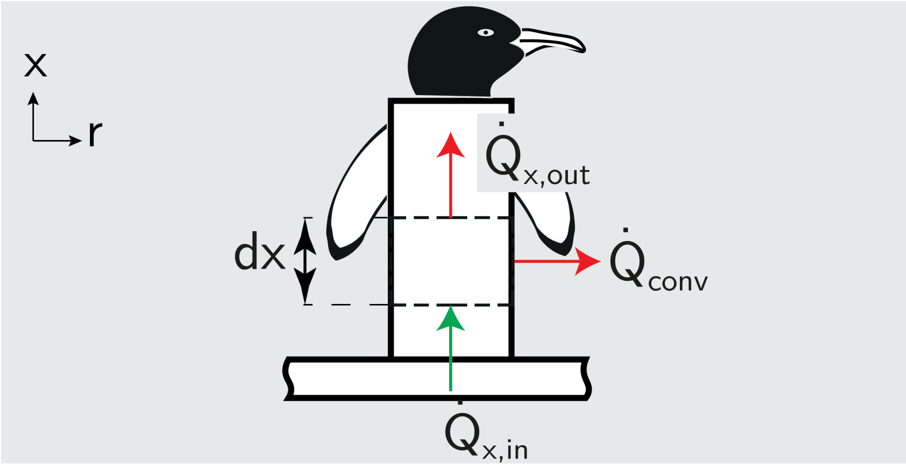
EB - Cond. - IE 11

Derive a homogeneous differential equation to describe the axial temperature distribution for the body of a penguin. Assume one-dimensional, steady-state heat transfer in the x-direction with no sources/sinks.



Homogenization:

$$\Theta \; = \; T(x) \; - \; T_\infty$$

$$m^2 = \; \tfrac{2 \cdot \alpha}{\lambda \cdot R}$$

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} - \dot{Q}_{conv}(x) = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \tfrac{\partial T}{\partial x} \; \rightarrow \dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \tfrac{\partial \Theta}{\partial x}$$

$$\dot{Q}_{x,out} \;\; = \;\; -\lambda \cdot \pi R^2 \cdot \tfrac{\partial T}{\partial x} + \tfrac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx \; \rightarrow \dot{Q}_{x,out} \;\; = \dot{Q}_{x,in} - \lambda \cdot \pi R^2 \cdot \tfrac{\partial^2 \Theta}{\partial x^2} dx$$

$$\dot{Q}_{conv} \;\; = \;\; \alpha \cdot 2\pi R dx \cdot (T(x) - T_\infty) \rightarrow \dot{Q}_{conv} \;\; = \alpha \cdot 2\pi R dx \cdot \Theta$$

Boundary Conditions:

$$\Theta(x = 0) \;\; = T_B - \; T_\infty$$

$$\Theta(x = L) \;\; = T_H - \; T_\infty$$

The first boundary condition results from the fact that $T(x = 0) = T_{\mathrm{B}}$ and the second one from the fact that $T(x = L) = T_{\mathrm{H}}$.