Q1

An average small car engine consumes 5 liters of diesel per 100 km of driving. Now, let's consider a scenario where we have a car traveling at an average speed of 60 km/h. During this operation, the car's engine generates an average of 21 kW of heat. Diesel has an LHV of 35.8 MJ per liter.

The outer surface of an engine is located in an area where oil leakage can occur, and if the leaked oil comes into contact with a hot surface having a temperature above its autoignition temperature, it can ignite spontaneously, posing a fire hazard. To address this concern, engineers have designed an engine cover made of an aluminum plate with a thickness of 2 cm and a thermal conductivity of 237 W/mK.

The inner surface of the engine cover, which is exposed to hot air, maintains a temperature of 330 ^{o}C. To prevent fire risks in the event of an oil leak onto the engine cover, a layer of thermal barrier coating (TBC) with a thermal conductivity of 1.1 W/mK is applied to the outer surface of the engine cover.

For this analysis, we consider the engine's surface area to be 1.1 m^2. The application of the TBC acts as a thermal insulator, reducing the rate of heat transfer from the hot inner surface to the outer surface of the engine cover. This insulation layer helps in lowering the temperature of the outer surface, thereby minimizing the risk of oil autoignition and potential fire hazards.

By implementing this thermal barrier coating, engineers aim to enhance the safety and reliability of the engine while ensuring optimal performance even in challenging operating conditions.

a Give the definition of efficiency.

b What is the efficiency of the car engine? State the assumptions that are made.

c Provide a sketch of the thermal network. Include all known temperatures, resistances, and the direction of the flow of heat. Explain each component.

d Determine the heat flux through the wall of the engine.

e Would a TBC layer of 8 mm in thickness be sufficient to keep the engine cover surface below the autoignition temperature of 200 °C to prevent fire hazards?

f Determine the temperature at the interface between the aluminium and TBC layer.

g Give a sketch of the temperature profile inside the layers of the car engine.

Note: Clearly indicate the change in temperature in the axial direction, the change in slope at the interface. Lastly, indicate the numerical value of the temperature at the interface

Solution Q1

a Give the definition of efficiency.

$$\eta = \frac{\text{useful work}}{\text{input energy}} \cdot 100\ \%$$

(1) Correct definition

b What is the efficiency of the car engine? State the assumptions that are made.

It is assumed that:

- all energy not being lost is used for power generation.

(0.5) Correct assumption

The consumption of diesel per unit of time can be calculated by the rate of diesel consumption per km (=5 per 100 km) and knowing the average velocity (=60 km per h):

$$\dot{V}_{\text{diesel}} = \frac{5}{100} \cdot 60 = 3\ \text{L/h} = 8.33 \cdot 10^{-4}\ \text{L/s}$$

(0.5) Correct calculation of diesel consumption per unit of time

The combustion energy is calculated as:

$$\dot{E}_{\text{combustion}} = \text{LHV}_{\text{diesel}} \cdot \dot{V}_{\text{diesel}} = \left(35.8 \cdot 10^{6}\right) \cdot \left(8.3 \cdot 10^{-4}\right) = 29.83\ \text{kW}$$

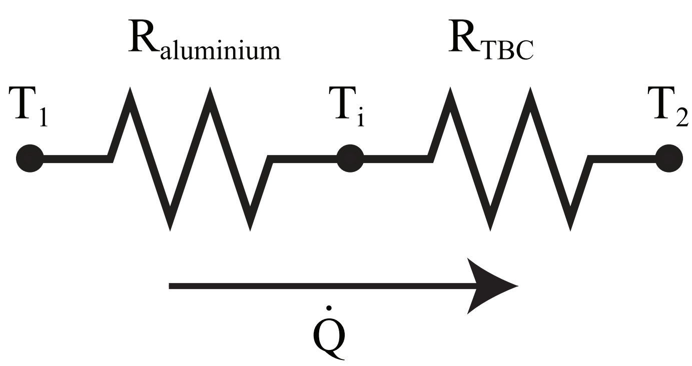
(0.5) Correct calculation of generation of heat

This yields an efficiency of:

$$\eta = \frac{\text{useful work}}{\text{input energy}} \cdot 100\ \% = \frac{\dot{E}_{\text{combustion}} - \dot{Q}}{\dot{E}_{\text{combustion}}} \cdot 100\ \% = \frac{29.83 - 21}{29.83} \cdot 100\ \% = 29.6\ \%$$

(0.5) Correct calculation of efficiency

c Provide a sketch of the thermal network. Include all known temperatures, resistances, and the direction of the flow of heat. Explain each component.



(1) Correct sketch

- T_1: Temperature at the interface between the aluminium and hot air inside the motor. Known to be 330°C.
- T_i: Temperature at the interface between the aluminium and TBC. Not known yet.
- T_2: Temperature at the interface between the TBC and cold air outside. Not known yet.
- $R_{\text{aluminium}}$: Thermal resistance induced by the aluminium layer due to conductive heat transfer. Not known yet.
- R_{TBC}: Thermal resistance induced by the TBC layer due to conductive heat transfer. Not known yet.
- $\dot{Q}$: Rate of heat transfer from the inside of the engine to the outside. Known to be 21 kW.

(1) Correct explanation of all components, (-0.5) per missing

d Determine the heat flux through the wall of the engine.

The heat flux is defined to be:

$$\dot{q}'' = \frac{\dot{Q}}{A} = \frac{21{,}000}{1.1} = 19.09\ \text{kW/m}^2$$

(1) Correct calculation of heat flux

e Would a TBC layer of 8 mm in thickness be sufficient to keep the engine cover surface below the autoignition temperature of 200 °C to prevent fire hazards?

To determine whether a TBC layer of 8 mm in thickness is sufficient to keep the engine cover surface below the autoignition temperature of 200 °C to prevent fire hazards it should be validated that $T_2 < 200\ ^{o}\mathrm{C}$.

First, all thermal resistances should be determined:

$$R_{\text{aluminium}} = \frac{x_{\text{aluminium}}}{k_{\text{aluminium}} \cdot A} = \frac{0.02}{237 \cdot 1.1} = 7.67 \cdot 10^{-5}\ \mathrm{K/W}$$

$$R_{\text{TBC}} = \frac{x_{\text{TBC}}}{k_{\text{TBC}} \cdot A} = \frac{0.008}{1.1 \cdot 1.1} = 6.61 \cdot 10^{-3}\ \mathrm{K/W}$$

(0.5) Correct calculation of conductive resistance

As the resistances stand in series, the total thermal resistance can be calculated as:

$$R_{\text{total}} = R_{\text{aluminium}} + R_{\text{TBC}} = \left(7.67 \cdot 10^{-5}\right) + \left(6.61 \cdot 10^{-3}\right) = 6.69 \cdot 10^{-3}\ \mathrm{K/W}$$

(0.5) Correct calculation of total resistance

From the thermal resistance theorem, it is known that:

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

So:

$$\rightarrow T_2 = T_1 - \dot{Q} \cdot R_{\text{total}} = 330 - 21{,}000 \cdot \left(6.69 \cdot 10^{-3}\right) = 189.55\ ^{o}\mathrm{C}$$

(0.5) Correct calculation of T_2

So yes, a TBC layer of 8 mm in thickness is sufficient to keep the engine cover surface below the autoignition temperature.

f Determine the temperature at the interface between the aluminium and TBC layer. From the thermal resistance theorem, it is known that:

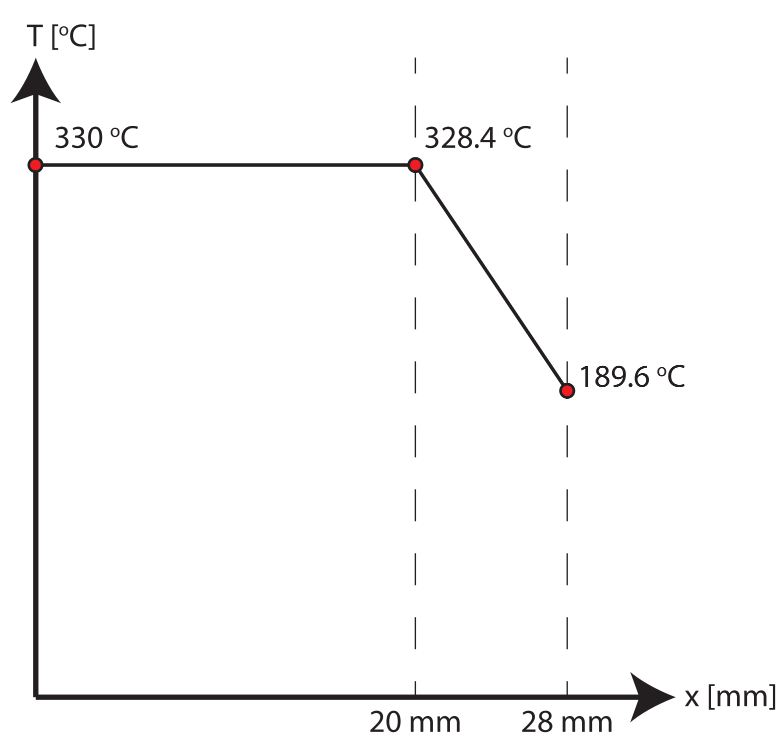
$$\dot{Q} = \frac{T_1 - T_{\text{i}}}{R_{\text{aluminium}}}$$

So:

$$\rightarrow T_{\text{i}} = T_1 - \dot{Q} \cdot R_{\text{aluminium}} = 330 - 21{,}000 \cdot \left(7.67 \cdot 10^{-5}\right) = 328.39\ ^{o}\mathrm{C}$$

(0.5) Correct calculation of T_{i}

g Give a sketch of the temperature profile inside the layers of the car engine.



