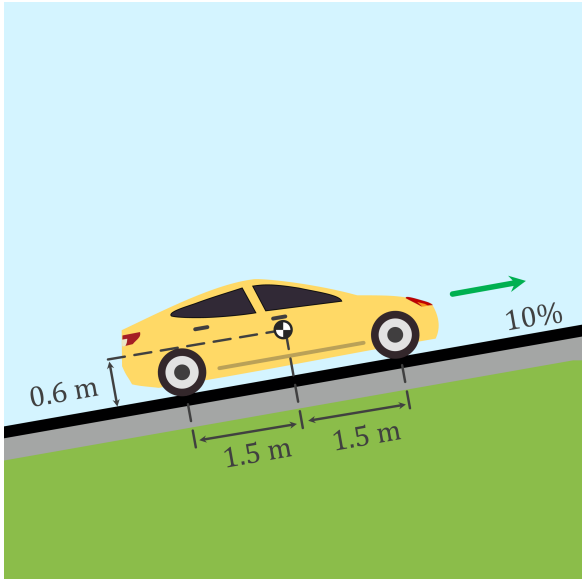
Electric Car on Incline



The rear-wheel drive electric car weighs 1800 kg and is accelerating at a rate of 4.11 m/s up the 10-percent incline. Calculate the resultant normal force N_1 under the front wheels, and the normal force N_2 under the rear driving wheels.

Using known expressions:

$$\sum F_x = m \cdot a_x \tag{1}$$

$$\sum F_y = m \cdot a_y \tag{2}$$

$$\sum M_G = I_G \cdot \alpha \tag{3}$$

Given:
Mass car: $m = 1800$ kg
Acceleration: $a = 4.11$ m/s^2
Incline: 10%

First we convert the 10% incline to an angle that can be used later. 10% means that for every 10 meters horizontal distance, there is 1 meter of vertical distance. This results in an angle of $\theta = \arctan \frac{1}{10} \approx 5.7°$.
Figure 1 shows a Free Body Diagram of the electric car. It shows the normal forces

N_1 and N_2, the friction force F in the opposite direction of the driving wheels, and the gravitational force F_g. Using this to fill in Equations 1, 2 and 3 gives:

$$\sum F_x = m \cdot a = F - m \cdot \sin\theta \cdot g \tag{4}$$

$$\sum F_y = m \cdot a_y = N_2 + N_1 - F_{g,y} = 0 \tag{5}$$

$$\sum M_G = I_G \cdot \alpha = 1.5N_1 - 1.5N_2 + 0.4F = 0 \tag{6}$$

Where $F_{g,y} = m \cdot g \cdot \cos\theta$. From Equation 4 F can be solved as follows.

$$\sum F_x = m \cdot a = F - m \cdot g \cdot \sin\theta \quad \Rightarrow \quad F = m \cdot a + m \cdot \sin\theta \cdot g \tag{7}$$

$$F \approx 7407 + 1757 = 9164N$$

Rewriting Equation 5 so N_1 is a function of N_2 gives:

$$\sum F_y = m \cdot a_y = N_2 + N_1 - F_{g,y} = 0 \quad \Rightarrow \quad N_1 = m \cdot g \cdot \cos\theta - N_2 \tag{8}$$

Inserting Equation 8 in Equation 6 gives:

$$1.5 \cdot (m \cdot g \cdot \cos\theta - N_2) - 1.5N_2 + 0.4F = 0 \quad \Rightarrow \quad 1.5 \cdot m \cdot g \cdot \cos\theta - 3N_2 + 0.4F = 0 \tag{9}$$

From this N_2 can be solved.

$$N_2 = 0.5 \cdot m \cdot g \cdot \cos\theta + \frac{2}{15}F \approx 8785 + 1222 = 10007N \tag{10}$$

Inserting N_2 in Equation 8 gives:

$$N_1 = m \cdot g \cdot \cos\theta - N_2 \approx 17570 - 10007 = 7563N \tag{11}$$

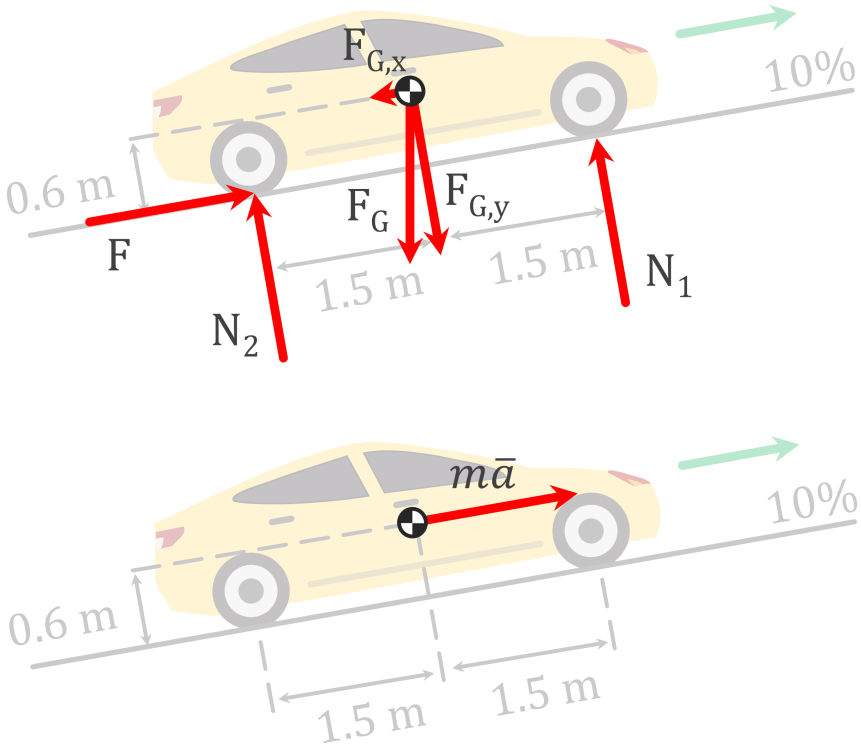


Figure 1: FBD of electric car on slope