

## 1 Exercise V-1a

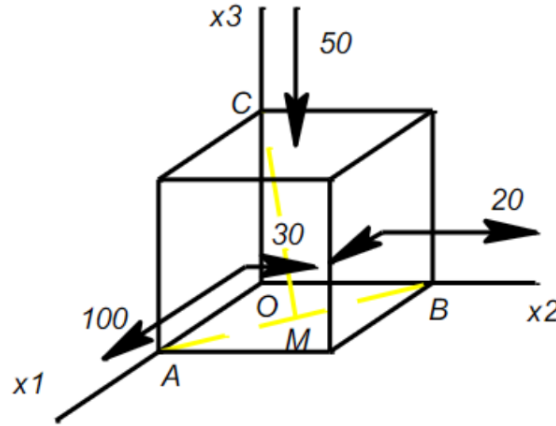
**Given:**  $E = 200 \text{ GPa}$ ,  $\nu = 0.25$

$OA = OB = a$  and  $OC = \frac{1}{2}\sqrt{2} \cdot a$

In this stress-state, the maximal principal stress must not be larger than: 150 MPa.

**Questions:**

a) Find  $\sigma_{ABC}$  and  $\tau_{ABC}$



## 2 Exercise V.4

**Given:**  $E = 2 \cdot 10^{11} \text{ Pa}$ ,  $\nu = 0.25$

Stress-state in point P:  $\sigma = \begin{bmatrix} 19 & -5 & -\sqrt{6} \\ -5 & 19 & -\sqrt{6} \\ -\sqrt{6} & -\sqrt{6} & 10 \end{bmatrix} \text{ MPa}$

**Questions:**

- Show that the principal stresses are 8, 16 and 24 MPa. Compute the directional cosines (transformation matrix entries) of the smallest eigen-stress.
- Compute the volumetric (isotropic) strain.
- What is the largest angle-change (not shear-strain) in P?
- Which material property is implicitly used in Hookes law?

### 3 Exercise V.12abc

In a linear elastic ( $E = 2 \cdot 10^5 MPa$ ,  $\nu = 0.25$ ) body under load, the strain-field is given (with four free parameters), with respect to the Cartesian  $x_1 - x_2 - x_3$  coordinate system as:

$$\sigma_{11}(x_1, x_2, x_3) = \sigma_0[20 + \alpha_1(\frac{x_1}{L}) - 10(\frac{x_2}{L}) + \alpha_2(\frac{x_1}{L})^2]$$

$$\sigma_{22}(x_1, x_2, x_3) = \sigma_0[10 + 8(\frac{x_1}{L}) + \beta_1(\frac{x_2}{L}) + \beta_2(\frac{x_2}{L})^2]$$

$$\sigma_{12}(x_1, x_2, x_3) = \sigma_0[12 - 10(\frac{x_1}{L}) + 7(\frac{x_2}{L}) - 8(\frac{x_1}{L})(\frac{x_2}{L})]$$

$$\sigma_{13}(x_1, x_2, x_3) = \sigma_{23}(x_1, x_2, x_3) = \sigma_{33}(x_1, x_2, x_3) = 0$$

with reference stress  $\sigma_0 = 1MPa$  and reference length  $L = 1m$ . Note that all stresses are independent on  $x_3$  and that the calculation in question (a) below is general with variables  $x_1, x_2$ , and  $x_3$ ; from question (b) on, use the point  $P(x_1 = 0, x_2 = 0, x_3 = 0)$ .

Questions:

a) Does the displacement field agree with the stress-equilibrium equations in absence of volume-forces? Which relations have to be valid for the four free parameters  $\alpha_1, \alpha_2, \beta_1, \beta_2$  due to stress equilibrium.

b) Compute the eigen-stresses in point P using linear algebra mathematics – not the circle of Mohr.

Describe and name the state of stress in point P (and in all other points in the body).

c) Compute the eigen-direction of the major eigen-stress.