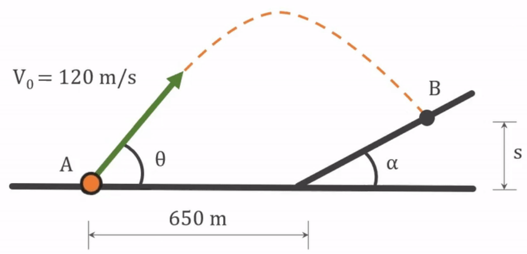
Trajectory of a Ball



A ball is launched from point A with the initial conditions shown. Find an expression for the horizontal displacement $x(t)$.

Neglect all air resistances.

Using known expressions (for constant acceleration):

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a\int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (at + v_0)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (at + v_0)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \tag{6}$$

Solution:

For the horizontal displacement in x-direction, equation (6) results in:

$$x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + s_{x,0} \tag{7}$$

Since $a_x = 0$ m/s^2 and $s_{x,0} = 0$ m, the resulting equation reduces to:

$$x(t) = v_{x,0}t \tag{8}$$

Where $v_{x,0} = v_0 \cos\theta$. Substituting for $s_{x,0}$ gives the final expression:

$$x(t) = v_0 t \cos\theta \tag{9}$$