

## 1.15 Heating and quenching of a sphere

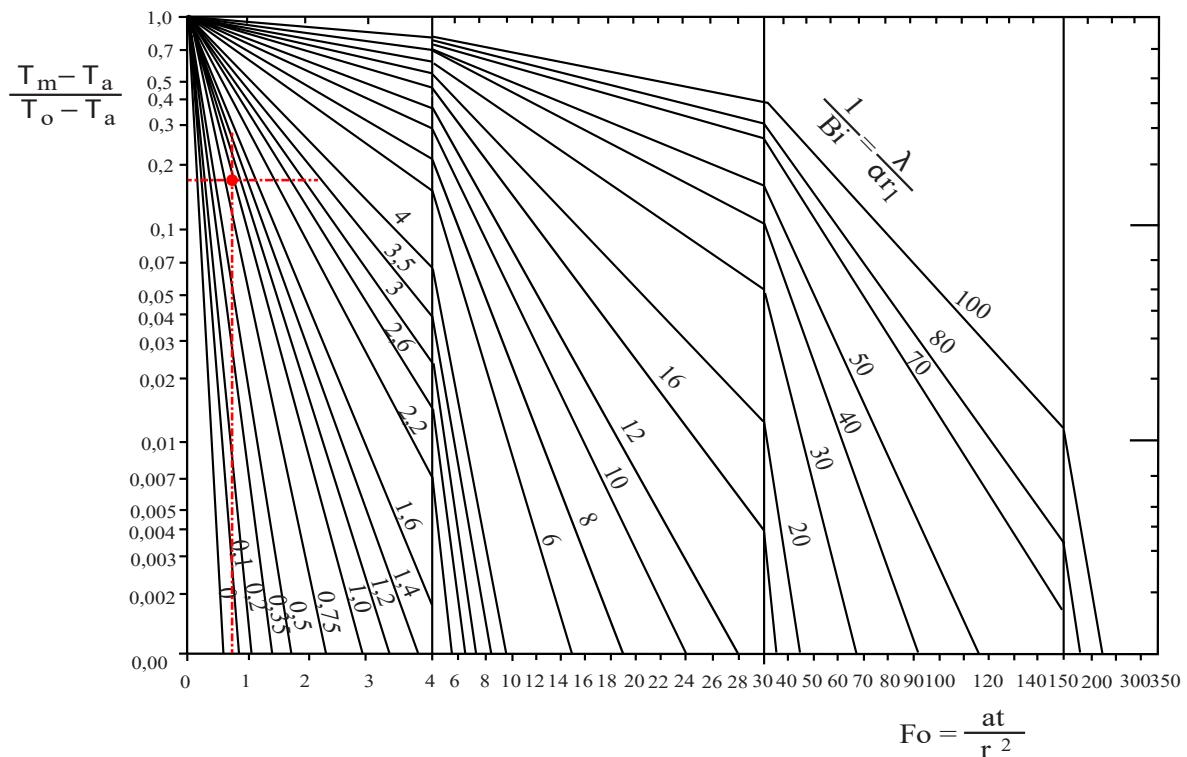
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a) Determine  $T_m$  after 3 minutes:

$$\frac{1}{Bi} = \frac{\lambda}{\alpha \cdot r_1} = \frac{1.52 \text{ [Wm}^{-1}\text{K}^{-1}]}{110 \text{ [Wm}^{-2}\text{K}^{-1}]\cdot 0.015 \text{ [m]}} = 0.9212 \quad (1.271)$$

$$Fo = \frac{a \cdot t}{r_1^2} = \frac{9.5 \cdot 10^7 \text{ [m}^2\text{s}^{-1}\text{]} \cdot 180 \text{ [s]}}{0.015^2 \text{ [m}^2\text{]}} = 0.76 \quad (1.272)$$

Using the Heisler diagram for the temperature in the centre of a sphere:



Results in:

$$\frac{T_m - T_a}{T_0 - T_a} \approx 0.18 \quad (1.273)$$

$$T_m = 0.18 (T_0 - T_a) + T_a = 0.18 (25 \text{ [}^\circ\text{C}] - 200 \text{ [}^\circ\text{C}]) + 200 \text{ [}^\circ\text{C}] \quad (1.274)$$

