# Energy & Heat Transfer

## Lecture 4

*By: Mohammad Mehrali*

# Study Materials

- **Slides**
- **Book : (**Y. A. Cengel& A. J. Ghajar. Heat and Mass Transfer: Fundamental & Application**)**
- **Heat Quiz**
- **Learning activities**

# Recap of lecture 3

Forced and natural/free convection:

$\dot{Q} = hA\Delta T$ $\dot{q} = h\,\Delta T$ Newton's cooling law

Nusselt Number: dimensionless variety of heat transfer coefficient *h*

$$\mathrm{Nu} = \frac{hL_c}{k}$$

With $L_c$ a characteristic length for the considered geometry

Determining the Nusselt Number: (empirical) correlations

- Forced convection: Nu as function of Re, Pr a.k.a.: Nu = *f*(Re, Pr)
- Natural convection: This lecture

⇒ Relation Nu, Re, Pr dependent on geometry and flow regime (laminar/ turbulent)

⇒ Nu, Re, Pr dimensionless numbers: reduction number of variables

# Recap of lecture 3

**Convection Resistance :**

$$\dot{Q} = hA\Delta T = \frac{\Delta T}{R_{conv}} \quad \text{met} \quad R_{conv} = \frac{1}{hA} \quad \text{(K/W)}$$

**Dimensionless Numbers :**

Nusselt Number: $$\text{Nu} = \frac{hL_c}{k}$$

Reynolds Number: $$\text{Re} = \frac{\rho U L_c}{\mu}$$

Prandtl Number: $$\text{P}r = \frac{\mu c_p}{k}$$

+ Background on boundary layers

# Recap of lecture 3

If $\dot{Q}$ must be found:

- Calculate at film temperature : $T_f = \dfrac{T_s + T_\infty}{2}$
- Pull out ingredients like *μ*, *ρ*, *k*, Pr from tables – like assignment bundle: air or given fluid) at $T_f = \dfrac{T_s + T_\infty}{2}$

- Calculate Re and choose apprioprate correlation based on geometry and Re
- Calculate Nu
- Derive *h* from it
- Fill out Newton's cooling

law: $\dot{Q} = hA\Delta T$

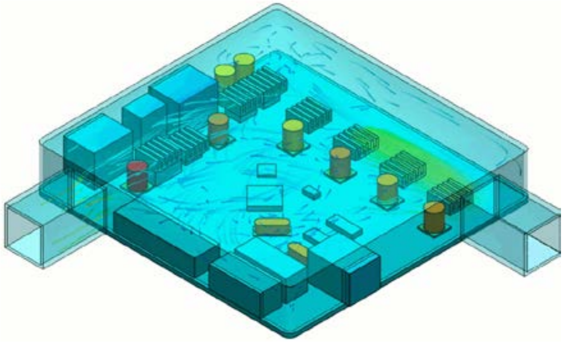
# Learning objectives lecture 4

## ○ Heat transfer through natural convection

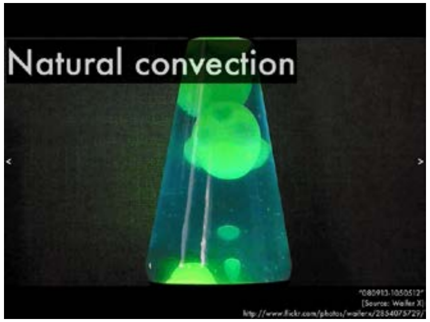
- ● **Natural Convection**
- ○ Grashof Number
- ○ Rayleigh Number
- ○ Nusselt Number
- ○ Using correlations for various configurations
- ○ Calculating natural convection with step-by-step plan

# TYPES OF CONVECTION (FROM LECTURE 3)

**Forced convection**



**Natural/free convection**



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[Source: Walfer X]
http://www.flickr.com/photos/walfers/2854075729/

Imposed flow (by pump, fan, …)

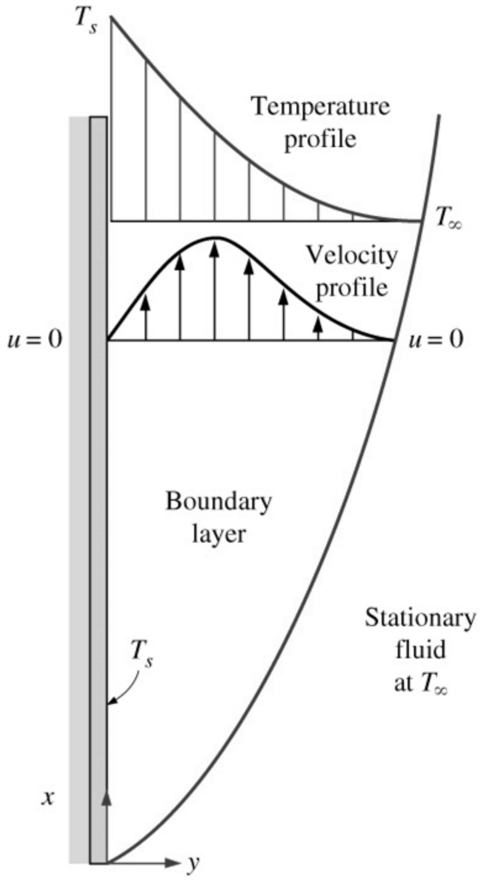
Temperature difference itself starts the flow

