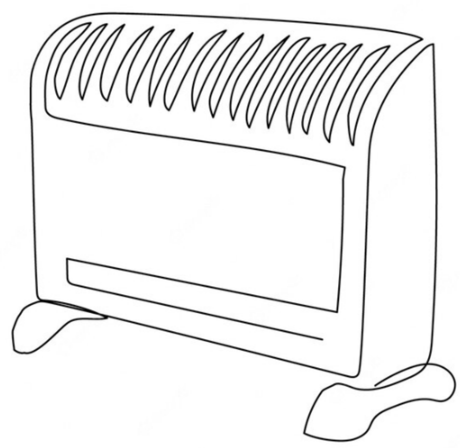
Assignment 1b

During the winter, people often use personal space heaters to provide them with much needed heat. One of these heaters is sketched in the figure below.



This particular heater has two settings, the lowest setting produces 750 W while the highest setting produces 1500 W.
In this assignment we will make several calculations on the heating of the air in a room. Please state the assumptions that you make, the values you have used and the correct dimensions.
Suppose that we want to heat a room of 50 m^3 under constant pressure, with a specific heat of $c_p = 1.005$ [kJ/kg] and a density of $\rho_{air} = 1.225$ [kg/m^3]. The initial temperature of the room is 12 ^{o}C and we turn the the heater on for 10 minutes.

a) What is the temperature increase of the air inside the room using the highest setting? What about the lowest setting?

b) If we want to heat the room up from 12 ^{o}C to 20 ^{o}C, how long would that take?

c) If the applied voltage to the wires is 230 V, what would be the current through the heating coil? Also determine the electrical resistance.

Without worrying about smoke, using a traditional fireplace is also an option to heat the room and add some ambiance to a room. Say wood releases 29 KJ/kg, and costs around 0.91 euro and 1 kWh of electricity costs 0.01 euro.

d) How many kg of wood would you have to burn to heat the room from 12 ^{o}C to 20 ^{o}C? Clearly state the assumptions you make.

e) Determine which of the methods is the cheapest. Does it matter which setting is used on the heater?

In the future we would like to heat this room by means of clean energy. One of the options is to use a heat pump, which uses electricity to pump up stored warm water. This water can then be used in radiators to warm the air in a room. Say the water is at 18 ^{o}C below ground, and is pumped up with a rate of 1.5 L/min.

f) How long does it take for the heatpump to heat up the room from 12 ^{o}C to 18 ^{o}C? Use $c_p = 4.182[kJ/kgK]$ and state all assumptions that you make.

g) Is it possible to heat this room to 21 ^{o}C with this heatpump? Please explain in detail.

Solution Assignment 1b

a) *What is the temperature increase of the air inside the room using the highest setting? What about the lowest setting?*

Given is is the power of the heater, 750 [W] and 1500 [W]. We should realize that this means either 750 [J/s] or 1500 [J/s], depending on the chosen setting. Also given:

$$V_{air} = 50\ [\text{m}^3], \qquad t = 10\ [\text{min}] = 600\ [\text{s}], \qquad T_{begin} = 12\ [^\circ\text{C}], \qquad \rho_{air} = 1.225\ [\text{kg/m}^3], \qquad c_p = 1.005\ [\text{kJ/kg}]$$

Using the density and the volume given, the mass of the air can be calculated:

$$m_{air} = \rho_{air} \cdot V_{air} = 1.225 \cdot 50 = 61.25\ [\text{kg}]$$

The energy that is released by the heater in 10 minutes is dependent on the chosen setting:

$$E_{low} = P_{low} \cdot t = 750 \cdot 600 = 450\ [\text{kJ}]$$
$$E_{high} = P_{high} \cdot t = 1500 \cdot 600 = 900\ [\text{kJ}]$$

The final temperature can be found by rewriting the equation of internal energy:

$$E = mc_p\Delta T \rightarrow E = mc_p(T_{end} - T_{begin})$$
$$\frac{E}{T_{end} - T_{begin}} = mc_p \rightarrow T_{end} - T_{begin} = \frac{E}{mc_p}$$
$$T_{end} = T_{begin} + \frac{E}{mc_p}$$

Then filling in all given and calculated values gives for the low and high setting:

$$\textbf{Low setting: } T_{end} = 12 + \frac{450}{61.25 \cdot 1.005} = 19.3\ [^\circ\text{C}]$$
$$\textbf{High setting: } T_{end} = 12 + \frac{900}{61.25 \cdot 1.005} = 26.6\ [^\circ\text{C}]$$

b) *If we want to heat the room up from 12 $^\circ$C to 20 $^\circ$C, how long would that take?*

First we need to calculate the energy needed to heat up the room from 12 $^\circ$C to 20 $^\circ$C:

$$E = mc_p\Delta T = 61.25 \cdot 1.005 \cdot (20 - 12) = 492.45\ [\text{kJ}]$$

Then dividing the required energy by the energy delivered at each setting:

$$\textbf{Low setting: } t = \frac{E_{required}}{P_{heater}} = \frac{492.45}{0.75} = 656.6\ [\text{s}] = 10.9\ [\text{min}]$$
$$\textbf{High setting: } t = \frac{E_{required}}{P_{heater}} = \frac{492.45}{1.5} = 328.3\ [\text{s}] = 5.5\ [\text{min}]$$

c) *If the applied voltage to the wires is 230 V, what would be the current through the heating coil? Also determine the electrical resistance.*

Remember that current can be calculated from the power law:

$$P = UI \rightarrow \frac{P}{U}$$

$$\textbf{Low setting: } I = \frac{P}{U} = \frac{750}{230} = 3.26\ [\text{A}]$$
$$\textbf{High setting: } I = \frac{P}{U} = \frac{1500}{230} = 6.52\ [\text{A}]$$

The resistance can then also be calculated:

$$\textbf{Low setting: } R = \frac{U}{I} = \frac{230}{3.26} = 70.55\ [\Omega]$$
$$\textbf{High setting: } R = \frac{U}{I} = \frac{230}{6.52} = 35.26\ [\Omega]$$

d) *How many kilos of wood would you have to burn to heat the room from 12 $^\circ$C to 20 $^\circ$C? Clearly state the assumptions you make.*

First the energy needed to heat up the room has to be calculated. We use the assumption that all energy released by the wood is absorbed by the air:

$$E_{needed} = m_{air}c_p\Delta T = 61.25 \cdot 1.005 \cdot (20 - 12) = 492.45\ [kJ]$$

Dividing this by the energy of burning 1 kg of wood:

$$m_{wood} = \frac{E_{needed}}{e_{released}} = \frac{492.45}{29} = 16.981\ [kg]$$

e) *Determine which of the methods is the cheapest.*

The costs of burning the wood are:

$$\text{cost} = m_{wood} \cdot €_{wood,kg} = 16.981 \cdot 0.91 = €15.45, -$$

And for the electricity:

$$\text{cost} = E_{electric} \cdot €_{elec.,kJ} = 492.45 \cdot 0.01 = €4.925, -$$

