Task 5 One-cylinder reciprocating engine for small motorcycle

Solution

Due to high rpm of the crankshaft of a small motorcycle it is required to balance inertial forces in the design of a one-cylinder reciprocating engine.

Derivation of the dynamically equivalent system of the connecting rod:

$$\begin{aligned} m_{21} &= m_2 \cdot \left(1 - \frac{l_{S2}}{l_2}\right) = 210\ g \\ m_{23} &= m_2 \cdot \frac{l_{S2}}{l_2} = 140\ g \\ Q_1 &= m_2 \cdot \left(1 - \frac{l_{S2}}{l_2}\right) + m_1 \cdot \frac{l_{S1}}{l_1} = 450\ g \\ Q_3 &= m_2 \cdot \frac{l_{S2}}{l_2} + m_3 = 320\ g \end{aligned} \tag{5.1}$$

Mass forces on the frame:

$$\begin{aligned} F_x = l_1 \cdot \omega_1^2 \cdot (Q_1 + Q_3) \cdot cos(\varphi) + l_1 \cdot \omega_1^2 \cdot Q_3 \\ \cdot (A_2 \cdot cos(2\varphi) - A_4 \cdot cos(4\varphi) \pm \cdots) \end{aligned} \tag{5.2}$$

$$F_y = l_1 \cdot \omega_1^2 \cdot Q_1 \cdot sin(\varphi) \tag{5.3}$$

Based on the polar diagrams of the first harmonic, compare following possibilities:

a) Balance of the first harmonic of the longitudinal force.

Inertial forces of the first harmonics:

$$F_{x1} = l_1 \cdot \omega_1^2 \cdot (Q_1 + Q_3) \cdot cos(\varphi) \tag{5.4}$$

$$F_{y1} = l_1 \cdot \omega_1^2 \cdot Q_1 \cdot sin(\varphi) \tag{5.5}$$

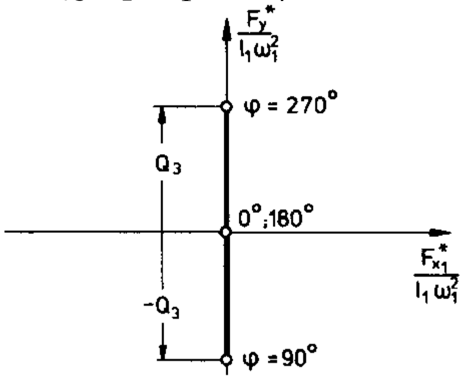
The first harmonic of the longitudinal force should be completely balanced by adding a balancing mass or counter weight to the crank cheek. Thus, the resultant first harmonic of the longitudinal force is zero:

$$\begin{aligned} F_{x1,res} &= F_{x1} + F_{a,x} \\ &= l_1 \cdot \omega_1^2 \cdot (Q_1 + Q_3) \cdot cos(\varphi) - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \cdot cos(\varphi) \\ &= 0 \end{aligned} \tag{5.6}$$

$$\Leftrightarrow m_{a1} = \frac{l_1}{l_{a1}} \cdot (Q_1 + Q_3) = 962{,}5\ g \tag{5.7}$$

For the transversal force it follows:

$$
\begin{aligned}
F_{y,res} = F_{y1} - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \cdot sin(\varphi) \\
&= l_1 \cdot \omega_1^2 \cdot Q_1 \cdot sin(\varphi) - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \cdot sin(\varphi) \\
&= \omega_1^2 \cdot sin(\varphi) \cdot (Q_1 \cdot l_1 - m_{a1} \cdot l_{a1}) \\
&= \omega_1^2 \cdot sin(\varphi) \cdot (Q_1 \cdot l_1 - Q_1 \cdot l_1 - Q_3 \cdot l_1) \\
&= -Q_3 \cdot l_1 \cdot \omega_1^2 \cdot sin(\varphi)
\end{aligned}
\tag{5.8}
$$



b) Balance of the first harmonic of the transversal force.