The temperature drop in the second layer is bigger than the temperature drop in the first layer.

Both lines should be straight.

The gradient of the second layer should be steeper than the gradient of the first layer at the intersection.

All temperatures should be at the correct location.

(2) For all correct criteria, (-0.5) per missing criteria

Q2

A refrigeration truck is in motion at a speed of 80 km/h. The truck's depiction is presented in the image below.

Figure 1: Refrigeration truck

The dimensions of the truck's upper section measure 3 meters in length and 2.1 meters in width. The truck interior houses stored ice cream, requiring temperature regulation. The internal air maintains a temperature of -7 °C, featuring a heat transfer coefficient of h_i = 10 W/m^{2}K. To insulate the roof, a sandwiched arrangement employs insulation (t_2 = 40 mm, k_2 = 0.0320 W/mK) nestled between two aluminum sheets (t_1 = t_3 = 3 mm, k_1 = k_3 = 180 W/mK). The ambient external temperature stands at 28°C.

a) Determine the Reynolds number for the roof of the passenger truck. Assume the average fluid properties to be T_f = 25$^\mathrm{o}$C. Please clearly indicate what properties or air are used and which assumptions have been made.

b) Determine at which length the laminar flow becomes turbulent.

c) Find an expression for the heat transfer coefficient h_o outside the truck. Also, explain what the heat transfer coefficient tells us.

d) Provide a sketch of the thermal network. Include all known temperatures, resistances, and the direction of the flow of heat. Explain each component

e) Determine the rate of heat transfer from the ambient through the top of the truck.

f) Determine the temperature T_s of the top surface of the truck roof. What would have been a good estimate for T_f in task a)?

g) Give a sketch of the temperature profile. The domain drawn should cover the inside temperature of -7 $^\mathrm{o}$C as well as the outside temperature of 28 $^\mathrm{o}$C.

Solution Q2

a) Determine the Reynolds number for the roof of the passenger truck. Assume the average fluid properties to be $T_f = 25^{o}$C. Please clearly indicate what properties or air are used and which assumptions have been made.

The assumptions that are being made are:

- The roof is perfectly flat.

(0.5) Correctly stated assumption

The properties of air are:

- $\rho = 1.184 \text{ kg/m}^3$
- $k = 0.02551 \text{ W/mK}$
- $\mu = 1.849 \cdot 10^{-5} \text{ kg/m} \cdot \text{s}$
- $\text{Pr} = 0.7296$

(0.5) Correct air properties

The Reynolds number can be determined as:

$$\text{Re}_\text{L} = \frac{\rho \cdot V \cdot L}{\mu} = \frac{1.184 \cdot 22.22 \cdot 3}{1.849 \cdot 10^{-5}} = 4.269 \cdot 10^6$$

where $V = 22.22$ m/s and the characteristic length is $L = 3$ m.

(0.5) Correct Reynolds number

b) Determine at which length the laminar flow becomes turbulent.

The flow becomes turbulent for a flat plate when $\text{Re}_\text{crit} = 5 \cdot 10^5$.

Rewriting the expression of the Reynolds number yields:

$$x_\text{crit} = \frac{\text{Re}_\text{crit} \cdot \mu}{\rho \cdot V} = \frac{\left(5 \cdot 10^5\right) \cdot \left(1.849 \cdot 10^{-5}\right)}{1.184 \cdot 22.22} = 0.35 \text{ m}$$

(0.5) Correct critical length

c) Find an expression for the heat transfer coefficient h_o outside the truck. Also, explain what the heat transfer coefficient tells us.

The heat transfer coefficient, often denoted as "h," is a crucial parameter in the field of heat transfer. It quantifies the rate of heat transfer between a solid surface and a fluid (liquid or gas) that is in contact with that surface. Essentially, the heat transfer coefficient tells us how efficiently heat is being exchanged between the surface and the fluid.

In more technical terms, the heat transfer coefficient is defined as the amount of heat transferred per unit area per unit temperature difference between the surface and the fluid.

(1) Correct definition heat transfer coefficient

Having determined the Reynolds number, it yields that the following correlation for the Nusselt number is applicable:

$$\text{Nu} = 0.037 \cdot \text{Re}_\text{L}^{4/5} \cdot \text{Pr}^{1/3} = 0.037 \cdot \left(4.269 \cdot 10^6\right)^{4/5} \cdot (0.7296)^{1/3} = 6.71 \cdot 10^3$$

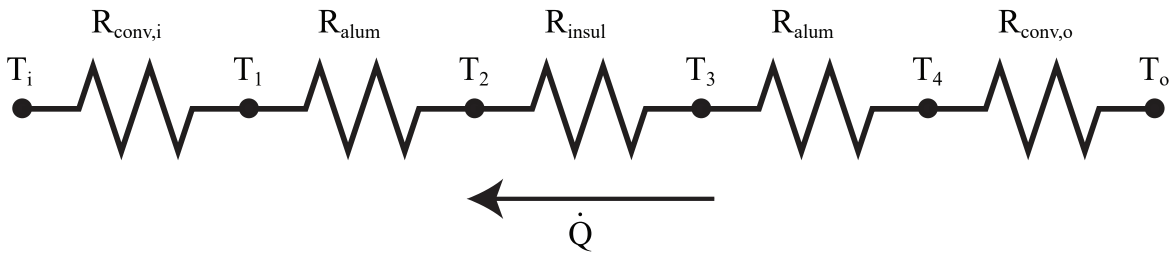
(1) Correct correlation Nusselt number and value

Which yields the heat transfer coefficient:

$$h_\text{o} = \frac{\text{Nu} \cdot k}{L} = \frac{\left(6.71 \cdot 10^3\right) \cdot 0.02551}{3} = 57.07 \text{ W/m}^2\text{K}$$

(0.5) Correct heat transfer coefficient

d) Provide a sketch of the thermal network. Include all known temperatures, resistances, and the direction of the flow of heat. Explain each component



(1) Correct sketch of thermal resistance network

- T_i: Temperature at the inside of the truck. Known to be -7^oC.
- T_o: Temperature at the outside ambient. Known to be 28^oC.
- T_1, T_2, T_3, T_4: Interface temperatures between different resistances. All are unknown at this point.
- $R_{conv,i}$: Thermal resistance induced by convection happening inside the truck. Not known yet.
- R_{alum}: Thermal resistance induced by the aluminum layer due to conductive heat transfer. Not known yet.
- R_{insul}: Thermal resistance induced by the insulating layer due to conductive heat transfer. Not known yet.
- $R_{conv,u}$: Thermal resistance induced by convection happening outside the truck. Not known yet.
- $\dot{Q}$: Rate of heat transfer from the outside ambient to the inside of the truck. Not known yet.