It does not matter which setting is used for the heater, since the same amount of energy needs to be transferred to the room. A higher setting will only heat the room up faster, since more energy per second is transferred.

f) *How long does it take for the heat pump to heat up the room from 12 ^oC to 18 ^oC? Please state all assumptions that you make.*

The heat pump delivers 1.5 [L/min], which is 0.025 [L/s] = 0.025 [kg/s]. First we calculate how much energy is transported by the water. For this, we assume that the water enters at 18 ^{o}C and leaves at 12 ^{o}C. This is not entirely correct, but for simplicity sake we will use this approximation:

$$e_{water} = m_{water} \cdot c_p \Delta T = 0.025 \cdot 4.182 \cdot (18 - 12) = 0.63\ [kJ/s]$$

We furthermore assume that all energy is absorbed by the air in the room. We then need to calculate the required energy to heat up the room, similar to previous questions:

$$E_{needed} = m_{air} c_p \Delta T = 61.25 \cdot 1.005 \cdot (18 - 12) = 369.34\ [kJ]$$

The time needed to heat up the room can then easily be calculated by dividing the required energy by the provided energy:

$$t = \frac{E_{needed}}{e_{water}} = \frac{369.34}{0.63} = 586.24\ [s] = 9.77\ [min]$$

g) *Is it possible to heat this room to 21 ^oC with this heat pump? Please explain in detail.*

No this is not possible. Since in heat exchange everything want to achieve equilibrium, once the air reaches the maximum temperature of the water, the temperature will no further increase. Even if the pump is kept running and the water keeps transporting energy, this energy will not be transported to the air. In this situation we have reached a steady state system.

Assignment 2

This assignment is dedicated to heat transfer by conduction. The equation for thermal conduction is given by

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x} \tag{1}$$

and

$$\dot{q} = -k\frac{\Delta T}{\Delta x} \tag{2}$$

It is important to know how to apply these equations, but it is just as important to understand how the equations work, so therefore:

a) Explain the difference between equation 1 and 2

b) What does each term ($\dot{Q}$, $\dot{q}$, A, k, $\frac{\Delta T}{\Delta x}$) represent in equation 1 and 2 and what are its corresponding units?

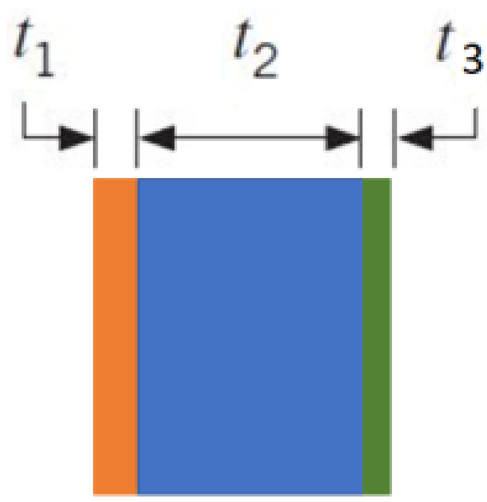
c) Why is there a minus sign in equation 1 and 2?

Have a look at this video about the heat transfer of various objects https://www.youtube.com/watch?v= hNGJ0WHXMyE (If the link is not working search for "veritasium misconceptions about heat" on Youtube). In this video, an example is given of the rate of heat transfer of a book and a hard drive. Most people are convinced that the hard drive is at a lower temperature than the book since it feels colder.

d) Equipped with your knowledge and equations about heat transfer, explain the differences and similarities between touching the book and the hard drive.

With the previous questions you have developed some insight into thermal conduction. Now it is time to apply this knowledge to the following case. In order to keep drinks hot or cold, thermoflasks use insulation. This minimizes the heat transfer between the liquid and the outside environment.

(a) Crossection of a thermosflask



(b) Schematic representation

Figure 1: Thermos flask to keep liquids hot or cold

Assume that the outside wall is made out of plastic, and the inside wall of the thermos flask is made out of aluminium, which is represented in figure 1b. In this schematic, the orange section represents the aluminium, the blue section represents the insulation material and the green section represents the plastic. The aluminium has a thickness of 2 mm and a thermal conductivity of 247 $Wm^{-1}K^{-1}$. The insulation, made out of mineral wool, has a thickness of 1 cm and a thermal conductivity of 0.047 $Wm^{-1}K^{-1}$. Finally, the plastic has a thickness of 4 mm and a thermal conductivity of 21 $Wm^{-1}K^{-1}$.
Assume that the thermos flask is filled with hot coffee at a temperature of 80 ^{o}C, and the outside temperature is quite chilly with 3 ^{o}C. The area of the thermos flask is 0.035 m^2.

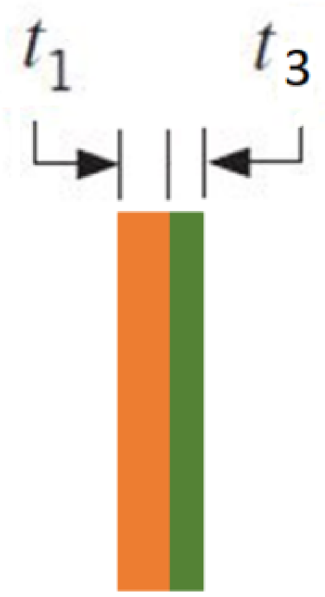
e) Sketch the temperature profile through the wall, and also explain the different slopes of temperature for the different layers.

f) Determine how much heat is conducted through the flask. Assume a height of 10 cm and that no heat is transferred through the bottom and top of the flask. Furthermore, the diameter of the water compartment, thus the inner diameter of the plastic hull equals 6 cm.

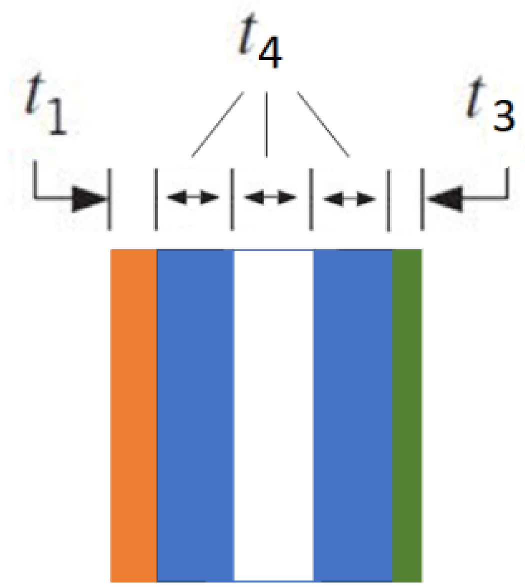
g) Imagine that there was no insulation used as depicted in figure 2a. How much **more** heat would be conducted compared to the insulated thermos flask?

h) As a (hopeful) improvement, one engineer suggests to decrease the thermal conductivity as well as the production costs of the flask by adding a layer of nitrogen in between two layers of insulation. The thermal conductivity of nitrogen is 0.02589 $Wm^{-1}K^{-1}$. All three sections of the insulating material, denoted as t_4 in figure 2b, have a combined thickness of 1.2 cm. Determine how much heat is lost through the wall.

i) If your answers at question f) and h) are different, how thick or thin can the nitrogen pocket be to get the same amount of heat conducted as your answer at question f)?