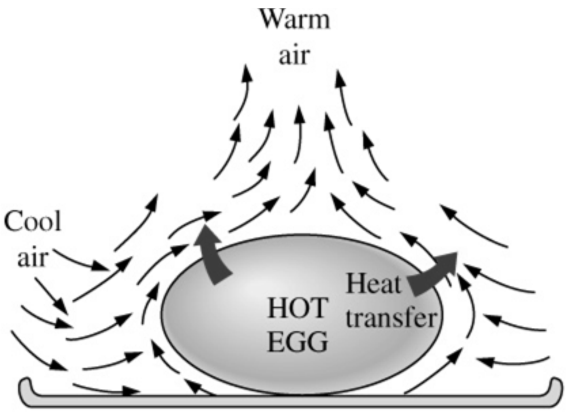
General: flow velocity and heat transfer rates are larger for forced convection

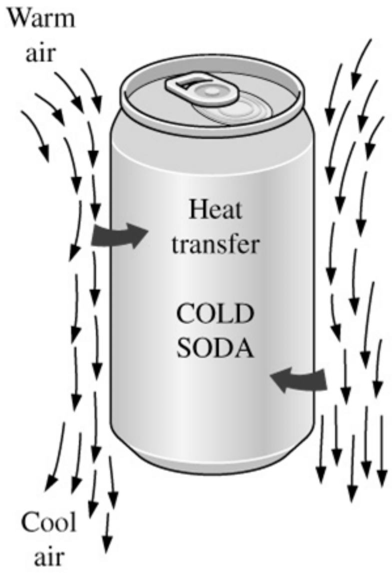
# Natural Convection

Also for natural convection velocity and thermal boundary layers!

- Velocity boundary layer is only area in which flow occurs
- **Hot** surface: **Upward flow**
- **Cold** surface: **Downward flow**







# Learning objectives lecture 4

## ○ Heat transfer through natural convection

- ● Natural Convection
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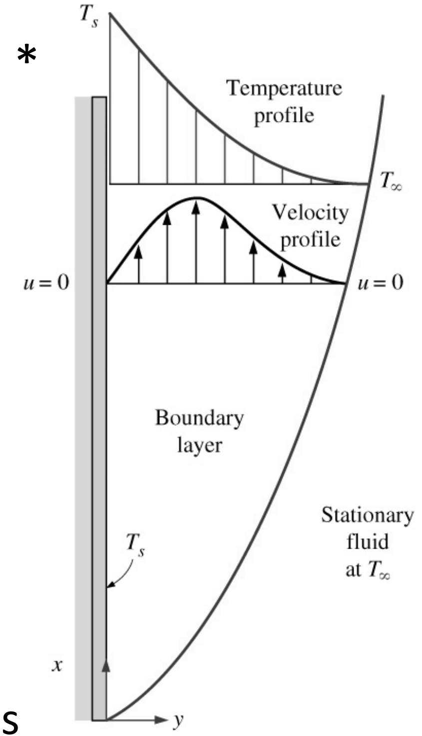
# Grashof Number

**Forced convection:** velocity $U$ imposed

**Natural convection**: velocity follows from temp.difference $T_s$ - $T_\infty$ *

$\Rightarrow$ Alternative for Reynolds number, with temp.diff. instead $U$:

Grashof number: $\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\underbrace{\nu^2}_{\text{Greek letter “nu”}}} \quad (-)$



The **Grashof number (Gr)** is a **dimensionless** number in fluid dynamics and heat transfer which approximates the **ratio of** the **buoyancy** to **viscous force** acting on a fluid.

*Choose $\Delta T$ positive for convenience

# Grashof Number

Grashof number: $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2} \quad (-)$

- Gravitational acceleration : **g = 9,81 m/s$^2$**
- Volume expansion coefficient ***β*** **(K$^{-1}$);** **for most gases:** $\beta = \dfrac{2}{\mathrm{T_s} + \mathrm{T_\infty}}$

  **(temperature in Kelvin; 0 ˚C = 273,15 K)**

- Length **L$_c$** characteristic for geometry **(length *L* for plate, diameter *D* for sphere/cylinder)**
- Kinematic viscosity : $\nu = \dfrac{\mu}{\rho}(\mathrm{m^2/s})$ at avg. Temp: $\mathrm{T_f} = \dfrac{\mathrm{T_s} + \mathrm{T_\infty}}{2}$

# Learning objectives lecture 4

## Heat transfer through natural convection

- ● Natural Convection
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- ● **Rayleigh Number**
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# Rayleigh Number

Laminar / turbulent:

↔

**Forced convection:**

Determined by Reynolds number : **Re**

**Natural convection:**

Determined by Grashof number **Gr**

Often combined with Prandtl number:

Rayleigh number **Ra = Gr · Pr**

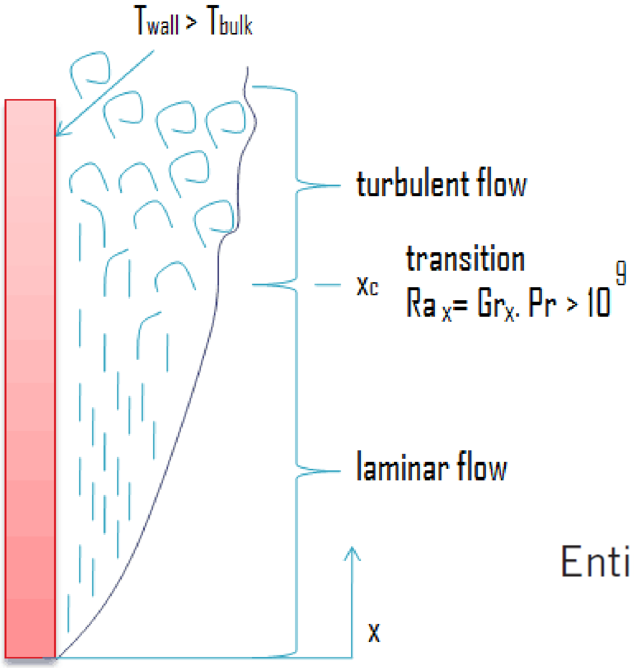
$$\text{Rayleigh Number : } \mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$$

**Rayleigh Number** is a dimensionless number associated with buoyancy-driven flow, also known as free or natural convection

# Rayleigh Number

Rayleigh Number : $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \dfrac{g\beta(T_s - T_\infty)L_c{}^3}{\nu^2}\mathrm{Pr}$

Vertical, flat plate (ex. radiator)



$\mathrm{Nu} = 0{,}59\ \mathrm{Ra}_L{}^{1/4}$ with $L_c = L$ $(10^4 < \mathrm{Ra}_L < 10^9)$

$\mathrm{Nu} = 0{,}1\ \mathrm{Ra}_L{}^{1/3}$ with $L_c = L$ $(10^{10} < \mathrm{Ra}_L < 10^{13})$

$10^9 < \mathrm{Ra}_L < 10^{10}$ : find intermediate

Entire range $\Rightarrow$

$$\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

(complex but more accurate)

# Learning objectives lecture 4

## ○ Heat transfer through natural convection

- ● Natural Convection
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# NUSSELT NUMBER

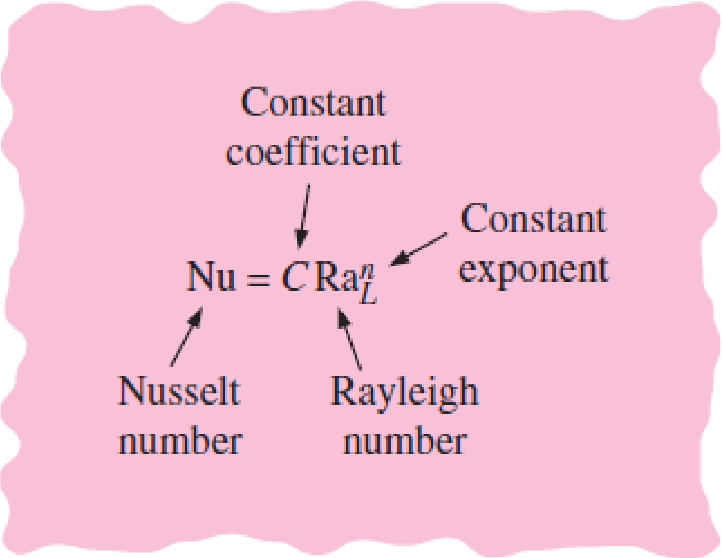
## Forced Convection

$$\frac{hL}{k} = a\left(\frac{\rho U L}{\mu}\right)^{b} {\frac{\mu c_p}{k}}^{c}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\mathrm{Nu} = a \cdot \quad \mathrm{Re}^{b} \cdot \quad \mathrm{Pr}^{c}$$

## Natural Convection

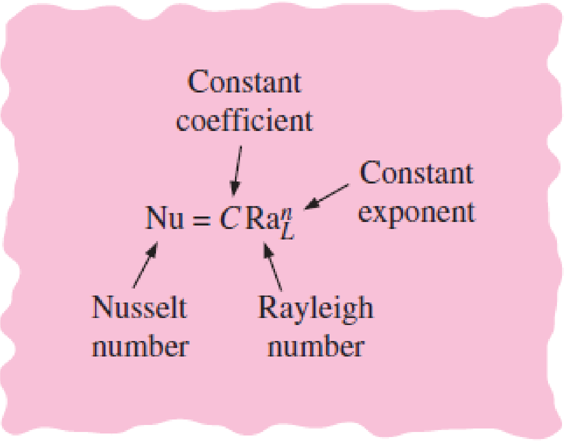


$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c{}^{3}}{\nu^{2}}\,\mathrm{Pr}$$

# NUSSELT NUMBER

Rayleigh Number : $$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c{}^3}{\nu^2}\mathrm{Pr}$$

**Rayleigh number,** which is the product of the Grashof and Prandtl numbers



**The values of the constants C and n depend on the**

- ✓ geometry of the surface
- ✓ the flow regime

**The value of n is usually**

- ✓ 0,25 for laminar flow
- ✓ 0,33 for turbulent flow.

