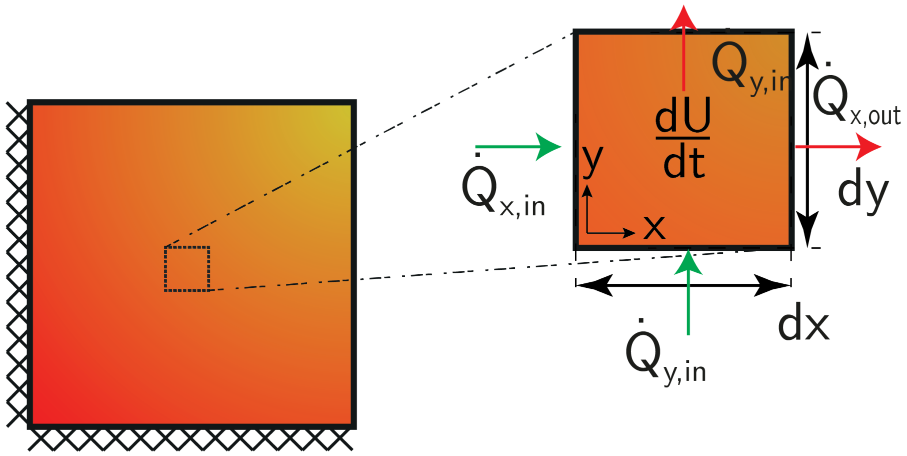
# EB - Cond. - IE 2

**Provide the governing two-dimensional energy balance in Cartesian coordinates to describe the heat conduction problem in the body without a heat source. Assume the process to be isochoric with transient conditions.**

## 1 Setting up the balance:

To derive the governing equation, an energy balance around a two-dimensional infinitesimal element is needed. Heat is conducted in and out of the element, while the inner energy changes over time.




Hence, the transient energy balance reads:

$$\frac{\partial U}{\partial t} = \dot{Q}_{\text{x,in}} - \dot{Q}_{\text{x,out}} + \dot{Q}_{\text{y,in}} - \dot{Q}_{\text{y,out}}.$$

## 2 Defining the elements within the balance:

The temporal change of inner energy is written as:

$$\frac{\partial U}{\partial t} = \rho \cdot c_v \cdot dx \cdot dy \cdot dz \cdot \frac{\partial T}{\partial t}.$$

The incoming rates of heat are described by Fourier's law of heat conduction:

$$\dot{Q}_{\text{x,in}} = -\lambda dy dz \frac{\partial T}{\partial x},$$

and:

$$\dot{Q}_{\text{y,in}} = -\lambda dx dz \frac{\partial T}{\partial y}.$$

The outgoing fluxes are approximated by the use of the Taylor series expansion:

$$\begin{aligned} \dot{Q}_{\text{x,out}} &= \dot{Q}_{\text{x,in}} + \frac{\partial \dot{Q}_{\text{x,in}}}{\partial x} \cdot dx \\ &= -\lambda dy dz \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-\lambda dy dz \frac{\partial T}{\partial x}\right) \cdot dx, \end{aligned}$$

and:

$$
\begin{aligned}
\dot{Q}_{\mathrm{y,out}} &= \dot{Q}_{\mathrm{y,in}} + \frac{\partial \dot{Q}_{\mathrm{y,in}}}{\partial y} \cdot dy \\
&= -\lambda dxdz \frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(-\lambda dxdz \frac{\partial T}{\partial y}\right) \cdot dy.
\end{aligned}
$$

3 **Inserting and rearranging:**

$$
\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_v}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)
$$