(a) Thermos flask without insulation



(b) Insulating layer with nitrogen

Figure 2: Schematic overview of no insulation and insulation using an nitrogen pocket

Solution Assignment 1: Thermosflask

a) *Explain the difference between equation 1 and 2*

The first equation already takes the total area into account which is transferring heat. The second one is per unit area. When area is already accounted for, we use a capital letter. When this is not the case, we use a lowercase letter. The dot above each of the variables ($\dot{Q}$, $\dot{q}$) means per unit time.

b) *What does each term ($\dot{Q}$, $\dot{q}$, A, k, $\frac{\Delta T}{\Delta x}$) represent in equation 1 and 2 and what are its corresponding units?*

- $\dot{Q}$ is the energy conducted per second [W]
- $\dot{q}$ is the energy conducted per area per second [W/m^2]
- A denotes the total area [m^2]
- k is the thermal conductivity of a material [W/mK]
- $\frac{\Delta T}{\Delta x}$ is the temperature gradient [K/m]

c *Why is there a minus sign in equation 1 and 2?*

Since the heat transfer is in opposite direction with regards to the temperature gradient, if we want a physically correct expression, we add the minus sign. This way we ensure that the energy is flowing from a hot body to a cold(er) body, instead of the other way around. Note that for the numerical value of the energy transferred it makes no difference, just the direction.

d) *Equipped with your knowledge and equations about heat transfer, explain the differences and similarities between touching the book and the hard drive.*

Which is the same for the book and the hard drive are the size, feel, look and perhaps weight. However, their thermal conductivity is different. This is what makes the hard drive feel colder to the touch. Since it has a higher thermal conductivity, our body transfers much more energy to the hard drive compared to the book. This is what we experience as 'colder'.

e) *Sketch the temperature profile through the wall, and also explain the different slopes of temperature for the different layers.*

f) *Determine how much heat is conducted through the flask.*

The equation for the thermal resistance of a cylinder is:

$$R = \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi L k}$$

For the first resistance, the plastic, we calculate the following resistance:

$$R_1 = R_{plastic} = \frac{\ln\left(\frac{0.068}{0.06}\right)}{2\pi \cdot 0.1 \cdot 21} = 0.0095\ [K/W]$$

For the insulation layer:

$$R_2 = R_{insulation} = \frac{\ln\left(\frac{0.088}{0.068}\right)}{2\pi \cdot 0.1 \cdot 0.047} = 8.731\ [K/W]$$

And finally for the layer of aluminium:

$$R_3 = R_{aluminium} = \frac{\ln\left(\frac{0.092}{0.088}\right)}{2\pi \cdot 0.1 \cdot 247} = 2.86 \cdot 10^{-4}\ [K/W]$$

All resistances are in series, thus the total resistance becomes:

$$R_{tot} = R_1 + R_2 + R_3 = 8.741\ [W/K]$$

PLAATJE RESISTANCE TOEVOEGEN The total heat conduction can then be calculated:

$$\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{(80-3)}{8.741} = 8.81\ [W]$$

g) *Imagine that there was no insulation used as depicted in figure 2a. How much **more** heat would be conducted compared to the insulated thermos flask?*

To find the answer, we only have to remove the resistance of the insulating layer from the previously found total resistance:

$$R_{tot} = 8.741 - 8.731 \approx 0.01 \; [W/K]$$

This gives again the heat conducted to be:

$$\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{(80-3)}{0.01} \approx 7700 \; [W]$$

The difference thus without an insulating layer present is approximately 7700 - 8.81 = 7691.2 [W], which is quite a difference.

h) *As a (hopeful) improvement, one engineer suggests to decrease the thermal conductivity as well as the production costs of the flask by adding a layer of nitrogen in between two layers of insulation. The thermal conductivity of nitrogen is 0.02589 $Wm^{-1}K^{-1}$. All three sections of the insulating material, denoted as t_4 in figure 2b, have a combined thickness of 1.2 cm. Determine how much heat is lost through the wall.*

Since the thickness of the layer of nitrogen appears to be equal to the two layers of insulation, we assume that all three layers have a thickness of 0.4 cm. With the given and known values for the thermal conductivity and equation this gives the following values:

$$R_1 = R_{plastic} = 0.00949 \; [W/K] \tag{3}$$

$$R_2 = R_{insulation} = \frac{\ln\left(\frac{0.076}{0.068}\right)}{2\pi \cdot 0.1 \cdot 0.0047} = 3.766 \; [W/K] \tag{4}$$

$$R_3 = R_{nitrogen} = \frac{\ln\left(\frac{0.084}{0.076}\right)}{2\pi \cdot 0.1 \cdot 0.02589} = 6.153 \; [W/K] \tag{5}$$

$$R_4 = R_{insulation} = \frac{\ln\left(\frac{0.092}{0.084}\right)}{2\pi \cdot 0.1 \cdot 0.047} = 3.08 \; [W/K] \tag{6}$$

$$R_5 = R_{aluminium} = \frac{\ln\left(\frac{0.096}{0.092}\right)}{2\pi \cdot 0.1 \cdot 247} = 2.74 \cdot 10^{-4} \; [W/K] \tag{7}$$

The total resistance is found by adding all resistances in series:

$$R_{tot} = R_1 + R_2 + R_3 + R_4 + R_5 = 13.74 \; [W/K]$$

Which results in a conducted heat of:

$$\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{(80-3)}{13.01} = 5.92 \; [W]$$

i) *If your answers at question f) and h) are different, how thick or thin can the nitrogen pocket be to get the same amount of heat conducted as your answer at question f)?*

The answer to this question can be found by realizing that for both situations the thermal resistance has to be the same, since there is no change in temperature. Therefore, we can say that R_{tot} is a function of $R_3 = R_{insulation}$, which is equal to the total resistance of question f):

$$R_{tot} = R_1 + R_2 + R_3 + R_4 + R_5 = 6.857 + R_3 = 13.01 \rightarrow R_3 = 1.884 \; [W/K]$$

Then we can rewrite the equation for thermal resistance to solve for the thickness of the nitrogen layer:

$$\frac{\ln\left(\frac{0.076+2t}{0.076}\right)}{2\pi \cdot 0.1 \cdot 0.02589} = 1.884 \tag{8}$$

$$\ln\left(\frac{0.076+2t}{0.076}\right) = 0.03065 \tag{9}$$

$$\frac{0.076+2t}{0.076} = \exp 0.03065 \tag{10}$$

$$0.076 + 2t = 0.076 \cdot 1.0311268 \tag{11}$$

$$t = 0.00118 \; [m] \tag{12}$$

Assignment 3

A passenger train is moving with a velocity of 80 km/h. This train is illustrated in the figure below.

