

Energy & Heat Transfer



Lecture 2

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RECAP OF LECTURE 1

- Work (W), energy (E), power (P)
- Work (W), Energy (E) in $J = N \cdot m = \frac{kg \cdot m}{s^2} \cdot m$
- Power : $P = \frac{\Delta E}{\Delta t}$ in Watt (W) = J / s
- Units kWh, kcal, hp,




RECAP OF LECTURE 1

- Potential energy : $E = F \cdot h$
 - Thermal energy : $Q = m \cdot c \cdot \Delta T$
 - c is specific heat capacity in J / (kg · K)
 - Electrical power: $P = U \cdot I$
 - Electrical energy: $E = P \cdot t = U \cdot I \cdot t$
 - Ohm's law: $U = I \cdot R$
- $$\left. \begin{array}{l} P = I^2 \cdot R \\ E = I^2 \cdot R \cdot t \end{array} \right\}$$




LEARNING OBJECTIVES LECTURE 2



Energy in general

-  Using an energy balance
-  Determine efficiency
-  Calculate losses for electric resistors

Conductive heat transfer

-  *Explaining* conduction principles
-  *Calculate* conductive heat transfer
-  Using *thermal resistances* and insulation values

LEARNING OBJECTIVES LECTURE 2

● Energy in general

● Using an energy balance

○ Determine efficiency

○ Calculate losses for electric resistors

○ Conductive heat transfer

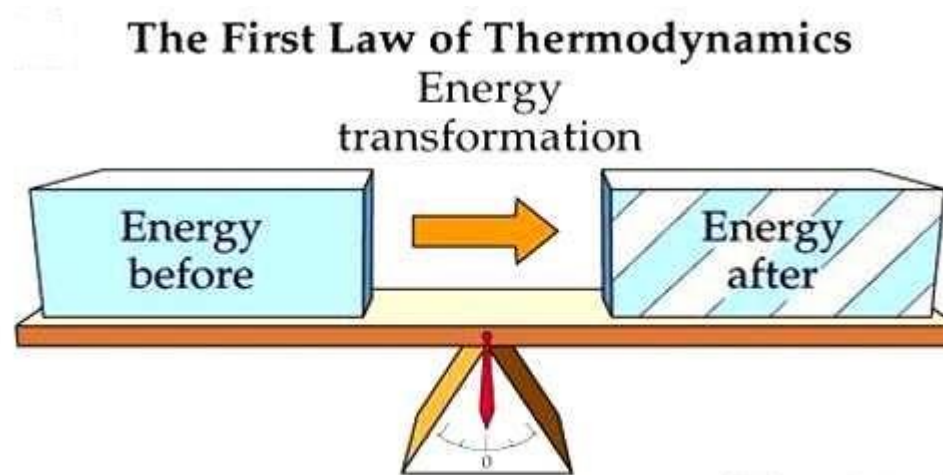
○ *Explaining* conduction principles

○ *Calculate* conductive heat transfer

○ Using *thermal resistances* and insulation values

ENERGY BALANCE

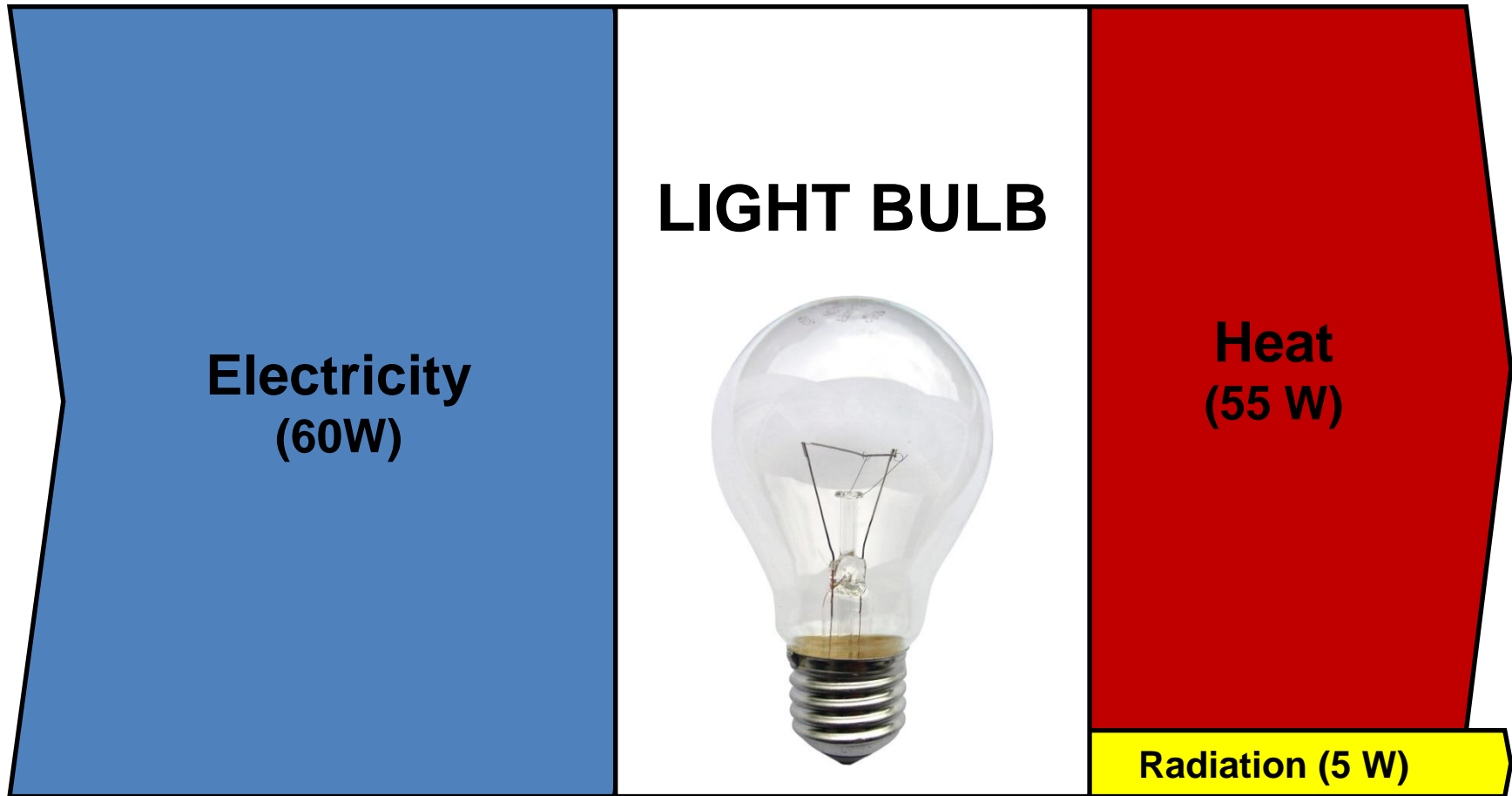
- Energy is always conserved!
- First law of Thermodynamics:



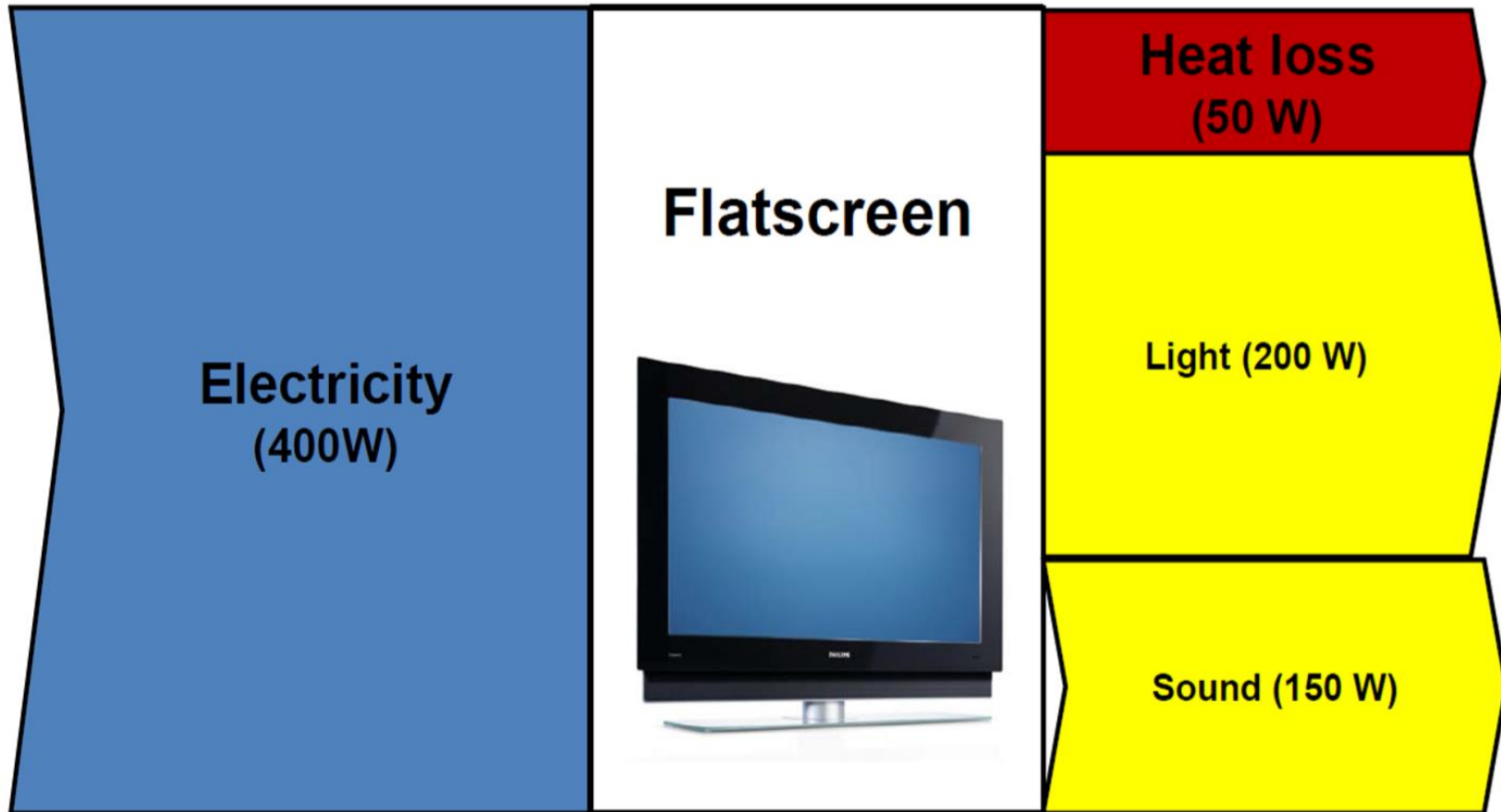
- https://www.youtube.com/watch?v=kP7q28wQ2P8&ab_channel=BBCEarthLab

ENERGY BALANCE

- Sankey diagram light bulb



ENERGY BALANCE



ENERGY BALANCE



LEARNING OBJECTIVES LECTURE 2

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- *Explaining* conduction principles
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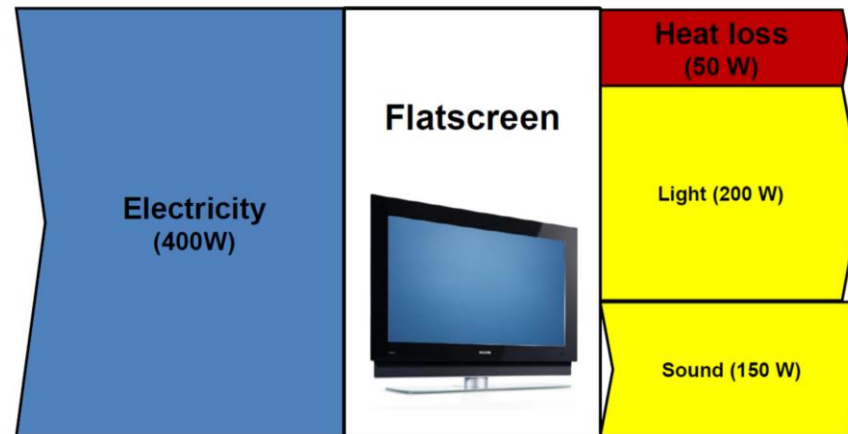
EFFICIENCY

- Efficiency = fraction of “useful” work/power used
 - What is defined as useful?
 - What is the reference?
- Efficiency $\eta = \frac{\text{useful work}}{\text{input energy}} = \frac{\text{useful power}}{\text{input power}} \quad (-)$
- Use a Sankey diagram!

EFFICIENCY

Efficiency definition not clear:

- What is useful? What is the input? Give the reasons for the choices made



Flatscreen: useful spent power?

So always explain how you have defined efficiency!

EFFICIENCY – PRACTICAL VALUES

Product	Efficiency	Remark
Solar panel	12%	Part of sun's radiation converted to electricity
Vacuum cleaner	20%	Part of electrical power converted to suction power
Petrol engine	35%	Part of chemical energy converted to kinetic energy
Electric kettle	84%	Part of electrical energy converted to useful heat in the water

Always state the used definition when talking about efficiency

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RESISTORS

Electric resistors: barrier in electric circuit
→ heat generation



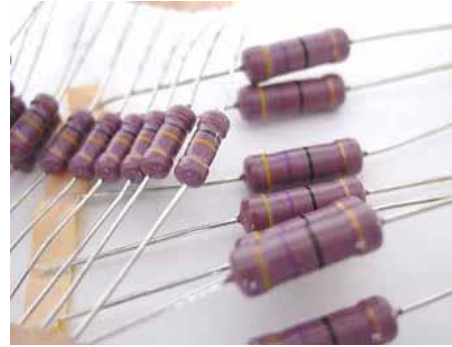
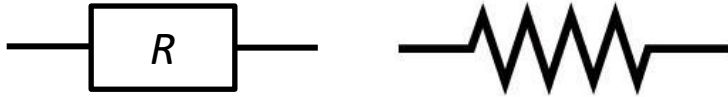
Heat: purpose



