}{\partial x} dx \tag{2.88}$$

Rearranging:

$$\boxed{\rightarrow 0 = \frac{\partial^2 T}{\partial x^2} - \frac{\dot{m}'' \cdot c}{\lambda} \cdot \frac{\partial T}{\partial x}} \tag{2.89}$$

4) Defining the boundary conditions:

As we are dealing with a 2nd order differential equation, we need 2 boundary conditions to be able to solve it.

At $x = 0$ the temperature is already specified, which gives us the first boundary condition:

$$\boxed{\rightarrow T(x = 0) = T_{\mathrm{F}}} \tag{2.90}$$

The other boundary condition can be obtained by setting up an energy balance at $x = \delta$ (see Figure sketched in step 1).

At this location we have the following energy balance:

$$\dot{Q}_{\mathrm{x}=\delta} + \dot{Q}_{\mathrm{S}} + \dot{H}_{x=\delta} - \dot{H}_{x=\delta} = 0 \tag{2.91}$$

Rewriting yields:

$$\Rightarrow \dot{Q}_{\mathrm{S}} = -\dot{Q}_{\mathrm{x}=\delta} = \lambda \cdot A \cdot \frac{\mathrm{d}T}{\mathrm{d}x}\Big|_{x=\delta} \tag{2.92}$$

Which yields in the following boundary condition:

$$\boxed{\rightarrow \frac{\mathrm{d}T}{\mathrm{d}x}\Big|_{x=\delta} = \frac{\dot{q}''_{\mathrm{s}}}{\lambda}} \tag{2.93}$$

5) Solving the differential equation:

As we are dealing with a linear differential equation, one can try to solve this equation by "guessing" that the temperature profile will be as follows:

$$T(x) = \exp(sx) \tag{2.94}$$

Differentiating once with respect to x:

$$\frac{\partial T}{\partial x} = s \exp(sx) \tag{2.95}$$

Differentiating twice with respect to x:

$$\frac{\partial^2 T}{\partial x^2} = s^2 \exp(sx) \tag{2.96}$$

Substitution of the 1st and 2nd derivative into the energy balance gives us:

$$0 = \frac{\partial^2 T}{\partial x^2} - \frac{\dot{m}'' \cdot c}{\lambda} \cdot \frac{\partial T}{\partial x} \tag{2.97}$$

$$s^2 \exp(sx) - \frac{\dot{m}'' \cdot c}{\lambda} \cdot s \exp(sx) = 0 \tag{2.98}$$

Rewriting:

$$\exp(sx)\left(s^2 - \frac{\dot{m}'' \cdot c}{\lambda} \cdot\right) = 0 \tag{2.99}$$

The standard solution would be $\exp(sx) = 0$, but we need a problem specific solution and therefore we say that $\exp(sx) \neq 0$, so:

$$s^2 - \frac{\dot{m}'' \cdot c}{\lambda} \cdot s = 0 \tag{2.100}$$

Which gives us:

$$s_1 = 0 \qquad \wedge \qquad s_2 = \frac{\dot{m}'' \cdot c}{\lambda} \tag{2.101}$$

Therefore our general solution will be:

$$T(x) = c_1 \exp(s_1 x) + c_2 \exp(s_2 x) \tag{2.102}$$

Which equals:

$$T(x) = c_1 + c_2 \exp\left(\frac{\dot{m}'' \cdot c}{\lambda} x\right) \tag{2.103}$$

Now we have a general solution, we should determine the integration constants c_1 and c_2 by use of the boundary conditions.

