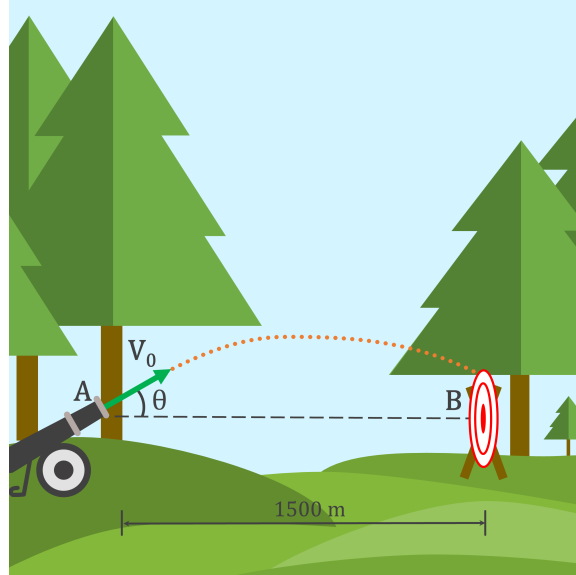


## Bullet on Target



A cannon fires a bullet from A toward a target B. Find an expression for the time  $t_{\text{end}}$  it takes for the bullet to reach the target, in terms of  $d$ ,  $v_0$  and  $\theta$ .

The target diameter is 2 m and the target centre is at the same altitude as the end of the cannon barrel. The bullet velocity at the end of the barrel 900 m/s, the distance between A and B is  $d = 1500$  m. Neglect all air resistances and assume that the bullet is directed along the vertical centreline of the target. Take  $g = 10 \text{ m/s}^2$

*Using known expressions (for constant acceleration):*

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \quad (1)$$

$$\int_{v_0}^{v(t)} dv = a \int_0^t dt \quad (2)$$

$$v(t) = at + v_0 \quad (3)$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt = (v_0 + at) dt \quad (4)$$

$$\int_{s_0}^{s(t)} ds = \int_0^t (v_0 + at) dt \quad (5)$$

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0 \quad (6)$$

*Given quantities:*

Distance A-B:  $d = 1500$  m

Gravitational acceleration:  $g = 10$  m/s<sup>2</sup>

Initial velocity:  $v_0 = 900$  m/s

Target diameter:  $D = 2$  m

*Solution:*

Filling in Equation (6) gives an relation for the  $x$ -position with respect to time. Where  $a = 0$  m/s<sup>2</sup> and  $x_0 = 0$  m, since there is no acceleration in the  $x$ -direction and the coordinate system is chosen at the end of the cannon barrel.

$$x(t_{\text{end}}) = v_{0,x}t_{\text{end}} = d \quad \Rightarrow \quad v_0t_{\text{end}} \cos \theta = d \quad (7)$$

Rewriting gives a relation for the time  $t_{\text{end}}$  with respect to  $\theta$  and  $v_0$ .

$$v_0t_{\text{end}} \cos \theta = d \quad \Rightarrow \quad t_{\text{end}} = \frac{s_{\text{AB}}}{v_0 \cos \theta} \quad (8)$$