1 Exercise V-7

Given is the displacement field:

$$u_1 = x_1 x_3$$
$$u_2 = -x_1 x_2$$
$$u_3 = x_1^2 - x_3^2$$

and material properties $E = 2$ (discuss the units, but drop them in calculations to save space) and $\upsilon = 0.25$.

Questions:

a. Compute the strain tensor (components).

Solutions:

a. The strain tensor is computed from the displacement field: $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = x_3$

$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = -x_1$

$\varepsilon_{33} = \frac{\partial u_3}{\partial x_3} = -2x_3$

$\varepsilon_{kk} = -x_1 - x_3$

$\gamma_{12} = 2\varepsilon_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} = 0 - x_2 = -x_2$

$\gamma_{23} = 2\varepsilon_{23} = \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} = 0 + 0 = 0$

$\gamma_{31} = 2\varepsilon_{31} = \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} = 2x_1 + x_1 = 3x_1$