From $T(x = 0) = T_\mathrm{F}$ we find that:

$$c_1 = T_\mathrm{F} - c_2 \tag{2.104}$$

From $\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{x=\delta} = \frac{\dot{q}''_\mathrm{s}}{\lambda}$ we find that:

$$\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{x=\delta} = c_2 \frac{\dot{m}'' \cdot c}{\lambda} \cdot \exp\left(\frac{\dot{m}'' \cdot c}{\lambda} \delta\right) = \frac{\dot{q}''_\mathrm{s}}{\lambda} \tag{2.105}$$

Which gives us:

$$\rightarrow c_2 = \frac{\dot{q}''_\mathrm{s}}{\dot{m}'' \cdot c} \exp\left(-\frac{\dot{m}'' \cdot c}{\lambda} \delta\right) \tag{2.106}$$

And therefore

$$c_1 = T_\mathrm{F} - \frac{\dot{q}''_\mathrm{s}}{\dot{m}'' \cdot c} \exp\left(-\frac{\dot{m}'' \cdot c}{\lambda} \delta\right) \tag{2.107}$$

Substitution of c_1 and c_2 into the expression of $T(x)$ gives:

$$T(x) = c_1 + c_2 \exp\left(\frac{\dot{m}'' \cdot c}{\lambda} x\right) \tag{2.108}$$

$$T(x) = T_\mathrm{F} - \frac{\dot{q}''_\mathrm{s}}{\dot{m}'' \cdot c} \exp\left(-\frac{\dot{m}'' \cdot c}{\lambda} \delta\right) + \frac{\dot{q}''_\mathrm{s}}{\dot{m}'' \cdot c} \exp\left(-\frac{\dot{m}'' \cdot c}{\lambda} \delta\right) \exp\left(\frac{\dot{m}'' \cdot c}{\lambda} x\right) \tag{2.109}$$

Rewriting:

$$\boxed{\rightarrow T(x) = T_\mathrm{F} - \frac{\dot{q}''_\mathrm{s}}{\dot{m}'' \cdot c} \cdot \exp\left(-\frac{\dot{m}'' \cdot c}{\lambda} \cdot \delta\right) \cdot \left(1 - \exp\left(\frac{\dot{m}'' \cdot c}{\lambda} \cdot x\right)\right)} \tag{2.110}$$

b) *Determine the maximum temperature $T_{\max}$ reached within the wall.*

6) Finding the maximum temperature:

The position of the maximum temperature can be found by taking the derivative of the temperature profile and finding its critical positions (equalling it to 0) or by reasoning that the fluid should be heated up when it enters the porous wall and the point where it leaves the wall, the maximum temperature should be. Which is $x = \delta$, So therefore:

$$T_{\max} = T(x=\delta) = T_F + \frac{\dot{q}_s''}{\dot{m}''\cdot c}\cdot\left(1-\exp\left(-\frac{\dot{m}''\cdot c\cdot\delta}{\lambda}\right)\right) \tag{2.111}$$

$$T_{\max} = -15\ [^{\circ}\mathrm{C}] + \frac{150\cdot 10^3\ [\mathrm{W/m^2}]}{0.6\ [\mathrm{kg/m^s\cdot s}]\cdot 1000\ [\mathrm{J/kg^{\circ}C}]}\cdot(1-\exp(-3.75)) \tag{2.112}$$

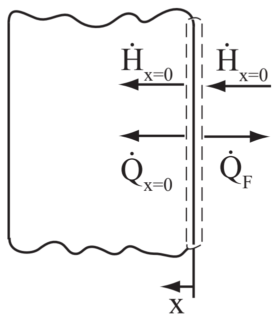
$$\boxed{\rightarrow T_{\max} = 229\ [^{\circ}\mathrm{C}]} \tag{2.113}$$

c) *Determine the heat flux $\dot{q}_F''$ per unit area, which is transmitted into the fluid at $x = 0$.*

The heat flux $\dot{q}_F''$ can be determined by setting up an energy balance exactly <u>at $x = 0$</u> or for the <u>entire wall</u>. Both methods lead to the same result.

1) Setting up an energy balance:

<u>At $x = 0$</u>



For the energy balance exactly at $x = 0$, one finds the following energy balance:

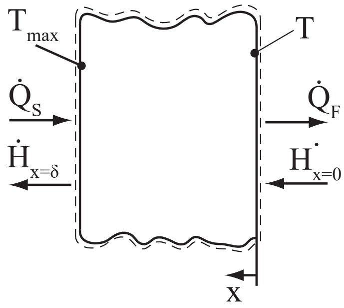
$$\cancel{\frac{\partial U}{\partial t}}^{\,0} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{2.114}$$

$$0 = \dot{H}_{x=0} - \dot{H}_{x=0} - \dot{Q}_{x=0} - \dot{Q}_F \tag{2.115}$$

Rewriting:

$$\boxed{\rightarrow \dot{Q}_F = -\dot{Q}_{x=0}} \tag{2.116}$$

Entire wall:



For the energy balance exactly around the entire wall, one finds the following energy balance:

$$\cancelto{0}{\frac{\partial U}{\partial t}} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \tag{2.117}$$

$$0 = \dot{Q}_{\text{s}} - \dot{Q}_{\text{F}} + \dot{H}_0 - \dot{H}_\delta \tag{2.118}$$

Rewriting:

$$\boxed{\rightarrow \dot{Q}_{\text{F}} = \dot{Q}_{\text{s}} + \dot{H}_{x=0} - \dot{H}_{x=\delta}} \tag{2.119}$$

2) Defining the fluxes:

The next steps will be performed only for the balance at x=0: The conductive flux transferred towards the wall can be described as:

$$\boxed{\rightarrow \dot{Q}_{\text{x=0}} = -\lambda \cdot A \cdot \frac{dT}{dx}|_{\text{x=0}}} \tag{2.120}$$

The rate of heat transferred into the fluid can be expressed in terms of the cross sectional area and the heat flux $\dot{q}''_{\text{F}}$ per unit area:

$$\boxed{\rightarrow \dot{Q}_{\text{F}} = A \cdot \dot{q}''_{\text{F}}} \tag{2.121}$$

3) Inserting and rearranging:

Inserting the found definitions into the energy balance at $x = 0$:

$$\dot{Q}_{\text{F}} = -\dot{Q}_{\text{x=0}} \tag{2.122}$$

$$\dot{q}''_{\text{F}} = -\lambda \cdot \frac{dT}{dx}|_{\text{x=0}} \tag{2.123}$$

Where substituting the derivative of the temperature profile with respect to x at $x = 0$ leads to:

$$\dot{q}''_{\text{F}} = \lambda \cdot \frac{\dot{q}''_{\text{s}}}{\dot{m}'' \cdot c} \cdot \exp\left(-\frac{\dot{m}'' \cdot c \cdot \delta}{\lambda}\right) \cdot \frac{\dot{m}'' \cdot c}{\lambda} \cdot 1 \tag{2.124}$$

$$\dot{q}''_{\mathrm{F}} = \dot{q}''_{\mathrm{s}} \cdot \exp\left(-\frac{\dot{m}'' \cdot c \cdot \delta}{\lambda}\right) \tag{2.125}$$

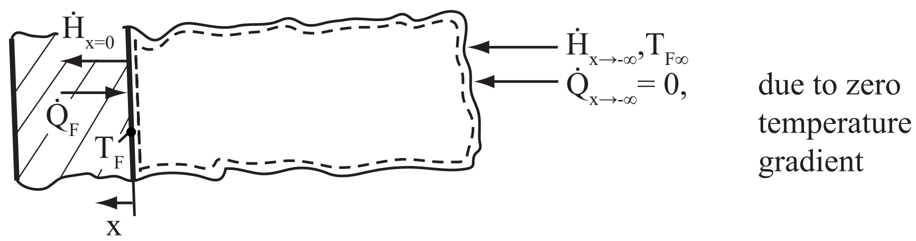
Which yields

$$\boxed{\rightarrow \dot{q}''_{\mathrm{F}} = 150 \cdot 10^3 \;\; \cdot \exp\left(-3.75\right) = 3.5 \cdot 10^3 [\mathrm{W/m^2}]} \tag{2.126}$$

d) *Which temperature $T_{\mathrm{F},\infty}$ does the fluid reach far away from the wall?*

$T_{\mathrm{F},\infty}$ can be determined by setting up an energy balance. The choice for the balance domain is chosen in identical manner as in subtask c). Again two possible approaches arise. One domain for $-\infty \leq x \leq 0$ or $-\infty \leq x \leq \delta$. Identical results will be obtained. In this solution only the approach for the domain $-\infty \leq x \leq 0$ will be discussed.