$\rightarrow T_m = 168.5 \text{ [}^\circ\text{C}]$

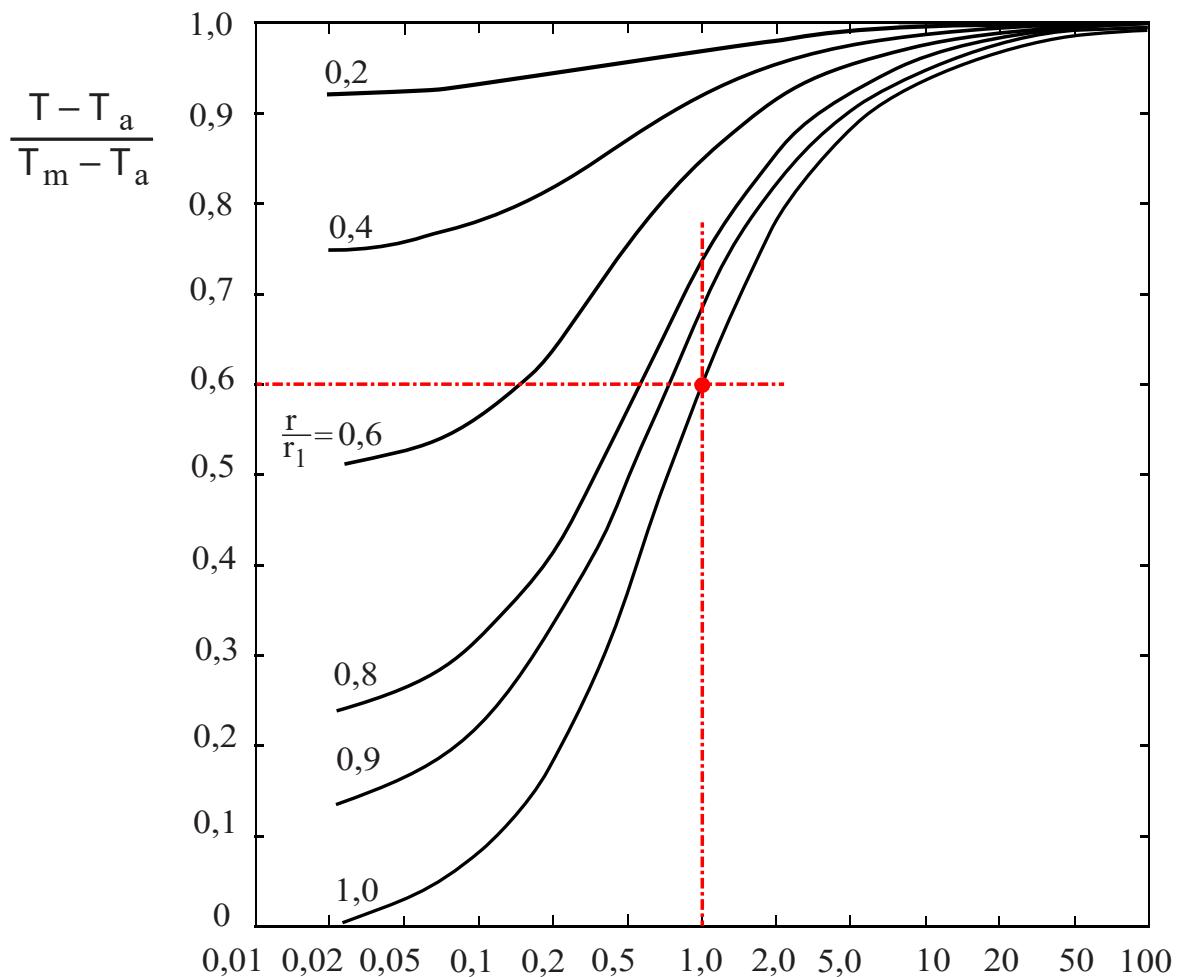
(1.275)

b) Determine  $t_1$ :

$$\frac{T - T_a}{T_m - T_a} = \frac{44.4 \text{ [°C]} - 30 \text{ [°C]}}{54 \text{ [°C]} - 30 \text{ [°C]}} = 0.6 \quad (1.276)$$

$$\frac{r}{r_1} = 1 \quad (1.277)$$

Using the Heisler diagram for the temperature distribution in a sphere to determine  $\frac{1}{Bi}$ : (Note that this diagram is only valid when  $Fo > 0.2$ )