Heat: “byproduct”

RESISTORS

Schematical symbols:



Resistance R determined by:

- Length of the wire (L)
- Through-flow area (A)
- Material (resistivity ρ)

Pouillet's law: $R = \rho \frac{L}{A}$

Not density:
 $\Omega \cdot \text{m} \neq \text{kg}/\text{m}^3$



Compare: flow
through straw

RESISTOR AS HEAT SOURCE



Resistor converts electric energy into thermal energy:

$$E = I^2 R \Delta t = Q \quad [\text{J}]$$

So electric power becomes thermal power:

Notation: \dot{Q} (“Heat-flow” or heat transfer rate)

$$P = I^2 R = \dot{Q} \quad [\text{J/s}] = [\text{W}]$$

SIDESTEP

Dot sign means: per unit time (time derivative)

$$\text{Power} = \frac{\text{Energy change}}{\text{Time}}$$

$$P = \frac{dE}{dt} \approx \frac{\Delta E}{\Delta t}$$

$$\dot{Q} = \frac{dQ}{dt} \approx \frac{\Delta Q}{\Delta t}$$

$$(\text{Velocity } v = \frac{dx}{dt} = \dot{x} \approx \frac{\text{displacement } \Delta x}{\Delta t})$$

$$(\text{Acceleration } a = \frac{dv}{dt} = \dot{v} = \ddot{x} \approx \frac{\text{velocity change } \Delta v}{\Delta t})$$



RESISTIVITY

$$R = \rho \frac{L}{A}$$

Material	ρ ($\Omega \cdot \text{m}$) at 20 ° C
Copper	$1,67 \cdot 10^{-8}$
Tungsten	$5,5 \cdot 10^{-8}$
Aluminum	$2,65 \cdot 10^{-8}$
Glass	$1 \cdot 10^{12}$
PVC	$1 \cdot 10^{14}$
Rubber	$1 \cdot 10^{15}$

} Conductors

} Insulators

CONDUCTORS / INSULATORS

Also thermal!



Insulators?



Conductors?

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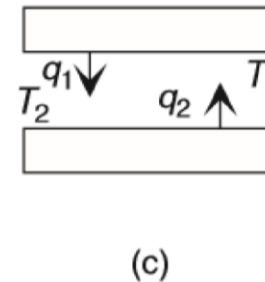
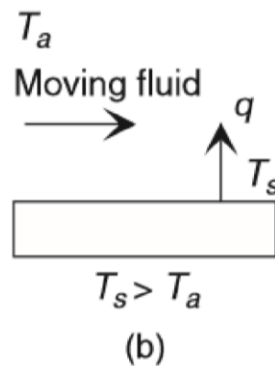
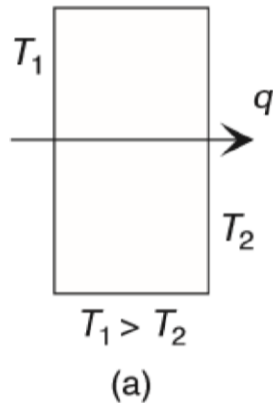
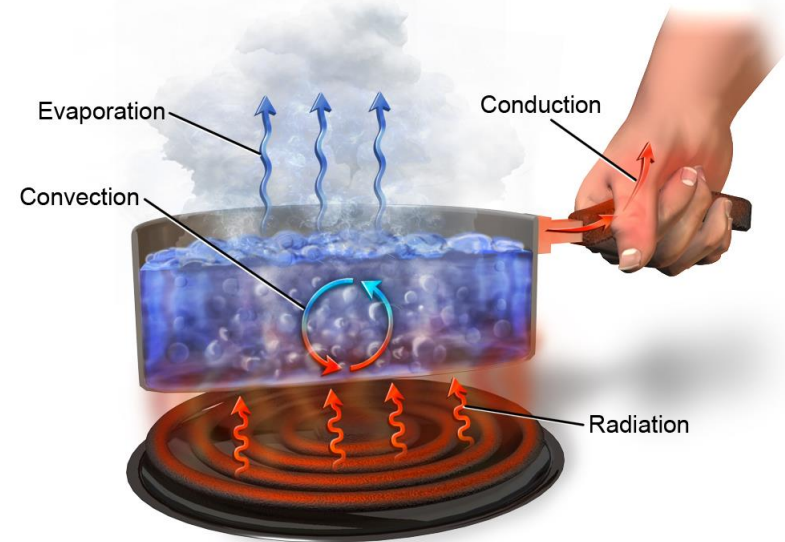
- *Explaining* conduction principles
- *Calculate* conductive heat transfer
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HEAT TRANSFER MECHANISMS

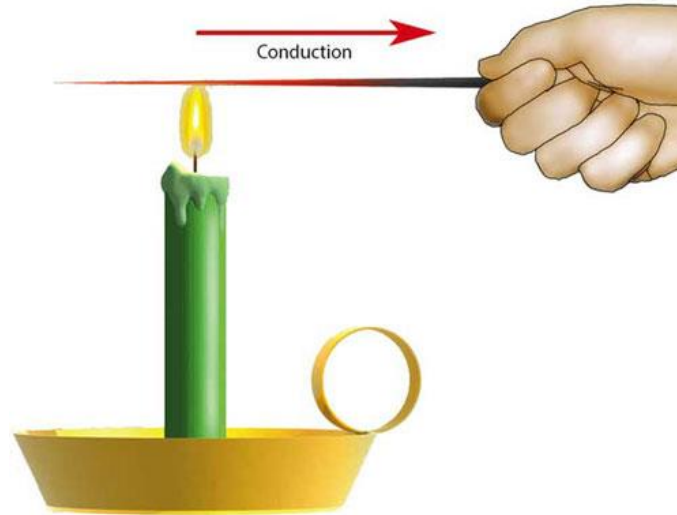
Representations of heat transfer modes:

- (a) **conduction** through a solid, liquid or gas,
- (b) **convection** from a surface to a moving fluid,
- (c) **radiation** between two surfaces

