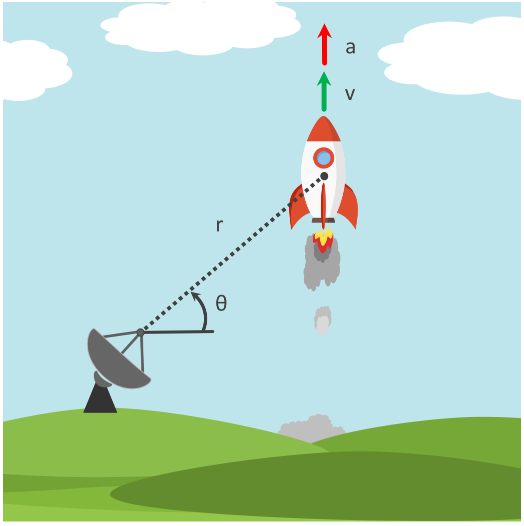
Rocket Tracking in Polar Coordinates



A rocket is fired vertically and tracked by a radar station. For the instant when $\theta = 60°$, the tracking data gives $r = 5000$ m, $\ddot{r} = 15$ m/s^2, and $\dot{\theta} = 0.02$ rad/s. Calculate the magnitude of the velocity and acceleration of the rocket at this position.

Hint: Use $v_\theta = r\dot{\theta}$ and $a_r = \ddot{r} - r\dot{\theta}^2$

Given quantities:

Distance: $r = 5000$ m
Acceleration: $\ddot{r} = 15$ m/s^2
Angle: $\theta = 60°$
Rate of change of the angle: $\dot{\theta} = 0.02$ rad/s

Solution:

Using the given expressions for v_θ and a_r, both can be calculated:

$$v_\theta = r\dot{\theta} = 5000 \cdot 0.02 = 100 \text{ m/s} \tag{1}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 15 - 5000 \cdot 0.02^2 = 13 \text{ m/s}^2 \tag{2}$$

Both v_θ and a_r can be drawn into the figure, this results in Figure 1. Using basic trigonometry for a right triangle, the velocity and acceleration can be expressed in terms of v_θ, a_r and θ.

$$v = \frac{v_\theta}{\cos\theta} = \frac{100}{\cos(60°)} = 200 \text{ m/s} \tag{3}$$

$$a = \frac{a_r}{\sin\theta} = \frac{13}{\sin(60°)} \approx 15.01 \text{ m/s}^2 \approx 15 \text{ m/s}^2 \tag{4}$$

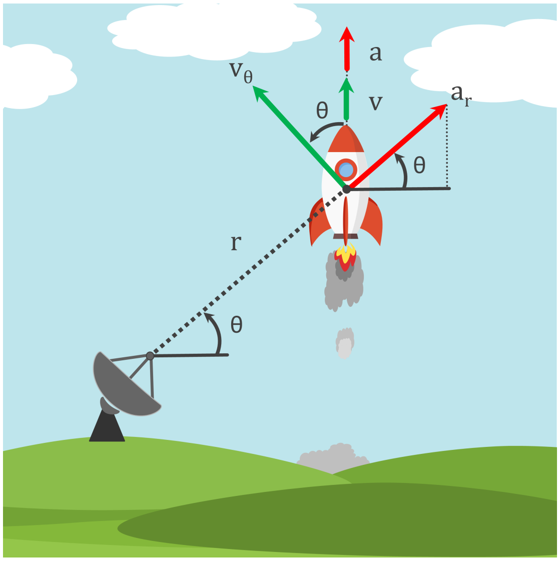


Figure 1: Rocket Tracking in Polar Coordinates, parallelograms.