(1) Correct explanation of all components, (-0.5) per missing criteria

e) Determine the rate of heat transfer from the ambient through the top of the truck. First, all resistances should be determined:

$$R_{conv,i} = \frac{1}{h_i \cdot A} = \frac{1}{10 \cdot (3 \times 1.8)} = 0.0185 \text{ K/W}$$

$$R_{alum} = \frac{t_1}{k_1 \cdot A} = \frac{0.003}{180 \cdot (3 \times 1.8)} = 3.086 \cdot 10^{-6} \text{ K/W}$$

$$R_{insul} = \frac{t_2}{k_2 \cdot A} = \frac{0.04}{0.032 \cdot (3 \times 1.8)} = 0.2315 \text{ K/W}$$

$$R_{conv,o} = \frac{1}{h_o \cdot A} = \frac{1}{57.0705 \cdot (3 \times 1.8)} = 0.0032 \text{ K/W}$$

(1) Correct thermal resistances, (-0.5) per missing criteria

The total thermal resistance yields from:

$$R_{total} = R_{conv,i} + R_{alum} + R_{insul} + R_{alum} + R_{conv,o} = 0.2533 \text{ K/W}$$

(0.5) Correct total thermal resistance

Using the thermal resistance theorem, one finds the rate of heat transfer:

$$\dot{Q} = \frac{T_o - T_i}{R_{total}} = \frac{28 - (-7)}{0.2533} = 138.21 \text{ W}$$

(1) Correct rate of heat transfer

f) Determine the temperature T_s of the top surface of the truck roof. The top surface $T_s = T_4$ yields from using the thermal resistance theorem:

$$\dot{Q} = \frac{T_o - T_s}{R_{conv.o}}$$

Where rewriting gives:

$$T_s = T_o - \dot{Q} \cdot R_{conv.o} = 27.55 \text{ }^o\text{C}.$$

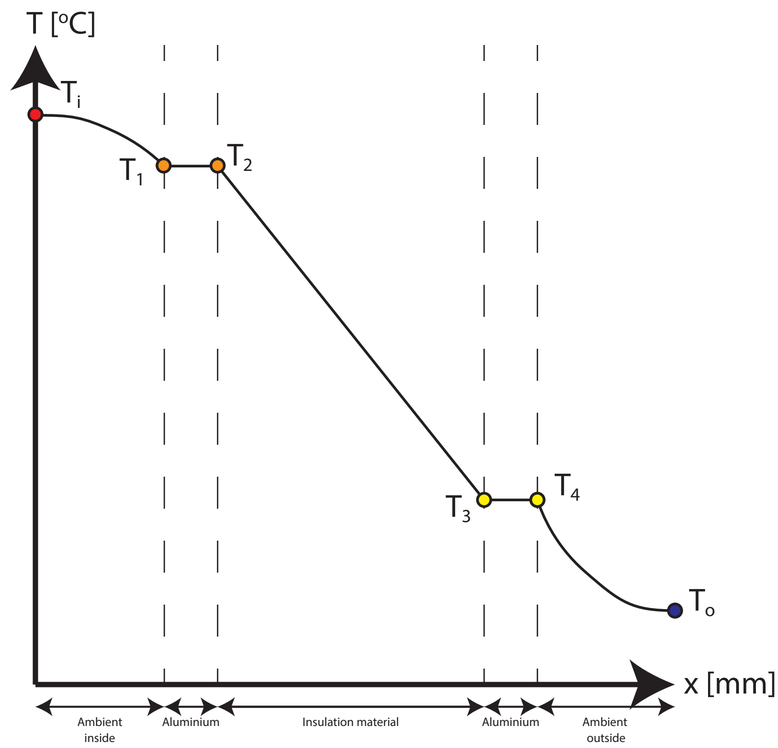
(1) Correct surface temperature

A good estimate for T_f would have been:

$$T_f = \frac{T_o + T_s}{2} = \frac{28 + 27.55}{2} = 27.78 \text{ }^o\text{C}.$$

(1) Correct average fluid temperature

g) Give a sketch of the temperature profile. The domain drawn should cover the inside temperature of -7 ^{o}C as well as the outside temperature of 28 ^{o}C.



The temperature in the ambient inside should start with a horizontal slope, which increases when moving in the positive x-direction.

The temperature drop in the aluminum layers is negligible.

The temperature drop in the insulation layer is the largest of all, with the steepest slope.

The temperature in the ambient outside should have a decrease in slope until it reaches the ambient temperature where the slope becomes horizontal.

(2) For all correct criteria, (-0.5) per missing criteria

Q3

During spring and autumn, in the evenings it can still get quite chilly. However, by using a so called terrace heater as illustrated below in figure 2a, it is possible to stay warm even during those chilly evenings.

(a) Terrace heater

(b) Forced convection heater

Figure 2: Two types of heaters

Another option is to use a different type of heater, which makes use of forced convection. An example is showed in figure 2b. We are interested in the difference between both options. Assume that the terrace is protected against the wind, such that there is no air flowing in or out of the terrace. The terrace has a total volume of 27 m^3.

For the comparison, we assume that the temperature of the lamp and coil in the forced heater are a constant 300 ^{o}C and the air inside the terrace is 12 ^{o}C. The velocity profile of the air around the lamp is illustrated in figure ... The properties of air can be found in the table 1.

a) Explain what the Grashof number physically represents

b) Explain the shape of the velocity profile in figure 3.

c) For the terrace lamp, determine the rate of heat transfer when the lamp is turned on. The lamp can be modeled as a cylinder with a diameter of 40 cm and a thickness of 1 cm.

Now we start using the forced convection heater. You may assume the fan is blowing horizontally over the coils with a speed of 2.0 m/s. All coils together can be modeled as a cylinder with a diameter of 1 cm and a length of 1.25 m.

d) Determine the rate of heat transfer of the forced convection heater.

For the following questions, we want to test your understanding of the subject. It is therefore not necessary to calculate anything, but try to reason while looking at the equations.

e) Can you explain the difference in the rates of heat transfer between the natural and forced convection situation?

f) Can you think of another reason why not only the heat transfer due to convection is of importance in case of the lamp?

g) Suppose we use a much bigger fan which is able to blow the air over the coil at a speed of 50 m/s. This will result in a turbulent flow over the coils. Explain the effect on the heat transfer coefficient, and why this effect takes place.

h) Now suppose that we also use the same heaters in the winter, where the temperatures of the surroundings drop to 0 ^{o}C. Can you (in words) explain what will happen to the heat transfer coefficient h, compared to spring/autumn conditions? Can you also explain what will happen to the required energy E_{tot} to heat up the air?