Mechanisms of Heat Transfer



HEAT TRANSFER MECHANISMS



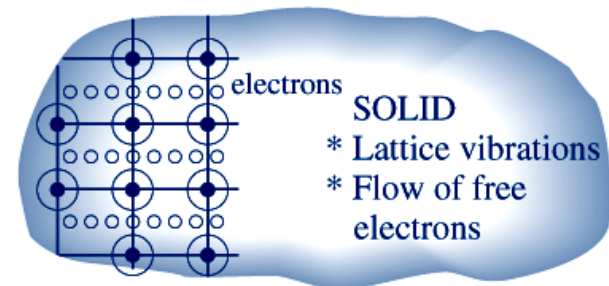
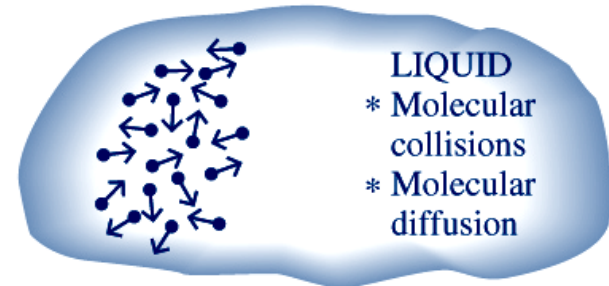
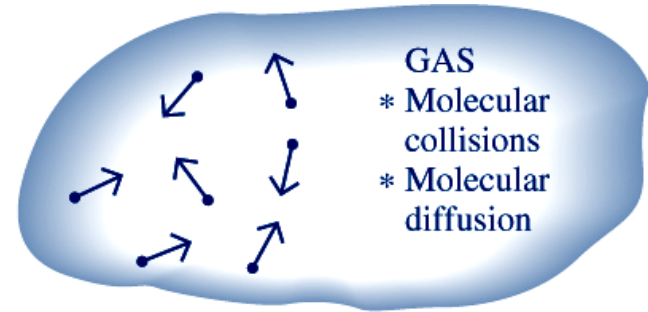
- Conduction
- Convection
- Radiation

CONDUCTION

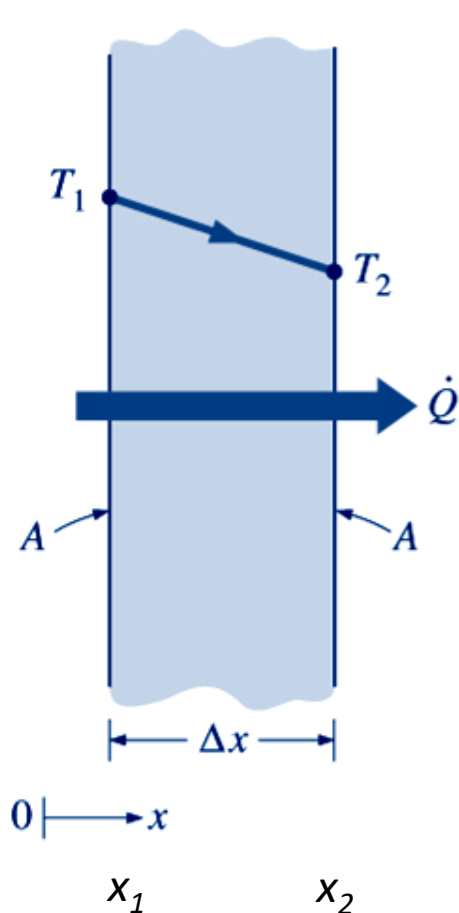
Conduction:

Transfer of thermal energy between (non flowing) molecules

- Always **from** more energetic **(warm)** **to** less energetic **(cold)** particles!



CONDUCTION(FOURIER'S LAW)



Cross section of part of a wall (almost flat)

Heat transfer rate : \dot{Q}

- **Steady state condition-1D**
- **Proportional** to area A
- **Proportional** to temp. Difference : $T_2 - T_1$
- **Inversely proportional** to thickness: $\Delta x = x_2 - x_1$
- **Dependent on material** \rightarrow thermal conductivity: k

$$\dot{Q} = -k A \frac{T_2 - T_1}{x_2 - x_1} = -k A \frac{\Delta T}{\Delta x} \quad (\text{W})$$

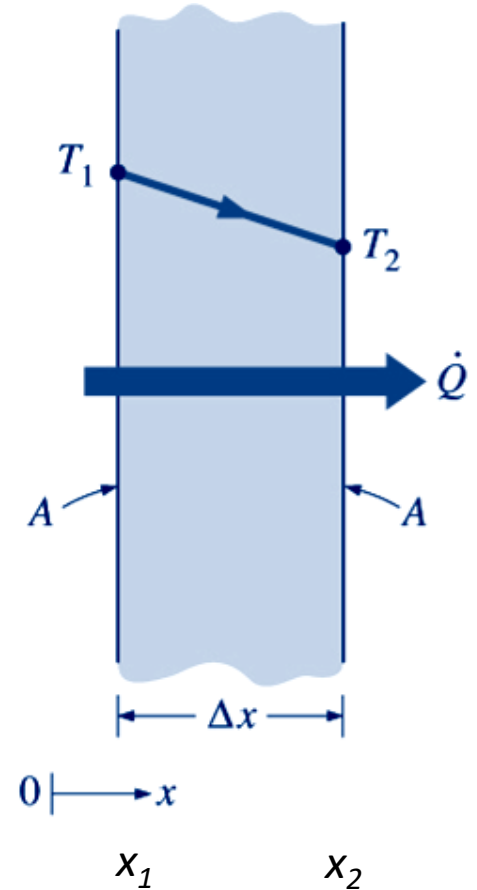
Negative sign since **heat flow** is **positive** at a negative temperature gradient.

CONDUCTION(FOURIER'S LAW)

$$\dot{Q} = -kA \frac{T_2 - T_1}{x_2 - x_1} = -kA \frac{\Delta T}{\Delta x} (\text{W})$$

- Fourier's law simplified for **plane surface**
- Per m²: **Heat flux**

$$\dot{q} = \frac{\dot{Q}}{A} = -k \frac{T_2 - T_1}{x_2 - x_1} = -k \frac{\Delta T}{\Delta x} (\text{W/m}^2)$$



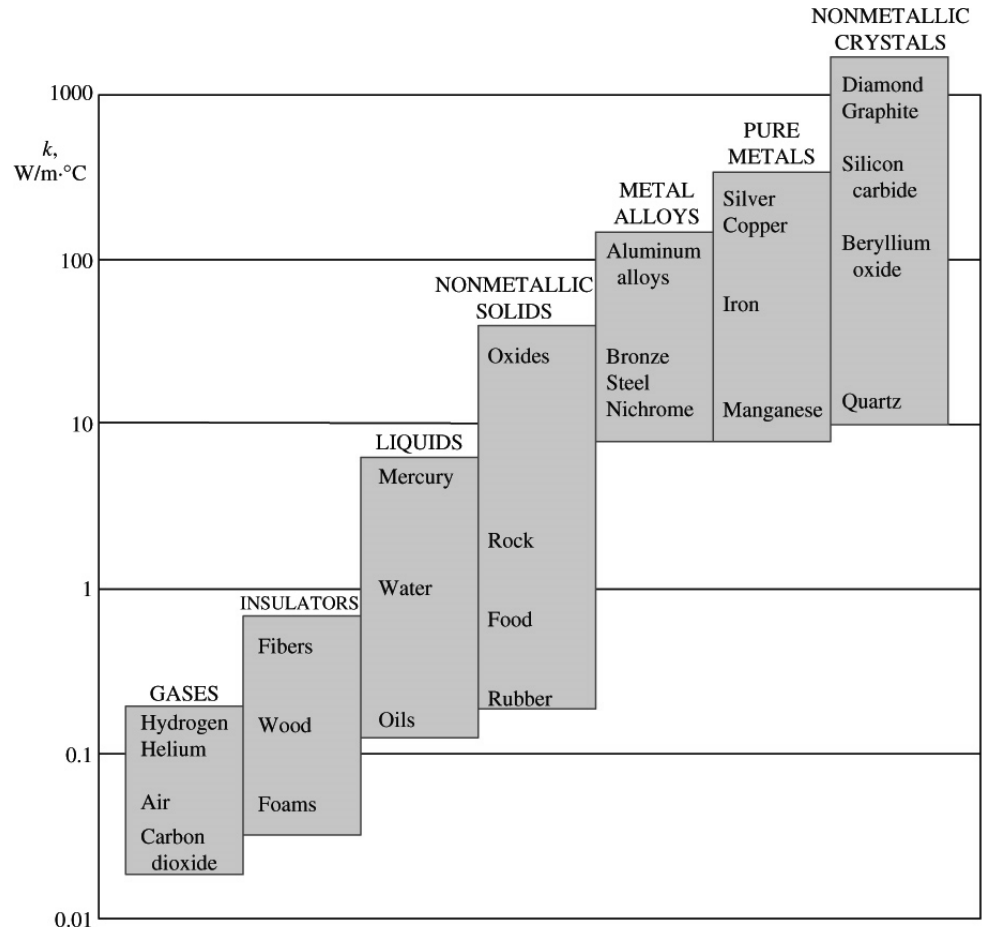
Thermal conductivity (k)