Figure 7: Dutch railway carriage

The top of this carriage, which is 3 meters wide and 10 meters long, is absorbing solar radiation at a rate of 250 W/m^2. The ambient temperature of the air is 30 ^{o}C. The roof of this carriage is insulated by using sandwiching isolation (t_2 = 40 mm, k_2 = 0.0320 $Wm^{-1}K^{-1}$) between two sheets of aluminium ($t_1 = t_3 = 3$ mm, $k_1 = k_3 = 180$ $Wm^{-1}K^{-1}$). The temperature inside of the passenger car is cooled using an air conditioning to a temperature of 21 ^{o}C.

a) Determine the Reynolds number for the roof of the passenger car. Assume that $T_f = 20^{o}$C. The roof can be modeled as a flat plate. Please clearly indicate what properties or air are used.

b) Determine at which length the laminar flow becomes turbulent.

The equilibrium equation of the roof of the passenger car is given below:

$$\dot{Q}_{sun} + \dot{Q}_{convection} = \dot{Q}_{conduction} \tag{16}$$

This equation can be used to determine the temperature of the roof.

c) Explain each term in the equilibrium equation and explain the effect of each type of heat transfer on the rooftop surface temperature

d) Give the equations for the conduction and convection part which are required to solve the equilibrium equation and give the dimensions. Make a sketch of the situation..

For part e), use the following properties of air

- ρ_{air} = 1.164 $[kg/m^3]$
- k_{air} = 0.02588 $[W/mK]$
- μ_{air} = $1.872 \cdot 10^{-}5$ $[kg/ms]$
- $Pr = 0.7282$

e) Determine the equilibrium temperature T_s of the top surface of the carriage. Assume the roof of the carriage to be perfectly insulated at its bottom side and the flow over the entire length of the roof to be turbulent. Hint: Determine the total resistance of the conduction part

Solution Assignment 3

a) *Determine the Reynolds number for the roof of the passenger car. Assume that $T_f = 20^oC$. The roof can be modeled as a flat plate. Please clearly indicate what properties or air are used.*

For $T_f = 20^o$C, air has the following properties:

- $\rho = 1.204$ [kg/m^3]
- $\mu = 1.825 \cdot 10^{-5}$ [kg/ms]

It is also given that the length of the roof is 10 m and the velocity of the train is 80 km/h = 22.2 m/s. The Reynolds number for a flat plate can be determined using the following formula:

$$Re = \frac{\rho U L_c}{\mu} = \frac{1.204 \cdot 22.2 \cdot 10}{1.825 \cdot 10^{-5}} = 14.65 \cdot 10^6$$

b) *Determine at which length the laminar flow becomes turbulent.*

A flow over a flat plate becomes turbulent when Re > 10^5, thus:

$$\frac{\rho U L_c}{\mu} > 10^5 \rightarrow L > \frac{10^5 \mu}{\rho U} = \frac{10^5 \cdot 1.825 \cdot 10^{-5}}{1.204 \cdot 22.2} \rightarrow L > 0.068\ [m]$$

c) *Explain each term in the equilibrium equation and explain the effect of each type of heat transfer on the rooftop surface temperature*

- $\dot{Q}_{sun}$ is the term that describes how much energy from the sun is landing on the carriage. This will also heat up the top surface of the carriage.
- $\dot{Q}_{conv}$ is the term that describes the heat transferred due to convection. This is happening at the top of the carriage surface, on the interface with the air.
- $\dot{Q}_{cond}$ is the term that describes the heat conduction from inside the carriage, through the two aluminium plates and insulation to the top of the carriage.

d) *Give the equations for the conduction and convection part which are required to solve the equilibrium equation and give the dimensions. Make a sketch of the situation.* The three terms in the equilibrium equation can

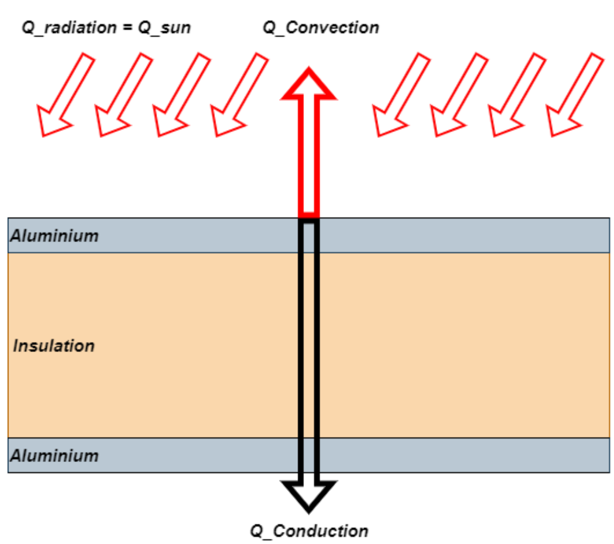


Figure 8: Caption

be defined as follows:

- $\dot{Q}_{sun} = 250 \cdot A = 250 \cdot 30 = 7500$ [W]
- $\dot{Q}_{conv} = h \cdot A \cdot \Delta T$ [W]
- $\dot{Q}_{cond} = \frac{\Delta T}{R_{tot}}$ where $R = \frac{\Delta x}{kA}$ [W]

e) *Determine the equilibrium temperature T_s of the top surface of the carriage. Assume the roof of the carriage to be perfectly insulated at its bottom side and the flow over the entire length of the roof to be turbulent. Hint: Determine the total resistance of the conduction part*

First we start with the $\dot{Q}_{cond}$ term. The resistance of both alumiumium plates are:

$$R_1 = R_3 = \frac{0.003}{180 \cdot 30} = 5.55 \cdot 10^{-7}\ [K/W]$$

The resistance of the insulation:

$$R_2 = \frac{0.04}{0.0320 \cdot 30} = 0.04167\ [K/W]$$

Since the aluminium plates have such a low resistance compared to the insulation, we approximate the resistance by $R_{tot} \approx R_2 = 0.04167$ [K/W]. We then find an expression for $\dot{Q}_{cond}$:

$$\dot{Q}_{cond} = \frac{T_A - T_s}{0.04167}$$

Where T_A is defined as the temperature of the air conditioning and T_s as the surface temperature of the carriage.
For the convection term we need to find an expression for the heat transfer coefficient h. We find the correct expression for the Nusselt number, and using the given properties of air we are able to calculate h:

$$Nu = 0.037 \cdot Re^{0.8} \cdot Pr^{\frac{1}{3}}$$
$$Nu = 0.037 \cdot (14.65 \cdot 10^6)^{0.8} \cdot 0.7282^{\frac{1}{3}}$$
$$Nu \approx 17982.85$$