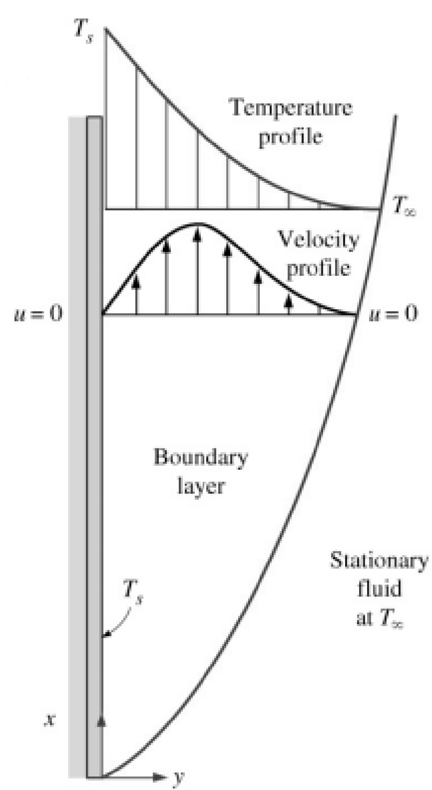


Figure 3: Velocity and temperature profile

Temperature ^{o}C	Density kg/m^3	Specific heat J/kgK	Thermal conductivity W/m·K	Thermal diffusivity m^2/s	Dynamic viscosity kg/m·s	Kinematic viscosity m^2/s	Prandtl number
0	1.292	1006	0.02364	$1.818 \cdot 10^{-5}$	$1.729 \cdot 10^{-5}$	$1.338 \cdot 10^{-5}$	0.7362
10	1.246	1006	0.02439	$1.944 \cdot 10^{-5}$	$1.778 \cdot 10^{-5}$	$1.426 \cdot 10^{-5}$	0.7336
20	1.204	1007	0.02514	$2.074 \cdot 10^{-5}$	$1.825 \cdot 10^{-5}$	$1.516 \cdot 10^{-5}$	0.7309
30	1.164	1007	0.02588	$2.208 \cdot 10^{-5}$	$1.872 \cdot 10^{-5}$	$1.608 \cdot 10^{-5}$	0.7282
40	1.127	1007	0.02662	$2.346 \cdot 10^{-5}$	$1.918 \cdot 10^{-5}$	$1.702 \cdot 10^{-5}$	0.7255
50	1.092	1007	0.02375	$2.487 \cdot 10^{-5}$	$1.963 \cdot 10^{-5}$	$1.798 \cdot 10^{-5}$	0.7228
60	1.059	1007	0.02808	$2.632 \cdot 10^{-5}$	$2.008 \cdot 10^{-5}$	$1.896 \cdot 10^{-5}$	0.7202
70	1.028	1007	0.02881	$2.780 \cdot 10^{-5}$	$2.052 \cdot 10^{-5}$	$1.995 \cdot 10^{-5}$	0.7177
80	0.9994	1008	0.02953	$2.931 \cdot 10^{-5}$	$2.096 \cdot 10^{-5}$	$2.097 \cdot 10^{-5}$	0.7154
90	0.9718	1008	0.03024	$3.086 \cdot 10^{-5}$	$2.139 \cdot 10^{-5}$	$2.201 \cdot 10^{-5}$	0.7132
100	0.9458	1009	0.03095	$3.243 \cdot 10^{-5}$	$2.181 \cdot 10^{-5}$	$2.306 \cdot 10^{-5}$	0.7111
120	0.8977	1011	0.03235	$3.565 \cdot 10^{-5}$	$2.264 \cdot 10^{-5}$	$2.522 \cdot 10^{-5}$	0.7073
140	0.8542	1013	0.03374	$3.898 \cdot 10^{-5}$	$2.345 \cdot 10^{-5}$	$2.745 \cdot 10^{-5}$	0.7041
160	0.8148	1016	0.03511	$4.241 \cdot 10^{-5}$	$2.420 \cdot 10^{-5}$	$2.975 \cdot 10^{-5}$	0.7014

Table 1: Air properties at 1 atm pressure

Solution Q3

a) *Explain what the Grashof number physically represents*

The Grashof number is a dimensionless number named after Franz Grashof. The Grashof number is defined as the ratio of the buoyant to a viscous force acting on a fluid in the velocity boundary layer. Its role in natural convection is much like that of the Reynolds number in forced convection.

b) *Explain the shape of the velocity profile in figure 9.*

At the interface between the fluid and the hot plate, $y = 0$, the air is stationery. This is due to the friction forces between plate and fluid. Moving in positive y-direction, the fluid rises due to buoyancy forces and increases in velocity. Moving even further in positive y-direction, the velocity decreases again, since the effect of the plate is no longer working on the fluid.

c) *For the terrace lamp, determine the rate of heat transfer when the lamp is turned on. The lamp can be modeled as a cylinder with a diameter of 40 cm and a thickness of 1 cm.*

Since we are dealing with natural convection, we have to use the Rayleigh number to define a Nusselt number, which in turn leads to a heat transfer coefficient. The Grashof number can be calculated as follows:

$$Gr = \frac{g\beta(T_{lamp} - T_{terrace})L_c^3}{\nu^2}$$

For β we use the expression

$$\beta = \frac{2}{T_s + T_\infty} = \frac{2}{573 - 285} = 0.002331\ [K^{-1}]$$

Using the correct data of air at a fluid temperature of $T_f = \frac{300+12}{2} = 156\,^\circ C \rightarrow 160\,^\circ C$ gives the following:

$$Ra = Gr \cdot Pr = \frac{9.81 \cdot 0.002331 \cdot (573 - 285) \cdot 0.01^3}{(2.975 \cdot 10^{-5})^2} \cdot 0.7014 \approx 5219$$

Finding the correct expression for the Nusselt number of a cylinder, and using the characteristic length $L_c = D$:

$$Nu_D = \left\{ 0.6 + \frac{0.387 \cdot Ra^{\frac{1}{6}}}{[1 + (\frac{0.559}{Pr})^{\frac{9}{16}}]^{\frac{8}{27}}} \right\}^2 \approx 3.75$$

Now we can find the heat transfer coefficient:

$$Nu_D = \frac{hD}{k} \rightarrow h = \frac{Nu \cdot k}{D} \approx 0.329\ [W/m^2K]$$

And finally we are able to determine the heat transfer:

$$\dot{Q} = hA\Delta T = 0.329 \cdot 0.01256 \cdot 288 = 1.19\ [W]$$

d) *Determine the rate of heat transfer of the forced convection heater*

To determine the heat transfer due to forced convection we start at the Reynolds number:

$$Re = \frac{\rho U L_c}{\mu} = \frac{\rho \cdot U \cdot D}{\mu} = \frac{0.8148 \cdot 2 \cdot 0.01}{2.420 \cdot 10^{-5}} \approx 673.4$$

Finding the correct relation for the Nusselt number as a function of the Reynolds and Prantl number:

$$Nu_D = 0.193 \cdot Re^{0.618} \cdot Pr^{\frac{1}{3}} = 0.193 \cdot 673.4^{0.618} \cdot 0.7014^{\frac{1}{3}} \approx 9.6$$

From this we take the same steps as previous question to find the heat transfer coefficient and the resulting heat transfer:

$$Nu_D = \frac{h \cdot D}{k} \rightarrow h = \frac{Nu \cdot k}{D} = \frac{29.04 \cdot 0.03511}{0.01} = 33.7\ [W/m^2K]$$

$$\dot{Q} = hA\Delta T = 101.95 \cdot 0.01256 \cdot 288 = 121.87\ [W]$$

e) *Can you explain the difference in the rates of heat transfer between the natural and forced convection situation?*

Since the cylinder has quite a small diameter, not much heat is transferred by natural convection. This is also to be expected. When the fan starts blowing, we create a turbulent flow around the cylinder which results in a much higher heat transfer.

f) *Can you think of another reason why not only the heat transfer due to convection is of importance in case of the lamp?*

The heaters radiate most of the heat away to the people sitting on the table. It therefore does not necessarily matter if there is wind blowing, since radiation does not need a medium to transfer heat.

g) *Suppose we use a much bigger fan which is able to blow the air over the coil at a speed of 50 m/s. This will result in a turbulent flow over the coils. Explain the effect on the heat transfer coefficient, and why this effect takes place*

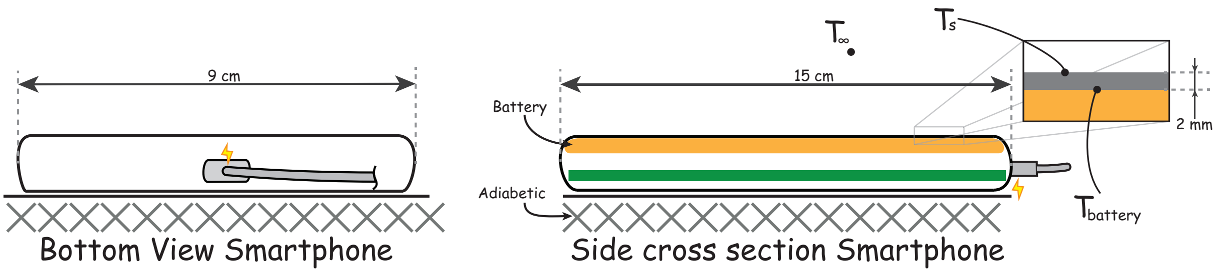
As was also already noted in question e), due to the turbulent flow the heat transfer coefficient increases. This is due to the fact that the boundary layer on the surface of the cylinder starts to mix the fluid (air) more violently due to the high velocity of the air, resulting in a higher transfer of heat.

h) *Now suppose that we also use the same heaters in the winter, where the temperatures of the surroundings drop to 0 °C. Can you (in words) explain what will happen to the heat transfer coefficient h, compared to spring/autumn conditions? Can you also explain what will happen to the required energy E_{tot} to heat up the air?*

The temperature will drop, thus the difference in temperature will become larger. This results in a larger potential to transfer heat, thus the heat transfer coefficient will also increase. This can also be observed by the definition of the Grashof number.

