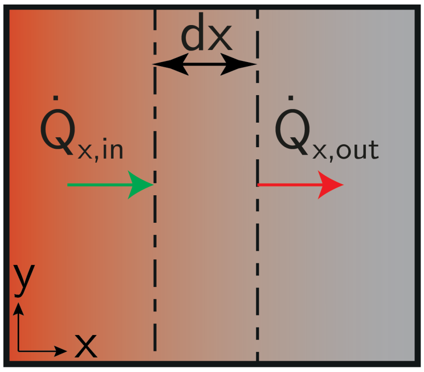
EB - Cond. - IE 6

Derive the energy balance and boundary conditions required to calculate the one-dimensional steady-state temperature profile inside the wall.

1 Setting up the balance:

To derive the one-dimensional steady-state temperature profile, an energy balance around an infinitesimal element is needed. Heat is conducted in and out of the element.



Hence, the steady-state balance reads:

$$\dot{Q}_{\text{x,in}} - \dot{Q}_{\text{x,out}} = 0,$$

the sum of the in- and outgoing fluxes should equal zero, because of steady-state conditions.

2 Defining the elements within the balance:

The ingoing flux described by use of Fourier's law:

$$\dot{Q}_{\text{x,in}} = -\lambda A \frac{\partial T}{\partial x},$$

and the outgoing flux is approximated by the use of the Taylor series expansion.

$$\begin{aligned} \dot{Q}_{\text{x,out}} &= \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{\text{x,in}}}{\partial x} \cdot dx \\ &= -\lambda A \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-\lambda A \frac{\partial T}{\partial x}\right) \cdot dx. \end{aligned}$$

3 Inserting and rearranging:

$$\frac{\partial^2 T}{\partial x^2} = 0.$$

4 Defining the boundary and/or initial conditions:

The first boundary condition yields from a local energy balance at $x = 0$:

$$
\begin{aligned}
-\lambda A \frac{\partial T(x=0)}{\partial x} &= \dot{q}'' A \\
\Rightarrow \frac{\partial T(x=0)}{\partial x} &= \frac{\dot{q}''}{\lambda}
\end{aligned}
$$

and the second from the given temperature at $x = L$:

$$
T(x = L) \;=\; T_1.
$$