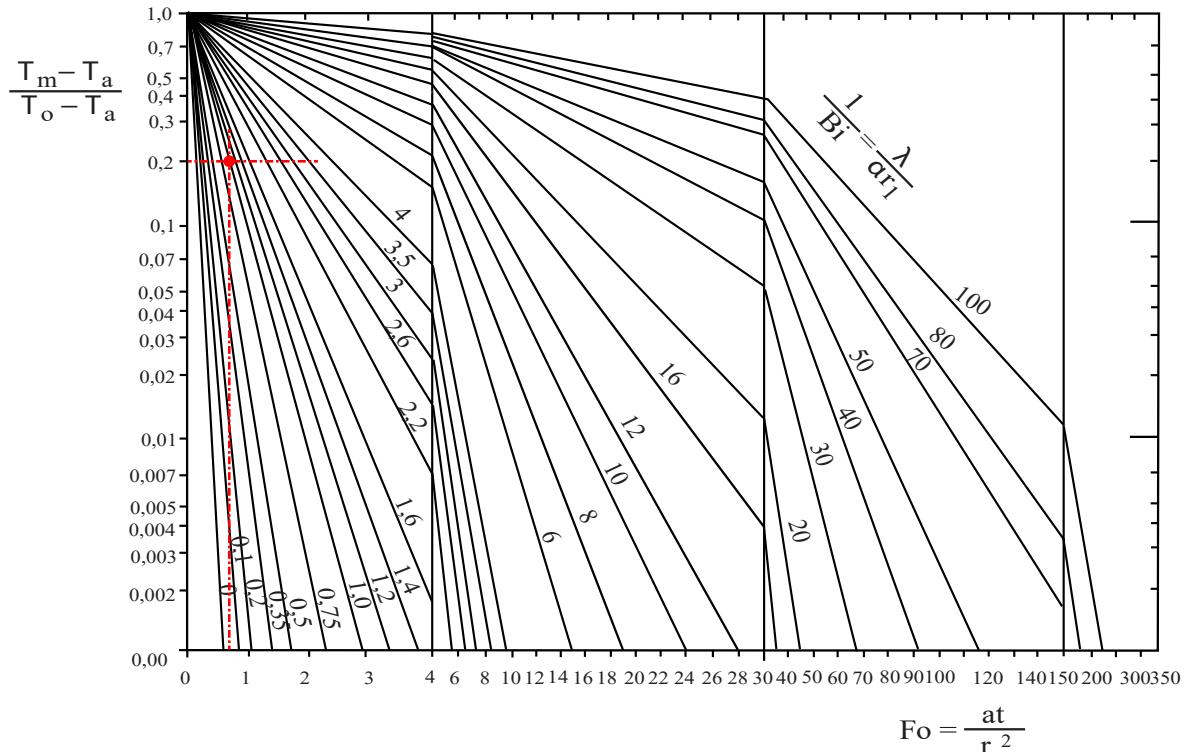


$$\frac{1}{Bi} = \frac{\lambda}{\alpha r_1}$$

$$\rightarrow \frac{1}{Bi} \approx 1 \quad (1.278)$$

$$\frac{T_m - T_a}{T_0 - T_a} = \frac{54 [^{\circ}\text{C}] - 30 [^{\circ}\text{C}]}{150 [^{\circ}\text{C}] - 30 [^{\circ}\text{C}]} = 0.2 \quad (1.279)$$

Using the Heisler diagram for the temperature in the centre of a sphere to determine Fo:

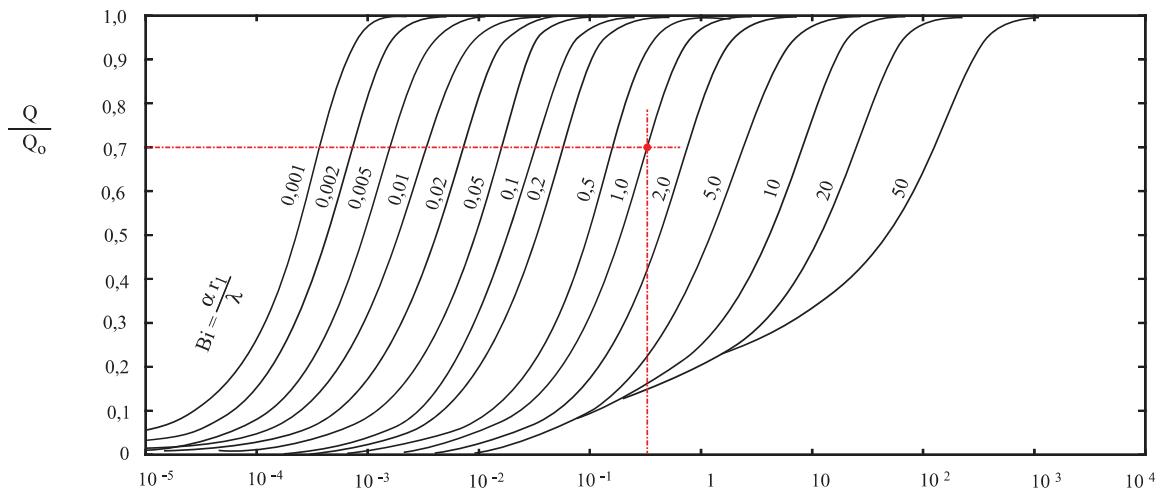


$$\rightarrow Fo \approx 0.7 \rightarrow t_1 \approx 165.79 [\text{s}] \quad (1.280)$$

c) Determine the dissipated heat  $Q$  at time instant  $t_1$ .

$$Bi^2 \cdot Fo = 0.7 \quad (1.281)$$

Using the Heisler diagram for the heat loss of a sphere:



$$Bi^2 \cdot Fo = -\frac{\alpha^2 t}{\rho c \lambda}$$

$$\frac{Q}{Q_o} \approx 0.92 \quad (1.282)$$

$$Q \approx 0.92 \cdot \left( \rho \cdot \frac{4}{3} \cdot \pi \cdot r_1^2 \cdot c_p \cdot (T_0 - T_a) \right) = \quad (1.283)$$

$$Q = 0.92 \cdot \left( 1.45 \cdot 10^3 \text{ [kg/m}^3] \cdot \frac{4}{3} \cdot \pi \cdot 0.015^2 \text{ [m}^2] \cdot 880 \text{ [J/kgK]} \cdot (150 - 30) \text{ [K]} \right) \quad (1.284)$$

$$\boxed{\rightarrow Q = 132.76 \text{ [kJ]}} \quad (1.285)$$