Q4

The battery of a smartphone is being charged, while the device is lying outside during the day. It is a windstill day and the sun is shining. The phone's dimensions are 15 cm x 9 cm and the smartphone is lying flat with the backside upwards towards the sky. The smartphone has a large battery that covers the entire backside of the device and the battery is only covered by a thin 2 mm plastic backplate. The battery is imperfect, and some of the energy supplied by the charging cable is wasted in heat. The backplate is heated by the radiation of the sun and the warmth of the battery. It has a temperature of 42 ^{o}C and only loses heat by convection. The outside temperature is 15 ^{o}C and the device absorbs an effective radiative heat flux of 79.6 W/m^2. Assume steady state-state heat (and energy) transfer.



a) Explain what the Grashof number physically represents.

b) Determine the convective heat transfer coefficient h of the backplate and calculate the convective heat flux to the environment $\dot{Q}_{\text{convection}}$

Most of the power supplied by the charging cable $\dot{E}_{cable}$ is stored in the battery. Therefore, the charging rate of the battery is $\dot{E}_{battery,stored}$. The rest of the electrical power supplied by the charging cable is wasted into heat $\dot{Q}_{battery,heat}$ and is lost fully by the backplate. The efficiency at which the battery is charged $\dot{\eta}_{battery} = \frac{\dot{E}_{battery,stored}}{\dot{E}_{cable}}$ is 95%.

c) Determine the conductive heat flux from the battery $\dot{Q}_{\text{battery,heat}}$
Hint:
$0 = \dot{Q}_{\text{battery,heat}} + \dot{Q}_{\text{radiation}} - \dot{Q}_{\text{convection}}$

d) Find the rate at which the charging cable $\dot{E}_{cable}$ supplies energy to the smartphone.
Hint:
The units of the electricity flows $\dot{E}$ and heat flows $\dot{Q}$ are the same; [W s^{-1}]!

e) Determine the temperature of the battery. The thermal conductivity of the plastic backplate is 0.12 W m^{-1} K^{-1}. It may be assumed that the temperature of the battery is constant.

The smartphone is accidentally nudged off the side off the table and now only hangs on the charging cable. The front of the smartphone is still adiabatic and the solar radiation on the backplate of the smartphone is still the same. Also, the charging rate has not changed. However, due to the now vertical position, the convective properties of the device have changed. The owner of the smartphone wants to know if this changes the temperature of the battery.

f) Recalculate the convective heat transfer coefficient h for this vertical scenario and compare it with the horizontal case. Argue, in words or a short additional calculation, how the difference in coefficients influences the temperature of the backplate and the battery of the smartphone. Will it be higher, lower or remain the same? You may neglect the change in the properties of the air.
Hint:
The surface temperature of the backplate is not known in the vertical case. You may - if you feel you need to - simplify the problem by neglecting this change when calculating the Grashof number as well (and thus use the horizontal surface temperature).

Solution Q4

a) *Explain what the Grashof number physically represents.* The Grashof number is a dimensionless number in fluid dynamics and heat transfer which approximates the ratio of the buoyancy to viscous force acting on a fluid.

(1) correct definition

b) *Determine the convective heat transfer coefficient h of the backplate and calculate the convective heat flux to the environment*

Average fluid properties:

$$T_f = \frac{T_s + T_{infty}}{2} = \frac{15+42}{2} \; [^oC] = 28.5 \; [^oC]$$

Properties of air at 28.5 [^{o}C]

$$k = 0.02551 \; [W/mK]$$

$$\nu = 1.562 \cdot 10^{-5} \; [m^2/s]$$

$$Pr = 0.7296$$

$$\beta = \frac{1}{25.5+273} \; [K^{-1}] = 0.0034 \; [W/mK]$$

Which yields the following Grashof number

$$Gr = \frac{g\beta\left(T_5 - T_{amb} L_c^2\right)}{\nu^2}$$

And thus the following Rayleigh number:

$$Ra = GrPr$$

(0.5) properties at the correct avg. temperature

Characteristic length of the described situation:

$$L_c = \frac{A_s}{p} = \frac{0.15 \cdot 0.09}{2 \cdot 0.15 + 2 \cdot 0.09} \; [m] = 0.0281 \; [m]$$

(0.5) correct characteristic length

Rayleigh number:

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right) L_c^3}{\nu^2} \cdot Pr = \frac{9.81 \; [m/s^2] \cdot 0.0033 \; [K^{-1}] \cdot (42-15) \; [K] \cdot 0.0281^3 \; [m^3]}{\left(1.5942 \cdot 10^{-5}\right)^2 \; [m^4/s^2]} \cdot 0.7286 = 5.60 \cdot 10^4$$

(0.5) correct determination of Ra

Nusselt number:

$$Nu_L = 0.59 \cdot {Ra_L}^{1/4} = 0.59 \cdot \left(5.60 \cdot 10^4\right)^{1/4} = 9.076 \vee 8.3071 \text{ When 0.54 in stead of 0.59}$$

(0.5) correct determination of Nu

Heat transfer coefficient:

$$h = \frac{Nu_L \cdot k}{L_c} = \frac{9.076 \cdot 0.0258 \; [W/mK]}{0.0281 \; [m]} = 8.316 \vee 7.6112 \; [W/m^2K]$$

(0.5) correct determination of h with the correct properties

Rate of heat loss by convection:

$$\dot{Q}_{convection} = h \cdot A_s \cdot (T_s - T_\infty) = 8.316 \; [W/m^2K] \cdot (2 \cdot 0.0135) \; [m^2] \cdot (42-15) \; [K] = 3.03 \vee 2.77 \; [W]$$

(0.5) correct determination of $\dot{Q}_{convection}$

c) *Determine the conductive heat flux from the battery* $\dot{Q}_{battery,heat}$

Rate of heat transfer towards the backplate by incident radiation:

$$\dot{Q}_{incident} = \dot{q}_{incident} \cdot A = 79.6 \; [W/m^2] \cdot (0.15 \cdot 0.09) \; [m^2] = 1.075 \; [W]$$

(1) correct determination of $\dot{Q}$

Rate of heat transfer from the battery to the outside of the backplate:

$$\dot{Q}_{\text{battery,heat}} = \dot{Q}_{\text{convection}} - \dot{Q}_{\text{incident}} = 3.03\ [\text{W}] - 1.075\ [\text{W}] = 1.96 \vee 1.70\ [\text{W}]$$

(0.5) correct determination of $\dot{Q}_{\text{battery,heat}}$

d) *Find the rate at which the charging cable $\dot{E}_{cable}$ supplies energy to the smartphone.*
Finding relationship of the efficiency and the energy fluxes.

$$\eta_{battery} = \frac{\dot{E}_{battery,stored}}{\dot{E}_{cable}} = 0.95$$

And;

$$\dot{E}_{cable} = \dot{E}_{battery,stored} + \dot{Q}_{battery,heat}$$

(0.5) Finding relationship of $\dot{E}_{cable}$

So ;

$$\dot{E}_{battery,stored} = \dot{E}_{cable} - \dot{Q}_{battery,heat}$$

Substituting;

$$\eta_{battery} = \frac{\dot{E}_{cable} - \dot{Q}_{battery,heat}}{\dot{E}_{cable}} = 1 - \frac{\dot{Q}_{battery,heat}}{\dot{E}_{cable}}$$

Finally;

$$\dot{E}_{cable} = \frac{\dot{Q}_{battery,heat}}{1-\eta_{battery}} = \frac{1.96}{1-0.95} = 39.1 \vee 34.0[\text{W}]$$

(1) correct determination of $\dot{E}_{cable}$

e) *Determine the temperature of the battery. The thermal conductivity of the plastic backplate is 0.12 W m^{-1} K^{-1}. It may be assumed that the temperature of the battery is constant.* Thermal resistance of the backplate:

$$R = \frac{t}{k \cdot A_s} = \frac{0.002}{0.12 \cdot 0.0135} = 1.234[\text{K/W}]$$

Conduction of heat trough the backplate;

$$\dot{Q}_{battery,heat} = \frac{T_{battery} - T_s}{R}$$

Rewritten to $T_{battery}$;

$$T_{battery} = \dot{Q}_{battery,heat} \cdot R + T_s = 1.96 * 1.234 + 42 = 44.4 \vee 44.1^{o}\text{C}$$

(1) correct determination of $T_{battery}$

f) *Recalculate the convective heat transfer coefficient h for this vertical scenario and compare it with the horizontal case. Argue, in words or a short additional calculation, how the difference in coefficients influence the temperature of the backplate and the battery of the smartphone. Will it be higher, lower or remain the same? You may neglect the change in the properties of the air.*

The Rayleigh number can be calculated with or without the $(T_s - T_{infty})$ term neglected. Neglecting this term will not influence the qualitative answer. It is most important to see that the characteristic length will change.

$$L_c = L = 0.15[m]$$

Then the the Rayleigh number is;

$$\text{Ra}_\text{L} = \frac{g\beta L_\text{c}^3}{\nu^2} \cdot \text{Pr} \cdot (T_\text{s} - T_\infty) = \frac{9.81\ [\text{m/s}^2] \cdot 0.0033\ [\text{K}^{-1}] \cdot 0.15^3\ [\text{m}^3]}{\left(1.5942 \cdot 10^{-5}\right)^2\ [\text{m}^4/\text{s}^2]} \cdot 0.7286\,(T_\text{s} - T_\infty)\,[\text{K}] = 4.32 \cdot 10^5\,(T_\text{s} - T_\infty)$$

Or when change in T_s is neglected in Grashof;

$$\text{Ra}_\text{L} = 1.16 \cdot 10^7$$

(1) correct determination of Ra_L

In both cases it is clear that the Nusselt relationship remains the same, depending on relationship used before;

$$Nu = 0.59 * Ra_L^{1/4}$$

(1) correct determination of Nu. Note: it now really is 0.59

Giving;

$$h = \frac{Nu \cdot k}{L_c} = \frac{15.13 \cdot (T_s - T_{infty})^{1/4} \cdot 0.258}{0.15} = 2.40 \cdot (T_s - T_{infty})^{1/4} \vee h = 5.47[\text{W/m}^2\text{K}]$$

(1) correct determination of h

There are multiple ways to show that the convective heat transfer coefficient has decreased. The conclusion should be that the temperature of the device should have increased to still have the same energy flow.

