Tutorial T04 – Elasticity Stress

Answer the following questions as they could come up in an exam.

1 Strain and stress relation

(Exercise V7 in old material before 2022)

Given is the displacement field:
$u_1 = x_1 x_3$, $u_2 = -x_1 x_2$, and $u_3 = x_1^2 - x_3^2$
and material properties $E = 2$ and $\upsilon = 0.25$
(discuss the units, but drop them in calculations to save space).

Questions:

a) Compute the stress tensor (components).
b) In the point $(x_1, x_2, x_3) = (0, 0, z_0)$ compute the eigen-stresses and maximum shear stress.

2 From strain to stress equilibrium

(Exercise V8 in old material before 2022)

In a homogenous body made of a linear elastic, isotropic material, the displacement field is given:

$u_1 = \frac{p}{E} a \left[\frac{x_2}{a} + 2\frac{x_1 x_2}{a^2} - \frac{x_2^2}{a^2} \right]$

$u_2 = \frac{p}{E} a \left[\frac{x_1}{a} + \alpha\frac{x_1^2}{a^2} + \beta\frac{x_1 x_2}{a^2} - 2\frac{x_2^2}{a^2} \right]$

$u_3 = 0$

with coordinates x_1 and x_2, and variables p, E, a with $\upsilon = 0.25$.

Questions:

In absence of volume forces, compute the magnitude of the parameters α and β, using the information that the stress field is in mechanical equilibrium.