**The value of C**

- ✓ is normally less than 1.

All fluid properties are to be evaluated at the film temperature Tf $=(T_s+T_{inv})/2$.

# Learning objectives lecture 4

○ **Heat transfer through natural convection**

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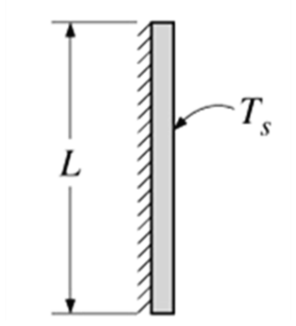
- **Using correlations for various configurations**

○ Calculating natural convection with step-by-step plan

# Free Convection Correlations

Vertical, flat plate

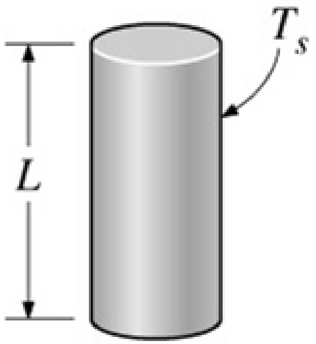


$\mathrm{Nu} = 0{,}59\ \mathrm{Ra}_L^{1/4}$ with $L_c = L$ $(10^4 < \mathrm{Ra}_L < 10^9)$

$\mathrm{Nu} = 0{,}1\ \mathrm{Ra}_L^{1/3}$ with $L_c = L$ $(10^{10} < \mathrm{Ra}_L < 10^{13})$

Entire range ⇒ $$\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

(complex but more accurate)

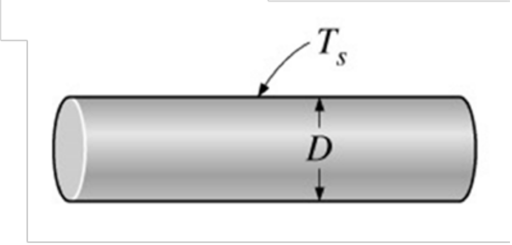


Vertical cylinder (e.g. can)

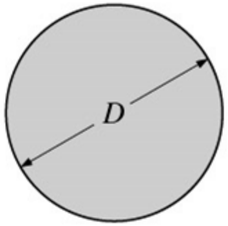
Treat a vertical plate with height $L$ and surface $\pi DL$, if diameter $D$ sufficiently large with respect to $L$ $\left(D \geq \frac{35L}{Gr_L^{1/4}}\right)$

# FREE CONVECTION CORRELATIONS

Horizontal cylinder (e.g. pipe)



$$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$



$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$$

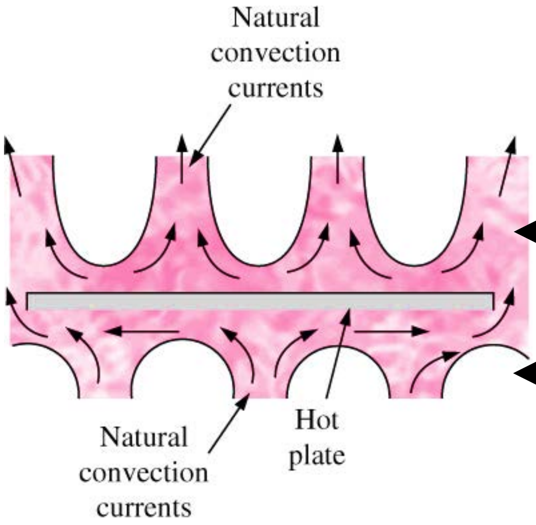
Sphere (e.g. light bulb)

**Notation:**
$\mathrm{Ra}_L$ is Ra with $L_c = L$ ;
$\mathrm{Ra}_D$ is Ra with $L_c = D$

# Free Convection Correlations

Horizontal plate: upward / downward flow perpendicular to surface→ plumes instead of adjacent boundary layer

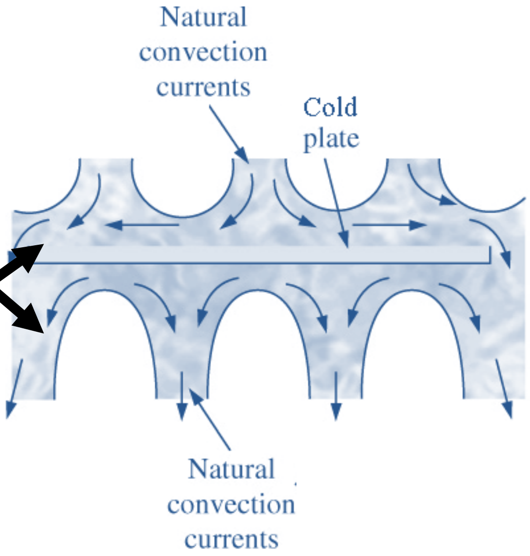
**Hot surface**



**Fluid away from surface**
Effective heat transfer
$\mathrm{Nu} = 0{,}54\ \mathrm{Ra}^{1/4}$ $(10^4 < \mathrm{Ra} < 10^7)$
$\mathrm{Nu} = 0{,}15\ \mathrm{Ra}^{1/3}$ $(10^7 < \mathrm{Ra} < 10^{11})$