(1) Right conclusion that the device has an increased temperature

Q5

You have been hired as a junior food processing engineer at a leading fruit processing company. Your first task is to analyze and optimize the drying process of mango slices to produce high-quality dried mangoes, a favorite snack enjoyed by millions around the world.

The food processing company wants to produce dried mango slices using the semicylindrical dryer. The dryer's long semicylindrical shape allows for a continuous and efficient drying process.

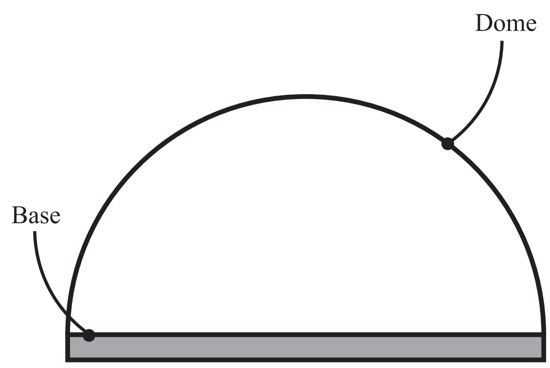


Figure 4: Cross section of the mango dryier

Given parameters:

- The dryer is a long, curved duct with a diameter of 1.5 meters.
- The base temperature is 370 Kelvin.
- The base emissivity is 0.5.
- The base acts as an opaque body.
- The dome temperature is 1000 Kelvin.
- The dome emissivity is 0.8.
- The dome does not reflect any radiation.
- The latent heat of vaporization for water is 2.3 MJ/kg
- $F_{\mathrm{B}\to\mathrm{B}} = 0$
- $F_{\mathrm{B}\to\mathrm{D}} = 1$
- $F_{\mathrm{D}\to\mathrm{B}} = \frac{2}{\pi}$
- $F_{\mathrm{D}\to\mathrm{D}} = 1 - \frac{2}{\pi}$

In this case study, we explore the process of drying organic materials, focusing on the production of dried mango slices. We will examine the application of a semicylindrical dryer designed to efficiently remove moisture from water-soaked mango slices while preserving their natural flavors and nutrients.

a Give the values of the emissivity, transmissivity, and reflectivity for the base and dome.

From now on, it can be assumed that all bodies act as black bodies. The temperatures and other parameters remain the same.

b Determine the wavelength that holds the maximum power coming off of the dome.

c Determine the net rate of heat transfer per unit length from the dome to the base.

d Determine the drying rate per unit length experienced by the wet mango slices as they pass through the semicylindrical dryer.

e Reflect on your given answer in d). Is it realistic? If not, what is the implication caused by the assumptions on the mango slices, and what about the oven itself?

f Your boss would like you to improve the drying rate by improving the design. Mention one **design improvement** and explain why this improves the drying rate.

Solution Q5

a Give the values of the emissivity, transmissivity, and reflectivity for the base and dome.

For the **base**:

It is given that:

$$\epsilon_{\mathrm{B}} = 0.5$$

Also, it is stated that the base is opaque, so:

$$\tau_{\mathrm{B}} = 0$$

(0.5) Correct reasoning of property

Using Kirchoff's law it yields that:

$$\epsilon_{\mathrm{B}} + \tau_{\mathrm{B}} + \rho_{\mathrm{B}} = 1$$

(0.5) Correct calculation of property

$$\rightarrow \rho_{\mathrm{B}} = 0.5$$

For the **dome**:

It is given that:

$$\epsilon_{\mathrm{D}} = 0.8$$

Also, it is stated that the dome does not reflect, so:

$$\rho_{\mathrm{D}} = 0$$

(0.5) Correct reasoning of property

Using Kirchoff's law it yields that:

$$\epsilon_{\mathrm{D}} + \tau_{\mathrm{D}} + \rho_{\mathrm{D}} = 1$$

$$\rightarrow \tau_{\mathrm{D}} = 0.2$$

(0.5) Correct calculation of property

b Determine the wavelength that holds the maximum power coming off of the dome.
Wien's displacement law calculates the wavelength with the maximum power as:

$$\lambda_{\mathrm{max,power}} = \frac{2897.8}{T_{\mathrm{dome}}} = \frac{2897.8}{1000} = 2.898\ \mu\mathrm{m}$$

(1) Correct calculation of wavelength

c Determine the net rate of heat transfer per unit length from the dome to the base.
The net rate of heat transfer between two black bodies can be calculated according to the expression:

$$\dot{Q}_{D\to B} = A_{\mathrm{D}} F_{\mathrm{D}\to\mathrm{B}} \sigma \left(T_{\mathrm{D}}^4 - T_{\mathrm{B}}^4\right) = \frac{\pi D}{2} \cdot \frac{2}{\pi} \cdot \left(5.67 \cdot 10^{-8}\right)\left(1000^4 - 370^4\right) = 83.6\ \mathrm{kW/m}$$

(1) Correct calculation of rate of heat transfer

d Determine the drying rate per unit length experienced by the wet mango slices as they pass through the semicylindrical dryer.
From an energy balance, it yields that:

$$\dot{Q}_{\mathrm{evaporation}} = \dot{Q}_{D\to B}$$

(1) Correct energy balance

Where:

$$\dot{Q}_{\mathrm{evaporation}} = \dot{m} h_{\mathrm{fg}}$$

(0.5) The correct definition of rate of heat transfer

Rewriting yields:

$$\dot{m} = \frac{\dot{Q}_{D\to B}}{h_{\mathrm{fg}}} = \frac{83.6 \cdot 10^3}{2.3 \cdot 10^6} = 0.0363\ \mathrm{kg/s \cdot m}$$

(0.5) Correct calculation of drying rate e Reflect on your given answer in d). Is it realistic? If not, what is the implication caused by the assumptions on the mango slices and what about the oven itself?

Also, the mango slices will heat up a bit as well, so not all heat is directly used for the evaporation of water. So it will turn out to have a slightly smaller evaporation rate.

(0.5) Correct reflection on assumed energy balance

The assumption of having only black bodies can be made when we are dealing with a perfect absorber, having no reflectivity, having a uniform temperature distribution, having a radiation-dominant scenario, having steady-state conditions, and having high temperatures. Reflection on one of these conditions and why they might not be a perfect approximation can be counted to be correct.

(0.5) Correct reflection on assuming black bodies

f Your boss would like you to improve the drying rate by improving the design. Mention one **design improvement** and explain why this improves the drying rate.

Any design improvement that will increase the rate of heat transfer from the dome to the base can be counted to be correct.

(1) Correct design improvement

Q6

During the summer, often steaks and potatoes are cooked on the BBQ. Before these can be enjoyed, they have to cool down to a comfortable temperature.

(a) Steak on a BBQ

(b) Potato

1. Based on which criteria can the lumped capacity model be applied?
2. For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why. And why will it give more accurate results for one than the other?

A steak ($k = 4$ W/mK, $\rho = 1006$ kg/m^3, $c_p = 2850$ J/kgK) with a diameter of 8 cm, which initially has a surface temperature of 70 ^{o}C, is located outside with an ambient temperature of 20 ^{o}C. It cools down due to convection ($h = 35$ W/m^{2}K).

3. Using the lumped capacity model, determine the time that it takes for the steak to cool down to 40 ^{o}C.
4. Evaluate the accuracy of the found answer in c).
5. Determine the amount of energy that the steak has lost, when cooled down from 70 ^{o}C to 40 ^{o}C.
6. Determine the maximum diameter of the potato, for which the lumped capacity model is still valid.
7. Provide a sketch of the temperature profile as a function of time, in the case that we would have let the potato cool down in the room for a very long time.

