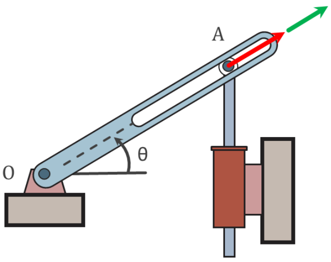
Slider Inside Radially Slotted Arm



Rotation of the radially slotted arm is governed by $\boldsymbol{\theta(t) = -3t + \frac{2}{15}t^3}$, where $\boldsymbol{\theta}$ is in radians and $\boldsymbol{t}$ is in seconds. Simultaneously, slider A is hydraulically moved, its distance from O is described by $\boldsymbol{r(t) = -3 + 2t^2}$. $\boldsymbol{t}$ is the time in seconds. Determine the magnitude of the velocity at $\boldsymbol{t = 3}$ s.

Using known expressions:

$$v_\theta = r\dot{\theta} \tag{1}$$

$$v_r = \dot{r} \tag{2}$$

$$v = \sqrt{{v_\theta}^2 + {v_r}^2} \tag{3}$$

Given:

Distance: $r(t) = -3 + 2t^2$
Angle: $\theta(t) = -3t + \frac{2}{15}t^3$
Time: $t = 3$ s

Solution:

Taking the time-derivative of $r(t)$ results in $\dot{r}(t)$:

$$\dot{r}(t) = 4t \tag{4}$$

Taking the time-derivative of $\theta(t)$ results in $\dot{\theta}(t)$:

$$\dot{\theta}(t) = -3 + \frac{6}{15}t^2 \tag{5}$$

Inserting $t = 3$ s results in $r(3) = -3 + 2 \cdot 3^2 = 15$ m, $\dot{r}(3) = 4 \cdot 3 = 12$ m/s and $\dot{\theta}(3) = -3 + \frac{6}{15} \cdot 3^2 = \frac{3}{5}$ rad/s. Now all all variables of Equation 2 and 1 have been found and values for v_r and v_θ at $t = 3$ s can be calculated.

$$v_\theta(3) = r(3)\dot{\theta}(3) \quad \Rightarrow \quad v_\theta = 15 \cdot \frac{3}{5} = 9 \text{ m/s} \tag{6}$$

$$v_r(3) = \dot{r}(3) \quad \Rightarrow \quad v_r = 12 \text{ m/s} \tag{7}$$

Combining this in Equation (3) results in a final answer for the total velocity:

$$v(3) = \sqrt{v_\theta(3)^2 + v_r(3)^2} \quad \Rightarrow \quad v = \sqrt{9^2 + 12^2} = 15 \text{ m/s} \tag{8}$$

HEATQUIZ