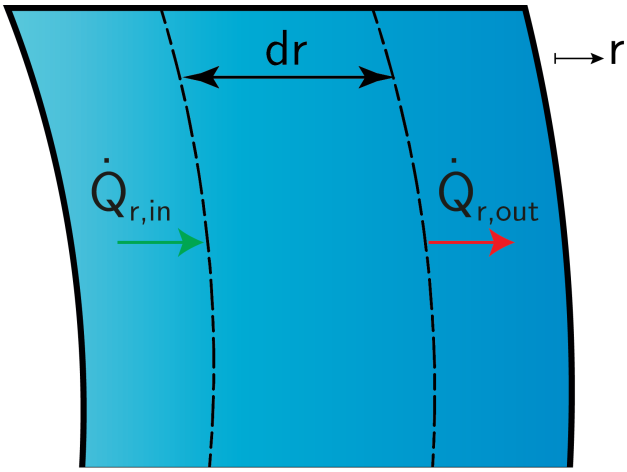
EB - Cond. - IE 7

Derive the energy balance and boundary conditions required to calculate the one-dimensional radial steady-state temperature profile inside the spherical wall.

1 Setting up the balance:

To derive the one-dimensional steady-state temperature profile, an energy balance around an infinitesimal element is needed. Heat is conducted in and out of the element.



Hence, the steady-state balance reads:

$$\dot{Q}_{\mathrm{r,in}} - \dot{Q}_{\mathrm{r,out}} = 0,$$

the sum of the in- and outgoing fluxes should equal zero, because of steady-state conditions.

2 Defining the elements within the balance:

The ingoing flux described by use of Fourier's law:

$$\dot{Q}_{\mathrm{r,in}} = -\lambda \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r},$$

and the outgoing flux is approximated by the use of the Taylor series expansion.

$$\begin{aligned}\dot{Q}_{\mathrm{r,out}} &= \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{\mathrm{r,in}}}{\partial r} \cdot dr \\ &= -\lambda \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(-\lambda \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right) \cdot dr.\end{aligned}$$

3 Inserting and rearranging:

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = 0.$$

4 Defining the boundary and/or initial conditions:

The boundary conditions yield from the given temperatures

$$T(r = r_1) \;=\; T_1,$$

and:

$$T(r = r_2) \;=\; T_2.$$