Note: clearly indicate the temperatures for t=0 and t→ ∞

Solution Q6

1. *Based on which criteria can the lumped capacity model be applied?*

$$\text{Bi} \leq 0.1 \text{ or } \text{Bi} \rightarrow 0$$

(1) correct criteria

2. *For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why. And why will it give more accurate results for one than the other?* We know:

$$\text{Bi} = \frac{hL_c}{k}$$

Simplifying both objects to spheres, L_c will be bigger for the steak. A steak and a potato have roughly similar thermal conductivity. Therefore it can be assumed that the lumped capacity model is more applicable for the potato, as the characteristic length is smaller. Besides, a potato has a more uniform temperature distribution as its properties are more constant throughout its body. A chicken contains parts such as bones and fat, where other properties for thermal conductivity will be found.

(1) correct explanation

3. *Using the lumped capacity model, determine the time that it takes for the potato to cool down to 50 °C.*

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p}t}$$

$$\rightarrow t = \frac{\rho V c_p}{hA_s} \ln\left(\frac{T_i - T_\infty}{T(t) - T_\infty}\right)$$

(0.5) correct definition for t

Where

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 0.03^3 \ \left[\text{m}^3\right] = 6.544 \cdot 10^{-5} \ \left[\text{m}^3\right]$$

$$A_s = 4\pi r^2 = 4\pi 0.03^2 \ \left[\text{m}^3\right] = 0.0079 \ \left[\text{m}^2\right]$$

(0.5) correct definitions for V and A_s

So:

$$\rightarrow t = \frac{1006 \ \left[\text{kg/m}^3\right] \cdot 6.544 \cdot 10^{-5} \ \left[\text{m}^3\right] \cdot 2850 \ \left[\text{J/kgK}\right]}{35 \ \left[\text{W/m}^2\text{K}\right] \cdot 0.0079 \ \left[\text{m}^2\right]} \ln\left(\frac{70-20}{40-20}\right) = 346.63 \ [\text{s}] = 5.78 \ [\text{min}]$$

(0.5) correct final answer for t

4. *Evaluate the accuracy of the found answer in c).*

$$\text{Bi} = \frac{hL_c}{k} = \frac{hV}{kA_s} = \frac{35 \ \left[\text{W/m}^2\text{K}\right] \cdot 6.544 \cdot 10^{-5} \ \left[\text{m}^3\right]}{4 \ [\text{W/mK}] \cdot 0.0079 \ \left[\text{m}^2\right]} = 0.0725 < 0.1$$

(0.5) correct evaluation

Therefore the lumped capacity model provides a high accuracy.

5. *Determine the amount of energy that the potato has lost, when cooled down from 90 °C to 50 °C.* Heat lost:

$$Q = V\rho c_p (T_i - T(t)) = 6.544 \cdot 10^{-5} \ \left[\text{m}^3\right] \cdot 1006 \ \left[\text{kg/m}^3\right] \cdot 2850 \ \left[\text{J/kgK}\right] (70-40) \ [\text{K}] = 5.629 \cdot 10^{30} \ [\text{J}]$$

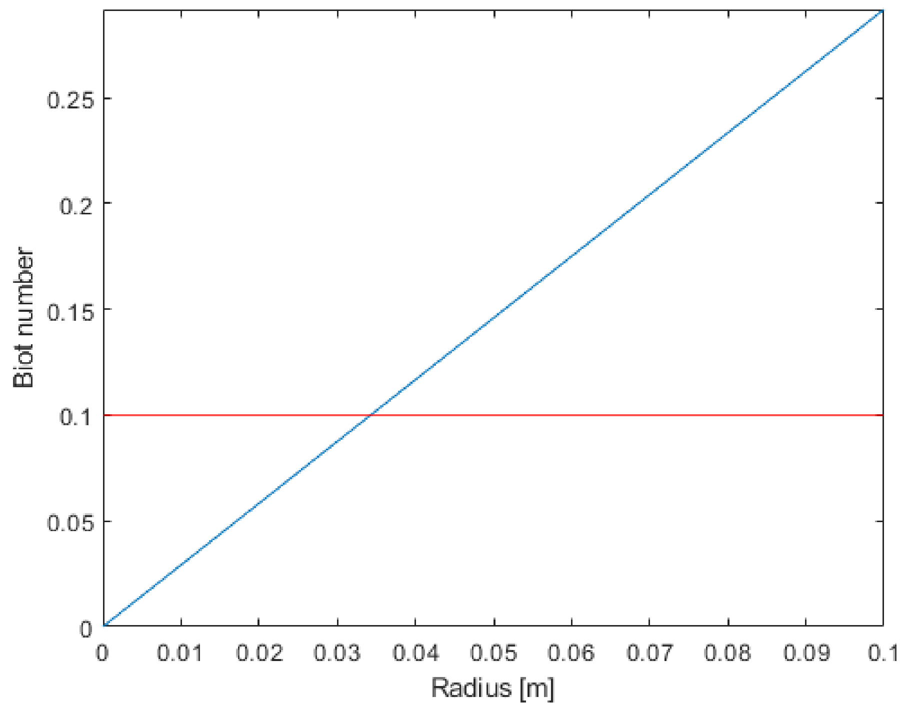
(1) correct determination of Q

6. *Determine the maximum diameter of the potato, for which the lumped capacity model is still valid.*

$$\text{Bi}(r) = \frac{hr}{3k}$$

(1) correct definition for Bi(r)

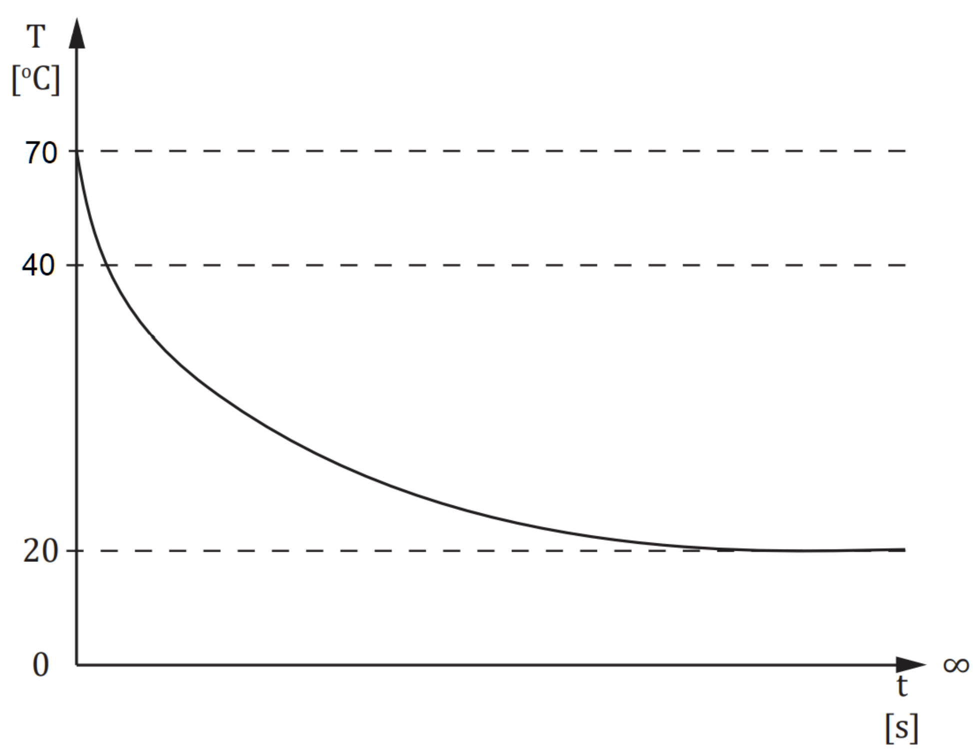
Plotting:



Intersection is at r = 3.4 cm (D=6.8 cm)

(1) correct point of intersection

7. *Provide a sketch of the temperature profile as a function of time, in the case that we would have let the potato cool down in the room for a very long time.*



- At t=0 T = 70 ^{o}C.
- T = 20 ^{o}C should be clearly defined.
- For t→ ∞ the temperature should approach the ambient temperature with a zero gradient slope.

(2) for all correct criteria, (-1) per missing criteria, (0) as minimum

Q7

One way of preparing an chicken egg is boiling it. Before these can be enjoyed, they have to cool down to a comfortable temperature.

(a) Boiled eggs

(b) Potato

1. Based on which criteria can the lumped capacity model be applied?
2. For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why you come to this conclusion. Are there differences in properties?