Realizing that the Nusselt number can also be defined as a function for h gives:

$$Nu = \frac{hL_c}{k} \rightarrow h = \frac{Nu \cdot k}{L_c} = \frac{17982.85 \cdot 0.02588}{10} = 46.54\ [W/m^2K]$$

Now that we have an expression for all terms, we can combine them in the equilibrium equation, and find an expression for the surface temperature T_s:

$$\begin{aligned}
\dot{Q}_{sun} + \dot{Q}_{convection} &= \dot{Q}_{conduction} \\
7500 + h \cdot A \cdot (T_A - T_s) &= \frac{T_A - T_s}{R_{tot}} \\
7500 + 46.54 \cdot 30 \cdot (21 - T_s) &= \frac{21 - T_s}{0.04167} \\
312.525 + 1221.77 - 58.17_s &= 21 - T_s \\
1513.26 &= 57.17 \cdot T_s \\
T_s &= 26.47\ [^\circ C]
\end{aligned}$$

Assignment 4

During spring and autumn, in the evenings it can still get quite chilly. However, by using a so called terrace heater as illustrated below in figure 13a, it is possible to stay warm even during those chilly evenings.

(a) Terrace heater

(b) Forced convection heater

Figure 13: Two types of heaters

Another option is to use a different type of heater, which makes use of forced convection. An example is showed in figure 13b. We are interested in the difference between both options. Assume that the terrace is protected against the wind, such that there is no air flowing in or out of the terrace. The terrace has a total volume of 27 m^3.

For the comparison, we assume that the temperature of the lamp and coil in the forced heater are a constant 300 ^oC and the air inside the terrace is 12 ^oC. The velocity profile of the air around the lamp is illustrated in figure ... The properties of air can be found in the table 5.

a) Explain what the Grashof number physically represents

b) Explain the shape of the velocity profile in figure 14.

c) For the terrace lamp, determine the rate of heat transfer when the lamp is turned on. The lamp can be modeled as a cylinder with a diameter of 40 cm and a thickness of 1 cm.

Now we start using the forced convection heater. You may assume the fan is blowing horizontally over the coils with a speed of 2.0 m/s. All coils together can be modeled as a cylinder with a diameter of 1 cm and a length of 1.25 m.

d) Determine the rate of heat transfer of the forced convection heater.

For the following questions, we want to test your understanding of the subject. It is therefore not necessary to calculate anything, but try to reason while looking at the equations.

e) Can you explain the difference in the rates of heat transfer between the natural and forced convection situation?

f) Can you think of another reason why not only the heat transfer due to convection is of importance in case of the lamp?

g) Suppose we use a much bigger fan which is able to blow the air over the coil at a speed of 50 m/s. This will result in a turbulent flow over the coils. Explain the effect on the heat transfer coefficient, and why this effect takes place.

h) Now suppose that we also use the same heaters in the winter, where the temperatures of the surroundings drop to 0 ^oC. Can you (in words) explain what will happen to the heat transfer coefficient h, compared to spring/autumn conditions? Can you also explain what will happen to the required energy E_{tot} to heat up the air?

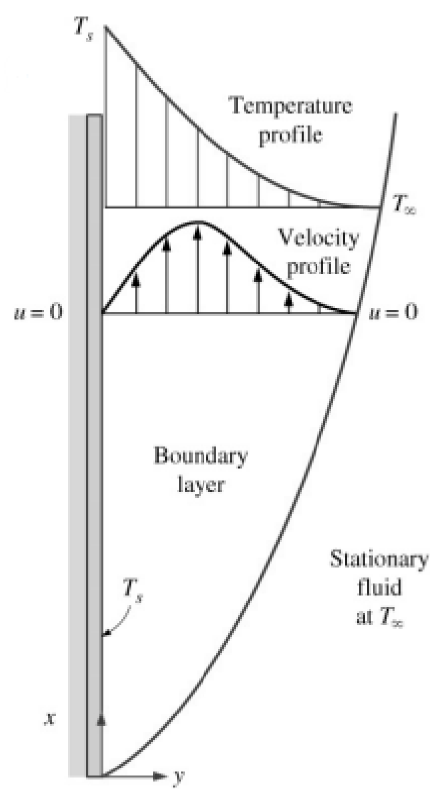


Figure 14: Velocity and temperature profile

Temperature ^oC	Density kg/m^3	Specific heat J/kgK	Thermal conductivity $W/m \cdot K$	Thermal diffusivity m^2/s	Dynamic viscosity $kg/m \cdot s$	Kinematic viscosity m^2/s	Prandtl number
0	1.292	1006	0.02364	$1.818 \cdot 10^{-5}$	$1.729 \cdot 10^{-5}$	$1.338 \cdot 10^{-5}$	0.7362
10	1.246	1006	0.02439	$1.944 \cdot 10^{-5}$	$1.778 \cdot 10^{-5}$	$1.426 \cdot 10^{-5}$	0.7336
20	1.204	1007	0.02514	$2.074 \cdot 10^{-5}$	$1.825 \cdot 10^{-5}$	$1.516 \cdot 10^{-5}$	0.7309
30	1.164	1007	0.02588	$2.208 \cdot 10^{-5}$	$1.872 \cdot 10^{-5}$	$1.608 \cdot 10^{-5}$	0.7282
40	1.127	1007	0.02662	$2.346 \cdot 10^{-5}$	$1.918 \cdot 10^{-5}$	$1.702 \cdot 10^{-5}$	0.7255
50	1.092	1007	0.02375	$2.487 \cdot 10^{-5}$	$1.963 \cdot 10^{-5}$	$1.798 \cdot 10^{-5}$	0.7228
60	1.059	1007	0.02808	$2.632 \cdot 10^{-5}$	$2.008 \cdot 10^{-5}$	$1.896 \cdot 10^{-5}$	0.7202
70	1.028	1007	0.02881	$2.780 \cdot 10^{-5}$	$2.052 \cdot 10^{-5}$	$1.995 \cdot 10^{-5}$	0.7177
80	0.9994	1008	0.02953	$2.931 \cdot 10^{-5}$	$2.096 \cdot 10^{-5}$	$2.097 \cdot 10^{-5}$	0.7154
90	0.9718	1008	0.03024	$3.086 \cdot 10^{-5}$	$2.139 \cdot 10^{-5}$	$2.201 \cdot 10^{-5}$	0.7132
100	0.9458	1009	0.03095	$3.243 \cdot 10^{-5}$	$2.181 \cdot 10^{-5}$	$2.306 \cdot 10^{-5}$	0.7111
120	0.8977	1011	0.03235	$3.565 \cdot 10^{-5}$	$2.264 \cdot 10^{-5}$	$2.522 \cdot 10^{-5}$	0.7073
140	0.8542	1013	0.03374	$3.898 \cdot 10^{-5}$	$2.345 \cdot 10^{-5}$	$2.745 \cdot 10^{-5}$	0.7041
160	0.8148	1016	0.03511	$4.241 \cdot 10^{-5}$	$2.420 \cdot 10^{-5}$	$2.975 \cdot 10^{-5}$	0.7014