**Fluid towards surface**
Reduced heat transfer
$\mathrm{Nu} = 0{,}27\ \mathrm{Ra}^{1/4}$ $(10^5 < \mathrm{Ra} < 10^{11})$

**Cold surface**



N.B.: Characteristic length
So actually: $\mathrm{Ra}_{A/p}$

$$L_c = \frac{\text{Area}}{\text{Perimeter}} = \frac{A}{p}$$

# Free Convection Correlations

**Plate at angle $\theta$ with vertical axis**

**($\theta < 60°$)**

Hot surface:

- Top side: advanced methods
- Bottom side: same as vertical plate, except for using $g \cos \theta$ instead of $g$:

$$\mathrm{Ra}_L = \mathrm{Gr}_L \cdot \mathrm{Pr} = \frac{\boxed{g \cos \theta}\, \beta (T_s - T_\infty) L^3}{\nu^2} \cdot \mathrm{Pr}$$

Cold surface: everything upside down

- Bottom side: advanced methods
- Top side: same as vertical plate, except for using $g \cos \theta$ instead of $g$

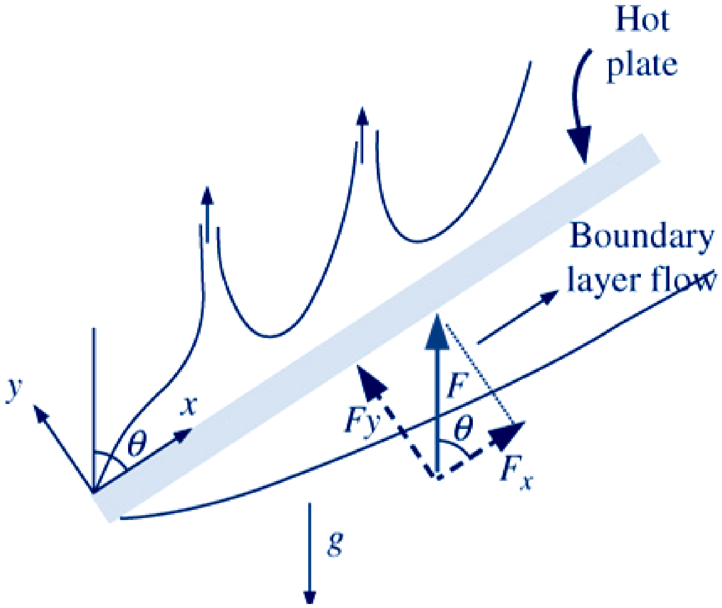


Plate at angle $\theta$ (ex. radiator in attic room)

Vertical plate:

$\mathrm{Nu} = 0{,}59\ \mathrm{Ra}_L^{1/4}$ ($10^4 < \mathrm{Ra}L < 10^9$)

$\mathrm{Nu} = 0{,}1\ \mathrm{Ra}_L^{1/3}$ ($10^{10} < \mathrm{Ra}L < 10^{13}$)