A egg (k = 3 W/mK, ρ = 1150 kg/m^3, c_p = 3600 J/kgK) with a diameter of 3 cm, which initially has a surface temperature of 95 $^\mathrm{o}$C, is located outside with an ambient temperature of 20 $^\mathrm{o}$C. It cools down due to convection (h = 40 W/m^{2}K).

3. Using the lumped capacity model, determine the time that it takes for the egg to cool down to 35 $^\mathrm{o}$C.
4. Evaluate the accuracy of the found answer in c).
5. Determine the amount of energy that the egg has lost, when cooled down from 95 $^\mathrm{o}$C to 35 $^\mathrm{o}$C.
6. Determine the maximum diameter of the egg, for which the lumped capacity model is still valid.
7. Provide a sketch of the temperature profile as a function of time, in the case that we would have let the egg cool down in the room for a very long time.

Note: clearly indicate the temperatures for t=0 and t$\rightarrow\infty$

Solution Q7

1. *Based on which criteria can the lumped capacity model be applied?*

$$\text{Bi} \leq 0.1 \text{ or } \text{Bi} \rightarrow 0$$

(1) correct criteria

2. *For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why you come to this conclusion. Are there differences in properties?* We know:

$$\text{Bi} = \frac{hL_c}{k}$$

Simplifying both objects to spheres, L_c will be bigger for the potato. A egg and a potato have roughly similar thermal conductivity as can be seen on `https://www.engineeringtoolbox.com/food-thermal-conductivity-d_2177.html`. Therefore it may be assumed that the lumped capacity model is more applicable for the egg, as the characteristic length is smaller. However, a potato has a more uniform temperature distribution as its properties are more constant throughout its body. A egg contains parts such the shell, egg white and yolk, where other properties for thermal conductivity will be found. Therefore, it might also be argued that the lumped model would be more suitable for the potato.

(1) correct explanation

3. *Using the lumped capacity model, determine the time that it takes for the egg to cool down to 35 ^{o}C*

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t}$$

$$\rightarrow t = \frac{\rho V c_p}{hA_s} \ln\left(\frac{T_i - T_\infty}{T(t) - T_\infty}\right)$$

(0.5) correct definition for t

Where

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 0.015^3 \ \left[\text{m}^3\right] = 1.414 \cdot 10^{-5} \ \left[\text{m}^3\right]$$

$$A_s = 4\pi r^2 = 4\pi 0.015^2 \ \left[\text{m}^3\right] = 0.0023 \ \left[\text{m}^2\right]$$

(0.5) correct definitions for V and A_s

So:

$$\rightarrow t = \frac{1150 \ \left[\text{kg/m}^3\right] \cdot 1.414 \cdot 10^{-5} \ \left[\text{m}^3\right] \cdot 3600 \ \left[\text{J/kgK}\right]}{40 \ \left[\text{W/m}^2\text{K}\right] \cdot 0.0023 \ \left[\text{m}^2\right]} \ln\left(\frac{95-20}{35-20}\right) = 797.1 \ [\text{s}] = 13.3 \ [\text{min}]$$

(0.5) correct final answer for t

4. *Evaluate the accuracy of the found answer in c).*

$$\text{Bi} = \frac{hL_c}{k} = \frac{hV}{kA_s} = \frac{40 \ \left[\text{W/m}^2\text{K}\right] \cdot 1.414 \cdot 10^{-5} \ \left[\text{m}^3\right]}{3 \ [\text{W/mK}] \cdot 0.0023 \ \left[\text{m}^2\right]} = 0.0820 < 0.1$$

(0.5) correct evaluation

Therefore the lumped capacity model provides a high accuracy.

5. *Determine the amount of energy that the egg has lost, when cooled down from 95 ^{o}C to 35 ^{o}C.* Heat lost:

$$Q = V\rho c_p (T_i - T(t)) = 1.414 \cdot 10^{-5} \ \left[\text{m}^3\right] \cdot 1150 \ \left[\text{kg/m}^3\right] \cdot 3600 \ \left[\text{J/kgK}\right] (95-35) \ [\text{K}] = 3512 \ [\text{J}]$$

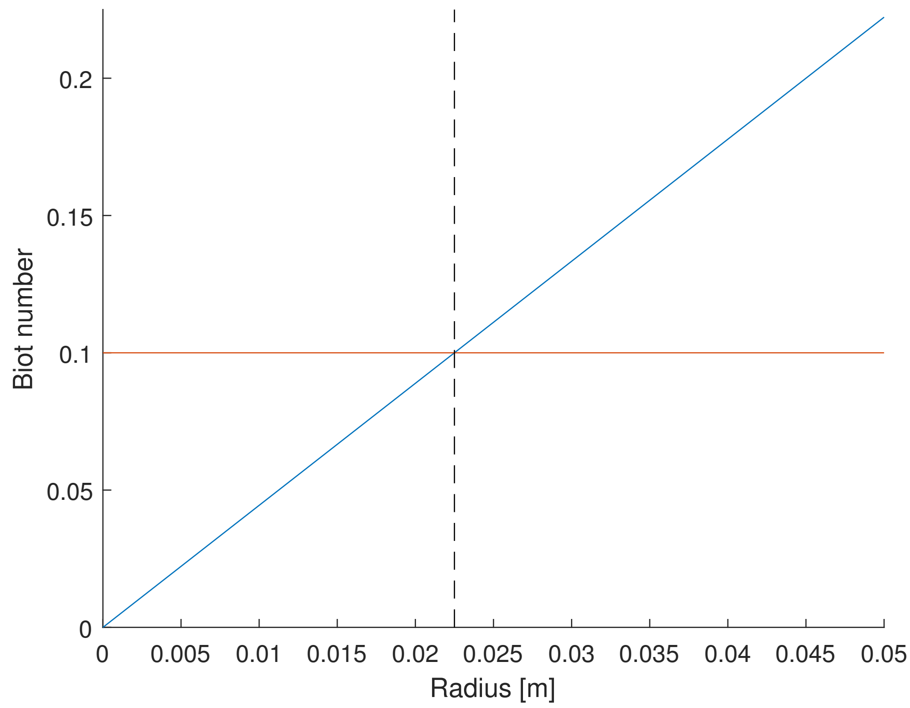
(1) correct determination of Q

6. *Determine the maximum diameter of the egg, for which the lumped capacity model is still valid.*

$$\text{Bi}(r) = \frac{hr}{3k}$$

(1) correct definition for Bi(r)

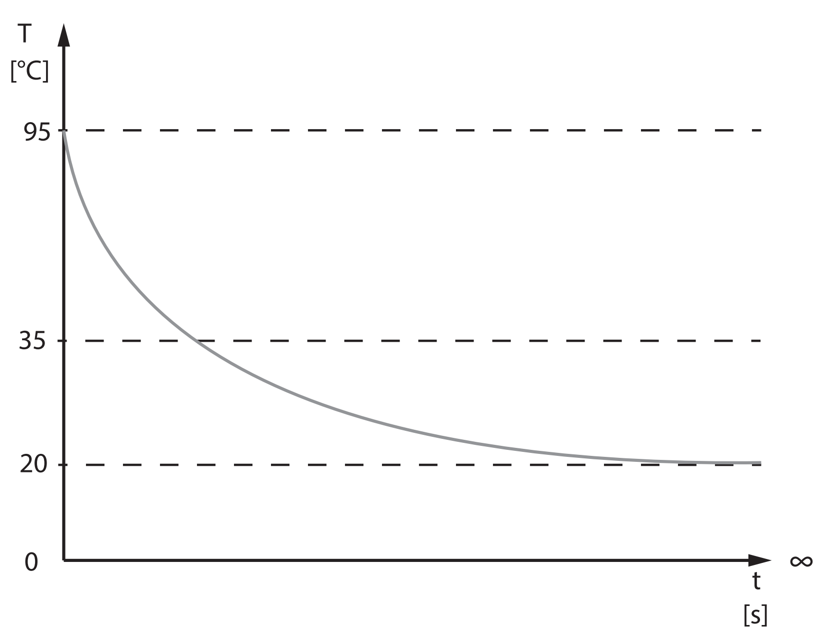
Plotting:



Intersection is at r = 2.25 cm (D=4.5 cm)

(1) correct point of intersection

7. *Provide a sketch of the temperature profile as a function of time, in the case that we would have let the egg cool down in the room for a very long time.*



- At t=0 T = 95 ^{o}C.
- T = 20 ^{o}C should be clearly defined.
- For t$\rightarrow \infty$ the temperature should approach the ambient temperature with a zero gradient slope.

(2) for all correct criteria, (-1) per missing criteria, (0) as minimum