Table 5: Air properties at 1 atm pressure

Solution Assignment 4b

a) *Explain what the Grashof number physically represents*

The Grashof number is a dimensionless number named after Franz Grashof. The Grashof number is defined as the ratio of the buoyant to a viscous force acting on a fluid in the velocity boundary layer. Its role in natural convection is much like that of the Reynolds number in forced convection.

b) *Explain the shape of the velocity profile in figure 9.*

At the interface between the fluid and the hot plate, $y = 0$, the air is stationery. This is due to the friction forces between plate and fluid. Moving in positive y-direction, the fluid rises due to buoyancy forces and increases in velocity. Moving even further in positive y-direction, the velocity decreases again, since the effect of the plate is no longer working on the fluid.

c) *For the terrace lamp, determine the rate of heat transfer when the lamp is turned on. The lamp can be modeled as a cylinder with a diameter of 40 cm and a thickness of 1 cm.*

Since we are dealing with natural convection, we have to use the Rayleigh number to define a Nusselt number, which in turn leads to a heat transfer coefficient. The Grashof number can be calculated as follows:

$$Gr = \frac{g\beta(T_{lamp} - T_{terrace})L_c^3}{\nu^2}$$

For β we use the expression

$$\beta = \frac{2}{T_s + T_\infty} = \frac{2}{573 - 285} = 0.002331\ [K^{-1}]$$

Using the correct data of air at a fluid temperature of $T_f = \frac{300+12}{2} = 156\,°C \to 160\,°C$ gives the following:

$$Ra = Gr \cdot Pr = \frac{9.81 \cdot 0.002331 \cdot (573 - 285) \cdot 0.01^3}{(2.975 \cdot 10^{-5})^2} \cdot 0.7014 \approx 5219$$

Finding the correct expression for the Nusselt number of a cylinder, and using the characteristic length $L_c = D$:

$$Nu_D = \left\{ 0.6 + \frac{0.387 \cdot Ra^{\frac{1}{6}}}{[1 + (\frac{0.559}{Pr})^{\frac{9}{16}}]^{\frac{8}{27}}} \right\}^2 \approx 3.75$$

Now we can find the heat transfer coefficient:

$$Nu_D = \frac{hD}{k} \to h = \frac{Nu \cdot k}{D} \approx 0.329\ [W/m^2K]$$

And finally we are able to determine the heat transfer:

$$\dot{Q} = hA\Delta T = 0.329 \cdot 0.01256 \cdot 288 = 1.19\ [W]$$

d) *Determine the rate of heat transfer of the forced convection heater*

To determine the heat transfer due to forced convection we start at the Reynolds number:

$$Re = \frac{\rho U L_c}{\mu} = \frac{\rho \cdot U \cdot D}{\mu} = \frac{0.8148 \cdot 12 \cdot 0.01}{2.420 \cdot 10^{-5}} \approx 4040.33$$

Finding the correct relation for the Nusselt number as a function of the Reynolds and Prantl number:

$$Nu_D = 0.193 \cdot Re^{0.618} \cdot Pr^{\frac{1}{3}} = 0.193 \cdot 4040.33^{0.618} \cdot 0.7014^{\frac{1}{3}} \approx 29.04$$

From this we take the same steps as previous question to find the heat transfer coefficient and the resulting heat transfer:

$$Nu_D = \frac{h \cdot D}{k} \to h = \frac{Nu \cdot k}{D} = \frac{29.04 \cdot 0.03511}{0.01} = 101.95\ [W/m^2K]$$

$$\dot{Q} = hA\Delta T = 101.95 \cdot 0.01256 \cdot 288 = 368.8\ [W]$$

e) *Can you explain the difference in the rates of heat transfer between the natural and forced convection situation?*

Since the cylinder has quite a small diameter, not much heat is transferred by natural convection. This is also to be expected. When the fan starts blowing, we create a turbulent flow around the cylinder which results in a much higher heat transfer.

f) *Can you think of another reason why not only the heat transfer due to convection is of importance in case of the lamp?*

The heaters radiate most of the heat away to the people sitting on the table. It therefore does not necessarily matter if there is wind blowing, since radiation does not need a medium to transfer heat.

g) *Suppose we use a much bigger fan which is able to blow the air over the coil at a speed of 50 m/s. This will result in a turbulent flow over the coils. Explain the effect on the heat transfer coefficient, and why this effect takes place*

As was also already noted in question e), due to the turbulent flow the heat transfer coefficient increases. This is due to the fact that the boundary layer on the surface of the cylinder starts to mix the fluid (air) more violently due to the high velocity of the air, resulting in a higher transfer of heat.

h) *Now suppose that we also use the same heaters in the winter, where the temperatures of the surroundings drop to 0°C. Can you (in words) explain what will happen to the heat transfer coefficient h, compared to spring/autumn conditions? Can you also explain what will happen to the required energy E_{tot} to heat up the air?*

The temperature will drop, thus the difference in temperature will become larger. This results in a larger potential to transfer heat, thus the heat transfer coefficient will also increase. This can also be observed by the definition of the Grashof number.

Assignment 5

Consider a circular terrace heater, which are used to warm persons sitting around it with the help of radiation, illustrated in figure 16. The heater hangs above the floor at a distance of 2 m, with a radius of 50 cm. The pack-

Figure 16: Terrace heater

aging states that it can radiate a surface area with a diameter of 4 m. Both the heater as the radiated surface can be considered as parallel planes. It is assumed that the heater has a surface temperature of 200 ^{o}C and transmits all of its energy as radiation. The surface on the ground is assumed to behave as a opaque surface, with an absorptivity of 0.7. The surrounding temperature is 12 ^{o}C.

a) Give a sketch of the two surfaces, their dimensions and their properties as stated in the introduction

b) Give values for the emissivity (ϵ), transmissivity (τ) and reflectivity (ρ) of surfaces 1 and 2

c) Determine the wavelength that holds the maximum power coming off of the heater, using Wien's displacement law.

d) Compute the **net** rate of heat transfer by radiation from the heater to the surface at the given temperatures

e) Based on the answer found in question d), how long will it take the bricks on the floor, which have a thickness of 3 cm and a specific heat of 840 [J/kgK] to heat up 1 ^{o}C?

f) Reflect on your given answer in question e). Is it realistic? What would change if instead of bricks we look at the persons sitting on the terrace?