# Free Convection Correlations

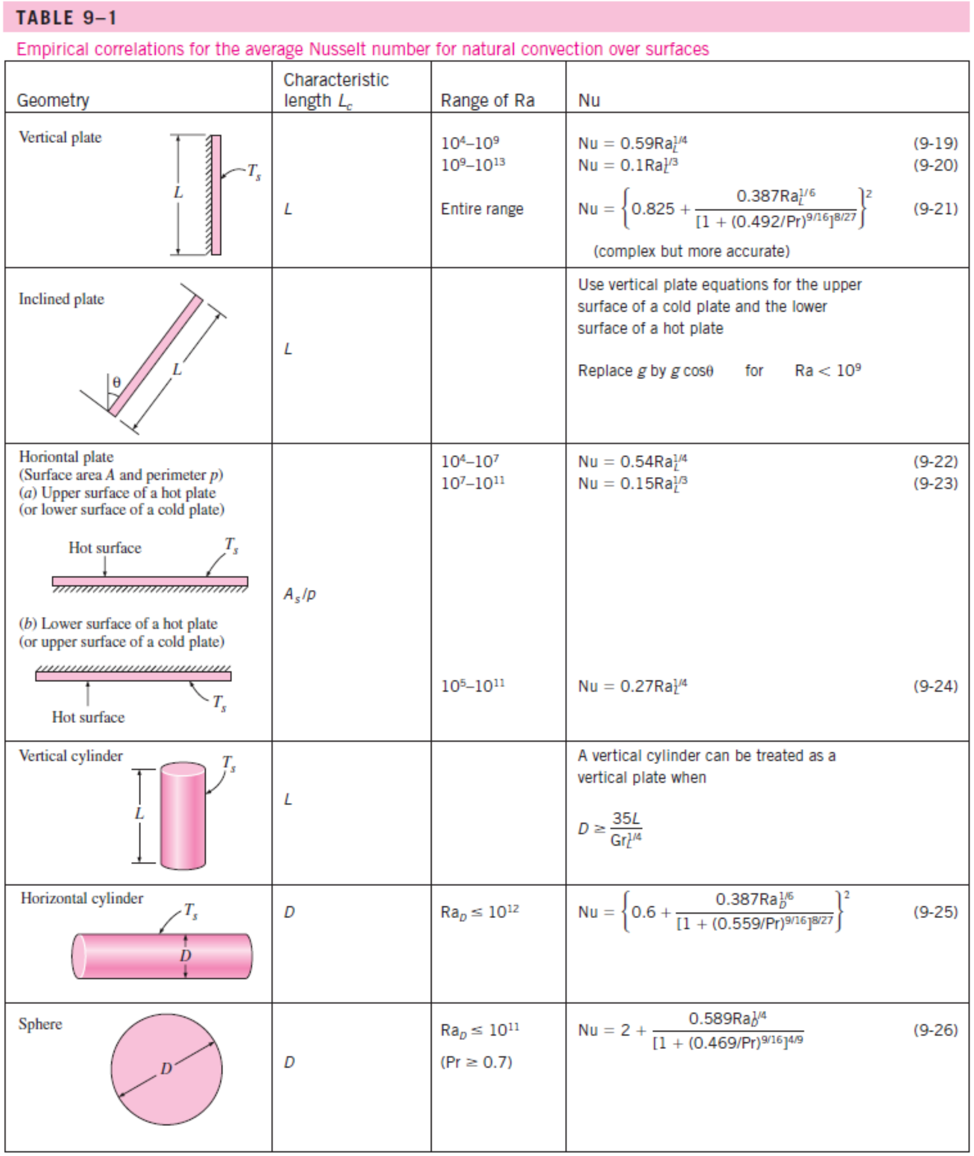


**TABLE 9–1**

Empirical correlations for the average Nusselt number for natural convection over surfaces

| Geometry | Characteristic length $L_c$ | Range of Ra | Nu |
|---|---|---|---|
| Vertical plate | $L$ | $10^4$–$10^9$<br>$10^9$–$10^{13}$<br>Entire range | $\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19)<br>$\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20)<br>$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21)<br>(complex but more accurate) |
| Inclined plate | $L$ | | Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate<br>Replace $g$ by $g\cos\theta$ for $\text{Ra} < 10^9$ |
| Horiontal plate<br>(Surface area $A$ and perimeter $p$)<br>(*a*) Upper surface of a hot plate (or lower surface of a cold plate)<br>Hot surface | $A_s/p$ | $10^4$–$10^7$<br>$10^7$–$10^{11}$ | $\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22)<br>$\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) |
| (*b*) Lower surface of a hot plate (or upper surface of a cold plate)<br>Hot surface | | $10^5$–$10^{11}$ | $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24) |
| Vertical cylinder | $L$ | | A vertical cylinder can be treated as a vertical plate when<br>$D \geq \frac{35L}{\text{Gr}_L^{1/4}}$ |
| Horizontal cylinder | $D$ | $\text{Ra}_D \leq 10^{12}$ | $\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25) |
| Sphere | $D$ | $\text{Ra}_D \leq 10^{11}$<br>$(\text{Pr} \geq 0.7)$ | $\text{Nu} = 2 + \frac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26) |

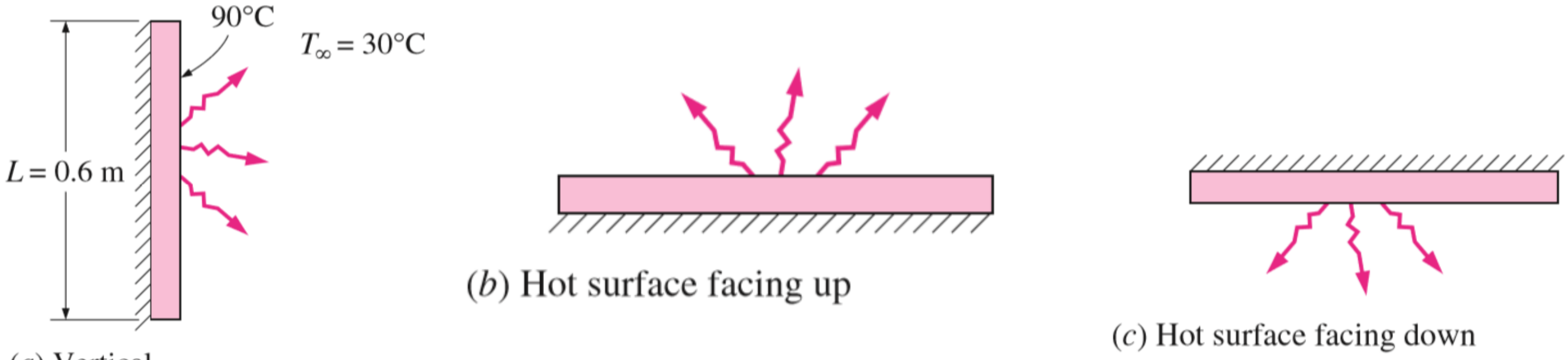
# Learning objectives lecture 4

## Heat transfer through natural convection

- Natural Convection
- Grashof Number
- Rayleigh Number
- Nusselt Number
- Using correlations for various configurations
- **Calculating natural convection with step-by-step plan**

# Example 1

## Cooling of a Plate in Different Orientations



(*a*) Vertical

(*b*) Hot surface facing up

(*c*) Hot surface facing down

- A: 0.6-m x 0.6-m thin square plate
- room temperature: 30˚C.
- One side of the plate is maintained at a temperature of 90˚C, while the other side is insulated
- Determine the rate of heat transfer from the plate by natural convection if the plate is

➢ (a) vertical,

➢ (b) horizontal with hot surface facing up, and

➢ (c) horizontal with hot surface facing down.