Unit: $\frac{\text{W}}{\text{m} \cdot \text{K}}$

⇒ The amount of power conducted through 1 m of material at a 1 K temperature difference

High k : Conductor

Low k : Insulator



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ASSIGNMENT: ELECTRIC KETTLE

What is the **efficiency** of an electric kettle when the water is almost boiling ($T=100\text{ }^{\circ}\text{C}$)?

Data/assumptions/simplifications:

- Wall temperature outside 50°C
- Diameter 10 cm
- Height 20 cm
- Plastic wall:
 - Thickness 2,7 mm
 - $k = 0,20\text{ W} / (\text{m} \cdot \text{K})$
- Neglect losses through top and bottom
- It can be treated as plane wall
- Electric power $P_{el} = 930\text{ W}$



$$\dot{Q} = -k A \frac{\Delta T}{\Delta x}$$

CONDUCTION FLAT WALL- SUMMARIZED

- Fourier's law for plane surface:

- Temperature difference $\Delta T = T_2 - T_1$ (K)
- Thickness $\Delta x = x_2 - x_1$ (m)
- Surface area A (m²)
- Thermal conductivity k (W / (m · K))

Heat transfer rate: $\dot{Q} = -k A \frac{\Delta T}{\Delta x}$ (W)

Heat flux: $\dot{q} = -k \frac{\Delta T}{\Delta x}$ (W / m²)

**Next: model heat flow as thermal resistance network
(Make it easier for calculations)**

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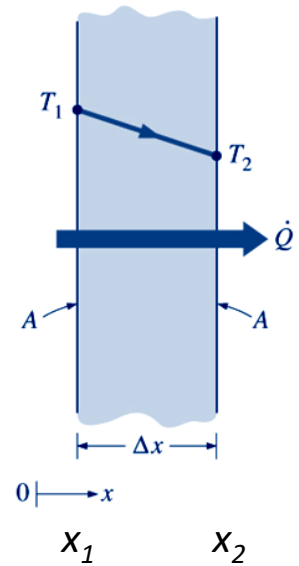
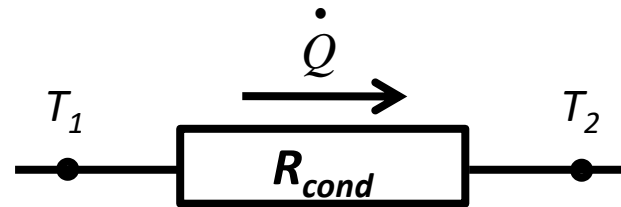
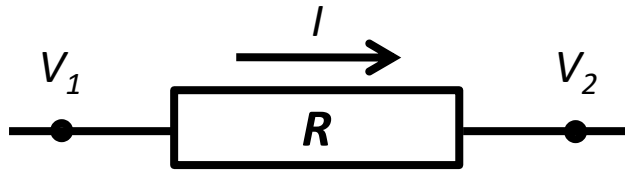
- *Explaining* conduction principles
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ANALOGY ELECTRICITY - HEAT

Electric resistor



Flat wall as thermal resistor



Voltage difference

$$V_1 - V_2$$



Temperature difference

$$T_1 - T_2$$

Current flow

$$I$$



Heat flow

$$\dot{Q}$$

Electrical resistance

$$R$$



Thermal resistance

$$R_{cond}$$

unit: Ω

unit: K/W

ANALOGY ELECTRICITY - HEAT

Heat conduction through plane wall

Fourier's law:

$$\begin{aligned}\dot{Q} &= -k A \frac{T_2 - T_1}{\Delta x} \\ &= +k A \frac{T_1 - T_2}{\Delta x} \\ &= \frac{T_1 - T_2}{\frac{\Delta x}{k A}}\end{aligned}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{cond}} \text{ with } R_{cond} = \frac{\Delta x}{kA} \text{ (K/W)}$$

Heat transfer in “Ohmic way”!

Electrical resistance

Ohm's law:

$$I = \frac{V_1 - V_2}{R}$$

$$V_1 - V_2 \quad \leftrightarrow \quad T_1 - T_2$$

$$I \quad \leftrightarrow \quad \dot{Q}$$

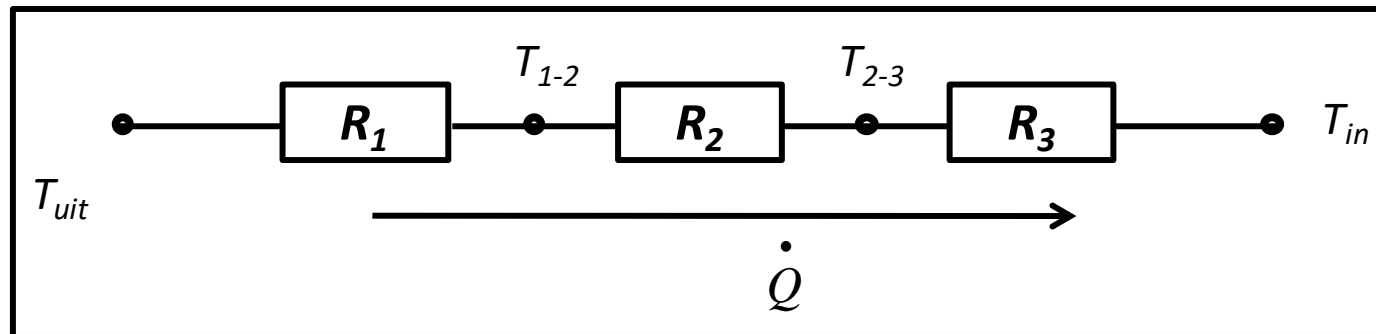
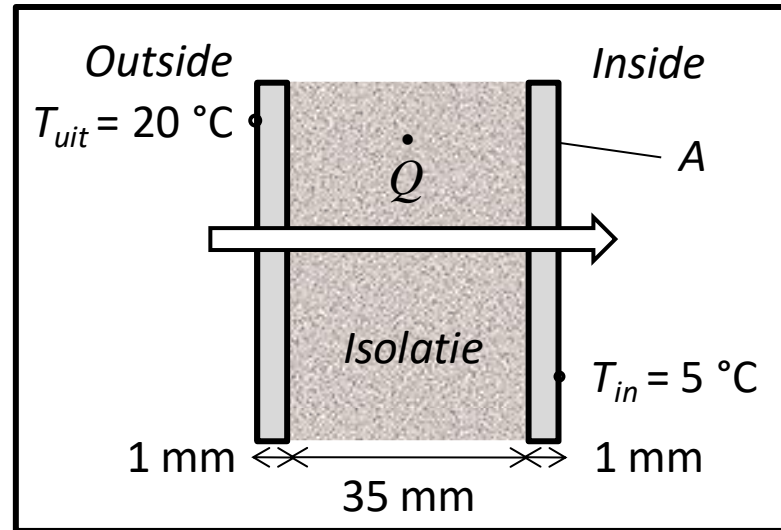
$$R \quad \leftrightarrow \quad R_{cond}$$

$$\Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{cond}}$$

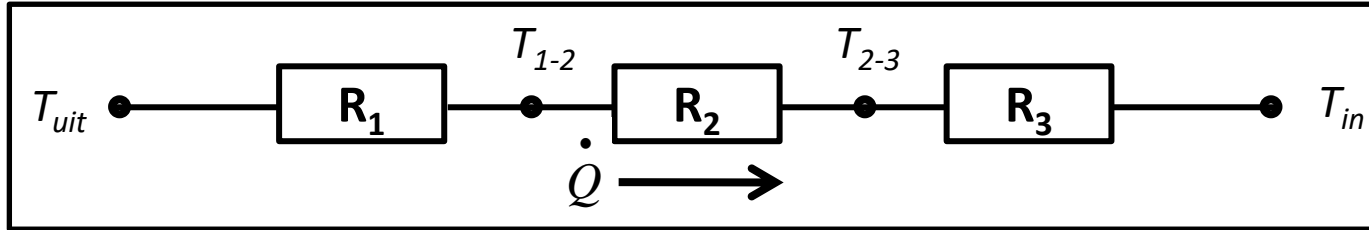
THERMAL RESISTANCE NETWORKS

Fridge wall: heat flow \dot{Q} through three resistors

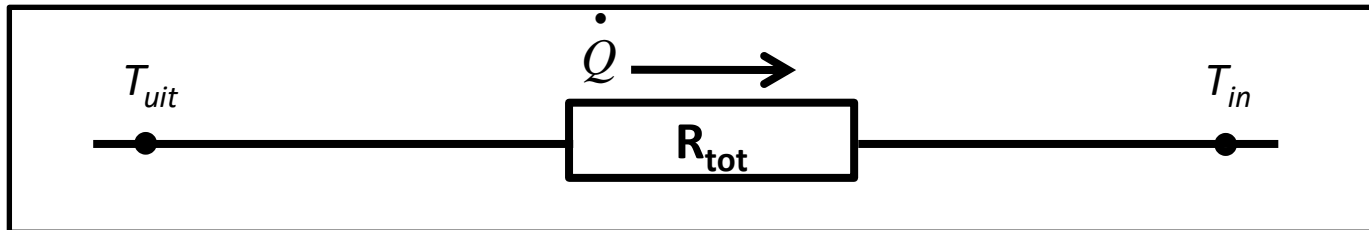
- Outer wall (R_1)
- Insulation (R_2)
- Inner wall (R_3)



THERMAL RESISTANCE NETWORKS



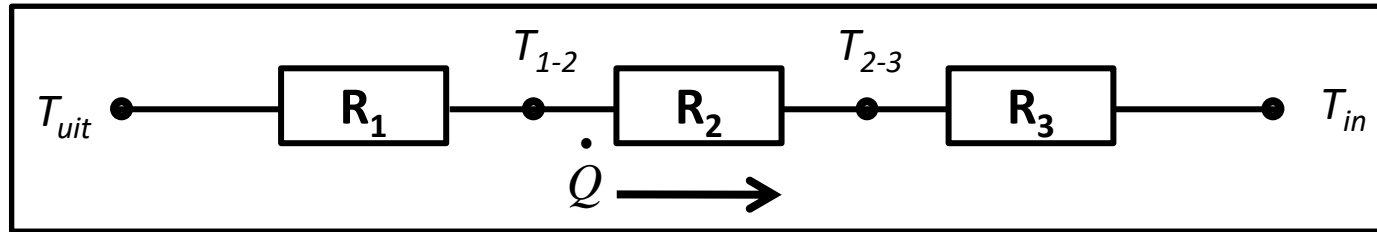
Total resistance $R_{tot} = R_1 + R_2 + R_3$ (series)



\Rightarrow heat flow
$$\dot{Q} = \frac{T_{uit} - T_{in}}{R_{tot}}$$

Resistors in series: total resistance equals the sum of the individual resistances

THERMAL RESISTANCE NETWORKS



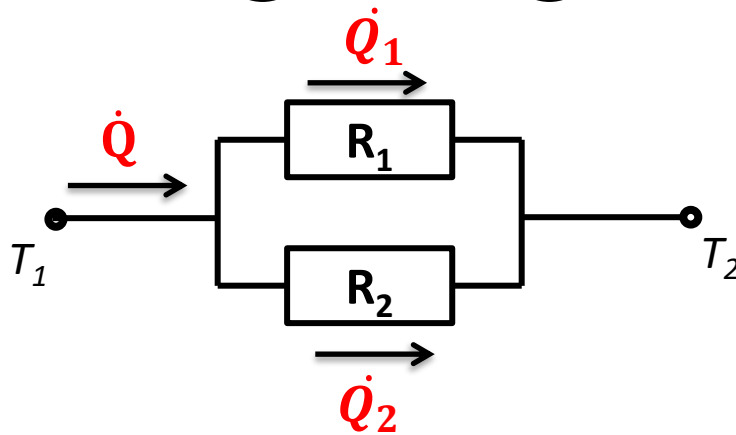
T_{1-2}, T_{2-3} ? Use: $\dot{Q} = \text{constant throughout the resistors}$

$$\begin{aligned}\dot{Q} &= \frac{T_{uit} - T_{in}}{R_{tot}} \\ &= \frac{T_{uit} - T_{1-2}}{R_1} = \frac{T_{1-2} - T_{2-3}}{R_2} = \frac{T_{2-3} - T_{in}}{R_3} = \frac{T_{uit} - T_{2-3}}{R_1 + R_2} = \frac{T_{1-2} - T_{in}}{R_2 + R_3}\end{aligned}$$

