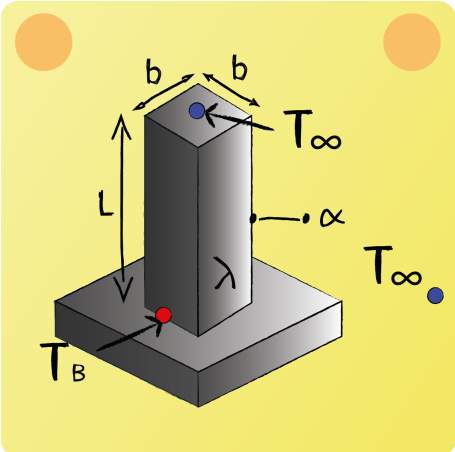
Lecture 12 Question 3

Derive an expression for the fin efficiency.



Definition of the fin efficiency:

$$\eta_{\mathrm{R}} = \frac{\dot{Q}_{\mathrm{cond,base}}}{\dot{Q}_{\mathrm{max}}}.$$

Actual heat transferred through the base:

$$\begin{aligned}\dot{Q}_{\mathrm{cond,base}} &= \lambda \cdot A_c \cdot \frac{\Theta_{\mathrm{B}} \cdot m}{\tanh(mL)} \\ &= \lambda \cdot b^2 \cdot \frac{\Theta_{\mathrm{B}} \cdot m}{\tanh(mL)}.\end{aligned}$$

Maximum rate of heat transfer in the case of a homogeneous body temperature:

$$\begin{aligned}\dot{Q}_{\mathrm{max}} &= A_{\mathrm{s}} \cdot \alpha \cdot \Theta_{\mathrm{B}} \\ &= 4b \cdot \alpha \cdot \Theta_{\mathrm{B}}.\end{aligned}$$

Thus the fin efficiency is written as:

$$\Rightarrow \eta_{\mathrm{R}} = \frac{\lambda \cdot b}{4 \cdot \alpha} \cdot \frac{1}{L} \cdot \frac{m}{\tanh(mL)},$$

where $m = \sqrt{\frac{4\alpha}{b\lambda}}$:

$$\eta_{\mathrm{R}} = \underbrace{\frac{\lambda \cdot b}{4 \cdot \alpha}}_{(1/m)^2} \cdot \frac{1}{L} \cdot \frac{m}{\tanh(mL)}.$$

Hence:

$$\eta_{\mathrm{R}} = \frac{1}{\sqrt{\frac{4\alpha}{b\lambda}}} \cdot \frac{1}{L} \cdot \frac{1}{\tanh(mL)}.$$