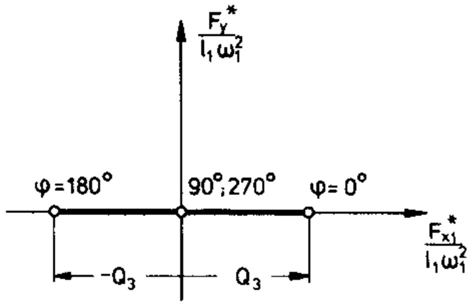
Since the transversal force solely consists of first harmonics terms, the transversal direction can be cancelled completely. Again a single counter weight is used:

$$F_{y,res} = l_1 \cdot \omega_1^2 \cdot Q_1 \cdot sin(\varphi) - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \cdot sin(\varphi) = 0 \tag{5.9}$$

$$\Leftrightarrow m_{a1} = \frac{l_1}{l_{a1}} \cdot Q_1 = 562{,}5\ g \tag{5.10}$$

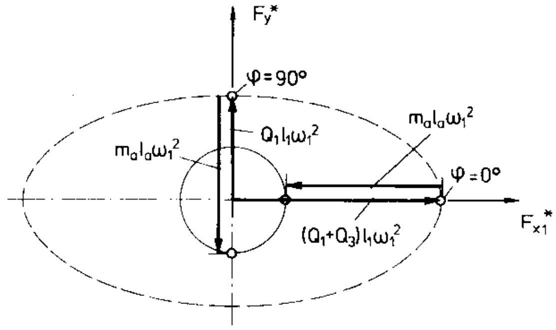
For the first harmonic of the longitudinal force it follows:

$$
\begin{aligned}
F_{x1,res} = l_1 \cdot \omega_1^2 \cdot (Q_1 + Q_3) \cdot cos(\varphi) - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \cdot cos(\varphi) \\
&= \omega_1^2 \cdot cos(\varphi) \cdot (Q_1 \cdot l_1 - Q_1 \cdot l_1 + Q_3 \cdot l_1) \\
&= Q_3 \cdot l_1 \cdot \omega_1^2 \cdot cos(\varphi)
\end{aligned}
\tag{5.11}
$$



c) Mean balance of the first longitudinal and transversal force harmonic.

The first longitudinal and transversal force harmonic should be balanced, resulting in the same magnitudes of the first harmonics independent from the crank angle:



For the mass forces it follows:

$$F_{x1,res} = l_1 \cdot \omega_1^2 \cdot (Q_1 + Q_3) \cdot cos(\varphi) - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \cdot cos(\varphi) \tag{5.12}$$

$$F_{y1,res} = l_1 \cdot \omega_1^2 \cdot Q_1 \cdot sin(\varphi) - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \cdot sin(\varphi) \tag{5.13}$$

According to the polar diagram, the mass force in x-direction for $\varphi = 0°$ is identical to the mass force in y-direction for $\varphi = 90°$. With

$$cos(\varphi = 0°) = 1 \tag{5.14}$$

$$sin(\varphi = 90°) = 1 \tag{5.15}$$

it follows:

$$F_{x1,res}(\varphi = 0°) = l_1 \cdot \omega_1^2 \cdot (Q_1 + Q_3) - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \tag{5.16}$$

$$F_{y1,res}(\varphi = 90°) = l_1 \cdot \omega_1^2 \cdot Q_1 - m_{a1} \cdot l_{a1} \cdot \omega_1^2 \tag{5.17}$$

From

$$|F_{x1,res}| = |F_{y1,res}| \tag{5.18}$$

two cases can be considered:

$$F_{x1,res} = F_{y1,res} \text{ und } F_{x1,res} = -F_{y1,res} \tag{5.19}$$

Case 1:

$$F_{x1,res} = F_{y1,res} \tag{5.20}$$

$$\Leftrightarrow Q_3 = 0 \tag{5.21}$$

This case is not feasible due to design reasons, i.e. the masses of the connecting rod and the piston cannot be neglected.

