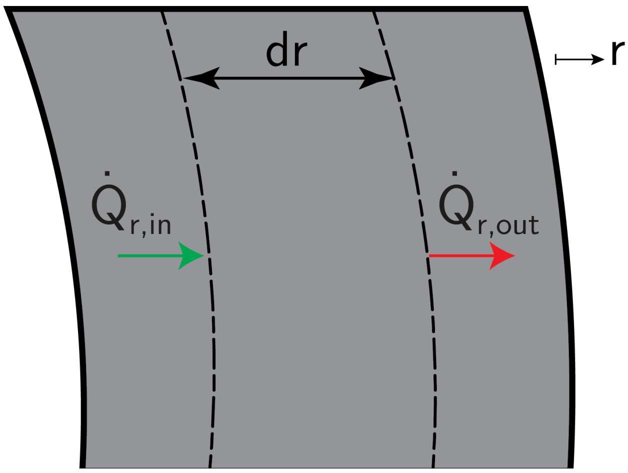
EB - Cond. - IE 9

Hot water flows through a long pipe of length L. The water temperature and external surface temperature of the pipe are constant and equal to T_∞ and T_1 respectively. Set up the energy balance for radial heat conduction in the pipe wall and give the appropriate boundary conditions.

1 Setting up the balance:

To derive the one-dimensional steady-state temperature profile, an energy balance around an infinitesimal element is needed. Heat is conducted in and out of the element.



Hence, the steady-state balance reads:

$$\dot{Q}_{\mathrm{r,in}} - \dot{Q}_{\mathrm{r,out}} = 0,$$

the sum of the in- and outgoing fluxes should equal zero, because of steady-state conditions.

2 Defining the elements within the balance:

The ingoing flux described by use of Fourier's law:

$$\dot{Q}_{\mathrm{x,in}} \quad = \quad -\lambda \cdot 2\pi r L \frac{\partial T}{\partial r},$$

and the outgoing flux is approximated by the use of the Taylor series expansion.

$$\begin{aligned}\dot{Q}_{\mathrm{r,out}} \quad &= \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{\mathrm{r,in}}}{\partial r} \cdot dr \\ &= -\lambda \cdot 2\pi r L \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(-\lambda \cdot 2\pi r L \frac{\partial T}{\partial r}\right) \cdot dr.\end{aligned}$$

3 Inserting and rearranging:

$$\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) = 0.$$

4 Defining the boundary and/or initial conditions:

The first boundary condition yields from a local energy balance at $r = r_1$:

$$-\lambda A \left.\frac{\partial T}{\partial x}\right|_{r=r_1} = \alpha A \left(T_\infty - T(r = r_1)\right)$$

$$\Rightarrow \left.\frac{\partial T}{\partial x}\right|_{r=r_1} = -\frac{\alpha}{\lambda} \left(T_\infty - T(r = r_1)\right),$$

and

$$T(r = r_2) \;=\; T_1.$$