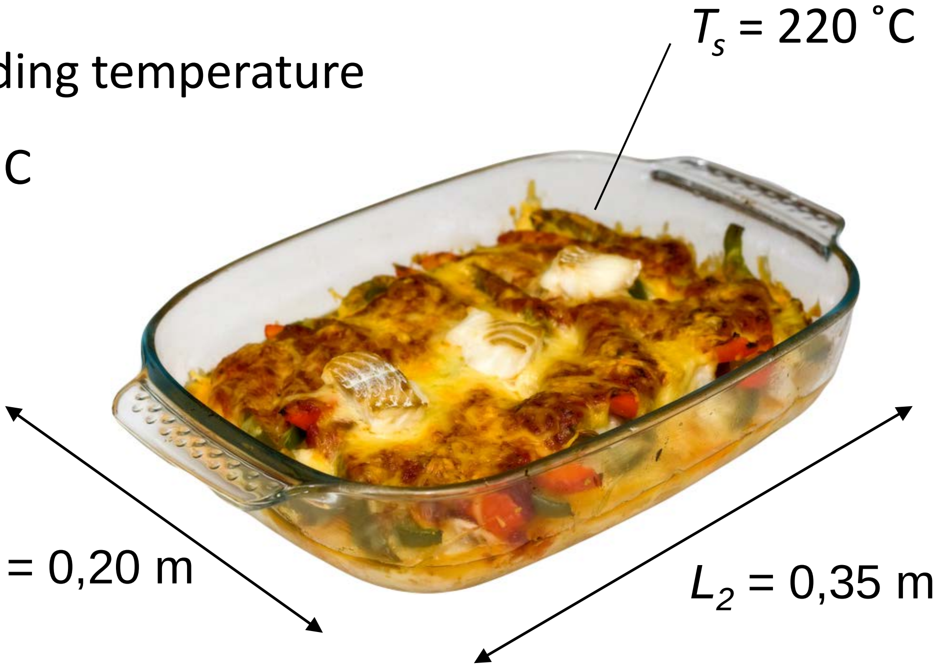
# Step-by-step plan natural convection

If $\dot{Q}$ must be found:

- Determine ingredients necessary for dimensionless no.
  (Pr, *k* and *v* at average temperature $\frac{T_s + T_\infty}{2}$ )
- Determine Ra: $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr}$
- Choose appropriate correlation based on geometry and Ra
- Determine Nu
- Resolve *h* from it
- Fill out Newton’s cooling law: $\dot{Q} = hA\Delta T$

# Example 2

## How large is the heat flow?


Surrounding temperature

$T_\infty = 20$ ˚C

$T_s = 220$ ˚C

$L_1 = 0{,}20$ m

$L_2 = 0{,}35$ m

*Natural convection*

# Example 3

Imagine: 25 W light bulb with diameter $D$ = 0,08 m releases 22,5 W of heat. Determine $T_s$ for $T_\infty$ = 20 ˚C

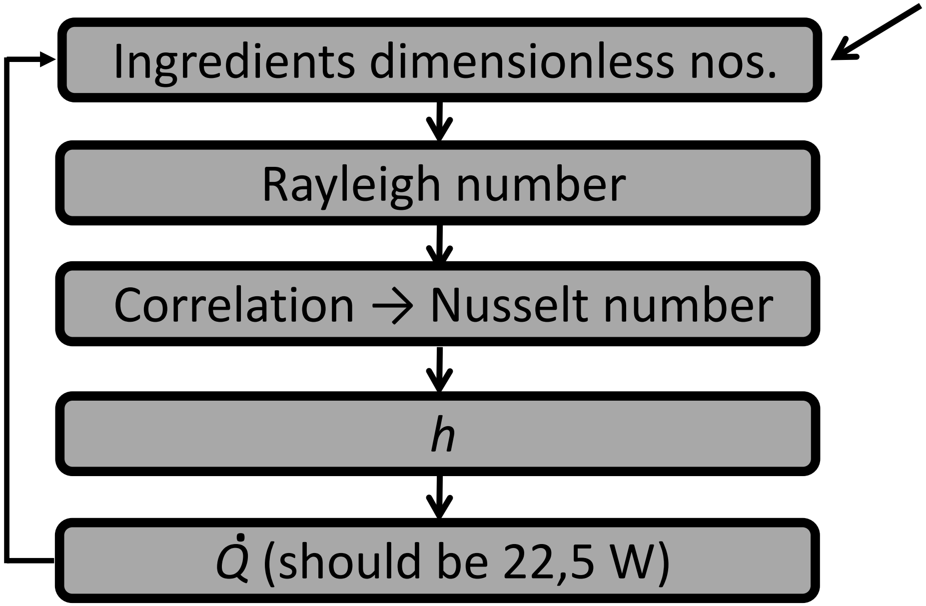
*Problem:*

To calculate $T_s$ ,$h$ is necessary ($\dot{Q} = hA\left(T_s - T_\infty\right)$) but $h$ depends on $T_s$ (because Nu, Ra, Pr are dependent on temperature) and that is the one we are looking for…

$$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$$

# Example 3

Problem: $T_s$ unknown → starting values for step-by-step plan unknown (Pr, $k$, $\nu$, …)



At what avg. temp? $\frac{T_s + T_\infty}{2}$ ?