Solution Assignment 5b

a) *Give a sketch of the two surfaces, their dimensions and their properties as stated in the introduction*

b) *Give values for the emissivity (ϵ), transmissivity (τ) and reflectivity (ρ) of surfaces 1 and 2*

The lamp radiates all the energy away, so we can say that the lamp has an emissivity of 1, $\epsilon = 1$. This in turn means that the absorptivity and the transmission are both equal to 0. The floor behaves as an opaque surface, thus the transmissivity is equal to 0. It is specified that the absorptivity is 0.7, thus the reflectivity is equal to 0.3.

c) *Determine the wavelength that holds the maximum power coming off of the heater, using Wien's displacement law.*

Wien's displacement law calculates the wavelength with the maximum power as:

$$\delta_{max,power} = \frac{2897.8}{T_{surface}} = \frac{2897.8}{200+273} = 6.13\ [\mu m]$$

d) *Compute the **net** rate of heat transfer by radiation from the heater to the surface at the given temperatures*

First we need to find the view factor of the lamp to the ground. Since they are two parallel planes, we can use a chart, illustrated in figure 17. We define the lamp as being body two and the floor as body one. First we compute both areas:

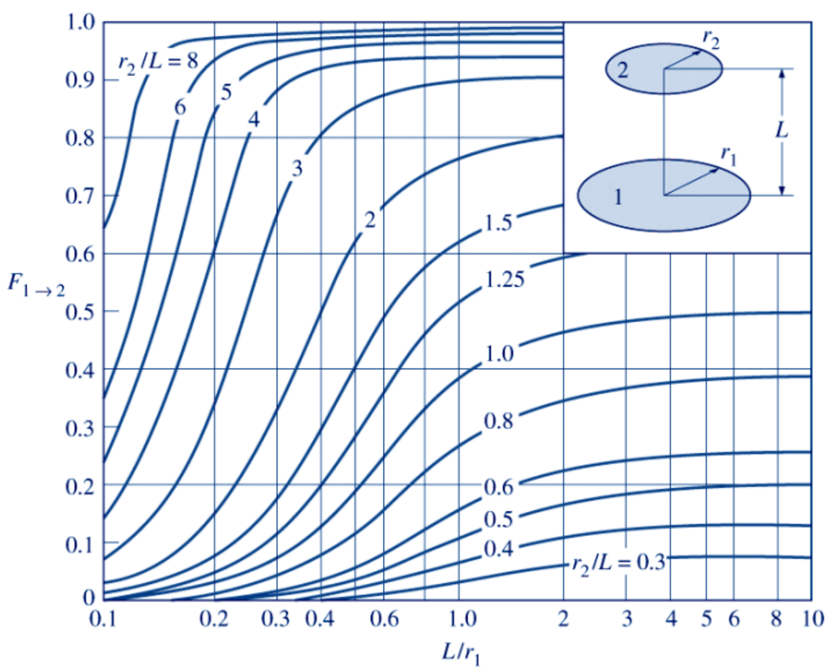


Figure 17: View factor for parallel planes

$$A_1 = \pi r_1^2 = \pi 2^2 = 12.56\ [\mathrm{m}^2] \tag{17}$$

$$A_2 = \pi r_2^2 = \pi 0.5^2 = 0.785\ [\mathrm{m}^2] \tag{18}$$

Filling in the values gives $F_{1\to 2} \approx 0.015$. Then using the reciprocity relation:

$$\begin{aligned} A_1 F_{1\to 2} &= A_2 F_{2\to 1} \\ F_{2\to 1} &= \frac{A_1}{A_2} F_{1\to 2} = \frac{12.56}{0.785} \cdot 0.015 = 0.24 \end{aligned}$$

To compute the net heat transfer to the floor, first we compute the energy radiated from the lamp. For this we use the following relation:

$$\begin{aligned} \dot{Q}_{lamp,emitted} &= A \cdot \sigma \cdot \epsilon \cdot T^4 \\ &= 0.785 \cdot 5.67 \cdot 10^{-8} \cdot 1 \cdot 573^4 \\ &= 4.7981 \cdot 10^3\ [\mathrm{W}] \approx 4.8\ [\mathrm{kW}] \end{aligned}$$

However, not all this energy hits the surface and is also absorbed. Thats why we multiply the emitted energy by the viewfactor of surface 2 to 1 and the absorptivity of surface 1:

$$\begin{aligned} \dot{Q}_{net} &= \dot{Q}_{lamp,emitted} \cdot F_{2\to 1} \cdot \alpha \\ &= 4.7981 \cdot 10^3 \cdot 0.24 \cdot 0.7 = 806.08\ [\mathrm{W}] \end{aligned}$$

e) *Based on the answer found in question d), how long will it take the bricks on the floor, which have a thickness of 3 cm and a specific heat of 840 [J/kgK] to heat up 1 ^{o}C? It is assumed that the bricks have a density of* $\rho = 2000$ $[kg/m^3]$

First we need to calculate how much energy it takes for the bricks to heat up 1 degree. This can be achieved by using the following formula:

$$E = m \cdot c_p \cdot \Delta T = m \cdot c_p \cdot 1$$

The mass can be calculated by multiplying the volume with the density of the bricks:

$$m = \rho \cdot V = \rho \cdot A \cdot t = 2000 \cdot 12.56 \cdot 0.03 = 753.6 \text{ [kg]}$$

The energy that the bricks thus need to heat up one degree is thus:

$$E = 753.6 \cdot 840 = 633,024 \text{ [kJ]}$$

To determine how long it takes the lamp to provide this much energy to the surface, thus heating it up 1 degree equals:

$$t = \frac{633,024 \cdot 10^3}{806.08} = 785.31 \text{ [s]} \approx 13 \text{ [min]}$$

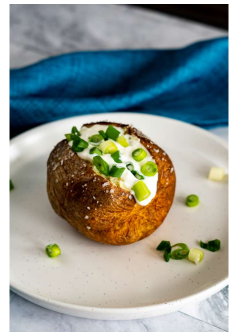
f) *Reflect on your given answer in question e). Is it realistic? What would change if instead of bricks we look at the persons sitting on the terrace?*

It looks quite realistic, however, the bricks would normally not heat up equally. This would thus also mean that the top layer of brick is warmer than the bottom layer. If persons were sitting on the terrace the view factor from the lamp to the persons would change, depending on where they are sitting they might receive more or less heat transfer.

Assignment 6b

During the summer, often steaks and potatoes are cooked on the BBQ. Before these can be enjoyed, they have to cool down to a comfortable temperature.

(a) Steak on a BBQ



(b) Potato

a) Based on which criteria can the lumped capacity model be applied?

b) For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why. And why will it give more accurate results for one than the other?