1) Setting up an energy balance:



The energy balance for the domain $-\infty \leq x \leq 0$ can be described as:

$$\cancel{\frac{\partial U}{\partial t}}^{\,0} = \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\mathrm{out}} \tag{2.127}$$

$$\boxed{\rightarrow 0 = \dot{Q}_{\mathrm{F}} - \dot{H}_{x=0} + \dot{H}_{x\rightarrow-\infty}} \tag{2.128}$$

2) Defining the fluxes:

The rate of heat transfer towards the fluid can be described as:

$$\boxed{\rightarrow \dot{Q}_{\mathrm{F}} = A \cdot \dot{q}''_{\mathrm{F}}} \tag{2.129}$$

The enthalpy flow crossing the boundary at $x = 0$ can be expressed as:

$$\boxed{\rightarrow \dot{H}_{\mathrm{x=0}} = A \cdot \dot{m}'' \cdot c \cdot T_{\mathrm{x=0}}} \tag{2.130}$$

The enthalpy flow crossing the boundary at $x \rightarrow -\infty$ can be expressed as:

$$\boxed{\rightarrow \dot{H}_{\mathrm{x}\rightarrow-\infty} = A \cdot \dot{m}'' \cdot c \cdot T_{\mathrm{F},\infty}} \tag{2.131}$$

3) Inserting and rearranging:

Inserting the defintions of the fluxes into the energy balance yields:

$$0 = \dot{Q}_{\mathrm{F}} - \dot{H}_{x=0} + \dot{H}_{x\rightarrow-\infty} \tag{2.132}$$

$$\dot{q}''_{\mathrm{F}} - \dot{m}'' \cdot c \cdot T_{\mathrm{x=0}} + \dot{m}'' \cdot c \cdot T_{\mathrm{F},\infty} \tag{2.133}$$

Rewriting yields (note that $T_{\mathrm{x=0}} = T_{\mathrm{F}}$):

$$T_{\mathrm{F},\infty} = -\frac{\dot{q}''_{\mathrm{F}}}{\dot{m}'' \cdot c} + T_{\mathrm{F}} \tag{2.134}$$

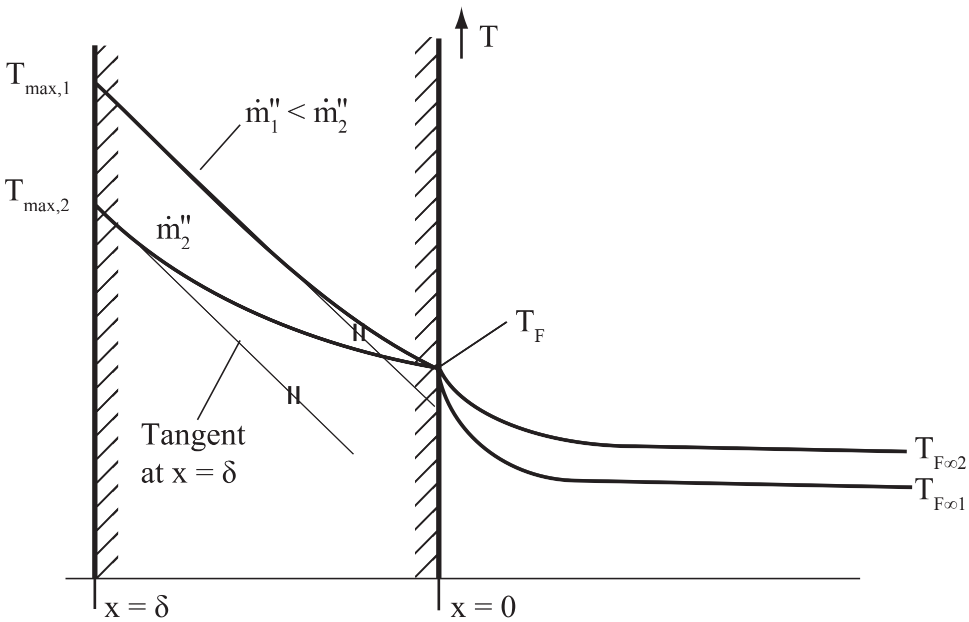
Inserting numerical values gives us:

$$\boxed{\rightarrow T_{\mathrm{F},\infty} = -\frac{3.5 \cdot 10^3\ [\mathrm{W/m^2}]}{0.6\ [\mathrm{kg/m^2 \cdot s}] \cdot 1000\ [\mathrm{J/kg^oC}]} - 15\ [^\circ\mathrm{C}] = -21\ [^\circ]\mathrm{C}} \tag{2.135}$$

e) *Sketch the temperature profiles for two different mass fluxes and mark each curve.*

