

W4-6-1 Determining pressure and temperature 1

Determine an expression for the temperature of a system for which the fundamental relation is: $U = U(S, V) = \left(\frac{\nu_o \theta}{R^2}\right) \frac{S^3}{NV}$, in which ν_o , θ , N and R are constants.

Tip: remember that you know that:

$$\left(\frac{\partial U}{\partial S}\right)_V = T \quad (1)$$

$U = \left(\frac{\nu_o \theta}{R^2}\right) \frac{S^3}{NV} = C \frac{S^3}{V}$, where $C = \frac{\nu_o \theta}{NR^2}$ is a constant.

Write $U = U(S, V)$ as the total differential: $dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$.

It is known that:

$$\left(\frac{\partial U}{\partial S}\right)_V = T \quad (2)$$

These two differentials can be determined for this function:

$$\left(\frac{\partial U}{\partial S}\right)_V = \frac{\partial}{\partial S} \left(\frac{CS^3}{V}\right) = \frac{3CS^2}{V} = T \quad \Rightarrow \quad T = \frac{\nu_o \theta}{R^2} \frac{3S^2}{NV} \quad (3)$$