⇒ Solve all unknown temperatures by comparing total and partial resistors in series, similar to voltage in an electrical circuit

⇒ **Next: parallel thermal resistors**

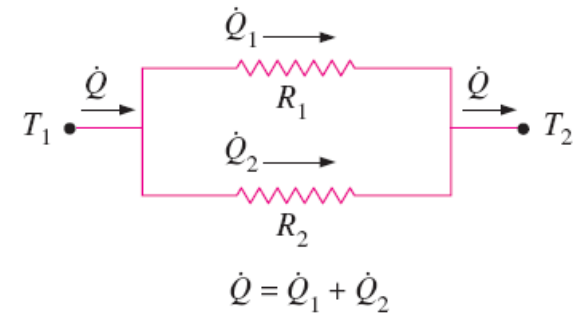
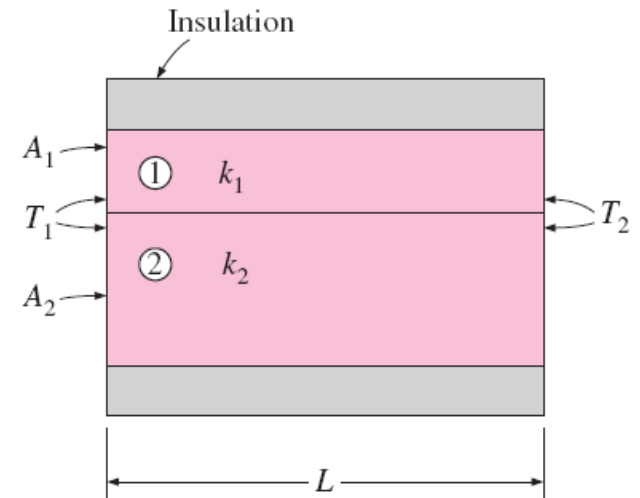
THERMAL RESISTANCE NETWORKS



$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_{\text{total}}}$$

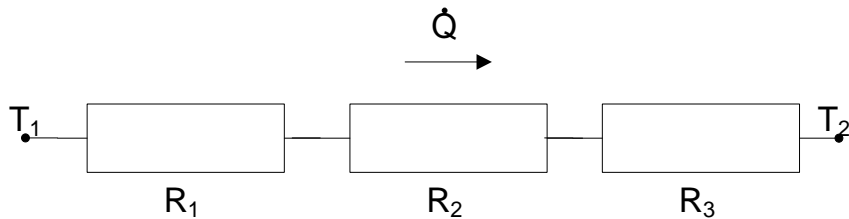
$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



THERMAL RESISTANCE NETWORKS

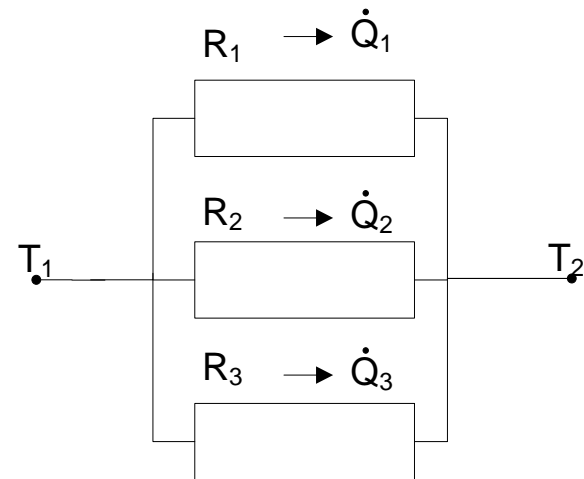
Series Resistors



$$R_{tot} = \sum_i R_i$$

(Add Resistors)

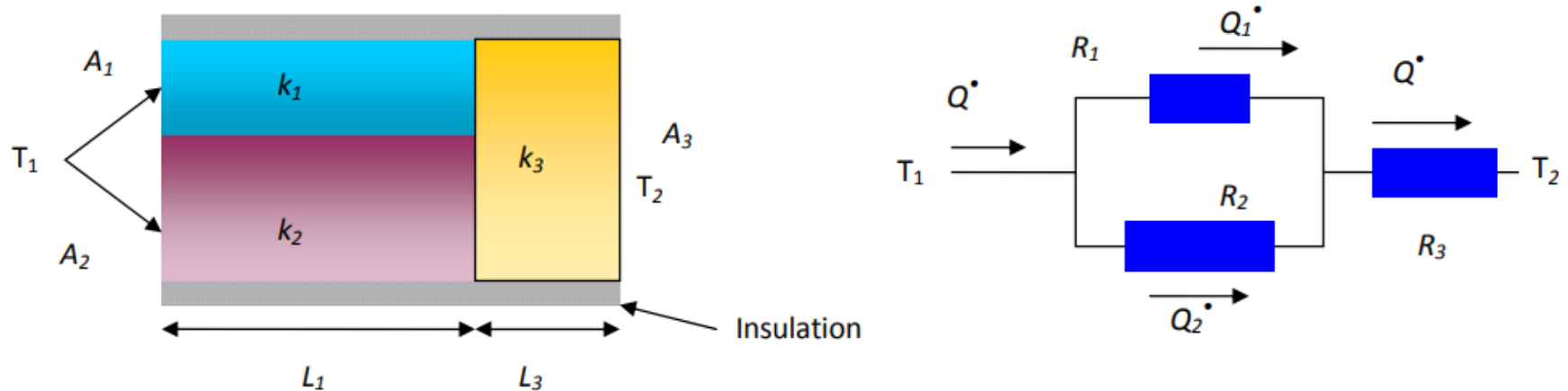
Parallel Resistors



$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

(Add Heat Flows)

THERMAL RESISTANCE NETWORKS



$$Q^{\bullet} = \frac{T_1 - T_{\infty}}{R_{total}}$$

$$R_{total} = R_{12} + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

RESISTANCE AND INSULATION

Thermal resistance plane surface:

$$R = \frac{\Delta x}{kA} \quad \left(\frac{\text{K}}{\text{W}} \right)$$

Alternative concept: insulation value
(building materials):

$$R - \text{value} = \frac{\Delta x}{k} = R \cdot A \quad \left(\frac{\text{m}^2 \cdot \text{K}}{\text{W}} \right)$$

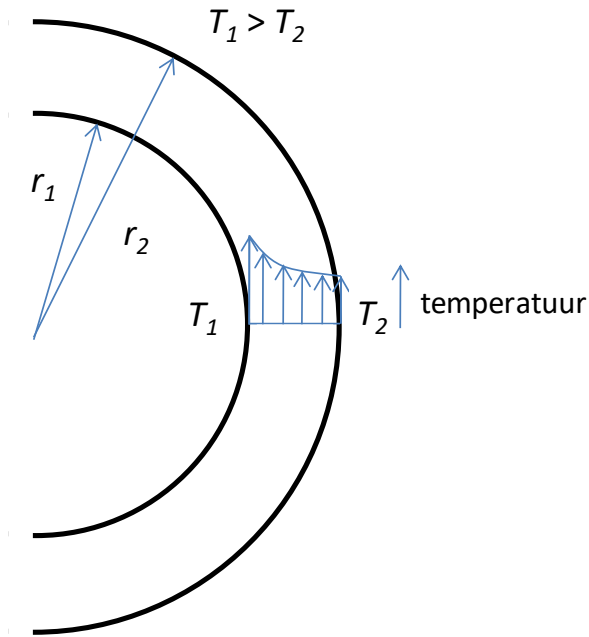
⇒ Insulation value is the heat resistance of
1 m² of material



Material	R-value (m ² · K / W)
Bricks (wall)	0,26
Rockwool 5 cm thick	1,45
Glasswool 11 cm thick	2,50

NB: Often the symbol *R* is used for both concepts. Look at the units!

