11.7 Exercises

Problem 11.1. *Consider a sphere in compressible flow. Far upstream from the sphere the pressure, Mach number and temperature are known: p_∞, M_∞, and T_∞. The pressure and temperature in the stagnation point are p_o and T_o.*

(a) Express p_o in terms of p_∞, M_∞.
(b) Express p_∞ in terms of p_o, M_∞.
(c) Express T_∞ in terms of T_o, M_∞.
(d) For measured p_o, T_o, p_∞, compute the velocity U_∞ far upstream of the sphere.

Problem 11.2. *(a) Show that $\frac{1}{p}\frac{Dp}{Dt} = \frac{D}{Dt}\ln p$.*
(b) What is the meaning of $\frac{Dp}{Dt}$?

Problem 11.3. *Consider steady flow with $\mu = 0$ and $k = 0$.*

(a) Show that the mass and energy equations reduce to

$$\frac{\partial}{\partial x_j}(\rho u_j) = 0$$

$$\frac{\partial}{\partial x_j}(\rho u_j H) = 0$$

(b) Show that these equations lead to $\frac{DH}{Dt} = 0$.
(c) What is the meaning of $\frac{DH}{Dt} = 0$?

Problem 11.4. *Show that in case of a perfect gas $p_t = \rho_t R T_t$.*

Problem 11.5. *For steady flow with $\mu = 0$ and $k = 0$ explain that the pressure along a streamline can not exceed the total pressure along that streamline.*

Problem 11.6. *For a thermally perfect gas show that $p = (\gamma - 1)\rho e$.*

Problem 11.7. *Let $p\rho^\gamma = const$, and let $p = p_o + p'$, $\rho = \rho_o + \rho'$, with p_o, ρ_o constants and p', ρ' small perturbations. Show that in the limit of $p'/p_o \to 0$, and $\rho_o/\rho' \to 0$ we have*

$$p' = a^2\rho', \quad a^2 \equiv \gamma\frac{p_o}{\rho_o}.$$

[Hint: use Taylor series $(1+\epsilon)^\alpha = 1 + \alpha\epsilon$ ...]