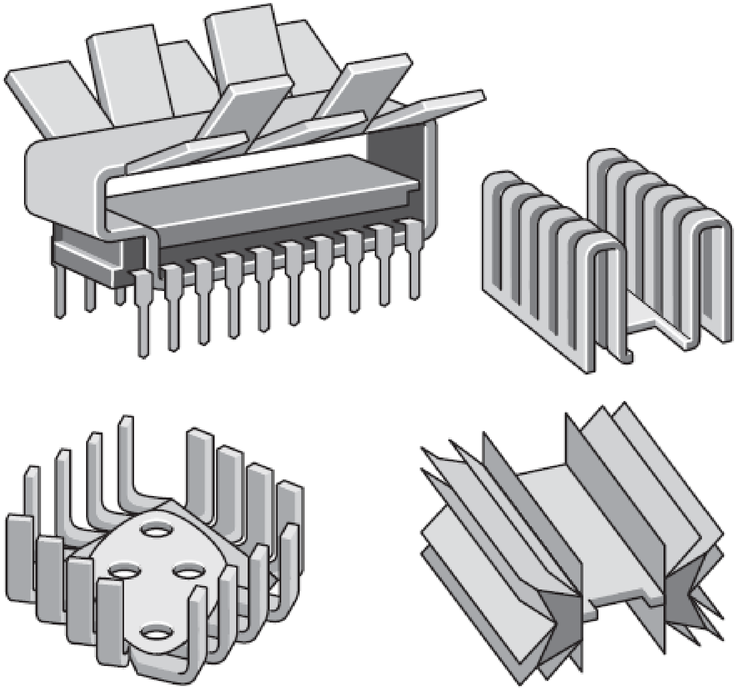
Unknown

<u>Approach: iterative solving</u>

- Choose an estimated $T_s$ to start with
- Follow steps and check whether $T_s$ yields the right heat flow
- Resulting heat flow ≠ 22,5 W?Adjust estimated $T_s$ and repeat all steps till you find a $T_s$ that (approximately) yields $\dot{Q}$ = 22,5 W

# Fins

What is Purpose of the Fins on Engine Surfaces ? Why design such a thing ?



Fins are extended surfaces and they increase the surface area leading to increase in heat transfer rate. Useful in cooling applications

# Summary natural convection

General (same as for forced convection)

$$\dot{Q} = hA\Delta T \quad \text{(W)}$$

Newton's cooling law

"Supporting" equations for *h*:

Nusselt number Nu as function of Rayleigh number Ra = Gr · Pr

Grashof number $\mathrm{Gr} = \dfrac{g\beta(T_s - T_\infty)L_c{}^3}{\nu^2}$

- Forced convection: Re ↔ natural convection: Gr
- Ra and Nu just like Re based upon characteristic length of geometry (indicate using subscript)

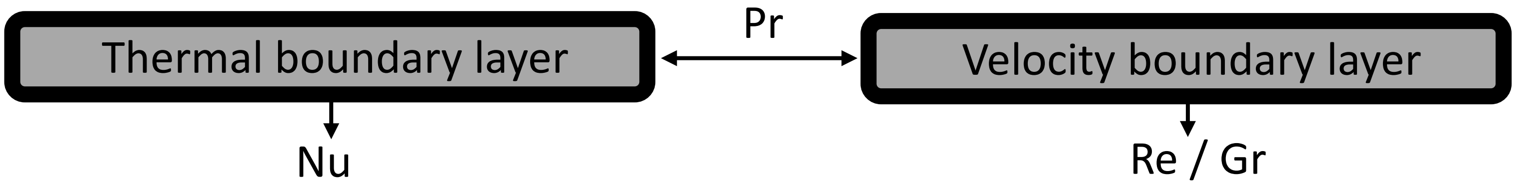
# General conclusion convection

- Calculate heat flow using Newton’s cooling law
- Determine heat transfer coefficient *h* in this using correlations between Nu (→ *h*) and other dimensionless nos.

⇒ step-by-step plans (+ iterative solving)

Other learning objectives convection: *flow phenomena*

- Being able to sum up what parameter are influencing *h*
- Knowing differences between laminar and turbulent ,
- Explaining how velocity and thermal boundary layers form
- Predicting/reasoning the temperature development



# REFERENCE

Chapter 9 of Heat Trasnfer, Cenegel.

# Feedback Session

- Next Lectorial : Forced Convection
- Questions? Feel free to ask!