A steak (k = 4 W/mK, ρ = 1006 kg/m^3, c_p = 2850 J/kgK) with a diameter of 8 cm, which initially has a surface temperature of 70 ^{o}C, is located outside with an ambient temperature of 20 ^{o}C. It cools down due to convection (h = 35 W/m^{2}K).

c) Using the lumped capacity model, determine the time that it takes for the steak to cool down to 40 ^{o}C.

d) Evaluate the accuracy of the found answer in c).

e) Determine the amount of energy that the steak has lost, when cooled down from 70 ^{o}C to 40 ^{o}C.

f) Determine the maximum diameter of the potato, for which the lumped capacity model is still valid.

g) Provide a sketch of the temperature profile as a function of time, in the case that we would have let the potato cool down in the room for a very long time.

Note: clearly indicate the temperatures for t=0 and t→ ∞

Solution: Assignment 6b

a) *Based on which criteria can the lumped capacity model be applied?*

$$\mathrm{Bi} \leq 0.1 \text{ or } \mathrm{Bi} \to 0$$

b) *For which of the two dishes is the lumped capacity model the most suitable to determine the cool down time? Explain why. And why will it give more accurate results for one than the other?*

We know:

$$\mathrm{Bi} = \frac{hL_c}{k}$$

Simplifying both objects to spheres, L_c will be bigger for the steak. A steak and a potato have roughly similar thermal conductivity. Therefore it can be assumed that the lumped capacity model is more applicable for the potato, as the characteristic length is smaller. Besides, a potato has a more uniform temperature distribution as its properties are more constant throughout its body. A chicken contains parts such as bones and fat, where other properties for thermal conductivity will be found.

c) *Using the lumped capacity model, determine the time that it takes for the potato to cool down to 50 °C.*

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p}t}$$

$$\to t = \frac{\rho V c_p}{hA_s}\ln\left(\frac{T_i - T_\infty}{T(t)-T_\infty}\right)$$

Where

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 0.03^3\ \left[\mathrm{m}^3\right] = 6.544\cdot 10^{-5}\ \left[\mathrm{m}^3\right]$$

$$A_s = 4\pi r^2 = 4\pi 0.03^2\ \left[\mathrm{m}^3\right] = 0.0079\ \left[\mathrm{m}^2\right]$$

So:

$$\to t = \frac{1006\ \left[\mathrm{kg/m}^3\right]\cdot 6.544\cdot 10^{-5}\ \left[\mathrm{m}^3\right]\cdot 2850\ \left[\mathrm{J/kgK}\right]}{35\ \left[\mathrm{W/m^2K}\right]\cdot 0.0079\ \left[\mathrm{m}^2\right]}\ln\left(\frac{70-20}{40-20}\right) = 346.63\ [\mathrm{s}] = 5.78\ [\mathrm{min}]$$

d) *Evaluate the accuracy of the found answer in c).*

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{hV}{kA_s} = \frac{35\ \left[\mathrm{W/m^2K}\right]\cdot 6.544\cdot 10^{-5}\ \left[\mathrm{m}^3\right]}{4\ [\mathrm{W/mK}]\cdot 0.0079\ \left[\mathrm{m}^2\right]} = 0.0725 < 0.1$$

Therefore the lumped capacity model provides a high accuracy.

e) *Determine the amount of energy that the potato has lost, when cooled down from 90 °C to 50 °C.*

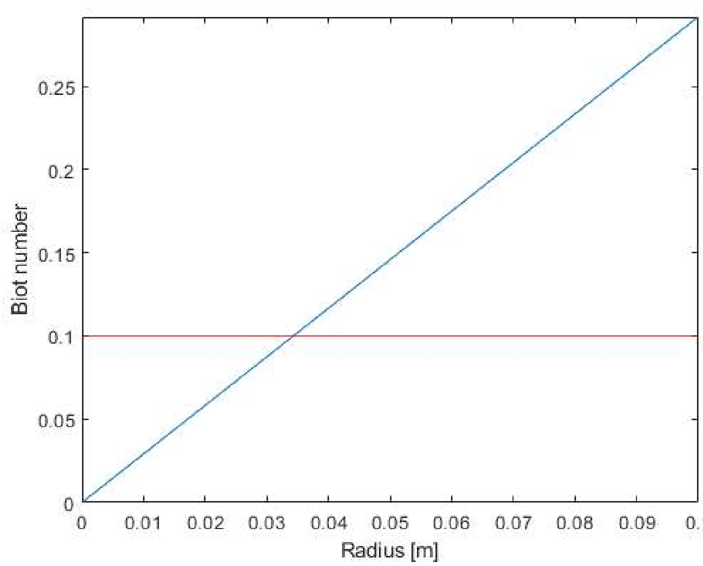
Heat lost:

$$Q = V\rho c_p\left(T_i - T(t)\right) = 6.544\cdot 10^{-5}\ \left[\mathrm{m}^3\right]\cdot 1006\ \left[\mathrm{kg/m}^3\right]\cdot 2850\ \left[\mathrm{J/kgK}\right](70-40)\ [\mathrm{K}] = 5.629\cdot 10^{30}\ [\mathrm{J}]$$

f) *Determine the maximum diameter of the potato, for which the lumped capacity model is still valid.*

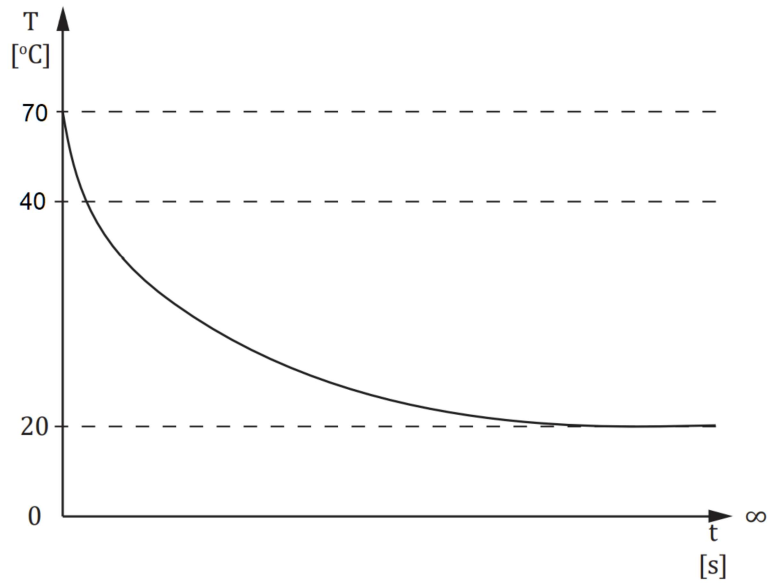
$$\mathrm{Bi}(r) = \frac{hr}{3k}$$

Plotting:



Intersection is at r = 3.4 cm (D=6.8 cm)

g) *Provide a sketch of the temperature profile as a function of time, in the case that we would have let the potato cool down in the room for a very long time.*



- At t=0 T = 70 °C.
- T = 20 °C should be clearly defined.
- For t→ ∞ the temperature should approach the ambient temperature with a zero gradient slope.