From the derived equation for the temperature profile inside the wall it can be found that the maximum temperature is higher for a smaller mass flux. But at $x = 0$ the temperature is fixed to T_{F}. Therefore inside the wall it is steeper for a smaller mass flux.

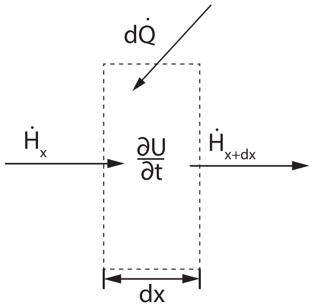
Furthermore, from the expression found for $T_{F,\infty}$ it can be seen that this becomes higher in the case that the mass flux is smaller. Still in both cases, it will have a zero-slope gradient for $x \rightarrow -\infty$.



2.8 Pipe flow - constant heat flux ★★

a) *Derive the transient differential energy balance for the averaged temperature in the fluid in axial direction.*

1) Setting up an energy balance:



By setting up an energy balance for an infinitesimal element the transient differential equation can be derived. The unsteady energy balance is described as:

$$\frac{\partial U}{\partial t} = \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\mathrm{out}} \tag{2.136}$$

$$\boxed{\rightarrow \frac{\partial U}{\partial t} = d\dot{Q} + \dot{H}_{\mathrm{x}} - \dot{H}_{\mathrm{x+dx}}} \tag{2.137}$$

2) Defining the fluxes:

The ingoing enthalpy flow can be described as:

$$\boxed{\rightarrow \dot{H}_{\mathrm{x}} = \dot{m} c_p T = \frac{\pi D^2}{4} \rho u c_p T} \tag{2.138}$$

For an infinitesimal element, the outgoing enthalpy flow can be approximated by use of Taylor series expansion:

$$\boxed{\rightarrow \dot{H}_{\mathrm{x+dx}} = \dot{H}_{\mathrm{x}} + \frac{\partial \dot{H}_{\mathrm{x}}}{\partial x} \cdot dx = \frac{\pi D^2}{4} \rho u c_{\mathrm{p}} \left(T + \frac{\partial T}{\partial x} dx \right)} \tag{2.139}$$

The rate of heat transfer towards the pipe can be expressed by the heat flux and the circumferential area:

$$\boxed{\rightarrow d\dot{Q} = \dot{q}'' \cdot A_{\mathrm{s}} = \dot{q}'' \pi D dx} \tag{2.140}$$

And lastly, the change of energy inside the system can be expressed by:

$$\boxed{\rightarrow \frac{\partial U}{\partial T} = mc_{\mathrm{p}}\frac{\partial T}{\partial t} = \frac{\pi D^2}{4} dx \rho c_{\mathrm{p}} \frac{\partial T}{\partial t}} \tag{2.141}$$

3) Inserting and rearranging:

Inserting the definition of the found fluxes yields:

$$\frac{\partial U}{\partial t} = d\dot{Q} + \dot{H}_{\mathrm{x}} - \dot{H}_{\mathrm{x+dx}} \tag{2.142}$$

$$\frac{\pi D^2}{4} dx \rho c_{\mathrm{p}} \frac{\partial T}{\partial t} = \dot{q}'' \pi D dx - \frac{\pi D^2}{4} \rho u c_{\mathrm{p}} \frac{\partial T}{\partial x} dx \tag{2.143}$$

Rearranging:

$$\boxed{\rightarrow \frac{\partial T}{\partial t} = \frac{4\dot{q}''}{\rho c_{\mathrm{p}} D} - u\frac{\partial T}{\partial x}} \tag{2.144}$$