Case 2:

$$F_{x1,res} = -F_{y1,res} \tag{5.22}$$

$$\Leftrightarrow l_1 \cdot (Q_1 + Q_3) - m_{a1} \cdot l_{a1} = -(l_1 \cdot Q_1 - m_{a1} \cdot l_{a1}) \tag{5.23}$$

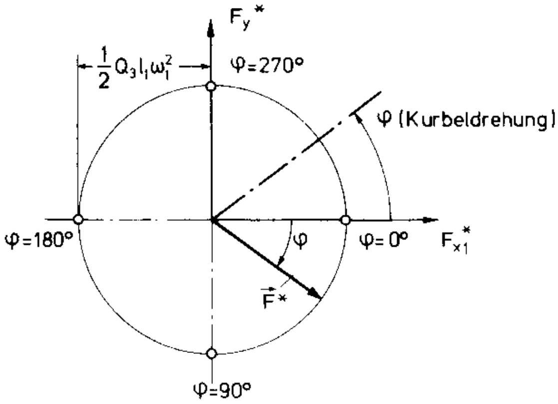
$$\Leftrightarrow m_{a1} = \frac{l_1}{l_{a1}} \cdot \left(Q_1 + \frac{1}{2} \cdot Q_3\right) = 762{,}5\ g \tag{5.24}$$

Thus, for the mass forces of the first harmonic it follows:

$$F_{x1,res} = \frac{1}{2} \cdot Q_3 \cdot l_1 \cdot \omega_1^2 \cdot cos(\varphi) \tag{5.25}$$

$$F_{y,res} = -\frac{1}{2} \cdot Q_3 \cdot l_1 \cdot \omega_1^2 \cdot sin(\varphi) \tag{5.26}$$

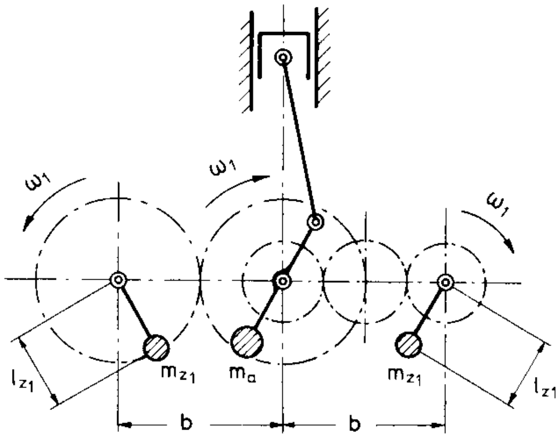
This means, the resultant mass force rotates into the opposite direction of the crank with identical speed. The following polar diagram illustrates the relations:



Comparison of possibilities a, b and c:

The first harmonic cannot be balanced completely by adding a single mass to thr crank. This can only be achieved by additional rotating balancing masses.

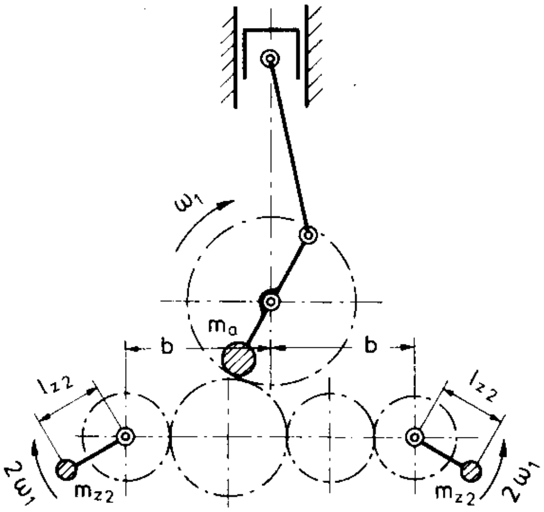
1st possibility:
The resultant first harmonic of the longitudinal force from b) can be canceled completely as shown in the following figure:



For the additional masses it holds:

$$m_{z1} = \frac{1}{2} \cdot Q_3 \cdot \frac{l_1}{l_{z1}} \tag{5.27}$$

2nd possibility:
The remaining longitudinal force of the second order from b) can be completely balanced as shown in the following figure:

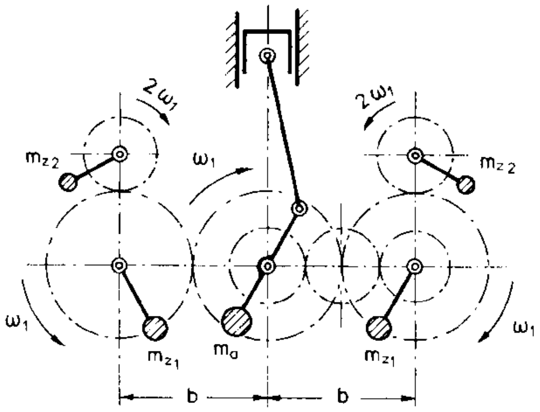


The additional masses m_{z2} need to rotate at double speed compared to the crank, i.e. $2\omega_1$, and in opposite direction to each other. The masses are:

$$m_{z2} = \frac{1}{8} \cdot Q_3 \cdot A_2 \cdot \frac{l_1}{l_{z2}} \tag{5.28}$$

3rd possibility:

Combining possibilities 1 and 2, the longitudinal and transversal forces of both first and second order can be balanced completely:

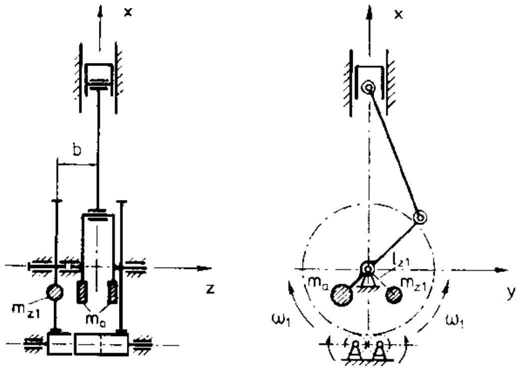


4th possibility:

The resultant force of the longitudinal and transversal force of the first harmonic of case c) can be completely balanced by a single additional mass

$$m_{z1} = \frac{1}{2} \cdot Q_3 \cdot \frac{l_1}{l_{z1}} \tag{5.29}$$

rotating in opposite direction at the same speed of the crank.



d) Complete balance of all harmonics by attaching balancing masses.

Now, additional masses can be attached to the connecting rod as well as to the crank. The distances are given in the task sheet.
A complete balancing of the mass forces can only be accomplished if the joint center of gravity of the connecting rod and the piston lies in A.

Therefore, the following mass is required:

$$m_{a2} \cdot l_{a2} = m_2 \cdot l_{S2} + m_3 \cdot l_2 \tag{5.30}$$

$$\Leftrightarrow m_{a2} = \frac{m_2 \cdot l_{S2} + m_3 \cdot l_2}{l_{a2}} = 800\ g \tag{5.31}$$

Moreover, the balancing mass at the crank helps to shift the center of gravity of the overall system into joint A_0:

$$m_{a1} \cdot l_{a1} = m_1 \cdot l_{S1} + (m_{a2} + m_2 + m_3) \cdot l_1 \tag{5.32}$$

$$\Leftrightarrow m_{a1} = \frac{m_1 \cdot l_{S1} + (m_{a2} + m_2 + m_3) \cdot l_1}{l_{a1}} = 1962{,}5\ g \tag{5.33}$$

In doing so, for the complete balancing an additional mass of 2762,5 g is required. Furthermore, a greater installation space is needed.