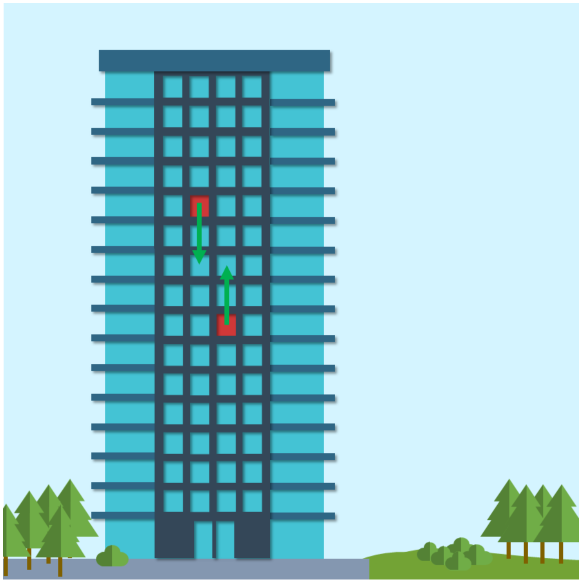
# Elevator in Building



**The elevator of a skyscraper rises 350 meters at a maximum speed of 22 km/h. Both the acceleration and deceleration have a constant magnitude of 0.25$g$ during this rise. After reaching its maximum speed, the elevator maintains this speed until it decelerates to perfectly come to a full stop at the top. Determine the duration $T$ of this full elevator rise.**

*Using known expressions (for constant acceleration)*:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}v = a\mathrm{d}t \tag{1}$$

$$\int_{v_0}^{v(t)} \mathrm{d}v = a \int_0^t \mathrm{d}t \tag{2}$$

$$v(t) = at + v_0 \tag{3}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}s = v\mathrm{d}t = (v_0 + at)\mathrm{d}t \tag{4}$$

$$\int_{s_0}^{s(t)} \mathrm{d}s = \int_0^t (v_0 + at)\,\mathrm{d}t \tag{5}$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6}$$

*Given quantities*:

Distance: $s = 350$ m

Acceleration: $a = 0.25g$

Velocity: $v = 22$ km/h $\approx 6.11$ m/s

*Solution*:

The easiest thing is to visualize the problem by drawing a graph of how the velocity changes over time (see Figure 1). The problem is divided in three parts; the first one is the acceleration of $a_{\mathrm{I}} = 0.25g$ of the elevator from standstill ($v_0 = 0$ m/s) to its maximum speed $v_{\mathrm{max}} = 6.11$ m/s, then there is a time the elevator has a constant speed of 6.11 m/s ($a_{\mathrm{II}} = 0$ m/s$^2$) and finally it decelerates with $a_{\mathrm{III}} = -0.25g$ to a standstill.

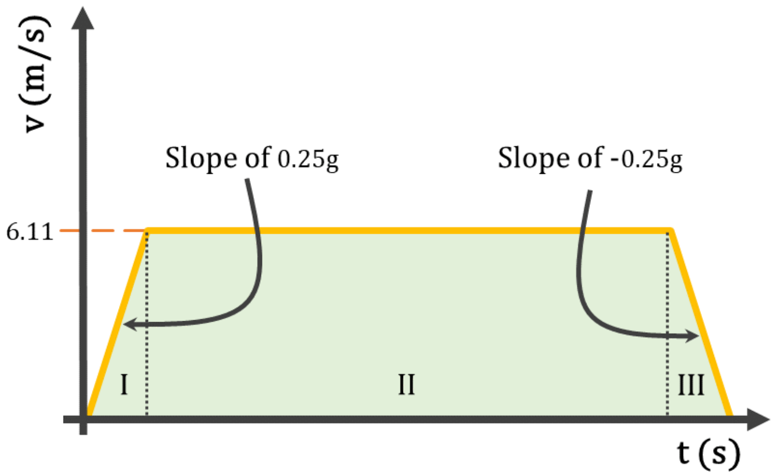



Figure 1: Visualization graph of changing velocity over time

Using Equation (3), the time $t$ for the elevator to reach its maximum constant velocity of 6.11 m/s from standstill ($v_0 = 0$ m/s) can be calculated.

$$t = \frac{v}{a} \quad \Rightarrow \quad t = \frac{6.11}{0.25 \cdot 9.81} \approx 2.49 \text{ s} \tag{7}$$

The traveled distance in this time is the area of section I of Figure 1. It can be directly seen from the figure or calculated using Equation 6, with $s_0 = 0$ m and $v_0 = 0$ m/s.

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 = \frac{1}{2}at^2 = \frac{0.25}{2}gt^2 \quad \Rightarrow \quad s(2.49) = \frac{0.25}{2} \cdot 9.81 \cdot 2.49^2 \approx 7.61 \tag{8}$$

The elevator both accelerates and decelerates, thus the total distance traveled during both combined is $2 \cdot 7.61 = 15.22$ m (area I and area II from the figure combined).

To calculate the time that the elevators travels at 6.11 m/s, we use the fact that the total length is 350 m (intended total area under the curve). Thus the distance the elevator travels at $v_{\text{max}} = 6.11$ m/s (or area II under the curve) is equal to $350 - 15.22 = 334.8$ m. Since the velocity is constant, the acceleration is 0 thus Equation 6 becomes:

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0 = v_{\text{max}} t \quad \Rightarrow \quad t = \frac{s(t)}{v_{\text{max}}} = \frac{334.8}{6.11} \approx 54.79 \text{ s} \tag{9}$$

Thus the total time becomes $T = 2t + 54.79 = 2 \cdot 2.49 + 54.79 = 59.77$ s.