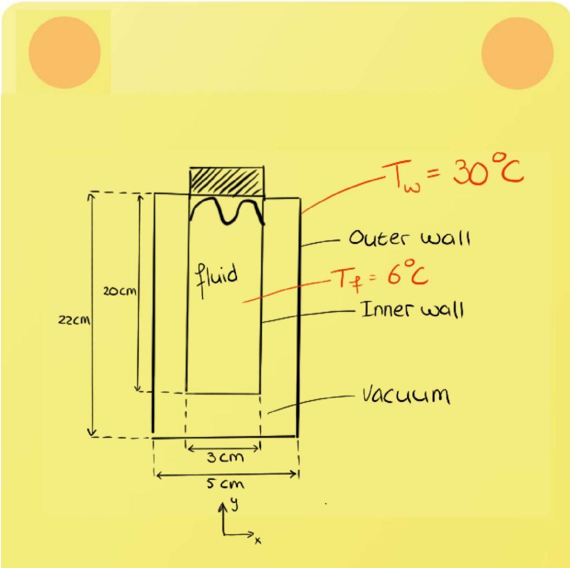
Exam Preparation - Radiation 2



A thermos flask can be used to keep drinks cool. The flask has two walls separated by a vacuum so as to minimize heat transfer by convection. For a day of hiking you take along your aluminium vacuum flask with a cold soda. At the start of the hike the soda is at a temperature of 6 °C. Between the two walls, i.e. inside the vacuum compartment, heat is transferred by radiation only. The problem can be considered as a two surface enclosure. Let the outside wall temperature be 30 °C. The flask has an inner diameter of 3 cm, an inner height of 20 cm and an outer diameter of 5 cm and an outer height of 22 cm. Assume that the walls behaves as a black body and that the fluid has the properties of water.

A) Determine the view factor Φ_{oi} from the outer wall to the inner wall.

B) Compute the net rate of heat transfer by radiation from the outer wall to the inner wall at the given temperatures.

C) Use your answer given at **B)** to estimate the time to heat up the soda by 1 degree Celsius.

Thermos flask. Given: all dimensions and temperatures:

$D_i = 3 \cdot 10^{-2}\,\mathrm{m}$
$D_o = 5 \cdot 10^{-2}\,\mathrm{m}$
$L_i = 20 \cdot 10^{-2}\,\mathrm{m}$
$L_i = 22 \cdot 10^{-2}\,\mathrm{m}$

Question: how much heat is going from outside to inside the heat beverage and how quickly does it heat up one degree?

Strategy: Assume a 2 surface enclosure:
$\dot{Q}_{oi} = A_o \Phi_o i \sigma (T_o^4 - T_i^4)$
Determine view factor $\rightarrow$ use summation rule and reciprocity rule.
Assume them to be black surfaces.
Time to heat up one degree $= \dot{Q}$

a) Summation rule: $\Phi_{oo} + \Phi_{oi} = 1$
Inside does not radiate onto itself: $\{Phi_{ii} = 0,$
$\Phi_{io} = 1$
$A_i \Phi_{io} = A_o \Phi_{oi}$
$\Rightarrow \Phi_{oi} = \frac{A_i \Phi_{io}}{A_o} = \frac{A_i}{A_o}$
$A_i = \pi D_i L_i + \frac{\pi}{4} D_i^2 = \pi 0.03 \cdot 20 \cdot 10^{-2} + \frac{\pi}{4} 0.03^2 = 0.01885 + 7.06 \cdot 10^{-4} = 0.0196\,\mathrm{m}^2$
$A_o = \pi D_o L_o + \frac{\pi}{4} D_o^2 = \pi 0.05 \cdot 22 \cdot 10^{-2} + \frac{\pi}{4} 0.05^2 = 3.77 \cdot 10^{-2} = 0.0377\,\mathrm{m}^2$
$\Phi_{oi} = \frac{A_i}{A_o} = \frac{0.0196}{0.0377} = 0.530$

b) $\dot{Q}_{rad} = \dot{Q}_{oi} = \Phi_{oi} A_o \sigma (T_o^4 - T_i^4) = 3.77 \cdot 10^{-2} \cdot 0.53 \cdot 5.67 \cdot 10^{-8} (303^4 - 279^4) = 2.69\,\mathrm{W}$

c) Energy to heat up 1 degree:
$Q_{heating} = m C_p \Delta T$
$m = \rho V = 9.999\,\mathrm{kg/m^3} \cdot \frac{\pi}{4} D_i^2 L_i = 1.41 \cdot 10^{-4}\,\mathrm{m}^3 \Rightarrow m = 0.14\,\mathrm{kg}$
$C_p = 4217 \mathrm{J/kgK}$
$\Rightarrow Q_{heating} = 0.14 \cdot 4217 \cdot 1 = 596\,\mathrm{W}$
Heating time: $\frac{\dot{Q}_{heating}}{\dot{Q}_{rad}} = \frac{596}{2.631} = 226.6\,\mathrm{s} = 3\,\mathrm{min\,and 46\,sec}$