CONDUCTION IN NON-PLANAR SURFACE



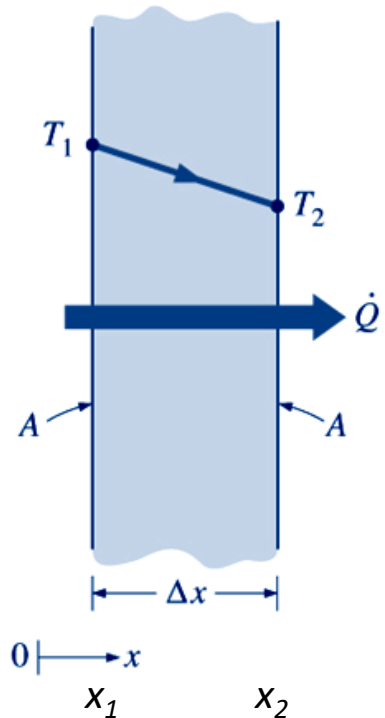
Small radius: small flow-through surface \rightarrow high resistance \rightarrow large temperature *gradient*

(In areas with a high resistance the temperature decreases more in the direction of the heat flow)

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

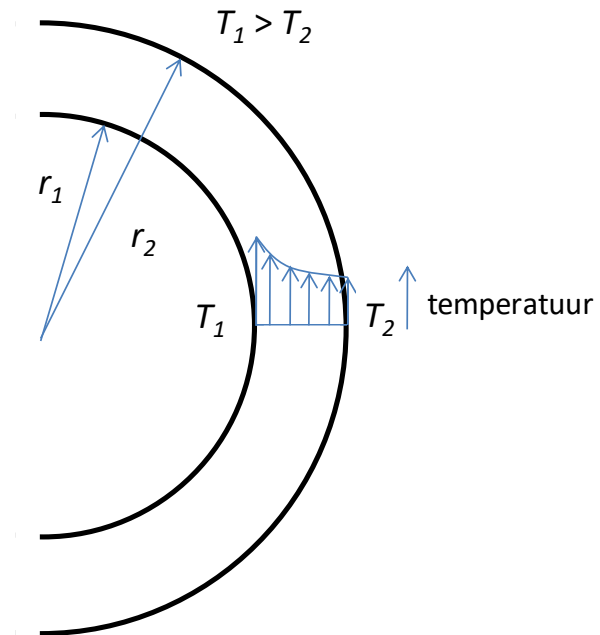
CONDUCTION IN NON-PLANAR SURFACE

Plane wall



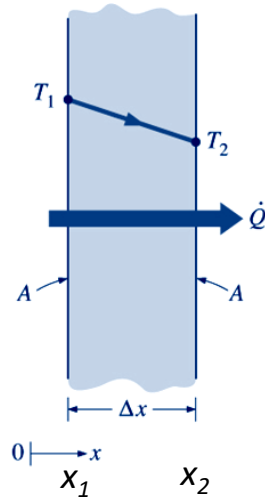
- Surface area constant over x
- Temperature profile linearly decreasing (Fourier's law)
- Temperature gradient (= 'slope') constant

Curved wall



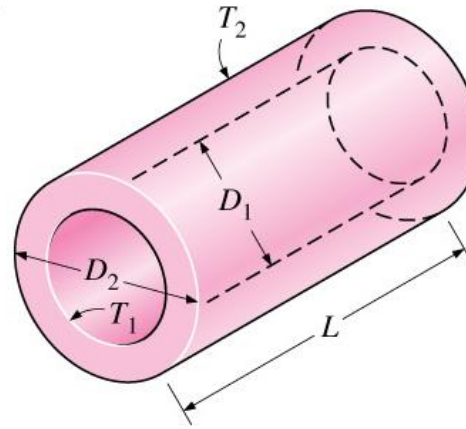
- Surface area increases with r
- Temperature profile decreasing concave up
- Temperature gradient smaller with increasing radius r

VARIOUS CONDUCTION RESISTANCES



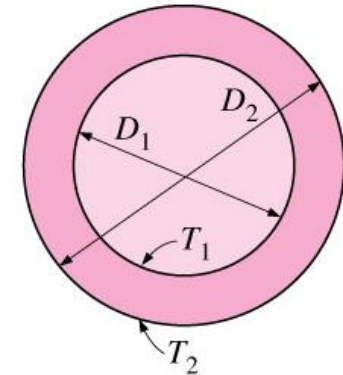
Plane wall

$$R = \frac{\Delta x}{kA}$$



Cilindrical pipe

$$R = \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi L k}$$



Spherical shell

$$R = \frac{D_2 - D_1}{2\pi k D_1 D_2}$$

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

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SUMMARY LECTURE 2 (1/2)

- Efficiency $\eta = \frac{\text{useful work}}{\text{inputted energy}} = \frac{\text{useful power}}{\text{inputted power}}$

Justify the applied definition!

- Heat transfer rate \dot{Q} (W); Heat flux: $\dot{q} = \dot{Q} / A$ (W/m²)

↑
“Q-flux”

- Heat dissipation electric resistor

$$\dot{Q} = I^2 R \quad (\text{W}) \quad \text{with} \quad R = \rho \frac{L}{A} \quad (\Omega)$$

SUMMARY LECTURE 2 (2/2)

- Conduction: heat transfer between molecules
- Different appearances of Fourier's law

– Plane surface: $\dot{Q} = -k A \frac{T_2 - T_1}{x_2 - x_1} = \frac{T_1 - T_2}{R}$ with $R = \frac{\Delta x}{kA}$ $(\frac{K}{W})$

– Cylindrical tube: $\dot{Q} = \frac{T_1 - T_2}{R}$ with $R = \frac{\ln(\frac{D_2}{D_1})}{2\pi L k}$

– Spherical shell: $\dot{Q} = \frac{T_1 - T_2}{R}$ with $R = \frac{D_2 - D_1}{2\pi k D_1 D_2}$

- Thermal resistors: for ease of calculation

- Insulation value: R -value = $R \cdot A$ $(\frac{m^2 \cdot K}{W})$