

2.13 Cooling of a copper rod

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Determine the time it takes for the copper rod to cool to an average temperature of 25 °C: The could possibly be solved by use of the lumped capacity model in the case of $Bi \ll 1$.

Therefore checking whether the lumped capacity model can be used:

$$Bi = \frac{\alpha \cdot V}{\lambda \cdot A_s} = \frac{200 \text{ W/m}^2\text{K} \cdot 0.02 \text{ [m]}}{4 \cdot} = \frac{\alpha \cdot d}{4 \cdot 399 \text{ [W/mK]}} = 0.025 \quad (2.225)$$

$$\boxed{\rightarrow Bi \ll 1} \quad (2.226)$$

The lumped capacity model can be found on the book of formulas, or it can be derived by use of an energy balance (steps 1-5).

1) Setting up an energy balance:

Energy balance around the copper rod:

$$\boxed{\rightarrow \frac{\partial U}{\partial t} = -\dot{Q}_{\text{conv}}} \quad (2.227)$$

2) Defining the fluxes:

Where the change of internal energy over the course of time can be expressed as:

$$\boxed{\rightarrow \frac{\partial U}{\partial t} = V \cdot \rho \cdot c_p \cdot \frac{\partial T}{\partial t}} \quad (2.228)$$

And the rate of heat loss by convection

$$\boxed{\rightarrow \dot{Q}_{\text{conv}} = \alpha \cdot A_s \cdot (T(t) - T_{\infty})} \quad (2.229)$$

3) Inserting and rearranging:

Inserting yields:

$$\frac{\partial U}{\partial t} = -\dot{Q}_{\text{conv}} \quad (2.230)$$

$$\boxed{\rightarrow V \cdot \rho \cdot c_p \cdot \frac{\partial T}{\partial t} = -\alpha \cdot A_s \cdot (T(t) - T_{\infty})} \quad (2.231)$$

4) Defining the initial condition:

In order to solve the equation, an initial condition should be given. This can be described in terms of the initial temperature of the copper rod, which is:

$$\boxed{\rightarrow T(t = 0) = T_0} \quad (2.232)$$

5) Solving the equation:

In order to solve the equation, an homogenized temperature should be introduced, which is:

$$\Theta^* = \frac{T(t) - T_0}{T_\infty - T_0} \quad (2.233)$$

And:

$$\frac{\partial \Theta^*}{\partial t} = \frac{\partial}{\partial t} \left(\frac{T(t) - T_0}{T_\infty - T_0} \right) = \frac{1}{T_\infty - T_0} \frac{\partial T}{\partial t} \quad (2.234)$$

And therefore the energy balance can be written to:

$$V \cdot \rho \cdot c_p \cdot \frac{\partial T}{\partial t} = -\alpha \cdot A_s \cdot (T(t) - T_\infty) \quad (2.235)$$

$$V \cdot \rho \cdot c_p \cdot \frac{1}{T_\infty - T_0} \cdot \frac{\partial T}{\partial t} = -\alpha \cdot A_s \cdot \frac{T(t) - T_\infty}{T_\infty - T_0} \quad (2.236)$$

$$\frac{\partial \Theta^*}{\partial t} = -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot (\Theta^* - 1) \quad (2.237)$$

Rewriting yields:

$$\frac{1}{\Theta^* - 1} \frac{\partial \Theta^*}{\partial t} = -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \quad (2.238)$$

Integration:

$$\ln |\Theta^* - 1| = -\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t + c_1 \quad (2.239)$$

$$\Theta^* = C_1 \cdot \exp\left(-\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t\right) + 1 \quad (2.240)$$

At $T(t = 0) = T_0$, thus:

$$\Theta^*(t = 0) = \frac{T_0 - T_0}{T_\infty - T_0} = 0 \quad (2.241)$$

And therefore:

$$C_1 = -1 \quad (2.242)$$

So:

$$\boxed{\rightarrow \Theta^*(t) = -\exp\left(-\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t\right) + 1} \quad (2.243)$$

6) Determining the time where the temperature of the rod is equal to 25 °C:

At $t = t_1$, the temperature of the rod is equal to 25 °C. Thus:

$$\Theta^*(t = t_1) = -\exp\left(-\frac{\alpha \cdot A_s}{V \cdot \rho \cdot c_p} \cdot t_1\right) + 1 = \frac{T(t = t_1) - T_0}{T_\infty - T_0} = 0.9375 \quad (2.244)$$

Rewriting yields:

$$t_1 = -\ln(0.0625) \frac{V \cdot \rho \cdot c_p}{\alpha \cdot A_s} \quad (2.245)$$

$$t_1 = -\ln(0.0625) \frac{d \cdot \rho \cdot c_p}{4 \cdot \alpha} = -\ln(0.0625) \frac{0.02 \text{ [m]} \cdot 8930 \text{ [kg/m}^3\text{]} \cdot 382 \text{ [J/kgK]}}{4 \cdot 200 \text{ [W/m}^2\text{K]}} \quad (2.246)$$

$$\boxed{\rightarrow t_1 = 236.45 \text{ [s]}} \quad (2.247)$$

2.14 The temperature delay

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a) *Determine Δt :*

1) **Setting up an energy balance:**

Energy balance:

$$\boxed{\rightarrow \frac{dU}{dt} = \dot{Q}_{\text{conv}}} \quad (2.248)$$

2) **Defining the fluxes:**

Where:

$$\boxed{\rightarrow \frac{dU}{dt} = m \cdot c_p \cdot \frac{dT}{dt}} \quad (2.249)$$

$$\boxed{\rightarrow \dot{Q}_{\text{conv}} = \alpha \cdot A_s \cdot \Delta T} \quad (2.250)$$

3) **Inserting and rearranging:**

Inserting yields:

$$\frac{dU}{dt} = \dot{Q}_{\text{conv}} \quad (2.251)$$

$$\boxed{\rightarrow m \cdot c_p \cdot \frac{dT}{dt} = \alpha \cdot A_s \cdot \Delta T} \quad (2.252)$$

For $t \rightarrow \infty$ the temperature increases linearly and therefore we can say: $\frac{dT}{dt} = \frac{\Delta T}{\Delta t}$
Substitution of $\frac{dT}{dt}$ results in:

$$m \cdot c_p \cdot \frac{\Delta T}{\Delta t} = \alpha \cdot A_s \cdot \Delta T \quad (2.253)$$

And for that reason:

$$\boxed{\rightarrow \Delta t = \frac{m \cdot c_p}{\alpha \cdot A_s}} \quad (2.254)$$